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## Abstract

Especially in markets with frequent price promotions, where consumers constantly have to form preferences over changing offers, product choice may depend on a relative assessment of prices and qualities. We show how such "relative thinking" profoundly influences firms' pricing and product-positioning strategies, compared to a benchmark with standard preferences. When competition is sufficiently intense, firms want to differentiate in the quality space, and they may find it advantageous to occupy the lower end of the quality spectrum. Such a strategy pays off particularly for firms with a small base of loyal customers, which then compete more aggressively for shoppers.



JEL Classification: D11, D22, L11, L15

Keywords: Relative thinking, Product Positioning, vertical differentiation, promotions, Sales, price competition



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# Pricing and Product Positioning with Relative Consumer Preferences

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October 5, 2020

**Abstract**

Especially in markets with frequent price promotions, where consumers constantly have to form preferences over changing offers, product choice may depend on a relative assessment of prices and qualities. We show how such "relative thinking" profoundly influences firms' pricing and product-positioning strategies, compared to a benchmark with standard preferences. When competition is sufficiently intense, firms want to differentiate in the quality space, and they may find it advantageous to occupy the lower end of the quality spectrum. Such a strategy pays off particularly for firms with a small base of loyal customers, which then compete more aggressively for shoppers.

*Keywords:* Relative Thinking, Product Positioning, Vertical Differentiation, Promotions, Sales, Price Competition

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# 1 Introduction

In promotion-intense markets, consumers continuously face new offerings and must reconsider their relative preferences, sometimes "on the spot" when facing promotions displayed on shelves. Various contributions in economics and marketing suggest that in such situations, consumers may be prone to "relative thinking", where their relative preferences over, for instance, lower prices or higher quality depend on their reference point, which is in turn shaped by those offers that they currently face.[1] For example, in a well-known experiment, Tversky and Kahneman (1981) document that 68% of subjects were willing to drive 20 minutes to save \$5 on a \$15 calculator, but only 29% of subjects wanted to do the same to save \$5 on a \$125 calculator, emphasizing the importance of relative differences.[2]

Various contributions in the literature have focused on a (monopolistic) firm's optimal strategy to design its range of products and prices so as to exploit the fact that consumers assess products contingent on the overall choice context.[3] Instead, our focus lies squarely on market competition, as we show that relative thinking gives rise to strategic considerations that would be absent if consumers were "absolute thinkers", with fixed preferences over attributes that would thus not depend on their choice context.[4] While in such a (standard) benchmark firms would not vertically differentiate themselves if consumers had the same or similar preferences, we find that under relative thinking, a firm's optimal product choice depends on the anticipated rival choice. Especially when competition is intense, occupying a lower-quality position becomes more profitable – and in particular so when a firm has few loyal customers.

[1]Cf. already Monroe (1973). Several authors have related this to Kahneman and Tversky's (1979) seminal Prospect Theory, e.g., Diamond and Sanyal (1990). The term "relative thinking" is shared with, for instance, Azar (2007) and Bushong et al. (forthcoming).

[2] See e.g. Azar (2007) and Bordalo et al. (2013) for overviews of experimental and empirical evidence supporting relative thinking and context-dependent preferences. Importantly, the experiments in Azar (2011) establish that relative thinking is not only prevalent when it comes to tradeoffs between paying a higher price and spending more time, as in Tversky and Kahneman (1981), but also more generally when consumers face price-quality tradeoffs.

[3]Contributions in this vein go back at least to Huber et al. (1982) or Simonson (1989), and have more recently been put forward by Dahremöller and Fels (2015) and Herweg et al. (2017), among others.

[4]We review the related literature at the end of this Introduction.

To capture competition with promotions we adapt the workhorse models of Varian (1980) and Narasimhan (1988), where we allow for (endogenous) differences in qualities and costs, and a differing number of loyal customers per firm. The intensity of competition is captured by the fraction of shoppers, who observe all offers on the market – and only they are actually susceptible to relative thinking, as only they can compare offers across firms. The degree of shopping in the market thus affects both the degree of competition as well as the importance of relative preferences. One of our contributions is thus to extend these seminal models to relative thinking, which proves surprisingly tractable.

To clarify ideas, we preview already here our model of consumer choice. For this take a product $i = 1$ with price $p_1$ and quality $v_1$, which a shopper compares to another product $i = 2$ with respective characteristics $p_2$ and $v_2$. We suppose that quality is suitably normalized so that, according to standard choice theory, an absolute thinker compares the net benefits $v_1 - p_1$ and $v_2 - p_2$.[5] For concreteness, let 2 be the higher-quality product, $v_2 > v_1$, so that also $p_2 > p_1$, as otherwise product 1 would be dominated by product 2 in all relevant aspects. Thus, an absolute thinker asks himself whether the absolute difference in quality $v_2 - v_1$ is worth the respective absolute difference in price $p_2 - p_1$ and chooses product 2 only if $v_2 - v_1 \geq p_2 - p_1$. We now turn to the implications of relative thinking, before referencing several foundations for this. There, the consumer compares *relative* differences in quality, i.e., that the quality of $i = 2$ is $100 \cdot \frac{v_2 - v_1}{v_1}$ *percent* higher, to those in prices, i.e., that the respective price is $100 \cdot \frac{p_2 - p_1}{p_1}$ *percent* higher. A simple transformation reveals that a relative thinker compares two products in terms of the respective ratios of quality to price, $\frac{v_1}{p_1}$ and $\frac{v_2}{p_2}$, that is, in terms of the respective "quality-per-dollar".[6]

One foundation of such preferences is in terms of salience, as applied also in Inderst and Obradovits (2020), which in turn adopts and modifies recent developments in Behavioral

[5] For the purpose of the subsequent analysis, we need not distinguish between a consumer's "true" (or "hedonic") or his "perceived" (or "normed") utility.

[6] The relative thinker prefers product 2 if $\frac{v_2 - v_1}{v_1} > \frac{p_2 - p_1}{p_1}$, which becomes $\frac{v_2}{v_1} > \frac{p_2}{p_1}$ and is thus equivalent to $\frac{v_2}{p_2} > \frac{v_1}{p_1}$.

Economics, notably by Bordalo et al. (2013).[7] With two products in the market, the average quality and average price are given by $v_\varnothing = \frac{1}{2}(v_1 + v_2)$ and $p_\varnothing = \frac{1}{2}(p_1 + p_2)$. Following Bordalo et al. (2013), suppose now that for product 2 its higher quality is salient when $\frac{v_2}{v_\varnothing} > \frac{p_2}{p_\varnothing}$, while when the converse holds strictly, its higher price is salient. It is straightforward to show that the same attribute is salient for either product. By stipulating that consumers compare products only on the salient attribute, we obtain exactly the same choice logic as previously.[8] This choice logic would finally also pertain when consumers derive a constant marginal utility from quality and maximize consumption with respect to a binding fixed budget constraint, motivated from a theory of mental accounting (Thaler (1985)).[9]

Our primary contribution is however not the motivation of such preferences. Rather, we show how with respect to firms' competitive strategies and empirical predictions, implications become markedly different when consumers exhibit relative preferences. In terms of profitability and pricing, relative thinking works unambiguously in favor of lower-quality products, which end up being discounted more often. While the equilibrium characterization with standard preferences would predict that products with different qualities and different regular prices exhibit the same maximum discount ("depth of promotion"), with relative thinking promotion depth is only proportionally the same across products, so that the largest observed promotion for low-quality products should be smaller in absolute terms. While these implications are of interest in their own right, our main focus lies on

[7] See also relatedly Koszegi and Szeidl (2013) and Bushong et al. (forthcoming). Other contributions, such as Azar (2008, 2014), already start with (variations of) a direct specification of relative thinking.

[8] Inderst and Obradovits (2020) solve also the intermediate case where consumers place positive weight on both attributes, but greater weight on the salient feature. Besides focusing on different issues (of loss leading), the analysis there is constrained to symmetry and uses a different timing.

[9] To see this, suppose that consumers choose quantities $x_i \geq 0$ so as to maximize $\sum_{i \in I} x_i(v_i - p_i)$ subject to the (binding) resource constraint $\sum_{i \in I} x_i p_i \leq E$ (see, for instance, equation (3.11) in Chandukala et al. (2007)). As already noted, $E$ could be motivated from a theory of mental accounting. When the constraint binds, we are indeed back to our specification of relative thinking. We note however that the multi-unit demand case is considerably more complex. While there the optimal choice is still (generically) a corner solution, with $x_i > 0$ only for the product where the respective "quality-per-dollar" $v_i/p_i$ is highest, in this case $x_i$ depends on $p_i$.

product choice, where we find that relative thinking has the most profound impact.

When consumers have the same or sufficiently similar preferences for quality, with standard preferences all firms position their product in the same way and their optimal choice is independent of their rival's. This is strikingly different with relative thinkers. Depending on the share of shoppers and on each firm's number of loyal customers, we identify circumstances where firms' product choices represent strategic substitutes and when they represent strategic complements. With relative thinking, the fact that some consumers shop and have a larger consideration set makes also their preferences different from those who only observe their "local" firm's offer, which provides scope for differentiation that would otherwise not exist. Even when high-quality products provide a higher absolute surplus, firms with a sufficiently small number of captive consumers may find it optimal to select low-quality products. In this case, with relative thinking but not otherwise, occupying the lower end of the quality spectrum may also generate higher profits, as this may render a firm a (much) stronger competitor for shoppers.

Our paper contributes to the growing field of Behavioral Industrial Organization examining the impact of various consumer biases on firm strategies and market outcomes.[10] In particular, we build on a classic literature in Industrial Organization and marketing studying firms' product-positioning strategies with vertically differentiated products (Shaked and Sutton (1982), Tirole (1988), Moorthy (1988), Motta (1993)), incorporating a workhorse model of pricing and promotions (Narasimhan (1988)) augmented by relative consumer preferences (Azar (2007), Koszegi and Szeidl (2013), Bordalo et al. (2013), Bushong et al. (forthcoming)). The most closely related work is given by Bordalo et al. (2016), who consider duopolistic competitors' quality choices and subsequent price competition in a market populated with salient thinkers (whose choice rule boils down to that of our relative thinkers, given the authors' assumptions on the intensity of consumers' salience

[10]See Spiegler (2011) for a textbook treatment and, e.g., Grubb (2015) and Heidhues and Kőszegi (2018) for recent surveys.

bias). Since consumers are homogeneous in their model, there is no scope for vertical differentiation, and perfect competition leads firms to choose the same product and to price at marginal cost in equilibrium, provided that they have access to the same set of production technologies (while otherwise, the disadvantaged firm is priced out of the market). We extend their analysis by allowing firms to have (asymmetric) bases of loyal customers, which induces more realistic (promotional) pricing patterns, leads to non-trivial tradeoffs regarding firms' product choice, and may give rise to vertical product differentiation in equilibrium.

Other work analyzing the impact of relative (or, more generally, non-standard) consumer preferences on firms' quality positioning in oligopolistic environments is surprisingly scarce. Recently, Apffelstaedt and Mechtenberg (forthcoming) have examined competing multiproduct retailers' optimal choice of product line when consumers do not (fully) anticipate that they may be manipulated through this product selection and therefore may fall victim to their own context-dependent preferences once at a store. Their model only considers symmetric firms and, like in Bordalo et al. (2016), competition is perfect, such that a desire to vertically differentiate, or interesting promotional strategies, do not arise.[11,12]

While not affected by context-dependent preferences, a share of consumers in Armstrong and Chen (2009) and Dubovik and Janssen (2012) choose which firm to frequent based only on firms' prices, but not on their (also endogenously chosen, but unobserved by these consumers) product qualities. In Armstrong and Chen (2009), such "inattentive" consumers naively buy from the cheapest firm, thereby often obtain worthless products, and may overall make a negative expected surplus in equilibrium. Instead, in Dubovik and Janssen (2012), all consumers are rational and make a positive surplus, also because they are assumed to observe product quality once at a store (and may subsequently re-

[11] As mentioned earlier, Dahremöller and Fels (2015) provide a similar analysis in a monopoly setting.

