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Learning in the Marriage Market: The Economics of Dating*

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Abstract

We develop a dynamic model of two-sided matching with search *and learning* frictions. Agents engage in a search for a potential partner and, upon meeting, may gradually acquire information about their compatibility as a couple, a process we refer to as dating. Dating is mutually exclusive and, as such, introduces a tradeoff between becoming better informed about one's compatibility with a potential partner and meeting other, more promising, potential partners. We derive a closed-form solution for the unique steady-state equilibrium when agents are ex-ante homogeneous, and characterize it when they are vertically heterogeneous. In the steady state, agents date for longer than is socially optimal, an inefficiency that is alleviated by a small degree of asymmetry in dating costs between partners. Furthermore, block segregation fails, yet matching is assortative – in a probabilistic sense we refer to as *single-crossing in marriage probabilities*. Motivated by recent advances in matching technologies in decentralized markets, we study the effects of improvements in search and learning technologies and show that they differ qualitatively.

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1 Introduction

Choosing the right partner – arguably one of the most important decisions in one's life – typically involves a great deal of uncertainty: no matter how promising a potential partner may appear, it is often difficult to immediately assess whether s/he will indeed make a suitable partner. As a result, before committing to a serious relationship, prospective couples often attempt to reduce such uncertainty by *dating*, that is, by spending time getting to know one another and learning about each other's suitability as a partner.[1] If dating is, at least to some extent, exclusive, dating decisions – whom to begin dating and when to break up – reflect a tradeoff between acquiring more information about a prospective partner's suitability and having the opportunity to search for more promising potential partners.

This paper develops an equilibrium model of matching with both search *and learning* frictions in which – in a departure from the existing literature on matching with search frictions[2] – potential partners need not immediately (and irreversibly) decide whether to accept or reject their match, but instead may date in order to gradually learn about its merits. Introducing dating into the classic search and matching marriage-market paradigm opens the door to new questions, such as the (in)efficiency of equilibrium dating and marriage decisions, and whether or not – and in what sense – dating gives rise to assortative matching. The model also allows us to study how dating and marriage patterns are affected by recent advances in search and learning technologies (e.g., the introduction of dating applications such as Tinder, Bumble, and Hinge), which have drastically altered the dating market in recent years by facilitating dating and thickening traditional matching markets. For example, the model can be used to study how changes in search and/or learning frictions affect the amount of time agents invest in each dating partner, and the number of partners individuals date before "tying the knot."

In our model, agents engage in a time-consuming and random search for partners. Every pair of agents are either compatible as a couple or not: the agents derive positive utility from marrying compatible partners, and incur a cost when they marry incompatible ones. As in the classic matching-with-frictions framework, individuals are characterized by a single characteristic, which, following Burdett and Coles (1997), we refer to as *pizzazz*. Unlike in Burdett and Coles (1997), in our model, the higher an agent's pizzazz, the larger the share of singles on the other side of the market with which the agent is compatible.[3] Specifically, we

[1] This type of learning is also common in other matching markets. For example, in many labor markets the hiring process includes a series of job interviews and/or a probationary period that allows workers and employers to learn about the prospects of their match.

[2] See Chade, Eeckhout and Smith (2017) for a comprehensive review of this literature.

[3] In the classic marriage market model with nontransferable utility, there is no uncertainty and pizzazz re-

assume that two agents with pizzazz x and y are compatible with probability xy. Throughout the paper, we assume that utility is nontransferable.

While pizzazz is observable, actual compatibility is not. The main novelty in our model is that when a couple meets, they may date for a while and learn about their compatibility before deciding whether to marry or not. Dating is exclusive, requires mutual consent, and may be broken off unilaterally by either one of the partners, at any time. Dating, in our model, is costly for two reasons. First, agents incur an exogenous flow cost while dating. Second, agents incur an endogenous time cost reflecting the forgone opportunity to search for new partners while dating. Building on the now-canonical learning technology introduced in Keller, Rady and Cripps (2005), we assume that while a pair of compatible agents date they *click* at a Poisson rate λ, whereas incompatible couples never click. Thus, once a couple clicks, they infer that they are compatible, and marry. As long as a click does not occur, both agents become more pessimistic about their compatibility until, at some point in time, one of them decides to break up. At that point, both agents return to the singles market to search for new potential partners.

We begin by analyzing the case in which all agents are homogeneous and have the same pizzazz. In a steady-state equilibrium, agents' dating decisions – for how long to date a potential partner – are optimal and the induced flows of agents between the single, dating, and married populations are balanced. We show that a steady-state equilibrium exists and is unique, and characterize it in closed form. This allows us to study how the search friction (i.e., the rate at which agents meet new potential partners) and the learning friction (i.e., the rate at which dating partners learn about their compatibility) each shape dating decisions. We show that when the speed of search increases, agents invest less time in dating each partner and date more partners before marrying. Intuitively, when it is easier to meet alternative partners, agents are less patient when a click does not occur in the early stages of their relationship. By contrast, when the speed of learning increases, agents invest more time in each relationship as the informational value of such investment increases.

In equilibrium, agents date for longer than is socially optimal. This inefficiency arises because the agents' dating behavior imposes a negative externality on third parties. While a pair of agents are dating, they take into account the fact that they forgo the possibility of meeting other, more promising, partners during this dating period, but they do not internalize the fact that other singles (who may view them as promising potential partners) cannot meet them during this time. This externality is reminiscent of the so-called "thick market" externality that emerges in search and matching models (Shimer and Smith, 2001).

flects actual payoffs. For example, in Burdett and Coles (1997), when a woman with pizzazz x marries a man with pizzazz y, the man obtains a payoff of x and the woman obtains a payoff of y.

Motivated by the wide range of social norms governing the division of dating costs between partners, we use our model to study the implications of such norms (e.g., "splitting the bill") on social welfare.[4] To examine this question, we modify the exogenous cost of dating such that singles on one side of the market incur a flow dating cost of $c + \Delta/2$ and singles on the other side of the market incur a flow dating cost of $c - \Delta/2$, with $\Delta > 0$ reflecting the level of dating cost asymmetry. We find that the total welfare is hump-shaped in Δ. In particular, increasing the cost asymmetry reduces the amount of time couples spend dating. When cost asymmetry is low, this corrects for the inefficient over-dating that arises in equilibrium. For higher levels of cost asymmetry, however, once dating times drop below the efficient length of time, such asymmetry becomes detrimental to social welfare. In other words, a small amount of asymmetry in dating costs is beneficial to social welfare, but too much asymmetry becomes detrimental.

After studying the homogeneous case, we then turn to study a market in which agents are vertically heterogeneous (that is, agents differ in their pizzazz). The existence of a steady-state equilibrium in such an environment is not immediate. Following Shimer and Smith (2000), we establish existence in the value function space.[5] However, while the existing proof methods are tailored to the binary nature of agents' decisions – whether to accept or reject each match – in our model agents make richer decisions, choosing whether and for how long to date their potential partners.[6] An additional complication arises from the fact that, in our model, agents transition between three different pools: singles, dating couples, and married couples. Our proof methodology therefore differs from existing ones. As discussed in Smith (2006), a critical distinction in the existing literature between the existence proof methodologies of transferable-utility and nontransferable-utility search models lies in the discontinuity of the value functions. An interesting feature of our model is that, even though utility is nontransferable, value functions are in fact continuous, as agents can continuously adjust their dating times. In this sense, dating replaces surplus division in smoothing the value functions.

The model with heterogeneous pizzazz allows us to study whether matching is assortative, and how sorting depends on search and learning frictions. We show that, in equilibrium, the time cost of dating is higher for high-pizzazz individuals, who are therefore more selective. In particular, we show that an agent with pizzazz x will date a potential partner until either a click occurs or the agent's belief about the couple's compatibility falls below a

[4] See, e.g., https://matadornetwork.com/abroad/splitting-the-bill-etiquette-around-the-world.

[5] See Lauermann and Nöldeke (2015), Manea (2017), and Lauermann, Nöldeke and Tröger (2020) for important generalizations of this approach.

[6] Formally, existing methods typically make use of "acceptance sets," which are not well defined in our probabilistic setting.

cutoff $q^{\star}(x)$ that is increasing in x. This implies that the higher the agent's pizzazz is, the more agents there are who are willing to start dating her/him, and the fewer agents there are whom s/he is willing to date.

In our setting, dating leads to the failure of block segregation, yet matching is assortative in a probabilistic sense. We define a new notion of assortative matching: *single-crossing in marriage probabilities*. This property is satisfied if, for any two agents with pizzazz x and x' such that $x' > x$, there exists a critical pizzazz level $y^{\star}$ such that an agent with pizzazz y has a higher probability of marrying the higher-pizzazz agent x' than s/he does of marrying the lower-pizzazz agent x if and only if $y > y^{\star}$. In particular, this property implies that high-pizzazz agents are more likely to marry other high-pizzazz agents, but (as in reality) on occasion may marry low-pizzazz agents.[7] We show that, in equilibrium, this single-crossing-in-marriage-probabilities property is satisfied.

As in the case of homogeneous pizzazz, when agents' pizzazz is heterogeneous, the search and learning frictions have important – but distinct – effects on dating and marriage outcomes, and, in particular, on sorting. We show that as search frictions vanish, agents date only agents of their own pizzazz. By contrast, as learning frictions vanish, agents are willing to date all potential partners. Thus, learning frictions and search frictions have opposite effects on the resolution of the tradeoff between dating and finding a more promising partner. This suggests that, on the one hand, advances in search technologies that facilitate meeting new potential partners (e.g., dating apps such as Tinder and Bumble) could make singles more picky and less willing to invest in each given potential partner. On the other hand, advances in learning technologies that make it easier for partners to get to know each other (e.g., dating apps such as Hinge and OkCupid, in which individuals provide and receive a great deal of information about one another) may make them more willing to invest time in dating, even when a potential partner is less promising at first sight.

While we assume that utility is nontransferable and use marriage-market and dating terminology, our results are relevant to other matching markets in which utility is typically transferable, such as the labor market and markets in which buyers and sellers trade bilaterally and engage in time-consuming searches. In these alternative applications, information acquisition takes different forms. For example, in the context of the labor market, information acquisition may take the form of job interviews (or probationary periods) during the hiring process, and, in the context of bilateral trade, it may take the form of due diligence procedures. In Section 3.2 we show that the homogeneous version of our model is equivalent to a setting in which utility is transferable and the agents' share of the surplus is determined via

[7] Such probabilistic sorting has been empirically observed by Bruch and Newman (2018), who document that, while unlikely, attracting the attention of someone out of one's league is entirely possible.

Nash bargaining. This equivalence suggests that hiring processes and probationary periods may be inefficiently long and due diligence procedures may be excessive.

The paper proceeds as follows. In the remainder of this section we discuss the related literature. Section 2 presents the model. Section 3 studies the model in the case where agents are ex-ante homogeneous, while Section 4 studies the case of vertical heterogeneity. Section 5 concludes. All proofs are relegated to the Appendix.

Related literature

This paper contributes to the matching-with-search-frictions literature by introducing a model in which agents acquire information about the prospects of a potential match before committing to that match. This literature explores the properties of equilibrium matching under various assumptions on the search technology, match payoffs, search costs, the ability to transfer utility, and the agents' rationality. Closely related papers in this literature include McNamara and Collins (1990), Smith (1992), Bergstrom and Bagnoli (1993), Morgan (1996), Burdett and Coles (1997), Eeckhout (1999), Bloch and Ryder (2000), Shimer and Smith (2000), Chade (2001, 2006), Adachi (2003), Atakan (2006); Smith (2006), Lauermann and Nöldeke (2014), Coles and Francesconi (2019), Lauermann, Nöldeke and Tröger (2020), and Antler and Bachi (2021). To the best of our knowledge, existing models in this literature preclude dating – typically by assuming that agents decide irreversibly whether or not to marry *immediately* upon meeting. An additional contribution to this literature is that, in our model, "beauty is in the eye of the beholder" – an inherent feature of the marriage market that, as discussed in Chade, Eeckhout and Smith's (2017) survey, is missing within this literature.[8]

With the exception of Chade (2006), the above papers also assume that when an agent meets a potential partner, s/he is immediately fully informed about the merits of a relationship between them. Chade incorporates information frictions into this framework by assuming that agents receive only a noisy signal about the payoffs from marrying a potential partner before making an irreversible decision whether or not to marry that partner. This leads to an *acceptance curse*: the merits of a relationship with a partner, conditional on that partner agreeing to the marriage, are lower than the unconditional merits of such a relationship.

