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Bartosz Mackowiak and Sebastian Schmidt

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Centre for Economic Policy Research
33 Great Sutton Street, London EC1V 0DX, UK
Tel: +44 (0)20 7183 8801
www.cepr.org

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JEL Classification: E31, E63, F45

Keywords: Fiscal theory of the price level, Monetary union, Fiscal rules, Eurobonds

Bartosz Mackowiak - bartosz.a.mackowiak@gmail.com
European Central Bank and CEPR

Sebastian Schmidt - econschmidt@gmail.com
European Central Bank and CEPR

Passive Monetary Policy and Active Fiscal Policy in a Monetary Union*

Bartosz Maćkowiak
European Central Bank and CEPR

Sebastian Schmidt
European Central Bank and CEPR

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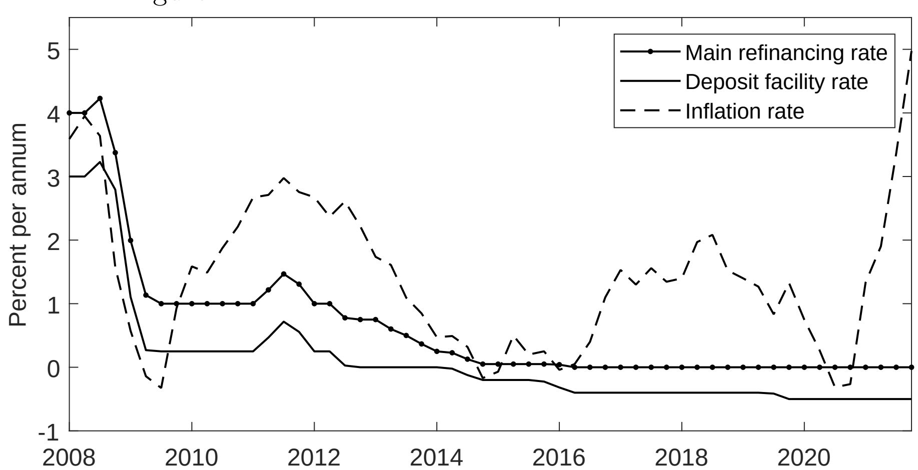
*Postal address: European Central Bank, 60640 Frankfurt, Germany; E-mail addresses: bartosz.a.mackowiak@gmail.com (B. Maćkowiak), sebastian.schmidt@ecb.europa.eu (S. Schmidt). The views expressed in this paper are solely those of the authors, and do not necessarily reflect the views of the European Central Bank. We thank for comments Jim Costain, Eric Leeper, Nigel McClung, and workshop participants in Athens, Barcelona, Mannheim, and Paris.

1 Introduction

How is the price level in a monetary union determined when the central bank keeps the nominal interest rate constant for an extended time, whether by choice or because of the lower bound constraint, and in parallel buys, or stands ready to buy, government bonds?

In the wake of the financial crisis of 2008, the European Central Bank cut its main refinancing rate and its deposit facility rate sharply, to 1 percent and 0.25 percent in 2009. The policy rates varied little in the subsequent *twelve years*. At the end of 2021, the policy rates stood at 0 percent and -0.50 percent, respectively. See Figure 1. In parallel, the ECB instituted four government bond buying programs. Under the Asset Purchase Program and the Pandemic Emergency Purchase Program, the ECB purchased about one-third of national public debt in the euro area by the end of 2021.[1]

Figure 1: **Euro area interest rates and inflation**



Euro area quarterly data, 2008Q1-2021Q4. Interest rates are quarterly averages. Inflation rate is the annual rate of percentage change of the end-of-quarter HICP price level.

While there may be different ways to model this episode, in this paper we use it to motivate a model of price level determination in a monetary union in which monetary policy is passive and fiscal policy is active.[2] We think that this is an interesting hypothesis

[1] The other programs were the Securities Markets Program and the Outright Monetary Transactions.

[2] Monetary policy in a monetary union is passive if the central bank's policy rate moves less than one-for-one with the union inflation rate, and active otherwise. Fiscal policy is passive if a change in the level of total public liabilities in the union begets a change in the present value of the sum of primary surpluses by

to consider.

The passive-money active-fiscal policy mix in a monetary union is interesting to study also because of an important result in monetary economics which states that "for the price level to be uniquely determined, fiscal policy must be seen to 'back' the price level" (Del Negro and Sims (2015), p. 2). "Fiscal backing" means that fiscal policy can be passive most of the time, even all the time on the rational expectations equilibrium path, but it must turn active in the face of a threat of an inflationary spiral or a deflation trap.[3] In a monetary union, there are multiple fiscal authorities, one for each member country of the union, and in addition there may be a common fiscal authority.[4] The question arises how to coordinate the different fiscal policies to back the price level.[5]

We study a model with multiple countries, a fiscal authority in each country, and a common monetary authority. In addition, there may be a common fiscal authority. In each country, a representative household receives an endowment good in every period. Each country has its own consumption-based price level because of home bias in consumption, and the price level of the union is a weighted average of the national price levels. The fiscal authorities collect lump-sum taxes and issue nominal debt. The central bank pegs the nominal interest rate.

Different combinations of national fiscal policies and common fiscal policy amount to active fiscal policy for the union as a whole. For each such combination, the price level of the union and the price level in each country are uniquely determined – there is "fiscal backing." At the same time, the effects of fiscal and monetary policy can be very different depending on which combination is in place. Thus, in a monetary union stating that "fiscal policy is active" is a very incomplete description of fiscal policy – one must give specifics.

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the same amount, regardless of the path of prices and the policy rate, and active otherwise.

[3]Obstfeld and Rogoff (1983), Benhabib et al. (2002), Woodford (2003), Chapter 2.4, and Sims (2013). See also Section 7 of this paper.

[4]In 2020, the member countries of the European Union authorized the European Commission to issue up to 750 billion euros of common debt.

[5]Sims (2013), p. 575, writes: "Institutions like those in the EMU that make it unclear where the fiscal backing would come from, or even whether it exists, are destabilizing; yet in normal times, because large fiscal-backing interventions do not occur in equilibrium, it is easy for the importance of fiscal backing to be lost sight of."

To begin, we suppose that each national fiscal authority maintains a constant primary surplus except for a one-time shock (for simplicity, there is no common fiscal authority). This specification is a natural extension to a monetary-union setting of the usual, constant surplus specification of active fiscal policy in a single-country model. A symmetric fiscal shock, a shock to primary surpluses that affects all national budgets equally, causes inflation or deflation throughout the union, leaving unchanged the relative price levels and consumption. An asymmetric fiscal shock changes the relative price levels and affects consumption differentially by country. For example, a fiscal expansion by a country increases consumption and the relative price level in that country, permanently, while consumption and the relative price levels in the other countries fall. Thus, in this benchmark configuration asymmetric deficits or surpluses *necessarily* cause cross-country wealth transfers and inflation differentials, *even though* direct cross-country budgetary transfers are ruled out in the model. Furthermore, a completely symmetric shock to the central bank's policy rate also changes the relative price levels and affects consumption differentially by country, causing cross-country wealth transfers, except in the knife-edge case when public debt has identical maturity in each country.

The intuition for these results is that the price level of the union adjusts to make the real value of *all* public debt in the union match the present value of the *sum* of all primary surpluses; after an asymmetric fiscal shock or after a monetary policy shock when public debt maturity differs by country, the union price level is "too high" for some countries and "too low" for others; and this is consistent with optimizing behavior of private agents and therefore a valid equilibrium.

The model divides the balance sheet of the common monetary authority into balance sheets of national central banks (NCBs), and a cross-country wealth transfer can show up as debt between NCBs that is rolled over forever.[6]

The benchmark passive-money active-fiscal policy mix may be a realistic, simple de-

[6]Debt between NCBs in the model corresponds to TARGET2 claims between NCBs in the euro area. Policy commentators and researchers have emphasized that TARGET2 claims between NCBs can lead to cross-country wealth transfers if there is a default or the euro area breaks up. See for instance Bassetto and Caracciolo (2021). We emphasize the same possibility as part of "normal" functioning of the euro area, without a default or break-up.

scription of some episodes. However, since cross-country wealth transfers and inflation differentials are part and parcel of this policy configuration, we doubt that it is a sustainable way to provide fiscal backing for price stability in a monetary union.

We propose a new configuration of national fiscal policies and a common fiscal authority that amounts to active fiscal policy for the union as a whole. We think that this configuration has desirable properties and therefore may be able to provide the fiscal backing sustainably. We include a common fiscal authority motivated by the idea that there may be a special role in this task for such an authority. The common fiscal authority in the model collects lump-sum taxes symmetrically from each country and issues "Eurobonds." Each national fiscal authority follows a standard passive feedback rule for its primary surplus, reacting to fluctuations in the real value of its own debt so as to stabilize that value regardless of the path of prices. The common authority "leans against" the response of the member countries to their debt. When the national policies tighten *in response* to higher national debt, the common policy loosens. When the national policies loosen *in response* to lower national debt, the common policy tightens. The *sum* of all primary surpluses, *including* the primary surplus of the common authority, does not respond to debt which makes fiscal policy active in the union as a whole. This yields price level determinacy, *even though* the national authorities follow standard passive feedback rules. Asymmetric deficits and surpluses or monetary policy shocks do not cause cross-country wealth transfers. At the same time, fiscal policy can produce inflationary or deflationary impulses at the level of the union as may be appropriate to stabilize the economy.

To emphasize, the common fiscal authority does not make cross-country budgetary transfers, targeting an appropriate *aggregate* fiscal policy stance for the union. The common authority can be modestly-sized, because its main role is to react only to the variation in the national surpluses that constitutes a response to national debt levels, and this component of the variation in the national surpluses is typically small or moderate.

We also study other configurations that amount to active fiscal policy for the union. In addition, we extend the model in several ways. For instance, we consider a version of the model in which, instead of pegging the nominal interest rate, the central bank follows a passive feedback rule for interest-rate setting. This version yields the more realistic prediction for some situations, including the recent euro area experience, that the policy rates vary to some degree over time instead of being literally constant. As another example, we study a version of the model in which default by a national fiscal authority occurs in equilibrium. This is an important extension because a public debt restructuring has already taken place in the euro area; furthermore, the presence of government bond spreads in the data is consistent with the possibility of similar events in the future, and this version of the model generates a government bond spread due to default risk.

This paper belongs to the literature on the fiscal theory of the price level, following Leeper (1991), Sims (1994), and Woodford (1994). While this literature is vast, very few papers within it study a monetary union. The recent textbook by Cochrane (2022), as comprehensive as it is, contains no model of a monetary union. The textbook discusses the ECB in the chapter on a “stand-alone” central bank that is disconnected from fiscal authorities and does not hold government bonds (or at least government bonds play a minor role in its operations). Cochrane concludes this discussion (Chapter 10.3, p. 301): “The ECB is becoming a more classic fiscal theory of the price level operation, money backed by collective general government surpluses (...).” One can see this paper as pursuing the implications of Cochrane’s conclusion.

The few fiscal-theory papers that study a monetary union include Bergin (2000), Sims (1997), Section VI, and Woodford (1998), Section 5. They all study homogeneous monetary unions with a single price level (the consumption basket is identical in each member country). They each find that when the central bank pegs the nominal interest rate and each national fiscal authority pursues a constant primary surplus policy, the price level is determinate. They each note that asymmetric fiscal shocks cause cross-country wealth transfers in this policy configuration. Skeptical about this feature, Sims proposes a feedback rule for national primary surpluses that makes union-wide fiscal policy active while ruling out such cross-country wealth transfers.[7] We study the Sims rule and argue in favor of our proposal. Recent papers with fiscal-theory monetary-union models include Jarociński and Maćkowiak (2018) and Bianchi, Melosi, and Rogantini Picco (2021). Both are applied

[7] Woodford sees this feature as a justification for passive fiscal policy, or Maastricht-type constraints on fiscal policy, in a monetary union.

papers that do not study the details of price level determination. Bonam and Hobijn (2021) study determinacy of equilibrium in a linearized homogenous monetary-union model with monetary-fiscal regime switches.

The next section presents the model. Section 3 considers the benchmark passive-money active-fiscal policy mix. Section 4 presents our proposal. Section 5 studies additional configurations that amount to active fiscal policy for the union. Section 6 presents the extensions of the model. In Section 7, we derive in the context of our monetary-union model the usual result that equilibrium is indeterminate if fiscal policy is always passive. As noted at the outset, this result provides additional motivation for studying active fiscal policy. Section 8 concludes. There is an online Appendix with supplementary material.

2 Model of a monetary union

We study a model of long-run price level determination in a heterogenous monetary union. By "long-run" we mean that the model abstracts from nominal rigidities that in the real world likely affect the short-run response of the price level to shocks. By "heterogenous" we mean that the model allows for home bias in consumption, which lets us study the determination of both the price level in the union and the price levels in the member countries.

The model consists of $I > 1$ countries that form a monetary union. Time is discrete, $t = 0, 1, 2, \ldots$. In each country i, $i = 1, ..., I$, there is a representative household ("household i") and a fiscal authority ("fiscal authority i"). There is a common monetary authority. We add a common fiscal authority in Section 4.

