DISCUSSION PAPER SERIES

DP17120
(v. 2022-12-15 19:58:11)

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INTERNATIONAL MACROECONOMICS AND FINANCE AND MONETARY ECONOMICS AND FLUCTUATIONS

CEPR

ISSN 0265-8003

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Discussion Paper DP17120
First Published N/A
This Revision 18 March 2022

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www.cepr.org

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Keywords: N/A

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Time-consistent implementation in macroeconomic games*

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December 2022

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1 Introduction

In many macroeconomic games, private agents act anticipating future government's policy. Such an order of actions may lead to self-fulfilling expectations and multiple equilibria depending on the government's future policy response. As it is well known, in such situations, the government can commit to a rule that describes its future response to private actions to rule out undesired self-fulfilling expectations. This is, for example, one possible rationale for the "no bailout clause" in the EU Treaty (Article 125). The fear is that private bailout expectations fuel excessive risk taking, making bailouts ex post necessary. In contrast, the commitment not to bail out is supposed to steer agents to play the "good" equilibrium without socially costly bailouts and rules out "bad" equilibria with excessive risk taking.

However, governments are not fully committed to rules. Sometimes, depending on private actions, sticking to the rule is too costly for governments. Even if enshrined in a treaty, the "no bailout clause" was, for instance, not sufficient to completely rule out bailouts in the euro area (see, among others, Gourinchas et al., 2020). In the extreme case of full discretion, governments may be tempted to respond to the private sector by confirming private sector's expectations, regardless of their past commitments. The resulting mutual feedback between private actions and the ex-post policy response may then lead to multiple equilibria.[1]

Still, rules may allow to do more than discretion. From simple speeches to more formal commitments such as contracts, laws, constitutions, treaties or the delegations to independent authorities, commitments have in common to make future deviations costly, e.g., from the simple embarrassment a policymaker may feel for breaking past promises[2] or the political costs of changing or breaching past legislations. But, to what extent the resulting limited commitment ability is sufficient to steer the private sector's expectations on a unique equilibrium? Alternatively, are multiple equilibria the unintended consequence of any limits to governments' commitment ability? If not, how rules should be designed to credibly exclude undesired self-fulfilling expectations? What would be, for example, a credible no-bailout clause?

To answer these questions, we consider a macroeconomic game between a large player—a government—and a large set of small agents—the private sector—that allows us to nest together the full spectrum of commitment abilities between full discretion and full commitment. We determine the conditions under which a government with a finite commitment ability can rule out equilibrium multiplicity. When it exists, we find the lowest commit-

[1] The literature review lists papers in different literatures obtaining such equilibrium multiplicity.

[2] As Woodford (2012) notes about the commitment to forward guidance announcements, "In practice, the most logical way to make such commitment achievable and credible is by publicly stating the commitment, in a way that is sufficiently unambiguous to make it embarrassing for policymakers to simply ignore the existence of the commitment when making decisions at a later time."

ment ability that the government needs to implement a unique equilibrium outcome—a situation that we refer to as (time-consistent) implementation. We describe the rules that can implement a unique equilibrium in a credible way, whenever possible. Finally, we derive implications for models of bailouts, inflation bias, and capital taxation.

More precisely, we add to a generic macroeconomic game an ex-ante stage in which the government first commits to a reaction function that specifies policy responses as a function of every possible private sector's aggregate action, following Schelling (1960) and, more recently, Bassetto (2005). Then, the private sector is competitive—i.e., each private agent optimizes given the expectation of what other agents do and the government's future response. Finally, the government optimally selects its policy response. The government's payoff depends on the aggregate private action and on the policy response. The government also incurs a cost if the policy response deviates from the one that would follow the application of the reaction function. This cost measures the extent to which the government is bound by its commitments, and we will refer to this cost as the government's commitment ability.[3] When this cost is zero, the government always chooses its ex-post best response, regardless of its reaction function—this is the case of *full discretion*. In the other polar case, when this cost is infinite, the government never deviates from its reaction function—which corresponds to the situation of *full commitment*.

Our main results are as follows. First, in many static games under rational expectations, we show that the government can implement its best time-consistent outcome with an arbitrarily small commitment ability. Yet, in this case, limited commitment ability has a strong impact on the design of credible rules: for example, a government can rule out bailout expectations by committing to a partial bailout close to, but below, the ex-post optimal bailout. Second, this general result is, however, not robust to the introduction of imperfect information, fixed costs, or repeated interactions. In these cases, a government with a low level of commitment ability fails to implement a unique equilibrium outcome in a credible way because the government has strong incentives to ex post deviate from the necessary policies to rule out equilibrium multiplicity.

Static game. We first identify that, in static macroeconomic games, the critical parameter for implementation is what we dub the cost of controllability (Section 2). Consider a private-sector action that the government wants to avoid. To rule it out, the government should pick a policy response that deters private agents from individually playing this action. There may exist many such responses, which may entail different costs for the government relative to its ex-post best response. The cost of controllability is the maximum over all undesired private actions of the minimal cost of deterrence. The government implements a unique equilibrium outcome when its commitment ability exceeds this cost of controllability. Under this condition, the government can credibly commit to

[3] We provide potential interpretations of this cost in Section 2.4.

a reaction function from which it will not deviate ex post and that prevents any undesired outcome to form as an equilibrium. Finally, we show that a larger commitment ability always improves the best equilibrium outcome in static settings.

To our surprise, the cost of controllability boils down to zero in the static versions of the banks bailout from Farhi and Tirole (2012), the capital taxation problem and the inflation bias model. This implies that an arbitrarily small commitment ability is sufficient to implement a unique equilibrium. The main reason for this result is that, in these static models, private agents' marginal utilities are continuous functions of government responses, and the government can, thus, deter any undesired private actions by committing to a policy response arbitrarily close to its ex-post best response. However, we show that, even with a zero cost of controllability, commitment ability still constrains the set of reaction functions that the government can use to obtain a unique equilibrium—and a larger commitment ability enlarges this set.

An application of these results in the banks' bailout example is that the central bank can rule out bailout expectations with an arbitrarily low level of commitment ability. To this end, the central bank can commit to off-equilibrium partial bailouts that are only slightly less generous than the bailout expected by the private sector. These off-equilibrium partial bailouts are sufficient to make suboptimal any private actions anticipating a bailout. Thus, committing never to bail out is not necessary—on top of being time-inconsistent for low commitment abilities. In the capital taxation example, inefficient equilibria under discretion, where capital is insufficiently accumulated and taxed at high rates, can be ruled out by off-equilibrium low-cost commitments, whereby the government commits to tax capital at a smaller rate compared to expectations.

We then explore limits to this strong implementation result and show in which situations higher commitment ability is needed to ensure implementation.

Limits in static settings. We first showcase potential limits to our implementation result in static settings (Section 3) by introducing either imperfect information or fixed costs.

First, we assume that the government can only observe private actions with some noise. We show that this noise prevents the government from using discontinuous policy response to implement a unique equilibrium. In the capital taxation example, because of this constraint, the government needs a sufficient commitment ability to implement a unique equilibrium even for arbitrarily small noises. But, imperfect information does not always overturn our benchmark result on implementation, which holds, for example, in the bailout example whatever the level of noise.

Second, we introduce two additional frictions that lead to a positive cost of controllability: a fixed cost in private agents' maximization problem and a discrete set of actions on the government's side. In both cases, the more expensive it is for private agents to pay the fixed cost or the bolder the government's actions is constrained to be because of discrete actions, the more commitment ability the government needs to implement a unique outcome.

Repeated game. We then show how our benchmark implementation result in static settings changes in repeated games (Section 4). In repeated setting, due to history-dependent strategies, the private sector can react to past policy decisions, giving incentives to the government to stick to its commitments. This is why the resulting reputation forces[4] are often considered in the literature as an endogenous substitute for commitment ability, but they are also known to produce multiple equilibria, including in macroeconomic settings (see the initial contribution by Barro and Gordon, 1983b). To study how reputation forces change the cost of controllability, we extend macroeconomic games (see Chari and Kehoe, 1990; Stokey, 1991; Ljungqvist and Sargent, 2018, among others) to include the commitment to a reaction function by the government at each stage game. In this repeated setting, deviating from the reaction function leads to a welfare cost –either contemporaneous or future– for the government that measures its commitment ability, as in the static setting.

We show that, compared with the static setting, only a larger commitment ability ensures the implementation of a unique equilibrium outcome. The higher cost of controllability results from reputation forces that make government's future payoffs dependent on its response: due to history-dependent private reactions, the private sector can shift to an inferior continuation equilibrium when the government sticks to its commitment and, in contrast, to a better continuation equilibrium when the government deviates. Such history-dependent reactions lead to reputation forces for the government not to keep its commitment and to deviate from its reaction function. To get a unique equilibrium, the commitment ability must be sufficiently large to overcome these reputation forces in addition to the incentives that we identified in the static setting. Finally, as reputation forces have an influence through government's future payoffs, more-patient governments need more commitment ability to obtain a unique outcome.

In fact, reputation forces are of no help to obtain a unique outcome as reputation forces disappear when the equilibrium outcome is unique. These forces provide incentives to the government by promising inferior future equilibrium outcomes in case of deviations. But inferior outcomes simply do not exist when there is only one equilibrium outcome. Such a result implies that there is potentially a tension between obtaining a unique equilibrium outcome and the existence of reputation forces, which may be helpful to sustain better outcomes.

[4] As in Ljungqvist and Sargent (2018) reputation forces refer to the incentives stemming from repeated interactions between a government and a continuum of atomistic private agents. In such macroeconomic games, individuals are not strategic but atomistic. Still, they may form history-dependent expectations that allow for trigger "strategies." In contrast with "reputation effects" in game theory, we do not assume any form of asymmetric information.

Perhaps paradoxically, we also obtain that greater commitment ability is not always beneficial. Reputation forces are endogenous: they depend on future equilibrium outcomes, which also depend on the government's commitment ability. To further investigate the set of equilibria as a function of the commitment ability, we construct a fixed-point algorithm to compute the set of equilibrium payoffs for a given commitment ability. We apply this algorithm to the Barro-Gordon model, which features equilibrium multiplicity in repeated settings.[5] We obtain a "non-monotonic" relationship between commitment ability and the set of equilibrium outcomes. When the commitment ability is low, a marginal increase in this ability expands the set of equilibrium outcomes: the best equilibrium outcome becomes even better and the worst becomes worse. There exists a point at which the commitment ability becomes sufficiently large to obtain a unique equilibrium: at this point, a further increase in the commitment ability leads the equilibrium set to shrink and the payoff for the government in the best equilibrium to decrease.[6]

Related literature. First, our paper is connected to the literature on the time-inconsistency of government policies, starting with Kydland and Prescott (1977) and Barro and Gordon (1983a). More recently, in a setting that is very close to ours, Dovis and Kirpalani (forthcoming) analyze the asymmetric information problem in which policymakers can either fully commit to rules or act under discretion. In contrast with their study, ours does not consider asymmetric information, and we focus on the ability of the government to implement a unique equilibrium outcome.

Within this literature, our paper is closer to the papers that, after Barro and Gordon (1983b), show that government's time-inconsistent policies lead to equilibrium multiplicity. Such a multiplicity of equilibria was obtained in multiple strands of the literature, in either static or dynamic settings.

In the literature on bailouts, the complementarity between private actions and bailout decisions is well known to produce multiple equilibria (see Schneider and Tornell, 2004). Farhi and Tirole (2012) build a static model in which the inability to commit not to bail out leads to additional inferior equilibria.[7] This is a finding that is shared by Keister (2016) in a setting close to Ennis and Keister (2009) with the difference that some bailout is desirable even in the best equilibrium. In this literature, our paper is closely related to Philippon and Wang (2021), who also investigate "how much" commitment ability is needed to avoid bailout expectations, but in a setting in which policy can be made

[5]In the static version of the inflation bias model, the equilibrium is unique even under discretion, so commitment ability is useless for implementing a unique outcome.

[6]More commitment ability is, then, not always desirable, echoing the literature on commitment vs. flexibility (see Amador et al., 2006, among others).

[7]Notice that Farhi and Tirole (2012) emphasize that potential credibility losses may lead to a fixed cost for bailouts. However, they do not investigate the implications of such credibility losses for the required "amount" of commitment ability that would rule out equilibrium multiplicity.

contingent on individual actions. Our finding that, at least in some models, the cost of controllability is small so that the coordination problem is easily solved, may be taken as a motive to focus on the effects of time-inconsistency on the best equilibrium, as in Chari and Kehoe (2016). However, as we emphasize, this result is not robust to considering deviations from rationality or, simply, reputation forces, as in repeated settings, where, absent a large commitment ability, multiple equilibria may emerge.

In monetary economics, following Barro and Gordon (1983b), a literature has explored the conditions for the existence of multiple equilibria due to the central bank's time-inconsistency—or "expectation traps" as dubbed by Chari et al. (1998), who show how trigger strategies may lead to multiple equilibria, in a setting endogenizing both the public and the private sectors' behaviors. Time-inconsistency also produces multiple equilibria in static settings (see Albanesi et al., 2003; King and Wolman, 2004; Armenter, 2008). With respect to this literature, our paper shows that such a multiplicity is a robust feature only when adding additional ingredients such as reputation forces or bounds to rationality, as, in static contexts, arbitrarily small commitment abilities are sufficient to rule out multiplicity. Finally, while Chari et al. (1998) show in a dynamic setting that the full commitment to next-period policies are sufficient to rule out multiple equilibria, our paper suggests that such a full commitment may have a potentially large welfare cost that the government may not prefer to afford.

In monetary economics, the literature on monetary rules is also confronted to the presence of multiple equilibria. Here, the government or the central bank is able to commit to rules but, depending on its features, the rule itself may not be sufficient to prevent multiple equilibria to form (see Sargent and Wallace, 1975; Taylor, 1999; Clarida et al., 2000; Loisel, 2009; Atkeson et al., 2010; Hall and Reis, 2016, among others). Our paper is in the tradition of this literature, which emphasized that rules should be state-contingent or, even, sophisticated—i.e., history dependent. This literature has already highlighted constraints on off-equilibrium policy actions—they should be at least feasible as emphasized by Bassetto (2005)—or, they allow for a continuation of an equilibrium as in Atkeson et al. (2010), where the off-equilibrium central bank action is to keep the quantity of money constant forever. Historically, a key motive for introducing rules was to solve time-consistency issues as emphasized by Kydland and Prescott (1977). Such a motive has also led to investigate principal-agent approaches, delegations and contract theory in monetary economics (see Rogoff, 1985; Walsh, 1995; Jensen, 1997; Halac and Yared, forthcoming, among others). Our contribution with respect to this literature is to allow the government to deviate from its commitments, consistently with Bilbiie (2011) and Cochrane (2011). This has two implications. First, we consider the optimal design of commitments as part of the "game" played between the government and the private sector. This leads us to consider and model the government's incentives (e.g., through reputation forces). Second, and more importantly, we investigate how limited commitment is not only a constraint on the design of rules but can also simply prevent the government to obtain a unique equilibrium. In dynamic settings as is the case of monetary economies, we show that obtaining a unique equilibrium requires a large commitment ability. We leave, however, to future research the investigation on how time-consistency limits the commitment to history-dependent rules in monetary economies.

The focus of the literature on taxation is usually more on the best equilibrium that can be sustained—see the recent contribution of Halac and Yared (2019). However, multiple equilibria may still emerge in frameworks such as those of Chari and Kehoe (1990) or Bassetto and Phelan (2008). In this literature, our paper is more closely connected to Farhi et al. (2012), who first investigate time-consistent capital taxation in a static model with an exogenous commitment ability and then consider the repeated setting to endogenize commitment ability. In contrast to their approach, we not only look at the best equilibrium outcome, but we investigate the full set of equilibria.

2 The static game

We start with a static game. The government first commits to a reaction function; then, a continuum of atomistic private agents make decisions; finally, the government acts. The government incurs a welfare cost if its ex-post action deviates from the reaction function it has committed to. The cost allows us to obtain a continuum of commitment abilities between full discretion and full commitment. We first show that, to solve the coordination problem, the degree of commitment ability has to be strictly above a key variable that measures the cost of controllability. We then show that, under mild conditions, the cost of controllability is zero, and we discuss the cases in which it can be strictly positive. Finally, we show how our findings apply to three standard macroeconomic games: inflation bias, bailouts and capital taxation. In the last two examples, which otherwise produce multiple equilibrium outcomes, any arbitrarily small commitment ability is sufficient to rule out equilibrium multiplicity.

2.1 The environment

Consider an economy populated by a continuum of identical private agents and a government. There are three stages. First, the government commits to a reaction function, $\bar{y}$, that maps any private-sector action $x \in X$ to a policy action $\bar{y}(x) \in Y$, where X and Y are compact subsets of $\mathbb{R}$. Second, each private agent chooses an action $\xi \in X$, whose average is $x \in X$. Finally, the government chooses an action $y \in Y$. As with feasibility conditions, private decisions constrain government actions, so any government action must belong to a non-empty closed subset $D(x) \subseteq Y$ that depends on the average private action x. Finally, we focus on pure strategies.[8]

Competitive outcome. For a given allocation (x, y), the payoff of a private agent to play $\xi \in X$ is $u(\xi, x, y)$, where u is strictly increasing, strictly concave and twice continuously differentiable in ξ. We define a competitive outcome as follows:

Definition 1 (Competitive outcome)**.** *A competitive outcome is an allocation $(x, y) \in X \times Y$ such that*

$$y \in D(x), \tag{1}$$

$$x = \arg\max_{\xi \in X} u(\xi, x, y). \tag{2}$$

We denote by C the set of competitive outcomes.

Condition (1) requires that the government action y is feasible given average private action x, and Condition (2) requires that, given the allocation (x, y), it is optimal for any individual to set $\xi = x$. As all private agents are alike, this implies that aggregate x is indeed x.

Finally, to avoid making the implementation problem trivial, we assume that the government cannot punish individual deviations directly. Thus, we assume that only the aggregate private outcome x, not individual decisions ξ, is public information.

Government. Before any action, the government commits to a reaction function $\bar{y}$. Such a reaction function corresponds to the commitment to take the action $\bar{y}(x)$ if the average private action is x. We assume that the government commits only to feasible actions; that is, for any $x \in X$, $\bar{y}(x) \in D(x)$.[9] We denote by $Y(X)$ this set of functions.

The government cannot fully commit to the actions implied by its reaction function and can renege on its past commitments. However, if the government plays an action inconsistent with its reaction function, $y \neq \bar{y}(x)$, it incurs a cost $\kappa > 0$. κ measures the government value of sticking to a promise and will be referred to as the *commitment ability*.[10] For a given reaction function $\bar{y}$ and an average private-sector action $x \in X$, the government selects an action $y \in D(x)$ by maximizing :

$$r(\bar{y}, x, y, \kappa) = \begin{cases} w(x, y), \text{ if } y = \bar{y}(x), \\ w(x, y) - \kappa, \text{ otherwise}, \end{cases} \tag{3}$$

[8]As this will become clearer, there is no loss of generality in doing so when the government plays last and is not indifferent between multiple actions given aggregate private-sector action x, which we will assume.

[9]Under full commitment ability, committing to an unfeasible action would lead to a violation of a feasibility constraint. See Bassetto (2005) for further discussion about the role of feasibility constraints in implementation problem. In the absence of commitment, the government can still select a feasible action, despite the commitment to an unfeasible action. However, such a commitment would have no effects on ex-post incentives to select one particular feasible action over another, and there is then no loss of generality in our assumption.

[10]As a benchmark, we consider a fixed cost. We discuss the effect of considering more-general cost functions in Section 2.2.

where w is a strictly concave and twice differentiable function in y. As a result of this assumption, the government always has a unique best response to any aggregate private action x.

The government selects its reaction function so as to maximize the ex-ante payoff $\bar{r}(\bar{y}, x, y)$ defined as $r(\bar{y}, x, y)$, but where w is replaced by $\bar{w}$:

$$\bar{r}(\bar{y}, x, y, \kappa) = \begin{cases} \bar{w}(x, y), \text{ if } y = \bar{y}(x), \\ \bar{w}(x, y) - \kappa, \text{ otherwise}, \end{cases} \tag{4}$$

where $\bar{w}$ is a strictly concave and twice differentiable function in y. By selecting the reaction function $\bar{y}$, the government affects its ex-post incentives through the commitment ability κ. When $\kappa = 0$, the reaction function $\bar{y}$ is immaterial.

Notice that when $\bar{w}(x, y) = w(x, y) = u(x, x, y)$, aside from the reneging cost, the government is benevolent but disregards individual deviations. Considering a different payoff function for the ex-ante choices by the government will be useful in some examples.
Timing and equilibrium. An equilibrium is characterized by three strategies. They are, in chronological order: the reaction function $\bar{y} \in Y(X)$; the private-sector strategy $\sigma^h : Y(X) \to X$ that maps reaction function $\bar{y}$ into aggregate private-sector action $x = \sigma^h(\bar{y})$; and the government strategy σ^g specifies the government action $y = \sigma^g(\bar{y}, x) \in D(x)$ given the private-sector action $x \in X$ and the reaction function $\bar{y} \in Y(X)$. Figure 1 below summarizes the timing of the game.

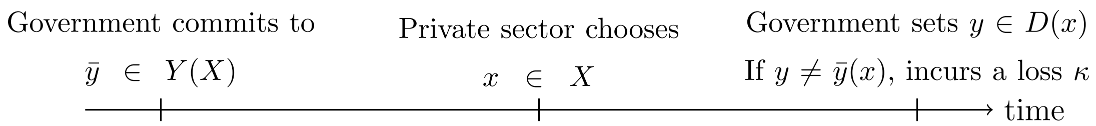


Figure 1: Timing of the game

To define an equilibrium, we proceed in two stages. First we define, if it exists, the continuation of an equilibrium given a reaction function. Second, we define the equilibrium itself. We do this as the continuation of an equilibrium may not necessarily form after any reaction function: the discontinuity in payoffs due to the cost κ may lead the government to favor ex-post actions that are not compatible with any competitive outcomes.

Let us first define the set of strategies for the private sector and the government's ex post action that allow for a continuation of an equilibrium after a reaction function:

Definition 2. *For a given reaction function $\bar{\eta} \in Y(X)$, the strategies $\{h, g\}$ forms a* ***continuation of an equilibrium*** *when:*

(i) $(h(\bar{\eta}), g(\bar{\eta}, h(\bar{\eta})))$ *is a competitive outcome; and*

(ii) $\forall x \in X$, $\forall \eta \in D(x)$, $r(\bar{\eta}, x, g(\bar{\eta}, x), \kappa) \geq r(\bar{\eta}, x, \eta, \kappa)$.

Conditions (i) and (ii) require that, given a reaction function $\bar{\eta}$, the strategies (h, g) constitute an equilibrium as in the standard models presented in Stokey (1991) or in chapter 21 in Ljungqvist and Sargent (2018). The only noticeable difference is that the reaction function matters for condition (ii) through the cost κ in r, which depends on the reaction function $\bar{\eta}$.[11]

Let us denote by $\mathcal{CE}(\bar{\eta})$ the set of strategies $\{h, g\}$ that form the continuation of an equilibrium given a reaction function $\bar{\eta} \in Y(X)$. As we mentioned, $\mathcal{CE}(\bar{\eta})$ may well be empty for some reaction functions and we then denote by $\hat{Y}(X)$ the subset of $Y(X)$ so that $\mathcal{CE}(\bar{\eta})$ is not empty.

We then define an equilibrium as follows:

Definition 3. *A (subgame perfect) equilibrium is a reaction function $\bar{y}$, a private sector strategy σ^h, and a government strategy σ^g such that, $\forall \bar{\eta} \in \hat{Y}(X)$:*

(i) $(\sigma^h, \sigma^g) \in \mathcal{CE}(\bar{\eta})$*; and*

(ii) $\bar{r}(\bar{y}, \sigma^h(\bar{y}), \sigma^g(\bar{y}, x), \kappa) \geq \bar{r}(\bar{\eta}, \sigma^h(\bar{\eta}), \sigma^g(\bar{\eta}, \sigma^h(\bar{\eta})), \kappa)$.