Information frictions have also been incorporated into search and matching models in the related context of the labor market.[9] Jovanovic (1984) and Moscarini (2005) develop a

[8]In their words, "[t]he literature frontier assumes a common evaluation of agents, without a hint that beauty is in the eye of the beholder, and this remains a major direction of future research." As they note, the trading model of Smith (1995) has this feature.

[9]Information frictions have been incorporated into decentralized matching models in other contexts as well.

theory of job turnover in which employers and workers make inferences about the productivity of their match from the output they observe. In their models, disappointing outcomes may lead workers to return to the search pool or move to a new job. Beyond the different context and questions, there are three important distinctions between our model and theirs. First, they assume that all workers are homogeneous, which precludes sorting. Second, they assume that utility is transferable, whereas we assume that it is not. Third, they assume that information arrives after a match is formed, whereas we assume that agents acquire information before deciding whether or not to match.

We study the implications of the recent decline in search and learning frictions on matching market sorting.[10] Similar questions were previously pursued by Eeckhout (1999) and Adachi (2003), who showed that when search frictions vanish, the equilibrium matching converges to a matching that is pairwise stable in the Gale and Shapley (1962) sense and hence is efficient. Under vertical heterogeneity and a supermodular production function, pairwise stability also implies positive assortative matching in the classic frictionless marriage model of Becker (1973). Lauermann and Nöldeke (2014) showed that equilibrium matching can converge to a matching that is not pairwise stable if (i) mixed strategies are allowed and (ii) there are at least two pairwise stable matchings.[11] Antler and Bachi (2021) established that vanishing search frictions lead to radically different results when agents' reasoning is coarse: in the frictionless limit agents search indefinitely and never marry.

The tradeoff between learning about an alternative and searching for other alternatives is also central to Fershtman and Pavan (2020). In contrast to the present paper, that paper focuses on the problem of a *single* decision maker who sequentially chooses between exploring existing alternatives and searching for additional options to explore.

2 The Model

We consider a marriage market with nontransferable utility. There is a unit mass of agents in the market, each of which is characterized by a single characteristic $x \in X \equiv [\underline{x}, \overline{x}]$, where $0 < \underline{x} < \overline{x} < 1$. Following Burdett and Coles (1997), we refer to this characteristic as *pizzazz*. In

For example, Lauermann and Wolinsky (2016), Lauermann, Merzyn and Virág (2018), and Mauring (2017) study two-sided search models in which buyers and/or sellers make inferences about an *aggregate state* from the terms of trade they encounter. Anderson and Smith (2010) show that information frictions can upset equilibrium sorting in a model without search frictions. In their model, agents choose a partner not only to maximize their current production, but also to signal their productivity to future partners.

[10]A parallel strand of the literature studies the implications of declining search frictions on product design, vertical differentiation, and growth in product and labor markets (e.g., Albrecht, Menzio and Vroman, 2021; Martellini and Menzio, 2021; Menzio, 2021).

[11]In Adachi (2003) (i) is violated and in Eeckhout (1999) (ii) is violated.

Section 3, we analyze a *homogeneous model* in which all agents have the same pizzazz, whereas in Section 4 we analyze a *heterogeneous model* in which agents are vertically heterogeneous. In the heterogeneous model agents' pizzazz is distributed according to a continuous density $g(x)$ that is bounded in $[\underline{g}, \overline{g}]$, with $0 < \underline{g} < \overline{g} < \infty$.

The market operates in continuous time and agents discount the future at a rate of $r > 0$. Agents transition between three states: singlehood, dating, and marriage. While an agent is single, s/he meets other single agents according to a quadratic search technology with parameter $\mu > 0$. That is, for any subset $Y \subseteq X$, if the measure of agents with pizzazz in Y in the singles pool is ν, then agent x meets such agents at a rate of $\mu\nu$.

We assume that any two agents are either compatible with one another or not, and that the ex-ante probability that a couple is compatible is determined by the pizzazz of both agents. In particular, the prior probability that a pair of agents with pizzazz x and y are compatible is xy. The compatibility (or lack thereof) of a couple determines the payoff each agent receives while the couple is married: the flow payoff while married to a compatible partner is normalized to 1, whereas the flow payoff while married to an incompatible partner is given by $-z < 0$. We assume that $z(1 - \overline{x}^2) > \overline{x}^2$, which implies that no couple will marry without first receiving (positive) information about their compatibility.

When a pair of agents meet, they can either begin dating or reject the match and return to the singles pool. While dating, a couple gradually learn about their compatibility. We make the following assumptions about the process of dating. First, we assume that dating requires the mutual consent of both agents. Thus, at any point in time each agent can unilaterally *break up* with her/his partner. Following a breakup, both agents immediately return to the singles market to search for new partners. Second, we assume that dating is exclusive; that is, while a couple are dating they are not matched with other potential partners. Finally, we assume that dating entails a flow cost of $c > 0$.

Dating couples receive information about their compatibility according to the following classic learning technology (e.g., Keller, Rady and Cripps, 2005). If the couple are compatible, they *click* according to an exponential distribution with arrival rate $\lambda > 0$. On the other hand, if the couple are incompatible, they will never click. Consequently, once a couple click, they infer that they are compatible and marry immediately. In the absence of a click, the couple gradually become more pessimistic about their compatibility. Thus, after dating for a sufficiently long period of time, one of the partners will choose to break up, at which point both partners return to the singles market.[12]

[12] It is straightforward to extend the model to one where a dating couple may also receive conclusive news that they are incompatible, as long as the arrival rate of this process is lower than λ, that is, as long as "no news is bad news."

As in Shimer and Smith (2000) and Smith (2006), we assume that married couples are subject to exogenous dissolution shocks that arrive according to an exponential distribution with arrival rate $\delta > 0$. Once a dissolution shock occurs, both agents return to the singles pool. Moreover, we assume that $c(r+\delta) < \lambda \underline{x}^2$, which prevents the exclusion of agents from the marriage market due to the cost of dating.

2.1 Preliminaries

Following the literature, we consider the steady-state equilibrium of this model (formal definition below). In a steady-state equilibrium, the agents' decisions – whether and for how long to date a given potential partner – are optimal given their beliefs, and the flows of individuals between the three states – singlehood, dating, and marriage – are balanced such that the distributions of singles, dating couples, and married couples are stationary. In the preliminary analysis below we formally describe the agents' value functions and the transitions between states.

The Dating Problem

The strategy of each agent in the steady state is a function that specifies the maximal amount of time that the agent is willing to date each potential partner. That is, agent x's strategy is a mapping $T_x : [\underline{x}, \overline{x}] \to \mathbb{R}_+$, where $T_x(y)$ is the maximal time that agent x is willing to date an agent with pizzazz y. Note that by setting $T_x(y) = 0$, agent x effectively rejects agent y immediately. Since dating requires mutual consent, after agents x and y meet, they will date for at most $\min\{T_x(y), T_y(x)\}$ units of time. In particular, if they click beforehand, they will marry; otherwise, they will separate after dating for $\min\{T_x(y), T_y(x)\}$ units of time.

We begin by conducting a partial-equilibrium analysis of the problem faced by an agent with pizzazz x when s/he meets a potential partner. Let $W_s(x)$ denote the steady-state continuation value of an agent with pizzazz x who is currently single – or simply the *value of singlehood*. Agent x's capital gain from marrying a compatible partner is $(1 - rW_s(x))/(r+\delta)$. Hence, agent x's capital gain from meeting a potential partner y, conditional on the latter's choice of maximal dating time, is

$$V_d(x;y) = \max_{T \in [0, T_y(x)]} \left\{ xy \int_0^T \lambda e^{-\lambda t} \left(e^{-rt} \frac{1 - rW_s}{r+\delta} - \frac{1-e^{-rt}}{r} (c + rW_s(x)) \right) dt \right.$$
$$\left. - \left(1 - xy(1 - e^{-\lambda T})\right) \frac{1 - e^{-rT}}{r} (c + rW_s(x)) \right\}. \quad (1)$$

The first term in this expression is agent x's expected gain in case s/he clicks with agent y while dating, and the second term represents the expected cost agent x incurs when they do not click and eventually separate without marrying.

By dating agent y for an additional dt units of time agent x will enjoy, with probability $\lambda q_t(x,y)$, a capital gain of $(1 - rW_s(x))/(r+\delta)$ from marriage, where $q_t(x,y)$ denotes the joint belief about the compatibility of agents x and y after they have dated for $t \geq 0$ units of time. While dating, however, agent x must forgo the flow value of singlehood, which is given by $rW_s(x)$, and must also incur the flow cost of dating, c. Hence, the marginal value of dating is positive if

$$\lambda q_t(x,y)\frac{1-rW_s(x)}{r+\delta} \geq rW_s(x) + c. \tag{2}$$

Standard arguments (e.g., Keller, Rady and Cripps, 2005) show that

$$\frac{q_t(x,y)}{1-q_t(x,y)} = e^{-\lambda t}\frac{xy}{1-xy}, \tag{3}$$

and that

$$\frac{dq_t}{dt} = -\lambda q_t(1-q_t).$$

Since $\frac{dq_t}{dt} < 0$, the marginal value of dating is decreasing, and, hence, the objective in (1) is concave.[13] It follows that, were agent x unconstrained by her/his dating partner's choice of dating time, s/he would like to continue dating that partner until the joint belief about compatibility reaches a critical belief. Equation (2) implies that this critical belief is

$$q^\star(x) \equiv \frac{rW_s(x)+c}{1-rW_s(x)} \times \frac{r+\delta}{\lambda}. \tag{4}$$

We refer to $q^\star(x)$ as agent x's *breakup threshold*.

Agent x's preferred dating time with anent y is the time it takes the joint belief to drop from xy to $q^\star(x)$. Note that agent x's breakup threshold is independent of (the pizzazz of) the partner that s/he is currently dating. Hence, from (3), the preferred dating time with agent y is

$$\frac{1}{\lambda}\log\left(\frac{xy(1-q^\star(x))}{(1-xy)q^\star(x)}\right).$$

Since dating times are nonnegative and dating requires mutual consent, x's preferred dating time with agent y may be infeasible, in which case x's optimal dating choice in (1) is set at the relevant boundary (i.e., 0 or $T_y(x)$).

[13] Antler, Bird and Oliveros (2021) show formally that the objective function in this type of learning problem is concave.

