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Abstract

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JEL Classification: E60, E61, E62

Keywords: Capital income tax, free trade, value-added taxes, border adjustment, origin- and destination-based taxation, production efficiency

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Optimal Cooperative Taxation in the Global Economy*

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ABSTRACT

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I Introduction

In the face of increased globalization, the design of the international tax system is a pressing concern for policy makers. The European Union, for example, has conducted extensive discussions on the feasibility and desirability of tax harmonization and has also discussed the possibility of forming a fiscal union. Awareness of how fiscal policy can be used to mimic tariffs is increasing. Recent discussion on the tax reform package in the United States – see, for example, Auerbach et al. (2017) – focused on the extent to which border adjustments were being proposed to mimic tariffs.

In this paper, we develop a dynamic international trade model and use it to ask how countries should cooperate on fiscal and trade policy when government expenditures must be financed with distorting taxes. We show that production efficiency is optimal so that goods, services, and capital should effectively flow freely across borders. We argue that residence-based income tax systems have advantages over source-based systems and that value-added taxes should be adjusted at the border. We show that the choice of tax system determines whether taxes on domestic activities can act as tariffs on international trade and thereby undermine international agreements on free trade. Thus, if trade agreements are to be effective, they must be supplemented by agreements on fiscal policy. We argue that integrating dynamic public finance with dynamic international trade provides insights that are often not obvious in static formulations.

Our dynamic trade model is based on Backus, Kehoe, and Kydland (1994), though as we argue, the results hold in a variety of other trade models. We use both the Ramsey and the Mirrlees approaches to optimal taxation. In the Ramsey approach, the tax system is exogenously given, while in the Mirrlees approach, the tax system is restricted only by informational constraints. In both cases, we study cooperative equilibria.

The bulk of our analysis is conducted using the Ramsey approach. We begin with a benchmark system that taxes consumption, labor income, and international trade and does not allow for direct transfers across governments. We show that every point on the Pareto frontier has production efficiency as long as countries are connected by direct or indirect trade links. We show that any such point can be implemented by setting trade taxes so that import tariffs are exactly offset by export subsidies and appropriately setting consumption and labor income taxes. We also show that wedges between marginal rates of substitution and marginal rates of transformation are not necessarily equated across countries, so that tax rates across countries need not be equal. In this sense, we show that tax harmonization is not necessarily optimal.

We extend the tax system to allow for lump-sum transfers across governments and show that the Ramsey outcomes can be implemented by setting trade taxes to zero. This result shows that the role of offsetting trade taxes is solely to redistribute resources across countries. We show that there is a point on the Pareto frontier where government-to-government transfers are zero. This result implies that if countries have chosen an allocation associated with this point, then even if they are prevented from making direct or indirect transfers to each other, no Pareto improvement is possible.

Adding to the benchmark system other widely used taxes, such as taxes on corporate income and returns to household assets, as well as value-added taxes, does not change the Pareto frontier. We also analyze alternative tax systems that exclude some of the taxes in our benchmark system. We do so because they are widely used, and because other taxes may be easier to administer.

We begin by considering a system that taxes only labor income, corporate income, and household asset income. It does not tax consumption or international trade but does allow for government-to-government transfers. We show that any point on the Pareto frontier with the benchmark system can be implemented by setting the corporate income tax to zero and appropriately choosing the other two taxes. We also show that non-zero corporate income taxation can act as a restriction on capital mobility. This result shows that free trade agreements need to be supplemented with agreements on fiscal policy.

We show that it is optimal to tax all types of inflation-adjusted household asset income – including interest, dividends and capital gains – at a common country-specific rate. We go on to show that tax systems that allow only labor and corporate income to be taxed cannot, in general, implement outcomes on the Pareto frontier. Since a uniform tax on household asset income is residence based, while a tax on corporate income is source based, our analysis implies that residence-based tax systems have advantages over source-based systems.

It is often argued that source-based systems – for example, those that use corporate income taxes – have administrative advantages. While we have not explicitly modelled administrative ease, our analysis shows that source-based taxes can be used to implement the Ramsey allocation. We exploit the idea that while the Ramsey allocation uniquely pins down wedges, it can be implemented with a variety of different tax systems. Consider, for example, corporate income taxes. We show that the Ramsey allocation can be implemented with such taxes as long as the tax base is changed to exclude investment expenses and the taxes are constant either over time or across countries. Other taxes will in general need to be adjusted appropriately. These insights help clarify issues in the design of optimal corporate income tax systems (see, for example, OECD 2007).

Value-added taxes are also widely used as part of tax systems in many countries. We examine the role of border adjustments in setting these taxes. The tax base of a value-added tax with border adjustment, referred to as *VAT with BA,* excludes revenues from exports and includes expenditures on imports. Since such a tax is equivalent to a consumption tax, a system that includes this tax, together with a tax on labor income, can implement any point on the Pareto frontier. The tax base of a value-added tax without border adjustment (*VAT without BA*) includes revenues from exports and excludes expenditures on imports. We show that a system that has a VAT without BA and a labor income tax introduces intertemporal trade wedges if, in the benchmark system, optimal consumption tax rates vary over time. Thus, in general, a system with a VAT without BA and a labor income tax cannot implement outcomes on the Pareto frontier. Furthermore, with this tax system, since VATs can effectively impose tariffs, trade agreements that are not supplemented with fiscal policy agreements can be ineffective. Taken together, these findings suggest that systems that allow for border adjustments are desirable.

These results shed light on apparent differences between the literature in public finance (see Auerbach et al. 2017) and that in international trade (see Grossman 1980, Feldstein and Krugman 1990, and Costinot and Werning 2018) on the desirability of border adjustments. The public finance literature has argued that border adjustments are desirable, while the international trade literature has argued that they are irrelevant. The international trade literature effectively considers uniform tax systems in the sense that the VAT tax rate is the same for all goods. We can think of our dynamic economy as a static economy with an infinite number of goods. If, in the benchmark system, optimal consumption tax rates are constant over time, then the associated VAT tax rate is the same for all goods so that, regardless of border adjustments, systems with VAT and labor income taxes can implement outcomes on the Pareto frontier. If, in the benchmark system, optimal consumption taxes vary over time, then the associated VAT rate is different for different goods, and the international trade results no longer apply. Our results help reconcile these differences and suggest that border adjustments are desirable in general. Barbiero et al. (2017) show that permanent changes in border adjustments are irrelevant if they are unanticipated, while they are not if anticipated. The difference between the two exercises is that the first change is uniform, while the second is not.

The analysis of border adjustments helps us compare destination-based with origin-based taxes on goods and services. A tax system is destination based if tax rates at the destination of use do not depend on the origin of production and is origin based if the tax rates do not depend on the destination of use. Value added taxes with border adjustment are destination based, and those without border adjustment are origin based. Thus, our results suggest that destination-based systems have advantages over origin-based systems.

Our result that the Ramsey equilibrium without transfers is production efficient differs from that in Keen and Wildasin (2004), who argue that such equilibria are, in general, production inefficient. The reason for this difference is that Keen and Wildasin impose restrictions on trade taxes. With their restrictions, it turns out that it may not be possible to simultaneously achieve production efficiency and the needed redistribution across countries. They require tariffs on each imported good to be the same, regardless of its origin of production. Likewise, export subsidies on a given good cannot depend on the destination of exports. In contrast, we explicitly allow trade taxes for each good to depend on the origin and the destination.

Restrictions on policies of the type in Keen and Wildasin are of very limited applied interest, since countries routinely impose different tariffs on the same physical good based on country of origin or destination. For example, groups of countries often enter into trade agreements with one another that impose different tariffs based on whether a given good is produced within or outside the group. In examining cooperative Ramsey equilibria, our view is that the set of allowable tax instruments should include widely used tax instruments. From this perspective, it seems unreasonable to exclude instruments that countries routinely use. In any event, even if we were to impose restrictions of the kind in Keen and Wildasin, it turns out that they would not be generically binding in our dynamic model.

In our discussion of alternative implementations, we have assumed that explicit transfers, rather than offsetting trade taxes, are used to redistribute resources across countries. We have done so because even though these are equivalent, our view is that transfers have advantages over offsetting trade taxes. This view is based on two considerations. First, adopting a policy of explicitly free trade for a wide variety of goods and services helps protect policy makers from lobbying by self-interested groups seeking to promote their own sectors. Olson (2009) has persuasively argued that countries often adopt tariffs that hurt the vast majority of their citizens because tariffs on individual sectors confer concentrated benefits on small groups and diffuse costs on large groups (see also Grossman and Helpman, 1994). The large group may have a free rider problem in overcoming the lobbying efforts of a particular small group. The large group may, however, be able to overcome the interest of many small groups by negotiating a free trade agreement that applies to all sectors. One example of a cooperative agreement that combines internal free trade with transfers is the European Union.

Second, we think of transfers as consisting of more than explicit monetary transfers. Countries agree to treaties on a variety of issues such as the environment, military cooperation, migrant flows, and the like. The kinds of agreements countries arrive at on these issues are linked to the kinds of agreements they arrive at on trade issues.

Our model can be extended to address issues of fiscal federalism. In this extension, countries are reinterpreted as states or provinces, and the public good provided by each country in our model is interpreted as a local public good. We can easily allow for national public goods that are provided by the federal government and for labor mobility. We conjecture that as long as taxes can depend on the origin and destination of mobile agents, production efficiency is still optimal.

We go on to argue that our results generalize to other international trade models such as Obstfeld and Rogoff (1995), Stockman and Tesar (1995), and Eaton and Kortum (2002). Finally, we show that our results extend to a Mirrlees-like environment in which the government can use nonlinear tax systems and the productivity of households is private information. We show that these outcomes also satisfy production efficiency so that free trade and unrestricted capital mobility are optimal.

For ease of exposition, we study a deterministic model. It is straightforward to extend the analysis to stochastic models in which productivity, government consumption, and other shocks generate fluctuations in the aggregates. All our results continue to hold in the stochastic model. In such models, optimal consumption tax rates will typically vary with the underlying state, even in the stochastic steady state. These fluctuations may be large if the underlying shocks are large. This observation strengthens the case for the desirability of household asset taxes over taxation of corporate income and for the desirability of border adjustments in VAT systems.

One purpose of this paper is to help integrate static trade theory into widely used dynamic macroeconomic models of public finance. Doing so provides insights that are often not obvious in static public finance formulations. We have clarified that while systems of taxation with and without border adjustment are equivalent in a class of static models, they are not equivalent in dynamic models. This clarification helps advance the discussion on the role of different ways of adjusting taxes at the border. Second, we have shown that dynamic models are useful in understanding the role that incentives play in the discussion of international asset income taxation. Third, the restrictions on policies that are binding in static models, as they are in Keen and Wildasin (2004), are not generically binding in dynamic extensions of those static models. Fourth, we can easily use the insights from the theory of repeated games - see our online Appendix F - to provide non-cooperative foundations for Pareto optimal outcomes. In this sense, cooperative Ramsey equilibria in a dynamic model are not merely benchmarks, but outcomes that can be attained even when countries cannot commit to cooperating. In contrast, in static models, cooperative outcomes typically cannot be attained in a non-cooperative setting.

II A Dynamic Trade Economy

The model is that in Backus, Kehoe, and Kydland (1994), with distorting taxes. There are N countries indexed by $i = 1, ..., N$. The preferences of a representative household in each country are over consumption c_{it}, labor n_{it}, and government consumption, g_{it},

$$(1) \quad U^i = \sum_{t=0}^{\infty} \beta^t \left[u^i \left(c_{it}, n_{it} \right) + h^i \left(g_{it} \right) \right].$$

We assume that u^i satisfies the usual properties and h^i is an increasing, concave, and differentiable function. We assume that the total endowment of time is normalized to be one. For much of what follows, we assume, without loss of generality, that government consumption is exogenously given.

Each country, i, produces a country-specific intermediate good, y_{it}, according to a production technology given by

$$(2) \quad \sum_{j=1}^{N} y_{ijt} = y_{it} = F^i \left(k_{it}, n_{it} \right),$$

where y_{ijt} denotes the quantity of intermediate goods produced in country i and used in country j, k_{it} is the capital stock, n_{it} is labor input, and F^i is constant returns to scale. Here the first subscript denotes the location of production, and the second subscript denotes the location of use. The intermediate goods produced by each country are used to produce a country-specific final good that can be used for private consumption, c_{it}; public consumption, g_{it}; and investment, x_{it}, according to

$$(3) \quad c_{it} + g_{it} + x_{it} \leq G^i \left(y_{1it}, ..., y_{Nit} \right),$$

where G^i is constant returns to scale. Capital accumulates according to the law of motion

$$(4) \quad x_{it} = k_{it+1} - (1 - \delta) k_{it},$$

so that the final goods resource constraint is

$$(5) \quad c_{it} + g_{it} + k_{it+1} - (1 - \delta) k_{it} \leq G^i \left(y_{1it}, ..., y_{Nit} \right).$$

Note that in this economy, only intermediate goods are traded across countries; final goods are not.

We use $G^i_{j,t}$ to denote the derivative of the intermediate goods production function of country i with respect to intermediate good of country j, in period t, and $u^i_{c,t}$ and $u^i_{n,t}$ to denote the marginal utilities of consumption and labor in period t. We then have that if lump-sum taxes as well as government to government transfers are available, the allocations on the Pareto frontier satisfy the following efficiency conditions:

$$-\frac{u^i_{ct}}{u^i_{nt}} = \frac{1}{G^i_{i,t}F^i_{nt}}, \tag{6}$$

$$\frac{u^i_{c,t}}{\beta u^i_{c,t+1}} = 1 - \delta + G^i_{i,t+1}F^i_{kt+1}; \tag{7}$$

and, for each pair of goods (j, l),

$$\frac{G^i_{j,t}}{G^i_{l,t}} \text{ is the same across countries } i; \tag{8}$$

and, for each good j,

$$\frac{G^i_{j,t}}{G^i_{j,t+1}}\left[G^i_{i,t+1}F^i_{k,t+1} + 1 - \delta\right] \text{ is the same across countries } i. \tag{9}$$

These conditions, together with the resource constraints, characterize the Pareto frontier.

These conditions mean that there are no intratemporal wedges (condition (6)), no intertemporal wedges (condition (9)), and no production distortions (conditions (7) and (8)). We say that an allocation is *statically production efficient* if it satisfies (8), *dynamically production efficient* if it satisfies (9), and simply *production efficient* if it satisfies both. Static production efficiency requires that the marginal rate of technical substitution for any pair of intermediate goods be equated across countries. Dynamic production efficiency requires that capital be allocated so as to equate the social rate of return on capital across the different countries.

Note, for future use, that the conditions above also imply the intertemporal consumption efficiency condition that, for all goods j,

$$(10) \quad \frac{u^i_{c,t}}{\beta u^i_{c,t+1}} \frac{G^i_{j,t}}{G^i_{j,t+1}} \text{ is the same across countries } i.$$

A. Equilibria with Consumption, Labor Income, and Trade Taxes

Consider now the economy with distorting taxes. Each government finances public consumption and initial debt with proportional taxes on consumption and labor income, τ^c_{it} and τ^n_{it}; trade taxes; and a tax on initial wealth, τ^W_i. The trade taxes consist of an export tax, τ^x_{ijt}, levied on exports shipped from country i to country j, and a tariff, τ^m_{ijt}, levied on imports shipped from country i to country j.

Firms

Each country has two representative firms. The *intermediate good firm* in each country uses the technology in (2) to produce the intermediate good using capital and labor, purchases investment goods, and accumulates capital according to (4). Let V_{i0} be the value of the firm in period zero after the dividend paid in that period, d_{i0}. The intermediate good firms maximize the value of dividends

$$(11) \quad V_{i0} + d_{i0} = \sum_{t=0}^{\infty} Q_t \left[p_{iit} y_{iit} + \sum_{j \neq i} \left(1 - \tau^x_{ijt}\right) p_{ijt} y_{ijt} - w_{it} n_{it} - q_{it} x_{it} \right],$$

subject to (2) and (4). Here, p_{ijt} is the price of the intermediate good produced in country i and sold in country j at t, w_{it} is the wage rate, and q_{it} is the price of the final good, all in units of a common world numeraire. The intertemporal price Q_t is the price of the numeraire at time t in units of the numeraire at zero ($Q_0 = 1$). Note that we assume that the intertemporal prices Q_t are the same across countries. This assumption captures the idea that world capital markets are fully integrated. Below, we explore policies that restrict international capital flows.

The *final goods firm* of country i chooses the quantities of intermediate goods to maximize the value of dividends:

$$\sum_{t=0}^{\infty} Q_t \left[q_{it} G^i \left(y_{iit}, y_{jit} \right) - p_{iit} y_{iit} - \sum_{j \neq i} \left(1 + \tau_{jit}^m \right) p_{jit} y_{jit} \right].$$

For future use, note that if we define r_{t+1}^f to be the return on one-period bonds in units of the numeraire between period t and $t+1$, then $Q_t / Q_{t+1} = 1 + r_{t+1}^f$ for $t \geq 0$.

Households

The household problem in country i is to maximize utility (1) subject to the budget constraint

$$\sum_{t=0}^{\infty} Q_t \left[\left(1 + \tau_{it}^c \right) q_{it} c_{it} - \left(1 - \tau_{it}^n \right) w_{it} n_{it} \right] \leq \left(1 - \tau_i^W \right) a_{i0}, \tag{12}$$

with

$$a_{i0} = V_{i0} + d_{i0} + Q_{-1} b_{i0} + \left(1 + r_0^f \right) f_{i0},$$

where a_{i0} denotes net holdings of assets by the household of country i; $Q_{-1} b_{i0}$ denotes holdings of domestic public debt in units of the numeraire, inclusive of interest; and $\left(1 + r_0^f \right) f_{i0}$ denotes holdings of claims on households in the other country, in units of the numeraire, also inclusive of interest. Without loss of generality, households within a country hold claims to the firms in that country as well as the public debt of its government. For most of our analysis, we assume that the initial net foreign asset position in real terms–namely, f_{i0}/q_{i0}– is fixed.

Governments

The budget constraint of the government of country i is given by

$$\begin{aligned} & \sum_{t=0}^{\infty} Q_t \left[\tau_{it}^c q_{it} c_{it} + \tau_{it}^n w_{it} n_{it} + \sum_{j \neq i} \tau_{jit}^m p_{jit} y_{jit} + \sum_{j \neq i} \tau_{ijt}^x p_{ijt} y_{ijt} - q_{it} g_{it} \right] \\ (13) \qquad = \; & Q_{-1} b_{i0} - \tau_i^W a_{i0}. \end{aligned}$$

Combining the budget constraints of the government and the household (with equality) in each country, we obtain the balance of payments condition:

$$(14)\quad \sum_{t=0}^{\infty} Q_t \sum_{j\neq i} \left[p_{ijt}y_{ijt} - p_{jit}y_{jit}\right] = -\left(1+r_0^f\right) f_{i,0},$$

for all i, and with $\sum_i \left(1+r_0^f\right) f_{i,0} = 0$ (see Appendix A for the derivation.[1])

A *competitive equilibrium* consists of allocations $\{c_{it}, n_{it}, y_{ijt}, k_{it+1}, x_{it}, g_{it}\}$; prices and initial dividends, $\{q_{it}, p_{ijt}, w_{it}, Q_t, V_{i0}, d_{i0}\}$; and policies $\left\{\tau_{it}^c, \tau_{it}^n, \tau_{ijt}^m, \tau_{ijt}^x\right\}$, τ_i^W, given k_{i0}, $Q_{-1}b_{i0}$, $\left(1+r_0^f\right) f_{i0}$, such that firms maximize value; households maximize utility subject to their budget constraints; the governments' budget constraints hold; the balance of payments conditions (14) hold; and markets clear in that (2), (3), and (4) hold.

We say that an allocation $\{c_{it}, n_{it}, y_{ijt}, k_{it+1}, x_{it}, g_{it}\}$ is *implementable* if it is part of a competitive equilibrium.

