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JEL Classification: C62, F31, G12

Keywords: volatility risk, foreign exchange disconnect, Risk Sharing

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1 Introduction

A key question in international economics concerns the connection between currency valuations and macroeconomic fundamentals. In a classical international setting with complete markets and time-additive power utility, Kollmann (1991) and Backus and Smith (1993) document a "disconnect" between the *levels* of foreign exchange rates and economic activity. Specifically, according to these models, exchange rate growth (Δe_t) should be perfectly correlated with the consumption differential across countries ($\Delta g_{i,t} - \Delta g_{j,t}$), whereas in the data this correlation ($\text{corr}(\Delta e_t, \Delta g_{i,t} - \Delta g_{j,t})$) is close to zero.

Naturally, the classical paradigm further implies a perfect correlation of the ex-ante conditional *volatilities* of the exchange rates and the consumption differentials ($\text{corr}(\sigma_t(\Delta e_t), \sigma_t(\Delta g_{i,t} - \Delta g_{j,t}))$).[1] In this paper, we are the first to study the empirical and theoretical significance of such a (dis)connect between the second moments of exchange rates and consumption. Both on empirical and theoretical grounds, we argue that the volatility evidence is informative about the economic channels active in foreign exchange markets above and beyond the traditional level correlations.

Specifically, we present novel empirical evidence for three aspects of the data. First, the correlation of the conditional volatilities tends to be larger than the correlation of the levels of consumption growth differentials and exchange rates. This evidence can be interpreted as suggesting that the volatility disconnect is smaller than the level disconnect. Second, even though the second moments are more correlated than the levels, their correlation is still small at about 20%, which is a clear contradiction to the implications of a traditional international macro-finance model with complete markets and time-additive preferences. To emphasize our find of a mild correlation between the volatilities, we will often use the expression "volatility (dis)connect" in foreign exchange markets. Third, there is a substantial amount of cross-country heterogeneity in the volatility discon-

[1] At a broader level, this is true for all the high-order moments.

nect. This variation is important because it can be linked to heterogeneous economic fundamentals across country pairs.

We examine four economic dimensions as possible determinants of the heterogeneity in the volatility correlations. We document that both the relative amount of *unexpected* growth risk and the relative country size are not significantly associated with changes in $\text{corr}(\sigma_t(\Delta e_t), \sigma_t(\Delta g_{i,t} - \Delta g_{j,t}))$ across country pairs. In contrast, both the relative unconditional volatility of *expected* output growth rates and the relative amount of time-varying volatility of each country pair matter. In particular, the riskiness of expected output growth rates features a negative association to the correlation between consumption and exchange rate's volatilities. In contrast, this association is positive when we look at the amount of volatility risk across countries, meaning the volatility disconnect becomes less evident.

The main objective of our subsequent economic analysis is to explain these novel empirical facts. Specifically, we consider an economy with two countries, each populated by one agent with Epstein and Zin (1991) preferences (henceforth EZ preferences). Each agent is endowed with the stochastic supply of one country-specific good, whose dynamics are characterized by the presence of time-varying volatility shocks. Preferences feature a bias for the consumption of the domestic good. Trade occurs in frictionless goods markets and in financial markets featuring a complete set of state- and date-contingent securities.

Preferences are calibrated so that our agents dislike volatility of their continuation utilities. Since continuation utilities are a reflection of the entire future streams of consumption, we say that agents dislike long-run consumption variance. When news shocks hit the economy, agents have an incentive to trade in order to reduce the uncertainty of their future utility. Specifically, a country affected by a positive news shock will receive a smaller share of resources and have lower volatility of continuation utility going forward, but it will also have higher short-run consumption volatility.

When news pertains to future expected growth rates, the international reallocation of resources results in an international exchange of both short-run and long-run consumption volatility across countries. That is, variances are characterized by negative comovements. We call this force the reallocation effect. News to output volatility, in contrast, produces a positive comovement in consumption volatilities across all countries: changes in output volatility spread in the cross section of countries, with the reallocation channel only partially mitigating the effects of local shocks on local consumption volatility.

Our model can account for a mild positive comovement between the volatility of consumption differentials and the volatility of exchange rate fluctuations due to two opposite forces. Volatility shocks tend to create a positive correlation between the two volatilities, as they increase the uncertainty of all the variables in the economy. Long-run shocks, in contrast, generate a large negative comovement. In a model without shocks to output volatility (e.g., Colacito and Croce (2013)), the volatility of the exchange rate and that of the international differential of consumption growth rates would be strongly negative because of the dominance of the reallocation channel. In contrast, exogenous output volatility shocks increase the conditional volatility of all macroeconomic aggregates and hence endogenously produce positive comovements. Under our benchmark calibration, these opposite forces end up producing a positive but moderate correlation between consumption differentials and exchange rate volatility.

Related literature. Our study is related to a large and growing body of literature that studies macroeconomic foundations for international financial markets' fluctuations (see, inter alia, Lustig and Verdelhan (2007), Farhi and Gabaix (2016), Verdelhan (2010), Stathopoulos (2017), Mueller, Stathopoulos, and Vedolin (2017), Della Corte, Riddiough, and Sarno (2016b), Heyerdahl-Larsen (2015), Pavlova and Rigobon (2007; 2010; 2013), Hassan (2013), Hassan, Mertens, and Zhang (2015; 2016)). The emphasis on the importance of long-lasting news for international asset prices is consistent with the international long-run risks literature (see, among others, Colacito (2008); Colacito and Croce (2013); and Bansal and Shaliastovich (2013)). Colacito, Croce, Liu, and Shaliastovich (2021) focus on volatility risk sharing in order to explain volatility pass through across countries. We adopt a similar model but we highlight a distinct margin of volatility risks in the data: the volatility disconnect.

Additionally, recent research has documented the relevance of second and higher-order moments for currency dynamics and their relation to economic fundamentals. Fernandez-Villaverde, Guerron-Quintana, Rubio-Ramirez, and Uribe (2011) study the role of time-varying volatility in the context of a small open economy setting. Compared to their analysis, we study the comovement of second moments resulting from optimal risk-sharing in a multi-country equilibrium setting. Zviadadze (2017) extracts a common stochastic volatility component in the U.S. macroeconomic and financial market data and analyzes the relationship between the term structure of currency carry trade and U.S. macroeconomic risk. Berg and Mark (2018) show that the cross-country high-minus-low conditional skewness of the unemployment gap is a measure of global macroeconomic uncertainty which is priced in currency excess returns. Liu and Shaliastovich (2021) relate movements in the dollar value and the currency risk premium to policy-related uncertainty in the U.S. Farhi, Fraiberger, Gabaix, Ranciere, and Verdelhan (2015), Lettau, Maggiori, and Weber (2014), and Chernov, Graveline, and Zviadadze (2018) study the role of downside risk for currency risk premia, and Gavazzoni, Sambalaibat, and Telmer (2013) highlight the importance of that non-Gaussian dynamics of the stochastic discount factors to reconcile the riskiness of currencies with the level of the interest rates. Fang and Liu (2021) study the effect of volatility on exchange rates through intermediary Value-at-Risk constraints. Naturally, these studies are part of a broader research which examines implications of time-varying uncertainty and volatility for the economic growth and asset prices; (see, among many others, Bloom (2009); Della Corte, Sarno, and Tsiakas (2011); Justiniano and Primiceri (2008); Jurado, Ludvigson, and Ng (2015); Kollmann (2016); and Gilchrist, Sim, and Zakrajsek (2014)). We contribute to this literature by analyzing the (dis)connect between the volatilities of the exchange rates and the macroeconomic fundamentals.

Finally, in the paper we focus on a frictionless risk-sharing setting with symmetric countries. We regard the introduction of frictions, heterogeneity, and market incompleteness into our model as an important direction for future research in this area (see, e.g., Maggiori (2017); Gabaix and Maggiori (2015); Ready, Roussanov, and Ward (2017); Lustig, Roussanov, and Verdelhan (2011; 2014); Sandulescu, Trojani, and Vedolin (2021); Lustig and Verdelhan (2019); Bakshi, Cerrato, and Crosby (2018)). These frictions may be important in addressing the empirical link with international capital flows (Gourinchas and Rey (2007), Gourio, Siemer, and Verdelhan (2014)).

Organization of the paper. In the next section we describe our empirical strategy and our novel findings concerning the volatility disconnect. Sections 3 and 4 describe our model and its implications. Section 5 concludes the paper.

2 Empirical Evidence

In this section we examine the empirical evidence on the comovement between the conditional volatilities of exchange rates and of the consumption growth differentials across countries. In the spirit of the literature which investigates a puzzling disconnect between the *level* of consumption differentials and the exchange rates (Backus and Smith (1993)), we refer to our findings as the *volatility (dis)connect* in foreign exchange markets. We link the magnitudes of the bilateral volatility disconnects to country characteristics, such as size and the amount of expected growth and volatility risk. Our novel evidence has important implications for understanding the risk sharing across countries, and it represents a challenge for many existing international finance models.

2.1 Data Description

Our empirical analysis is based on the cross section of the following $N = 17$ major industrialized countries: Australia, Belgium, Canada, Denmark, France, Germany, Italy, Japan, the Netherlands, New Zealand, Norway, Portugal, Spain, Sweden, Switzerland, the United Kingdom and the United States. From 1999 onward, the data for the Eurozone countries are collapsed into a single Euro unit. We collect the national accounts, population, and CPI data for these countries from the Organization for Economic Cooperation and Development (henceforth OECD) database. The exchange rates, quoted as the US dollar price of the foreign currency, are from the Global Financial Database. The price-dividend ratios are from the MSCI. The macroeconomic data are seasonally adjusted and real.

