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Double Auctions and Transaction Costs*

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Abstract

Transaction costs are omnipresent in markets but are often omitted in economic models. We show that the presence of transaction costs can fundamentally alter incentive and welfare properties of Double Auctions, a canonical market organization. We further show that transaction costs can be categorized into two types. Double Auctions with homogeneous transaction costs—a category that includes fixed fees and price based fees—preserve the key advantages of Double Auctions without transaction costs: markets with homogeneous transaction costs are asymptotically strategyproof, and there is no efficiency-loss due to strategic behavior. In contrast, double auctions with heterogeneous transaction costs—such as spread fees—lead to complex strategic behavior (price guessing) and may result in severe market failures. Allowing for aggregate uncertainty, we extend these insights to market organizations other than Double Auctions.

Keywords: Double Auction, Transaction Costs, Incentives, Efficiency, Robustness.

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1 Introduction

There is almost no trade without transaction costs such as taxes, commissions, fees, or transportation costs. Any difference between what the buyer pays and what the seller receives is a transaction cost. Yet, these costs are often omitted in the strategic analyses of markets.[1] Is the omission of transaction costs affecting the analysis? We show that the answer differ across different cost structures. For some costs, such as fixed transaction costs and price fees, the omission does not substantially affect the strategic properties of the market, but for other costs, such as spread fees, it does.

The canonical structures of transaction costs include fixed transaction costs, price fees, and spread fees; our analysis throws light on the impact of these transaction costs as well as others.[2] A *fixed transaction cost* charged to a trader depends only on whether the trader participates in trade. Examples range from handling fees that are often related to overhead costs, to transaction costs related to packaging and shipping. A *price fee* is a percentage of the price. Examples range from stamp duties set by governments, Tobin taxes as levied in Sweden and Latin America, the 'buyer's premium' charged by art auction houses, to 'service fees' or 'final value fees' as charged by Airbnb, eBay, Uber and Lyft, etc. A *spread fee* is a percentage of the difference between bid and ask. Examples range from commissions charged by car dealers, to bid-ask spreads present in stock exchanges such as the New York Stock Exchange (NYSE). Notably, stock exchanges, including NYSE, run opening auctions at the start of each trading day, which closely resemble Double Auctions with transaction costs.[3]

This paper aims to fill this gap. In Double Auctions (DAs) with general transaction costs, we investigate participants' strategic behavior and resulting market efficiency. We characterize optimal strategic behavior, and identify which classes of transaction costs preserve, and which do not, the desirable properties of DAs–asymptotic truthfulness and efficiency–known from the theory of DAs without transaction costs (c.f., e.g., Rustichini et al. 1994). We also analyze the robustness of said properties to market participants having misspecified beliefs and aggregate uncertainty.

Our main insight is that the aforementioned desirable properties – truthfulness, efficiency, robustness to misspecified beliefs and aggregate uncertainty – of DA markets are determined by whether the transaction costs are *homogeneous* or *heterogeneous*. We say that a transaction cost is homogeneous if, conditional on a market participant trading in the market, the participant's impact on the transaction cost they pay vanishes as the market grows large; else we say that the transaction cost is heterogeneous.[4] Price fees are examples of homogeneous transaction costs as, in the limit, the

[1]Below we discuss the notable exceptions among the studies of Double Auctions, the market organization we focus on: the analysis of efficiency under fixed fees in Tatur (2005), market entry under fixed fees in Marra (2019), and platform revenues with fixed and price fees in Chen and Zhang (2020).

[2]Our analysis does not hinge on whether these transaction costs cover the cost of trading infrastructure or additional services (such as transport or insurance).

[3]During a trading day stock exchanges run quasi-continuous markets, which can be thought off as open-bid Double Auctions in contrast to the standard (sealed-bid) Double Auction.

[4]The two types of transaction costs are close to partitioning but do not completely partition the set of possible

market participants impact on the fee vanishes (and, relatedly, all participants who trade pay the same fee). Spread fees are examples of heterogeneous transaction costs as, in the limit, the spread and hence the fee paid depends on the trading participant's action. Not surprisingly, under homogeneous transaction costs, the traders behave similarly to traders in markets with no transaction costs and they are approximately *truthful* in large markets. In contrast, heterogeneous transaction costs distort incentives fundamentally, and, asymptotically, lead to what we call *price-guessing* behavior whereby traders bid close to estimated market prices in order to try to minimize their transaction cost.

Homogeneous transaction costs lead to some unavoidable welfare losses in finite markets that are due to strategic behavior and possible direct loss due to unprofitability of trades whose surplus is insufficient to cover the cost. Because truthfulness emerges in the limit, in large markets the outcomes are not much affected when the transaction costs are small; and the same obtains even when agents have misspecified beliefs.

In contrast, in large markets, heterogeneous transaction costs lead to no loss due to strategic behavior, but again may lead to a direct loss as described above. However, even slight belief misspecification often leads to substantive market failure. The risk of market failure occurs for all heterogeneous transaction costs, and the degree of inefficiency does not vanish with decreasing size of the transaction cost.

Allowing for aggregate uncertainty, we show that the aforementioned results qualitatively hold true and extend beyond Double Auctions. In particular, our insights remain valid in any market organization in which the participants believe that they have no influence on market prices.

Related literature

We know a lot about strategic behavior in DAs without transaction costs as these mechanisms have been extensively studied.[5] Since the formal definition of the setting as one characterized by two-sided incomplete information (Chatterjee and Samuelson, 1983), the analysis of DAs focused on large markets because of the empirical relevance of this setting, and because in finite-size markets Myerson and Satterthwaite (1983) showed that there generally exists no budget-balanced, incentive-compatible, and individually rational mechanism that is Pareto efficient.[6]

In large DA markets, participants have incentives to be increasingly truthful, which results in asymptotic efficiency (Roberts and Postlewaite 1976, Rustichini et al. 1994, Cripps and Swinkels 2006, Reny and Perry 2006, Azevedo and Budish 2019); any given participant's influence on the market price vanishes in larger markets, and market participants place increasing weight on maximizing their trading probability (as opposed to influencing the price), which they do by bidding close to truthfully.

transaction costs; see Section 4.3. We study both large finite and continuum models. In continuum models, the definition simplifies and, conditional on trade, the homogeneous transaction cost paid by a participant is the same irrespective of the action of the participant, while the heterogeneous transaction cost depend on participant's actions.

[5] See Friedman and Rust (1993) for a survey of the DA as a market mechanism in history, theory and practice.

[6] The impossibility hinges on the quasilinearity of the preferences, which we also assume; see Garratt and Pycia (2016).

Rustichini et al. (1994) established this key insight for DAs with independent private values (c.f. Satterthwaite and Williams 1989b). Their work assumes existence of symmetric equilibria, which was later established by Fudenberg et al. (2007) under correlated but conditionally independent private values.[7]

We know much less about DAs with transaction costs, except for the case of fixed transaction costs. Tatur (2005) analyzes incentives and efficiency in DAs but only with fixed transaction costs; unlike us he does not require budget balance. Chen and Zhang (2020) study revenues in linear equilibria of DAs with transaction costs; they allow transaction costs to depend on the size of individual trade but not on price, bid-ask spread, nor other parameters of the market schemes. Marra (2019) studies market entry in DAs with fixed transaction costs. Noussair et al. (1998) provides experimental evidence that fixed transaction costs lead to efficiency loss. Fixed transaction costs have also been the focus in the finance literature on limit order books [Colliard and Foucault 2012, Foucault et al. 2013, Malinova and Park 2015].[8] Where this literature focuses on specific (fixed) transaction costs, we look at transaction costs more generally and our classification has no counterpart in the literature. Our general incentive, efficiency, and robustness results are also new.

Our analysis also contributes to the burgeoning literature on market behavior in the presence of misspecified beliefs. The impact of misspecified beliefs on mechanism design has been analyzed by many authors, c.f., e.g., Ledyard (1978), Wilson (1987), Chung and Ely (2007), Bergemann and Morris (2005), Chassang (2013), Bergemann et al. (2015), Carroll (2015), Wolitzky (2016), Carroll (2017), Madarász and Prat (2017), Li (2017), Boergers and Li (2019), Pycia and Troyan (2019). The main thrust of this literature is that robustness to misspecification requires the mechanism to be simple. The impact of heterogeneous, misspecified, beliefs on Walrasian markets has been analyzed e.g., by Harrison and Kreps (1978) and Eyster and Piccione (2013).[9] We contribute to the studies of misspecified models by analyzing how misspecification impacts the efficiency of DAs with transaction costs.

[7] They also generalized the convergence results of Rustichini et al. (1994). Earlier work on equilibrium existence in DAs includes Chatterjee and Samuelson (1983), Wilson (1985), Leininger et al. (1989), Satterthwaite and Williams (1989a), Williams (1991), and Cripps and Swinkels (2006). See also Jackson and Swinkels (2005) who studied equilibrium existence in a broad class of private value auctions that includes DAs.

[8] See also Shi et al. (2013) who study a numerical model of marketplace competition with transaction costs.

[9] See also, e.g., (Heidhues et al., 2018) who study overconfidence in markets and (de Clippel and Rozen, 2018) who study the misperception of tastes.

2 Example

In the example, we consider a special case of our general model. We assume there is a continuum of traders on each side of the market. One of the main results of our paper is that there are two qualitatively different categories of transaction costs. In the example, we focus on two common transaction costs that are representative of these categories: price fees and spread fees. We make the exposition parallel to the structure of the general results so that the reader can easily read it as both a preview and an illustration of the general results.

Model (cf. Section 3)

The market (cf. Section 3.1)**.** We consider a two-sided infinite market with a unit mass of buyers and sellers who are interested in either buying or selling an indivisible good. *Types*, giving the *gross value* of the item to a trader i, are uniformly distributed with $t_i \in T = [1, 2]$. The *utility* of each trader is the sum of the gross value of the object (if they have it) and their money holdings, normalized such that a trader who does not participate in the mechanism has utility 0.

The mechanism (cf. Section 3.2)**.** Every trader i submits an *action* $a_i(t_i) \in \mathbb{R}^{\geq 0}$ representing a buyer's bid and a seller's ask. Given all actions, the *double auction* selects subsets of buyers and sellers involved in trade at a unique *market price* P^*. The market price is selected to balance demand and supply, which are the total mass of buyers and sellers, who, given their actions, weakly prefer trading over not trading at that price. Additionally, every trader involved in trade has to pay a *transaction cost*. In the example, we consider our representative transaction costs, price and spread fees. A *price fee* is given by a fixed percentage $\phi \in [0, 1]$ of the market price and a *spread fee* is given by a fixed percentage $\phi \in [0, 1]$ of the spread between the action of a trader and the market price.

Beliefs and aggregate uncertainty (cf. Section 3.3 for beliefs and Section 7 for aggregate uncertainty)**.** We assume that traders know the market mechanism, but have incomplete information about the market environment, that is the distribution of gross values and market behavior of other traders. Both market sides may have incorrect and heterogeneous *beliefs*, and *aggregate uncertainty*. We work with traders' beliefs over actions. In an infinite market—as considered in the example—this simplifies to considering beliefs that are directly over the market price. Suppose that all buyers believe the market price to be $\beta \in [1, 2]$ and all sellers believe it to be $\sigma \in [1, 2]$. Traders might be uncertain about the market price and believe that it is distributed according to a *Beta*-distribution over $[1, 2]$, with mean equal to β respectively σ.

Incentives (cf. Section 4)

Truthfulness (cf. Section 4.1)**.** In a DA without transaction costs bidding one's gross value is the only action that (1) never results in a loss, (2) dominates all less aggressive actions (that is higher for the buyer and lower for the seller), and (3) is not dominated by any more aggressive action. If transaction costs are due, bidding one's gross value may no longer satisfy these properties. We define the *net value*, t_b^Φ of a buyer with gross value $t_b(t_s)$ as the largest action satisfying (1)-(3). In analogy, for a seller with gross value t_s, the net value is the smallest action satisfying (1)-(3). With no transaction costs, the net value is the gross value, and motivated by this we say that bidding is *truthful* if the trader bids their net value. To illustrate the concepts of net values and truthfulness, let us consider price and spread fees. With price fees, for a buyer with gross value t_b, the net value is $t_b^\Phi = {t_b}/{(1+\phi)}$ and for a seller with gross value t_s, the net value is $t_b^\Phi = {t_s}/{(1-\phi)}$. With positive price fees, trading at the market price equal to gross value results in negative utility while trading at the price equal to net value results in the utility of 0. With spread fees, the net values are equal to the gross values, that is, $t_b^\Phi = t_b$ and $t_s^\Phi = t_s$. A trader is indifferent between trading and not trading if the market price is equal to their gross value.

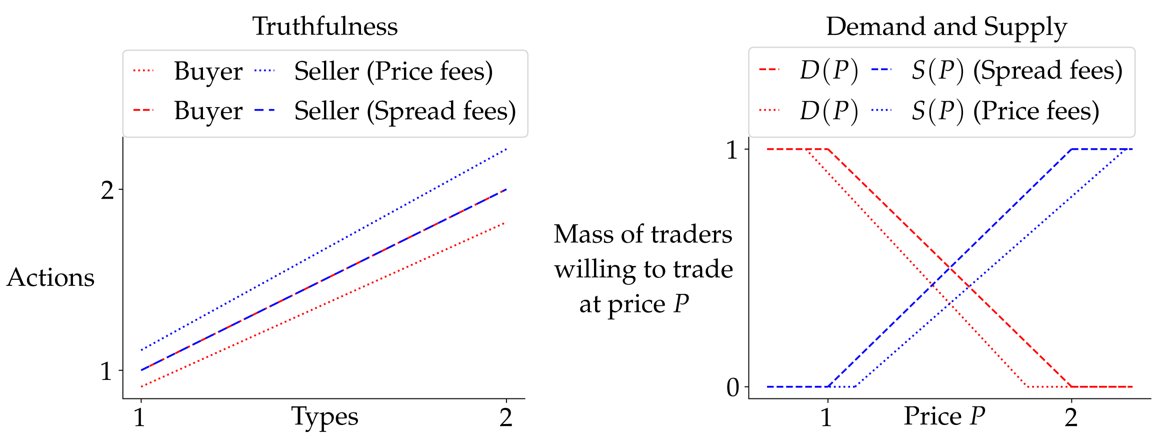

Figure 1: *Left.* Truthful strategy profiles for a 10% price and any spread fee. *Right.* Demand and supply functions, if traders act truthfully, again with a 10% price and any spread fee.

Probability of trade (cf. Section 4.2)**.**

Without uncertainty, a buyer believes to trade, if their bid is above the market price. Similarly, a seller believes to trade, if their action is below the market price. If their action is equal to the market price they believe to be involved in *tie-breaking* and trade with some probability. In the presence of uncertainty, the probability to be involved in trade is a continuous function in a trader's action. Decreasing the *aggressiveness* of an action, that is the distance to truthfulness, increases the probability of being involved in trade.

Profitability of trade (cf. Section 4.3)**.**

In an infinite market, a trader cannot influence the market price and hence also a price fee is independent of a trader's action. In contrast, a spread fee is directly influenced by the action of a trader and decreases, if a trader reports a more aggressive action that is closer to the market price.

As a general analysis shows, a trader's influence on their transaction cost or its lack plays a crucial role in determining their optimal strategy.

Optimal behavior (cf. Section 4)

Optimal behavior maximizes the expected utility of a trader given their beliefs by finding the right amount of aggressiveness to balance probability and profitability of trade. In the absence of tie-breaking, optimal strategies exist. With tie-breaking, existence of optimal strategies depends on the nature of the transaction cost.

Truthfulness is optimal for price fees (cf. Section 5.1)**.** As a trader cannot influence their payment, in order to maximize expected utility, it is optimal to maximize trading probability as long as the involvement in trade is individually rational. This is achieved by a trader truthfully bidding his net value. Note that truthfulness is independent of beliefs and uncertainty.

Price-guessing is optimal for spread fees (cf. Section 5.2)**.** In the absence of uncertainty, it is optimal to bid the market price, if this is individually rational given a trader's gross value and there is no tie-breaking. We call this behavior *price-guessing*. If there is uncertainty or tie-breaking, the trade-off between decreasing the transaction cost and increasing the probability of trade is non-trivial and depends on beliefs. However, if the uncertainty is sufficiently small, the incentive on the former outweighs the latter and it is optimal to bid close to the estimated market price. Note that price-guessing crucially depends on beliefs and uncertainty.

Figure 2: *Left.* Best responses as a function of the gross value, for a 10% price fee. *Right.* Best responses as a function of the gross value, for a 100% spread fee with deterministic beliefs $\beta = \sigma = 1.5$ without uncertainty (solid lines) and uncertain beliefs according to $Beta(5,5)$ (dotted lines). For comparison, the diagonal line coincides with reporting the gross value. For price fees, the best responses coincide with the net values. For spread fees, best responses constitute price-guessing for 'in-the-market' gross values and truthfulness otherwise, if there is no uncertainty. Uncertainty diminishes price-guessing.

Market performance (cf. Section 6)

Metrics (cf. Section 6.1)**.** The *trading volume* Tv is the mass of traders, who are involved in trade. The *trading excess* Ex measures for the two market sides the difference in mass of traders, who are willing to trade at the market price. The *individual gains of trade* for a trader i is the difference between their gross value and the market price. Integrating the individual gains of trade over all traders involved in trade results in the *gains of trade* G. If agents report their net values, call G_Φ the *achievable gains of trade*. In the absence of fees this coincides with reporting their gross value, achieving the *maximum gains of trade*, G_{id}. The *total loss* is the difference $L = G_{id} - G$, which measures how much gains of trade are lost due to fee considerations and strategic behavior. We split it up into $L = L_d + L_s$, where $L_d = G_{id} - G_\Phi$ is the *direct loss* due to fee constraints and $L_s = G_\Phi - G$ is the *strategy-induced loss*. Denote by Tc the *total transaction cost*. The *surplus generated by the traders* is the difference between the gains of trade and the collected fees: $Su = G - Tc$. G_{id} can then be decomposed into total fees, total surplus generated by the traders and the loss: $G_{id} = Su + Tc + L_d + L_s$.

Market outcome for price fees (cf. Section 6.2)**.** Independent of beliefs and uncertainty, truthfulness is optimal. The market price does not depend on the fee parameter and is equal to $P^* = {}^3\!/_2$. The trading volume $Tv = {}^{(1-3\phi)}\!/_2$ decreases linearly in ϕ with maximal trading volume without price fees equal to ${}^1\!/_2$ and full market failure occurring at $\phi = {}^1\!/_3$. Trading excess is equal to 0, so no tie-breaking is needed. The maximum gains of trade are $G_{id} = {}^1\!/_4$ and can be decomposed as follows: The realized gains of trade are equal to $G = {}^{(1-9\phi^2)}\!/_4$. There is no strategy-induced loss, as traders report truthfully. The direct loss is equal to ${}^{9\phi^2}\!/_4$, which is strictly increasing in the fee. The realized gains of trade can further be decomposed into surplus and transaction costs, that is $G = Tc + S$ with $Tc = {}^{(3\phi-9\phi^2)}\!/_4$ and $S = {}^{(1-6\phi-9\phi^2)}\!/_4$. From a market maker's point of view, transaction costs are maximized at $\phi = {}^1\!/_6$, where individuals' fee payments and market volume are balanced. At this point, 25% of the true gains of trade are lost, 50% are transaction costs and 25% remain as surplus to the traders. The second column of Figure 3 shows the decomposition of the maximum gains of trade as a function of the fee parameter ϕ.

Market outcome for spread fees (cf. Section 6.3)**.** The optimal behavior in DAs with spread fees is dependent on beliefs and uncertainty. Without uncertainty, price-guessing is optimal. With uncertainty, traders might deviate from price-guessing: Traders with profitable gross values are less aggressive, while traders with gross value close to the true market price might submit actions that are more aggressive. We show that depending on the beliefs β and σ about the market price, market outcomes range from full efficiency (with different decomposition of the true gains of trade into surplus and total fees) to complete market failure. Note that inefficiency is only due to strategic behavior, as spread fees do not lead to a direct loss. Furthermore, depending on the beliefs, uncertainty can either improve or worsen the market outcome, both from traders and the market maker's perspective.

To illustrate the range of possibilities, we analyze five different belief scenarios:

1. **Calibrated beliefs** ($\beta = \sigma = {}^3\!/_2$). The market is fully efficient. There are no transaction costs, as there is no bid-ask spread for traders involved in trade. Uncertainty leads to a strategy-induced loss and transaction costs.

2. **Homogeneous bias** ($\beta = \sigma \neq 1.5$)**.** The market is not fully efficient. The strategy-induced loss is increasing in the distance between $\beta = \sigma$ and ${}^3\!/_2$. Similar to calibrated beliefs, there are no transaction costs. Uncertainty diminishes the strategy-induced loss and leads to positive transaction costs.

3. **Conservative bias** ($\beta \geq 1.5 \geq \sigma$)**.** The market is fully efficient. The transaction costs decrease, if traders act more aggressive and β, σ approach ${}^3\!/_2$. Uncertainty decreases the transaction costs and adds a strategy-induced loss.

4. **Aggressive bias** ($\sigma \geq 1.5 \geq \beta$)**.** Complete market failure occurs. There is no trade, leading to zero transaction costs and surplus. Uncertainty lessens this effect, as traders are less aggressive, leading to trade, and hence some transaction costs and surplus.

5. **Mixed bias** ($1.5 \geq \beta \geq \sigma$)**.**[10] The market is not fully efficient. The loss is increasing in σ, more aggressive price-guessing by sellers leads to an efficiency loss. The transaction costs depends on the spread $\beta - \sigma$ and are generated entirely by buyers. Uncertainty leads to greater transaction costs and less strategy-induced loss.

A table with the full analysis of market characteristics is relegated to the Appendix. The third and fourth column of Figure 3 show the decomposition of the true gains of trade as a function of the fee parameter ϕ for examples of the five belief scenarios with or without aggregate uncertainty.

[10] The case $\beta \geq \sigma \geq 1.5$ is analogous.

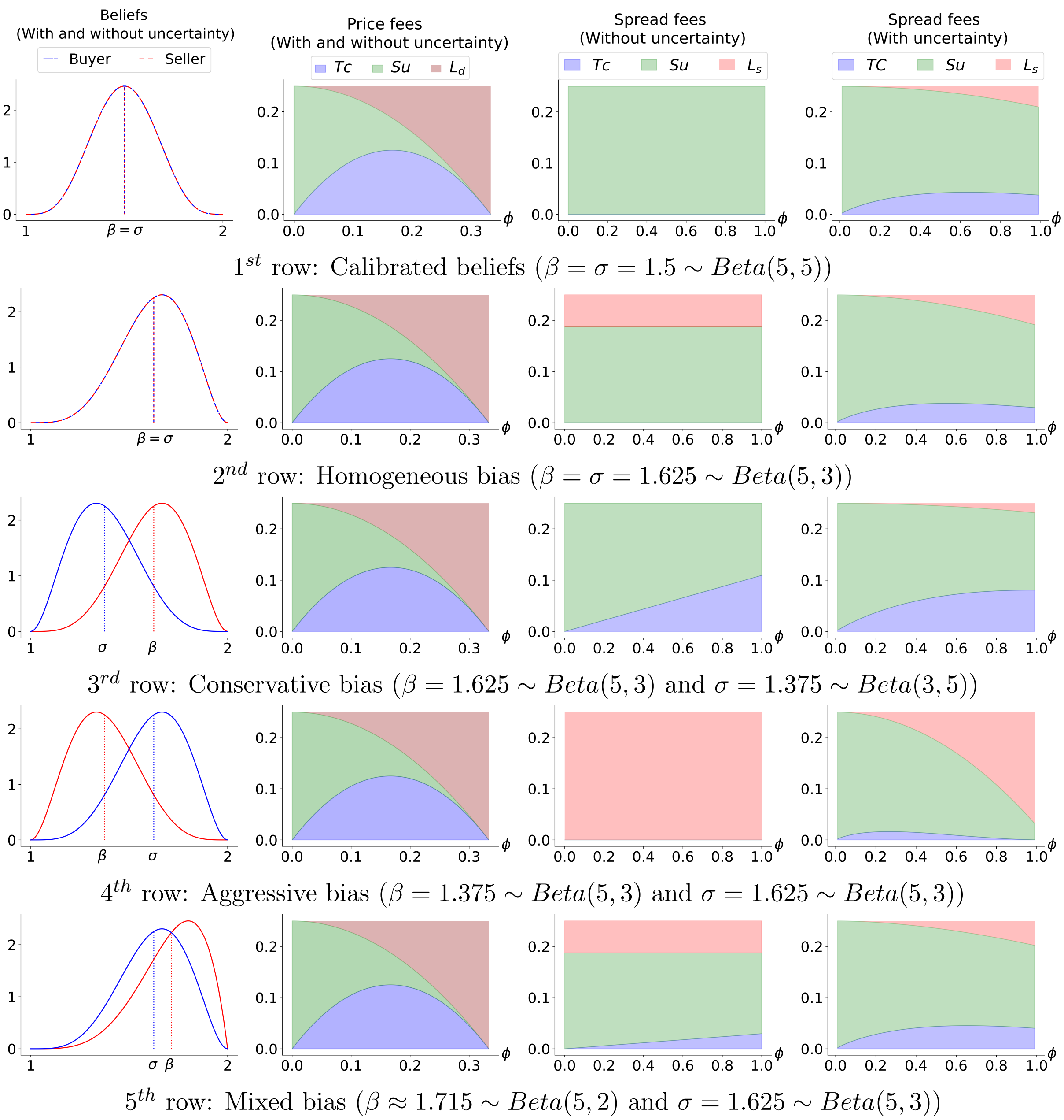

Figure 3: Decomposition of the maximum gains of trade of an infinite uniform market into total transaction costs Tc (blue), surplus Su (green), direct loss L_d (dark-red) and strategy-induced loss L_s (light-red) as a function of price (2^{nd} column, independent of uncertainty) or spread fee ϕ (3^{rd} column without uncertainty and 4^{th} column with uncertainty), if traders best respond to their beliefs. The first column in each row shows the beliefs as indicated in the sub-captions.

3 The model

3.1 The market

We consider a two-sided market populated by traders belonging to sets $\mathcal{B}, \mathcal{S} \subset \mathbb{R}$ of buyers ($b \in \mathcal{B}$) and sellers ($s \in \mathcal{S}$). Traders are interested in either buying or selling an indivisible good. We consider both the *finite* case, with m buyers $\mathcal{B} = \{1, 2, ..., m\}$ and n sellers $\mathcal{S} = \{1, 2, ..., n\}$, and the *infinite* case, with $\mathcal{B} \subset \mathbb{R}$ and $\mathcal{S} \subset \mathbb{R}$ being two closed intervals. Denote the *distributions* of buyers and sellers on $\mathcal{B}$ and $\mathcal{S}$ by μ_B and μ_S.[11] By $R = \frac{\mu_S(\mathcal{S})}{\mu_B(\mathcal{B})}$ we denote the ratio of sellers to buyers.

We are particularly interested in large markets. We say that a property $\mathcal{P}$ holds *in sufficiently large finite markets*, if there exist $m, n \geq 1$ such that $\mathcal{P}$ holds in any finite market with at least m buyers and n sellers. If the property also holds in infinite markets, we say that it holds *in sufficiently large markets*.

Every trader $i \in \mathcal{B} \cup \mathcal{S}$ has a *type* $t_i \in T = [\underline{t}, \overline{t}] \subset \mathbb{R}^{\geq 0}$ giving valuation, reservation price or gross value. T is called the *type space*. Denote by $t_B : \mathcal{B} \to T$, $t_S : \mathcal{S} \to T$ Borel-functions that assign a type to each trader. Let μ_B^t and μ_S^t be the push-forward measures of μ_B and μ_S with respect to t_B and t_S, that is, $\mu_B^t(\cdot) = \mu_B(t_B^{-1}(\cdot))$ and $\mu_S^t(\cdot) = \mu_S(t_S^{-1}(\cdot))$. We call these the *type distributions*. They are σ-additive and finite measures on T, and specify the mass of traders with types inside any measurable subset of T. Write $t = (t_i, t_{\text{-}i})$, where t_i is trader i's type and $t_{\text{-}i}$ is the type distribution of all traders excluding trader i.[12]

Types are distributed according to continuous probability density functions (f_B^t, f_S^t) that have full support on the type space T.[13] In finite markets, we assume that traders' types are independent random variables on the type space T and that they are identically distributed for each of the two market sides. Let (F_B^t, F_S^t) be the corresponding pair of cumulative distribution functions of types. Realizations of these random variables $t_b^1, ..., t_b^m$ and $t_s^1, ..., t_s^n$ induce type distributions, the random empirical measures $\mu_B^t = \sum_{j=0}^m \delta_{t_b^j}$ and $\mu_S^t = \sum_{k=0}^n \delta_{t_s^k}$. Letting n and m tend to infinity, normalized versions of the random empirical measures converge uniformly to measures with densities f_B^t and f_S^t.[14] In an infinite market, we scale these measures by $\mu_B(\mathcal{B})$ and $\mu_S(\mathcal{S})$ to achieve the market ratio $R = \mu_S(\mathcal{S})/\mu_B(\mathcal{B})$. Let $(\Omega, \mathcal{F}, \mathbb{P})$ be the probability space describing the randomness of sampling type distributions. Denote by $\mathbb{E}[\cdot]$ the *expectation* with respect to the probability measure $\mathbb{P}$.

Every trader i submits an *action* $a_i \in \mathbb{R}^{\geq 0}$ representing a buyer's *bid* and a seller's *ask*. Denote by $a_B : \mathcal{B} \to A_B$ with $a_B(b) = a_b$ and by $a_S : \mathcal{S} \to A_S$ with $a_S(s) = a_s$ Borel-functions that assign an action for each trader. Let the *action distributions* μ_B^a and μ_S^a be two induced σ-additive and finite measures on $\mathbb{R}^{\geq 0}$ with support in the *action spaces* $A_B = [\underline{a}_B, \overline{a}_B]$ and $A_S = [\underline{a}_S, \overline{a}_S]$. That is, $\mu_B^a(\cdot) = \mu_B(a_B^{-1}(\cdot))$ and $\mu_S^a(\cdot) = \mu_S(a_S^{-1}(\cdot))$. Write $a = (a_i, a_{\text{-}i})$, where a_i is trader i's action and $a_{\text{-}i}$

[11] These are counting measures for finite markets and Lebesgue-measures for infinite markets.

[12] In finite markets, t is obtained by adding a point mass at t_i to $t_{\text{-}i}$. In infinite markets, single traders do not change the type distributions.

[13] This is a common assumption in the literature, c.f. Rustichini et al. (1994); Azevedo and Budish (2019).

[14] A discussion of this result can be found in Vapnik and Chervonenkis (2015).

is the action distribution of all traders excluding trader i.[15] We will sometimes consider *strategies* $a_i : T \to A_i$, where $a_i(t_i)$ specifies the action given i's type. Given type distributions t, strategies of traders induce action distributions a as the push-forward measure of the type distributions.