Next, we characterize the competitive equilibria. To do so, we rearrange the first-order conditions of households and firms to obtain (details are in Appendix A)

$$(15)\quad -\frac{u_{c,t}^i}{u_{n,t}^i} = \frac{(1+\tau_{it}^c)}{(1-\tau_{it}^n)\, G_{i,t}^i F_{n,t}^i},$$

$$(16)\quad \frac{u_{c,t}^i}{\beta u_{c,t+1}^i} = \frac{(1+\tau_{it}^c)}{\left(1+\tau_{it+1}^c\right)} \left[G_{i,t+1}^i F_{k,t+1}^i + 1 - \delta\right];$$

for each pair of goods (j, l),

$$(17)\quad \frac{\left(1-\tau_{jit}^x\right)(1+\tau_{lit}^m)\, G_{j,t}^i}{\left(1+\tau_{jit}^m\right)(1-\tau_{lit}^x)\, G_{l,t}^i} \text{ is the same for all } i;$$

and, for each good j,

$$(18)\quad \frac{\left(1+\tau_{jit+1}^m\right)\left(1-\tau_{jit}^x\right) G_{j,t}^i}{\left(1-\tau_{jit+1}^x\right)\left(1+\tau_{jit}^m\right) G_{j,t+1}^i} \left[G_{i,t+1}^i F_{k,t+1}^i + 1 - \delta\right] \text{ is the same for all } i.$$

Comparing these conditions with the ones for the Pareto frontier with lump-sum tax-

[1]For future reference, all of the appendices are online.

ation – (6), (7), (8), and (9) – we have that the consumption and labor taxes create an intratemporal wedge, as can be seen in (15), and that time-varying consumption taxes create intertemporal wedges, as can be seen in (16). The consumption and labor income taxes do not affect the production efficiency conditions (17) and (18). We say that the economy has *no trade wedges* if the trade taxes are set so that (17) coincides with (8) and (18) coincides with (9). One example of such an economy sets trade taxes so that $\tau_{jit}^m = -\tau_{jit}^x$. In our proposition below, we will construct trade taxes with this property. If the economy has trade wedges, then trade taxes distort production efficiency.

We can also use (16) and (18) to write the intertemporal consumption condition

$$\text{(19)} \quad \frac{\left(1+\tau_{jit+1}^m\right)\left(1-\tau_{jit}^x\right)}{\left(1-\tau_{jit+1}^x\right)\left(1+\tau_{jit}^m\right)} \frac{\left(1+\tau_{it+1}^c\right)}{\left(1+\tau_{it}^c\right)} \frac{G_{j,t}^i}{G_{j,t+1}^i} \frac{u_{c,t}^i}{\beta u_{c,t+1}^i} \text{ is the same for all } i,$$

which makes clear how time-varying ratios of consumption and trade taxes distort this intertemporal margin across households in different countries.

For future use, it is helpful to express the balance of payments condition in terms of the allocations and policies. Straightforward algebra (shown in Appendix A) yields that the balance of payments condition can be written as

$$\begin{aligned} & \sum_{t=0}^{\infty} \frac{1}{\Pi_{s=0}^t \left[G_{i,s}^i F_{k,s}^i + 1 - \delta\right]} \sum_{j \neq i} \left[\frac{G_{i,t}^i y_{ijt}}{\left(1-\tau_{ijt}^x\right)} - \frac{G_{j,t}^i y_{jit}}{\left(1+\tau_{jit}^m\right)} \right] \\ \text{(20)} \qquad & = -\left(1+r_0^f\right) \frac{f_{i0}}{q_{i0}}. \end{aligned}$$

Necessary conditions for implementability

In order to characterize the Ramsey equilibrium, we begin with a partial characterization of the set of implementable allocations for a given path of government consumption, $\{g_{it}\}$. A necessary condition for an allocation $\{c_{it}, n_{it}, y_{ijt}, k_{it+1}, x_{it}\}$ and period zero policies and prices, $\left\{\tau_i^W, \tau_{i0}^c, q_{i0}\right\}$, given $\{k_{i0}, b_{i0}, f_{i0}\}$, to be implementable as a competitive equilibrium is that they must satisfy the resource constraints (2), (3), (4), and the implementability conditions

$$(21) \quad \sum_{t=0}^{\infty} \left[\beta^t u^i_{c,t} c_{it} + \beta^t u^i_{n,t} n_{it}\right] = \mathcal{W}_{i0},$$

where

$$(22) \quad \mathcal{W}_{i0} = \frac{\left(1-\tau_i^W\right) u^i_{c,0}}{\left(1+\tau^c_{i0}\right)} \left[\left(1-\delta+G^i_{i,0} F^i_{k,0}\right) k_{i0} + Q_{-1} \frac{b_{i0}}{q_{i,0}} + \left(1+r^f_0\right) \frac{f_{i,0}}{q_{i,0}}\right].$$

The proof of the following proposition is standard and is omitted.

Proposition 1 (Necessary conditions for implementation): Any implementable allocation and period zero policies and prices must satisfy the implementability constraint for each country (21) and the resource constraints (2), (3), and (4).

This proposition is useful in developing the relaxed Ramsey problem described below. Note that we are not including other conditions of the competitive equilibrium; in particular, we omit the balance of payments condition, (20).

B. Cooperative Ramsey Equilibria

Here, we ask how fiscal policy and trade policy should be conducted when governments can cooperate in setting these policies. We show that the Ramsey allocations are production efficient as long as countries are connected through trade links. Such production efficiency in general requires tariffs on imports that are exactly offset by export subsidies.

We assume that households in each country must be allowed to keep an exogenous value of initial wealth $\bar{\mathcal{W}}_i$, measured in units of utility. Specifically, we impose the following restriction on the policies:

$$(23) \quad \mathcal{W}_{i0} \geq \bar{\mathcal{W}}_i,$$

which we refer to as the *wealth restriction*. With this restriction, policies, including initial policies, can be chosen arbitrarily, but the household must receive a value of initial wealth in utility terms of $\bar{\mathcal{W}}_i$. This restriction implicitly limits the extent of confiscation of initial wealth. In particular, it limits the tax rates on initial wealth τ_i^W. Chari, Nicolini, and Teles

(2020) offer a rationalization and a defense of restrictions of this kind in a closed economy (see also Armenter 2008 for an analysis in a closed economy with such a restriction).

Formally, a *(cooperative) Ramsey equilibrium* is a competitive equilibrium that is not Pareto dominated by any other competitive equilibrium. The *Ramsey allocation* is the associated allocation.

Ramsey problem

The Ramsey problem is to choose allocations, prices and policies to maximize a weighted sum of utilities of the households of the N countries,

$$(24) \quad \sum_{i=1}^{N} \omega^i U^i,$$

over the set of competitive equilibria satisfying the wealth restriction, (23).

Relaxed Ramsey problem

Next we state and prove that the Ramsey allocations satisfy production efficiency. To do so, it is convenient to consider a *relaxed Ramsey problem,* which consists of choosing allocations and period zero policies to maximize the planner's objective, (24), subject to the implementability constraints, (21); the initial wealth condition, (23); and the resource constraints (2), (3), and (4). Note that these conditions are the necessary conditions described in proposition 1.

We now show that the solution to the relaxed Ramsey problem can be implemented as a Ramsey equilibrium, as long as countries are connected in a way we make precise below. To do so, we will construct policies that, together with the allocations associated with the relaxed Ramsey problem, constitute a competitive equilibrium. In particular, we will show that the balance of payments condition (20) is satisfied. Choose period zero policies to satisfy the initial wealth condition, (23), and the tax rates on consumption and labor to satisfy (15) and (16). Prices are set to satisfy the firms' and households' first-order conditions, and the household budget constraints are satisfied because the implementability conditions are imposed. The government budget constraints are implied by the other equilibrium conditions.

Choose the trade taxes so that $\tau^x_{ijt} = -\tau^m_{ijt}$. With these offsetting trade taxes, there are no trade wedges, as required by the production efficiency conditions (8) and (9).[2] The balance of payments condition (20) becomes

$$(25)\quad \sum_{t=0}^{\infty} \frac{1}{\Pi_{s=0}^{t}\left[G^i_{i,s}F^i_{k,s} + 1 - \delta\right]} \sum_{j\neq i} \left[\frac{G^i_{i,t}y_{ijt}}{\left(1-\tau^x_{ijt}\right)} - \frac{G^i_{j,t}y_{jit}}{\left(1-\tau^x_{jit}\right)}\right] = -\left(1 + r^f_0\right)\frac{f_{i0}}{q_{i0}} \text{ for all } i.$$

To develop the assumptions needed to ensure that (25) is satisfied, we use some ideas from graph theory. We say that a pair of countries i, j is *directly linked* if there exists some t such that $y_{ijt} \neq 0$ or $y_{jit} \neq 0$. Countries are *indirectly linked* if for any pair of countries i, j, there is some sequence of direct links between i and j. (In terms of language from graph theory, countries are *nodes*, direct links are *edges,* and indirect links are *paths*). The countries are *connected* if any pair of countries i and j is directly or indirectly linked. It should be clear that generically, the economies studied here will be connected.

In Appendix B we show that if countries are connected, it is possible to construct trade taxes so that (25) is satisfied.[3] We then have the following proposition:

Proposition 2 (Ramsey allocations are production efficient): If at the solution to the relaxed Ramsey problem, countries are connected, then the Ramsey allocation satisfies production efficiency.

Remark 1: Thus far, we have taken the perspective that goods are indexed by both their physical attributes and the location of their production and destination. We have done so to capture the idea that countries impose trade taxes that depend on those different characteristics. For example, the United States imposes tariffs on car parts imported from Mexico that are very different from those on identical parts imported from Brazil. Indeed, the entire literature on trade diversion is about different tariffs on identical goods imported from different countries.

[2] Since the trade taxes exactly offset each other, the price received by producers of the intermediate good in the exporting country is the same as the price paid by users of the intermediate good in the importing country. Since the prices do not depend on the route, there are no arbitrage opportunities in exporting an intermediate good to a third country and re-exporting it to the final destination country.

[3] Recall that we have assumed initial foreign assets are fixed in real terms. If, instead, we had assumed that initial foreign assets were fixed in nominal terms, an additional channel that would be available to help satisfy (25) is to appropriately choose q_{i0} for each country. These choices can be thought of as choices of the initial exchange rates.

While a realistic formulation demands that we index goods in the way we do, in order to relate our results to the literature, we turn now to a more restrictive formulation in which goods are indexed only by their physical attributes. We restrict policies so that import tariffs can depend only on the identity of the good and of the importing country, and not on the identity of the origin country. Similarly, we restrict export subsidies to depend only on the identity of the good and the exporting country.

In Appendix B, we develop a simple static example with more countries than goods and show that the Ramsey allocation with these restrictions does not satisfy production efficiency. The reason is that with these restrictions, trade taxes cannot be chosen to simultaneously satisfy production efficiency and (20). We go on to show, in a dynamic extension of this simple example, that the Ramsey allocations do satisfy production efficiency. The reason now is that since taxes are allowed to be different across time, we effectively have more goods than countries and therefore have enough degrees of freedom to simultaneously satisfy production efficiency and (20).

These restrictions help us understand the result in Keen and Wildasin (2004) that with more countries than goods, the Ramsey allocation can be production inefficient. Simply put, Keen and Wildasin impose the restrictions that tariffs cannot depend on the origin/destination pair of trade. In our view, restrictions of this type are of very limited applied interest, given that countries routinely impose different tariffs on the same physical good based on country of origin or destination.

Remark 2: Proposition 2 requires that countries be connected at the solution to the relaxed Ramsey problem. If at the solution to the relaxed Ramsey problem, countries are not connected, then the relaxed Ramsey can be implemented with government to government transfers, as described below, and cannot be implemented by offsetting trade taxes. A small perturbation of the original environment will ensure that, in this case, the countries are connected. Thus, we do not view the requirement of connectedness as very stringent.

Remark 3: Inspection of (25) makes clear that this equation can be satisfied by many paths of trade taxes. Indeed, these trade taxes can be chosen in a continuum of ways so that there is indeterminacy. This indeterminacy result can also hold even if we restrict trade taxes so that they do not depend on the origin/destination pair. See Maggi and Rodríguez-Clare

(2005) for a proof of indeterminacy with such a restriction.

We now characterize the Ramsey equilibrium in greater detail. To do so, it is useful to define

$$v^i\left(c_{it}, n_{it}; \varphi^i\right) = u^i\left(c_{it}, n_{it}\right) + \varphi^i\left[u^i_{c,t}c_{it} + u^i_{n,t}n_{it}\right],$$

where $\omega^i\varphi^i$ is the multiplier of the implementability condition (21). The relaxed Ramsey problem then reduces to maximize

$$\sum_{i=1}^{N}\omega^i\left[\sum_{t=0}^{\infty}\beta^t\left[v^i\left(c_{it}, n_{it}; \varphi^i\right) + h^i\left(g_{it}\right)\right] - \varphi^i\bar{\mathcal{W}}_i\right],$$

subject to the resource constraints (2) and (5).

The solution of the relaxed Ramsey problem satisfies

$$(26)\quad -\frac{v^i_{c,t}}{v^i_{n,t}} = \frac{1}{G^i_{i,t}F^i_{n,t}},$$

$$(27)\quad \frac{v^i_{c,t}}{\beta v^i_{c,t+1}} = 1 - \delta + G^i_{i,t+1}F^i_{kt+1},$$

together with the production efficiency conditions, (8) and (9).

These equations imply an analog of the intertemporal consumption efficiency condition

$$(28)\quad \frac{v^i_{c,t}}{\beta v^i_{c,t+1}}\frac{G^i_{j,t}}{G^i_{j,t+1}} \text{ is the same across countries } i.$$

In Appendix C, we report the first-order conditions for the optimal levels of government consumption.

Note that

$$v^i_{c,t} = u^i_{c,t}\left[1 + \varphi^i\left(1 + \sigma^{in}_t - \sigma^{inc}_t\right)\right] \text{ and } v^i_{n,t} = u^i_{n,t}\left[1 + \varphi^i\left(1 + \sigma^i_t - \sigma^{icn}_t\right)\right],$$

where σ^i_{cct} and σ^i_{nnt} are the own elasticities of the marginal utilities of consumption and labor; and σ^i_{nct} and σ^{icn}_{cnt} are the cross elasticities. In general, these own and cross elasticities depend on the allocations and vary across countries and over time. Thus, in general, the ratios of the derivatives of the v^i functions do not coincide with the marginal rates of substitution, and the wedges vary across countries. In particular,

$$(29)\quad \frac{-v^i_{c,t}/v^i_{n,t}}{-u^i_{c,t}/u^i_{n,t}} \neq \frac{-v^j_{c,t}/v^j_{n,t}}{-u^j_{c,t}/u^j_{n,t}} \text{ and } \frac{v^i_{c,t}/\beta v^i_{c,t+1}}{u^i_{c,t}/\beta u^i_{c,t+1}} \neq \frac{v^j_{c,t}/\beta v^j_{c,t+1}}{u^j_{c,t}/\beta u^j_{c,t+1}}.$$

Inspecting (26) and (27) and their analogs in the competitive equilibrium, (15) and (16), we find that the taxes on consumption and labor that implement the Ramsey equilibrium are given by

$$(30)\quad \frac{-u^i_{c,t}/u^i_{n,t}}{-v^i_{c,t}/v^i_{n,t}} = \frac{(1+\tau^c_{it})}{(1-\tau^n_{it})},$$

$$(31)\quad \frac{u^i_{c,t}/\beta u^i_{c,t+1}}{v^i_{c,t}/\beta v^i_{c,t+1}} = \frac{(1+\tau^c_{it})}{\left(1+\tau^c_{it+1}\right)}.$$

Remark 4: Inspecting (29), (30) and (31) we see that tax wedges are, in general, not equated across countries. In this sense, tax harmonization is not necessarily optimal.

Remark 5: Comparing (26) and (27) with (30) and (31), we see that the tax wedges are, in general, time varying. Thus, taxes will generally also be time varying.[4]

Thus far, we have considered the general case of $N > 2$ countries. Having more than two countries is essential in understanding the role of restrictions on policies discussed in Remark 1. In the remainder of the paper, in order to reduce the notational burden, we restrict our analysis to two countries. It should be clear that all our results go through for $N > 2$ countries.

Allowing for transfers

Thus far, we have not allowed governments to make lump-sum transfers to each other. Here, we allow for such transfers and show that with transfers, efficient allocations can be supported by policies that set all tariffs and export taxes to zero and use only consumption and labor income taxes. This result clarifies that the role of offsetting trade tariffs and

[4]See Chari, Nicolini, and Teles (2020) for relationships between these results and those in the closed economy literature.

subsidies is solely redistributive across governments. Without loss of generality, we assume that country i makes a (net) transfer T_{i0} to the other country. The budget constraint for the government of country i, (13), becomes

$$\begin{aligned} & \sum_{t=0}^{\infty} Q_t \left[\tau_{it}^c q_{it} c_{it} + \tau_{it}^n w_{it} n_{it} + \sum_{j \neq i} \tau_{jit}^m p_{jit} y_{jit} + \sum_{j \neq i} \tau_{ijt}^x p_{ijt} y_{ijt} - q_{it} g_{it} \right] \\ (32) \qquad = \; & Q_{-1} b_{i0} - T_{i0} - \tau_i^W a_{i0}. \end{aligned}$$

Since transfers made by one government are received by the other, we have

$$(33) \quad \sum_{i=1}^{2} T_{i0} = 0.$$

Combining the budget constraints of the government and the household (with equality) in each country, we obtain a rewritten version of the balance of payments condition, (14):

$$(34) \quad \sum_{t=0}^{\infty} Q_t \sum_{j \neq i} \left[p_{ijt} y_{ijt} - p_{jit} y_{jit} \right] = -\left(1 + r_0^f\right) f_{i,0} - T_{i0},$$

for all i.

The Ramsey problem in this case is to choose policies, allocations, and initial transfers. Since transfers appear only in the balance of payments condition (34), we can simply drop constraint (34), and the Ramsey problem with transfers coincides with the relaxed Ramsey problem. This result implies that the offsetting trade taxes in the Ramsey equilibrium without transfers can be replaced by explicit transfers. The level of the transfers is uniquely pinned down by the welfare weights of the Ramsey problem. In this sense, the offsetting trade taxes play a purely redistributive role. We summarize this result in the following proposition.

Proposition 3 (Optimality of explicit free trade): If governments can make transfers to each other, the Ramsey allocations can be implemented by setting all trade taxes to zero.

For all the results that follow, unless explicitly stated to the contrary, we assume that direct transfers are available and that trade taxes are not used for redistribution across countries.[5]

The logic behind propositions 2 and 3 can be extended to show that restrictions on capital mobility are not efficient. To see this result, consider, for example, allowing the Ramsey planner to impose capital controls. One way of allowing for capital controls is to impose constraints on the foreign assets that residents of country i can hold, so that households face additional constraints of the form $f_{it} \leq \bar{f}_{it}$, where $\bar{f}_{it}$ is chosen by the planner. Using the evolution of household wealth, we can alternatively represent the constraints as

$$\begin{aligned} & \sum_{s=0}^{\infty} Q_{t+s} \left[q_{it+s} \left(1+\tau_{it+s}^{c}\right) c_{it+s} - \left(1-\tau_{it+s}^{n}\right) w_{it+s} n_{it+s} \right] \\ \leq \;& V_{it} + d_{it} + Q_{t-1} b_{it} + \left(1+r_0^f\right) \bar{f}_{it}, \; t \geq 1. \end{aligned} \tag{35}$$

These are additional constraints on the Ramsey problem that can be written in terms of the allocations. Thus, in the solution to the cooperative Ramsey problem, it is optimal to set $\bar{f}_{it}$ to be sufficiently large so that these additional constraints are never binding. The same logic applies to any other restrictions on capital mobility, including taxes on capital flows. In this sense, the logic behind proposition 2 implies that unrestricted capital mobility is optimal in a cooperative Ramsey equilibrium.

We next turn to proving an analog of the first welfare theorem. We will show that there is a pair of welfare weights, ω^1 and ω^2, such that the government-to-government transfers are zero. Without loss of generality, let $\omega^1 = \omega \in [0,1]$ and $\omega^2 = 1-\omega$. Let $T^i(\omega)$ denote the transfers to country i under the Ramsey allocation associated with welfare weight ω.

Proposition 4 (Optimality of production efficiency with zero transfers): Assume $\bar{\mathcal{W}}_i$ and f_{i0} are sufficiently close to zero for $i = 1, 2$. There exists a weight $\omega \in [0, 1]$ such that transfers are zero: $T^i(\omega) = 0$, $i = 1, 2$.

Proof: Under our differentiability and interiority assumptions, the transfer functions $T^i(\omega)$, $i = 1, 2$ are continuous. In Appendix C, we show that $T^1(0) \leq 0$ and $T^1(1) \geq 0$. The result follows from the intermediate value theorem. ■

Remark: The same theorem holds with more than two countries. In this case, we

[5] If direct transfers are not available, all the results below go through with trade taxes used solely for redistributive purposes.

can apply the argument in Negishi (1972) to prove the result.