The partial equilibrium analysis allows us to abstract away from the choices of other agents. However, in the full-blown equilibrium of the model, agent x's choice of whether or not to continue dating agent y is relevant only if agent y chooses to continue dating agent x. As in many other two-sided matching models, the mutual consent requirement can sustain an equilibrium in which any two agents reject one another. The matching-with-frictions literature typically assumes that an agent always accepts any match with other agents whose pizzazz is strictly greater than her/his reservation value – an assumption that precludes this type of equilibrium. In this paper, we make the analogous assumption that agent x chooses to continue dating agent y as long as the marginal value of dating y is positive, that is, as long as $q_t(x,y) \geq q^\star(x)$.[14] The above assumption implies that, in equilibrium, agent x's strategy is fully characterized by the breakup threshold $q^\star(x)$ or, alternatively, by the maximum of the preferred dating time and zero

$$T^\star_x(y) = \max\left\{0, \frac{1}{\lambda}\log\left(\frac{xy(1-q^\star(x))}{(1-xy)q^\star(x)}\right)\right\}. \tag{5}$$

The Value of Singlehood

The capital gain from dating, $V_d(x;y)$, can be used to represent the value function $W_s(x)$ of an agent with pizzazz x. While searching for a partner, the agent obtains a flow payoff of zero. Upon being matched with a potential partner, s/he derives a capital gain from the possibility of dating her/him, which depends on the latter's pizzazz and is given by $V_d(x;y)$. Hence, the flow value of singlehood, $rW_s(x)$, satisfies

$$rW_s(x) = \mu\int_{\underline{x}}^{\overline{x}} V_d(x;y)u(y)dy, \tag{6}$$

where $u(y)$ denotes the (steady-state) measure of agents with pizzazz y in the singles market. This representation of the value of singlehood is analogous to the standard representation in the literature, in which agents' value of singlehood is expressed as the expected flow gain from marrying partners in their "acceptance set," where the value of a match is replaced by the capital gain from dating derived above.

[14]Since in our setting, in the absence of a click, the drift of beliefs is strictly monotone and the distribution of the time at which a click occurs is continuous, it is irrelevant how agents resolve their indifference when (2) holds with equality.

Balanced Flow

We now characterize the balanced-flow condition for the case where the distribution of pizzazz is continuous; the balanced-flow condition in the homogeneous version of the model is a special case of the one derived below. At each moment in time, the outflow of agents with pizzazz x from the singles market is given by the measure of such agents who meet a partner y such that both x and y are willing to date one another for a strictly positive amount of time. Note that agents date for a positive amount of time if and only if they marry with positive probability. The balanced flow condition is easier to state using marriage probabilities than using dating times, and so we represent the outflow of agents with pizzazz x from the singles pool by

$$\mu u(x) \int_{\{y:\alpha(x,y)>0\}} u(y)dy,$$

where $\alpha(x,y)$ is defined as the probability that x and y's dating ends in marriage.

There are two cases in which an agent with pizzazz x returns to the singles market after dating an agent with pizzazz y. First, agents x and y may break up after dating without clicking. The probability that this event occurs is $1-\alpha(x,y)$, and hence the inflow of agents with pizzazz x that return to the singles market due to failed dating is given by

$$\mu u(x) \int_{\{y:\alpha(x,y)>0\}} u(y)(1-\alpha(x,y))dy.$$

Second, while agent x is married, s/he may return to the singles market due to a dissolution shock. Denote by $d(x,y)$ the measure of agents with pizzazz x who are dating an agent with pizzazz y. The set of agents with pizzazz x who are dating is then

$$d(x) = \int_{\{y:\alpha(x,y)>0\}} d(x,y)dy. \tag{7}$$

Thus, the measure of agents with pizzazz x that are married is $g(x)-u(x)-d(x)$, and so the inflow of agents with pizzazz x that return to the singles market due to dissolution shocks is

$$\delta(g(x)-d(x)-u(x)).$$

By equating the inflow and outflow of agents with pizzazz x derived above and rearranging, it follows that the flow of agents with pizzazz x into and out of singlehood is balanced if

$$\mu u(x) \int_{\{y:\alpha(x,y)>0\}} \alpha(x,y)u(y)dy = \delta(g(x)-d(x)-u(x)). \tag{8}$$

Note that the LHS of (8) is the inflow to marriage, and the RHS of (8) is the outflow from marriage. Hence, (8) guarantees that the flows into and out of marriage are also balanced. Thus, when (8) holds for every x, the distributions of singles, dating couples, and married couples are stationary.

Steady-State Equilibrium

We can now formally define a steady-state equilibrium in our model.

Definition 1 *A* steady-state equilibrium *consists of a tuple* $\langle W_s(\cdot), q^\star(\cdot), u(\cdot), d(\cdot)\rangle$ *that consists of the value functions of single agents, the agents' breakup thresholds, the measure of unmatched agents, and the measure of dating agents, such that* (4), (6), *and* (8) *hold.*

In other words, in a steady-state equilibrium, (i) agents set their breakup thresholds optimally given their beliefs and the measure of unmatched and dating agents, and (ii) the flows between singlehood, dating, and marriage are balanced.

3 Homogeneous Pizzazz

In this section we analyze the homogeneous version of our model in which all agents have pizzazz $x_0 \in X$. Note that although ex-ante agents are symmetric, ex-post they are not, as some couples are compatible and others are not. Thus, couples have an incentive to date in order to learn about their compatibility before deciding whether or not to marry.

The ex-ante symmetry in the homogeneous model allows us to obtain a closed-form solution for the unique steady-state equilibrium and derive its comparative statics with respect to the speed of search, μ, and the speed of learning, λ. Moreover, it enables us to establish that dating is excessively long in equilibrium and adapt the model to discuss the welfare implications of an uneven division of the costs of dating between men and women. We conclude this section by showing that, when agents are homogeneous, there is an equivalence between our model in which utility is nontransferable and one in which utility is transferable. Hence, the analysis in this section is also applicable to matching markets where utility is transferable (e.g., labor markets).

Our first result establishes that there exists a unique steady-state equilibrium. To establish this result we leverage the fact that if all agents have the same pizzazz, then the equilibrium is pinned down by a single breakup threshold, $q^\star \equiv q^\star(x_0)$, that characterizes the behavior of all agents in the market.

Proposition 1 *There exists a unique steady-state equilibrium. This equilibrium is given by the unique solution of the equation*

$$\frac{(1+l_0)r(\lambda+r)(l^\star(\lambda-c(\delta+r))-c(\delta+r))}{(1+c)\lambda\left(r(l_0-l^\star)-\lambda l^\star\left(1-\left(\frac{l^\star}{l_0}\right)^{r/\lambda}\right)\right)} = \frac{2\mu}{1+\sqrt{1+\frac{4\mu}{\delta\lambda(1+l_0)}\left((\delta+\lambda)(l_0-l^\star)+\delta\log\left(\frac{l_0}{l^\star}\right)\right)}}, \quad (9)$$

where $l^\star \equiv \frac{q^\star}{1-q^\star}$ *and* $l_0 \equiv \frac{x_0^2}{1-x_0^2}$.

To solve for the unique equilibrium in closed form, we utilize the fact that in the homogeneous model, all agents choose the same dating time and, hence, the equilibrium can be analyzed *as if* agents are not affected by the dating choices of their partners. This independence enables us to represent the value of singlehood, W_s, the capital gain from dating, V_d, and the mass of agents in the singles market, u, as a function (of the likelihood ratio) of the agents' breakup threshold $q^\star$. These representations can then be used to transform Equation (6) into Equation (9), and to show that the latter equation has a unique solution.

The challenge in this proof is due to the fact that the mass of agents in the singles pool is endogenous. Longer dating times (i.e., lower breakup thresholds) reduce the number of available singles for two reasons. First, longer dating means that more couples end up marrying and leaving the pool of singles/daters. Second, since dating and search are mutually exclusive, more dating implies that, within this joint pool, there are more agents that are dating and fewer that are single. Thus, the more time agents spend dating each potential partner the fewer singles there are in the pool, which, in turn, increases the incentive to date each potential partner for a longer time. The tractability of the homogeneous model enables us to show that, despite these reinforcing effects, the steady-state equilibrium is unique.

We now utilize our closed-form solution to derive comparative statics and study how the number of partners agents date before marrying and the probability of marriage conditional on dating vary with changes in the search and learning technologies. Note that these observables are determined by the agents' breakup threshold $q^\star$ as follows: from (5), the probability of marriage conditional on dating is

$$\alpha^\star = x_0^2(1-e^{-\lambda T^\star(x_0,x_0)}) = \frac{x_0^2-q^\star}{1-x_0^2},$$

and the expected number of partners that an agent dates before marriage is $1/\alpha^\star$.

We start by establishing that improvements in the matching technology raise the agents' breakup threshold.

Proposition 2 *The breakup threshold $q^\star$ is strictly increasing in the speed of search, μ.*

Intuitively, when μ increases, the amount of time it takes to meet a new potential partner decreases. This makes the outside option of going back to the singles pool more attractive and leads agents to reduce the amount of time they invest in dating (i.e., raise $q^\star$). This, in turn, increases the mass of singles and further decreases the amount of time an agent must wait between breaking up with his/her current partner and meeting the next potential partner, which reinforces the incentive to break up sooner.

Next, we show that improvements in the learning technology lower the agents' breakup threshold. We derive this result for the case in which the dissolution rate δ is low, which is consistent with the casual observation that, by and large, the length of time for which a couple are married is of a larger order of magnitude than the length of time for which they were dating each other.

Proposition 3 *If the dissolution rate δ is sufficiently small, then the breakup threshold $q^\star$ is strictly decreasing in the speed of learning, λ.*

An increase in λ means compatible couples are more likely to click in each unit of time that they spend dating. This, in turn, increases the marginal value of dating and leads agents to lower their breakup threshold. However, since learning is faster, it is unclear if agents will spend more or less time dating before their belief reaches this lower threshold. If this change leads to a reduction in dating times then, as explained above, the size of the singles pool may increase, raising the value of being single and thereby creating an incentive to break up sooner. The proof shows that if δ is small then the latter, indirect effect, is weaker than the direct effect on the breakup threshold.

Propositions 2 and 3 show that improvements in search technologies and in learning technologies have different effects on observable dating and marriage patterns. Proposition 2 states that as the speed of search becomes faster – as facilitated by dating apps such as Tinder and Bumble – each couple dates for less time and is less likely to marry. An immediate implication of this is that advances in search technology increase the number of partners that an agent is expected to date before marrying, which is consistent with recent trends in dating markets. On the other hand, Proposition 3 states that as the speed of learning becomes faster, dating is more likely to be successful and lead to marriage, and hence agents will, on average, date fewer partners before marrying. This prediction is consistent with the marketing strategy of some dating applications – e.g., Hinge and OkCupid – that is built on the idea that high-quality information about potential partners increases the probability of forming a serious relationship and avoiding the so-called dating apocalypse.

3.1 Efficiency, Over-dating, and the Division of Dating Costs

In our model, the steady-state equilibrium is inefficient. In particular, agents date for longer than is socially optimal. To gain intuition for this effect, consider an agent who contemplates breaking up with a potential partner whom s/he is currently dating. The agent takes into account the exogenous cost of dating and the option value of being single (i.e., forgoing the possibility of meeting other, more promising, singles). However, s/he does not internalize the fact that while s/he is with her/his current partner s/he is unavailable to other potential partners for whom s/he may be a good match. The social perspective takes this additional consideration into account and, therefore, equilibrium dating times are excessively long relative to the socially optimal ones.

Formally, consider a social planner who does not know the compatibility of different agents and has to choose the agents' breakup thresholds. The planner's objective is to maximize the agents' aggregate welfare.

Proposition 4 *The agents' equilibrium breakup threshold $q^{\star}$ is strictly lower than the socially efficient breakup threshold.*

The externality underlying Proposition 4 is reminiscent of the "thick market" externality that emerges in search-and-matching models (e.g., Shimer and Smith, 2001): when an agent does not search s/he recognizes that s/he forgoes the opportunity of matching with others, but fails to take into account that others cannot match with her/him. However, unlike in these models, the magnitude of this externality in our model follows from costly information acquisition. To see this, note that as the learning friction vanishes, the equilibrium outcome becomes efficient: compatible agents marry instantaneously and incompatible agents break up instantaneously.

The inefficiency of dating choices rests on the assumption that agents have the same dating costs. We illustrate this by briefly considering a variant of our baseline model in which there are two sides to the market, one in which agents have a dating cost of $c + \Delta/2$ and one in which agents have a dating cost of $c - \Delta/2$. In this model, Δ represents the asymmetry between the agents, which may reflect social norms or gender imbalance. Using this modification of the model, we show that a small amount of asymmetry in the cost of dating can facilitate efficient outcomes in the marriage market.