[12] Relatedly, Azar (2014) also considers competition between two (horizontally differentiated) multiproduct firms selling one low and one high-quality product each, where consumers care both about absolute *and* relative price differences to (exogenously given) reference prices for either product. However, firms' product selection is not modeled.

ject to buy). Both articles show that quality and price dispersion arises in equilibrium, however firms' incentives to strategically differentiate in quality space are not explicitly studied. Indeed, qualities and prices are set simultaneously in both models, and firms are clearly indifferent between every quality-price pair that they choose from in the examined mixed-strategy equilibria.

The rest of this paper is organized as follows. Section 2 introduces the competitive context, in Section 3 we derive pricing and promotional strategies, and in Section 4 we consider the equilibrium in firms' quality choice. Section 5 concludes. The Appendix contains proofs as well as an analysis of the benchmark case with standard preferences.

## 2 Firm Strategies and Consumer Choice

We consider a market comprised of two risk-neutral, profit-maximizing firms $i = 1, 2$ selling one product each, where products are completely characterized by their real-valued qualities $v_i > 0$ and constant marginal costs $c_i \in (0, v_i)$. As we illustrate in Appendix C, our approach is however not constrained to two firms. For given products, firms compete by simultaneously choosing prices $p_i$.

There is a total mass 1 of consumers in the market. Following Narasimhan (1988), we assume that for each firm $i$, there is a loyal consumer base of size $\alpha_i > 0$ that considers only the respective firm's offer.[13] We suppose that there is always also a positive share of shoppers who are considering all offers, $1 - \alpha_1 - \alpha_2 > 0$. For the purpose of our analysis, we stay agnostic as to why only this fraction of consumers compares offers, for instance, because they have sufficiently low costs of information acquisition or switching (which are outside our model).

Consumers' (true) utility is additively separable in quality and money, such that when a consumer considers a product in isolation (thus evaluates it like an "absolute thinker"),

[13] The assumption that $\alpha_i > 0$ for $i = 1, 2$ ensures that both firms will always be active in equilibrium and avoids cumbersome case distinctions.

his valuation is $v_i - p_i$. For shoppers who compare different offers, we however assume that they are prone to relative thinking. As already discussed in the Introduction, this amounts to the following choice criterion: A consumer then strictly prefers firm $i$'s product over firm $j$'s if and only if $\frac{v_i}{p_i} > \frac{v_j}{p_j}$. All consumers have an outside option of value zero, which implies that a consumer who is loyal to firm $i$ will buy there if and only if $p_i \leq v_i$. For simplicity, it may be assumed that also shoppers reject any product $i$ for which $p_i > v_i$, for example, because they view the outside option as (virtual) product with $v_0 = p_0 > 0$. This is however inconsequential for the equilibrium characterization: there exists no pricing equilibrium in which any firm $i$ prices above $v_i$ with positive probability,[14] and all of the to-be-characterized equilibria persist when shoppers simply choose the product $i$ for which $v_i/p_i$ is highest, regardless of whether $p_i \leq v_i$ or $p_i > v_i$.

For the baseline game analyzed in the subsequent section, we assume that firms' products are exogenously fixed, such that we only need to pin down the pricing equilibrium arising through competition between two firms characterized by $(v_1, c_1, \alpha_1)$ and $(v_2, c_2, \alpha_2)$. In Section 4, we proceed to endogenize firms' product choice by enlarging the game with an initial stage in which they simultaneously decide which product to offer. For tractability and to be able to clearly highlight the strategic incentives, we assume that firms can only choose between two different (high- and low-quality) products. Hence, they select their product from some exogenously given set $\{(v_H, c_H), (v_L, c_L)\}$, where $v_H > v_L$ and $c_H > c_L$.[15]

[14] Suppose to the contrary that any firm $i$'s highest price in its pricing support, $\overline{p}_i$, satisfied $\overline{p}_i > v_i$. Then, since this firm could guarantee a positive profit by setting $p_i = v_i$ and selling (at least) to its loyal consumers, it follows that when pricing at $\overline{p}_i$, firm $i$ must have a positive probability to sell to shoppers (as it clearly cannot sell to its loyal consumers). Due to shoppers' relative preferences, this requires that $\overline{p}_j \geq \frac{v_j}{v_i}\overline{p}_i$. Since by assumption $\overline{p}_i > v_i$, this in turn implies that $\overline{p}_j > v_j$, which means that also firm $j$ could not sell to its loyal consumers at $\overline{p}_j$. By the same logic as for firm $i$, we thus obtain that also $\overline{p}_i \geq \frac{v_i}{v_j}\overline{p}_j$ must hold. Put together, this pins down the relation between firms' upper price bounds as $\overline{p}_j = \frac{v_j}{v_i}\overline{p}_i$. Given this, each firm could only sell when pricing at its candidate upper price bound when its rival did the same. But since it cannot be the case that *both* firms have a mass point at these candidate upper price bounds satisfying $\overline{p}_j = \frac{v_j}{v_i}\overline{p}_i$ (due to the arising profitability of marginally undercutting one's own mass point to avoid ties), at least one firm's equilibrium profit would have to be zero, leading to a contradiction.

[15] An alternative assumption would be that firms can freely choose product qualities from some interval

# 3 Pricing and Promotions

In the following, we first provide an intuitive discussion of the derivation of the (mixed-strategy) pricing equilibrium for given product choices $v_i$. Subsequently, we give an explicit and full characterization. In Section 4, this will be used to endogenize product choice, which is the main contribution of our analysis.

We start by noting that, for similar reasons as in Varian (1980) and Narasimhan (1988), no pure-strategy pricing equilibrium exists. This is because in a hypothetical pure-strategy equilibrium, either the shoppers strictly prefer one firm's offer (as $v_i/p_i > v_j/p_j$) or they are indifferent between both firms' offers (as $v_i/p_i = v_j/p_j$). However, in the former case, it pays for firm $i$ to slightly increase its price,[16] while in the latter case, provided that at least one firm's price lies above its marginal cost, such a firm has an incentive to marginally decrease its price to attract all shoppers (rather than to share them with its rival). But marginal-cost pricing cannot be part of an equilibrium either, as firms may always profitably sell to their loyal consumers.

To describe the mixed-strategy equilibrium in pricing and promotions, we use the following notation. Each firm $i \in \{1, 2\}$ in the market will choose a price from a distribution $F_i(p_i)$ with support $[\underline{p}_i, \overline{p}_i]$. As we will show, at most the highest price $\overline{p}_i$ is selected with strictly positive probability, which we denote by $\gamma_i$ ("mass point"). The probability

$V = [\underline{v}, \overline{v}]$ (where $0 < \underline{v} < \overline{v}$), with corresponding constant marginal costs $c(v_i)$ such that $c(\underline{v}) \in (0, \underline{v})$ and $c'(v_i) > 0$ for all $v_i \in V$. However, apart from special cases, such as when there is a (necessarily unique) $v_i \in V$ which maximizes both $v_i - c_i$ and $v_i/c_i$, a full equilibrium characterization is difficult with asymmetric shares of loyal consumers and shoppers who are relative thinkers – and neither equilibrium uniqueness nor existence (in pure product strategies) is guaranteed. On the other hand, letting without loss of generality $\alpha_2 \leq \alpha_1$, it is straightforward to show that an efficient equilibrium in product choice, meaning that both firms choose a product $v_i$ for which $v_i - c(v_i)$ is maximized, exists if and only if

$$\max_{v_i} \left( v_i - c(v_i) - \left( \frac{1 - \alpha_1 - \alpha_2}{1 - \alpha_2} \right) \frac{\hat{v} - c(\hat{v})}{\hat{v}} v_i \right) \leq (\hat{v} - c(\hat{v})) \frac{\alpha_1}{1 - \alpha_2},$$

where $\hat{v}$ is the lowest $v_i$ in the set $\hat{V} = \arg\max_{v_i \in V} (v_i - c(v_i))$. If the above inequality is violated, firms necessarily differentiate their product qualities, and product choice is inefficient from a social perspective. Further details are available from the authors upon request.

[16] In particular, this is true since necessarily $v_i > p_i$: if it were to hold that $v_i \leq p_i$, then also $v_j < p_j$, but this cannot be part of an equilibrium because firm $j$ would make zero profit (even though it could guarantee a positive profit by selling to its loyal consumers at $p_j = v_j$).

$g_i = 1-\gamma_i$ thus denotes the *frequency* of firm $i$'s promotions, while the difference $d_i = \overline{p}_i - \underline{p}_i$ denotes the maximum *depth* of its respective promotional discount.

For the derivation of the pricing equilibrium, it may first be observed that firm $i$, with a mass $\alpha_i$ of loyal consumers, can guarantee a profit of $\pi_{i,min} = (v_i - c_i)\alpha_i$ by setting $p_i = v_i$ and (at least) selling to all of its loyal consumers at the maximal price $v_i$. When choosing any lower price, the firm can at best hope to additionally attract the mass $1-\alpha_i-\alpha_j$ of shoppers for sure, which would give a profit of $\pi_{i,max}(p_i) = (p_i - c_i)[\alpha_i + (1-\alpha_i-\alpha_j)] = (p_i - c_i)(1-\alpha_j)$ (i.e., the firm would attract all consumers apart from the mass $\alpha_j$ loyal to firm $j$). Solving $\pi_{i,max}(p_i) < \pi_{i,min}$ for $p_i$ therefore gives the set of all prices $p_i$ which are strictly dominated for $i$, such that in any pricing equilibrium, only prices for which $\pi_{i,max}(p_i) \geq \pi_{i,min}$ may be sampled. The following must therefore hold for all prices in firms' equilibrium supports:

$$p_i \geq \underline{\underline{p}}_i \equiv c_i + (v_i - c_i)\frac{\alpha_i}{1-\alpha_j}. \tag{1}$$

Due to shoppers' relative thinking, firm $i$ attracts them rather than firm $j$ whenever $\frac{v_i}{p_i} > \frac{v_j}{p_j}$, that is, whenever $p_i < \frac{v_i}{v_j}p_j$. But since firm $j$ never prices below $\underline{\underline{p}}_j$, firm $i$ need not go all the way down to its lowest non-dominated price $\underline{\underline{p}}_i$ to guarantee to attract the shoppers if it holds that $\underline{\underline{p}}_i < \frac{v_i}{v_j}\underline{\underline{p}}_j$: in this case, it suffices for firm $i$ to price at (or $\epsilon$ below) $\frac{v_i}{v_j}\underline{\underline{p}}_j$ to be certain to attract them. The inequality $\underline{\underline{p}}_i < \frac{v_i}{v_j}\underline{\underline{p}}_j$ can now be reduced to[17]

$$\left(1-\frac{c_i}{v_i}\right)(1-\alpha_i) > \left(1-\frac{c_j}{v_j}\right)(1-\alpha_j). \tag{2}$$

This is more likely to be satisfied when firm $i$ has a lower mass $\alpha_i$ of loyal customers (such that attracting the shoppers is more important for $i$) and when its quality relative to cost

[17] Rewriting the inequality as $\underline{\underline{p}}_i/v_i < \underline{\underline{p}}_j/v_j$, this becomes $\frac{c_i}{v_i}+\left(1-\frac{c_i}{v_i}\right)\frac{\alpha_i}{1-\alpha_j} < \frac{c_j}{v_j}+\left(1-\frac{c_j}{v_j}\right)\frac{\alpha_j}{1-\alpha_i}$. Adding $-1$ to both sides and factoring $1-\frac{c_{(\cdot)}}{v_{(\cdot)}}$ on either side then gives $\left(1-\frac{c_i}{v_i}\right)\left(\frac{\alpha_i}{1-\alpha_j}-1\right) < \left(1-\frac{c_j}{v_j}\right)\left(\frac{\alpha_j}{1-\alpha_i}-1\right)$, from which (2) easily follows.