Households. Let C_{it} denote consumption of household i in period t. Household i maximizes the expected discounted sum of utility. The household's utility function $U(C_{it})$ is strictly increasing and strictly concave. There are I traded goods. In every period household i receives an endowment of good i, $Y_{it} > 0$. The household derives utility from consumption of all goods. Preferences allow for home bias according to

$$C_{it} = \left(\sum_{j=1}^{I} \gamma_{ij}^{\frac{1}{\theta}} C_{ijt}^{\frac{\theta-1}{\theta}} \right)^{\frac{\theta}{\theta-1}} \quad (1)$$

where C_{ijt} is consumption of good j by household i in period t, $\theta > 0$, $\gamma_{ij} > 0$, and $\sum_j \gamma_{ij} = 1$ for each i. The consumption-based price level for country i is

$$P_{it} = \left(\sum_{j=1}^{I} \gamma_{ij} W_{jt}^{1-\theta} \right)^{\frac{1}{1-\theta}} \tag{2}$$

where W_{jt} is the price of good j in period t.

National fiscal authorities. Household i pays lump-sum taxes to, or receives lump-sum transfers from, fiscal authority i. The fiscal authorities issue nominal debt. Let B_{ij}^{H} denote bonds of fiscal authority j held by household i. Let Q_{jt} be the price of bond j in period t. Following Woodford (2001), bond j issued in period t pays ρ_j^k euros $k+1$ periods later, for each $k \geq 0$, where $\rho_j \in (0,1)$ is a parameter. In Section 6, we extend the model to include default on government bonds.[8]

Common monetary authority. The central bank of the union issues the monetary base (the euro). In the baseline model, the monetary base consists entirely of reserves that pay interest. In Section 6, we consider the case in which the monetary base consists of interest-paying reserves and currency that provides liquidity services and does not pay interest. Let $H_{it} \geq 0$ denote reserves held by household i in period t. Reserves in the model are one-period debt. Let R_t be the interest rate on reserves in period t.

Flow budget constraints. The flow budget constraint of household i reads

$$\sum_{j=1}^{I} \left(1 + \rho_j Q_{jt}\right) B_{ijt-1}^{H} + R_{t-1} H_{it-1} + W_{it} Y_{it} = P_{it} C_{it} + W_{it} S_{it} + \sum_{j=1}^{I} Q_{jt} B_{ijt}^{H} + H_{it} \tag{3}$$

where S_{it} is a lump-sum tax paid by household i in period t (S_{it} can be strictly negative, in which case it is a lump-sum transfer).

Let B_i^{CB} denote bonds of fiscal authority i held by the central bank, and let $B_i = \sum_j B_{ji}^{H} + B_i^{CB}$ denote all bonds of fiscal authority i. Fiscal authority i faces the flow budget constraint

$$\left(1 + \rho_i Q_{it}\right) B_{it-1} = W_{it} S_{it} + W_{it} Z_{it} + Q_{it} B_{it} \tag{4}$$

[8] As is common in fiscal-theory papers, the model focuses on wealth effects of fiscal policy in a setting with lump-sum taxes, abstracting from other effects of fiscal policy that arise in the presence of government consumption and investment or distorting taxes.

where Z_{it} is a remittance to fiscal authority i from the central bank in period t. This formulation rules out direct transfers from the fiscal authority in one country to the fiscal authority in another country.

The flow budget constraint of the common monetary authority reads

$$R_{t-1}\sum_i H_{it-1} - \sum_i (1+\rho_i Q_{it}) B_{it-1}^{CB} + \sum_i W_{it} Z_{it} = \sum_i H_{it} - \sum_i Q_{it} B_{it}^{CB}. \quad (5)$$

Thus, in the baseline model the central bank holds only government bonds. At the end of Section 3, we discuss the case in which the central bank holds private debt.

When we sum equation (4) across i and combine the sum with equation (5), we obtain the flow budget constraint of the public sector in the union:

$$\sum_j (1+\rho_j Q_{jt}) \sum_i B_{ijt-1}^H + R_{t-1}\sum_i H_{it-1} = \sum_i W_{it} S_{it} + \sum_j Q_{jt} \sum_i B_{ijt}^H + \sum_i H_{it}. \quad (6)$$

Resource constraints. The resource constraint for each good i in every period reads

$$Y_{it} = \sum_j C_{jit}. \quad (7)$$

Price level of the union and relative prices. We assume that the price level of the union is defined according to

$$P_t = \prod_{i=1}^{I} P_{it}^{n_i}, \quad (8)$$

where $n_i > 0$ for each i and $\sum_i n_i = 1$. Real GDP of the union is defined according to $Y_t = \left(\sum_i W_{it} Y_{it}\right)/P_t$. It is convenient to define for each country i the relative price level $\tilde{P}_{it} \equiv P_{it}/P_t$; furthermore, for each good j, the relative price of that good is $\tilde{W}_{jt} \equiv W_{jt}/P_t$.

Optimality conditions. Let λ_{it} denote the period t marginal value of wealth to household i. For each household i, define the period t stochastic discount factor for period $t+k$, $k \geq 0$:

$$\Theta_{itk} \equiv \beta^k \frac{\lambda_{it+k}}{\lambda_{it}} \frac{\tilde{P}_{it}}{\tilde{P}_{it+k}} \quad (9)$$

where $\beta \in (0,1)$ is a parameter. The first-order conditions imply that the following relations hold in equilibrium for each household i:

$$U'(C_{it}) = \lambda_{it} \quad (10)$$

$$\frac{1}{P_t} = E_t\left(\Theta_{it1}\frac{R_t}{P_{t+1}}\right) \tag{11}$$

$$\frac{Q_{jt}}{P_t} = E_t\left[\Theta_{it1}\frac{(1+\rho_j Q_{jt+1})}{P_{t+1}}\right],\ j=1,...,I \tag{12}$$

$$C_{ijt} = \gamma_{ij}\left(\frac{\tilde{W}_{jt}}{\tilde{P}_{it}}\right)^{-\theta} C_{it},\ j=1,...,I. \tag{13}$$

The following transversality condition holds in equilibrium for each household i:

$$\lim_{T\to\infty} E_t\left[\Theta_{itT}\left(\frac{\sum_j Q_{jT}B^H_{ijT} + H_{iT}}{P_T}\right)\right] = 0. \tag{14}$$

We solve forward the budget constraint of the public sector in the union, equation (6). Using first-order conditions (11)-(12) for any household $h = 1,\ldots,I$ and a no-arbitrage condition,[9] we arrive at the relation:

$$\begin{aligned}&\frac{\sum_j (1+\rho_j Q_{jt})\left(\sum_i B^H_{ijt-1}\right) + R_{t-1}\sum_i H_{it-1}}{P_t}\\ =\ &\sum_{k=0}^{T} E_t\left[\Theta_{htk}\left(\sum_i \tilde{W}_{it+k}S_{it+k}\right)\right] + E_t\left[\sum_i \Theta_{itT}\left(\frac{\sum_j Q_{jt+T}B^H_{ijt+T} + H_{it+T}}{P_{t+T}}\right)\right]\end{aligned} \tag{15}$$

The first term on the right-hand side of equation (15) uses the stochastic discount factor of any household h, while the second term is a sum involving the stochastic discount factors of all households. Summing transversality condition (14) across the households implies that in equilibrium

$$\lim_{T\to\infty} E_t\left[\sum_i \Theta_{itT}\left(\frac{\sum_j Q_{jT}B^H_{ijT} + H_{iT}}{P_T}\right)\right] = 0. \tag{16}$$

It follows from equations (15)-(16) that

$$\frac{\sum_j (1+\rho_j Q_{jt})\left(\sum_i B^H_{ijt-1}\right) + R_{t-1}\sum_i H_{it-1}}{P_t} = \sum_{k=0}^{\infty} E_t\left[\Theta_{itk}\left(\sum_j \tilde{S}_{jt+k}\right)\right], \tag{17}$$

where we defined $\tilde{S}_{jt} \equiv \tilde{W}_{jt}S_{jt}$ as the period t primary budget surplus of fiscal authority $j = 1,\ldots,I$ expressed in units of GDP of the union. Equation (17) will be important below, like in any fiscal-theory model. There are differences compared with the usual single-country representative-agent case. On the left-hand side, liabilities of the public sector *in the union*

[9] Let X_{t+k} denote the real value of a portfolio of bonds and reserves in any period $t+k$, $k \geq 0$. The no-arbitrage condition is $E_t(\Theta_{itk}X_{t+k}) = E_t(\Theta_{jtk}X_{t+k})$ for any pair of households i and j.

as a whole appear. The right-hand side is the expectation of the present value of the *sum* of primary surpluses of *all* fiscal authorities in the union. With heterogenous households and incomplete risk sharing equation (17) must hold for each household i: if the surpluses are exogenous, then the price level of the union, the bond prices, *and* the path of the stochastic discount factor for each household i (consumption and the relative price level in country i) adjust in equilibrium.

Finally, it will sometimes be useful to think of the balance sheet of the common monetary authority as the sum of I balance sheets of national central banks (NCBs), where NCB i is associated with country i. Household i holds reserves at NCB i. NCB i makes remittances to fiscal authority i.[10] Let B_{ij}^{CB} denote bonds of fiscal authority j held by NCB i. We allow for one-period debt between the NCBs remunerated at the same interest rate as reserves (in the context of the euro area, this debt corresponds to TARGET2 claims between the NCBs). The flow budget constraint of NCB i reads

$$R_{t-1}\left(H_{it-1}-\sum\nolimits_j T_{ijt-1}\right)-\sum\nolimits_j\left(1+\rho_j Q_{jt}\right)B_{ijt-1}^{CB}+W_{it}Z_{it}=H_{it}-\sum\nolimits_j T_{ijt}-\sum\nolimits_j Q_{jt}B_{ijt}^{CB} \tag{18}$$

where T_{ijt} denotes the period t net claim of NCB i on NCB j. Summing equation (18) across i, using $\sum_i\sum_j T_{ijt}=0$ and $B_{it}^{CB}=\sum_j B_{jit}^{CB}$, yields equation (5).

We focus on the case when $I=2$ (two countries, two goods) and the endowments are constant over time, $Y_{it}=Y_i>0$. We solve for variables in the model in every period $t\geq 0$, taking as given initial, period -1 holdings of bonds and reserves. We assume perfect foresight, except that there may be a one-time unanticipated shock in period 0.

3 Benchmark passive-money active-fiscal policy mix

We follow the tradition in macroeconomics of specifying rules for policy instruments and studying the behavior of the economy given those rules (e.g., Leeper (1991), Taylor (2001),

[10] This is a simplification of the euro area institutional setup. In reality, the Eurosystem is the sum of the NCBs and the ECB. 80 percent of government bonds purchased in the Asset Purchase Program and the Pandemic Emergency Purchase Program are held by the NCBs, and 20 percent are held by the ECB (with gains or losses from the ECB government bond portfolio shared among the NCBs).

Woodford (2003), Part I). In this section, following Leeper (1991), we consider a benchmark specification of active fiscal policy and passive monetary policy.

National fiscal policies. We assume that each fiscal authority i maintains a constant primary surplus, $\tilde{S}_{it} = \tilde{S}_i > 0$, $t \geq 0$. This specification is a natural extension to a monetary-union setting of the usual, constant surplus specification of active fiscal policy in a single-country model.[11]

Monetary policy. Let $\Pi_t \equiv P_t/P_{t-1}$ denote the union inflation rate in period t. We suppose that the central bank chooses an inflation target $\Pi \geq 1$ and pays a time-invariant interest rate on reserves $R_t = R$, $t \geq 0$, consistent with this target, $R = \Pi/\beta$. In Section 6, we consider the case of a more general passive monetary policy where the nominal interest rate responds weakly to inflation.

In addition to interest rate policy as just specified, monetary policy in this model includes also balance sheet policy (e.g., a rule for determining the quantity of reserves, the central bank's government bond holdings, and remittances). The specifics of balance sheet policy turn out to be inessential here, and we will discuss them later. One general aspect of balance sheet policy is essential, however. By specifying constant national primary surpluses and ruling out default, we are in effect assuming that if, for whatever reason, private agents were to refuse to purchase newly issued government bonds, the central bank would create reserves and purchase the bonds. Private agents believe the central bank's promise and in equilibrium they purchase newly issued government bonds.[12] The central bank's promise holds given the fiscal policies that we have assumed *including* the numerical values, introduced below, of the surpluses of each fiscal authority. The central bank's promise is not a blank check; it does not hold for arbitrary numerical values of the surpluses. The closest real-world analogy may be the OMT program of the ECB in which conditions for government bond purchases by the central bank had been specified and no actual purchases

[11] It turns out to be simpler to solve the model assuming that $\tilde{S}_{it}$ (expressed in units of GDP of the union) rather than S_{it} (expressed in units of GDP of country i) is time-invariant. To maintain a constant $\tilde{S}_{it}$ fiscal authority i is assumed to vary S_{it} in response to fluctuations in the relative price $\tilde{W}_{it}$.

[12] Analogously, private agents believe the central bank's promise not to default on maturing reserves, and to convert reserves into currency and vice versa (in the version of the model in which the monetary base consists of reserves and currency, Section 6.3).

were made. In Section 6, we extend the model to include default on government bonds.

Equilibrium. A perfect-foresight equilibrium can be defined as follows.