An equilibrium is a reaction function and strategies for the private sector and the government so that, given the reaction function, the strategies form the continuation of an equilibrium (Condition (i)) and the reaction function leads to the best ex-ante payoff for the government (Condition (ii)). Both conditions only require that, at each node of the game, actions are optimally selected as is the case in subgame perfect equilibria. Several additional comments are in order.

First, as we mentioned, the continuation of an equilibrium may not form after any reaction function. We do not consider this situation when assessing (ii) in the Definition 3, implicitly putting an arbitrarily low payoff to the absence of a continuation of an equilibrium.

Second, it is crucial in this definition that the private sector should play consistently with a competitive outcome after any reaction function, at least when possible –$(\sigma^h, \sigma^g) \in \mathcal{CE}(\bar{\eta})$. Individual optimality effectively puts a constraint on the private-sector strategy σ^h that has to react to the government's reaction function. Otherwise, without this constraint, the government would not be able to induce the private sector to change its behavior, as the private sector would be somehow able to "commit" to deviate from the competitive outcome to prevent the government from making certain commitments.

[11] Notice that the government cannot play unfeasible actions, as we always require (both in- and off-equilibrium) that $y \in D(x)$ with x the action played by private agents. As a result and, as in Bassetto (2005), the government cannot rule out a competitive outcome by playing (or promising to play) unfeasible actions in our setting.

Finally, we are interested in the set of *equilibrium outcomes*—and not necessarily the precise strategy profile that leads to such an equilibrium outcome—and the properties of this set as a function of the commitment ability parametrized by κ. More formally:

Definition 4. *Let us consider an equilibrium* $(\bar{y}, \sigma^h, \sigma^g)$*. Denoting by* $x = \sigma^h(\bar{y})$ *and* $y = \sigma^g(\bar{y}, x)$*, the resulting allocation* $(x, y) \in X \times Y$ *is an* ***equilibrium outcome****.*

$\Theta(\kappa)$ *denotes the set of such equilibrium outcomes and* $v(\kappa)$ *denotes the set of equilibrium (ex-ante) payoffs for the government.*

In particular, an allocation $(x, y) \in \Theta(0)$ *is a* ***Nash outcome****.*

One of our first tasks in the next subsection will be to move from Definition 4 to characterizations of the set of equilibrium outcomes. In what follows, we make the following assumption:

Assumption 1. *There exists at least one Nash outcome—*$\Theta(0)$ *is not empty.*

As we will see, this assumption is necessary for the set of equilibrium outcomes, $\Theta(\kappa)$, to be non-empty for any $\kappa > 0$.

Coordination problems and implementation. We can define what we shall refer to as a coordination problem using the set of equilibrium outcomes $\Theta(0)$.

Definition 5. *The government faces a* ***coordination problem*** *when there exist multiple equilibrium outcomes under discretion; that is,* $\Theta(0)$ *contains more than one equilibrium outcome.*

Given a commitment ability κ*, the government* ***implements*** *an allocation* (x, y) *if the set of equilibrium outcomes satisfies* $\Theta(\kappa) = \{(x, y)\}$*.*

A coordination problem is solved for a commitment ability κ when $\Theta(\kappa)$ is a singleton, in which case we also talk about implementation. The minimum for κ such that $\Theta(\kappa)$ is a singleton is the measure of the commitment ability required to solve the coordination problem. Notice that a coordination problem may be immaterial for the government when the different equilibrium outcomes lead to the same ex ante payoff for the government. In contrast, a coordination problem is payoff-relevant for the government when $\inf v(0) < \sup v(0)$.

Ramsey allocation. Finally, an allocation of interest is the Ramsey allocation, which we denote by (x^R, y^R). It corresponds to the best competitive outcome given the *ex ante* objective function of the government –assuming that the government does not incur the reneging cost, κ:

$$\bar{w}(x^R, y^R) = \max_{(x,y) \in C} \bar{w}(x, y). \tag{5}$$

This allocation coincides with the standard definition of Ramsey allocation when the government is benevolent ($\bar{w} = u$).

Notice that the Ramsey allocation (x^R, y^R) is not necessarily in the set of equilibrium outcomes $\Theta(\kappa)$, as playing y^R after x^R may be time-inconsistent for the government. In contrast, when (x^R, y^R) belongs to the set of equilibrium outcomes –the government optimally plays y^R after the private sector plays x^R given the reaction function–, the Ramsey allocation may not be the unique equilibrium outcome.[12] Only in the case in which (x^R, y^R) is the only equilibrium outcome will we say that the government can implement the Ramsey allocation. This is obviously the ideal situation from the government's ex-ante perspective.

2.2 Implementation under limited commitment ability

In this subsection, we investigate the equilibrium set as a function of the commitment ability κ along the continuum $[0, \infty)$. Our main result is that implementation depends only on three simple objects: first, the best time-consistent competitive outcome; second, the *controllability*, which is the fact that the set of policy actions that deter private agents from expecting an inferior outcome is non-empty; third, the cost of playing such actions—the *cost of controllability*. When this latter cost is lower than the commitment ability, the government can solve the coordination problem and implement the best time-consistent competitive outcome.

To obtain these results, our main task is to convert the definition of an equilibrium (Definition 3) that is about a strategy profile into characterizations of which allocations can be an equilibrium outcome.

Best time-consistent competitive outcome. Let $(x^\kappa, y^\kappa) \in C$ be the competitive outcome that maximizes the government's ex ante welfare, under the constraint that the government prefers playing y^κ instead of deviating and paying the cost κ—that is playing y^κ is time consistent after x^κ. Formally:

$$\max_{(x,y)\in C} \bar{w}(x, y) \tag{6}$$

$$\text{s.t. } w(x, y) \geq w(x, \eta) - \kappa, \forall \eta \in D(x). \tag{7}$$

As we will show, the best time-consistent competitive outcome is always the best equilibrium outcome.[13]

Notice that (x^κ, y^κ) coincides with the Ramsey allocation (x^R, y^R) when $\kappa \geq \bar{\kappa}$, where $\bar{\kappa} \in \mathbb{R}^+$ is the lowest cost such that the constraint (7) is not binding for the Ramsey allocation:

$$\bar{\kappa} = \max_{\eta \in D(x^R)} w(x^R, \eta) - w(x^R, y^R). \tag{8}$$

[12] As Chari and Kehoe (2016) note, in this case, the Ramsey allocation is only *weakly* implemented.

[13] (x^κ, y^κ) exists. (x^0, y^0) exists, as $\Theta(0)$ is not empty following Assumption 1. As a result, the set of allocation $(x, y) \in C$ satisfying the constraint (7) is not empty. As this set is a compact set and $\bar{w}$ a continuous function, it admits an upper bound.

The threshold $\bar{\kappa}$ measures the government's temptation to deviate from the Ramsey policy y^R when the private sector has played x^R.

Controllability. What should the government commit to in order to control the private sector and force it to play the action consistent with the best constrained outcome x^κ? In principle, private agents can play any action x that is consistent with a competitive outcome $(x, y) \in C$. To rule out a given private action x, the government should then commit to and stick to an action y such that (x, y) is not a competitive outcome. By definition, this means that the action y should make private agents better off playing $\xi \neq x$ if the aggregate private action is $x \neq x^\kappa$. Formally, for any $x \in X$, the government should select an action in the set:

$$\mathcal{Y}(x) = \{y \in D(x) |\ \exists \xi \in X, u(\xi, x, y) > u(x, x, y)\}.$$

For any $x \neq x^\kappa$, the government *can* discourage private agents from playing x if this set $\mathcal{Y}(x)$ is non-empty.

Assumption 2 (Controllability)**.** *For any $x \in X \setminus \{x^\kappa\}$, the set $\mathcal{Y}(x)$ is non-empty.*

Under this assumption, the government *can* control the private sector's actions and force it to play x^κ. But even if it *can*, it may be unable to credibly commit to all these policy actions because such actions are (ex post) costly. Take an aggregate private action $x \neq x^\kappa$. To deter that action, the government has to commit to an action $y \in \mathcal{Y}(x)$. Playing that action is costly to the extent that it is not the ex-post optimal action—i.e., the action that maximizes ex-post welfare $y^*(x) = \arg\max_{y \in D(x)} w(x, y)$. The resulting cost is then $w(x, y^*(x)) - w(x, y)$. Naturally, this cost is 0 when the ex-post optimal action is sufficient to prevent private agents from playing x, that is, $y^*(x) \in \mathcal{Y}(x)$. When the commitment ability κ is larger than this cost, the government will stick to its commitment and play y, and the expectation of such an action will deter agents from playing x.

Of course, for each $x \in X$, the government selects the action that minimizes its cost so that we can consider $\inf_{y \in \mathcal{Y}(x)} w(x, y^*(x)) - w(x, y)$. Finally, the government has to find such actions for all $x \in X \setminus \{x^\kappa\}$, so that the cost to deter any action that differs from x^κ is:

Definition 6 (Cost of controllability)**.** *Let $\rho \geq 0$ be the cost of controllability with:*

$$\rho \equiv \sup_{x \in X \setminus \{x^\kappa\}} \inf_{y \in \mathcal{Y}(x)} [w(x, y^*(x)) - w(x, y)], \tag{9}$$

where $y^*(x) = \arg\max_{y \in D(x)} w(x, y)$ is the ex-post best action of the government with $\kappa = 0$.

The cost of controllability refers to the maximum cost that the government has to tolerate ex-post to control the private sector. The cost of controllability is well defined under Assumption 2 as, otherwise, no policy action may exist to deter the private sector from playing some private-sector action $x \neq x^\kappa$. The controllability assumption also implies that the cost of controllability is finite ($\rho < \infty$).[14]

The equilibrium set as a function of commitment ability. We can describe the equilibrium set $\Theta(\kappa)$ using the best time-consistent competitive outcome (x^κ, y^κ) and the cost of controllability ρ:

Proposition 1. *Under Assumption 2, the equilibrium set $\Theta(\kappa)$ is such that:*

(i) ***Best time-consistent competitive outcome:*** $(x^\kappa, y^\kappa) \in \Theta(\kappa)$. *The set of equilibrium payoffs for the government $v(\kappa)$ is a compact set and $\bar{w}(x^\kappa, y^\kappa) = \max v(\kappa)$.*

(ii) ***Coordination:*** *If $\kappa > \rho$, the best equilibrium outcome (x^κ, y^κ) is implementable. $\kappa \geq \rho$ is a necessary condition for implementation.*

(iii) *Otherwise, when $\kappa < \rho$, the welfares in the best and the worst equilibrium outcomes, $\bar{w}(x^\kappa, y^\kappa)$ and $v_{worst}(\kappa) = \min v(\kappa)$, are weakly increasing in κ.*

Proof. See Appendix D.1 □

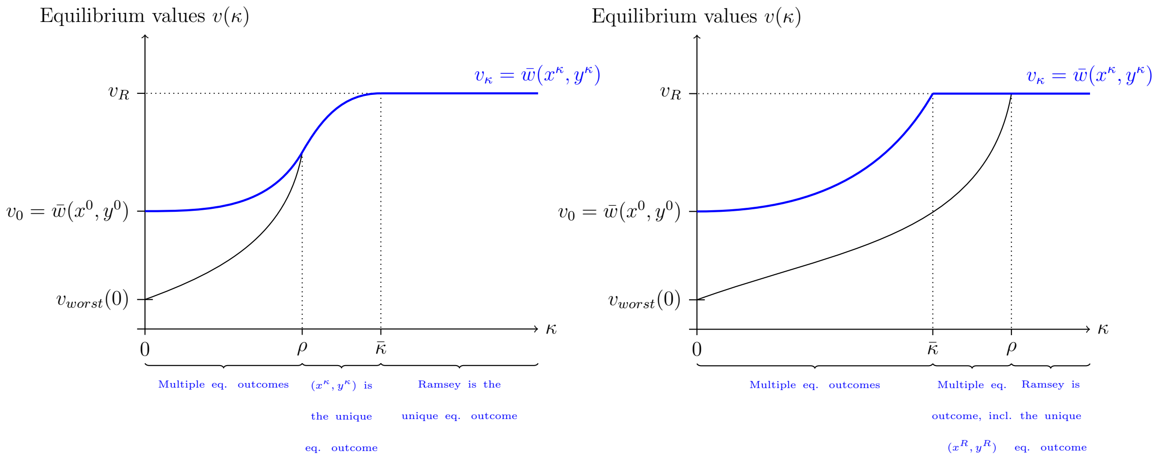
As a result of this proposition, the main variable to examine in determining the extent to which a government can solve a coordination problem is ρ: a unique equilibrium obtains when $\kappa > \rho$, and this equilibrium is the best time-consistent competitive outcome (x^κ, y^κ) defined above.

The proof goes as follows. On the one hand, the government can commit to a reaction function such that the response to x^κ is y^κ—i.e., $\bar{y}(x^\kappa) = y^\kappa$. Such a commitment implies that the deviation from y^κ would lead to a cost κ for the government, thus making y^κ a time-consistent action for the government. On the other hand, when $\rho < \kappa$, the government commits to play $\bar{y}(x) \in \mathcal{Y}(x)$, which discourages agents from playing x whenever $x \neq x^\kappa$, as in Bassetto (2005). $\rho < \kappa$ ensures that $\bar{y}(x)$ can be chosen so that the government is ex-post better off playing that action rather than anything else.

The proposition also clarifies that the best time-consistent competitive outcome is always an equilibrium outcome, one that leads to the best equilibrium payoff for the government.

Finally, the proposition provides some comparative statics on the equilibrium set as a function of commitment ability. Figure 2 summarizes the result of Proposition 1 in the two cases in which $\rho \leq \bar{\kappa}$ and $\rho > \bar{\kappa}$, where $\bar{\kappa}$ is the minimum level of commitment ability such that the Ramsey allocation is an equilibrium outcome and $(x^\kappa, y^\kappa) = (x^R, y^R)$. As the figure illustrates, increasing κ improves the equilibrium outcomes, not only the best one but also the worst one.

[14]That controllability implies $\rho < \infty$ results from X and Y being compact sets and w being a continuous function.



Case $\bar{\kappa} \leq \rho$ Case $\bar{\kappa} > \rho$

Figure 2: Best and worst equilibrium payoffs as a function of κ.

In the two graphs, we plot the best (v_κ) and the worst (v_{worst}) equilibrium payoffs for the government as a function of commitment ability (κ). The left-hand panel corresponds to the case in which $\bar{\kappa}$ –the minimum commitment ability required to achieve the Ramsey outcome– is below the cost of controllability ρ. The right-hand panel corresponds the case in which $\bar{\kappa}$ is above ρ.

Full commitment ability. A direct implication of Proposition 1 is when the government can fully commit—i.e. $\kappa = \infty$.

Corollary 2 (Bassetto (2005); Atkeson et al. (2010))**.** *Suppose that $\kappa = \infty$ and Assumption 2 is satisfied. Then, the government implements the Ramsey outcome, $\Theta(\infty) = \{(x^R, y^R)\}$.*

When $\kappa = \infty$, κ is larger than $\bar{\kappa}$. The Ramsey allocation is an equilibrium outcome and, by definition, the best one. When $\kappa = \infty$ and Assumption 2 is satisfied, the cost of controllability is well defined and κ is larger than ρ: the government can credibly rule out any inferior outcomes.

No additional commitment needed to solve the coordination problem. The critical variable for the ability of the government to solve the coordination problem is ρ. This variable captures the welfare cost for the government to engage in actions that discourage individual private agents from playing something other than the government desires.

When is a given private action $x \in X$ difficult to rule out? Inspecting the definition of ρ in Definition 6 reveals that discouraging agents from playing actions x that are not part of any competitive outcomes is not costly at all as playing the ex-post optimal action is already sufficient to deter private agents from playing x—formally, $y^*(x) \in \mathcal{Y}(x)$. More generally, any action x—whether part of a competitive outcome or not—such that the ex-post optimal policy does not lead to a competitive outcome ($y^*(x) \in \mathcal{Y}(x)$), is costless to deter. The remaining question is, thus, whether or not it is costly to deter an action $x \in X$ such that $y^*(x) \notin \mathcal{Y}(x)$.

In many examples, private agents' marginal utility responds to government actions. In this case, we show that it is easy to rule out not only competitive outcomes that are not Nash outcomes but also Nash outcomes themselves:

Proposition 3. **(i)** *The government implements* (x^κ, y^κ) *if and only if it can credibly deter the private sector from playing the actions* $x \in X$ *different from* x^κ *and consistent with a Nash outcome.*

(ii) *Suppose that any Nash outcome* $(x, y) \in \Theta(0)$ *is interior and* $u_{13}(x, x, y) \neq 0$*; then,* $\rho = 0$ *and* $\Theta(\kappa) = \{(x^\kappa, y^\kappa)\}$ *for any* $\kappa > 0$*.*

Proof. See Appendix D.2 □

Formally, a Nash outcome $(x, y) \in \Theta(0)$ is interior when there exists an open interval $I(x) \subset D(x)$ containing y and when $x \in \mathring{X}$, the interior of X.

The proposition first clarifies that only the private-sector actions belonging to a Nash outcome may be costly to deter—that is, contribute to the cost of controllability ρ.

Second, the proposition shows that, if all the Nash outcomes are *interior* and if the marginal utility of private agents locally depends on the policy action, then the government can deter all the actions associated with a Nash outcome and, hence, the government can solve the coordination problem at no cost.

To see that, take an interior Nash outcome (x, y) such that $u_{13}(x, x, y) > 0$. As x is interior, the marginal utility of an individual agent is zero for the allocation ($u_1(x, x, y) = 0$), as, otherwise, this agent would be better off by setting $\xi \neq x$. Given that $u_{13}(x, x, y) > 0$, the government can select an action slightly above the ex-post best action y and increase the individual agent's marginal utility above 0; that is, $u_1(x, x, y) > 0$. In turn, the individual agent is better off increasing its action ξ above x to maximize utility, which is feasible since the action x belongs to the interior of X. Finally, the policy action is almost not costly since it can be selected arbitrarily close to the ex-post best action. The same reasoning applies when $u_{13}(x, x, y) < 0$.

When a Nash outcome is not interior, the cost of controllability may be high ($\rho > 0$) even if $u_{13} \neq 0$. The government may be unable to depart from its ex-post best action locally in the right direction (in the above paragraph, choosing an action y slightly above the ex-post optimal one $y^*(x)$) because of a feasibility constraint on its action or because the private sector cannot move away from the aggregate private action x locally in the right direction (in the above paragraph, choosing ξ slightly above x). In such a case, and under the controllability assumption, only a costly policy action may succeed in deterring private agents from playing the undesired action $x \neq x^\kappa$, leading to a high cost of controllability.

In the next subsection, we illustrate how Proposition 3 applies in different examples, such as the Farhi and Tirole (2012) model of bailouts and the capital taxation problem.

2.3 Examples

The model laid out above can encompass multiple macroeconomic situations under discretion ($\kappa = 0$). In this subsection, we describe some of these situations and illustrate how time-inconsistency may or may not lead to coordination problems—see Table 1 for a summary. The reader interested only in the general results may skip this subsection.

To start with, the model of bailouts by Farhi and Tirole (2012) illustrates how time-inconsistency leads to a coordination problem. This happens even though the Ramsey outcome is an equilibrium outcome. Second, the model of inflation bias from Barro and Gordon (1983a) illustrates that, despite time-inconsistency, only one equilibrium outcome may arise, even though this equilibrium outcome is inferior to the Ramsey outcome. In Section 4, we use this model in a repeated context. Finally, under some conditions, a simple model of capital taxation, as in Chari and Kehoe (1990), illustrates a situation of time-inconsistency leading to a coordination problem where the Ramsey outcome is not an equilibrium outcome.

	Time-inconsistent Ramsey allocation	Coordination Problem
Inflation bias (Barro and Gordon, 1983)	✓	
Bailouts (Farhi and Tirole, 2012)		✓
Capital taxation (Chari and Kehoe, 1990)	✓	✓

Table 1: Coordination problem and time-consistency of the Ramsey allocation.

2.3.1 Bailout problem.

The environment. Consider a bailout problem as in Farhi and Tirole (2012), from which we borrow the notations. There are three periods $t \in \{0, 1, 2\}$. The economy is populated by risk-neutral bankers, deep-pocket risk-neutral investors and a government.

At date 0, bankers receive an endowment A, and they can invest in a risky investment opportunity and borrow short-term. At date 1, with probability α, the investment opportunity yields $(\pi + \rho_1)i$ for i invested at date 0, among which $(\pi + \rho_0)i$ can be pledged to investors with $\rho_0 < 1$. With probability $1 - \alpha$, investment yields only πi at date 1 and $\rho_1 j$ at date 2 with $j \leq i$ the amount of resources reinvested at date 1. In this case, only $\rho_0 j$ can be pledged to investors. The returns on investment opportunities are perfectly correlated across bankers. At date 0, bankers optimally set a contingent short-term debt contract equal to πi in the absence of crisis and di otherwise.

The government sets the real rate of interest. Between date 0 and date 1, as well as between date 1 and date 2 when investment is successful, the government optimally selects an interest rate equal to 1. In the event of a crisis, the government sets an interest rate $R \leq 1$ between date 1 and date 2 to maximize its objective function:

$$-L(R) - \frac{(1-R)\rho_0 j}{R} + \beta j \tag{10}$$

with $L(R)$ a deadweight loss associated with setting interest rates below 1. L satisfies $L(R) \geq 0$, $L(1) = L'(1) = 0$ and L is decreasing on $[\rho_0, 1]$. The second member of (10) corresponds to the subsidy from savers to borrowing banks at a rate below 1. The last member stands for the gain due to higher date-1 reinvestment—by convention, $j = i$ in the case of a successful investment. The date-0 objective of the government is the expectation of the date-1 objective function. In this model, bankers play first by selecting investment i and short-term debt d. Then, the government plays R at date 1 (in the event of a crisis), and then the bankers decide to reinvest if needed.

When investment is unsuccessful and needs reinvestment, bankers optimally select reinvestment j so that:

$$j = \min\left\{\frac{\pi - d}{1 - \frac{\rho_0}{R}}; 1\right\} i. \tag{11}$$

At date 0, this leads bankers to select investment i so that:

$$i = \frac{A}{1 - \pi - \alpha\rho_0 + (1-\alpha)\xi}, \tag{12}$$

where $\xi \equiv \pi - d$ is the liquidity ratio; that is, ξi is the banker's cash-flow available at date 1 in case of a crisis net of debt repayment di. ξ maximizes

$$(\rho_1 - \rho_0)(\alpha i + (1-\alpha)j) = (\rho_1 - \rho_0)\left(\frac{\alpha + (1-\alpha)\frac{\xi}{1-\rho_0/R}}{1 - \pi - \alpha\rho_0 + (1-\alpha)\xi}\right) A. \tag{13}$$

Mapping with the general model. Farhi and Tirole (2012)'s model is a game between bankers at date 0 and the government's intervention in the case of unsuccessful investment at date 1. Notice that the individual banker's action ξ does not depend on other actions, except through the dependence of the policy rate R on the aggregate bankers' decisions. Using ξ as defined above, we can then map this model to our general setting as follows:

$$\xi \in [0, 1-\rho_0], \;\; x = \xi, y = R \;\; D(x) = [\rho_0, 1].$$

We can express the objective function (10) as a function of $y = R$ and $x = \xi$ by using (11) and (12). This defines $w(x, y)$. Finally, (13) defines $u(\xi, x, y)$.

In this example, the objective function relevant ex ante for the government differs from the one relevant ex post due to the uncertainty around the success of the investment at date 1. Ex ante, this objective function is:

$$\bar{w}(x,y) = \alpha\beta i(x) + (1-\alpha)\left(-L(R) - \frac{(1-R)\rho_0 j(x)}{R} + \beta j(x)\right).$$

First, under the assumption that β is sufficiently small ($\beta \leq 2-\alpha-\pi-\rho_0$), the Ramsey allocation is such that $R=1$ (Proposition 1 in Farhi and Tirole (2012)). The Ramsey allocation is an equilibrium outcome in this model. This happens under the condition that $(\beta+\rho_0-1)A/(1-\pi-\alpha\rho_0) \geq L(\rho_0)$ (Corollary 1 in Farhi and Tirole (2012)). Second, there are multiple other equilibria with bailout, $R<1$ (see Proposition 2 in their paper): this model features a coordination problem, even though time-inconsistency does not prevent the Ramsey allocation from being an equilibrium outcome.