The benchmark sample is quarterly from 1971Q1 to 2019Q4.[2] As is common in the literature, we focus on a period of substantial financial integration across major industrialized countries (see, among others, Quinn (1997), Obstfeld (1998), Taylor (2002), and Quinn and Voth (2008)). The sample stops in 2019 to exclude the impact of economic disruptions due to the COVID pandemic.

Table 1 provides descriptive statistics for the consumption and GDP growth and the real exchange rates against the US dollar across countries. The mean real consumption and output growth rates are about 2.2% per annum, and their volatilities average just under 1.9%. Consistent with the literature, foreign exchange rates are quite volatile, with the average standard deviation of 11.2% on an annual basis.

2.2 Foreign Exchange Disconnect

To examine the connection between the foreign exchange markets and macroeconomic fundamentals, the literature traditionally considers the correlations between the levels

[2] Due to data availability and quality issues, the data for Belgium, New Zealand, Norway, and Spain start in 1980Q4.

Table 1: Summary Statistics

	consumption		GDP		exchange rate	
	mean	s.d.	mean	s.d.	mean	s.d.
Average	2.15	1.85	2.17	1.86	0.30	11.17
Australia	3.16	1.49	3.01	1.78	0.25	10.45
Belgium	1.64	0.93	1.80	1.19	-1.03	12.89
Canada	2.93	1.50	2.65	1.59	-0.45	6.16
Denmark	1.52	3.53	1.87	2.12	0.50	11.15
Euro	1.12	0.72	1.38	1.20	-0.23	9.98
France	2.08	1.31	2.04	1.06	0.81	11.48
Germany	1.77	1.79	1.88	1.88	1.16	12.65
Italy	1.74	1.49	1.59	1.67	0.46	11.01
Japan	2.23	2.33	2.33	2.17	0.98	12.29
Netherlands	1.83	1.91	2.20	2.19	1.28	12.40
New Zealand	2.85	2.73	2.79	3.28	0.34	11.84
Norway	2.65	2.20	2.28	2.51	-0.77	10.81
Portugal	2.31	2.72	2.41	2.34	1.74	11.70
Spain	1.85	1.76	2.22	1.55	-0.73	11.69
Sweden	1.84	2.61	2.05	2.20	-0.80	11.05
Switzerland	1.54	0.92	1.64	1.48	1.40	12.34
UK	2.62	2.05	2.16	1.79	0.14	10.04
US	2.99	1.27	2.77	1.56		

Notes: This table shows summary statistics for consumption growth, GDP growth, and change in real exchange rate, respectively. 'Average' refers to simple averages of key moments for the 17 countries. Macroeconomic variables are real and seasonally adjusted. Exchange rates are real. Means and standard deviations are annualized, in percentages. Quarterly observations are from the 1971:Q1–2019:Q4 sample.

of consumption growth differentials across countries, $\Delta\tilde{g}_{ij,t} = \Delta g_{i,t} - \Delta g_{j,t}$, and the real exchange rates $\Delta e_{ij,t}$ defined as the value of currency j in units of currency i. Kollmann (1991) and Backus and Smith (1993) find that such correlations are small in the data, contrary to classical models of foreign exchange markets in which exchange rate changes are perfectly correlated with consumption growth differentials. We show the evidence for these bilateral correlations in the right panel of Table 2.

Specifically, for every country i in our sample we tabulate the average of the Backus-Smith correlations with the remaining $N-1$ countries, $\frac{1}{N-1}\sum_{j\neq i} Corr(\Delta\tilde{g}_{ij,t}, \Delta e_{ij,t})$, as well as the first and fourth quintiles of the correlation distributions. Consistent with the

Table 2: Foreign Exchange Disconnect

	$Corr(\sigma_t(\Delta\tilde{g}_{ij,t}), \sigma_t(\Delta e_{ij,t}))$			$Corr(\Delta\tilde{g}_{ij,t}, \Delta e_{ij,t})$		
	mean	1st	4th	mean	1st	4th
Average	0.21	-0.05	0.50	0.04	-0.04	0.12
Australia	0.01	-0.39	0.34	0.05	-0.01	0.16
Belgium	0.26	-0.01	0.62	0.13	0.05	0.23
Canada	0.16	-0.07	0.44	0.01	-0.07	0.08
Denmark	0.31	-0.02	0.59	0.14	0.08	0.21
Euro	0.36	0.22	0.62	-0.12	-0.25	0.01
France	0.22	-0.15	0.71	0.05	-0.06	0.18
Germany	0.26	-0.06	0.56	0.05	-0.04	0.12
Italy	0.13	-0.04	0.43	0.04	-0.03	0.12
Japan	0.02	-0.23	0.28	0.05	-0.01	0.10
Netherlands	0.26	-0.03	0.63	0.01	-0.07	0.08
New Zealand	0.57	0.48	0.71	0.14	0.08	0.21
Norway	0.17	-0.03	0.45	0.05	-0.03	0.12
Portugal	0.29	-0.09	0.54	0.10	0.04	0.16
Spain	-0.12	-0.42	0.06	-0.05	-0.13	0.07
Sweden	0.06	-0.21	0.35	-0.02	-0.09	0.04
Switzerland	0.39	0.14	0.68	0.05	-0.04	0.12
UK	0.28	0.06	0.50	0.05	0.00	0.12
US	0.17	-0.05	0.47	-0.08	-0.14	-0.01

Notes: This table shows correlations between the levels (right panel) and conditional volatilities (left panel) of consumption growth differentials ($\Delta\tilde{g}_{ij,t} \equiv \Delta g_{i,t} - \Delta g_{j,t}$) and the change in the real exchange rate ($\Delta e_{ij,t}$), respectively. 'mean' refers to simple averages of correlations for each country with the remaining ones. '1st' and '4th' show the first and fourth quintiles of the correlations. 'Average' is the average of the moments across countries. Quarterly observations are from the 1971:Q1–2019:Q4 sample.

literature, the Backus-Smith correlations are essentially zero in our sample.

We take the next step and examine the co-movements between the conditional variances, as opposed to the levels, of consumption growth differentials and the exchange rates. To extract the conditional volatility of the series of interest, z_t, we consider the following econometric specification:

$$\begin{aligned} z_t &= \mu(1-\rho) + \rho z_{t-1} + e^{\sigma_t(z)/2}\eta_t, \\ \sigma_t(z) &= \mu_\sigma(1-\nu) + \nu\sigma_{t-1}(z) + \sigma_w w_t, \end{aligned} \tag{2.1}$$

where $\sigma_t(z)$ is a latent process equal to the logarithm of the variance of the shock to z_t. The innovations η_t and w_t are Gaussian shocks to the level and the volatility of z_t, respectively. The parameters ρ and ν govern the persistence of z_t and $\sigma_t(z_t)$, respectively, whereas μ and μ_σ represent the average level and volatility of z_t and $\sigma_t(z_t)$, respectively. The parameter σ_w captures the volatility of volatility. According to our specification, the variance of z_t is guaranteed to take on positive values. In the remainder of this manuscript we refer to σ_t as either log-volatility or volatility interchangeably.[3]

For any pair of countries i and j, we extract the time-varying volatilities of the consumption differentials $\sigma_t(\Delta\tilde{g}_{ij,t})$ and the foreign exchange rate $\sigma_t(\Delta e_{ij,t})$, and compute the unconditional correlation between the two:

$$\text{Volatility Correlation}_{ij} \equiv Corr(\sigma_t(\Delta\tilde{g}_{ij,t}), \sigma_t(\Delta e_{ij,t})) \;\; \forall i \neq j. \tag{2.2}$$

The volatility correlations provide an intuitive measure of the volatility (dis)connect in foreign exchange markets: close-to-one correlations indicate a high degree of connectedness between the volatilities of exchange rates and the consumption fundamentals, while low correlations suggest the disconnect between the two.

Our empirical estimates of these correlations across countries are reported in the left panel of Table 2. We highlight three main findings. First, in the majority of the cases the correlations between the variances are larger than the correlations between the levels of the consumption growth differentials and the exchange rates; that is, the volatility disconnect is smaller than the level disconnect. The average of all the pairwise volatility correlations is 0.2, relative to 0.04 for the levels. The fourth quintile of the volatility correlations is 0.50 on average and it can be as high as 0.70 for some countries (France,

[3]Colacito *et al.* (2021), Cogley and Sargent (2005) and Primiceri (2005), among others, entertain similar econometric specifications for macroeconomic volatility, and Della Corte, Sarno, and Tsiakas (2009) for financial volatility modeling. We estimate the system of equations (2.1) using the Bayesian methods in Kim, Shephard, and Chib (1998). We fit our volatility specification to each country variable separately, and condition our estimates on the entire history of data. Appendix A provides additional details.

New Zealand, Switzerland). These estimates are higher than their level counterparts: the fourth quintile of the average level correlation is about 0.1 and it is under 0.3 at the country level.

Second, while the volatility correlations are larger than the level correlations, they are still quite below one. This specific value is an important reference point because typical classical models would predict that exchange rate and consumption differentials should be perfectly correlated. Stated simply, if foreign exchange rates were proportional to consumption growth differentials in each period and in each state of the world, both the level and volatility correlations would be equal to one. In contrast to this theoretical prediction, the conditional volatility of exchange rates is not perfectly connected to the volatility of the fundamentals in the data. To emphasize the joint findings of a modest but far from perfect correlation between the volatilities, we refer to the evidence as "volatility (dis)connect" in foreign exchange markets. In Figure 1, we depict our point estimates for both the level and the volatility correlation across countries. The resulting scatter plot confirms that the volatility disconnect is distinct from the level disconnect in the data.

