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Opportunism in Vertical Contracting: A Dynamic Perspective*

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Abstract

This paper proposes a dynamic approach to modeling opportunism in bilateral vertical contracting between an upstream monopolist and competing downstream firms. Unlike previous literature on opportunism which has focused on games in which the upstream firm makes simultaneous secret offers to the downstream firms, we model opportunism as a consequence of asynchronous recontracting in an infinite-horizon continuous-time model. We find that the degree of opportunism depends on the absolute and relative reaction speeds of the different bilateral upstream-downstream firm pairs and on the firms' discount rate. Patience, fast reaction speeds, and asymmetries in reaction speeds across upstream-downstream pairs are shown to alleviate the opportunism problem. Our results are relevant for vertical merger policy and for competition policy on vertical restraints.

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1 Introduction

It is well-known that an upstream firm may suffer from an opportunism problem when it deals with multiple competing downstream firms through bilateral contracts. Although the upstream firm wants to restrict the total quantity of its input in order to maintain high prices and profits, it may be unable to commit not to engage in opportunistic moves that increase the bilateral surplus with one downstream firm at the expense of other firms. Therefore, the upstream firm may be unable to fully exert its market power, even if it enjoys a monopoly position in the upstream market. This opportunism problem, which was first analyzed by Hart and Tirole (1990), O'Brien and Shaffer (1992), and McAfee and Schwartz (1994), occupies a central place in the literature on vertical contracting and has been invoked as an explanation for vertical mergers and vertical restraints such as exclusive dealing that allow the supplier to restore its market power (see Rey and Tirole (2007) for an overview).[1]

The leading approach to modeling opportunism in vertical contracting is to assume that an upstream monopolist makes simultaneous *secret* offers to competing downstream firms. Under the commonly adopted equilibrium refinement that a downstream firm holds passive beliefs when it receives an out-of-equilibrium offer, that is, the downstream firm does not revise its beliefs about the offers received by its rivals upon reception of an 'unexpected' offer, the upstream firm is unable to fully exert its market power in equilibrium in this case. If the upstream monopolist makes simultaneous public offers, on the other hand, then the monopoly outcome arises in equilibrium.

This paper proposes a different approach to modeling the opportunism problem in vertical contracting. Our approach is dynamic in nature, capturing the notion that opportunism arises because an upstream "monopolist might gain by recontracting with another firm" once "the initial firm is somewhat locked in" (McAfee and Schwartz (1994, p. 210)). Although recontracting has long been recognized as an important source of opportunism, previous attempts to model the opportunism problem dynamically are scarce. McAfee and Schwartz (1994)'s seminal paper on opportunism in vertical contracting does consider a

[1]Important contributions on the opportunism problem and the various solutions to it include Rey and Vergé (2004), Marx and Shaffer (2004), White (2007), Montez (2015), Reisinger and Tarantino (2015), and Gaudin (2019).

game in which the upstream monopolist makes sequential offers (see Section II of their paper), but, as they explain, this game fails to capture that "*all* firms will be leery that the monopolist might recontract with their rivals" (McAfee and Schwartz (1994, p. 218), emphasis added). This weakness of the sequential-offers model prompts them to consider a game with simultaneous secret offers, which has since become the workhorse model of vertical contracting with opportunism in the literature. Our paper, in contrast, models recontracting explicitly, using an infinite-horizon model in which each downstream firm anticipates future recontracting between the upstream firm and its rival, as well as between the upstream firm and itself.

In addition to capturing the inherently dynamic recontracting explanation for opportunism, our dynamic modeling approach has a number of advantages over the standard secret-offers approach. First, the dynamic approach allows us to obtain new comparative statics results. It delivers predictions about the *degree* of opportunism and how it varies with the key parameters of our model—the discount rate and the absolute and relative recontracting reaction speeds of the various bilateral upstream-downstream pairs. This is useful for vertical merger policy and competition policy on vertical restraints, because when the supplier's opportunism problem is worse, the competitive damage from strategies such as vertical intergration that restore the supplier's monopoly power will be worse as well (and the supplier's incentive to use such strategies will be stronger).

Second and relatedly, our model yields less stark, and thus perhaps more realistic, predictions about the degree of opportunism. As a function of the different parameter values, our theory can explain steady-state outcomes that lie between the boundary cases of (i) the integrated monopoly outcome (no opportunism), which would arise in equilibrium under simultaneous public offers, and (ii) the pairwise-proof outcome (full opportunism), which would arise in equilibrium under simultaneous secret offers and passive beliefs.

Third, no assumptions about out-of-equilibrium beliefs are needed to characterize the degree of opportunism in our dynamic model. The equilibrium outcome of a game with simultaneous secret offers, on the other hand, is highly sensitive to which out-of-equilibrium belief refinement is adopted, with different assumptions often leading to radically different equilibrium outcomes.

Our setting embeds a simple Cournot-style model of bilateral contracting between one upstream firm and two competing downstream firms, similar to the one set out by Rey and Tirole (2007), into an infinite-horizon continuous-time framework. The upstream firm (henceforth also called supplier) gets to make new contract offers to the downstream firms (henceforth also called retailers) according to independent Poisson processes, one for each retailer. The independence of the Poisson processes implies that recontracting occurs asynchronously, that is, the probability of simultaneous offers is zero. Moreover, because a higher Poisson rate implies that a retailer's contract reacts more quickly (in expectation) to changes in the other retailer's contract, the Poisson rate that governs the arrival of recontracting events between a supplier-retailer pair has a natural interpretation as the reaction speed of that retailer's contract. Contracts specify the quantity supplied by the supplier and a fixed fee to be paid by the downstream firm per unit of time, and we focus on Markov Perfect Equilibria (MPE).[2]

The supplier's Markov strategy involves dynamic quantity reaction functions that specify how the quantity that the supplier offers to a retailer depends on the quantity that the supplier currently supplies to the retailer's rival. We characterize the first-order conditions that differentiable dynamic quantity reaction functions must satisfy in a Markov Perfect Equilibrium, and use them to derive a series of results about the equilibrium steady-state quantities (assuming existence of a stable equilibrium with differentiable dynamic quantity reaction functions). For the case of linear demand functions, we establish the existence of a unique MPE with linear dynamic quantity reaction functions. This linear MPE is shown to be dynamically stable, with the retailers selling symmetric quantities in the equilibrium steady state when the reaction speeds are symmetric, and the retailer whose contract reacts faster selling a larger quantity than its rival when the reaction speeds are asymmetric.

The analysis yields three broad takeaways about the degree of opportunism, as measured by the (inverse of the) aggregate quantity sold in the equilibrium steady state. First, patience reduces opportunism. Second, a faster average reaction speed reduces opportunism. And third, asymmetry in reaction speeds reduces opportunism.

The intuition for the first two insights can be understood as follows. When making an offer to retailer D_i, the supplier internalizes only the effect on D_i's own variable profit in the

[2]The restriction to Markov strategies reduces the large multiplicity of equilibria, and it allows us to focus on the key trade-off between short-term incentives to behave opportunistically and longer-term incentives to achieve collective surplus maximization.

time interval until the next recontracting with the retailer's rival D_{-i}, but it internalizes the future effects on *all* retailers' variable profits from the next recontracting with D_{-i} onwards (through its current and anticipated future fixed fees). Greater patience makes the supplier care relatively more about profits earned after the next recontracting with the rival retailer, the time for which it internalizes the effects of its current offer on total surplus, thus weakening the supplier's incentive to behave opportunistically. Similarly, fast reaction speed reduces the expected length of time until the next recontracting with the rival retailer, that is, the length of time during which effects on the rival retailer's variable profits are ignored, thereby weakening the supplier's incentive to behave opportunistically.[3]

Nonetheless, under symmetric reaction speeds, some degree of opportunism prevails in the equilibrium steady state even in the limit, when the discount rate goes to zero or the reaction speed goes to infinity. Formally, the steady-state quantity is bounded below by a quantity strictly above the per-firm monopoly quantity in any stable symmetric equilibrium with differentiable dynamic quantity reaction functions. Asymmetries in reaction speeds across supplier-retailer pairs, however, can further reduce the extent of opportunism, and even eliminate it in a limiting case given the assumption that retailers sell perfect substitutes. We find that (under some technical conditions) in any equilibrium steady state, the aggregate quantity is close to the monopoly quantity when the reaction speed of one retailer is close to zero and either the discount rate is close to zero or the reaction speed of the other retailer is large enough. Moreover, in the linear-demand case, the aggregate equilibrium steady-state quantity in the unique MPE with linear dynamic quantity reaction functions is falling in the degree of asymmetry across reaction speeds, for any given average reaction speed and discount rate.

Our results have implications for vertical merger policy and for competition policy on vertical restraints. Models of vertical contracting with opportunism have been used to understand anticompetitive effects of vertical mergers and vertical restraints by noting that if secret offers and passive beliefs are observed, then vertical intergration and contract provisions that eliminate the opportunism problem reduce total output and harm downstream

[3]This result may appear surprising in the sense that fast reaction speed can be thought of as a lack of commitment to refrain from recontracting. However, as our setting makes it clear, opportunism arises not due to a lack of commitment to long contracts, but due to a lack of commitment on the terms of future contracts.

consumers. Our model shows that secret offers are not needed for opportunism to arise, and that vertical mergers and opportunism-avoiding contract provisions can have anticompetitive effects even when contracts are public. Moreover, our results offer guidance on when vertical mergers and opportunism-avoiding contract provisions are likely to be more harmful, namely when firms are impatient, there are long time gaps between recontracting, and reaction speeds are symmetric.

The paper is organized as follows. In Section 2, we discuss the connections of our paper to the related literatures on dynamic vertical contracting and on dynamic oligopoly games. In Section 3, we describe the model setup. Section 4 offers a brief summary of the analysis and results in the benchmark case of simultaneous offers in a static game. Section 5 contains our analysis. In Section 5.1, we derive the equilibrium conditions, and we prove the existence of a unique MPE with linear dynamic quantity reaction functions in the linear-demand case. In Section 5.2, we derive limit results and comparative static results on the equilibrium steady-state quantities for the case of symmetric reaction speeds. In Section 5.3, we extend these results, analyzing the implications of asymmetries in reaction speeds across supplier-retailer pairs for the equilibrium steady state. Section 6 discusses strategies the supplier could use to restore its monopoly power. In Section 7, we discuss the policy implications of the results and some directions for further research. The appendix contains all proofs that are omitted from the main text.

2 Related literature

In addition to the aforementioned literature on opportunism in static models of vertical contracting, our paper contributes to the literatures on dynamic vertical contracting and on dynamic oligopoly games.

Dynamic vertical contracting Although the literature on vertical contracting is vast, previous attemps to model the opportunism problem dynamically are scarce. McAfee and Schwartz (1994), Marx and Shaffer (2004), and Bedre-Defolie (2012) consider models in which a supplier makes public sequential offers to competing retailers, so the supplier has an incentive to behave opportunistically with the later retailer(s) in the sequence. However, sequential-offer models fail to capture that all retailers may be wary of future opportunistic behavior, and they impose a strong ex ante asymmetry between firms. Our infinite-horizon model is more general, encompassing sequential offers as a special limiting case when the reaction speeds are allowed to be asymmetric.

Dequiedt and Martimort (2015) consider a model in which offers are made simultaneously but that nonetheless has a dynamic flavor, because the supplier is allowed to move last, choosing each retailer's quantity from a menu contract accepted by the retailer. Offers are public but retailers have private information. The supplier's ability to choose quantities can thus be thought of as allowing the supplier to dynamically "respond to new information as it arrives." We focus on forcing contracts instead, and introduce dynamics explicitly by considering repeated asynchronous contracting in an infinite-horizon model. Opportunism arises due to the supplier's inability to commit to the terms of future contracts in our setting, whereas Dequiedt and Martimort (2015) focus on a new form of "informational" opportunism due to private information on the retailer side.[4]

Lee and Fong (2013) analyze Markov perfect equilibria of an infinite-horizon dynamic seller-buyer network formation game with transfers. However, the focus of their work is on network formation (who trades with whom), whereas we are interested in the severity of the opportunism problem that a monopolistic supplier faces. Closer to our setting, Farrell (2019, Section 5) proposes to analyze a symmetric alternating-offers model of vertical contracting between an upstream monopolist and competing downstream firms. However, the formal analysis of the dynamic model in his paper is highly incomplete. It does not include a characterization of the Markov perfect equilibrium strategies and steady-state quantities, nor any comparative statics results. Moreover, Farrell (2019)'s main focus are partially exclusionary contracts rather than the opportunism problem.[5]

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[4]Segal and Whinston (2003) consider menu contracts, from which the principal chooses after the agents have made their acceptance decisions, in a model without private information. They show that when marginal production costs are nonincreasing (as we will assume), allowing menu contracts does not restrict the equilibrium set in a game in which a principal makes simultaneous secret offers to competing agents: Any profile of quantities and transfers such that total surplus is nonnegative can be sustained as a Weak Perfect Bayesian Equilibrium in this case (see Proposition 6 in their paper).

[5]The broader literature on dynamic vertical contracting also includes work on dynamic common agency games (Bergemann and Välimäki (2003), Pavan and Calzolari (2009)), in which multiple principals make offers to a common agent rather than the opposite scenario considered in this paper, and research on the implications of vertical market structure and contracting for firms' ability to sustain collusion in a repeated game (Nocke and White (2007), Piccolo and Miklós-Thal (2012), Gilo and Yehezkel (2020)).

Dynamic oligopoly and asynchronous moves This paper is also related to the literature on dynamic oligopoly, and more broadly the literature on dynamic games with asynchronous moves. In a series of seminal articles on dynamic oligopoly, Maskin and Tirole (1987, 1988a,b) analyze Markov Perfect Equilibria of repeated games in which duopolists make alternating moves.[6] Our continuous-time model in which contract timing is governed by independent Poisson processes is inspired by the micro-foundation that Maskin and Tirole (1988a, Section 4) propose for such games.

A key difference to the models considered by Maskin and Tirole and other work on dynamic oligopoly is that we consider a vertical industry structure with a supplier that makes asynchronous offers to two competing firms (who then accept or reject and compete in the market), instead of two competing firms that make asynchronous strategic decisions. Unsurprisingly, this leads to qualitatively different results. For instance, under symmetry the equilibrium steady-state quantity lies below the static Cournot quantity in our setting, whereas Maskin and Tirole (1987) obtain the opposite result in a dynamic game of symmetric Cournot competition with linear demand functions.

An important feature of our dynamic model is that contract offers are asynchronous. Asynchronous moves have also been analyzed in repeated coordination games (Lagunoff and Matsui (1997)), and more recently in asynchronous revision games where players prepare some actions at the beginning and then obtain revision opportunities according to independent Poisson processes until some predetermined deadline (Kamada and Kandori (2012), Calcagno et al. (2014)). Ambrus and Lu (2015) analyze a continuous-time finite-horizon coalitional bargaining game in which opportunities to make offers arrive asynchronously according to independent Poisson processes, until an agreement is reached. While these papers share the asynchronous-moves assumption and some important modeling ingredients with our research, they differ substantially from our work in motivation, focus, and analysis.

[6] Other important contributions to this literature include Eaton and Engers (1990) who consider dynamic price competition with differentiated goods, De Fraja (1993) who analyzes the impact of staggered wage bargaining on wages in an oligopolistic industry, Davies (1991) who explores a dynamic entry deterrence model where two firms alternate in making price and entry choices, and Pastine and Pastine (2002) who consider dynamic competition when duopolists choose advertisement and price levels and there are consumption externalities among buyers. Jun and Vives (2004) analyze a dynamic duopoly model with adjustment costs.

3 Setting

We embed a simple Cournot-style model of vertical contracting, similar to the one set out by Rey and Tirole (2007), into a continuous-time dynamic setting. Consider a vertical structure with one upstream supplier, U, and two competing downstream firms D_i ($i = A, B$, also called retailers). The downstream firms purchase an input from the supplier, transform it into a final good using a one-to-one technology, and sell the final good to consumers. Upstream marginal costs are constant and equal to $c \geq 0$, downstream marginal costs are constant and normalized to zero.

Consumers have an inverse demand curve $P(Q) : \mathbb{R}_+ \to \mathbb{R}$ for the product, where $Q = q_A + q_B$ denotes the total quantity put on the market by the downstream firms. We make the following assumptions:

A1 $P(Q)$ is continuous and strictly decreasing for all $Q \geq 0$, and twice continuously differentiable for all $Q > 0$.

A2 $P'(Q) + P''(Q)Q < 0$ for all $Q > 0$.

A3 $P(0) > c$ and $\lim_{Q\to\infty} P(Q) < c$.

A1 and A3 implies that there exists a unique quantity $\overline{Q}_c > 0$ such that price is equal to total marginal cost: $P\left(\overline{Q}_c\right) = c$.[7] This quantity will be useful because it represents a natural upper bound to impose on quantities in order to obtain bounded action spaces.

The instantaneous variable profit of downstream firm D_i (gross of any payments to the supplier) is given by

$$\pi(q_i, q_{-i}) = q_i P(q_A + q_B),$$

where q_i denotes D_i's own quantity, and q_{-i} its competitor's quantity. We use subscripts to denote derivatives, e.g., $\pi_2(q_i, q_{-i}) = \frac{\partial \pi(q_i, q_{-i})}{\partial q_{-i}}$. Assumptions A1-A2 imply that $\pi_{11}(q_i, q_{-i}) = \frac{\partial^2 \pi(q_i, q_{-i})}{\partial q_i^2} < 0$ and $\pi_{12}(q_i, q_{-i}) = \frac{\partial^2 \pi(q_i, q_{-i})}{\partial q_i \partial q_{-i}} < 0$.

The instantaneous industry profit, that is, the sum of all three firms' profits, is given by

$$\Pi(q_A + q_B) = (q_A + q_B)(P(q_A + q_B) - c).$$

[7] Note that for $c = 0$, A3 implies that price must be negative for high quantities. This assumption is not crucial for our results. See Footnote 16 in section 5.1, where we discuss the implications of restricting prices to be non-negative in the linear-demand case.

We denote by $R^C(q_{-i}) = \arg\max_q (q(P(q+q_{-i})-c)) = \arg\max_q (\Pi(q+q_{-i}) - \pi(q_{-i},q))$ the Cournot reaction function given the marginal production cost c. The per-firm Cournot quantity q^C is defined by $q^C = R^C(q^C)$. The quantity that maximizes industry profits is denoted by $Q^M = \arg\max_Q \Pi(Q)$, and we let $R^M(q_{-i}) = \arg\max_q \Pi(q+q_{-i}) = Q^M - q_{-i}$ denote the "monopoly reaction function" and $q^M = Q^M/2 = R^M(q^M)$ the quantity per downstream firm when they split the total monopoly quantity equally.

Time is continuous and infinite, indexed by $t \in [0,\infty)$, and all firms discount future payoffs at a rate $r > 0$. A contract between U and D_i consists of a vector $(q_i, f_i) \in [0, \overline{Q}_c] \times \mathbb{R}$ that specifies a flow of input quantity q_i from the supplier to the retailer and a fixed payment f_i from the retailer to the supplier per unit of time. For simplicity, we assume that the supply contracts are quantity-fixing, that is, they fix how much quantity the retailer transforms into the final output and sells to consumers per unit of time.[8] The absence of a contract between U and D_i is equivalent to $(q_i, f_i) = (0,0)$.

D_i's flow payoff given the current contracts is thus $\pi(q_i, q_{-i}) - f_i$, and U's flow payoff is

$$f_A + f_B - c(q_A + q_B).$$

The sum of the three firms' flow payoffs is equal to the industry profit $\Pi(q_A + q_B)$.

The timing of contracts is governed by two independent Poisson processes with rates $\lambda_A > 0$ and $\lambda_B > 0$, respectively. For a small time interval Δt, the probability that the current contract between U and D_i terminates and recontracting occurs is $\lambda_i \Delta t$. In this event, U instantaneously makes a new offer to D_i, and D_i immediately accepts or rejects the offer.[9] If D_i accepts (rejects) an offer (q_i, f_i), its quantity becomes q_i (zero) per unit of

[8] An alternative would be that after purchasing their input quantities, the downstream firms play the Bertrand-Edgeworth game of downstream price competition with capacity constraints. In this alternative specification, both retailers find it optimal to transform all their input and set their price at $P(q_A, q_B)$ for any $(q_A, q_B) \in [0, \overline{Q}_c]^2$ if the following assumptions hold: the upstream marginal cost c is high enough, stockpiling is infeasible (i.e., a retailer whose contract specifies quantity q_i can sell at most $q_i \Delta t$ in a time interval Δt), and the retailers can adjust their prices instantaneously in response to a change in either retailer's contract. Our assumptions on contracts (quantity-transfer pairs, and quantity fixing) are in line with those in Segal (1999)'s general analysis of contracting with externalities between one principal and several agents.

[9] The assumption that time lapses between offers, but not between offers and acceptance decisions, goes back to the classic bargaining models of Stahl (1972) and Rubinstein (1982). It has also been adopted more recently by Ambrus and Lu (2015) in a continuous-time bargaining model where proposal opportunities arrive according to a Poisson process over time. The assumption helps us to focus on the opportunism problem and how the supplier's contract with one retailer reacts to changes in its contract with the other retailer.

time until the contract terminates and the next recontracting between U and D_i occurs.

We will refer to λ_i as the "reaction speed" of D_i's contract. A higher λ_i (shorter commitment length) means that recontracting with D_i occurs more frequently, therefore the bilateral pair $U - D_i$ can react quickly to changes in the contract between U and D_{-i}. Conversely, a lower λ_i (longer commitment length) means that recontracting with D_i occurs less frequently, therefore the bilateral pair $U - D_i$ reacts less quickly to changes in the contract between U and D_{-i}.

We focus on (stationary) pure Markov strategies. The state variable when U is about to make an offer to D_i is the quantity in U's current contract with D_{-i}.[10] Formally, a Markov strategy of U is given by a pair of mappings $(R_i, F_i)_{i=A,B}$ where $(R_i(q_{-i}), F_i(q_{-i}))$ is the contract offered to D_i when U currently sells quantity q_{-i} to D_{-i}. We will refer to (R_A, R_B) as the *dynamic quantity reaction functions*, because they capture how the quantity in U's contract with one retailer reacts to the quantity in the competing retailer's contract. The quantity action spaces are restricted to a bounded set; specifically, we assume that $R_i : \left[0, \overline{Q}_c\right] \to \left[0, \overline{Q}_c\right]$ for all i.[11] The fixed fee offers are allowed to take on any value in $\mathbb{R}$. For D_i, a pure Markov strategy is given by a function $M_i(q, f; q_{-i}) \in \{0, 1\}$, where $M_i(q, f; q_{-i}) = 1$ (resp. $M_i(q, f; q_{-i}) = 0$) means that D_i accepts (resp. rejects) the offer (q, f) when U currently sells quantity q_{-i} to D_{-i}. All actions are public.