Definition 1 *A perfect-foresight equilibrium is a sequence of allocations* $\{C_{it}\}$ *and prices* $\left\{P_t, \tilde{P}_{it}, \tilde{W}_{it}, Q_{it}\right\}$, $i = 1, 2$, $t \geq 0$, *that solves the following system of equations:*

$$\frac{\sum_j (1+\rho_j Q_{j0}) \left(\sum_i B^H_{ij,-1}\right) + R_{-1} \sum_i H_{i,-1}}{P_0} = \sum_{k=0}^{\infty} \beta^k \left(\sum_i \tilde{S}_{ik}\right) \tag{19}$$

$$\frac{\sum_j (1+\rho_j Q_{j0}) B^H_{ij,-1} + R_{-1} H_{i,-1}}{P_0} = \sum_{k=0}^{\infty} \beta^k \left(\tilde{P}_{ik} C_{ik} - \tilde{W}_{ik} Y_i + \tilde{S}_{ik}\right), \ i = 1, 2 \tag{20}$$

$$Y_j = \sum_i \gamma_{ij} \left(\frac{\tilde{W}_{jt}}{\tilde{P}_{it}}\right)^{-\theta} C_{it}, \ j = 1, 2 \tag{21}$$

$$1 = \beta R_t \frac{P_t}{P_{t+1}} \tag{22}$$

$$\frac{Q_{it}}{P_t} = \beta \frac{1 + \rho_i Q_{it+1}}{P_{t+1}}, \ i = 1, 2 \tag{23}$$

$$\tilde{P}_{it} = \left(\gamma_{i1} \tilde{W}_{1t}^{1-\theta} + \gamma_{i2} \tilde{W}_{2t}^{1-\theta}\right)^{\frac{1}{1-\theta}}, \ i = 1, 2 \tag{24}$$

$$1 = \tilde{P}_{1t}^{n_1} \tilde{P}_{2t}^{n_2} \tag{25}$$

given policy rules for $\left\{R_t, \tilde{S}_{it}\right\}$, *endowments* Y_i, *and initial conditions* $B^H_{ij,-1}, R_{-1} H_{i,-1}$, $i, j = 1, 2$.

The first equation in this definition is the period 0 version of equation (17), where on the right-hand side we use the fact that in a perfect-foresight equilibrium $\Theta_{itk} = \beta^k$ for any i, t, and k.[13] The second equation is the intertemporal budget constraint of household i in period 0.[14] The subsequent equations hold in every period $t \geq 0$. The third equation is the period resource constraint for good j, equation (7), after substituting optimality condition (13). The fourth and fifth equations are optimality conditions (11)-(12). It follows from these two equations that in a perfect-foresight equilibrium with the assumed policies each

[13] Θ_{itk} depends on β, on the ratio of current and future marginal utilities of household i, and on the ratio of current and future relative prices levels in country i (equation (9)). The marginal utility and the relative price level can change in period 0 if there is a shock, but the ratios remain equal to 1.

[14] The intertemporal budget constraint of household i follows from equations (3), (11)-(12), and (14).

bond price is constant over time, $Q_i = 1/(R - \rho_i)$. The sixth equation gives the relative price level in country i (equation (2)). The last equation follows from the definition of the price level of the union (equation (8)).[15]

Note that in this monetary-union model equation (19) must hold for the public sector in the union as a whole and equation (20) must hold for each household $i = 1, 2$. By contrast, optimizing behavior of private agents does not imply a similar relation for the public sector *in each country* (fiscal authority i plus NCB i). For an individual country, the real value of its public debt may or may not equal the present value of the country's primary budget surplus, depending on fiscal policy.

Existence and uniqueness. In the case when the trade elasticity θ equals 1, we can prove analytically that a perfect-foresight equilibrium exists and is unique. Thus, passive monetary policy and active fiscal policy determine a unique price level in a monetary union of heterogenous countries and a unique price level in each country.

Proposition 1 *Suppose that $\theta = 1$ (a unit trade elasticity). The common monetary authority sets a time-invariant interest rate on reserves, $R = \Pi/\beta$. Each fiscal authority i maintains a time-invariant primary surplus, $\tilde{S}_i > 0$. There exists a unique perfect-foresight equilibrium.*

Proof: See Appendix A.

When $\theta \neq 1$ we solve the model numerically, and always find a unique equilibrium. We solve for P_0 from equation (19) and then use the bisection method to solve the rest of the model. The details of the solution method are in Appendix B.

Parameterization. The baseline parameterization (Table 1) assumes that the two countries have the same size ($n_1 = n_2 = 0.5$) and are also otherwise symmetric. One period in the model equals one quarter. The chosen values of β, Π, and ρ_i imply an annual real interest rate of 2 percent, the same nominal interest rate, a net inflation target of zero percent, and a debt duration of about 20 quarters. To specify the degree of home bias (equations (1)-(2)), we assume that parameters satisfy $\gamma_{11} = 1 - n_2\nu$, $\gamma_{12} = n_2\nu$, $\gamma_{21} = n_1\nu$,

[15] Given a solution for the variables stated in the definition of a perfect-foresight equilibrium, we can solve for consumption of individual goods $\{C_{1jt}, C_{2jt}\}$, $j = 1, 2$, in every period from equation (13).

and $\gamma_{22} = 1 - n_1\nu$, where $\nu \in [0,1]$ is a measure of home bias (full home bias is $\nu = 0$, no home bias is $\nu = 1$), and we set $\nu = 0.3$ as in Gopinath et al. (2020). Following the same paper, we choose a trade elasticity of $\theta = 2$.[16] We set $\tilde{S}_i = 0.02n_i$ in each country (a primary surplus of 2 percent of GDP), which with $\beta = 0.995$ implies a ratio of national debt to annualized GDP of 100 percent. For comparison, the government debt-to-GDP ratio in the euro area in 2020 was 98 percent. Initial net foreign assets equal zero in line with the symmetry assumption. As an initial condition for bonds and reserves, we set $\sum_j (1 + \rho_j Q_j) B^H_{ij,-1} + R_{-1} H_{i,-1} = 2$ for each household i. We can then solve for the variables stated in the definition of a perfect-foresight equilibrium, and, from the household's flow budget constraint (3), we can compute the equilibrium portfolio $\sum_j Q_j B^H_{ijt} + H_{it}$ for each household i in every period $t \geq 0$. We will comment below on the equilibrium evolution of bonds and reserves.

Table 1: **Baseline parameterization**

Parameter	Value	Economic interpretation
n_i	0.5	Relative size country $i = 1, 2$
Y_i	n_i	GDP country i
β	0.995	Subjective discount factor
Π	1	Gross inflation target at union level
ρ_i	0.95	Debt duration country i
ν	0.3	Home bias parameter
θ	2	Trade elasticity parameter
$\tilde{S}_i$	$0.02n_i$	Primary surplus country i, in units of GDP of the union

Baseline equilibrium. Figure 2 plots the equilibrium paths of key variables for the parameterization in Table 1, in the absence of shocks (lines with points). The price level of the union, P_t, and the price level in each country, P_{1t} and P_{2t}, are uniquely determined. P_t is constant over time (and happens to be equal to 1 given the numerical assumptions we have made). Thus, the union inflation rate is time-invariant and equals the central bank's target, $\Pi_t = \Pi = 1$. Furthermore, $P_{it} = P_t$ for each i in every period ($\tilde{P}_{1t} = \tilde{P}_{2t} = 1$). Similarly,

[16] These values of ν and θ are standard in the open economy literature. See also Galí (2015), Chapter 8.

consumption of each household is constant over time, and consumption of household 1 equals consumption of household 2.

In the rest of this section, we consider policy experiments where in each experiment there is a one-time unanticipated shock in period 0. We compare the effects of the shock to the baseline equilibrium (the equilibrium in the absence of shocks).

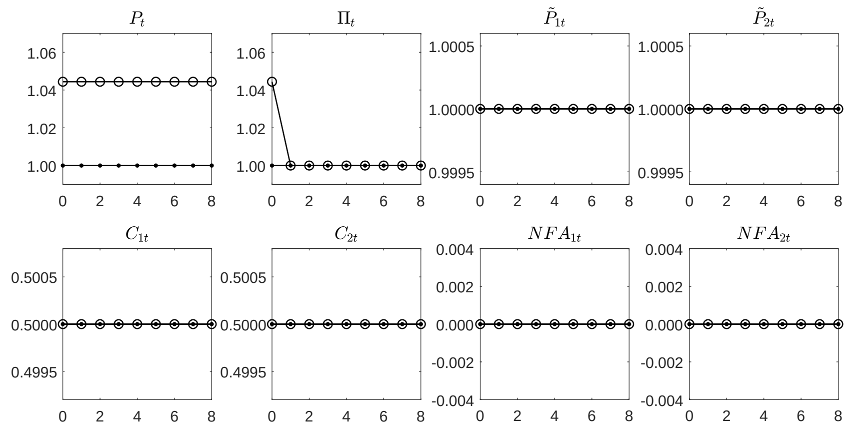
A symmetric fiscal expansion. Suppose that in period 0 the primary surplus of each fiscal authority i falls by an equal amount, $\tilde{S}_{i0} = -0.15n_i$ (a primary deficit of 15 percent of quarterly GDP). This is a symmetric fiscal expansion.[17]

Figure 2 shows the resulting equilibrium (lines with circles). The price level of the union jumps in period 0, $\Pi_0 > \Pi = 1$ (there is no persistent change in inflation because the central bank maintains a constant interest rate on reserves). The relative price levels and consumption are unaffected. A fall in the present value of primary surpluses implies that households' lifetime income net of taxes rises. See the right-hand side of equation (20). Households are wealthier at a given price level, and they attempt to increase consumption. With endogenous output and a nominal rigidity, this wealth effect could raise production in the short run. Here the attempt to increase consumption causes a jump in the union price level such that equation (19) holds with the new, smaller right-hand side. The equilibrium price level of the union is unique because for any other price level households' wealth would grow at rate β^{-1}, eventually violating transversality condition (14). Since the shock is symmetric, there is no change in consumption or the relative price levels (the country price levels jump by the same amount as the union price level). The same intuition helps understand why equilibrium is unique in the baseline (Proposition 1 and the lines with points in Figure 2). Note also that a symmetric fiscal *contraction* has analogous effects.

How much inflation in the union there is depends on the change in the *sum* of primary surpluses of *all* fiscal authorities. The period 0 inverse price level can be expressed analytically, $P_0^{-1} = \beta - (1-\beta)\,7.5 \simeq 1.04^{-1}$, where $7.5 = 0.15/0.02$ is the ratio of the sum of the period 0 primary deficits, $-\sum_i \tilde{S}_{i0}$, to the sum of the steady-state primary surpluses, $\sum_i \tilde{S}_i$. This follows from dividing equation (19) in the equilibrium with the fiscal expansion

[17] We model the fiscal expansion as a "policy shock" only for simplicity; one could model the deficits as a response to "the business cycle" (a non-policy shock that lowers endowments).

Figure 2: **Baseline and symmetric fiscal expansion**



Lines with points: baseline (no shocks). Lines with circles: symmetric fiscal expansion.

by equation (19) in the baseline equilibrium and recalling that $P_0 = 1$ in the baseline. Thus, the magnitude of the wealth effect – and hence also the strength of the inflationary impulse – simply depend on the current aggregate fiscal policy stance in the union relative to the average or steady-state stance.

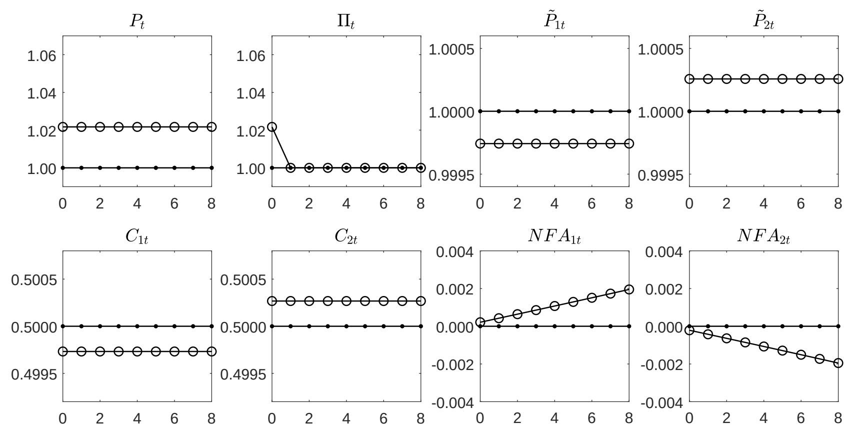
We can also write $P_0^{-1} = 1 - \left(\sum_i \tilde{S}_i - \sum_i \tilde{S}_{i0}\right)/X$, where X is the left-hand side of equation (19), or households' wealth (the real value of their asset holdings), in the baseline equilibrium.[18] This expression shows that to determine the strength of the inflationary impulse, we can simply compare the change in the aggregate fiscal policy stance (the aggregate tax cut or tax increase) with households' initial or steady-state wealth. Think of households as receiving a transfer (or having to pay a higher tax) in period 0 and asking themselves "how large is the transfer (the extra tax) as a fraction of our wealth?"