In Appendix C, we explain the role of the technical assumption that $\bar{\mathcal{W}}_i$ is sufficiently close to zero and make clear that for a (potentially large) neighborhood of promised wealth around zero, the proposition goes through.

This proposition says that there is a vector of welfare weights such that no transfers (or trade taxes) are needed to implement the cooperative Ramsey equilibrium. Thus, if the economy starts off at this Ramsey allocation, no Pareto improvement is possible.

C. Allowing for Distributional Considerations

In the model above, we abstract from the distributional effects of policies within each country. In this section, we briefly address those considerations, allowing for the possibility that different agents may be differently affected by trade policies. For simplicity, we consider only two worker types with equal mass. The production function in country i is described by

$$y_{i1t} + y_{i2t} = y_{it} = F^i\left(k_{it}, n_{it}^a, n_{it}^b\right),$$

where n_{it}^a and n_{it}^b are the labor hours of agents a and b in country i. Notice that with this production function, the relative wages of the two agents are endogenous and are a function of trade policies. A special case in which the relative wage is exogenous is when the two agent types differ only in their efficiency units but are perfect substitutes in production, as in $F^i\left(k_{it}, n_{it}^a + \eta_i n_{it}^b\right)$. The preferences of type a agents are

$$U^{ia} = \sum_{t=0}^{\infty} \beta^t \left[u^{ia}\left(c_{it}^a, n_{it}^a\right) + h^i\left(g_{it}\right)\right]$$

and are similar for type b agents.

An allocation in this economy consists of consumption and labor allocations for each household a and b,$\{c_{it}^a, n_{it}^a\}$ and $\left\{c_{it}^b, n_{it}^b\right\}$, and aggregate allocations for each country $\{y_{ijt}, k_{it+1}, x_{it}\}$. The market clearing condition for the final good is

$$c_{it}^a + c_{it}^b + g_{it} + x_{it} \leq G^i\left(y_{1it}, y_{2it}\right),$$

and the capital accumulation equation is (4).

We start by allowing for a version of our benchmark system with transfers across countries. In this version, the government in each country can impose taxes that are specific to each type of agent on consumption and labor income and initial wealth. The planner must also respect the wealth constraints for each type of agent in each country.

In this case, it is straightforward to show that proposition 2 holds, so that restrictions on trade and capital mobility are not optimal.

Suppose instead that the tax rates on the two types of agents are restricted so that they are the same within a country. Then, the following additional implementability conditions must be imposed:

$$-\frac{u^i_{c^a,t}F^i_{n^a,t}}{u^i_{n^a,t}} = -\frac{u^i_{c^b,t}F^i_{n^b,t}}{u^i_{n^b,t}},$$

and

$$\frac{u^i_{c^a,t}}{\beta u^i_{c^a,t+1}} = \frac{u^i_{cb,t}}{\beta u^i_{c^b,t+1}}.$$

With these extra restrictions, production efficiency may not be optimal.[6]

III Alternative Implementations

Thus far, we have considered tax systems that include taxes on consumption, labor income, and trade. Here, we discuss a variety of other tax systems, including taxes on the income from different assets and value-added taxes. As mentioned above, we assume that direct transfers are available to redistribute across countries. Trade taxes are used only if needed to achieve efficiency.

Our analysis is motivated by the observation that these alternative tax systems are widely used in practice. We show that no tax system can yield higher welfare than the tax system with only consumption and labor income taxes. We show that a variety of tax systems can implement the Ramsey allocation associated with those taxes. Furthermore, some tax

[6]For particular cases in which production efficiency is still optimal, see the preference and technology specifications in the closed economy model of Chari, Nicolini and Teles (2020).

systems do yield lower welfare.

A. Taxes on Corporate Income and Asset Returns

Here, we consider a tax system that consists of taxes on labor income, on corporate income, and on the returns of households' asset holdings. We assume that tax rates on household asset income are the same for all assets. We show that the Ramsey outcome can be implemented with zero taxation of corporate income and with suitably chosen taxes on household asset income. We also show that non-zero corporate income taxation can impose intertemporal trade wedges. This result shows that free trade agreements need to be supplemented with agreements on fiscal policy.

We now describe the problems of the firms and the household in each country and define a competitive equilibrium.

Firms

The representative intermediate good firm in each country produces and invests in order to maximize the present value of dividends, $V_{i0} + d_{i0} = \sum_{t=0}^{\infty} Q_t d_{it}$, where Q_t is the pretax discount factor. Dividends, d_{it}, in units of the numeraire, are given by

$$\begin{aligned} d_{it} &= p_{it}F(k_{it}, n_{it}) - w_{it}n_{it} - \tau_{it}^k \left[p_{it}F(k_{it}, n_{it}) - w_{it}n_{it} - q_{it}\delta k_{it}\right] \\ &\quad - q_{it}\left[k_{it+1} - (1-\delta)k_{it}\right], \end{aligned} \tag{36}$$

where τ_{it}^k is the tax rate on corporate income net of depreciation. Here, we have specified the tax base for corporate income in the standard way. Below, we describe an alternative tax base, which allows for investment expenses to be deducted.

Note that one of the first-order conditions of the firm's problem is

$$\frac{Q_t q_{it}}{Q_{t+1} q_{it+1}} = 1 + \left(1 - \tau_{it+1}^k\right)\left(\frac{p_{it+1}}{q_{it+1}} F_{k,t+1}^i - \delta\right). \tag{37}$$

We use this condition in the proof of proposition 5, below.

The problem of the final good firm is as it was before.

Households

Here we explicitly allow for sequential trading by households. In each period, households choose consumption, labor supply, and holdings of domestic and foreign bonds and equity in domestic firms. For simplicity, and without loss of generality, we assume that households cannot hold foreign equity.

The tax base for bond income, expressed in real terms, is given by $(r_t^f - (q_{it} - q_{it-1})/q_{it-1})(f_{it} + b_{i}$
This way of defining the base ensures that taxes are levied only on real income. The tax base for equity income taxation is given by

$$d_{it} + V_{it} - V_{it-1} - (q_{it} - q_{it-1}) V_{it-1}/q_{it-1}$$

per share. Note that this tax base includes dividends received in the current period and accrued capital gains generated by changes in the price of equity, as well as an adjustment to ensure that the base is expressed in real terms.

The flow of funds constraint in period $t \geq 1$ for the household in country i in units of the numeraire is then given by

$$\begin{aligned}(38) \qquad & q_{it}c_{it} + b_{it+1} + f_{it+1} + V_{it}s_{it+1} \\ = \; & (1 - \tau_{it}^n) w_{it}n_{it} + \left[1 + r_t^f - \tau_{it}\left(r_t^f - \frac{q_{it} - q_{it-1}}{q_{it-1}}\right)\right](f_{it} + b_{it}) \\ & + (V_{it} + d_{it}) s_{it} - \tau_{it}\left(d_{it} + V_{it} - V_{it-1} - \frac{(q_{it} - q_{it-1}) V_{it-1}}{q_{it-1}}\right) s_{it}.\end{aligned}$$

The period zero constraint needs to be adjusted by the wealth tax and is given in Appendix D. Note that since domestic and foreign bond income are taxed at the same rate in each country, the pretax returns on bonds are the same in the two countries.

The household's problem is to maximize utility (1), subject to (38) and the relevant budget constraint at period zero and no-Ponzi conditions, $\lim_{T\to\infty} Q_{iT+1}b_{iT+1} \geq 0$ and $\lim_{T\to\infty} Q_{iT+1}f_{iT+1} \geq 0$, where Q_{it}/Q_{it+1} is the return on bonds net of taxes given by

$$\frac{Q_{it}}{Q_{it+1}} = (1 - \tau_{it+1})\left(1 + r_{t+1}^f\right) + \tau_{it+1}\frac{q_{it+1}}{q_{it}} \text{ with } Q_{i0} = 1.$$

Using the no-Ponzi-scheme condition, we can consolidate the budget constraints of the household, (38) and the period zero budget constraint into the single budget constraint,

$$\sum_{t=0}^{\infty} Q_{it} \left[q_{it}c_{it} - (1-\tau_{it}^{n}) w_{it}n_{it}\right] = \left(1-\tau_i^W\right) a_{i0},$$

where the expression for the initial wealth, a_{i0}, is given in Appendix D.

It is straightforward to show that the consolidated budget constraint reduces to the same implementability constraint, (21), with $\mathcal{W}_{i0}$ given in Appendix D. It is also straightforward to show that any allocation that satisfies the implementability and resource constraints can be implemented as a competitive equilibrium.[7]

The Ramsey problem is then to maximize (24) subject to the implementability constraints (21), with the wealth restriction (23) and the resource constraints. It is immediate that the Ramsey allocation in the economy with consumption and labor income taxes coincides with the one in the economy considered here.

Next, we show that it is optimal to set corporate income taxes to zero and that, in general, asset taxes are needed to implement the Ramsey outcome. It is straightforward to show that if we use the first-order conditions of the firms, any competitive equilibrium satisfies static production efficiency. Next, we turn to conditions that implement dynamic production efficiency. Using (37) for both countries, as well as the final good firms' conditions, we obtain a version of the interest rate parity condition:

$$\begin{aligned} (39) \qquad & \frac{G^1_{j,t}}{G^1_{j,t+1}}\left[1+\left(1-\tau^k_{1t+1}\right)\left(G^1_{1,t+1}F^1_{k,t+1}-\delta\right)\right] \\ = \; & \frac{G^2_{j,t}}{G^2_{j,t+1}}\left[1+\left(1-\tau^k_{2t+1}\right)\left(G^2_{2,t+1}F^2_{k,t+1}-\delta\right)\right], \text{ for } j=1,2. \end{aligned}$$

Comparing this condition with (9), we see that setting both corporate income taxes to zero ensures dynamic production efficiency. It is also clear that, if one of the countries sets the corporate income tax equal to zero, the other country can impose wedges on intertemporal trade by setting a tax rate different from zero. Comparing (18) with (39), we see that these wedges are similar to the ones that arise with time varying tariffs. We summarize this result

[7] If there were multiple consumption goods in each period, consumption taxes on those goods might be needed.

in the following proposition.

Proposition 5 (Corporate income taxes and capital mobility): Non-zero corporate income taxes introduce wedges in international intertemporal trade.

Next, we show that asset taxes are needed to implement the Ramsey allocation. To do so, we use the households' and firms' first-order conditions to obtain

$$(40)\quad -\frac{u^i_{c,t}}{u^i_{n,t}} = \frac{1}{\left(1-\tau^n_{it}\right) G^i_{i,t} F^i_{n,t}}$$

and

$$(41)\quad \frac{u^i_{c,t}}{\beta u^i_{c,t+1}} = 1 + \left(1-\tau_{it+1}\right)\left(1-\tau^k_{it+1}\right)\left(G^i_{i,t+1} F^i_{k,t+1} - \delta\right).$$

In general, the solution to the Ramsey problem requires time-varying intertemporal distortions. Thus, implementing the Ramsey outcome with the system considered here requires asset taxes, given that the corporate income tax is set to zero. If we set the asset taxes to zero, it is in general not possible to choose the corporate income tax rates in both countries to satisfy both (39) and (41) at the Ramsey allocation.

We summarize these results in the following proposition.

Proposition 6 (Common tax on domestic equity and foreign returns): The Ramsey outcome can be implemented with labor income taxes and asset taxes and by setting the corporate income taxes to zero. In general, the Ramsey outcome cannot be implemented with only labor income taxes and corporate income taxes.

Remark 1: These results are quite different from those in a closed economy. In a closed economy, household asset taxes and corporate income taxes distort capital accumulation in the same way. Thus, it is possible to support the Ramsey allocations with labor income taxes and corporate income taxes or, equivalently, with labor income taxes and household asset taxes. In the open economy, a system with corporate income taxes distorts dynamic production efficiency by distorting the allocation of capital across countries in addition to distorting capital accumulation. In this sense, a system with corporate income taxes is dominated by a system with household asset taxes.

Remark 2: Note that we have assumed that the tax rates on domestic and foreign asset income are the same. If these tax rates are allowed to be different, then the Ramsey equilibrium can be implemented by setting them so that they coincide.

Remark 3: Our analysis allows for a comparison of residence-based and source-based tax systems. In our model, a residence-based system is one in which all household asset income is taxed at a rate that is independent of where the income is generated but can depend on where the household resides. A source-based system is one in which income is taxed where it is generated–namely, at a point of production. A corporate income tax is an example of a source-based system. Since we have argued that household asset taxes have advantages over corporate income taxes, we have shown that residence-based tax systems have advantages over source-based systems.

Remark 4: In our discussion so far, we have assumed that investment expenditures are not deductible in calculating the base for corporate income taxation. An alternative formulation is to allow investment expenditures to be deductible. In this case, the dividends are given by

$$\begin{aligned} d_{it} &= p_{it}F(k_{it}, n_{it}) - w_{it}n_{it} - q_{it}\left[k_{it+1} - (1-\delta)k_{it}\right] \\ &\quad -\tau_{it}^{k}\left[p_{it}F(k_{it}, n_{it}) - w_{it}n_{it} - q_{it}\left(k_{it+1} - (1-\delta)k_{it}\right)\right] \end{aligned}$$

If we adapt the arguments on the optimal taxation of capital income in a closed economy (see, for example, Chari, Nicolini, and Teles (2020)), it is possible to show that a constant corporate income tax, with suitably time-varying consumption and labor taxes, can implement the Ramsey allocation. Interestingly, in the global economy, a tax rate that is the same across countries, but varies over time, can also be part of the implementation of the Ramsey allocation (see Appendix D for a proof of these results). This remark helps clarify that, as is well known, the Ramsey allocation can be implemented in a variety of ways, so that Ramsey outcomes pin down wedges, rather than specific patterns of taxes.

Our analysis clarifies the discussion on the desirability of corporate income taxation. The conventional wisdom in this area is well summarized in the 2007 OECD Volume *Fundamental Reform of Corporate Income Tax*:[8]

[8] We thank a referee for suggesting that we include this clarification.

"The main reason for imposing a corporate tax is that the tax plays an important withholding function, acting as a 'backstop' to the personal income tax. The corporate tax might be needed to avoid excessive income shifting between labour income and capital income. The corporate tax also acts as a withholding tax on equity income earned by non-resident shareholders, which might otherwise escape taxation in the source country. Moreover, governments might levy a corporate income tax because firms earn location-specific rents and/or because capital is not perfectly mobile."

We view the backstop function as suggesting that it might be administratively easier to collect taxes at the source. In this case, our discussion in Remark 4 suggests that corporate income taxes on a properly defined base that are constant over time or the same across countries, together with other suitably designed taxes, can implement Ramsey outcomes. Second, to the extent that, for administrative reasons, taxes on asset income earned by non-residents are taxed, our analysis suggests that one way of implementing the Ramsey allocation is to have these revenues rebated to the destination country along suitable adjustments in taxation in the destination country. Third, our environment can be easily adapted to allow for fixed factors in production, as long as the income from these factors is taxed at a rate that meets the initial wealth constraint. In this case, the corporate income tax could be one way of raising taxes on fixed factors, but it will require the base to exclude investment expenses.

We have considered a decentralization in which investment decisions are made by firms. Much of the macroeconomics literature considers decentralizations in which investment decisions are made by households and firms simply rent capital and labor from households. It is possible to show that with this decentralization, the same Ramsey outcomes can be supported by a tax system under which household assets are taxed at a rate that may vary across countries but is uniform across asset types.

B. Border-Adjusted Value-Added Taxes and Labor Income Taxes

Consider next an economy in which consumption taxes are replaced by value-added taxes levied on firms with border adjustment. Border adjustment means that firms in a country do not pay value-added taxes on exports and cannot deduct imports. Taxes on assets are set to zero, but labor income taxes are not. The value-added taxes are denoted by

τ_{it}^{v}. We refer to the system with value-added taxes with border adjustment as a *VAT with BA system.*

The intermediate goods firm now maximizes

$$(42)\quad \sum_{t=0}^{\infty} Q_t \left[(p_{i1t} y_{i1t} + p_{i2t} y_{i2t}) - w_{it} n_{it} - q_{it} x_{it} \right] - \sum_{t=0}^{\infty} Q_t \tau_{it}^{v} \left[p_{iit} y_{iit} - q_{it} x_{it} \right],$$

subject to (2) and (4), where p_{ijt} is the price of the intermediate good produced in country i and sold in country j. Note that the final goods firm pays taxes on the good when it is sold domestically, but not when it is exported. In this sense, taxes are adjusted at the border.

The final goods firm now maximizes

$$(43)\quad \sum_{t=0}^{\infty} Q_t \left[q_{it} G^{i} \left(y_{1it}, y_{2it} \right) - p_{1it} y_{1it} - p_{2it} y_{2it} \right] - \sum_{t=0}^{\infty} Q_t \tau_{it}^{v} \left[q_{it} G^{i} \left(y_{1it}, y_{2it} \right) - p_{iit} y_{iit} \right].$$

This firm is able to deduct the input produced domestically, but not the one imported. Thus, taxes are again adjusted at the border. The household problem is the same as it was above, except that the consumption taxes are set to zero.

In Appendix E, we show that the equilibrium conditions in this economy with VAT with BA are

$$(44)\quad -\frac{u_{c,t}^{i}}{u_{n,t}^{i}} = \frac{1}{\left(1 - \tau_{it}^{n}\right)\left(1 - \tau_{it}^{v}\right) G_{i,t}^{i} F_{n,t}^{i}},\ t \geq 0,$$

$$(45)\quad u_{c,t}^{i}\left(1 - \tau_{it}^{v}\right) = \left(1 - \tau_{it+1}^{v}\right) \beta u_{c,t+1}^{i} \left[G_{i,t+1}^{i} F_{k,t+1}^{i} + 1 - \delta) \right],\ t \geq 0,$$

together with the production efficiency conditions. Clearly, these conditions coincide with those in an economy with only consumption and labor income taxes if the value-added taxes satisfy

$$(46)\quad 1 + \tau_{it}^{c} = \frac{1}{1 - \tau_{it}^{v}}.$$

The proposition follows.

Proposition 7 (Value-added taxes with border adjustment): A value-added tax system with border adjustment, including labor income taxes, is equivalent to a system that taxes consumption and labor and has no tariffs.

Since the Ramsey allocation can be implemented by a system that taxes only consumption and labor, this proposition implies that the Ramsey allocations can be implemented by a value-added tax system with border adjustments. In this sense, a value-added tax system with border adjustments has desirable features.

Consider an environment where countries agree to free trade and commit to using a VAT with BA system, but they are free to set tax rates as they see fit. Notice that in this non-cooperative setting, they will not be able to use fiscal policy to impose trade wedges. Thus, the design of tax systems affects the extent to which fiscal policy is trade policy. Of course, countries could always use fiscal policy to affect international terms of trade. This proposition establishes that if countries are constrained to adopt VAT with border adjustments, they cannot introduce trade wedges.

C. Value-Added Taxes without Border Adjustment: The Role of Tariffs

Consider next an economy just like the one in the previous section, except that value-added taxes are levied on firms without border adjustment. This means that the taxation of intermediate goods will be origin based. Here, we allow for trade taxes, because as it turns out, they may be needed to achieve efficiency. We refer to the system with value-added taxes without border adjustment and with trade taxes as a *VAT without BA system.* We will show that this system with trade taxes set to zero cannot in general implement the Ramsey allocation. We show that the system with non-zero trade taxes can implement the Ramsey allocation.

The intermediate goods firm in country 1 now maximizes

$$\sum_{t=0}^{\infty} Q_t \left[(1 - \tau_{1t}^v) \left(p_{11t} y_{11t} + (1 - \tau_{12t}^x) \, p_{12t} y_{12t} - q_{1t} x_{1t} \right) - w_{1t} n_{1t} \right],$$

subject to (2) and (4).

The final goods firm in country 1 now maximizes

$$\sum_{t=0}^{\infty} Q_t \left(1-\tau_{1t}^{v}\right) \left[q_{1t} G^1 \left(y_{11t}, y_{21t}\right) - p_{11t} y_{11t} - \left(1+\tau_{21t}^{m}\right) p_{21t} y_{21t}\right].$$

Firms in country 2 solve similar problems.