The asymmetry in dating costs creates different dating incentives for each side of the market. In particular, the agents for whom dating is more costly are more choosy – i.e., have a higher breakup threshold – than their counterparts on the other side of the market.[15] Since dating requires mutual consent, an increase in Δ, just like an increase in c, reduces the maximal amount of time each couple may date. For small Δ, such an increase brings the equilibrium dating patterns closer to the social optimum, whereas for large Δ it moves them further away from it. We conclude that the aggregate welfare in the market is hump-shaped in the asymmetry parameter Δ and that, for some $\Delta^\star > 0$, equilibrium dating is efficient.

This finding is illustrated in Figure 1, which depicts social welfare (in the left-hand panel) and the value of singlehood for agents with high and low dating costs (in the right-hand panel) as a function of the asymmetry Δ in dating costs. Note that the value of singlehood of the agents on the side with the higher cost of dating is decreasing in Δ, but, for low values of Δ, this is more than compensated for by the increase in the value of singlehood of the agents on the other side of the market.

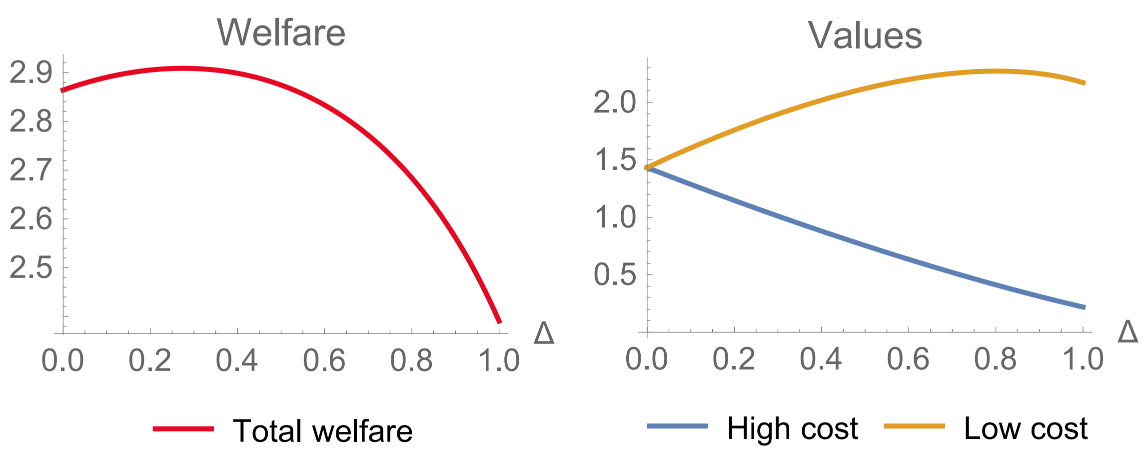

Figure 1: Welfare and the values of singlehood as a function of dating-cost asymmetry Δ, for $c = \mu = \lambda = l_0 = 1$ and $r = \delta = \frac{1}{10}$.

Note that the increase in efficiency is attained due to the increase in the agents' "choosiness." In the variant of our model described above, agents on one side of the market become more choosy because of the increase in their cost of dating. However, there are other mechanisms that may lead to such behavior. For example, if agents are slightly overoptimistic about their outside option (e.g., slightly overestimate μ), then their equilibrium behavior will also

[15] That $q^\star$ is increasing in c can be established formally by implicit differentiation of Equation (9).

be closer to the social optimum.

3.2 Transferable Utility

While we use the marriage market terminology throughout the paper, the analysis in this section has implications for other decentralized matching markets in which utility is transferable, such as the labor market and markets in which buyers and sellers trade bilaterally. In fact, the homogeneous version of our model is equivalent to a model in which agents' utility is transferable under the assumption that the flow surplus from a marriage between compatible partners is equal to 2 and that bargaining over the surplus generated when a couple marries is settled via the Nash bargaining solution. To see the equivalence, note that the symmetry between the agents implies that they have the same outside option, which, under the Nash bargaining solution, implies that their flow payoff from being married to a compatible agent is equal to 1.

In the context of the labor market, dating can be interpreted as a process of hiring in which both parties acquire information about their compatibility (e.g., via job interviews). Thus, the results of this section suggest that hiring processes are inefficiently long due to the mechanism uncovered in Proposition 4. Moreover, Proposition 2 suggests that recent developments in search technologies (e.g., social networks such as LinkedIn) that allow firms and workers to meet at a higher rate may lead them to invest less in learning about the compatibility of each potential match. On the other hand, Proposition 3 suggests that advances in learning technologies that improve the way workers and firms gather information about their compatibility would make them invest more in exploring the merits of each potential match.

4 Heterogeneous Pizzazz

In this section, we analyze the case in which agents are vertically heterogeneous. That is, their pizzazz is distributed continuously over the interval X. This heterogeneity enables us to examine classic questions in two-sided matching about equilibrium sorting that, by construction, are moot in the homogeneous version of our model. We start this section by examining whether equilibrium outcomes result in an assortative matching and, if so, in what sense. Subsequently, we investigate how changes in search and learning technologies affect equilibrium sorting. We conclude this section by establishing the existence of a steady-state equilibrium.

4.1 Assortative Matching

Since Becker (1973), a fundamental question in matching theory is who marries whom and why. As a benchmark, consider the famous block segregation result that arises in matching markets in which agents are vertically heterogeneous, utility is nontransferable, and preferences are multiplicatively separable.[16] Block segregation is an extreme version of assortative matching in which, in equilibrium, the set of agents is partitioned into classes, and all agents marry agents who belong to the same class as they do. In our setting, block segregation fails due to the *strict* monotonicity of the value of singlehood and, hence, of the break up threshold.

The strict monotonicity of $W_s(\cdot)$ follows from the combination of two properties. First, all agents use breakup thresholds and, second, beliefs about compatibility drift downward over time. This combination implies that a high-pizzazz agent can mimic the dating times of a low-pizzazz agent with *every* potential partner. Since a high-pizzazz agent is (strictly) more likely to be compatible with any given partner than a low-pizzazz agent, the probability of a click with any given partner under the mimicking strategy is greater for the high-pizzazz agent than for the low-pizzazz agent. It follows that $W_s(x)$ is strictly increasing in x and, since $W_s(x)$ reflects agent x's prospects in the singles market, $q^\star(x)$ is strictly increasing in x as well (by Equation (4)). The next result follows directly.

Lemma 1 *$W_s(\cdot)$ and $q^\star(\cdot)$ are strictly increasing.*

The monotonicity of the breakup threshold leads to the failure of block segregation in our model. In equilibrium, agents with pizzazz x must be willing to date agents with a slightly lower pizzazz for some amount of time, that is, $q^\star(x) < x^2$. Otherwise, agents with pizzazz x reject (i.e., date for zero units of time) all agents of their pizzazz and below and, since $q^\star(x)$ is monotone, are rejected by all agents that are above their pizzazz. Hence, such agents remain single forever. However, remaining single forever guarantees a payoff of 0 and rules out $q^\star(x) \geq x^2$ as being part of an equilibrium. Thus, in equilibrium, if agents were partitioned into classes, those in the bottom part of their class would be willing to date agents at the top of the class below theirs. We conclude that agents with relatively close pizzazz do not have entirely disjointed sets of marriage partners (see the criticism on the discontinuous aspect of the block segregation result in Smith, 2006 and Chade, Eeckhout and Smith, 2017).

While the block segregation result does not hold in our setting, matching is still assortative in a probabilistic sense. In particular, for any $x'', x' \in X$ such that $x'' > x'$, the difference

[16] This result has been established by multiple authors in various settings. See Chade, Eeckhout and Smith (2017) for a comprehensive review of these papers.

$\alpha^\star(x'', y) - \alpha^\star(x', y)$ satisfies a single-crossing property in y. That is, high-pizzazz agents are more likely to marry high-pizzazz agents, whereas low-pizzazz agents are more likely to marry other low-pizzazz agents.

Formally, for any matching probability function α and $x'', x' \in X$, denote the set of agents who marry x'' or x' with positive probability by

$$A_\alpha(x'', x') = \{y : \alpha(x'', y) > 0 \text{ or } \alpha(x', y) > 0\}.$$

Definition 2 (Single-crossing in matching probabilities) *A matching probability function $\alpha(\cdot, \cdot)$ satisfies the single-crossing property if for every $x'', x' \in X$ such that $x'' > x'$, there exists $y^\star \in X$ such that for any $y \in A_\alpha(x'', x')$ it holds that $\alpha(x'', y) > \alpha(x', y)$ if and only if $y > y^\star$.*

Proposition 5 *Every steady-state equilibrium matching probability function satisfies the single-crossing property.*

To gain intuition for this result, consider the agents x'' and x', where $x' < x''$. Recall that the agents' breakup thresholds are monotone in pizzazz, and so for every couple that breaks up, it is the higher-pizzazz agent who breaks up with her/his potential partner. This means that an agent with pizzazz y such that $y > x''$ dates agent x'' for longer than s/he dates agent x'. Together with the fact that higher-pizzazz agents are more likely to be compatible with their partner, this implies that $\alpha(x'', y) \geq \alpha(x', y)$ for these agents. The more challenging case is the one in which the potential partner y has a pizzazz of $y < x''$. In this case, we show that the probability of marriage is supermodular in y, x, and $q^\star$, which leads to the single-crossing property of match probabilities.

4.2 Search and Learning Frictions

In recent years, the way individuals seek new partners has undergone drastic changes due to the introduction of new dating apps that have replaced more traditional matching channels.[17] This trend affects the speed of search as well as the difficulty of learning about the potential partner. In this section, we explore the implications of such advances in matching technologies and, in particular, their effect on equilibrium sorting. To capture these recent trends, we characterize the steady-state equilibrium when either the learning friction vanishes or the search friction vanishes.[18]

[17] According to a recent survey by Pew (2019) the majority of couples who started a relationship in the past decade met online.

[18] In the matching-with-frictions literature, comparative static results typically take the form of either limit results or use a simplifying "cloning" assumption. In this paper, we pursue the former approach.

Proposition 6 *As learning frictions vanish, agents are willing to date everyone:* $\lim_{\lambda\to\infty} q^\star(x) = 0$ *for every* $x \in X$.

To gain intuition for this result, consider an agent x who contemplates dating a potential partner y for an additional dt units of time. By dating agent y, this agent incurs the direct cost of dating and the indirect cost of having to forgo the opportunity of meeting other potential partners in that span of time. In a short span of time dt, both of these costs are flow costs that are bounded from above by c and $1/r$, respectively. The benefit of dating agent y stems from the probability that a click will occur while x and y date, in which case the agents will marry and enjoy a (strictly positive) capital gain of $(1 - rW_s(x))/(r + \delta)$. When learning frictions vanish, the probability that a compatible couple clicks during dt units of time converges to one. Thus, as long as agent x believes that s/he is compatible with agent y with strictly positive probability, s/he finds it optimal to continue dating y.

Proposition 6 implies that in the limit as $\lambda \to \infty$, any pair of agents (x, y) that meet are willing to date one another, and if they are compatible, they will indeed marry. It follows that in this limiting case $\alpha(x, y) = xy$. Moreover, in this limiting case the mass of dating couples is zero as agents instantaneously learn whether they are compatible or not. These two observations enable us to characterize how the distribution of pizzazz in the singles pool relates to the distribution of pizzazz in the general population. In particular, by plugging $\alpha(x, y) = xy$ and $d(x) = 0$ into the balanced flow condition (8) and rearranging, it can be shown that low-pizzazz agents are overrepresented in the singles pool in the sense that the ratio $u(x)/g(x)$ is strictly decreasing in x. Intuitively, when every compatible couple that meets ends up marrying, agents that are more likely to be compatible with other agents – namely, high-pizzazz agents – leave the singles market more quickly than those that are less likely to be compatible with others – namely, low-pizzazz agents. This, in turn, leads to the overrepresentation of low-pizzazz agents in the singles pool.