An asymmetric fiscal expansion. Suppose that in period 0 the surplus of fiscal authority 2 falls by the same amount as before, $\tilde{S}_{20} = -0.15n_2$, while the surplus of fiscal authority 1 remains the same as in the baseline, $\tilde{S}_{10} = 0.02n_1$. This is an asymmetric fiscal expansion.

Figure 3 plots the equilibrium (lines with circles). The price level of the union jumps in period 0, $\Pi_0 > \Pi = 1$, by less than in the case of the symmetric fiscal expansion. From equation (19), $P_0^{-1} = \beta + (1-\beta)\, n_1 - (1-\beta)\, 7.5n_2 \simeq 1.02^{-1}$. The inflationary impulse

[18] As in the previous paragraph, we use the fact that $P_0 = 1$ in the baseline.

Figure 3: **Asymmetric fiscal expansion**



Lines with points: baseline (no shocks). Lines with circles: fiscal expansion in country 2.

depends on the ratio of the period 0 primary deficit to the steady-state primary surplus of the expanding country *and* on the size of the expanding country. Table 2 summarizes the fiscal policy experiments from this section. The table includes an example of the same asymmetric fiscal expansion in which the expanding country is small, $n_1 = 0.95$ and $n_2 = 0.05$. In this case the jump in the price level of the union is small, $P_0^{-1} = \beta + (1-\beta)\, n_1 - (1-\beta)\, 7.5 n_2 \simeq 1.002^{-1}$ (we are holding constant the ratio of the period 0 primary deficit to the steady-state primary surplus of the expanding country).

An asymmetric fiscal expansion changes permanently the relative price levels and consumption (Figure 3). The relative price level in the expanding country 2, $\tilde{P}_2$, rises while the relative price level in country 1, $\tilde{P}_1$, falls. The relative price levels react because consumption changes and there is home bias. Consumption in country 2 rises while consumption in country 1 falls.[19] Household 2 is richer; the real value of its assets has fallen by less than the present value of its taxes. Household 1 is poorer; the real value of its assets has fallen with its taxes unchanged. With home bias, this increases the relative demand for good 2 and decreases the relative demand for good 1. One can make similar statements about the fiscal authorities: The new union price level is "too high" for fiscal authority 1; the real value of its debt has fallen with its surpluses unchanged. The new union price level is "too

[19]The absolute changes in the relative price level and consumption vary inversely with the size of the country. See Table 2.

low" for fiscal authority 2; the real value of its debt has fallen by less than the present value of its surpluses. Again, an asymmetric fiscal *contraction* has analogous effects. In either case, the price level of the union adjusts to make the real value of *all* public debt in the union match the present value of the *sum* of all surpluses.

Table 2: **Summary of fiscal policy experiments from Section 3**

Experiment	Fiscal shock		Effects (permanent, in %)				
(FE = fiscal expansion)	$\tilde{S}_{10}/\tilde{S}_1$	$\tilde{S}_{20}/\tilde{S}_2$	P	$\tilde{P}_1$	$\tilde{P}_2$	C_1	C_2
Symmetric FE	-7.5	-7.5	4.44	0	0	0	0
Asymmetric FE	0	-7.5	2.17	-0.03	0.03	-0.05	0.05
Asymmetric FE, $n_2 = 0.05$	0	-7.5	0.21	-0.00	0.05	-0.00	0.10
FE matched by fiscal contraction	9.5	-7.5	0	-0.05	0.05	-0.11	0.11
Asymmetric FE, $\nu = 0.1, \theta = 0.75$	0	-25	6.95	-1.99	2.03	-0.36	0.35

Note: "Effects" refers to changes relative to baseline (no shocks).

The figures show how net foreign assets evolve.[20] There is no change in net foreign assets after a symmetric shock (Figure 2). After an asymmetric expansion in country 2, the net claims of country 1 on country 2 grow forever (Figure 3); discounted with β, they converge to a positive constant. The real value of public debt in the expanding country 2 exceeds the present value of the country's surpluses. The public sector in country 2 borrows from the public sector in country 1, where the real value of public debt falls short of the present value of the surpluses, and the new debt is rolled over forever. Thus, a wealth transfer from household 1 to household 2 is taking place, via the public sectors, *even though* direct cross-country budgetary transfers are ruled out (equation (4)).

To gain further insight, it is helpful to solve forward the balance-of-payments identity of country 1 in a perfect-foresight equilibrium. In the case of $I = 2$, assuming for simplicity that $B^{CB}_{ijt} = 0$ for $j \neq i$ and $t \geq 0$ (NCB i holds only debt of fiscal authority i), we obtain:

$$\frac{(1 + \rho_2 Q_2)\, B^H_{12t-1} + RT_{12t-1} - (1 + \rho_1 Q_1)\, B^H_{21t-1}}{P_t}$$

[20]To compute the net foreign assets of country i in period $t \geq 0$ we use the equation $NFA_{it} = \tilde{W}_{it} Y_{it} - \tilde{P}_{it} C_{it} + \beta^{-1} NFA_{it-1}$, except that below we adjust the last term appropriately if there is a capital gain or loss due to a shock (there are no such capital gains or losses in the experiments considered so far).

$$= \sum_{k=0}^{T} \beta^k \left(\tilde{P}_{1t+k} C_{1t+k} - \tilde{W}_{1t+k} Y_1 \right) + \beta^T \left(\frac{Q_2 B^H_{12t+T} + T_{12t+T} - Q_1 B^H_{21t+T}}{P_{t+T}} \right).$$

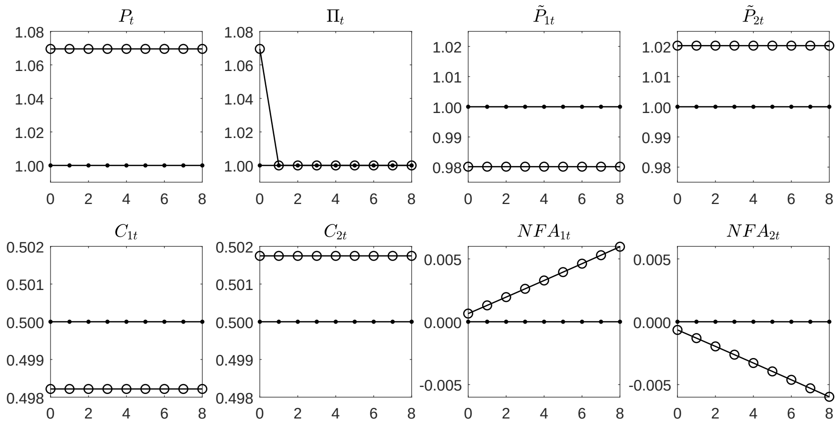
After an asymmetric fiscal expansion in country 2, the limit of the last term as time goes to infinity equals a strictly positive number. Household 1 consumes less than the present value of its endowment, while household 2 consumes more than the present value of its endowment. One feasible outcome is that $\beta^t Q_2 B^H_{12t}$ is strictly positive in the limit (household 1 is lending to fiscal authority 2 and, in parallel, fiscal authority 1 is lending to household 1). Another feasible outcome is that $\beta^t Q_2 B^H_{21t}$ is strictly negative in the limit (household 2 is borrowing from fiscal authority 1 and, in parallel, lending to fiscal authority 2). Yet another feasible outcome is that $\beta^t T_{12t}$ is strictly positive in the limit (NCB 1 is lending to NCB 2 while, in parallel, fiscal authority 1 is making a transfer to NCB 1 and NCB 2 is making a transfer to fiscal authority 2). Each outcome is consistent with the transversality conditions (equation (14) for household 1 and for household 2). While the model does not determine gross asset positions and therefore which outcome occurs, this is irrelevant for the price level of the union, the relative price levels, and consumption in each country, all of which are determinate. We may find it unrealistic that a national fiscal authority will maintain positive net claims on private agents, either domestically or in another country in the union. However, the outcome in which a fraction of net claims between the NCBs is rolled over forever seems perfectly possible. As a reference, at the end of 2020 the national central banks of Germany, Italy, and Spain had net TARGET2 claims equal to 0.34, -0.31, and -0.45 of country GDP, respectively.[21]

A fiscal expansion matched by a fiscal contraction. Suppose that fiscal authority 1 responds to the expansion by fiscal authority 2 by raising its own surplus so as to preserve price stability ($\Pi_0 = \Pi$), $\tilde{S}_{10} = 0.02n_1 + 0.17n_2$ (thus, $\sum_i \tilde{S}_{i0} = 0.02$ as in the steady state). This is a fiscal expansion in country 2 matched by a fiscal contraction in country 1. The absolute changes in the relative price levels and consumption are now greater than before (the cross-country wealth transfer is larger than before). See Table 2.

A symmetric fiscal expansion is "paid for" by unanticipated inflation. An asymmetric

[21] Additional feasible outcomes arise if we abandon the assumption that $B^{CB}_{ijt} = 0$ for $j \neq i$ and $t \geq 0$. For example, NCB 1 could then be lending to fiscal authority 2 with, in parallel, fiscal authority 1 making a transfer to NCB 1.

Figure 4: **Asymmetric fiscal expansion, stronger relative price effects**



Lines with points: baseline (no shocks). Lines with circles: fiscal expansion in country 2.

fiscal expansion is financed in part by unanticipated inflation and in part by taxpayers in the other country. A fiscal expansion matched by a fiscal contraction is paid for *entirely* by taxpayers in the contracting country.

Stronger relative price effects. The effects of an asymmetric fiscal expansion strengthen compared with Figure 3 if there is more home bias, the trade elasticity is smaller, or the fiscal shock is larger. Figure 4 shows an example with $\nu = 0.1$, $\theta = 0.75$, and $\tilde{S}_{20} = -0.5n_2$ (compare with $\nu = 0.3$, $\theta = 2$, and $\tilde{S}_{20} = -0.15n_2$ in Figure 3). In Figure 4 the relative price level in the expanding country 2, $\tilde{P}_2$, rises by as much as 2 percent while the relative price level in country 1, $\tilde{P}_1$, falls also by 2 percent. With a stronger home bias, a given cross-country wealth transfer leads to a larger change in the relative demand for goods. With a smaller trade elasticity, a given change in the relative demand requires a larger movement in the relative prices. And a bigger fiscal shock produces a greater wealth transfer. See also Table 2.

Non-zero initial net foreign assets. In Appendix C, we relax the assumption that initial net foreign assets equal zero. We suppose that in period -1 country 1 is a net creditor of country 2 (household 1 holds some bonds of fiscal authority 2). After the same *symmetric* fiscal expansion as before, there is now a wealth transfer from household 1 (the net creditor) to household 2 (effectively the net debtor). Consumption in country 1 falls while consumption in country 2 rises, permanently, compared with the baseline equilibrium.

The relative price levels change accordingly.[22] The net foreign assets of country 1 jump down and those of country 2 jump up in period 0; they do not grow or shrink forever as in Figures 3-4, because in this equilibrium the real value of public debt matches the present value of the surpluses country by country. Here we observe the usual wealth transfer caused by unanticipated inflation in a model with imperfect risk sharing and nominal debt. The same cross-country wealth transfer would arise if fiscal policy were passive in both countries and the unanticipated inflation resulted from a non-policy shock. Furthermore, after the same *asymmetric* fiscal expansion as before, household 2 gains and household 1 loses *for two reasons*: the standard revaluation effect due to the initial non-zero net foreign asset position, and the wealth transfer due to the benchmark passive-money active-fiscal policy mix. See Appendix C for more details.

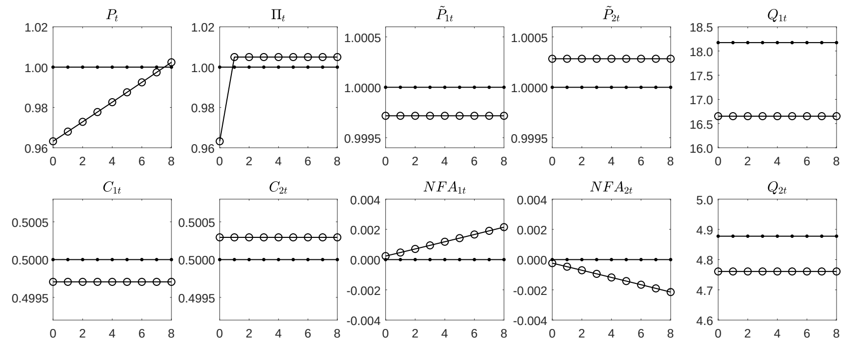
Monetary policy shock. Consider a one-time unanticipated change in the common monetary policy (again, we make the assumption that initial net foreign assets equal zero). In period 0, the central bank raises its inflation target from $\Pi = 1$ to $\Pi = 1.005$. To implement the new inflation objective, the central bank increases the interest rate on reserves from $R = 1/\beta$ to $R = 1.005/\beta$ ($R_t = R$, $t \geq 0$). Equation (22) implies that in the new equilibrium $\Pi_t = 1.005$ for $t \geq 1$. P_0 follows from equation (19): since the bond prices fall at $t = 0$ in expectation of the future inflation, and the bond prices enter the numerator on the left-hand side of equation (19), P_0 (which is in the denominator) *decreases*. At $t = 0$, households (in both countries) are poorer at a given price level and they attempt to lower consumption. The attempt to lower consumption leads to a drop in the union price level.[23] The relative price levels and consumption are unchanged. See Appendix C for a figure.