The equilibrium set under limited commitment ability. Our general results imply that, in this model, for any positive—and potentially arbitrarily low—$\kappa>0$, the Ramsey allocation is the unique equilibrium outcome. Indeed, the inspection of the objective function of bankers (13) indicates that $u_{13}<0$ and that the Nash outcomes, with the exception of the Ramsey outcome, are interior. As a result, the conditions of Proposition 3 are satisfied.

But how does this work in practice and how can the government deter bankers from anticipating a bailout? Figure 3 illustrates what the government may do.

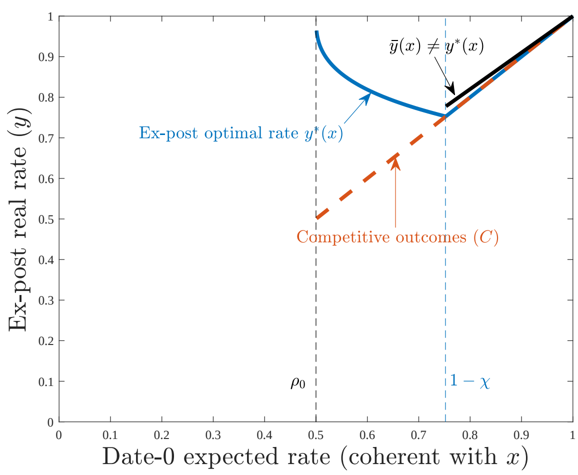


Figure 3: Obtaining a unique equilibrium in Farhi and Tirole (2012) model of bailouts

In this graph, we plot the set of competitive outcomes (C), the ex-post optimal response by the government as a function of private-sector actions x ($y^*(x)$) and, finally, one reaction function $\bar{y}$ that allows the government to select a unique equilibrium outcome, when this reaction function differs from the ex-post optimal reaction function y^*. Notice that such a reaction function $\bar{y}$ allows the government to implement the Ramsey allocation.

In this model, the ex-post optimal policy is to set the interest rate R at the level at which the private sector expected it, at least when this interest rate does not deviate much from the Ramsey level $R=1$. The cost of decreasing R is continuous in R, while there is a fixed gain related to refinancing projects. This leads to a continuum of equilibria. To prevent this, the government simply commits to a bailout policy very close to the one expected—$R = R^e + \epsilon$—where R^e is the expected rate by the private sector. Then, the government calibrates ϵ as a function of its commitment ability κ: $w(x(R^e), R + \epsilon) \leq w(x(R^e), R) + \kappa$. As can be observed in Figure 3, such a bailout policy prevents any equilibrium to form where $R < 1$.

How realistic are such partial bailouts? Implementing this solution may be quite involved, first because it may require eliciting the private sector's expectations and, second, because it also requires that private agents are sufficiently responsive to small variations in the expected policy. However, in other contexts, such as the rescue of a sovereign like Greece during the euro area sovereign debt crisis, a form of partial bailouts was put in place with private-sector involvement (PSI). Our result suggests that committing to a limited private-sector involvement may be sufficient to rule out expectations of bailout in equilibrium. In addition, such a low-cost commitment to partial bailouts make non-necessary Farhi and Tirole (2012)'s solution, which requires regulating bankers by imposing a cap on short-term debt or, equivalently, a liquidity requirement at date-0, whenever the government cannot perfectly commit.

2.3.2 Inflation bias problem.

The environment. Consider the inflation bias model à la Barro and Gordon (1983a). First, the government commits to a reaction function that maps aggregate households' inflation expectations to a level of inflation; second, households form expectations about current inflation, the average of which is π^e; then, the government picks the inflation rate π; finally, output z is realized:

$$z = \lambda(\pi - \pi^e), \tag{14}$$

where $\lambda > 0$ is the slope of the Phillips curve. As in Bassetto (2019), the objective of each household is to minimize its forecasting error (all nominal contracts that households and, more generally, the private sector can write are a source of costs for forecast errors):

$$u(\xi, \pi) = -(\xi - \pi)^2. \tag{15}$$

Therefore, a competitive outcome in this setting is simply one with $\xi = \pi = \pi^e$, and so $z = 0$ in any competitive outcome.

The government seeks to minimize the loss function:

$$v(z, \pi) = -(z - y^*)^2 - \alpha(\pi)^2, \tag{16}$$

where $\alpha > 0$ is the relative weight of the government's inflation stabilization objective and y^* is the source of the inflation bias.

Mapping with the general model. We can map this example to the general setting as follows:

$$\xi \in [\underline{\pi}; \bar{\pi}], \; x = \pi^e \in [\underline{\pi}; \bar{\pi}], \; y = \pi \in [\underline{\pi}; \bar{\pi}], \; D(x) = [\underline{\pi}; \bar{\pi}],$$
$$u(\xi, x, y) = -(\xi - y)^2, \text{ and } w(x, y) = -(\lambda(y - x) - y^*)^2 - \alpha y^2.$$

The best competitive outcome (the Ramsey allocation) is $(z^R, \pi^R) = (0, 0)$. Under discretion ($\kappa = 0$), there exists a unique equilibrium outcome, the pair $(z_m, \pi_m) = (0, \frac{\lambda}{\alpha} y^*)$. When $y^* > 0$, this equilibrium outcome differs from the Ramsey allocation. Under discretion, the inflation-bias model does not suffer from a coordination problem, even though time-inconsistency prevents the Ramsey allocation from being an equilibrium outcome.
The equilibrium set under limited commitment ability. In this model, there is a unique Nash outcome. The same is true for any positive commitment ability—note that $u_{13} = -2$ and the conditions of Proposition 3 are satisfied. The uniqueness of the equilibrium outcome can be observed in the left panel of Figure 4, where the set of competitive outcomes intersects the set of the ex-post optimal policy only once.

However, the Nash outcome does not coincide with the Ramsey outcome. The best inflation outcome π^κ with $\kappa > 0$ is a strictly decreasing function of κ for $\kappa \in [0; \bar{\kappa}]$, and then $\pi^\kappa = 0$ for any $\kappa \geq \bar{\kappa}$. To further illustrate this outcome, the dashed blue line in the right panel of Figure 4 plots π^κ as a function of the commitment ability κ, for the following parameters: $\alpha = 1$, $\lambda = 0.5$, and $y^* = 0,005$. The red line reports the Nash outcome, $\pi_N = \frac{\lambda}{\alpha} y^*$, and the yellow line reports the Ramsey allocation, $\pi_R = 0$. We observe that the inflation rate in the best equilibrium outcome, x_κ, goes continuously from the Nash level to the Ramsey level when κ increases.

2.3.3 Optimal capital taxation problem.

The environment. Consider a two-period taxation problem (adapted from Fischer, 1980; Chari and Kehoe, 1990; Bassetto, 2005). Time is discrete and indexed by $t = \{1, 2\}$. The economy is populated by a continuum of households and a government. At date 1, each household receives an endowment ω and decides to consume $\tilde{c}_1$ or to invest ξ in a linear saving technology, which yields $R(k)\xi > 1$ units of goods at date 2, where $R(k)$ is a decreasing function of aggregate capital k. At date 2, households work and we denote by l the corresponding number of hours. We assume that the marginal product of labor is 1. The government can tax the return of capital at a tax rate δ and labor income at a rate τ. Then, each household consumes the after-tax return of its investment and labor. Households value the consumption profile $(\tilde{c}_1, \tilde{c}_2)$ and labor $\tilde{l}$ using the utility function $\tilde{u}(\tilde{c}_1, \tilde{c}_2, \tilde{l})$. The government decides on taxes so as to maximize households' utility subject to the constraint to finance an exogenous amount of public expenditures G—its budget constraint is $G \geq \delta R k + \tau L$.

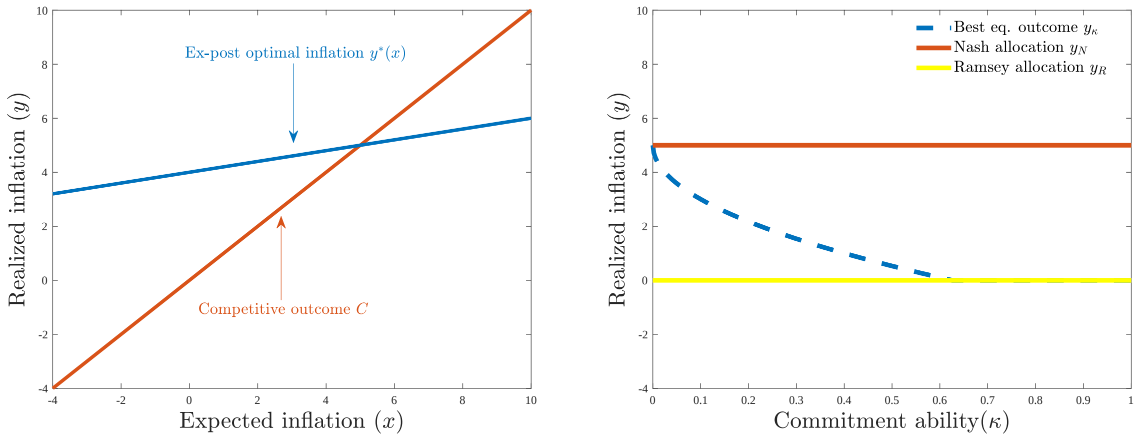


Figure 4: Equilibrium outcome in the Barro-Gordon model and equilibrium realized inflation as a function of the commitment ability κ

In the left-hand panel, we plot the set of competitive outcomes and the ex-post optimal actions $(y^*(x))$ as a function of private agents' average action x. In the right-hand panel, we plot the equilibrium realized inflation as a function of the commitment ability κ, along with realized inflation in the Ramsey outcome and in the Nash outcome.

The timing is as follows. First, the government commits to a reaction function $\bar{\delta}$, which maps a date-1 aggregate saving k to a (promised) tax rate $\delta = \bar{\delta}(k)$. Then, households consume (in aggregate) c_1 and invest k. The government selects the tax on capital income δ and the tax on labor τ. If the government sets a tax rate different from $\delta = \bar{\delta}(k)$, it incurs a reneging cost κ. Finally, the households choose l and consume c_2.

Households select date-1 saving ξ for a given aggregate capital k and expected tax rates (δ, τ) as follows:

$$\max_{\xi, l, c_1, c_2} \tilde{u}(c_1, c_2, l)$$
$$\text{s.t. } c_1 \leq \omega - \xi \text{ and } c_2 \leq R(k)\xi(1-\delta) + l(1-\tau).$$

A competitive outcome must be such that $\xi = k$.

Under discretion ($\kappa = 0$), at date 2—that is, given k and c_1—the government selects tax rates (δ, τ) by maximizing:

$$\max_{\delta, \tau, c_2, l} \tilde{u}(c_1, c_2, l)$$
$$\text{s.t. } -\frac{u_l}{u_{c_2}} = 1 - \tau \text{ and } G \leq \delta R(k)k + \tau L.$$

The first constraint corresponds to the optimal consumption-leisure household's decision, the second to the government budget constraint. These two constraints and the equality $l = L$ implicitly define the labor tax $\tau(\delta, k)$ and the individual labor decision $l(\delta, k)$ as a function of the capital tax rate δ and the aggregate date-1 saving k. Notice that these functions are unaffected by the presence of a reneging cost. The date-2 decision by the government to tax labor and the households' decisions to consume and to work are non-strategic. The strategic interaction concerns date-1 private saving decisions and date-2 capital taxation.

Mapping with the general model. The model can be linked to the general setting defined above as follows.

$$\xi \in [0;\omega],\ x = k \in [0;\omega],\ y = \delta \in [0,1],\ D(x) = [0;1],$$
$$\text{and } u(\xi, x, y) = \tilde{u}(\omega - \xi, R\xi(1-y) + (1-\tau(y,x))l(y,x), l(y,x)).$$

From the date-2 perspective, taxing capital is not distortive, while taxing labor is. Therefore, under discretion, the government taxes capital as much as needed to finance government expenditures. Under discretion, there exists an equilibrium in which households expect a tax rate $\delta = 1$ and do not save. There exists at least another equilibrium in which taxes on capital do not prevent households from saving when government's expenditures are low enough and the return on capital is high enough.

In the end, time-inconsistency in the capital taxation model both prevents the Ramsey allocation from being an equilibrium outcome and leads to a coordination problem as multiple equilibria emerge.

The equilibrium set under limited commitment ability. In this example, our general results help to rule out only interior Nash outcomes that are inferior for the government and the Nash outcome where capital is taxed at 100% is not an interior Nash outcome. Despite this, an arbitrarily small commitment ability is sufficient to rule out all inferior Nash outcomes.

Let us start with our general results. The controllability assumption depends on the combination of second derivates of $\tilde{u}$ as follows:

$$u_{13}(x,x,y) = \frac{\partial l}{\partial y}\left(R(1-y)\tilde{u}_{22} - \tilde{u}_{12} + R(1-y)\tilde{u}_{23} - \tilde{u}_{13}\right) - R\tilde{u}_2. \tag{17}$$

When the utility function $\tilde{u}$ is separable in each argument, u_{13} is of the sign of $\frac{\partial l}{\partial y}R(1-y)\tilde{u}_{22} - R\tilde{u}_2$, which is strictly negative when $\tilde{u}$ is strictly increasing and strictly concave in c_2: an increase in the tax rate y decreases saving ξ. Under these assumptions, all interior Nash outcomes can be ruled out with an arbitrarily small commitment ability—but, this is not the case of the outcome where capital is fully taxed.

Let us illustrate how *all* the inferior Nash outcomes can be ruled out, including the one where capital is fully taxed. To this purpose, we calibrate the model as follows: $u(c_1, c_2, l) = 1/4\log(c_1) + \beta(c_2 - l^2/2)$. Parameters are set to $R = \beta = 1$, $\omega = 1$, and $G = 0.1$.

Figure 5 plots the set of competitive outcomes (in red) and the ex-post optimal policy (in blue)—the set of Nash outcomes is the intersection of the two. The best outcome is the one where capital is less taxed and savings are high. In black, we plot the reaction function that allows the government to rule out the two inferior outcomes. As we already discussed, this reaction function needs to differ from the ex-post optimal policy only for the inferior outcomes. In this case, the government commits to a capital tax rate lower than the ex-post optimal rate.

As can be observed, the interior Nash outcome can be ruled out by committing to an arbitrarily close tax rate, which makes the ex-post cost of such a commitment arbitrarily small—thus requiring only an arbitrarily small commitment ability. However, the same logic cannot be applied to the Nash outcome where capital is fully taxed: the tax rate should be reduced by a substantial amount to push individual households to save. Yet, such an important reduction in the tax rate is not costly for the government: when aggregate capital is 0, modifying the tax rate yields 0 variations in the capital tax income received by the government.

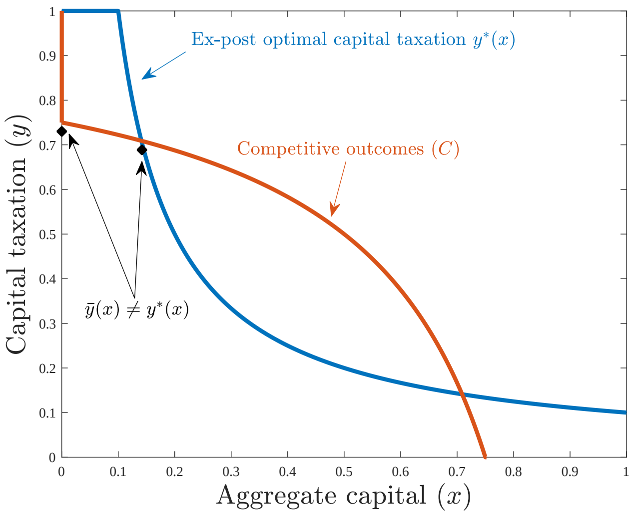


Figure 5: Equilibrium capital stock and capital taxation

This graph plots the set of competitive outcome (C) and the ex-post optimal action $y^*(x)$ as a function of private agents' action x. In addition, we add to this graph the reaction function $\bar{y}$ that is sufficient to rule out all equilibria except the best one, when this reaction function differs from the ex-post optimal reaction function y^*.

2.4 Interpreting κ?

To conclude this section, let us discuss what may be behind the cost κ.

Reputation loss. A first interpretation of the cost κ is that this cost captures the reputation loss associated with a deviation from past announcements (see Dovis and Kirpalani, forthcoming, for such a recent model or reputation loss in macroeconomics). Suppose, indeed, that there are two types of policymakers, one that can perfectly commit to reaction functions and the other one that cannot and sets its policy under discretion. When the type of the policymaker cannot be observed, private agents form beliefs about its type. In this environment, playing something else than the reaction function $\bar{y}(x)$ leads the discretion-type policymaker to reveal its type and to lose any gain from being pooled with the commitment-type policymaker.

Remark. Whereas reputation loss may provide micro-foundations for commitment ability, we show in Section 4 that the repetition of the static game does not—the dynamic incentives in the repeated game setting is also sometimes called reputation in macroeconomics.

Political institutions. A second interpretation of the cost κ is the cost due to the decision process needed to deviate from a reaction function. Such a deviation may require to change a law or a regulation, requiring a potentially costly political and bureaucratic process—for example, as a result from disagreement between political parties (e.g., Piguillem and Riboni, 2021, in the context of fiscal policy). Such kind of costs may exist when decisions are made by committees, as it is the case in public institutions such as central banks (see, for example, Riboni, 2010).

Judicial institutions. A third interpretation of the cost is the fact that legislations may put constraints on the degree of discretion of policymakers. This is, for example, the case of the Administrative Procedure Act of 1946 in the US, which requires that regulations by agencies should not be "arbitrary and capricious, an abuse of discretion". Especially under the "Hard Look" doctrine, this requires agencies to sufficiently motivate their decisions to change their set course of actions.

Intrinsic preference. The potential embarrassment of the policymaker—when deviating from past commitments—emphasized by Woodford (2012) may be related to an intrinsic preference by the policymaker to stick to commitments. Also, such a preference may be shared by private agents so that a deviation by the policymaker may result into a welfare loss, that the policymaker may internalize.

Cognitive cost/bounded rationality. Finally, the cost associated with a deviation may simply be the cost of acquiring and processing information to find the optimal deviation.

3 Limits to implementation in static settings

This section documents three situations in static settings in which, contrasting with our benchmark result (Proposition 3), a non-negligible commitment ability is needed for implementation: imperfect information, fixed costs and discrete action sets. For the sake of exposition, we derive our results in the capital taxation problem but they can be easily extended to the general setup of Section 2.

3.1 Imperfect information

In this subsection, we show that more commitment ability may be needed for implementation when the government imperfectly observes the private sector action. The main intuition of this result is that, in such a configuration, the expectations of government action are continuous in the private action, x, preventing the government from adopting a deterring strategy that is not ex-post costly.

Suppose that, when the government selects the level of capital taxation, δ, the government imperfectly observes the level of capital $k + \epsilon$ where ϵ is a noise on a bounded compact support $|\epsilon| < \bar{\epsilon}$ including 0.[15] Then, the government observes the true level of capital and sets the labor tax so that the labor and capital income tax revenues cover the government spending G.

Imperfect information crucially modifies the Euler equation at date 1. Instead of correctly expecting the future tax level δ, the private agents take date-1 decision based on their expectations of future capital income tax, $\mathbb{E}_\epsilon \delta(k+\epsilon)$, over all possible noises ϵ:

$$\tilde{u}_1 = \mathbb{E}_\epsilon \left[\beta R(k)(1 - \delta(k+\epsilon))\tilde{u}_2 \right]. \tag{18}$$

The government deters the private agents from investing a level of capital $k \in (0, \omega)$, if the above equation is not verified for $\xi = k$, that is, if the ex-post return on capital is not consistent with optimal consumption smoothing. Using the same calibration as in Section 2, this condition boils down to:

$$\frac{1}{4}\frac{1}{1-k} \neq (1 - \mathbb{E}_\epsilon \delta(k+\epsilon)). \tag{19}$$

Since δ is bounded, the left-hand-side member is continuous in k. The government cannot count on a discontinuity in its actions for implementation contrary to the baseline case with perfect information because the expected action is always continuous. This constraint makes the commitment ability required for implementation higher than its perfect information counterpart:

Proposition 4. *In the capital taxation problem with imperfect information, there exists a threshold $\kappa_\epsilon > 0$ such that when the noise is small enough ($\bar{\epsilon} \to 0$), for any commitment ability $\kappa > \kappa_\epsilon$ there exists a unique equilibrium outcome. For strictly lower levels, multiple equilibrium outcomes exist.*

Proof. See Appendix D.3. □

Introducing noise leads to substantially different results compared with Proposition 3. A strictly positive commitment ability is required to implement a unique equilibrium, even in the limit case where noise is close to 0. The threshold κ_ϵ plays a similar role than the cost of controllability ρ in Proposition 3, but depends on the ex-post incentives to deviate not only around Nash equilibria but also far away from them. This difference is due to the fact that private agents expect a government action that is continuous in x which forces the government to count on continuous reaction function to implement a unique equilibrium.

[15] Since $k \in [0, \omega]$, the support of the noise has to be capital dependent at least for capital levels close to 0 and ω. But this will not play an important role in the result and the noise could be allowed only for interior levels of capital. See Appendix D.3 for a formal definition.

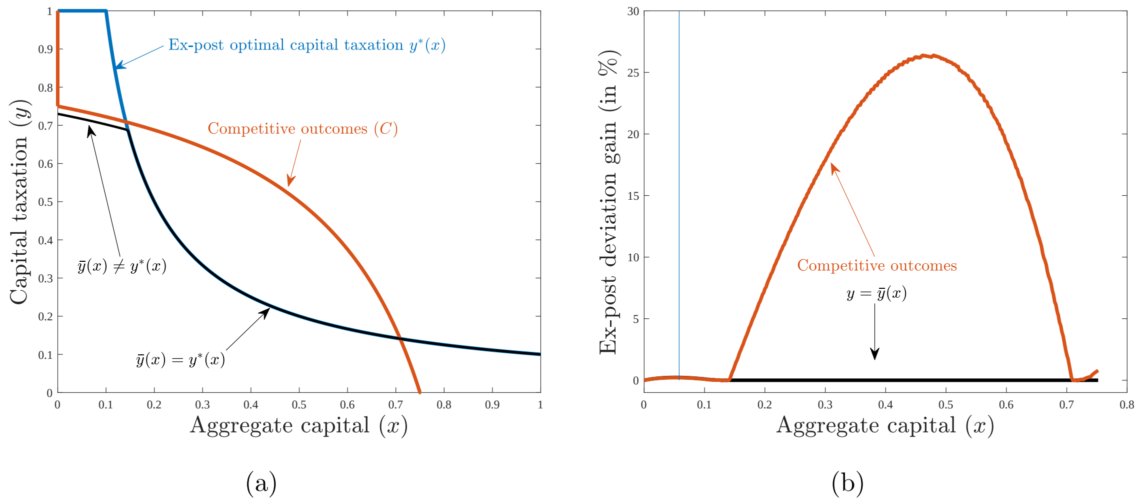


Figure 6: Equilibrium capital stock and capital taxation when the noise is close to 0.

This graph plots the set of competitive outcome (C), the ex-post optimal action $y^*(x)$ as a function of private agents' action x and the reaction function $\bar{y}$ that is sufficient to rule out all equilibria except the best one, when the size of the noise is close to 0.

To better understand these results, panel (a) in Figure 6 plots the set of competitive outcomes, the ex-post optimal policy action and the reaction function required to obtain a unique equilibrium. As we noticed, noise requires the reaction function to be a continuous function of private actions. As a result, the reaction function is sometimes far from the ex post optimal policy action. The commitment ability that is required to implement a unique outcome is then the largest difference in welfare terms between these two curves.