$\frac{v_i}{c_i}$ is higher (such that $i$'s product is better suited to attract the shoppers).

Intuitively, firm $i$ for which (2) holds[18] will compete more aggressively, since, keeping in mind the profit it could guarantee from its loyal consumers, it would be willing to offer the shoppers a strictly higher perceived utility $u_i = v_i/p_i$ than firm $j$ to attract them for sure. This is precisely what happens in equilibrium: firm $i$ always promotes its product by pricing strictly below $v_i$, while firm $j$ sets the "regular" price $p_j = v_j$, catered only to its loyal consumers, with positive probability $\gamma_j > 0$ and discounts its product only with the remaining probability. Firm $i$ is also advantaged in the sense that it can guarantee a higher profit than its minimal (min-max) profit $\pi_{i,min}$ by setting $p_i = \frac{v_i}{v_j}\underline{\underline{p}}_j > \underline{\underline{p}}_i$. And indeed, it turns out that firm $i$'s equilibrium profit is exactly given by $\pi_i = \pi_i\big(\frac{v_i}{v_j}\underline{\underline{p}}_j\big) = (\frac{v_i}{v_j}\underline{\underline{p}}_j - c_i)(1-\alpha_j)$, while firm $j$'s equilibrium profit just matches its min-max profit, $\pi_j = \pi_{j,min} = (v_j - c_j)\alpha_j$. In what follows, we will refer to some firm $i$ for which (2) holds as being *more aggressive*. This also implies that the respective firm makes a higher equilibrium profit than its min-max profit.

The pricing equilibrium is now such that the firms randomize over the same set of (perceived) shopper utilities $u_{(\cdot)} = v_{(\cdot)}/p_{(\cdot)}$, with firm $i$ (for which (2) holds) drawing prices from the support $[\frac{v_i}{v_j}\underline{\underline{p}}_j, v_i)$ and firm $j$ drawing prices from the support $[\underline{\underline{p}}_j, v_j]$. Clearly, any price in firms' respective pricing supports must yield the same expected profit (of $\pi_i$ and $\pi_j$, as provided above). The size of firm $j$'s mass point on $v_j$, $\gamma_j$, is finally pinned down to ensure that when firm $i$ prices marginally below $v_i$, its resulting expected profit

$$\pi_i(v_i - \epsilon) = (v_i - c_i)[\alpha_i + \gamma_j(1 - \alpha_i - \alpha_j)]$$

matches its profit $\pi_i\big(\frac{v_i}{v_j}\underline{\underline{p}}_j\big)$ when choosing the lowest (or any other) price in its equilibrium support. Proposition 1 gives the precise characterization.

[18]Clearly, there is always (at least) one firm $i \in \{1, 2\}$ for which (2) holds weakly – and generically, it holds strictly for exactly one firm.

**Proposition 1** *Suppose without loss of generality that condition (2) holds weakly. Then there exists a unique pricing equilibrium such that firm $i$ randomizes over prices $p_i \in [\underline{p}_j \frac{v_i}{v_j}, v_i)$, while firm $j$ randomizes over prices $p_j \in [\underline{p}_j, v_j]$ and chooses $p_j = v_j$ with probability*

$$\gamma_j = 1 - \left(\frac{1-\alpha_j}{1-\alpha_i}\right)\left(\frac{1-\frac{c_j}{v_j}}{1-\frac{c_i}{v_i}}\right). \tag{3}$$

*Firm profits are given by $\pi_i = (v_i - c_i)[\alpha_i + (1-\alpha_i-\alpha_j)\gamma_j]$ and $\pi_j = (v_j - c_j)\alpha_j$. The CDF over $[\underline{p}_i, \overline{p}_i)$ is given by*

$$F_i(p) = 1 - \frac{\frac{\pi_j}{p\frac{v_j}{v_i} - c_j} - \alpha_j}{1-\alpha_i-\alpha_j}$$

*and that over $[\underline{p}_j, \overline{p}_j)$ by*

$$F_j(p) = 1 - \frac{\frac{\pi_i}{p\frac{v_i}{v_j} - c_i} - \alpha_i}{1-\alpha_i-\alpha_j}.$$

Comparing promotion probabilities and the boundaries of the respective supports, we have the following immediate corollary, which summarizes some of our previous discussion:

**Corollary 1** *When condition (2) holds, firm $i$ promotes strictly more often, with $g_i - g_j = \gamma_j > 0$. For firm $i$, the (maximum) depth of promotions is*

$$d_i = \frac{v_i}{v_j}(v_j - c_j)\left(1 - \frac{\alpha_j}{1-\alpha_i}\right),$$

*while that for firm $j$ is*

$$d_j = (v_j - c_j)\left(1 - \frac{\alpha_j}{1-\alpha_i}\right),$$

*so that $d_i/v_i = d_j/v_j$.*

With relative thinking, relative promotional depth of high- and low-quality products is the same. With standard preferences, this would intuitively be different, as the shoppers would need to be indifferent between the two products when they were priced at the respective lower boundaries of firms' pricing supports – so that ultimately, each product would have the same depth of promotion (cf. Appendix B for the full characterization). In fact, while with standard preferences the range of prices is the same regardless of the product's quality and thus maximum price, with relative thinking the absolute price range is larger for the more expensive (higher-quality) product.

From the preceding characterization we finally obtain the following immediate comparative results. When firm $i$'s base of loyal consumers $\alpha_i$ increases, it becomes (weakly) less likely that firm $i$ promotes its product (i.e., $g_i$ decreases). When instead the rival firm's base of loyal consumers $\alpha_j$ increases, it becomes (weakly) more likely that firm $i$ promotes its product (i.e., $g_i$ increases). These predictions are however not different to those under standard preferences (cf. Narasimhan (1988)).

# 4 Product Choice

Taking into account the equilibrium outcomes from arbitrary pricing subgames determined above, we now study firms' equilibrium product choice when they may, in the first stage of the correspondingly modified game, select between two different products characterized by $(v_H, c_H)$ and $(v_L, c_L)$, with $v_H > v_L > 0$ and $c_H > c_L > 0$.

Suppose first that $v_L - c_L > v_H - c_H$, so that a firm focusing only on captive consumers, whose preferences are not distorted by relative thinking, would obviously prefer the low-quality product, simply as this would enable the firm a strictly higher margin. Note now that, as is easy to see, from $v_L - c_L > v_H - c_H$ it follows also that $\frac{v_L}{c_L} > \frac{v_H}{c_H}$.[19] When priced at costs, shoppers would therefore choose the lower-quality product. Intuitively, when $v_L - c_L > v_H - c_H$, this thus makes the lower-quality version the "stronger" product both for targeting loyal consumers and shoppers, implying that in this case both firms should

[19] $v_L - c_L > v_H - c_H$ is equivalent to $c_L \left(\frac{v_L}{c_L} - 1\right) > c_H \left(\frac{v_H}{c_H} - 1\right)$, which, as $c_L < c_H$, clearly implies that $\frac{v_L}{c_L} > \frac{v_H}{c_H}$.

choose low quality. The symmetric picture arises when now both $v_H - c_H > v_L - c_L$ and $\frac{v_H}{c_H} \geq \frac{v_L}{c_L}$ hold, albeit now the former condition does not imply the latter. We summarize these results as follows:

**Proposition 2** *If $v_L - c_L > v_H - c_H$, which implies $\frac{v_L}{c_L} > \frac{v_H}{c_H}$, then $v_1 = v_2 = v_L$;*[20] *if $v_H - c_H > v_L - c_L$ and $\frac{v_H}{c_H} \geq \frac{v_L}{c_L}$, then $v_1 = v_2 = v_H$.*

In what follows, we thus consider the (residual) case where $v_H - c_H > v_L - c_L$ and $\frac{v_H}{c_H} < \frac{v_L}{c_L}$. That is, one product variant is stronger when the firm would want to target captive consumers and one is stronger when it wants to target shoppers whose larger consideration set implies that they compare offers in relative terms. We now first consider firms' best responses, before solving for the equilibrium in product choice.

## 4.1 Best Responses

Deriving best responses is clearly a prerequisite to derive the (Nash) equilibrium in product choice, but it is also of independent interest, as sometimes only one firm may be in a position to choose or change the positioning of its product, while this does not apply to its rival. For instance, the rival may face technical limitations, and only one firm may have the knowledge or patents that allow to "upgrade" its product's quality. For these reasons, we also devote some space to first deriving a firm's "best response".

**Optimal product choice when rival chooses low quality.**

**Lemma 1** *Suppose that $v_H - c_H > v_L - c_L$ and $\frac{v_H}{c_H} < \frac{v_L}{c_L}$. When firm $i$ anticipates that its rival chooses low quality, $v_j = v_L$, its optimal response is described as follows: First, for any given loyal share of its rival $\alpha_j$, firm $i$ may only choose low quality if its own loyal share $\alpha_i$ is sufficiently small; second, now for given own loyal share $\alpha_i$, firm $i$ may only*

[20]If $v_L - c_L = v_H - c_H$, which still implies that $\frac{v_L}{c_L} > \frac{v_H}{c_H}$, it is again always an equilibrium that both firms choose $v_L$. However, it is also an equilibrium that the firm with the larger mass of loyal consumers chooses $v_H$, while its rival chooses $v_L$.

*choose low quality if its rival's loyal share takes on intermediate values, that is, when it lies between a lower and an upper boundary.*

*Formally,* $v_i = v_L$ *is strictly more profitable if:*

$$\alpha_i < \begin{cases} \tilde{\alpha}_{i,L}(\alpha_j)\ (< \alpha_j) & \textit{for} \quad 0 < \alpha_j \leq \underline{\alpha}_{j,L} \\ \hat{\alpha}_{i,L}(\alpha_j)\ (< \alpha_j) & \textit{for} \quad \underline{\alpha}_{j,L} \leq \alpha_j < \overline{\alpha}_{j,L}, \end{cases}$$ [21]

*where*

$$\tilde{\alpha}_{i,L}(\alpha_j) = 1/2 - \sqrt{1/4 - \alpha_j(1-\alpha_j)\frac{v_L - c_L}{v_H - c_H}} \tag{4}$$

$$\hat{\alpha}_{i,L}(\alpha_j) = 1 - \alpha_j \left( \frac{(v_H - v_L)(v_L - c_L)}{c_H v_L - c_L v_H} \right) \tag{5}$$

$$\underline{\alpha}_{j,L} = \frac{c_H v_L - c_L v_H}{(v_H - v_L)(v_L - c_L) + \frac{v_L}{v_H}(c_H v_L - c_L v_H)} > 0 \tag{6}$$

$$\overline{\alpha}_{j,L} = \frac{c_H v_L - c_L v_H}{(v_H - v_L)(v_L - c_L)} \in (\underline{\alpha}_{j,L}, 1). \tag{7}$$

*Otherwise, it is more profitable to choose* $v_i = v_H$*.*

The characterization of Lemma 1 is illustrated in the left-hand panel of Figure 1. The shaded area depicts the parameter range for which the best response of firm $i$ is to choose low quality as well. We next provide intuition for this characterization.

Here, the role of firm $i$'s own loyal share is particularly intuitive. Loyal consumers effectively only consider the absolute value of the firm's product, simply as they do not compare different offers. From $v_H - c_H > v_L - c_L$, a firm can thus extract more value from loyal consumers when it offers the high-quality product. The low-quality product is however the better choice when the firm wants to attract shoppers. Therefore, choosing the low-quality product is only optimal when a firm has sufficiently few loyal customers and there are sufficiently many shoppers available in the market, i.e., when $\alpha_i$ is sufficiently

[21] $\tilde{\alpha}_{i,L}(\underline{\alpha}_{j,L}) = \hat{\alpha}_{i,L}(\underline{\alpha}_{j,L})$.