The first-order conditions of the household and firms can be used to obtain (44), (45), and

$$(47)\quad \frac{\left(1+\tau_{21t}^{m}\right)}{\left(1-\tau_{21t}^{x}\right)} \frac{G_{2,t}^2}{G_{2,t}^1} = \frac{\left(1-\tau_{12t}^{x}\right)}{\left(1+\tau_{12t}^{m}\right)} \frac{G_{1,t}^2}{G_{1,t}^1},$$

$$(48)\quad \frac{\left(1+\tau_{12t}^{m}\right)}{\left(1-\tau_{12t}^{x}\right)} \frac{\left(1-\tau_{12t+1}^{x}\right)}{\left(1+\tau_{12t+1}^{m}\right)} \frac{\left(1-\tau_{1t+1}^{v}\right)}{\left(1-\tau_{1t}^{v}\right)} \frac{G_{1,t}^1}{G_{1,t+1}^1} \left[G_{1,t+1}^1 F_{k,t+1}^1 + 1 - \delta\right]$$
$$= \frac{\left(1-\tau_{2t+1}^{v}\right)}{\left(1-\tau_{2t}^{v}\right)} \frac{G_{1,t}^2}{G_{1,t+1}^2} \left[G_{2,t+1}^2 F_{k,t+1}^2 + 1 - \delta\right].$$

Using (45) and (48), we obtain the analog to (19),

$$(49)\quad \frac{\left(1+\tau_{12t}^{m}\right)/\left(1-\tau_{12t}^{x}\right)}{\left(1+\tau_{12t+1}^{m}\right)/\left(1-\tau_{12t+1}^{x}\right)} \frac{u_{c,t}^1}{\beta u_{c,t+1}^1} = \frac{u_{c,t}^2}{\beta u_{c,t+1}^2} \frac{G_{1,t}^2/G_{1t}^1}{G_{1,t+1}^2/G_{1t+1}^1}.$$

We use these conditions to show that if trade taxes are constrained to be zero in both countries, it is not possible to implement the Ramsey outcome for general preferences. Recall from (28) that in a Ramsey outcome,

$$\frac{v_{c,t}^1}{\beta v_{c,t+1}^1} \frac{G_{j,t}^1}{G_{j,t+1}^1} = \frac{v_{c,t}^2}{\beta v_{c,t+1}^2} \frac{G_{j,t}^2}{G_{j,t+1}^2},$$

so that, from (29), we see that in general,

$$(50)\quad \frac{u_{c,t}^1}{\beta u_{c,t+1}^1} \frac{G_{j,t}^1}{G_{j,t+1}^1} \neq \frac{u_{c,t}^2}{\beta u_{c,t+1}^2} \frac{G_{j,t}^2}{G_{j,t+1}^2}.$$

Comparing (49) and (50), we see that with zero trade taxes, it is not possible to implement the Ramsey outcome in general.

Once we allow for tariffs, it is possible to implement the Ramsey outcome. To ensure static production efficiency, tariffs have to compensate each other so that (47) coincides with (8). To ensure dynamic production efficiency, the tariffs have to suitably vary over time so as to undo the distortions arising from time-varying VATs. One implementation of the Ramsey outcome has

$$(51)\quad \frac{1-\tau_{1t}^{v}}{1-\tau_{2t}^{v}} = \frac{1-\tau_{21t}^{x}}{1+\tau_{21t}^{m}} = \frac{1+\tau_{12t}^{m}}{1-\tau_{12t}^{x}}.$$

It is straightforward to verify that with these policies, it is possible to implement the Ramsey allocation. This implementation has value-added tax rates that are the same as the ones in the VAT with BA system and an effective export subsidy on good 2 and an effective import tax on good 1 of the same magnitude as the ratio of the two VATs, $(1-\tau_{1t}^{v})\,/\,(1-\tau_{2t}^{v})$. Trade taxes chosen in this fashion do not distort static production efficiency, and they correct for the dynamic production inefficiencies induced by time-varying VATs.

We state these results in the following proposition.

Proposition 8 (Value-added taxes without border adjustment): Suppose trade taxes are constrained to be zero in both countries. Then, for general preferences, the Ramsey allocation cannot be implemented with a tax system with labor income taxes and value-added taxes without border adjustment. If trade taxes are unconstrained, then the Ramsey allocation can be implemented with consumption taxes replaced by value-added taxes and tariffs.

Remark 1: The equilibrium conditions in this system illustrate the sense in which fiscal policy is trade policy. Consider an environment, paralleling our earlier discussion, where countries agree to free trade and commit to using a VAT without BA system, being free to set tax rates as they see fit. In this non-cooperative setting, countries will be able to use fiscal policy to impose trade wedges. To see this result, note from (48)that time-varying value-added taxes impose intertemporal trade wedges. This observation implies that countries have an incentive to use time-varying taxes to directly affect international terms of trade, just as in the optimal tariff literature. These findings reinforce the observation that the design of tax systems affects the extent to which fiscal policy is trade policy.

Proposition 8 is connected to results in international trade. Some of the literature in international economics (e.g., Grossman 1980, Feldstein and Krugman 1990, Costinot and Werning 2018) has argued that VAT systems with border adjustment are equivalent to VAT systems without such adjustment, holding trade taxes constant. The version of the result applicable to our analysis (see Grossman 1980) is that a uniform value-added tax with border adjustment is equivalent to a uniform value-added tax without border adjustment, in the sense that, taking international prices as given, an individual country can achieve the same allocations with either system. (The theorem requires qualifications regarding the availability of initial wealth taxes to ensure that the government's budget is balanced and international lump-sum transfers to ensure that the balance of payments condition is satisfied.)

The key requirement in Grossman's version of the theorem is that value-added taxes are the same across all goods. If value-added taxes differ across goods, then the two systems are not in general equivalent. We can think of our dynamic economy as a static economy with an infinite number of goods. Suppose that the dynamic economy has constant value-added taxes over time. Then, in the reinterpreted static economy, value-added taxes are the same across all goods. Inspecting the marginal conditions with BA–namely, (8) and (9)–and those without BA–namely, (47) and (48)–we see that the same allocations can be supported by a VAT with BA system and a VAT without BA system, with no tariffs in either case. Suppose next that in the dynamic economy, value-added taxes vary over time, so that in the reinterpreted static economy, value-added taxes are different across goods. Then, inspecting the same conditions, we see that the two systems are not equivalent in the absence of tariffs.

Our results can also be used to compare destination- versus origin-based systems. To see this comparison, recall that a destination-based system is one in which tax rates do not depend on origin, and an origin-based system is one in which tax rates do not depend on destination. In the case of value-added taxes with border adjustment, the goods leave the country untaxed and are taxed at the single value-added tax rate in the destination country. In this sense, the VAT with BA system is destination-based. With value-added taxes without border adjustments, goods are taxed at the single rate of the origin country, so that a VAT without BA system is origin-based. Our results imply that if countries are restricted from imposing trade taxes, then a destination-based system dominates an origin-based system.

D. Lerner Symmetry

The arguments in the previous section make clear that any competitive equilibrium allocation in a VAT without BA system and no trade taxes can be implemented in a VAT with BA system with the same VAT rates and trade taxes chosen according to (51). The trade taxes are an effective import tariff and an export subsidy of the same magnitude. The results regarding the conditions under which VAT with and without border adjustments are equivalent are related to Lerner symmetry. In a static two-good economy, Lerner symmetry asserts that an import tariff is equivalent to an export tax. To understand the relationship with our results, we begin with the following lemma, which establishes a version of Lerner symmetry for our dynamic model.

Lemma 1 (Lerner (a)symmetry): The competitive equilibrium allocations of an economy with trade taxes given by τ_{12t}^{x} and τ_{21t}^{m} coincide with the competitive equilibrium allocations with trade taxes $\hat{\tau}_{12t}^{x}$ and $\hat{\tau}_{21t}^{m}$ satisfying $(1-\hat{\tau}_{12t}^{x}) = \kappa_t\,(1-\tau_{12t}^{x})$ and $(1+\hat{\tau}_{21t}^{m}) = \kappa_t\,(1+\tau_{21t}^{m})$, if and only if $\kappa_t = \kappa_s$ for all t and s, provided initial wealth taxes or international transfers are chosen appropriately.

The proof of the lemma is in Appendix B. In proving this lemma, we use only the properties that any competitive equilibrium must satisfy and do not use any properties of the Ramsey allocation. The key idea is that a change in trade taxes, even if it is offsetting within a period, is neutral if the change in taxes is uniform across periods. Such a uniform change leaves after-tax relative prices unaffected and leaves allocations unaffected if international transfers and wealth taxes are adjusted appropriately. If the change is not uniform across periods, allocations will change.

This lemma helps in understanding our results on value-added taxes. Consider starting with a VAT without BA system in which trade taxes are zero and the VAT rates are set at the same level as in a VAT with BA system. If the VAT rates vary over time, the VAT without BA economy has dynamic production inefficiency. Adding trade taxes–which vary over time, as given in (51)–restores dynamic production efficiency to this economy. This restoration is possible only because trade taxes are not uniform over time.

These results shed light on some of those in the literature. For example, Barbiero et al. (2017) show that in an economy with sticky prices and no capital, permanent changes in tax systems similar to the ones studied here have no effects on allocations. This result is similar to our finding that uniform changes in trade taxes have no real effects. Barbiero et al. (2017) also show that anticipated changes in tax systems have real effects. This result is similar to our result that nonuniform changes in trade taxes may lead to changes in allocations. As another example, Costinot and Werning (2018) show that uniform changes in trade taxes have no effect on allocations.

We have shown that uniform changes in import tariffs and export subsidies leave domestic relative prices unaffected. We turn now to the question of the determination of the level of prices. If domestic prices are denominated in a world numeraire, as in our model, a uniform change in trade taxes of magnitude κ proportionately raises all domestic prices, so that $\hat{p}_{11t} = \kappa p_{11t}$, $\hat{q}_{1t} = \kappa q_{1t}$, $\hat{w}_{1t} = \kappa w_{1t}$. In Lemma 2 below, we show that if domestic prices are denominated in terms of a domestic numeraire, then a change in the exchange rate between the domestic and the world numeraires can achieve the needed adjustment without having to change domestic prices at all. We let tildes denote prices in terms of a domestic numeraire and E_t denote the exchange rate between the domestic and the world numeraire measured as units of domestic numeraire per world numeraire; for example, $\tilde{p}_{11t} = E_t p_{11t}$.

Lemma 2 (Exchange rate adjustment): Consider a competitive equilibrium of an arbitrary economy. Consider now an alternative economy with the same international prices in world numeraire–Q_t, p_{12t}, p_{21t}–and the same domestic prices–in domestic currency, $\tilde{q}_{1t}, \tilde{p}_{11t}, \tilde{w}_{1t}$–in which allocations, domestic policies, and the exchange rate are denoted by carets. Suppose now policies in the alternative economy satisfy $1 - \hat{\tau}^x_{12t} = \kappa\,(1 - \tau^x_{12t})$ and $1 + \hat{\tau}^m_{21t} = \kappa\,(1 + \tau^m_{21t})$. There is an equilibrium in the alternative economy, with the same allocations and domestic policies, and with exchange rates given by $\hat{E}_t = E_t/\kappa$, provided initial wealth taxes or international transfers are chosen appropriately.

Next we turn to the needed adjustment in the initial wealth taxes or international transfers. In order to understand the needed adjustment, suppose now that foreign assets, denominated in the world numeraire, f_{10}, are fixed. In Appendix B, we show that in this case, only the initial wealth tax may have to be adjusted. There is no need to adjust international transfers to satisfy the balance of payments condition for country 1 in (14). If, instead, domestic and foreign assets of country 1, b_{10} and f_{10}, are denominated in the domestic numeraire, there is no need to adjust the initial wealth tax, but international transfers may need to be adjusted.

IV Remarks on the Generality of the Results

In this section, we argue that our results generalize to other models of international trade and models of nonlinear taxation.

Other models of international trade

Thus far we have concentrated on one widely used model of international trade–namely, that in Backus, Kehoe, and Kydland (1994). This focus allowed us to derive explicit expressions for the optimal wedges and allowed for a detailed analysis of alternative tax systems. Here, we show that our propositions continue to hold in two other widely used models of international trade.

Consider, for example, the economy in Stockman and Tesar (1995). This economy has two countries. Consumers in each country derive utility from a traded good produced in their country, a traded good produced in the other country, and a non-traded good. For simplicity, we consider a static version of their model. In this version, consumers' utility in, say, country 1 is given by

$$\frac{1}{1-\sigma}\left[\left(c_1^{\theta}c_2^{1-\theta}\right)^{\mu}+d_1^{\mu}\right]^{\frac{1-\sigma}{\mu}}v(n_1),$$

where c_1 is the consumption of the traded good produced in country 1, c_2 is the consumption of the traded good produced in country 2, and d_1 is the amount of non-traded good produced in country 1. The preferences in country 2 are described in a similar fashion. Goods are produced with a linear technology that uses labor. Mapping this economy into ours requires only adding for each country an intermediate good that captures the role of the non-traded good. To develop this map, consider a technology for producing the single final good given by

$$C_1 = \left[\left(y_{11}^{\theta} y_{21}^{1-\theta}\right)^{\mu} + d_1^{\mu}\right]^{\frac{1}{\mu}}.$$

with preferences given by

$$\frac{1}{1-\sigma} C_1^{1-\sigma} v(n_1),$$

and similarly for country 2. It is straightforward to show, with offsetting trade taxes, that proposition 2 holds for the reinterpreted Stockman-Tesar economy. Our other results also continue to hold.

Consider, next, a model with a continuum of goods on the interval $[0,1]$, like the one in Obstfeld and Rogoff (1995).[9] Goods indexed $[0,n]$ are produced in country 1, and the remaining goods are produced in country 2. We assume that these goods are produced by competitive firms. Preferences over the final consumption good, C_{it}, and labor, n_{it}, for residents of country i are given by

$$(52) \quad \sum_{t=0}^{\infty} \beta^t \left[\frac{C_{it}^{1-\sigma}}{1-\sigma} - v(n_{it})\right],$$

where

$$(53) \quad C_{it} = \left[\int c_{it}(z)^{\frac{\theta-1}{\theta}} dz\right]^{\frac{\theta}{\theta-1}}.$$

Here, $c_{it}(z)$ is consumption of good z in country i. Labor is the only input of production, and one unit of labor produces one unit of each of these goods, so that the technology for, say, country 1 is $y_{1t}(z) = n_{1t}(z)$, $z \in [0,n]$, where $y_{it}(z)$ is the amount of the good z produced in country i and $n_{it}(z)$ is the labor used in the production of good z in country i, together with

$$n_{1t} = \int_0^n n_{1t}(z)\, dz.$$

[9]Obstfeld and Rogoff (1995) have monopolistic competition and sticky prices. We consider a version of their model without these features.

The technology is defined in a similar way for country 2.

To map this economy into our setup, we need to extend our economy to allow for a continuum of traded intermediate goods. Let $y_{ijt}(z)$ denote the intermediate good of type z produced in country i and used in country j. The technology for producing the single final non-traded consumption good is given for, say, country 1, by

$$(54) \quad C_{1t} = \left[\int_0^n y_{11t}(z)^{\frac{\theta-1}{\theta}} \, dz + \int_n^1 y_{21t}(z)^{\frac{\theta-1}{\theta}} \, dz \right]^{\frac{\theta}{\theta-1}},$$

and similarly for country 2. The technology for producing these intermediate goods is the same as in Obstfeld and Rogoff (1995). Preferences are given by (52). It should be clear that all our propositions extend in a straightforward manner to this economy. A similar extension is possible for the economy in Eaton and Kortum (2002).

The logic behind these arguments suggests that our propositions also extend to other widely used models with perfect competition. Our results do not immediately generalize to trade models in which firms have monopoly power, such as those in Helpman and Krugman (1985) and Melitz (2003), or in which there are externalities, as in Alvarez, Buera, and Lucas (2013). These environments require corrective tax and subsidy instruments, even without the need to finance government expenditures with distorting taxes. We conjecture that with the appropriate tax and subsidy instruments, production efficiency is still optimal.

Nonlinear taxation

Here we briefly show how proposition 2 generalizes to environments with nonlinear taxation. We consider a Mirrlees-like environment in which we set up a mechanism design problem, then discuss how the resulting allocations can be implemented as a competitive equilibrium with nonlinear taxes on consumption and labor income and also trade taxes.

Consider a version of our benchmark model with a continuum of households in each country in the unit interval. Household k in country i is indexed by a parameter θ_i^k. This parameter is constant over time and determines the effective units of labor supplied by household k in country i. Specifically, a household of type θ_i^k that supplies n_t units of labor supplies $l_t = \theta_i^k n_t$ units of effective labor. The distribution of household types is given by $H_i\left(\theta_i^k\right)$.

The cooperative planner observes consumption and effective labor by each household but not the household type. An allocation in this economy consists of allocations for each household $\left\{c_t\left(\theta_i^k\right), l_t\left(\theta_i^k\right)\right\}$ and aggregate allocations for each country $\{y_{ijt}, k_{it+1}, x_{it}\}$. The resource constraints are the analogs of (2) and (3),

$$(55)\quad y_{i1t} + y_{i2t} = y_{it} = F^i\left(k_{it}, \int l_t\left(\theta_i^k\right) dH_i\left(\theta_i^k\right)\right),$$

$$(56)\quad \int c_t\left(\theta_i^k\right) dH_i\left(\theta_i^k\right) + g_{it} + x_{it} \leq G^i\left(y_{1it}, y_{2it}\right),$$

and (4). The utility of household of type θ_i^k is given by

$$(57)\quad U^i\left(\theta_i^k\right) = \sum_{t=0}^{\infty} \beta^t \left[u^i\left(c_t\left(\theta_i^k\right), \frac{l_t\left(\theta_i^k\right)}{\theta_i^k}\right) + h^i\left(g_{it}\right)\right].$$

An allocation is *incentive compatible* if

$$(58)\quad \sum_{t=0}^{\infty} \beta^t u^i\left(c_t\left(\theta_i^k\right), l_t\left(\theta_i^k\right)/\theta_i^k\right) \geq \sum_{t=0}^{\infty} \beta^t u^i\left(c_t\left(\hat{\theta}_i^k\right), l_t\left(\hat{\theta}_i^k\right)/\theta_i^k\right)$$

for all θ_i^k, $\hat{\theta}_i^k$. An allocation is *incentive feasible* if it is incentive compatible and resource feasible in that it satisfies the resource constraints.

An allocation is a cooperative Mirrlees outcome if it maximizes

$$\omega^1 \int U^1\left(\theta_1^k\right) dJ_1\left(\theta_1^k\right) + \omega^1 \int U^2\left(\theta_2^k\right) dJ_2\left(\theta_2^k\right)$$

over the set of incentive feasible allocations, where $J_i\left(\theta_i^k\right)$ is a distribution that represents a combination of the underlying distribution H and Pareto weights over households of different types.

Suppose now that the preferences of households are separable in that

$$(59)\quad u^i\left(c_t\right) - v\left(\frac{l_t}{\theta_i^k}\right).$$

It is straightforward to show that the Mirrleesian allocation can be supported as a competitive equilibrium with nonlinear consumption and labor income taxes. Trade taxes may be needed to redistribute resources across countries as in proposition 2.

Using the same logic as that in Atkinson and Stiglitz (1976), Golosov, Kocherlakota, and Tsyvinski (2003), and Werning (2007), we have the following proposition, provided countries are connected through trade links.

Proposition 9 (Production efficiency): The Mirrleesian outcomes satisfy production efficiency so that free trade and unrestricted capital mobility are optimal.

In this formulation, workers differ from one another along a single dimension–namely, the parameter θ_i^k, which determines the effective units of labor supplied by a worker. If they differ along multiple dimensions–say, because they differ in their comparative advantage in working in the various sectors–then the planning problem becomes a multidimensional screening problem and the analysis becomes more complicated. See Hosseini and Shourideh (2018) and Costinot and Werning (2018) for analyses of optimal trade taxation with restricted systems.

V Concluding Remarks

We characterize cooperative Ramsey allocations in the global economy. We show that effective free trade and unrestricted capital mobility are optimal. In the benchmark model, Ramsey allocations can be supported by time-varying taxes on consumption and labor income. We study alternative implementations of the Ramsey allocation, including taxation of equity returns and foreign asset returns as well as corporate income. We show that it is optimal to tax all types of household assets at the same country-specific rate and not to tax corporate income. We show that border adjustments are desirable if in the benchmark model, it is optimal to have time-varying consumption taxes and trade taxes are not to be used. We clarify apparently conflicting views in the public finance and trade literatures regarding the desirability of border adjustments. We show that our results hold in a variety of trade models, and we extend our results to nonlinear tax systems.