This leads to the second difference with Proposition 3: as illustrated by panel (b) in Figure 6, the largest commitment ability is reached for a level of capital x that is not a Nash outcomes and, in contrast, the distance in welfare terms between the reaction function and the ex post optimal action is 0 for Nash outcomes—even if the reaction function implies an action substantially different that the ex post optimal one when $x = 0$, the welfare cost is 0. From Proposition 3 in perfect information, this second point would imply a cost of controllability equal to 0—and a required commitment ability of 0. With noise, one has also to consider incentives to deviate away from Nash outcomes: the government has to stick to the reaction function $\bar{y}$ and not deviate from it even away from Nash outcomes, or it will create discontinuities—here, jumps from $\bar{y}$ to y^*—which would introduce multiple equilibria.

A general result? Does noise always lead to a strictly positive required commitment ability? The answer is, in general, no. This is the case only when, under perfect information, the reaction function has to be discontinuous to obtain a unique equilibrium. In contrast, when this reaction function can be continuous, introducing some noise does not modify the results of Proposition 3. An example of such a situation is the bailout problem as illustrated by Figure 7.

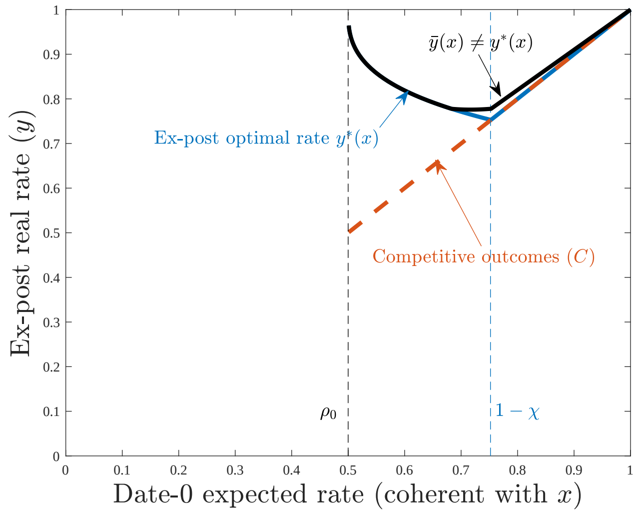


Figure 7: Equilibrium capital stock and capital taxation when the noise is close to 0.

The graph on the left plots the set of competitive outcome (C), the ex-post optimal action $y^*(x)$ as a function of private agents' action x and the reaction function $\bar{y}$ that is sufficient to rule out all equilibria except the best one, when the size of the noise is sufficiently close to 0. The graph on the right reports the relative welfare gain in percentage of an ex-post optimal deviation from competitive outcome and from the reaction function $\bar{y}$ from the right graph.

3.2 Fixed cost

In this subsection, we include a fixed cost for agents to depart from the aggregate action x. More precisely, in the capital taxation problem, we assume that investors pay a cost θ when they choose an action ξ that differs from the aggregate one x. A competitive outcome is then any couple (x, y) such that $y \in D(x)$ and

$$u(x, x, y) \geq \max_{\xi \in X} u(\xi, x, y) - \theta. \tag{20}$$

The cost θ can be viewed either as the cognitive cost associated with the computation of the best action[16] or a cost paid when an investor action deviates from others' choice —for example, a social norm. Anyway, such a cost leads agents to prefer to comply with the aggregate action rather than taking its best action when the optimizing gain is lower than the cost θ. This new force raises the ex-post cost for the government to deter undesired Nash outcomes: choosing an action close to the ex-post optimal action is no more sufficient. We therefore get the following result:

Proposition 5. *The cost of controllability cost ρ_θ, associated with the cognitive cost θ, increases with θ and $\rho_\theta > 0$ whenever $\theta > 0$.*

[16]In Appendix A.3, we present a potential foundation of such cost based on bounded rationality.

Proof. See Appendix A.3. □

To deter the low-capital Nash outcome, the government must rely on a capital tax that sufficiently differs from the ex-post optimal one. Otherwise, when the aggregate capital coincides with the low-capital Nash level, the cognitive cost θ dominates the gain for an individual investor from optimizing its investment and deviating from the aggregate level ($\xi \neq x$).

3.3 Discrete action sets

The cost of controllability can also be strictly positive when agents face discrete choices. In this subsection, we modify the capital taxation problem to allow for a discrete set of actions by the private-sector or by the government. In both cases, the cost of controllability ρ depends on the distance between two discrete values: the ability to implement a single equilibrium outcome only depends on "how" discrete the set of actions is.[17]

Indivisible capital. We first consider a discrete set of investment levels: $\xi \in \{k_0, \ldots, k_n\}$ where $n > 3$. For simplicity, we assume that there exist three levels $k_0 = 0$, k_p, and k_q with $p < q < n$ that correspond to the three levels of capital associated with the no-, low-, and high-capital Nash equilibria in the model with a continuous set of actions as described in Section 2.3.3. Naturally, these three levels of capital k_0, k_p, and k_q also belong to the set of Nash outcomes with a discrete set of private actions.

In this case, the cost of controllability is higher thaH812n the one in the continuous action set benchmark. When the private-sector action set is discrete, the government must deviate sufficiently from $y^*(k_p)$ such that the best private action ξ anticipating k_p is not k_p. More formally, this requires to find an action y such that, there exists $q \neq p$ so that:

$$u(k_q, k_p, y) > u(k_p, k_p, y). \tag{21}$$

On the other hand, the action y is costly for the government as it requires to deviate from $y^*(k_p)$ and the cost is $w(k_p, y^*(k_p)) - w(k_p, y)$, thus requiring a strictly positive commitment ability κ.

Discrete set of tax levels. An alternative source of discretization is on the government side. Let us assume that the government selects a capital tax income in a discrete set $\delta \in \{\delta_0, ..., \delta_n\}$ with δ_0, δ_p, and δ_q denoting the three equilibrium tax levels in the absence

[17] The reason why actions are discrete is beyond the scope of this paper. To make things more palatable, on the government side, one can have in mind political economy constraints that force public authority to favor bold policy movements. On the private sector side, discrete choices can come from information frictions such as rational inattention or physical constraints.

of commitment ability and continuous set of policy actions. In this case, the cost of controllability is at least:

$$\min_{r\neq p} w(k_p,\delta_p) - w(k_p,\delta_r) > 0.$$

Therefore as for discrete private actions, discrete set of tax levels raises the cost of controllability compared with the continuous case.

4 The repeated game

We now consider the infinitely repeated version of the static game and, again, investigate how the equilibrium set depends on the commitment ability κ. In particular, we determine the conditions under which the the coordination problem is solved. We show that the private sector's reactions to government decisions lead the commitment ability required to implement a unique outcome to be larger than in the static case. We apply our findings to the repeated inflation bias problem.[18]

4.1 The environment

This subsection describes the repeated version of our static game, the strategies of the agents and the equilibrium definition.

The repeated game. Each date $t \in \{0, 1, ...\}$ corresponds to a static game with the intra-period timing of actions of Section 2. First, the government commits to a reaction function $\bar{y}_t \in Y(X)$. Then, private agents choose an action $\xi_t \in X$, and $x_t \in X$ is the aggregation of all individual private agents' decisions. Finally, the government plays an action $y_t \in D(x_t)$.

We denote by h_t the history of public actions until the end of date t which includes the reaction function to which the government commits, the aggregate action played by the private agents and the action played by the government. We also denote by H_t the set of all possible histories until the end of date t. Histories are defined recursively as follows: $h_t = \{h_{t-1}, \bar{y}_t, x_t, y_t\}$ and $h_{-1} = \emptyset$. As in the static game, individual private actions are private information.[19]

The payoff for an individual private agent playing a sequence $\{\xi_t\}_{t=0}^{\infty}$ is $\sum_{t\geq 0} \beta^t u(\xi_t, x_t, y_t)$, where $\beta \in [0, 1]$ is the private agents' discount factor and the payoff function u is strictly concave and continuously differentiable on $X \times X \times Y$.

[18]While our main analysis is the strict repetition of the static game, we investigate in Appendix B two situations in which the cost incurred by deviating from the commitment to a reaction function has some intertemporal aspect: first, deviating from the reaction function implies welfare costs over multiple periods ; second, the cost is itself history-dependent. In addition, in our repeated game, the government announces its reaction function at each date. We discuss alternatives in which the government announces all its reaction functions at date 0 in Appendix B.1.

[19]See Chari and Kehoe (1990) for the foundation of such games as anonymous games.

The payoff of the government is the discounted sum of the instantaneous flow of utility $r(\bar{y}_t, x_t, y_t, \kappa)$:

$$V_\tau^g\left(\{\bar{y}_t, x_t, y_t\}_{t\geq 0}\right) = \sum_{t\geq\tau} \beta^t r(\bar{y}_t, x_t, y_t, \kappa), \tag{22}$$

where the instantaneous flow of utility $r(\bar{y}_t, x_t, y_t, \kappa)$ is given by:

$$r(\bar{y}_t, x_t, y_t, \kappa) = \begin{cases} w(x_t, y_t), \text{ if } y_t = \bar{y}_t(x_t) \\ w(x_t, y_t) - \kappa_t, \text{ otherwise,} \end{cases} \tag{23}$$

where $\kappa \geq 0$ is the cost of deviating from the action consistent with the reaction function $\bar{y}_t(x_t)$ at date t, and w is a strictly concave and twice differentiable function. For exposition purposes, the government payoffs before and after the private-sector action are supposed to be the same: $w(.,.) = \bar{w}(.,.)$.

Competitive outcome and Ramsey allocation. An allocation $\{(x_t, y_t)\}_{t=0}^{\infty}$ is a competitive outcome when $y_t \in D(x_t)$ for any $t \geq 0$ and:

$$\{x_t\}_{t=0}^{\infty} = \arg \max_{\{\xi_t\}_{t=0}^{\infty}} \sum_{t\geq 0} \beta^t u(\xi_t, x_t, y_t). \tag{24}$$

Because the game is the repetition of a static game, an allocation $\{(x_t, y_t)\}_{t=0}^{\infty}$ is a competitive outcome if and only if (x_t, y_t) is a competitive outcome at any date t; that is, $y_t \in D(x_t)$ and

$$x_t = \arg \max_{\xi_t} u(\xi_t, x_t, y_t). \tag{25}$$

We denote by $\mathcal{C}$ the set of competitive outcomes in a given period.

The Ramsey allocation is an allocation $\{(x_t^R, y_t^R)\}_{t\geq 0}$ that solves the Ramsey problem:

$$\{(x_t^R, y_t^R)\}_{t\geq 0} = \arg \max_{(x_t,y_t)\in C,\ t\geq 0} \sum_{t\geq 0} \beta^t w(x_t, y_t), \tag{26}$$

where we assume that $w(x_t, y_t) = u(x_t, x_t, y_t)$. In words, a Ramsey allocation is a competitive outcome that reaches the government's highest payoff. Given the repeated structure of the game, the Ramsey allocation is the repetition of the static Ramsey allocation: $\{(x_t^R, y_t^R)\}_{t\geq 0} = \{(x^R, y^R)\}_{t\geq 0}$.

A Nash allocation is an allocation $\{(x_t^N, y_t^N)\}_{t\geq 0}$ such that, at each date t, the allocation (x_t^N, y_t^N) is a Nash equilibrium outcome of the static game. We denote by V_d the lowest payoff for the government among the set of Nash allocations. Notice that this lowest payoff is achieved by the repeated worst static Nash equilibrium outcome, whose per-period payoff is denoted by w^d.

Strategies and equilibrium definition. For the government, a commitment strategy $\bar{\sigma}^g = \{\bar{\sigma}_t^g\}_{t\geq 0}$ is a sequence that, at each date $t \geq 0$, maps a history h_{t-1} to a reaction function $\bar{y}_t \in Y(X)$: $\bar{\sigma}_t^g : H_{t-1} \rightarrow Y(X)$, for each $t \geq 0$.

For the aggregate of private agents, a strategy $\sigma^h = \{\sigma_t^h\}_{t\geq 0}$ is a sequence that, at each date t, maps a history $\{h_{t-1}, \bar{y}_t\}$ to an aggregate action $x_t \in X$: $\sigma_t^h : H_{t-1} \times Y(X) \to X$, for each $t \geq 0$. For the government, a strategy $\sigma^g = \{\sigma_t^g\}_{t\geq 0}$ is a sequence that, at each date, maps a history $\{h_{t-1}, \bar{y}_t, x_t\}$ into an action $y_t \in D(X)$: $\sigma_t^h : H_{t-1} \times Y(X) \times X \to D(X)$, for each $t \geq 0$. Finally, we denote by $V_t^g(\sigma|_{(h_{t-1})})$ the payoff for the government at date t implied by a history $\{h_{t-1}\}$ and a strategy profile σ.

We can now define an equilibrium. As in the static setting, we first define the continuation of an equilibrium. At date t, after any history h_{t-1} and for all $\bar{\eta} \in Y(X)$, the set of continuation strategies (h, g) at date t is $\mathcal{CE}(h_{t-1}, \bar{\eta})$ such that:

(i) $(h(h_{t-1}, \bar{\eta}), g(h_{t-1}, \bar{\eta}, h(h_{t-1}, \bar{\eta})))$ is a competitive outcome;

(ii) for any $x \in X$ and for any $\eta \in D(x)$,

$$r(\bar{\eta}, x, g(h_{t-1}, \bar{\eta}, x), \kappa) + \beta V_{t+1}^g(\sigma|_{(h_{t-1}, \bar{\eta}, x, g(h_{t-1}, \bar{\eta}, x))}) \geq r(\bar{\eta}, x, \eta, \kappa) + \beta V_{t+1}^g(\sigma|_{(h_{t-1}, \bar{\eta}, x, \eta)}). \tag{27}$$

Definition 7 (Subgame perfect equilibrium)**.** *A strategy profile $\sigma = (\bar{\sigma}^g, \sigma^h, \sigma^g)$ is a subgame perfect equilibrium if, for any $t \geq 0$ and for any history $h_{t-1} \in H_{t-1}$, the following conditions are satisfied: for all $\bar{\eta} \in Y(X)$ such that $\mathcal{CE}(h_{t-1}, \bar{\eta})$ is not empty,*

(i) $(\sigma^h, \sigma^g) \in \mathcal{CE}(h_{t-1}, \bar{\eta})$*;*

(ii) *denoting by $\bar{y}_t = \bar{\sigma}_t^g(h_{t-1})$, $x_t = \sigma^h(h_{t-1}, \bar{y}_t)$, $y_t = \sigma^g(h_{t-1}, \bar{y}_t, x_t)$, $x(\bar{\eta}) = \sigma^h(h_{t-1}, \bar{\eta})$ and $y(\bar{\eta}) = \sigma^g(h_{t-1}, \bar{\eta}, x(\bar{\eta}))$,*

$$V_t^g(\sigma|_{(h_{t-1}, \bar{y}_t, x_t, y_t)}) \geq V_t^g(\sigma|_{(h_{t-1}, \bar{\eta}, x(\bar{\eta}), y(\bar{\eta}))}). \tag{28}$$

We say that $\{x_t, y_t\}_{t\geq 0}$ is an equilibrium outcome of the strategy profile σ. We denote by $\Theta^\infty(\kappa)$ the set of equilibrium outcomes for a given cost κ and by $V(\kappa)$ the set of equilibrium payoffs for the government.

Definition 7 extends Definition 3 to repeated games. The first condition requires that, for any reaction function where this is possible, a continuation of an equilibrium forms. The second condition requires that there does not exist a better reaction function than the one selected.

Remark. Definition 7 coincides with a standard subgame perfect equilibrium in the absence of commitment ability ($\kappa = 0$) (as in Ljungqvist and Sargent, 2018). In this case, the commitment to a reaction function is immaterial, and $\Theta^\infty(0)$ is simply the set of all the subgame perfect equilibrium outcomes in the game with reaction functions.

Coordination problem. As for the static game, the government faces a *coordination problem* when $\Theta(0)$ contains multiple equilibrium outcomes. An allocation $\{x_t, y_t\}_{t\geq 0}$ is *implementable* when $\Theta^\infty(\kappa) = \{\{x_t, y_t\}_{t\geq 0}\}$.

There are at least two situations in which multiple equilibria may emerge. First, the static game admits multiple Nash outcomes. Second, dynamic incentives lead to multiple equilibria, which are standard in repeated settings, at least when the discount factor is sufficiently close to 1 and when the static game admits at least one inferior Nash outcome.[20]

4.2 Equilibrium characterization and dynamic incentives

In this subsection, we first characterize the set of equilibrium payoffs for the government. We then use these results to describe the incentives for the government to rule out undesired private actions in a dynamic context.

The set of equilibrium payoffs. We follow the approach by Abreu et al. (1986, 1990) to characterize the set of equilibrium payoffs. To this purpose, we define the worst payoff that a competitive outcome can achieve:

$$V_m = \frac{\inf_{(x,y)\in C} w(x,y)}{1-\beta}$$

and denote by (x^m, y^m) a competitive outcome that achieves the worst payoff V_m. Given assumptions on the sets X and Y and on the function w, $V_m > -\infty$.[21]

We can now characterize the set of equilibrium payoffs:

Lemma 6. *The set of equilibrium payoffs for the government, $V(\kappa)$, is a convex and compact set; that is, there exist $(V_{worst}(\kappa), V_{best}(\kappa)) \in \mathbb{R}^2$ such that $V(\kappa) = [V_{worst}(\kappa), V_{best}(\kappa)]$.*

In addition, $V_m \leq V_{worst}(\kappa) \leq V_{best}(\kappa) \leq V_{Ramsey}$.

Proof. See Appendix D.4 □

Extending Ljungqvist and Sargent (2018) –based on Abreu et al. (1986, 1990)– to our setting requires that we take into account the reaction function by the government and the resulting discontinuity of payoffs. We show that, with these two additions, the set of equilibrium payoffs is still compact—a key point, here, is that there is only one discontinuity due to the welfare loss.

At first glance, the second part of the Lemma may seem trivial, as it requires only that the worst equilibrium payoff, V_{worst}, exceeds the payoff of the repetition of the worst competitive outcome (x^m, y^m). However, one has to remember that the per-period payoff

[20]Such a Folk theorem result does not only imply that better outcomes can be sustained as equilibria including the Ramsey one, it also implies that multiple equilibria emerge, as the repetitions of inferior Nash outcomes are also equilibrium outcomes of the repeated game.

[21]Notice that V_m is potentially below the repetition of the worst static Nash outcome, as we do not require (x_m, y_m) to be a static Nash outcome.

of the government can be lower than $w(x^m, y^m)$, as it can also have to pay the cost κ so that the worst per-period payoff can be as low as $w(x^m, y^m) - \kappa$.

Incentives to solve coordination problems. What are the resulting incentives for the government to steer the private sector to play a given $\hat{x} \in X$ instead of $x \neq \hat{x}$? Again, as in the static environment, the government has to play some action $y \in \mathcal{Y}(x)$ to prevent the private sector from collectively playing $x \neq \hat{x}$. In a static setting, this incentive is simply the difference in the payoff of the government with respect to playing the ex-post optimal action $w(x, y) - w(x, y^*(x))$. In a dynamic setting, these incentives do not depend only on the current payoff of the government but also on the future consequences for the government:

$$w(x, y) + \beta V_t^g \left(\sigma|_{h_{t-1},\bar{y}_t,x,y}\right) \geq w(x, y^*(x)) - \kappa + \beta V_t^g \left(\sigma|_{h_{t-1},\bar{y}_t,x,y^*(x)}\right), \tag{29}$$

where $y^*(x) = \arg\max_{\tilde{y}}[w(x, \tilde{y}) - \kappa + \beta V_t^g \left(\sigma|_{h_{t-1},\bar{y}_t,x,\tilde{y}}\right)]$ and the reaction function is such that $\bar{y}_t(x) = y \neq y^*(x)$ if $y^*(x) \notin \mathcal{Y}(x)$.

The incentive for the government to stick to the action y is a function of the system of expectations that are entailed in the strategy profile σ and, more precisely, a function of how this strategy profile modifies future government payoffs—$V_t^g \left(\sigma|_{h_{t-1},\bar{y}_t,x,y}\right)$ and $V_t^g \left(\sigma|_{h_{t-1},\bar{y}_t,x,y^*(x)}\right)$. As σ should lead to an equilibrium, the continuation of σ as well and, hence, the payoffs for the government can take any value in $V(\kappa)$, as implied by Lemma 6.

We can construct a strategy profile σ such that (29) rewrites:

$$\kappa \geq \underbrace{w(x, y^*(x)) - w(x, y)}_{\text{Static incentive}} + \underbrace{\beta \left(V_{best}(\kappa) - V_{worst}(\kappa)\right)}_{\text{Dynamic incentive}}. \tag{30}$$

That is, the strategy profile "punishes" the government for playing y and, instead, "rewards" the government for deviating from its reaction function. This condition is not only necessary, but given that $V_{best}(\kappa) - V_{worst}(\kappa)$ is the maximum dynamic incentive, this condition is also sufficient for the government to be always better off playing y. Taking (30) over all the x different from the the desired one and noticing that $V_{best}(\kappa) - V_{worst}(\kappa)$ does not depend on x, we obtain that

$$\kappa \geq \rho + \beta \left(V_{best}(\kappa) - V_{worst}(\kappa)\right). \tag{31}$$

The commitment ability κ has to be large enough so that the government prefers to stick to its commitment to a reaction function even at the cost of the private sector's reaction. As in the literature on repeated games (see Abreu, 1988), to consider the effects of this private sector's reaction, it is sufficient to look at the extremum values ($V_{best}(\kappa)$ and $V_{worst}(\kappa)$). In particular, this means that the reaction of the private sector does not "fit the crime"—it is not a function of the deviation itself. As a result, following through on the reaction function comes at some sort of a fixed cost.

However, dynamic incentives are not exogenous objects, and they depend on equilibrium objects—$V_{best}(\kappa)$ and $V_{worst}(\kappa)$. These objects are themselves functions of what the government can do in the future. Furthermore, the commitment ability κ enters the inequality both on the left- and right-hand sides, so that the way the commitment ability κ impacts the ability of the government to obtain a unique equilibrium is not straightforward. We investigate the feedback between dynamic incentives and the ability to rule out equilibria in the next subsection.

4.3 The equilibrium set

Let us describe the set of equilibrium outcomes as a function of parameters κ and β. We start with a situation in which the Ramsey allocation is an equilibrium outcome only due to static incentives, formally, when $\kappa \geq \bar{\kappa}$. We then extend our results to the case in which $\kappa < \bar{\kappa}$. In what follows, for simplicity, we make the assumption that the cost of controllability in the static version of the game (ρ) is zero to focus on the role of reputation forces only.

Case $\kappa \geq \bar{\kappa}$. In this case, the Ramsey allocation is an equilibrium outcome, so the best payoff for the government is $V_{best}(\kappa) = w(x^R, y^R)/(1-\beta)$.[22] Looking at (31), the main question regarding implementation is then about $V_{worst}(\kappa)$.

But how do we handle (31) to think about multiplicity of equilibria, given that this inequality includes equilibrium objects such as $V_{worst}(\kappa)$? In particular, if there is only one equilibrium outcome, it should be that $V_{best}(\kappa) = V_{worst}(\kappa)$ and (31) boil down to $\kappa \geq 0$—an inequality that is trivially satisfied for any strictly positive commitment ability κ.

But we should also make sure that multiple equilibrium outcomes do not arise. In this case, $V_{worst}(\kappa) < V_{best}(\kappa)$. If this difference is sufficiently large so that (31) is not satisfied, the commitment ability κ is not sufficient for preventing the emergence of multiple equilibrium outcomes. As we already know from Lemma 6, the worst payoff $V_{worst}(\kappa)$ is bounded below and the best payoff corresponds to the repetition of the Ramsey outcome. Taken together, it is easy to conclude that with a sufficiently high commitment ability, only one equilibrium exists.