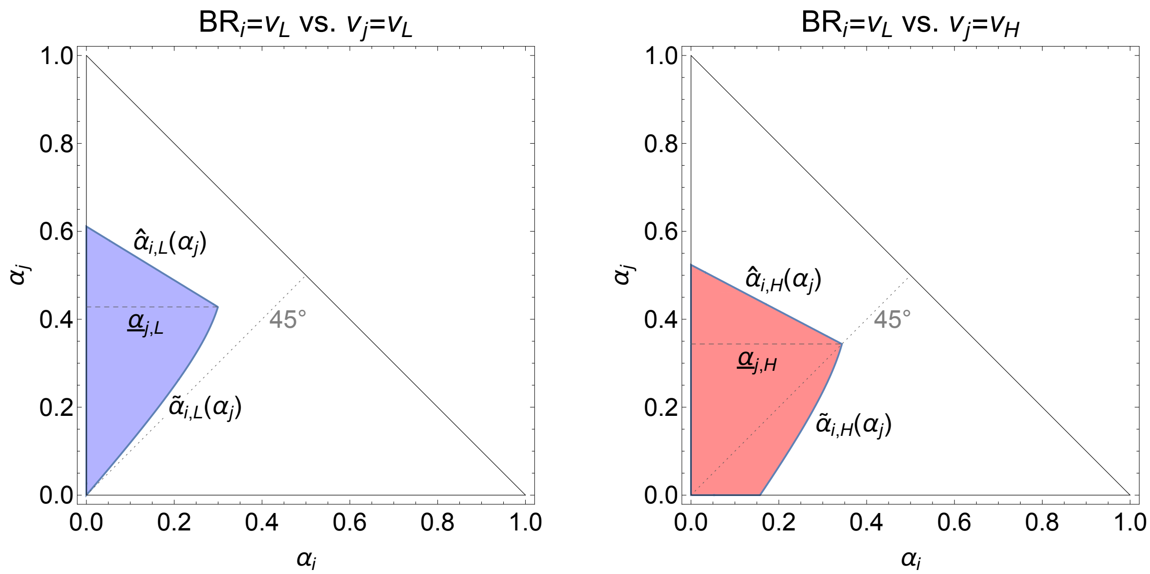


Figure 1: Illustration of regions in $(\alpha_i, \alpha_j)$-space in which firm $i$ finds it optimal to choose $v_L$, given $v_j = v_L$ (left panel) and $v_j = v_H$ (right panel). The parameters used are $v_H = 1$, $c_H = 0.65$, $v_L = 0.7$, $c_L = 0.4$.

small and $\alpha_j$ is not too large. In Figure 1 this can be seen as, for a given value of its rival's loyal share $\alpha_j$ (i.e., moving on a horizontal line), the shaded area lies to the left of the respective curve ($\alpha_i < \tilde{\alpha}_{i,L}(\alpha_j)$, or $\alpha_i < \hat{\alpha}_{i,L}(\alpha_j)$), but only when $\alpha_j$ is not too large ($\alpha_j < \overline{\alpha}_{j,L}$).

Somewhat less immediate is the impact that a change of the rival's loyal share has on firm $i$'s product choice. From Lemma 1 (and Figure 1) we know that $\alpha_j$ has a non-monotonic impact on firm $i$'s best response, and we turn next to the rationale for this.

When $\alpha_j$ is high, this also means that there are, ceteris paribus, few shoppers in the market. By the preceding observations, it is then not profitable for firm $i$ to choose low quality.[22] When $\alpha_j$ is instead relatively low, while there may then be sufficient shoppers available to make choosing the low quality attractive for firm $i$, this also means that firm $j$ will itself aggressively pursue shoppers. This in turn renders shoppers, relative to loyal

[22]Note that we presently do not ask whether, in this case, the choice of $v_j = v_L$ is optimal for firm $j$. We turn to a characterization of the equilibrium below. Recall however that either firm may be restricted (e.g., for technological reasons) to a particular product variant.

consumers, overall less attractive also for firm $i$. The interplay of the two described forces leads to the outcome described in Lemma 1: For a given own share of loyal consumers, provided that it is then indeed sometimes profitable to choose the low-quality product (which requires that $\alpha_i$ is not too large), this is only the case when the loyal share $\alpha_j$ of the firm's rival takes on intermediate values. Graphically, holding now fixed a value of $\alpha_i$, we see in Figure 1 (now by remaining on a vertical line) that the respective range of values $\alpha_j$ in the shaded area is indeed an interior interval.

Note at this point also that $\alpha_i < \alpha_j$ needs to hold to make the low-quality product optimal for firm $i$ when $v_j = v_L$. In other words, it is never optimal for firm $i$ to challenge its rival and choose as well the low-quality product to compete for shoppers when its rival has fewer loyal consumers and would thus remain more aggressive.

**Optimal product choice when rival chooses high quality.** It turns out that, while following the same logic, the explicit characterization of the best response of firm $i$ when $v_j = v_H$ (instead of $v_j = v_L$) is slightly more involved, while still qualitatively similar.

**Lemma 2** *Suppose that $v_H - c_H > v_L - c_L$ and $\frac{v_H}{c_H} < \frac{v_L}{c_L}$. When firm $i$ anticipates that $v_j = v_H$, its optimal response is qualitatively identical to that characterized in Lemma 1 (where $v_j = v_L$) and depicted in the right-hand panel of Figure 1.*

*Formally, $v_i = v_L$ is now strictly more profitable if:*

$$\alpha_i < \begin{cases} \tilde{\alpha}_{i,H}(\alpha_j)\ (\geq \alpha_j) & \textit{for} \quad 0 < \alpha_j \leq \underline{\alpha}_{j,H} \\ \hat{\alpha}_{i,H}(\alpha_j)\ (\leq \alpha_j) & \textit{for} \quad \underline{\alpha}_{j,H} \leq \alpha_j < \overline{\alpha}_{j,H}, \end{cases}$$ [23]

[23] $\tilde{\alpha}_{i,H}(\underline{\alpha}_{j,H}) = \hat{\alpha}_{i,H}(\underline{\alpha}_{j,H}) = \underline{\alpha}_{j,H}$.

*where*

$$\tilde{\alpha}_{i,H}(\alpha_j) = \frac{1+(1-\alpha_j)\left(\frac{c_H\frac{v_L}{v_H}-c_L}{v_H-c_H}\right)}{2} - \sqrt{\left[\frac{1-(1-\alpha_j)\left(\frac{c_H\frac{v_L}{v_H}-c_L}{v_H-c_H}\right)}{2}\right]^2 - (1-\alpha_j)\alpha_j\frac{v_L}{v_H}} \tag{8}$$

$$\hat{\alpha}_{i,H}(\alpha_j) = 1-\alpha_j\left(\frac{(v_H-v_L)(v_H-c_H)}{c_Hv_L-c_Lv_H}\right) \tag{9}$$

$$\underline{\alpha}_{j,H} = \frac{c_Hv_L-c_Lv_H}{(v_H-v_L)(v_H-c_H)+c_Hv_L-c_Lv_H} > 0 \tag{10}$$

$$\overline{\alpha}_{j,H} = \frac{c_Hv_L-c_Lv_H}{(v_H-v_L)(v_H-c_H)} \in (\underline{\alpha}_{j,H}, 1). \tag{11}$$

*Otherwise, it is more profitable to choose* $v_i = v_H$*.*

Again, choosing $v_i = v_L$ is only strictly optimal when firm $i$'s loyal share $\alpha_i$ is sufficiently low ($\alpha_i < \tilde{\alpha}_{i,H}(\alpha_j)$, or $\alpha_i < \hat{\alpha}_{i,H}(\alpha_j)$), provided that firm $j$'s loyal share is not too large ($\alpha_j < \overline{\alpha}_{j,H}$) such that there are sufficiently many shoppers in the market. Note now however that in difference to Lemma 1, when $v_j = v_H$ instead of $v_j = v_L$, it may be strictly profitable for firm $i$ to choose low quality even though it has more loyal customers than its rival, $\alpha_i > \alpha_j$ (compare with Figure 1).[24] This is so as even when the rival has fewer loyal customers, the fact that only firm $i$ has then the relatively stronger product may still make firm $i$ more aggressive and thus induces firm $i$ to promote more and firm $j$ to promote less, which is a prerequisite to make $v_i = v_L$ optimal.

Figure 2 compares the two best responses for $v_i$ when the rival has either low or high quality. There, the two light shaded areas (blue and red) depict the areas where the two best responses $v_i = v_L$ to $v_j \in \{v_L, v_H\}$ do not overlap. The darker shaded area (purple) depicts the region where $v_i = v_L$ is a best response to either rival product. We can thus observe that the rival's choice of low quality $v_j = v_L$ makes it *relatively* more profitable that also firm $i$ chooses low quality, compared to when $v_j = v_H$, when the rival's share of

[24]Precisely, this holds if and only if $\alpha_j < \underline{\alpha}_{j,H}$ and $\alpha_i$ falls in the non-empty interval $(\alpha_j, \tilde{\alpha}_{i,H}(\alpha_j))$.

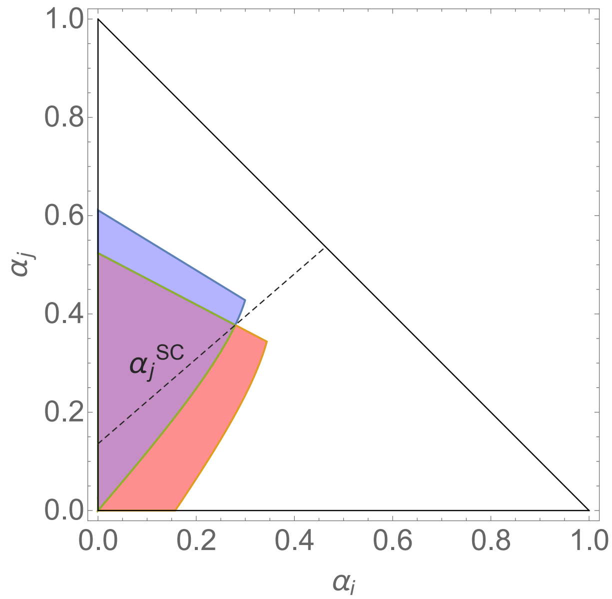


Figure 2: Comparison of firm $i$'s best-response regions in $(\alpha_i, \alpha_j)$-space. The parameters used are $v_H = 1$, $c_H = 0.65$, $v_L = 0.7$, $c_L = 0.4$.

loyal consumers $\alpha_j$ is high (as in the light blue region, $v_i = v_L$ is only a best response to $v_j = v_L$ but not to $v_j = v_H$). Instead, when $\alpha_j$ is low, then firm $i$ finds it *relatively* less profitable to choose low quality when this is chosen by its rival (as in the light red region, $v_i = v_L$ is only a best response to $v_j = v_H$ but not to $v_j = v_L$). Before commenting on this result, we now make this more precise by referring to the concepts of strategic substitutes and complements.

Consider the difference in firm $i$'s profit with low quality instead of high quality. Then firms' product strategies are (strict) strategic complements when this difference is higher in case also firm $j$ chooses low quality: $\pi_i(v_L, v_L) - \pi_i(v_H, v_L) > \pi_i(v_L, v_H) - \pi_i(v_H, v_H)$.[25] Instead, firms' product strategies are (strict) strategic substitutes when this difference is higher in case firm $j$ chooses high quality: $\pi_i(v_L, v_H) - \pi_i(v_H, v_H) > \pi_i(v_L, v_L) - \pi_i(v_H, v_L)$. Now the dotted line in Figure 2 precisely separates the areas where firms' product strategies are strategic complements and substitutes, both for ease of exposition considered only from the perspective of some firm $i$:

[25]In light of the preceding discussion we find it more instructive to express the difference in this way, rather than by the possibly more standard subtraction of low-quality profits from high-quality profits.