A strategy profile is called a *Markov Perfect Equilibrium* (MPE) if it is a subgame perfect equilibrium in Markov strategies. We restrict attention to equilibria in which all offers are accepted ($M_i(R_i(q_{-i}), F_i(q_{-i}); q_{-i}) = 1$ for all q_{-i} and all i), which is without loss of generality because an offer that would be rejected can be replaced by an accepted null contract $(q_i, f_i) = (0, 0)$ without any impact on expected present discounted payoffs.

[10] One may wonder why the state when U makes an offer to D_i excludes the fixed fee f_{-i} in U's current contract with D_{-i}, although this fixed fee affects U's (and D_{-i}'s) payoff until the next recontracting with D_{-i}. The reason is that U's preferences over continuation strategies are independent of f_{-i}. The fixed fee f_{-i} does not affect D_i's payoffs when deciding whether to accept or reject an offer (q_i, f_i) from U, which implies that U's continuation payoff functions for the same q_{-i} but two different values of f_{-i} are positive affine transformations of one another. Applying the criterion set out in Maskin and Tirole (2001), Markov strategies therefore only depend on the quantity that U currently supplies to the rival retailer in our context.

[11] The assumption that the quantity action spaces are bounded ensures that the value functions are well defined and that dynamic programming techniques are applicable. Our approach in the analysis will be to ignore the upper and lower bounds on quantity when solving the supplier's optimization problem and to verify ex post that indeed $R_i(q_{-i}) \in \left[0, \overline{Q}_c\right]$ for all $q_{-i} \in \left[0, \overline{Q}_c\right]$ and i.

Henceforth, a Markov perfect equilibrium is also referred to simply as *equilibrium*, and we will say that an equilibrium is linear if the equilibrium dynamic quantity quantity reaction functions are linear. All firms are risk neutral and seek to maximize expected present discounted payoffs.

Remarks The results in our main model do not hinge on the assumption that contract offers are publicly observed. They would remain unchanged if the offers were secret. As the two Poisson processes according to which contracts terminate are independent, the probability that the supplier makes simultaneous offers to the two downstream firms is zero. Moreover, since contracts are quantity-forcing, a downstream firm can infer its rival's quantity from its own variable profit. When a downstream firm receives a contract offer, it would thus be aware of the quantity in its rival's current contract even if that contract offer had been privately observed. The issue of beliefs about the supplier's offer to another downstream firm, which is central to the analysis when the suuplier makes secret simultaneous offers, is therefore moot in our setting.

It is also worth noting that the stochastic nature of the contract timing is not important for our results. Given that the firms are risk neutral, formally the model is equivalent to one in which all contracts have length $\Delta_A + \Delta_B$, where Δ_i $(i = A, B)$ denotes the time lag with which the bilateral pair $U - D_i$ reacts to a change in D_{-i}'s quantity.[12] By the Poisson property, the probability of contract termination is independent of a contract's age in our setting. Therefore, as in a model with alternating offers and deterministic contract lengths, only the quantity that the rival retailer is committed to under its current contract, and not contract age, is relevant for the negotiation between the supplier and a retailer. The critical feature of our model is that recontracting events are asynchronous, not that they are stochastic.

[12]Specifically, the setting with deterministic reactions lags Δ_A and Δ_B is analogue to our model when $e^{-r\Delta_i} = \frac{\lambda_i}{r+\lambda_i}$ for each i (or, in a discrete-time version of the model with deterministic reaction lags, when $\delta^{T_i} = \frac{\lambda_i}{r+\lambda_i}$, where T_i denotes the number of periods before i's contract reacts and δ the discount rate). See also Maskin and Tirole (1988a) and Lagunoff and Matsui (1997, Section 3) for discussions of various microfoundations of dynamic games with asynchronous moves.

4 Benchmark: Static Model

Before we analyze the dynamic game, let us set out the benchmark of simultaneous offers in a static game. Consider a game in which U makes simultaneous contract offers of the form (q_i, f_i) to the retailers, and the retailers simultaneously and independently decide whether to accept or reject these offers. The payoffs are the same as the flow payoffs in our dynamic setting, namely $\pi(q_i, q_{-i}) - f_i$ for D_i and $f_A + f_B - c(q_A + q_B)$ for U, with $(q_i, f_i) = (0, 0)$ if D_i rejects U's offer.

Public offers When the supplier's offers are publicly observed by the retailers, the supplier can fully exert its market power and obtains the entire monopoly profit in (a subgame perfect) equilibrium. For instance, U can achieve this by offering the contract $\left(q^M, \pi\left(q^M, q^M\right)\right)$ to each retailer. Both retailers will accept and together they will sell the monopoly quantity. The intuition for why the monopoly outcome arises in equilibrium is that the supplier internalizes the effects on *all* retailers' profits when making offers: Any change in the quantity offered to D_i affects the fixed fee that the supplier can obtain from retailer D_{-i} by an amount equal to the effect of the change on D_{-i}'s variable profit.

Secret offers When D_i cannot observe the contract offered to D_{-i}, the (perfect Bayesian) equilibrium of the game is sensitive to D_i's beliefs about the contract offered to D_{-i} when D_i receives an out-of-equilibrium offer. A sensible and widely-used assumption in Cournot settings like the one we consider is that retailers hold *passive beliefs*, whereby a retailer that receives an out-of-equilibrium continues to believe that its rival was offered the equilibrium contract.[13]

Let $(\widehat{q}_A, \widehat{q}_B)$ denote the equilibrium quantities. With passive beliefs, retailer D_i is willing to accept an offer (q, f) if and only if $f \leq \pi(q, \widehat{q}_{-i})$. The equilibrium offer to D_i must therefore maximize the bilateral surplus of the pair $U - D_i$ given $\widehat{q}_{-i}$, which in our setting means that D_i's equilibrium quantity must be the Cournot best response to $\widehat{q}_{-i}$:

$$\widehat{q}_i = R^C(\widehat{q}_{-i}) = \arg\max_{q_i} \left(\Pi(q_i + \widehat{q}_{-i}) - \pi(\widehat{q}_{-i}, q_i)\right).$$

[13]The passive beliefs refinement is appealing in Cournot-like settings because U has no incentive to change the offer to D_{-i} when it changes the offer to D_i. See Hart and Tirole (1990), Rey and Vergé (2004), or Rey and Tirole (2007) for more detailed discussions.

In the unique equilibrium given passive beliefs, the retailers thus sell the Cournot quantities $\left(q^C, q^C\right)$ and the supplier earns $\Pi\left(2q^C\right) < \Pi\left(Q^M\right)$. Intuitively, the supplier is unable to fully exert its market power because when making an offer to D_i, it does *not* internalize the negative effect that a higher q_i has on D_{-i}'s variable profit.[14]

Different beliefs can lead to different equilibrium outcomes. In particular, the monopoly outcome arises in equilibrium under symmetric beliefs, whereby a retailer that receives an out-of-equilibrium offer believes that its rival was offered the same contract.[15,16]

The simultaneous-offer games analyzed in the literature thus all feature one of two polar outcomes in our setting. Either the monopoly outcome arises in equilibrium (when offers are public or offers are secret and beliefs symmetric) or opportunism leads to the Cournot competition outcome in equilibrium (when offers are secret and beliefs passive or wary). Other than through changes in parameters that affect the Cournot or the monopoly outcome, these models do not accommodate varying degrees of opportunism.

5 Dynamic Model

5.1 Equilibrium conditions and existence

To solve for a Markov Perfect Equilibrium, we define four value functions. Given the Markov strategies $((R_A, F_A, R_B, F_B), M_A, M_B)$, let $W_A(q_B, f_B)$ denote the expected present discounted value of U's profits when U is about to make an offer to D_A, the other retailer's

[14]Some papers in the vertical contracting literature (e.g., O'Brien and Shaffer, 1992) use the "contract equilibrium" concept pioneered by Cremer and Riordan (1987), which requires contracts to be pairwise stable (i.e., each contract must maximize bilateral surplus given the contracts of other retailers) but does not rule out multi-lateral deviations. In the model with Cournot competition and quantity-fixing contracts considered here, the quantities in a passive-beliefs perfect Bayesian equilibrium coincide with the quantities in such a contract equilibrium, hence this alternative solution concept would lead to the same conclusion.

[15]The third belief refinement in the literature are wary beliefs, first introduced by McAfee and Schwartz (1994). Wary beliefs coincide with passive beliefs in our setting with quantity-fixing contracts, hence the Cournot outcome would remain the equilibrium outcome under this alternative belief refinement. See In and Wright (2018) for a more general analysis of "endogenous signaling games" that offers a game-theoretic foundation for wary beliefs in vertical contracting games.

[16]In fact, in our setting, an equilibrium with any non-negative quantities $(\widehat{q}_A, \widehat{q}_B)$ such that $\widehat{q}_A + \widehat{q}_B < \overline{Q}_c$ can be sustained by appropriately defined out-of-equilibrium beliefs. To see this, suppose that if D_i receives an offer $q_i \neq \widehat{q}_i$, D_i believes that D_{-i} was offered the contract $\left(\overline{Q}_c - q_i, 0\right)$ and that D_{-i} holds similar beliefs (which makes accepting $\left(\overline{Q}_c - q_i, 0\right)$ optimal for D_{-i}). The highest fixed fee D_i is willing to pay is thus equal to $\pi\left(q_i, \overline{Q}_c - q_i\right) = q_i c$, which makes a deviation unprofitable for the supplier.

current contract is (q_B, f_B), and all firms play according to their Markov strategies henceforth. $W_B(q_A, f_A)$ is defined symmetrically. Let $V_A(q, f; q_B)$ denote the expected present discounted value of D_A's profits when D_A accepts the contract offer (q, f) it just received, U currently supplies q_B to the other retailer, and all firms play according to their Markov strategies henceforth. $V_B(q, f; q_A)$ is defined symmetrically.

U's optimization problem when it is about to make an offer to D_i can then be written as[17]

$$W_i(q_{-i}, f_{-i}) = \max_{(q,f)} \left(\frac{f + f_{-i} - c\,(q + q_{-i})}{r + \lambda_i + \lambda_{-i}} + \frac{\lambda_i}{r + \lambda_i + \lambda_{-i}} W_i(q_{-i}, f_{-i}) + \frac{\lambda_{-i}}{r + \lambda_i + \lambda_{-i}} W_{-i}\,(q, f) \right) \tag{1}$$

$$\text{s.t. } V_i(q, f; q_{-i}) \geq V_i(0, 0; q_{-i}).$$

The first term in the objective function captures the discounted present value of U's profits in the time interval until the next recontracting. The second and third term capture the discounted present value of U's continuation profits from the next recontracting onwards, taking into account that either the contract with D_i or the contract with D_{-i} can terminate first. Morover, the supplier is constrained by the condition that D_i finds it optimal to accept U's offer (q, f). Note that it is without loss of generality to require D_i's acceptance in U's optimization problem, because any offer that D_i's strategy would reject can be replaced by an offer $(0, 0)$ such that D_i is indifferent between acceptance and rejection.

We first show that the retailer's acceptance condition boils down to an upper bound on the fixed fee:

Lemma 1 *It is optimal for D_i to accept offer (q, f) when the rival's current quantity is q_{-i}, i.e., $V_i(q, f; q_{-i}) \geq V_i(0, 0; q_{-i})$, if and only if*

$$f \leq f_i(q; q_{-i}) \equiv \frac{r + \lambda_i}{r + \lambda_i + \lambda_{-i}} \pi\,(q, q_{-i}) + \frac{\lambda_{-i}}{r + \lambda_i + \lambda_{-i}} \pi\,(q, R_{-i}\,(q))\,.$$

The fixed payment $f_i(q; q_{-i})$ extracts the present discounted value of all variable profits that D_i earns (in expectation) during its current contract. Capturing the possibility of a reaction in the rival's contract before D_i's own contract terminates, it is a weighted average of D_i's variable profit given its rivals' current quantity q_{-i} and D_i's variable profit given its rival's quantity $R_{-i}\,(q)$ after recontracting between U and D_{-i}. The weights depend

[17] Details on how to derive the objective function in (1) can be found in Appendix B.

on the relative reaction speeds of the two supplier-retailer pairs, with a smaller λ_i and a higher λ_{-i} increasing the relative weight of $\pi\left(q, R_{-i}\left(q\right)\right)$, and on the discount rate, with a higher discount rate increasing the weight of the profit prior to the possible recontracting with D_{-i}. Variable profits beyond the current contract do not matter for the retailer's acceptance decision, because these profits will be fully extracted by the fixed fees in future contracts.

The objective function in the supplier's optimization problem in (1) is strictly increasing in f, as can be seen by noting that $\frac{\partial W_{-i}(q,f)}{\partial f} = \frac{1}{r+\lambda_i} > 0$. It follows that $f = f_i(q; q_{-i})$ at the solution of the supplier's problem. Substituting the binding constraint into the objective function, solving for $W_i(q_{-i}, f_{-i})$, and denoting by $\overline{W}_i(q_{-i}) = W_i(q_{-i}, f_{-i}) - \frac{f_{-i}}{r+\lambda_{-i}}$ the supplier's value function net of the fixed payments already committed to in the past, the supplier's problem can be rewritten as

$$\overline{W}_i(q_{-i}) = \frac{1}{r+\lambda_{-i}} \max_q \left(\pi\left(q, q_{-i}\right) - c\left(q + q_{-i}\right) + \frac{\lambda_{-i}}{r+\lambda_i}\pi\left(q, R_{-i}\left(q\right)\right) + \lambda_{-i}\overline{W}_{-i}\left(q\right)\right). \tag{2}$$

The strategies $\left(\left(R_A, F_A, R_B, F_B\right), M_A, M_B\right)$ form a Markov Perfect equilibrium if and only if there exist value functions $\left(\overline{W}_A, \overline{W}_B\right)$ such that, for every i and q_{-i}, (2) holds,

$$R_i\left(q_{-i}\right) \in \arg\max_q \left(\pi\left(q, q_{-i}\right) - c\left(q + q_{-i}\right) + \frac{\lambda_{-i}}{r+\lambda_i}\pi\left(q, R_{-i}\left(q\right)\right) + \lambda_{-i}\overline{W}_{-i}\left(q\right)\right),$$

$F_i\left(q_{-i}\right) = f_i(R_i\left(q_{-i}\right); q_{-i})$, and $M_i\left(q, f; q_{-i}\right) = 1$ if and only if $f \leq f_i(q; q_{-i})$.

For the remainder of the analysis, we will focus on characterizing the equilibrium dynamic quantity reaction functions (R_A, R_B), with the implicit understanding that $F_i\left(q_{-i}\right) = f_i(R_i\left(q_{-i}\right); q_{-i})$ and $M_i\left(q, f; q_{-i}\right) = 1$ if and only if $f \leq f_i(q; q_{-i})$.

Our first result on the equilibrium dynamic quantity reaction functions is that they must be downward sloping (if they exist). As in dynamic oligopoly models, this result follows from $\pi_{12} < 0$ (see Maskin and Tirole (1987, 1988a,b) or Vives (2005)).

Lemma 2 *When an equilibrium exists, the equilibrium dynamic quantity reaction functions are downward sloping:* $R_i\left(q\right) \leq R_i\left(q'\right)$ *if* $q > q'$, *for* $i \in \{A, B\}$.

Assuming that an equilibrium has differentiable dynamic quantity reaction functions, we can use the first-order conditions of the supplier's optimization problem to further characterize these functions. For each $i = A, B$, the first-order condition of the problem in (2) is that, at $q = R_i(q_{-i})$,

$$\pi_1(q, q_{-i}) - c + \frac{\lambda_{-i}}{r+\lambda_i}\left(\pi_1(q, R_{-i}(q)) + \pi_2(q, R_{-i}(q)) R'_{-i}(q)\right) + \lambda_{-i}\frac{d\overline{W}_{-i}(q)}{dq} = 0.$$

By the envelope theorem,

$$\frac{d\overline{W}_{-i}(q)}{dq} = \frac{1}{r+\lambda_i}\left(\pi_2(R_{-i}(q), q) - c\right),$$

which implies that the first-order condition for each $i = A, B$ can be written as follows: at $q = R_i(q_{-i})$,

$$\begin{aligned} 0 = {} & -c + \frac{r+\lambda_i}{r+\lambda_i+\lambda_{-i}}\pi_1(q, q_{-i}) \\ & + \frac{\lambda_{-i}}{r+\lambda_i+\lambda_{-i}}\left[\pi_1(q, R_{-i}(q)) + \pi_2(q, R_{-i}(q)) R'_{-i}(q) + \pi_2(R_{-i}(q), q)\right]. \end{aligned} \tag{3}$$

The first-order condition in (3) has the following interpretation. Consider U's contract offer to D_A given that D_B's current contract specifies a quantity q_B. At $q = R_A(q_B)$, a small change Δq in the quantity that U and D_A agree upon must have zero effect on the present discounted joint profit of the bilateral pair $U - D_A$. This effect can be decomposed as follows. First, there is a direct effect $-c\Delta q$ on upstream costs until U's next recontracting with D_A. Second, D_A's variable profit is affected, both at its rival's current quantity q_B and after a possible reaction in the rival's quantity during D_A's contract. At the rival's current quantity, the only effect on D_A's variable profit is the direct effect $\pi_1(q, q_B)\Delta q$, but after a reaction in D_B's contract there is both a direct effect $\pi_1(q, R_B(q))\Delta q$ and an indirect effect $\pi_2(q, R_B(q)) R'_B(q)\Delta q$ due to the marginal reaction in D_B's quantity. Third, there is the direct effect $\pi_2(R_B(q), q)\Delta q$ on D_B's variable profit after a reaction of D_B's contract during D_A's contract. This change in D_B's variable profit is part of the present discounted joint profit of $U - D_A$, because it will be fully extracted by U through the fixed fee in its next contract with D_B. From the envelope theorem, Δq has no additional first order effects. In particular, the indirect effect of Δq on D_B's variable profit due to the marginal reaction in D_B's quantity in case of recontracting is not first order, because U's recontracting offer to D_B internalizes this change.[18]

[18] In contrast, U's recontracting offer to D_B will *not* internalize the effect on D_A's variable profit in the time interval until the next recontracting with D_A, therefore the indirect effect $\pi_2(q, R_B(q)) R'_B(q)$ appears in the first-order condition for U's offer to D_A.

The first-order conditions are necessary for an equilibrium given differentiable dynamic quantity reaction functions, but they are not sufficient. For the case of linear demand functions, however, we can show that there exists a unique set of linear dynamic quantity reaction functions such that the necessary and sufficient condition of U's dynamic optimization problem are satisfied:

Proposition 1 *Suppose $P(Q) = 1 - Q$.*[19,20] *For any parameter values $r, \lambda_A, \lambda_B > 0$ and $c \in [0, 1)$, there exists a unique linear MPE. This MPE is dynamically stable, i.e., for any history, quantities converge to a steady state.*

The next two subsections further analyze the properties of the steady state in a dynamically stable equilibrium. We first consider the case of symmetric reaction speeds and then turn to the implication of asymmetries in reaction speeds. In each case, we first derive results for the general model assuming equilibrium existence and differentiability of the dynamic quantity reaction functions, and then we analyze the comparative statics of the steady-state equilibrium quantities in the unique linear MPE under linear demand.

5.2 Symmetric reaction speeds

If $\lambda_A = \lambda_B = \lambda$, the equilibrium dynamic quantity reaction functions depend on $\frac{r}{\lambda}$, the discount rate scaled by the recontracting rate, but not on r and λ separately. Given differentiability, this can be seen directly from the first-order condition, which can be written as follows: at $q = R_i(q_{-i})$,

$$0 = -c + \frac{\frac{r}{\lambda} + 1}{\frac{r}{\lambda} + 2}\pi_1(q, q_{-i}) + \frac{1}{\frac{r}{\lambda} + 2}\left(\pi_1(q, R_{-i}(q)) + \pi_2(q, R_{-i}(q)) R'_{-i}(q) + \pi_2(R_{-i}(q), q)\right).$$

[19]Our specification allows for negative prices, but that is not essential for the results. The alternative specification $P(Q) = \max\{1 - Q, 0\}$ would yield the same equilibrium for high enough c, because the equilibrium dynamic reaction functions in our unrestricted specification satisfy $R_i(q_{-i}) + q_{-i} < 1$ for all $q_{-i} \in [0, \overline{Q}_c]$ and i if c is large enough. In the case of small c, where $R_i(q_{-i}) + q_{-i} > 1$ for q_{-i} close to $\overline{Q}_c = 1 - c$ (although on path aggregate quantity never exceeds 1, that is, $R_i(R_{-i}(q)) + R_{-i}(q) < 1$ for all $q \in [0, \overline{Q}_c]$), restricting the quantity action space to $[0, Q^M]$ re-establishes the equivalence between the two specifications, because the equilibrium dynamic reaction functions in our specification satisfy $R_i(q_{-i}) + q_{-i} < 1$ and $R_i(q_{-i}) \in (0, Q^M)$ for all $q_{-i} \in [0, Q^M]$ and i.

[20]Using $P(Q) = 1 - Q$ rather than $P(Q) = a - bQ$ is without loss of generality. Given that the marginal cost c is constant and a free parameter, setting $a = 1$ amounts to a choice of measurement units for output, and setting $b = 1$ is a normalization of the market size.

Intuitively, what matters for the firms' present discounted payoffs is not the absolute value of the discount rate but the discount rate relative to the expected time between recontracting events.

The next proposition uses the result that the equilibrium dynamic reaction functions must be downward-sloping together with the (local) stability condition for the steady-state quantities, $R'_A(q^e_B)\,R'_B(q^e_A) < 1$,[21] to establish upper and lower bounds on the equilibrium steady-state quantity in a symmetric equilibrium:

Proposition 2 *Suppose $\lambda_A = \lambda_B = \lambda$. In any dynamically stable equilibrium with symmetric differentiable dynamic quantity reaction functions and a symmetric steady state (when such an equilibrium exists),*

(i) *the steady-state quantity $q^e < q^C$.*

(ii) *the steady-state quantity $q^e \geq \underline{q}\left(\frac{r}{\lambda}\right)$, where the lower bound $\underline{q}\left(\frac{r}{\lambda}\right) \in \left(q^M, q^C\right)$ is uniquely defined by*

$$\Pi'\left(2\underline{q}\right) - \pi_2\left(\underline{q}, \underline{q}\right) + \frac{1}{\frac{r}{\lambda}+1}\Pi'\left(2\underline{q}\right) = 0,$$

strictly increasing in $\frac{r}{\lambda}$, and has limits

$$\lim_{\frac{r}{\lambda}\to 0} \underline{q}\left(\frac{r}{\lambda}\right) \in \left(q^M, q^C\right) \quad \textit{and} \quad \lim_{\frac{r}{\lambda}\to\infty} \underline{q}\left(\frac{r}{\lambda}\right) = q^C.$$

Proof. Let $\lambda_A = \lambda_B = \lambda$, and suppose that a dynamically stable equilibrium with differentiable $R_A = R_B = R$ and a symmetric steady state (q^e, q^e) exists. The first-order condition (3) evaluated at (q^e, q^e) is

$$-c + \frac{r+\lambda}{r+2\lambda}\pi_1\left(q^e, q^e\right) + \frac{\lambda}{r+2\lambda}\left(\pi_1\left(q^e, q^e\right) + \pi_2\left(q^e, q^e\right) R'\left(q^2\right) + \pi_2\left(q^e, q^e\right)\right) = 0$$

for each $i = A, B$, which, using $\Pi'(2q^e) = \pi_1(q^e, q^e) + \pi_2(q^e, q^e) - c$, can be rewritten as

$$\Pi'\left(2q^e\right) - \pi_2\left(q^e, q^e\right) + \frac{1}{\frac{r}{\lambda}+1}\left(\Pi'\left(2q^e\right) + \pi_2\left(q^e, q^e\right) R'\left(q^e\right)\right) = 0. \tag{4}$$

We first prove part (ii), and then part (i), of the proposition.