The above discussion is summarized in the following corollary.

Corollary 1 *In the limiting case where* $\lambda \to \infty$*, in the steady-state equilibrium:*

1. *Dating is non-assortative: all agents are willing to date one another.*
2. *If a couple of agents with pizzazz x and y meet, then they marry with probability xy.*
3. *Higher-pizzazz agents are underrepresented in the singles pool relative to the population. In particular,*

$$\frac{g(x)}{u(x)} = 1 + x\frac{\mu\eta}{\delta},$$

where $\eta > 0$ denotes the average pizzazz of agents in the singles pool.

The next result considers the case in which learning frictions exist, but search frictions vanish.

Proposition 7 *As search frictions vanish, agents are only willing to date agents of their own pizzazz and above:* $\lim_{\lambda \to \infty} q^\star(x) = x^2$ *for every* $x \in X$*.*

To gain intuition for this result suppose, for now, that as μ goes to infinity, agent x meets singles of every possible pizzazz instantaneously. In this case, agent x has no reason to continue dating a potential partner after the probability that they are compatible has dropped below x^2: s/he can break up with the partner, and instantaneously start dating a new partner with pizzazz x (recall that, in equilibrium, $q^\star(x) \leq x^2$). Thus, in the $\mu \to \infty$ limit, agent x's breakup threshold $q^\star(x)$ converges to x^2.

The main challenge in the proof is showing that $u(x)$ does not vanish as the search frictions vanish, which validates the above assumption that an agent meets singles of every possible pizzazz instantaneously. Loosely speaking, $u(x)$ can vanish only if there is a positive measure of agents that date x in equilibrium. However, if there is a positive measure of agents whose pizzazz is greater than x, then none of them should agree to date x, as they have instantaneous access to more promising potential partners. Similarly, if there is a positive measure of agents whose pizzazz is lower than x, then agent x should refuse to date almost all of them, as s/he has instantaneous access to more promising potential partners. It follows that there cannot be a positive measure of agents whom x dates, and so $u(x)$ does not vanish when search frictions vanish.

Proposition 7 implies that as search frictions vanish, every agent x is willing to date singles with pizzazz x and higher and refuses to date agents with lower pizzazz. Thus, the induced matching is fully assortative in the agents' pizzazz: agents with pizzazz x only marry one another.

A second implication of Proposition 7 is that agents break up unless they click instantaneously. This means that agents invest very little in exploring the merits of each potential relationship. It also means that agents date a large number of singles before marrying.

The above discussion is summarized in the following corollary.

Corollary 2 *In the limiting case where* $\mu \to \infty$*, in the steady-state equilibrium:*

- *There is full assortative matching: agents marry only agents of their own pizzazz.*
- *The amount of time each couple dates before marrying goes to zero.*
- *The average number of partners each agent dates before marrying goes to infinity.*

4.3 Equilibrium Existence

We now turn to establish the existence of a steady-state equilibrium in our model. Following the approach pioneered by Shimer and Smith (2000), we prove existence in the value function space. While existence proofs in the literature typically rely on matching and acceptance sets, which reflect binary accept/reject choices, in our model the agents make richer decisions, i.e., choose for how long to date their potential partners. Our proof methodology must therefore differ from existing ones. Another challenge stems from the fact that, in our model, agents transition between three states – singlehood, dating, and marriage – rather than just two states.

We prove this result by invoking a fixed point argument. This requires establishing that the mappings (i) from value functions to the probabilities with which each pair of agents marry, and (ii) from these marriage probabilities to the distribution of agents in the singles market, are continuous. The latter is the analog of Shimer and Smith's (2000) fundamental matching lemma. Equilibria are then fixed points of an appropriately defined mapping for which, using Schauder's fixed point theorem, we establish that a fixed point indeed exists.

Theorem 1 *A steady-state equilibrium exists. Moreover,* $W_s(\cdot)$ *and* $q^\star(\cdot)$ *are (Lipschitz) continuous.*

As noted in Smith (2006), a critical distinction between the existence proof methodologies of transferable-utility and nontransferable-utility search models lies in the discontinuity of the value functions. In our model, while utility is nontransferable, value functions are in fact continuous. Since agent y uses a breakup threshold and the prior belief that a couple (x, y) is compatible is continuous in x, it follows that if agent x and $\tilde{x}$ have similar pizzazz, the spans of time for which any agent y is willing to date them are similar as well. This means that by mimicking one another's dating strategy, agents x and $\tilde{x}$ obtain similar payoffs. It follows that $W_s(x)$ must be close to $W_s(\tilde{x})$ if the difference in the pizzazz of agents x and $\tilde{x}$ is small. In this sense, the continuous choice of dating times replaces the continuous surplus division in smoothing the value functions.

5 Concluding Remarks

We introduce a model of two-sided matching with search and learning frictions in which agents can spend time together to learn about their compatibility before marrying. We show that the search and learning frictions have radically different effects on the equilibrium dating and marriage outcomes, and the manner in which agents sort themselves. On the one hand, advances in search technologies reduce the amount of time agents spend dating each partner, increase the number of partners whom they date, and result in more sorting in equilibrium. On the other hand, advances in learning technologies reduce the number of partners whom agents date and result in less sorting in equilibrium. Our analysis shows that equilibrium dating is inefficient – results in over-dating – and illustrates how different social norms pertaining to the division of dating costs between partners may correct for this inefficiency. Finally, we show that equilibrium matching satisfies a new notion of assortative matching: single-crossing in matching probabilities.

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A Proofs

Proof of Proposition 1. *Characterization of the Equilibrium.* In the first part of the proof we show that the steady-state equilibria are represented by the solutions of Equation (9). To do so, we express all equilibrium outcomes and values as a function of $l^\star$, the likelihood ratio of the breakup threshold, and then use Equation (6) to characterize the equilibrium.

We begin by deriving the relation between the equilibrium breakup threshold, $l^\star$, the optimal dating time, $T^\star$, and the equilibrium probability that dating leads to marriage, $\alpha^\star$. The learning technology implies that $\frac{x_0^2}{1-x_0^2}e^{-\lambda t} = \frac{q_t}{1-q_t}$. Hence, the optimal dating time $T^\star$ satisfies

$$\frac{x_0^2}{1-x_0^2}e^{-\lambda T^\star} = \frac{q^\star}{1-q^\star} \Rightarrow T^\star = \frac{1}{\lambda}\log\left(\frac{l_0}{l^\star}\right). \tag{10}$$

Using (10), we obtain that the probability that dating leads to marriage is

$$\alpha^\star \equiv x_0^2(1-e^{-\lambda T^\star}) = \frac{l_0 - l^\star}{1+l_0}.$$

Next, we derive the value of singlehood, W_s, as a function of $l^\star$. From Equation (4) it follows that

$$l^\star \equiv \frac{q^\star}{1-q^\star} = \frac{(\delta+r)(c+rW_s)}{\lambda - c(\delta+r) - rW_s(\delta+\lambda+r)}.$$

Rearranging, we obtain that

$$W_s = \frac{\lambda l^\star - c(l^\star+1)(\delta+r)}{(l^\star+1)r(\delta+r)+\lambda l^\star r}. \tag{11}$$

Our next step is to derive the capital gain from dating. The expected discounted time at which a click occurs, conditional on a click occurring, is

$$\sigma^\star \equiv \int_0^{T^\star} e^{-rt}\frac{\lambda e^{-\lambda t}}{1-e^{-\lambda T^\star}}dt = \frac{\lambda}{\lambda+r}\frac{\left(l_0 - l^\star\left(\frac{l_0}{l^\star}\right)^{-\frac{r}{\lambda}}\right)}{(l_0-l^\star)}.$$

The expected cost of dating is the product of the flow cost of acting $c_r W_s$ and the expected discounted amount of time for which the couple will date, $\kappa^\star$. The latter term is given by

$$\begin{aligned}\kappa^\star &\equiv (1-\alpha^\star)\frac{1-e^{-rT^\star}}{r} + \alpha^\star\int_0^{T^\star}\frac{(1-e^{-rt})}{r}\frac{\lambda e^{-\lambda t}}{1-e^{-\lambda T^\star}}dt \\ &= \frac{\lambda + r(l_0+1) - (\lambda + r(l^\star+1))\left(\frac{l_0}{l^\star}\right)^{-r/\lambda}}{r(\lambda+r)(l_0+1)}.\end{aligned}$$

Hence, the expected cost of dating is

$$\kappa^\star(c+rW_s) = \frac{(1+c)\lambda l\left((l_0+1)r+\lambda - ((l^\star+1)r+\lambda)\left(\frac{l_0}{l}\right)^{-\frac{r}{\lambda}}\right)}{(1+l_0)r(\lambda+r)(\lambda l^\star + (l^\star+1)(\delta+r))}.$$

The capital gain from dating can be represented by

$$V_d = \alpha^\star\sigma^\star\frac{1-rW_s}{\delta+r} - \kappa^\star(c+rW_s).$$

Note that

$$\alpha^\star\sigma^\star\frac{1-rW_s}{\delta+r} = \frac{(1+c)\lambda(l^\star+1)^2\left(l_0 - l^\star\left(\frac{l_0}{l^\star}\right)^{-\frac{r}{\lambda}}\right)}{(1+l_0)(\lambda+r)(\lambda l^\star+(l^\star+1)(\delta+r))},$$

and so it follows that we can express the capital gain from dating in equilibrium by

$$V_d = \frac{(1+c)\lambda\left(l_0 r + l^\star\left(\lambda\left(\frac{l_0}{l^\star}\right)^{-\frac{r}{\lambda}} - (\lambda+r)\right)\right)}{(l_0+1)r(\lambda+r)(\lambda l^\star + (l^\star+1)(\delta+r))}.$$

The final step of this part of the proof is to represent the mass of agent in the singles market as a function of $l^\star$. The mass of dating agents is given by

$$d = \mu u^2 \int_0^{T^\star} 1 - x_0^2(1-e^{-\lambda t})dt = \frac{\mu u^2\left(\log\left(\frac{l_0}{l^\star}\right) + l_0 - l^\star\right)}{\lambda(1+l_0)}.$$

Hence, the balanced flow condition $\alpha\mu u^2 = \delta(1-d-u)$ becomes

$$1-u = \frac{\mu u^2}{\lambda(1+l_0)}\left((1+\frac{\lambda}{\delta})(l_0-l^\star) + \delta\log\left(\frac{l_0}{l^\star}\right)\right).$$

This equation has a single positive root that is given by

$$u^\star = \frac{2}{1+\sqrt{1+\frac{4\mu}{\delta\lambda(1+l_0)}\left((\delta+\lambda)(l_0-l^\star)+\delta\log\left(\frac{l_0}{l^\star}\right)\right)}}. \tag{12}$$

By Equation (6), in equilibrium, $rW_s = \mu u V_d$. Plugging in the representations of W_s, V_d, and μ as a function of $l^\star$ that were derived above, we have that Equation (6) is equivalent to Equation (9).

Uniqueness of the Equilibrium. The first step in showing that Equation (9) has a unique solution is to show that it has at least one solution, and that the number of its solutions is odd. Recall that $l^\star \in [0, l_0]$ and observe that i) the limit of the LHS (resp., RHS) of Equation (9) as $l^\star \to l_0$ from below, diverge to ∞ (resp., converge to a finite number), and ii) the limit of the LHS (resp., RHS) of Equation (9) as $l^\star \to 0$ from above converge to a negative number (resp., to zero). Since both sides of Equation (9) are continuous in $l^\star$, the intermediate value theorem implies that Equation (9) has a solution. Moreover, the number of solutions must be odd, as each solution is a point at which there is a change in the sign of the difference between the LHS and RHS of Equation (9).[19]

Next, we show that the number of solutions to Equation (9) is at most two, which, together with the above claim, implies there it has a unique solution. Recall that $l^\star = l_0 e^{-\lambda T^\star}$.