**Lemma 3** *Suppose that $v_H - c_H > v_L - c_L$, $\frac{v_H}{c_H} < \frac{v_L}{c_L}$, and define*

$$\alpha_j^{SC}(\alpha_i) = \left(\alpha_i + \frac{c_H \frac{v_L}{v_H} - c_L}{v_H - c_H}\right) / \left(1 + \frac{c_H \frac{v_L}{v_H} - c_L}{v_H - c_H}\right) > \alpha_i. \tag{12}$$

*Then, product strategies represent for firm $i$ strategic complements if $\alpha_j > \alpha_j^{SC}(\alpha_i)$ and strategic substitutes if $\alpha_j < \alpha_j^{SC}(\alpha_i)$.*

The observation in Lemma 3 needs to be put into a wider perspective. As we already commented, given that consumers' (true) preferences are in fact homogeneous, with standard preferences there would be no scope for product differentiation, and both firms would choose the same quality, notably the high quality in the present case (where $v_H - c_H > v_L - c_L$). And the rival's anticipated choice would be irrelevant for a firm's own "best response". But this is no longer the case when consumers are relative thinkers. Then, when the high-quality product is relatively weaker with $\frac{v_H}{c_H} < \frac{v_L}{c_L}$, the firm's optimal product choice *does* depend on its rival's product choice. When $\alpha_j$ is low relative to $\alpha_i$, so that firm $j$ is likely to aggressively pursue shoppers, choosing low quality is less likely to be optimal for firm $i$ when firm $j$ has also low quality. As put in Lemma 3, firms' product strategies are then strategic substitutes. They are however strategic complements when $\alpha_j$ is high relative to $\alpha_i$: Firm $i$ is then more likely to choose low quality as well when firm $j$ chooses low quality, as it otherwise risks losing its competitive advantage vis-à-vis shoppers.

## 4.2 Equilibrium Product Strategies

Note again that so far we have taken one firm's (that is, firm $j$'s) product choice as given. As discussed above, this may sometimes represent a realistic scenario. Now, however, we suppose that both firms can choose product quality. We are thus interested in characterizing all pure-strategy (Nash) equilibria where firms may choose $v_i$ from the set $\{v_H, v_L\}$. To save space, without loss of generality we now restrict consideration to the case where

$\alpha_1 \geq \alpha_2$. In Figure 3 this is captured by the fact that we only need to consider parameter values $(\alpha_1, \alpha_2)$ "below" the line where $\alpha_2 = \alpha_1$.

**Proposition 3** *Suppose that $v_H - c_H > v_L - c_L$, $\frac{v_H}{c_H} < \frac{v_L}{c_L}$, and, without loss of generality, $\alpha_1 \geq \alpha_2$. Then, when both firms can choose whether to offer low or high quality, the equilibrium product strategies, as depicted in Figure 3, are characterized as follows:*

*i)* ***Area I****: If there are altogether few shoppers as the (appropriately weighted) sum of $\alpha_1$ and $\alpha_2$ is high, both firms choose high quality ($v_1 = v_2 = v_H$). Formally, this is the case if $\alpha_2 \geq \hat{\alpha}_{2,H}(\alpha_1)$, where $\hat{\alpha}_{2,H}(\alpha_1)$ is given by (9) for $i = 2$, $j = 1$.*

*ii)* ***Area II****: If there are sufficiently many shoppers as the (appropriately weighted) sum of $\alpha_1$ and $\alpha_2$ is not too high, and if the loyal share of firm 2 is sufficiently smaller than that of firm 1, only firm 2 chooses low quality ($v_1 = v_H, v_2 = v_L$). Formally, this is the case if $\alpha_2 < \hat{\alpha}_{2,H}(\alpha_1)$,[26] and $\alpha_2 < \tilde{\alpha}_{2,L}(\alpha_1)$ or $\alpha_1 > \tilde{\alpha}_{1,H}(\alpha_2)$ (or both), where $\tilde{\alpha}_{2,L}(\alpha_1)$ is given by (4) for $i = 2$, $j = 1$ and $\tilde{\alpha}_{1,H}(\alpha_2)$ is given by (8) for $i = 1$, $j = 2$.*

*iii)* ***Area III****: Otherwise, i.e., if there are sufficiently many shoppers and the two firms' loyal shares are sufficiently similar, there exist two pure-strategy equilibria, where in each of them one firm chooses high and the other firm low quality ($v_1 = v_H, v_2 = v_L$ or $v_1 = v_L, v_2 = v_H$). Formally, this is the case if $\alpha_2 \geq \tilde{\alpha}_{2,L}(\alpha_1)$ and $\alpha_1 \leq \tilde{\alpha}_{1,H}(\alpha_2)$.*

The characterization of Areas I and II is particularly intuitive and relies on our preceding discussion of the role of shoppers vs. that of a firm's share of loyal customers in determining the optimality of either the (absolutely stronger) high-quality product or the (relatively stronger) low-quality product. The characterization of Area III, where there exist multiple equilibria, in turn follows from the observation in Lemma 3 that firms' product strategies can be strategic substitutes. That is, in the considered parameter region, where firms' loyal shares are not too different, each firm would choose low quality if and only if its rival chooses high quality. The firm that then ends up with the low-quality product

[26] The product combination ($v_1 = v_H$, $v_2 = v_L$) is also an equilibrium if $\alpha_2 = \hat{\alpha}_{2,H}(\alpha_1)$, although it then coexists with the equilibrium of Area I where $v_1 = v_2 = v_H$.

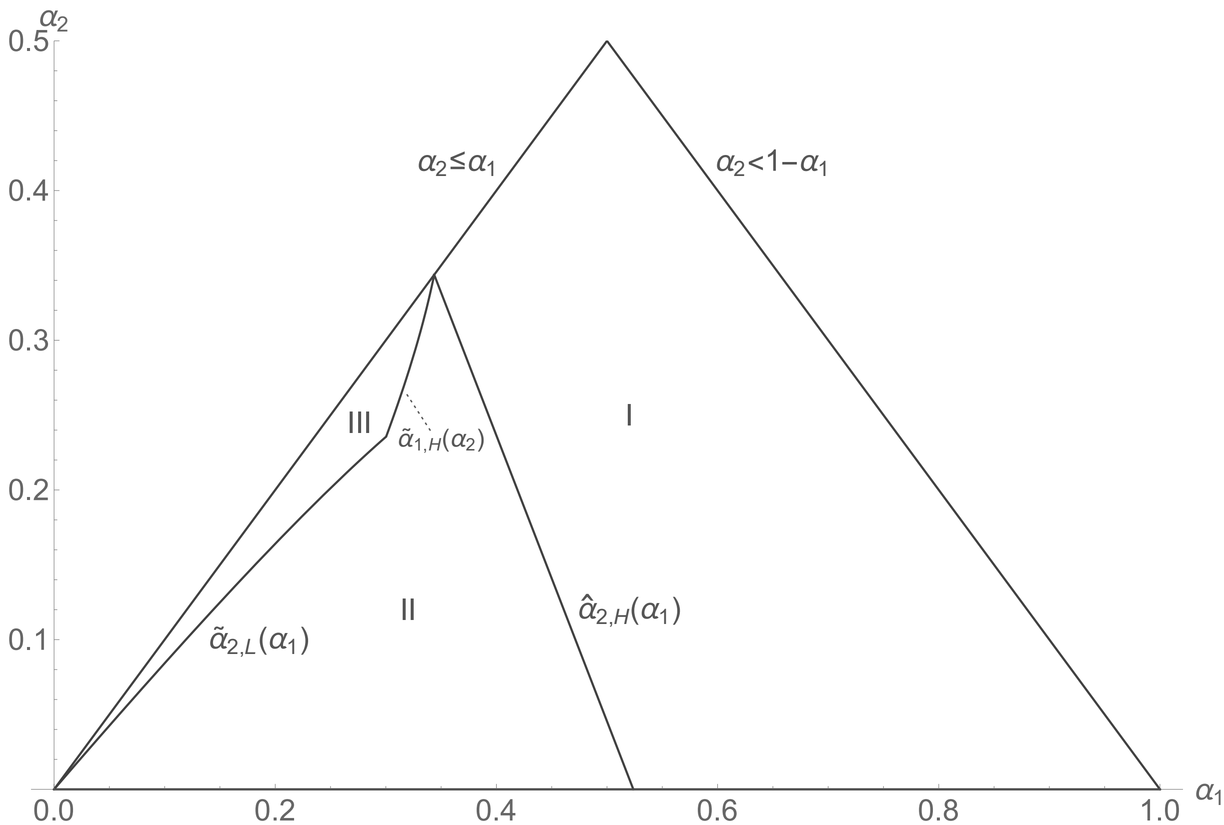


Figure 3: Illustration of product-choice equilibrium regions for $\alpha_1 \geq \alpha_2$. The parameters used are $v_H = 1$, $c_H = 0.65$, $v_L = 0.7$, $c_L = 0.4$.

will promote its product more frequently, thereby capturing (on average) a larger share of shoppers. As this is more profitable than choosing instead the high-quality product, while then the rival promotes (its low-quality product) more frequently, in this case a firm strictly profits from occupying the low-quality ("discount") space.

**Corollary 2** *In Area III of Figure 3 there exist two pure-strategy equilibria, where in each of them one of the firms chooses high quality and the other chooses low quality. There, the firm with the low-quality product makes a strictly higher expected profit.*

Both in Area II and in Area III we thus have equilibria where firms choose different products. Apart from Area III with quite similar loyal shares where firm 1 (with $\alpha_1 \geq \alpha_2$) may also have low quality in equilibrium, when firms choose different qualities, it is the firm with the larger loyal share of consumers that should have the higher quality. It is important to emphasize that this does not follow from an assumption that a firm with a higher quality can command over greater consumer loyalty. The causality is instead reversed. In fact, taking now the firm with a lower loyal share, it pays for this firm to choose the low-quality product as this provides *relatively* more value for its cost and is thus more profitable when competing for shoppers who are relative thinkers.

**Corollary 3** *Firms with a lower share of loyal consumers are more likely to choose low quality and firms with a higher share more likely to choose high quality. Precisely, supposing still without loss of generality that $\alpha_1 \geq \alpha_2$, we have: i) when there exists an equilibrium where $v_1 = v_L$ then there also exists an equilibrium where $v_2 = v_L$ (while the converse does not hold) and ii) when there exists an equilibrium where $v_2 = v_H$ then there also exists an equilibrium with $v_1 = v_H$ (while the converse does not hold).*

# 5 Concluding Remarks

The focus of this paper is on the strategic implications for product competition when consumers are relative thinkers. As then consumers' consideration set also affects to what extent relative thinking matters, we find that this provides scope for vertical differentiation that would otherwise not exist. Importantly, this generates strategic interactions that would be absent under standard preferences, and we show how this can make firms' product qualities both strategic complements and substitutes. One prediction of our model is that a firm with a smaller segment of captive customers is more likely to position its product at the lower end of the quality spectrum and that it is subsequently more likely to promote its product.

We explicitly model a context with frequent price promotions, in which shoppers are constantly forced to reassess their choices. In markets where promotions are less common and consumers repeatedly face the same set of offerings, relative thinking may be less relevant. The importance of relative thinking, in our model, derives however also from the degree to which consumers shop, as only shoppers compare (new) offers, while loyal consumers are supposed to simply frequent, say, always the same retailer. A side insight of our model is thus that the (strategic) relevance of relative thinking, or possibly also of other behavioral traits, depends crucially on other intermediating factors, such as consumers' shopping habits.