[21] The equilibrium reaction functions define a difference equation from $(q_{A,t}, q_{B,t-1})$ to $(q_{A,t+2}, q_{B,t+1}) = (R_A(R_B(q_{A,t})), R_B(R_A(q_{B,t-1})))$. Then, its Jacobian matrix evaluated at a steady state (q^e_A, q^e_B) has two equal real eigenvalues $R'_B(q^e_A)R'_A(q^e_B)$, which must be strictly less than one for (local) stability. See Vives (1999) for more details.

Part (ii): By Lemma 2, $R'(q^e) \leq 0$. Since $\pi_2(q,q) \leq 0$ for all $q \geq 0$, (4) and $R'(q^e) \leq 0$ imply that

$$0 \geq \Pi'(2q^e) - \pi_2(q^e, q^e) + \frac{1}{\frac{r}{\lambda}+1}\Pi'(2q^e). \tag{5}$$

The function $G(q) \equiv \Pi'(2q) - \pi_2(q,q) + \frac{1}{\frac{r}{\lambda}+1}\Pi'(2q)$ is strictly decreasing in q, because both $\Pi'(2q) - \pi_2(q,q)$ and $\Pi'(2q)$ are strictly decreasing in q. Moreover, by the definitions of q^M and q^C, $G(q^M) = -\pi_2\left(q^M, q^M\right) > 0$ and $G(q^C) = \frac{1}{\frac{r}{\lambda}+1}\Pi'\left(2q^C\right) < 0$. Hence, for every $\frac{r}{\lambda} > 0$, there exists a unique $\underline{q}\left(\frac{r}{\lambda}\right) \in \left(q^M, q^C\right)$ such that

$$\Pi'\left(2\underline{q}\right) - \pi_2\left(\underline{q}, \underline{q}\right) + \frac{1}{\frac{r}{\lambda}+1}\Pi'\left(2\underline{q}\right) = 0, \tag{6}$$

and (5) implies that $q^e \geq \underline{q}\left(\frac{r}{\lambda}\right)$.

Applying the implicit function theorem to (6), we obtain that

$$\frac{d\underline{q}}{d\left(\frac{r}{\lambda}\right)} \overset{sign}{=} -\frac{1}{\left(\frac{r}{\lambda}+1\right)^2}\Pi'\left(2\underline{q}\right),$$

which is strictly positive because $\underline{q} > q^M$ and thus $\Pi'\left(2\underline{q}\right) < 0$. Moreover, it follows from (6) that $\lim_{\frac{r}{\lambda}\to\infty} \underline{q}\left(\frac{r}{\lambda}\right) = q^C$ and $\lim_{\frac{r}{\lambda}\to 0} \underline{q}\left(\frac{r}{\lambda}\right) \in \left(q^M, q^C\right)$.

Part (i): By the stability condition and Lemma 2, $R'(q^e) > -1$. Moreover, the result from part (ii) that $q^e > q^M$ implies that $\pi_2(q^e, q^e) < 0$. (4) and $R'(q^e) > -1$ thus imply that

$$0 < \left(1 + \frac{1}{\frac{r}{\lambda}+1}\right)\left(\Pi'(2q^e) - \pi_2(q^e, q^e)\right). \tag{7}$$

The function $\Pi'(2q) - \pi_2(q,q)$ is strictly decreasing in q and equal to zero for $q = q^C$. Hence, (7) implies that $q^e < q^C$. ■

Proposition 2 implies that any symmetric steady-state quantity (if it exists) lies strictly between q^M and q^C. The intution for this finding is as follows. Consider U's contract offer to D_A. When making an offer, the supplier internalizes only the effect on D_A's own variable profit in the time interval until the next recontracting with D_B, but it internalizes the effect on *all* retailers' variable profits from the next recontracting with D_B onwards (because the fixed fee offered to D_A takes into account expected changes in D_A's variable profit due to recontracting with D_B before D_A's own contract terminates, and similarly the fixed fees in future contracts extract all expected future changes in the retailers' variable profits). Hence, it is intuitive that the steady state in the dynamic model falls in between the benchmark cases of simultaneous secret offers with passive beliefs, in which the supplier only internalizes the effect on D_A's own variable profits when making an offer to D_A and each retailer sells q^C in equilibrium, and the case of simultaneous public offers, in which the supplier internalizes the effects on all retailers variable profits when making an offer and each retailer sells q^M in the symmetric equilibrium.[22]

By this reasoning, one might expect that $\lim_{\frac{r}{\lambda}\to\infty} \underline{q}\left(\frac{r}{\lambda}\right) = q^C$ and $\lim_{\frac{r}{\lambda}\to 0} \underline{q}\left(\frac{r}{\lambda}\right) = q^M$, but the latter part of this intuition turns out to be wrong. While indeed $\lim_{\frac{r}{\lambda}\to\infty} \underline{q}\left(\frac{r}{\lambda}\right) = q^C$, we find that $\lim_{\frac{r}{\lambda}\to 0} \underline{q}\left(\frac{r}{\lambda}\right) > q^M$. Although the short-term gain from making an offer that raises bilateral profits at the expense of industry profit goes to zero when reactions become near instantaneous ($\lambda \to \infty$),[23] if an equilibrium exists its steady-state quantity must nonetheless lie strictly above the monopoly quantity. Intuitively, that the short-term gain from making an opportunistic offer goes to zero is not enough to guarantee that the supplier has no incentive to behave opportunistically, because the expected present discount value of the losses in future industry profits triggered by an opportunistic move also goes to zero in this limit case. In a dynamically stable equilibrium, quantities converge back to the steady-state level after a deviation from it (and do so "fast" for $\lambda \to \infty$), hence a deviation from a candidate steady-state $\left(q^M, q^M\right)$ does not have a persistent negative effect on industry profits.[24] For $\frac{r}{\lambda} \to \infty$, on the other hand, the short-term gain from an opportunistic move remains positive while the present discounted value of long-term effects goes to zero, hence the supplier makes offers that maximize bilateral profits in equilibrium.

Consistent with these intuitions, we obtain the following comparative statics results on the steady-state quantity in the linear-demand case:

[22]This insight may seem to suggest that $R^M(q_{-i}) < R(q_{-i}) < R^C(q_{-i})$ (which is equivalent to $q^M < q^e < q^C$ for $q_{-i} = q^e$) for all q_{-i}, but that intuition is incorrect. As will be illustrated later, $R^M(q_{-i}) > R(q_{-i})$ for small q_{-i} in the linear-demand case. Intuitively, maximizing current industry profits given q_{-i} is different from maximizing *future* industry profits, because the quantity offered to D_i today triggers reactions in future recontracting offers and thus dynamically affects the quantities of both retailers.

[23]Or, in the case of $r \to 0$, the weight put on the short-term gain goes to zero.

[24]The observation that both the short-term gain from an opportunistic offer and the net present value of the effects on future industry profits go to zero for $\frac{r}{\lambda} \to 0$ also holds for a deviation from a candidate equilibrium steady-state with a symmetric quantity $q^e > q^M$. However, intuitively the supplier's incentive to behave opportunistically is smaller when the hypothetical steady-state quantity is larger, because (i) the maximal gain in bilateral profits that $U - D_i$ can obtain is smaller for larger q_{-i}, and (ii) since industry profits are concave, the negative effect of a marginal increase in total quantity on industry profits is larger when the aggregate quantity is farther above q^M.

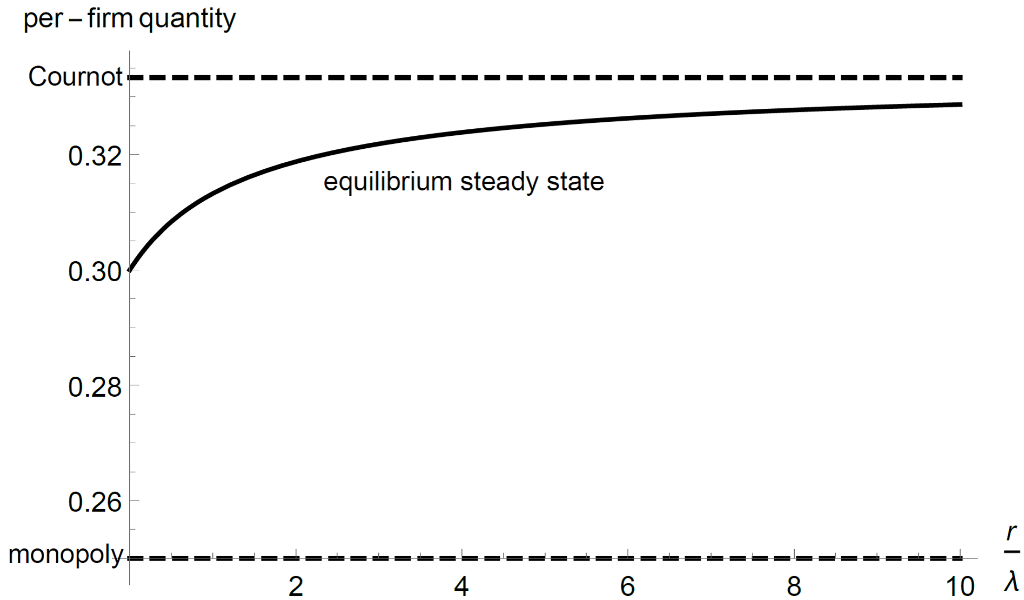

Figure 1: The solid line shows the equilibrium steady-state quantity q^e as a function of $\frac{r}{\lambda}$, the dashed lines the per-firm Cournot quantity (q^C) and the per-firm symmetric monopoly quantity (q^M). All quantities are computed for $P(Q) = 1 - Q$ and $c = 0$.

Proposition 3 *Suppose $P(Q) = 1 - Q$, $\lambda_A = \lambda_B = \lambda$, and $c \in [0, 1)$. Then, the unique linear MPE is symmetric ($R_A = R_B$) with steady-state quantities $q_A^e = q_B^e = q^e$. The steady-state quantity q^e is strictly increasing in $\frac{r}{\lambda}$, with $\lim_{\frac{r}{\lambda}\to 0} q^e = \frac{3(1-c)}{10} \in (q^M, q^C)$, and $\lim_{\frac{r}{\lambda}\to\infty} q^e = \frac{1-c}{3} = q^C$.*

Figure 1 illustrates the comparative statics results of Proposition 3. The equilibrium steady-state quantity is smaller, that is, the opportunism problem is less severe, for greater patience (lower r) and faster reaction speed (higher λ). Intuitively, faster reaction speed alleviates the opportunism problem, for a given discount rate, because it decreases the length of time during which each supplier-retailer pair can gain from opportunistic moves at the expense of the rival retailer, and greater patience alleviates the opportunism problem, for a given reaction speed, because firms attach less weight to the short-term profit gains from opportunistic moves. As also illustrated in the figure, the equilibrium steady state approaches the Cournot quantity as $\frac{r}{\lambda}$ approaches infinity, but remains bounded strictly above the per-firm monopoly quantity as $\frac{r}{\lambda}$ approaches zero.

It is worth noting that although the aggregate quantity always lies between Q^M and $2q^C$

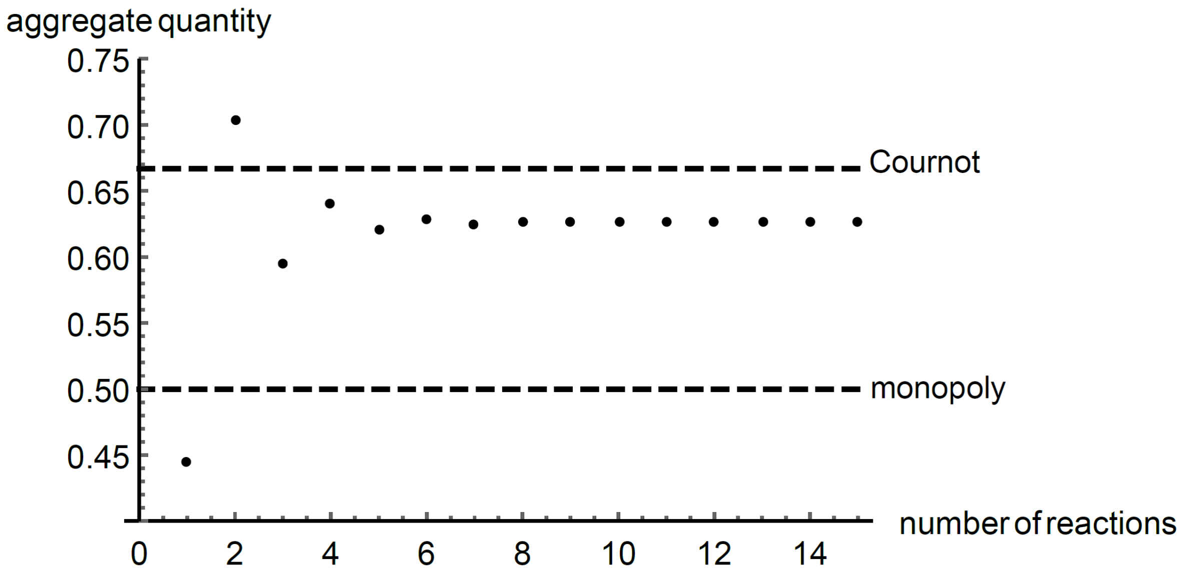

Figure 2: The dots show the sequence of aggregate quantities $0 + R(0)$, $R(R(0)) + R(0)$, $R(R(0)) + R(R(R(0)))$, etc. given the dynamic quantity reaction function R in the equilibrium from Proposition 3. The dashed lines show the aggregate Cournot quantity ($2q^C$) and the monopoly quantity (Q^M). All quantities are computed for $P(Q) = 1 - Q$ and $c = 0$.

in the equilibrium steady state, out of steady state the aggregate quantity can lie outside of this range. This is illustrated in Figure 2, which shows how aggregate quantity changes with each reaction in the rival retailer's contract starting from the state $q_{-i} = 0$.

5.3 Asymmetric reaction speeds

This section considers the implications of asymmetries in reaction speeds. Specifically, we assume that D_B's contract reacts faster than D_A's contract, i.e., that

$$\lambda_A \leq \lambda_B,$$

which is without loss of generality but simplifies the exposition.

In order to disentangle the effects of asymmetry from effects of changes in the aggregate recontracting rate, our comparative statics exercises will focus on the following two parameters. First, the mean reaction speed, denoted by

$$\lambda_M = \frac{\lambda_A + \lambda_B}{2},$$

and second, the degree of asymmetry in reaction speeds, defined as

$$\mu = \frac{\lambda_M - \lambda_A}{\lambda_M} \in [0, 1) .$$

The reaction speeds of the two supplier-retailer pairs can be expressed as functions of these two parameters as follows:

$$\begin{aligned}\lambda_A &= (1-\mu)\,\lambda_M, \\ \lambda_B &= (1+\mu)\,\lambda_M.\end{aligned}$$

For a small time interval Δq, the probability of a contract termination occuring is then given by $2\lambda_M \Delta q$, independently of the degree of asymmetry μ. Raising μ, in turn, makes the reaction speeds more asymmetric without changing the mean recontracting rate.

Given these notations, the first-order equilibrium conditions in (3) can be rewritten as functions of $\frac{r}{\lambda_M}$ and μ as follows: at $q = R_A(q_B)$,

$$0 = -c + \frac{\frac{r}{\lambda_M} + 1 - \mu}{\frac{r}{\lambda_M} + 2}\pi_1(q, q_B) + \frac{1+\mu}{\frac{r}{\lambda_M} + 2}\left(\pi_1(q, R_B(q)) + \pi_2(q, R_B(q))\, R_B'(q) + \pi_2(R_B(q), q)\right), \tag{8}$$

and at $q = R_B(q_A)$,

$$0 = -c + \frac{\frac{r}{\lambda_M} + 1 + \mu}{\frac{r}{\lambda_M} + 2}\pi_1(q, q_A) + \frac{1-\mu}{\frac{r}{\lambda_M} + 2}\left(\pi_1(q, R_A(q)) + \pi_2(q, R_A(q))\, R_A'(q) + \pi_2(R_A(q), q)\right). \tag{9}$$

Using these first-order conditions together with the result that the dynamic quantity reaction functions are non-increasing and the local stability condition, our next proposition establishes a series of insights about the steady-state quantities under asymmetric reaction speeds without imposing linear demand.

Proposition 4 *Suppose $\lambda_A \leq \lambda_B$. In any dynamically stable equilibrium with twice differentiable dynamic quantity reaction functions that have uniformly bounded first and second derivatives (when such an equilibrium exists),*

(i) *the steady-state quantities (q_A^e, q_B^e) satisfy $q_i^e < R^C\left(q_{-i}^e\right)$ for at least one $i \in \{A, B\}$;*

(ii) *the aggregate steady-state quantity* $q_A^e + q_B^e \geq \underline{Q}\left(\frac{r}{\lambda_M}, \mu\right)$*, where the lower bound* $\underline{Q}\left(\frac{r}{\lambda_M}, \mu\right) \in \left(Q^M, 2q^C\right)$ *is uniquely defined by*

$$\Pi'\left(\underline{Q}\right) - \pi_2\left(\frac{1}{2}\underline{Q}, \frac{1}{2}\underline{Q}\right) + \frac{\frac{r}{\lambda_M} + 1 + \mu^2}{\left(\frac{r}{\lambda_M} + 1 - \mu\right)\left(\frac{r}{\lambda_M} + 1 + \mu\right)}\Pi'\left(\underline{Q}\right) = 0,$$

strictly decreasing in μ*, strictly increasing in* $\frac{r}{\lambda_M}$*, and has limits*

$$\begin{aligned} \lim_{\mu \to 0} \underline{Q}\left(\frac{r}{\lambda_M}, \mu\right) &= 2\underline{q}\left(\frac{r}{\lambda_M}\right) > Q^M, \; \lim_{\mu \to 1} \lim_{\frac{r}{\lambda_M} \to 0} \underline{Q}\left(\frac{r}{\lambda_M}, \mu\right) = Q^M, \text{ and} \\ \lim_{\frac{r}{\lambda_M} \to \infty} \underline{Q}\left(\frac{r}{\lambda_M}, \mu\right) &= 2q^C; \end{aligned}$$

(iii) *for any given* $\epsilon > 0$*, the steady-state quantities satisfy*

$$q_A^e < \epsilon \text{ and } \left|q_B^e - Q^M\right| < \epsilon$$

if $\frac{r}{\lambda_M}$ *is close enough to* 0 *and* μ *is close enough to* 1*.*

The key new insight in Proposition 4 is that asymmetry in reaction speeds can lead to a lower aggregate quantity in the equilibrium steady state (when an equilibrium exists). When the reaction speeds are symmetric, the aggregate steady-state quantity is bounded below by an amount strictly above the monopoly quantity, as shown in part (ii) of the proposition. When the degree of reaction speed asymmetry is sufficiently large (and $\frac{r}{\lambda_M}$ is close enough to zero), however, the aggregate steady-state quantity is arbitrarily close to the monopoly quantity, as shown in part (iii) of the proposition.

Consistent with this, our next proposition shows that in the linear-demand case, the aggregate steady-state quantity is falling in the degree of reaction speed asymmetry in the unique equilibrium with linear dynamic quantity reaction functions.

Proposition 5 *Suppose* $P(Q) = 1 - Q$*,* $c \in [0, 1)$*, and* $\lambda_A \leq \lambda_B$*. The steady-state quantities* (q_A^e, q_B^e) *in the unique linear MPE vary with* $\frac{r}{\lambda_M}$ *and* μ *as follows:*

(i) *The aggregate quantity* $q_A^e + q_B^e$ *is strictly increasing in* $\frac{r}{\lambda_M}$*.*

(ii) *The aggregate quantity* $q_A^e + q_B^e$ *is strictly decreasing in* μ*,* q_A^e *is strictly decreasing in* μ*, and* q_B^e *is strictly increasing in* μ*.*

(iii) $\lim_{\mu\to 1}\lim_{\frac{r}{\lambda_M}\to 0}(q_A^e, q_B^e) = (0, Q^M)$.

Figures 3 and 4 provide graphical illustrations of these results. First, as illustrated in Figure 3, reaction speed asymmetry alleviates the supplier's opportunism problem: For a given mean reaction speed, the aggregative quantity in the steady state is lower for higher degrees of asymmetry. Second, and consistent with the results in the symmetric case, for a given degree of asymmetry, the aggregate steady-state quantity is smaller when the firms are more patient (smaller r) or the mean reaction speed is higher (greater λ_M). Third, as illustrated in Figure 4, at the steady state the retailer whose contract reacts faster sells a larger quantity than the retailer whose contract reacts more slowly. Finally, as $\frac{r}{\lambda_M} \to 0$ and $\mu \to 1$, the steady state approaches the "exclusive dealing outcome" in which one retailer sells the entire monopoly quantity and its rival sells zero.

The intuition behind these findings can be understood as follows. First, keeping the future dynamic quantity reaction functions fixed, a faster reaction of D_B's contract weakens the supplier's incentive to behave opportunistically when making offers to D_A, because it leaves less time for the bilateral pair $U - D_A$ to "free-ride" on D_B's variable profits. And similarly, keeping the future dynamic quantity reaction functions fixed, a slower reaction of D_A's contract raises the supplier's incentive to behave opportunistically when making offers to D_B, because it means that the bilateral pair $U - D_B$ can "free-ride" on D_A's variable profits for a longer time.

Second, the gap between the quantity $R^C(q_{-i})$ that maximizes $\Pi(q, q_{-i}) - \pi(q_{-i}, q)$ and the quantity $R^M(q_{-i})$ that maximizes $\Pi(q, q_{-i})$, which can be thought of as the extent of the conflict between collective surplus maximization and bilateral surplus maximization, is rising in q_{-i}.[25] For $q_{-i} = 0$, the conflict vanishes altogether, as $R^C(0) = R^M(0) = Q^M$. Hence, although reaction speed asymmetry strengthens the incentive for opportunism with D_B for a *given* current quantity q_A, when q_A is small the conflict between collective surplus maximization and bilateral surplus maximization between the supplier and D_B is weak to begin with. By selling a small quantity to D_A, exploiting the weakened incentive to behave opportunistically when making offers to D_A, the supplier thus also weakens its own incentive to behave opportunistically when making the next contract offer to D_B.

[25] Formally, $R^C(q_{-i}) - R^M(q_{-i})$ is strictly increasing for all $q_{-i} > 0$ because $\frac{\partial R^M}{\partial q_{-i}} = -1$ while $\frac{\partial R^C}{\partial q_{-i}} \in (-1, 0)$ by assumptions A1 and A2.