[19]Equation (9) cannot have a continuum of solutions. To see this, observe that both the LHS and RHS of this equation belong to C^∞, and so if the LHS and RHS were equal over any nonempty subinterval of $[0, l_0]$ they would be equal on all of $[0, l_0]$.

Plugging this expression into Equation (9) and rearranging yields

$$-(l_0+1)r(\lambda+r)\left(\delta+\psi(T^\star)\right)\left(c(\delta+r)\left(l_0+e^{\lambda T^\star}\right)-\lambda l_0\right)=$$
$$2(c+1)\delta\lambda\mu l_0\left(r\left(e^{\lambda T^\star}-1\right)+\lambda\left(e^{-rT^\star}-1\right)\right), \quad (13)$$

where

$$\psi(T)=\sqrt{\frac{\delta\left(\delta\lambda(l_0+1)+4\mu(\delta+\lambda)\left(l_0-l_0e^{-\lambda T}\right)+4\delta\lambda\mu T\right)}{\lambda(l_0+1)}}.$$

The RHS of Equation (13) is convex in $T^\star$ since its second derivative is

$$2(c+1)\delta\lambda^2\mu l_0 re^{-rT}\left(\lambda e^{T(\lambda+r)}+r\right)>0.$$

On the other hand, the LHS of Equation (13) is concave. To see this, note that the second derivative with respect to $T^\star$ of the LHS is

$$-(l_0+1)r(\lambda+r)\left\{c\lambda^2(\delta+r)e^{\lambda T^\star}\left(\delta+\Psi(T^\star)\right)+\frac{4c\delta\lambda\mu(\delta+r)\left(l_0(\delta+\lambda)+\delta e^{\lambda T^\star}\right)}{(l_0+1)\Psi(T^\star)}\right.$$
$$\left.+\left(\lambda l_0-c(\delta+r)\left(l_0+e^{\lambda T^\star}\right)\right)\left(\frac{4\delta^2\mu^2\left(\delta+l_0(\delta+\lambda)e^{-\lambda T^\star}\right)^2}{(l_0+1)^2\left(\Psi(T^\star)\right)^3}+\frac{2\delta\lambda\mu l_0(\delta+\lambda)e^{-\lambda T^\star}}{(l_0+1)\Psi(T^\star)}\right)\right\}.$$

The term outside the curly brackets is negative, whereas all terms inside the curly brackets are unambiguously positive with the exception of $\left(\lambda l_0-c(\delta+r)\left(l_0+e^{\lambda T^\star}\right)\right)$. However, plugging $e^{\lambda T^\star}=\frac{l_0}{l^\star}$ into that expression and rearranging shows that this expression is positive if and only if

$$\lambda q^\star\frac{1}{r+\delta}>c.$$

This inequality states that the marginal value of dating is greater than the direct cost of dating. Hence, in the relevant range of cutoff beliefs, this inequality must hold. Since a convex function and a concave function can have at most two intersections, it follows that there are are most two solutions to (9), which completes the proof. ■

Proof of Proposition 2. Recall that in the proof of Proposition 1 we showed that the difference between the LHS of Equation (9) and the RHS of Equation (9) crosses zero exactly one time, and, moreover, that it does so from below. Hence, to establish this comparative static result it is sufficient to show that increasing μ: i) does not decrease the LHS of Equation (9) , and ii) increases the RHS of Equation (9).

Point i) is immediate as the LHS of Equation (9) is independent of μ. To establish point ii), we can differentiate the RHS w.r.t. to μ and obtain that this derivative is

$$\frac{\partial RHS}{\partial \mu} = \frac{1}{\sqrt{\frac{4\mu(\delta+\lambda)(l_0-l^\star)+4\delta\mu\log\left(\frac{l_0}{l^\star}\right)+\delta\lambda(l_0+1)}{\delta\lambda(l_0+1)}}} > 0.$$

■

Proof of Proposition 3. The equilibrium is given by the solution to Equation (9). Thus, it can also be represented by the root of the function

$$F(l) = (l_0+1)r(\lambda+r)(c(l+1)(\delta+r)-\lambda l) + \frac{2(c+1)\delta\lambda\mu\left(l_0 r - l\left(\lambda - \lambda\left(\frac{l}{l_0}\right)^{r/\lambda} + r\right)\right)}{\delta + \sqrt{\frac{\delta\left(4\mu(\delta+\lambda)(l_0-l)+4\delta\mu\log\left(\frac{l_0}{l}\right)+\delta\lambda(l_0+1)\right)}{\lambda(l_0+1)}}}.$$

From the implicit function theorem,

$$\frac{\partial l^\star}{\partial \lambda} = -\frac{\frac{\partial F}{\partial \lambda}(l^\star)}{\frac{\partial F}{\partial l}(l^\star)}.$$

From the proof of Proposition 1 it follows that $\frac{\partial F}{\partial l}(l^\star) \neq 0$, and so $l^\star$ is nonmonotone in λ only if $\frac{\partial F}{\partial \lambda}$ can equal zero. The limit as $\delta \to 0$ of the derivative of the second term of F w.r.t. λ is equal to zero. The limit as $\delta \to 0$ of the derivative of the first term of F w.r.t. λ is equal to

$$\frac{l_0}{l}(l_0+1)r(c(l+1)r - l(2\lambda+r)),$$

an expression that equals zero only if

$$c = q(1+2\lambda\frac{1}{r}).$$

However, agents learn only if $c \leq \lambda q \frac{1}{r}$, and so this derivative is bounded away from zero. Since F is continuously differentiable in δ, it follows that, for sufficiently small δ, $l^\star$ is strictly monotone in λ.

Finally, we show that $l^\star$ is decreasing in λ. The value of singlehood is bounded from above by $\frac{\mu}{\mu+r}\frac{1}{r}$, as the first term of this product is the expected discount factor at the time an agent is matched with a potential partner if every agent in the population is single, and the second term is the value of a marriage that lasts indefinitely. Hence, $W_s(x)$ is bounded away from $\frac{1}{r}$ uniformly over λ, and so Equation (4) implies that $\lim_{\lambda \to \infty} l^\star(x) = 0$. Since $l^\star \geq 0$, it follows that $l^\star$ is decreasing in λ. ■

Proof of Proposition 4. Consider an agent who dates all partners for T units of time. Denote by $W_d(T, \tilde{T})$ this agent's continuation utility at the moment s/he meets a potential partner, given that all other agents date for $\tilde{T}$ units of time. This value can be written as

$$W_d(T, \tilde{T}) = \overline{v}_d(T) + \sigma(T) \frac{\mu u(\tilde{T})}{\mu u(\tilde{T}) + r} W_d(T, \tilde{T}),$$

where $\overline{v}_d(T)$ is the actual payoff while the two agents are together (i.e., a flow cost of c while the agents are dating and a flow benefit of 1 while they are married), $\sigma(T)$ is the expected discount factor when the agent breaks up (before or after marriage) with her/his partner, and $\frac{\mu u(\tilde{T})}{\mu u(\tilde{T})+r}$ is the expected discounting over the amount of time the agent will wait between the time at which s/he breaks up with the current partner and the time s/he will meet the next partner. Rearranging yields

$$W_d(T, \tilde{T}) = \frac{r + \mu u(\tilde{T})}{r + \mu u(\tilde{T})(1 - \sigma(T))} \overline{v}_d(T).$$

On the one hand, in the equilibrium of the homogeneous model, the agent acts as if s/he is unconstrained by the choice of her/his partners. Hence, s/he takes $\tilde{T}$ as given, and chooses T optimally. The necessary and sufficient first-order condition of this problem is

$$\frac{\overline{v}_d'(T)}{1 - \frac{\sigma(T)}{\frac{r}{\mu u(\hat{T})} + 1}} + \frac{\overline{v}_d(T)\sigma'(T)}{\left(\frac{r}{\mu u(\hat{T})} + 1\right)\left(1 - \frac{\sigma(T)}{\frac{r}{\mu u(\hat{T})} + 1}\right)^2} = 0.$$

Note that if $\tilde{T}$ is the equilibrium dating time, then the agent will also choose $T = \tilde{T}$, which justifies the assumption that the agent is unconstrained by the choice of her/his potential partners. Hence, in equilibrium, it holds that

$$\frac{\overline{v}_d'(\hat{T})}{1 - \frac{\sigma(\hat{T})}{\frac{r}{\mu u(\hat{T})} + 1}} + \frac{\overline{v}_d(\hat{T})\sigma'(\hat{T})}{\left(\frac{r}{\mu u(\hat{T})} + 1\right)\left(1 - \frac{\sigma(\hat{T})}{\frac{r}{\mu u(\hat{T})} + 1}\right)^2} = 0.$$

On the other hand, the social planner maximizes a weighted average of the continuation utility of agents that are single, agents that are dating, and agents that are married. As a first step, we solve the social planner's problem if s/he assigns a weight of one to the utility of agents that are dating. Formally, the social planner's problem of maximizing the continuation value of agents who have just met can be written as follows.

$$W_d(\hat{T}, \hat{T}) = \frac{r + \mu u(\hat{T})}{r + \mu u(\hat{T})(1 - \sigma(\hat{T})}\overline{v}_d(\hat{T})).$$

The necessary and sufficient first-order condition of this problem is

$$\frac{r\sigma(\hat{T})\overline{v}_d(\hat{T})u'(\hat{T})}{\mu u(\hat{T})^2\left(\frac{r}{\mu u(\hat{T})}+1\right)^2\left(1-\frac{\sigma(\hat{T})}{\frac{r}{\mu u(\hat{T})}+1}\right)^2} + \left\{\frac{\overline{v}_d'(\hat{T})}{1-\frac{\sigma(\hat{T})}{\frac{r}{\mu u(\hat{T})}+1}} + \frac{\overline{v}_d(\hat{T})\sigma'(\hat{T})}{\left(\frac{r}{\mu u(\hat{T})}+1\right)\left(1-\frac{\sigma(\hat{T})}{\frac{r}{\mu u(\hat{T})}+1}\right)^2}\right\}.$$

Note that the term inside the curly brackets is identical to the agents' FOC, whereas the term outside the curly brackets is negative since u is decreasing in T. To see that u is decreasing in T, recall that it is increasing in $l^\star$ (by Equation (12)). It follows that, for these weights, the social planner would instruct agents to date for less time than they would choose to date by themselves.

The continuation utility of a married agent is given by the flow utility s/he will derive while married, and the continuation value of a single agent discounted by the exogenous time at which her/his marriage will break up. The continuation utility of a agent that is single is $\frac{\mu u(T)}{\mu u(T)+r}$ times the continuation utility of an agent upon meeting a potential partner. Since $\frac{\mu u(T)}{\mu u(T)+r}$ is decreasing in T, it follows that if the social planer assigns a positive weight to the welfare of single or married agents, then s/he will choose a shorter dating time than $\hat{T}$, which, in turn, implies that equilibrium breakup threshold is strictly below the socially efficient one. ■

Proof of Proposition 5. First consider potential partners with pizzazz $y \geq x''$. Since $q^\star(\cdot)$ is increasing, for both the couple (y, x'') and the couple (y, x') agent y is the one that breaks up with her/his partner, and, moreover, s/he does so when the belief about the couple's compatibility drops to $q^\star(y)$. It follows that, for all $y \in A_\alpha(x', x'')$ such that $y > x''$, agent y will date agent x'' for strictly longer than s/he dates agent x', and so $\alpha(y, x') > \alpha(y, x'')$.