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# 6 Appendix A: Proofs

**Proof of Proposition 1.** The subsequent proof is relatively short as we can rely on well-known arguments from Varian (1980) and Narasimhan (1988). Shoppers strictly prefer firm $i$'s offer over firm $j$'s if and only if $\frac{v_i}{p_i} > \frac{v_j}{p_j}$, that is, $p_j > p_i \frac{v_j}{v_i}$. Consequently, the respective firm profits become

$$\pi_i(p; F_j(\cdot)) = (p - c_i)\left[\alpha_i + (1 - \alpha_i - \alpha_j)\left(1 - F_j\big(p\frac{v_j}{v_i}\big)\right)\right] \tag{13}$$

and

$$\pi_j(p; F_i(\cdot)) = (p - c_j)\left[\alpha_j + (1 - \alpha_i - \alpha_j)\left(1 - F_i\big(p\frac{v_i}{v_j}\big)\right)\right]. \tag{14}$$

Given the respective definition of $F_j(\cdot)$ (for (13)) and $F_i(\cdot)$ (for 14)), we can confirm, first, that firms realize the same profit, $\pi_i$ and $\pi_j$, for all prices in the respective supports; second, that profits are strictly lower for prices outside the respective supports; and, third, that the distribution functions $F_i(\cdot)$ and $F_j(\cdot)$ are indeed well behaved. Note here, in particular, that firm $j$'s mass point at $v_j$ is also well-defined, since $\gamma_j \geq 0$ follows from the assumption that condition (2) holds weakly.

Finally, for the arguments that support uniqueness we can directly refer to Narasimhan (1988), who proved for symmetric qualities $q_i = q_j$ (and standard preferences) that both firms must randomize over convex supports ("no gaps") and that there can be at most one mass point – and if so, only for one firm and then at the upper boundary of its support.[27] These necessary characteristics of any equilibrium then immediately imply the respective characterization. ∎

**Proof of Proposition 2.** This is obvious when noting that firm $i \in \{1, 2\}$'s expected

[27] An application of these arguments can be obtained from the authors upon request.

profit can, irrespective of whether (2) holds or not, always be written as

$$\pi_i(v_i, v_j) = (v_i - c_i)[\alpha_i + (1 - \alpha_i - \alpha_j)\max\{\gamma_j(v_i, v_j), 0\}],$$

where $\gamma_j(v_i, v_j)$ is given by (3). When now $v_L - c_L > v_H - c_H$, such that also $\frac{v_L}{c_L} > \frac{v_H}{c_H}$, $\pi_i(v_L, \cdot) > \pi_i(v_H, \cdot)$ due to $v_L - c_L > v_H - c_H$ and as $\gamma_j(v_L, \cdot) > \gamma_j(v_H, \cdot)$ due to $\frac{v_L}{c_L} > \frac{v_H}{c_H}$. Hence, choosing $v_L$ is a strictly dominant strategy for either firm. When instead $v_H - c_H > v_L - c_L$ and $\frac{v_H}{c_H} \geq \frac{v_L}{c_L}$, it holds that $\pi_i(v_H, \cdot) > \pi_i(v_L, \cdot)$ due to $v_H - c_H > v_L - c_L$ and as $\gamma_j(v_H, \cdot) \geq \gamma_j(v_L, \cdot)$ due to $\frac{v_H}{c_H} \geq \frac{v_L}{c_L}$. Hence, choosing $v_H$ is a strictly dominant strategy for either firm. ■

**Proof of Lemma 1.** Recall for the considered lemma that it is assumed that $v_j = v_L$. When $\alpha_i \geq \alpha_j$, $v_i = v_L$ would imply $\pi_i(v_L) = (v_L - c_L)\alpha_i < (v_H - c_H)\alpha_i \leq \pi_i(v_H)$, where we have now made explicit the dependency on firm $i$'s product choice. Consider next the case $\alpha_i < \alpha_j$, where from $\pi_i = (v_i - c_i)[\alpha_i + (1 - \alpha_i - \alpha_j)\gamma_j]$ and $\gamma_j(v_L, v_L) = 1 - \frac{1-\alpha_j}{1-\alpha_i}$ (compare with (3)) it can easily be established that $\pi_i(v_L) = (v_L - c_L)\frac{\alpha_j(1-\alpha_j)}{1-\alpha_i}$. If $i$ chooses $v_H$, there are two cases to consider. First, consider the case where $i$ is then more aggressive, which holds when $\left(1 - \frac{c_H}{v_H}\right)(1 - \alpha_i) > \left(1 - \frac{c_L}{v_L}\right)(1 - \alpha_j)$ and thus when

$$\alpha_i < 1 - (1 - \alpha_j)\left(\frac{v_L - c_L}{v_H - c_H}\right)\frac{v_H}{v_L} \equiv \check{\alpha}_{i,L}(\alpha_j) < \alpha_j.$$

Then, we we have profits of

$$\begin{aligned}\pi_i(v_H) &= (v_H - c_H)[\alpha_i + (1 - \alpha_i - \alpha_j)\gamma_j(v_H, v_L)] \\ &= (v_H - c_H)(1 - \alpha_j) - (1 - \alpha_i - \alpha_j)\left(v_H - c_L\frac{v_H}{v_L}\right)\frac{1 - \alpha_j}{1 - \alpha_i}.\end{aligned}$$

Second, if $\alpha_i > \check{\alpha}_{i,L}(\alpha_j)$, we have $\pi_i(v_H) = (v_H - c_H)\alpha_i$. We treat both cases in turn.

Case (A): $\alpha_i < \check{\alpha}_{i,L}(\alpha_j)$. Then $i$ strictly prefers $v_L$ over $v_H$ if and only if

$$(v_L - c_L)\frac{\alpha_j(1-\alpha_j)}{1-\alpha_i} > (v_H - c_H)(1-\alpha_j) - (1-\alpha_i-\alpha_j)\left(v_H - c_L\frac{v_H}{v_L}\right)\frac{1-\alpha_j}{1-\alpha_i},$$

which is equivalent to

$$\alpha_i < 1 - \alpha_j\left(\frac{(v_H - v_L)(v_L - c_L)}{c_H v_L - c_L v_H}\right) = \hat{\alpha}_{i,L}(\alpha_j).$$

Hence, if both $\alpha_i < \hat{\alpha}_{i,L}(\alpha_j)$ and $\alpha_i < \check{\alpha}_{i,L}(\alpha_j)$, firm $i$ strictly prefers $v_L$. Comparing these two constraints, the first is stricter if

$$\alpha_j > \frac{c_H v_L - c_L v_H}{(v_H - v_L)(v_L - c_L) + \frac{v_L}{v_H}(c_H v_L - c_L v_H)} = \underline{\alpha}_{j,L}.$$

Hence, case (A) can be split up into two subcases. First, if $\alpha_j > \underline{\alpha}_{j,L}$, firm $i$ strictly prefers $v_L$ over $v_H$ if and only if $\alpha_i < \hat{\alpha}_{i,L}(\alpha_j)$. Second, if $\alpha_j \leq \underline{\alpha}_{j,L}$, firm $i$ strictly prefers $v_L$ over $v_H$ if and only $\alpha_i < \check{\alpha}_{i,L}(\alpha_j)$. Note finally that for $\alpha_j \geq \frac{c_H v_L - c_L v_H}{(v_H - v_L)(v_L - c_L)} = \overline{\alpha}_{j,L} \in (\underline{\alpha}_{j,L}, 1)$, the inequality $\alpha_i < \hat{\alpha}_{i,L}(\alpha_j)$ requires that $\alpha_i < 0$, which can never be satisfied.

Case (B): $\alpha_i \geq \check{\alpha}_{i,L}(\alpha_j)$. Then $i$ strictly prefers $v_L$ over $v_H$ if and only if

$$(v_L - c_L)\frac{\alpha_j(1-\alpha_j)}{1-\alpha_i} > (v_H - c_H)\alpha_i,$$

or

$$g(\alpha_i) \equiv \alpha_i^2 - \alpha_i + \left(\frac{v_L - c_L}{v_H - c_H}\right)\alpha_j(1-\alpha_j) > 0.$$

Note that $g(\alpha_i)$ is strictly convex in $\alpha_i$, with $g(0) > 0$, $g(\alpha_j) < 0$, and $g(1) > 0$. Hence, the critical $\alpha_i$ below which choosing $v_L$ becomes profitable is given by the lower root to

$g(\alpha_i)$, which equals

$$1/2 - \sqrt{1/4 - \alpha_j(1-\alpha_j)\frac{v_L - c_L}{v_H - c_H}} = \tilde{\alpha}_{i,L}(\alpha_j) < \alpha_j.$$

We thus have that for $\alpha_i \in [\breve{\alpha}_{i,L}(\alpha_j), \tilde{\alpha}_{i,L}(\alpha_j))$ firm $i$ strictly prefers $v_L$ over $v_H$ in case (B). This interval is only non-empty if $\tilde{\alpha}_{i,L}(\alpha_j) > \breve{\alpha}_{i,L}(\alpha_j)$, or, after inserting and rearranging,

$$\sqrt{1/4 - \alpha_j(1-\alpha_j)\frac{v_L - c_L}{v_H - c_H}} < (1-\alpha_j)\left(\frac{v_L - c_L}{v_H - c_H}\right)\frac{v_H}{v_L} - 1/2. \tag{15}$$

It is easy to see that the expression under the root is strictly positive, so the LHS is well-defined and strictly positive. If the RHS is not positive, which is true if $\alpha_j \geq 1 - 1/2\left(\frac{v_H - c_H}{v_L - c_L}\right)\frac{v_L}{v_H}$, the inequality cannot be satisfied. Hence, this obtains the requirement that

$$\alpha_j < 1 - 1/2\left(\frac{v_H - c_H}{v_L - c_L}\right)\frac{v_L}{v_H}. \tag{16}$$

If this is true, such that the RHS of inequality (15) is strictly positive, after simplifying expressions, the inequality transforms to

$$\alpha_j < \frac{c_H v_L - c_L v_H}{(v_H - v_L)(v_L - c_L) + \frac{v_L}{v_H}(c_H v_L - c_L v_H)} = \underline{\alpha}_{j,L}. \tag{17}$$

One can see that (17) is stricter than (16), which implies that $\alpha_j < \underline{\alpha}_{j,L}$ is necessary in order for $i$ to strictly prefer $v_L$ in the considered case (B). (Note that this is the same critical value of $\alpha_j$ as in case (A) above.) To sum up, in case (B), firm $i$ strictly prefers to choose $v_L$ over $v_H$ if and only if $\alpha_j < \underline{\alpha}_{j,L}$ and $\alpha_i \in [\breve{\alpha}_{i,L}(\alpha_j), \tilde{\alpha}_{i,L}(\alpha_j))$.

Finally, we can combine cases (A) and (B). If $\alpha_j < \underline{\alpha}_{j,L}$, firm $i$ strictly prefers $v_L$ if $\alpha_i < \breve{\alpha}_{i,L}(\alpha_j)$ (case A) or if $\alpha_i \in [\breve{\alpha}_{i,L}(\alpha_j), \tilde{\alpha}_{i,L}(\alpha_j))$ (case B), i.e., if and only if $\alpha_i < \tilde{\alpha}_{i,L}(\alpha_j)$. If $\alpha_j \in [\underline{\alpha}_{j,L}, \overline{\alpha}_{j,L})$, firm $i$ never finds it optimal to choose $v_L$ in case (B), while it finds it strictly optimal to do so in case (A) if and only if $\alpha_i < \hat{\alpha}_{i,L}(\alpha_j) < \alpha_j$. Lastly, if $\alpha_j \geq \overline{\alpha}_{j,L}$, firm $i$ never finds it optimal to choose $v_L$.