Finally, our data show that there is a substantial amount of cross-country heterogeneity in the volatility correlations. Their cross-country first and fourth quintiles are -0.05 and 0.50, respectively. When focusing on the level correlations, we have a smaller range [-0.04; 0.12]. The quintile ranges get even more extreme for individual country estimates. In the next section, we link this cross-sectional heterogeneity in the amount of volatility disconnect to key economic characteristics of our countries.

2.3 Economic Determinants

We examine the connection between our documented volatility disconnect and several key characteristics of our countries in the data. There are four major factors that we consider in the paper, which are motivated by the prior evidence in the international

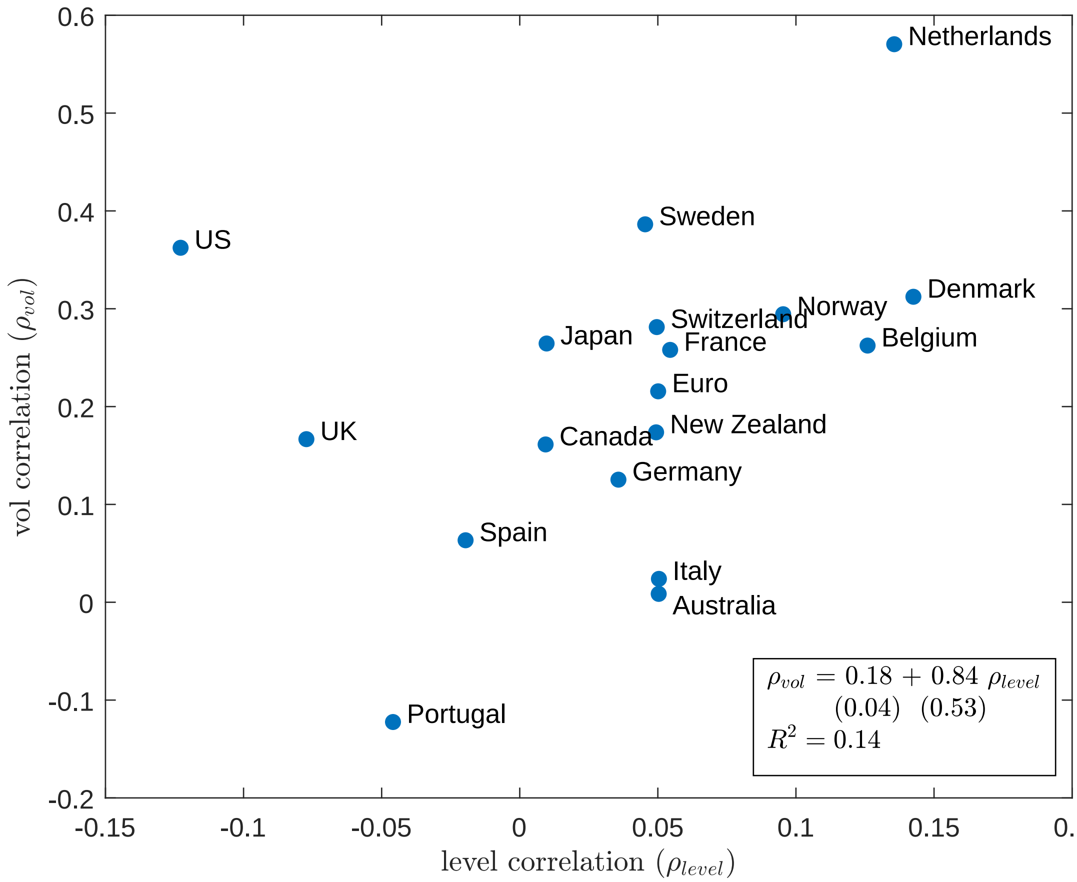


Figure 1 - **Level and Volatility Disconnect.** This figure shows scatterplots of the average level correlations (ρ_{level}) and the average volatility correlation (ρ_{vol}). The regression result is reported in the box.

finance literature. First, we consider the relative size of the country, defined as the average share of its PPP-adjusted real GDP. Second, we measure the standard deviation of unexpected growth shocks. Our third and fourth characteristics correspond to the country-level standard deviation of expected growth and volatility news shocks, respectively.

To measure expected growth risk, we adopt a standard predictive approach in the literature and project a four-quarter ahead GDP growth in each country on the local and US price-dividend ratio.[4] We take the fitted value from the projection as a proxy for

[4]This is similar to the predictive regression approach in Colacito, Croce, Gavazzoni, and Ready (2018), Bansal and Shaliastovich (2013), Bansal, Kiku, and Yaron (2012), and Colacito and Croce

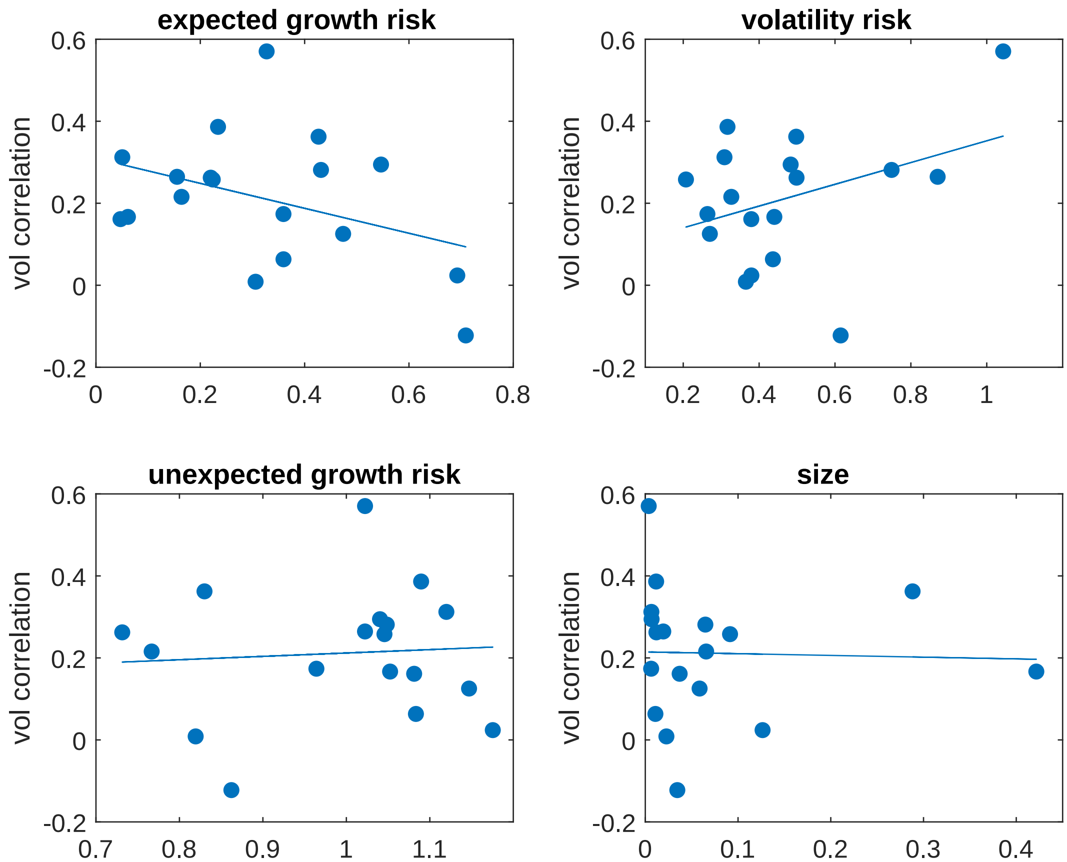


Figure 2 - **Economic Determinants of Volatility Disconnect.** This figure shows scatterplots of the average volatility correlations and each of the following four country characteristics: the long-run expected growth risk; the volatility risk; the unexpected growth risk; and the relative size of the country.

the expected GDP growth, and its unconditional volatility as a measure of the country's amount of long-run expected growth risk. The unconditional volatility of the residual of the predictive regression is our proxy for the amount of the unexpected growth risk. To estimate the quantity of volatility risk, we apply our econometric specification (2.1) to the GDP growth in each country. The variance of the volatility process implied by the estimation, $\sigma_w/\sqrt{1-\nu^2}$, defines our measure of the volatility risk in each country.

Figure 2 presents illustrative evidence about the relation between these factors and the amount of volatility disconnect. Each panel of this figure shows a scatter plot of

(2011).

the average of correlations between the volatilities of the country consumption growth differential and of its foreign exchange rate and each of the four country characteristics. First, the data does not show any size effect: the relationship between the volatility disconnect and country size is effectively flat (bottom-right panel of Figure 2). The same applies to the amount of unexpected growth risk (bottom-left panel of Figure 2). At the same time, the evidence indicates a positive (negative) association between the volatility correlations and the amount of volatility (expected growth) risk (top panels of Figure 2). The relations of the volatility disconnect to the economic factors constitute novel empirical facts and are important targets for our economic analysis.

We use statistical regressions to formally assess the connections between the volatility disconnect and both long-run risk and volatility risk. Specifically, we run a cross-sectional regression of the volatility correlations between country i and j on a characteristic of interest f:

$$Corr(\sigma_t(\Delta\tilde{g}_{ij,t}), \sigma_t(\Delta e_{ij,t})) = constant + \beta(f_i + f_j) + residual. \tag{2.3}$$

In this specification, we use the estimates of the volatility disconnect between all the country pairs, and not just the average of each country against all other countries as displayed in Figure 2. This strategy allows to significantly increase the number of observations. Further, the loadings on country i and j are restricted to be the same, because the correlations are symmetric.