Figure 5 shows the equilibrium in the case when debt maturity differs by country, $\rho_1 = 0.95$ and $\rho_2 = 0.8$ (vs. $\rho_1 = \rho_2 = 0.95$ before). Now debt of fiscal authority 1 has a longer maturity than debt of fiscal authority 2, and therefore Q_{1t} falls *by more* than Q_{2t} at $t = 0$.

[22] The baseline equilibrium (the equilibrium in the absence of shocks) also changes. Consumption of household 1 (the net creditor) is higher, consumption of household 2 (the net debtor) is lower, the price level in country 1 is higher, and the price level in country 2 is lower compared with the case of zero initial net foreign assets. The price level of the union is unchanged. See Appendix C.

[23] This is a case of the "stepping on a rake" effect from Sims (2011), where an interest rate increase causes disinflationary pressure followed by inflationary pressure.

Figure 5: **Monetary policy shock with asymmetric debt duration**



Lines with points: baseline (no shocks). Lines with circles: increase in monetary policy rate when $\rho_2 < \rho_1$.

P_0 adjusts to make the real value of all public liabilities match the present value of the sum of all surpluses, but now the new union price level is "too high" for fiscal authority 1 and "too low" for fiscal authority 2. Consumption and the relative price level in country 1 fall while consumption and the relative price level in country 2 rise, permanently. The mechanism is the same as after an asymmetric fiscal shock.[24]

The general principle is that under the benchmark passive-money active-fiscal policy configuration *if* a shock moves the price level of the union, the bond prices, and the present values of the surpluses in such a way that the real value of public debt fails to match the present value of the surplus country by country, *then* the shock causes a cross-country wealth transfer via the public sectors, even if initial cross-country asset positions equal zero.

National debt and the central bank's balance sheet. We can compute the evolution of bond and reserve holdings in equilibrium if we make some assumptions about the specifics of balance sheet policy. To calculate the path of debt of fiscal authority i, B_{it}, we can use the solution for the variables stated in the definition of a perfect-foresight equilibrium (consumption levels and prices) and the flow budget constraint of fiscal authority i, equation (4); we also need to make an assumption about how remittances are set.[25] Next,

[24]In this experiment, unlike in the previous experiments, initial gross foreign asset positions matter, even if net positions equal zero, because the two bond prices react differently to the shock. To focus on the channel of interest here, we assume that initial gross foreign asset positions equal zero.

[25]In the baseline model, the central bank does not earn seigniorage. The initial capital of NCB i equals $\sum_j (1+\rho_j Q_j) B^{CB}_{ij,-1} + R_{-1}T_{ij,-1} - R_{-1}H_{i,-1}$. As an example, if we assume $\sum_j (1+\rho_j Q_j) B^{CB}_{ij,-1} = R_{-1}H_{i,-1}$ and $T_{ij,-1} = 0$ for each i (i.e., the initial capital of NCB i equals zero), we can simply specify that

to compute the path of the central bank's bond holdings, we assume that the common monetary authority chooses bond holdings B_{it}^{CB} for each i and $t \geq 0$ and instructs the NCBs to implement that path of bond holdings. As an example, consider the policy rule "$B_{it}^{CB} = \max(\delta B_{it}, 0)$, where $\delta \in (0,1)$ is a parameter, and NCB i holds only debt of fiscal authority i ($B_{ijt}^{CB} = 0$ for $i \neq j$ in every period)." Thus, the central bank holds a fraction δ of government bonds (we use the max operator because we have not restricted B_{it} to be non-negative while reserves must be non-negative).[26] Finally, we can calculate the path of $\sum_i H_{it}$ from the flow budget constraint of the common monetary authority, equation (5).[27]

The central bank holds private debt. Suppose the central bank holds private debt and does not hold government bonds ($B_{ijt}^{CB} = 0$ for each i, j, and t). Specifically, NCB i makes one-period loans to household i at the interest rate R. Let D_i denote debt of household i held by the central bank. The equilibrium conditions are the same as before, except that in each equation where initial *net* claims of household i on the public sector appear one must subtract $RD_{i,-1}$. In particular, one must subtract $R\sum_i D_{i,-1}$ from the numerator on the left-hand side of equation (19). If we adjust the initial conditions appropriately (household i owns the government bonds previously held by the central bank, but household i is also indebted to the central bank so that the household's net assets are unchanged), the baseline equilibrium paths of the price level of the union, the relative price levels, and consumption in each country are *the same* as before. It is sometimes claimed that active fiscal policy requires the central bank to "monetize" government bonds in equilibrium. This claim is incorrect. The central bank need not hold any government bonds in equilibrium.

remittances satisfy $Z_{it} = 0$ for each i and $t \geq 0$.

[26] For instance, we can set $\delta = 0.25$ (the Eurosystem purchased, via the APP and PEPP, about 25 percent of national public debt in the euro area by the end of 2020). One could make δ time-varying. The assumption "NCB i holds only debt of fiscal authority i" is also consistent with these two asset purchase programs.

[27] We need to make another assumption to compute the path of reserves for each household i (each NCB i), H_{it}, separately (as opposed to the sum $\sum_i H_{it}$). For example, if we suppose that $T_{ijt} = 0$ for each i, j and $t \geq 0$ (no net claims between the NCBs), we can calculate H_{it} for each i and $t \geq 0$ from the flow budget constraint of NCB i, equation (18), and we can compute $\sum_j B_{ijt}^H$ for each i and $t \geq 0$ from the flow budget constraint of household i, equation (3). Even then the model determines the sum $\sum_j B_{ijt}^H$ but not $B_{i1t}^H, \ldots, B_{iIt}^H$ individually. To pin down all asset holdings, in future work one could assume that reserves and government bonds provide liquidity services in different amounts (and bonds of fiscal authority i provide a different convenience yield to household i than to household j).

Summary of Section 3. In a monetary union, the benchmark passive-money active-fiscal policy mix determines a unique price level for the union and a unique price level for each country – there is fiscal backing for price stability. Fiscal expansions or contractions move the price level of the union. When they are asymmetric, they also *necessarily* change relative price levels and shift wealth between the countries in the union. Even a completely symmetric shock to the common monetary policy does the same, except in the knife-edge case when public debt has identical maturity in each country.

The benchmark passive-money active-fiscal policy mix may be a realistic, simple description of some episodes. However, since cross-country wealth transfers and inflation differentials are part and parcel of this policy configuration, we doubt that it is a sustainable way to provide the fiscal backing.

4 A proposal to implement active fiscal policy in a monetary union

In a monetary union, there are multiple fiscal authorities and different combinations of their policies can amount to active fiscal policy for the union as a whole. In this section, we propose a configuration that we think has desirable properties and therefore may be able to provide the fiscal backing sustainably. We are motivated by the idea that there may be a special role in this task for a common fiscal authority. We maintain from Section 3 the assumption that the central bank pegs the interest rate on reserves, $R = \Pi/\beta$.

National fiscal authorities. In contrast to Section 3, each national fiscal authority i sets its primary surplus according to a *standard passive* feedback rule

$$\tilde{S}_{it} = \phi_{it} + \phi_B \frac{Q_{it-1}B_{it-1}}{P_{t-1}} \tag{26}$$

where ϕ_B satisfies the usual condition for passive fiscal policy, $0 < \beta^{-1} - \phi_B < 1$. The intercept in equation (26) is time-invariant, $\phi_{it} = \phi_i < 0$, except that in period 0 the intercept may be subject to a one-time unanticipated shock. This is a simple way to model a fiscal expansion or contraction, analogous to Section 3 (one could introduce a business cycle shock and a response of the surplus to that shock). Since ϕ_B is strictly greater than the steady-state net real interest rate $\beta^{-1}-1$, the real value of debt $Q_{it}B_{it}/P_t$ converges to a constant, $-\phi_i/\left(\phi_B+1-\beta^{-1}\right)>0$, from any initial condition regardless of the path of the price level.

Common fiscal authority. We introduce a common fiscal authority that issues "Eurobonds" and faces the flow budget constraint

$$(1+\rho Q_t)\,F_{t-1}=\sum\nolimits_i W_{it}S_{it}^F+Q_tF_t \tag{27}$$

where F denotes Eurobonds, Q_t is the period t price of a Eurobond, $\rho\in(0,1)$ is a parameter that governs the duration of Eurobonds, and S_{it}^F is a lump-sum tax collected by the common authority from household i in period t (S_{it}^F can be strictly negative, in which case it is a lump-sum transfer). Eurobonds are held by households and may be held by the central bank; in general, $F_t=\sum_i F_{it}^H+\sum_i F_{it}^{CB}$, where F_{it}^H are Eurobonds held by household i and F_{it}^{CB} are Eurobonds held by NCB i in period t. It is convenient to define $\tilde{S}_{it}^F\equiv\tilde{W}_{it}S_{it}^F$ and $\tilde{S}_t^F\equiv\sum_i\tilde{S}_{it}^F$.

The common fiscal authority follows a policy that consists of two ingredients. First, $\tilde{S}_{it}^F=n_i\tilde{S}_t^F$ for each i in every period, which implies that the common tax or transfer is always *symmetric* by country – the common authority does not make cross-country budgetary transfers. Second, the common authority follows the feedback rule

$$\tilde{S}_t^F=\phi_t^F-\phi_B\frac{\sum_i Q_{it-1}B_{it-1}}{P_{t-1}} \tag{28}$$

where the intercept is time-invariant, $\phi_t^F=\phi^F$, except that in period 0 the intercept may be subject to a one-time unanticipated shock. Equations (26) and (28) imply that $\sum_i\tilde{S}_{it}+\tilde{S}_t^F=\sum_i\phi_{it}+\phi_t^F$ for each t. Thus, the sum of the surpluses, including the common surplus, is independent of debt which makes fiscal policy active at the level of the union. This follows because the common policy "leans against" the response of the member countries to their debt. When the national surpluses rise *in response* to higher national debt, the common policy expands. When the national surpluses fall *in response* to lower national debt, the common policy contracts. The "leaning-against-the-national-policies" effect can be modest: the value of ϕ_B can be just above $\beta^{-1}-1$, and therefore fluctuations in the common surplus can be small or moderate in relation to GDP of the union.[28] Moreover,

[28] This statement is simplified because we assume the same coefficient on debt, ϕ_B, in feedback rule (26)

there is no particular requirement for the size of the steady-state common surplus.[29]

It is straightforward to add the common fiscal authority to the model. For example, in the flow budget constraint of household i (equation (3)) we add the term $(1+\rho Q_t)\, F^H_{it-1}$ on the left-hand side and the term $Q_t F^H_{it}$ on the right-hand side. Similarly, in the flow budget constraint of NCB i (equation (18)) we subtract the term $(1+\rho Q_t)\, F^{CB}_{it-1}$ on the left-hand side and the term $Q_t F^{CB}_{it}$ on the right-hand side. The transversality condition of household i now reads

$$\lim_{T\to\infty} E_t\left[\Theta_{itT}\left(\frac{\sum_j Q_{jT}B^H_{ijT}+Q_T F^H_{iT}+H_{iT}}{P_T}\right)\right]=0,$$

instead of equation (14). The definition of a perfect-foresight equilibrium is the same as in Section 3, except that: (i) equation (19) changes to

$$\frac{(1+\rho Q_0)\sum_i F^H_{i,-1}+\sum_j(1+\rho_j Q_{j0})\left(\sum_i B^H_{ij,-1}\right)+R_{-1}\sum_i H_{i,-1}}{P_0}=\sum_{k=0}^{\infty}\beta^k\left(\sum_i \tilde{S}_{ik}+\tilde{S}^F_k\right), \tag{29}$$

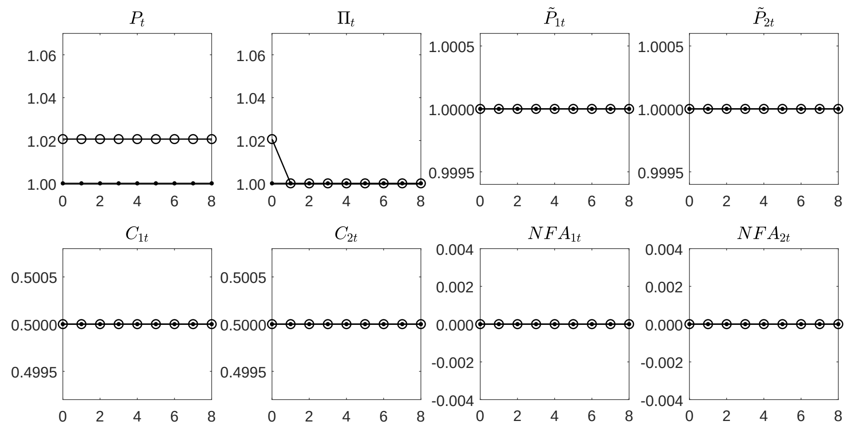
(ii) equation (20) changes in an analogous way, (iii) there is a new equilibrium condition for pricing Eurobonds, analogous to equation (23), and (iv) initial conditions $F^H_{i,-1}$ for each i must be specified.