Next consider potential partners with pizzazz $y \leq x''$. For such potential partners, x'' is the one that breaks up with y. Hence, x'' will date such y's with positive probability if and only if $y > \frac{q^\star(x'')}{x''}$. Hence, the set of such agents that marry agent x'' with positive probability is an interval with an upper bound of x''. If the sets of potential partners with $y < x''$ whom x'' and x' marry with strictly positive probability do not intersect, then the proposition is established. In the remainder of the proof, we therefore assume that such an intersection exists and focus on potential partners in $A_\alpha(x', x'')$ with pizzazz $y < x''$.

If $\alpha(x'', y) > \alpha(x', y)$ for all $y \in A_\alpha(x', x'')$ such that $y \leq x''$, then the proposition is established. Otherwise, there exists $y^\star < x''$ for which $\alpha(x', y^\star) = \alpha(x'', y^\star) > 0$. To see this, note that if agents x and y date until their belief over their compatibility is q, then the maximal length of time for which they will date is given by the solution to $e^{-\lambda T(q)} \frac{xy}{1-xy} = \frac{q}{1-q}$. Hence the probability that they marry is

$$\alpha(x, y, q) = \begin{cases} xy(1 - e^{-\lambda T(q)}) = \frac{xy-q}{1-q} & \text{, if } xy > q \\ 0 & \text{, otherwise} \end{cases}. \tag{14}$$

The following lemma, which is used in the remainder of the proof and is established formally at the end of this proof, establishes the Lipschitz continuity of $W_s(x)$ and $q^\star(x)$.

Lemma A.1 *$W_s(x)$ and $q^\star(x)$ are Lipschitz continuous.*

Since $q^\star(\cdot)$ is continuous it follows that the probability that a couple marries is also continuous in the pizzazz of both agents. The existence of such a $y^\star$ follows from the intermediate value theorem. The fact that $y^\star < x''$ follows from the fact that since both the couple (x'', x'') and the couple (x'', x') use the breakup threshold $q^\star(x'')$, the former couple is more likely to marry than the latter couple.

To conclude the proof, we show that $\alpha(x', y) \geq \alpha(x'', y)$ for every potential partner $y \in A_\alpha(x', x'')$ with pizzazz $y \leq y^\star$. For such y, the breakup threshold of the couple (x'', y) is $q^\star(x'')$. Thus, by Equation (14), if a couple (x'', y) date, they marry with probability

$$\alpha(x'', y) = \frac{x''y - q^\star(x'')}{1 - q^\star(x'')}.$$

Hence, for any $y < y^\star$ that x'' dates for a strictly positive amount of time, it holds that

$$\frac{d\alpha(x'', y)}{dy} = \frac{x''}{1 - q^\star(x'')}.$$

Similarly, the probability that a dating couple (x', y) will marry is

$$\alpha(x', y) = \frac{x'y - q^\star(\max\{x', y\})}{1 - q^\star(\max\{x', y\})}.$$

Since $q^\star(\cdot)$ is monotone, it is differentiable almost everywhere. Hence, for almost all

$y < y^\star$, it holds that

$$\frac{d\alpha(x',y)}{dy} = \frac{x'}{1-q^\star(\max\{x',y\})} - \frac{1-x'y}{(1-q^\star(\max\{x',y\}))^2}\frac{dq^\star(\max\{x',y\})}{dy}.$$

Since $x'' > \max\{x',y\}$ and $q^\star(\cdot)$ is increasing, it follows that $\frac{d\alpha(x'',y)}{dy} > \frac{d\alpha(x',y)}{dy}$ for all $y < y^\star$ at which $q^\star$ is differentiable. By Lemma A.1, $q^\star$ is Lipschitz continuous and hence absolutely continuous, which, in turn, implies that α is also absolutely continuous. Since, by definition, $\alpha(y^\star, x'') = \alpha(y^\star, x')$, the proposition follows from the fundamental theorem of calculus. ■

Proof of Lemma A.1. Since $W_s(x)$ is increasing (Lemma 1), in order to show that it is Lipschitz continuous it suffices to show that there exists $\bar{K}$ such that $W_s(x+\epsilon) - W_s(x) \leq \bar{K}\cdot\epsilon$ for any $x, \epsilon > 0$ such that $x, x+\epsilon \in [\underline{x}, \overline{x}]$.

Denote $\Delta(x,\epsilon,y) = V_d(x+\epsilon;y) - V_d(x;y)$. From Equation (6) it holds that

$$\begin{aligned} W_s(x+\epsilon) - W_s(x) =& \frac{\mu}{r}\int_{\underline{x}}^{\overline{x}} \Delta(x,\epsilon,y)u(y)dy \\ =& \frac{\mu}{r}\left(\underbrace{\int_{\underline{x}}^{x} \Delta(x,\epsilon,y)u(y)dy}_{I_1} + \underbrace{\int_{x}^{\overline{x}} \Delta(x,\epsilon,y)u(y)dy}_{I_2}\right). \end{aligned}$$

Next, we show that $I_n \leq K_n \cdot \epsilon$ for $n = 1, 2$ and an appropriately defined $K_n > 0$. From Equation (4) it follows that $q^\star$ is increasing in W_s, and, hence, increasing in the agent's pizzazz. It follows that when dating a partner with pizzazz $y < x$, both agent x and agent $x+\epsilon$ will be the ones that decide when to break up with that partner. Hence, with respect to such partners, the dating times that agent $x+\epsilon$ chooses are also feasible for x. Note that for any dating strategy, the effect of increasing the ex-ante probability that the pair of agents are compatible by ϵ is bounded from above by the gain when the cost of dating is zero, $T = \infty$, and the marriage lasts indefinitely. This gain is given by $\epsilon\frac{\lambda}{\lambda+r}\frac{1}{r}$. Moreover, the discounted number of partners that an agent can date is bounded from above by the discounted number of partners that the agent would meet if s/he were to date all partners for zero units of time. It is straightforward to show that this upper bound is given by $\frac{\mu}{r}$. It follows that were agent x to use the dating strategy of agent $x+\epsilon$, the difference in their values would be at most $\frac{\lambda}{\lambda+r}\frac{1}{r}\frac{\mu}{r}$. Since $\int_{\underline{x}}^{x} u(y)dy$ is less than the measure of agents in the population with pizzazz

$y \leq x$, it follows that

$$I_1 \leq \left(Pr(y \leq x)\lambda \frac{\mu}{r^2}\right) \cdot \epsilon \leq \left(\lambda \frac{\mu}{r^2}\right) \cdot \epsilon \equiv K_1 \cdot \epsilon.$$

When agent x dates a partner with pizzazz $y > x$, the monotonicity of $q^\star$ and W_s implies that agent y will be the one to break off the dating relation. Moreover, after the couple $(y, x + \epsilon)$ has been dating so that $q_t(y, x+\epsilon) = xy$, the amount of time they will continue to date is at most the amount of time that the couple (x, y) would date. It follows that the difference in the dating times between the couples x, y and $x + \epsilon, y$ is at most the amount of time it takes q_t to drift down by ϵy, which, in turn, is bounded from above by the time it takes beliefs to drift down by ϵ.

Since $\frac{dq_t}{dt} = -\lambda q_t(1-q_t)$, an upper bound on the difference in dating times is $\frac{\epsilon}{\min_{q_t} \lambda q_t(1-q_t)}$. Note that this bound is finite, since $q_t \leq \overline{x}^2 < 1$ and, in optimum, all agents terminate dating no later than when $q_t = q_{min}$, where $q_{min}\lambda\frac{1}{r} = c$. Finally, note that the marginal value of increasing the dating time is bounded from above by $\lambda\frac{1}{r}$. Since $\int_x^{\overline{x}} u(y)dy$ is less than the measure of agents in the population with pizzazz $y \geq x$, it follows that

$$I_2 \leq \left(\frac{Pr(y > x)}{\min_{q \in \{\frac{cr}{\lambda}, \overline{x}^2\}} \lambda q(1-q)} \frac{\lambda}{r}\right) \cdot \epsilon \leq \left(\frac{1}{\min_{q \in \{\frac{cr}{\lambda}, \overline{x}^2\}} rq(1-q)}\right) \cdot \epsilon \equiv K_2 \cdot \epsilon.$$

Therefore, $W_s(\cdot)$ is Lipschitz continuous with modulus $\bar{K} = \frac{\mu}{r}(K_1 + K_2)$.

The strict monotonicity and Lipschitz continuity of $q^\star$ then follows immediately from condition (4) and the observation that $rW_s \leq r\frac{\mu}{r+\mu}\frac{1}{r} < 1$, where the first inequality follows from the fact that rW_s is bounded from above by r times the expected discounted time at which the next partner is met, $\frac{\mu}{r+\mu}$, and the payoff from a marriage that lasts forever, $\frac{1}{r}$. ■

Proof of Proposition 6. Since agents do not marry before clicking with their partner and the arrival rate of clicks is at most $\overline{x}^2\lambda$, the expected discounted amount of time that agents are married (with all partners that they marry) is bounded away from $\frac{1}{r}$ uniformly over μ. Thus, for any μ, $rW_s(x)$ is bounded away from 1 for every $x \in [\underline{x}, \overline{x}]$. This result follows directly from the definition of $q^\star$ in (4). ■

Proof of Corollary 1. The first part follows directly from Proposition 6. For the second part, plug $d(x) = 0$ and $\alpha(x, y) = xy$ into (8) to obtain

$$x\mu u(x)E^s = \delta(g(x) - u(x)).$$

Rearranging this expression yields the desired result. ■

Proof of Proposition 7. **Step 1**. In this step, we connect the measure of pairs of agents (x, y) that are dating and the probability that such a couple will eventually marry ($\alpha(x, y)$). We then use this connection to derive $u(x)$ as a function of $\alpha(x, y)$.

Denote by T the maximal time a couple (x, y) will date given their breakup thresholds. The probability that such a couple will still be dating after $t < T$ units of time is given by $1 - xy(1 - e^{-\lambda t})$. Hence,

$$d(x, y) = \mu u(x)u(y) \int_0^T (1 - xy(1 - e^{-\lambda t}))dt = \mu u(x)u(y) \left(\frac{xy\left(1 - e^{-\lambda T}\right)}{\lambda} + T(1 - xy) \right).$$

Moreover, recall that if agents x and y date for at most T units of time, then the probability that they marry is $\alpha(x, y) = xy(1 - e^{-\lambda T})$, and so $T = \frac{1}{\lambda} \log\left(\frac{xy}{xy - \alpha(x,y)}\right)$. It follows that the measure of agents with pizzazz x that are dating is

$$d(x) = \int_X \mu u(x)d(x, y)u(y)dy = u(x) \int_X \frac{\mu}{\lambda} \left(\alpha(x, y) + (1 - xy) \log\left(\frac{xy}{xy - \alpha(x, y)}\right)\right) u(y)dy.$$

Rearranging the balanced-flow condition (8) yields

$$u(x) \left(1 + \frac{\mu}{\delta} u(x) \int_X \alpha(x, y)u(y)dy\right) = g(x) - d(x).$$

Plugging in the expression for $d(x)$ and rearranging yields

$$u(x) = \frac{g(x)}{1 + \mu \int_X \left(\frac{\lambda+\delta}{\lambda\delta}\alpha(x, y) + \frac{1-xy}{\lambda} \log\left(\frac{xy}{xy-\alpha(x,y)}\right)\right) u(y)dy}. \tag{15}$$

Step 2. Assume by way of contradiction that $\liminf_{\mu\to\infty} u(x) = 0$ for some x. From Equation (15) it follows that there exist $Y \subset X$ with strictly positive measure such that (i) $\alpha(x, y) > 0$ for every $y \in Y$, and (ii) $\liminf_{\mu\to\infty} \mu u(y) = \infty$ for every $y \in Y$. However, this implies that agents $y \in Y$ also instantaneously meet singles with pizzazz y. Recall that, in equilibrium, agents with the same pizzazz agree to date each other. Hence, for every $y \in Y$ such that $x < y$, it holds that $\alpha(x, y) = 0$. Similarly, since agent x can instantaneously start dating any $y \in Y : y < x$, it follows that in the limit, for almost all $y \in Y : y < x$, we have $\alpha(x, y) = 0$. Hence, any $Y \subset X$ that satisfies properties (i) and (ii) must be of measure zero, in contradiction to the assumption that Y has a positive measure.