We report the loadings on expected growth and volatility risk in Table 3; we exclude size and unexpected growth risk because they do not feature a significant relation with the volatility disconnect (Figure 2). Consistent with Figure 2, the volatility correlations load negatively on the expected growth risk and positively on the amount of volatility risk. Both effects have a sizeable statistical significance. Our evidence strengthens in the multivariate specification of our regression, i.e., when we simultaneously consider long-

Table 3: Foreign Exchange Disconnect

A. Vol correlation $Corr(\sigma_t(\Delta\tilde{g}_{ij,t}), \sigma_t(\Delta e_{ij,t}))$

long-run risk	(t-stat)	vol risk	(t-stat)	R^2
-0.08	(-3.00)			0.07
		0.29	(3.50)	0.06
				0.07
-0.09	(-3.46)	0.33	(4.19)	0.15

B. Level correlation $Corr(\Delta\tilde{g}_{ij,t}, \Delta e_{ij,t})$

long-run risk	(t-stat)	vol risk	(t-stat)	R^2
-0.01	(-1.08)			0.01
		0.02	(0.68)	0.00
				0.00
-0.01	(-1.20)	0.03	(0.83)	0.01

Notes: Panel A shows cross-sectional regression results of each country pair's volatility correlations ($Corr(\sigma_t(\Delta\tilde{g}_{ij,t}), \sigma_t(\Delta e_{ij,t}))$) and the expected growth risk and/or the volatility risk. Panel B shows cross-sectional regression results of each country pair's Backus-Smith correlation ($Corr(\Delta\tilde{g}_{ij,t}, \Delta e_{ij,t})$) and the expected growth risk and/or the volatility risk. t-statisitcs are based on heteroskedasticity consistent standard errors.

run and volatility risk. As suggested by the regression results, the amount of expected growth and volatility risk can explain 15% of the volatility disconnect across countries.

The bottom panel of Table 3 shows the corresponding evidence for the Backus and Smith (1993) level correlations. The level correlations show much weaker statistical relation to the considered factors. Only a negative effect of expected growth risk has a t-statistic larger than one.

In the next section, we show that our novel evidence on the level and cross-country differences in the volatility (dis)connect is a challenge to the existing equilibrium risk-sharing model with time-additive preferences. In contrast, when agents have recursive preferences, news about both future growth rates and future volatility are priced, and thus they can jointly determine the extent of volatility disconnect in a manner consistent with the data.

3 Model

The economy consists of two countries, home (h) and foreign (f), and two goods, X and Y. Agents' preferences are defined over consumption aggregates of the two goods as follows.

Consumption aggregate. Let x_t^i and y_t^i denote the consumption of good X and good Y in country $i \in \{h, f\}$ at date t. Let $\alpha \in (0, 1)$. The consumption aggregates in the home and foreign countries are

$$C_t^h = \left(x_t^h\right)^{\alpha} \left(y_t^h\right)^{1-\alpha} \quad \text{and} \quad C_t^f = \left(x_t^f\right)^{1-\alpha} \left(y_t^f\right)^{\alpha}, \tag{3.1}$$

respectively. The parameter α captures the degree of bias of the consumption of each representative agent. In what follows we assume that the home country is endowed with good X, while the foreign country is endowed with good Y. Following some of the international macrofinance articles surveyed by Lewis (2011), we assume that α is larger than 0.5. This allows us to build consumption home bias into the model.

Preferences. As in Epstein and Zin (1993), agents' preferences are recursive but not time separable:

$$U_t^i = \left[(1-\delta) \cdot \left(C_t^i\right)^{1-1/\psi} + \delta E_t \left[\left(U_{t+1}^i\right)^{1-\gamma}\right]^{\frac{1-1/\psi}{1-\gamma}}\right]^{\frac{1}{1-1/\psi}}, \quad \forall i \in \{h, f\}. \tag{3.2}$$

The coefficients γ and ψ measure the relative risk aversion (RRA) and the IES, respectively.

In contrast to the constant RRA case, these preferences allow agents to be risk averse in future utility as well as future consumption. The extent of such utility risk aversion depends on the preference for early resolution of uncertainty, measured by $\gamma - 1/\psi > 0$. To better highlight this feature of the preferences, we focus on the ordinally equivalent transformation

$$V_t = \frac{U_t^{1-1/\psi}}{1-1/\psi}$$

and approximate it with respect to $\theta \equiv \frac{\gamma-1/\psi}{1-1/\psi}$ around $\theta_0 = 1$:

$$\begin{aligned} V_t &= (1-\delta)\frac{C_t^{1-1/\psi}}{1-1/\psi} + \delta E_t\left[V_{t+1}^{1-\theta}\right]^{\frac{1}{1-\theta}} \\ &\approx (1-\delta)\frac{C_t^{1-1/\psi}}{1-1/\psi} + \delta E_t\left[V_{t+1}\right] - \frac{\delta}{2}\frac{\theta}{E_t\left[V_{t+1}\right]}Var_t\left[V_{t+1}\right]. \end{aligned} \tag{3.3}$$

Note that the sign of $\left(\frac{\theta}{E_t[V_{t+1}]}\right)$ depends on the sign of $(\gamma - 1/\psi)$. When $\gamma = 1/\psi$, the agent is utility-risk neutral and preferences collapse to the standard time-additive case. When the agent prefers early resolution of uncertainty, that is, when $\gamma > 1/\psi$, the coefficient θ is positive: uncertainty about continuation utility reduces welfare and generates an incentive to trade off future expected utility, $E_t[V_{t+1}]$, for future utility risk, $Var_t[V_{t+1}]$.

This mean-variance trade-off is absent when agents have standard time-additive preferences, and it represents the most important element of our analysis, given our focus on the propagation of uncertainty shocks.

Since there is a one-to-one mapping between utility, U_t^i, and lifetime wealth, that is, the value of a perpetual claim to consumption, $W_{c,t}^i$,

$$U_t^i = \left[(1-\delta)(C_t^i + W_{c,t}^i)\right]^{\frac{1}{1-1/\psi}}, \quad \forall i \in \{h, f\}, \tag{3.4}$$

the optimal risk-sharing scheme can also be interpreted in terms of the mean-variance trade-off of wealth. For this reason, in what follows we use the terms "wealth" and "continuation utility" interchangeably.

Endowments. We choose to endow each country with a stochastic supply of its most-preferred good. Endowments are specified in the spirit of Colacito and Croce (2013), with the important difference of accounting also for time-varying risk:

$$\begin{aligned}\Delta \log X_t &= \mu_x + z_{1,t-1} + e^{\sigma_{x,t}/2}\sigma\varepsilon_{x,t} - ci_{t-1} \\ \Delta \log Y_t &= \mu_y + z_{2,t-1} + e^{\sigma_{y,t}/2}\sigma\varepsilon_{y,t} + ci_{t-1},\end{aligned} \tag{3.5}$$

where the process $ci_t \equiv \tau \log(X_t/Y_t)$ with $\tau \in (0,1)$ introduces cointegration and guarantees the existence of the equilibrium, and the components z_1 and z_2 are highly persistent AR(1) processes,

$$z_{j,t} = \rho z_{j,t-1} + \sigma_z \varepsilon_{j,t}, \forall j \in \{1,2\}. \tag{3.6}$$

Throughout the paper, we refer to $\varepsilon_{1,t}$ and $\varepsilon_{2,t}$ as long-run shocks, due to their long-lasting impact on the growth rates of the two endowments. Similarly, we call $\varepsilon_{x,t}$ and $\varepsilon_{y,t}$ short-run shocks.

We focus on time-varying short-run risk, as captured by the following process:

$$\sigma_{j,t} = \rho_\sigma \sigma_{j,t-1} + \sigma_{sr}\varepsilon_{\sigma j,t}, \forall j \in \{x,y\}. \tag{3.7}$$

Shocks are jointly log-normal:

$$\xi_t \equiv \begin{bmatrix} \varepsilon_{1,t} & \varepsilon_{2,t} & \varepsilon_{x,t} & \varepsilon_{y,t} & \varepsilon_{\sigma 1,t} & \varepsilon_{\sigma 2,t} \end{bmatrix} \sim \quad i.i.d.N(\mathbf{0}, \Sigma),$$

and the matrix Σ is assumed to be block-diagonal to allow for cross-country correlation of shocks of the same type.

Markets. At each date, trade occurs in a complete set of one-period-ahead claims to state-contingent consumption. Financial and goods markets are assumed to be frictionless. The budget constraints of the two agents can be written as

$$
\begin{aligned}
x_t^h + p_t y_t^h + \int_{\zeta^{t+1}} A_{t+1}^h\left(\zeta^{t+1}\right) Q_{t+1}(\zeta^{t+1}) &= A_t^h + X_t \\
x_t^f + p_t y_t^f + \int_{\zeta^{t+1}} A_{t+1}^f\left(\zeta^{t+1}\right) Q_{t+1}(\zeta^{t+1}) &= A_t^f + p_t Y_t,
\end{aligned} \tag{3.8}
$$

where p_t denotes the relative price of goods X and Y (the terms of trade), $A_t^i\left(\zeta^t\right)$ denotes country i's claims to time t consumption of good X, and $Q_{t+1}(\zeta^{t+1})$ gives the price of one unit of time $t+1$ consumption of good X contingent on the realization of ζ^{t+1} at time $t+1$. In equilibrium, the market for international state-contingent claims clears, implying that $A_t^h + A_t^f = 0, \forall t$. In our analysis, all assets are denominated in units of the numeraire good. We regard the extension to a setup in which the currency of denomination of international assets matters as an important direction for future research (see Maggiori, Neiman, and Schreger (2020) and Du, Pflueger, and Schreger (2020)).