Parameterization. Suppose that the steady-state common primary surplus equals 5 percent of the sum of the national surpluses, $\tilde{S}^F=0.05\sum_i \tilde{S}_i=0.001$ (recall from Table 1 that $\tilde{S}_i=0.02n_i$). Accordingly, the ratio of Eurobonds to annual GDP of the union equals 5 percent. For comparison, in 2020 the member countries of the European Union authorized the European Commission to issue up to 750 billion euros of debt while GDP of the euro area was about 12 trillion euros in 2019. We set $\phi_B=0.05$, which implies a half-life of a deviation of the real value of national debt from steady-state of about 4 years. We make other numerical assumptions such that the equilibrium in the absence of shocks is identical to the baseline equilibrium in Section 3 (except that households pay an additional

for each country, but a similar statement could be made if the feedback rule had a different coefficient on debt by country so long as none of the coefficients was much greater than $\beta^{-1}-1$. The required changes in the common surplus in response to a shock become larger with the size of these coefficients and with the magnitude of the shock.

[29] We assume that $\sum_i \phi_i+\phi^F>0$ to guarantee that the sum of the national surpluses and the common surplus is strictly positive in the steady state.

Figure 6: **Asymmetric fiscal expansion with Eurobonds**



Lines with points: baseline (no shocks). Lines with circles: fiscal expansion in country 2.

lump-sum tax to the common fiscal authority).[30] We also assume in the numerical analysis that Eurobonds have the same duration as national debt ($\rho = 0.95$) and they are all held by households.

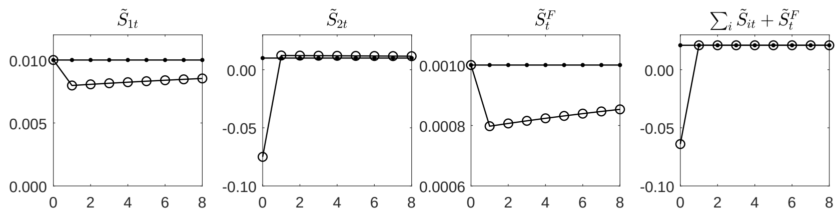
A symmetric fiscal expansion. Consider a one-time unanticipated shock to each ϕ_{i0} and ϕ_0^F such that $\tilde{S}_{i0} = -0.15n_i$, $i = 1, 2$, and $\tilde{S}_0^F = -0.0075$. This is the familiar symmetric fiscal expansion; in particular, the ratio of the period 0 primary deficit to the steady-state primary surplus, for each fiscal authority, is the same as before, $-\tilde{S}_{i0}/\tilde{S}_i = 0.15n_i/0.02n_i = 7.5$, $i = 1, 2$, and $-\tilde{S}_0^F/\tilde{S}^F = 0.0075/0.001 = 7.5$. We find that the equilibrium is *identical* to Section 3 (Figure 2, lines with circles).

An asymmetric fiscal expansion. Consider a one-time unanticipated shock to ϕ_{20} such that $\tilde{S}_{20} = -0.15n_2$. This is the same asymmetric fiscal expansion as in Section 3 (Figure 3). Figure 6 shows the equilibrium (lines with circles), which is *very different* from Section 3: there is now no cross-country wealth transfer, and there are no changes in the relative price levels or consumption.[31] In period 0, the real value of debt of country 2 rises (country 2 issues more debt than the unanticipated inflation "pays for") while the real value

[30] In particular, we choose the value of ϕ_i (equation (26)) appropriately.

[31] The increase in P_0 is somewhat smaller than in Section 3, because a given deficit reduces the sum of the surpluses by less when the common fiscal authority contributes to the sum. See Table 3 which summarizes the fiscal policy experiments from this section.

Figure 7: **Asymmetric fiscal expansion with Eurobonds**



Lines with points: baseline (no shocks). Lines with circles: fiscal expansion in country 2.

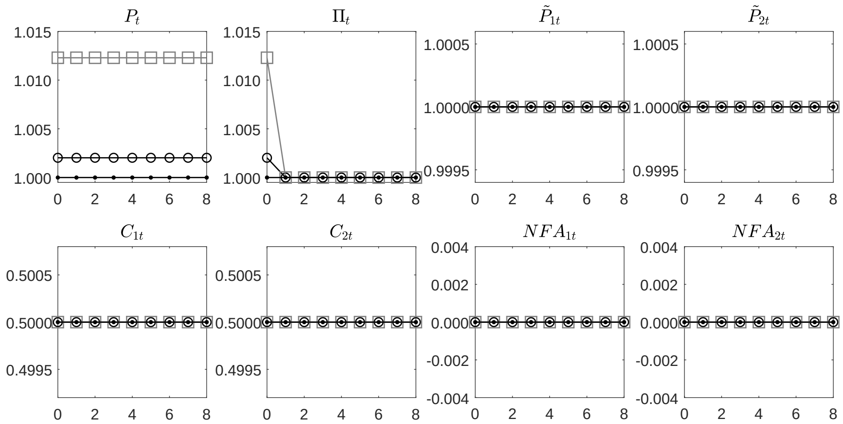
of debt of country 1 falls. This is the same outcome as after the asymmetric fiscal expansion in Section 3. The subsequent developments, however, stand in contrast to Section 3: the surplus in country 2 rises above its steady-state value for some time, following feedback rule (26); in parallel, the surplus in country 1 *and* the common surplus decrease. See Figure 7. The fall in the surplus of country 1 is a response to the decrease in the real value of national debt, consistent with feedback rule (26). The fall in the common surplus reflects the leaning against the response of the member countries to their debt, following feedback rule (28). To emphasize, the cut in the common surplus is *symmetric* by country – it is *not* a transfer to country 2 – and its purpose is to maintain an appropriate fiscal policy stance for the union as a whole.

If the common authority or country 1, or both, tighten in period 0 in response to the expansion in country 2 ($\phi_0^F > \phi^F$ or $\phi_{10} > \phi_1$, or both), then the period 0 inflation rate falls relative to Figure 6 and there is still no cross-country wealth transfer.

Following the financial crisis of 2008, after an initial period when fiscal policies of the euro area member countries were expansionary, the fiscal policies turned contractionary in response to higher national debt levels, which arguably weakened the recovery and contributed to the 2012-13 recession.[32] With this policy configuration, the aggregate euro area fiscal policy stance could have remained more supportive of the recovery without affecting the responsibility of the member countries to stabilize their own debt. In the future, the opposite situation may arise in which national fiscal policies loosen, despite high and rising inflation, in response to declining national debt-to-GDP ratios. With this policy configu-

[32] Corsetti et al. (2019), p. 23, write: "The primary budget balance for the euro area as a whole improved in each year between 2009 and 2015, from 3.5 percent of GDP in 2009 to 0.3 percent of GDP in 2015, including in 2012 and 2013, two years in which euro area output contracted."

Figure 8: **Expansion by common fiscal authority**



Lines with points: baseline (no shocks). Lines with circles: expansion by the common fiscal authority. Lines with squares: same expansion with a higher steady-state primary surplus.

ration, the aggregate euro area fiscal policy stance would then remain more supportive of price stability.

An expansion by the common fiscal authority. Consider a one-time unanticipated shock to ϕ_0^F such that $\tilde{S}_0^F = -0.0075$ (the ratio of the period 0 common deficit to the steady-state common surplus is $-\tilde{S}_0^F/\tilde{S}^F = 7.5$). This is an expansion by the common authority on its own. Figure 8 shows the equilibrium (lines with circles). The inflationary impulse for the union depends on the change in the sum of primary surpluses of all fiscal authorities, the common authority *and* the national authorities. See equation (29). Here, $P_0^{-1} = \beta + (1-\beta)\,0.0125/0.021 = 1.002^{-1}$ because $\tilde{S}_0^F + \sum_i \tilde{S}_i = 0.0125$ and $\tilde{S}^F + \sum_i \tilde{S}_i = 0.021$. The inflationary impulse (0.2 percent) is *much smaller* than from the symmetric fiscal expansions considered before (where it exceeded 4 percent). The reason is that the steady-state common surplus is small, 5 percent of the sum of the national surpluses.

Additional Eurobonds could be issued and common taxes raised in the steady state. In the real world, however, an increase in steady-state taxation would be distortionary (and we would be contemplating a sizable increase in steady-state taxation). Consider a different route, without a change in steady-state taxation. Suppose that additional Eurobonds, worth 25 percent of annual GDP of the union, are issued and swapped with the central bank against its holdings of national debt (previously, we assumed that the central bank holds

25 percent of national debt (footnote 26) and the national debt-to-GDP ratios equal 100 percent). Furthermore, coincident with the bond swap, the common fiscal authority acquires the right to tax directly in order to back the newly issued Eurobonds while the national fiscal authorities stop making payments on their bonds held by the common authority. The common surplus increases to $\tilde{S}^F = 0.001 + 0.005 = 0.006$, where 0.005 is 25 percent of the sum of the national surpluses before the swap, and the national surpluses decrease by 25 percent. The equilibrium in the absence of shocks is unchanged, except that Eurobonds equal 30 percent instead of 5 percent and national debt equals 75 percent instead of 100 percent, as a ratio of annual GDP.[33] Suppose that $\tilde{S}_0^F = -0.045$ (again, the ratio of the period 0 common deficit to the steady-state common surplus is $-\tilde{S}_0^F/\tilde{S}^F = 7.5$). Figure 8 shows the equilibrium (lines with squares). The inflationary impulse becomes *much stronger*, $P_0^{-1} = \beta - (1-\beta)\, 0.03/0.021 \simeq 1.012^{-1}$ (1.2 percent vs. 0.2 percent before). The reason is that the sum of the period 0 surpluses of all fiscal authorities is much smaller, $\tilde{S}_0^F + \sum_i \tilde{S}_i = -0.03$, for the same ratio of the period 0 common deficit to the steady-state common surplus.

We do not think it essential that the common fiscal authority be able to provide a non-trivial inflationary or deflationary impulse by itself. If this feature was considered desirable, however, we conclude that the common authority would have to be sizable (its steady-state primary surplus would have to be sizable).

Table 3: **Summary of fiscal policy experiments from Section 4**

Experiment	Fiscal shock			Effects (permanent, in %)
(FE = fiscal expansion)	$\tilde{S}_{10}/\tilde{S}_1$	$\tilde{S}_{20}/\tilde{S}_2$	$\tilde{S}_0^F/\tilde{S}^F$	P
Symmetric FE	-7.5	-7.5	-7.5	4.44
Asymmetric FE	0	-7.5	0	2.07
FE by common authority	0	0	-7.5	0.20
same, more Eurobonds	0	0	-7.5	1.23

Note: Changes relative to baseline (no shocks); the relative price levels and consumption do not change.

Monetary policy shock. Reconsider the same monetary policy shock as in Section 3 in the case when debt maturity differs by country. In this policy configuration, in contrast

[33] We decrease the value of ϕ_i (equation (26)) by 25 percent for each i.

to Section 3, the shock has *no effect* on consumption or the relative price levels. There is no cross-country wealth transfer because fiscal rule (26) guarantees that the present value of the surplus in each country matches the real value of the country's public debt. See Appendix C for a figure.

Summary of Section 4. The policy configuration proposed here determines a unique price level for the union and a unique price level for each country, *even though* each national fiscal authority follows a standard passive feedback rule. Moreover, asymmetric fiscal expansions or contractions do *not* cause cross-country wealth transfers or inflation differentials. The same is true of monetary expansions or contractions. Therefore, this policy configuration may be a sustainable way to provide fiscal backing for price stability in a monetary union.

The policy configuration has other potentially desirable properties. The common fiscal authority can be modest in size, and it does not make cross-country budgetary transfers. Symmetric fiscal interventions produce inflationary or deflationary impulses at the level of the union, even though national fiscal policies follow a standard passive feedback rule. Such interventions, which the common authority can coordinate, may be appropriate in the real world in response to symmetric non-policy shocks. Finally, asymmetric fiscal interventions also cause, to some extent, inflationary or deflationary impulses at the level of the union. This leaves some room for national fiscal policy to respond to asymmetric non-policy shocks at the same time as cross-country wealth transfers are ruled out.

Comment. We have modeled the common fiscal authority as being able to tax directly, as opposed to having to rely on a commitment by the member countries to raise some additional tax revenue and hand it over to the common authority. While the distinction has little consequence in this model, in practice the arrangement would likely be more stable if the common authority had some direct ability to tax. We have also assumed that the common taxes and transfers are symmetric by country. The symmetry is inessential from the viewpoint of price level determination for the union, but in its absence cross-country budgetary transfers would take place via the common authority.

5 Other ways to implement active fiscal policy in a monetary union

In this section, we consider additional ways to implement active fiscal policy in a monetary union. Throughout, we continue to assume that the central bank pegs the interest rate on reserves, $R = \Pi/\beta$.