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A Competitive Equilibrium with Consumption, Labor and Trade Taxes

The first-order conditions of the household's problem include

(A.1) $-\dfrac{u^i_{c,t}}{u^i_{n,t}} = \dfrac{(1+\tau^c_{it})\, q_{it}}{(1-\tau^n_{it})\, w_{it}},$

(A.2) $u^i_{c,t} = \dfrac{Q_t q_{it}\,(1+\tau^c_{it})}{Q_{t+1} q_{it+1}\left(1+\tau^c_{it+1}\right)} \beta u^i_{c,t+1},$

for all $t \geq 0$, where $u^i_{c,t}$ and $u^i_{n,t}$ denote the marginal utilities of consumption and labor in period t. Note that (A.2) can be used to recover the familiar interest rate parity condition,

$$\frac{u^i_{c,t}\left(1+\tau^c_{it+1}\right) e^i_{t+1}}{\beta u^i_{c,t+1}\,(1+\tau^c_{it})\, e^i_t} \text{ is the same for all } i$$

where $e^i_t \equiv q_{it}/q_{1t}$ denotes the price of the final goods in country i in units of final goods in, say, country 1–namely, the bilateral real exchange rate relative to country 1.

The first-order conditions of the firms' problems are, for all i and all $t \geq 0$,

(A.3) $p_{iit} F^i_{n,t} = w_{it},$

(A.4) $\dfrac{Q_t}{Q_{t+1}} = \dfrac{p_{iit+1}}{q_{it}} F^i_{k,t+1} + \dfrac{q_{it+1}}{q_{it}}\,(1-\delta)\,,$

where $F^i_{n,t}$ and $F^i_{k,t}$ denote the marginal products of capital and labor in period t,

(A.5) $p_{iit} = \left(1-\tau^x_{ijt}\right) p_{ijt},\ i \neq j,$

(A.6) $q_{it} G^i_{i,t} = p_{iit},$

(A.7) $q_{it} G^i_{j,t} = \left(1+\tau^m_{jit}\right) p_{jit}$, and $i \neq j$.

If we combine the household's and firm's equilibrium conditions, it can be shown that the value of the firm in (11) is

$$V_{i0} + d_{i0} = q_{i0}\left[1-\delta + G^i_{i,0} F^i_{k,0}\right] k_{i0}.$$

We can obtain the familiar condition that the returns on capital adjusted for the real exchange rates are equated across countries. To obtain this condition, note that (A.4) and (A.6) can be combined to obtain that

$$\left[G^i_{i,t+1}F^i_{k,t+1} + 1 - \delta\right] \frac{e^i_t}{e^i_{t+1}} \text{ is the same for all } i.$$

We now derive the equilibrium conditions (15)-(18). Using conditions (A.1), (A.3), and (A.6), we obtain (15). Using (A.2), (A.4), and (A.6), we obtain (16).

Using (A.7),

$$\frac{G^i_{j,t}}{G^i_{l,t}} = \frac{\left(1+\tau^m_{jit}\right) p_{jit}}{\left(1+\tau^m_{lit}\right) p_{lit}},$$

and (A.5),

$$\frac{p_{jjt}}{p_{llt}} = \frac{\left(1-\tau^x_{jit}\right) p_{jit}}{\left(1-\tau^x_{lit}\right) p_{lit}},$$

we have that

$$\frac{\left(1-\tau^x_{jit}\right)\left(1+\tau^m_{lit}\right) G^i_{j,t}}{\left(1+\tau^m_{jit}\right)\left(1-\tau^x_{lit}\right) G^i_{l,t}} = \frac{p_{jjt}}{p_{llt}}$$

is the same for all i, which is condition (17).

Using (A.4), (A.6), and (A.7), we have

$$\frac{Q_t}{Q_{t+1}} = \frac{\left(1+\tau^m_{jit+1}\right) p_{jit+1} G^i_{j,t}}{\left(1+\tau^m_{jit}\right) p_{jit} G^i_{j,t+1}} \left[G^i_{i,t+1}F^i_{k,t+1} + 1 - \delta\right],$$

and using (A.5), we obtain that

$$\frac{Q_t p_{jjt}}{Q_{t+1} p_{jjt+1}} = \frac{\left(1+\tau^m_{jit+1}\right)\left(1-\tau^x_{jit}\right) G^i_{j,t}}{\left(1-\tau^x_{jit+1}\right)\left(1+\tau^m_{jit}\right) G^i_{j,t+1}} \left[G^i_{i,t+1}F^i_{k,t+1} + 1 - \delta\right]$$

is the same for all i, which is (18).

A.1 Balance of Payments Conditions

Here, we show that in an economy with two countries $i = 1, 2$, the following balance of payments conditions must hold:

$$\sum_{t=0}^{\infty} Q_t \left[p_{ijt}y_{ijt} - p_{jt}y_{jit}\right] = -\left(1 + r_0^f\right) f_{i,0}, \text{ for } j \neq i,$$

with $\left(1 + r_0^f\right) f_{1,0} + \left(1 + r_0^f\right) f_{2,0} = 0$.

The budget constraints of the household and the government, with equality, in each country,

$$\sum_{t=0}^{\infty} Q_t \left[q_{it}\left(1 + \tau_{it}^c\right) c_{it} - \left(1 - \tau_{it}^n\right) w_{it}n_{it}\right] = \left(1 - \tau_i^W\right) a_{i0},$$

$$a_{i0} = V_{i0} + d_{i0} + Q_{-1}b_{i0} + \left(1 + r_0^f\right) f_{i0},$$

and

$$\sum_{t=0}^{\infty} Q_t \left[\tau_{it}^c q_{it}c_{it} + \tau_{it}^n w_{it}n_{it} + \tau_{ijt}^x p_{ijt}y_{ijt} + \tau_{ijt}^m p_{jit}y_{jit} - q_{it}g_{it}\right] + \tau_i^W a_{i0} = Q_{-1}b_{i0},$$

imply

$$\sum_{t=0}^{\infty} Q_t \left[q_{it}c_{it} + q_{it}g_{it} - \tau_{ijt}^x p_{ijt}y_{ijt} - \tau_{ijt}^m p_{jit}y_{jit} - w_{it}n_{it}\right] = V_{i0} + d_{i0} + \left(1 + r_0^f\right) f_{i0}.$$

Using the expression for the value of the intermediate good firm,

$$V_{i0} + d_{i0} = \sum_{t=0}^{\infty} Q_t \left[p_{iit}y_{iit} + \left(1 - \tau_{ijt}^x\right) p_{ijt}y_{ijt} - w_{it}n_{it} - q_{it}x_{it}\right],$$

we get

$$\begin{aligned} & \sum_{t=0}^{\infty} Q_t \left[q_{it}c_{it} + q_{it}g_{it} - \tau_{ijt}^x p_{ijt}y_{ijt} - \tau_{ijt}^m p_{jit}y_{jit}\right] \\ = & \sum_{t=0}^{\infty} Q_t \left[p_{iit}y_{iit} + \left(1 - \tau_{ijt}^x\right) p_{ijt}y_{ijt} - q_{it}x_{it}\right] + \left(1 + r_0^f\right) f_{i0}, \end{aligned}$$

or

$$\sum_{t=0}^{\infty} Q_t \left[q_{it}\left(c_{it}+g_{it}+x_{it}\right)-p_{iit}y_{iit}-p_{ijt}y_{ijt}-\tau_{ijt}^{m}p_{jit}y_{jit}\right]=\left(1+r_0^f\right)f_{i0}.$$

Using the zero profits condition of the final good firms,

$$\sum_{t=0}^{\infty} Q_t \left[q_{it}\left(c_{it}+g_{it}+x_{it}\right)-p_{iit}y_{iit}-\left(1+\tau_{jit}^{m}\right)p_{jit}y_{jit}\right]=0,$$

we have

$$\sum_{t=0}^{\infty} Q_t \left[p_{jit}y_{jit}-p_{ijt}y_{ijt}\right]=\left(1+r_0^f\right)f_{i0}, \text{ for } i=1,2, \text{ and } i\neq j,$$

which is the balance of payments condition, with $\left(1+r_0^f\right)f_{10}+\left(1+r_0^f\right)f_{20}=0$.

Using the final goods firms' conditions, (A.5)-(A.7), repeated here,

$$p_{iit}=\left(1-\tau_{ijt}^{x}\right)p_{ijt},\ i\neq j,$$

$$q_{it}G_{i,t}^{i}=p_{iit},$$

$$q_{it}G_{j,t}^{i}=\left(1+\tau_{jit}^{m}\right)p_{jit}, \text{ and } i\neq j.$$

together with the household's intertemporal condition, (A.2),

$$u_{c,t}^{i}=\frac{Q_t q_{it}\left(1+\tau_{it}^{c}\right)}{Q_{t+1}q_{it+1}\left(1+\tau_{it+1}^{c}\right)}\beta u_{c,t+1}^{i},$$

we obtain the balance of payments condition,

$$\sum_{t=0}^{\infty}\frac{\left(1+\tau_{i0}^{c}\right)}{\left(1+\tau_{it}^{c}\right)}\frac{\beta^{t}u_{c,t}^{i}}{u_{c,0}^{i}}\left[\frac{G_{j,t}^{i}y_{jit}}{\left(1+\tau_{jit}^{m}\right)}-\frac{G_{i,t}^{i}y_{ijt}}{\left(1-\tau_{ijt}^{x}\right)}\right]=\left(1+r_0^f\right)\frac{f_{i0}}{q_{i0}}, \text{ for } i=1,2, \text{ and } i\neq j,$$

where

$$\frac{u_{c,0}^{i}\left(1+\tau_{it}^{c}\right)}{\beta u_{c,t}^{i}\left(1+\tau_{i0}^{c}\right)}=\Pi_{s=0}^{t}\left[G_{i,s}^{i}F_{k,s}^{i}+1-\delta\right],$$

$$\sum_{t=0}^{\infty} \frac{1}{\Pi_{s=0}^{t}\left[G_{i,s}^{i}F_{k,s}^{i}+1-\delta\right]}\left[\frac{G_{j,t}^{i}y_{jit}}{\left(1+\tau_{jit}^{m}\right)}-\frac{G_{i,t}^{i}y_{ijt}}{\left(1-\tau_{ijt}^{x}\right)}\right]=\left(1+r_0^f\right)\frac{f_{i0}}{q_{i0}}, \text{ for } i=1,2, \text{ and } i\neq j.$$

B Production Efficiency

B.1 Proof of Proposition 2

Here, we show that trade taxes can be chosen to satisfy both production efficiency and the balance of payments conditions (25) . We begin by setting the trade taxes so that $\tau_{ijt}^{x}=-\tau_{ijt}^{m}$, to satisfy production efficiency. We define variables $\varkappa_{ijt}$, g_{ijt}, and h_{jit} as

$$\varkappa_{ijt}\equiv\frac{1}{\left(1-\tau_{ijt}^{x}\right)},\ g_{ijt}\equiv\frac{1}{\Pi_{s=0}^{t}\left[G_{i,s}^{i}F_{k,s}^{i}+1-\delta\right]}G_{i,t}^{i}y_{ijt},\ \text{and } h_{jit}\equiv\frac{1}{\Pi_{s=0}^{t}\left[G_{i,s}^{i}F_{k,s}^{i}+1-\delta\right]}G_{j,t}^{i}y_{jit}$$

and rewrite (25) as

$$\text{(B.1)}\quad \sum_{t=0}^{\infty}\sum_{j\neq i}\left[\varkappa_{ijt}g_{ijt}-\varkappa_{jit}h_{jit}\right]=R_i \text{ for all } i,$$

where R_i is the right-hand side of (25). Proving proposition 2 amounts to finding $\varkappa_{ijt}$ that satisfy $(B.1)$. We find it useful to restate definitions from graph theory.

Definition 1: (Direct link) We say that there is direct link between a pair of countries (i,j) if there exists some t such that either $g_{ijt}\neq 0$ or $h_{jit}\neq 0$.

Definition 2: (Indirect link) We say that a pair of countries (i,j) is indirectly linked if there is a sequence of countries $\{i,..,j\}$ such that every pair of consecutive elements in the sequence is directly linked.

Definition 3: (Connectedness) Countries are connected if for every pair of countries, there is a direct or indirect link between them.

Definition 4: (Complete cover) A sequence is a complete cover if

1. every country is an element of the sequence;
2. every pair of consecutive countries in the sequence is directly linked.

Remark 1: Notice that sequences that are a complete cover may contain the same country several times.

Lemma 1: If countries are connected, there exists a finite complete covering.

Proof: Consider a sequence that begins with country 1 and ends with country 2. Such a link exists because the countries are connected. Append to the sequence a sequence that begins with country 2 and ends with country 3. Proceed in this fashion until we end with country N.

We measure the length of a sequence by the number of elements in it. Since a finite complete covering exists, it immediately follows that there is a shortest finite complete cover.[10]

Lemma 2: The first country in a shortest complete cover appears only once in the sequence.

Proof: Suppose the first country appears more than once. Then, consider a new sequence that omits the first element. This new sequence is a complete cover, since all countries appear on it and are connected. ■

We now describe an algorithm to construct a set of policies $\{\varkappa_{ijt}\}$ for all i, j, t that satisfy $(B.1)$. The first main step is to fix a shortest complete cover for countries 1 to N and to relabel the countries so that the first element in this complete cover is relabeled as country 1 and the second element as country 2. Since country 1 has a direct link with country 2, either $g_{1,2,t} \neq 0$ or $h_{2,1,t} \neq 0$ for some t. Set $\varkappa_{1jt} = 1$ and $\varkappa_{j1t} = 1$ for all $j > 2$, and set $\varkappa_{12t}$ and $\varkappa_{21t}$ so as to satisfy the balance of payment condition for country 1. Note that $\varkappa_{12t}$ and $\varkappa_{21t}$ appear only in the balance of payment condition for countries 1 and 2. Thus the balance of payments condition for country 2 can be written as

$$\text{(B.2)} \quad \sum_{t=0}^{\infty} \sum_{j>2} \left[\varkappa_{2jt} g_{2jt} - \varkappa_{j2t} h_{j2t}\right] = R_2 - \left[\varkappa_{12t} g_{12t} - \varkappa_{21t} h_{21t}\right],$$

and the balance of payment conditions for all other countries are suitably adjusted.

Consider a new sequence that is obtained from the given shortest complete cover, but omitting country 1. This sequence is a complete cover for countries 2 to N, but it may not be the shortest complete cover for these $N-1$ countries.

The second main step in the algorithm is to fix a new shortest complete cover for these $N-1$ countries. Notice that this implies relabeling the remaining $N-1$ countries. Suppose

[10] Clearly, there may be many shortest complete covers.

that in the first stage of the procedure, country 2 becomes, say, country l with this relabeling. Repeat the procedure within the first main step to construct policies for the first element in this new sequence, recognizing that the balance of payment condition for country l (which was labeled 2 in the first stage of the procedure) is now given by the analogue of $(B.2)$, and all other conditions are suitably adjusted.

Proceeding in this fashion, we construct policies for all the countries that satisfy both production efficiency and $(B.1)$.

B.2 Restrictions on Trade Taxes and Efficiency

Here, we consider restrictions on trade taxes similar to the ones imposed in Keen and Wildasin (2004). We first consider a static version of our economy with two goods and four countries. Countries 1, 2, and 3 produce good 1, while country 4 produces good 2. The static model has no capital and no assets, and labor is inelastically supplied. We then consider a dynamic version in which for simplicity, we also ignore capital.

The static economy

Assume that at the relaxed Ramsey allocation, countries 1, 2, and 3 export good 1 to country 4, which also exports good 2 to countries 1, 2, and 3. Countries 1, 2, and 3 do not directly trade with each other. Thus, the countries are connected.

Using the notation in the appendix above, the balance of payment conditions $(B.1)$ for countries 1 to 3 can be written as

$$\begin{aligned}
&\text{(B.3)} \quad [\varkappa_{14}g_{14} - \varkappa_{41}h_{41}] &= R_1 \\
&\text{(B.4)} \quad [\varkappa_{24}g_{24} - \varkappa_{42}h_{42}] &= R_2 \\
&\text{(B.5)} \quad [\varkappa_{34}g_{34} - \varkappa_{43}h_{43}] &= R_3,
\end{aligned}$$

where $\varkappa_{ij} = 1/\left(1 - \tau^x_{ijt}\right)$. Walras law implies that the balance of payment condition for country 4 will be satisfied.

We first show that if no restrictions are imposed on the policy terms $\varkappa_{ij}$, they can be chosen to satisfy all balance of payment conditions. This is just an application to this particular case of the proof of proposition 2. To do so, first note that as countries 1 and 4 have a direct link, then either $g_{14} \neq 0$, or $h_{41} \neq 0$. Then, set the corresponding $\varkappa_{14}$ and $\varkappa_{41}$ so as to satisfy $(B.3)$. Set $\varkappa_{12} = \varkappa_{21} = \varkappa_{13} = \varkappa_{31} = 1$. Countries 2 and 3 also have a direct link with 4, so proceed accordingly.

Remark: Each country has three policy instruments. These are three export subsidies/taxes (the import tariffs are then pinned down by the production efficiency conditions $\tau^x_{ij} = -\tau^m_{ij}$). The twelve available instruments, together with the connectedness assumption, ensure that they are enough to satisfy the balance of payments conditions.

Restrictions on trade taxes

Now, we impose the restriction that trade taxes imposed by any given country can only depend on the physical characteristics of the goods and not on the origin-destination pair. This restriction is imposed on export taxes, τ^x_{ij} so

(B.6) $\tau^x_{ij} = \tau^x_i$ for $j = 1, 2, 3, \text{and } 4,$

which implies that there are only four instruments, $\varkappa_1, \varkappa_2, \varkappa_3$, and $\varkappa_4$.

The restriction is also imposed on tariffs. This implies that

(B.7) $\tau^m_{14} = \tau^m_{24} = \tau^m_{34} = \tau^m_4.$

But production efficiency requires that

(B.8) $\tau^x_{14} = -\tau^m_{14},\ \tau^x_{24} = -\tau^m_{24}$ and $\tau^x_{34} = -\tau^m_{34}$.

If we combine $(B.7)$ and $(B.8)$,

$$\tau^x_{14} = \tau^x_{24} = \tau^x_{34} = \tau^m_4,$$

which implies two additional restrictions

$$\varkappa_1 = \varkappa_2 = \varkappa_3 \equiv \varkappa'.$$

These restrictions reduce the number of independent policy instruments to two, $\varkappa_4$ and $\varkappa'$, which in general will not be sufficient to satisfy the balance of payment conditions $(B.3)$ - $(B.5)$.

The dynamic economy

Consider now an economy that consists of repeating the economy above an infinite number of periods. The balance of payments conditions are given by $(B.1)$ and repeated here:

$$\begin{aligned}
\sum_{t=0}^{\infty}\left[\varkappa_{14t}g_{14t}-\varkappa_{41t}h_{41t}\right] &= R_1 \\
\sum_{t=0}^{\infty}\left[\varkappa_{24t}g_{24t}-\varkappa_{42t}h_{42t}\right] &= R_2 \\
\sum_{t=0}^{\infty}\left[\varkappa_{34t}g_{34t}-\varkappa_{43t}h_{43t}\right] &= R_3.
\end{aligned}$$

We maintain the restriction that trade taxes cannot depend on the origin-destination pair. Thus, following the analysis of the static case, we have that

$$\begin{aligned}
\varkappa_{41t} &= \varkappa_{42t}=\varkappa_{43t}=\varkappa_{4t} \\
\varkappa_{14t} &= \varkappa_{24t}=\varkappa_{34t}=\varkappa_t',
\end{aligned}$$

which implies that there are two independent instruments each period, $\varkappa_{4t}$ and $\varkappa_t'$. If we impose these restrictions, the balance of payment conditions can be written

$$\begin{aligned}
\sum_{t=0}^{\infty}\left[\varkappa_{4t}g_{14t}-\varkappa_t'g_{41t}\right] &= R_1 \\
\sum_{t=0}^{\infty}\left[\varkappa_{4t}g_{24t}-\varkappa_t'g_{42t}\right] &= R_2 \\
\sum_{t=0}^{\infty}\left[\varkappa_{4t}g_{34t}-\varkappa_t'g_{43t}\right] &= R_3,
\end{aligned}$$

so there are now an infinite number of instruments to satisfy the three conditions.