We compare actions with respect to their *aggressiveness*, which refers to the amount of a bid's (or ask's) misrepresentation: A buyer's bid a_b^1 is *(strictly) more aggressive* than a_b^2, write $\underset{(\succ)}{\succsim}$, if $a_b^1 \underset{(>)}{\geq} a_b^2$ and similarly a seller's offer a_s^1 is *(strictly) more aggressive* than a_s^2, write $\underset{(\succ)}{\succsim}$, if a_s^1 is (strictly) less than a_s^2.

The *utility* of each trader is the sum of the gross value of the object (if they have it) and their money holdings, normalized such that a trader who does not trade has utility 0. A buyer b involved in trade makes a payment, $P_b(a_b, a_{\text{-}b})$, in order to obtain an item and their resulting utility is $u_b\left(t_b, a_b, a_{\text{-}b}\right) = t_b - P_b\left(a_b, a_{\text{-}b}\right)$. Similarly, a seller s involved in trade receives a payment ,$P_s(a_s, a_{\text{-}s})$, for their item and their utility is $u_s\left(t_s, a_s, a_{\text{-}s}\right) = P_s\left(a_s, a_{\text{-}s}\right) - t_s$.

3.2 The mechanism

Given action distributions a, the *k-double auction with transaction costs* (or simply *DA with transaction costs*) selects a *market outcome*, which consists of an allocation $A^*(a) = \mathcal{B}^*(a) \cup \mathcal{S}^*(a)$ identifying subsets of traders $\mathcal{B}^*(a) \subset \mathcal{B}$ and $\mathcal{S}^*(a) \subset \mathcal{S}$ who will be involved in trade, a market price $P^*(a)$ paid or received by all trading agents, and *transaction costs* $\Phi\left(a\right) = \Phi_i(a)_{i \in \mathcal{B} \cup \mathcal{S}} = (\Phi_i(a), \Phi_{-i}(a))$ for all active traders.[16]

To specify, how the k-DA with transaction costs selects the allocation and the market price, consider the set of traders whose actions are (strictly) above or below price P; for a relation $\mathcal{R} \in \{\geq, >, =, <, \leq\}$, we therefore introduce the shorthand notations $\mathcal{B}_{\mathcal{R}}(P) = \{b \in \mathcal{B} : a_b \mathcal{R} P\}$ and $\mathcal{S}_{\mathcal{R}}(P) = \{s \in \mathcal{S} : a_s \mathcal{R} P\}$. *Demand* and *supply* at a price P are defined as $D(P) = \mu_B(\mathcal{B}_{\geq}(P))$ and $S(P) = \mu_S(\mathcal{S}_{\leq}(P))$, that is, by the mass of all traders who, given their actions, weakly prefer trading over not trading at P.

For $k \in [0, 1]$ the market price is set as

$$P^*(a) = k \cdot \min \mathcal{P}^{MC}(a) + (1 - k) \cdot \max \mathcal{P}^{MC}(a),$$

where $\mathcal{P}^{MC}(a)$ is the set of *market clearing prices* that equilibrate demand and supply.[17] Given

[15] In finite markets a is obtained by adding a point mass to $a_{\text{-}i}$. In infinite markets, single traders do not influence the action distributions.

[16] Whenever the dependence on the action distribution is clear, we will simply write P^*, $\mathcal{B}^*$ and $\mathcal{S}^*$. When focusing on a single trader with action a_i, we will write, e.g., $P^*(a_i, a_{\text{-}i})$.

[17] Analytic properties of demand and supply, as well as a detailed account of market-clearing prices are formulated in Appendix A.1, and proven for the k-DA without transaction costs in finite and infinite markets in Jantschgi et al. (2022).

$P^*(a)$, the following allocations are carried out:

$$\mathcal{S}^*(a) = \mathcal{S}_<(P^*(a)) \cup \tilde{\mathcal{S}}(a) \text{ and } \mathcal{B}^*(a) = \mathcal{B}_>(P^*(a)) \cup \tilde{\mathcal{B}}(a),$$

where $\tilde{\mathcal{B}}(a) \subset \mathcal{B}_=(P^*(a))$ (respectively $\tilde{\mathcal{S}}(a) \subset \mathcal{S}_=(P^*(a))$) are uniform random sets selecting players to balance trade in case there is trading excess.[18]

Given that each trader involved in trade pays a transaction cost $\Phi_i(a_i, a_{\text{-}i})$, the total payment for a buyer b and to a seller s are equal to $P_b = P^*(a_b, a_{\text{-}b}) + \Phi_b(a_b, a_{\text{-}b})$ and $P_s = P^*(a_s, a_{\text{-}s}) - \Phi_s(a_s, a_{\text{-}s})$.

We make the assumption that the fee payments are well-behaved, that is, for a trader i and any distribution of other actions $a_{\text{-}i}$ we require that the function $a_i \mapsto P_i(a_i, a_{\text{-}i})$ is continuous and increasing. Concretely, for a buyer bidding more aggressively leads to a lower payment and for a seller bidding more aggressively leads to a higher payment. Note that, the function $a_i \mapsto P^*(a_i, a_{\text{-}i})$ is continuous and increasing in a_i, see Appendix A.3.3. Therefore, for the latter to hold it is sufficient that the transaction cost is continuous and increasing.

Commonly observed transaction costs, that also satisfy the above assumption, are *constant* transaction costs, and *price* and *spread* fees: given a constant $c_i \geq 0$ and a percentage $\phi_i \in [0, 1]$, a transaction cost Φ_i is a *constant transaction cost* if $\Phi_i(a) = c_i$, a *price fee* if $\Phi_i(a) = \phi_i P^*(a)$, and a *spread fee* if $\Phi_i(a) = \phi_i |P^*(a) - a_i|$.[19]

3.3 Beliefs

We assume traders know the market mechanism, but have incomplete information regarding the number of other traders, the distribution of gross values, market behavior of other traders, and what transaction costs are charged. In particular, traders may have heterogeneous and incorrect beliefs. Trader i has beliefs regarding the *market environment* $\mathcal{M}_i$ with transaction cost Φ_i, buyer to seller ratio $R_i = \mu_S(\mathcal{S}_i)/\mu_B(\mathcal{B}_i)$, and other traders' action distributions.[20] Regarding the latter, actions of other traders are assumed to be distributed according to probability densities $f^a_{B,i}$ and $f^a_{S,i}$ that have full support on action spaces $A_{B,i} = [\underline{a}_{B,i}, \overline{a}_{B,i}]$ and $A_{S,i} = [\underline{a}_{S,i}, \overline{a}_{S,i}]$.[21,22] In finite markets, traders' actions are assumed to be independent random variables, identically distributed for each of the two market sides. Let $(F^a_{B,i}, F^a_{S,i})$ be the pair of corresponding C^1 distribution functions. Realizations of

[18] That is for all $b \in \mathcal{B}_=(P^*(a))$ it holds that $\mathbb{P}[b \in \tilde{\mathcal{B}}(a)] \equiv$ *const* (respectively for all $s \in \mathcal{S}_=(P^*(a))$ it holds that $\mathbb{P}[s \in \tilde{\mathcal{S}}(a)] \equiv$ *const*). See Appendix A.2 for details regarding the allocation and tie-breaking.

[19] If $\phi_i = 0$ or $c_i = 0$, the setting simplifies to the classical k-DA without transaction costs. Further, for spread fees, if $\phi_i = 1$ a trader has to pay their bid/ask.

[20] Directly assuming beliefs over actions permits beliefs about distributions of gross values and strategies of other traders, but also more general beliefs.

[21] If trader i believes that types are distributed according to (F^t_B, F^t_S) and all traders use a symmetric strategy profile (a_B, a_S), where both strategies are strictly increasing C^1-functions, then actions are distributed according to $F^t_B(a_B^{-1}(\cdot))$ on $A_{B,i}$ and $F^t_S(a_S^{-1}(\cdot))$ on $A_{S,i}$.

[22] We assume that $\overline{a}_{S,i} \geq \overline{a}_{B,i} > t^\Phi_i > \underline{a}_{S,i} \geq \underline{a}_{B,i}$. That is, the action spaces intersect, which means that there are both buyers and sellers who are in and out of the market, so that a trader believes that being truthful (Section 4.1) ensures competing with both buyers and sellers.

these random variables induce random empirical action distributions μ_B^a and μ_S^a. Similar to type distributions, letting n and m tend to infinity, normalized versions of the random empirical measures converge uniformly to measures with densities $f_{B,i}^a$ and $f_{S,i}^a$. Scaling these absolutely continuous measures by $\mu_B(\mathcal{B})$ and $\mu_S(\mathcal{S})$ to achieve the market ratio $R = {}^{\mu_S(\mathcal{S})}/_{\mu_B(\mathcal{B})}$ results in beliefs about action distributions in infinite markets. In such markets, we additionally allow for general beliefs about deterministic action distributions μ_B^a and μ_S^a, that may not be absolutely continuous.

Given the beliefs of trader i, let $(\Omega_{\text{-}i}, \mathcal{F}_{\text{-}i}, \mathbb{P}_{\text{-}i})$ be the probability space describing the randomness of the action distribution $a_{\text{-}i}$. Denote by $\mathbb{E}_{\text{-}i}[\cdot]$ the *expectation* with respect to the probability measure $\mathbb{P}_{\text{-}i}$.

4 Incentives

A trader has several incentives. First, they don't want to be loss-making. Second, they want to maximise the probability of trade. Third, they want to maximize the utility when trading. In particular, note that the second and third are in competition, as increasing one's aggressiveness, on the one hand may favourably influence the payment, but, on the other hand decreases the probability of being involved in trade.

4.1 Truthfulness

Without transaction costs, if trader i bids their gross value ($a_i(t_i) = t_i$), they maximize the probability to be involved in trade, conditional on guaranteeing ex-post individual rationality.[23] Such behavior is often called *truthful* because a trader reveals their type. Buyers prefer not to trade at market prices above their gross value, and sellers prefer not to trade at market prices below their gross value. Indeed, bidding gross values represents the maximal bids that constitute undominated actions for buyers, and similarly the minimal asks that constitute undominated actions for sellers.[24]

In the presence of transaction costs, actions may have to be more aggressive than gross values in order to guarantee ex-post individual rationality, and bidding gross values may be dominated. For some transaction costs, e.g., for constant and price fees, bidding ones gross value would result in negative utility when the market price is equal to the gross value. Taking transaction costs into account, we define a buyer's *net value*, t_b^Φ, as the supremum of the set of undominated and ex-post individually rationality. Similarly, we define a seller's *net value*, t_s^Φ, as the infimum of the latter set. If the net value does not exist the trader has no action guaranteeing ex-post individually rational actions (see Appendix A.4 for pathological examples, where the net value does not exist).

In the presence of transaction costs, we say that a trader is *truthful* if they bid their net value. Recall, that without transaction costs the net value is the gross value. Moreover, we say that an

[23]An action a_i is ex-post individually rational, if for all $a_{\text{-}i}$ it holds that $u_i(t_i, a_i, a_{\text{-}i}) \geq 0$.
[24]We say that an action a_i^1 dominates an action a_i^2, if for all $a_{\text{-}i}$ it holds that $u_i(t_i, a_i^1, a_{\text{-}i}) \geq u_i(t_i, a_i^2, a_{\text{-}i})$.

action a_i is *(strictly) individually rational*, if it is (strictly) smaller than the net value for buyers and (strictly) greater than the net value for sellers.

Next, we show that for an important class of transaction costs (that includes constant, price and spread fees), the net value exists and is analytically well-behaved. Consider that transaction cost only depends on the action of a trader and the market price, that is $\Phi_i(a_i, a_{\text{-}i}) = \Phi_i(a_i, P^*(a_i, a_{\text{-}i})$. Suppose that $P^* \mapsto P_i(a_i, P^*)$ is increasing, $a_i \mapsto P_i(a_i, a_i)$ is strictly increasing and both are continuous. Define the sets of gross values that allow for profitable trade, $T_b^+ = \{t_b : \exists a_b : t_b - a_b - \Phi_b(a_b, a_b) > 0\}$ and $T_s^+ = \{t_s : \exists a_s : a_s - t_s - \Phi_s(a_s, a_s) > 0\}$.

Proposition 1 (Existence of net values)**.** *Consider the latter class of transaction costs. For $t_i \in T_i^+$, the net value exists and it is undominated and ex-post individually rational. It is continuous and strictly increasing in the gross value and given by the unique solution of the equation $t_b - x - \Phi_{bb}(x, x) = 0$ for a buyer and $x - t_s - \Phi_s(x, x) = 0$ for a seller.*

Proof details are relegated to Appendix B.1. According to Proposition 1, for such transaction costs the net value is the unique action, at which a trader is indifferent between trading and not trading, when the market price is equal to their action. For constant, price and spread fees, this characterization allows to express the net value as a function of the gross value and the fee parameter.

Corollary 2 (Net values for constant fees, price, and spread fees)**.** *For constant fees, the net value shifts the gross value, that is, $t_b^\Phi = \max(0, t_b - c_b)$ and $t_s^\Phi = t_s + c_s$. Similarly, for price fees the net value scales the gross value, that is, $t_b^\Phi = {}^{t_b}/{1+\phi_b}$ and $t_s^\Phi = {}^{t_s}/{1-\phi_s}$. By contrast, for spread fees the gross value equals the net value.*

Proof details are relegated to Appendix B.2. To exclude pathological scenarios we will assume that the net value exists, is strictly increasing, and continuous in the gross value (thus, including the transaction costs considered in Proposition 1).

4.2 Probability of trade

Consider trader i's *probability of trading*, $\mathbb{P}_{\text{-}i}[i \in A^*(a_i, a_{\text{-}i})]$. In finite markets, the function $a_i \mapsto \mathbb{P}_{\text{-}i}[i \in A^*(a_i, a_{\text{-}i})]$ is continuous and can be expressed in terms of $F^a_{S,i}$ and $F^a_{B,i}$.[25] In an infinite market, trader i believes that the market price is equal to the unique solution of the equation $\mu_S(\mathcal{S})F^a_{S,i}(\cdot) = \mu_B(\mathcal{B})(1 - F^a_{B,i}(\cdot))$. Call this solution the *critical value* P_i^∞.[26] The probability of trading is equal to 1, if trader i's action is less aggressive than P_i^∞. If their action is equal to P_i^∞ they believe to be involved in tie-breaking and trade with some probability between 0 and 1. If their action is more aggressive, trader i believes that they are not involved in trade.

The critical value is also of central importance for the study of trading probabilities in large finite markets. If trader i's beliefs about others' behaviours are correct, then they can compute the

[25] This is proven in Appendix A.3.2 and Appendix A.7 (see Equations (28) and (29)).

[26] Existence and uniqueness are proven in Appendix B.3.

market price with increasing accuracy as the market grows. With increasing numbers of traders on both market sides the variance of the realized market price decreases and it converges to the critical value. The probability of trading then converges to a step function at the critical value P_i^∞.

Proposition 3 (Predictability of trade)**.** *Consider trader i with action a_i. For every $\epsilon > 0$, in sufficiently large markets, the probability of trade for i is (1) bounded from below by $1-\epsilon$ if a_i is strictly less aggressive than the critical value P_i^∞ and (2) bounded from above by ϵ if a_i is strictly more aggressive than the critical value P_i^∞.*

In the omitted case, when $a_i = P_i^\infty$, the trading probability in finite markets is determined by the action distributions and lies strictly between 0 and 1.[27]

Proof Outline. In infinite markets, the statement follows directly from the model. Growing market size in finite markets is formalized with respect to a single parameter. Consider a sequence of strictly increasing market sizes $(m(l), n(l))_{l\in\mathbb{N}}$ with $m(l), n(l) = \Theta(l)$ and $|R - \frac{n(l)}{m(l)}| = \mathcal{O}(l^{-1})$ for $R \in (0,\infty)$.[28] A buyer b is involved in trade, if their action a_b is greater (or equal, if they win tie-breaking) than at least $m(l)$ actions of other traders, that is $\mathbb{P}_{\text{-}b}\left[b \in A^*(a_b, a_{\text{-}b})\right] = \mathbb{P}_{\text{-}b}\left[a_b \geq a_{\text{-}b}^{m(l)}\right]$. The probability that the action of any other buyer and seller is below a_b is $p_{a_b} = F_{B,b}(a_b)$ and $q_{a_b} = F_{S,b}(a_b)$. If $X_i^{p_{a_b}}$ and are Bernoulli random variables with parameters p_{a_b} and q_{a_b}, then the total number of traders with actions below a_b has the same distribution as the sum $S_l^{a_b} = \sum_{i=1}^{m(l)-1} X_i^{p_{a_b}} + \sum_{i=1}^{n(l)}$. It follows that $\mathbb{P}_{\text{-}b}\left[b \in A^*(a_b, a_{\text{-}b})\right] = \mathbb{P}[S_l^{a_b} \geq m(l)] = 1 - \mathbb{P}[S_l^{a_b} \leq m(l) - 1]$. By the Berry-Esseen Theorem (Tyurin, 2012) an appropriately normalized version of $S_l^{a_b}$ converges in distribution to a standard normal random variable with CDF Φ. We show that there exists a sequence $(A_{a_b}(l))_{l\in\mathbb{N}} = \Theta(\sqrt{l})$ with $|\mathbb{P}[S_l^{a_b} \leq m(l)-1] - \Phi(A_{a_b}(l))| \in \mathcal{O}(l^{-\frac{1}{2}})$. For $a_b \prec P_b^\infty$ we show for sufficiently large l that $A_{a_b}(l) < 0$, which yields that $A_{a_b}(l) \in \Theta(-\sqrt{l})$. Using a concentration inequality for a standard Gaussian random variable gives $\Phi(A_{a_b}(l)) \in \mathcal{O}(e^{-l})$. It therefore holds that $\mathbb{P}[S_l^{a_b} \leq m(l)-1] = \mathcal{O}(l^{-\frac{1}{2}})$. The statement for $a_b \succ P_b^\infty$ and for sellers can be derived analogously. Proof details are relegated to Appendix B.4. □

We sometimes focus on *in-the-market* gross values that is gross values t_i such that $t_i^\Phi \prec P_i^\infty$. Traders with such gross values are able to submit individually rational actions that make them likely to be involved in trade when the market is sufficiently large. By contrast, for an *out-of-the-market* trader, that is, one with gross value $t_i^\Phi \succ P_i^\infty$, the probability of trade, when acting individually rationally, vanishes in large markets.

[27]E.g., for uniform action distributions and equally many buyers and sellers, the trading probability is independent of the market size and equal to $\frac{1}{2}$; we provide more details in the proof of point 2 of Theorem 9.

[28]If there exists a parameter l, such that for every $l' \geq l$ Proposition 3 holds in markets with $m(l')$ buyers and $n(l')$ sellers, then the statement also holds in sufficiently large finite markets.

4.3 Profitability of trade

We now turn to the *expected utility conditional on trading.* Write $\mathbb{E}_{\text{-}i}\left[\cdot|i \in A^*\left(a_i, a_{\text{-}i}\right)\right]$ for the conditional expectation of trader i given their beliefs. Recall, that we assume that payments are monotone in the aggressiveness of one's action. Further, payments are composed of the market price and a transaction cost. For the former, it is known from Rustichini et al. (1994), that in large markets traders have vanishing influence on the market price. On the other hand, this is not necessarily the case for transaction costs. To this end, a classification of transaction costs into two broad classes turns out to be useful.

Definition (Homogeneous vs. heterogeneous transaction costs)**.** Two actions a_i^1 and a_i^2, such that a_i^1 is less aggressive than a_i^2 and both are less aggressive than the critical value, that is $a_i^1 \prec a_i^2 \prec P_i^\infty$, lead to *asymptotically different transaction costs*, if there exists $\epsilon > 0$ such that in sufficiently large markets

$$\mathbb{E}_{\text{-}i}\left[\Phi_i(a_i^1, a_{\text{-}i})|i \in A^*\left(a_i^1, a_{\text{-}i}\right)\right] - \mathbb{E}_{\text{-}i}\left[\Phi_i(a_i^2, a_{\text{-}i})|i \in A^*\left(a_i^2, a_{\text{-}i}\right)\right] \geq \epsilon. \tag{1}$$

Otherwise, the two actions lead to *asymptotically equal transaction costs.* Φ_i is *heterogeneous*, if every two such actions $a_i^1 \prec a_i^2 \prec P_i^\infty$ lead to asymptotically different transaction costs. A transaction cost Φ_i is called *homogeneous*, if for every $\epsilon > 0$ in sufficiently large markets

$$\sup_{a_i^1 \prec a_i^2 \prec P_i^\infty} \mathbb{E}_{\text{-}i}\left[\Phi_i\left(a_i^1, a_{\text{-}i}\right)|i \in A^*\left(a_i^1, a_{\text{-}i}\right)\right] - \mathbb{E}_{\text{-}i}\left[\Phi_i\left(a_i^2, a_{\text{-}i}\right)|i \in A^*\left(a_i^2, a_{\text{-}i}\right)\right] \leq \epsilon. \tag{2}$$

In an infinite market, the definitions simplify: For heterogeneity, the conditional expected transaction cost is strictly monotone for actions a_i that are less aggressive than the critical value P_i^∞. For homogeneity, the conditional expected transaction cost is constant for such actions. Homogeneity and heterogeneity are not mutually exclusive, as one can construct transaction costs that are homogeneous in some price regions and heterogeneous at others. However, focusing on these two cases (rather than on hybrids) allows us to study the key strategic differences that in fact yield completely opposing behavior. In particular, the two canonical examples of transaction costs, price and spread fees, fall under the two definitions: Price fees are homogeneous, and spread fees are heterogeneous.

5 Optimal behavior

Best responses maximize individual expected utility given beliefs. Optimal behavior thus finds the right amount of aggressiveness, balancing the opposing forces of increasing the probability of trade versus increasing the utility when trading.[29] Given trader i's market environment $\mathcal{M}_i$ and gross

[29] A detailed analysis of this trade-off for price and spread fees in finite markets via first order conditions can be found in Appendix A.6.

value t_i, an action a_i is an ϵ-*best response* if $\mathbb{E}_{\text{-}i}\left[u_i\left(t_i, a_i, a_{\text{-}i}\right)\right] \geq \sup_{a_i' \in \mathbb{R}} \mathbb{E}_{\text{-}i}\left[u_i\left(t_i, a_i', a_{\text{-}i}\right)\right] - \epsilon$. For $\epsilon = 0$ a_i is a best response.

The analysis of best responses includes the special case of *symmetric Bayesian Nash equilibria*. If all buyers use the same strictly increasing and continuous strategy a_B and all sellers use the same strictly increasing and continuous strategy a_S, call (a_B, a_S) a *symmetric strategy profile*. Given type distributions, the corresponding action distributions are given by $\mu_B^a(\cdot) = \mu_B\left(t_B^{-1}(a_B^{-1}(\cdot))\right)$ and $\mu_S^a(\cdot) = \mu_S\left(t_S^{-1}(a_S^{-1}(\cdot))\right)$. Assume that beliefs over action distributions originate from beliefs over gross value distributions and over the symmetric strategy profiles of the other traders (a_B, a_S). If, for every trader and every gross value, the action specified by these strategies are ϵ-best responses, then the strategy profile constitutes a symmetric ϵ-Bayesian Nash equilibrium.[30]

Proposition 4 (Existence of best responses)**.** *Suppose the market environment is finite or, if not, tie-breaking is a probability zero event. Then a best response exists for trader i.*

In infinite markets the no-tie-breaking assumption matters. In its absence, a best response might not exist for a trader i with $t_i \prec P_i^\infty$. This is the case, for example, when spread fees are charged. Under spread fees, it is not optimal to bid P_i^∞ (or more aggressive) due to the risk of loosing out on trading. But for any less aggressive bid, bidding slightly more aggressively would lead to a higher payoff.

Proof Outline. We show that a best response is necessarily located in a compact action space. Given the continuity assumption of the payment, it follows that the expected utility is continuous in the action a_i and therefore attains a maximum by the Extreme Value Theorem. Proof details are relegated to Appendix B.5. □

The following theorem is a first indication that transaction costs have significant strategic consequences.

Theorem 5 (Asymptotically equal transaction costs)**.** *Let T^* be the set of gross values of trader i at which bidding the critical value is strictly individually rational. If trader i is best responding, then the expected transaction costs of any two types $t, t' \in T^*$ are asymptotically equal.*

For homogeneous transaction costs this result holds by definition. For heterogeneous transaction costs, the result is non-trivial and will be useful in later analyses (see Section 5.2).

Proof Outline. Assume that two actions $a_i^1 \prec a_i^2 \prec P_i^\infty$ lead to asymptotically different transaction costs. We show that in sufficiently large markets, a trader can increase their expected utility, when switching from action a_i^1 to a_i^2, proving that a_i^1 is not a best response. Formally, as $a_i^1 \prec a_i^2 \prec P_i^\infty$, Proposition 3 yields that for every $\epsilon_1 > 0$, $\mathbb{P}_{\text{-}i}\left[i \in A^*(a_i^1, a_{\text{-}i})\right], \mathbb{P}_{\text{-}i}\left[i \in A^*(a_i^2, a_{\text{-}i})\right] \geq 1 - \epsilon_1$ in

[30] Therefore all of the results that we shall present in this paper about best responses directly apply to the study of symmetric Bayesian Nash equilibria.

sufficiently large markets. The difference in trading probability between a_i^1 and a_i^2 is then upper bounded by ϵ_1. If ϵ_1 is sufficiently small, the loss in trading probability and possible influence on the market price is compensated by a decrease in expected transaction cost by at least some $\epsilon_2 > 0$ because transactions are assumed to be asymptotically different. For sufficiently small ϵ_1, the difference in expected utility between actions a_i^1 and a_i^2 is negative, if the market is sufficiently large, proving that a_i^1 is indeed not a best response. Proof details are relegated to Appendix B.6. □

5.1 Truthfulness is approximately optimal with homogeneous transaction costs

Strategic misrepresentation is driven by the incentive to influence market price and transaction cost. Reporting truthfully maximizes one's trading probability, while remaining individually rational. In large markets, the influence on the market price is vanishing 'faster' than the influence on one's trading probability, which is what drives the asymptotic truthfulness result in the literature, see Rustichini et al. (1994). Therefore, if the influence on one's own transaction cost is also vanishing 'fast' enough, then it is close to optimal to maximize one's trading probability by reporting truthfully. This is the case for homogeneous transaction costs, such as constant or price fee.

Theorem 6 (In large markets with homogeneous transaction costs truthfulness is an approximate best response)**.** *If the transaction cost is homogeneous and trader i's best response is uniformly bounded away from the critical value P_i^∞, then for every $\epsilon > 0$, in sufficiently large markets, truthfulness is an ϵ-best response.*

Proof Outline. Consider a best response a_i of trader i. If $a_i \prec t_i^\Phi$, then t_i^Φ is a best response by weak domination. Suppose now that $a_i \succ t_i^\Phi$. By assumption, there exists $\delta > 0$, such that in sufficiently large markets, (i) $a_i \prec P_i^\infty - \delta$ or (ii) $a_i \succ P_i^\infty + \delta$ holds. If (i) holds, then Proposition 3 implies that $\mathbb{P}_{\text{-}i}\left[i \in A^*(a_i, a_{\text{-}i})\right]$ converges to zero as the market gets large. Therefore for all $\epsilon > 0$ the expected utility of a_i is then upper bounded by ϵ, which also proves that that the net value is an ϵ-best response, because it leads to a non-negative expected utility. If (ii) holds, consider $\mathbb{E}_{\text{-}i}[u_i(t_i, a_i, a_{\text{-}i})] - \mathbb{E}_{\text{-}i}[u_i(t_i, t_i^\Phi, a_{\text{-}i})]$. We split the difference into two components and show that for every $\forall\epsilon > 0$ both components are less or equal than $\frac{\epsilon}{2}$ if the market is sufficiently large: (a) Difference in expected transaction costs and (b) Terms corresponding to a classical DA without transaction costs. To bound (a), we can use Proposition 3 and homogeneity. For (b), we will use that for a DA without transaction costs truthfulness is an ϵ-best response in sufficiently large markets, see Theorem 7.2 with price fees equal to zero. Proof details are relegated to Appendix B.7. □

Price fees. Fixing a specific transaction cost allows sharper results than Theorem 6. In particular, for a price fee, any best response can be explicitly shown to be close to truthful in large markets.

Theorem 7 (In large markets with price fees best responses are approximately truthful and truthfulness is an approximate best response)**.** *If the fee is a price fee, then for every $\epsilon > 0$ it holds* *that (1) in sufficiently large markets truthfulness is an ϵ-best response and (2) in sufficiently large finite markets all best responses are ϵ-truthful.*

Proof Outline. Consider a buyer b. For (2), a best response satisfies the first order condition $\frac{d\mathbb{E}_{\text{-}b}[u_b(t_b,a_b,a_{\text{-}b})]}{da_b} = 0$, see Appendix A.6. Explicit calculations yield that there exists a constant $\kappa > 0$, such that $t_b - (1+\phi_b)\,a_b \leq \kappa q(n,m)$, with $q(m,n) = \max\left\{\frac{1}{n}\left(1+\frac{m}{n}\right), \frac{1}{m}\left(1+\frac{n}{m}\right)\right\} = O(\max(m,n)^{-1})$, from which the statement follows.[31] For (1), we estimate $\mathbb{E}_{\text{-}b}\left[u_b(t_b, t_b^{\Phi}, a_{\text{-}b})\right] - \mathbb{E}_{\text{-}b}\left[u_b(t_b, a_b, a_{\text{-}b})\right]$, where a_b denotes the best response. This difference is shown to be upper bounded by $-2k(1+\phi_b)|t_b^{\Phi} - a_b|$. It follows from (2) that $\forall \delta > 0$ it holds in sufficiently large finite markets that $t_b^{\Phi} - a_b \leq \delta$. If for a given $\epsilon > 0$, $\delta > 0$ is chosen such that $\delta \leq \frac{\epsilon}{2k(1+\phi_b)}$, it holds that t_b^{Φ} is ϵ-close to a best response a_b in sufficiently large finite markets. In infinite markets, the expected utility is deterministic and truthfulness is a best response, as the only strategic incentive is to be involved in trade. Proof details are relegated to Appendix B.8. □

5.2 Price-guessing is approximately optimal with heterogeneous transaction costs

If a trader can influence their transaction cost, then there remains a (non-vanishing) incentive to act strategically in large markets. Moreover, given a trader will almost certainly trade as long as their action meets the required threshold of the critical value, the incentive to influence their transaction cost asymptotically outweighs the concern of loosing out on the deal. Therefore, it is optimal to bid close to the critical value that corresponds to the predicted price, which is why we shall call such behavior *Price-Guessing*. While our analysis only covers the case of a trader for whom bidding the critical value is individually rational, the case of traders for whom it is not is discussed in Proposition 18.