Prices. The stochastic discount factor in consumption aggregate units is

$$
M_{t+1}^i = \delta \left(\frac{C_{t+1}^i}{C_t^i}\right)^{-\frac{1}{\psi}} \left(\frac{U_{t+1}^{i1-\gamma}}{E_t\left[U_{t+1}^{i1-\gamma}\right]}\right)^{\frac{1/\psi-\gamma}{1-\gamma}}. \tag{3.9}
$$

Since markets are assumed to be complete, the log growth rate of the real exchange rate is

$$
\Delta e_t = \log M_t^f - \log M_t^h \tag{3.10}
$$

and the relative price of the two goods is $p_t = \frac{(1-\alpha)x_t^h}{\alpha y_t^h}$.

Allocations. Under complete markets, we can compute efficient allocations by solving the associated Pareto problem. The planner attaches date 0 nonnegative Pareto weights

$\mu^h = \mu$ and $\mu^f = 1 - \mu$ to the consumers and chooses the sequence of allocations $\left\{x_t^h, x_t^f, y_t^h, y_t^f\right\}_{t=0}^{+\infty}$ to maximize

$$\Lambda = \mu \cdot U_0^h + (1 - \mu) \cdot U_0^f,$$

subject to the following sequence of economy-wide feasibility constraints:

$$\begin{aligned} x_t^h + x_t^f &= X_t \\ y_t^h + y_t^f &= Y_t, \quad \forall t \geq 0, \end{aligned}$$

where the state-dependent notation is omitted for the sake of clarity. In characterizing the equilibrium, we follow Anderson (2005) and formulate the problem using the ratio of time-varying pseudo-Pareto weights, $S_t = \mu_t/(1 - \mu_t)$, as an additional state variable. This technique enables us to take into account the nonseparability of the utility functions.

The first-order necessary conditions imply the following allocations:

$$\begin{aligned} x_t^h = \alpha X_t \left[1 + \frac{(1-\alpha)(S_t - 1)}{1 - \alpha + \alpha S_t}\right], \qquad x_t^f = (1-\alpha) X_t \left[1 - \frac{\alpha(S_t - 1)}{1 - \alpha + \alpha S_t}\right] \\ y_t^h = (1-\alpha) Y_t \left[1 + \frac{\alpha(S_t - 1)}{\alpha + (1-\alpha) S_t}\right], \qquad y_t^f = \alpha Y_t \left[1 - \frac{(1-\alpha)(S_t - 1)}{\alpha + (1-\alpha) S_t}\right], \end{aligned} \tag{3.11}$$

where

$$S_t = S_{t-1} \cdot \frac{M_t^h}{M_t^f} \cdot \left(\frac{C_t^h / C_{t-1}^h}{C_t^f / C_{t-1}^f}\right), \quad \forall t \geq 1 \tag{3.12}$$

and $S_0 = 1$, as we start the economy from an identical allocation of wealth and endowments. This is consistent with the ergodic distribution of the model, which implies that on average the two countries consume an identical share of world resources because of symmetry.

We make three remarks. First, S_t is a key driver of the share of world consumption allocated to the home country, SWC_t,

$$SWC_t = \frac{x_t^h + p_t y_t^h}{X_t + p_t Y_t} = \frac{S_t}{1 + S_t}. \tag{3.13}$$

The higher S_t is, the larger is the home country. Second, as in Colacito and Croce (2013), when the home country receives good news for the endowment of good X, there is a persistent reduction in the domestic share of world consumption. This counter-cyclical adjustment is consistent with equation (3.12): as good news for the supply of good X relative to good Y materializes, the home country experiences a drop in its marginal utility. Therefore, it is optimal to reallocate resources to the foreign country. In the decentralized economy, the home country optimally substitutes part of its current consumption with exports to its foreign trading partner. Third, S_t introduces an endogenous time-varying volatility term into consumption growth, since allocations are nonlinear functions of this component. In section 4.2, we discuss the importance of this channel in the context of our explanation of the volatility disconnect anomaly.

3.1 Calibration and Solution Method

We calibrate the model as in Colacito *et al.* (2021). In what follows, we report our benchmark calibration in table 4 and provide a description of our calibration strategy in order to make our study self-contained.

We set the intertemporal elasticity of substitution to 1.5, as in Colacito and Croce (2013). Because of the presence of volatility risk, we can obtain a volatile stochastic discount factor with a risk aversion coefficient of 7, a value particularly conservative in this literature. The subjective discount factor is chosen so as to keep the average annual risk-free rate close to 1% when possible.

The consumption home bias is set to 0.96, a number that falls in the middle of the range observed for our countries. For example, in our sample the US home bias is 0.95,

Table 4: Calibration

Description	Parameter	Value
Panel A: Standard Parameters		
Relative Risk Aversion	γ	7
Intertemporal Elasticity of Substitution	ψ	1.50
Subjective Discount Factor	δ^4	0.9825
Degree of Home Bias	α	0.96
Mean of Endowment Growth	$\mu \cdot 4$	2.00%
Short-Run Risk Volatility	$\sigma \cdot \sqrt{4}$	1.87%
Long-Run Risk Autocorrelation	ρ^4	0.953
Relative Long-Run Risk Volatility	σ_z/σ	6.90%
Cross-correlation of Short-Run Shocks	ρ_X	0.15
Cross-correlation of Long-Run Shocks	ρ_z	0.90
Panel B: Time-Varying Short-Run Risk		
Persistence of Short-Run Volatility	ρ_σ	0.90
		[0.85; 0.95]
Volatility of Short-Run Volatility	σ_{sr}	0.15
		[0.12; 0.15]
Cross-correlation of Short-Run Volatility	ρ_{σ,σ^*}	0.30
		[0.23; 0.51]
Short-Run Volatility Correlation with	$\rho_{\sigma,\Delta y}$	-0.10
Short-Run Shocks		[-0.19; -0.01]

Notes: All parameters are calibrated at quarterly frequency. In panel B, the entries for the data are from the VAR specified in Colacito *et al.* (2021). Numbers in brackets denote the 95% credible intervals. Data are from the OECD dataset and refer to G-17 countries. The sample spans the post–Bretton Wood period, 1971:q1–2013:q4.

as imports comprise an average of 5% of US consumption goods (Erceg, Guerrieri, and Gust (2008)). Balta and Delgado (2009) document a stronger consumption home bias for the European countries in our dataset and suggest a value of $\alpha = 0.97$. Setting $\lambda = 0.97$ would improve our quantitative results, as it would make our risk-sharing channel even more relevant. We prefer to work with $\alpha = 0.96$ in order to obtain conservative results.

Annualized average output growth is set to 2%, consistent with the empirical findings in table 1. Unconditional volatilities are calibrated to produce an unconditional output volatility of 1.90%, as in the data. The long-run components are calibrated in the spirit of the international long-run risk literature, as they are both highly persistent and correlated across countries (Colacito and Croce (2013)). Since we set $\sigma_z/\sigma = 0.07\%$, the implied consumption growth rate is almost *i.i.d.*, as in the data. Short-run output growth shocks, in contrast, are as poorly cross-country correlated as output growth in our dataset (see table 1).

In table 4, panel B, we report the parameters that govern the volatility process of short-run shocks, that is, the novel and most important element of our investigation. These parameters are calibrated to be consistent with the empirical evidence in Colacito *et al.* (2021).

Consistent with our data, volatility shocks are as poorly correlated across countries as short-run growth shocks. We allow for negative within-country correlation between volatility and short-run growth shocks so that higher volatility is associated with economic slowdowns. Conditional volatilities are as persistent as in the data.

Given these parameters, we use perturbation methods to solve our system of equations. We compute an approximation of the third order of our policy functions using the dynare++ package. As documented in Colacito and Croce (2013), a third-order approximation is required to capture endogenous time-varying volatility due to the adjustments of the pseudo-Pareto weights. All variables included in our dynare++ code are expressed in log-units.

Both the calibration and the solution methods are standard in the literature. In what follows we discuss only the performance of our model for the dynamics of conditional volatilities, that is, the main objective of our investigation. For commonly targeted unconditional moments, we refer the reader to table B1 in the appendix. For the sake of completeness, this table also shows the same moments for the case in which we abstract away from volatility shocks, and for the setting with CRRA preferences.