To characterize a sufficient condition for the relaxed Ramsey allocation to be implementable, set $\varkappa'_t = \varkappa_{4t} = 1$ for all $t > 1$. Then, the conditions can be written as

$$\begin{aligned}
\sum_{t=0}^{1} [\varkappa_{4t} g_{14t} - \varkappa'_t h_{41t}] + \sum_{t=2}^{\infty} [g_{14t} - h_{41t}] &= R_1 \\
\sum_{t=0}^{1} [\varkappa_{4t} g_{24t} - \varkappa'_t h_{42t}] + \sum_{t=2}^{\infty} [g_{24t} - h_{42t}] &= R_2 \\
\sum_{t=0}^{1} [\varkappa_{4t} g_{34t} - \varkappa'_t h_{43t}] + \sum_{t=2}^{\infty} [g_{34t} - h_{43t}] &= R_3,
\end{aligned}$$

or by properly defining R'_1,

$$\begin{aligned}
\sum_{t=0}^{1} [\varkappa_{4t} g_{14t} - \varkappa'_t h_{41t}] &= R'_1 \\
\sum_{t=0}^{1} [\varkappa_{4t} g_{24t} - \varkappa'_t h_{42t}] &= R'_2 \\
\sum_{t=0}^{1} [\varkappa_{4t} g_{34t} - \varkappa'_t h_{43t}] &= R'_3.
\end{aligned}$$

This can be written as

$$\begin{bmatrix} g_{140} & h_{410} & g_{141} & h_{411} \\ g_{240} & h_{420} & g_{241} & h_{421} \\ g_{340} & h_{430} & g_{341} & h_{431} \end{bmatrix} \begin{bmatrix} \varkappa_{40} \\ -\varkappa'_0 \\ \varkappa_{41} \\ \varkappa'_1 \end{bmatrix} = \begin{bmatrix} R'_1 \\ R'_2 \\ R'_3 \end{bmatrix},$$

or, in matrix notation,

$$G\varkappa = R.$$

A sufficient condition for the relaxed Ramsey allocation to be implementable as a Ramsey equilibrium is that the matrix G be of rank 3. It is obvious that the choice of the first two periods was arbitrary, so it is required only that there exist two different periods for which the condition above holds. This argument can clearly be extended to have an arbitrary number of countries N, so that we have the following proposition.

Proposition B1: Consider a dynamic economy like the one above, extended to have $N-1$ type-1 countries. Consider the infinite-dimensional matrix formed by the coefficients g_{ijt} and h_{jit}. Suppose that there exist $N-1$ periods so that the submatrix G, induced by considering the coefficients for only these $N-1$ periods, has rank $N-1$. Then, the solution to the relaxed Ramsey problem can be implemented as a Ramsey equilibrium.

C Optimality of Explicit Free Trade with Zero Transfers

In this appendix, we show that a cooperative Ramsey solution is implemented with zero transfers across countries. We use consumption and labor income taxes, set trade taxes to zero, and solve for the optimal level of government consumption. Note that (13) can be written as

$$\left[\sum_{t=0}^{\infty} Q_t q_{it} g_{it} + Q_{-1} b_{i0}\right] - \left[\sum_{t=0}^{\infty} Q_t \left(\tau_{it}^c q_{it} c_{it} + \tau_{it}^n w_{it} n_{it}\right) + \tau_i^W a_{i0}\right] = T_{i0}.$$

The Ramsey problem is to maximize

$$\sum_{i=1}^{2} \omega^i U^i,$$

subject to the conditions

$$\sum_{t=0}^{\infty} \left[\beta^t u_{c,t}^i c_{it} + \beta^t u_{n,t}^i n_{it}\right] \geq \bar{\mathcal{W}}_i$$

$$c_{it} + g_{it} + k_{it+1} - (1-\delta) k_{it} \leq G^i\left(\overline{y}_{it}\right)$$

$$\sum_j y_{ijt} \leq F^i\left(k_{it}, n_{it}\right).$$

Let λ^i, ε_{it}, and δ_{it} be the multipliers on these three conditions. We prove the proposition for the case in which $\bar{\mathcal{W}}_i = 0$ for $i = 1, 2$. The result follows by continuity.

Proposition C1: Let $\mathcal{W}_{10} = 0$. Then there exists a weight ω^1 small enough such that $T_{10} < 0$.

Proof: The first-order conditions of the Ramsey problem include

$$\omega^1 h'(g_{1t}) = \varepsilon_{1t}.$$

Thus, as $\omega^1 \to 0$, $g_{1t} \to 0$ for all t.

Preliminary result 1.

The first-order conditions for an interior solution are

$$\omega^1 \beta^t u^1_{ct} + \lambda^1 \beta^t u^1_{ct} + \lambda^1 \beta^t \left[u^1_{cct} c_{1t} + u^1_{cnt} n_{1t}\right] = \varepsilon_{1t}$$

$$\omega^1 \beta^t u^1_{nt} + \lambda^1 \beta^t u^1_{nt} + \lambda^1 \beta^t \left[u^1_{nct} c_{1t} + u^1_{nnt} n_{1t}\right] = -\delta_{1t} F^1_{nt}$$

$$\varepsilon_{1t} G^1_{1t} = \delta_{1t}$$

$$\varepsilon_{1t} G^1_{2t} = \delta_{2t}$$

$$\varepsilon_{2t} G^2_{1t} = \delta_{1t}$$

$$\varepsilon_{2t} G^2_{2t} = \delta_{2t}$$

$$\varepsilon_{1t} = \varepsilon_{1t+1} (1 - \delta) + \delta_{1t+1} F^1_{kt}.$$

Now, replace δ_{1t} and multiply the first-order conditions by quantities

$$\omega^1 \beta^t u^1_{ct} c_{1t} + \lambda^1 \beta^t u^1_{ct} c_{1t} + \lambda^1 \beta^t \left[u^1_{cct} c^2_{1t} + u^1_{cnt} n_{1t} c_{1t}\right] = \varepsilon_{1t} c_{1t}$$

$$\omega^1 \beta^t u^1_{nt} n_{1t} + \lambda^1 \beta^t u^1_{nt} n_{1t} + \lambda^1 \beta^t \left[u^1_{nct} c_{1t} n_{1t} + u^1_{nnt} n^2_{1t}\right] = -\varepsilon_{1t} G^1_{1t} F^1_{nt} n_{1t}.$$

Add them up:

$$\beta^t \left[u^1_{ct} c_{1t} + u^1_{nt} n_{1t}\right] \left[\omega^1 + \lambda^1\right] + \lambda^1 \beta^t \left[u^1_{cct} c^2_{1t} + 2u^1_{cnt} n_{1t} c_{1t} + u^1_{nnt} n^2_{1t}\right] = \varepsilon_{1t} \left[c_{1t} - G^1_{1t} F^1_{nt} n_{1t}\right].$$

Add over time:

$$\left[\omega^1 + \lambda^1\right] \sum_{t=0}^{\infty} \beta^t \left[u^1_{ct} c_{1t} + u^1_{nt} n_{1t}\right] + \lambda^1 \sum_{t=0}^{\infty} \beta^t \left[u^1_{cct} c^2_{1t} + 2u^1_{cnt} n_{1t} c_{1t} + u^1_{nnt} n^2_{1t}\right] = \sum_{t=0}^{\infty} \varepsilon_{1t} \left[c_{1t} - G^1_{1t} F^1_{nt} n_{1t}\right].$$

Note that, since the multiplier λ^1 is non-negative and the function u is concave, the term

$$\lambda^1 \sum_{t=0}^{\infty} \beta^t \left[u^1_{cct} c^2_{1t} + 2u^1_{cnt} n_{1t} c_{1t} + u^1_{nnt} n^2_{1t}\right]$$

is negative.[11] It follows that

$$\text{(C.1)} \quad \left[\omega^1 + \lambda^1\right] \mathcal{W}_{i0} > \sum_{t=0}^{\infty} \varepsilon_{1t} \left[c_{1t} - G^1_{1t} F^1_{nt} n_{1t}\right].$$

Preliminary result 2.

We relate the term on the right-hand side,

$$\sum_{t=0}^{\infty} \varepsilon_{1t} \left[c_{1t} - G^1_{1t} F^1_{nt} n_{1t}\right],$$

to a term involving the present value of trade balances.

Owing to constant returns to scale, the Euler theorem implies

$$\text{(C.2)} \quad c_{1t} + g_{1t} + k_{1t+1} - (1-\delta) k_{1t} = G^1 (y_{11t}, y_{21t}) = G^1_{1t} y_{11t} + G^1_{2t} y_{21t}$$

$$\text{(C.3)} \quad y_{11t} + y_{12t} = F^1 (k_{1t}, n_{1t}) = F^1_{kt} k_{1t} + F^1_{nt} n_{1t}.$$

The trade balance (in units of the intermediate good produced in country 1) satisfies

$$y_{21t} q_{2t} = y_{12t} q_{1t} - TB_{1t} q_{1t},$$

or dividing by q_{1t},

$$y_{21t} \frac{q_{2t}}{q_{1t}} = y_{12t} - TB_{1t}.$$

[11] The non-negativity of the multiplier is directly implied by the Kuhn-Tucker conditions once we allow each government to make non-negative lump-sum transfers to the private agents. We omitted those transfers from the problem for simplicity.

But in a Ramsey allocation $\frac{q_{2t}}{q_{1t}} = \frac{G_2^1}{G_1^1}$, so

$$y_{21t}\frac{G_{2t}^1}{G_{1t}^1} = y_{12t} - TB_{1t}.$$

If we replace in $(C.2)$ above,

$$\begin{aligned} c_{1t} + g_{1t} + k_{1t+1} - (1-\delta)\,k_{1t} &= G_{1t}^1 y_{11t} + G_{1t}^1 y_{12t} - G_{1t}^1 TB_{1t} \\ &= G_{1t}^1\,(y_{11t} + y_{12t}) - G_{1t}^1 TB_{1t}, \end{aligned}$$

and using $(C.3)$,

$$c_{1t} + g_{1t} + k_{1t+1} - (1-\delta)\,k_{1t} = G_{1t}^1 F_{kt}^1 k_{1t} + G_{1t}^1 F_{nt}^1 n_{1t} - G_{1t}^1 TB_{1t},$$

so

$$c_{1t} - G_{1t}^1 F_{nt}^1 n_{1t} = G_{1t}^1 F_{kt}^1 k_{1t} - G_{1t}^1 TB_{1t} - g_{1t} - [k_{1t+1} - (1-\delta)\,k_{1t}]\,.$$

Multiplying each term by ε_{1t} and adding up for all t,

$$\sum_{t=0}^{\infty} \varepsilon_{1t}\left[c_{1t} - G_{1t}^1 F_{nt}^1 n_{1t}\right] = \sum_{t=0}^{\infty} \varepsilon_{1t}\left[G_{1t}^1 F_{kt}^1 k_{1t} - G_{1t}^1 TB_{1t} - g_{1t} - [k_{1t+1} - (1-\delta)\,k_{1t}]\right].$$

Recall that the first-order condition with respect to k_{1t+1} implies

$$-\varepsilon_{1t} + \left[G_{1t+1}^1 F_{kt+1}^1 + (1-\delta)\right]\varepsilon_{1t+1} = 0,$$

so we obtain the preliminary result 2:

$$\text{(C.4)}\quad \sum_{t=0}^{\infty} \varepsilon_{1t}\left[c_{1t} - G_{1t}^1 F_{nt}^1 n_{1t}\right] = -\sum_{t=0}^{\infty} \varepsilon_{1t}\left[G_{1t}^1 TB_{1t} + g_{1t}\right] + \left[G_{10}^1 F_{k0}^1 + (1-\delta)\right]\varepsilon_{10} k_{10}.$$

Proof: Using $(C.4)$ with $(C.1)$, and noting that when $\omega^1 \to 0$, $g_{1t} \to 0$ for all t, we obtain

$$\left[\omega^1 + \lambda^1\right] \mathcal{W}_{10} > -\sum_{t=0}^{\infty} \varepsilon_{1t} G_{1t}^1 TB_{1t} - \left[G_{10}^1 F_{k0}^1 + (1-\delta)\right] \varepsilon_{10} k_{10},$$

or

$$\text{(C.5)} \quad \sum_{t=0}^{\infty} \varepsilon_{1t} G_{1t}^1 TB_{1t} = \sum_{t=0}^{\infty} \delta_{1t} TB_{1t} > -\left[\omega^1 + \lambda^1\right] \mathcal{W}_{10} + \left[G_{10}^1 F_{k0}^1 + (1-\delta)\right] \varepsilon_{10} k_{10}.$$

As we assumed that $\mathcal{W}_{10} = 0$, it follows that

$$\sum_{t=0}^{\infty} \delta_{1t} TB_{1t} > \left[G_{10}^1 F_{k0}^1 + (1-\delta)\right] \varepsilon_{10} k_{10}.$$

As the right-hand side is positive, this equation implies that

$$\sum_{t=0}^{\infty} \delta_{1t} TB_{1t} > 0.$$

The δ_{1t} are the multipliers of constraints

$$(\delta_{1t}) y_{11t} + y_{12t} \leq F^1\left(k_{1t}, n_{1t}\right),$$

which is the value for the planner of the intermediate goods. Because of production efficiency, the private and social values of the intermediate goods are the same, so the present value of the trade balance is positive, which means that the transfer is negative since f_{i0} are zero.

Remark: Equation $(C.4)$ makes clear that, given that $\omega^1 \to 0$, a weaker sufficient condition is

$$-\left[\omega^1 + \lambda^1\right] \mathcal{W}_{10} + \left[G_{10}^1 F_{k0}^1 + (1-\delta)\right] \varepsilon_{10} k_{10} \geq 0,$$

or

$$\lambda^1 \mathcal{W}_{10} \leq \left[G_{10}^1 F_{k0}^1 + (1-\delta)\right] \varepsilon_{10} k_{10},$$

which is weaker than the one assumed in the proposition. This condition, however, involves multipliers, which are endogenous.

To understand the role of restricting the value for $\mathcal{W}_{10}$, imagine that it takes a value that is higher than the present value of current plus all future national incomes in country 1, when all taxes are set to zero and all government expenditures are set to zero. Any feasible allocation therefore requires transfers of resources from country 2 to country 1, independently of the values of the weights ω^i. This logic also makes clear that there are high enough values for $\mathcal{W}_{10}$ and $\mathcal{W}_{20}$ such that the set of implementable allocations is empty.

Thus far we have focused on interior allocations. It is possible to extend the proof to situations in which the solution is at corner; details are available upon request.

D Taxes on Assets

In this appendix, we show that it is possible to implement the solution of the Ramsey problem in Section III.A as a competitive equilibrium.

We consider a system with income taxation of labor and assets, including a corporate income tax. We consider a common tax on the household's returns from foreign assets and on equity returns including capital gains.

We now describe the problems of the firms and the household in each country and define a competitive equilibrium. We maintain the assumption that ownership of firms is domestic, but we will see that this is without loss of generality.

Firm

The representative intermediate good firm in each country produces and invests in order to maximize the present value of dividends, $V_{i0} + d_{i0} = \sum_{t=0}^{\infty} Q_t d_{it}$. Dividends, in units of the numeraire, d_{it}, are given by

$$\text{(D.1)}\quad d_{it} = p_{it}F(k_{it}, n_{it}) - w_{it}n_{it} - \tau_{it}^k \left[p_{it}F(k_{it}, n_{it}) - w_{it}n_{it} - q_{it}\delta k_{it}\right] - q_{it}\left[k_{it+1} - (1-\delta)k_{it}\right],$$

where τ_{it}^k is the tax rate on capital income net of depreciation.

The first-order conditions of the firm's problem are now $p_{iit}F^i_{n,t} = w_{it}$, together with

$$\text{(D.2)} \quad \frac{Q_t q_{it}}{Q_{t+1} q_{it+1}} = 1 + \left(1 - \tau^k_{it+1}\right)\left(\frac{p_{it+1}}{q_{it+1}} F^i_{k,t+1} - \delta\right).$$

Substituting for d_{it} from $(D.1)$ and using the firm's first-order conditions, it is easy to show that the present value of the dividends at time zero in units of the numeraire is given by

$$\text{(D.3)} \quad V_{i0} + d_{i0} = \sum_{t=0}^{\infty} Q_t d_{it} = \left[1 + \left(1 - \tau^k_{i0}\right)\left(\frac{p_{i0}}{q_{i0}} F_{ik,0} - \delta\right)\right] p_{i0} k_{i0}.$$

The problem of the final good firm is as it was before.

Households

The flow of funds constraint in period t for the household in country i in units of the numeraire is given by

$$\begin{aligned} \text{(D.4)} \qquad & q_{it}c_{it} + b_{it+1} + f_{it+1} + V_{it}s_{it+1} \\ = \quad & \left(1 - \tau^n_{it}\right) w_{it} n_{it} + \left[1 + r^f_t - \tau_{it}\left(r^f_t - \frac{q_{it} - q_{it-1}}{q_{it-1}}\right)\right](b_{it} + f_{it}) \\ & + (V_{it} + d_{it})\, s_{it} - \tau_{it}\left(d_{it} + V_{it} - V_{it-1} - \frac{(q_{it} - q_{it-1})\, V_{it-1}}{q_{it-1}}\right) s_{it}. \end{aligned}$$

In period 0, the constraint is

$$\begin{aligned} \text{(D.5)} \qquad & q_{i0}c_{i0} + b_{i1} + f_{i1} + V_{i0}s_{i1} \\ = \quad & \left(1 - \tau^n_{i0}\right) w_{i0} n_{i0} + \left(1 - \tau^W_i\right)\left[1 + r^f_0 - \tau_{i0}\left(r^f_0 - \frac{q_{i0} - q_{i-1}}{q_{i-1}}\right)\right](b_{i0} + f_{i0}) \\ & \left(1 - \tau^W_i\right)\left[(V_{i0} + d_{i0})\, s_{i0} - \tau_{i0}\left(d_{i0} + V_{i0} - V_{i-1} - \frac{(q_{i0} - q_{i-1})\, V_{i-1}}{q_{i-1}}\right) s_{i0}\right]. \end{aligned}$$

Dividends and capital gains are taxed at rate τ_{it} with an allowance for numeraire inflation. Returns on domestic and foreign bonds are also taxed at the same rate, τ_{it}, also with an allowance for numeraire inflation.

The household's problem is to maximize utility (1), subject to $(D.4)$; $(D.5)$; and no-Ponzi-scheme conditions, $\lim_{T\to\infty} Q_{iT+1} b_{iT+1} \geq 0$ and $\lim_{T\to\infty} Q_{iT+1} f_{iT+1} \geq 0$ with

(D.6) $\dfrac{Q_{it}}{Q_{it+1}} = (1 - \tau_{it+1}) \left(1 + r^f_{t+1}\right) + \tau_{it+1} \dfrac{q_{it+1}}{q_{it}}$ with $Q_{i0} = 1$.

The first-order conditions of the household's problem in each country are, for $t \geq 0$,

(D.7) $-\dfrac{u^i_{c,t}}{u^i_{n,t}} = \dfrac{q_{it}}{(1 - \tau^n_{it})\, w_{it}}$,

(D.8) $u^i_{c,t} = \dfrac{Q_{it} q_{it}}{Q_{it+1} q_{it+1}} \beta u^i_{c,t+1}$,

and

(D.9) $\dfrac{Q_{it}}{Q_{it+1}} = \dfrac{(V_{it+1} + d_{it+1}) - \tau_{it+1} \left(V_{it+1} - V_{it} + d_{it+1} - \frac{q_{it+1} - q_{it}}{q_{it}} V_{it}\right)}{V_{it}}$.