5.1 The Sims rule

We return to the setup without a common fiscal authority as in Section 3. We assume that each national fiscal authority follows a feedback rule similar to Sims (1997), Section VI. Specifically, fiscal authority $i = 1, 2$ sets its primary surplus according to

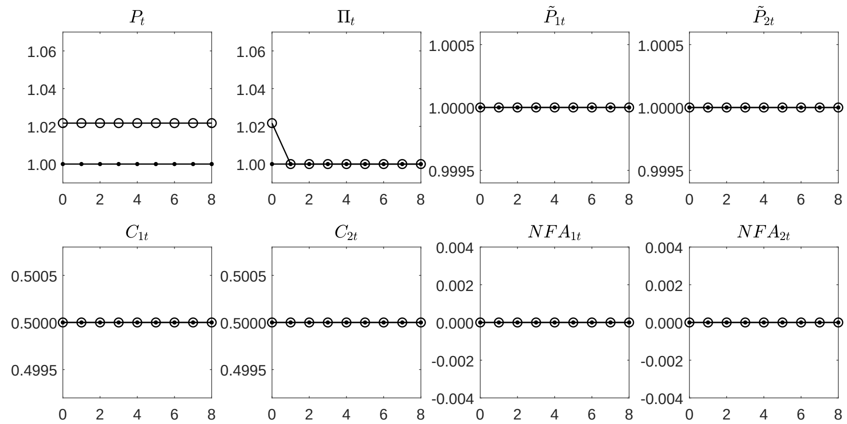
$$\tilde{S}_{it} = \phi_{it} + \phi_B \left(\frac{Q_i B_{it-1}}{P_{t-1}} - \eta_i \frac{\sum_j Q_j B_{jt-1}}{P_{t-1}} \right) \tag{30}$$

where $\eta_i > 0$, $\sum_i \eta_i = 1$, and $0 < \beta^{-1} - \phi_B < 1$. The intercept in equation (30) is time-invariant, $\phi_{it} = \phi_i > 0$, except that in period 0 the intercept may be subject to a one-time unanticipated shock. The Sims rule requires each fiscal authority to respond, not to the real value of its own debt, but to its deviation from an intended share in the sum for the union. Equation (30) implies that $\sum_i \tilde{S}_{it} = \sum_i \phi_{it}$ for each t, which means that the sum of the surpluses is independent of debt. We set $\eta_i = \left(Q_i B_{i,-1} / \sum_j Q_j B_{j,-1} \right)$, $\phi_i = 0.02 n_i$, $i = 1, 2$, and $\phi_B = 0.05$. The equilibrium in the absence of shocks is then identical to the baseline equilibrium in Section 3 (Figure 2, lines with points).

A symmetric fiscal expansion. Suppose that $\phi_{i0} = -0.15 n_i$, $i = 1, 2$. This is the same symmetric fiscal expansion as in Section 3. We find that the equilibrium is identical to Section 3 (Figure 2, lines with circles).

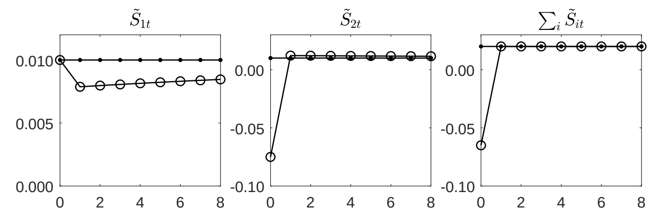
An asymmetric fiscal expansion. Suppose that $\phi_{20} = -0.15 n_2$. This is the same asymmetric fiscal expansion as in Section 3 (Figure 3). Figure 9 shows the equilibrium (lines with circles), which is very different from Section 3 and very similar to Section 4. The surplus in country 2 rises for some time, starting in period 1, to pay for a part of the period 0 expansion. In parallel, the surplus in country 1 falls, as country 1 responds to the expansion in country 2 by expanding itself. See Figure 10. Consequently, in Figure 9 the

Figure 9: **Asymmetric fiscal expansion with the Sims rule**



Lines with points: baseline (no shocks). Lines with circles: fiscal expansion in country 2.

Figure 10: **Asymmetric fiscal expansion with the Sims rule**



Lines with points: baseline (no shocks). Lines with circles: fiscal expansion in country 2.

price level of the union follows the same path as in Figure 3, but no cross-country wealth transfer occurs. If country 1 tightens in period 0 in response to the expansion by country 2 ($\phi_{10} > \phi_1$), then the period 0 inflation rate falls relative to Figure 9 and there is still no cross-country wealth transfer.

Discussion. The Sims rule is attractive. Fiscal policy in the union as a whole is active, implying price level determinacy, and, at the same time, asymmetric deficits and surpluses do not cause cross-country wealth transfers. However, feedback rule (30) is quite different from standard passive fiscal policy. Feedback rule (30) requires a country to vary its primary surplus in response to a surplus or deficit in another country *even if* the real value of the country's own debt is unchanged. Furthermore, in the euro area with nineteen countries, *all nineteen* national fiscal authorities would need to respond appropriately to one another's budget balance.

5.2 One active fiscal agent

We consider the case when a single fiscal authority maintains a constant primary surplus and the other fiscal authorities are passive.

Suppose that the single active fiscal agent is the common fiscal authority. We are in the setup of Section 4, except that the common authority maintains a constant primary surplus, $\tilde{S}^F_t = \tilde{S}^F > 0$. We set $\tilde{S}^F = 0.001$ (Eurobonds equal 5 percent of annual GDP of the union). The equilibrium in the absence of shocks is identical to the baseline equilibrium in Section 4. There is a unique price level for the union and a unique price level in each country, even though only a single fiscal agent is active.

An expansion by the active fiscal agent. Consider an expansion by the common fiscal authority. In period 0 the common surplus falls, $\tilde{S}^F_0 = -0.0075$ (the ratio of the period 0 common deficit to the steady-state common surplus is $-\tilde{S}^F_0/\tilde{S}^F = 7.5$). The inflationary effect is the same as in the *symmetric* fiscal expansion from Section 3 (Figure 2, lines with circles), $P_0^{-1} = \beta - (1-\beta)\,\tilde{S}^F_0/\tilde{S}^F = \beta - (1-\beta)\,7.5 \simeq 1.04^{-1}$. The inflationary impulse depends only on the *ratio* of the period 0 deficit to the steady-state surplus *of the common authority*; the inflationary impulse is independent of the size of the common fiscal authority (of its steady-state primary surplus). For any other P_0, households' holdings of Eurobonds rise (or fall) in real terms at rate β^{-1}, eventually violating transversality condition (14), regardless of the initial size of such holdings in relation to initial households' wealth.

This result is more general in the sense that the single active fiscal agent can also be a national fiscal authority. Return to the model without a common fiscal authority as in Section 3. Suppose that fiscal policy in country 2 is active and fiscal policy of country 1 is passive. Specifically, fiscal authority 2 follows the same policy as in Section 3 ($\tilde{S}_{2t} = \tilde{S}_2 > 0$) while fiscal authority 1 follows feedback rule (26). In period 0 the surplus of fiscal authority 2 falls, $\tilde{S}_{20} = -0.15 n_2$ (the ratio of the period 0 deficit to the steady-state surplus is $-\tilde{S}_{20}/\tilde{S}_2 = 0.15 n_2/0.02 n_2 = 7.5$). This is the same asymmetric fiscal expansion as in Section 3. The inflationary effect is identical to the *symmetric* fiscal expansion from Section 3 (Figure 2, lines with circles), $P_0^{-1} = \beta - (1-\beta)\,\tilde{S}_{20}/\tilde{S}_2 = \beta - (1-\beta)\,7.5 \simeq 1.04^{-1}$. The inflationary impulse depends only on the *ratio* of the period 0 deficit to the steady-state surplus *of the active fiscal authority*; the inflationary impulse is independent of country size (of the steady-state primary surplus of the active fiscal authority).

Discussion. We are uncomfortable with "one active fiscal agent" as a policy prescription. Outside of Section 5.2, the wealth effect of fiscal policy always depends on households' wealth *as a whole* (in Section 3, we showed that to determine the effect of fiscal policy on the union price level, we can compare the change in the aggregate fiscal policy stance with the initial or steady-state wealth of households in the union). In Section 5.2, by contrast, households react to a component of their wealth that can be arbitrarily small (the result from Section 3 does not apply). Households recognize that even an arbitrarily small component of their wealth will *eventually* become very large if they fail to adjust spending – and they adjust spending appropriately without delay, which causes inflation or deflation throughout the union. In the future, it will be interesting to study how robust this result is to removing the standard assumption of perfect forward-looking behavior. We conjecture that without perfect forward-looking behavior, the union price level will respond with delay under this policy configuration unless Eurobonds are a sizable component of households' wealth to begin with. The case when the single active fiscal agent is a national fiscal authority is even less attractive, because then a single country determines on its own the price level in the entire union – something the other countries are unlikely to accept.[34]

6 Extensions

In this section, we present several extensions of the model.

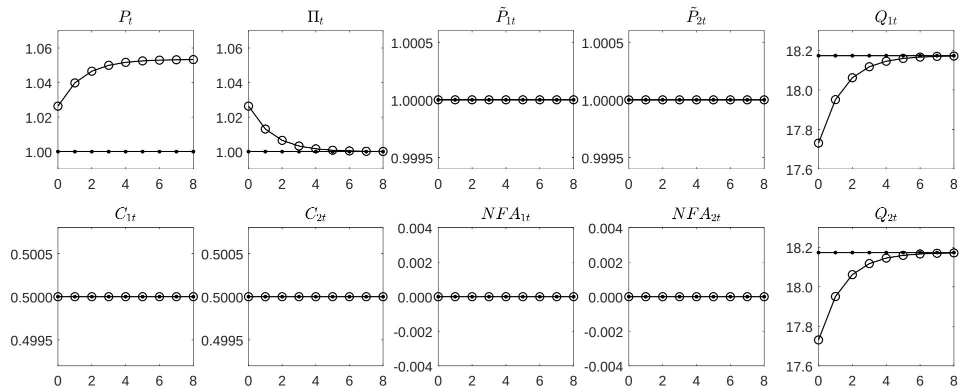
6.1 An interest-rate feedback rule

Suppose that instead of pegging the interest rate, the common monetary authority follows a Taylor-type feedback rule, $R_t = \max\left\{\left(\Pi^{1-\alpha}/\beta\right)\Pi_t^{\alpha}, 1\right\}$, where $\alpha \leq 1$ so that monetary policy remains passive.[35] Fiscal policy is as in Section 3. Figure 11 shows the effects of the same symmetric fiscal expansion as in Section 3 (Figure 2) with identical numerical

[34]With $I > 2$ countries, one could also consider the case in which multiple but not all countries pursue a constant-surplus policy. On backward-looking behavior in a fiscal-theory model, see Sims (2016).

[35]The interest rate feedback rule includes a lower bound, which is formally justified in the version of the model with currency that does not pay interest (Section 6.3).

Figure 11: **Symmetric fiscal expansion with an interest rate feedback rule**



Lines with points: baseline (no shocks). Lines with circles: symmetric fiscal expansion.

assumptions and $\alpha = 0.5$. There is now a persistent increase in inflation, and the bond prices decrease. Since the adjustment involves both a period 0 price level jump and a fall in the bond prices, the initial price level jump is smaller than in Section 3. Furthermore, the initial price level jump is smaller if debt maturity is longer. This can be either because the central bank holds a smaller fraction of government bonds than in Section 3 (and therefore households hold more government bonds and fewer reserves) or because the parameter ρ_i is larger (Table 1). Another interesting feature of this equilibrium is that the central bank makes a capital loss in period 0. The central bank has long-term assets and short-term liabilities, and, since the period 0 shock causes persistent inflation, the real value of the assets falls more than the real value of the liabilities.[36]

6.2 Default risk

In this subsection, we study the effects of partial default by a national fiscal authority. To make the analysis more interesting, we abandon perfect foresight and suppose that agents attach a strictly positive probability to default in the future, and then in one state of the

[36]Previously, we assumed that remittances equal zero in every period (footnote 25). We must use a different remittance rule here. Let $\tilde{Z}_{it} \equiv \tilde{W}_{it} Z_{it}$. From any period in which there is a shock (here period 0), we can assume a constant remittance $\tilde{Z}_i$ for each NCB i such that the present value of remittances equals that period's after-the-shock value of the NCB's assets minus liabilities. See Appendix B for more details. Note that the present value of remittances can be negative.

world default occurs in equilibrium while in another state of the world default is avoided. The central bank pegs the interest rate on reserves as in Section 3.

Setup. To introduce default into the model, think of an agent who enters period $t \geq 0$ holding one unit of a bond of national fiscal authority j. If default occurs in period t, the bond "shrinks" to Δ_{jt} where $\Delta_{jt} \in (0,1)$ ($\Delta_{jt} = 1$ if there is no default). The bond price Q_{jt} now reflects default risk: the term $1 + \rho_j Q_{jt+1}$ inside the expectation operator on the right-hand side of optimality condition (12) gets multiplied by Δ_{jt+1}. Each term in the sum $\sum_j (1 + \rho_j Q_{jt})$ in the numerator on the left-hand side of equation (17) gets multiplied by Δ_{jt}: if there is a shock to the right-hand side of this equilibrium condition, the adjustment can involve a combination of a price level jump, default, and a change in bond prices due to expected future inflation or expected future default.