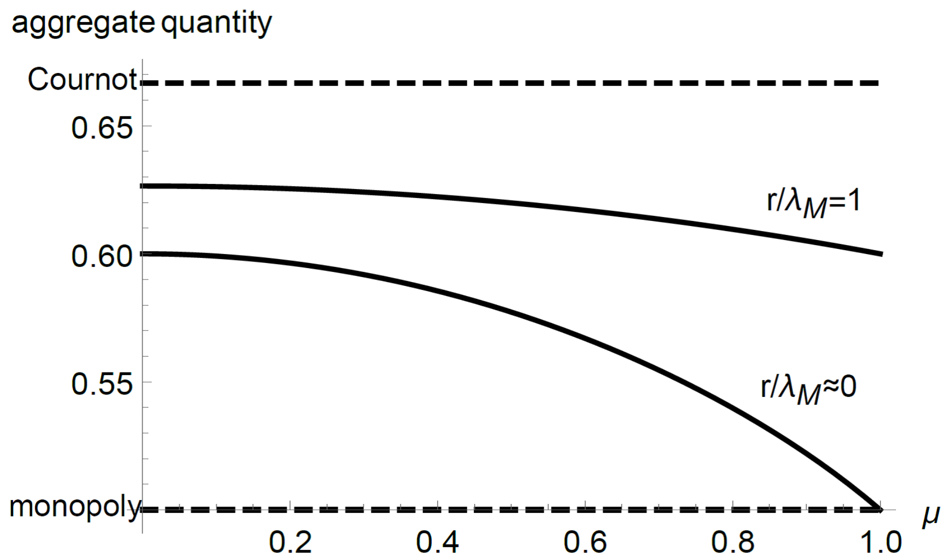

Figure 3: The solid lines show the aggregate equilibrium steady-state quantity $q_A^e + q_B^e$ as a function of the reaction speed asymmetry μ for two different values of $\frac{r}{\lambda_M}$, the dashed lines the aggregate Cournot quantity $(2q^C)$ and the monopoly quantity (Q^M). All quantities are computed for $P(Q) = 1 - Q$ and $c = 0$.

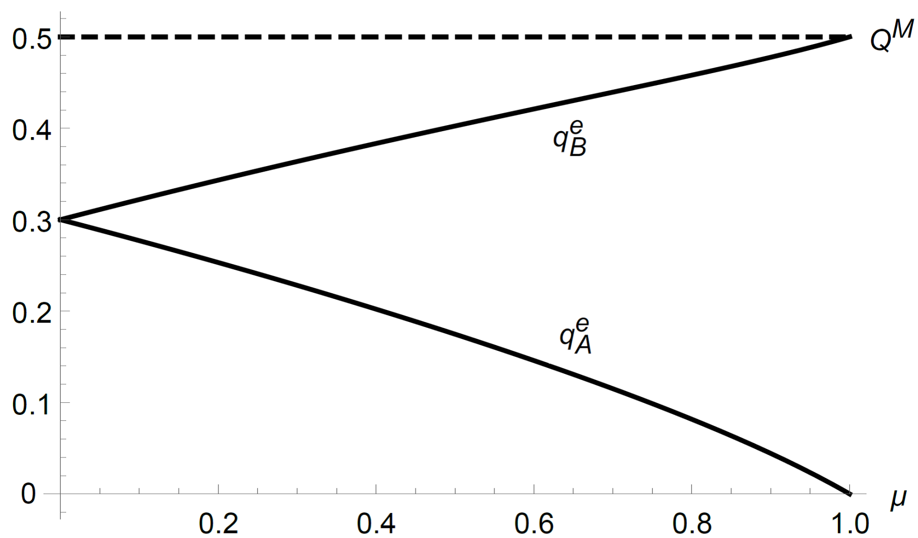

Figure 4: The solid lines show the equilibrium steady-state quantities q_A^e and q_B^e as a function of the reaction speed asymmetry μ, given $\frac{r}{\lambda_M} \approx 0$. The dashed line indicates the monopoly quantity Q^M. All quantities are computed for $P(Q) = 1 - Q$ and $c = 0$.

Overall, this intuition suggests that, consistent with our results, the aggregate steady-state quantity will be smaller under reaction speed asymmetry than under symmetry, and that it will be allocated asymmetrically across retailers, with the retailer whose contract reacts faster (D_B) selling a larger quantity than its rival. In the limit case where $\mu \to 1$ and $\frac{r}{\lambda_M} \to 0$, D_A's quantity converges to zero, eliminating the conflict between collective surplus maximization and bilateral surplus maximization when U contracts with D_B. Anticipating the slow future reaction of D_A's contract (since $\lambda_A \to 0$ when $\mu \to 1$), the supplier therefore optimally offers the quantity $R^C(0) = R^M(0) = Q^M$ to D_B. The supplier also has no profitable deviation to offering a larger quantity to D_A in this limit case, because doing so would harm future industry profits and these negative effects are persistent when $\lambda_A \approx 0$.

More generally, the steady state in the limit case where $\mu \to 1$ and $\frac{r}{\lambda_M} \to 0$ corresponds to the equilibrium outcome of a sequential-move game in which U first offers a contract to D_A and then to D_B, and the second retailers (D_B) observes the first retailer's (D_A's) contract. The contract offered to the second retailer (D_B) then maximizes the bilateral surplus of the supplier and the second retailer given the contract accepted by the first retailer, while the contract offered to the first retailer (D_A) maximizes the bilateral surplus of the supplier and the first retailer anticipating that the later contract will maximize the bilateral surplus of the supplier and the second retailer (D_B). In our setting, this means that the supplier offers $R^C(q_A)$ to D_B, and, anticipating this, offers 0 to D_A, which leads to quantities $\left(0, Q^M\right)$ and a total profit of Π^M. While the result that the opportunism problem is fully solved in the limit clearly hinges on the retailers selling perfect substitutes, the findings that reaction speed asymmetry and patience alleviate the opportunism problem hold more broadly, as Figure 5 illustrates in the context of a Cournot model with differentiated goods.[26]

[26]The same qualitative results also hold in a model of differentiated Bertrand competition with linear demands when supply contracts fix the retailer's downstream price. The result that reaction speed asymmetry alleviates the opportunism problem also does not depend on the particular way in which asymmetry was parametrized. In particular, they also hold when asymmetry is measured by $\lambda_B - \lambda_A$ while keeping constant $\frac{1}{\lambda_A} + \frac{1}{\lambda_B}$, the expected time it takes for a retailer's quantity to change out of steady state (i.e., the expected time until D_i's quantity q_i changes to $R_i(R_{-i}(q_i))$). Similarly, the result also holds in a model with deterministic reaction lags where all contracts have length $\Delta_A + \Delta_B$, and asymmetry is measured by $|\Delta_A - \Delta_B|$ while keeping $\Delta_A + \Delta_B$ fixed.

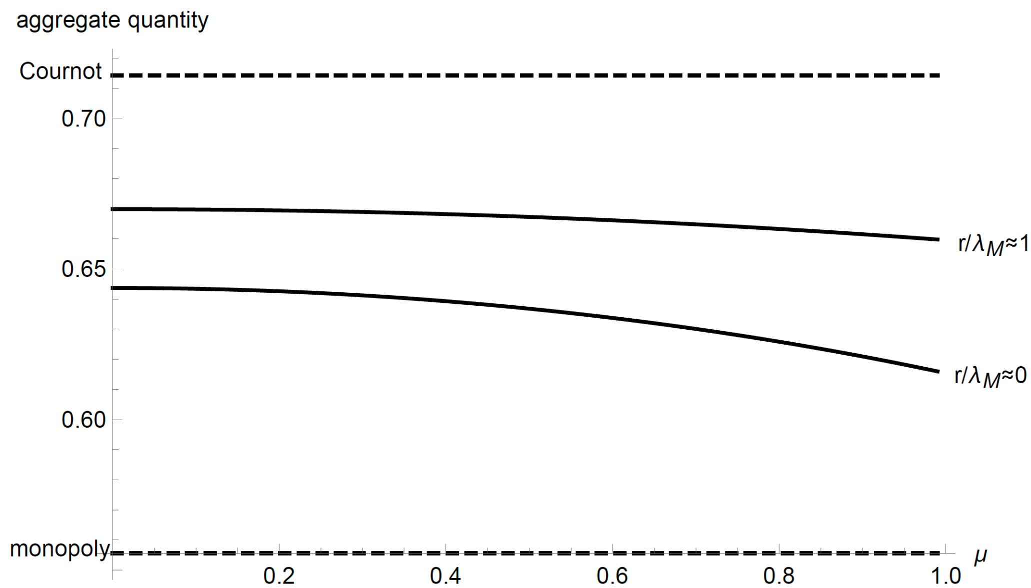

Figure 5: Illustration of the aggregate steady-state quantity under differentiated Cournot competition, letting $P_i(q_i, q_{-i}) = 1 - q_i - 0.8q_{-i}$ and $c = 0$. The solid lines show the aggregate equilibrium steady-state quantity $q_A^e + q_B^e$ as a function of the reaction speed asymmetry μ for two different values of $\frac{r}{\lambda_M}$, the dashed lines the aggregate Cournot quantity $(2q^C)$ and the aggregate monopoly quantity $(2q^M)$.

6 Restoring Monopoly Power

The extent of opportunism that prevails in the absence of any strategies used by the supplier to restore its monopoly power has been the focus of our analysis so far. We now discuss various strategies the supplier can use to overcome its opportunism problem in our dynamic setting with asynchronous recontracting.

Vertical integration As in simultaneous secret-offer models, vertical integration helps the upstream monopolist to overcome its opportunism problem in our dynamic setting. To see this, suppose U is vertically intergrated with D_A. Given a contract (q_B, f_B) signed with D_B, the vertical integrated firm then sets its quantity at

$$R^C(q_B) = \arg\max_q (\pi(q, q_B) - cq) = \arg\max_q (\Pi(q + q_B) - \pi(q_B, q)).$$

Anticipating this, the highest fixed fee D_B is willing to accept in a contract with quantity q_B is $\pi\left(q_B, R^C(q_B)\right)$. Thus, the integrated firm's flow profit becomes

$$\Pi\left(R^C(q_B) + q_B\right),$$

which is maximized and equal to Π^M at $q_B = 0$. Hence, the vertically integrated upstream monopolist forecloses the non-integrated downstream firm and earns monopoly profits in equilibrium.[27]

Opt-out contracts One way to protect retailers against opportunistic moves by the supplier in our dynamic context is to allow each retailer to "opt out" of its current contract, that is, to stop selling and stop paying the supplier for the remaining duration of the contract.[28] To see why, suppose supplier D_{-i} has signed a contract (q_{-i}, f_{-i}) with an opt-out clause, and U offers a contract with quantity q_i to D_i, which is accepted. Then, D_{-i} will want to continue selling and paying the fixed fee if $\pi(q_{-i}, q_i) > f_{-i}$, but will prefer to exercise its opt-out clause if $\pi(q_{-i}, q_i) < f_{-i}$. The supplier therefore loses the fixed payment from D_{-i} if it makes an opportunistic offer to D_i that pushes D_{-i}'s variable profit below

[27] Of course, this "extreme" conclusion depends on the absence of an alternative supplier and the assumption that downstream firms sell undifferentiated final goods (see, e.g., Rey and Tirole, 2007, and Reisinger and Tarantino, 2015).

[28] We are grateful to Volker Nocke for this suggestion.

f_{-i}. Thus, opt-out contracts, by protecting retailers against negative flow payoffs, limit the supplier's ability to profit from opportunistic offers.

Indeed, the supplier can earn Π^M, the maximum industry profit, by offering opt-out contracts such that at any point in time only one of the retailers is active and sells the monopoly quantity Q^M. To see this, suppose U offers an opt-out contract $\left(Q^M, \pi\left(Q^M, 0\right)\right)$ at every recontracting opportunity. Then, since $\pi\left(Q^M, Q^M\right) < \pi\left(Q^M, 0\right)$, D_{-i} will exercise its opt-out clause if D_i accepts U's offer, which makes it optimal for D_i to accept the offer and sell Q^M until the next recontracting between U and D_{-i}, at which point D_i will opt out of its current contract. And given that D_{-i} currently has an opt-out contract $\left(Q^M, \pi\left(Q^M, 0\right)\right)$, the supplier cannot profitably deviate to making a different offer to D_i even in the limit case $r \to \infty$ (in which opportunism was shown to be most severe in the absence of opt-out clauses), because D_{-i} will opt out if $q_i > 0$ and the bilateral profit with either one of the retailers cannot exceed $\pi\left(Q^M, 0\right) = \Pi^M$.

For a more formal analysis of opt-out contracts, see Appendix B, where we prove that for $r \to \infty$, there exists a MPE in which for any history, U eventually offers the opt-out contract $\left(Q^M, \pi\left(Q^M, 0\right)\right)$ at every recontracting opportunity.

Exclusive dealing To see how a commitment to deal exclusively with one of the retailers can help the supplier to restore its monopoly power, suppose that U makes the following offers at every recontracting opportunity: it offers an exclusive contract $\left(Q^M, \pi\left(Q^M, 0\right)\right)$ to D_A, and the null contract $(0, 0)$, which is equivalent to no offer, to D_B. An exclusive contract commits the supplier not to offer a strictly positive quantity to the other retailer for the duration of the contract. It is then optimal for D_A to accept $\left(Q^M, \pi\left(Q^M, 0\right)\right)$, because (i) given that $q_B = 0$, upon acceptance D_A will earn variable profits of $\pi\left(Q^M, 0\right)$ until the next recontracting between U and D_B, and (ii) the exclusivity clause ensures D_A that it will continue to earn variable profits of $\pi\left(Q^M, 0\right)$ even if U recontracts with D_B before D_A's own contract expires. Hence, D_A is guaranteed variable profits that cover the fixed payment for the entire duration of its contract. U cannot profitably deviate from these offers. When making an offer to D_B, U's hands are tied by its previous commitment to deal exclusively with D_A. And when making an offer to D_A, U cannot profitably deviate because first, conditional on using an exclusivity clause, offering $\left(Q^M, \pi\left(Q^M, 0\right)\right)$ is clearly optimal, and second, dropping the exclusivity clause reduces the fixed fee D_A is willing to pay for any q_A without increasing industry profits from the next recontracting onwards (which are already at the maximum level).

For a more formal analysis of exclusive dealing contracts, see Appendix B, where we prove that for $r \to \infty$, there exists a MPE in which for any history, the aggregate quantity eventually becomes Q^M when the supplier can offer exclusivity clauses. In our analysis, the dynamic transition from a state without exclusive contracts to a state in which the supplier's good is distributed exclusively by one of the downstream firms is achieved by allowing the supplier to commit not to offer D_i's rival a new contract with a strictly positive quantity before termination of D_i's contract (even if the rival firm sells a strictly positive quantity at the time D_i's contract is signed).

7 Concluding Remarks

We have analyzed a dynamic model of bilateral contracting between one supplier and multiple competing downstream firms. In our setting, each downstream firm anticipates future recontracting between the supplier and its rival (as well as itself), and supplier suffers from opportunism even if it can make public contract offers. The proposed dynamic model offers an alternative to simultaneous secret-offers models of opportunism in vertical contracting. Although characterizing equilibria tends to be more difficult in the dynamic model, we have shown that a unique equilibrium in simple linear strategies, with closed-form solutions for the strategies and equilibrium steady-state quantities, exists under linear demand. Moreover, the dynamic model overcomes a key weakness of simultaneous secret-offers models, the sensitivity of the equilibrium outcome to out-of-equilibrium beliefs. The dynamic model also unifies existing literature, by offering a setting that generates the equilibrium outcomes of existing models, with either simultaneous or sequential offers, as special limit cases.

Our results are relevant for vertical merger policy as well as for competition policy on vertical restraints. First, the results show that secret offers are not needed for opportunism to arise, and that vertical mergers and opportunism-avoiding contract provisions like exclusive dealing or opt-out clauses can have anticompetitive effects even when contracts are public.

Second, the results offer guidance on when vertical mergers and opportunism-avoiding contract provisions are likely to be more harmful. We have found that the degree of opportunism is greater when firms are impatient, there are long time gaps between recontracting, and reaction speeds are symmetric. This, in turn, suggests that vertical mergers and opportunism-avoiding contract provisions are likely to be more harmful for downstream consumers (and more attractive for suppliers) when firms are impatient, time gaps between recontracting are long, and reaction speeds are symmetric. Proxies for the model's key parameters that can potentially be observed and used by competition authorities to help assess the likely competitive harm include the average duration of downstream firm i's contracts (as a proxy for $\frac{1}{\lambda_i}$) and the interest rate (as a proxy for r). Moreover, useful information to assess the degree of asymmetries in reaction speeds would include observed asymmetries in the time gap between a change in downstream firm A's supply terms followed by a change in downstream firm B's supply terms versus the opposite order, and evidence of otherwise similar downstream firms being treated asymmetrically prior to a vertical merger.

There are several interesting issues that we leave for future research. First, an important yet difficult direction for future research is to endogenize the speed of contract reactions and the associated contract durations. Second, the analysis could be extended to allow for three or more downstream firms. Third, other forms of non-linear vertical contracts could be considered, which would open up the possibility of a retailer adjusting the quantity it orders from the supplier in reponse to a change in its rival's contract.

Appendix A: Proofs

Proof of Lemma 1. Let $V_i(s) = V_i(R_i(s), F_i(s); s)$ and recall that $M_i(R_i(s), F_i(s); s) = 1$ for all i and s. We have that

$$\begin{aligned} V_i(q,f;s) &= \frac{\pi(q,s) - f}{r+\lambda_i+\lambda_{-i}} + \frac{\lambda_i}{r+\lambda_i+\lambda_{-i}} V_i(s) + \frac{\lambda_{-i}}{r+\lambda_i+\lambda_{-i}} \widetilde{V}_i(q,f), \\ \widetilde{V}_i(q,f) &= \frac{\pi(q, R_{-i}(q)) - f}{r+\lambda_i} + \frac{\lambda_i}{r+\lambda_i} V_i(R_{-i}(q)). \end{aligned}$$

The first term in $V_i(q,f;s)$ captures the flow profit that D_i earns until the next contract termination occurs. The second term captures the case in which D_i's own contract terminates before D_{-i}'s contract. The third term captures the case in which D_{-i}'s contract terminates first. In the latter case, D_i earns flow profit $(\pi(q, R_{-i}(q)) - f)$ in the time interval between the termination of D_{-i}'s contract and the termination of its own current contract,[29] and an expected discounted profit of $V_i(R_{-i}(q))$ thereafter.[30]

Substituting $\widetilde{V}_i(q,f)$ into $V_i(q,f;s)$ yields

$$\begin{aligned} V_i(q,f;s) &= -\frac{f}{r+\lambda_i} + \frac{\pi(q,s)}{r+\lambda_i+\lambda_{-i}} + \frac{\lambda_{-i}}{(r+\lambda_i+\lambda_{-i})(r+\lambda_i)} \pi(q, R_{-i}(q)) \\ &\quad + \frac{\lambda_i \lambda_{-i}}{(r+\lambda_i+\lambda_{-i})(r+\lambda_i)} V_i(R_{-i}(q)) + \frac{\lambda_i}{r+\lambda_i+\lambda_{-i}} V_i(s). \end{aligned}$$

Firm D_i prefers acceptance over rejection if $V_i(q,f;s) \geq V_i(0,0;s)$, which holds if and only if

$$f \leq \frac{r+\lambda_i}{r+\lambda_i+\lambda_{-i}} \pi(q,s) + \frac{\lambda_{-i}}{r+\lambda_i+\lambda_{-i}} \pi(q, R_{-i}(q)) + \frac{\lambda_i \lambda_{-i}}{r+\lambda_i+\lambda_{-i}} \left[V_i(R_{-i}(q)) - V_i(R_{-i}(0)) \right]. \quad (10)$$

The supplier's objective function in (1) is strictly increasing in f, because $\frac{\partial W_{-i}(q,f)}{\partial f} = \frac{1}{r+\lambda_i} > 0$. Hence, the retailer's acceptance condition must be binding in equilibrium, otherwise U could increase its profit by offering a contract with the same quantity but a higher fixed payment. It follows that

$$V_i(s) \equiv V_i(R_i(s), F_i(s); s) = \frac{\lambda_i \lambda_{-i}}{(r+\lambda_i+\lambda_{-i})(r+\lambda_i)} V_i(R_{-i}(0)) + \frac{\lambda_i}{r+\lambda_i+\lambda_{-i}} V_i(s),$$

which can be rewritten as

$$V_i(s) = \frac{\lambda_i \lambda_{-i}}{(r+\lambda_i)(r+\lambda_{-i})} V_i(R_{-i}(0)).$$

Since this must hold for all s including $s = R_{-i}(0)$, we can conclude that

$$V_i(s) = 0 \text{ for all } s.$$

Hence, (10) simplifies to $f \leq f_i(q; q_{-i}) \equiv \frac{r+\lambda_i}{r+\lambda_i+\lambda_{-i}} \pi(q, q_{-i}) + \frac{\lambda_{-i}}{r+\lambda_i+\lambda_{-i}} \pi(q, R_{-i}(q))$. ■

[29] Note that if D_{-i}'s contract terminates within this interval, D_{-i} will be offered and accept a contract with quantity $R_{-i}(q)$ again, hence D_i's flow profit remains constant until its own contract terminates.

[30] For details on how to derive the weights in $V_i(q,f;s)$ and $\widetilde{V}_i(q)$, see Appendix B, which presents the ommitted details for the analysis of the supplier's objective function.