Theorem 8 (In large markets with heterogeneous transaction costs best responses are close to price guessing)**.** *If the transaction cost is heterogeneous and bidding the critical value P_i^{∞} is strictly individually rational for trader i, then for every $\epsilon > 0$, in sufficiently large markets, all best responses of i are in an ϵ-neighbourhood of the critical value P_i^{∞}.*

Proof Outline. Consider a buyer with action $a_b > P_b^{\infty}$. We show that if $a_b - P_b^{\infty} \geq \epsilon$, then the difference in expected utility from playing a_b versus $P_b^{\infty} + \frac{\epsilon}{2}$ is strictly negative in sufficiently large markets, proving that a_b is then not a best response. Similar to the proof of Theorem 5, we show that in such markets, the buyer will be involved in trade with high probability with both actions. Using that the transaction cost is heterogeneous, the decrease of the transaction cost when switching to the more aggressive action $P_b^{\infty} + \frac{\epsilon}{2}$ outweighs the decrease in trading probability. Proof details are relegated to Appendix B.9. □

[31] A similar proof technique has been used to show that Bayesian Nash equilibria are approximately truthful in DAs without fees, see Rustichini et al. (1994, Theorem 3.1).

Spread fees. As a spread fee depends linearly on a trader's action, it is an example of a heterogeneous transaction cost. A best response exists given the spread fee is continuous and must be close to the critical value. However, an analogous statement to Theorem 7.2, i.e., the utility at the critical value is close to optimal, is not true in general. We show that there exist markets, such that bidding the critical value is in general not ϵ-optimal in large markets.

Theorem 9 (In large markets with spread fees best responses are close, but not necessarily equal, to the critical value)**.** *If the fee is a strictly positive spread fee, then a best response exists for a trader i in finite and infinite markets without tie-breaking. Further, if bidding the critical value is strictly individually rational, then (1) for every $\epsilon > 0$, in sufficiently large markets, all best responses of i are in an ϵ-neighbourhood of the critical value P_i^∞ and (2) for sufficiently small $\epsilon > 0$, there exist beliefs F, such that in sufficiently large finite markets the critical value P_i^∞ is not an ϵ-best response for i.*

Proof Outline. We show that the expected fee is and therefore the expected utility is continuous in a_i. The existence of a best response again follows as in Theorem 7. Consider a buyer b with $t_b^\Phi > P_i^\infty$. (1) is proven in complete analogy to Theorem 6.1. For (2), consider beliefs such that the number of traders is equal to l for both market sides, where beliefs are uniformly distributed over $A_B = A_S = [0,1]$. It follows that $P_b^\infty = \frac{1}{2}$. We prove that for every $l > 1$ it holds that $\mathbb{P}_{\text{-}b}[b \in \mathcal{B}^*(P_b^\infty, a_{\text{-}b})] = \frac{1}{2}$. Therefore, for every bid $a_b > P_b^\infty$ and for every $\epsilon > 0$, it follows from Proposition 3 that the buyer can increase their trading probability by $\frac{1}{2} - \epsilon$ when switching from P_b^∞ to a_b. If a_b is chosen close to P_b^∞, then this outweighs the increase in spread fee payment. Proof details are relegated to to Appendix B.10. □

5.3 Best responses and Bayesian Nash equilibria for price versus spread fees

Consider a finite markets with sizes (i) 2×2 (that is, two buyers and two sellers) and (ii) 5×5 in the presence of either a price fee $\phi_i = 0.1$ or a spread fee $\phi_i = 1$, and $k = 0.5$. Figure 4 shows best response strategies (*Top.*) for uniform beliefs over others' actions in $[1,2]$ and a symmetric Bayesian Nash Equilibrium (*Bottom*) for uniform beliefs over gross values in $[1,2]$ for price fees (*Left.*) and spread fees (*Right.*).

In line with Theorem 7.1, optimal strategic behavior converges to truthfulness with growing market size, if price fees are charged. In a small market (2×2), traders have an incentive to be more aggressive and misrepresent their net value, as can be measured by the distance between their respective best response (dashed red/blue lines) and the net value (solid black lines). In contrast, and in line with Theorem 7.1, the best responses (dotted red/blue line) in the larger market (5×5) are approaching truth-telling.

Note that in line with Theorem 9.1, best responses converge towards price-guessing with growing market size if spread fees are charged. In a small market with two buyers and two sellers traders have an incentive to be aggressive and misrepresent their true net value in order to influence the price and reduce their fee payment. In line with implications from Theorem 9, best responses in a larger market with five buyers and sellers (dotted line) do not approach truth-telling, if $t_i \prec P_i^\infty$. Instead traders remain aggressive as they aim to reduce their fee payment. In contrast, their influence on the price diminishes which results in traders approximating the critical value P_i^∞ provided it is individually rational.

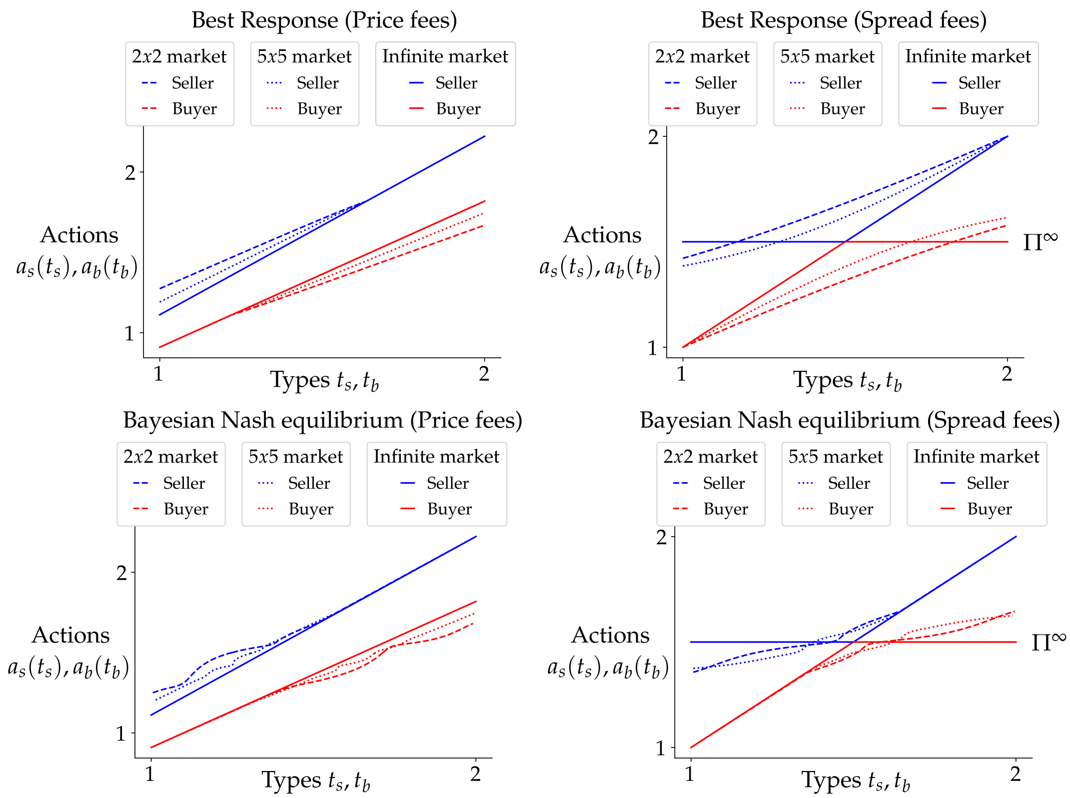

Figure 4: Best responses for uniform beliefs over actions (top) and a symmetric Bayesian Nash equilibrium for uniform beliefs over types (bottom) for buyers (red) and sellers (blue) as functions of their gross value for 2×2 (dashed lines) and 5×5 (dotted lines) markets with price fee $\phi_i = 0.1$ (left) or spread fee $\phi_i = 1$ (right).

6 Market performance

In this section, we evaluate efficiency of market outcomes under homogeneous and heterogeneous transaction costs when traders adopt best responses as were characterized in the previous section. We show that homogeneous transaction costs cause an inefficiency that scales with the size of the transaction fee and gets smaller in larger markets, while heterogeneous transaction costs result in knife-edge results with either no or substantial efficiency loss that does not vanish asymptotically and does not scale in their size.

In our analysis, we shall speak of traders having *belief systems* F about the market, allowing for heterogeneous beliefs in the population. F consists of two mappings, M_B, M_S, from type space T into the space of market environments, with $M_i(t_i)$ denoting what trader i with type t_i believes.

We assume that traders choose their action according to their type and beliefs. We therefore analyze symmetric strategy profiles (a_B, a_S), that is all buyers with gross value t_b bid $a_B(t_b)$ and all sellers with gross value t_s ask for $a_S(t_s)$.

First, we introduce various metrics that will be used to evaluate market outcomes.[32]

6.1 Metrics

The *trading volume* at market price P^* is $V(P^*) = \min(D(P^*), S(P^*))$ and the *trading excess* is $Ex(P^*) = |D(P^*) - S(P^*)|$. If $D(P^*) > \mathcal{S}(P^*)$, call $Ex(P^*)$ *demand excess*, and if $\mathcal{S}(P^*) > D(P^*)$, call it *supply excess*.

The individual *gains of trade* for a buyer b with gross value t_b are $t_b - P^*$. Similarly, for a seller s with gross value t_s, the gains of trade are $P^* - t_s$.[33] Given the actions of all traders, the *realized gains of trade* G are $G = \int_{\mathcal{B}^*} (t_b - P^*)\, d\mu_B(b) + \int_{\mathcal{S}^*} (P^* - t_s)\, d\mu_S(s)$. If agents report truthfully, the realized gains of trade are maximized by market clearing at G_Φ, called the *achievable gains of trade*. In the absence of transaction costs this coincides with reporting their gross value, achieving the *maximum total gains of trade*, G_{id}.

The *total transaction costs* are $Tc = \int_{\mathcal{B}^*} \Phi_b d\mu_B(b) + \int_{\mathcal{S}^*} \Phi_s d\mu_S(s)$.

By the *surplus generated by the traders* we refer to the difference between the total gains of trade and the total transaction costs: $Su = G - Tc$. Similarly, by *loss* we refer to $L = G_{id} - G$, which measures how much gains of trade are lost due to fee considerations and strategic behavior. The loss can be split up into the *direct loss* $L_\phi = G_{id} - G_\Phi$, that is due to transaction costs, and the *strategy-induced loss* $L_F = G_\Phi - G$.

G_{id} can therefore be decomposed into total fees, total surplus generated by the traders and the loss: $G_{id} = Su + Tc + L = Su + Tc + L_\phi + L_F$.

We measure the impact of strategic behavior on the market outcome as how much of the achievable—subject to individual rationality given fee considerations—gains of trade are realized in expectation. We refer to $E_\Phi = {}^{\mathbb{E}[G]}/_{\mathbb{E}[G_\Phi]}$ as the *expected achievable efficiency ratio*, where the expectations are taken with respect to type distributions and tie-breaking.

6.2 Market outcome for homogeneous transaction costs

Given homogeneous transaction costs Φ, we know that truthfulness is asymptotically optimal and, for price fees, ϵ-truthfulness emerges (see Theorems 6 and 7). The following theorem analyzes the efficiency of large markets, if traders act approximately truthful.

[32] Note that in these metrics we omit dependencies on types and action distributions, because those will not be varied when evaluated.

[33] We focus on individually rational strategies $a_B(t_b) \le t_b$ and $a_S(t_s) \ge t_s$, so that the individual gains of trade are non-negative.

Theorem 10 (In large markets with homogeneous transaction costs, independent of the belief system, strategic behavior leads to almost full achievable efficiency)**.** *If the transaction cost is homogeneous, then for all* $\zeta \in [0, 1)$*, in sufficiently large markets, for sufficiently small* $\epsilon > 0$ *and a continuous and strictly increasing symmetric strategy profile that is* ϵ*-truthful, the achievable efficiency ratio* $\mathbb{E}[E_\Phi]$ *is greater or equal than* ζ*.*

Proof Outline. We prove that for every $\xi > 0$ $\frac{\mathbb{E}[G_\Phi - G]}{\mathbb{E}[G_\Phi]} \leq \xi$. First, consider large finite markets.[34] We bound the numerator by showing that $\mathbb{E}[G_\Phi] \in \Theta\left(\min(m, n)\right)$. Next, we will bound the numerator $\mathbb{E}[G_\Phi - G]$. Denote by t^Φ a sample of $n + m$ net values. Denote by μ the distribution of the market price $\Pi(t^\Phi)$ and by $L(t^\Phi) = G_\Phi - G$ the total value of trades that inefficiently fail to occur given t^Φ and strategy profile (a_B, a_S). It holds that $\mathbb{E}\left[L(t^\Phi)\right] = \int_{-\infty}^{\infty} \mathbb{E}\left[L(t^\Phi)|\Pi(t^{\Phi,(m)})\right] d\mu(\Pi(t^{\Phi,(m)}))$. The latter can be bounded by $O(\min(m, n)^{\frac{1}{2}} + \min(m, n) \cdot \epsilon))$, thus yielding the result. In infinite markets, we prove that for a regular strategy profile G can be represented as a continuous and deterministic function $G(\cdot)$ evaluated at the trading volume V. If strategies converge to truthfulness, then demand and supply converge uniformly to D_Φ and S_Φ. This implies that also the market price and trading volume converge to P^*_Φ and V_Φ. As the achievable efficiency ratio is equal to ${}^{G(V)}/_{G(V_\Phi}$, the statement follows. Proof details are relegated to Appendix B.11. □

6.3 Market outcome for heterogeneous transaction costs

Efficiency results change when heterogeneous transaction costs Φ are charged. Given beliefs F, denote by $P^\infty(t_i)$ the guess of the critical value of trader i with gross value t_i. Our characterizations of best responses under heterogeneous transaction costs imply that price-guessing behavior approximates optimal strategic behavior for traders expecting to be in the market in large markets (see Theorems 8 and 9).[35] In contrast to price-taking which leads to full efficiency, price-guessing can lead to arbitrary efficiency outcomes.

Theorem 11 (In large markets, depending on the belief system, strategic behavior can lead to substantive market failure)**.** *For a heterogeneous transaction cost, for every* $\epsilon \geq 0$ *and for every* $\zeta \in [0, 1]$*, there exist beliefs* F *and a continuous symmetric strategy profile, which is* ϵ*-close to price-guessing, such that the achievable efficiency ratio is less or equal than* ζ *in sufficiently large markets.*

In infinite markets, the theorem can be strengthened such that the efficiency ratio is equal to ζ. That is, depending on the belief system, optimal strategic behavior can lead to any level of efficiency.

Proof Outline. Suppose that all buyers and all sellers identify the same critical value, that is for every $t_b \in T$ $P^\infty(t_b) = P^\infty_B$ and for every $t_s \in T$ $P^\infty(t_s) = P^\infty_S$. Suppose that $P^\infty_B < P^\infty_S$ and

[34]The proof follows the methods from Rustichini et al. (1994, Theorem 3.2).

[35]Recall best responses were such that traders chose actions equal to their belief of the critical value if this is individually rational, and are truthful otherwise.

traders act as price-guessers. For any realization of gross values, no profitable trade is possible and $G = 0$, which implies the result. In infinite markets, we have that for a regular strategy profile, G can be represented as a continuous function $G(\cdot)$ evaluated at V with $G(V_\Phi) = G_\Phi$ and $G(0) = 0$.[36] $E = {}^{G}\!/_{G_\Phi}$ can be represented as the continuous function $E(V) = {}^{G(V)}\!/_{G_\Phi}$ with $E(V_\Phi) = 1$ and $E(0) = 0$. For every $V \in [0, V_\Phi]$ and every $\epsilon \geq 0$, we construct beliefs, such that a symmetric strategy profile, which is ϵ-close to price-guessing, leads to this trading volume. The result follows from the Intermediate Value Theorem. Proof details are relegated to Appendix B.12. □

7 Aggregate uncertainty

A key feature of large DAs is that traders have vanishing influence on the market price above or below which they are involved in trade. Indeed, in the continuum model, traders have no influence on the market price. In order to isolate the effect of aggregate uncertainty we shall from now onward assume this feature, namely that the market price is exogenous. Note that, by doing so, a trader's beliefs no longer depend on the fact that they face a k-DA, as the key considerations of trading probability and profitability are now only dependent on the exogenous, possibly uncertain market price. Indeed, this allows to extend our results from the k-DA with transaction costs to any market organization in which the participants believe that the market price is exogenous. Examples for other market institutions include first- and second-price auctions or Vickrey (VCG) mechanisms, as well as markets where players believe to have no influence on the market price for bounded rationality reasons.[37]

Suppose, a trader believes to have no influence on the market price P^*. The mechanism then essentially consists of allowing buyers above the market price and sellers below the market price to trade (with an appropriate tie-breaking probability at the market price). Note that, this also allows to study posted-price mechanisms with a single buyer. As before, we assume that a trader i's payment P_i is composed of the market price P^* and a transaction cost $\Phi_i(a_i, P^*)$ that may depend on the market price and on their action.As in Section 3 and Section 4.1, we require that $a_i \mapsto P_i(a_i, a_{\text{-}i})$ and $P^* \mapsto P_i(a_i, P^*)$ are increasing, $a_i \mapsto P_i(a_i, a_i)$ is strictly increasing and all of them are continuous. As before, their utility is the sum of the gross value of the object (if they have it) and their money holdings, normalized such that a trader who does not trade has utility zero. Finally, net values are defined as before and always exists as the transaction cost only depends on the market price and one's own action (see Proposition 1).

Suppose that the market price P^* is distributed according to a CDF F_{P^*} on $[\underline{P^*}, \overline{P^*}] \subset \mathbb{R}^{\geq 0}$ with $\underline{P^*} \leq \overline{P^*}$ and corresponding probability measure $\mathbb{P}_{P^*}$. Trader i has incomplete information regarding

[36]This was shown in the proof of Theorem 10.

[37]For example, consider the first-price auction, where the highest bidder receives the item and pays their bid. Here, to fit into our model framework the market price is given by the second-highest bid and a 100% spread-fee is charged to the highest bidder.

the market price distribution and may thus have inaccurate beliefs. Suppose, trader i believes that the market price is distributed according to some CDF $F_{P^*,i}$. We assume these distributions have convex support $[\underline{P_i^*}, \overline{P_i^*}]$ with either $\underline{P_i^*} < \overline{P_i^*}$ and continuous density function $f_{P^*,i} > 0$, or $\underline{P_i^*} = \overline{P_i^*}$ corresponding to deterministic beliefs.[38] Additionally, traders also hold individual beliefs about the tie-breaking probability $p_i \in [0, 1]$. Denote by $\mathbb{P}_i$ the probability measure corresponding to trader i's beliefs. Finally, trader i may be more or less certain about their beliefs, which, for some degree $\delta \geq 0$, we measure by δ-aggregate uncertainty as follows: given $\delta \geq 0$, there exists a price P_i^*, such that $\mathbb{P}_i\left[P^* \in [P_i^* - \delta, P_i^* + \delta]\right] \geq 1 - \delta$.[39]

For transaction costs that only depend on the action of a trader and the market price, we simplify the definition of homogeneous and heterogeneous transaction costs: A transaction cost Φ_i is *homogeneous* if the function $a_i \mapsto \Phi_i(a_i, P^*)$ is constant, write $\Phi_i(P^*)$. Examples include price and constant fees. A transaction cost Φ_i is *heterogeneous* if the function $a_i \mapsto \Phi_i(a_i, P^*)$ is strictly increasing for buyers and strictly decreasing for sellers.[40] Spread fees are an example of a heterogeneous transaction cost.

We now analyze best responses (analogous definition to Section 5) and efficiency to extend our results from Section 4 and Section 6. As the market price is exogenous, that is, traders can not influence it the main strategic considerations reduce to maximizing the trading probability and minimizing the transaction costs.

For homogeneous transaction costs, our results from Theorems 6 and 10 directly extend:

Theorem 12 (For an exogenous market price and homogeneous transaction costs, truthfulness is a best response and fully efficient)**.** *Consider a homogeneous transaction cost and δ-uncertainty. For every $\delta \geq 0$, truthfulness is a best response, leading to achievable efficiency ratio equal to* 1*. Moreover, for $\delta > 0$, truthfulness is the unique best response.*

The proof is relegated to Appendix B.13.

For heterogeneous transaction costs, Theorems 8 and 11 also have their natural counterparts. In contrast to homogeneous transaction costs, beliefs (in particular about tie-breaking) have a non-negligible impact on strategic incentives.

Theorem 13 (For an exogenous market price and heterogeneous transaction costs best responses approximate price-guessing, which, dependent on beliefs, leads to any efficiency level)**.** *Consider a heterogeneous transaction cost, δ-uncertainty, and assume that for trader i bidding the critical value P_i^∞ is strictly individually rational. Then, best responses depend on the beliefs:*

1. *$\delta > 0$ **(Uncertainty):** A best response exists. If δ is sufficiently small, best responses approximate price-guessing. There exist beliefs, such that for all best responses the achievable efficiency ratio is* 0.

[38] If beliefs are not deterministic, we assume as before that the net value t_i^Φ of trader i lies in the support of P^*.
[39] $\delta = 0$ describes the case of deterministic beliefs, which corresponds to the limit case of our DA model.
[40] This is equivalent to the condition that $a_i \mapsto P_i(a_i, P^*)$ is strictly increasing.

2. $\delta = 0$ ***(Deterministic):*** *If there is no tie-breaking, price-guessing is the unique best response. Else, if there is tie-breaking, no best response exists and for sufficiently small $\epsilon > 0$, ϵ-best responses approximate price-guessing. Further, if the market price has a continuous and strictly positive density function, for every $\zeta \in [0, 1]$, there exist beliefs, such that the achievable efficiency ratio of best responses is equal to ζ.*

The proof is relegated to Appendix B.14.

8 Conclusion

Large markets, in particular large DAs, have been shown to be asymptotically efficient. However, much of the preexisting literature on the topic has abstracted away from transaction costs. Our paper brings the importance of transaction costs to the spotlight—they may fundamentally change incentives. In fact, transaction cost considerations may become more important in larger markets, not less important, unlike strategic considerations related to prices. Different transaction cost types—more so than their levels—have drastically different implications for incentives. In particular, spread fees, or heterogeneous transaction costs more generally, even if small and charged implicitly, may alter bid/ask behavior and result in substantial market inefficiency.

Studying DAs with transaction costs allows to understand many market mechanisms beyond the DA without transaction costs. In particular, our results qualitatively hold true, as long as the influence on the market price is small or vanishes for large markets. This holds, for example, true for first and second price auctions.

Our results raise several natural empirical questions. What are the cost of strategic transaction cost avoidance in markets, e.g., those we discussed in the Introduction? Are more experienced, more sophisticated, more informed traders better at avoiding transaction costs? The type and level of transaction cost charged may have substantive efficiency and fairness consequences, and is an important question for regulators and intermediaries (e.g., platforms and brokers).

References

Azevedo, E.M. and E. Budish (2019), "Strategy-proofness in the Large." *Review of Economic Studies*, 86, 81–116.

Bartle, R.G. and D.R. Sherbert (2000), *Introduction to Real Analysis*, volume 2. Wiley New York.

Bergemann, D., B. Brooks, and S. Morris (2015), "The limits of price discrimination." *American Economic Review*, 105, 921–57.

Bergemann, D. and S. Morris (2005), "Robust mechanism design." *Econometrica*, 73, 1771–1813.

Boergers, T. and J. Li (2019), "Strategically simple mechanisms." *Econometrica*, 87, 2003–2035.

Carroll, G. (2015), "Robustness and linear contracts." *American Economic Review*, 105, 536–63.

Carroll, G. (2017), "Robustness and separation in multidimensional screening." *Econometrica*, 85, 453–488.

Chassang, S. (2013), "Calibrated incentive contracts." *Econometrica*, 81, 1935–1971.

Chatterjee, K. and W. Samuelson (1983), "Bargaining under incomplete information." *Operations Research*, 31, 835–851.

Chen, D. and A.L. Zhang (2020), "Subsidy schemes in double auctions."

Chung, K.-S. and J. C. Ely (2007), "Limited foundations of dominant-strategy mechanisms." *The Review of Economic Studies*, 74, 447–476.

Colliard, J.-E. and T. Foucault (2012), "Trading fees and efficiency in limit order books." *The Review of Financial Studies*, 24, 3389–3421.

Cripps, M.W. and J.M. Swinkels (2006), "Efficiency of large double auctions." *Econometrica*, 74, 47–92.

de Clippel, G. and K. Rozen (2018), "Consumer theory with misperceived tastes." *Working paper*.

Eyster, E. and M. Piccione (2013), "An approach to asset pricing under complete and diverse perceptions." *Econometrica*, 81, 1483–1506.

Foucault, T., O. Kadan, and E. Kandel (2013), "Liquidity cycles and make/take fees in electronic markets." *The Journal of Finance*, 68, 299–341.

Friedman, D. and J. Rust (1993), *The double auction market: institutions, theories, and evidence*. Westview Press.

Fudenberg, D., M. Mobius, and A. Szeidl (2007), "Existence of equilibrium in large double auctions." *Journal of Economic Theory*, 133, 550 – 567.

Garratt, R. and M. Pycia (2016), "Efficient bilateral trade." *Unpublished Paper, UCLA.[535]*.

Harrison, J. M. and D. M. Kreps (1978), "Speculative investors behavior in a stock market with heterogeneous expectations." *Quarterly Journal of Economics*, 92, 323–336.

Heidhues, P., B. Kőszegi, and P. Strack (2018), "Unrealistic expectations and misguided learning." *Econometrica*, 86, 1159–1214.

Jackson, M.O. and J.M. Swinkels (2005), "Existence of equilibrium in single and double private value auctions." *Econometrica*, 73, 93–139.

Jantschgi, S., H.H. Nax, B.S.R. Pradelski, and M. Pycia (2022), "On Market Prices in Double Auctions." *Working Paper.*

Ledyard, J.O. (1978), "Incentive compatibility and incomplete information." *Journal of Economic Theory*, 18, 171–189.

Leininger, W., P.B. Linhart, and R. Radner (1989), "Equilibria of the sealed-bid mechanism for bargaining with incomplete information." *Journal of Economic Theory*, 48, 63–106.

Li, S. (2017), "Obviously strategy-proof mechanisms." *American Economic Review*, 107, 3257–3287.

Madarász, K. and A. Prat (2017), "Sellers with misspecified models." *The Review of Economic Studies*, 84, 790–815.

Malinova, K. and A. Park (2015), "Subsidizing liquidity: The impact of make/take fees on market quality." *The Journal of Finance*, 70, 509–536.

Marra, M. (2019), "Pricing and fees in auction platforms with two-sided entry." Technical report, Sciences Po.

Myerson, R.B. and M.A. Satterthwaite (1983), "Efficient mechanisms for bilateral trading." *Journal of Economic Theory*, 29, 265–281.

Noussair, C., S. Robin, and B. Ruffieux (1998), "The effect of transaction costs on double auction markets." *Journal of Economic Behavior & Organization*, 36, 221–233.

Pycia, M. and P. Troyan (2019), "A theory of simplicity in games and mechanism design." In *ACM Conference on Economics and Computation EC'19.*

Reny, P.J. and M. Perry (2006), "Toward a strategic foundation for rational expectations equilibrium." *Econometrica*, 74, 1231–1269.

Roberts, D.J. and A. Postlewaite (1976), "The incentives for price-taking behavior in large exchange economies." *Econometrica: journal of the Econometric Society*, 115–127.

Rustichini, A., M.A. Satterthwaite, and S.R. Williams (1994), "Convergence to efficiency in a simple market with incomplete information." *Econometrica*, 1041–1063.

Satterthwaite, M.A. and S.R. Williams (1989a), "Bilateral trade with the sealed bid k-double auction: Existence and efficiency." *Journal of Economic Theory*, 48, 107 – 133.

Satterthwaite, M.A. and S.R. Williams (1989b), "The rate of convergence to efficiency in the buyer's bid double auction as the market becomes large." *The Review of Economic Studies*, 56, 477–498.

Shi, B., E.H. Gerding, P. Vytelingum, and N.R. Jennings (2013), "An equilibrium analysis of market selection strategies and fee strategies in competing double auction marketplaces." *Autonomous Agents and Multi-Agent Systems*, 26, 245–287.

Tatur, T. (2005), "On the trade off between deficit and inefficiency and the double auction with a fixed transaction fee." *Econometrica*, 73, 517–570.

Tyurin, I.S. (2012), "A refinement of the remainder in the lyapunov theorem." *Theory of Probability and its Applications*, 56.

Vapnik, V.N. and A.Y. Chervonenkis (2015), *On the uniform convergence of relative frequencies of events to their probabilities*, 11–30.

Williams, S.R. (1991), "Existence and convergence of equilibria in the buyer's bid double auction." *The Review of Economic Studies*, 58, 351–374.

Wilson, R. (1985), "Incentive efficiency of double auctions." *Econometrica*, 53, 1101–1115.

Wilson, R. (1987), "Game theoretic approaches to trading processess. in t. bewley, ed., advances in economic theory: Fifth world congress."

Wolitzky, A. (2016), "Mechanism design with maxmin agents: Theory and an application to bilateral trade." *Theoretical Economics*, 11, 971–1004.

A Additional results

A.1 Demand, supply, and market-clearing prices

We clarify how the k-DA chooses the market price. For a detailed treatment of the k-DA and the proofs of Lemmas 14, 15, and 16 see Jantschgi et al. (2022).

Recall the following notation: For a relation $\mathcal{R} \in \{\geq, >, =, <, \leq\}$, define $\mathcal{B}_{\mathcal{R}}(P) = \{b \in \mathcal{B} : t_b \mathcal{R} P\}$ and $\mathcal{S}_{\mathcal{R}}(P) = \{s \in \mathcal{S} : t_s \mathcal{R} P\}$.

Definition (Demand and supply functions)**.** The *demand* and *supply functions* at price P are defined as $D(P) = \mu_B(\mathcal{B}_{\geq}(P))$ and $S(P) = \mu_S(\mathcal{S}_{\leq}(P))$, that is, by the mass of all traders who weakly prefer trading over not trading at price P.

We next define a special class of action distributions, which arise in infinite markets, e.g., if they are interpreted as the limit of finite markets where actions are modelled as independent random variables. Say that action distributions μ_B^a and μ_S^a are *continuous*, if they are equivalent to the Lebesgue-measure on A_B and A_S and their densities f_B and f_S are continuous, that is $\mu_B^a(A) = \int_A f_B(x)dx$ and $\mu_S^a(A) = \int_A f_S(x)dx$ for $A \subset \mathbb{R}$.

Lemma 14 (Analytic properties of demand and supply functions)**.** *The demand function is non-increasing, left-continuous with right limits. The supply function is non-decreasing, right-continuous with left limits. It holds that $D(P+) = \mu_B(\mathcal{B}_>(P))$ and $S(P-) = \mu_S(\mathcal{S}_<(P))$. If action distributions are continuous, then demand is continuous and strictly decreasing on A_B and supply is continuous and strictly increasing on A_S.*

The following concept corresponds to prices that equilibrate demand and supply.