The following proposition goes beyond the existence of such a commitment ability threshold, but it also provides bounds to this threshold as a function of the model's parameters:

Proposition 7. *There exists $\kappa^I \in \mathbb{R}^+$ such that the Ramsey allocation is implementable if and only if $\kappa \geq \max\{\kappa^I, \bar{\kappa}\}$. The threshold κ^I satisfies:*

$$\kappa^I \in \left[\frac{\beta}{1-\beta}(w(x^R, y^R) - w(x^d, y^d)); \frac{\beta}{1-\beta}(w(x^R, y^R) - w(x^m, y^m))\right], \tag{32}$$

[22] As we show in Lemma D.10 in the Appendix, $\kappa \geq \bar{\kappa}$ is a sufficient condition for the Ramsey allocation to be an equilibrium outcome.

where the allocation (x^d, y^d) is the worst Nash outcome of the static game and (x^m, y^m) is the worst competitive outcome.

Otherwise, if $\kappa^I > \bar{\kappa}$ and $\bar{\kappa} \leq \kappa < \kappa^I$, the Ramsey allocation is an equilibrium outcome but is not implementable.

Proof. See Appendix D.5 □

The commitment ability κ has to be larger than the threshold κ^I to prevent other equilibria from arising. As we will explain below, κ^I measures the extent to which the government can be incentivized by the private sector's reactions *not to stick* to its past commitments. When $\kappa < \kappa^I$, the government may be tempted to comply with the private sector's views and multiple equilibria may then arise. The threshold κ^I is, thus, the analogue of the cost of controllability ρ in the repeated game.

Proposition 7 gives bounds to κ^I depending on β highlighting the role of reputation forces. For the time being, let us give some intuition about how we obtain the bounds.

First, the threshold κ^I has to be larger than $\beta/(1-\beta)(w(x^R, y^R) - w(x^d, y^d))$. Otherwise, we can construct an equilibrium outcome other than the Ramsey allocation. To this purpose, we build a "stick-and-carrot" strategy profile where the initial stage is the worst static Nash outcome in which the government is forced to renege on its reaction function with a payoff $w(x^d, y^d) - \kappa$—*the stick*—and the continuation is the Ramsey allocation—*the carrot*. We show that this strategy profile is an equilibrium. The payoff of this "stick and carrot" strategy also provides an upper bound to $V_{worst}(\kappa)$, as it corresponds to an equilibrium payoff:

$$V_{worst}(\kappa) \leq w(x^d, y^d) - \kappa + \frac{\beta}{1-\beta} w(x^R, y^R).$$

Second, the threshold κ^I has to be smaller than $\beta/(1-\beta)(w(x^R, y^R) - w(x^m, y^m))$. From Lemma 6, we obtain a lower bound $V_{worst}(\kappa) \geq V_m = 1/(1-\beta) w(x^m, y^m)$. Using this upper bound for $V_{worst}(\kappa)$, (31) thus implies that, when $\kappa \geq \beta/(1-\beta)\left(w(x^R, y^R) - w(x^m, y^m)\right)$, there exists only one equilibrium outcome. This latter condition is then sufficient for the implementation of the Ramsey allocation when $\kappa \geq \bar{\kappa}$.

Finally, the existence of κ^I between these two values results from a monotonicity argument.

Reputation forces make implementation more difficult. Compared with the static setting, the condition for the implementation of the Ramsey outcome in a repeated setting is stronger, as not only $\kappa \geq \bar{\kappa}$ but also $\kappa > \kappa^I$ in order to prevent any other equilibrium from forming: the coordination problem is more difficult to solve due to reputation forces.

This has two implications. First, in terms of commitment ability, the set in which the government can implement the Ramsey allocation for a given $\kappa \geq \bar{\kappa}$ in the static game is a superset of the same set in the repeated game version. Second, as the threshold κ^I

is bounded below by something that is increasing with the discount factor β, stronger reputation forces (as captured by a higher β) may lead to more-difficult implementation. In fact, in the limit where $\beta \to 1$, implementation is impossible.

In the end, if reputation forces and commitment ability are substitutes for making the Ramsey allocation an equilibrium outcome, as put forward by Proposition D.10, Proposition 7 shows that reputation forces crowd out commitment ability—and so, they are complements—when it comes to implementation.

Strictly positive commitment ability is usually necessary for implementation. The condition $\kappa^I \geq \beta/(1-\beta)(w(x^R, y^R) - w(x^d, y^d))$ has direct implications for our examples. In all these models, the worst static Nash outcome usually differs from the Ramsey outcome, implying that κ^I is strictly positive. As a result, in all these examples, implementation in a repeated setting requires a higher commitment ability.

Inflation bias. In this example, (x^d, y^d) is the unique static equilibrium outcome when $\kappa = 0$. When $y^* > 0$ and $\lambda > 0$, this outcome differs from the Ramsey outcome (x^R, y^R) and $w(x^R, y^R) > w(x^d, y^d)$.

Bailouts. In this example, there exist multiple static Nash outcomes, including not only the Ramsey one, but also the worst ones. Again, $w(x^R, y^R) > w(x^d, y^d)$. In this context, this means that repeated interactions may be a robust source of coordination failures leading to bailouts.[23]

Capital taxation. In this example, the worst static Nash outcome is full taxation of capital and no investment, which generically differs from the Ramsey outcome. Again, $w(x^R, y^R) > w(x^d, y^d)$.

Case $\kappa < \bar{\kappa}$. On the one hand, if only one equilibrium outcome exists, it is necessarily the best static outcome $\{(x^\kappa, y^\kappa)\}_{t\geq 0}$. Indeed, the government cannot benefit from dynamic incentives to sustain a better outcome, and it has to rely only on its commitment ability as in the static setting and (x^κ, y^κ) is the best outcome that it can sustain in this case.

On the other hand, the existence of other equilibria depends on (31). But in contrast with the case $\kappa \geq \bar{\kappa}$, both $V_{best}(\kappa)$ and $V_{worse}(\kappa)$ may depend on κ, and whether there exists only one equilibrium outcome is a matter of how $\beta\,(V_{best}(\kappa) - V_{worse}(\kappa))$ increases as a function of κ.

As this is a question that depends on both the exact model that we consider and its parametrization, in Section 4.4, we develop an algorithm to compute the set of equilibrium payoffs. We then apply our method to the inflation bias model where we obtain that $V_{best}(\kappa) - V_{worse}(\kappa)$ increases, but at a lower speed compared with κ, so that the equilibrium outcome is also unique for sufficiently high values of κ but below $\bar{\kappa}$.

[23] Notice that Proposition 7 in Farhi and Tirole (2012) shows in an overlapping generation version of their model that bailouts themselves can distort future investment decisions, leading to a form of fixed cost of bailouts. Here, the fixed cost is about *not bailing out* and stems from trigger strategies.

4.4 Computing the equilibrium set and an application

In this subsection, we use a fixed point algorithm generalizing Ljungqvist and Sargent (2018) to compute the set of equilibrium payoffs V when the government commits to a reaction function (see Appendix C for the derivation of the algorithm). We focus on the inflation bias example. Our main result is that the set of equilibrium outcomes is non-monotonic with respect to commitment ability κ due to the presence of reputation forces.

Figure 8 shows the set of equilibrium inflation (left panel) and payoffs, V, (right panel) with respect to the reneging cost, κ. The figure has been computed in the case $\bar{\kappa} > \kappa^I$, in which case there exists only one equilibrium outcome when $\kappa > \bar{\kappa}$.

Several observations emerge. First, when the commitment ability κ is low enough, there are multiple equilibrium outcomes. With the calibration that we are using, the Ramsey outcome is not an equilibrium outcome for low values of κ. By raising κ, the best equilibrium outcome improves, but at the cost of decreasing the payoff in the worst equilibrium outcome. Second, when the commitment ability exceeds some threshold κ,[24] there is only one equilibrium outcome that coincides with the unique static equilibrium outcome (x^κ, y^κ), but the associated payoff is lower than the one with slightly lower commitment ability. Third, when κ is large enough, the Ramsey allocation is the unique equilibrium outcome.

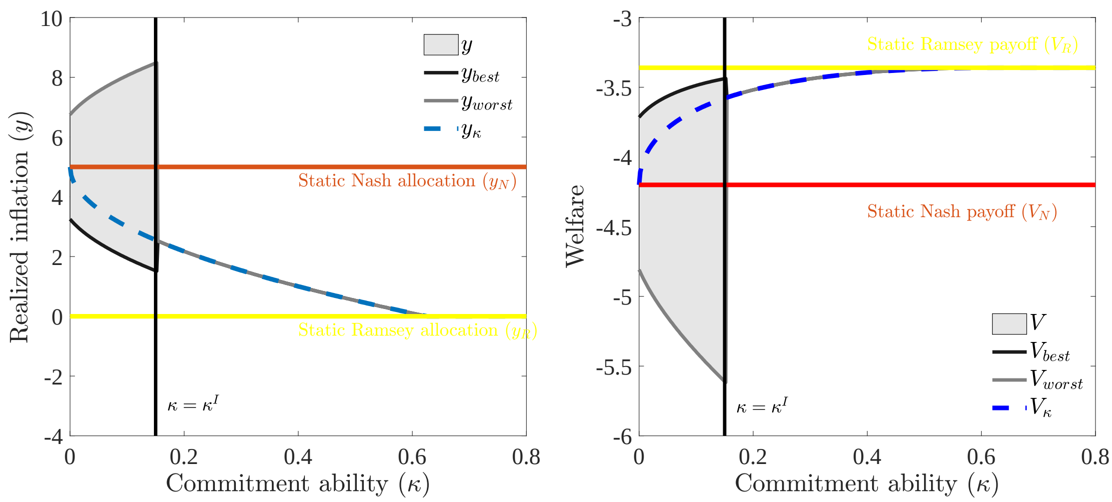


Figure 8: Inflation and welfare as a function of the commitment ability κ

These are the set of equilibrium inflation rates (left-hand panel) and welfare (right-hand panel) as a function of the commitment ability κ. In red, we plot the (unique) static Nash outcome; in blue (dashed line), the best static outcome (x^κ, y^κ); in yellow, the Ramsey outcome. The vertical black line is the value $\kappa = \kappa^I$ such that there exists a unique equilibrium outcome for any greater commitment ability. The black plain line is the best equilibrium outcome and the grey is the worst equilibrium outcome.

[24]Note that this threshold may not be κ^I, which concerns the implementation of the Ramsey outcome.

Notice that the set of equilibrium outcomes is non-monotonic as a function of commitment ability: more precisely, the boundaries of V, $\bar{w}$ and $\underline{w}$ are not monotonic with respect to κ. In particular, more commitment ability does not necessarily reduce the set of equilibrium outcomes, as we observed that sometimes the equilibrium set expands with κ. We also notice that more commitment ability does not necessarily mean that the best payoff for the government increases because, when the set of equilibrium payoffs collapses to a singleton, w_κ, the collapse can be accompanied by a reduction of welfare compared to the best equilibrium payoff with a slightly lower κ.

In the end, while it is always optimal to have a high degree of commitment ability (here, $\kappa > \max(\kappa^I, \bar{\kappa})$), it is not clear that a higher degree of commitment ability always improves welfare.

The forces behind the non-monotonicity. Two main economic forces explain the non-monotonicity of the set of equilibrium payoffs: on the one hand, there is the direct effect of commitment ability as in the static environment and, on the other hand, the indirect effect through reputation forces.

First, as in the static environment, a higher commitment ability κ allows the government to sustain a better equilibrium outcome, for given reputation forces (fix future value functions). This direct effect increases both $V_{best}(\kappa)$ and $V_{worst}(\kappa)$.

Second, by modifying the set of equilibrium payoffs, a change in κ also modifies reputation forces. These reputation forces, which affect government's incentives through future payoffs, are a function of the difference between the best equilibrium payoff V_{best} and the worst equilibrium payoff $V_{worst}(\kappa)$. In our case, this difference is first increasing as a function of the commitment ability κ, as can be observed in Figure 9. Increasing reputation forces can sustain better equilibrium outcomes (i.e., increases $V_{best}(\kappa)$). It can also sustain worst equilibrium outcomes (lower $V_{worst}(\kappa)$): future payoffs can be used as a carrot to push the government to comply with a bad current outcome—as in the stick-and-carrot strategy.

When κ is low enough, the indirect effect through reputation forces dominates: $V_{best}(\kappa) - V_{worst}(\kappa)$ increases, thus leading $V_{best}(\kappa)$ to increase and $V_{worst}(\kappa)$ to decrease as a function of κ.

However, in our example, $V_{best}(\kappa) - V_{worst}(\kappa)$ increases at a slower pace than κ, as illustrated by Figure 9, so there exists a point where $\kappa \geq \beta\,(V_{best}(\kappa) - V_{worst}(\kappa))$. This point is κ^I, according to Proposition 7, and it is such that (31) is satisfied for any greater $\kappa \geq \kappa^I$ and only one equilibrium outcome exists ($\rho = 0$ in this example, as emphasized in Section 2): the commitment ability through κ is sufficient to overcome reputation forces to push the government to stick to its commitments. In turn, that only one equilibrium outcome exists leads to $V_{best}(\kappa) = V_{worst}(\kappa)$. As reputation forces disappear, the (unique) equilibrium outcome corresponds to the static outcome (x^κ, y^κ) and is increasing as a

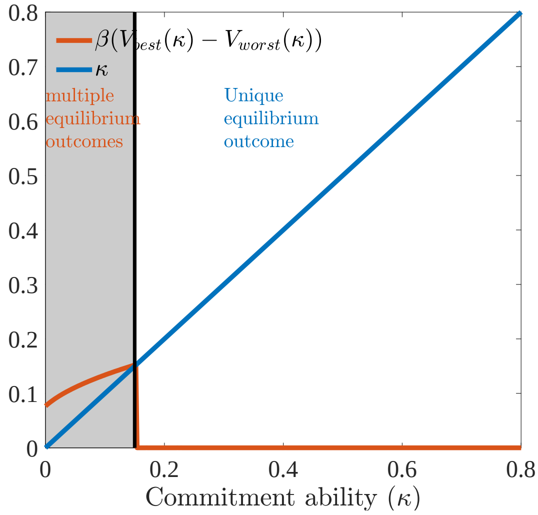


Figure 9: Reputation forces ($V_{best} - V_{worst}$) as a function of the commitment ability

This graph plots in red the difference between the payoff associated with the best equilibrium outcome and with the worst equilibrium outcome discounted by β as a function of the commitment ability κ; and in blue the commitment ability κ itself. The grey area corresponds to values of the commitment ability κ consistent with multiple equilibrium outcomes and, in white, to values consistent with a unique equilibrium outcome.

function of the commitment ability, as in the static environment.

5 Conclusion

In this paper, we investigate the ability of a government to implement a unique equilibrium outcome when its commitment ability is limited. We find that, surprisingly, implementation does not require large commitment ability in a relatively large set of static games. We then determine some limits of this benchmark result and find that a larger commitment ability may be needed for implementation when information is imperfect, when (public or private) agents face fixed costs and when interactions are repeated.

On top of the quantification of the commitment ability required for implementation, our results also give insights about the design of commitments and, especially, the importance of designing commitments that are ex-post credible in and out-of equilibrium. Interestingly, in general, designing credible rules is relatively simple as it simply relies on slight deviation from the ex-post optimal policy when this ex-post optimal policy is consistent with private decisions—a competitive outcome—and to follow the ex-post optimal policy otherwise.

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A Extensions – static game – For online publication

In this Appendix, we show auxiliary results connected to the static game studied in Section 2.

A.1 Reneging in equilibrium?

An important but simple question is whether the government deviates from its reaction function in equilibrium. An implication from Proposition 1 is that such deviation does not occur when there is implementation—that is, there exists a unique equilibrium:

Proposition A.1 (The government does what it promised)**.** *Suppose that the best equilibrium outcome* (x^κ, y^κ) *is implementable. Then, in any equilibrium* $(\bar{y}, \sigma^h, \sigma^g)$*,* $\sigma^g(\bar{y}, \sigma^h(\bar{y})) = \bar{y}(\sigma^h(\bar{y}))$*.*

Proof. Let us assume that there exists a unique equilibrium outcome. From Proposition 1, this equilibrium outcome is necessarily (x^κ, y^κ). Let us suppose that there exists an equilibrium such that $\bar{y}(x^\kappa) \neq y^\kappa$ with $x^\kappa = \sigma^h(\bar{y})$ and $y^\kappa = \sigma^g(\bar{y}, x^\kappa)$. The government may improve its ex ante payoff by selecting the same reaction function but with $\bar{y}(x^\kappa) = y^\kappa$. As the equilibrium outcome is always (x^κ, y^κ), the government's payoff shifts from $w(x^\kappa, y^\kappa) - \kappa$ to $w(x^\kappa, y^\kappa)$, which is strictly better. □

The intuition behind this result is that the government always prefers to opt for a reaction function that saves on the cost κ. An important reminder is that a unique equilibrium outcome does not necessarily imply that the equilibrium is unique, as multiple strategies, implying multiple off-equilibrium actions, may implement this outcome. However, Proposition A.1 implies that equilibrium actions are the same across all equilibria.

Let us also contrast such a result with what can happen when multiple equilibria arise. In such a case, the expectation of an inferior equilibrium outcome may lead the government to deviate and pay the cost κ. More precisely, the commitment to a suboptimal reaction function may be used as a coordination device on the best equilibrium outcome and any change in such a commitment may lead the private sector to coordinate on the inferior equilibrium: when the cost κ is sufficiently low, the government prefers to pay this cost and to benefit from the best equilibrium outcome rather than not paying the cost and suffering from the inferior equilibrium outcome.

A.2 More general welfare cost functions.

In this paragraph, we extend the results of Proposition 1 to reneging cost of the form $\bar{\kappa}(y, y')$ where y' is the actual action played by the government and y is the action that the government committed to. The only assumption that we make on this function is that $\bar{\kappa}(y, y) = 0$ for all feasible y. With this notation, our assumption of a fixed welfare cost can be written as $\bar{\kappa}(y, y') = \kappa 1_{y'=y}$.

As with a fixed welfare cost, we define the best allocation that can be sustained by the function $\bar{\kappa}$ as $(x^{\bar{\kappa}}, y^{\bar{\kappa}}) \in C$ solving:

$$\max_{(x,y)\in C} \bar{w}(x,y) \tag{33}$$

$$\text{s.t. } w(x,y) \geq w(x,\eta) - \bar{\kappa}(y,\eta), \forall \eta \in D(x). \tag{34}$$

Proposition A.2. *Under Assumption 2, the best equilibrium outcome $(x^{\bar{\kappa}}, y^{\bar{\kappa}})$ is implementable when*

$$\sup_{x \neq x^{\bar{\kappa}}} \inf_{y \in \mathcal{Y}(x)} [w(x, y^*(x)) - w(x,y) - \bar{\kappa}(y, y^*(x))] \leq 0.$$

Proof. The proof closely follows the proof of Proposition 1. □

In particular, note that, when $\bar{\kappa}$ and w are continuous and differentiable with respect to the first and the second variable respectively, a sufficient condition for Proposition A.2 to be satisfied is that: (i) the second item of Proposition 3 is satisfied and in particular $u_{13}(x,x,y) \neq 0$ in any Nash outcome (x,y); (ii) for any $x \neq x^{\bar{\kappa}}$ such that x belongs to at least one competitive outcome, the partial derivative $\partial_1 \bar{\kappa}(y^*(x), y^*(x))$ is greater than than the partial derivative $\partial_2 w(x, y^*(x))$; that is, the marginal cost of deviating from the reaction function is larger than the marginal gain around the ex-post best response. As we assume that Nash outcomes are all interior, this second condition boils down to $\partial_1 \bar{\kappa}(y^*(x), y^*(x)) \neq 0$: the marginal cost of deviating should simply be strictly positive.

A.3 Bounded rationality

In this subsection, we study a simple departure from full rationality in the static game described in Section 2. Whereas there is a unique way to model rationality, there are multiple ways to depart from it. Here we simply assume that it is costly for agents to process information and select their individual best response—we borrow such costs from Reis (2006), who introduced them to model inattentive producers, among others. We show that such a deviation from full rationality exacerbates the coordination problem by leading to more equilibria and makes the coordination problem harder to solve, individual actions being more costly to control.

A.3.1 Model

The model is similar to the static model described in Section 2, except that private agents are not fully rational. Private agents pay a cost θ to maximize their utility and take their best action. Whether to pay this cost then depends on their prior beliefs about the gain to maximize. When agents do not pay this cost, they play an action taken from their prior beliefs about the right action to play.

Private sector. Let us denote by $\Delta(x, y)$ the expected gain to maximize over its individual action ξ rather than playing an alternative action (defined below) by an agent expecting the allocation to be (x, y). The function Δ maps an aggregate action x, a policy action y to a positive real number.[25]

Consider, then, the optimal decision of an individual agent for a given (expected) aggregate action $x \in X$ and (expected) policy action $y \in D(x)$. Two situations may arise.

If $\Delta(x, y) \geq \theta$, the private agent selects $\xi \in X$ so as to maximize $u(\xi, x, y)$, as in the competitive outcome under rational expectations:

$$\xi^* = \arg \max_{\xi \in X} [u(\xi, x, y) - \theta].$$

Notice that the cognitive cost θ, being separable, does not alter the optimal choice when the agent chooses to select the optimal action. Otherwise, when

$$\Delta(x, y) < \theta, \tag{35}$$

the private agent plays its prior belief $\Xi(x, y)$ about what constitutes the optimal action given aggregate action, x and policy action $y \in D(x)$.

Definition A.1 (Competitive outcome under bounded rationality). *Given Δ and Ξ, an allocation (x, y) is a competitive outcome under bounded rationality if:*

$$x \in X,\ \ y \in D(x), \Xi(x, y) = x \ \textit{and}\ \Delta(x, y) < \theta; \tag{36}$$

$$\textit{or}\ (x, y) \in C \ \textit{and}\ \Delta(x, y) \geq \theta. \tag{37}$$

We denote this set of competitive outcomes by $C_\theta(\Delta, \Xi)$.

Equilibrium definition. Given a belief function Δ, the definition of an equilibrium is unchanged and described by Definition 3 where the definition of a competitive outcome is modified along the lines described above.

Four remarks are in order. First, when a private agent chooses to select the optimal action, it chooses the optimal unconstrained action, as in Section 2. The limits to rationality do not play any role in this case. Second, the agent computes the expected gain using the strategy profile; that is, $y = \sigma^g(\bar{y}, x)$. Private agents perfectly understand the future policy action, but can prefer not paying attention to it if they perceive that the utility gain from computing and choosing the optimal action is too low. Third, the equilibrium set depends on both belief functions Δ and Ξ. Fourth, we do not micro-found the belief functions here; hence, they are a priori given and not equilibrium objects. For instance, these functions can be arbitrarily far from the true optimizing gain. One extreme case is when the perceived gain is always zero ($\Delta = 0$). In such a case, the private agents always play their a priori action $\xi = \Xi(x, y)$. Still, a reaction function can matter through the policy y, as the reaction function $\bar{y}$ modifies ex-post incentives of the government. In an even more extreme case, when the private agents always comply with the aggregate if they choose their a priori action ($\Delta(x, y) = x$); then, any aggregate action $x \in X$ is part of an equilibrium outcome.

[25]Note that we implicitly assume here that agents have symmetric prior beliefs.

Prior beliefs. The set of equilibria depends on prior beliefs Δ and Ξ. In what follows, we study a special case in which these functions satisfy:

Assumption A.1 (Prior functions)**.** *For any $x \in X$ and $y \in D(x)$, prior functions Δ and Ξ satisfy:*

$$\Delta(x, y) = \max_{\xi \in X} u(\xi, x, y) - u(x, x, y) \text{ and } \Xi(x, y) = x. \tag{38}$$

The restriction on Δ simply means that private agents perfectly anticipate the gain to optimize: the limit to rationality relies only on finding the action that actually maximizes the utility function. The restriction on Ξ means that, when private agents choose to follow their prior belief about the best action, agents play their expected aggregate action x. Our understanding is that there is no loss of generality to focus on this particular Ξ function: with alternative functions, one would have to find fixed points of $\Xi(\xi, y) = \xi$ for each potential y. Furthermore, private agents may also anticipe the gain to optimize with some noise, which, if they are risk-averse, would be equivalent to a lower perceived gain to optimize.