Proof of Lemma 2. The proof strategy follows that of Lemma 1 in Maskin and Tirole (1988a). Suppose (in negation) that there exist $q_{-i} > q'_{-i}$ such that $R_i(q_{-i}) > R_i(q'_{-i})$. By the definition of R_i, we have that

$$\pi(R_i(q_{-i}), q_{-i}) - c(R_i(q_{-i}) + q_{-i}) + \frac{\lambda_{-i}}{r+\lambda_i}\pi(R_i(q_{-i}), R_{-i}(R_i(q_{-i}))) + \lambda_{-i}\overline{W}_{-i}(R_i(q_{-i}))$$
$$\geq \pi(R_i(q'_{-i}), q_{-i}) - c(R_i(q'_{-i}) + q_{-i}) + \frac{\lambda_{-i}}{r+\lambda_i}\pi(R_i(q'_{-i}), R_{-i}(R_i(q'_{-i}))) + \lambda_{-i}\overline{W}_{-i}(R_i(q'_{-i})), \tag{11}$$

and similarly,

$$\pi(R_i(q'_{-i}), q'_{-i}) - c(R_i(q'_{-i}) + q'_{-i}) + \frac{\lambda_{-i}}{r+\lambda_i}\pi(R_i(q'_{-i}), R_{-i}(R_i(q'_{-i}))) + \lambda_{-i}\overline{W}_{-i}(R_i(q'_{-i}))$$
$$\geq \pi(R_i(q_{-i}), q'_{-i}) - c(R_i(q_{-i}) + q'_{-i}) + \frac{\lambda_{-i}}{r+\lambda_i}\pi(R_i(q_{-i}), R_{-i}(R_i(q_{-i}))) + \lambda_{-i}\overline{W}_{-i}(R_i(q_{-i})). \tag{12}$$

Adding (11) to (12), we obtain that

$$\pi(R_i(q_{-i}), q_{-i}) - \pi(R_i(q'_{-i}), q_{-i}) \geq \pi(R_i(q_{-i}), q'_{-i}) - \pi(R_i(q'_{-i}), q'_{-i}),$$

which can be rewritten as

$$\int_{q'_{-i}}^{q_{-i}} \int_{R_i(q'_{-i})}^{R_i(q_{-i})} \pi_{12}(x, y)dxdy \geq 0.$$

This is a contradiction because $\pi_{12} < 0$. ■

Proof of Proposition 1. Suppose that $P(Q) = 1 - Q$ and $c \in [0, 1)$. We look for an equilbrium with linear dynamic quantity reactions functions of the form

$$R_i(q_{-i}) = \alpha_i - \beta_i q_{-i},$$

where $\beta_i \geq 0$ for each i. Given linearity of the dynamic reaction functions, the first-order conditions (3) simplify as follows: for $q = R_i(q_{-i})$,

$$1 - c - q_{-i} - 2q + \frac{\lambda_{-i}}{r+\lambda_i}(1 - 2\alpha_{-i} - c + (3\beta_{-i} - 2)q) = 0,$$

or, equivalently,

$$R_i(q_{-i}) = \frac{1}{2 - \frac{\lambda_{-i}}{r+\lambda_i}(3\beta_{-i} - 2)}\left(1 - c + \frac{\lambda_{-i}}{r+\lambda_i}(1 - 2\alpha_{-i} - c) - q_{-i}\right). \tag{13}$$

Setting the right-hand side of (13) equal to $\alpha_i - \beta_i q_{-i}$ for each $i = A, B$, we obtain that either $(\beta_A, \beta_B) = (\beta_A^*, \beta_B^*)$ or $(\beta_A, \beta_B) = (\beta_A^{**}, \beta_B^{**})$, where

$$\beta_i^* = \frac{7\lambda_i + \lambda_{-i} + 4r - \sqrt{(\lambda_A + \lambda_B + 4r)^2 + 12\lambda_A\lambda_B}}{12\lambda_i} > 0, \tag{14}$$

and

$$\beta_i^{**} = \frac{7\lambda_i + \lambda_{-i} + 4r + \sqrt{(\lambda_A + \lambda_B + 4r)^2 + 12\lambda_A\lambda_B}}{12\lambda_i} > 0. \tag{15}$$

We first show that the solution (β_A^*, β_B^*) gives rise to a dynamically stable equilibrium, and then rule out an equilibrium in which $(\beta_A, \beta_B) = (\beta_A^{**}, \beta_B^{**})$. From the definition of β_i^*,

$$\beta_i^* < \frac{7\lambda_i + \lambda_{-i} + 4r - \lambda_A - \lambda_B - 4r}{12\lambda_i} = \frac{1}{2}.$$

This observation implies that the second-order conditions of the supplier's optimization problem hold when $(\beta_A, \beta_B) = (\beta_A^*, \beta_B^*)$. The second derivative of the supplier's objective function is $-2 + \frac{\lambda_{-i}}{r+\lambda_i}\left(3\beta_{-i} - 2\right)$, which is strictly negative for $\beta_{-i} = \beta_{-i}^* < \frac{1}{2}$. Hence, the necessary conditions in (13) are also sufficient.

Given the slope parameters, we can characterize the intercepts of the dynamic quantity reaction functions. First, one can check from (13) that the following system of linear equations should be satisfied:

$$\alpha_i^* = \frac{(1-c)(r+\lambda_i) + \lambda_{-i}(1 - 2\alpha_{-i}^* - c)}{2(r+\lambda_i) - \lambda_{-i}(3\beta_{-i}^* - 2)}.$$

Solving for the intercepts yields

$$\alpha_i^* = \frac{(1-c)\left(\lambda_A + \lambda_B + r\right)\left(7\lambda_i + \lambda_{-i} + 4r - \sqrt{\left(\lambda_A + \lambda_B + 4r\right)^2 + 12\lambda_A\lambda_B}\right)}{2\lambda_i\left(7\lambda_A + 7\lambda_B + 10r - \sqrt{\left(\lambda_A + \lambda_B + 4r\right)^2 + 12\lambda_A\lambda_B}\right)}, \tag{16}$$

for each $i = A, B$.

Letting $R_i^*(q_{-i}) = \alpha_i^* - \beta_i^* q_{-i}$, we check that $R_i^*(q_{-i}) \in [0, \overline{Q}_c] = [0, 1-c]$ for all $q_{-i} \in [0, 1-c]$ and $i = A, B$, i.e., that it was innocuous to ignore the lower and upper bound on the action space in the supplier's optimization problem. To economize on notation, let us define

$$x = (\lambda_A + \lambda_B + 4r)^2 + 12\lambda_A\lambda_B.$$

We then obtain that

$$R_i^*(0) = \frac{(1-c)\left(\lambda_A + \lambda_B + r\right)\left(7\lambda_i + \lambda_{-i} + 4r - \sqrt{x}\right)}{2\lambda_i\left(7\lambda_A + 7\lambda_B + 10r - \sqrt{x}\right)},$$

and

$$R_i^*(\overline{Q}_c) = R_i^*(1-c) = \frac{(1-c)\left(\sqrt{x} - \lambda_A - \lambda_B - 4r\right)\left(7\lambda_i + \lambda_{-i} + 4r - \sqrt{x}\right)}{12\lambda_i\left(7\lambda_A + 7\lambda_B + 10r - \sqrt{x}\right)}.$$

It is immediate that $R_i^*(\overline{Q}_c) \geq 0$, and hence $R_i^*(q_{-i}) \geq 0$ for all $q_{-i} \in [0, \overline{Q}_c]$. Moreover, $R_i^*(0) \leq 1-c$ is equivalent to

$$\begin{aligned}
&\left(\lambda_A + \lambda_B + r\right)\left(7\lambda_i + \lambda_{-i} + 4r - \sqrt{x}\right) \leq 2\lambda_i\left(7\lambda_A + 7\lambda_B + 10r - \sqrt{x}\right)\\
\iff &\left(\lambda_{-i} - \lambda_i + r\right)\left(7\lambda_A + 7\lambda_B + 10r - \sqrt{x}\right) \leq 6\left(\lambda_{-i} + r\right)\left(\lambda_A + \lambda_B + r\right),
\end{aligned}$$

which is true since $6\lambda_A + 6\lambda_B + 6r \geq 7\lambda_A + 7\lambda_B + 10r - \sqrt{x}$ and $\lambda_{-i} + r \geq \lambda_{-i} - \lambda_i + r$.

We can conclude that there exists an equilibrium with steady-state quantities (q_A^e, q_B^e) that satisfy $R_A^*(q_B^e) = q_A^e$ and $R_B^*(q_A^e) = q_B^e$ and are given by

$$(q_A^e, q_B^e) = \left(\frac{\alpha_A^* - \beta_A^*\alpha_B^*}{1 - \beta_A^*\beta_B^*}, \frac{\alpha_B^* - \beta_B^*\alpha_A^*}{1 - \beta_A^*\beta_B^*}\right). \tag{17}$$

The steady state is dynamically stable because the slopes of the reaction functions are less than one in absolute value.

It remains to rule out the existence of an equilibrium with slope parameters $(\beta_A, \beta_B) = (\beta_A^{**}, \beta_B^{**})$, which would give rise to an unstable dynamic path because $\beta_A^{**}\beta_B^{**} > 1$.[31] Computing the intercepts of the dynamic quantity reaction functions that correspond to $(\beta_A^{**}, \beta_B^{**})$, we obtain

$$\alpha_i^{**} = \frac{(1-c)(\lambda_A + \lambda_B + r)\left(7\lambda_i + \lambda_{-i} + 4r + \sqrt{(\lambda_A + \lambda_B + 4r)^2 + 12\lambda_A\lambda_B}\right)}{2\lambda_i\left(7\lambda_A + 7\lambda_B + 10r + \sqrt{(\lambda_A + \lambda_B + 4r)^2 + 12\lambda_A\lambda_B}\right)}. \tag{20}$$

Denoting $R_i^{**}(q_{-i}) = \alpha_i^{**} - \beta_i^{**}q_{-i}$, it is easy to check $R_i^{**}\left(\overline{Q}_c\right) < 0$. Thus, there is no MPE in which the dynamic quantity reaction functions are (R_A^{**}, R_B^{**}), because the restriction that $R_i : \left[0, \overline{Q}_c\right] \to \left[0, \overline{Q}_c\right]$ would be violated.

It is worth noting that this equilibrium inexistence result is not an artifact of the restriction to non-negative quantities. First, if quantities were allowed to take on negative values and the supplier were to follow (R_A^{**}, R_B^{**}), its present discounted payoff when contracting with one retailer would tend to $+\infty$ and its present discounted payoff when contracting with the other retailer would tend to $-\infty$. Hence, the value functions would not be well defined. Second, an equilibrium with piece-wise linear dynamic quantity reaction functions of the form $R_i(q_{-i}) = \max\{\alpha_i^{**} - \beta_i^{**}q_{-i}, 0\}$ does not exist either, as shown in the next lemma.[32] ■

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[31]Since each β_i^{**} is strictly increasing in r,

$$\begin{aligned}\beta_A^{**}\beta_B^{**} &> \frac{\left(7\lambda_A + \lambda_B + \sqrt{(\lambda_A + \lambda_B)^2 + 12\lambda_A\lambda_B}\right)\left(\lambda_A + 7\lambda_B + \sqrt{(\lambda_A + \lambda_B)^2 + 12\lambda_A\lambda_B}\right)}{144\lambda_A\lambda_B} \\ &= \frac{1}{3} + \frac{(\lambda_A + \lambda_B)^2 + (\lambda_A + \lambda_B)\sqrt{(\lambda_A + \lambda_B)^2 + 12\lambda_A\lambda_B}}{18\lambda_A\lambda_B},\end{aligned} \tag{18}$$

where the last equality is obtained by straightforward algebra. From (18), $\beta_A^{**}\beta_B^{**} > 1$ if

$$\frac{(\lambda_A + \lambda_B)^2 + (\lambda_A + \lambda_B)\sqrt{(\lambda_A + \lambda_B)^2 + 12\lambda_A\lambda_B}}{18\lambda_A\lambda_B} \geq \frac{2}{3},$$

which can be rewritten as

$$(\lambda_A + \lambda_B)^2 + (\lambda_A + \lambda_B)\sqrt{(\lambda_A + \lambda_B)^2 + 12\lambda_A\lambda_B} - 12\lambda_A\lambda_B \geq 0. \tag{19}$$

For a given $\lambda_T = \lambda_A + \lambda_B$, the derivative of the left-hand-side of (19) with respect to $\lambda_A\lambda_B$ is

$$\frac{6\lambda_T}{\sqrt{\lambda_T^2 + 12\lambda_A\lambda_B}} - 12 < 0.$$

Hence, if (19) holds for $\lambda_A = \lambda_B = \frac{\lambda_T}{2}$, then it also holds for all other $\lambda_A, \lambda_B > 0$ such that $\lambda_A + \lambda_B = \lambda_T$. For $\lambda_A = \lambda_B = \frac{\lambda_T}{2}$, (19) becomes $\lambda_T^2 + \lambda_T\sqrt{\lambda_T^2 + 3\lambda_T^2} - 3\lambda_T^2 \geq 0$, which holds with equality. Thus, $\beta_A^{**}\beta_B^{**} > 1$.

[32]In contrast, since $R_i^*(q_{-i}) > 0$ for all $q_{-i} \in \left[0, \overline{Q}_c\right]$, an equilibrium in which $R_i(q_{-i}) = \max\{R_i^*(q_{-i}), 0\}$ exists and exhibits the same dynamics and steady-state quantities as the equilibrium charaterized in this proof.

Lemma A1 *Suppose $P(Q) = 1 - Q$ and $c \in [0,1)$. There does not exist a MPE in which the dynamic quantity reaction functions are*

$$R_i(q_{-i}) = \max\{\alpha_i^{**} - \beta_i^{**} q_{-i}, 0\} \text{ for each } i,$$

where α_i^{**} and β_i^{**} are as given in (20) and (15), respectively.

Proof of Lemma A1. Suppose (in negation) that there exists a MPE with the dynamic reaction functions

$$R_i(q_{-i}) = \max\{\alpha_i^{**} - \beta_i^{**} q_{-i}, 0\}$$

for each $i = A, B$. Moreover, without loss of generality, suppose that $\lambda_A \geq \lambda_B$.

For all $i = A, B$,

$$\alpha_i^{**} - \beta_i^{**} q_{-i} = 0 \iff q_{-i} = \overline{q} \equiv \frac{6(1-c)(\lambda_A + \lambda_B + r)}{7(\lambda_A + \lambda_B) + 10r + \sqrt{x}}.$$

It is easy to check that

$$\alpha_B^{**} > \overline{q},$$

and thus $R_B(0) > \overline{q}$, if and only if

$$24\lambda_B(\lambda_A - \lambda_B + 2r) > 0,$$

which is true for $\lambda_A \geq \lambda_B$.

Since $R_A(q_B) = 0$ for all $q_B \geq \overline{q}$, we have that for all $q_B \geq \overline{q}$,

$$\overline{W}_A(q_B) = -\frac{cq_B}{r + \lambda_B} + \frac{\lambda_B}{r + \lambda_B} \bar{W}_B(0).$$

Now consider U's optimal offer to D_B when D_A's current quantity is 0. According to the postulated equilibrium strategies, $R_B(0) = \alpha_B^{**} > \overline{q}$, hence

$$\begin{aligned}
\overline{W}_B(0) &= \frac{\pi(\alpha_B^{**}, 0) - c\alpha_B^{**}}{r + \lambda_A} + \frac{\lambda_A}{(r + \lambda_A)(r + \lambda_B)} \pi(\alpha_B^{**}, 0) + \frac{\lambda_A}{r + \lambda_A} \overline{W}_A(\alpha_B^{**}) \\
&= \frac{\pi(\alpha_B^{**}, 0) - c\alpha_B^{**}}{r + \lambda_A} + \frac{\lambda_A}{(r + \lambda_A)(r + \lambda_B)} \pi(\alpha_B^{**}, 0) + \frac{\lambda_A}{r + \lambda_A} \left(-\frac{c\alpha_B^{**}}{r + \lambda_B} + \frac{\lambda_B}{r + \lambda_B} \overline{W}_B(0) \right).
\end{aligned}$$

Now consider a one-shot deviation to offering $\alpha_B^{**} - \epsilon$, where $\alpha_B^{**} - \epsilon \geq \overline{q}$. U's deviation profit is

$$\begin{aligned}
\widetilde{W}_B(\alpha_B^{**} - \epsilon, 0) &\equiv \frac{\pi(\alpha_B^{**} - \epsilon, 0) - c(\alpha_B^{**} - \epsilon)}{r + \lambda_A} + \frac{\lambda_A}{(r + \lambda_A)(r + \lambda_B)} \pi(\alpha_B^{**} - \epsilon, 0) + \frac{\lambda_A}{r + \lambda_A} \overline{W}_A(\alpha_B^{**} - \epsilon) \\
&= \frac{\pi(\alpha_B^{**} - \epsilon, 0) - c(\alpha_B^{**} - \epsilon)}{r + \lambda_A} + \frac{\lambda_A}{(r + \lambda_A)(r + \lambda_B)} \pi(\alpha_B^{**} - \epsilon, 0) \\
&\quad + \frac{\lambda_A}{r + \lambda_A} \left(-\frac{c(\alpha_B^{**} - \epsilon)}{r + \lambda_B} + \frac{\lambda_B}{r + \lambda_B} \overline{W}_B(0) \right).
\end{aligned}$$

Comparing U's deviation profit to its equilibrium profit, we obtain that

$$\widetilde{W}_B(\alpha_B^{**} - \epsilon, 0) - \overline{W}_B(0) = \left(\frac{\lambda_A + \lambda_B + r}{(r + \lambda_A)(r + \lambda_B)} \right) \epsilon \left(c + 2\alpha_B^{**} - (1 + \epsilon) \right).$$

Thus, U has a profitable deviation if there exists an $\epsilon > 0$ such that $\alpha_B^{**} - \epsilon \geq \overline{q}$ and

$$c > 1 + \epsilon - 2\alpha_B^{**} \iff \alpha_B^{**} > \frac{1 - c + \epsilon}{2}.$$

If $\alpha_B^{**} > \frac{1-c}{2}$, then both inequalities are satisfied for small enough $\epsilon > 0$. Denoting $x = (\lambda_A + \lambda_B + 4r)^2 + 12\lambda_A\lambda_B$, we obtain that $\alpha_B^{**} > \frac{1-c}{2}$ if and only if

$$(\lambda_A + \lambda_B + r)(7\lambda_B + \lambda_A + 4r + \sqrt{x}) > \lambda_B(7\lambda_A + 7\lambda_B + 10r + \sqrt{x}),$$

which can be simplified to

$$7\lambda_A + 7\lambda_B + 10r + \sqrt{x} > 6\lambda_A + 6\lambda_B + 6r,$$

which is true. Hence, U has a strictly profitable deviation. ■

Proof of Proposition 3. Let $\lambda_A = \lambda_B = \lambda$, and suppose that $P(Q) = 1 - Q$ and $c \in [0, 1)$. From the expressions for α_i^* and β_i^* in (16) and (14) in the proof of Proposition 1, it is immediate that $\alpha_A^* = \alpha_B^* = \alpha^*$ and $\beta_A^* = \beta_B^* = \beta^*$. Specifically, letting $\tilde{r} \equiv \frac{r}{\lambda}$, we obtain that

$$\beta^* = \frac{8\lambda + 4r - \sqrt{(2\lambda + 4r)^2 + 12\lambda^2}}{12\lambda} = \frac{2 + \tilde{r} - \sqrt{1 + \tilde{r} + \tilde{r}^2}}{3},$$

and

$$\begin{aligned} \alpha^* &= \frac{(1-c)(2\lambda + r)\left(8\lambda + 4r - \sqrt{(2\lambda + 4r)^2 + 12\lambda^2}\right)}{2\lambda\left(14\lambda + 10r - \sqrt{(2\lambda + 4r)^2 + 12\lambda^2}\right)} \\ &= (1-c)(2+\tilde{r})\left(\frac{2 + \tilde{r} - \sqrt{1 + \tilde{r} + \tilde{r}^2}}{7 + 5\tilde{r} - 2\sqrt{1 + \tilde{r} + \tilde{r}^2}}\right). \end{aligned}$$

Given that the dynamic quantity reaction functions are symmetric, the steady-state quantities are symmetric as well and given by

$$q^e = \frac{\alpha^*}{1 + \beta^*} = (1-c)(2+\tilde{r})\, 3 \frac{2 + \tilde{r} - \sqrt{1 + \tilde{r} + \tilde{r}^2}}{\left(7 + 5\tilde{r} - 2\sqrt{1 + \tilde{r} + \tilde{r}^2}\right)\left(5 + \tilde{r} - \sqrt{1 + \tilde{r} + \tilde{r}^2}\right)},$$

which can be simplified to

$$q^e = \frac{(1-c)(2+\tilde{r})\, 3}{19 + 8\tilde{r} + \sqrt{1 + \tilde{r} + \tilde{r}^2}}. \tag{21}$$

The symmetric steady-state quantity q^e is strictly increasing in $\tilde{r}$ because

$$\begin{aligned} \frac{\partial q^e}{\partial \tilde{r}} &\overset{sign}{=} 19 + 8\tilde{r} + \sqrt{1 + \tilde{r} + \tilde{r}^2} - (2 + \tilde{r})\left(8 + \frac{1 + 2\tilde{r}}{2\sqrt{1 + \tilde{r} + \tilde{r}^2}}\right) \\ &= 3 + \sqrt{1 + \tilde{r} + \tilde{r}^2} - \frac{2\left(1 + \tilde{r} + \tilde{r}^2\right)}{2\sqrt{1 + \tilde{r} + \tilde{r}^2}} - \frac{3\tilde{r}}{2\sqrt{1 + \tilde{r} + \tilde{r}^2}} \\ &= 3\left(1 - \frac{\tilde{r}}{2\sqrt{1 + \tilde{r} + \tilde{r}^2}}\right) > 0. \end{aligned}$$

Next, observe that for $\tilde{r} = 0$, (21) simplifies to $q^e = \frac{3(1-c)}{10}$. It remains to show that q^e converges to $q^C = \frac{1-c}{3}$ as $\tilde{r}$ tends to $+\infty$. From (21),

$$\begin{aligned}
\lim_{\tilde{r}\to\infty} q^e &= 3(1-c)\lim_{\tilde{r}\to\infty}\left(\frac{2+\tilde{r}}{19+8\tilde{r}+\sqrt{1+\tilde{r}+\tilde{r}^2}}\right) \\
&= 3(1-c)\lim_{\tilde{r}\to\infty}\left(\frac{\frac{2}{\tilde{r}}+1}{\frac{19}{\tilde{r}}+8+\sqrt{\frac{1+\tilde{r}}{\tilde{r}^2}+1}}\right) \\
&= 3(1-c)\frac{1}{9},
\end{aligned}$$

which is equal to the Cournot quantity q^C. ■

Proof of Proposition 4. Let $\tilde{r} = \frac{r}{\lambda_M}$ thoughout the proof.