Definition ((Strong) market clearing prices)**.** P is a market-clearing price if $D(P) \geq S(P)$ and $D(P+) \leq S(P)$ (*type I*) or $S(P) \geq D(P)$ and $S(P-) \leq D(P)$ (*type II*). P is a *strong market-clearing price* if $D(P) = S(P)$. Denote the set of all market-clearing prices by $\mathcal{P}^{MC}$ and the set of all strong market-clearing prices by $\mathcal{P}^{SMC}$.

Using the analytical properties of demand and supply, we can characterize the topology of the set of (strong) market clearing prices.

Lemma 15 (Topology of $\mathcal{P}^{SMC}$ and $\mathcal{P}^{MC}$)**.** *The set $\mathcal{P}^{SMC}$ is a convex subset of T. Every strong market-clearing price is a market-clearing price (of type I and II). The set of market-clearing prices is non-empty, convex and closed. The set $\mathcal{P}^{MC} \setminus \mathcal{P}^{SMC}$ has Lebesgue-measure zero. More precisely, if $\mathcal{P}^{SMC} \neq \emptyset$, then $\mathcal{P}^{MC} = \overline{\mathcal{P}^{SMC}}$, and if $\mathcal{P}^{SMC} = \emptyset$, then $\mathcal{P}^{MC}$ is a singleton.*
If action distributions are continuous, and $\overline{a}_S > \underline{a}_B$, then there exists a unique market clearing price with positive trading volume and $\mathcal{P}^{SMC} = \mathcal{P}^{MC}$.

In finite markets the mechanism described in Section 3 coincides with the classical k-DA (Rustichini et al., 1994), for which an explicit formula for the set of market-clearing prices is given. Let $a^{(m)}$ be the m'th smallest action in the set of all actions a.

Lemma 16. *In finite markets with m buyers and n sellers $\mathcal{P}^{MC} = [a^{(m)}, a^{(m+1)}]$. If $a^{(m)} \neq a^{(m+1)}$, then for every $P \in (a^{(m)}, a^{(m+1)})$ it follows that $P \in \mathcal{P}^{SMC}$.*

A.2 Allocation and tie-breaking

If the k-DA results in a strong market-clearing price P^*, that is $D(P^*) = S(P^*)$, then no tie-breaking is needed. The allocation is set as $\mathcal{B}^* = \mathcal{B}_{\geq}(P^*)$ and $\mathcal{S}^* = \mathcal{S}_{\leq}(P^*)$, which *balances trade*, that is $\mu_B(\mathcal{B}^*) = \mu_S(\mathcal{S}^*)$. Therefore, the allocation consists of all traders, who weakly prefer trading over not trading at P^*.

Next, suppose that the k-DA results in a market clearing price of type I, which is not a strong market clearing price. Then, $D(P^*) > S(P^*)$ and $D(P^*+) \leq S(P^*)$. Set $\mathcal{S}^* = \mathcal{S}_{\leq}(P^*)$, that is all sellers who, given their action, weakly prefer trading over not trading are involved in trade. Consider the set of all buyers who strictly prefer to trade at P^*, that is $\mathcal{B}_>(P^*)$. It follows from Lemma 14 that $D(P^*+) = \mu_B(\mathcal{B}_>(P^*))$. Let $x = S(P^*) - \mu_B(\mathcal{B}_>(P^*)) \geq 0$ and let $\tilde{\mathcal{B}}$ be a subset of $\mathcal{B}_=(P^*)$ with μ_B-measure equal to x. Such a set exists because $D(P^*) = \mu_B(\mathcal{B}_\geq(P^*)) = \mu_B(\mathcal{B}_>(P^*)) + \mu_B(\mathcal{B}_=(P^*)) \geq S(P^*)$ and $D(P^*+) = \mu_B(\mathcal{B}_>(P^*)) \leq S(P^*)$. Set $\mathcal{B}^* = \mathcal{B}_>(P^*) \cup \tilde{\mathcal{B}}$. That is, all buyers who strictly prefer to trade at P^* are involved in trade, together with a subset of traders with bid equal to P^* that are indifferent in order to balance trade.

Finally, if a market clearing price of type II is chosen, the allocation is set analogously: $\mathcal{B}^* = \mathcal{B}_\geq(P^*)$ and $\mathcal{S}^* = \mathcal{S}_<(P^*) \cup \tilde{\mathcal{S}}$, where $\tilde{\mathcal{S}}$ is a subset of $\mathcal{S}_=(P^*)$ that balances trade.

Suppose that $\tilde{B}$ (respectively $\tilde{S}$) are chosen uniformly at random, this ensures fairness. That is, they are random compact sets such that for all $b \in \mathcal{B}_=(P^*)$ it holds that $\mathbb{P}[b \in \tilde{\mathcal{B}}] \equiv {}^{\mu_B(\tilde{\mathcal{B}})}/_{\mu_B(\mathcal{B})}$ (respectively for all $s \in \mathcal{S}_=(P^*)$ it holds that $\mathbb{P}[s \in \tilde{\mathcal{S}}] \equiv {}^{\mu_S(\tilde{\mathcal{S}})}/_{\mu_S(\mathcal{S})}$). The existence of uniform random sets is discussed in Jantschgi et al. (2022).

A.3 Explicit formulas

In this section we derive explicit formulas for some of the concepts introduced in the model in Section 3 that will be used in subsequent proofs. We will sometimes differentiate between finite markets with m buyers and n sellers and infinite markets with market ratio R.

Throughout this section, consider a buyer b with gross value t_b and bid a_b, and a seller s with gross value t_s and ask a_s. Let a denote an action distribution. Recall that in a finite market, $a^{(k)}$ denotes the k'th smallest element in the set of all taken actions.

A.3.1 Involvement in trade

Finite markets. If $a_b < a_{\text{-}b}^{(m)}$, then it is strictly smaller than the $m+1$'st smallest element in the set of all actions a (including a_b) and buyer b is not involved in trade, because their bid is below the market price. If $a_b > a_{\text{-}b}^{(m)}$, then it is at least the $m+1$'st largest element and therefore sufficient to be involved in trade. If $a_b = a_{\text{-}b}^{(m)}$, then the buyer might be subject to tie-breaking.

If $a_s > a_{\text{-}s}^{(m)}$, then it is at least the $m+1$'st smallest element in the set of all actions (including a_s) and seller s is not involved in trade, because their ask was above the market price. If $a_s < a_{\text{-}s}^{(m)}$, then it is at most the m'th smallest action and therefore sufficient to be involved in trade. If $a_s = a_{\text{-}s}^{(m)}$s, then the seller might be subject to tie-breaking.

Infinite markets. If there exists no demand excess, then a buyer is involved in trade, if $a_b \geq P^*(a)$. If $a_b < P^*(a)$, then the buyer is not involved in trade. If there exists demand excess, it is generated by bids at $P^*(a)$. If $a_b > P^*(a)$, then the buyer is involved in trade. If $a_b = P^*(a)$, then the buyer might be subject to tie-breaking.

If there exists no supply excess, then the seller is involved in trade, if $a_s \leq P^*(a)$. If $a_s > P^*(a)$, then the seller is not involved in trade. If there exists supply excess, it is generated by asks at $P^*(a)$. If $a_s < P^*(a)$, then the seller is involved in trade. If $a_s = P^*(a)$, then the seller might be subject to tie-breaking.

We can now express the probability of trade, given the beliefs of a trader.

A.3.2 Trading probabilities given beliefs

Finite markets. Given the belief that actions are random variables with continuous distribution, tie-breaking is a probability zero event in finite markets. It follows from Appendix A.3.1 that

$$\mathbb{P}_{a_{\text{-}b}}\left[b \in \mathcal{B}^*(a_b, a_{\text{-}b})\right] = \mathbb{P}_{a_{\text{-}b}}\left[a_b \geq a_{\text{-}b}^{(m)}\right] \quad \text{and} \quad \mathbb{P}_{a_{\text{-}s}}\left[s \in \mathcal{S}^*(a_s, a_{\text{-}s})\right] = \mathbb{P}_{a_{\text{-}s}}\left[a_s \leq a_{\text{-}s}^{(m)}\right]. \tag{3}$$

In Appendix A.7, explicit formulas for such probabilities are derived in a more general context (see Equations (28) and (29)).

Infinite markets. If there exists no demand excess at P^*, then

$$\mathbb{P}_{a_{\text{-}b}}\left[b \in \mathcal{B}^*(a_b, a_{\text{-}b})\right] = \begin{cases} 1 & a_b \geq P^*(a), \\ 0 & \text{else.} \end{cases} \tag{4}$$

Suppose that there is strictly positive demand excess. That is $\mu_B(\mathcal{B}_{\geq}(P^*(a))) = V(a) + x$ and $\mu_B(\mathcal{B}_{>}(P^*(a))) = V(a) - y$ for $x > 0$ and $y \geq 0$ (see Appendix A.2). Then,

$$\mathbb{P}_{a_{\text{-}b}}\left[b \in \mathcal{B}^*(a_b, a_{\text{-}b})\right] = \begin{cases} 1 & a_b > P^*(a), \\ \frac{y}{x+y} & a_b = P^*(a), \\ 0 & \text{else.} \end{cases}. \tag{5}$$

If there exists no supply excess, then

$$\mathbb{P}_{a_{\text{-}s}}\left[s \in \mathcal{S}^*(a_s, a_{\text{-}s})\right] = \begin{cases} 1 & a_s \leq P^*(a), \\ 0 & \text{else.} \end{cases} \tag{6}$$

Suppose that there is strictly positive supply excess. That is $\mu_S(\mathcal{S}_{\leq}(P^*(a))) = V(a) + x$ and $\mu_S(\mathcal{S}_{<}(P^*(a))) = V(a) - y$ for $x > 0$ and $y \geq 0$. Then,

$$\mathbb{P}_{a_{\text{-}s}}\left[s \in \mathcal{S}^*(a_s, a_{\text{-}s})\right] = \begin{cases} 1 & a_s < P^*(a), \\ \frac{y}{x+y} & a_s = P^*(a), \\ 0 & \text{else.} \end{cases} \tag{7}$$

Note that in the presence of strictly positive trading excess, traders believe that if they are involved in tie-breaking in an infinite market, then they have a fair chance of being involved in trade.

A.3.3 Market Price

Finite markets. Recall that by Lemma 16 $P^*(a) = ka^{(m)} + (1-k)a^{(m+1)}$. Interpreting the market price as a function of a single action yields

$$P^*\left(a_b, a_{\text{-}b}\right) = \begin{cases} (1\text{-}k)a_{\text{-}b}^{(m)} + ka_b & \text{if } a_{\text{-}b}^{(m)} \leq a_b \leq a_{\text{-}b}^{(m+1)}, \\ (1\text{-}k)a_{\text{-}b}^{(m)} + ka_{\text{-}b}^{(m+1)} & \text{else.} \end{cases} \tag{8}$$

$$P^*\left(a_s, a_{\text{-}s}\right) = \begin{cases} (1\text{-}k)a_s + ka_{\text{-}s}^{(m)} & \text{if } a_{\text{-}s}^{(m\text{-}1)} \leq a_s \leq a_{\text{-}s}^{(m)}, \\ (1\text{-}k)a_{\text{-}s}^{(m\text{-}1)} + ka_{\text{-}s}^{(m)} & \text{else.} \end{cases} \tag{9}$$

Note that $P^*\left(a_b, a_{\text{-}b}\right)$ depends only on $a_{\text{-}b}^{(m)}$ and $a_{\text{-}b}^{(m+1)}$, and $P^*\left(a_s, a_{\text{-}s}\right)$ depends only on $a_{\text{-}s}^{(m\text{-}1)}$ and $a_{\text{-}s}^{(m)}$. In some proofs, this dependence will be of importance and we will, for example, write $P^*\left(a_b, a_{\text{-}b}^{(m)}, a_{\text{-}b}^{(m+1)}\right)$ instead of $P^*\left(a_b, a_{\text{-}b}\right)$.

In addition, for a trader i, we will in some proofs consider $\tilde{P^*}\left(a_i, a_{\text{-}i}\right)$, which is equal to the market price, if i is involved in trade, and zero otherwise.

Infinite markets. In an infinite market, a single trader cannot influence the market price. It therefore holds for a trader i and for all actions a_i and a_i' that $P^*(a_i, a_{\text{-}i}) = P^*(a_i', a_{\text{-}i})$. By abuse of notation, we will in some proofs write $P^*(a_{\text{-}i})$.

A.3.4 Utility functions

For a buyer the utility of being involved in trade is equal to the difference between their gross value and the market price minus the additional transaction cost:

$$u_b\left(t_b, a_b, a_{\text{-}b})\right) = \begin{cases} t_b\text{-}P^*\left(a_b, a_{\text{-}b}\right)\text{-}\Phi_b\left(a_b, a_{\text{-}b}\right) & b \in \mathcal{B}^*, \\ 0 & \text{else.} \end{cases} \tag{10}$$

For a seller the utility of being involved in trade is equal to the difference between the market price and their gross value minus the additional transaction cost:

$$u_s\left(t_s, a_s, a_{\text{-}s})\right) = \begin{cases} P^*\left(a_s, a_{\text{-}s}\right)\text{-}t_s\text{-}\Phi_s\left(a_s, a_{\text{-}s}\right) & s \in \mathcal{S}^*, \\ 0 & \text{else.} \end{cases} \tag{11}$$

Finite markets. Let $\mu_b\left(a_{\text{-}b}\right)$ denote the distribution of $a_{\text{-}b}$ according to the beliefs of trader b. It holds that

$$\begin{gathered}\mathbb{E}_{\text{-}b}\left[u_b(t_b, a_b, a_{\text{-}b})\right] = \\ \int\limits_{\{a_b \geq a_{\text{-}b}^{(m)}\}} \left(t_b\text{-}P^*\left(a_b, a_{\text{-}b}\right)\text{-}\Phi_b\left(a_b, a_{\text{-}b}\right)\right) d\mu_b\left(a_{\text{-}b}\right) = \\ t_b \cdot \mathbb{P}_{\text{-}b}[b \in \mathcal{B}^*(a_b, a_{\text{-}b})] - \int\limits_{[\underline{a}_{S,b}, \overline{a}_{S,b}]^2} \tilde{P^*}\left(a_b, a_{\text{-}b}^{(m)}, a_{\text{-}b}^{(m+1)}\right) d\mu_b(a_{\text{-}b}^{(m)}, a_{\text{-}b}^{(m+1)}) - \mathbb{E}_{\text{-}b}\left[\Phi_b(a_b, a_{\text{-}b})\right]\end{gathered} \tag{12}$$

Note that both $a_{\text{-}b}^{(m)}$ and $a_{\text{-}b}^{(m+1)}$ have support in $[\underline{a}_{S,b}, \overline{a}_{S,b}]$. That is because $a_{\text{-}b}$ consists of $m-1$ bids and n asks. So there must be at least one ask below or equal to $a_{\text{-}b}^{(m)}$.

Let $\mu_s\left(a_{\text{-}s}\right)$ denote the distribution of $a_{\text{-}s}$ according to the beliefs of a seller s. It holds that

$$\begin{gathered}\mathbb{E}_{\text{-}s}\left[u_s(t_s, a_s, a_{\text{-}s})\right] = \\ \int\limits_{\{a_s \leq a_{\text{-}s}^{(m)}\}} \left(P^*\left(a_s, a_{\text{-}s}\right)\text{-}t_s - \Phi_s\left(a_s, a_{\text{-}s}\right)\right) d\mu_s\left(a_{\text{-}s}\right) = \\ \int\limits_{[\underline{a}_{B,s}, \overline{a}_{B,s}]^2} \tilde{P^*}\left(a_s, a_{\text{-}s}^{(m\text{-}1)}, a_{\text{-}s}^{(m)}\right) d\mu_s(a_{\text{-}s}^{(m\text{-}1)}, a_{\text{-}s}^{(m)}) - t_s \cdot \mathbb{P}_{\text{-}s}[s \in \mathcal{S}^*(a_s, a_{\text{-}s})] - \mathbb{E}_{\text{-}s}\left[\Phi_s(a_s, a_{\text{-}s})\right].\end{gathered} \tag{13}$$

Note that both $a_{\text{-}s}^{(m\text{-}1)}$ and $a_{\text{-}s}^{(m)}$ have support in $[\underline{a}_{B,s}, \overline{a}_{B,s}]$.

Infinite markets. The expectation is only concerned with tie-breaking, as both the market price and the transaction cost are deterministic. Therefore,

$$\mathbb{E}_{\text{-}b}\left[u_b(t_b, a_b, a_{\text{-}b})\right] = \left(t_b - P^*(a_b, a_{\text{-}b}) - \Phi_b(a_b, a_{\text{-}b})\right) \mathbb{P}_{\text{-}b}\left[b \in \mathcal{B}^*(a_b, a_{\text{-}b})\right] \tag{14}$$

and

$$\mathbb{E}_{\text{-}s}\left[u_s(t_s, a_s, a_{\text{-}s})\right] = \left(P^*(a_s, a_{\text{-}s}) - t_s - \Phi_s(a_s, a_{\text{-}s})\right) \mathbb{P}_{\text{-}s}\left[s \in \mathcal{S}^*(a_s, a_{\text{-}s})\right]. \tag{15}$$

Difference in expected utility for actions a_i^1 and a_i^2 in finite markets In multiple proofs, we will estimate the difference in expected utility in finite markets for two actions a_i^1 and a_i^2. The following lemma yields an upper bound:

Lemma 17. *For bids $a_b^1 > a_b^2$ and for asks $a_s^1 < a_s^2$ it holds that*

$$\begin{gathered}\mathbb{E}_{\text{-}b}\left[u_b\left(t_b, a_b^1, a_{\text{-}b}\right)\right] - \mathbb{E}_{\text{-}b}\left[u_b\left(t_b, a_b^2, a_{\text{-}b}\right)\right] \leq \\ t_b\left(\mathbb{P}_{\text{-}b}\left[b \in \mathcal{B}^*(a_b^1, a_{\text{-}b})\right] - \mathbb{P}_{\text{-}b}\left[b \in \mathcal{B}^*(a_b^2, a_{\text{-}b})\right]\right) - \left(\mathbb{E}_{\text{-}b}\left[\Phi_b\left(a_b^1, a_{\text{-}b}\right)\right] - \mathbb{E}_{\text{-}b}\left[\Phi_b\left(a_b^2, a_{\text{-}b}\right)\right]\right).\end{gathered} \tag{16}$$

and

$$
\begin{aligned}
&\mathbb{E}_{\text{-}s}\left[u_s\left(t_s, a_s^1, a_{\text{-}s}\right)\right] - \mathbb{E}_{\text{-}s}\left[u_s\left(t_s, a_s^2, a_{\text{-}s}\right)\right] \\
&\leq 2\overline{a}_{B,s}\left(1 - \mathbb{P}_{\text{-}s}\left[s \in \mathcal{S}^*(a_s, a_{\text{-}s})\right]\right) - t_s\left(\mathbb{P}_{\text{-}s}\left[s \in \mathcal{S}^*(a_s^1, a_{\text{-}s})\right] - \mathbb{P}_{\text{-}s}\left[s \in \mathcal{S}^*(a_s^2, a_{\text{-}s})\right]\right) \\
&\quad - \left(\mathbb{E}_{\text{-}s}\left[\Phi_s\left(a_s^1, a_{\text{-}s}\right)\right] - \mathbb{E}_{\text{-}s}\left[\Phi_s\left(a_s^2, a_{\text{-}s}\right)\right]\right).
\end{aligned} \tag{17}
$$

The proof of this Lemma is relegated to Appendix B.15.

A.4 Discussion of truthfulness for pathological transaction costs

The set of undominated actions might be empty. First, the net value might not exist, as the set of undominated actions can be empty. Consider a seller s with a positive gross value t_s. If a price fee $\phi_s = 1$ is charged, then any involvement in trade results in a loss for the seller. Therefore any action a_s is dominated by a greater action a'_s, proving that there does not exist an undominated action.

The net value might be dominated. Second, the net value might be dominated, as the set of undominated actions might be open. Consider a buyer b with positive gross value t_b. If a constant fee $c_b > t_b$ is charged, any involvement in trade results in a loss for a buyer. It is therefore optimal to not be involved in trade. Formally, this would mean to submit a negative action $a_b \in [-\infty, 0)$. But $a_b = 0$ is dominated by any negative action, as a buyer can still be involved in trade, if all other traders submit 0, and the buyer wins tie-breaking. Therefore the supremum of the set of undominated and ex-post individually rational actions is not attained as a maximum.

The maximal undominated action might not be ex-post individually rational. Third, the maximal undominated action might not be ex-post individually rational. Consider a buyer with gross value t_b and a fee Φ_b that is equal to zero, unless for one action distribution $a'_{\text{-}b}$, where the buyer is involved in trade with action t_b and the fee is greater than t_b. The largest undominated action is equal to t_b, as this action dominates all larger actions actions, but is not dominated by smaller actions. But it is not ex-post individually rational, because $u_b(t_b, a_b, a'_{\text{-}b}) < 0$.

A.5 Out-of-the market gross values

We sometimes focus on *in-the-market* gross values that is gross values t_i such that $t_i^\Phi \prec P_i^\infty$. Traders with such gross values are able to submit individually rational actions that make them likely to be involved in trade when the market is sufficiently large. By contrast, for an *out-of-the-market* trader, that is, one with gross value $t_i^\Phi \succ P_i^\infty$, the probability of trade, when acting individually rationally, vanishes in large markets. Observe that bidding the critical value P_i^∞ is individually rational for for in-the-market traders but not for out-of-the-market traders.

Proposition 18 (For out-of-the-market gross values, truthfulness is close to optimal)**.** *If bidding the critical value P_i^∞ is not individually rational for trader i, then for every $\epsilon > 0$, in sufficiently large markets, truthfulness is an ϵ-best response.*

Proof Outline. As $t_i^\Phi \succ P_i^\infty$, the best response for a trader is more aggressive than P_i^∞. But for any such action, it follows from Proposition 3 that the trading probability gets arbitrarily small in sufficiently large markets. Therefore, for any $\epsilon > 0$, the expected utility of a best response is less or equal than ϵ, and, as truthfulness leads to a non-negative expected utility by assumption, it is an ϵ-best response in sufficiently large markets. Proof details are relegated to Appendix B.16. □

A.6 Strategic incentives for price and spread fees

This section contains a detailed discussion of the opposing strategic incentives for price and spread fees in finite markets: (i) Utility when trading, versus (ii) probability of trading.[41]

Recall that a trader i believes that actions are distributed in intervals $A_{B,i} = [\underline{a}_{B,i}, \overline{a}_{B,i}]$ and $A_{S,i} = [\underline{a}_{S,i}, \overline{a}_{B,i}]$ with the assumption that $\overline{a}_{S,i} \geq \overline{a}_{B,i} > t_i^\Phi > \underline{a}_{S,i} \geq \underline{a}_{B,i}$.

Consider a buyer b with action a_b. We can omit the analysis of $a_b > \overline{a}_{B,b}$ and $a_b < \underline{a}_{S,b}$; for the first, such an action is by assumption not individually rational and strictly dominated by t_b^Φ, for the second, any action below $\underline{a}_{S,b}$ has probability of trade equal to 0, because no seller is believed to submit an action below it. Therefore, the expected utility at such a bid is equal to 0. We therefore consider $a_b \in [\underline{a}_{S,b}, \overline{a}_{B,b}]$.

As the market price depends only on a_b, $a_{\text{-}b}^{(m)}$ and $a_{\text{-}b}^{(m+1)}$. For ease of notation, let $y = a_{\text{-}b}^{(m)}$ and $z = a_{\text{-}b}^{(m+1)}$ and denote by $e(y,z)$ the joint density of y and z given the beliefs of buyer b.

Price fees. The expected utility of a buyer is of the form

$$\begin{aligned}\mathbb{E}_{\text{-}b}\left[u_b(t_b, a_b, a_{\text{-}b})\right] = &\int_{a_b}^{\overline{a}_{S,i}} \int_{\underline{a}_{S,b}}^{a_b} \left(t_b - (1+\phi_b)\left(ka_b + (1-k)\,y\right)\right) e(y,z) dydz + \\ &\int_{\underline{a}_{S,b}}^{a_b} \int_{\underline{a}_{S,b}}^{z} \left(t_b - (1+\phi_b)\left(kz + (1-k)\,y\right)\right) e(y,z) dydz.\end{aligned} \tag{18}$$

The expected utility is continuously differentiable as a function of a_b over the interval $[\underline{a}_{S,b}, \overline{a}_{S,b}]$. Straightforward computation using Leibniz's rule for differentiation under the integral sign yields

$$\frac{d\mathbb{E}_{\text{-}b}\left[u_b(t_b, a_b, a_{\text{-}b})\right]}{da_b} = \left(t_b - (1+\phi_b)\,a_b\right) f_y(a_b) - (1+\phi_b)\,k\mathbb{P}_{\text{-}b}\left[y \leq a_b \leq z\right], \tag{19}$$

[41]This section is closely related to methods used in Rustichini et al. (1994) to analyze strategic incentives in k-DAs without transaction costs.

where $f_y(a_b)$ denotes the density function of y. If $a_b \in (\underline{a}_{S,b}, \overline{a}_{S,b})$ maximizes the expected utility, then the first order condition

$$\frac{d\mathbb{E}_{\text{-}b}\left[u_b(t_b, a_b, a_{\text{-}b})\right]}{da_b} = 0 \tag{20}$$

holds. $f_y(a_b)$ is equal to $\frac{d\mathbb{P}_{\text{-}b}[y \leq a_b]}{da_b}$. A formula for $\mathbb{P}_{\text{-}b}[y \leq a_b]$ is stated in Appendix A.7. Therefore, we can explicitly state the first order condition in terms of distribution and density functions, see Equation (24) below. The first order condition for a seller can be derived in analogy, see Equation (25) below.

Spread fees. The expected utility of a buyer is of the form

$$\begin{aligned}\mathbb{E}_{\text{-}b}\left[u_b(t_b, a_b, a_{\text{-}b}\right] = &\int_{a_b}^{\overline{a}_{S,b}} \int_{\underline{a}_{S,b}}^{a_b} \left(t_b - \phi_b a_b - (1-\phi_b)\left(k a_b + (1-k)\, y\right)\right) e(y,z) dy dz + \\ &\int_{\underline{a}_{S,b}}^{a_b} \int_{\underline{a}_{S,b}}^{z} \left(t_b - \phi_b a_b - (1-\phi_b)\left(k z + (1-k)\, y\right)\right) e(y,z) dy dz.\end{aligned} \tag{21}$$

The expected utility is continuously differentiable as a function of a_b over the interval $[\underline{a}_{S,b}, \overline{a}_{S,b}]$. Straightforward computation using Leibniz's rule for differentiation under the integral sign yields

$$\frac{d\mathbb{E}_{\text{-}b}\left[u_b(t_b, a_b, a_{\text{-}b}\right]}{da_b} = \left(t_b - a_b\right) f_y(a_b) - \phi_b \mathbb{P}_{\text{-}b}\left[y \leq a_b\right] - (1-\phi_b)\, k \mathbb{P}_{\text{-}b}\left[y \leq a_b \leq x\right]. \tag{22}$$

where $f_y(a_b)$ denotes the density function of y. If $a_b \in (\underline{a}_{S,b}, \overline{a}_{S,b})$ maximizes the expected utility, then the first order condition

$$\frac{d\mathbb{E}_{\text{-}b}\left[u_b(t_b, a_b, a_{\text{-}b})\right]}{da_b} = 0 \tag{23}$$

holds. $f_y(a_b)$ is equal to $\frac{d\mathbb{P}_{\text{-}b}[y \leq a_b]}{da_b}$. A formula for $\mathbb{P}_{\text{-}b}[y \leq a_b]$ is stated in Appendix A.7. Therefore, we can explicitly state the first order condition in terms of distribution and density functions, see Equation (24) below. The first order condition for a seller can be derived in analogy, see Equation (25) below.

First Order Conditions To explicitly state the first order conditions, we introduce additional notation: Define $a_{i,j}$ as an action distribution for i buyers and j sellers. In this notation, a as defined in Section 3 corresponds to $a_{m,n}$ and for any buyer b and seller s, $a_{\text{-}b}$ and $a_{\text{-}s}$ correspond to $a_{m-1,n}$ and $a_{m,n-1}$. Denote again by $a_{i,j}^{(l)}$ its l'th smallest element.

We say that an action a_b satisfies the *buyer's first order condition* for gross value t_b if

$$\left.\begin{matrix}(t_b\text{-}\,(1{+}\phi_b)\,a_b)\\ \\ (t_b\text{-}a_b)\end{matrix}\right\}\cdot\left(n\mathbb{P}_{\text{-}b}\left[a^{(m\text{-}1)}_{m\text{-}1,n\text{-}1}\le a_b\le a^{(m)}_{m\text{-}1,n\text{-}1}\right]f_{S,b}(a_b){+}\,(m\text{-}1)\,\mathbb{P}_{\text{-}b}\left[a^{(m\text{-}1)}_{m\text{-}2,n}\le a_b\le a^{(m)}_{m\text{-}2,n}\right]f_{B,b}(a_b)\right)=$$
$$\begin{cases}(1+\phi_b)\,k\mathbb{P}_{\text{-}b}\left[a^{(m)}_{m\text{-}1,n\text{-}1}\le a_b\le a^{(m+1)}_{m\text{-}1,n}\right] & \text{for price fees}\\ \phi_b\mathbb{P}_{\text{-}b}\left[a^{(m)}_{m,n\text{-}1}\le a_b\right]+(1\text{-}\phi_b)\,k\mathbb{P}_{\text{-}b}\left[a^{(m)}_{m\text{-}1,n}\le a_b\le a^{(m+1)}_{m\text{-}1,n}\right] & \text{for spread fees}\end{cases}. \tag{24}$$

We say that an action a_s satisfies the *seller's first order condition* for gross value t_s if

$$\left.\begin{matrix}((1\text{-}\phi_s)\,a_s\text{-}t_s)\\ \\ (a_s\text{-}t_s)\end{matrix}\right\}\cdot\left((n\text{-}1)\,\mathbb{P}_{\text{-}s}\left[a^{(m\text{-}1)}_{m,n\text{-}2}\le a_s\le a^{(m)}_{m,n\text{-}2}\right]f_{S,s}(a){+}m\mathbb{P}_{\text{-}s}\left[a^{(m\text{-}1)}_{m\text{-}1,n\text{-}1}\le a_s\le a^{(m)}_{m\text{-}1,n\text{-}1}\right]f_{B,s}(a)\right)=$$
$$\begin{cases}(1\text{-}\phi_s)\,(1\text{-}k)\,\mathbb{P}_{\text{-}s}\left[a^{(m\text{-}1)}_{m,n\text{-}1}\le a_s\le a^{(m)}_{m,n\text{-}1}\right] & \text{for price fees}\\ \phi_s\mathbb{P}_{\text{-}s}\left[a^{(m)}_{m,n\text{-}1}\ge a_s\right]+(1\text{-}\phi_s)\,(1\text{-}k)\,\mathbb{P}_{\text{-}s}\left[a^{(m\text{-}1)}_{m,n\text{-}1}\le a_s\le a^{(m)}_{m,n\text{-}1}\right] & \text{for spread fees}\end{cases}. \tag{25}$$

Interpretation of a buyer's first order condition. Despite the extensive and complex form of the condition, it has a natural interpretation: It balances between the probability of trade and the utility when trading.