Under Assumption A.1, the set of competitive outcomes C_θ is a superset of C converging to C when θ tends to 0 and can be rewritten as the set of allocations (x, y) such that:

$$y \in D(x) \text{ and } u(x, x, y) \geq \max_{\xi \in X} u(\xi, x, y) - \theta.$$

Best constrained outcome and Ramsey outcome. As under full rationality, let us first define $(x_\theta^\kappa, y_\theta^\kappa) \in C_\theta$ as the best competitive outcome such that the government has no ex-post incentive to deviate. We can also define the Ramsey allocation (x_θ^R, y_θ^R), as the best competitive outcome.

Notice that these two allocations may differ from the ones under full rationality and lead to higher ex-ante government payoffs, as the cost θ extends the set of competitive outcomes.

A.3.2 Cost of controllability

Here, we investigate the equilibrium set under limited commitment ability ($\kappa < \infty$). As under full rationality, the critical measure is the cost of controllability of private actions ρ (see Proposition 1). With the deviation from full rationality that we consider, this cost depends on θ:

$$\rho_\theta = \sup_{x \in X} \inf_{y \in \mathcal{Y}_\theta(x)} \left(w(x, y^*(x)) - w(x, y) \right), \tag{39}$$

$$\text{where } y^*(x) = \arg \max_{y \in D(x)} w(x, y), \tag{40}$$

$$\text{and } \mathcal{Y}_\theta(x) = \{y \in D(x) | \exists \xi \in X, \; u(\xi, x, y) > u(x, x, y) + \theta\}. \tag{41}$$

The only difference with respect to ρ is that y has to be selected in $\mathcal{Y}_\theta(x)$, which depends on θ as it can be observed in (41). To move individual actions away from x, a policy decision should lead to a utility gain larger than θ.

Proposition A.3. *The cost of controllability ρ_θ increases with θ and $\rho_\theta > 0$ when $\theta > 0$.*

Proof. The proof closely follows the proof of Proposition 1.

That ρ_θ is increasing in θ results from (i) the fact that C_θ expands with θ and, (ii) the fact for a given θ and for each $(x, y) \in C_\theta$, $\mathcal{Y}_{\theta'}(x) \subset \mathcal{Y}_\theta(x)$ if $\theta' > \theta$.

Finally, for $\theta > 0$, $\rho_\theta > 0$ results from the fact that it is always costly for the government to deter a Nash outcome when $\theta > 0$ as explained after the Proposition in the main text. To see that, take a Nash outcome (x, y). To deter such an action x, the government should play an action $y' \in D(x)$ such that $\max_{\xi \in X} u(\xi, x, y') > u(x, x, y') + \theta$. As u is continuous it implies that the government has to play an action far away from $y = y^*(x)$ leading to cost for the government, w being strictly concave. Notice that, if there is a a unique Nash outcome that coincides with the best equilibrium outcome, one can redo the same reasoning with an allocation very close to the Nash outcome and show that the government necessarily needs to incur a cost to rule out such an allocation. Therefore $\rho_\theta > 0$ for $\theta > 0$.

□

Given Assumption A.1, the cost of controllability is the one under full rationality when $\theta = 0$ ($\rho_0 = \rho$). The larger the departure from full rationality (θ), the higher the commitment ability (ρ_θ) required to implement a unique outcome. Finally, this proposition shows that when rationality is bounded ($\theta > 0$), the cost of controllability cannot be zero ($\rho_\theta > 0$). The government needs some commitment ability to implement a unique equilibrium outcome as it needs to commit to costly policy action ex-post.

The intuition of the proposition is as follows. Suppose that (x, y) is a Nash outcome that the government would like to rule out through the commitment to its reaction function. To deter the action x, the government should play an action $y' \in D(x)$ such that $\max_{\xi \in X} u(\xi, x, y') > u(x, x, y') + \theta$. By continuity of u, the action that the government has to commit to deter private agents from playing the action x is necessarily far away from $y = y^*(x)$ entailing cost for the government, w being strictly concave. Compared to Proposition 3, this situation corresponds to $u_{13} = 0$ in a neighborhood of Nash outcomes.

Remark. Under full commitment ability, when $\kappa = \infty$, implementation requires that $\mathcal{Y}_\theta(x)$ is non-empty for any $x \in X$. This is the same condition as under full rationality (see Corollary 2).

We now apply Proposition A.3 to the inflation bias problem, which features a coordination problem with bounded rationality but not with full rationality.

Application to the inflation bias problem. Let us show how the cost θ modifies the inflation bias problem. First, households do not pay the cost θ when the inflation rate π and the aggregate expectation π^e satisfy:

$$(\pi^e - \pi)^2 \leq \theta$$

– that is, when π^e is between $\pi - \sqrt{\theta}$ and $\pi + \sqrt{\theta}$. Otherwise, households pay the cost θ and $\xi = \pi$ so that $\pi^e = \pi$ as well. Therefore, for a given θ, the set of competitive outcomes C_θ is

$$\left\{(\pi^e, \pi), \pi \in [\underline{\pi}, \bar{\pi}], \pi^e \in [\max\{\pi - \sqrt{\theta}, \underline{\pi}\}, \min\{\bar{\pi}, \pi + \sqrt{\theta}\}]\right\}.$$

Second, the cost θ leads to a coordination problem under discretion. When $\kappa = 0$, the best response for government to π^e, $\pi^*(\pi^e)$ solves:

$$\pi^*(\pi^e) = \arg \max_{\pi \in [\underline{\pi}, \bar{\pi}]} -(\lambda(\pi - \pi^e) - y^*)^2 - \alpha(\pi)^2.$$

When interior, the difference between $\pi^*(\pi^e)$ and π^e satisfies:

$$\pi^*(\pi^e) - \pi^e = \frac{\lambda y^* - \alpha \pi^e}{\alpha + \lambda^2},$$

which, in addition to the optimality of private agents, leads to the following set of equilibrium outcomes:[26]

$$\pi^e \in \left[-\theta \frac{\alpha + \lambda^2}{\alpha} + \frac{\lambda}{\alpha} y^*; \theta \frac{\alpha + \lambda^2}{\alpha} + \frac{\lambda}{\alpha} y^*\right] \text{ and } \pi = \frac{\lambda y^* + \lambda^2 \pi^e}{\alpha + \lambda^2}. \tag{42}$$

Interestingly, when θ is large enough, the Ramsey allocation achieves the highest possible welfare with $\pi = 0$ and $\pi^e = -y^*/\lambda$ and is an equilibrium outcome under discretion. In such a case, there is no time-consistency problem. In other words, the distortion due to the cost θ may help the government to achieve the highest possible welfare which is unreachable under rational expectations.

Let us turn to the implementation problem under limited commitment ability ($\kappa > 0$). For a given competitive outcome (π^e, π), the government's minimal action to push agents to change their decision is π', such that $\pi' < \pi^e - \sqrt{\theta}$ or $\pi' > \pi^e + \sqrt{\theta}$, which defines:

$$\mathcal{Y}_\theta(\pi^e) = \left\{\pi | (\pi - \pi^e)^2 > \theta\right\}.$$

On the other hand, given private sector expectation π^e and a cost κ, the government does not deviate from its commitment $\bar{\pi}(\pi^e)$ whenever $\pi = \bar{\pi}(\pi^e)$ satisfies:

$$-(\lambda(\pi - \pi^e) - y^*)^2 - \alpha(\pi)^2 + \kappa \geq -(\lambda(\pi^*(\pi^e) - \pi^e) - y^*)^2 - \alpha(\pi^*(\pi^e))^2. \tag{43}$$

The left-hand panel of Figure 10 plots the set of competitive outcomes as defined by (42) in orange; the borders of the area where the government does not deviate from commitments as defined by (43) are the blue lines; and a candidate reaction function is in black. In this graph, the intersection between the set of competitive outcomes and the set of allocations implied by the reaction function constitutes the set of equilibrium outcomes (the star at the bottom left and the segment). Each allocation of the set of equilibrium outcomes is indexed by a level of expected inflation, which is a particular private-sector strategy, σ^h. Increasing the commitment ability κ moves the star to the bottom left, thereby lowering equilibrium inflation and contracting the

[26] We assume here that the boundaries $\underline{\pi}$ and $\bar{\pi}$ are sufficiently large so that they do not constrain the equilibrium outcomes.

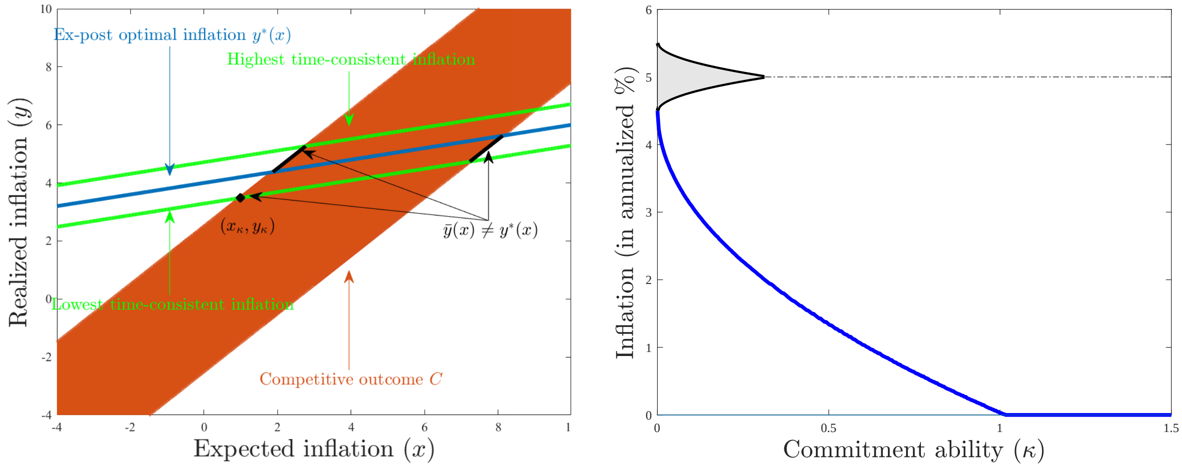

Figure 10: Best responses and equilibrium inflation as a function of the commitment ability κ.

The left-hand panel plots realized inflation—which is the action of the government y—as a function of expected inflation—which is the action of the private sector. In orange, we plot the set of competitive outcomes; in blue, the ex post optimal action $y^*(x)$; in green, the highest and lowest time-consistent inflation rates; in black, a time-consistent reaction function $\bar{y}$ that accepts (x^κ, y^κ) as an equilibrium outcome—we plot this reaction function whenever it is different from $y^*(x)$. The intersection between the reaction function and the competitive outcome describes the set of equilibrium outcomes. The right-hand panel plots the set of equilibrium inflation rates as a function of the commitment ability (κ). Increasing κ moves the highest inflation rate up and the lowest rate down so that $\bar{y} \neq y^*(x)$ for a larger set of values of x in the orange area.

segment, thus shrinking the set of equilibrium inflation rates belonging to the segment in the interior of the competitive outcome set.

The right-hand panel of Figure 10 plots the equilibrium set as a function of κ. It is the union of two subsets. First, the blue curve stands for the best equilibrium outcome (x_κ, y_κ) (the star in the left-hand panel) that converges to the Ramsey level of inflation $\pi = 0$ (and $\pi^e = -y^*/\lambda$). The grey area depicts a second subset of equilibrium outcomes when commitment ability is sufficiently low (the segment in the left-hand panel). It is a subset of competitive outcomes around the Nash allocation under full rationality (dashed black line). This second set monotonously shrinks with κ and completely disappears when κ is sufficiently large—when the government can deter private agents from expecting the Nash inflation level.

B Extensions – Repeated game – For online publication

B.1 Date-0 strategy

An alternative presentation of the repeated game would be that the government announces at date 0 a collection of reaction functions $\bar{y}(h^t, x)$ for any history h_t. However, there are two possibilities regarding this initial strategy of the government, depending on whether the government can change this strategy in the future.

Reoptimizing over the strategy. A first possibility is that the government announces all its reaction function at date-0 but can deviate from them in the future. In this case, we are back to the analysis in the core of the text, in which the government announces its reaction function sequentially.

Sticking to the initial strategy. The second possibility is that the government announces all its reaction functions—after any history of actions. In this case, the government then commits not to renege on reaction functions, even though its actions can ex post deviate from what is announced by the reaction function. In this case, we show that implementation of the Ramsey outcome at date 0 can more easily obtain: on the one hand, the government can commit to future reaction functions that create additional cost of deviations to its initial deviations but it can also strategically keep some reputation forces to make it easier to sustain the Ramsey outcome. This result stems from the commitment technology on reaction functions. This commitment on reaction functions also creates an asymmetry between date 0 and the following periods. That being said, the full analysis of this game remains beyond the scope of this paper.

Strategies and equilibrium definition. The main difference when the government is unable to reoptimize over its strategy over reaction function $\bar{\sigma}^g$ is that the history that agents take into account only contains the actions played by the private agents x_t and the action played by the government y_t, so that h_t is defined recursively by $h_t = \{h_{t-1}, x_t, y_t\}$. Again, H_t denotes the set of potential histories up to period t.

Let us denote by $R(X) \subseteq Y(X)$ the subset of of $Y(X)$ such that $\bar{y} \in R(X)$ if and only if $\{x, (x, \bar{y}(x)\} \in \mathcal{CE}\}$ is not empty—that is the set of reaction functions consistent with competitive equilibria. We denote by R the set of announcement strategies where reaction functions are selected from $R(X)$.

For the aggregate of private agents, a strategy $\sigma^h = \{\sigma^h_t\}_{t\geq 0}$ is a sequence that, at each date t, maps a announcement strategy of the government and a history $\{h_{t-1}\}$ to an aggregate action $x_t \in X$: $\sigma^h_t : R \times H_{t-1} \to X$, for each $t \geq 0$.

The government's strategy $\sigma^g = \{\sigma^g_t\}_{t\geq 0}$ is a sequence that, at each date, maps a history $\{h_{t-1}, , x_t\}$ and an announcement strategy into an action $y_t \in D(X)$: $\sigma^h_t : R \times H_{t-1} \times X \to D(X)$, for each $t \geq 0$. We denote by $V^g_t(\sigma|_{(h_{t-1})})$ the payoff for the government at date t implied by a history $\{h_{t-1}\}$ and a strategy profile σ. The strategy profile is the collection of strategies $\{\bar{\sigma}^g, \sigma^h, \sigma^g\}$.

We can now define an equilibrium.

Definition B.1. *A strategy profile $\sigma = (\bar{\sigma}^g, \sigma^h, \sigma^g)$ is an equilibrium if, for any $t \geq 0$ and for any history $h_{t-1} \in H_{t-1}$, the following conditions are satisfied:*

For all $\eta \in R$, for all period t and histories h_{t-1}

(i) $(\sigma^h(\eta, h_{t-1}), \sigma^g(\eta, h_{t-1})$ *is a competitive equilibrium,*

(ii) *for all $y \in D(\sigma^h(\eta, h_{t-1}))$,*

$$r(\eta, \sigma^h(\eta, h_{t-1}), \sigma^g(\eta, h_{t-1}), \kappa) + \beta V^g_{t+1}(\sigma|_{h_{t-1}, \sigma^h(\eta,h_{t-1}), \sigma^g(\eta,h_{t-1})}) \geq \cdots$$
$$\cdots r(\eta, \sigma^h(\eta, h_{t-1}), y, \kappa) + \beta V^g_{t+1}(\sigma|_{h_{t-1}, \sigma^h(\eta,h_{t-1}), y})$$

$\bar{\sigma}^g$ *is such that: for any* $\eta \in R$, $\tilde{\sigma} = \{\eta, \sigma^g, \sigma^h\}$ *is such that:*

$$V_0^g(\sigma|_{h_0}) \geq V_0^g(\tilde{\sigma}|_{h_0}).$$

Equilibrium set as a function of κ. Let us investigate how the equilibrium set evolves as a function of κ. To this end, let us look at the incentives for the government to stick to its reaction function $\bar{y}$ in any given period. For a given x_t, the government sticks to $\bar{y}(x_t)$ whenever:

$$u(x_t, \bar{y}(x_t)) + \beta V_t^g\left(\{h_{t-1}, x_t, \bar{y}(x_t)\}\right) \geq \max_y u(x, y) - \kappa + V_t^g\left(\{h_{t-1}, x_t, y\}\right). \tag{44}$$

Under the following assumption:

Assumption B.1. *For any* $x \in X$*, there exists* $y \in D(x)$ *so that* $u(x, y)$ *is arbitrarily low.*

we can establish the following result

Proposition B.1. *For* $\kappa \geq \kappa^{I,0}$*, the government can implement a unique equilibrium outcome.*

Whenever $\max_y u(x^R, y^R) - u(x^R, y) \leq \bar{\kappa}^0/(1-\beta)$*, and* $\kappa \leq \kappa^{I,0}$*, the government uniquely implements the Ramsey outcome at date 0.*

We have $\kappa^{I,0} < \kappa^I$ *and* $\bar{\kappa}^0 < \bar{\kappa}$*.*

Before going to the proof, several comments are in order. First, $\kappa^{I,0}$ and $\bar{\kappa}^0$ are lower than their counterparts without commitment on reaction functions as the government can create deviation costs in the future through contingent future reaction functions. Second, it can also decide to keep multiple equilibria in the future, in which case $\bar{\kappa}^0$ is even lower. Indeed multiple equilibria in the future (starting at date 1) does not mean multiple equilibria at date 0 and the government is, here, only concerned by its date-0 welfare.

Proof. Let us first show the existence of $\bar{\kappa}$. First, we have:

$$V_t^g\left(\{h_{t-1}, x_t, \bar{y}(x_t)\}\right) \geq V_{worst}(0) - \frac{\kappa}{1-\beta}$$

However, as the government is always better off announcing the policy leading to V_{worst}, $V_t^g\left(\{h_{t-1}, x_t, \bar{y}(x_t)\}\right) \geq V_{worst}(0)$. On the other hand, after deviating from its reaction function, as the government can always announce a sequence of reactions functions so that it is better off deviating at each date t:

$$V_t^g\left(\{h_{t-1}, x_t, y\}\right) \leq V_{best} - \frac{\kappa}{1-\beta}.$$

Plugging these inequalities into (44), we find

$$u(x_t, \bar{y}(x_t)) - u(x_t, y^*(x_t)) \geq \beta V_{best} - \frac{\kappa}{1-\beta} - \beta V_{worst}$$

As in the benchmark case, $\bar{y}(x_t)$ can be selected arbitrarily close to $y^*(x_t)$ and:

$$\frac{\kappa}{1-\beta} \geq \beta\left(V_{best}(0) - V_{worst}(0)\right)$$

The existence of $\kappa^{I,0}$ stems from this inequality. Notice that $\kappa^{I,0} < \kappa^I$ as defined in the main text.

Whether the government can sustain the Ramsey outcome at date 0 amounts to the following equation:

$$u(x^R, y^R) + \beta V \geq u(x^R, y) - \kappa/(1-\beta) + \beta V_{worst}$$

where V is a continuation payoff in $[V_{worst}, V_{best}]$. On the right hand side, the government does not only deviate from its reaction function at date 0 but commits to deviate at any further date by announcing contingent reaction functions.

□

B.2 Alternative forms of costs

In this Appendix, we investigate two potential extensions of our repeated setting in which the cost has some intertemporal aspects. In the first extension, the government incurs a cost over multiple periods if it deviates from its reaction function. We show that this situation is, in fact, analogous to a repeated game setting with only contemporaneous welfare cost. In the second extension, the cost of deviation may depend on the past. In this latter situation, we show that if the cost of deviation is sufficiently high, the government implements a unique equilibrium outcome.

That costs of deviations may span over multiple periods and/or depend on past history are natural features when thinking about political costs that may take time to unfold or credibility losses –e.g., due to the revelation of the type of policymaker in case of a deviation.

Multi-period costs. Let us first introduce multi-period costs: when the government deviates from its reaction function in period t, the resulting cost does not accrue only in the current period but over multiple periods—this may entail the situation in which all the cost of deviation stems only from future periods.

We consider the sequence $\{\kappa_t\}_{t\geq 0}$. The payoff of the government given a sequence $\{\bar{y}_t, x_t, y_t\}_{t\geq\tau}$:

$$V_\tau^g\left(\{\bar{y}_t, x_t, y_t\}_{t\geq\tau}\right) = \sum_{t\geq\tau} \beta^t \left(w(x_t, y_t) - 1_{\bar{y}_t(x_t)\neq y_t} \sum_{k\geq 0} \beta^k \kappa_k \right).$$

At each period t following a private-sector action x, if the actual action played y_t differs from the reaction function $\bar{y}_t(x_t)$, the government faces a sequence of costs κ_t. More precisely, it incurs a cost κ_0 at date t, κ_1 at date $t+1$ and so on. The date-t discounted sum of these costs is $\sum_{k\geq 0} \beta^k \kappa_k$ and the previous repeated game is a special case where $\kappa_k = 0$ for any $k > 0$. This game can be equivalently recasted as a repeated game with a cost $\kappa = \sum_{k\geq 0} \beta^k \kappa_k$ with the payoff of the government being:

$$V_\tau^g\left(\{\bar{y}_t, x_t, y_t\}_{t\geq\tau}\right) = \sum_{t\geq\tau} \beta^t \left(w(x_t, y_t) - 1_{\bar{y}_t(x_t)\neq y_t}\kappa \right) = \sum_{t\geq\tau} \beta^t r(\bar{y}_t, x_t, y_t, \kappa),$$

with $r(\bar{y}_t, x_t, y_t, \kappa) = w(x_t, y_t) - \kappa 1_{\bar{y}_t(x_t)\neq y_t}$.

Therefore, we can apply the results obtained with contemporaneous costs to repeated settings with a sequence of costs. What eventually matters is the present value of all future costs incurred following one deviation κ.

History-dependent costs. Commitment ability could also depend on the past and especially on past policy decisions. For simplicity, we focus on contemporaneous costs only. At time t, after a history h_{t-1}, deviating from the reaction function leads to a cost $\kappa_t(h_{t-1}) \geq 0$. For any $t \geq 0$, the cost function $\kappa_t(.)$ maps a history of events until date t to a positive real number. Using this cost function, we can redefine the payoff after a history h_{t-1} by:

$$\sum_{k \geq t} r(\bar{y}_k, x_k, y_k, \kappa_k(h_{k-1})).$$

In this dynamic environment and under some conditions on the cost function $\kappa_t(.)$, we still obtain sufficient conditions for implementation:

Proposition B.2. *Suppose that for any $t \geq 0$, $\kappa_t(.)$ takes value in a bounded set $[\underline{\kappa}, \overline{\kappa}]$ with $\overline{\kappa} < \infty$. As a result, if $\underline{\kappa}$ is large enough, there exists a unique equilibrium outcome.*

Proof. Equilibrium payoffs are bounded. Thus, we can follow the same reasoning as in the repeated game to observe that, as, after any history h_{t-1}, $\kappa(h_{t-1}) > \underline{\kappa}$, we have $\kappa(h_{t-1}) \geq \beta\left(V_{max} - V_{min}\right)$, with V_{max} the upper bound of the set of payoffs. V_{min} and V_{max} exist as the set of payoff is bounded, between $1/(1-\beta)\left(\min_{(x,y)\in C} w(x,y) - \overline{\kappa}\right)$ and $1/(1-\beta)\max_{(x,y)\in C} w(x,y)$. □

C A method to obtain the set of equilibrium payoffs – Repeated game – For online publication

Let us start with a superset of equilibrium payoffs; for instance: $W_0 = [\underline{w}_0, \bar{w}_0]$, with $\underline{w}_0 = V^m$ and $\bar{w}_0 = V^R = u(x^R, y^R)/(1-\beta)$. Using W_0, we can compute the boundaries of $W_1 = B(W_0)$, where B is the mapping defined in Appendix 6 and, afterwards, iterate up to the convergence toward $W = B(W) = V$. Proposition C.1 provides the way to compute $\underline{w}_{n+1}$ and $\bar{w}_{n+1}$ given $\underline{w}_n$ and $\bar{w}_n$.