Part (i): The first-order conditions evaluated at a steady state (q_A^e, q_B^e) can be written as

$$\begin{aligned}
R_B'(q_A^e) &= -\frac{(\tilde{r}+2)\,\Pi'(Q^e)-(\tilde{r}+1-\mu)\,\pi_2(q_B^e,q_A^e)}{(1+\mu)\,\pi_2(q_A^e,q_B^e)}, \\
R_A'(q_B^e) &= -\frac{(\tilde{r}+2)\,\Pi'(Q^e)-(\tilde{r}+1+\mu)\,\pi_2(q_A^e,q_B^e)}{(1-\mu)\,\pi_2(q_B^e,q_A^e)},
\end{aligned}$$

where $Q^e = q_A^e + q_B^e$ denotes the aggregate quantity. The stability condition $R_A'(q_B^e)R_B'(q_A^e) < 1$ therefore implies that

$$\frac{\left((\tilde{r}+2)\,\Pi'(Q^e)-(\tilde{r}+1+\mu)\,\pi_2(q_A^e,q_B^e)\right)\left((\tilde{r}+2)\,\Pi'(Q^e)-(\tilde{r}+1-\mu)\,\pi_2(q_B^e,q_A^e)\right)}{(1-\mu)\,\pi_2(q_B^e,q_A^e)\,(1+\mu)\,\pi_2(q_A^e,q_B^e)} < 1,$$

or, equivalently,

$$\begin{aligned}
&\left((\tilde{r}+1-\mu)\left(P(Q^e)-c+q_A^eP'(Q^e)\right)+(1+\mu)\left(P(Q^e)-c+Q^eP'(Q^e)\right)\right) \\
&\times\left((\tilde{r}+1+\mu)\left(P(Q^e)-c+q_B^eP'(Q^e)\right)+(1-\mu)\left(P(Q^e)-c+Q^eP'(Q^e)\right)\right) \\
&<(1+\mu)(1-\mu)\left(P'(Q^e)\right)^2 q_A^e q_B^e.
\end{aligned} \tag{22}$$

Now suppose (in negation) that $q_A^e \geq R^C(q_B^e)$ and $q_B^e \geq R^C(q_A^e)$. By the definition of the Cournot reaction function, we then have that $P(Q^e)-c+q_A^eP'(Q^e) \leq 0$ and $P(Q^e)-c+q_B^eP'(Q^e) \leq 0$. Moreover, adding up these two conditions yields $P(Q^e)-c+\frac{Q^e}{2}P'(Q^e) \leq 0$, and thus

$$P(Q^e)-c+Q^eP'(Q^e) \leq \frac{Q^e}{2}P'(Q^e) \leq 0,$$

It follows that the left-hand side of (22) is greater than or equal to

$$(1+\mu)(1-\mu)\frac{(Q^e)^2}{4}\left(P'(Q^e)\right)^2. \tag{23}$$

We now show that (23) is greater than or equal to the right-hand side of (22), thereby establishing a contradiction. Since $q_A^e + q_B^e = Q^e$ and $q_i^e \geq 0$, there exists an $\alpha \in [0,1]$ such that $q_A^e = \alpha Q^e$, $q_B^e =$

$(1-\alpha)\,Q^e$, and thus $q_A^e q_B^e = \alpha\,(1-\alpha)\,(Q^e)^2$. As $\alpha\,(1-\alpha) \leq \frac{1}{4}$ for all $\alpha \in [0,1]$, it follows that the right-hand side of (22) is smaller than or equal to

$$(1+\mu)(1-\mu)\left(P'\left(Q^e\right)\right)^2 \frac{\left(Q^e\right)^2}{4}. \tag{24}$$

Since (23) is equal to (24), condition (22) is violated. Hence, it cannot be that $q_A^e \geq R^C(q_B^e)$ and $q_B^e \geq R^C(q_A^e)$ in a stable equilibrium with differentiable dynamic quantity reaction functions.

Part (ii): From $R_B' \leq 0$, $R_A' \leq 0$, and the first-order conditions, we obtain that

$$\begin{aligned}
\frac{\tilde{r}+1-\mu}{\tilde{r}+2}\left(\Pi'\left(q_A^e+q_B^e\right)-\pi_2\left(q_B^e,q_A^e\right)\right)+\frac{1+\mu}{\tilde{r}+2}\Pi'\left(q_A^e+q_B^e\right) &\leq 0,\\
\frac{\tilde{r}+1+\mu}{\tilde{r}+2}\left(\Pi'\left(q_A^e+q_B^e\right)-\pi_2\left(q_A^e,q_B^e\right)\right)+\frac{1-\mu}{\tilde{r}+2}\Pi'\left(q_A^e+q_B^e\right) &\leq 0,
\end{aligned}$$

which can be rewritten as

$$\begin{aligned}
q_B^e\frac{P'\left(Q^e\right)}{\Pi'\left(Q^e\right)} &\leq \frac{\tilde{r}+2}{\tilde{r}+1-\mu},\\
q_A^e\frac{P'\left(Q^e\right)}{\Pi'\left(Q^e\right)} &\leq \frac{\tilde{r}+2}{\tilde{r}+1+\mu}.
\end{aligned}$$

Adding up these two conditions yields

$$\frac{Q^e P'\left(Q^e\right)}{\Pi'\left(Q^e\right)} \leq \frac{2\left(\tilde{r}+2\right)\left(\tilde{r}+1\right)}{\left(\tilde{r}+1+\mu\right)\left(\tilde{r}+1-\mu\right)}. \tag{25}$$

The left-hand side of (25) is strictly decreasing in Q^e, because

$$\frac{\partial}{\partial Q}\left(\frac{QP'\left(Q\right)}{\Pi'\left(Q\right)}\right) = \frac{\left(P'(Q)+QP''(Q)\right)\left(P(Q)-c\right)-Q\left(P'(Q)\right)^2}{\left(\Pi'(Q)\right)^2} < 0, \tag{26}$$

where the inequality follows from assumption $A2$ and $P\left(Q^e\right) \geq c$. Moreover, $\lim_{Q^e\to Q^M}\frac{Q^e P'(Q^e)}{\Pi'(Q^e)} = \infty$ and $\frac{2q^C P'\left(2q^C\right)}{\Pi'\left(2q^C\right)} = 2$, while the right-hand side of (25) lies strictly above 2 for all $\tilde{r} > 0$ and $\mu \in [0,1)$. It follows that, for each $\tilde{r} > 0$ and $\mu \in [0,1)$, there exists a unique $\underline{Q}\left(\tilde{r},\mu\right) \in \left(Q^M, 2q^C\right)$ such that

$$\frac{\underline{Q}P'\left(\underline{Q}\right)}{\Pi'\left(\underline{Q}\right)} = \frac{2\left(\tilde{r}+2\right)\left(\tilde{r}+1\right)}{\left(\tilde{r}+1-\mu\right)\left(\tilde{r}+1+\mu\right)}, \tag{27}$$

which can be rewritten as

$$\Pi'\left(\underline{Q}\right)-\pi_2\left(\frac{1}{2}\underline{Q},\frac{1}{2}\underline{Q}\right)+\frac{\tilde{r}+1+\mu^2}{\left(\tilde{r}+1-\mu\right)\left(\tilde{r}+1+\mu\right)}\Pi'\left(\underline{Q}\right) = 0. \tag{28}$$

The right-hand side of (27) is strictly increasing in μ, and strictly decreasing in $\tilde{r}$. Given (26), it follows that $\underline{Q}\left(\tilde{r},\mu\right)$ is strictly decreasing in μ, and strictly increasing in $\tilde{r}$.

For $\mu = 0$, (28) coincides with the condition that defines the lower bound $\underline{q}\left(\tilde{r}\right)$ in the symmetric case (see Proposition 2), hence

$$\underline{Q}\left(\tilde{r},0\right) = 2\underline{q}\left(\tilde{r}\right) > Q^M.$$

For $\tilde{r} = 0$, the right-hand side of (27) is equal to $\frac{4}{(1-\mu)(1+\mu)}$, which converges to infinity as μ tends to one. It follows that

$$\lim_{\mu \to 1} \lim_{\tilde{r} \to 0} \underline{Q}(\tilde{r}, \mu) = Q^M.$$

Finally, as $\tilde{r}$ tends to ∞, the right-hand side of (27) converges to 2, which implies that

$$\lim_{\tilde{r} \to \infty} \underline{Q}(\tilde{r}, \mu) = 2q^C.$$

This completes the proof of part (ii).

Part (iii): We define $\mathcal{E}$ as the set of equilibrium dynamic reaction functions (R_A, R_B) such that R_i's are twice continuously differentiable, and $\sup|R_i'|$ and $\sup|R_i''|$ are bounded by K. Although the set $\mathcal{E}$ can depend on the choice of bounds $K > 0$, as well as the parameters r, λ_M, and μ, we omit them to simplify the notations. In particular, it will be clear in the proof that, given equilibrium existence, the specific choice of $K > 0$ does not affect our limit result.[33]

Our first intermediate result is that R_B approaches the Cournot reaction function when the degree of asymmetry μ converges to 1.

Lemma A2 *Let $\epsilon > 0$ be given. There exists $\bar{\mu} < 1$ such that*

$$\max_{q_A \in [0, \bar{Q}_c]} |R_B(q_A) - R^C(q_A)| < \epsilon,$$

for any $(R_A, R_B) \in \mathcal{E}$ when $\mu \geq \bar{\mu}$.

Proof of Lemma A2. Let $\epsilon > 0$ be given. Recall that for $q = R_B(q_A)$,

$$\Pi'(q + q_A) - \pi_2(q_A, q) + \frac{1-\mu}{\tilde{r} + 1 + \mu}\left(\Pi'(R_A(q) + q) + \pi_2(q, R_A(q))R_A'(q)\right) = 0,$$

while for $q = R^C(q_A)$,

$$\Pi'(q + q_A) - \pi_2(q_A, q) = 0.$$

Now, we choose $\bar{\mu} < 1$ such that $\mu \geq \bar{\mu}$ implies

$$\left(\frac{1-\mu}{\tilde{r} + 1 + \mu}\right) \cdot \max|\Pi' + \pi_2 \cdot K| \leq \epsilon \cdot \min|\pi_{11}|.$$

Then, we have for any $q_A \in [0, \bar{Q}_c]$ and $(R_A, R_B) \in \mathcal{E}$ with $\mu \geq \bar{\mu}$,

$$\left|\pi_1(R_B(q_A), q_A) - \pi_1(R^C(q_A), q_A)\right| \leq \epsilon \cdot \min|\pi_{11}|.$$

By the mean-value theorem, there exists z between $\min\{R^C(q_A), R_B(q_A)\}$ and $\max\{R^C(q_A), R_B(q_A)\}$ such that

$$\left(R_B(q_A) - R^C(q_A)\right)\pi_{11}(z, q_A) = \pi_1\left(R_B(q_A), q_A\right) - \pi_1\left(R^C(q_A), q_A\right),$$

[33] The existence of equilibria with bounded first and second derivaties will be satisfied in the case of linear demand, where $|R_i^{*\prime}| \leq \frac{1}{2}$ for all i and all parameter values $r > 0$, $\lambda_M > 0$, and $\mu \in [0, 1)$.

and hence

$$|R_B(q_A) - R^C(q_A)||\pi_{11}(z, q_A)| \leq \epsilon \cdot \min |\pi_{11}|.$$

We conclude that $\max_{q_A \in [0, \bar{Q}_c]} |R_B(q_A) - R^C(q_A)| < \epsilon$ for any $(R_A, R_B) \in \mathcal{E}$ with $\mu \geq \bar{\mu}$. ■

Our next intermediate result is that the slope of reaction function R_B also converges to that of the Cournot reaction function as μ approaches 1.

Lemma A3 *Let $\epsilon > 0$ be given. There exists $\bar{\mu} < 1$ such that*

$$\max_{q_A \in [0, \bar{Q}_c]} |R'_B(q_A) - R^{C\prime}(q_A)| < \epsilon,$$

for any $(R_A, R_B) \in \mathcal{E}$ when $\mu \geq \bar{\mu}$.

Proof of Lemma A3. Let $\epsilon > 0$ be given. Recall that for $q = R_B(q_A)$,

$$\Pi'(q + q_A) - \pi_2(q_A, q) + \frac{1-\mu}{\tilde{r} + 1 + \mu} \left(\Pi'(R_A(q) + q) + \pi_2(q, R_A(q))R'_A(q)\right) = 0,$$

while for $q = R^C(q_A)$,

$$\Pi'(q + q_A) - \pi_2(q_A, q) = 0.$$

By the implicit function theorem,

$$R'_B(q_A) = -\frac{\pi_{12}(R_B(q_A), q_A)}{H(R_B(q_A), q_A)}, \text{ and } R^{C\prime}(q_A) = -\frac{\pi_{12}(R^C(q_A), q_A)}{\pi_{11}(R^C(q_A), q_A)},$$

where

$$H(q, q_A) = \pi_{11}(q, q_A) + \frac{1-\mu}{\tilde{r} + 1 + \mu} \tilde{H}(q),$$

for some $\tilde{H}(q)$ such that there exists an upper bound $K' > 0$ with $\max |\tilde{H}| < K'$ for any $(R_A, R_B) \in \mathcal{E}$. Together with Lemma A2, this implies that for any given $\epsilon' > 0$, there exists $\bar{\mu}' < 1$ such that for any $(R_A, R_B) \in \mathcal{E}$ with $\mu \geq \bar{\mu}'$,

$$\max_{q_A \in [0, \bar{Q}_c]} |H(R_B(q_A), q_A) - \pi_{11}(R^C(q_A), q_A)| \leq \epsilon'. \tag{29}$$

To see this, note first that

$$\begin{aligned}
&|H(R_B(q_A), q_A) - \pi_{11}(R^C(q_A), q_A)| \\
\leq &|\pi_{11}(R_B(q_A), q_A) - \pi_{11}(R^C(q_A), q_A)| + \frac{1-\mu}{\tilde{r} + 1 + \mu} K' \\
\leq &\max |P''| \cdot |R_B(q_A) - R^C(q_A)| + \max |R^C| \cdot |P''(R_B(q_A) + q_A) - P''(R^C(q_A) + q_A)| \\
&+ 2|P'(R_B(q_A) + q_A) - P'(R^C(q_A) + q_A)| + \frac{1-\mu}{\tilde{r} + 1 + \mu} K'.
\end{aligned}$$

Since P'' and P' are continuous functions on a compact interval, they are uniformly continuous. Thus, by Lemma A2, we can choose $\bar{\mu}' < 1$ such that for any $(R_A, R_B) \in \mathcal{E}$ with $\mu \geq \bar{\mu}'$,

$$|H(R_B(q_A), q_A) - \pi_{11}(R^C(q_A), q_A)| < \epsilon'.$$

Now, notice that

$$\begin{aligned}
&|R_B'(q_A) - R^{C\prime}(q_A)| \\
=&\left|\frac{\pi_{12}(R_B(q_A), q_A)}{H(R_B(q_A), q_A)} - \frac{\pi_{12}(R^C(q_A), q_A)}{\pi_{11}(R^C(q_A), q_A)}\right| \\
=&\left|\frac{\pi_{12}(R_B(q_A), q_A)\pi_{11}(R^C(q_A), q_A) - H(R_B(q_A), q_A)\pi_{12}(R^C(q_A), q_A)}{H(R_B(q_A), q_A)\pi_{11}(R^C(q_A), q_A)}\right|.
\end{aligned}$$

Observe that the denominator is strictly bounded away from zero when μ beccomes arbitrarily close to 1. To see this, note that for each $q_A \in [0, \bar{Q}]$,

$$\begin{aligned}
&|H(R_B(q_A), q_A)\pi_{11}(R^C(q_A), q_A)| \\
=&\left|\left(H\left(R_B(q_A), q_A\right) - \pi_{11}\left(R^C(q_A), q_A\right)\right)\pi_{11}(R^C(q_A), q_A) + \pi_{11}\left(R^C(q_A), q_A\right)^2\right| \\
\geq&\pi_{11}(R^C(q_A), q_A)^2 - \left|H(R_B(q_A), q_A) - \pi_{11}(R^C(q_A), q_A)\right|\left|\pi_{11}(R^C(q_A), q_A)\right|.
\end{aligned}$$

Then, from (29) and the fact that $\min|\pi_{11}^2| > 0$, we have $\bar{\mu}'' > 0$ such that the minimum of the right-hand side is bounded below from some $K'' > 0$ for any $(R_A, R_B) \in \mathcal{E}$ with $\mu \geq \bar{\mu}''$. Therefore, we have

$$\begin{aligned}
&\left|R_B'(q_A) - R^{C\prime}(q_A)\right| \\
\leq&\frac{1}{K''}\left|\pi_{12}(R_B(q_A), q_A)\pi_{11}(R^C(q_A), q_A) - H(R_B(q_A), q_A)\pi_{12}(R^C(q_A), q_A)\right|.
\end{aligned}$$

Since $\pi_{12}(R_B(q_A), q_A)$ and $H(R_B(q_A), q_A)$ approach to $\pi_{12}(R^C(q_A), q_A)$ and $\pi_{11}(R^C(q_A), q_A)$ respectively as μ converges to 1, we can apply the same argument before and find $\bar{\mu} > 0$ such that

$$\max_{q_A \in [0, \bar{Q}_c]}\left|R_B'(q_A) - R^{C\prime}(q_A)\right| < \epsilon,$$

for any $(R_A, R_B) \in \mathcal{E}$ with $\mu \geq \bar{\mu}$. ■

Given Lemmas A2 and A3, we now prove the statement in part (iii). Let $\epsilon > 0$ be given. Recall that the first-order conditions evaluated at steady-state quantities (q_A^e, q_B^e) are given by, for $R_A(q_B^e) = q_A^e$,

$$(\tilde{r} + 1 - \mu)\left(\Pi'\left(Q^e\right) - \pi_2\left(q_B^e, q_A^e\right)\right) + (1 + \mu)\left(\Pi'\left(Q^e\right) + \pi_2\left(q_A^e, q_B^e\right)R_B'\left(q_A^e\right)\right) = 0,$$

and for $R_B(q_A^e) = q_B^e$,

$$(\tilde{r} + 1 + \mu)\left(\Pi'\left(Q^e\right) - \pi_2\left(q_A^e, q_B^e\right)\right) + (1 - \mu)\left(\Pi'\left(Q^e\right) + \pi_2\left(q_B^e, q_A^e\right)R_A'\left(q_B^e\right)\right) = 0.$$

By Lemma A3, we have $\max|R_B'| \leq L$ for some $L \in (0, 1)$ for any $(R_A, R_B) \in \mathcal{E}$ with μ being close enough to 1, because the Cournot reaction function has a slope whose absolute value is strictly less than than 1. As a result, we obtain

$$\begin{aligned}
0 &= (\tilde{r} + 1 - \mu)\left(\Pi'\left(Q^e\right) - \pi_2\left(q_B^e, q_A^e\right)\right) + (1 + \mu)\left(\Pi'\left(Q^e\right) + \pi_2\left(q_A^e, q_B^e\right)R_B'\left(q_A^e\right)\right) \\
&\leq (\tilde{r} + 1 - \mu)\left(\Pi'\left(Q^e\right) - \pi_2\left(q_B^e, q_A^e\right)\right) + (1 + \mu)\left(\Pi'\left(Q^e\right) - L\pi_2\left(q_A^e, q_B^e\right)\right) \\
&= (\tilde{r} + 1 - \mu)\left(\Pi'\left(Q^e\right) - \pi_2\left(q_B^e, q_A^e\right)\right) + (1 + \mu)\left(\pi_1\left(q_B^e, q_A^e\right) - c\right) + (1 + \mu)(1 - L)\pi_2\left(q_A^e, q_B^e\right),
\end{aligned}$$

which is equivalent to

$$0 \leq \left(\frac{\tilde{r}+1-\mu}{1+\mu}\right)\left(\Pi'\left(Q^e\right)-\pi_2\left(q_B^e, q_A^e\right)\right)+\pi_1\left(q_B^e, q_A^e\right)-c+(1-L)\,\pi_2\left(q_A^e, q_B^e\right).$$

Now, choose $\bar{\mu}<1$ and $\bar{r}>0$ such that

$$\left(\frac{\tilde{r}+1-\mu}{1+\mu}\right)\left(\Pi'\left(Q^e\right)-\pi_2\left(q_B^e, q_A^e\right)\right) \leq \frac{\epsilon\,(1-L)\min|P'|}{2},$$

and

$$\pi_1(q_B^e, q_A^e)-c=\pi_1\left(R_B(q_A^e), q_A^e\right)-c \leq \frac{\epsilon(1-L)\min|P'|}{2},$$

for any $(R_A, R_B) \in \mathcal{E}$ with $\mu \geq \bar{\mu}$ and $\tilde{r} \leq \bar{r}$, where the latter inequality follows from Lemma A2. As a result, we have

$$\begin{aligned}0 &\leq \left(\frac{\tilde{r}+1-\mu}{1+\mu}\right)\left(\Pi'(Q^e)-\pi_2(q_B^e, q_A^e)\right)+\pi_1(q_B^e, q_A^e)-c+(1-L)\pi_2(q_A^e, q_B^e)\\ &\leq (1-L)\left(\epsilon\min|P'|+P'(Q^e)q_A^e\right),\end{aligned}$$

implying

$$0 \leq \epsilon\min|P'|+P'(Q^e)q_A^e.$$

Therefore,

$$\left(-P'(Q^e)\right)q_A^e \leq \epsilon\min|P'| \iff q_A^e \leq \frac{\epsilon\min|P'|}{|P'(Q^e)|}<\epsilon.$$

By Lemma A2, we also obtain

$$|q_B^e-Q^M|=|R_B(q_A^e)-R^C(0)|<\epsilon,$$

provided that μ is sufficiently close to 1. This completes the proof of part (iii). ■

Proof of Proposition 5. Letting $\tilde{r}=\frac{r}{\lambda_M}$ and $\tilde{x}=(2+4\tilde{r})^2+12(1-\mu^2)$, the intercepts and slopes of the equilibrium dynamic quantity reactions (R_A^*, R_B^*) from the proof of Proposition 1 become

$$\alpha_A^*=\frac{(1-c)\,(2+\tilde{r})\left(2+4\tilde{r}+6\,(1-\mu)-\sqrt{\tilde{x}}\right)}{2\,(1-\mu)\left(14+10\tilde{r}-\sqrt{\tilde{x}}\right)},$$

$$\alpha_B^*=\frac{(1-c)\,(2+\tilde{r})\left(2+4\tilde{r}+6\,(1+\mu)-\sqrt{\tilde{x}}\right)}{2\,(1+\mu)\left(14+10\tilde{r}-\sqrt{\tilde{x}}\right)},$$

$$\beta_A^*=\frac{1}{2}-\frac{\sqrt{\tilde{x}}-(2+4\tilde{r})}{12\,(1-\mu)},$$

$$\beta_B^*=\frac{1}{2}-\frac{\sqrt{\tilde{x}}-(2+4\tilde{r})}{12\,(1+\mu)}.$$

The steady-state quantities are given by

$$(q_A^e, q_B^e)=\left(\frac{\alpha_A^*-\alpha_B^*\beta_A^*}{1-\beta_A^*\beta_B^*}, \frac{\alpha_B^*-\alpha_A^*\beta_B^*}{1-\beta_A^*\beta_B^*}\right).$$

We first prove part (iii), and then parts (i) and (ii) of the proposition.

Part (iii): Note that

$$\lim_{\tilde{r}\to 0}\beta_B^* = \frac{1}{2} - \frac{\sqrt{4+12(1-\mu^2)}-2}{12\,(1+\mu)},$$

and

$$\lim_{\mu\to 1}\lim_{\tilde{r}\to 0}\beta_B^* = \frac{1}{2}.$$

Similarly, one can check

$$\lim_{\tilde{r}\to 0}\beta_A^* = \frac{1}{2} - \frac{\sqrt{4+12(1-\mu^2)}-2}{12\,(1-\mu)}.$$

Applying the L'Hospital's rule, we have

$$\lim_{\mu\to 1}\lim_{\tilde{r}\to 0}\beta_A^* = 0.$$

On the other hand,

$$\lim_{\tilde{r}\to 0}\alpha_B^* = \frac{2\,(1-c)\left(2+6\,(1+\mu)-\sqrt{4+12\,(1-\mu^2)}\right)}{2\,(1+\mu)\left(14-\sqrt{4+12\,(1-\mu^2)}\right)},$$

and

$$\lim_{\mu\to 1}\lim_{\tilde{r}\to 0}\alpha_B^* = \frac{1-c}{2} = Q^M.$$

Similarly,

$$\lim_{\tilde{r}\to 0}\alpha_A^* = \frac{2\,(1-c)\left(2+6\,(1-\mu)-\sqrt{4+12\,(1-\mu^2)}\right)}{2\,(1-\mu)\left(14-\sqrt{4+12\,(1-\mu^2)}\right)},$$

and by L'Hospital's rule,

$$\lim_{\mu\to 1}\lim_{\tilde{r}\to 0}\alpha_A^* = 0.$$

The result follows from evaluating $(q_A^e, q_B^e) = \left(\frac{\alpha_A^*-\alpha_B^*\beta_A^*}{1-\beta_A^*\beta_B^*}, \frac{\alpha_B^*-\alpha_A^*\beta_B^*}{1-\beta_A^*\beta_B^*}\right)$ at these limit values.