In particular, an incremental increase Δa_b in a buyer's bid has two opposing effects: If the bid a_b does not include the buyer amongst those who trade, then by increasing it to $a_b + \Delta a_b$, the buyer may surpass other actions and be involved in trade. If the bid a_b is sufficient to include the buyer in trade, then increasing their bid by Δa_b may lead to an increase in market price and their fee.

In Equation (24), the left-hand side of the equation describes the gain from increasing one's trading probability. The sum in brackets times Δa_b is the probability that the buyer enters the set of buyers who trade as they incrementally raise their bid by Δa_b. The first term in the sum is the marginal probability of acquiring an item by passing a seller's offer and the second term is the marginal probability of acquiring an item by passing another buyer's bid. For a price fee the profit from such a trade is between $t_b - (1 + \phi_b)a_b$ and $t_b - (1 + \phi_b)a_b - (1 + \phi_b)\Delta a_b$. Therefore, the marginal expected profit for a buyer who raises their bid is $t_b - (1 + \phi_b)a_b$ times the term in the brackets. In analogy, for a spread fee the marginal expected profit for a buyer who raises their bid is $t_b - \phi_b a_b$ times the term in the brackets.

Next, in Equation (24), the right-hand side of the equation describes the buyer's marginal execpted loss from increasing their bid above a_b. $\mathbb{P}_{\text{-}b}\left[a^{(m)}_{m\text{-}1,n}\le a_b\le a^{(m+1)}_{m\text{-}1,n}\right]$ is the probability that a buyer who increases their bid by Δa_b increases the market price by $k(1 + \phi_b)\Delta a_b$ for a price fee and by $k(1 - \phi_b)\Delta a_b$ for a spread fee. Additionally, for a spread fee $\mathbb{P}_{\text{-}b}\left[a^{(m)}_{m\text{-}1,n}\le a_b\right]$ is the probability that a buyer who increases their bid by Δa_b increases the part of the charged fee depending on their bid by $\phi_b\Delta a_b$.

The interpretation for a seller is symmetric and thus omitted.

A.7 Probabilities in the first order conditions

In this section we derive explicit formulas for the probabilities arising in the first order conditions in Equations (24) and (25), that are also used in the proof of Theorem 7 in Appendix B.8. Instead of deriving expressions for all different probabilities, note that for general n, m, l all of them can be expressed as one of the following three probabilities for different n, m, l: (i) $\mathbb{P}_{\text{-}i}\left[a_{m,n}^{(l)} \leq a_i \leq a_{m,n}^{(l+1)}\right]$, (ii) $\mathbb{P}_{\text{-}i}\left[a_{m,n}^{(l)} \leq a_i\right]$ and (iii) $\mathbb{P}_{\text{-}i}\left[a_{m,n}^{(l)} \geq a_i\right]$.

For (i) it is the probability that action a_i lies between the l'th and $l+1$'st smallest element in a set of m bids and n asks. The probability that another buyer submits an action smaller or equal a_i is $F_{B,i}^a(a_i)$. The probability that a buyer submits an action greater or equal a_i is therefore $1 - F_{B,i}^a(a_i)$. Replace $F_{B,i}^a$ by $F_{S,i}^a$ for sellers. The event that exactly l bids and asks are below a_i can be split up in the following way: Suppose that i buyers and j sellers bid and offer less or equal than a_i. $i+j$ must be equal to l. Assuming that there are m buyers and n sellers in total, this means that exactly $m-i$ buyers and $n-j$ sellers bid and offer more than a_i. Selecting i buyers and j sellers, the probability that exactly $i+j=l$ bids and offers are below or equal to a_i is

$$F_{B,i}^a\left(a_i\right)^i F_{S,i}^a\left(a_i\right)^j \left(1\text{-}F_{B,i}^a\left(a_i\right)\right)^{m\text{-}i} \left(1\text{-}F_{S,i}^a\left(a_i\right)\right)^{n\text{-}j}, \tag{26}$$

because the actions of traders are assumed to be independent. There are $\binom{m}{i}$ possibilities to choose i buyers and $\binom{n}{j}$ possibilities to choose j sellers. Therefore, the total probability that exactly l traders submit below a_i is equal to

$$\mathbb{P}_{\text{-}i}\left[a_{m,n}^{(l)} \leq a_i \leq a_{m,n}^{(l+1)}\right] = \sum_{\substack{i+j=l\\0\leq i\leq m\\0\leq j\leq n}} \binom{m}{i}\binom{n}{j} F_{B,i}^a\left(a_i\right)^i F_{S,i}^a\left(a_i\right)^j \left(1\text{-}F_{B,i}^a\left(a_i\right)\right)^{m\text{-}i} \left(1\text{-}F_{S,i}^a\left(a_i\right)\right)^{n\text{-}j}. \tag{27}$$

For (ii) it is the probability that a_i is greater than the l'th action. That is, for some $k \in [l, m+n]$ the number of offers below a_i is exactly equal to k. Summing over k yields that

$$\mathbb{P}_{\text{-}i}\left[a_{m,n}^{(l)} \leq a_i\right] = \sum_{k=l}^{n+m} \sum_{\substack{i+j=k\\0\leq i\leq m\\0\leq j\leq n}} \binom{m}{i}\binom{n}{j} F_{B,i}^a\left(a_i\right)^i F_{S,i}^a\left(a_i\right)^j \left(1\text{-}F_{B,i}^a\left(a_i\right)\right)^{m\text{-}i} \left(1\text{-}F_{S,i}^a\left(a_i\right)\right)^{n\text{-}j}. \tag{28}$$

For (iii), because distributions are assumed to be atomless $\mathbb{P}_{\text{-}i}\left[a_{m,n}^{(l)} = a_i\right] = 0$. It therefore holds that

$$\mathbb{P}_{\text{-}i}\left[a_{m,n}^{(l)} \geq a_i\right] = 1 - \mathbb{P}_{\text{-}i}\left[a_{m,n}^{(l)} \leq a_i\right], \tag{29}$$

which was computed above.

A.8 Market performance in the infinite market with spread fees

Consider the infinite market with type space $T = [1, 2]$, μ_B^t and μ_S^t the Lebesgue-measures from ??. Assume that a symmetric spread fee, that is, $\phi_b = \phi_s = \phi$ is charged. Best responses divide the population into price-guessers choosing actions at the critical value and price-takers. We suppose all buyers identify the critical value at $\beta \in [1, 2]$, and all sellers at $\sigma \in [1, 2]$. The following table gives different measures describing the outcome in a market with and without fees.

	Case (i)	**Case (ii)**	**Case (iii)**	**Case (iv)**
Buyer strategy $a_B(t_b)$	β if $t_b \geq \beta$ and t_b if $t_b < \beta$			
Seller strategy $a_S(t_s)$	σ if $t_s \leq \sigma$ and t_s if $t_s > \sigma$			
Demand $D(P)$	$2 - P$ if $P \leq \beta$ and 0 if $P > \beta$			
Supply $S(P)$	0 if $P < \sigma$ and $P - 1$ if $P \geq \sigma$			
Market Price P^*	$3/2$	σ	β	$\in (\beta, \sigma)$
Market Volume Q^*	$1/2$	$2 - \sigma$	$\beta - 1$	0
Market Excess Ex^*	0	$2\sigma - 3$	$3 - 2\beta$	0
Max. Gains of Trade G_{id}	$1/4$			
Gains of Trade	$1/4$	$\frac{3\sigma-\sigma^2-2}{2(\sigma-1)}$	$\frac{3\beta-\beta^2-2}{2(2-\beta)}$	0
Transaction Costs	$\phi\Big((2\text{-}\beta)(\beta\text{-}{}^3\!/\!_2)+\frac{(\beta\text{-}3/2)^2}{2}$ $+(\sigma\text{-}1)({}^3\!/\!_2\text{-}\sigma)+\frac{(3/2\text{-}\sigma)^2}{2}\Big)$	$\phi\Big((2\text{-}\beta)(\beta\text{-}\sigma)$ $+\frac{(\beta\text{-}\sigma)^2}{2}\Big)$	$\phi\Big((1\text{-}\sigma)(\beta\text{-}\sigma)$ $+\frac{(\beta\text{-}\sigma)^2}{2}\Big)$	0
Surplus	$G - Tc$	$G - Tc$	$G - Tc$	0
Loss	0	$\frac{2\sigma^2\text{-}5\sigma+3}{4(\sigma\text{-}1)}$	$\frac{2\sigma^2\text{-}7\beta+6}{4(2\text{-}\sigma)}$	${}^1\!/\!_4$

B Proofs

B.1 Proof of Proposition 1

Proof. Consider a buyer b with gross value $t_b \in T_b^+$. First, we prove that there exists a unique solution to the equation $t_b - x - \Phi_b(x, x) = 0$. Because $t_b \in T_b^+$, there exists an action a_b such that $t_b - a_b - F_b(a_b, a_b) > 0$. Furthermore, for $a_b > t_b$, it holds that $t_b - a_b - F_b(a_b, a_b) < 0$. Because the function $x \mapsto t_b - x - F_b(x, x)$ is continuous and strictly decreasing, there exists a unique zero point by the Intermediate Value theorem.

Existence. Next, we show that this solution x is equal to the net value t_b^Φ, by proving that x is undominated, it dominates every larger action a_b, it is ex-post individually rational, and no larger action a_b is ex-post individually rational. Consider $a_b > x$. If $a_{\text{-}b}$ is such that buyer b is not involved in trade with x and a_b, then the utility is equal to 0 for both actions. If $a_{\text{-}b}$ is such that b is involved in trade with both actions, then it follows that $u_b(t_b, x, a_{\text{-}b}) \geq u_b(t_b, a_b, a_{\text{-}b})$, because the fee is monotone. If $a_{\text{-}b}$ is such that b is only involved in trade with a_b, then then the market price $P^*(a_b, a_{\text{-}b})$ is greater or equal than x. It holds that $u_b(t_b, a_b, a_{\text{-}b}) \leq u_b(t_b, P^*(a_b, a_{\text{-}b}), a_{\text{-}b}) = t_b - P^*(a_b, a_{\text{-}b}) - \Phi_b(P^*(a_b, a_{\text{-}b}), P^*(a_b, a_{\text{-}b})) \leq t_b - x - \Phi_b(x, x) = 0$. The first inequality follows from the monotonicity of the fee, the second inequality follows, because the map $a_i \mapsto P_i(a_i, a_i)$ is strictly increasing, and the final equality follows from the definition of x. Therefore a_b is dominated by x. Consider $a_b < x$. We show that there exists $a_{\text{-}b}$ such that $u_b(t_b, x, a_{\text{-}b}) > u_b(t_b, a_b, a_{\text{-}b})$. Take $a_{\text{-}b}$, such that buyer b is involved in trade only with x and the market price is strictly less than x. It holds that $u_b(t_b, x, a_{\text{-}b}) = t_b - P^*(x, a_{\text{-}b}) - \Phi_b(x, P^*(x, a_{\text{-}b}) > t_b - x - \Phi_b(x, x) = 0$. The inequality follows from regularity of the fee. Therefore x is not dominated by a_b. To show that x is ex-post individually rational, take any distribution of actions $a_{\text{-}b}$. If buyer b is involved in trade with x, it holds that $P^*(x, a_{\text{-}b}) \leq x$ and therefore $u_b(t_b, x, a_{\text{-}b}) = t_b - P^*(x, a_{\text{-}b}) - \Phi_b(x, P^*(x, a_{\text{-}b})) \geq t_b - x - \Phi_b(x, x) = 0$, where the inequality follows from regularity. Finally, we show that $a_b > x$ is not ex-post individually rational. Take $a_{\text{-}b}$, such that buyer b is involved in trade with a_b and $P^*(a_b, a_{\text{-}b}) > x$. It holds that $u_b(t_b, a_b, a_{\text{-}b}) \leq u_b(t_b, P^*(a_b, a_{\text{-}b}), a_{\text{-}b}) = t_b - P^*(a_b, a_{\text{-}b}) - \Phi_b(P^*(a_b, a_{\text{-}b}), P^*(a_b, a_{\text{-}b})) < t_b - x - \Phi_b(x, x) = 0$, where the first inequality follows from monotonicity, and the second one follows, because the map $a_i \mapsto P_i(a_i, a_i)$ is strictly increasing. This finally proves that $x = t_b^{\Phi}$. Therefore, the net value exists and the supremum is attained as a maximum.

Continuity. It was proven above that the net value exists on T_b^+ and is equal to the unique zero point of the function $x \mapsto t_b - x - \Phi_b(x, x)$. Because this function is strictly increasing and continuous, the zero point continuously depends on the gross value t_b.

Monotonicity. The map $t_b \mapsto t_b - x - \Phi_b(x, x)$ is strictly increasing. Therefore, the zero point of the map $x \mapsto t_b - x - \Phi_b(x, x)$ is strictly increasing in t_b.

The statement for sellers can be proven analogously. □

B.2 Proof of Corollary 2

Proof. Consider a buyer b.

Spread fees. It holds that $\Phi_b(a_b, a_{\text{-}b}) = \phi_b(a_b - P^*(a_b, a_{\text{-}b}) = F_b(a_b, P^*(a_b, a_{\text{-}b})$ with the function $F_b(x, y) = \phi_b(x - y)$. It holds that the map $y \mapsto y + F_b(x, y) = \phi_b x + (1 - \phi_b)y$ is increasing, the map $x \mapsto x + F_b(x, x) = x$ is strictly increasing in y and both are continuous. Therefore spread fees satisfy the conditions of Proposition 1. For any tb, there exists a unique solution of $t_b - t_b^{\Phi} - F_b(t_b^{\Phi}, t_b^{\Phi}) = 0$. It is given by $t_b^{\Phi} = t_b$, proving that the net value equals the gross value.

Price fees. It holds that $\Phi_b(a_b, a_{\text{-}b}) = \phi_b P^*(a_b, a_{\text{-}b} = F_b(a_b, P^*(a_b, a_{\text{-}b})$ with the function $F_b(x, y) = \phi_b y$. It holds that the maps $y \mapsto y + F_b(x, y) = (1 + \phi_b)y$ and $x \mapsto x + F_b(x, x) = x$ are strictly increasing and continuous. Therefore price fees satisfy the conditions of Proposition 1. The unique solution of $t_b - t_b^{\Phi} - F_b(t_b^{\Phi}, t_b^{\Phi}) = 0$ is given by $t_b^{\Phi} = \frac{t_b}{1+\phi_b}$, proving that the net value scales the gross value.

Constant fees. It holds that $\Phi_b(a_b, a_{\text{-}b}) = c_b = F_b(a_b, P^*(a_b, a_{\text{-}b}))$ with the function $F_b(x, y) = c_b$. It holds that the maps $y \mapsto y + F_b(x, y) = y + c_b$ and $x \mapsto x + F_b(x, x) = x + c_b$ are continuous and strictly increasing in y. Therefore constant fees satisfy the conditions of Proposition 1. There exists a solution to $t_b - t_b^\Phi - F_b(t_b^\Phi, t_b^\Phi) = 0$, if $t_b \geq c_b$. It is given by $t_b^\Phi = t_b - c_b$, proving that the net value shifts the gross value.

The statement for sellers can be proven analogously. $\square$

B.3 Proof that the critical value P_i^∞ exists and is unique

Proof. At the point $\underline{a}_{S,i}$, it holds that $F_{B,i}^a(\underline{a}_{S,i}) < 1$. That is because $F_{B,i}^a$ has a strictly positive density $f_{B,i}^a$ on $[\underline{a}_{B,i}, \overline{a}_{B,i}]$ and $\underline{a}_{S,i} < \overline{a}_{B,i}$ by assumption. Second, it holds that $F_{S,i}^a(\underline{a}_{S,i}) = 0$, because the corresponding density $f_{S,i}^a$ has support $[\underline{a}_{S,i}, \overline{a}_{B,i}]$. Therefore, at $\underline{a}_{S,i}$, it holds that

$$F_{B,i}^a(\underline{a}_{S,i}) + R_i F_{S,i}^a(\underline{a}_{S,i}) < 1. \tag{30}$$

A similar argument yields that at the point $\overline{a}_{B,i}$, it holds that $F_{B,i}^a(\overline{a}_{B,i}) = 1$ and $F_{S,i}^a(\overline{a}_{S,i}) > 0$. This implies that

$$F_{B,i}^a(\overline{a}_{B,i}) + R_i F_{S,i}^a(\overline{a}_{B,i}) > 1. \tag{31}$$

Because $F_{B,i}^a$ and $F_{S,i}^a$ are both continuous, it follows from the Intermediate Value Theorem, that there exists $P_i^\infty \in (\underline{a}_{S,i}, \overline{a}_{B,i})$ with

$$F_{B,i}^a(P_i^\infty) + R_i F_{S,i}^a(P_i^\infty) = 1. \tag{32}$$

Because both $F_{B,i}^a$ and $F_{S,i}^a$ are strictly monotone on $(\underline{a}_{S,i}, \overline{a}_{B,i})$, the uniqueness of P_i^∞ follows. $\square$

B.4 Proof of Proposition Proposition 3

Proof. For trader i, consider a sequence of strictly increasing market sizes $(m(l), n(l))_{l \in \mathbb{N}}$ with $m(l), n(l) = \Theta(l)$ and $|R - \frac{n(l)}{m(l)}| = \mathcal{O}(l^{-1})$ for $R \in (0, \infty)$.[42]

Consider a buyer b. It follows from Appendix A.3 that $\mathbb{P}_{\text{-}b}\left[b \in \mathcal{B}^*(a_b, a_{\text{-}b})\right] = \mathbb{P}_{\text{-}b}\left[a_b \geq a_{\text{-}b}^{m(l)}\right]$. This is equal to the probability that at least $m(l)$ actions are below a_b in a sample of actions from $m(l) - 1$ buyers and $n(l)$ sellers. Let $p_{a_b} = F_{B,b}(a_b) \in (0, 1)$ be the probability that another buyer's bid is below a_b. In analogy, define $q_{a_b} = F_{S,b}(a_b) \in (0, 1)$ for sellers. For $i > 0$ let $X_i^{p_{a_b}}$ denote an independent Bernoulli random variable with parameter p_{a_b} and for $j > 0$ let $X_j^{q_{a_b}}$ denote an

[42]This means that both market sides are assumed to have linear growth with respect to a single parameter l, such that neither side of the market dominates the other asymptotically and the ratio of buyers to sellers converges and fluctuates only slightly in finite markets.

independent Bernoulli random variable with parameter q_{a_b}. Define

$$S_l^{a_b} = \sum_{i=1}^{m(l)-1} X_i^{p_{a_b}} + \sum_{j=1}^{n(l)} X_j^{q_{a_b}}. \tag{33}$$

$S_l^{a_b}$ has the same distribution as the number of traders in a sample of $m(l)-1$ buyers and $n(l)$ sellers, whose actions are less or equal than a_b. It follows that

$$\mathbb{P}_{\text{-}b}\left[b \in \mathcal{B}^*(a_b, a_{\text{-}b})\right] = \mathbb{P}[S_l^{a_b} \geq m(l)] = 1 - \mathbb{P}[S_l^{a_b} \leq m(l)-1]. \tag{34}$$

Next, we will show that a properly normalized version of $S_l^{a_b}$ converges in distribution to a standard normal random variable. This follows as an application of the following version of the Berry-Esseen theorem, see Tyurin (2012):

Theorem 19 (Berry-Esseen)**.** *Suppose $X_1, X_2, ...$ is a sequence of independent random variables with (i) $\mu_i = \mathbb{E}[X_i] < \infty$, (ii) $\sigma_i^2 = \mathbb{E}[(X_i - \mu_i)^2] < \infty$ and (iii) $\rho_i = \mathbb{E}[|X_i - \mu_i|^3] < \infty$. Set $r_n = \sum_{i=1}^n \rho_i$, $s_n^2 = \sum_{i=1}^n \sigma_i^2$, $F_n(x) = \mathbb{P}\left[\frac{\sum_{i=1}^n (X_i - \mu_i)}{\sqrt{s_n^2}} \leq x\right]$ and let $\Phi(x)$ be the distribution function of a standard random variable. There exists a constant $C = 0.5591$ such that for all $x \in \mathbb{R}$*

$$|F_n(x) - \Phi(x)| \leq \frac{C r_n}{s_n^3} \tag{35}$$

In order to apply Theorem 19, we rewrite $S_l^{a_b}$ as a single sum of random variables and check all requirements. Define $Y_i^{p_{a_b}} = \sum_{j=0}^{m(i)-m(i-1)} X_{i,j}^{p_{a_b}}$ for $i \leq l-1$ and $Y_l^{p_{a_b}} = \sum_{j=1}^{m(l)-m(l-1)-1} X_{i,j}^{p_{a_b}}$ with $X_{i,j}^{p_{a_b}}$ independent Bernoulli random variables with parameter p_{a_b}. In analogy, define $Y_i^{q_{a_b}} = \sum_{j=1}^{n(i)-n(i-1)} X_{i,j}^{q_{a_b}}$ for $i \leq l$ independent Bernoulli random variables with parameter q_{a_b} and $Z_i^{a_b} = Y_i^{p_{a_b}} + Y_i^{q_{a_b}}$. This yields that in distribution

$$S_l^{a_b} \stackrel{d}{=} \sum_{i=1}^{l} Z_i^{a_b}. \tag{36}$$

Recall that a Bernoulli random variable with parameter p has expectation p and variance $p(1-p)$. Using linearity of expectation and, because the random variables are independent, linearity of variance, it holds for $i < l$, that the random variables satisfy (i) and (ii) in Theorem 19, i.e.

$$\begin{aligned} \mu_i &= (m(i) - m(i-1))\, p_{a_b} + (n(i) - n(i-1))\, q_{a_b} < \infty, \\ \sigma_i^2 &= (m(i) - m(i-1))\, p_{a_b}(1 - p_{a_b}) + (n(i) - n(i-1))\, q_{a_b}(1 - q_{a_b}) < \infty. \end{aligned} \tag{37}$$

For $i = l$ it holds that

$$\begin{aligned}\mu_l &= \left(m\left(l\right) - m\left(l-1\right) - 1\right) p_{a_b} + \left(n\left(l\right) - n\left(l-1\right)\right) q_{a_b} < \infty,\\ \sigma_l^2 &= \left(m\left(l\right) - m\left(l-1\right) - 1\right) p_{a_b}\left(1 - p_{a_b}\right) + \left(n\left(l\right) - n\left(l-1\right)\right) q_{a_b}\left(1 - q_{a_b}\right) < \infty.\end{aligned} \tag{38}$$

Furthermore, for $i < l$ it holds that

$$\begin{aligned}\rho_i &= \mathbb{E}\left[\left|\sum_{j=0}^{m(i)-m(i-1)} X_{i,j}^{p_{a_b}} + \sum_{j=0}^{n(i)-n(i-1)} X_{i,j}^{q_{a_b}} - \left(m\left(i\right) - m\left(i-1\right)\right) p_{a_b} - \left(n\left(i\right) - n\left(i-1\right)\right) q_{a_b}\right|^3\right]\\ &\leq \left(\left(m\left(i\right) - m\left(i-1\right)\right)\left(1 - p_{a_b}\right) + \left(n\left(i\right) - n\left(i-1\right)\right)\left(1 - q_{a_b}\right)\right)^3\\ &\leq K < \infty.\end{aligned} \tag{39}$$

The first inequality in Equation (39) holds, because $X_{i,j}^{p_{a_b}} \leq 1$ and $X_{i,j}^{q_{a_b}} \leq 1$ almost surely. The second inequality follows for some finite $K > 0$ from the assumption $\sup_{i \geq 1} m(i) - m(i-1) < \infty$ and $\sup_{i \geq 1} n(i) - n(i-1) < \infty$. In analogy, for $i = l$ it holds that

$$\rho_l \leq K < \infty, \tag{40}$$

which proves that requirement (iii) is fulfilled. Finally, it holds that

$$s_l^2 = (m(l) - 1) p_{a_b}(1 - p_{a_b}) + n(l) q_{a_b}(1 - q_{a_b}). \tag{41}$$

Next, define the sequence $(A_{a_b}(l))_{l \in \mathbb{N}}$ via

$$\begin{aligned}A_{a_b}(l) &= \frac{m\left(l\right) - 1 - \left(\left(m\left(l\right) - 1\right) p_{a_b} + n\left(l\right) q_{a_b}\right)}{\sqrt{\left(m(l) - 1\right) p_{a_b}\left(1 - p_{a_b}\right) + n(l) q_{a_b}\left(1 - q_{a_b}\right)}}\\ &= \sqrt{m(l)}\frac{\left(1 - \frac{1}{m(l)}\right) - \left(\left(1 - \frac{1}{m(l)}\right) p_{a_b} + \frac{n(l)}{m(l)}\right) q_{a_b})}{\sqrt{\left(1 - \frac{1}{m(l)}\right) p_{a_b}\left(1 - p_{a_b}\right) + \frac{n(l)}{m(l)} q_{a_b}\left(1 - q_{a_b}\right)}}.\end{aligned} \tag{42}$$

Theorem 19 now implies that

$$|\mathbb{P}[\leq m(l) - 1] - \Phi(A_{a_b}(l))| \leq \frac{C r_l}{s_l^3} \leq \frac{C K l}{\left(s_l^2\right)^{3/2}} = \mathcal{O}(l^{-\frac{1}{2}}). \tag{43}$$

It follows from Equation (42) that $|A_{a_b}(l)| = \Theta(\sqrt{l})$. We now argue that for $a_b > P_b^\infty$ and sufficiently large l, $A_{a_b}(l) < 0$. This follows, if we show that for sufficiently large l

$$\left(1 - \frac{1}{m(l)}\right) - \left(\left(1 - \frac{1}{m(l)}\right) p_{a_b} + \frac{n(l)}{m(l)} q_{a_b}\right) < 0. \tag{44}$$

Given that a_b is strictly greater than the critical value P_b^∞, there exists $\delta > 0$, such that $p_{a_b} + R q_{a_b} =$

$1+\delta$. By adding and subtracting Rq_{a_b} it follows that Equation (44) is equivalent to

$$1 - \frac{1}{m(l)}(1-p_{a_b}) - (1+\delta) + (R - \frac{n(l)}{m(l)})q_{a_b} < 0 \tag{45}$$

and therefore to

$$R - \frac{n(l)}{m(l)} < \frac{1}{q_{a_b}}(\delta + \frac{(1-p_{a_b})}{m(l)}). \tag{46}$$

Because it is assumed that $|R - \frac{n(l)}{m(l)}| = \mathcal{O}(\frac{1}{l})$, Equation (44) holds for sufficiently large l. This implies that $A_{a_b}(l) = \Theta(-\sqrt{l})$. A standard concentration inequality for a standard Gaussian random variable Z and $x > 0$ using the Chernoff bound gives

$$\mathbb{P}\,|Z| \geq x] \leq 2\exp\left(\frac{-x^2}{2}\right) \tag{47}$$

It follows that

$$\Phi(A_{a_b}(l)) = \mathcal{O}(e^{-l}). \tag{48}$$

Equation (43) therefore implies that $\mathbb{P}[S_l^{a_b} \leq m(l) - 1] = \mathcal{O}(l^{-\frac{1}{2}})$. Recalling Equation (34) finishes the proof. The statements for $a_b < P_b^\infty$ and for sellers can be proven analogously. □

B.5 Proof of Proposition 4

Proof. Consider a buyer b with private type t_b.

Finite Markets. As was shown in Equation (12), the expected utility is of the form

$$\mathbb{E}_{\text{-}b}\left[u_b(t_b, a_b, a_{\text{-}b})\right] = t_b \cdot \mathbb{P}_{\text{-}b}\left[b \in \mathcal{B}^*(a_b, a_{\text{-}b})\right] - \mathbb{E}_{\text{-}b}\left[P^*(a_b, a_{\text{-}b})\right] - \mathbb{E}_{\text{-}b}\left[\Phi_b(a_b, a_{\text{-}b})\right]. \tag{49}$$

First, we will show that the expected utility is continuous in a_b.[43] The first term $t_b \cdot \mathbb{P}_{\text{-}b}\left[b \in \mathcal{B}^*(a_b, a_{\text{-}b})\right]$ is continuous by Equation (3) and Equation (28). To show that the expected market price is continuous, consider $\mathbb{E}_{\text{-}b}\left[P^*(a_b'', a_{\text{-}b})\right] - \mathbb{E}_{\text{-}b}\left[P^*(a_b', a_{\text{-}b})\right]$ for two bids $a_b'' > a_b'$ as $a_b'' - a_b'$ approaches zero. The buyer increases the expected market price when raising their bid if (1) they are involved in trade at a_b'', but not at a_b' or (2) a_b' influences the market price. For (1), the market price is at most a_b'' and for (2) the change in market price is at most $a_b'' - a_b'$. This implies that

$$\begin{gathered}\mathbb{E}_{\text{-}b}\left[P^*(a_b'', a_{\text{-}b})\right] - \mathbb{E}_{\text{-}b}\left[P^*(a_b', a_{\text{-}b})\right] \leq \\ a_b''\left(\mathbb{P}_{\text{-}b}\left[b \in \mathcal{B}^*(a_b'', a_{\text{-}b})\right] - \mathbb{P}_{\text{-}b}\left[b \in \mathcal{B}^*(a_b', a_{\text{-}b})\right]\right) + (a_b'' - a_b').\end{gathered} \tag{50}$$

[43]The same proof strategy for continuity is used in Williams (1991) for the expected utility in a buyer's bid DA without fees in the context of Bayesian Nash equilibria.

The continuity of $\mathbb{E}_{\text{-}b}\left[P^*(\cdot, a_{\text{-}b})\right]$ therefore follows from the continuity of $\mathbb{P}_{\text{-}b}\left[b \in \mathcal{B}^*(\cdot, a_{\text{-}b})\right]$. For the expected transaction cost, it holds that

$$\mathbb{E}_{\text{-}b}\left[\Phi_b(a_b, a_{\text{-}b})\right] = \int_{a_b \geq a_{\text{-}b}^{(m)}} \Phi_b(a_b, a_{\text{-}b}) d\mu(a_{\text{-}b}). \tag{51}$$

By assumption, the map $a_b \mapsto \Phi_b(a_b, a_{\text{-}b})$ is continuous. Therefore Equation (51) implies that the map $a_b \mathbb{E}_{\text{-}b}\left[\Phi_b(a_b, a_{\text{-}b})\right]$ is continuous as well. Therefore, the expected utility is indeed continuous in a_b.