4 Main Results

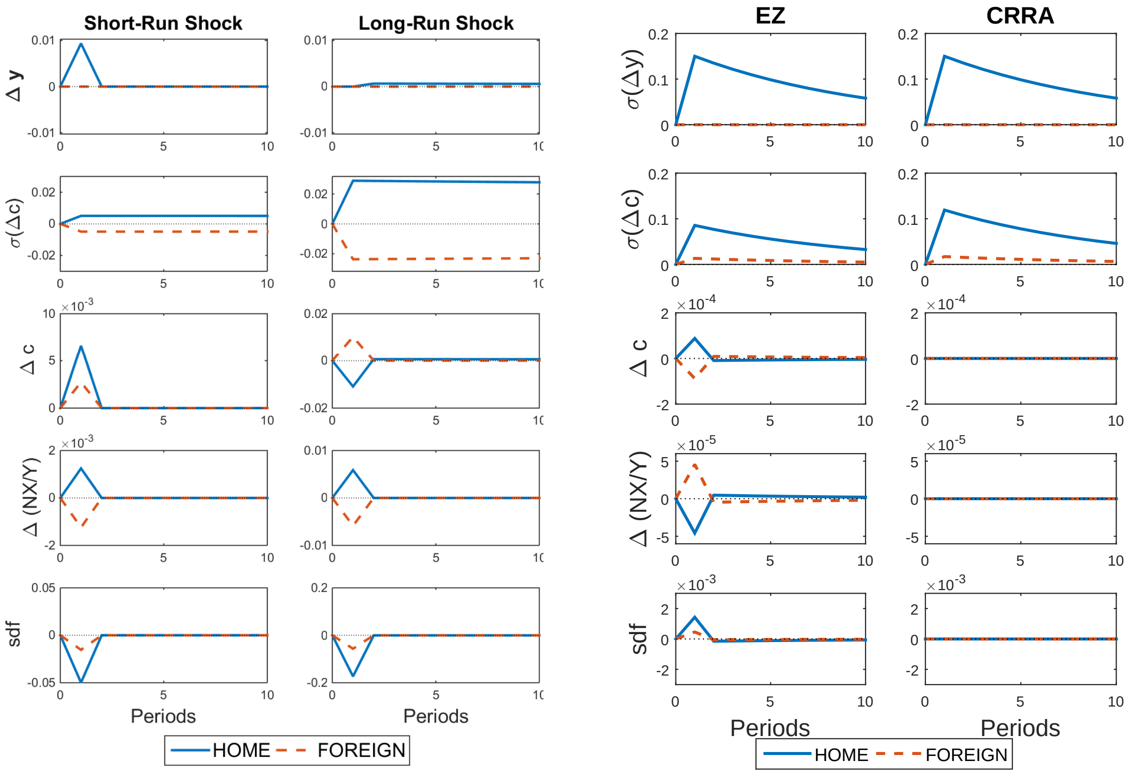
In this section, we present the main results of our theoretical analysis. We start by describing the risk-sharing motives of both level and volatility shocks. To our knowledge, we are the first to connect recursive risk sharing to the empirical volatility (dis)connect. We then assess the quantitative performance of our model by means of simulations and show that a frictionless recursive risk-sharing scheme can rationalize our empirical findings.

4.1 Risk-Sharing Motives

Risk sharing of level shocks. In figure 3(a), we report the response of the variables of interest to a short-run level shock (left panels) and to a long-run level shock (right panels) to the growth rate of the endowment of the home country. Note that on impact the short-run shock is sizeably larger than the long-run shock (figure 3(a), first row of panels). However, the long-run shock is highly persistent, and it ultimately affects the growth rate of the home endowment for a large number of periods.

Consistent with Colacito and Croce (2013), the growth rates of consumption increase in both countries in response to a positive short-run shock, whereas they move in opposite directions in response to a positive long-run shock (figure 3(a), third row of panels). The asymmetric response of consumption growth rates to a long-run endowment shock is the result of the agents' extreme sensitivity to persistent news to the growth rates of their endowments.

When a shock of this nature materializes, the home country's marginal utility drops substantially (figure 3(a), bottom-right panel). To restore the equality of the marginal utilities of consumption across countries, an international redistribution of resources must take place. Specifically, the home country increases its exports, while the foreign country increases its imports (figure 3(a), fourth row of panels). Equivalently, the ratio



(a) Level Shocks (EZ only)

(b) Vol Shock (EZ vs CRRA)

Figure 3 - **Impulse Responses.** Panel (a) shows the percentage impulse response functions of output growth (Δy), consumption growth volatility ($\sigma(\Delta c)$), consumption growth (Δc), change of net-export–output ratio ($\Delta NX/Y$), and stochastic discount factors (sdf) to a shock to the home endowment for both the home country (solid line) and the foreign country (dashed line). Level shocks materialize only in the home country, and only at time 1. Shocks are not orthogonalized; we consider a positive σ shock in the short-run, and a positive σ_x shock for the long-run. In panel (b) we consider an endowment volatility shock which is orthogonalized within and across countries, i.e., it affects only the home country and it does not change the growth rate level. All parameters are calibrated to the quarterly values reported in Table 4.

of the pseudo-Pareto weights S_t declines, as dictated by equation (3.12).

Since the long-run shock is a pure news shock, that is, a shock that results in a larger amount of home endowment only in future time periods, the international redistribution of resources takes place through a drop in home consumption and an increase in foreign consumption. As pointed out in Colacito and Croce (2013), this immediate response of the consumption level simultaneously comes with an opposite swap of long-run consumption variance (as measured by $\sigma_t(U_{t+1})$). Specifically, the home country optimally reduces its current consumption share, $S_t/(1-S_t)$, in exchange for a reduction in $\sigma_t(U_{t+1})$. Consistent with equation (3.3), the reduction of long-term uncertainty improves welfare.

Risk sharing of vol shocks. Figure 3(b) shows the response of our main set of variables of interest to a volatility shock in the home country. For comparability, we report the responses from both our benchmark model and a model with standard time-additive CRRA preferences.

We first point out that qualitatively, the responses of consumption, net exports, and stochastic discount factors in the model with EZ preferences are the mirror image of those obtained for a positive long-run endowment shock, since a positive volatility shock is a negative news shock.

Second, we note that the relative response of the volatilities of consumption growth rates in the two countries differs across the two preference specifications. With CRRA preferences, volatility news shocks are not directly priced and hence marginal utilities do not move. There is no reallocation of resources across countries, and as a result the increase in volatility of the domestic endowment is almost entirely absorbed by domestic consumption.

Without volatility shocks (Colacito and Croce, 2013), the endogenous response of volatilities to long-run shocks dominates and results in a counterfactual negative correlation between exchange rate and consumption differential conditional volatility. Our recursive risk-sharing of volatility shocks overcomes this problem.

4.2 Risk Sharing and the Volatility Disconnect Anomaly

In table 5, we compare our empirical findings on the disconnect between exchange rates and consumption differentials to our simulation results. In the top panel, we show that

Table 5: Foreign Exchange Disconnect and Risk Sharing

	Data	Model			
	Aver.	Bench-mark	No TVV ($\sigma_\sigma = 0$)	No LRR ($\sigma_z = 0$)	SRR only ($\sigma_\sigma = \sigma_z = 0$)
Levels Disconnect					
$Corr(\Delta\tilde{g}_t, \Delta e_t)$	0.04 [-0.04; 0.12]	-0.37	-0.39	1.00	1.00
Volatility Disconnect					
$Corr(\sigma_t(\Delta\tilde{g}_t), \sigma_t(\Delta e_t))$	0.21 [-0.05; 0.50]	0.34	-0.80	1.00	1.00

Notes: This table reports key moments for real consumption growth differentials ($\Delta\tilde{g} = \Delta c - \Delta c^*$) and exchange rate growth (Δe). Conditional log-volatilities are denoted by σ_t. The empirical moments are obtained by estimating equation (2.1) country by country, as detailed in section 2.2. The data refer to G-17 countries and are described in section 2.1. For each country, we compute the moments of interest over the post–Bretton Wood period, 1971:Q1–2013:Q4, as detailed in section 2.1. For each moment, we report (i) its GDP-weighted average across countries; and (ii) its first and fourth cross-country quintiles in squared brackets The entries from the model are obtained from 100 repetitions of small samples. Our benchmark quarterly calibration is reported in table 4.

our benchmark model delivers a lack of perfect positive correlation between consumption growth differentials and exchange rate in the data. As in the model with constant volatility (Colacito and Croce (2013)), news shocks are sufficient to break the perfect correlation of the consumption differentials and the exchange rate. Consistent with the observation in Colacito and Croce (2013), in a model with short-run risk only the optimal allocations are very similar across EZ and CRRA preferences. Hence the right-most column in table 5 can also be interpreted as capturing the case of CRRA preferences. Not surprisingly, in this setting the Backus and Smith (1993) anomaly is back.

The model with the only short-run shocks also delivers a perfect positive correlation between the conditional variances of consumption differentials and the exchange rate (bottom portion of table 5, rightmost column). Interestingly, this correlation switches to large and negative in the recursive utility model without time-varying volatilities

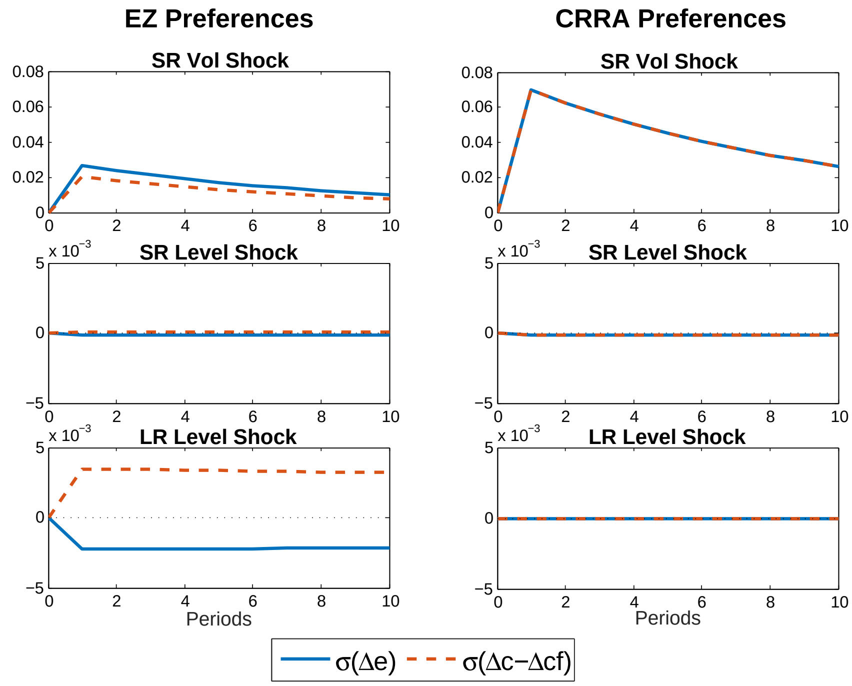


Figure 4 - **Impulse Response Functions and Volatility Disconnect.** This figure shows the percentage response of the volatility of consumption growth differentials (dashed line) and exchange rate growth rate volatility (thick line) to a volatility shock in the home country (top panels), a short-run shock in the home country (middle panels), and a long-run shock in the home country (bottom panels). The left (right) panels report the response functions for our benchmark model with EZ (CRRA) preferences.