Condition $(D.9)$ implies that

$$1 + r^f_{t+1} = \frac{V_{it+1} + d_{it+1}}{V_{it}}.$$

This condition on the two returns can be written, using $1 + r^f_{t+1} = \frac{Q_t}{Q_{t+1}}$, as

$$Q_t V_{it} = Q_{t+1} V_{it+1} + Q_{t+1} d_{it+1}.$$

Imposing that $\lim_{T\to\infty} Q_{T+1} V_{iT+1} = 0$, then

$$V_{it} = \sum_{s=0}^{\infty} \frac{Q_{t+1+s}}{Q_t} d_{it+1+s}.$$

The present value of dividends for the households of country i is a different expression from the one above because they pay taxes on the asset income. Using (D.9), we have that

$$V_{i0} = \sum_{t=0}^{\infty} \left(1 - \hat{\tau}^a_{it+1}\right) Q_{it+1} d_{it+1},$$

where $1-\hat{\tau}^a_{it+1} = \Pi^t_{s=0}\left(1-\hat{\tau}_{is+1}\right)$, and $1-\hat{\tau}_{it+1} = \frac{(1-\tau_{it+1})}{\left(1-\tau_{it+1}\frac{q_{it+1}Q_{it+1}}{q_{it}Q_{it}}\right)}$. The values are the same, since $\left(1-\hat{\tau}^a_{it+1}\right)Q_{it+1} = Q_{t+1}$. This condition is obtained from (D.6).

The value of the firm for the households in country i, including the dividends in period 0, is

$$\text{(D.11)} \qquad V_{i0}+d_{i0}-\tau_{i0}\left(V_{i0}+d_{i0}-\frac{q_{i0}V_{i-1}}{q_{i-1}}\right)$$
$$= \left(1-\tau_{i0}\right)\left(V_{i0}+d_{i0}\right)+\tau_{i0}\frac{q_{i0}V_{i-1}}{q_{i-1}}.$$

Notice that the market price of the firm before dividends, $V_{i0}+d_{i0}$, is a linear function of the value for the firm for the households of each country, so that the solution of the maximization problem of the firm also maximizes shareholder value. That would also be the case if the stocks were held by the households of the foreign country. This means that the restriction that firms are owned by the domestic households is without loss of generality.

Using the no-Ponzi-games condition, the budget constraints of the household, (D.4) and (D.5), can be consolidated into the single budget constraint,

$$\sum_{t=0}^{\infty} Q_{it}\left[q_{it}c_{it}-\left(1-\tau^n_{it}\right)w_{it}n_{it}\right] = \left(1-\tau^W_i\right)a_{i0},$$

where

$$\text{(D.12)} \quad a_{i0} = \left(1-\tau_{i0}\right)\left(V_{i0}+d_{i0}\right)+\tau_{i0}\frac{q_{i0}V_{i-1}}{q_{i-1}}+\left[1+r^f_0-\tau_{i0}\left(1+r^f_0-\frac{q_{i0}}{q_{i-1}}\right)\right]\left(b_{i0}+f_{i0}\right).$$

Using (D.3) as well as $s_0=1$, the initial asset holdings in (D.12) can be written as

$$a_{i0} = \left(1-\tau_{i0}\right)q_{i0}\left[1+\left(1-\tau^k_{i0}\right)\left(G^i_{i,0}F_{ik,0}-\delta\right)\right]k_{i0}+\tau_{i0}\frac{q_{i0}V_{i-1}}{q_{i-1}}$$
$$+\left[1+r^f_0-\tau_{i0}\left(1+r^f_0-\frac{q_{i0}}{q_{i-1}}\right)\right]\left(b_{i0}+f_{i0}\right).$$

The interest rate parity condition is obtained from

$$\frac{Q_t}{Q_{t+1}} = \frac{q_{it+1}}{q_{it}} \left[1 + \left(1 - \tau^k_{it+1}\right) \left(\frac{p_{it+1}}{q_{it+1}} F^i_{k,t+1} - \delta \right) \right]$$

for $i = 1, 2$, or

$$\frac{q_{1t+1}}{q_{1t}} \left[1 + \left(1 - \tau^k_{1t+1}\right) \left(\frac{p_{1t+1}}{q_{1t+1}} F^1_{k,t+1} - \delta \right) \right] = \frac{q_{2t+1}}{q_{2t}} \left[1 + \left(1 - \tau^k_{2t+1}\right) \left(\frac{p_{2t+1}}{q_{2t+1}} F^2_{k,t+1} - \delta \right) \right].$$

Using the first-order conditions of the firms to replace the relative prices of the intermediate and final goods, it follows that

$$\text{(D.13)} \qquad \begin{aligned} & \frac{G^1_{j,t}}{G^1_{j,t+1}} \left[1 + \left(1 - \tau^k_{1t+1}\right) \left(G^1_{1,t+1} F^1_{k,t+1} - \delta \right) \right] \\ = \; & \frac{G^2_{j,t}}{G^2_{j,t+1}} \left[1 + \left(1 - \tau^k_{2t+1}\right) \left(G^2_{2,t+1} F^2_{k,t+1} - \delta \right) \right], \text{ for } j = 1, 2. \end{aligned}$$

To get production efficiency–that is, to satisfy (9)–we need to either set the two tax rates to zero or pick τ^k_{1t+1} and τ^k_{2t+1} according to

$$\begin{aligned} & \tau^k_{1t+1} \left(G^1_{1,t+1} F^1_{k,t+1} - \delta \right) \\ = \; & \tau^k_{2t+1} \left(G^1_{1,t+1} F^1_{k,t+1} - \delta - \left(\frac{G^1_{j,t+1}/G^2_{j,t+1}}{G^1_{j,t}/G^2_{j,t}} - 1 \right) \right), \text{ for } j = 1, 2. \end{aligned}$$

Using the intertemporal condition of the household (D.8) and

$$\frac{Q_{it}}{Q_{it+1}} = \left(1 - \tau_{it+1}\right) \frac{Q_t}{Q_{t+1}} + \tau_{it+1} \frac{q_{it+1}}{q_{it}}$$

obtained from (D.6), together with $\frac{Q_t}{Q_{t+1}} = 1 + r^f_{t+1}$, and combining them with the firm's condition (D.2), together with the first-order conditions of firms' production decisions, we obtain

$$\text{(D.14)} \quad \frac{u^i_{c,t}}{\beta u^i_{c,t+1}} = 1 + \left(1 - \tau_{it+1}\right) \left(1 - \tau^k_{it+1}\right) \left(G^i_{i,t+1} F^i_{k,t+1} - \delta \right).$$

The marginal conditions in this economy can be summarized by

$$\text{(D.15)} \quad -\frac{u^i_{c,t}}{u^i_{n,t}} = \frac{1}{(1-\tau^n_{it})\, G^i_{i,t} F^i_{n,t}},$$

as well as the intertemporal condition (D.14), the interest rate parity condition (D.13), and condition (8), for all $t \geq 0$.

The Ramsey allocation can be implemented with a (possibly time-varying) common tax on home and foreign assets. Corporate income taxes in both countries either must be set to zero or must be set according to the difference in real returns in the goods of the two countries to ensure production efficiency. In this economy with a common tax on domestic equity and foreign returns, firms use a common price to value dividends. If relaxed, the restriction that firms are owned by the domestic residents would not change the implementable allocations.

D.1 Corporate Income Taxes with Deductibility

Here, we consider an implementation with taxes on assets in which the corporate income taxes allow for the deduction of investment expenses. We will show that, as long as the tax rate is the same across countries or constant over time, the Ramsey allocation can be implemented with such taxes.

The representative intermediate good firm in each country produces and invests in order to maximize the present value of dividends, $V_{i0} + d_{i0} = \sum_{t=0}^{\infty} Q_t d_{it}$, where Q_t is the pretax discount factor. Dividends, d_{it}, in units of the numeraire, are now given by

$$d_{it} = \left(1-\tau^k_{it}\right)\left[p_{it}F(k_{it}, n_{it}) - w_{it}n_{it}\right] - \left(1-\tau^k_{it}\right) q_{it}\left[k_{it+1} - (1-\delta)k_{it}\right],$$

where τ^k_{it} is the tax rate on corporate income net of investment expenses.

The first-order conditions of the firm's problem are now $p_{iit}F^i_{n,t} = w_{it}$, together with

$$\frac{Q_t}{Q_{t+1}}\frac{q_{it}}{q_{it+1}} = \frac{\left(1-\tau^k_{it+1}\right)}{\left(1-\tau^k_{it}\right)}\left[\frac{p_{it+1}}{q_{it+1}}F_k(k_{it+1}, n_{it+1}) - (1-\delta)\right].$$

This implies the following interest-rate parity condition:

$$\frac{q_{it+1}\left(1-\tau_{it+1}^{k}\right)}{q_{it}\left(1-\tau_{it}^{k}\right)}\left[\frac{p_{it+1}}{q_{it+1}}F_k(k_{it+1},n_{it+1})-(1-\delta)\right] \text{ has to be the same across } i.$$

The profit maximization conditions for the final goods producers are, for all i,

$$p_{ii,t} = p_{ij,t} \equiv p_{i,t},\ i \neq j,$$

$$q_{i,t}G_{i,t}^{i} = p_{ii,t},$$

$$q_{i,t}G_{j,t}^{i} = p_{ji,t},\ i \neq j.$$

This implies

$$q_{i,t}G_{j,t}^{i} = p_{j,t}, \text{ for all } i \text{ and } j.$$

It follows that the interest rate parity condition can be written as

$$\frac{G_{j,t}^{i}\left(1-\tau_{it+1}^{k}\right)}{G_{j,t+1}^{i}\left(1-\tau_{it}^{k}\right)}\left[G_{i,t+1}^{i}F_k(k_{it+1},n_{it+1})-(1-\delta)\right] \text{ has to be the same across } i.$$

The dynamic production efficiency condition is satisfied if τ_{it}^{k} is the same across countries or if it is constant over time.

The households conditions are

$$u_{c,t}^{i} = \frac{Q_{it}q_{it}}{Q_{it+1}q_{it+1}}\beta u_{c,t+1}^{i},$$

with

$$\frac{Q_{it}}{Q_{it+1}} = \left(1-\tau_{it+1}\right)\frac{Q_t}{Q_{t+1}} + \tau_{it+1}\frac{q_{it+1}}{q_{it}}.$$

These conditions, together with

$$\frac{Q_t}{Q_{t+1}}\frac{q_{it}}{q_{it+1}} = \frac{\left(1-\tau_{it+1}^{k}\right)}{\left(1-\tau_{it}^{k}\right)}\left[\frac{p_{it+1}}{q_{it+1}}F_k(k_{it+1},n_{it+1})-(1-\delta)\right],$$

imply

$$\frac{u^i_{c,t}}{\beta u^i_{c,t+1}} = (1-\tau_{it+1})\frac{\left(1-\tau^k_{it+1}\right)}{\left(1-\tau^k_{it}\right)}\left[G^i_{i,t+1}F_k(k_{it+1},n_{it+1})-(1-\delta)\right]+\tau_{it+1}.$$

E Value-Added Taxes

E.1 Algebra for Border-Adjusted VAT

Here, we display the algebra needed to prove proposition 7. The first-order conditions of the household's problem now include

(E.1) $-\frac{u^i_{c,t}}{u^i_{n,t}} = \frac{q_{it}}{(1-\tau^n_{it})\,w_{it}},\ t\geq 0$

and

(E.2) $u^i_{c,t} = \frac{Q_t q_{it}}{Q_{t+1}q_{it+1}}\beta u^i_{c,t+1},\ t\geq 0.$

The first-order conditions of the firms' problems for an interior solution are

(E.3) $p_{iit}\,(1-\tau^v_{it})\,F^i_{n,t} = w_{it}$

(E.4) $Q_t q_{it}\,(1-\tau^v_{it}) = Q_{t+1}p_{iit+1}\left(1-\tau^v_{it+1}\right)F^i_{k,t+1} + Q_{t+1}q_{it+1}\left(1-\tau^v_{it+1}\right)(1-\delta))$

(E.5) $p_{iit}\,(1-\tau^v_{it}) = p_{ijt}$, for $j\neq i$

(E.6) $q_{it}G^i_{i,t} = p_{iit}$

(E.7) $q_{it}\,(1-\tau^v_{it})\,G^i_{j,t} = p_{jit}$, for $j\neq i$.

The households' and firms' conditions can be manipulated to obtain (44) and (45), together with (8) and (9).

Conditions (E.1), (E.3), and (E.6) can be used to obtain (44). Conditions (E.2), (E.4), and (E.6) can be used to obtain (45). To see that the conditions (E.3)-(E.7) imply

(8) and (9), note that $(E.5)$-$(E.6)$ imply

$$q_{it}\left(1-\tau_{it}^{v}\right)G_{j,t}^{i}=p_{jit}=p_{jjt}\left(1-\tau_{jt}^{v}\right)$$

and

$$q_{it}G_{i,t}^{i}=p_{iit}=\frac{p_{ijt}}{\left(1-\tau_{it}^{v}\right)},$$

implying

$$\frac{G_{j,t}^{i}}{G_{i,t}^{i}}=\frac{p_{jit}}{p_{ijt}}=\frac{p_{jjt}\left(1-\tau_{jt}^{v}\right)}{q_{jt}\left(1-\tau_{jt}^{v}\right)G_{i,t}^{j}}=\frac{G_{j,t}^{j}}{G_{i,t}^{j}}.$$

Note also that $(E.3)$ and $(E.6)$ imply

$$\frac{Q_t}{Q_{t+1}}=\frac{q_{it+1}\left(1-\tau_{it+1}^{v}\right)}{q_{it}\left(1-\tau_{it}^{v}\right)}\left[G_{i,t+1}^{i}F_{k,t+1}^{i}+1-\delta\right],$$

so that

$$\frac{q_{it+1}\left(1-\tau_{it+1}^{v}\right)}{q_{it}\left(1-\tau_{it}^{v}\right)}\left[G_{i,t+1}^{i}F_{k,t+1}^{i}+1-\delta\right]=\frac{q_{jt+1}\left(1-\tau_{jt+1}^{v}\right)}{q_{jt}\left(1-\tau_{jt}^{v}\right)}\left[G_{j,t+1}^{j}F_{k,t+1}^{j}+1-\delta\right].$$

Since, from $(E.5)$ - $(E.6)$,

$$q_{jt}\left(1-\tau_{jt}^{v}\right)G_{i,t}^{j}=p_{ijt}=p_{iit}\left(1-\tau_{it}^{v}\right)=q_{it}G_{i,t}^{i}\left(1-\tau_{it}^{v}\right),\text{ for } j\neq i,$$

we obtain

$$\frac{G_{i,t}^{i}}{G_{i,t+1}^{i}}\left[G_{i,t+1}^{i}F_{k,t+1}^{i}+1-\delta\right]=\frac{G_{i,t}^{j}}{G_{i,t+1}^{j}}\left[G_{j,t+1}^{j}F_{k,t+1}^{j}+1-\delta\right].$$

Comparing the four equilibrium conditions, (44) - (9), with the corresponding ones in the economy with consumption, labor income, and trade taxes, (15)-(18), we obtain proposition 7.

E.2 Algebra for VAT Without BA

The first-order conditions in the economy with VAT without border adjustments (with trade taxes) include the households' conditions $(E.1)$ and $(E.2)$, which are the same as with border adjustments, repeated here,

$$-\frac{u^i_{c,t}}{u^i_{n,t}} = \frac{q_{it}}{(1-\tau^n_{it})\, w_{it}},\ t \geq 0,$$

and

$$u^i_{c,t} = \frac{Q_t q_{it}}{Q_{t+1} q_{it+1}} \beta u^i_{c,t+1},\ t \geq 0;$$

and the first-order conditions for the final good firms, $(E.3)$ and $(E.4)$, which are also the same as with border adjustments,

$$p_{iit}\,(1-\tau^v_{it})\, F^i_{n,t} = w_{it},$$

$$\frac{Q_t}{Q_{t+1}} = \frac{p_{iit+1}\left(1-\tau^v_{it+1}\right)}{q_{it}\,(1-\tau^v_{it})} F^i_{k,t+1} + \frac{q_{it+1}\left(1-\tau^v_{it+1}\right)}{q_{it}\,(1-\tau^v_{it})}\,(1-\delta));$$

as well as the conditions for the intermediate good firms, $(A.5)$-$(A.7)$, repeated here,

$$p_{iit} = \left(1-\tau^x_{ijt}\right) p_{ijt},\ i \neq j,$$

$$q_{it} G^i_{i,t} = p_{iit},$$

$$q_{it} G^i_{j,t} = \left(1+\tau^m_{jit}\right) p_{jit},\ i \neq j.$$

In order to show that these conditions can be written as (44)-(48), note first that (44) and (45) can be obtained as in the case with border adjustments, using $(E.1)$, $(E.2)$, $(E.3)$, $(E.4)$, and $(A.6)$. In order to obtain (47), note that $(A.5)$-$(A.7)$ imply

$$\frac{q_{it} G^i_{j,t}}{q_{it} G^i_{i,t}} = \frac{\left(1+\tau^m_{jit}\right) p_{jit}}{p_{iit}} = \frac{p_{jjt}}{\left(1-\tau^x_{ijt}\right) p_{ijt}} = \frac{q_{jt} G^j_{j,t}}{q_{jt} G^j_{i,t}},\ i \neq j.$$

Condition (48) is obtained using $(E.4)$ and $(A.6)$, so that

$$\frac{q_{it+1}\left(1-\tau_{it+1}^{v}\right)}{q_{it}\left(1-\tau_{it}^{v}\right)}\left[G_{i,t+1}^{i}F_{k,t+1}^{i}+1-\delta\right]=\frac{q_{jt+1}\left(1-\tau_{jt+1}^{v}\right)}{q_{jt}\left(1-\tau_{jt}^{v}\right)}\left[G_{j,t+1}^{j}F_{k,t+1}^{j}+1-\delta\right],$$

and from $(A.5)$-$(A.7)$,

$$q_{jt}G_{i,t}^{j}=\left(1+\tau_{ijt}^{m}\right)p_{ijt}=\frac{\left(1+\tau_{ijt}^{m}\right)q_{it}G_{i,t}^{i}}{\left(1-\tau_{ijt}^{x}\right)},\ i\neq j,$$

so that

$$\frac{\left(1-\tau_{it+1}^{v}\right)}{\left(1-\tau_{it}^{v}\right)}\left[G_{i,t+1}^{i}F_{k,t+1}^{i}+1-\delta\right]=\frac{\frac{\left(1+\tau_{ijt+1}^{m}\right)G_{i,t+1}^{i}}{\left(1-\tau_{ijt}^{x}\right)G_{i,t+1}^{j}}\left(1-\tau_{jt+1}^{v}\right)}{\frac{\left(1+\tau_{ijt}^{m}\right)G_{i,t}^{i}}{\left(1-\tau_{ijt}^{x}\right)G_{i,t}^{j}}\left(1-\tau_{jt}^{v}\right)}\left[G_{j,t+1}^{j}F_{k,t+1}^{j}+1-\delta\right],\ i\neq j.$$

E.3 Border Adjustments and Lerner Symmetry

Lemma 1 We start by proving Lemma 1. Consider that country 1 introduces an import tariff, τ_{21t}^{m}, and an export tax on all goods, τ_{12t}^{x}. The conditions for the household and firms in country 1 are

(E.8) $-\frac{u_{c,t}^{1}}{u_{n,t}^{1}}=\frac{(1+\tau_{1t}^{c})\,q_{1t}}{(1-\tau_{1t}^{n})\,w_{1t}},$

(E.9) $\frac{u_{c,t}^{1}}{(1+\tau_{1t}^{c})}=\frac{Q_{t}q_{1t}}{Q_{t+1}q_{1t+1}}\frac{\beta u_{c,t+1}^{1}}{\left(1+\tau_{1t+1}^{c}\right)},$

(E.10) $F_{n,t}^{1}=\frac{w_{1t}}{p_{11t}},$

(E.11) $\frac{Q_{t}}{Q_{t+1}}=\frac{p_{11t+1}}{q_{1t}}F_{k,t+1}^{1}+\frac{q_{1t+1}}{q_{1t}}\left(1-\delta\right),$

(E.12) $G_{1,t}^{1}=\frac{p_{11t}}{q_{1t}},$

(E.13) $p_{11t}=(1-\tau_{12t}^{x})\,p_{12t},$

(E.14) $q_{1t}G_{2,t}^{1}=(1+\tau_{21t}^{m})\,p_{21t}.$

The proof of Lemma 1 follows by inspecting the first-order conditions above, $(E.8)$

through $(E.14)$, as well as the household budget constraints written as (21) and (22) and satisfied with an appropriate choice of $\hat{\tau}_1$,

$$\mathcal{W}_{10} = (1-\hat{\tau}_1)\frac{u^1_{c,0}}{(1+\tau^c_{10})}\left[\left(1-\delta+G^1_{1,0}F^1_{k,0}\right)k_{10}+Q_{-1}\frac{b_{10}}{\hat{q}_{10}}+\left(1+r^f_0\right)\frac{f_{1,0}}{\hat{q}_{10}}\right].$$

Conditions $(E.8)$ through $(E.14)$ are satisfied in the economy with $(1-\hat{\tau}^x_{12t}) = \kappa_s(1-\tau^x_{12t})$ and $(1+\hat{\tau}^m_{21t}) = \kappa_s(1+\tau^m_{21t})$ with $\hat{p}_{11t} = \kappa_s p_{11t}$, $\hat{q}_{1t} = \kappa_s q_{1t}$, $\hat{w}_{1t} = \kappa_s w_{1t}$ for $\kappa_t = \kappa_s$ Here, we have assumed that b_{10} and $f_{1,0}$ are fixed in units of the world numeraire. Notice that the proof goes through even if these initial conditions are fixed in real terms. The higher price of the final good in country 1 (and the price of the imported good after the tariff, together with the price of the exported good after the subsidy) reduces the value of domestic and foreign assets, so that the government must compensate that with a lower tax on initial wealth $\hat{\tau}_1$. There is no need to adjust transfers to satisfy the balance of payments condition for country $i=1$ in (14).