Every bid $a_b < \underline{a}_{S,b}$ results in zero utility, as the buyer is almost surely not involved in trade. For every bid $a_b > t_b^\Phi$, it follows from weak domination ex post that the expected utility for a_b is smaller or equal than for $t_b^\Phi \leq t_b$. If $t_b^\Phi \leq \underline{a}_{S,b}$, then t_b^Φ is a best response with expected utility equal to zero. Otherwise, in order to compute a best response, it is sufficient to consider the interval $\left[\underline{a}_{S,b}, t_b^\Phi\right]$. Because the expected utility is a continuous function on this compact set, it follows from the Extreme Value Theorem that the expected utility attains a maximum. Therefore, a best response exists.

Infinite Markets. It was shown in Appendix A.3 that the expected utility is of the form

$$\mathbb{E}_{\text{-}b}\left[u_b(t_b, a_b, a_{\text{-}b})\right] = (t_b - P^*(a_b, a_{\text{-}b}) - \Phi_b(a_b, a_{\text{-}b})) \cdot \mathbb{P}_{\text{-}b}\left[b \in \mathcal{B}^*(a_b, a_{\text{-}b})\right]. \tag{52}$$

In an infinite market, the market price $P^*(a_b, a_{\text{-}b})$ and the fee $\Phi_b(a_b, a_{\text{-}b})$ are deterministic. By assumption, $\Phi_b(a_b, a_{\text{-}b})$ is continuous in the action a_b. By Appendix A.3 it holds that

$$\mathbb{P}_{\text{-}b}\left[b \in \mathcal{B}^*(a_b, a_{\text{-}b})\right] = \begin{cases} 1 & a_b \geq P^*(a) \\ 0 & \text{else} \end{cases}, \tag{53}$$

if there is no tie-breaking. If $t_b^\Phi < P^*(a)$, then buyer b has no undominated action with positive probability of trade. Therefore t_b^Φ is a best response with expected utility equal to zero. If $t_b^\Phi = P^*(a)$, then the only undominated action with positive probability of trade is t_b^Φ. If this results in a strictly positive utility, then it is a best response. If not, then any bid below $P^*(a)$ is a best response. Therefore, consider the case $t_b^\Phi > P^*(a)$. If there is no tie-breaking, then the trading probability is constant and equal to 1 on the compact set $[P^*(a), t_b^\Phi]$. Note that any bid above t_b^Φ is not a best response by weak domination. By similar arguments as before, the expected utility on this interval is equal to $t_b - P^*(a_b, a_{\text{-}b}) - \Phi_b(a_b, a_{\text{-}b})$ and therefore a continuous function. The Extreme Value Theorem implies again that the maximum is attained and a best response exists.

The statement for sellers can be proven analogously. $\square$

B.6 Proof of Theorem 5

Proof. Consider a buyer b and two actions $a_b^1 > a_b^2 > P_b^\infty$ that lead to asymptotically different transaction costs. We will prove that in sufficiently large markets a buyer can improve their expected utility when switching from action a_b^1 to a_b^2. This in turn implies that best responses for two different gross values must lead to asymptotically equal transaction costs. Otherwise, there is a buyer with a certain gross value, who has an incentive to change their action in sufficiently large markets to increase their expected utility.

By assumption, there exists $\epsilon > 0$ such that in sufficiently large markets

$$\mathbb{E}_{\text{-}b}\left[\Phi_b(a_b^1, a_{\text{-}b}) | b \in \mathcal{B}^*\left(a_b^1, a_{\text{-}b}\right)\right] - \mathbb{E}_{\text{-}i}\left[\Phi_b(a_b^2, a_{\text{-}b}) | b \in A^*\left(a_b^2, a_{\text{-}b}\right)\right] \geq \epsilon. \tag{54}$$

We will show that in sufficiently large markets a_b^1 cannot be a best response. By contradiction, assume that it was a best response for some gross value t_b. The expected utility $\mathbb{E}_{\text{-}b}\left[u_b\left(t_b, a_b^1, a_{\text{-}b}\right)\right]$ is greater or equal than 0, otherwise it is trivially not a best response. We will prove that in sufficiently large markets

$$\mathbb{E}_{\text{-}b}\left[u_b\left(t_b, a_b^1, a_{\text{-}b}\right)\right] - \mathbb{E}_{\text{-}b}\left[u_b\left(t_b, a_b^2, a_{\text{-}b}\right)\right] < 0, \tag{55}$$

which proves that a_b^1 is not a best response in such markets, because a_b^2 increases the expected utility.

Using the law of total expectation, the expected difference in transaction costs can be lower bounded by

$$\begin{aligned}
&\mathbb{E}_{\text{-}b}\left[\Phi_b\left(a_b^1, a_{\text{-}b}\right)\right] - \mathbb{E}_{\text{-}b}\left[\Phi_b\left(a_b^2, a_{\text{-}b}\right)\right] \\
&= \mathbb{E}_{\text{-}b}\left[\Phi_b(a_b^1, a_{\text{-}b}) | b \in \mathcal{B}^*\left(a_b^1, a_{\text{-}b}\right)\right] \mathbb{P}_{\text{-}b}\left[b \in \mathcal{B}^*(a_b^1, a_{\text{-}b})\right] - \mathbb{E}_{\text{-}b}\left[\Phi_b(a_b^2, a_{\text{-}b}) | b \in \mathcal{B}^*\left(a_b^2, a_{\text{-}b}\right)\right] \mathbb{P}_{\text{-}b}\left[b \in \mathcal{B}^*(a_b^2, a_{\text{-}b})\right] \\
&\geq \mathbb{P}_{\text{-}b}\left[b \in \mathcal{B}^*(a_b^2, a_{\text{-}b})\right]\left(\mathbb{E}_{\text{-}b}\left[\Phi_b(a_b^1, a_{\text{-}b}) | b \in \mathcal{B}^*\left(a_b^1, a_{\text{-}b}\right)\right] - \mathbb{E}_{\text{-}b}\left[\Phi_b(a_b, a_{\text{-}b}) | b \in \mathcal{B}^*\left(a_b^2, a_{\text{-}b}\right)\right]\right)
\end{aligned} \tag{56}$$

The inequality from the last line follows from the monotonicity of the trading probability, which implies

$$\mathbb{P}_{\text{-}b}\left[b \in \mathcal{B}^*(a_b^1, a_{\text{-}b})\right] \geq \mathbb{P}_{\text{-}b}\left[b \in \mathcal{B}^*(a_b^2, a_{\text{-}b})\right]. \tag{57}$$

It follows from Proposition 3 that for every γ it holds in sufficiently large markets that $\mathbb{P}_{\text{-}b}\left[b \in \mathcal{B}^*(a_b^2, a_{\text{-}b})\right] \geq 1 - \gamma$. Combining this with the assumption of asymptotically different transaction costs yields that in sufficiently large markets

$$\mathbb{E}_{\text{-}b}\left[\Phi_b\left(a_b^1, a_{\text{-}b}\right)\right] - \mathbb{E}_{\text{-}b}\left[\Phi_b\left(a_b^2, a_{\text{-}b}\right)\right] \geq (1-\gamma)\epsilon. \tag{58}$$

Using Equation (16) in Lemma 17 it holds in sufficiently large markets that

$$\mathbb{E}_{\text{-}b}\left[u_b\left(t_b, a_b^1, a_{\text{-}b}\right)\right] - \mathbb{E}_{\text{-}b}\left[u_b\left(t_b, a_b^2, a_{\text{-}b}\right)\right] \leq t_b\gamma - (1-\gamma)\epsilon. \tag{59}$$

If we now choose $\gamma < \frac{\epsilon}{t_b + \epsilon}$, the difference in expected utility is strictly negative, thus contradicting that a_b^1 is a best response. The statement for sellers can be proven analogously. $\square$

B.7 Proof of Theorem 6

Proof. Consider a buyer b with gross value t_b, such that the best response a_b is uniformly bounded away from the critical value. That is, there exists $\delta > 0$, such that in sufficiently large markets either (i) $a_b \leq P_b^\infty - \delta$ or (ii) $a_b \geq P_b^\infty + \delta$. It suffices to prove that for every $\epsilon > 0$ it holds in sufficiently large markets that

$$\mathbb{E}_{\text{-}b}\left[u_b(t_b, t_b^\Phi, a_{\text{-}b})\right] - \mathbb{E}_{\text{-}b}\left[u_b(t_b, a_b, a_{\text{-}b})\right] \geq -\epsilon, \tag{60}$$

which implies that truthfulness is an ϵ-best response. If it holds that $t_b^\Phi \leq a_b$, it holds that

$$\mathbb{E}_{\text{-}b}\left[u_b(t_b, t_b^\Phi, a_{\text{-}b})\right] = \mathbb{E}_{\text{-}b}\left[u_b(t_b, a_b, a_{\text{-}b})\right], \tag{61}$$

because t_b^Φ weakly dominates every larger bid and since a_b is a best response, the expected utilities must be equal. Therefore, assume that $t_b^\Phi > a_b$.

If (i) holds, then Proposition 3 implies that for all $\gamma > 0$ $\mathbb{P}_{\text{-}b}\left[b \in \mathcal{B}^*(a_b, a_{\text{-}b})\right] \leq \gamma$ holds in sufficiently large markets. If $\gamma < \frac{\epsilon}{t_b}$ it follows that

$$\mathbb{E}_{\text{-}b}\left[u_b(t_b, a_b, a_{\text{-}b})\right] \leq t_b\gamma \leq \epsilon. \tag{62}$$

By assumption it also holds that

$$\mathbb{E}_{\text{-}b}\left[u_b(t_b, t_b^\Phi, a_{\text{-}b})\right] \geq 0. \tag{63}$$

Combining Equations (62) and (63) yields Equation (60).

If (ii) holds, then

$$\begin{aligned}\mathbb{E}_{\text{-}b}\left[u_b(t_b, t_b^\Phi, a_{\text{-}b})\right] - \mathbb{E}_{\text{-}b}\left[u_b(t_b, a_b, a_{\text{-}b})\right] \geq \\ t_b^\Phi\left(\mathbb{P}_{\text{-}b}\left[b \in \mathcal{B}^*(t_b^\Phi, a_{\text{-}b})\right] - \mathbb{P}_{\text{-}b}\left[b \in \mathcal{B}^*(a_b, a_{\text{-}b})\right]\right) - \left(\mathbb{E}_{\text{-}b}\left[P^*(t_b^\Phi, a_{\text{-}b})\right] - \mathbb{E}_{\text{-}b}\left[P^*(a_b, a_{\text{-}b})\right]\right) \\ - \left(\mathbb{E}_{\text{-}b}\left[\Phi_b(t_b^\Phi, a_{\text{-}b})\right] - \mathbb{E}_{\text{-}b}\left[\Phi_b(a_b, a_{\text{-}b})\right]\right),\end{aligned} \tag{64}$$

because by assumption $t_b^\Phi \leq t_b$. It follows from Theorem 7 that for a DA without transaction costs for every $\epsilon_1 > 0$ truthfulness is an ϵ_1-best response in sufficiently large markets. Assume that a buyer has gross value equal to t_b^Φ. It therefore holds in sufficiently large markets that for any other bid, i.e., also the best response a_b for gross value t_b

$$\begin{aligned}t_b^\Phi\left(\mathbb{P}_{\text{-}b}\left[b \in \mathcal{B}^*(t_b^\Phi, a_{\text{-}b})\right] - \mathbb{P}_{\text{-}b}\left[b \in \mathcal{B}^*(a_b, a_{\text{-}b})\right]\right) - \left(\mathbb{E}_{\text{-}b}\left[P^*(t_b^\Phi, a_{\text{-}b})\right] - \right. \\ \left. \mathbb{E}_{\text{-}b}\left[P^*(a_b, a_{\text{-}b})\right]\right) \geq -\epsilon_1.\end{aligned} \tag{65}$$

Using the law of total expectation, the expected difference in transaction costs in Equation (65) is equal to

$$\begin{aligned}
&\mathbb{E}_{\text{-}b}\left[\Phi_b(t_b^{\Phi}, a_{\text{-}b})\right] - \mathbb{E}_{\text{-}b}\left[\Phi_b(a_b, a_{\text{-}b})\right] \\
&= \mathbb{E}_{\text{-}b}\left[\Phi_b(t_b^{\Phi}, a_{\text{-}b}) | b \in \mathcal{B}^*\left(t_b^{\Phi}, a_{\text{-}b}\right)\right] \mathbb{P}_{\text{-}b}\left[b \in \mathcal{B}^*(t_b^{\Phi}, a_{\text{-}b})\right] - \\
&\mathbb{E}_{\text{-}b}\left[\Phi_b(a_b, a_{\text{-}b}) | b \in \mathcal{B}^*\left(a_b, a_{\text{-}b}\right)\right] \mathbb{P}_{\text{-}b}\left[b \in \mathcal{B}^*(a_b, a_{\text{-}b})\right].
\end{aligned} \tag{66}$$

Because both actions are by assumption greater or equal than $P_b^{\infty} + \delta$, for every $\gamma > 0$ it holds in sufficiently large markets that $\mathbb{P}_{\text{-}b}\left[b \in \mathcal{B}^*(t_b^{\Phi}, a_{\text{-}b})\right], \mathbb{P}_{\text{-}b}\left[b \in \mathcal{B}^*(a_b, a_{\text{-}b})\right] \geq 1 - \gamma$. It therefore holds that

$$\mathbb{P}_{\text{-}b}\left[b \in \mathcal{B}^*(t_b^{\Phi}, a_{\text{-}b})\right] - \mathbb{P}_{\text{-}b}\left[b \in \mathcal{B}^*(t_b^{\Phi}, a_{\text{-}b})\right] \leq \gamma. \tag{67}$$

This implies that in sufficiently large markets

$$\begin{aligned}
&\mathbb{E}_{\text{-}b}\left[\Phi_b(t_b^{\Phi}, a_{\text{-}b})\right] - \mathbb{E}_{\text{-}b}\left[\Phi_b(a_b, a_{\text{-}b})\right] \leq \\
&\mathbb{P}_{\text{-}b}\left[b \in \mathcal{B}^*(a_b, a_{\text{-}b})\right]\left(\mathbb{E}_{\text{-}b}\left[\Phi_b(t_b^{\Phi}, a_{\text{-}b}) | b \in \mathcal{B}^*\left(t_b^{\Phi}, a_{\text{-}b}\right)\right] - \mathbb{E}_{\text{-}b}\left[\Phi_b(a_b, a_{\text{-}b}) | b \in \mathcal{B}^*\left(a_b, a_{\text{-}b}\right)\right]\right) + \\
&\gamma \mathbb{E}_{\text{-}b}\left[\Phi_b(t_b^{\Phi}, a_{\text{-}b}) | b \in \mathcal{B}^*\left(t_b^{\Phi}, a_{\text{-}b}\right)\right].
\end{aligned} \tag{68}$$

Homogeneity of transaction costs implies that for every $\epsilon_2 > 0$ the first term in Equation (68) is less or equal than ϵ_2 and for every $\epsilon_3 > 0$ the second term can be chosen to be less or equal than ϵ_3 in sufficiently large markets by choosing $\gamma \leq \frac{\epsilon_3}{\mathbb{E}_{\text{-}b}\left[\Phi_b(t_b^{\Phi}, a_{\text{-}b}) | A^*\left(b, t_b^{\Phi}\right)\right]}$. If ϵ_1, ϵ_2 and ϵ_3 are chosen such that their sum is less or equal than ϵ, plugging Equations (65) and (68) into Equation (64) yields that in sufficiently large markets

$$\mathbb{E}_{\text{-}b}\left[u_b(t_b, t_b^{\Phi}, a_{\text{-}b})\right] - \mathbb{E}_{\text{-}b}\left[u_b(t_b, a_b, a_{\text{-}b})\right] \geq -(\epsilon_1 + \epsilon_2 + \epsilon_3) \geq -\epsilon, \tag{69}$$

which completes the proof. The statement for sellers can be proven analogously. □

B.8 Proof of Theorem 7

Proof. Consider a buyer b with private type t_b.

Best responses are close to truthfulness (2). We will show that there exists a constant $\kappa > 0$, such that

$$t_b - (1 + \phi_b)\, a_b \leq \kappa q(n, m), \tag{70}$$

with $q(m, n) = \max\left\{\frac{1}{n}\left(1 + \frac{m}{n}\right), \frac{1}{m}\left(1 + \frac{n}{m}\right)\right\} = O(\max(m, n)^{-1})$, from which the statement follows. It was proven in Appendix A.6, that a best response a_b necessarily satisfies the first order condition in Equation (24), which implies the following bound:

$$t_b - (1 + \phi_b)\, a_b \leq \frac{(1 + \phi_b)\, k \mathbb{P}_{\text{-}b}\left[a_{m-1,n}^{(m)} \leq a_b \leq a_{m-1,n}^{(m+1)}\right]}{(m-1) \mathbb{P}_{\text{-}b}\left[a_{m-2,n}^{(m-1)} \leq a_b \leq a_{m-2,n}^{(m)}\right] f_{B,b}(a_b)}. \tag{71}$$

It can be proven analogous to Rustichini et al. (1994, Appendix) that

$$\frac{\mathbb{P}_{\text{-}b}\left[a_{m-1,n}^{(m)} \leq a_b \leq a_{m-1,n}^{(m+1)}\right]}{\mathbb{P}_{\text{-}b}\left[a_{m-2,n}^{(m-1)} \leq a_b \leq a_{m-2,n}^{(m)}\right]} \leq 2\left[F_{B,b}(a_b) + \frac{n}{m}\frac{(1-F_{B,b}(a_b))\,F_{S,b}(a_b)}{1-F_{S,b}(a_b)}\right]. \tag{72}$$

Defining

$$\tau_b \equiv 2 \max_{x\in[\underline{a}_{S,b},\overline{a}_{B,b}]}\left\{\frac{F_{B,b}(x)}{f_{B,b}(x)}, \frac{(1-F_{B,b}(x))\,F_{S,b}(x)}{f_{B,b}(x)\,(1-F_{S,b}(x))}\right\} \tag{73}$$

yields that

$$t_b - (1+\phi_b)\,a_b \leq \frac{\tau_b k\,(1+\phi_b)}{m-1}\left[1+\frac{n}{m}\right]. \tag{74}$$

To obtain the bounds in the theorem, note that $\frac{n}{n-1}$ and $\frac{m}{m+1}$ are both less than 2. Setting $\kappa \equiv 2\tau_b k$ proves the statement for buyers. For a seller s with private type t_s an analogous argument yields

$$(1-\phi_s)\,a_s - t_s \leq \frac{\tau_s(1\text{-}k)(1-\phi_s))}{n-1}\left[1+\frac{m}{n}\right] \tag{75}$$

for τ_s with

$$\tau_s \equiv 2\max\left\{\frac{1-F_{S,s}(x)}{f_{S,s}(x)}, \frac{(1-F_{B,s}(x))\,F_{S,s}(x)}{f_{S,s}(x)F_{B,s}(x)}\right\}. \tag{76}$$

Truthfulness is an ϵ-best response. We start by estimating the difference in utility when a buyer switches from a bid a_b^1 to a smaller bid a_b^2, i.e., $\mathbb{E}_{\text{-}b}\left[u_b(t_b, a_b^1, a_{\text{-}b})\right] - \mathbb{E}_{\text{-}b}\left[u_b(t_b, a_b^2, a_{\text{-}b})\right]$. The expected utility is not dependent on the entirety of $a_{\text{-}b}$, but only on $a_{\text{-}b}^{(m)}$ and $a_{\text{-}b}^{(m+1)}$. We consider all six possible cases for the realizations of $a_{\text{-}b}^{(m)}$ and $a_{\text{-}b}^{(m+1)}$ with respect to $a_b^1 > a_b^2$.

		$u_b(t_b, a_b^1, a_{\text{-}b})$	$u_b(t_b, a_b^2, a_{\text{-}b})$
I	$a_b^1 \geq a_b^2 \geq a_{\text{-}b}^{(m+1)} \geq a_{\text{-}b}^{(m)}$	$t_b\text{-}(1+\phi_b)\left(ka_{\text{-}b}^{(m+1)}+(1\text{-}k)a_{\text{-}b}^{(m)}\right)$	$t_b\text{-}(1+\phi_b)\left(ka_{\text{-}b}^{(m+1)}+(1\text{-}k)a_{\text{-}b}^{(m)}\right)$
II	$a_b^1 \geq a_{\text{-}b}^{(m+1)} \geq a_b^2 \geq a_{\text{-}b}^{(m)}$	$t_b\text{-}(1+\phi_b)\left(ka_{\text{-}b}^{(m+1)}+(1\text{-}k)a_{\text{-}b}^{(m)}\right)$	$t_b\text{-}(1+\phi_b)\left(ka_b^2+(1\text{-}k)a_{\text{-}b}^{(m)}\right)$
III	$a_b^1 \geq a_{\text{-}b}^{(m+1)} \geq a_{\text{-}b}^{(m)} \geq a_b^2$	$t_b\text{-}(1+\phi_b)\left(ka_{\text{-}b}^{(m+1)}+(1\text{-}k)a_{\text{-}b}^{(m)}\right)$	0
IV	$a_{\text{-}b}^{(m+1)} \geq a_b^1 \geq a_b^2 \geq a_{\text{-}b}^{(m)}$	$t_b\text{-}(1+\phi_b)\left(ka_b^1+(1\text{-}k)a_{\text{-}b}^{(m)}\right)$	$t_b\text{-}(1+\phi_b)\left(ka_b^2+(1\text{-}k)a_{\text{-}b}^{(m)}\right)$
V	$a_{\text{-}b}^{(m+1)} \geq a_b^1 \geq a_{\text{-}b}^{(m)} \geq a_b^2$	$t_b\text{-}(1+\phi_b)\left(ka_b^1+(1\text{-}k)a_{\text{-}b}^{(m)}\right)$	0
VI	$a_{\text{-}b}^{(m+1)} \geq a_{\text{-}b}^{(m)} \geq a_b^1 \geq a_b^2$	0	0

Analogously, we consider the difference in utilities:

		$u_b(t_b, a_b^1, a_{\text{-}b}) - u_b(t_b, a_b^2, a_{\text{-}b})$
I	$a_b^1 \geq a_b^2 \geq a_{\text{-}b}^{(m+1)} \geq a_{\text{-}b}^{(m)}$	0
II	$a_b^1 \geq a_{\text{-}b}^{(m+1)} \geq a_b^2 \geq a_{\text{-}b}^{(m)}$	$\text{-}k(1+\phi_b)\left(a_{\text{-}b}^{(m+1)}\text{-}a_b^2\right)$
III	$a_b^1 \geq a_{\text{-}b}^{(m+1)} \geq a_{\text{-}b}^{(m)} \geq a_b^2$	$t_b\text{-}(1+\phi_b)\left(ka_{\text{-}b}^{(m+1)}+(1\text{-}k)a_{\text{-}b}^{(m)}\right)$
IV	$a_{\text{-}b}^{(m+1)} \geq a_b^1 \geq a_b^2 \geq a_{\text{-}b}^{(m)}$	$\text{-}k(1+\phi_b)\left(a_b^1\text{-}a_b^2\right)$
V	$a_{\text{-}b}^{(m+1)} \geq a_b^1 \geq a_{\text{-}b}^{(m)} \geq a_b^2$	$t_b\text{-}(1+\phi_b)\left(ka_b^1+(1\text{-}k)a_{\text{-}b}^{(m)}\right)$
VI	$a_{\text{-}b}^{(m+1)} \geq a_{\text{-}b}^{(m)} \geq a_b^1 \geq a_b^2$	0

We want to lower bound $\mathbb{E}_{\text{-}b}\left[u_b(t_b, a_b^1, a_{\text{-}b})\right] - \mathbb{E}_{\text{-}b}\left[u_b(t_b, a_b^2, a_{\text{-}b})\right]$. It is therefore sufficient to lower bound the expression in **II** and **IV**, since they are negative and neglect the positive difference in the other cases. In order to prove truthfulness is close to optimal, consider $a_b^1 = t_b^\Phi$ and $a_b^2 = a_b$ a best response. We show that for any $\epsilon > 0$ it holds in sufficiently large finite markets the difference in expected utility is bounded from below by $-\epsilon$. Because best responses are ϵ-close to truthfulness in sufficiently large finite markets, it holds in such markets that for all $\delta > 0$ $t_b^\Phi - a_b \le \delta$. Therefore the difference in **II** and **IV** is lower bounded by $-k(1+\phi_b)\delta$. It follows that

$$\begin{aligned} &\mathbb{E}_{\text{-}b}\left[u_b(t_b, t_b^\Phi, a_{\text{-}b})\right] - \mathbb{E}_{\text{-}b}\left[u_b(t_b, a_b, a_{\text{-}b})\right] \le \\ &-k(1+\phi_b)\delta\left(\mathbb{P}\left[\mathbf{II}\right] + \mathbb{P}\left[\mathbf{IV}\right]\right) \le -2k(1+\phi_b)\delta. \end{aligned} \tag{77}$$

If for a given $\epsilon > 0$, $\delta > 0$ is chosen such that $\delta \le \frac{\epsilon}{2k(1+\phi_b)}$, it holds in sufficiently large finite markets that t_b^Φ is ϵ-close to a best response a_b. In infinite markets, the expected utility is equal to

$$\mathbb{E}[u_b(t_b, a_b, a_{\text{-}b})] = \begin{cases} t_b - (1+\phi_b)P^* & \text{if } a_b \ge P^*, \\ 0 & \text{if } a_b < P^*. \end{cases} \tag{78}$$

If $t_b^\Phi \ge P^*$, then the expected utility is equal to $t_b - (1+\phi_b)P^* > 0$, and therefore a best response. If $t_b^\Phi \le P^*$, then the expected utility is equal to 0. Because every action $a_b > t_b^\Phi$ is dominated, t_b^Φ is again a best response. Therefore truthfully reporting t_b^Φ is a best response. The statement for sellers can be proven analogously. □

B.9 Proof of Theorem 8

Proof. Consider a buyer b with a gross value t_b, such that $t_b^\Phi > P_b^\infty$. First, we show that in sufficiently large markets an action $a_b^1 < P_b^\infty$ is not a best response. We show that there exists an action $a_b^2 > P_b^\infty$ such that in sufficiently large markets

$$\mathbb{E}_{\text{-}b}\left[u_b(t_b, a_b^2, a_{\text{-}b})\right] - \mathbb{E}_{\text{-}b}\left[u_b(t_b, a_b^1, a_{\text{-}b})\right] > 0, \tag{79}$$

which implies that a_b^1 is not a best response. Because the net value is by assumption continuous and strictly increasing in the gross value, there exists a gross value $t_b' < t_b$, such that $t_b^\Phi > t_b^{\Phi\prime} > P_b^\infty$. Denote the difference between t_b^Φ and $t_b^{\Phi\prime}$ by $\delta > 0$. It holds that

$$\mathbb{E}_{\text{-}b}\left[u_b(t_b, t_b^{\Phi\prime}, a_{\text{-}b})\right] = \mathbb{E}_{\text{-}b}\left[u_b(t_b', t_b^{\Phi\prime}, a_{\text{-}b})\right] + \delta \ge \delta, \tag{80}$$

because the net value is assumed to be ex-post individually rational. Note that this inequality holds for every market size. To prove Equation (119), it therefore suffices to show that for $a_b^1 < P_b^\infty$ it holds in sufficiently large markets that

$$\mathbb{E}_{\text{-}b}\left[u_b(t_b, a_b, a_{\text{-}b})\right] < \delta. \tag{81}$$

We can upper bound the expected utility by neglecting the expected market price and the expected fee and get that

$$\mathbb{E}_{\text{-}b}\left[u_b(t_b, a_b^1, a_{\text{-}b})\right] \leq t_b \mathbb{P}_{\text{-}b}\left[b \in \mathcal{B}^*(a_b^1, a_{\text{-}b})\right]. \tag{82}$$

Proposition 3 implies that for any $\gamma > 0$ it holds in sufficiently large markets that $\mathbb{P}_{\text{-}b}\left[b \in \mathcal{B}^*(a_b, a_{\text{-}b})\right] \leq \gamma$. If we choose $\gamma < \frac{\delta}{t_b}$, the statement follows. We therefore consider an action a_b that is ϵ-distant to the critical value, that is, there exists $\epsilon > 0$ such that $a_b - P_b^\infty \geq \epsilon$. We will prove that in sufficiently large markets it holds that

$$\mathbb{E}_{\text{-}b}\left[u_b(t_b, a_b, a_{\text{-}b})\right] - \mathbb{E}_{\text{-}b}\left[u_b(t_b, P_b^\infty + \epsilon/2, a_{\text{-}b})\right] < 0, \tag{83}$$

which proves that a_b is not a best response in sufficiently large markets. Therefore, best responses must be ϵ-close, but above the critical value in sufficiently large markets. Using the law of total expectation, the expected difference in transaction cost can be lower bounded by

$$\begin{gathered}
\mathbb{E}_{\text{-}b}\left[\Phi_b\left(a_b, a_{\text{-}b}\right)\right] - \mathbb{E}_{\text{-}b}\left[\Phi_b\left(P_b^\infty + \epsilon/2, a_{\text{-}b}\right)\right] = \\
\mathbb{E}_{\text{-}b}\left[\Phi_b(a_b, a_{\text{-}b}) | b \in \mathcal{B}^*\left(a_b, a_{\text{-}b}\right)\right] \mathbb{P}_{\text{-}b}\left[b \in \mathcal{B}^*(a_b, a_{\text{-}b})\right] - \\
\mathbb{E}_{\text{-}b}\left[\Phi_b(P_b^\infty + \epsilon/2, a_{\text{-}b}) | b \in \mathcal{B}^*\left(P_b^\infty + \epsilon/2, a_{\text{-}b}\right)\right] \mathbb{P}_{\text{-}b}\left[b \in \mathcal{B}^*(P_b^\infty + \epsilon/2, a_{\text{-}b})\right] \geq \\
\mathbb{P}_{\text{-}b}\left[b \in \mathcal{B}^*(P_b^\infty + \epsilon/2, a_{\text{-}b})\right] \left(\mathbb{E}_{\text{-}b}\left[\Phi_b(a_b, a_{\text{-}b}) | b \in \mathcal{B}^*\left(a_b, a_{\text{-}b}\right)\right] - \right. \\
\left. \mathbb{E}_{\text{-}b}\left[\Phi_b(P_b^\infty + \epsilon/2, a_{\text{-}b}) | b \in \mathcal{B}^*\left(P_b^\infty + \epsilon/2, a_{\text{-}b}\right)\right]\right)
\end{gathered} \tag{84}$$

The inequality on the last line holds because the trading probability is monotone, which implies $\mathbb{P}_{\text{-}b}\left[b \in \mathcal{B}^*(a_b, a_{\text{-}b})\right] \geq \mathbb{P}_{\text{-}b}\left[b \in \mathcal{B}^*(P_b^\infty + \epsilon/2, a_{\text{-}b})\right]$. It follows from Proposition 3 that for every γ it holds in sufficiently large markets that $\mathbb{P}_{\text{-}b}\left[b \in \mathcal{B}^*(P_b^\infty + \epsilon/2, a_{\text{-}b})\right] \geq 1 - \gamma$. Combining this with the assumption of heterogeneity of transaction costs yields that there exists $\delta > 0$ such that it holds in sufficiently large markets that

$$\mathbb{E}_{\text{-}b}\left[\Phi_b\left(a_b, a_{\text{-}b}\right)\right] - \mathbb{E}_{\text{-}b}\left[\Phi_b\left(P_b^\infty + \epsilon/2, a_{\text{-}b}\right)\right] \geq (1 - \gamma)\delta. \tag{85}$$

Using Equation (16) from Lemma 17, it therefore holds in sufficiently large markets that

$$\mathbb{E}_{\text{-}b}\left[u_b\left(t_b, a_b, a_{\text{-}b}\right)\right] - \mathbb{E}_{\text{-}b}\left[u_b\left(t_b, P_b^\infty + \epsilon/2, a_{\text{-}b}\right)\right] \leq t_b\gamma - (1 - \gamma)\delta. \tag{86}$$

If we now choose $\gamma < \delta/t_b + \delta$, the difference is strictly smaller than 0, which proves that a_b is not a best response in sufficiently large markets.