(our 'No TVV' case), which is the model analyzed by Colacito and Croce (2013). The predictions of both of these restricted models are at odds with the data: the empirical estimates suggest a positive but weak correlation of about 20-30%. Our full model, on the other hand, delivers a positive and mild correlation of about 34%, which is close to the data. These findings highlight the role of the recursive utility and output volatility shocks to resolve the volatility disconnect anomaly. To explain the economic mechanisms behind the results, we consider separate impact of volatility and level shocks on the conditional variances of consumption differentials and the exchange rate. These responses are depicted in figure 4.

Volatility shock. A volatility shock in the home country produces a *positive* comovement between the volatility of the exchange rate and that of the differential of consumption growth rates. This is because the two countries share the risk associated with an increase in macroeconomic uncertainty, as explained in the previous section. Hence, in the absence of level shocks, we would have a perfect connection between exchange rate and consumption differential volatility. This is true both in the recursive utility and the CRRA model.

Short-run shocks. We note that short-run shocks are irrelevant in this context, as they result in a negligible response of the two volatilities, since investors' marginal utilities are not particularly sensitive to this type of shock (figure 4, middle-left panel). Hence it is not surprising that in the last column of table 5 the model featuring only short-run shocks implies a counterfactually perfect connection between exchange rate and consumption differentials volatility.

Long-run shocks. In contrast to short-run shocks, in a recursive-utility environment a long-run shock to the home country generates a significant *negative* comovement between the two volatilities and lowers their unconditional correlation (figure 4, bottom-left panel). In a model with CRRA preferences, long-run shocks have no impact on the two conditional volatilities. Hence, all the effect is driven by volatility shocks, which leads to a perfect positive correlation between the conditional volatilities of consumption differential and the exchange rates.

To explain the origin of this negative comovement, it is useful to decompose the variance of the consumption differential growth rate into its subcomponents:

$$\begin{aligned} Var_t(\Delta c_{t+1} - \Delta c^*_{t+1}) &= Var_t(\Delta c_{t+1}) + Var_t(\Delta c^*_{t+1}) \\ &\quad -2 \cdot \sqrt{Var_t(\Delta c_{t+1}) \cdot Var_t(\Delta c^*_{t+1})} \cdot corr_t(\Delta c_{t+1}, \Delta c_{t+1}). \end{aligned} \tag{4.1}$$

At the equilibrium, the conditional correlation of consumption growth rates is almost time invariant.[5] As a result, the dynamics of the variance of consumption differentials is mostly determined by the sum of the variances of the consumption growth rates across countries, as depicted in the left panel of Figure 5. Because of the convexity of the short-run volatility frontier, the sum of the variances of the growth rates of consumption is increasing in wealth inequality, that is, it is U-shaped with respect to the log-ratio of the Pareto weights (figure 5, left panel). As a result, starting from an equal distribution of wealth, $\sigma_t(\Delta c_{t+1} - \Delta c^*_{t+1})$ increases upon the arrival of a long-run shock (figure 4, bottom-left panel).

Given our assumption of complete markets, the variance of the exchange rate growth can be decomposed as follows:

$$\begin{aligned} Var_t\left(\Delta e_{t+1}\right) &= Var_t(\Delta m_{t+1} - \Delta m^*_{t+1}) = Var_t(\Delta m_{t+1}) + Var_t(\Delta m^*_{t+1}) \\ &\quad -2\sqrt{Var_t(\Delta m_{t+1}) \cdot Var_t(\Delta m^*_{t+1})} \cdot corr_t(\Delta m_{t+1}, \Delta m_{t+1}). \end{aligned}$$

In a model with long-run growth news, most of the volatility of the stochastic discount rates is driven by the continuation utilities. Colacito *et al.* (2021) show that the utility variance frontier is linear, meaning that the drop in the conditional volatility of the utility of one country is almost entirely offset by the increase in volatility of the other country. As a result, $Var_t(\Delta m_{t+1}) + Var_t(\Delta m^*_{t+1})$ is close to being time invariant and the conditional volatility of the exchange rate is mostly explained by the endogenous time variation in the correlation of the stochastic discount factors (figure 5, right panel).

With recursive preferences, the reallocation prompted by long-run shocks keeps the continuation utilities of the two agents aligned to each other, that is, it introduces a

[5] This correlation is driven by the positive comovement between the short-run shock of a country and the adjustment in the share of consumption of the other country. In equilibrium, this correlation increases modestly in wealth inequality.

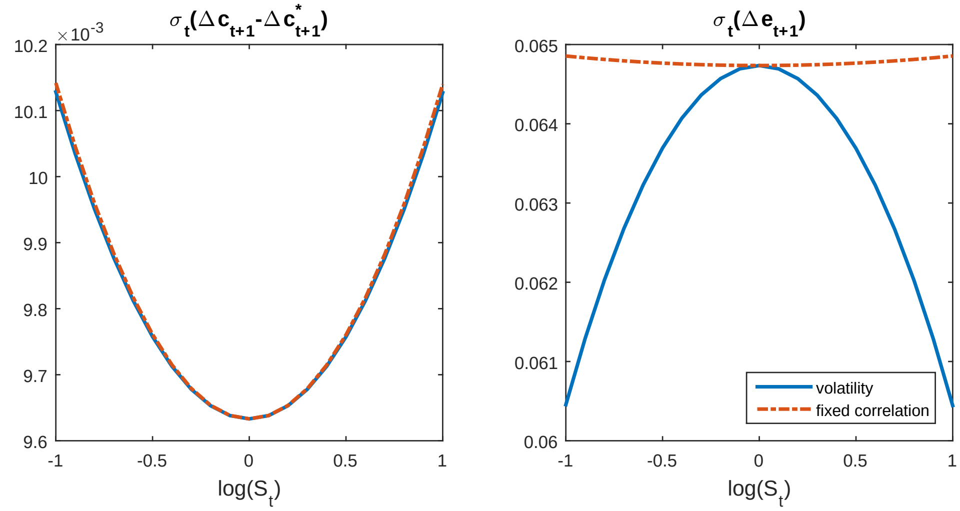


Figure 5 - **Conditional Volatilities Disconnect.** The left panel plots the conditional volatility of the difference between the growth rate of consumption in the home and foreign countries, $\sigma_t\left(\Delta c_{t+1} - \Delta c^*_{t+1}\right)$. The right panel depicts the conditional volatility of the growth rate of the exchange rate, $\sigma_t\left(\Delta e_{t+1}\right)$. Both volatilities are plotted against the logarithm of the ratio of the pseudo-Pareto weights, S_t. Across all cases, both the exogenous long-run components and the exogenous volatility processes are fixed at their unconditional mean. In each panel, the solid line refers to the conditional volatility obtained at the equilibrium, whereas the dashed line refers to the conditional volatility obtained by holding the correlations fixed at their unconditional mean in equations (4.1)–(4.2).

positive cross-country comovement of continuation utilities and hence stochastic discount factors.[6] Because our utility function satisfies the Inada's conditions, the strength of the reallocation channel is enhanced when one of the two countries is small. Equivalently, the correlation of the stochastic discount factors increases with wealth inequality. As a result, the exchange rate volatility has an inverse U-shape with respect to the log-ratio of the Pareto weights (see the right panel of figure 5). Thus starting from an equal distribution of wealth, the impulse response of the exchange rate volatility is negative, in sharp contrast to the response of the volatility of the consumption differentials.

[6]When a country receives good news for the long run, its utility increases immediately, reflecting the total discounted impact of the news. The other country benefits from the international redistribution of resources, which determines an increase in its share of consumption. Given the persistent nature of the consumption shares, the other country also experiences an increase in the present value of its consumption and, thus, its utility. As a consequence, the extent of comovement of the continuation utilities (and of the stochastic discount factors in general) increases.

4.3 Matching the cross section

Given the way in which we have constructed Figure 2, we analyze the cross-sectional implications of our model by solving it using different values for short-run risk, long-run risk and volatility risk. This is equivalent to running a comparative statics analysis with respect to the parameters σ, σ_z and σ_{sr}, respectively. The cross-section of country size is obtained by initializing the time zero ratio of pseudo Pareto weights (S) to different values. We compare our empirical regressions with the theoretical ones obtained from our model and depict them in figure 4.3.

Consistent with our recursive risk-sharing scheme, countries featuring more expected growth risk tend to have a stronger disconnect between their exchange rate conditional vol and the vol of their consumption differentials, meaning that $\text{corr}(\sigma_t(\Delta e), \sigma_t(\Delta \tilde{g}))$ declines. In contrast, countries facing more fundamental volatility risk tend to have a smaller disconnect, meaning that $\text{corr}(\sigma_t(\Delta e), \sigma_t(\Delta \tilde{g}))$ increases.