Since $\liminf_{\mu\to\infty} u(x) > 0$, it holds that $\lim_{\mu\to\infty} \mu u(x) = \infty$. This, in turn, by the argument used above, implies that $\lim_{\mu\to\infty} q^{\star}(x) = x^2$. ■

Proof of Theorem 1. To establish the existence of a steady-state equilibrium, we show that 1) value functions have a continuous impact on the probability that any two agents marry, 2) marriage probabilities have a continuous impact on the distribution of agents in the singles market, and 3) the value functions are given by a fixed point of a continuous operator.

Define the family $\mathcal{F}$ of functions from X to $[0, \overline{W}]$, where $\overline{W} = \frac{\mu}{(r+\mu+\delta)r}$, that are weakly increasing and Lipschitz continuous with modulus $K^{\star}$. This subset of $C[0,1]$ is nonempty, bounded, closed, and convex. Note that $\overline{W}$ is an upper bound on any agent's value, since this is the expected value from meeting compatible partners at rate μ while single, marrying them immediately, and returning to the market once the marriage is hit by a dissolution shock. We endow this family of functions with the sup norm $||W_s|| = sup_{x\in X}|W_s(x)|$.

As explained in the proof of Lemma A.1, every agent will break up with her/his partner once the belief about compatibility is $q_{\min} = \frac{rc}{\lambda}$, regardless of the value of being single. Therefore, for any pair of agents, by Equation (14) it holds that $\alpha(x,y) \in [0, \frac{xy-q_{min}}{1-q_{min}}]$. Hence, we denote by $\alpha^{W_s}(x,y) \to [0, \frac{xy-q_{min}}{1-q_{min}}]$ a mapping that specifies the probability that any pair of agents that meet will marry, when they behave according to the breakup thresholds given by (4), and given the value functions W_s. We endow the space of matching probabilities with the sup norm. Finally, we denote by $q^{W_s}(x)$ the breakup threshold for agent x that is optimal (i.e., given by Equation (4)) when the value of singlehood is $W_s(x)$.

Lemma A.2 *α^{W_s} is continuous in W_s.*

Proof of Lemma A.2. From Equation (14) it follows that

$$\alpha^{W_s}(x,y) = \begin{cases} \frac{xy-\max\{q^{W_s}(x),q^{W_s}(y)\}}{1-\max\{q^{W_s}(x),q^{W_s}(y)\}} & \text{, if } \max\{q^{W_s}(x), q^{W_s}(y)\} \geq xy \\ 0 & \text{, otherwise} \end{cases}.$$

Since $W_s \in [0, \overline{W}]$, condition (4) implies that $q^{W_s}(x) \in [c\frac{r+\delta}{\lambda}, \frac{c(\delta+\mu+r)+\mu}{\lambda}]$, and that $\frac{dq^{W_s}(x)}{dW_s(x)}$ is uniformly bounded from above. It is straightforward to verify that α is concave and decreasing in $\max\{q^{W_s}(x), q^{W_s}(y)\}$. Hence, for any $x, y \in X$, the absolute value of the derivative of α with regard to $\max\{q^{W_s}(x), q^{W_s}(y)\}$ is bounded from above by the absolute value of its derivative at $\max\{q^{W_s}(x), q^{W_s}(y)\}$, which is given by $\frac{1}{1-xy}$. Since X is compact and bounded away from one, it follows that α is continuous in the sup norm. ■

Let u_α and d_α denote, respectively, the measure of agents in the singles market and the measure of agents that are dating, as functions of the marriage probabilities α. We endow both measures with the sup norm.

Lemma A.3 *u_α is continuous in α.*

Proof of Lemma A.3. Fix a matching function α. The maximal amount of time for which a couple (x, y) dates is

$$T^\star(x,y) = -\frac{1}{\lambda} log\left(1 - \frac{\alpha(x,y)}{xy}\right),$$

and so the measure of couples with pizzazz (x, y) that are dating in the steady state is

$$d(x,y) = \mu u(x)u(y) \int_0^{T^\star(x,y)} (1 - xy(1 - e^{-\lambda t}))dt. \tag{16}$$

Combining these two equations yields

$$d(x,y) = \mu u(x)u(y)\frac{\alpha(x,y) - (1-xy)\log(1 - \frac{\alpha(x,y)}{xy})}{\lambda}.$$

Combining this equation with Equations (7) and (8) yields

$$\mu u(x) \int_X \alpha(x,y)u(y)dy = \delta\left(g(x) - \int_X \left(\mu u(x)u(y)\frac{\alpha(x,y) - (1-xy)\log(1-\frac{\alpha(x,y)}{xy})}{\lambda}\right)dy - u(x)\right).$$

Rearranging this condition, we have[20]

$$\int_X u(y)\left\{\alpha(x,y)(\frac{\mu}{\delta} + \frac{\mu}{\lambda}) - \frac{\mu}{\lambda}(1-xy)\log\left(\frac{xy - \alpha(x,y)}{xy}\right)\right\}dy = \frac{g(x)}{u(x)} - 1. \tag{17}$$

Consider how changing the value of α by at most ϵ (for any element in its domain) impacts the term in curly brackets in Equation (17). The change in the first term inside the curly brackets is at most

$$\epsilon(\frac{\mu}{\delta} + \frac{\mu}{\lambda}).$$

Since $\alpha(x,y) \leq \frac{xy - q_{min}}{1-q_{min}}$ it follows that $xy - \alpha(x,y) \geq q_{min}\frac{1-xy}{1-q_{min}}$, and so the change in the

[20] $u(x)$ is positive for any x since all dating times are bounded, and since marriages dissolve at rate $\delta > 0$.

second term inside the curly brackets is at most

$$\frac{\mu(1-q_{min})}{\lambda q_{min}(1-\overline{x})}\epsilon.$$

Moreover, note that since $u_\alpha(x) \in [0, \overline{g}]$ for any α, the change in $u_\alpha(x)$ due to any change in α is at most $\overline{g}$. It follows that such a change in α can change the LHS of Equation (17) by at most

$$\overline{g}\left(\frac{\mu}{\delta}+\frac{\mu}{\lambda}+\frac{\mu(1-q_{min})}{\lambda q_{min}(1-\overline{x})}\right)\epsilon.$$

Therefore, u_α is continuous in the sup norm. ■

In equilibrium, value functions satisfy

$$rW_s(x) = \mu \int_X (W_d^{W_S}(x;y) - W_s(x))u^{W_s}(y)dy,$$

where $W_d^{W_S}(x,y)$ is agent x's *continuation utility* upon meeting agent y given W_s, and $u^{W_s}(y)$ are the densities in the singles market that are consistent with value functions W_s. Adding the expectation of $W_s(x)$ to both sides and rearranging, we define the operator Γ by

$$\Gamma W_s(x) = \frac{\mu}{r+\mu\overline{u}^{W_s}} \int_X W_d^{W_S}(x;y)u^{W_s}(y)dy, \tag{18}$$

where $\overline{u}^{W_s} = \int_X u^{W_s}(y)dy$. Note that a fixed point of this operator is an equilibrium.

Lemma A.4 *If $K^\star$ is sufficiently large, then $W_s \in \mathcal{F}$ implies that $\Gamma(W_s) \in \mathcal{F}$.*

Poof of Lemma A.4. First, we show that $\Gamma W_s(x)$ lies in $[0, \overline{W}]$. Since $W_d^{W_S}$ is nonnegative, it is immediate that $\Gamma W_s(x) \geq 0$. An upper bound on agent x's value upon meeting agent y is given by her/his value from marrying a compatible partner immediately, separating when the dissolution shock occurs, and then being matched again to agent y according to a Poisson process with arrival rate μ. This is the case since an upper bound on the flow value is the expected value between the point in time when a couple marries and the point in time when each of the agents meets her/his next partner, and the equilibrium matching rate is less than μ. That is,

$$W_d^{W_S}(x;y) \leq \frac{1}{r+\delta} + \frac{\delta}{r_\delta}\frac{\mu}{r+\mu}W_d^{W_S}(x;y),$$

which implies that

$$W_d^{W_S}(x;y) \leq \frac{\mu+r}{r(\delta+\mu+r)}.$$

It follows that

$$\Gamma W_s(x) \le \frac{\mu \overline{u}^{W_s}}{r + \mu \overline{u}^{W_s}} \frac{\mu + r}{r(\delta + \mu + r)} \le \overline{W}.$$

Next, we must show that $\Gamma W_s(x)$ is monotone. This follows from the fact that an agent with pizzazz $x_2 > x_1$ can perfectly duplicate the dating time of agent x_1 with any potential partner, and that $W_s(x)$ is assumed to be increasing, as it is an element of $\mathcal{F}$.

Finally, we must show that for sufficiently large $K^\star$, $\Gamma W_s(x)$ is Lipschitz continuous with modulus $K^\star$. Since ΓW_s is increasing in x, it suffices to show that $\Gamma W_s(x+\epsilon) - \Gamma W_s(x) \le \epsilon K^\star$. Note that

$$W_d^{W_s}(x;y) = V_d^{W_s}(x;y) + W_s(x),$$

where $V_d^{W_s}(x;y)$ is agent x's capital gain from meeting agent y, given value functions W_s. Using this representation, it follows that

$$\Gamma W_s(x+\epsilon) - \Gamma W_s(x) = \frac{\mu}{r + \mu \overline{u}^{W_s}} \int_X (V_d^{W_s}(x+\epsilon, y) - V_d^{W_s}(x,y)) u^{W_s}(y) dy + \frac{\mu}{r + \mu \overline{u}^{W_s}} \int_X (W_s(x+\epsilon) - W_s(x)) u^{W_s}(y) dy.$$

Since W_s is increasing and continuous by assumption, the same arguments used in the proof of Lemma A.1 show that $V_d^{W_s}(x,y)$ is Lipschitz continuous with modulus $\bar{K}$, where $\bar{K}$ is a function of the model parameters that is independent of $K^\star$. Thus,

$$\Gamma W_s(x+\epsilon) - \Gamma W_s(x) \le \frac{\mu}{r + \mu \overline{u}^{W_s}} \int_X (\overline{K}\epsilon + K^\star \epsilon) u^{W_s}(y) dy \le (\frac{\mu}{r+\mu} \overline{K} + \frac{\mu}{r+\mu} K^\star)\epsilon.$$

Hence, if $K^\star$ is sufficiently large, then ΓW_s is Lipschitz continuous with modulus $K^\star$, in which case $\Gamma : \mathcal{F} \to \mathcal{F}$. ∎

Lemma A.5 *The operator Γ is continuous.*

Proof of Lemma A.5. If W_s^1 and W_s^2 are close under the sup norm, then the maximal dating times of any couple are close under these two value functions. Since small changes in W_s also induce small changes in u^{W_s} (by Lemmas A.2 and A.3), it follows that small changes in W_s have a small impact on ΓW_s. ∎

We have shown that $\mathcal{F}$ is closed, bounded, convex, and nonempty, and that Γ is a continuous mapping from $\mathcal{F}$ to $\mathcal{F}$. Moreover, since $\mathcal{F}$ is family of Lipschitz continuous functions with the same modulus, it is equicontinuous. Thus, Schauder's fixed point theorem (Theorem 17.4 in Stokey and Lucas, 1989) establishes that Γ has a fixed point.

The Lipschitz continuity of $W_s(\cdot)$ and $q^\star(\cdot)$ has already been established in Lemma A.1.

■