The statement for sellers can be proven analogously. $\square$

B.10 Proof of Theorem 9

Proof. To prove that best responses are in an ϵ-neighbourhood of the critical value in sufficiently large markets , consider a buyer b with gross value t_b, such that $t_b^\Phi > P_b^\infty$. It follows analogous to Appendix B.9 that in sufficiently large markets an action $a_b^1 < P_b^\infty$ is not a best response. We therefore consider an action $a_b > P_b^\infty$. That is, there exists $\epsilon > 0$ such that $a_b - P_b^\infty \geq \epsilon$. We will prove that in sufficiently large markets

$$\mathbb{E}_{\text{-}b}\left[u_b(t_b, a_b, a_{\text{-}b})\right] - \mathbb{E}_{\text{-}b}\left[u_b(t_b, P_b^\infty + \epsilon/2, a_{\text{-}b})\right] < 0, \tag{87}$$

which proves that a_b is not a best response in such markets. Therefore, best responses must be ϵ-close, but above the critical value in sufficiently large markets. For two bids $a_b^1 > a_b^2$ Lemma 17 implies in the presence of a spread fee that

$$\begin{aligned} &\mathbb{E}_{\text{-}b}\left[u_b\left(t_b, a_b^1, a_{\text{-}b}\right)\right] - \mathbb{E}_{\text{-}b}\left[u_b\left(t_b, a_b^2, a_{\text{-}b}\right)\right] \\ &\leq \left(t_b - \phi_b a_b^1\right)\mathbb{P}_{\text{-}b}\left[b \in \mathcal{B}^*(a_b^1, a_{\text{-}b})\right] - \left(t_b - \phi_b a_b^2\right)\mathbb{P}_{\text{-}b}\left[b \in \mathcal{B}^*(a_b^2, a_{\text{-}b})\right]. \end{aligned} \tag{88}$$

Now set $a_b^1 = a_b$ and $a_b^2 = P_b^\infty + \epsilon/2$. It follows from Proposition 3 that for any $\gamma > 0$ it holds in sufficiently large markets that $\mathbb{P}_{\text{-}b}\left[b \in \mathcal{B}^*(a_b, a_{\text{-}b})\right], \mathbb{P}_{\text{-}b}\left[b \in \mathcal{B}^*(P_b^\infty + \epsilon/2, a_{\text{-}b})\right] \geq 1 - \gamma$ and therefore also

$$\mathbb{P}_{\text{-}b}\left[b \in \mathcal{B}^*(a_b, a_{\text{-}b})\right] \leq \mathbb{P}_{\text{-}b}\left[b \in \mathcal{B}^*(P_b^\infty + \epsilon/2, a_{\text{-}b})\right] + \gamma. \tag{89}$$

Combining Equations (88) and (89) implies that it holds in sufficiently large markets that

$$\begin{aligned} &\mathbb{E}_{\text{-}b}\left[u_b\left(t_b, a_b, a_{\text{-}b}\right)\right] - \mathbb{E}_{\text{-}b}\left[u_b\left(t_b, P_b^\infty + \epsilon/2, a_{\text{-}b}\right)\right] \\ &\leq -\phi_b(1-\gamma)(a_b - (P_b^\infty + \epsilon/2)) + \gamma\left(t_b - \phi_b a_b\right). \end{aligned} \tag{90}$$

By assumption, it holds that $a_b - (P_b^\infty + \epsilon/2) \geq \epsilon/2$, which yields

$$\begin{aligned} &\mathbb{E}_{\text{-}b}\left[u_b\left(t_b, a_b, a_{\text{-}b}\right)\right] - \mathbb{E}_{\text{-}b}\left[u_b\left(t_b, P_b^\infty + \epsilon/2, a_{\text{-}b}\right)\right] \\ &\leq -\phi_b(1-\gamma)\frac{\epsilon}{2} + \gamma\left(t_b - \phi_b a_b\right) \leq -\phi_b(1-\gamma)\frac{\epsilon}{2} + \gamma t_b. \end{aligned} \tag{91}$$

If γ is chosen such that $\gamma < \frac{\phi_b \epsilon}{2t_b + \phi_b \epsilon}$ holds, then in sufficiently large markets

$$\mathbb{E}_{\text{-}b}\left[u_b\left(t_b, a_b, a_{\text{-}b}\right)\right] - \mathbb{E}_{\text{-}b}\left[u_b\left(t_b, P_b^\infty + \epsilon/2, a_{\text{-}b}\right)\right] < 0, \tag{92}$$

which implies that a_b is not a best response in such markets.

Next, we prove that for sufficiently small $\epsilon > 0$, there exist beliefs, such that the critical value is not an ϵ-best response in sufficiently large finite markets. Consider a buyer b with gross value $t_b^\Phi > P_b^\infty$ in a sequence of market environment with $m(l) = l$, $n(l) = l$, $T = [0, 1]$ and uniformly distributed beliefs over actions for both buyers and sellers. In this case, the critical value P_b^∞ is equal to $\frac{1}{2}$. By assumption, there exists $\epsilon > 0$, such that $t_b = P_b^\infty + \epsilon$ for $\epsilon > 0$. We will show that in sufficiently large markets

$$\mathbb{E}_{\text{-}b}\left[u_b(t_b, P_b^\infty + \epsilon/4, a_{\text{-}b})\right] - \mathbb{E}_{\text{-}b}\left[u_b(t_b, P_b^\infty, a_{\text{-}b})\right] > 0, \tag{93}$$

which proves that P_b^∞ is not a best response. In order to estimate the difference in expected utility for two bids $a_b^1 > a_b^2$, we use a table similar to the one in Appendix B.8:

		$u_b(t_b, a_b^1, a_{\text{-}b})$	$u_b(t_b, a_b^2, a_{\text{-}b})$
I	$a_b^1 \geq a_b^2 \geq a_{\text{-}b}^{(m+1)} \geq a_{\text{-}b}^{(m)}$	$t_b\text{-}\phi_b a_b^1\text{-}(1\text{-}\phi_b)\left(ka_{\text{-}b}^{(m+1)}+(1\text{-}k)a_{\text{-}b}^{(m)}\right)$	$t_b\text{-}\phi_b a_b^2\text{-}(1\text{-}\phi_b)\left(ka_{\text{-}b}^{(m+1)}+(1\text{-}k)a_{\text{-}b}^{(m)}\right)$
II	$a_b^1 \geq a_{\text{-}b}^{(m+1)} \geq a_b^2 \geq a_{\text{-}b}^{(m)}$	$t_b\text{-}\phi_b a_b^1\text{-}(1\text{-}\phi_b)\left(ka_{\text{-}b}^{(m+1)}+(1\text{-}k)a_{\text{-}b}^{(m)}\right)$	$t_b\text{-}\phi_b a_b^2\text{-}(1\text{-}\phi_b)\left(ka_b^2+(1\text{-}k)a_{\text{-}b}^{(m)}\right)$
III	$a_b^1 \geq a_{\text{-}b}^{(m+1)} \geq a_{\text{-}b}^{(m)} \geq a_b^2$	$t_b\text{-}\phi_b a_b^1\text{-}(1\text{-}\phi_b)\left(ka_{\text{-}b}^{(m+1)}+(1\text{-}k)a_{\text{-}b}^{(m)}\right)$	0
IV	$a_{\text{-}b}^{(m+1)} \geq a_b^1 \geq a_b^2 \geq a_{\text{-}b}^{(m)}$	$t_b\text{-}\phi_b a_b^1\text{-}(1\text{-}\phi_b)\left(ka_b^1+(1\text{-}k)a_{\text{-}b}^{(m)}\right)$	$t_b\text{-}\phi_b a_b^2\text{-}(1\text{-}\phi_b)\left(ka_b^2+(1\text{-}k)a_{\text{-}b}^{(m)}\right)$
V	$a_{\text{-}b}^{(m+1)} \geq a_b^1 \geq a_{\text{-}b}^{(m)} \geq a_b^2$	$t_b\text{-}\phi_b a_b^1\text{-}(1\text{-}\phi_b)\left(ka_b^1+(1\text{-}k)a_{\text{-}b}^{(m)}\right)$	0
VI	$a_{\text{-}b}^{(m+1)} \geq a_{\text{-}b}^{(m)} \geq a_b^1 \geq a_b^2$	0	0

Analogously, we consider the difference in utilities:

		$u_b(t_b, a_b^1, a_{\text{-}b}) - u_b(t_b, a_b^2, a_{\text{-}b})$
I	$a_b^1 \geq a_b^2 \geq a_{\text{-}b}^{(m+1)} \geq a_{\text{-}b}^{(m)}$	$-\phi_b\left(a_b^1 - a_b^2\right)$
II	$a_b^1 \geq a_{\text{-}b}^{(m+1)} \geq a_b^2 \geq a_{\text{-}b}^{(m)}$	$-\phi_b\left(a_b^1 - a_b^2\right) - k(1-\phi_b)\left(a_{\text{-}b}^{(m+1)} - a_b^2\right)$
III	$a_b^1 \geq a_{\text{-}b}^{(m+1)} \geq a_{\text{-}b}^{(m)} \geq a_b^2$	$t_b - \phi_b a_b^1 - (1-\phi_b)\left(ka_{\text{-}b}^{(m+1)} + (1\text{-}k)a_{\text{-}b}^{(m)}\right)$
IV	$a_{\text{-}b}^{(m+1)} \geq a_b^1 \geq a_b^2 \geq a_{\text{-}b}^{(m)}$	$-\phi_b\left(a_b^1 - a_b^2\right) - k\left((1-\phi_b)\left(a_b^1 - a_b^2\right)\right)$
V	$a_{\text{-}b}^{(m+1)} \geq a_b^1 \geq a_{\text{-}b}^{(m)} \geq a_b^2$	$t_b - \phi_b a_b^1 - (1-\phi_b\left(ka_b^1 + (1\text{-}k)a_{\text{-}b}^{(m)}\right)$
VI	$a_{\text{-}b}^{(m+1)} \geq a_{\text{-}b}^{(m)} \geq a_b^1 \geq a_b^2$	0

In order to obtain a lower bound on the expected difference in utility, we bound all five non-zero terms from below. We set $a_b^1 = P_b^\infty + \epsilon/4$ and $a_b^2 = P_b^\infty$, which implies that there difference is equal to $\epsilon/4$. The expressions in **I**, **II** and **IV** are therefore greater or equal than $-\epsilon/4$. For **III** and **V**, the lower bound $t_b - (P_b^\infty + \epsilon/4) = \frac{3\epsilon}{4}$ holds, because $t_b = P_b^\infty + \epsilon$. Combining these bounds with the probabilities of each event, the following inequality holds:

$$\begin{aligned}
&\mathbb{E}_{\text{-}b}\left[u_b(t_b, P_b^\infty + \epsilon/4, a_{\text{-}b})\right] - \mathbb{E}_{\text{-}b}\left[u_b(t_b, P_b^\infty, a_{\text{-}b})\right] \geq \\
&-\frac{\epsilon}{4}\mathbb{P}_{\text{-}b}\left[P_b^\infty \geq a_{\text{-}b}^{(m)}\right] + \frac{3\epsilon}{4}\mathbb{P}_{\text{-}b}\left[P_b^\infty + \epsilon/4 \geq a_{\text{-}b}^{(m)} \geq P_b^\infty\right] = \\
&-\frac{\epsilon}{2}\mathbb{P}_{\text{-}b}\left[P_b^\infty \geq a_{\text{-}b}^{(m)}\right] + \frac{3\epsilon}{4}\left(\mathbb{P}_{\text{-}b}\left[a_{\text{-}b}^{(m)} \leq P_b^\infty + \epsilon/4\right] - \mathbb{P}\left[a_{\text{-}b}^{(m)} \leq P_b^\infty\right]\right)
\end{aligned} \tag{94}$$

By definition $a_{\text{-}b}^{(m)}$ is the m'th smallest submission in a set of $m-1$ bids and n asks. Since buyer b assumes that those are uniformly distributed and that there are $m(l) = l$ and $n(l) = l$ many buyers and sellers, it follows from order statistics that $a_{\text{-}b}^{(m)} \sim Beta(l, l)$. This distribution is symmetric on $[0, 1]$ for every l and therefore at the critical value $P_b^\infty = \frac{1}{2}$, it holds that $\mathbb{P}_{\text{-}b}\left[a_{\text{-}b}^{(m)} \leq P_b^\infty\right] = \frac{1}{2}$.

Furthermore, it follows from Proposition 3 that for any $\gamma > 0$ it holds in sufficiently large markets that $\mathbb{P}\left[a^{(m)}_{\text{-}b} \leq P^{\infty}_{b} + {}^{\epsilon}\!/_{4}\right] \geq 1 - \gamma$. It follows that

$$\begin{aligned}\mathbb{E}_{\text{-}b}\left[u_b(t_b, P^{\infty}_b + {}^{\epsilon}\!/_{4}, a_{\text{-}b})\right] - \mathbb{E}_{\text{-}b}\left[u_b(t_b, P^{\infty}_b, a_{\text{-}b})\right] \geq \\ -\frac{\epsilon}{8} + \frac{3\epsilon}{4}\left(\frac{1}{2} - \gamma\right),\end{aligned} \tag{95}$$

which is positive if γ is chosen to be smaller than $\frac{1}{3}$. The statement for sellers can be proven analogously. $\square$

B.11 Proof of Theorem 10

Proof. Recall that $E_{\Phi} = \frac{\mathbb{E}[G]}{\mathbb{E}[G_{\Phi}]}$. Because the allocation balances trade, that is $\mu_B(\mathcal{B}^*) = \mu_S(\mathcal{S}^*)$, it holds that

$$\mathbb{E}[G] = \mathbb{E}\left[\int\limits_{\mathcal{B}^*} (t_b - P^*)\, d\mu_B(b) + \int\limits_{\mathcal{S}^*} (P^* - t_s)\, d\mu_S(s)\right] = \mathbb{E}\left[\int\limits_{\mathcal{B}^*} t_b d\mu_B(b) - \int\limits_{\mathcal{S}^*} t_s d\mu_S(s)\right]. \tag{96}$$

B.11.1 Finite Markets

In finite markets, the integral representation of the gains of trade simplifies to the following sum:

$$\mathbb{E}[G] = \mathbb{E}\left[\sum_{\mathcal{B}^*} t_b - \sum_{\mathcal{S}^*} t_s\right] \tag{97}$$

To show that $E_{\Phi} \geq 1 - \zeta$, it suffices to prove that $\frac{\mathbb{E}[G_{\Phi} - G]}{\mathbb{E}[G_{\Phi}]} \leq \zeta$.[44] We start by lower bounding the denominator. We consider $\min(n, m)$ buyer-seller pairs. The expected gains of trade $\max(t_b - t_s, 0)$ of such a pair is equal to some $\alpha > 0$. It therefore holds that $\mathbb{E}[G_{\Phi}] \geq \alpha \cdot \min(m, n)$. However the value of trade is bounded above by $\beta = \overline{a}_B - \underline{a}_S$, proving that $\mathbb{E}[G_{\Phi}] \leq \beta \cdot \min(m, n)$ and therefore $\mathbb{E}[G_{\Phi}] = \Theta(\min(m, n))$.

In a next step, we bound the numerator $\mathbb{E}[G_{\Phi} - G]$. Consider that traders use the symmetric strategy pair (a_B, a_S) with strictly increasing and continuous strategies, that are ϵ-close to truthful. Let F^{Φ}_B and F^{Φ}_S be the distribution functions of net values on $A_B = [\underline{a}_B, \overline{a}_B] \subset \mathbb{R}^{\geq 0}$ and $A_S = [\underline{a}_S, \overline{a}_S] \subset \mathbb{R}^{\geq 0}$. Denote by t^{Φ} a sample of $n + m$ net values. Denote by μ the distribution of the market price $\Pi(t^{\Phi})$ and by $L(t^{\Phi})$ the total value of trades that inefficiently fail to occur given t^{Φ} and the strategies in use. It holds that

$$\mathbb{E}[G_{\Phi} - G] = \mathbb{E}\left[L(t^{\Phi})\right] = \int_{-\infty}^{\infty} \mathbb{E}\left[L(t^{\Phi}) | \Pi(t^{\Phi,(m)})\right] d\mu(\Pi(t^{\Phi,(m)})). \tag{98}$$

We will bound the value of this integral over (i) $(-\infty, \underline{a}_S + \delta)$, (ii) $[\underline{a}_S + \delta, \overline{a}_B - \delta]$, and (iii) $[\overline{a}_B - \delta, \infty]$

[44] The following proof is based on methods from Rustichini et al. (1994).

for some $\delta > 0$. δ is chosen small enough, such that $\underline{a}_S + \delta < P^\infty$ and $\overline{a}_B - \delta > P^\infty$, where P^∞ denotes the critical value of F_B^Φ and F_S^Φ. The same proof-technique as in Proposition 3 shows that

$$\mathbb{P}[\Pi(t^{\Phi,(m)}) \leq \underline{a}_S + \delta], \mathbb{P}[\Pi(t^{\Phi,(m)}) \geq \overline{a}_B - \delta] \in O(\min(m,n)^{-\frac{1}{2}}). \tag{99}$$

Because it holds that $\mathbb{E}[L(t^\Phi)|\Pi(t^{\Phi,(m)})] \leq \beta \min(n,m)$, where $\beta = \overline{a}_B - \underline{a}_S$ we get that the integral in Equation (98) over (i) and (iii) is $O(\min(m,n)^{\frac{1}{2}})$. Next we bound the integral over (ii). Consider any symmetric strategy profile $a = (a_B, a_S) \in \Upsilon_{\Phi,F}^{\epsilon,opt}$ for some $\epsilon > 0$. Given a realization of net values t^Φ, consider the set of values, if traders use the symmetric strategy-profile a, denote the set of actions by t^a. If a is ϵ-close to truthfulness, it holds that

$$t^{\Phi,(m)} - \epsilon \leq t^{a,(m)} \leq t^{\Phi,(m)} + \epsilon. \tag{100}$$

The value of a missed trade is at most some constant $\zeta > 0$. A buyer with gross value t_b and a seller with gross value t_s fail to trade when using strategies a_B and a_S, but would trade when being truthful, if $t_b^\Phi \geq t_s^\Phi$, $a_B(t_b) \leq P^*(t^a) \leq t^{a,(m)}$ and $a_S(t_s) \geq P^*(t^a) \geq t^a$. We bound the expected number of missed trades conditional on $P^*(t^a)$. It is bounded by the expected number of net values in the 2ϵ-neighbourhood of $P^*(t^a)$. This is bounded by fixing $P^*(t^a)$ and summing over the number i of buyers with net values above or equal to $P^*(t^a)$. These i values are independently distributed according to $\frac{F_B^\Phi(\cdot) - F_B^\Phi(P^*(t^a))}{1 - F_B^\Phi(P^*(t^a))}$ with density $\frac{f_B(\cdot)}{1 - F_B^\Phi(P^*(t^a))}$. Similarly, the remaining $n - i$ net values of sellers above or equal to $P^*(t^a)$ are independently distributed according to $\frac{F_S^\Phi(\cdot) - F_S^\Phi(P^*(t^a))}{1 - F_S^\Phi(P^*(t^a))}$ with density $\frac{f_S(\cdot)}{1 - F_S^\Phi(P^*(t^a))}$. Because $P^*(t^a) \leq \overline{a}_B - \delta$ (case (ii)) and f_B and f_S are continuous, the densities are bounded from above by some number $\alpha(F_B^\Phi, F_S^\Phi, \delta)$ that is independent of m. Conditional upon $P^*(t^a)$, the expected number of net values above and within 2ϵ of $P^*(t^a)$ is thus no more than $n \cdot 2\epsilon \cdot \alpha(F_B^\Phi, F_S^\Phi, \delta)$. A similar argument shows that for some $\beta(F_B^\Phi, F_S^\Phi, \delta)$ the expected number of net values below and within 2ϵ of t^a is no more than $m \cdot 2\epsilon \cdot \beta(F_B^\Phi, F_S^\Phi, \delta)$. Thus the expected number of missed trades conditional on t^a is bounded by $\min(n,m) \cdot \epsilon \cdot \gamma(F_B^\Phi, F_S^\Phi, \delta)$. Therefore $\mathbb{E}[L(t^\Phi)|t^{\Phi,(m)}] \leq \min(m,n) \cdot \zeta \cdot \epsilon \cdot \gamma(F_B^\Phi, F_S^\Phi, \delta)$. Finally, we have that

$$\begin{aligned} &\frac{\mathbb{E}[G_\Phi - G]}{\mathbb{E}[G_\Phi]} = \\ &\frac{\int_{(i)+(iii)} \mathbb{E}\left[L(t^\Phi)|\Pi(t^{\Phi,(m)})\right] d\mu(\Pi(t^{\Phi,(m)}))}{\mathbb{E}[G_\Phi]} + \frac{\int_{(ii)} \mathbb{E}\left[L(t^\Phi)|\Pi(t^{\Phi,(m)})\right] d\mu(\Pi(t^{\Phi,(m)}))}{\mathbb{E}[G_\Phi]}. \end{aligned} \tag{101}$$

Recall that the denominator is of order $\Theta(\min(m,n))$. The numerator of the first term is of order $O(\min(m,n)^{\frac{1}{2}})$. Therefore the whole term is of order $O(\min(m,n)^{-\frac{1}{2}})$, so it goes to zero in sufficiently large market. The numerator of the second term is of order $O(\min(m,n) \cdot \epsilon)$. Therefore the second term is of order $O(\epsilon)$. Therefore, for any $\gamma > 0$ and for any sequence of ϵ that goes to zero, $\frac{\mathbb{E}[G_\Phi - G]}{\mathbb{E}[G_\Phi]} \leq \gamma$ in sufficiently large finite markets.

B.11.2 Infinite Markets

We consider again a symmetric strategy profile (a_B, a_S) that is strictly increasing, continuous and ϵ-close to truthfulness.

Observation 20. *Demand and supply are continuous. Further, demand is strictly decreasing on A_B and supply is strictly increasing on A_S.*

Proof of Observation 20. It holds that

$$D(P) = \begin{cases} 0 & \text{if } P < \underline{a}_B \\ \mu_B^t([a_B^{-1}(P), \bar{t}]) & \text{if } P \in A_B \\ \mu_B^t(\Theta) & \text{if } P > \bar{a}_B \end{cases} \quad \text{and} \quad S(P) = \begin{cases} \mu_S^t(\Theta) & \text{if } P < \underline{a}_S \\ \mu_B^t([\underline{t}, a_B^{-1}(P)]) & \text{if } P \in A_S \\ 0 & \text{if } P > \bar{a}_S \end{cases}, \tag{102}$$

from which the observation directly follows. □

Observation 21. *If $\underline{a}_S < \bar{a}_B$, then there exists a unique market price, which lies in the interval $[\underline{a}_S, \bar{a}_B]$ equating positive demand and supply. Otherwise, if $\underline{a}_S \geq \bar{a}_B$, then the trading volume is equal to zero. Note that in both cases, there is zero market excess, implying that the gains of trade, G, are deterministic.*

Proof of Observation 21. This follows from Observation 20 and the Intermediate Value Theorem. □

Observation 22. *The realized gains of trade G can be represented as a continuous function $G(\cdot)$ evaluated at the trading volume V.*

Proof of Observation 22. Let $\mathcal{B}^*$ and $\mathcal{S}^*$ be the allocation and denote by $T_B^* = t_B(\mathcal{B}^*)$ and $T_S^* = t_S(\mathcal{S}^*)$ the set of gross values involved in trade. First, note that

$$G = \int_{T_B^*} x d\mu_B^t(x) - \int_{T_S^*} x d\mu_S^t(x). \tag{103}$$

Using that gross values are assumed to be continuously distributed, it holds that

$$G = \int_{\Theta_B^*} x f_B(x) dx - \int_{\Theta_S^*} x f_S(x) dx, \tag{104}$$

where f_B and f_S are the strictly positive and continuous Radon-Nikodym derivatives. Because of the strict monotonicity of strategies, the traders with the most profitable gross values are involved in trade. Therefore T_B^* is of the form $[a, \bar{t}]$ for some $a \in T$ and T_S^* is of the form $[\underline{t}, b]$ for some $b \in T$. If the trading volume $V = 0$, then $a = \bar{t}$ and $b = \underline{t}$. If $V > 0$, then $a < \bar{t}$ and $b > \underline{t}$. It therefore holds that

$$G = \int_a^{\bar{t}} x f_B(x) dx - \int_{\underline{t}}^b x f_S(x) dx. \tag{105}$$

Next, we prove that a and b can be expressed as continuous functions of the trading volume V. Because the allocation balances trade, it holds that $\mu_B^t(T_B^*) = \mu_S^t(T_S^*) = V$. Let $F_B(x) = \int_{\underline{t}}^{x} f_B(x)dx$ denote the anti-derivative of f_B, which is a continuous and increasing function. We can write $\mu_B^t(T_B^*) = \int_a^{\bar{t}} d\mu_B^t = \mu_B^t(T) - F_B(a)$ and $\mu_S^t(T_S^*) = \int_{\underline{t}}^{b} d\mu_S^t = F_S(b)$. This yields

$$a(V) = \begin{cases} \bar{t} & \text{if} V = 0 \\ F_B^{-1}(\mu_B^t(T) - V) & \text{if } 0 < V < \mu_B^t(T) \\ \underline{t} & \text{if} V = \mu_B^t(\Theta) \end{cases} \quad \text{and} \quad b(V) = \begin{cases} \underline{t} & \text{if} V = 0 \\ F_S^{-1}(V) & \text{if } 0 < V < \mu_S^t(T) \\ \bar{t} & \text{if} V = \mu_S^t(T) \end{cases}. \tag{106}$$

$a(V)$ is continuous on $(0, \mu_B^t(T))$, because F_B is continuous and strictly decreasing on T. Because $\lim_{x \uparrow \bar{t}} F_B(x) = 0$ and $\lim_{x \downarrow \underline{t}} F_B(x) = \mu_B^t(T)$, the continuity of $a(V)$ extends to $V = 0$ and $V = \mu_B^t(T)$. Analogous reasoning yields that $b(V)$ is continuous on $[0, \mu_S^t(t)]$. The corresponding gains of trade can therefore be represented as

$$G = \int_{a(V)}^{\bar{t}} x f_B(x)dx - \int_{\underline{t}}^{b(V)} x f_S(x)dx. \tag{107}$$

Because the integrands $xf_B(x)$ and $xf_S(x)$ are continuous in x, it follows that G is continuous in V. □

Observation 23. *Consider two symmetric, strictly increasing and continuous strategy profiles $a^1 = (a_B^1, a_S^1)$ and $a^2 = (a_B^2, a_S^2)$, such that for all $t \in T$ it holds that $a_{B1}(t) \succcurlyeq a_{B2}(t)$ and $a_{S1}(t) \succ a_{S2}(t)$. Then it holds that $G_{a_1} \geq G_{a_2}$.*

Proof of Observation 23. By Appendix A.1 and Observation 21, for both strategy profiles the trading volume TV is equal to demand and supply at their unique crossing point. It follows from Equation (102) that $\forall P \; D_{a^1}(P) \geq D_{a^2}(P)$ and $S_{a^1}(P) \geq \mathcal{S}_{a^2}(P)$ hold, which implies that $V_{a^1} \geq V_{a^2}$. The observation now follows from Equation (107). □

Define the symmetric strategy profile a_n, which is equal to $t_b^\Phi - \frac{1}{n}$ and $t_s^\Phi + \frac{1}{n}$. Denote by the subscripts n and Φ market characteristics, when traders use a_n and truthfulness respectively.

Assume that the trading volume V_Φ at the market price P_Φ^* is strictly positive, that is $\underline{a}_{S\Phi} < \overline{a}_{B\Phi}$. Otherwise, it holds that $G_\Phi = 0$ and therefore also $G_n = 0$.

Observation 24. *For sufficiently large n, there exists a unique market price P_n^* with trading volume $V_n > 0$.*

Proof of Observation 24. According to Observation 20, demand $D_n(P)$ is continuous in P and strictly decreasing on an interval $A_{Bn} = [\underline{a}_{Bn}, \overline{a}_{Bn}]$. Supply $S_n(P)$ is continuous in P and strictly increasing on an interval $A_{Sn} = [\underline{a}_{Sn}, \overline{a}_{Sn}]$. $\underline{a}_{Bn}$ is for example the action of a buyer with gross value $\underline{t}$. Because $\lim_{n \to \infty} a_n(x) = x$, we can choose n large enough, such that also $\underline{a}_{Sn} < \overline{a}_{Bn}$. A unique market price $P_n^* \in [\underline{a}_{Sn}, \overline{a}_{Bn}]$ with trading volume $V_n > 0$ exists by Observation 21. □

Observation 25. *It holds that*

$$\lim_{n\to\infty}\sup_{P\in\Theta}|D_n(P)-D_\Phi(P)|=0 \quad \textit{and} \quad \lim_{n\to\infty}\sup_{P\in\Theta}|S_n(P)-S_\Phi(P)|=0. \tag{108}$$

Proof of Observation 25. Because larger n leads to a less aggressive strategy profile a_n, it follows that for fixed P $D_n(P) \leq D_{n+1}(P)$ and $S_n(P) \leq S_{n+1}(P)$. Furthermore, it holds that $\lim_{n\to\infty} D_n(P) = D_\Phi(P)$ and $\lim_{n\to\infty} S_n(P) = S_\Phi(P)$. Because D_Φ and S_Φ are continuous on Θ, the observation follows from Dini's theorem (Bartle and Sherbert, 2000, p.238). □

Observation 26. $\forall\delta_1 > 0$ *and sufficiently large n, it holds that* $|P_\Phi^* - P_n^*| \leq \delta_1$.