As in the data, both short-run risk, σ, and our endogenous country size variable, S_t, play no relevant role. Equivalently, even though size is an important endogenous determinant of both exchange rate volatility and consumption differentials volatility, it is irrelevant for their unconditional correlation. In short, size affects conditional second moments to a similar extent and hence it does not alter their unconditional correlation.

5 Conclusion

In this paper, we provide novel empirical evidence regarding the disconnect between the volatility of consumption differentials and the volatility of exchange rates. We show that these findings constitute a puzzle from the standpoint of both a frictionless model with CRRA preferences and a model with recursive preferences as in Colacito and Croce (2013). We then develop a frictionless general equilibrium model featuring long-run growth news shocks, volatility shocks, and two countries populated by agents with recur-

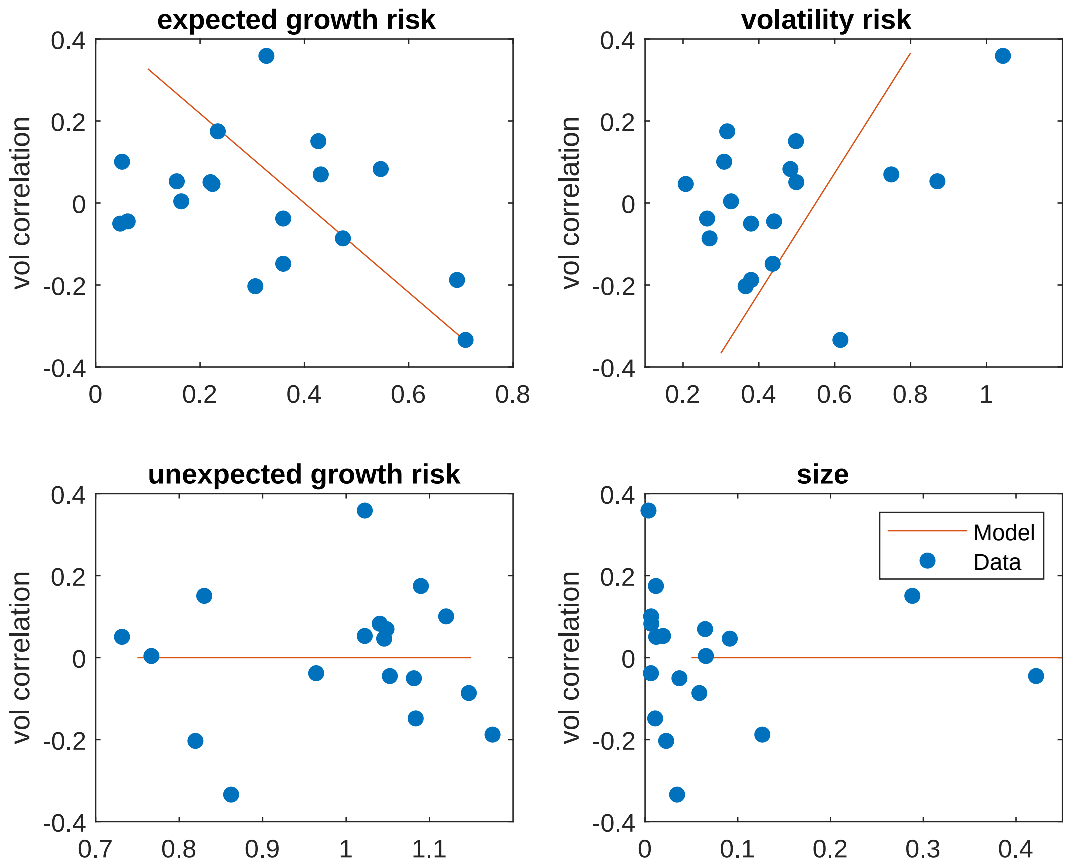


Figure 6 - **Economic Determinants of Volatility Disconnect.** This figure shows scatterplots of the average volatility correlations and each of the following four country characteristics: the long-run expected growth risk; the volatility risk; the short-run unexpected growth risk; and the relative size of the country. The model fitted line is from model simulations with different parameter values of the expected growth risk (σ_z), the volatility risk (σ_{sr}), or the unexpected growth risk (σ), respectively. The cross-section of country size is obtained by initializing the time zero ratio of pseudo Pareto weights (S) to different values. The correlations are express in terms of deviation from the median country.

sive preferences and demonstrate that our model can replicate these empirical findings.

Furthermore, the model reproduces key features of the cross section of countries. Specifically, we show that countries with more (less) volatility (long-run risk) risk feature a smaller disconnect between the conditional vol of their exchange rate and that of their consumption differentials.

Future developments should focus on extending this setting to international real business cycle models in an effort to better understand the role of international investment flows and international frictions in the origination and international propagation of volatility shocks. Trading frictions, portfolio composition, and market incompleteness are other promising avenues for future research.

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Appendix A. Volatility Estimation

We use an auxiliary mixture sampler to estimate the model specified in (2.1) and extract latent volatility components, following Kim, Shephard, and Chib (1998). Specifically, we rewrite the observation equation,

$$log((z_t - \mu - \rho z_{t-1})^2) = \sigma_t + log(\eta_t^2). \tag{A1}$$

The distribution of $log(\eta_t^2)$ can be well approximated by a mixture of Gaussian distributions:

$$p(log(\eta_t^2)) = \Sigma_{i=1}^n \pi_i \varphi(\eta_t; \mu_{,\eta i}, \sigma_{\eta,i}^2), \tag{A2}$$

where φ is the probability density function of a Gaussian distribution with mean $\mu_{\eta,i}$ and standard deviation $\sigma_{\eta,i}$. In the Markov Chain Monte Carlo procedure, $s_t \in [1,\ T]$ is drawn to indicate one Gaussian distribution to sample $log(\eta_t^2)$. Conditioning on s_t, the model is in Gaussian linear state-space form, and a standard forward-filtering, backward-sampling scheme can be applied. The algorithm thus takes the form:

1. Initialize $\mu,\ \rho,\ \mu_\sigma,\ \nu,\ \sigma_\omega,\ s_t$
2. Sample σ_t from $p(\sigma_t|z, \mu,\ \rho,\ \mu_\sigma,\ \nu,\ \sigma_\omega,\ s_t, z)$
3. Sample s_t from $p(s_t = i) \propto \pi_i \varphi(log((z_t - \mu - \rho z_{t-1})^2); \sigma_t + \mu_{\eta,i}, \sigma_{\eta,i}^2)$
4. Sample $\mu,\ \rho,\ \mu_\sigma,\ \nu,\ \sigma_\omega$ from $p(\mu,\ \rho,\ \mu_\sigma,\ \nu,\ \sigma_\omega|\sigma_t,\ z)$
5. Repeat 2–4 until convergence

In our empirical implementation the priors are very loose: $\mu \sim N(0, 100)$, $\rho \sim N(0, 10^2)$, $\mu_\sigma \sim N(-10, 10^2), \nu \sim N(0.9, 0.1^2)$, and $\sigma_\omega \sim IG(2, 0.3)$. We sample 20,000 times and discard the first 5,000. The posterior mean of σ_t is the volatility used in the empirical analysis.

Appendix B. Standard Moments from the Model

In table B1, we focus on unconditional moments typically targeted in the international finance literature. Our benchmark calibration conforms well with our data, both with and without volatility shocks. The adoption of CRRA preferences generates well-known puzzles: (i) the market price of risk is excessively low; (ii) the risk-free rate is too high; and (iii) international trade is modest. In our model the net exports are not as volatile as in our G17 dataset, but they are twice as volatile compared to the CRRA case.

Table B1: Standard Unconditional Moments

	G-17 Data		Model		
	Avg.	Quintiles [1^{st}; 4^{th}]	Bench-mark	No TVV ($\sigma_\sigma = 0$)	CRRA ($\gamma = 7$)
$corr(\Delta c, \Delta c^*)$	0.25	[0.13; 0.33]	0.29	0.28	0.74
$\sigma(\Delta c)(\%)$	1.67	[1.34; 2.47]	1.92	1.89	1.64
$\sigma(\Delta c)/\sigma(\Delta y)$	0.88	[0.57; 0.82]	0.97	0.97	0.83
$ACF1(\Delta c)$	0.17	[-0.16; 0.31]	0.06	0.06	0.08
$\sigma(M)/E(M)(\%)$	–	–	47.75	47.74	11.48
$\sigma(\Delta e)(\%)$	10.50	[10.2; 11.4]	13.82	13.67	8.30
$E(r^f)(\%)$	1.35	[1.44; 2.41]	2.18	2.20	14.89
$\sigma(r^f)(\%)$	1.79	[1.61; 2.27]	0.33	0.33	3.46
$corr(r^f, r^{f*})$	0.51	[0.37; 0.56]	0.89	0.89	0.98
$\sigma(\Delta(NX/Y))/\sigma(\Delta y)$	0.70	[0.67; 0.97]	0.36	0.37	0.16

Notes: This table reports key moments for real consumption (C), output (Y), the exchange rate (E), the risk-free rates (R^f), the net-export-to-output ratio (NX/Y), and the stochastic discount factor (M). Small letters refer to log-units; changes are denoted by 'Δ'; foreign variables are marked by '*'. We denote expectation, standard deviation, correlation, and first order auto-correlation by E, σ, $corr$, and $ACF1$, respectively. The data refer to G-17 countries and are described in section 2.1. For each country, we compute the moments of interest over the post–Bretton Wood period, 1971:Q1–2013:Q4, as detailed in section 2.1. For each moment, we report (i) its GDP-weighted average across countries; and (ii) its first and fourth cross-country quintiles. The entries from the model are obtained from 100 repetitions of small samples. Our benchmark quarterly calibration is reported in table 4.