Part (i): Note that the aggregate steady-state quantity is given by

$$Q^e = \left(\frac{12\,(1-c)\,(2+\tilde{r})}{14+10\tilde{r}-\sqrt{\tilde{x}}}\right)\left(\frac{36(1-\mu^2)-(2+4\tilde{r}-\sqrt{\tilde{x}})^2}{108\,(1-\mu^2)-12\left(2+4\tilde{r}-\sqrt{\tilde{x}}\right)-\left(2+4\tilde{r}-\sqrt{\tilde{x}}\right)^2}\right).$$

To simplify notations, let us write $X = 2+4\tilde{r}-\sqrt{\tilde{x}} < 0$. Then, we have

$$\tilde{x} = (2+4\tilde{r}-X)^2 \iff 12(1-\mu^2) = X^2 - 2X(2+4\tilde{r}),$$

and so

$$\begin{aligned}
Q^e &= \left(\frac{12\,(1-c)\,(2+\tilde{r})}{6\,(2+\tilde{r})+X}\right)\left(\frac{2X^2-6X\,(2+4\tilde{r})}{8X^2-18X\,(2+4\tilde{r})-12X}\right)\\
&= \left(\frac{12\,(1-c)\,(2+\tilde{r})}{12+6\tilde{r}+X}\right)\left(\frac{X-6-12\tilde{r}}{4X-24-36\tilde{r}}\right)\\
&= 12(1-c)\cdot\left(6+\frac{X}{2+\tilde{r}}\right)^{-1}\cdot\left(4+\frac{12\tilde{r}}{X-6-12\tilde{r}}\right)^{-1}.
\end{aligned}$$

Since

$$\begin{aligned}&\left(6+\frac{X}{2+\tilde{r}}\right)\cdot\left(4+\frac{12\tilde{r}}{X-6-12\tilde{r}}\right)\\=&24+4\left(\frac{18\tilde{r}}{X-6-12\tilde{r}}+\frac{X}{2+\tilde{r}}+\frac{3\tilde{r}X}{(2+\tilde{r})(X-6-12\tilde{r})}\right),\end{aligned}$$

it sufficies to show that

$$\frac{18\tilde{r}}{X-6-12\tilde{r}}+\frac{X}{2+\tilde{r}}+\frac{3\tilde{r}X}{(2+\tilde{r})(X-6-12\tilde{r})}$$

is strictly decreasing in $\tilde{r}$. One can check that the denominator of its first-order derivative is

$$(2+\tilde{r})^2\sqrt{4\left(1+\tilde{r}+\tilde{r}^2\right)-3\mu^2}\left(2+4\tilde{r}+\sqrt{4\left(1+\tilde{r}+\tilde{r}^2\right)-3\mu^2}\right)^2>0,$$

and the numerator is

$$\begin{aligned}&-3\Big(8-6\mu^4+2\tilde{r}\left(10+7\tilde{r}+3\tilde{r}^2\right)+\mu^2\left(10+13\tilde{r}^2+16\tilde{r}\right)\\&+2\left(2+4\tilde{r}+3\tilde{r}^2+10\mu^2+8\tilde{r}\mu^2\right)\sqrt{4\left(1+\tilde{r}+\tilde{r}^2\right)-3\mu^2}\Big)<0.\end{aligned}$$

Therefore, we conclude that Q^e is strictly increasing in $\tilde{r}$.

Part (ii): Finally, we prove part (ii). Remark that if Q^e is strictly decreasing in μ, and q_B^e is strictly increasing in μ, then q_A^e must be strictly decreasing in μ. Therefore, it sufficies to show the monotonicity of Q^e and q_B^e with respect to μ.

Recall that with $X=2+4\tilde{r}-\sqrt{\tilde{x}}$,

$$Q^e=\left(\frac{12\,(1-c)\,(2+\tilde{r})}{12+6\tilde{r}+X}\right)\left(\frac{X-6-12\tilde{r}}{4X-24-36\tilde{r}}\right).$$

Thus, Q^e is strictly decreasing in μ if and only if

$$\frac{(12+6\tilde{r}+X)\,(4X-24-36\tilde{r})}{X-6-12\tilde{r}}$$

is strictly increasing in μ. Since $\frac{\partial X}{\partial\mu}>0$, it sufficies to show that

$$\frac{\partial}{\partial X}\left(\frac{(12+6\tilde{r}+X)\,(4X-24-36\tilde{r})}{X-6-12\tilde{r}}\right)>0,$$

which is equivalent to

$$\begin{aligned}&(4X-24-36\tilde{r}+4\,(12+6\tilde{r}+X))\,(X-6-12\tilde{r})-(12+6\tilde{r}+X)\,(4X-24-36\tilde{r})>0\\\Longleftrightarrow\;&(4X-24-36\tilde{r})\,(X-6-12\tilde{r}-12-6\tilde{r}-X)+4\,(12+6\tilde{r}+X)\,(X-6-12\tilde{r})>0\\\Longleftrightarrow\;&18\,(1+\tilde{r})\,(24+36\tilde{r}-4X)>4\,(6+12\tilde{r}-X)\,(12+6\tilde{r}+X)\\\Longleftrightarrow\;&\left(\frac{3}{2}\,(24+36\tilde{r}-4X)\right)(12\,(1+\tilde{r}))>(24+48\tilde{r}-4X)\,(12+6\tilde{r}+X)\,.\end{aligned}$$

Note that the first term in the left-hand side is greater than the first term in the right-hand side. To see this,

$$\frac{3}{2}\,(24+36\tilde{r}-4X)-24-48\tilde{r}+4X=12+6\tilde{r}-2X>0,$$

where the inequality follows from $X < 0$. Similarly, the second term in the left-hand side is greater than the second term in the right-hand side, because

$$12\,(1+\tilde{r}) - 12 - 6\tilde{r} - X = 6\tilde{r} - X > 0.$$

We conclude that the aggregate quantity Q^e is strictly decreasing in μ.

Now, we show that q_B^e is strictly increasing in μ, which completes the proof of part (ii). Note that

$$q_B^e = \left(\frac{6(1-c)(2+\tilde{r})}{14+10\tilde{r}-\sqrt{\tilde{x}}}\right)\left(\frac{36(1-\mu^2)+12\mu(\sqrt{\tilde{x}}-(2+4\tilde{r}))-(\sqrt{\tilde{x}}-(2+4\tilde{r}))^2}{108(1-\mu^2)+12(\sqrt{x}-(2+4\tilde{r}))-(\sqrt{x}-(2+4\tilde{r}))^2}\right).$$

Using our previous notation $X = 2+4\tilde{r}-\sqrt{\tilde{x}} < 0$, it is rewritten as

$$\begin{aligned} q_B^e &= \left(\frac{6(1-c)(2+\tilde{r})}{12+6\tilde{r}+X}\right)\left(\frac{2X^2-6X(2+4\tilde{r})-12\mu X}{8X^2-48X-72\tilde{r}X}\right) \\ &= \left(\frac{6(1-c)(2+\tilde{r})}{12+6\tilde{r}+X}\right)\left(\frac{X-3(2+4\tilde{r})-6\mu}{4X-24-36\tilde{r}}\right). \end{aligned}$$

Thus, it suffices to show that

$$\frac{X-3(2+4\tilde{r})-6\mu}{(12+6\tilde{r}+X)\,(4X-24-36\tilde{r})}$$

is strictly increasing in μ. Letting $X' = \frac{\partial X}{\partial \mu}$, one can check that

$$\frac{\partial}{\partial \mu}\left(\frac{X-3(2+4\tilde{r})-6\mu}{(12+6\tilde{r}+X)\,(4X-24-36\tilde{r})}\right) > 0$$

is equivalent to

$$\begin{aligned} & X'\,(4X-24-36\tilde{r})\,(12+6\tilde{r}+X-X+3(2+4\tilde{r})+6\mu) \\ & -X'\,(48+24\tilde{r}+4X)\,(X-6-12\tilde{r}-6\mu) - 6\,(12+6\tilde{r}+X)\,(4X-24-36\tilde{r}) > 0 \\ \iff & X'\,((4X-24-36\tilde{r})\,(18+18\tilde{r}+6\mu) - (48+24\tilde{r}+4X)\,(X-6-12\tilde{r}-6\mu)) \\ & > 6\,(12+6\tilde{r}+X)\,(4X-24-36\tilde{r}) \\ \iff & X'\left(\frac{3+3\tilde{r}+\mu}{12+6\tilde{r}+X} - \frac{X-6-12\tilde{r}-6\mu}{6(X-6-9\tilde{r})}\right) < 1, \end{aligned}$$

where the last equivalence follows from dividing each side by $6\,(4X-24-36\tilde{r})\,(12+6\tilde{r}+X)$, which is negative. Since $\frac{\partial X}{\partial \mu} = \frac{12\mu}{\sqrt{\tilde{x}}} = \frac{12\mu}{2+4\tilde{r}-X}$, it can be shown to be equivalent to

$$\begin{aligned} & (2\mu(6+6\mu+12\tilde{r}-X)+(2+4\tilde{r}-X)(6+9\tilde{r}-X))\,(12+6\tilde{r}+X) \\ & > 12\mu(6+9\tilde{r}-X)(3+3\tilde{r}+\mu). \end{aligned}$$

Noting that $12\mu = 2\cdot 6\mu$, we show that each term in the left-hand side is greater than each term in the right-hand side. More precisely, note first that

$$\begin{aligned} & 12+6\tilde{r}+X > 2(3+3\tilde{r}+\mu) = (6+6\tilde{r}+2\mu) \\ \iff & 6+X > 2\mu \\ \iff & 2(2+4\tilde{r})(6-2\mu)+(6-2\mu)^2 > 12(1-\mu^2), \end{aligned}$$

where the last equivalence follows from $X = 2 + 4\tilde{r} - \sqrt{\tilde{x}}$. Certainly, the left-hand side is strictly greater than the right-hand side.

In addition, we have

$$\begin{aligned}
&2\mu(6 + 6\mu + 12\tilde{r} - X) + (2 + 4\tilde{r} - X)(6 + 9\tilde{r} - X) > 6\mu(6 + 9\tilde{r} - X) \\
\Longleftrightarrow\ &2\mu(6 + 6\mu + 12\tilde{r} - X) > (6 + 9\tilde{r} - X)(6\mu - 2 - 4\tilde{r} + X) \\
\Longleftrightarrow\ &2\mu(6 + 9\tilde{r} - X + 6\mu + 3\tilde{r}) > (6 + 9\tilde{r} - X)(6\mu - 2 - 4\tilde{r} + X) \\
\Longleftrightarrow\ &2\mu(6\mu + 3\tilde{r}) > (6 + 9\tilde{r} - X)(4\mu - 2 - 4\tilde{r} + X) \\
\Longleftrightarrow\ &6\mu(2\mu + \tilde{r}) > (6 + 9\tilde{r} - X)(4\mu - 2 - 4\tilde{r} + X).
\end{aligned}$$

Now, substituting $X = 2 + 4\tilde{r} - \sqrt{\tilde{x}}$, it reduces to

$$\begin{aligned}
&6\mu(2\mu + \tilde{r}) > (4 + 5\tilde{r} + \sqrt{\tilde{x}})(4\mu - \sqrt{\tilde{x}}) \\
\Longleftrightarrow\ &6\mu(2\mu + \tilde{r}) - (4 + 5\tilde{r} + \sqrt{\tilde{x}})(4\mu - \sqrt{\tilde{x}}) > 0. \qquad (30)
\end{aligned}$$

One can check that the left-hand side is strictly decreasing in μ:

$$\begin{aligned}
&\frac{\partial}{\partial\mu}\left(6\mu(2\mu + \tilde{r}) - (4 + 5\tilde{r} + \sqrt{\tilde{x}})(4\mu - \sqrt{\tilde{x}})\right) \\
=&-\frac{2\left(16(1 + \tilde{r} + \tilde{r}^2) - 24\mu^2 + 3\mu(4 + 5\tilde{r}) + (8 + 7\tilde{r})\sqrt{4(1 + \tilde{r} + \tilde{r}^2) - 3\mu^2}\right)}{\sqrt{4(1 + \tilde{r} + \tilde{r}^2) - 3\mu^2}} \\
<&0.
\end{aligned}$$

Thus, it suffices to show that the inequality (30) is satisfied at $\mu = 1$:

$$\begin{aligned}
&6(2 + \tilde{r}) - (4 + 5\tilde{r} + 2 + 4\tilde{r})(4 - 2 - 4\tilde{r}) > 0 \\
\Longleftrightarrow\ &6(2 + \tilde{r}) - (6 + 9\tilde{r})(2 - 4\tilde{r}) = 36\tilde{r}^2 + 12\tilde{r} > 0.
\end{aligned}$$

Thus, we conclude that q_B^e is strictly increasing in μ. ■

Appendix B: Omitted details and additional results

Derivation of the objective function in (1) Let $\tau_i \in \mathbb{R}_+$ be the random variable capturing the length of time until termination of U and D_i's current contract ($i = A, B$). Given the independent Poisson assumption, the probability of $\tau_i \leq t$ is given by $G_i(t) = 1 - e^{-\lambda_i t}$, with corresponding density $g_i(t) = \lambda_i e^{-\lambda_i t}$. The objective function in (1) can then be written as

$$\begin{aligned}
&\mathbb{E}\left[\mathbf{1}_{\tau_i \leq \tau_{-i}} \int_{t=0}^{\tau_i} e^{-rt}dt + \mathbf{1}_{\tau_i > \tau_{-i}} \int_{t=0}^{\tau_{-i}} e^{-rt}dt\right](f + f_{-i} - c(q + q_{-i})) \\
&+ \mathbb{E}\left[\mathbf{1}_{\tau_i \leq \tau_{-i}} e^{-r\tau_i}\right] W_i(q_{-i}, f_{-i}) + \mathbb{E}\left[\mathbf{1}_{\tau_i > \tau_{-i}} e^{-r\tau_{-i}}\right] W_{-i}(q, f), \qquad (31)
\end{aligned}$$

where

$$\begin{aligned}
\mathbb{E}\left[\mathbf{1}_{\tau_i \leq \tau_{-i}} e^{-r\tau_i}\right] &= \int_{\tau_{-i}=0}^{\infty} \int_{\tau_i=0}^{\tau_{-i}} e^{-r\tau_i} g_i(\tau_i) g_{-i}(\tau_{-i}) d\tau_i d\tau_{-i} \\
&= \lambda_i \lambda_{-i} \int_{\tau_{-i}=0}^{\infty} e^{-\lambda_{-i}\tau_{-i}} \left(\int_{\tau_i=0}^{\tau_{-i}} e^{-(r+\lambda_i)\tau_i} d_{\tau_i} \right) d_{\tau_{-i}} \\
&= \frac{\lambda_i}{r+\lambda_i+\lambda_{-i}}, \qquad (32)
\end{aligned}$$

and

$$\begin{aligned}
\mathbb{E}\left[\mathbf{1}_{\tau_i \leq \tau_{-i}} \int_{t=0}^{\tau_i} e^{-rt} dt\right] &= \frac{1}{r}\left(\mathbb{E}\left[\mathbf{1}_{\tau_i \leq \tau_{-i}}\right] - \mathbb{E}\left[\mathbf{1}_{\tau_i \leq \tau_{-i}} e^{-r\tau_i}\right]\right) \\
&= \frac{1}{r}\left(\int_{\tau_{-i}=0}^{\infty} G_i\left(\tau_{-i}\right) g_{-i}\left(\tau_{-i}\right) d_{\tau_{-i}} - \int_{\tau_{-i}=0}^{\infty} \int_{\tau_i=0}^{\tau_{-i}} e^{-r\tau_i} g_i\left(\tau_i\right) d\tau_i g_{-i}\left(\tau_{-i}\right) d\tau_{-i} \right) \\
&= \frac{1}{r}\left(\frac{\lambda_i}{\lambda_i+\lambda_{-i}} - \frac{\lambda_i}{r+\lambda_i+\lambda_{-i}} \right) \\
&= \frac{\lambda_i}{\left(r+\lambda_i+\lambda_{-i}\right)\left(\lambda_i+\lambda_{-i}\right)}. \qquad (33)
\end{aligned}$$

Similarly,

$$\begin{aligned}
\mathbb{E}\left[\mathbf{1}_{\tau_i > \tau_{-i}} e^{-r\tau_{-i}}\right] &= \frac{\lambda_{-i}}{r+\lambda_i+\lambda_{-i}}, \qquad (34) \\
\mathbb{E}\left[\mathbf{1}_{\tau_i > \tau_{-i}} \int_{t=0}^{\tau_{-i}} e^{-rt} dt\right] &= \frac{\lambda_{-i}}{\left(r+\lambda_i+\lambda_{-i}\right)\left(\lambda_i+\lambda_{-i}\right)}. \qquad (35)
\end{aligned}$$

Substituting (32) to (35) into (31) yields the expression in the text.

Opt-out contracts

Proposition 6 *Suppose contracts can contain opt-out clauses. Then, for $\frac{r}{\lambda_M} \to \infty$, there exists a MPE in which after any history, the aggregate quantity eventually becomes Q^M.*

Proof. Let $\frac{r}{\lambda_M} \to \infty$. We first prove equilibrium existence and then dynamic stability. Each contract offer can contain an opt-out clause, and we use the indicator variable $o \in \{0,1\}$ to denote whether a contract includes an opt-out clause ($o = 1$) or not ($o = 0$). The state when U makes an offer to D_i now consists of a triplet (q_{-i}, f_{-i}, o_{-i}), where the fixed fee is included because when $o_{-i} = 1$, f_{-i} matters for D_{-i}'s decision whether to opt out of its current contract or continue selling q_{-i}. We restrict attention to states such that $f_{-i} \leq \pi\left(q_{-i}, 0\right)$, since it would have been optimal for D_{-i} to reject the contract (q_{-i}, f_{-i}, o_{-i}) otherwise. Since $\frac{r}{\lambda_M} \to \infty$, each firm must myopically maximize its payoff in equilibrium.

Let us define $\bar{q}\left(q_{-i}, f_{-i}\right)$ as follows: if $q_{-i} = 0$, then $\bar{q}\left(q_{-i}, f_{-i}\right) = 0$ for all f_{-i}; and if $q_{-i} > 0$, then $\bar{q}\left(q_{-i}, f_{-i}\right)$ is the unique solution to $\bar{q} : f_{-i} = \pi\left(q_{-i}, \bar{q}\right)$.[34] Note that $\bar{q}\left(q_{-i}, f_{-i}\right) > 0$ implies $q_{-i} > 0$. Also, recall that $Q^M = R^C(0) \geq R^C\left(q_{-i}\right)$ for all q_{-i}, with a strict inequality for $q_{-i} > 0$.

Consider the following strategy profile:

[34] Since $f_{-i} \leq \pi\left(q_{-i}, 0\right)$ and $\pi_2 < 0$ for $q_{-i} > 0$, there exists a unique solution $\bar{q}\left(q_{-i}, f_{-i}\right) \geq 0$ for all $q_{-i} > 0$.

(i) Given D_i accepts (q, f, o), D_{-i} opts out if and only if $o_{-i} = 1$ and $q \geq \bar{q}(q_{-i}, f_{-i})$.

(ii) Given U offers (q, f, o) to D_i,

- if $q \geq \bar{q}(q_{-i}, f_{-i})$ and $o_{-i} = 1$, then D_i accepts if and only if $\pi(q, 0) \geq f$.
- if $q < \bar{q}(q_{-i}, f_{-i})$ or $o_{-i} = 0$, then D_i accepts if and only if $\pi(q, q_{-i}) \geq f$.

(iii) U's offer to D_i is given by

- $\left(Q^M, \pi\left(Q^M, 0\right), 1\right)$ if $\bar{q}(q_{-i}, f_{-i}) \leq Q^M$ and $o_{-i} = 1$,
- $(\bar{q}(q_{-i}, f_{-i}), \pi(\bar{q}(q_{-i}, f_{-i}), 0), 1)$ if $\bar{q}(q_{-i}, f_{-i}) > Q^M$, $o_{-i} = 1$, and $\Pi(\bar{q}(q_{-i}, f_{-i})) \geq f_{-i} + \pi\left(R^C(q_{-i}), q_{-i}\right) - c\left(R^C(q_{-i}) + q_{-i}\right)$
- $\left(R^C(q_{-i}), \pi\left(R^C(q_{-i}), q_{-i}\right), 1\right)$ otherwise.

We can show that this is an equilibrium by backward induction. First, consider D_{-i}'s decision whether to opt out when D_i accepts (q, f, o) and $o_{-i} = 1$. D_{-i} obtains 0 if it opts out and $\pi(q_{-i}, q) - f_{-i}$ if it continues selling, hence opting out if and only if $q \geq \bar{q}(q_{-i}, f_{-i})$ is optimal. Next, consider D_i's acceptance decision when it is offered (q, f, o). If $q \geq \bar{q}(q_{-i}, f_{-i})$ and $o_{-i} = 1$, acceptance is followed by D_{-i} opting out, thus accepting if and only if $\pi(q, 0) \geq f$ is optimal for D_i. If $q < \bar{q}(q_{-i}, f_{-i})$ or if $o_{-i} = 0$, then D_{-i} will not opt out after D_i's contract acceptance, and thus accepting if and only if $\pi(q, q_{-i}) \geq f$ is optimal for D_i.