Proof of Observation 26. P_Φ^* is unique and equates demand and supply, and it was proven above that P_n^* has the same properties for sufficiently large n. Define the two continuous functions

$$F_\Phi(P) = D_\Phi(P) - S_\Phi(P) \quad \text{and} \quad F_n(P) = D_n(P) - S_n(P). \tag{109}$$

It holds that P_Φ^* is the unique zero of $F_\Phi(\cdot)$ and P_n^* is the unique zero of $F_n(\cdot)$. Because of the strict monotonicity of D_Φ and S_Φ, for every $\delta_1 > 0$ it holds that F_Φ is strictly negative at $P_\Phi^* + \delta_1$ and strictly positive at $P_\Phi^* + \delta_1$. Therefore, for small δ_1, there exists $\gamma_1 > 0$, such that

$$F_\Phi(P_\Phi^* + \delta_1) \leq -\gamma_1 \quad \text{and} \quad F_\Phi(P_\Phi^* - \delta_1) \geq \gamma_1. \tag{110}$$

We will now prove that for every $\gamma_2 > 0$ the distance between F_Φ and F_n at the two points $P_\Phi^* + \delta_1$ and $P_\Phi^* - \delta_1$ is smaller or equal than γ_2, if n is chosen sufficiently large. We have that

$$\begin{aligned} |F_\Phi(P) - F_n(P)| &= |D_\Phi(P) - S_\Phi(P) - D_n(P) + S_n(P)| \\ &\leq |D_\Phi(P) - D_n(P)| + |S_\Phi(P) - S_n(P)|. \end{aligned} \tag{111}$$

If δ_1 is chosen small enough, such that $P_\Phi^* + \delta_1$ and $P_\Phi^* - \delta_1$ are in Θ, then the uniform convergence observation from above implies that for every $\gamma_2 > 0$ and sufficiently large n

$$|F_\Phi(P_\Phi^* + \delta_1) - F_n(P_\Phi^* + \delta_1)| \leq \gamma_2 \quad \text{and} \quad |F_\Phi(P_\Phi^* - \delta_1) - F_n(P_\Phi^* - \delta_1)| \leq \gamma_2 \tag{112}$$

If γ_2 is chosen to be strictly less than γ_1, it follows that also

$$F_n(P_\Phi^* + \delta_1) < 0 \quad \text{and} \quad F_n(P_\Phi^* - \delta_1) > 0. \tag{113}$$

This then implies that P_n^*, which is the unique zero of F_n, lies in the interval $(P_\Phi^* - \delta_1, P_\Phi^* + \delta_1)$, which proves the observation. □

Observation 27. $\forall\delta_2 > 0$ *and sufficiently large n, it holds that* $|V_\Phi - V_n| \leq \delta_2$.

Proof of Observation 27. V_Φ is equal to $D_\Phi(P^*_\Phi)$ and V_n is equal to $D_n(P^*_n)$. By adding and subtracting $D_n(P^*_\Phi)$ and using the triangle-inequality, we get that

$$|V_\Phi - V_n| \leq |D_\Phi(P^*_\Phi) - D_n(P^*_\Phi)| + |D_n(P^*_n) - D_n(P^*_\Phi)|. \tag{114}$$

The first term on the right-hand side is less or equal than $\frac{\delta_2}{2}$ for sufficiently large n by Observation 25. For the second term, note that D_n is a continuous function. Observation 26 implies that for sufficiently large n, such that the distance between P^*_Φ and P^*_n gets small enough, the second term is also bounded from above by $\frac{\delta_2}{2}$, which proves the observation. □

Observation 28. *For all $\delta_3 > 0$ and sufficiently large n, it holds that $|G_\Phi - G_n| \leq \delta_3$.*

Proof of Observation 28. Because reporting the net value is by assumption a continuous and increasing function, it was proven above that G_Φ and G_n can be represented as a continuous function $G(\cdot)$ evaluated at the two points V_Φ and V_n. If n is chosen sufficiently large, Observation 27 and the continuity of $G(\cdot)$ imply that the distance between V_Φ and V_n gets small enough to ensure that $G_n = G(V_n)$ is close to $G_\Phi = G(V_\Phi)$. □

Observation 29. *For all $\zeta > 0$ and sufficiently large n, it holds that $E_n \geq 1 - \zeta$.*

Proof of Observation 29. For the efficiency ratio E_n, it holds that

$$E_n = \frac{G_n}{G_\Phi} = 1 - \frac{G_\Phi - G_n}{G_\Phi}. \tag{115}$$

If n is now chosen large enough, such that by Observation 28 $|G_\Phi - G_n| \leq \zeta G_\Phi$, the statement follows. □

Observation 30. $\forall \zeta > 0$*, there exists* $\epsilon \in (0, 1]$*, such that for a strictly increasing and continuous strategy profile* (a_B, a_S)*, which is* ϵ*-close to truthfulness,* $E_a \geq 1 - \zeta$ *holds.*

Proof of Observation 30. Define $\epsilon_n = \frac{1}{n}$. By Observation 23, it holds that $G_{\epsilon_n} \leq G_a$. Therefore, if n is sufficiently large, it holds that $E_a \geq E_\epsilon \geq 1 - \zeta$. □

Observation 30 concludes the proof for infinite markets. □

B.12 Proof of Theorem 11

Proof. For finite markets, we construct the following beliefs F. Assume that all buyers believe that they are facing the same market environment, independent of their gross value, which implies that they have the same belief about the critical value, that is $\forall t_b \in T$ it holds that $P^\infty(t_b) = P^\infty_B$. In analogy, assume that all sellers have the same beliefs, implying that $\forall t_s \in T$ it holds that $P^\infty(t_s) = P^\infty_S$. Suppose that $P^\infty_B < P^\infty_S$. For any $\epsilon \geq 0$, consider the strategy-profile corresponding

to price-guessing (ρ_B, ρ_S). Recall that for this strategy-profile, a buyer's and seller's action are equal to P_B^∞ and P_S^∞ respectively, if it is individually rational, and truthful otherwise. That is all buyers submit an action less or equal to P_B^∞ and all sellers submit an action greater or equal to P_S^∞. Therefore, for any realization of gross values, no profitable trade is possible and the gains of trade are equal to zero almost surely. Therefore, the achievable efficiency ratio is equal to zero.
For infinite markets, it was proven in Observation 22 in Appendix B.11 that for continuous and strictly increasing strategy profile in an infinite market, the gains of trade G can be represented as a continuous function $G(\cdot)$ evaluated at V with $G(V_\Phi) = G_\Phi$ and $G(0) = 0$. Therefore the efficiency ratio $E = \frac{G}{G_\Phi}$ can be represented as the continuous function $E(V) = \frac{G(V)}{G_\Phi}$. For $V = V_\Phi$ the efficiency ratio is equal to one, for $V = 0$, the efficiency ratio is equal to zero. If we show that for every $V \in [0, V_\Phi]$, it is possible to construct increasing strategies, such that the trading volume is equal to V, the theorem follows from the Intermediate Value Theorem, because for every $\zeta \in [0, 1]$, there exists $V \in [0, V_\Phi]$ with $E(V) = \zeta$. One possible construction is as follows: For $a, b \geq 0$, consider beliefs F such that $P^\infty(t_b) = t_b^\Phi - a$ and $P^\infty(t_s) = t_s^\Phi + b$. For any $\epsilon \geq 0$, consider the strategy-profile (ρ_B, ρ_S), which is continuous and strictly increasing. Note that for every trader, their belief about the critical value is individually rational. For any $V \in [0, V_\Phi]$, choose $a \geq 0$, such that $D(P_\Phi^*) = D_\Phi(\rho_B^{-1}(P_\Phi^*)) = D_\Phi(P_\Phi^* + a) = V$. Such a constant exists in $[0, \bar{t} - P_\Phi^*)]$ by the Intermediate Value Theorem, because D_Φ is continuous and decreasing on T with $D_\Phi(P_\Phi^*) = V_\Phi$ and $D_\Phi(P_\Phi^* + (\bar{t} - P_\Phi^*)) = V_\Phi$. Next, choose $\tilde{P}$ as a price with $S_\Phi(\tilde{P}) = V$. This price exists in $[\underline{t}, P_\Phi^*]$ by the Intermediate Value Theorem, because S_Φ is continuous and increasing on T with $S_\Phi(\underline{t}) = 0$ and $S_\Phi(P_\Phi^*)) = V_\Phi$. If we set $b = P_\Phi^* - \tilde{P} \geq 0$, then $S(P_\Phi^*) = S_\Phi(\tilde{P}) = V$, which proves that the market price is equal to P_Φ^* and the trading volume is equal to V. This finishes the proof. □

B.13 Proof of Theorem 12

Proof. Consider a buyer b with gross value t_b and action a_b. First, suppose that $\delta > 0$. Tie-breaking is a probability zero event. The expected utility is equal to

$$\mathbb{E}_b[u_b(t_b, a_b, P^*)] = \int_{\underline{P^*}}^{a_b} (t_b - x - \Phi_b(x))\, f_{P^*}(x)dx. \tag{116}$$

Recall from Proposition 1 that $t_b - t_b^\Phi - \Phi_b(t_b^\Phi) = 0$. By assumption, the map $x \mapsto x + \Phi_b(x)$ is strictly increasing. Therefore, for $x \in [\underline{P^*}, t_b^\Phi)$, the integrand is strictly greater than zero. For $x \in (t_b^\Phi, \overline{P^*}]$, the integrand is strictly negative. Hence, the expected utility is maximized at the unique point $a_b = t_b^\Phi$.[45] The function $a_b \mapsto \mathbb{E}_b[u_b(t_b, a_b, P^*)]$ is continuous, increasing on $[\underline{P^*}_b, t_b^\Phi]$ and decreasing on $[t_b^\Phi, \overline{P^*}_b]$. ϵ-best responses therefore approximate t_b^Φ. As truthfulness is the unique best response a_b, it holds that $E_\Phi = \frac{\mathbb{P}_P^*[b \in \mathcal{B}^*(a_b, P^*)]}{\mathbb{P}_P^*[b \in \mathcal{B}^*(t_b^\Phi, P^*)]} = \frac{\mathbb{P}_P^*[b \in \mathcal{B}^*(t_b^\Phi, P^*)]}{\mathbb{P}_P^*[b \in \mathcal{B}^*(t_b^\Phi, P^*)]} = 1$.

[45] Alternatively, this can be proven via the first order condition by differentiating the expected utility using Leibniz's rule and setting the derivative zero.

Second, suppose that $\delta = 0$. The expected utility is of the form

$$\mathbb{E}_b[u_b(t_b, a_b, P^*)] = \begin{cases} t_b - P^* - \Phi_b(P^*) & \text{if } a_b > P^* \\ p_b(t_b - P^* - \Phi_b(P^*)) & \text{if } a_b = P^* \\ 0 & \text{if } a_b < P^*, \end{cases} \tag{117}$$

where $p_b \in [0, 1]$ depends on tie-breaking beliefs. If $t_b^\Phi > P^*$, then the expected utility is equal to $t_b - P^* - \Phi_b(P^*) > t_b - t_b^\Phi - \Phi_b(t_b^\Phi) = 0$, and therefore a best response. If $t_b^\Phi \leq P^*$, then the expected utility is equal to 0, regardless of tie-breaking assumptions. Because every action $a_b > t_b^\Phi$ is dominated, t_b^Φ is again a best response. Therefore truthfully reporting t_b^Φ is a best response for every gross value and as argued above, the efficiency ratio of truthfulness is equal to 1. The proof for sellers is analogous. □

B.14 Proof of Theorem 13

Proof. Consider a buyer b with gross value t_b and action a_b. Suppose that $t_b^\Phi > P_b^*$. First, consider $\delta > 0$. Tie-breaking is a probability zero event. The expected utility is equal to

$$\mathbb{E}_b[u_b(t_b, a_b, P^*)] = \int_{\underline{P^*}}^{a_b} (t_b - x - \Phi_b(a_b, x))\, f_{P^*}(x) dx. \tag{118}$$

The expected utility is continuous in a_b on $[\underline{P^*}, \overline{P^*}]$ and attains a maximum by the Extreme Value Theorem, which proves the existence of a best response.

First, we show that an action $a_b^1 < P_b^*$ is not a best response. We show that there exists an action $a_b^2 > P_b^*$ such that

$$\mathbb{E}_b\left[u_b(t_b, a_b^2, a_{\text{-}b})\right] - \mathbb{E}_b\left[u_b(t_b, a_b^1, a_{\text{-}b})\right] > 0, \tag{119}$$

which implies that a_b^1 is not a best response. Because the net value is by assumption continuous and strictly increasing in the gross value, there exists a gross value $t_b' < t_b$, such that $t_b^\Phi > t_b^{\Phi\prime} > P_b^\infty$. Denote the difference between t_b^Φ and $t_b^{\Phi\prime}$ by $\delta > 0$. It holds that

$$\mathbb{E}_{\text{-}b}\left[u_b(t_b, t_b^{\Phi\prime}, a_{\text{-}b})\right] = \mathbb{E}_{\text{-}b}\left[u_b(t_b', t_b^{\Phi\prime}, a_{\text{-}b})\right] + \delta \geq \delta, \tag{120}$$

because the net value is assumed to be ex-post individually rational.

We therefore consider an action a_b with $a_b - P_b^* \geq \epsilon$ for some $\epsilon > 0$. We will show that if the aggregate uncertainty δ is sufficiently small, then a_b is not a best response, proving that best responses must be ϵ-close to P_b^*. More specifically, we prove that a buyer can increase their expected utility when switching to $P_b^* + \epsilon/2$. For $\delta < \epsilon/2$ it holds that

$$\begin{aligned}
&\mathbb{E}_b[u_b(t_b, a_b, P^*)] - \mathbb{E}_b[u_b(t_b, P_b^* + \epsilon/2, P^*)] = \\
&\int_{\underline{P^*}}^{a_b} (t_b - x - \Phi_b(a_b, x))\, d\mu_{P^*}(x) - \int_{\underline{P^*}}^{P_b^*+\epsilon/2} (t_b - x - \Phi_b(P_b^\infty + \epsilon/2, x))\, d\mu_{P^*}(x) = \\
&\int_{P_b^*+\epsilon/2}^{a_b} (t_b - x)\, d\mu_{P^*}(x) - \left(\int_{\underline{P^*}}^{P_b^*+\epsilon/2} (\Phi_b(a_b, x) - \Phi_b(\epsilon/2, x))\, d\mu_{P^*}(x) + \int_{P_b^*+\epsilon/2}^{a_b} \Phi_b(a_b, x) d\mu_{P^*}(x) \right).
\end{aligned} \tag{121}$$

Note that for any two actions $a_b^1 \geq a_b^2$ there exists a constant $\gamma > 0$, such that for all $P \in [\underline{P^*}, a_b^2]$ it holds that $\Phi_b(a_b^1, P) - \Phi_b(a_b^2, P) \geq \gamma$. That is because the map $a_b \mapsto \Phi_b(a_b, P)$ is strictly increasing on $[\underline{P^*}, a_b]$. Therefore, for fixed actions a_b^1 and a_b^2 the continuous function $P \mapsto \Phi_b(a_b^1, P) - \Phi_b(a_b^2, P)$ is strictly positive on the compact interval $[\underline{P^*}, a_b^2]$ and attains a strictly positive minimum by the Extreme Value theorem. Consider the constant $\gamma > 0$ that corresponds to $a_b^1 = a_b$ and $a_b^2 = P_b^* + \epsilon/2$. Together with δ-aggregate uncertainty, we get that

$$\int_{\underline{P^*}}^{P_b^*+\epsilon/2} (\Phi_b(a_b, x) - \Phi_b(P_b^* + \epsilon/2, x))\, d\mu_{P^*}(x) \geq (1 - \delta)\gamma. \tag{122}$$

Moreover it holds that

$$\int_{P_b^*+\epsilon/2}^{a_b} (t_b - x)\, d\mu_{P^*}(x) \leq \delta t_b \quad \text{and} \quad \int_{P_b^*+\epsilon/2}^{a_b} \Phi_b(a_b, x) d\mu_{P^*}(x) \geq 0. \tag{123}$$

Combining Equations (121) to (123) yields

$$\mathbb{E}_b[u_b(t_b, a_b, P^*)] - \mathbb{E}_b[u_b(t_b, P_b^* + \epsilon/2, P^*)] \leq t_b\delta - (1 - \delta)\gamma. \tag{124}$$

If $\delta < \frac{\gamma}{t_b+\gamma}$, then the difference in expected utility is strictly negative, proving that a_b is not a best response. This implies that best responses are ϵ-close to P_b^* if δ is sufficiently small.

Next, we construct beliefs such that the efficiency of best responses is zero. Suppose again that $t_b^\Phi > P_b^*$. For sufficiently small δ, best responses are ϵ-close to P_b^∞. It holds that $t_b^\Phi > \underline{P^*}$ and suppose that beliefs are such that $P_b^* < \underline{P^*}$. That is, the buyer's prediction of the market price is not in the actual support of the market price, but their net value is. For small ϵ, $P_b^* + \epsilon < \underline{P^*}$. Therefore, the buyer is involved in trade with positive probability K when bidding truthful, but is almost surely not involved in trade with their best response a_b, which is ϵ-close to P_b^*. It follows that $E_\Phi = \frac{\mathbb{P}_{P^*}[b \in \mathcal{B}^*(a_b, P^*)]}{\mathbb{P}_{P^*}[b \in \mathcal{B}^*(t_b^\Phi, P^*)]} = \frac{0}{K} = 0$.

Second, suppose that $\delta = 0$. The expected utility is of the form

$$\mathbb{E}_b[u_b(t_b, a_b, P^*)] = \begin{cases} t_b - P^* - \Phi_b(a_b, P^*) & \text{if } a_b > P^* \\ c_b(t_b - P^* - \Phi_b(a_b, P^*)) & \text{if } a_b = P^* \\ 0 & \text{if } a_b < P^* \end{cases}, \tag{125}$$

where $p_b \in [0, 1]$ depends on tie-breaking assumptions. Consider a market without tie-breaking, that is $p_b = 1$. The minimum of $\Phi_b(\cdot, P^*)$ on $[P^*, \infty)$ is attained at P^*. Therefore, the best response is equal to P^*, if $t_b^\Phi \geq P^*$. With tie-breaking, that is $p_b \in [0, 1)$, the fee payment $\Phi_b(\cdot, P^*)$ decreases when a_b approximates P^*. However, because $\Phi_b(a_b, P^*)$ is continuous, there exists $\epsilon > 0$, such that

$$t_b - P^* - \Phi_b(P^* + \epsilon, P^*) > p_b\left(t_b - P^* - \Phi_b(P^*, P^*)\right). \tag{126}$$

It follows that P^* is not a best response. Furthermore, because for any $a_b^1 > a_b^2 > P^*$ it holds that $\Phi_b(a_b^1, P^*) > \Phi_b(a_b^2, P^*)$ and therefore also $\mathbb{E}_b[u_b(t_b, a_b^1, P^*)] < \mathbb{E}_b[u_b(t_b, a_b^2, P^*)]$, no best response exists, but ϵ-best responses approximate P^*.

Finally, suppose that F_{P^*} has a continuous density function $f_{P^*} > 0$ on $[\underline{P^*}, \overline{P^*}]$. For all $\zeta \in [0, 1]$, we construct beliefs, such that the efficiency of best responses is equal to ζ. First, $p_b = 1$, that is the buyer believes that there is no tie-breaking. Then the unique best response is equal to their deterministic belief P_b^∞ of the market price. Therefore, for any value x, beliefs can be constructed, such that the best response is equal to x. The efficiency ratio is then equal to $E_\Phi = \frac{\mathbb{P}_{P^*}[b \in \mathcal{B}^*(x, P^*)]}{\mathbb{P}_{P^*}[b \in \mathcal{B}^*(t_b^\Phi, P^*)]} = \frac{1 - F_{P^*}(x)}{1 - F_{P^*}(t_b^\Phi)}$ with $1 - F_{P^*}(t_b^\Phi)$ and therefore continuous for $x \in [\underline{P^*}, \overline{P^*}]$. If x is equal to $\underline{P^*}$, the efficiency ratio is equal to 0, and if it is equal to t_b^Φ, the efficiency ratio is equal to 1. By the Intermediate Value Theorem, $\forall \zeta \in [0, 1]$ there exists $x \in [\underline{P^*}, \overline{P^*}]$, such that $E_\Phi = x$. The proof for sellers is analogous. □

B.15 Proof of Lemma 17

Proof. Recall that $\tilde{P}^*$ denotes the market price, if a trader is involved in trade, and zero otherwise.

For a buyer b with private type t_b, Equation (12) yields that

$$\begin{gathered} \mathbb{E}_{\text{-}b}\left[u_b\left(t_b, a_b^1, a_{\text{-}b}\right)\right] - \mathbb{E}_{\text{-}b}\left[u_b\left(t_b, a_b^2, a_{\text{-}b}\right)\right] = \\ t_b\left(\mathbb{P}_{\text{-}b}\left[b \in \mathcal{B}^*(a_b^1, a_{\text{-}b})\right] - \mathbb{P}_{\text{-}b}\left[b \in \mathcal{B}^*(a_b^2, a_{\text{-}b})\right]\right) - \\ \int_{[\underline{a}_{S,b}, \overline{a}_{S,b}]^2} \left(\tilde{P}^*\left(a_b^1, a_{\text{-}b}^{(m)}, a_{\text{-}b}^{(m+1)}\right) - \tilde{P}^*\left(a_b^2, a_{\text{-}b}^{(m)}, a_{\text{-}b}^{(m+1)}\right)\right) d\mu(a_{\text{-}b}^{(m)}, a_{\text{-}b}^{(m+1)}) - \\ \left(\mathbb{E}_{\text{-}b}\left[\Phi_b\left(a_b^1, a_{\text{-}b}\right)\right] - \mathbb{E}_{\text{-}b}\left[\Phi_b\left(a_b^2, a_{\text{-}b}\right)\right]\right). \end{gathered} \tag{127}$$

Note that the integral in the difference above is non-negative, because $\tilde{P}^*(a_b, a_{\text{-}b}^{(m)}, a_{\text{-}b}^{(m+1)})$ is increasing in a_b for fixed $a_{\text{-}b}^{(m)}$ and $a_{\text{-}b}^{(m+1)}$. Equation (16) follows by neglecting the term corresponding to the change in expected market price.

For a seller s with private type t_s, Equation (13) yields

$$\begin{aligned}&\mathbb{E}_{\text{-}s}\left[u_s\left(t_s,a_s^1,a_{\text{-}s}\right)\right]-\mathbb{E}_{\text{-}s}\left[u_s\left(t_s,a_s^2,a_{\text{-}s}\right)\right]=\\ &\int_{[\underline{a}_{B,s},\overline{a}_{B,s}]^2}\left(\tilde{MP}\left(a_s^1,a_{\text{-}s}^{(m\text{-}1)},a_{\text{-}s}^{(m)}\right)-\tilde{P^*}\left(a_s^2,a_{\text{-}s}^{(m\text{-}1)},a_{\text{-}s}^{(m)}\right)\right)d\mu(a_{\text{-}s}^{(m\text{-}1)},a_{\text{-}s}^{(m)})-\\ &t_s\left(\mathbb{P}_{\text{-}s}\left[s\in\mathcal{S}^*(a_s^1,a_{\text{-}s})\right]-\mathbb{P}_{\text{-}s}\left[s\in\mathcal{S}^*(a_s^2,a_{\text{-}s})\right]\right)-\left(\mathbb{E}_{\text{-}s}\left[\Phi_s\left(a_s^1,a_{\text{-}s}\right)\right]-\mathbb{E}_{\text{-}s}\left[\Phi_s\left(a_s^2,a_{\text{-}s}\right)\right]\right).\end{aligned}\tag{128}$$

$t_s\left(\mathbb{P}_{\text{-}s}\left[s\in\mathcal{S}^*(a_s^1,a_{\text{-}s})\right]-\mathbb{P}_{\text{-}s}\left[s\in\mathcal{S}^*(a_s^2,a_{\text{-}s})\right]\right)\geq 0$ holds, because the trading probability is decreasing for a seller in their ask. To see that the integral in Equation (128) is bounded from above by $2t_s\left(1-\mathbb{P}_{\text{-}s}\left[s\in\mathcal{S}^*(a_s^2,a_{\text{-}s})\right]\right)$, we split up the integral into all six possible cases for the realizations of and $a_{\text{-}s}^{(m\text{-}1)}$ with respect to $a_s^1<a_s^2$. which is shown in the following table.[46]

		$\tilde{P^*}\left(a_s^1,a_{\text{-}s}^{(m\text{-}1)},a_{\text{-}s}^{(m)}\right)$	$\tilde{MP}\left(a_s^2,a_{\text{-}s}^{(m\text{-}1)},a_{\text{-}s}^{(m)}\right)$
I	$a_{\text{-}s}^{(m)}\geq a_{\text{-}s}^{(m\text{-}1)}\geq a_s^2\geq a_s^1$	$ka_{\text{-}s}^{(m)}+(1\text{-}k)a_{\text{-}s}^{(m\text{-}1)}$	$ka_{\text{-}s}^{(m)}+(1\text{-}k)a_{\text{-}s}^{(m\text{-}1)}$
II	$a_{\text{-}s}^{(m)}\geq a_s^2\geq a_{\text{-}s}^{(m\text{-}1)}\geq a_s^1$	$ka_{\text{-}s}^{(m)}+(1\text{-}k)a_{\text{-}s}^{(m\text{-}1)}$	$ka_{\text{-}s}^{(m)}+(1\text{-}k)a_s^2$
III	$a_s^2\geq a_{\text{-}s}^{(m)}\geq a_{\text{-}s}^{(m\text{-}1)}\geq a_s^1$	$ka_{\text{-}s}^{(m)}+(1\text{-}k)a_{\text{-}s}^{(m\text{-}1)}$	0
IV	$a_{\text{-}s}^{(m)}\geq a_s^2\geq a_s^1\geq a_{\text{-}s}^{(m\text{-}1)}$	$ka_{\text{-}s}^{(m)}+(1\text{-}k)a_s^1$	$ka_{\text{-}s}^{(m)}+(1\text{-}k)a_s^2$
V	$a_s^2\geq a_{\text{-}s}^{(m)}\geq a_s^1\geq a_{\text{-}s}^{(m\text{-}1)}$	$ka_{\text{-}s}^{(m)}+(1\text{-}k)a_s^1$	0
VI	$a_s^2\geq a_s^1\geq a_{\text{-}s}^{(m)}\geq a_{\text{-}s}^{(m\text{-}1)}$	0	0

For **I**, **II**, **IV** and **VI**, the difference between $\tilde{P^*}\left(a_s^1,a_{\text{-}s}^{(m\text{-}1)},a_{\text{-}s}^{(m)}\right)$ and $\tilde{P^*}\left(a_s^2,a_{\text{-}s}^{(m\text{-}1)},a_{\text{-}s}^{(m)}\right)$ is less or equal than 0. It follows that

$$\begin{aligned}&\int_{[\underline{a}_{B,s},\overline{a}_{B,s}]^2}\left(\tilde{P^*}\left(a_s^1,a_{\text{-}s}^{(m\text{-}1)},a_{\text{-}s}^{(m)}\right)-\tilde{P^*}\left(a_s^2,a_{\text{-}s}^{(m\text{-}1)},a_{\text{-}s}^{(m)}\right)\right)d\mu(a_{\text{-}s}^{(m\text{-}1)},a_{\text{-}s}^{(m)})\leq\\ &\int_{\mathbf{III}}(ka_{\text{-}s}^{(m)}+(1-k)a_{\text{-}s}^{(m\text{-}1)})d\mu_s^*(a_{\text{-}s}^{(m\text{-}1)},a_{\text{-}s}^{(m)})\\ &+\int_{\mathbf{V}}(ka_{\text{-}s}^{(m)}+(1-k)a_s^1)d\mu(a_{\text{-}s}^{(m\text{-}1)},a_{\text{-}s}^{(m)}).\end{aligned}\tag{129}$$

Because both integrands in Equation (129) are less or equal than $\overline{a}_{S,s}$, it follows that

$$\begin{aligned}&\int_{[a_s,\underline{s}]^2}\left(\tilde{P^*}\left(a_s^1,a_{\text{-}s}^{(m\text{-}1)},a_{\text{-}s}^{(m)}\right)-\tilde{P^*}\left(a_s^2,a_{\text{-}s}^{(m\text{-}1)},a_{\text{-}s}^{(m)}\right)\right)d\mu(a_{\text{-}s}^{(m\text{-}1)},a_{\text{-}s}^{(m)})\\ &\leq\overline{a}_{S,s}\mathbb{P}[\mathbf{III}]+\overline{a}_{S,s}\mathbb{P}[\mathbf{V}]\\ &\leq 2\overline{a}_{S,s}\mathbb{P}[a_{\text{-}s}^{(m)}<a_s^2]=2\overline{a}_{S,s}\left(1-\mathbb{P}_{\text{-}s}\left[(s,a_s^2\in\mathcal{S}^*\right]\right),\end{aligned}\tag{130}$$

which finishes the proof. □

[46]Different to $\tilde{P^*}_b\left(a_b,y,z\right)$ it holds that $\tilde{P^*}_s\left(a_s,y,z\right)$ is not increasing in a_s for fixed y and z.

B.16 Proof of Proposition 18

Proof. Consider a buyer b with gross value t_b, such that $t_b^\Phi < P_b^\infty$. A best response a_b with $a_b \leq t_b^\Phi$ must exist. That is because if there is a best response a_b with $a_b > t_b^\Phi$, the expected utilities must be equal, as the net value dominates all larger actions, proving that t_b^Φ is a best response as well. By the monotonicity of the trading probability, it then holds that

$$\mathbb{P}_{\text{-}b}\left[b \in \mathcal{B}^*(a_b, a_{\text{-}b})\right] \leq \mathbb{P}_{\text{-}b}\left[b \in \mathcal{B}^*(t_b^\Phi, a_{\text{-}b})\right]. \tag{131}$$

For all $\gamma > 0$, it holds by Proposition 3 that in sufficiently large markets $\mathbb{P}_{\text{-}b}\left[b \in \mathcal{B}^*(t_b^\Phi, a_{\text{-}b})\right] \leq \gamma$. The expected utility is upper bounded by neglecting the payment of market price and fee, that is the gross value times the probability of trade:

$$\mathbb{E}_{\text{-}b}\left[u_b(t_b, a_b, a_{\text{-}b})\right] \leq t_b\gamma. \tag{132}$$

Choose $\gamma \leq \frac{\epsilon}{t_b}$. This implies that $ISLM$, the expected utility of a best response is upper bounded by ϵ. The expected utility of truthfulness is non-negative by assumption. This implies that truthfulness is an ϵ-best response. The statement for sellers can be proven analogously. $\square$