Proposition C.1. *Given $W = [\underline{w}, \bar{w}]$, $W' = B(W) = [\underline{w}', \bar{w}']$ where $\underline{w}'$ and $\bar{w}'$ solve:*

$$\bar{w}' = \max_{(x,y)\in C} u(x,y) + \beta \bar{w}, \tag{45}$$

$$s.t.\ \ u(x,y) + \beta\bar{w} \geq u(x, y^*(x)) - \kappa + \beta\underline{w}. \tag{46}$$

$$u(x,y) + \beta\bar{w} \geq u(x_{\kappa-\beta(\bar{w}-\underline{w})}, y_{\kappa-\beta(\bar{w}-\underline{w})}) + \beta\underline{w}, \tag{47}$$

$$\underline{w}' = \min_{(x,y)\in C, w\in[\underline{w},\bar{w}]} u(x,y) + \beta w, \tag{48}$$

$$s.t.\ \ u(x,y) + \beta w \geq u(x, y^*(x)) - \kappa + \beta\underline{w}, \tag{49}$$

$$u(x,y) + \beta w \geq u(x_{\kappa-\beta(\bar{w}-\underline{w})}, y_{\kappa-\beta(\bar{w}-\underline{w})}) + \beta\underline{w}. \tag{50}$$

Proof. See appendix D.6. □

Note that the allocation $(x_{\kappa-\beta(\bar{w}-\underline{w})}, y_{\kappa-\beta(\bar{w}-\underline{w})})$ is the competitive outcome that achieves the highest welfare and is compatible with a reaction function $\bar{\eta}$ as defined in item (ii) of Definition D.1. Under the assumption of controllability, we can construct a $\bar{\eta} \in Y(X)$ as in item (ii) that is only consistent with this competitive outcome. Therefore, the competitive outcome that is a candidate for the worst and the best equilibrium payoffs should lead to a higher welfare than the competitive outcome $(x_{\kappa-\beta(\bar{w}-\underline{w})}, y_{\kappa-\beta(\bar{w}-\underline{w})})$.

While the worst payoff $\underline{w}'$ results from multiple forces, the best payoff $\bar{w}'$ is easier to compute. Take the maximization problem (46) subject to (46). The solution is

$$u(x_{\kappa-\beta(\bar{w}-\underline{w})}, y_{\kappa-\beta(\bar{w}-\underline{w})}) + \beta\bar{w},$$

which satisfies (47). Therefore $\underline{w}' = u(x_{\kappa-\beta(\bar{w}-\underline{w})}, y_{\kappa-\beta(\bar{w}-\underline{w})}) + \beta\bar{w}$ and the second constraint (47) is slack. For a given $\bar{w} - \underline{w}$, the upper bound of W' thus increases with κ.

D Proofs – For online publication

D.1 Proof of Proposition 1.

Points (i) and (iii). Let us start by showing that the set of equilibrium payoffs is a compact set, that (x^κ, y^κ) is an equilibrium outcome that achieves the best equilibrium payoff and that both the worst and the best payoffs are weakly increasing with κ.

To this purpose, let us show the following lemma. Let $\mathcal{S}_\kappa = \{(x, y) \in C, w(x, y^*(x)) - w(x, y) \leq \kappa\}$ be the set of competitive outcomes that can be ex-post sustained by the government if the reaction function is such that $\bar{y}(x) = y$. Let $\mathcal{S}^x_\kappa = \{x \in X, \exists y \in D(x), (x, y) \in S_\kappa\}$ be the set of private-sector actions that belong to at least one allocation in $\mathcal{S}_\kappa$.

Lemma D.1. *An allocation (x, y) is an equilibrium outcome if and only if*

(i) $(x, y) \in C$,

(ii) $\bar{w}(x, y) \geq \min_{x' \in \mathcal{S}^x_\kappa} \max_{y' | (x', y') \in \mathcal{S}_\kappa} \bar{w}(x', y')$.

Proof. Suppose that (x, y) is an equilibrium outcome. This means that there exists $\sigma = (\bar{y}, \sigma^g, \sigma^h)$ such that σ leads to (x, y). By definition, $(x, y) \in C$ and $w(x, y^*(x)) - w(x, y) \leq \kappa$. As a result, $(x, y) \in \mathcal{S}_\kappa$.

Let us show that condition (ii) is also satisfied. Indeed, suppose it is not. Without loss of generality, we can focus on $\bar{y}(x) = y$. In this case, for all $x' \in \mathcal{S}^x_\kappa$, there exists y' such that $(x', y') \in \mathcal{S}_\kappa$, such that $\bar{w}(x', y') > \bar{w}(x, y)$. Using these values of x' and y', we can build a reaction function $\bar{\eta}$ such that $\bar{\eta}(x') = y'$ on $\mathcal{S}^x_\kappa$—outside this set, we can take $\bar{\eta}$ in an arbitrary manner as no equilibrium can form. The strategy profile σ should define the actions after $\bar{\eta}$: $\sigma^h(\bar{\eta}) = x'$ and $\sigma^g(\bar{\eta}, \sigma^g(\bar{\eta})) = y'$.

As $\bar{w}(x, y)$ is the payoff associated with σ, that is $\bar{r}(\bar{y}, x, y)$ and the payoff associated with $\bar{\eta}$ is $\bar{r}(\bar{\eta}, x', y') = \bar{w}(x', y')$ (notice that we have constructed $\bar{\eta}$ so that $\bar{\eta}(x') = y'$), which implies that there exists a reaction function $\bar{\eta}$ that allows to do better than $\bar{y}$, a contradiction.

Conversely, suppose that (x, y) satisfies conditions (i) and (ii) from Lemma D.1. Let us show that it is an equilibrium outcome. Let us consider the reaction function such that $\bar{y}(x) = y$ and the strategies so that $\sigma^h(\bar{y}) = x$ and $\sigma^g(\bar{y}, \sigma^h(\bar{y}) = y$. As $(x, y) \in \mathcal{S}_\kappa$, $\sigma^h(\bar{y}) = x$ and $\sigma^g(\bar{y}, \sigma^h(\bar{y}) = y$ form a competitive outcome and y is optimal after $\sigma^h(\bar{y})$.

Let us consider $\bar{\eta} \in Y(X)$ such that $\mathcal{CE}(\bar{\eta})$ is non-empty. For any $x' \in \mathcal{S}^x_\kappa$, $\bar{w}(x', \bar{\eta}(x'))$. By definition, $\bar{w}(x', \bar{\eta}(x')) \leq \max_{y',(x',y') \in \mathcal{S}_\kappa} \bar{w}(x', y')$. As $\sigma^h(\bar{\eta})$ can be chosen in $\mathcal{S}^x_\kappa$ and then $\sigma^g(\bar{\eta}, x')$ can be selected such that $(x', \sigma^g(\bar{\eta}, x')) \in \mathcal{S}_\kappa$, with

$$\bar{r}(\bar{\eta}, \sigma^h(\bar{\eta}), \sigma^g(\bar{\eta}, \sigma^h(\bar{\eta}))) \leq \min_{x' \in \mathcal{S}^x_\kappa} \max_{y',(x',y') \in \mathcal{S}_\kappa} \bar{w}(x', y').$$

Using condition (ii), the right-hand term can be bounded above by $\bar{w}(x, y)$, which is the payoff associated with $\bar{r}(\bar{y}, x, y)$, thus showing that there exists a strategy profile such that it is optimal to play $\bar{y}$. As a result, (x, y) is an equilibrium outcome.

□

From Lemma D.1, v is a compact as $\mathcal{S}_\kappa$ is a compact and condition (ii) involves an inequality that is not strict. Furthermore, $(x^\kappa, y^\kappa) \in \Theta(\kappa)$: indeed, it belongs to $\mathcal{S}_\kappa$ and leads to the highest payoff in $\mathcal{S}_\kappa$ so that (ii) is trivially satisfied.

The worst equilibrium outcome. To show that the worst equilibrium outcome is an increasing function of κ. Let us consider the worst equilibrium outcome $v_{worst}(\kappa)$ for a given κ. The corresponding strategies are $y(.)$, σ^g and σ_h. For a larger κ, let us note that these strategies still satisfy condition (i) of the definition of an equilibrium but, given the larger commitment ability, the government may sustain something at least better. In particular, this means that something worse that $v_{worst}(\kappa)$ is not an equilibrium outcome.

Point (ii). The following lemma shows under which conditions the best equilibrium outcome is implementable.

Lemma D.2. *The best equilibrium outcome (x^κ, y^κ) is implementable if and only if there exists a reaction function $\bar{y}$ such that,*

(i) $\bar{y}(x^\kappa) = y^\kappa$,

(ii) $\forall x' \neq x^\kappa$, $(x', \bar{y}(x')) \notin C$,

(iii) $\forall (x', y') \in C$, $(x', y') \neq (x^\kappa, y^\kappa)$, $w(x', y') - \kappa < w(x', \bar{y}(x'))$.

Proof. First, suppose that there exists a reaction function $\bar{y}$ such that $\bar{y}(x^\kappa) = y^\kappa$ and

$$\forall x' \neq x^\kappa, \ (x', \bar{y}(x')) \notin C; \tag{51}$$

$$\forall (x', y') \in C, \ (x', y') \neq (x^\kappa, y^\kappa), \ w(x', y') - \kappa \leq w(x', \bar{y}(x')). \tag{52}$$

First, notice that committing to such a reaction function is part of an equilibrium that leads to an equilibrium outcome (x^κ, y^κ). Furthermore, there does not exist any alternative equilibrium outcome. Indeed, suppose that such an equilibrium outcome exists $(x', y') \neq (x^\kappa, y^\kappa)$. This means that there exists a reaction function $\bar{\eta}$ such that $\bar{\eta}(x) = y$ and a strategy profile σ^h and σ^g such that $\sigma^h(\bar{\eta}) = x$ and $\sigma^g(\bar{\eta}, \sigma^h(\bar{\eta})) = y$ and $\bar{\eta}$ is played in equilibrium. However, this strategy profile should be also such that $\sigma^h(\bar{y}) = x^\kappa$ and $\sigma^g(\bar{y}, \sigma^h(\bar{y})) = y^\kappa$. Indeed, from Condition (52), the government is better off to stick to $\bar{y}$ when having committed to it and (x^κ, y^κ) is the only continuation of an equilibrium that can form after $\bar{y}$. Finally, as (x^κ, y^κ) leads to the highest payoff, the government strictly prefers to play $\bar{y}$ instead of $\bar{\eta}$.

Let now prove the reciprocal. Suppose that (x^κ, y^κ) is implementable. Let us show that there exists a reaction function satisfying the three conditions of Lemma D.2.

(x^κ, y^κ) is an equilibrium outcome, so there exists $\bar{y}$ such that $(\bar{y}, \sigma^h, \sigma^g)$ is an equilibrium.

First, condition (i) is satisfied. Either $\bar{y}(x^\kappa) = y^\kappa$ or, if not, we can consider another $\bar{y}'$ that coincides with $\bar{y}$ except for $x = x^\kappa$, where $\bar{y}'(x^\kappa) = y^\kappa$ and this alternative reaction function yields a strictly higher payoff as the cost κ is not incurred in equilibrium.

Second, suppose that conditions (ii) and (iii) are not satisfied. Let us show there exists a competitive outcome (x, y) that the government cannot rule out with any reaction function. Indeed, if the reciprocal of Lemma D.2 is not true, for all reaction functions $\bar{\eta}$, either there exists $x \neq x^\kappa$ such that $(x, \bar{\eta}(x)) \in C$ or there exists y such that (x, y) is such that $w(x, y) - \kappa > w(x, \bar{\eta}(x))$, where $(x, y) \in C$.

The latter case is possible if and only if the ex post optimal action $y^*(x)$ satisfies $w(x, y^*(x)) - \kappa > w(x, \bar{\eta}(x))$. Otherwise, the government can select the reaction function to be $y^*(x)$ to rule out x. As a result, we can consider the strategy for the private sector to play $\sigma^h(\bar{\eta}) = x(\bar{\eta}) \neq x^\kappa$ and for the government to play $\sigma^g(\bar{\eta}, .) = y^*(.)$. This is so that the continuation of an equilibrium after the commitment to a reaction function intended to lead to (x^κ, y^κ) is $(x(\bar{\eta}), y(\bar{\eta}))$ with $y(\bar{\eta}) = \bar{\eta}(x(\bar{(\eta)}))$ if $(x(\bar{\eta}), \bar{\eta}(x(\bar{(\eta)}))) \in C$ or $y(\bar{\eta}) = y^*(x(\bar{(\eta)}))$ otherwise.

Finally, let us note that, anticipating a private sector strategy that would lead to an inferior outcome than (x^κ, y^κ), the government may simply commit to another reaction function but that cannot lead for sure to (x^κ, y^κ), so that another equilibrium outcome exists, contradicting implementation.

□

First, a simple application of Lemma D.2 shows that if the best equilibrium outcome is implementable for some $\underline{\kappa} > 0$, then it is also implementable for any $\kappa \geq \underline{\kappa}$.

Second, Lemma D.2 shows that when $\kappa > \rho$ the best equilibrium outcome is implementable. Construct $\bar{y} \in Y(X)$ that satisfies the three conditions. Define $\bar{y}$ such that $\bar{y}(x^\kappa) = y^\kappa$. Item (i) is satisfied. Lemma D.2 and the definition of ρ mean that, for any $x \in X$, we can find $\bar{y}(x) \in \mathcal{Y}(x)$ such that $w(x, y^*(x)) - \rho \leq w(x, \bar{y}(x))$. Hence, $(x, \bar{y}(x)) \notin \mathcal{C}$ which leads to Item (ii). Since $y^*(x)$ is the best response to x, for any couple $y \in D(x)$ such that $(x, y) \in \mathcal{C}$, $w(x, y) - \rho \leq w(x, \bar{y}(x))$. Then $\kappa > \rho$ implies Item (iii). Therefore, the best equilibrium outcome is implementable.

Reciprocally, suppose that the best equilibrium outcome is implementable. Thus for any $x \neq x^\kappa \in X$, we can build a $\bar{y}(x) \in \mathcal{Y}(x)$ and $w(x,y) - \kappa < w(x, \bar{y}(x))$ for any $(x,y) \in \mathcal{C}$. Thus, it is also true for $y = y^*(x)$ and hence, $w(x, y^*(x)) - \kappa < w(x, \bar{y}(x))$. This proves that $\kappa \geq \rho$.

D.2 Proof of Proposition 3.

The first point of Proposition 3 directly results from the definition of ρ. Take $x \neq x^\kappa$ that is not consistent with a Nash outcome, that means, such that $y^*(x) \notin C$. therefore the cost of deterring the private sector from playing this action x is zero as $y^*(x) \in \mathcal{Y}(x)$. So if the government can credibly deter the private agents from playing actions $x \neq x^\kappa$ that are consistent with a Nash outcome, the government implements (x^κ, y^κ). This condition means that for any $x \neq x^\kappa$ there exists a policy action y in $\mathcal{Y}(x)$ such that:

$$w(x,y) > w(x, y^*(x)) - \kappa. \tag{53}$$

Let us now prove the second point of Proposition 3. In this proof, we show that the conditions of the proposition ensures the existence of a reaction function satisfying the conditions of Lemma D.2, which is sufficient to guarantee implementation.

For any $x \neq x^\kappa$, such that $(x, y^*(x)) \notin C$, the reaction function will be simply the best response $y^*(x)$ as explained in the first point of the Proposition. Let now consider the other cases: all the actions $x \neq x^\kappa$ such that $(x, y^*(x)) \in C$. We assume that all the actions of these Nash outcomes are interior actions.

Since we assume that there exists an open interval around the best response $y^*(x)$ in $D(x)$, we can perturb the best response in $D(x)$ by a small amount $\epsilon(x) \geq 0$ such that:

$$\exists \xi \in X, u(\xi, x, y^*(x) + \epsilon(x)) > u(x, x, y^*(x) + \epsilon(x)), \tag{54}$$

$$\text{and } |w(\tilde{x}, y^*(x) + \epsilon(x)) - w(x, y^*(x))| \leq \kappa/2. \tag{55}$$

In interior competitive outcomes, $u_1(x,x,y) = 0$. That $u_{13}(x,x,y) \neq 0$ and x in the interior of X means that there exists a neighborhood for private actions ξ around x such that $u_1(\xi, x, y)$ is either strictly positive or negative. This explains why we can pick a function ϵ satisfying the first inequality. The second inequality is possible simultaneously by continuity of w.

Now construct the reaction function $\bar{y} \in Y(X)$ as follows:

$$\bar{y}(x) = y^\kappa, \text{ if } x = x^\kappa$$

$$\bar{y}(x) = y^*(x) + \epsilon(x), \text{ otherwise,}$$

where $\epsilon(x) = 0$ when $(x, y^*(x)) \notin C$ or equivalently $y^*(x) \in \mathcal{Y}(x)$.

Equation (54) means that for any $x' \neq x^\kappa$, $(x', \bar{y}(x')) \notin C$; Equation (55) ensures that the third item of Lemma D.2 is verified. Therefore, $\bar{y}$ satisfies the three conditions of Lemma D.2. The best equilibrium outcome is thus implementable.

D.3 Proof of Proposition 4.

Without loss of generality, let define the distribution of the noise as follows. For any level of capital k, ϵ is uniformely distributed over $[\min(-k, -\bar{\epsilon}), \max(\omega - k, \bar{\epsilon})]$.

Take a noise limit $\bar{\epsilon}$ as given.

Notice first that if there is implementation for a commitment ability κ, then it is also true for higher levels.

Second, the Euler equation is not satisfied if:

$$\frac{1}{4}\frac{1}{1-k} \neq (1 - \mathbb{E}_\epsilon \delta(k + \epsilon)). \tag{56}$$

For capital levels strictly above the low-capital Nash equilibrium outcome, the government can rely on the reaction function $\bar{y} = y * (k + \epsilon)$. For levels of noise sufficiently small, the Euler equation will be verified only for $x = x^\kappa$ for $\bar{\epsilon} \to 0$ as in the perfect information case.

Then, to prevent other equilibria to form the government should rely on a reaction function that satisfies (56) for any level of capital k. But as the right-hand-side member is continuous in k it requires to make sure that the right-hand-side member is always strictly below or strictly above the left-hand-side member. The expected tax $E_\epsilon \delta(k + \epsilon)$ cannot be above the one that satisifies the Euler equation because for k tends to 0 the tax rate that satisifes the savers' optimality condition is $\delta = 1$. Therefore, the only possible tax policy that supports implementation is one with a capital income tax below the one that satisfies the Euler equation. But this tax rate is far away from the ex-post optimal tax which is above (for instance for low enough level of capital $\kappa \to 0$, $y^* = 1$). Therefore, implementation requires a sufficiently large commitment ability.

D.4 Proof of Proposition 6.

In this proof, we closely follow Ljungqvist and Sargent (2018)'s proof that we extend to our setting.

Lemma D.3. *When $V_m = \left(\inf_{(x,y)\in C} w(x,y)\right)/(1-\beta) > -\infty$, the set of payoffs V is bounded.*

Proof. The worst per-period payoff for the government is $w(x^m, y^m) - \kappa$ and so, the worst payoff for the government is $(w(x^m, y^m) - \kappa)/(1 - \delta) > -\infty$ as $\delta \in (0, 1)$. As a result the set of equilibrium payoffs V is bounded below. Furthermore it is bounded above by the definition of the Ramsey allocation. So that V is bounded. □

Lemma D.4. *Consider a strategy profile σ. We denote by $\bar{y} = \bar{\sigma}_0^g$, and, for any $\bar{\eta} \in Y(X)$, $x(\bar{\eta}) = \sigma_0^h(\bar{\eta})$ and $y(\bar{\eta}) = \sigma_0^g(\bar{\eta}, x(\bar{\eta}))$. σ is a subgame perfect equilibrium if and only if:*

(i) *For any $(\bar{\eta}, \gamma, \epsilon) \in Y(X) \times X \times D(\gamma)$, $\sigma|_{\bar{\eta},\gamma,\epsilon}$ is a subgame perfect equilibrium.*

(ii) *For any $\bar{\eta} \in Y(X)$, if they exist, $x(\bar{\eta})$ and $y(\bar{\eta})$ satisfy:*

- *$(x(\bar{\eta}), y(\bar{\eta}))$ is a competitive outcome,*

- $\forall \eta \in D(x(\bar{\eta})),\ r(\bar{\eta}, x(\bar{\eta}), y(\bar{\eta})) + \beta V_g(\sigma|_{\bar{\eta}, x(\bar{\eta}), y(\bar{\eta})}) \geq r(\bar{\eta}, x(\bar{\eta}), \eta) + \beta V_g(\sigma|_{(\bar{\eta}), x(\bar{\eta}), \eta})$.

(iii) *$\forall \bar{\eta} \in Y(X)$ such that $x(\bar{\eta})$ and $y(\bar{\eta})$ exist:*

$$r(\bar{y}, x(\bar{y}), y(\bar{y})) + \beta V_g(\sigma|_{\bar{y}, x(\bar{y}), y(\bar{y})}) \geq r(\bar{\eta}, x(\bar{\eta}), y(\bar{\eta})) + \beta V_g(\sigma|_{\bar{\eta}, x(\bar{\eta}), y(\bar{\eta})}).$$

Proof. Suppose that σ is an subgame perfect equilibrium. From the definition of a SPE, we obtain conditions (ii) and (iii) from the definition at date 0. Condition (i) results from the fact that σ is a subgame perfect equilibrium if and only if it is subgame perfect equilibrium after any history.

Suppose that conditions (i), (ii) and (iii) are satisfied. Condition (i) implies that the definition of a subgame perfect equilibrium is satisfied for any date $t \geq 1$. Conditions (ii) and (iii) imply that the definition of a subgame perfect equilibrium is satisfied at date 0. □

Lemma D.5. *For a given competitive outcome $(x, y) \in \mathcal{C}$ and a reaction function $\bar{y} \in Y(X)$, there exists a subgame perfect equilibrium σ such that $\bar{y} = \bar{\sigma}_0^g$, $x = \sigma_0^h(\bar{y})$ and $y = \sigma_0^g(\bar{y}, x)$, if and only if there exist $v(.,.) : Y(X) \times Y \to V$ such that:*

$$\forall \eta \in D(x), r(\bar{y}, x, y) + \beta v(\bar{y}, y) \geq r(\bar{y}, x, \eta) + \beta v(\bar{y}, \eta),$$

$\forall \bar{\eta} \in Y(X)$, if there exists $(x(\bar{\eta}), y(\bar{\eta})) \in \mathcal{C}$ such that

$$\forall \eta \in D(x(\bar{\eta})), r(\bar{\eta}, x(\bar{\eta}), y(\bar{\eta})) + \beta v(\bar{\eta}, y(\bar{\eta})) \geq r(\bar{\eta}, x(\bar{\eta}), \eta) + \beta v(\bar{\eta}, \eta),$$

then:

$$r(\bar{y}, x, y) + \beta v(\bar{y}, y) \geq r(\bar{\eta}, x(\bar{\eta}), y(\bar{\eta})) + \beta v(\bar{\eta}, y(\bar{\eta})).$$

Proof. Let us consider a competitive outcome (x, y) and a reaction function $\bar{y}$. Suppose that there exists a subgame perfect equilibrium σ such that $x = \sigma_0^h(\bar{y})$ and $y = \sigma_0^g(\bar{y}, x)$. From Lemma D.4, for any $(\bar{\eta}, \gamma, \epsilon) \in Y(X) \times X \times D(\gamma)$, $\sigma|_{\bar{\eta}, \gamma, \epsilon}$ is a subgame perfect equilibrium. Let us denote by $v(\bar{\eta}, \epsilon) = V_g\left(\sigma|_{\bar{\eta}, x(\bar{\eta}), \epsilon}\right)$. Using this, we obtain the conditions of Lemma D.5 from conditions (ii) and (iii) in Lemma D.4.