Finally, consider the contract offer stage. First, note that since firms myopically maximize payoffs, offering an opt-out clause ($o = 1$) is always (weakly) optimal. It remains to show that is is optimal for U to offer the quantities and fixed fees in the specified strategy profile. We divide our analysis into three cases :

Case 1(a). $0 \leq \bar{q}(q_{-i}, f_{-i}) \leq R^C(q_{-i})$ and $o_{-i} = 1$

Let us consider offers that are accepted. For $q \geq \bar{q}(q_{-i}, f_{-i})$, U will optimally set $f = \pi(q, 0)$ since D_{-i} will opt out. Thus, it is optimal for U to offer $\left(Q^M, \pi\left(Q^M, 0\right), 1\right)$ in this case, which yields a payoff $\Pi\left(Q^M\right)$ for U. On the other hand, if $q < \bar{q}(q_{-i}, f_{-i})$, then $\bar{q}(q_{-i}, f_{-i}) > 0$ and so $q_{-i} > 0$. Clearly, U will optimally set $f = \pi(q, q_{-i})$, since D_{-i} will not opt out. Thus, U's payoff from offering a contract $(q, \pi(q, q_{-i}), 1)$ with $q < \bar{q}(q_{-i}, f_{-i})$ is strictly lower than

$$\begin{aligned}
& f_{-i} + \pi(\bar{q}(q_{-i}, f_{-i}), q_{-i}) - c(\bar{q}(q_{-i}, f_{-i}) + q_{-i}) \\
=& \pi(q_{-i}, \bar{q}(q_{-i}, f_{-i})) + \pi(\bar{q}(q_{-i}, f_{-i}), q_{-i}) - c(\bar{q}(q_{-i}, f_{-i}) + q_{-i}) \\
=& \Pi(q_{-i} + \bar{q}(q_{-i}, f_{-i})) \leq \Pi\left(Q^M\right).
\end{aligned}$$

where the first equality follows from the definition of $\bar{q}(q_{-i}, f_{-i})$ for $q_{-i} > 0$. Thus, it is optimal for U to offer $\left(Q^M, \pi\left(Q^M, 0\right), 1\right)$, which induces D_{-i} to opt out and yields payoff $\Pi\left(Q^M\right)$ for U.[35]

[35] We can ignore rejection cases: if U offers a contract that is rejected, then U obtains $f_{-i} - cq_{-i}$. In this case, if $q_{-i} > 0$, then $f_{-i} - cq_{-i} = \pi(q_{-i}, \bar{q}(q_{-i}, f_{-i})) - cq_{-i} \leq \pi(q_{-i}, 0) - cq_{-i} = \Pi(q_{-i}) \leq \Pi\left(Q^M\right)$. If $q_{-i} = 0$, then U obtains $f_{-i} = 0 < \Pi\left(Q^M\right)$.

Case 1(b). $R^C(q_{-i}) < \bar{q}(q_{-i}, f_{-i}) \leq Q^M$ and $o_{-i} = 1$

Let us consider offers that are accepted, as there is again no incentive to offer a contract that is rejected for the same reasons as in Case 1. For $q \geq \bar{q}(q_{-i}, f_{-i})$, U will optimally set $f = \pi(q, 0)$ since D_{-i} will opt out. Thus, conditional on $q \geq \bar{q}(q_{-i}, f_{-i})$, the optimal contract is given by $\left(Q^M, \pi\left(Q^M, 0\right), 1\right)$, in which case U obtains $\Pi\left(Q^M\right)$. On the other hand, consider $q < \bar{q}(q_{-i}, f_{-i})$. Clearly, U will optimally set $f = \pi(q, q_{-i})$ since D_{-i} will not opt out. Thus, conditional on $q < \bar{q}(q_{-i}, f_{-i})$, the optimal contract is given by $\left(R^C(q_{-i}), \pi\left(R^C(q_{-i}), q_{-i}\right), 1\right)$, in which case U obtains

$$\begin{aligned}
& f_{-i} + \pi\left(R^C(q_{-i}), q_{-i}\right) - c\left(R^C(q_{-i}) + q_{-i}\right) \\
=& \pi\left(q_{-i}, \bar{q}(q_{-i}, f_{-i})\right) + \pi\left(R^C(q_{-i}), q_{-i}\right) - c\left(R^C(q_{-i}) + q_{-i}\right) \\
<& \pi\left(q_{-i}, R^C(q_{-i})\right) + \pi\left(R^C(q_{-i}), q_{-i}\right) - c\left(R^C(q_{-i}) + q_{-i}\right) \\
=& \Pi\left(q_{-i} + R^C(q_{-i})\right) \\
\leq& \Pi\left(Q^M\right).
\end{aligned}$$

Thus, it is optimal for U to offer $\left(Q^M, \pi\left(Q^M, 0\right), 1\right)$, which induces D_{-i} to opt out and yields payoff $\Pi\left(Q^M\right)$ to U.

Together, Cases 1(a) and 1(b) imply that if $\bar{q}(q_{-i}, f_{-i}) \leq Q^M$ and $o_{-i} = 1$, then offering $\left(Q^M, \pi\left(Q^M, 0\right), 1\right)$ is optimal for U.

Case 2. $Q^M < \bar{q}(q_{-i}, f_{-i})$ and $o_{-i} = 1$, or $o_{-i} = 0$

Let us consider offers that are accepted. For $q \geq \bar{q}(q_{-i}, f_{-i})$ and $o_{-i} = 1$, U will optimally set $f = \pi(q, 0)$ since D_{-i} will opt out. Thus, the optimal contract is given by $(\bar{q}(q_{-i}, f_{-i}), \pi(\bar{q}(q_{-i}, f_{-i}), 0), 1)$ in this case, and U obtains $\Pi(\bar{q}(q_{-i}, f_{-i}))$. On the other hand, if either $q < \bar{q}(q_{-i}, f_{-i})$ and $o_{-i} = 1$ or $o_{-i} = 0$, then it is clearly optimal for U to set $f = \pi(q, q_{-i})$, because D_{-i} will not opt out in these cases. Thus, if either $q < \bar{q}(q_{-i}, f_{-i})$ and $o_{-i} = 1$ or if $o_{-i} = 0$, the optimal contract is given by $\left(R^C(q_{-i}), \pi\left(R^C(q_{-i}), q_{-i}\right), 1\right)$ and U obtains $f_{-i} + \pi\left(R^C(q_{-i}), q_{-i}\right) - c\left(R^C(q_{-i}) + q_{-i}\right)$.

Hence, for $o_{-i} = 1$, it is optimal for U to offer $(\bar{q}(q_{-i}, f_{-i}), \pi(\bar{q}(q_{-i}, f_{-i}), 0), 1)$ if

$$\Pi(\bar{q}(q_{-i}, f_{-i})) \geq f_{-i} + \pi\left(R^C(q_{-i}), q_{-i}\right) - c\left(R^C(q_{-i}) + q_{-i}\right),$$

and offer $\left(R^C(q_{-i}), \pi\left(R^C(q_{-i}), q_{-i}\right), 1\right)$ otherwise. Note that we can ignore rejection cases since, by offering $\left(R^C(q_{-i}), \pi\left(R^C(q_{-i}), q_{-i}\right), 1\right)$, U can guarantee its payoff

$$\begin{aligned}
& f_{-i} + \pi\left(R^C(q_{-i}), q_{-i}\right) - c\left(R^C(q_{-i}) + q_{-i}\right) \\
\geq& f_{-i} - cq_{-i} + \pi\left(R^C(q_{-i}), q_{-i}\right) - cR^C(q_{-i}) \\
\geq& f_{-i} - cq_{-i}
\end{aligned}$$

where the last term is the payoff when an offer is rejected.

This proves equilibrium existence. Next, we show that given the above equilibrium strategies, the aggregate eventually becomes Q^M for any history. We again consider two cases separately.

Case 1. Transition from $(q_{-i}, f_{-i}, 1)$ such that $0 \leq \bar{q}(q_{-i}, f_{-i}) \leq Q^M$.

In the equilibrium, U will offer $\left(Q^M, \pi\left(Q^M, 0\right), 1\right)$ to D_i, which is accepted and induces D_{-i} to opt out.

Suppose U makes an offer to D_{-i} at the next recontracting opportunity. Since D_i's existing contract will be $\left(Q^M, \pi\left(Q^M, 0\right), 1\right)$, we have $\bar{q}\left(Q^M, \pi\left(Q^M, 0\right)\right) = 0$. Therefore, U will offer $\left(Q^M, \pi\left(Q^M, 0\right), 1\right)$ to D_{-i}, and D_i will opt out. Now suppose U makes an offer to D_i again at the next recontracting opportunity. Then, since D_{-i} has opted out, the state is $(0,0)$, which implies that $\bar{q}(0,0) = 0$ and thus that U will offer $\left(Q^M, \pi\left(Q^M, 0\right), 1\right)$ to D_i again.[36] Note that in either case, one firm always opts out; the aggregate quantity is Q^M; and U obtains $\Pi\left(Q^M\right)$.

Case 2. Transition from $(q_{-i}, f_{-i}, 1)$ such that $\bar{q}(q_{-i}, f_{-i}) > Q^M$.

In the equilibrium, U's offer to D_i is either (i) $(\bar{q}(q_{-i}, f_{-i}), \pi(\bar{q}(q_{-i}, f_{-i}), 0), 1)$, which is accepted and induces D_{-i} to opt out, or (ii) $\left(R^C(q_{-i}), \pi\left(R^C(q_{-i}), q_{-i}\right), 1\right)$, which is accepted and does not induce D_{-i} to opt out.

Consider case (i) first. Suppose U makes an offer to D_{-i} at the next recontracting opportunity. Since D_i's existing contract will be $(\bar{q}(q_{-i}, f_{-i}), \pi(\bar{q}(q_{-i}, f_{-i}), 0), 1)$, we have $\bar{q}(\bar{q}(q_{-i}, f_{-i}), \pi(\bar{q}(q_{-i}, f_{-i}), 0)) = 0$. Thus, the optimal offer and subsequent equilibrium path are as given in Case 1. Now suppose Umakes an offer to $D_i$again at the next recontracting opportunity. Then, since D_{-i}has opted out, its existing contract is $(0,0,1)$, which implies $\bar{q}(0,0) = 0$. Thus, the optimal offer and subsequent equilibrium path are again as given in Case 1.[37]

Next, consider the case (ii). If U makes an offer to D_i again at the next recontracting opportunity, then it will offer the same contract because D_{-i} has not opted out and the state has remained the same. Now suppose that U makes an offer to D_{-i} at the next recontracting opportunity. Since D_i's contract is $\left(R^C(q_{-i}), \pi\left(R^C(q_{-i}), q_{-i}\right), 1\right)$, we have $\bar{q}\left(R^C(q_{-i}), \pi\left(R^C(q_{-i}), q_{-i}\right)\right) = q_{-i}$, that is, D_i will opt out if and only if U offers a quantity greater than q_{-i} to D_{-i}. Thus, if $q_{-i} \leq Q^M$, U will offer $\left(Q^M, \pi\left(Q^M, 0\right), 1\right)$ to D_{-i} and the subsequent path is as given in Case 1. If $q_{-i} > Q^M$, then the offer and subsequent equilibrium path are again as given in Case 2. If Case 2(i) applies, the transition to Case 1 occurs at the next recontracting opportunity. If Case 2(ii) applies again, then at the next recontracting opportunity with D_{-i}, U offers $\left(R^C\left(R^C(q_{-i})\right), \pi\left(R^C\left(R^C(q_{-i})\right), R^C(q_{-i})\right), 1\right)$, which again does not induce D_i to opt out. Now,

$$\bar{q}\left(R^C\left(R^C(q_{-i})\right), \pi\left(R^C\left(R^C(q_{-i})\right), R^C(q_{-i})\right)\right) = R^C(q_{-i}).$$

Since $R^C(q_{-i}) < R^C(0) = Q^M$ for all $q_{-i} > 0$ and $\bar{q}(q_{-i}, f_{-i}) > Q^M$ implies $q_{-i} > 0$, we know that $R^C(q_{-i}) < Q^M$. Hence, at the next recontracting opportunity with D_i , U offers $\left(Q^M, \pi\left(Q^M, 0\right), 1\right)$ and D_{-i} opts out, and the subsequent equilibrium path is again as given in Case 1.

In summary, starting at a state that belongs to Case 2, the transition to Case 1 occurs after a finite number of alternations of offers to D_A and D_B, from which point onwards the aggregate flow quantity is equal to Q^M. ■

[36] This implicitly assumes that a downstream cannot opt back in a contract after opting out of it, but the results would unchanged if opting back in were allowed. The state would be $(q_{-i}, f_{-i}, 1)$ in this case, and given $\bar{q}(q_{-i}, f_{-i}) \leq Q^M$, U would again offer $\left(Q^M, \pi\left(Q^M, 0\right), 1\right)$.

[37] If D_{-i} could opt back in after opting out, we would be back to Case 2(i) and the transition would to Case 1 would occur at the next recontracting between U and D_{-i}.

Exclusive contracts We define a contract between U and D_i as exclusive if the contract commits U not to offer any contract with a strictly positive quantity to D_{-i} while the contract between U and D_i is in place. If $q_{-i} = 0$ at the time when U and D_i sign an exclusive contract, this implies that $q_{-i} = 0$ for the entire duration of their contract. If $q_{-i} > 0$ at the time when U and D_i sign an exclusive contract, the exclusivity commitment implies that if D_{-i}'s contract terminates before D_i's contract, D_{-i} will be offered $q_{-i} = 0$. In other words, D_i will become the exclusive downstream reseller in the time between termination of its rival's contract and termination of its own contract.

Proposition 7 *Suppose contracts can contain exclusivity clauses. Then, for* $\frac{r}{\lambda_M} \to \infty$*, there exists a MPE in which after any history, the aggregate quantity eventually becomes* Q^M*.*

Proof. Let $\frac{r}{\lambda_M} \to \infty$. We first prove equilibrium existence and then dynamic stability. Each contract offer can contain an exclusivity clause, and we use the indicator variable $e \in \{0, 1\}$ to denote whether a contract includes an exclusivity clause ($e = 1$) or not ($e = 0$). The state when U makes an offer to D_i now consists of the vector (q_{-i}, e_{-i}), and a contract between U and D_i is a vector (q_i, f_i, e_i). Since $\frac{r}{\lambda} \to \infty$, each firm myopically maximizes its current payoff in equilibrium.

Consider the following strategy profile:

(i) U's offer to D_i in state (q_{-i}, e_{-i}) is

- $(0, 0, 0)$ if $e_{-i} = 1$
- $\left(R^C(q_{-i}), \pi\left(R^C(q_{-i}), q_{-i}\right), 1\right)$ if $e_{-i} = 0$

(ii) D_i accepts U's offer in state (q_{-i}, e_{-i}) if and only if $\pi(q, q_{-i}) \geq f$.

We can show that this is an equilibrium by backward induction. First, since D_i maximizes its current payoff, it is optimal for D_i to accept (q, f, o) if $\pi(q, q_{-i}) \geq f$ and reject otherwise. Second, consider the contract offer stage. If $e_{-i} = 1$, U is bound to offering $q = 0$ to D_i, hence the highest fixed fee consistent with contract acceptance is zero. Moreover, since U maximizes current payoff, it is indifferent between offering an exclusivity clause or not to D_i. Thus, offering $(0, 0, 0)$ is optimal for U given $e_{-i} = 1$. If $e_{-i} = 0$, U's current payoff is maximized by offering the quantity $R^C(q_{-i})$ that maximizes the bilateral current profit of U and D_i and extracting D_i's entire variable profit via the fixed fee. Moreover, for any quantity and fixed fee offered, U is again indifferent between offering an exclusivity clause or not to D_i. Hence, offering $\left(R^C(q_{-i}), \pi\left(R^C(q_{-i}), q_{-i}\right), 1\right)$ is optimal for U in state $(q_{-i}, 0)$.

Next, we show that given the above equilibrium strategies, the aggregate eventually becomes Q^M for any history. First, consider any state (q_{-i}, e_{-i}) with $e_{-i} = 1$. According to the equilibrium strategy profile, U offers $(0, 0, 0)$ to D_i in such a state. If D_i's contract terminates before D_{-i}'s contract, the state remains $(q_{-i}, 1)$, and U will offers $(0, 0, 0)$ to D_i again. Once D_{-i}'s contract terminates, the state at the recontracting between U and D_{-i} is $(0, 0)$ and U will offer $\left(R^C(0), \pi\left(R^C(0), 0\right), 1\right) = \left(Q^M, \pi\left(Q^M, 0\right), 1\right)$ to D_{-i}. Hence, starting from any state (q_{-i}, e_{-i}) with $e_{-i} = 1$, the aggrate quantity will be Q^M from the next recontracting between U and D_{-i} onwards.

Second, consider any state (q_{-i}, e_{-i}) with $e_{-i} = 0$. According to the equilibrium strategy profile, U offers $\left(R^C(q_{-i}), \pi\left(R^C(q_{-i}), q_{-i}\right), 1\right)$ to D_i in such a state. If D_i's contract terminates before D_{-i}'s contract, the state remains $(q_{-i}, 0)$, and U will offer $\left(R^C(q_{-i}), \pi\left(R^C(q_{-i}), q_{-i}\right), 1\right)$ to D_i again. Once D_{-i}'s contract terminates, the state at the recontracting between U and D_{-i} is $\left(R^C(q_{-i}), 1\right)$, and hence, as shown in previous paragraph, the aggregate quantity will be Q^M after one more round of alternating contract reactions. ■

References

Ambrus, A. and Lu, S. "A continuous-time model of multilateral bargaining." *American Economic Journal: Microeconomics*, Vol. 7 (2015), pp. 208-249.

Bedre-Defolie, Ö. "Vertical coordination through renegotiation." *International Journal of Industrial Organization*, Vol. 30 (2012), pp. 553-563.

Bergemann, D. and Välimäki, J. "Dynamic common agency." *Journal of Economic Theory*, Vol. 111 (2003), pp. 23-48.

Calcagno, R., Kamada, Y., Lovo, S. and Sugaya, T. "Asynchronicity and coordination in common and opposing interest games." *Theoretical Economics*, Vol. 9 (2014), pp. 409-434.

Cremer, J. and Riordan, M. "On governing multilateral transactions with bilateral contracts." *RAND Journal of Economics*, Vol. 18 (1987), pp. 436-451.

Davies, S. "Dynamic price competition, briefly sunk costs, and entry deterrence." *RAND Journal of Economics*, Vol. 22 (1991), pp. 519-530.

De Fraja, G. "Staggered vs. synchronised wage setting in oligopoly." *European Economic Review*, Vol. 37 (1993), pp. 1507-1522.

Dequiedt, V. and Martimort, D. "Vertical contracting with informational opportunism." *American Economic Review*, Vol. 105 (2015), pp. 2141-2182.

Eaton, J. and Engers, M. "Intertemporal price competition." *Econometrica*, Vol. 58 (1990), pp. 637-659.

Farrell, J. "Limiting and Taxing Competition Through Vertical Contracts: Anticompetitive Vertical Restraints Made Simple." *Working Paper*, 2019. Downloaded at: `https://www.law.northwestern.edu/research-faculty/clbe/events/antitrust/documents/farrell\_anticompetitive\_vertical\_restraints.pdf`.

Gaudin, G. "Vertical relations, opportunism, and welfare." *RAND Journal of Economics*, Vol. 50 (2019), pp. 343-358.

Gilo, D. and Yehezkel, Y. "Vertical collusion." *RAND Journal of Economics*, Vol. 51 (2020), pp. 133-157.

Hart, O. and Tirole, J. "Vertical integration and market foreclosure." *Brookings papers on economic activity. Microeconomics*, Vol. 1990 (1990), pp. 205-286.

In, Y. and Wright, J. "Signaling private choices." *Review of Economic Studies*, Vol. 85 (2018), pp. 558-580.

Jun, B. and Vives, X. "Strategic incentives in dynamic duopoly." *Journal of Economic Theory*, Vol. 116 (2004), pp. 249-281.

Kamada, Y. and Kandori, M. "Asynchronous revision games." *Working Paper*, 2012. Downloaded at: `https://economics.yale.edu/sites/default/files/kamada-120829_0.pdf`.

Lagunoff, R. and Matsui, A. "Asynchronous choice in repeated coordination games." *Econometrica*, Vol. 65 (1997), pp. 1467-1477.

Lee, R. and Fong, K. "Markov perfect network formation: An applied framework for bilateral oligopoly and bargaining in buyer-seller networks." *Working Paper*, 2013. Downloaded at: `http://www.people.fas.harvard.edu/~robinlee/papers/MPNENetworkFormation.pdf`.

Marx, L. and Shaffer, G. "Opportunism in multilateral vertical contracting: nondiscrimination, exclusivity, and uniformity: comment." *American Economic Review*, Vol. 94 (2004), pp. 796-801.

Maskin, E. and Tirole, J. "A theory of dynamic oligopoly, III: Cournot competition." *European Economic Review*, Vol. 31 (1987), pp. 947-968.

Maskin, E. and Tirole, J. "A theory of dynamic oligopoly, I: Overview and quantity competition with large fixed costs." *Econometrica*, Vol. 56 (1988a), pp. 549-569.

Maskin, E. and Tirole, J. "A theory of dynamic oligopoly, II: Price competition, kinked demand curves, and Edgeworth cycles." *Econometrica*, Vol. 56 (1988b), pp. 571-599.

Maskin, E. and Tirole, J. "Markov perfect equilibrium: I. Observable actions." *Journal of Economic Theory*, Vol. 100 (2001), pp. 191-219.

McAfee, R. and Schwartz, M. "Opportunism in multilateral vertical contracting: Nondiscrimination, exclusivity, and uniformity." *American Economic Review*, Vol. 84 (1994), pp. 210-230.

Montez, J. "Controlling opportunism in vertical contracting when production precedes sales." *RAND Journal of Economics*, Vol. 46 (2015), pp. 650-670.

Nocke, V. and White, L. "Do vertical mergers facilitate upstream collusion?" *American Economic Review*, Vol. 97 (2007), pp. 1321-1339.

O'Brien, D. and Shaffer, G. "Vertical control with bilateral contracts." *RAND Journal of Economics*, Vol. 23 (1992), pp. 299-308.

Pastine, I. and Pastine, T. "Consumption externalities, coordination, and advertising." *International Economic Review*, Vol. 43 (2002), pp. 919-943.

Pavan, A. and Calzolari, G. "Sequential contracting with multiple principals." *Journal of Economic Theory*, Vol. 144 (2009), pp. 503-531.

Piccolo, S. and Miklós-Thal, J. "Colluding through suppliers." *RAND Journal of Economics*, Vol. 43 (2012), pp. 492-513.

Reisinger, M. and Tarantino, E. "Vertical integration, foreclosure, and productive efficiency." *RAND Journal of Economics*, Vol. 46 (2015), pp. 461-479.

Rey, P. and Tirole, J. "A primer on foreclosure." *Handbook of industrial organization*, Vol. 3 (2007), pp. 2145-2220.

Rey, P. and Vergé, T. "Bilateral control with vertical contracts." *RAND Journal of Economics*, Vol. 35 (2004), pp. 728-746.

Rubinstein, A. "Perfect equilibrium in a bargaining model." *Econometrica*, Vol. 50 (1982), pp. 97-109.

Segal, I. "Contracting with externalities." *The Quarterly Journal of Economics*, Vol. 114 (1999), pp. 337-388.

Segal, I. and Whinston, M. "Robust predictions for bilateral contracting with externalities." *Econometrica*, Vol. 71 (2003), pp. 757-791.

Stahl, I. "Bargaining theory." Stockholm: Stockholm School of Economics, 1972.

Vives, X. "Oligopoly pricing: old ideas and new tools." Cambridge, Mass.: MIT press, 1999.

Vives, X. "Complementarities and games: New developments." *Journal of Economic Literature*, Vol. 43 (2005), pp. 437-479.

White, L. "Foreclosure with incomplete information." *Journal of Economics & Management Strategy*, Vol. 16 (2007), pp. 507-535.