Lemma 2 Let tildes denote prices in terms of domestic currency. Let E_t denote domestic currency per numeraire. Then, for example, $\tilde{p}_{11t} = E_t p_{11t}$. Now, when we multiply all the trade policy terms by κ, it is equivalent to letting $\hat{E}_t = \frac{E_t}{\kappa}$ (if $\kappa > 1$, the domestic currency appreciates) and leaving all domestic prices denoted in domestic currency unaffected.

Then, conditions $(E.8)$ through $(E.14)$ can be written as

$$-\frac{u^1_{c,t}}{u^1_{n,t}} = \frac{(1+\tau^c_{1t})\,\tilde{q}_{1t}}{(1-\tau^n_{1t})\,\tilde{w}_{1t}},$$

$$\frac{u^1_{c,t}}{(1+\tau^c_{1t})} = \frac{Q_t}{Q_{t+1}}\frac{\tilde{q}_{1t}}{\tilde{q}_{1t+1}}\frac{e_{t+1}}{e_t}\frac{\beta u^1_{c,t+1}}{\left(1+\tau^c_{1t+1}\right)},$$

$$F^1_{n,t} = \frac{\tilde{w}_{1t}}{\tilde{p}_{11t}},$$

$$\frac{Q_t}{Q_{t+1}} = \frac{\tilde{p}_{11t+1}}{\tilde{q}_{1t}}\frac{e_t}{e_{t+1}}F^1_{k,t+1} + \frac{\tilde{q}_{1t+1}}{\tilde{q}_{1t}}\frac{e_t}{e_{t+1}}(1-\delta),$$

$$G^1_{1,t} = \frac{\tilde{p}_{11t}}{\tilde{q}_{1t}},$$

$$\tilde{p}_{11t} = E_t(1-\tau^x_{12t})\,p_{12t},$$

$$\tilde{q}_{1t}G^1_{2,t} = E_t\left(1+\tau^m_{21t}\right)p_{21t}.$$

The proof of Lemma 2 follows by inspecting the first-order conditions above, as well as the household budget constraints written as (21) and (22) and satisfied with an appropriate choice of $\hat{\tau}_1$, as long as foreign assets are denominated in the world numeraire, so as to satisfy

$$\mathcal{W}_{10} = \left(1-\hat{\tau}_1\right)\frac{u^1_{c,0}}{\left(1+\tau^c_{10}\right)}\left[\left(1-\delta+G^1_{1,0}F^1_{k,0}\right)k_{10} + Q_{-1}\frac{b_{10}}{\tilde{q}_{10}} + \left(1+r^f_0\right)\frac{f_{1,0}}{\tilde{q}_{10}}\frac{e_0}{\kappa}\right].$$

There is no need to adjust transfers to satisfy the balance of payments condition for country $i=1$ in (14).

Suppose now that net foreign assets were denominated in the domestic numeraire. The value of initial wealth is given by

$$\mathcal{W}_{10} = \left(1-\tau_1\right)\frac{u^1_{c,0}}{\left(1+\tau^c_{10}\right)}\left[\left(1-\delta+G^1_{1,0}F^1_{k,0}\right)k_{10} + Q_{-1}\frac{b_{10}}{\tilde{q}_{1,0}} + \left(1+r^f_0\right)\frac{f_{1,0}}{\tilde{q}_{1,0}}\right].$$

Note that in this case, there is no change in the real value of domestic public debt and foreign assets, so that there is no need to change τ_1. On the other hand, there is a need to change the level of international transfers, since the balance of payments condition is now

$$\sum_{t=0}^{\infty} Q_t\left[p_{12t}y_{12t} - p_{21t}y_{21t}\right] = -\left(1+r^f_0\right)\frac{f_{1,0}\kappa}{E_0} - \hat{T}_{10}.$$

Since the foreign assets are denominated in domestic currency, they are now worth more in units of foreign currency, and country 1 would have to receive lower transfers.

Nonuniform changes in trade taxes

We start by taking international prices p_{21t}, p_{12t}, and Q_t and allocations as given. We multiply the trade taxes in country 1, $(1+\tau^m_{21t})$ and $(1-\tau^x_{12t})$, by $\kappa_t > 0$. The equilibrium conditions become

$$-\frac{u^1_{c,t}\left(1-\tau^n_{1t}\right)}{u^1_{n,t}\left(1+\tau^c_{1t}\right)} = \frac{q_{1t}}{w_{1t}},$$

$$\frac{u^1_{c,t}}{(1+\tau^c_{1t})} = \frac{q_{1t}Q_t}{q_{1t+1}Q_{t+1}} \frac{\beta u^1_{c,t+1}}{\left(1+\tau^c_{1t+1}\right)},$$

$$F^1_{n,t} = \frac{w_{1t}}{p_{11t}},$$

$$\frac{Q_t}{Q_{t+1}} = \frac{p_{11t+1}}{q_{1t}} F^1_{k,t+1} + \frac{q_{1t+1}}{q_{1t}} (1-\delta),$$

$$\frac{1}{(1-\tau^x_{12t})\, p_{12t}} = \frac{\kappa_t}{p_{11t}},$$

$$G^1_{1,t} = \frac{p_{11t}}{q_{1t}},$$

$$\frac{G^1_{2,t}}{p_{21t}\,(1+\tau^m_{21t})} = \frac{\kappa_t}{q_{1t}}.$$

In order for κ_t to be neutral, it must be that $\frac{\kappa_t}{q_{1t}}$, $\frac{\kappa_t}{p_{11t}}$, $\frac{q_{1t}}{w_{1t}}$, and $\frac{q_{1t}}{q_{1t+1}}$ are kept constant. This can happen only if $\kappa_t = \kappa$.

Changes in trade taxes may also be neutral if both countries change them in particular ways. To see this, let both countries multiply $\left(1+\tau^m_{jit}\right)$ and $\left(1-\tau^x_{ijt}\right)$ by κ_{it}, for $i = 1, 2$ and $j \neq i$. The equilibrium conditions can be written as

$$-\frac{u^i_{c,t}}{u^i_{n,t}} = \frac{(1+\tau^c_{it})}{(1-\tau^n_{it})\, G^i_{i,t} F^i_{n,t}},$$

$$\frac{u^i_{c,t}}{\beta u^i_{c,t+1}} = \frac{(1+\tau^c_{it})}{\left(1+\tau^c_{it+1}\right)} \left[G^i_{i,t+1} F^i_{k,t+1} + 1 - \delta\right],$$

$$\frac{G^1_{2,t}}{G^1_{1t}} = \frac{\kappa_{1t}\,(1+\tau^m_{21t})}{\kappa_{1t}\,(1-\tau^x_{12t})} \frac{\kappa_{2t}\,(1+\tau^m_{12t})}{\kappa_{2t}\,(1-\tau^x_{21t})} \frac{G^2_{2t}}{G^2_{1,t}},$$

$$\frac{\kappa_{2t}\,(1+\tau^m_{12t})}{\kappa_{2t+1}\left(1+\tau^m_{12t+1}\right)} \frac{\kappa_{1t+1}\left(1-\tau^x_{12t+1}\right)}{\kappa_{1t}\left(1-\tau^x_{12t+1}\right)} \frac{G^1_{1t}}{G^1_{1t+1}} \left[G^1_{1,t+1} F^1_{k,t+1} + 1 - \delta\right] = \frac{G^2_{1,t}}{G^2_{1,t+1}} \left[G^2_{2,t+1} F^2_{k,t+1} + 1 - \delta\right].$$

If the adjustments are such that $\frac{\kappa_{1t+1}}{\kappa_{1t}} = \frac{\kappa_{2t+1}}{\kappa_{2t}}$, the policy is neutral. The nominal intertemporal price, $\frac{Q_t}{Q_{t+1}}$, is adjusting by the same amount, $\frac{\kappa_{1t+1}}{\kappa_{1t}}$.

F Non-Cooperative Foundations of Cooperative Equilibria

Here, we provide explicit non-cooperative foundations for the cooperative Ramsey equilibria in our dynamic environment.

We begin by describing a static model that is a two-country version of our dynamic model. The static model has no capital, no assets, and no government consumption; labor is inelastically supplied.

The households in each country i have preferences over consumption of the country specific final good c_i, labor n_i, and public consumption g_i, $u^i(c_i, n_i) + h^i(g_i)$. Firms in country i produce a country-specific intermediate good y_i, according to

$$\text{(F.1)} \quad \sum_{j=1}^{N} y_{ij} = y_i = F^i n_i,$$

where y_{ij} denotes the quantity of intermediate goods produced in country i and used in country j and F^i is a parameter. The technology for producing the final good is

$$\text{(F.2)} \quad c_i + g_i \leq G^i(y_{1i}, y_{2i}),$$

where G^i is constant returns to scale.

If lump-sum taxes in each country, as well as transfers across countries, are available, the allocations on the Pareto frontier satisfy the following efficiency conditions:

$$\text{(F.3)} \quad -\frac{u_c^i}{u_n^i} = \frac{1}{G_i^i F_n^i},$$

$$\text{(F.4)} \quad \frac{G_j^i}{G_i^i} \text{ is the same across countries } i,\ j \neq i,$$

which, together with the resource constraints, characterize the Pareto frontier.

Consider now the economy with distorting labor income taxes, τ_i^n; taxes levied on exports shipped from country i to country j, τ_{ij}^x; and a tariff, τ_{ij}^m, levied on imports shipped from country i to country j.

Firms

Each country has two representative firms. The *intermediate good firm* in each country uses the technology in $(F.1)$ to produce the intermediate good using labor. The intermediate good firm maximizes profits given by

(F.5) $p_{ii}y_{ii} + \left(1 - \tau_{ij}^{x}\right) p_{ij}y_{ij} - w_i n_i$, for $j \neq i$

subject to $(F.1)$. Here, p_{ij} is the price of the intermediate good produced in country i and sold in country j and w_i is the wage rate, all in units of a common world numeraire.

The *final goods firm* of country i chooses the quantities of intermediate goods to maximize profits,

$$q_i G^i\left(y_{ii}, y_{ji}\right) - p_{ii}y_{ii} - \left(1 + \tau_{ji}^{m}\right) p_{ji}y_{ji}, \text{for } j \neq i.$$

Households

The household problem in country i is to maximize utility subject to the budget constraint

(F.6) $q_i c_i - \left(1 - \tau_i^{n}\right) w_i n_i \leq 0.$

Governments

The budget constraint of the government of country i is given by

(F.7) $\tau_i^{n} w_i n_i + \tau_{ji}^{m} p_{ji}y_{ji} + \tau_{ij}^{x} p_{ij}y_{ij} = q_i g_i,\ j \neq i.$

Combining the budget constraints of the government and the household (with equality) in each country, we obtain the balance of payments condition of country i:

(F.8) $p_{ij}y_{ij} - p_{ji}y_{ji} = 0,\ j \neq i.$

A *competitive equilibrium* is defined in the usual fashion.

Next, we characterize the competitive equilibrium. To do so, note that the first-order conditions of the household's problem include

$$\text{(F.9)} \quad -\frac{u_c^i}{u_n^i} = \frac{q_i}{(1-\tau_i^n)\, w_i}.$$

The first-order conditions of the firms' problems are, for all i, $p_{ii}F^i = w_i$,

$$\text{(F.10)} \quad p_{ii} = \left(1-\tau_{ij}^x\right) p_{ij}, \; i \neq j,$$

$$\text{(F.11)} \quad q_i G_i^i = p_{ii},$$

$$\text{(F.12)} \quad q_i G_j^i = \left(1+\tau_{ji}^m\right) p_{ji}, \; i \neq j.$$

The first-order conditions can be rearranged as

$$\text{(F.13)} \quad -\frac{u_c^i}{u_n^i} = \frac{1}{(1-\tau_i^n)\, G_i^i F^i},$$

$$\text{(F.14)} \quad \frac{G_j^i}{G_i^i} = \frac{\left(1+\tau_{ji}^m\right)\left(1+\tau_{ij}^m\right)}{\left(1-\tau_{ji}^x\right)\left(1-\tau_{ij}^x\right)} \frac{G_j^j}{G_i^j}, \; i \neq j.$$

The balance of payments condition can be written as

$$\text{(F.15a)} \quad \frac{G_i^i y_{ij}}{1-\tau_{ijt}^x} - \frac{G_j^i y_{ji}}{1+\tau_{ji}^m} = 0.$$

Next we define and characterize the non-cooperative equilibrium of a game. The timing is that the two governments simultaneously choose their policies. Given these policies, we then have a competitive equilibrium in which households and firms optimize and prices clear markets. Let $\pi_i = \left\{\tau_i^n, \tau_{ij}^m, \tau_{ij}^x\right\}$ denote the policies chosen by the government of country i, and let $\pi = (\pi_1, \pi_2)$. Let $x(\pi) = (x_1(\pi), x_2(\pi))$ denote the resulting competitive equilibrium allocations for the two countries, $x_1(\pi)$ and $x_2(\pi)$, and let $p(\pi)$ denote the associated prices. The government of country i chooses π_i to maximize

$$\text{(F.16)} \quad u^i\left(c_i(\pi_1, \pi_2), n_i(\pi_1, \pi_2)\right) + h^i\left(g_i(\pi_1, \pi_2)\right)$$

subject to its budget constraint,

$$\tau_i^n w_i(\pi_1,\pi_2) n_i(\pi_1,\pi_2) + \tau_{ji}^m p_{ji}(\pi_1,\pi_2) y_{ji}(\pi_1,\pi_2) + \tau_{ij}^x p_{ij}(\pi_1,\pi_2) y_{ij}(\pi_1,\pi_2)$$
$$= q_i(\pi_1,\pi_2) g_i(\pi_1,\pi_2), \; j \neq i, \tag{F.17}$$

taking as given the policies of the other country.

A non-cooperative equilibrium consists of policies, π^*, and allocations and pricing rules, $x(\pi), p(\pi)$, such that for each i, taking π_j^* as given for $j \neq i$, π_i^* maximizes $(F.16)$ over the set of policies, and for all π, $(\pi, x(\pi), p(\pi))$ is a competitive equilibrium.

Proposition F1: Non-cooperative equilibria of the static game do not satisfy production efficiency. The proof is by contradiction. Suppose country 2 sets all trade taxes to zero; then, country 1 can improve its welfare by deviating from zero trade taxes.

The proof of this proposition follows the standard logic in the optimal tariff literature.

For future use, let $z^s = (\pi^*, x^*, p^*)$ and $u^{s,i}$ denote the equilibrium outcomes and utilities in the non-cooperative equilibrium of the static economy.

Dynamic formulation

Consider now an infinite repetition of the static economy. In this infinite repetition, neither consumers nor governments can borrow or lend across periods. The only link between periods is strategic. To develop these strategic links, let h_t denote the history of policies and allocations, up to the beginning of period t. These histories are recursively defined by starting at the null history and constructing h_{t+1} as follows. Let the history for private agents be denoted by $h_{p,t} = (h_t, \pi_t)$, where $\pi_t = (\pi_{1,t}, \pi_{2,t})$. Let $h_{t+1} = (h_{p,t}, x_t, p_t)$, where x_t and p_t denote allocations and prices in period t.

A strategy for government i is given by a sequence of functions $\sigma_{i,t}(h_t)$, which maps histories into period t policies, with $\sigma_t = (\sigma_{1,t}, \sigma_{2,t})$. Allocation and pricing rules are denoted by sequences of functions $x_t(h_{p,t})$ and $p_t(h_{p,t})$, which map histories for private agents into allocations and prices. Strategies, allocations, and pricing rules induce future histories from past histories in the natural way. For example, induced history $h_{p,t}$ from some arbitrary history h_t is given by $h_{p,t} = (h_t, \sigma_t(h_t))$, and the induced history h_{t+1} from some arbitrary history $h_{p,t}$ is given by $h_{t+1} = (h_{p,t}, x_t(h_{p,t}), p_t(h_{p,t}))$. Let $V_t^i(h_t)$ denote the discounted utility for the residents of country i associated with the strategies, allocations, and pricing rules.

A *sustainable equilibrium* of this game consists of strategies, allocation rules, and pricing rules such that (1) for all periods t and for all histories $h_{p,t}$, the induced allocations and prices are a competitive equilibrium; and (2) for all periods t and for all histories h_t, the strategy for, say, government 1 in period t, maximizes

$$\text{(F.18)}\quad u^1\left(c_{1,t}\left(h_t, \pi_{1,t}, \pi_{2,t}\right), n_{1,t}\left(h_t, \pi_{1,t}, \pi_{2,t}\right)\right) + h^1\left(g_{1,t}\left(h_t, \pi_{1,t}, \pi_{2,t}\right)\right) + \beta V_{t+1}^1\left(h_{t+1}\right),$$

subject to the analog of the budget constraint for the static case, (F.17), where $\pi_{2,t} = \sigma_{2,t}\left(h_t\right)$, $h_{p,t} = \left(h_t, \pi_{1,t}, \sigma_{2,t}\left(h_t\right)\right)$, and $h_{t+1} = \left(h_{p,t}, x_t\left(h_{p,t}\right), p_t\left(h_{p,t}\right)\right)$.

A sustainable outcome is defined as an infinite sequence of policies, allocations and prices, $\{\pi_t, x_t, p_t\}_{t=0}^{\infty}$, induced from the null history by a sustainable equilibrium.

Next, we provide a characterization of the set of sustainable outcomes. We restrict ourselves to equilibria that can be sustained by reversion to static outcomes. Formally, we restrict ourselves to equilibria such that for all histories h_{t+1},

$$V_{t+1}^i\left(h_{t+1}\right) \geq \frac{u^{s,i}}{1-\beta}.$$

These equilibria are the analogs of equilibria in repeated games that are sustained by reversion to the Nash equilibria of the static game.[12]

We then have the following lemma.

Lemma F1: Characterization of sustainable equilibria

An arbitrary sequence $\{\pi_t, x_t, p_t\}_{t=0}^{\infty}$ is a sustainable outcome if and only if (1) it is a competitive equilibrium; and (2) for all periods r, and for, say, country 1

[12]While we could prove the theorem for other equilibria as well, using the techniques of Abreu, Pearce, and Stachetti (1990), the proof described here is simpler to follow.

$$\sum_{t=r}^{\infty} u^1 \left(c_{1,t}, n_{1,t}\right) + h^1 \left(g_{1,t}\right)$$

$$\geq \quad u^1 \left(c_1^s \left(\hat{\pi}_{1,t}, \pi_{2,t}\right), n_1^s \left(\hat{\pi}_{1,t}, \pi_{2,t}\right)\right) + h^1 \left(g_{1,t}^s \left(\hat{\pi}_{1,t}, \pi_{2,t}\right)\right) + \frac{\beta u^{s,1}}{1-\beta}, \tag{F.19}$$

for all $\hat{\pi}_{1,t}$, where $c_1^s(\cdot,\cdot)$, $n_1^s(\cdot,\cdot)$, and $g_{1,t}^s(\cdot,\cdot)$ are the functions associated with the static equilibrium.

The proof of this lemma is a straightforward adaptation of the arguments in Chari and Kehoe (1990). We then have the following proposition:

Proposition F2: Sustainability of the cooperative Ramsey equilibrium

There is some $\tilde{\beta} < 1$ such that for all $\beta \geq \tilde{\beta}$, the cooperative Ramsey outcome is a sustainable outcome.