Having characterized the respective parameter regions for which $v_i = v_L$ or $v_i = v_H$ is optimal, we turn to the comparative analysis in firms' loyalty share. Note first that the assertion for $\alpha_i$ follows immediately from the preceding characterization. With respect to the comparative analysis in $\alpha_j$, note first that for $\alpha_j = 0$, $i$'s best response is always to choose $v_H$. Next, note that for $\alpha_j \geq \underline{\alpha}_{j,L}$, the boundary $\hat{\alpha}_{i,L}(\alpha_j)$ is a linearly decreasing function in $\alpha_j$, with $\hat{\alpha}_{i,L}(\overline{\alpha}_{j,L}) = 0$ and $\hat{\alpha}_{i,L}(\underline{\alpha}_{j,L}) = \tilde{\alpha}_{i,L}(\underline{\alpha}_{j,L})$ (as is easy to check). To show that, now for given $\alpha_i$, the respective set of values $\alpha_j$ for which $v_i = v_L$ is optimal is indeed convex, it is sufficient to show that the boundary $\tilde{\alpha}_{i,L}(\alpha_j)$ is strictly quasi-concave (cf. also Figure 1). (Note that it need not be strictly monotonic, which would only be the case when $\alpha_j < 1/2$, which is however not implied by $\alpha_j < \underline{\alpha}_{j,L}$.) To simplify expressions, let $k = \frac{v_L - c_L}{v_H - c_H} \in (0,1)$, such that $\tilde{\alpha}_{i,L}(\alpha_j) = 1/2 - [1/4 - \alpha_j(1-\alpha_j)k]^{1/2}$. Then it is straightforward to establish that $\tilde{\alpha}''_{i,L}(\alpha_j)$ has the same sign as

$$\frac{k(1-2\alpha_j)^2}{1/2 - 2\alpha_j(1-\alpha_j)k} - 2,$$

which can be further simplified to

$$\frac{k-1}{1/2 - 2\alpha_j(1-\alpha_j)k} < 0,$$

where the inequality follows from $k < 1$ and $\alpha_j(1-\alpha_j) \leq 1/4$. Hence, $\tilde{\alpha}_{i,L}(\alpha_j)$ is strictly concave in $\alpha_j$, which completes the proof. ■

**Proof of Lemma 2.** Recall for the considered lemma that it is assumed that $v_j = v_H$. We first consider two separate cases: (A) $\alpha_i < \alpha_j$ and (B) $\alpha_i \geq \alpha_j$.

Case (A): $\alpha_i < \alpha_j$. In this case, firm $i$ will be more aggressive irrespective of whether it chooses $v_H$ or $v_L$. It thus strictly prefers $v_L$ over $v_H$ if and only if

$$\begin{aligned}\pi_i(v_L) &= (v_L - c_L)[\alpha_i + (1-\alpha_i-\alpha_j)\gamma_j(v_L, v_H)] \\ &= (v_L - c_L)\alpha_i + (1-\alpha_i-\alpha_j)\left(v_L - c_H\frac{v_L}{v_H}\right)\left(\frac{1-\frac{c_L}{v_L}}{1-\frac{c_H}{v_H}} - \frac{1-\alpha_j}{1-\alpha_i}\right) \\ &> (v_H - c_H)\frac{\alpha_j(1-\alpha_j)}{1-\alpha_i} = \pi_i(v_H),\end{aligned}$$

which is equivalent to

$$\alpha_i < 1 - \alpha_j\left(\frac{(v_H - v_L)(v_H - c_H)}{c_H v_L - c_L v_H}\right) = \hat{\alpha}_{i,H}(\alpha_j).$$

Hence, if the above inequality holds (together with $\alpha_i < \alpha_j$), then firm $i$ strictly prefers $v_L$ over $v_H$. Put differently, $i$ strictly prefers $v_L$ if $\alpha_i < \min\{\alpha_j, \hat{\alpha}_{i,H}(\alpha_j)\}$. Solving $\alpha_j < \hat{\alpha}_{i,H}(\alpha_j)$ for $\alpha_j$ gives

$$\alpha_j < \frac{c_H v_L - c_L v_H}{(v_H - v_L)(v_H - c_H) + c_H v_L - c_L v_H} = \underline{\alpha}_{j,H} \in (0, 1/2).$$

Thus we can split the case $\alpha_i \leq \alpha_j$ into two subcases. First, if $\alpha_j \leq \underline{\alpha}_{j,H}$, the stricter constraint is given by $\alpha_i < \alpha_j$, such that firm $i$ always strictly prefers $v_L$ over $v_H$ (given that $\alpha_i < \alpha_j$). Second, if $\alpha_j > \underline{\alpha}_{j,H}$, the stricter constraint is given by $\alpha_i < \hat{\alpha}_{i,H}(\alpha_j)$, such that $i$ strictly prefers $v_L$ if and only if $\alpha_i < \hat{\alpha}_{i,H}(\alpha_j)$ (given that $\alpha_i < \alpha_j$). Note moreover that for $\alpha_j \geq \frac{c_H v_L - c_L v_H}{(v_H - v_L)(v_H - c_H)} = \overline{\alpha}_{j,H} \in (\underline{\alpha}_{j,H}, 1)$, the inequality $\alpha_i < \hat{\alpha}_{i,H}(\alpha_j)$ requires that $\alpha_i < 0$, which can never be satisfied.

Case (B): $\alpha_i \geq \alpha_j$. We know that a necessary condition that $v_L$ is preferred over $v_H$ is that firm $i$ becomes more aggressive if it chooses $v_L$ (as otherwise, $\pi_i(v_H) = (v_H - c_H)\alpha_i > \pi_i(v_L) = (v_L - c_L)\alpha_i$). Suppose that this is the case. Then $i$ strictly prefers $v_L$ over $v_H$ if and only if

$$\pi_i(v_L) = (v_L - c_L)\alpha_i + (1-\alpha_i-\alpha_j)\left(v_L - c_H\frac{v_L}{v_H}\right)\left(\frac{1-\frac{c_L}{v_L}}{1-\frac{c_H}{v_H}} - \frac{1-\alpha_j}{1-\alpha_i}\right) > (v_H - c_H)\alpha_i,$$

which transforms to

$$(1-\alpha_j)\left(c_H\frac{v_L}{v_H} - c_L\right) + \left(v_L - c_H\frac{v_L}{v_H}\right)\frac{\alpha_j(1-\alpha_j)}{1-\alpha_i} > (v_H - c_H)\alpha_i$$

and finally

$$f(\alpha_i) \equiv \alpha_i^2 - \alpha_i\left[1 + (1-\alpha_j)\frac{c_H\frac{v_L}{v_H} - c_L}{v_H - c_H}\right] + (1-\alpha_j)\left[\frac{c_H\frac{v_L}{v_H} - c_L}{v_H - c_H} + \alpha_j\frac{v_L}{v_H}\right] > 0.$$

Note that $f(\alpha_i)$ is strictly convex in $\alpha_i$, with $f(0) > 0$ and $f(1) > 0$. Moreover, at the critical $\alpha_i$ below which firm $i$ becomes more aggressive when choosing $v_L$, $\check{\alpha}_{i,H}(\alpha_j) = 1 - \frac{1-\frac{c_H}{v_H}}{1-\frac{c_L}{v_L}}(1-\alpha_j)$, it clearly holds that $f(\check{\alpha}_{i,H}(\alpha_j)) < 0$ (compare with Inequality (6)). Hence, the critical $\alpha_i$ below which choosing $v_L$ becomes profitable is given by the lower root to $f(\alpha_i)$,

$$\alpha_i < \frac{1 + (1-\alpha_j)\left(\frac{c_H\frac{v_L}{v_H} - c_L}{v_H - c_H}\right)}{2} - \sqrt{\left[\frac{1 + (1-\alpha_j)\left(\frac{c_H\frac{v_L}{v_H} - c_L}{v_H - c_H}\right)}{2}\right]^2 - (1-\alpha_j)\left[\frac{c_H\frac{v_L}{v_H} - c_L}{v_H - c_H} + \alpha_j\frac{v_L}{v_H}\right]},$$

which simplifies to

$$\alpha_i < \frac{1 + (1-\alpha_j)\left(\frac{c_H\frac{v_L}{v_H} - c_L}{v_H - c_H}\right)}{2} - \sqrt{\left[\frac{1 - (1-\alpha_j)\left(\frac{c_H\frac{v_L}{v_H} - c_L}{v_H - c_H}\right)}{2}\right]^2 - (1-\alpha_j)\alpha_j\frac{v_L}{v_H}} = \tilde{\alpha}_{i,H}(\alpha_j).$$

To sum up, with $\alpha_i \geq \alpha_j$, firm $i$ strictly prefers $v_L$ over $v_H$ if and only if $\alpha_i \in$

$[\alpha_j, \tilde{\alpha}_{i,H}(\alpha_j))$. This range of $\alpha_i$'s is only non-empty if $\tilde{\alpha}_{i,H}(\alpha_j) > \alpha_j$. Let now

$$z = \frac{1 + (1-\alpha_j)\left(\frac{c_H \frac{v_L}{v_H} - c_L}{v_H - c_H}\right)}{2},$$

so that substituting $z$ in the requirement $\tilde{\alpha}_{i,H}(\alpha_j) > \alpha_j$ transforms this to

$$z - \sqrt{(1-z)^2 - (1-\alpha_j)\alpha_j \frac{v_L}{v_H}} > \alpha_j,$$

or

$$z - \alpha_j > \sqrt{(1-z)^2 - (1-\alpha_j)\alpha_j \frac{v_L}{v_H}}.$$

Since clearly $z > 1/2$ and $\alpha_j < 1/2$ due to $\alpha_i + \alpha_j < 1$ and $\alpha_i \geq \alpha_j$, both sides are strictly positive[28], and we may square both sides of the inequality. It thus has to hold that

$$z^2 - 2z\alpha_j + \alpha_j^2 > (1-z)^2 - (1-\alpha_j)\alpha_j \frac{v_L}{v_H},$$

which, after expanding $(1-z)^2$ and simplifying, becomes

$$2z(1-\alpha_j) > 1 - \alpha_j^2 - (1-\alpha_j)\alpha_j \frac{v_L}{v_H}.$$

Dividing both sides by $1-\alpha_j$ and noting that $1-\alpha_j^2 = (1-\alpha_j)(1+\alpha_j)$, the condition boils down to

$$2z > 1 + \alpha_j - \alpha_j \frac{v_L}{v_H}.$$

[28]To see that the expression under the root, $D \equiv (1-z)^2 - (1-\alpha_j)\alpha_j \frac{v_L}{v_H}$, is non-negative (such that the root is indeed well-defined), observe first that $D$ is strictly increasing in $c_L$. Since we have assumed throughout that $v_H - c_H > v_L - c_L$, at worst it holds that $c_L = v_L - (v_H - c_H)$. Substituting $v_L - (v_H - c_H)$ for $c_L$ in $D$ and simplifying yields $D \geq \left(\frac{\alpha_j - (1-\alpha_j)\frac{v_L}{v_H}}{2}\right)^2 \geq 0$.

Substituting back $z$ and simplifying yields

$$(1-\alpha_j)\left(\frac{c_H\frac{v_L}{v_H}-c_L}{v_H-c_H}\right)>\alpha_j\left(1-\frac{v_L}{v_H}\right),$$

which is linear in $\alpha_j$. Solving the above inequality for $\alpha_j$ subsequently reveals that the interval $[\alpha_j, \tilde{\alpha}_{i,H}(\alpha_j))$ is non-empty if and only if $\alpha_j < \underline{\alpha}_{j,H}$, i.e., the same critical value of $\alpha_j$ as in case (A) above. Summarizing the above results for case (B), if $\alpha_i \geq \alpha_j$, $i$ finds it strictly optimal to choose $v_L$ if and only if $\alpha_j < \underline{\alpha}_{j,H}$ and $\alpha_i \in [\alpha_j, \tilde{\alpha}_{i,H}(\alpha_j))$.