A future asymmetric fiscal expansion with default risk. The two national fiscal authorities run a constant-surplus policy as in Section 3 (for simplicity, there is no common fiscal authority). Let us modify the asymmetric fiscal expansion from Section 3 as follows. In period 0, a news shock arrives and agents learn that in period $T \geq 1$ the surplus in country 2 will fall, $\tilde{S}_{2T} = -0.3n_2$ (this is a deficit twice the size of the deficit in Section 3). Let $k \in \{D, N\}$ denote the state in period T, where D is the state in which fiscal authority 2 defaults and N is the state in which it does not default. The probability of state D is $d \in (0,1)$. In period T all uncertainty is resolved. Equilibrium condition (17) in period T in state k can be written

$$\frac{(1+\rho_1 Q_1)\sum_i B^H_{i1,T-1} + (1+\rho_2 Q_2)\,\Delta_k \sum_i B^H_{i2,T-1} + R\sum_i H_{iT-1}}{P_{T|k}} = \sum_{t=T}^{\infty} \beta^{t-T} \sum_i \tilde{S}_{it} \quad (31)$$

where $P_{T|k}$ is the price level of the union in period T in state k, $\Delta_k \equiv \Delta_{jt|k}$ for $j = 2$, $t = T$, and state k (hence, $\Delta_k = \Delta_D \in (0,1)$ in state D and $\Delta_k = \Delta_N = 1$ in state N), and Q_i is the price of bond i in periods $t \geq T$ for $i = 1, 2$ (Q_i does not depend on k because the interest rate on reserves is constant and the probability of default at $t \geq T + 1$ is zero). We set $T = 4$ and $d = 0.5$. We must also specify a value of Δ_D. The arrival of the news about the deficit in country 2 sets off a process of deliberation and bargaining, which we do not model, that takes time and has an uncertain outcome. In state N the deficit is "accepted" by policy makers in the union, including the central bank. In state D the deficit is "rejected," in which case fiscal authority 2 defaults with a haircut implying that the price level of the union in period 4 equals the baseline in the absence of any disturbances. Given the parameterization in Table 1, this yields $\Delta_D = 0.884$.[37]

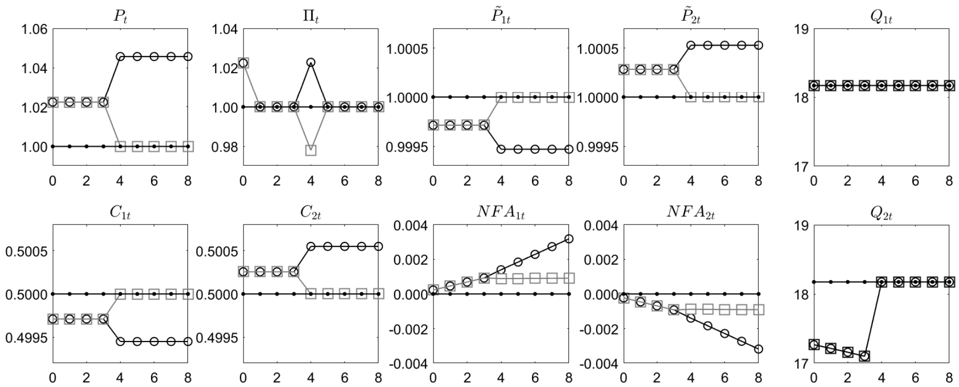
Since we have abandoned perfect foresight, we need to make some additional assumptions to solve the model. The stochastic discount factor of household i in equilibrium no longer depends only on β and on time (footnote 13). Therefore, we assume a particular utility function, $U(C_{it}) = \ln(C_{it})$, $i = 1, 2$. Furthermore, the bond price of country 2 is now affected by default risk, which implies that initial gross foreign asset positions matter for the equilibrium outcome. For simplicity, we suppose that initial gross foreign asset positions equal zero. In addition, since there is now a shock in period T in addition to a shock in period 0, in principle gross foreign asset positions in period T also matter. We make an assumption about how these evolve between period 0 and period $T-1$, for a given path of the current account determined by the model.[38]

New insights from the model with default risk. The lines with circles in Figure 12 show the path of the economy if state N occurs; the lines with squares show the path of the economy if state D occurs. Since the realization of the state is determined in period $T = 4$, the two paths diverge in period 4 (as usual, the lines with points show the baseline equilibrium in the absence of shocks). To gain intuition, note that state N is qualitatively the same as the asymmetric fiscal expansion from Section 3 (a fiscal expansion by country 2 "paid for" by inflation in the union and a wealth transfer from country 1). Accordingly, in period $T = 4$ in state N the price level of the union rises, consumption of household 1 and the relative price level in country 1 fall, and consumption of household 2 and the relative price level in country 2 increase; furthermore, the net claims of country 1 on country 2 grow at rate β^{-1}. In period 0, agents understand that all this will happen with probability $1-d$. As a consequence, *already in period zero*, the price level of the union rises, consumption of household 1 and the relative price level in country 1 fall to some extent, and consumption of

[37]One could make another assumption. One could assume that in the default state the policy makers "allow" some inflation relative to the baseline (they "accept" a fraction of the country 2 deficit).

[38]Specifically, we assume that if the current account is unbalanced between period 0 and period $T-1$, the imbalance is financed by debt of fiscal authority 2 held by household 1, B^H_{12t}. See Appendix D for the details of how we solve the model with default risk.

Figure 12: **Asymmetric fiscal expansion with default risk**



Lines with points: baseline (no shocks). Lines with circles: expansion in country 2 with no default. Lines with squares: same expansion with default.

household 2 and the relative price level in country 2 increase to some extent; furthermore, the net claims of country 1 on country 2 begin to grow.

Thus, a *future* asymmetric fiscal expansion causes inflation throughout the union, relative price changes, and a cross-country wealth transfer. Furthermore, *a change in the probability of default* (in this case from 0 to $d > 0$) also triggers union-wide inflation or deflation, relative price changes, and a cross-country wealth transfer.

The model now features a government bond spread: the bond price Q_{2t} declines relative to the bond price Q_{1t} at $t = 0$ because of the default risk. Default, when it occurs, triggers *deflation* in the union. Moreover, in the default state there is a wealth transfer from country 1 to country 2 for the usual reason: household 1 holds some bonds of fiscal authority 2 (a current account imbalance in favor of country 1 between period 0 and period $T-1$ has been financed by household 1 lending to fiscal authority 2); the default makes household 1, who is a creditor, poorer than in the baseline. In this example, however, household 1 is richer in state D than in state N: household 1 holds only a small fraction of bonds of fiscal authority 2, and consequently the household's capital loss in state D turns out to be smaller than its loss in state N. Since the cross-country wealth transfer is smaller in state D than in state N, state D undoes, to some extent, the relative price changes and consumption changes that took place in period 0, whereas state N exacerbates them. The general lesson here is that, while we are used to thinking of sovereign default as triggering a cross-country wealth transfer, replacing default with inflation as the adjustment mechanism in a monetary union can *also* lead to a cross-country wealth transfer, possibly a *greater* one.[39]

6.3 Currency

In Appendix E, we consider a version of the model in which the monetary base consists of reserves and currency that provides liquidity services and does not pay interest. The main objective is to show that abstracting from currency in the baseline model was only a simplification. The important substantive conclusions were unaffected.

To motivate why households hold currency that does not pay interest, similarly to Del Negro and Sims (2015), we assume that consumption involves transaction costs and holding currency reduces them. As in Section 3, we focus on the baseline perfect-foresight equilibrium with a time-invariant interest rate on reserves, $R_t = R$, and a time-invariant primary surplus in each country, $\tilde{S}_{it} = \tilde{S}_i > 0$. We now interpret primary surpluses as inclusive of seigniorage revenues from currency issuance. If we adjust initial conditions appropriately (e.g, household i's initial holdings of currency rise from zero to a strictly positive number while the household's initial holdings of reserves fall, in such a way that the household's net assets are unchanged), the equilibrium paths of the price level of the union, the relative price levels, and consumption (inclusive of the transaction costs) in each country are *the same* as in the model without currency. With seigniorage revenues ($R = \Pi/\beta > 1$), the lump-sum taxes are lower in equilibrium than in the model without currency. See Appendix E for the details.

6.4 Non-traded goods

We have studied a model with I countries and I traded goods, in which the home bias in consumption preferences is the source of country heterogeneity. In Appendix F, we solve a version of the model with a single traded good and I non-traded goods. The source of country heterogeneity is a technological constraint that prevents some goods from being

[39] It is also interesting that NCB 2, which holds bonds of fiscal authority 2, makes a capital loss in period 0 due to the decline in the bond price; and in period T makes another capital loss from default in state D or a capital gain in state N when the bond price rises.

traded internationally. Numerical solutions are very similar to the baseline model. As an example, we consider an asymmetric fiscal expansion in country 2 as in Section 3. Like in Section 3, the relative price level and consumption in the expanding country 2 rise while the relative price level and consumption in country 1 fall, permanently. The relative price levels change because the prices of non-traded goods change. The asymmetric fiscal expansion increases demand for the non-traded good in country 2 and decreases demand for the non-traded good in country 1. See Appendix F for the details.

7 Equilibria with passive fiscal policy

If fiscal policy is passive, the price level of the union and the price level in each country are indeterminate. In this section, we summarize why this standard result holds in our monetary-union model.[40] More details are in Appendix G.

We use the version of the model with currency introduced in Section 6.3. With currency that does not pay interest, the central bank is constrained to make the gross interest rate on reserves weakly greater than 1 in every period, $R_t \geq 1$.[41] As before, we solve for perfect-foresight equilibria taking as given initial, period -1 asset holdings. We assume that each national fiscal authority i sets its primary surplus according to passive feedback rule (26) (for simplicity, there is no common fiscal authority).

To begin, suppose that the common monetary authority sets the interest rate on reserves according to an active Taylor-type rule $R_t = \max\left\{\left(\Pi^{1-\alpha}/\beta\right)\Pi_t^{\alpha}, 1\right\}$, where $\alpha > 1$. The initial price level of the union P_0 is then indeterminate and, in addition, the path of the inflation rate Π_t for $t \geq 0$ is indeterminate. The price level in each country in the union is also indeterminate. There exists an equilibrium in which $\Pi_t = \Pi$ in every period $t \geq 0$ (P_0 is indeterminate). There also exist equilibria in which $\Pi_0 > \Pi$, $\Pi_t > \Pi_{t-1}$ for each $t \geq 1$, and Π_t goes to infinity in the limit as time goes to infinity. The "deficiency" of passive

[40] For a closed-economy analysis, see Sargent and Wallace (1975) and Woodford (1995) in the case when monetary policy pegs the nominal interest rate and Benhabib, Schmitt-Grohé, and Uribe (2001) and Cochrane (2011) in the case when monetary policy is active.

[41] If the central bank makes the gross interest rate on reserves strictly smaller than 1, private agents do not hold reserves and the effective net nominal interest rate is zero.

fiscal policy here is that it expands (lowers surpluses) as the real value of debt falls along an inflationary path. In addition, there exist equilibria in which $\Pi_0 < \Pi$, $\Pi_0 \geq \beta$, $\Pi_t \leq \Pi_{t-1}$ for each $t \geq 1$, and Π_t converges to β in finite time. Here the "deficiency" of passive fiscal policy is that it contracts (raises surpluses) as the economy moves to the deflationary steady state.[42]

Next, consider the case when monetary policy is passive. Specifically, $R_t = R = \Pi/\beta$ for each $t \geq 0$. In this case, P_0 and Π_0 are indeterminate while $\Pi_t = \Pi$ for each $t \geq 1$. Inflation is indeterminate in the short run; in the long run, inflation is equal to the central bank's target. Put differently, expected inflation is equal to the central bank's target while actual inflation is indeterminate.

The literature shows that when monetary policy follows an active rule like in this section, equilibrium uniqueness can be achieved when fiscal policy is passive but becomes active if the economy starts on a path leading to an inflationary spiral or a deflation trap (Obstfeld and Rogoff (1983), Benhabib et al. (2002), Woodford (2003), Chapter 2.4, Sims (2013)). The same result goes through in this monetary-union model for any of the active fiscal configurations we examined.[43] We believe this result provides an important reason to study active fiscal policy and to think about *which configuration* of active fiscal policy is most appealing in a monetary union.

8 Conclusions

We extended the fiscal theory of the price level to the case of a heterogenous monetary union. There are different ways to implement active fiscal policy in a monetary union, and the effects of fiscal and monetary policy can be very different depending on how active fiscal policy is implemented. A key question for policy becomes which configuration of active fiscal policy to choose. Active fiscal policy for the union can be achieved when national fiscal

[42] If active monetary policy is coupled with active fiscal policy for the union, as in Sections 3-5, then P_0 is determined for the same reason as in Sections 3-5; union inflation is constant at Π, explodes, or converges to β, for the reason given in this paragraph.

[43] As Sims (2013), Section III.B, emphasizes, the active component of fiscal policy can come into play only off the rational expectations equilibrium path or can be always in play, even in equilibrium, but be negligibly small in size.

policies are completely standard and passive while a modestly-sized common fiscal authority follows what appears to be a sensible feedback rule.

We mentioned in the paper ideas for future research (convenience yield from reserves and government bonds, less-than-perfect forward-looking behavior). Allowing for nominal rigidities, and for endogenous output and real interest rate, is another promising direction for further work. We would not want to maintain that monetary policy is passive and fiscal policy is active all the time in the real world, or that they should be. Future research should further develop models in which the monetary-fiscal regime changes over time or in some states of the world, building on Davig and Leeper (2007), Bianchi and Ilut (2017), and others.

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