Conversely, let us consider a reaction function $\bar{y} \in Y(X)$ and a competitive equilibrium $(x, y) \in \mathcal{C}$ satisfying the conditions of Lemma D.5. As $v(.,.)$ takes value in V, there exists a continuation strategy starting at date 2 that is an equilibrium. The conditions of Lemma D.5 also ensure that the strategy is an equilibrium at date 1 as they allow to satisfy conditions (ii) and (iii) in Lemma D.4.

□

Let us introduce a bunch of useful definitions:

Definition D.1. *Let $W \subseteq \mathbb{R}$. $(\bar{y}, x, y, w(.,.))$ is said to be admissible with respect to W if $\bar{y} \in Y(X)$, $(x, y) \in \mathcal{C}$ and $w(.,.) : Y(X) \times X \to W$ such that:*

(i) $\forall \eta \in D(x), r(\bar{y}, x, y) + \beta w(\bar{y}, y) \geq r(\bar{y}, x, \eta) + \beta w(\bar{y}, \eta)$.

(ii) *For any $\bar{\eta}$ in the subset of $Y(X)$ for which at least one competitive outcome $(x', y') \in \mathcal{C}$ satisfies $\forall \eta \in D(x'), r(\bar{\eta}, x', y') + \beta w(\bar{\eta}, y') \geq r(\bar{\eta}, x', \eta) + \beta w(\bar{\eta}, \eta)$, there exists one such competitive outcome such that:*

$$r(\bar{y}, x, y) + \beta w(\bar{y}, y) \geq r(\bar{\eta}, x', y') + \beta w(\bar{\eta}, y').$$

Let $B(W)$ be the set of possible values $w = r(\bar{y}, x, y) + \beta w(\bar{y}, y)$ associated with an admissible $(\bar{y}, x, y, w(.,.))$.

W is said to be self-generating when $W \subseteq B(W)$.

Lemma D.6. *The following properties are true:*

(i) *Monotonicity of B: if $W \subseteq W' \subseteq \mathbb{R}$, then $B(W) \subseteq B(W')$.*

(ii) *Self-generation: if $W \subset \mathbb{R}$ is bounded and self-generating, then $B(W) \subseteq V$.*

Proof. Monotonicity. It can be directly obtained by using $w_1(.,.) \in Y(X) \times Y \to W$ that supports $w \in B(W)$ to support $w \in B(W')$.

Self-generation. Suppose that $W \subseteq B(W)$. Let's choose $w \in B(W)$ and let us show that it is in V.

As $w \in B(W)$, there exists $(\bar{y}, x, y, w_1(.,.))$ such that $(x, y) \in \mathcal{C}$ and $w_1(.,.) \in Y(X) \times Y \to W$ and

$$\forall \eta \in D(x), w = r(\bar{y}, x, y) + \beta w_1(\bar{y}, y) \geq r(x, \eta) + \beta w_1(\bar{y}, \eta),$$

and, $\forall \bar{\eta} \in Y(X)$ such that $\exists (x(\bar{\eta}), y(\bar{\eta})) \in \mathcal{C}$ satisfying:

$$\forall \eta \in D(x(\bar{\eta})), w = r(\bar{\eta}, x(\bar{\eta}), y(\bar{\eta})) + \beta w_1(\bar{\eta}, y(\bar{\eta})) \geq r(x(\bar{\eta}), \eta) + \beta w_1((\bar{\eta}), \eta),$$

we have:

$$w = r(\bar{y}, x, y) + \beta w(\bar{y}, y) \geq r(\bar{\eta}, x(\bar{\eta}), y(\bar{\eta})) + \eta w(\bar{\eta}, y(\bar{\eta})).$$

Let set $\sigma_1 = (\bar{y}, x, y)$ and $w_1 = w(\bar{y}, y)$.

As $w_1 \in W \subseteq B(W)$, there exist outcomes $(\tilde{\bar{y}}, \tilde{x}, \tilde{y})$ such that $(\tilde{x}, \tilde{y}) \in \mathcal{C}$ and a function $\tilde{w_1}(.,.) \in Y(X) \times Y \to W$ that satisfy:

$$\forall \eta \in D(\tilde{x}), w_1 = r(\tilde{\bar{y}}, \tilde{x}, \tilde{y}) + \beta \tilde{w}_1(\tilde{\bar{y}}, \tilde{y}) \geq r(\tilde{\bar{y}}, \tilde{x}, \eta) + \beta \tilde{w}_1(\tilde{\bar{y}}, \eta),$$

and, $\forall \bar{\eta} \in Y(X)$ such that there exists $(x(\bar{\eta}), y(\bar{\eta})) \in \mathcal{C}$ and

$$\forall \eta \in D(x(\bar{\eta})), w_1 = r(\bar{\eta}, x(\bar{\eta}), y(\bar{\eta})) + \beta \tilde{w}_1(\bar{\eta}, y(\bar{\eta})) \geq r(\bar{\eta}, x(\bar{\eta}), \eta) + \beta \tilde{w}_1(\bar{\eta}, \eta),$$

we have:

$$w_1 = r(\tilde{\bar{y}}, \tilde{x}, \tilde{y}) + \beta \tilde{w}_1(\bar{y}, \tilde{y})) \geq r(\bar{\eta}, x(\bar{\eta}), y(\bar{\eta})) + \beta w(\bar{\eta}, y(\bar{\eta})).$$

Set the first-period outcome in period 2 equal to $(\tilde{\bar{y}}, \tilde{x}, \tilde{y})$, that is $(\sigma|_{(\bar{y},x,y)})_1 = (\tilde{\bar{y}}, \tilde{x}, \tilde{y})$.

This way, we can construct a sequence of continuation values with first-period outcomes. As noted by Ljungqvist and Sargent (2018), the strategy profile that we have built is optimal in the sense that, given continuation values, one-period deviations are not optimal. As a result, $V_g(\sigma) = w \in V$.

□

Lemma D.7. *Let W be a compact. $B(W)$ is a compact set.*

Proof. Let us first partition $B(W)$ as follows. Let $\mathcal{I}(W) \subseteq B(W)$ such that any $w \in B(W)$ satisfy:

(i) there exists $\bar{y}$, x,y and $w(.,.)$ such that $(\bar{y}, x, y, w(.,.))$ is admissible with respect to W and $\bar{y}(x) = y$,

(ii) $w = r(\bar{y}, x, y) + \beta w(\bar{y}, y)$.

By construction, we have:

$$B(W) = \mathcal{I}(W) \cup \complement_{B(W)}\mathcal{I}(W).$$

Payoffs are continuous on $\mathcal{I}(W)$ and $\complement_{B(W)}\mathcal{I}(W)$ respectively. As a result, $\mathcal{I}(W)$ and $\complement_{B(W)}\mathcal{I}(W)$ are compact sets. As it is the finite union of compact sets, $B(W)$ is a compact set. □

Let us now conclude. Let us assume that $w(x^m, y^m) > -\infty$. Lemma D.3 implies that V is bounded. From Lemma D.4, we have that $V \subseteq B(V)$ so that V is self-generating and bounded. Using Lemma D.6, we obtain that $B(V) \subseteq V$ and $B(V) = V$.

Finally, let us prove the following Lemma using the proof of Theorem 4 in Abreu et al. (1990):

Lemma D.8. *V is a compact.*

Proof. We have that V is bounded and self generating $(V \subseteq B(V))$. Let us denote by $cl(V)$ the closure of V. Given that V is bounded, $cl(V)$ is a compact and $V \subseteq cl(V)$.

By monotonicity, $V = B(V) \subseteq B(cl(V))$. As $B(.)$ maps compact sets into compact sets (Lemma D.7), $B(cl(V))$ is compact. As $cl(V)$ is the smallest closed set (and thus compact) that contains V, one can also infer that $cl(V) \subseteq B(cl(V))$, which implies that $cl(V)$ is bounded and self-generating. From Lemma D.6, we infer that $cl(V) \subseteq V$, which implies that $V = cl(V)$ and V is compact. □

Another consequence of the proof of Lemma D.6 is also that we can build a subgame perfect equilibrium for any $v \in V$, as V is bounded and self-generating. As a result, V is closed and bounded and so, it is a compact.

Let us conclude the proof of the Proposition with the following lemma which gives the second part of the results.

Lemma D.9. $V_m \leq V_{worst} \leq V_{best} \leq V_{Ramsey}$.

Proof. First, we show that the lowest level of payoff V_{worst} is higher than $\frac{w(x_m,y_m)}{1-\beta}$. We denote by σ the equilibrium that reaches this lowest payoff.

Consider a reaction function $\bar{\eta}$ that consists for the government to always pick an action such that for any $x \in X$, $(x, \bar{\eta}(x)) \in C$. Since σ is an equilibrium, item (iii) of the SPE definition applied for this particular reaction function $\bar{\eta}$ and leads to:

$$V_{worst} \geq V_0^g(\sigma|_{(\bar{\eta}, x_0(\bar{\eta}), y_0(\bar{\eta}))}).$$

In addition, item (ii) of the SPE definition for the policy action $\eta = \bar{\eta}(x_0(\bar{\eta}))$ yields the following inequality:

$$V_0^g(\sigma|_{(\bar{\eta}, x_0(\bar{\eta}), y_0(\bar{\eta}))}) \geq w(x_0(\bar{\eta}), \bar{\eta}(x_0(\bar{\eta}))) + \beta V_1^g(\sigma|_{(\bar{\eta}, x_0(\bar{\eta}), \bar{\eta}(x_0(\bar{\eta})))}).$$

The first right-hand-side member is greater than $w(x_m, y_m)$ because $(x_0(\bar{\eta}), \bar{\eta}(x_0(\bar{\eta}))) \in C$ by item (i) of the SPE definition and the second right-hand-side member is greater than V_{worst} because this is the lowest equilibrium payoff. Therefore,

$$V_{worst} \geq w(x_m, y_m) + \beta V_{worst} \text{ and hence } V_{worst} \geq \frac{w(x_m, y_m)}{1-\beta}.$$

Second, $V_{best} \leq \frac{w(x_R,y_R)}{1-\beta}$ because at any date the equilibrium outcome is a competitive equilibrium and hence leads to a per-period payoff lower than the Ramsey allocation's one. □

D.5 Proof of Proposition 7

The Ramsey allocation is an equilibrium outcome when $\kappa \geq \bar{\kappa}$. The following lemma provides conditions under which the Ramsey allocation is an equilibrium outcome:

Lemma D.10. *The Ramsey allocation is an equilibrium outcome if and only if V_{worst} satisfies:*

$$\beta \left(\frac{w(x^R, y^R)}{1-\beta} - V_{worst} \right) + \kappa \geq \max_{\eta} w(x^R, \eta) - w(x^R, y^R). \tag{57}$$

Proof. Suppose the Ramsey allocation is an equilibrium outcome and denote by $\sigma^R = (\bar{\sigma}^g, \sigma^h, \sigma^g)$ the equilibrium. At date 0, item (ii) of SPE definition for $\bar{\eta} = \bar{\sigma}^g$ and for any $\eta \in D(x^R) \neq \bar{\sigma}_0^g(x^R) = y^R$ leads to:

$$w(x^R, y^R) + \frac{\beta}{1-\beta} w(x^R, y^R) \geq r(\bar{\sigma}_0^g, x^R, \eta) + V_1^g(\sigma|_{(\bar{\sigma}_0^g, x^R, \eta)}),$$

where $r(\bar{\sigma}_0^g, x^R, \eta) = w(x^R, \eta) - \kappa$; therefore, by definition of the lowest equilibrium payoff V_{worst}:

$$w(x^R, y^R) + \frac{\beta}{1-\beta} w(x^R, y^R) \geq \max_{\eta \in D(x^R) \cup y^R} w(x^R, \eta) - \kappa + \beta V_{worst}.$$

Therefore,

$$\beta \left(\frac{w(x^R, y^R)}{1-\beta} - V_{worst} \right) + \kappa \geq \max_{\eta \in D(x^R) \cup y^R} w(x^R, \eta) - w(x^R, y^R).$$

Notice that this inequality is obviously also valid for $\eta = x^R$ thus the exclusion of $\eta = x^R$ is unnecessary.

Prove now the reciprocal implication. Take the carrot-and-stick strategy profile defined as –as long as in the past there is no deviation– at any date $t \geq 0$:[27]

$$\begin{aligned}
&\bar{\sigma}_t^g(x^R) = y^R \text{ and } \bar{\sigma}_t^g(x) = \arg\max_{y \in D(x)|(x,y)\in C} w(x,y) \text{ otherwise,}\\
&\sigma_t^h(\bar{\sigma}_t^g) = x^R \text{ and } \sigma_t^h(\bar{\eta}) = x_0^{worst} \text{ otherwise,}\\
&\sigma^g(\bar{\sigma}_t^g, x) = \bar{\sigma}_t^g(x) \text{ and } \sigma^g(\bar{\eta}, x_0^{worst}) = y_{worst} \text{ otherwise.}
\end{aligned}$$

After a deviation, the strategy simply replicates the worst equilibrium of the repeated game. We do not define it entirely but we know that there exists such a continuation strategy since the worst equilibrium is an equilibrium of the repeated game.

It suffices then to check that this strategy is an equilibrium. First, if $\bar{\eta} \neq \bar{\sigma}_t^g$, then the resulting allocation is $(x_0^{worst}, y_0^{worst})$ which is a competitive outcome. If $\bar{\eta} = \bar{\sigma}_t^g$, then the resulting allocation is (x^R, y^R) which is also a competitive outcome. Item (i) of the SPE definition is verified.

Second, if $\bar{\eta} \neq \bar{\sigma}_t^g$, item (iii) is verified since $V^R \geq V_{worst}$ and item (ii) is verified since the worst outcome $(x_0^{worst}, y_0^{worst})$ and its continuation is an equilibrium outcome.

Third, it remains to show that when $\bar{\eta} = \bar{\sigma}_t^g$ item (ii) is satisfied for any $\eta \in D(x^R)$ which is exactly inequality (57). □

The Ramsey allocation is an equilibrium outcome when the (potential) gains from deviating from the Ramsey allocation are dominated by the costs either due to reputation forces—captured by $\beta\left(\frac{w(x^R,y^R)}{1-\beta} - V_{worst}\right)$—or due to commitment ability—captured by the reneging cost κ. Inequality (57) highlights that reputation forces and commitment ability are *substitutes*: they do not crowd out each other and both contribute to solve the time-consistency problem. Reputation forces appear when discounting is positive ($\beta > 0$) and other equilibrium outcomes exist that are worse than the Ramsey allocation. In this latter case, $V_{worst} < w(x^R, y^R)/(1-\beta)$. Finally, Proposition D.10 implies that, in the absence of reputation forces, the Ramsey allocation is an equilibrium outcome if and only if $\kappa \geq \bar{\kappa}$, with $\bar{\kappa}$ defined as in the static case (Section 2). On the contrary, when β tends to 1, the time-consistency problem is solved even without any commitment ability ($\kappa = 0$).

Necessity. We first prove that the Ramsey allocation is an equilibrium outcome and implementable only if $\kappa \geq \bar{\kappa}$ (see Lemma D.10).

Suppose $\kappa < \frac{\beta}{1-\beta}(w(x^R, y^R) - w(x^d, y^d))$, where (x^d, y^d) is the *worst* static Nash equilibrium. Take the strategy defined at date 0 as follows:

[27] For simplicity we omit the whole history in the notations.

$$\bar{\sigma}_0^g = y^R$$
$$\sigma_0^h = x_d$$
$$\sigma_0^g(x^d) = y^d$$

The continuation strategy is a strategy sustaining the Ramsey equilibrium outcome if $y_0 = y^d$ and the reaction function was $\bar{\sigma}_0$; otherwise, the continuation strategy is to follow the date-0 strategy (as in a self-enforcing strategy).

This strategy is an equilibrium. A date 0, item (i) of SPE definition is satisfied since the date-0 outcome is a competitive outcome ($(x^d, y^d) \in C$); Item (ii) is satisfied if: for any $\bar{\eta}$ such that $\bar{\eta}(x^d) \neq y^d$ and any $\eta \in D(x)$:

$$V_0 = w(x^d, y^d) - \kappa + \beta V^R \geq w(x^d, \bar{\eta}(x^d)) + \beta V_0, \tag{58}$$

where V_0 is the date-0 value. Notice that playing y^d is necessary better than any other deviation from $\bar{\eta}(x^d)$ that is the reason why this is the only inequality to check when $\bar{\eta}(x^d) \neq y^d$. For $\bar{\eta}(x^d) = y^d$, the inequality to check is:

$$w(x^d, y^d) + \beta V_0 \leq w(x^d, \eta) - \kappa + \beta V^R, \tag{59}$$

which is clearly always satisfied by definition of y^d.

Finally, check item (iii) of SPE definition requires to compare payoffs for different $\bar{\eta}$; given the private sector strategy this only requires to compare the case $\bar{\eta}(x^d) = y^d$ and $\bar{\eta}(x^d) \neq y^d$:

$$V_0 \geq w(x^d, y^d) + \beta V_0. \tag{60}$$

Therefore $V_0 \geq w(x^d, y^d)/(1-\beta)$, and all the above inequalities are satisfied if $\kappa \leq \frac{\beta}{1-\beta}(w(x^R, y^R) - w(x^d, y^d))$. We can build such a strategy for any Nash equilibrium but that is for the worst Nash equilibrium (x^d, y^d) that the threshold is the highest for κ. Under such assumption, the strategy is an equilibrium and hence (x^R, y^R) is an equilibrium outcome but not implementable.

Sufficiency. Assume that $\kappa > \max\left(\bar{\kappa}, \frac{\beta}{1-\beta}(w(x^R, y^R) - w(x^m, y^m))\right)$. In this case, Proposition D.10 shows that the Ramsey allocation is an equilibrium outcome. As in Proposition 3, consider the reaction function $\bar{y}$ such that $\bar{y}(x^R) = y^R$ and $\bar{y}(x) \notin \{y' | x = \arg\max_\xi u(\xi, x, y')\}$ defined in the proof of Proposition 3. It is easy to check that this reaction function is part of an equilibrium where the private sector's strategy is to play x^R and the government strategy σ^g is to stick to the reaction function if $\bar{\eta} = \bar{y}$ and to play the best response given x^R and $\bar{\eta}$ otherwise.

Let us prove that there is no other equilibrium outcome. We proceed by contradiction.

Suppose that the strategy profile σ' is an equilibrium such that, at some date t, the equilibrium outcome (x_t, y_t) is such that $x_t \neq x^R$.

First, show that for $\bar{\eta} = \bar{y}$ and for any history h_{t-1}, $(\sigma_t^h(h_{t-1}, \bar{\eta}), \sigma^g(h_{t-1}, \bar{\eta}, \sigma_t^h(\bar{\eta}))) = (x^R, y^R)$. Suppose that for an history, this equality is not verified, then it means that there exists a competitive equilibrium $(x, y) \neq (x^R, y^R)$ and two equilibrium payoffs V_1 and V_2 in V such that:

$$w(x, y) - \kappa + \beta V_1 \geq w(x, \bar{y}(x)) + \beta V_2.$$

Lemma D.9 shows that $V_1 \leq \frac{w(x^R,y^R)}{1-\beta}$ and $V_2 \geq \frac{w(x^m,y^m)}{1-\beta}$ therefore

$$\frac{\beta}{1-\beta}(w(x^R, y^R) - w(x^m, y^m)) \geq \kappa + w(x, \bar{y}(x)) - w(x, y), \tag{61}$$

which is impossible given the construction of $\bar{y}$, and the assumption on κ —$w(x, \bar{y}(x))$ can be as close as we want to $w(x, y)$. This shows that the Ramsey allocation is always the outcome resulting from $\bar{y}$.

So item (iii) of SPE definition cannot be verified for σ' since there exists a reaction function leading to the repeated Ramsey allocation. Thus σ' is not an equilibrium and the Ramsey allocation is the unique equilibrium outcome.

Existence of a threshold. Suppose that for some κ, the Ramsey allocation is the unique equilibrium outcome. Then for any higher reneging cost $\kappa' \geq \kappa$ the Ramsey allocation is also the unique equilibrium outcome (this can be shown through a similar proof as above). With the previous two results, it leads to the existence of

$$\kappa^I \in \left[\frac{\beta}{1-\beta}(w(x^R, y^R) - w(x^d, y^d)), \frac{\beta}{1-\beta}(w(x^R, y^R) - w(x^m, y^m))\right]$$

such that the Ramsey allocation is the unique equilibrium outcome for any κ above κ^I and not the unique equilibrium outcome below (assuming $\kappa \geq \bar{\kappa}$).

D.6 Proof of Proposition C.1

Take $W = [\underline{w}, \bar{w}]$ and compute the boundaries $W' = B(W) = [\underline{w}', \bar{w}']$.

We obtain the maximum $\bar{w}' = \max r(\bar{y}, x, y) + \beta w(\bar{y}, y)$ over any admissible 4-uple $(\bar{y}, x, y, w(.,.))$ with respect to W. To get the highest payoff $\bar{w}'$, one should pick the highest admissible payoff for $w(\bar{y}, y) = \bar{w}$ while selecting the lowest one in case of deviation $w(\bar{y}, \eta) = \underline{w}$ for any $\eta \neq \bar{y}(x)$. Therefore the first condition (item (i) of Definition D.1)) on the highest payoff reads as:

$$u(x, y) + \beta \bar{w} \geq u(x, y^*(x)) - \kappa + \beta \underline{w}.$$

For the highest payoff, the $\bar{y}$ should be chosen such that there is no deviation $y = \bar{y}(x)$, and hence $r(\bar{y}, x, y) = u(x, y)$.

First, if $\kappa < \beta(\bar{w} - \underline{w})$, then one can choose $w(.,.)$ such that there is no alternative $\bar{\eta} \neq \bar{y}$ satisfying $(x', y') \in \mathcal{C}$ and $\forall \eta \in D(x')$, $u(x', y') + \beta w(\bar{\eta}, y') \geq u(x', \eta) - \kappa + \beta w(\bar{\eta}, \eta)$. But since $\kappa < \beta(\bar{w} - \underline{w})$, such couple does not exist because $u(x', y') + \beta \bar{w} < u(x', y^*(x') + \epsilon) - \kappa + \beta \underline{w}$ where ϵ is positive if $y' = y^*(x')$. Finally, in this case, there is no additional constraint on the best payoff $\bar{w}'$.

Otherwise, let us show that the additional constraint is:

$$u(x,y) + \beta \bar{w} \geq u(x_{\kappa-\beta(\bar{w}-\underline{w})}, y_{\kappa-\beta(\bar{w}-\underline{w})}) + \beta \underline{w}.$$

where $(x_{\kappa-\beta(\bar{w}-\underline{w})}, y_{\kappa-\beta(\bar{w}-\underline{w})})$ is the best constrained static outcome when the commitment ability is $\max\{0, \kappa - \beta(\bar{w} - \underline{w})\}$.

To show that, first notice that any competitive outcome $(x', y') \in \mathcal{C}$ that satisfies:

$$u(x,y) + \beta \underline{w} \geq u(x', y^*(x')) - \kappa + \beta \bar{w},$$

can be implemented (in the static sense) with a well-chosen reaction function $\bar{\eta}$.

Finally, with the exact same reasoning, we find that the lowest payoff admissible with respect to W is:

$$\begin{aligned} \underline{w}' = & \min_{(x,y)\in C, w \in [\underline{w}, \bar{w}]} u(x,y) + \beta w, \\ \text{subject to } & u(x,y) + \beta w \geq u(x, y^*(x)) - \kappa + \beta \underline{w}, \\ & u(x,y) + \beta w \geq u(x_{\kappa-\beta(\bar{w}-\underline{w})}, y_{\kappa-\beta(\bar{w}-\underline{w})}) + \beta \underline{w}. \end{aligned}$$