Finally, we can combine cases (A) and (B). If $\alpha_j < \underline{\alpha}_{j,H}$, firm $i$ finds it either optimal to choose $v_L$ if $\alpha_i < \alpha_j$ (case A), or if $\alpha_i \in [\alpha_j, \tilde{\alpha}_{i,H}(\alpha_j))$ (case B). Thus, firm $i$ strictly prefers $v_L$ if and only if $\alpha_i < \tilde{\alpha}_{i,H}(\alpha_j)$. If $\alpha_j \in [\underline{\alpha}_{j,H}, \overline{\alpha}_{j,H})$, firm $i$ never finds it optimal to choose $v_L$ in case (B), while it finds it strictly optimal to do so in case (A) if and only if $\alpha_i < \hat{\alpha}_{i,H}(\alpha_j) < \alpha_j$. Lastly, if $\alpha_j \geq \overline{\alpha}_{j,H}$, firm $i$ never finds it optimal to choose $v_L$.

Having characterized the respective parameter regions for which $v_i = v_L$ or $v_i = v_H$ is optimal, we turn to the comparative analysis in firms' loyalty share. Note first that the assertion for $\alpha_i$ follows immediately from the preceding characterization. With respect to the comparative analysis in $\alpha_j$, note first that for $\alpha_j \geq \underline{\alpha}_{j,H}$ the boundary $\hat{\alpha}_{i,H}(\alpha_j)$ is a linearly decreasing function in $\alpha_j$, with $\hat{\alpha}_{i,H}(\overline{\alpha}_{j,H}) = 0$. Since it also holds that $\tilde{\alpha}_{i,H}(\underline{\alpha}_{j,H}) = \hat{\alpha}_{i,H}(\underline{\alpha}_{j,H})$, in order to show that the respective set of values $\alpha_j$ for which $v_i = v_L$ is optimal is indeed convex, it is sufficient to show that the boundary $\tilde{\alpha}_{i,L}(\alpha_j)$ is strictly quasiconcave (cf. also Figure 1). (Note again that it need not be strictly monotonic.).

To simplify expressions, let $m \equiv (c_H\frac{v_L}{v_H} - c_L)/(v_H - c_H)$, such that

$$\tilde{\alpha}_{i,H}(\alpha_j)=\frac{1+(1-\alpha_j)m}{2}-\sqrt{\frac{(1-m+\alpha_j m)^2}{4}-(1-\alpha_j)\alpha_j\frac{v_L}{v_H}}.$$

Note further that $m \in (0, 1-\frac{v_L}{v_H})$, as follows from the requirements that $v_H - c_H >$

$v_L - c_L$ and $\frac{v_H}{c_H} < \frac{v_L}{c_L}$. Then, it is straightforward to establish that $\tilde{\alpha}''_{i,H}(\alpha_j)$ has the same sign as

$$\frac{\left[\alpha_j\left(\frac{1}{2}m^2 + 2\frac{v_L}{v_H}\right) + \frac{1}{2}m(1-m) - \frac{v_L}{v_H}\right]^2}{(1-m+\alpha_j m)^2 - 4\alpha_j(1-\alpha_j)\frac{v_L}{v_H}} - \frac{1}{2}\left(\frac{1}{2}m^2 + 2\frac{v_L}{v_H}\right).$$

A tedious calculation reveals that the above expression is equal to

$$\frac{v_L}{v_H^2}\left(\frac{v_L - v_H(1-m)}{(1-m+\alpha_j m)^2 - 4\alpha_j(1-\alpha_j)\frac{v_L}{v_H}}\right).$$

The nominator of the fraction in brackets is clearly negative, since $m < 1 - \frac{v_L}{v_H}$. We thus want to show that the denominator of this fraction is strictly positive. For this, note first that the denominator is strictly decreasing in $m$. Since $m < 1 - \frac{v_L}{v_H}$, the denominator is bounded from below by

$$\left[\frac{v_L}{v_H}(1-\alpha_j) + \alpha_j\right]^2 - 4\alpha_j(1-\alpha_j)\frac{v_L}{v_H} = \left[\alpha_j - (1-\alpha_j)\frac{v_L}{v_H}\right]^2 \geq 0.$$

Hence, $\tilde{\alpha}_{i,H}(\alpha_j)$ is indeed concave in $\alpha_j$, which completes the proof. ■

**Proof of Lemma 3.** We denote the respective profits, depending on the choice of products, by $\pi_i(v_i, v_j)$. By definition, from firm $i$'s perspective, product quality is a (weak) strategic complement if and only if

$$\pi_i(v_H, v_H) - \pi_i(v_L, v_H) \geq \pi_i(v_H, v_L) - \pi_i(v_L, v_L). \tag{18}$$

(That is, we consider here the more "standard" expression where we subtract profits when $v_i = v_H$ (high quality) from profits when $v_i = v_L$ (low quality).) We consider the following four cases, which together comprise all possibilities:

Case (A) $\alpha_i \geq \check{\alpha}_{i,H}(\alpha_j) = 1 - (1-\alpha_j)\frac{1-\frac{c_H}{v_H}}{1-\frac{c_L}{v_L}} > \alpha_j$. In this subregion, $\alpha_i$ is so large relative to $\alpha_j$ that firm $i$ is always less aggressive, irrespective of firm $i$'s and $j$'s product choice.

Hence, we have that $\pi_i(v_H, v_H) = \pi_i(v_H, v_L) = (v_H - c_H)\alpha_i$ and $\pi_i(v_L, v_H) = \pi_i(v_L, v_L) = (v_L - c_L)\alpha_i$, from which it trivially follows that the converse of (18) holds weakly.

Case (B) $\alpha_i \in [\alpha_j, \check{\alpha}_{i,H}(\alpha_j))$. In this subregion, $\alpha_i$ is moderately large, such that firm $i$ is more aggressive if and only if $i$ chooses $v_L$ while $j$ chooses $v_H$. Hence,

$$\pi_i(v_H, v_H) = (v_H - c_H)\alpha_i, \pi_i(v_L, v_H) = (v_L - c_L)\alpha_i + (1 - \alpha_i - \alpha_j)(v_H - c_H)\frac{v_L}{v_H}\left(\frac{1 - \frac{c_L}{v_L}}{1 - \frac{c_H}{v_H}} - \frac{1 - \alpha_j}{1 - \alpha_i}\right),$$

and $\pi_i(v_H, v_L) = \pi_i(v_L, v_L) = (v_L - c_L)\alpha_i$, so that the converse of (18) holds weakly when $\pi_i(v_L, v_H) \geq \pi_i(v_L, v_L)$. This follows immediately from $\alpha_i < \check{\alpha}_{i,H}(\alpha_j)$.

Case (C) $\alpha_i < \check{\alpha}_{i,L}(\alpha_j) = 1 - (1 - \alpha_j)\left(\frac{v_L - c_L}{v_H - c_H}\right)\frac{v_H}{v_L} < \alpha_j$. In this subregion, $\alpha_i$ is so low relative to $\alpha_j$ that firm $i$ is always more aggressive, irrespective of firm $i$'s and $j$'s product choice. Hence, we have that $\pi_i(v_H, v_H) = (v_H - c_H)\frac{\alpha_j(1-\alpha_j)}{1-\alpha_i}$,

$$\begin{aligned}
\pi_i(v_L, v_H) &= (v_L - c_L)\alpha_i + (1 - \alpha_i - \alpha_j)(v_H - c_H)\frac{v_L}{v_H}\left(\frac{1 - \frac{c_L}{v_L}}{1 - \frac{c_H}{v_H}} - \frac{1 - \alpha_j}{1 - \alpha_i}\right) \\
&= (1 - \alpha_j)\left(c_H\frac{v_L}{v_H} - c_L\right) + \frac{\alpha_j(1 - \alpha_j)}{1 - \alpha_i}\left(v_L - c_H\frac{v_L}{v_H}\right), \\
\pi_i(v_H, v_L) &= (v_H - c_H)\alpha_i + (1 - \alpha_i - \alpha_j)(v_L - c_L)\frac{v_H}{v_L}\left(\frac{1 - \frac{c_H}{v_H}}{1 - \frac{c_L}{v_L}} - \frac{1 - \alpha_j}{1 - \alpha_i}\right) \\
&= (1 - \alpha_j)\left(c_L\frac{v_H}{v_L} - c_H\right) + \frac{\alpha_j(1 - \alpha_j)}{1 - \alpha_i}\left(v_H - c_L\frac{v_H}{v_L}\right),
\end{aligned}$$

and $\pi_i(v_L, v_L) = (v_L - c_L)\frac{\alpha_j(1-\alpha_j)}{1-\alpha_i}$. We now rewrite (18) as $\pi_i(v_H, v_H) + \pi_i(v_L, v_L) \geq \pi_i(v_H, v_L) + \pi_i(v_L, v_H)$, or

$$\frac{\alpha_j(1 - \alpha_j)}{1 - \alpha_i}\left[(v_H - c_H) + (v_L - c_L)\right] \geq (1 - \alpha_j)\left[c_H\frac{v_L}{v_H} - c_L + c_L\frac{v_H}{v_L} - c_H\right]$$
$$+\frac{\alpha_j(1 - \alpha_j)}{1 - \alpha_i}\left[v_L - c_H\frac{v_L}{v_H} + v_H - c_L\frac{v_H}{v_L}\right].$$

After collecting terms and canceling out $(1-\alpha_j)$, this becomes

$$\frac{\alpha_j}{1-\alpha_i}\left[c_H\frac{v_L}{v_H}-c_L+c_L\frac{v_H}{v_L}-c_H\right]\geq c_H\frac{v_L}{v_H}-c_L+c_L\frac{v_H}{v_L}-c_H.$$

Since $c_H\frac{v_L}{v_H}-c_L+c_L\frac{v_H}{v_L}-c_H<0$ due to $\frac{v_H}{c_H}<\frac{v_L}{c_L}$, this is equivalent to $\alpha_j\leq 1-\alpha_i$, which is indeed satisfied.

Case (D) $\alpha_i\in[\breve{\alpha}_{i,L}(\alpha_j),\alpha_j)$. In this last remaining subregion, $\alpha_i$ is moderately low, such that firm $i$ is less aggressive if and only if $i$ chooses $v_H$ and $j$ chooses $v_L$. Hence, $\pi_i(v_H,v_H)=(v_H-c_H)\frac{\alpha_j(1-\alpha_j)}{1-\alpha_i}$,

$$\pi_i(v_L,v_H)=(1-\alpha_j)\left(c_H\frac{v_L}{v_H}-c_L\right)+\frac{\alpha_j(1-\alpha_j)}{1-\alpha_i}\left(v_L-c_H\frac{v_L}{v_H}\right),$$

$\pi_i(v_H,v_L)=(v_H-c_H)\alpha_i$, and $\pi_i(v_L,v_L)=(v_L-c_L)\frac{\alpha_j(1-\alpha_j)}{1-\alpha_i}$. Now rewriting (18) as $\pi_i(v_H,v_H)+\pi_i(v_L,v_L)\geq\pi_i(v_H,v_L)+\pi_i(v_L,v_H)$, inserting the above profit expressions, multiplying by $1-\alpha_i$, and collecting terms, this holds if and only if

$$h(\alpha_i)\equiv\alpha_i^2-\alpha_i\left[1-(1-\alpha_j)\frac{c_H\frac{v_L}{v_H}-c_L}{v_H-c_H}\right]-(1-\alpha_j)^2\frac{c_H\frac{v_L}{v_H}-c_L}{v_H-c_H}+(1-\alpha_j)\alpha_j\geq 0,$$

i.e., $\alpha_i$ must lie (weakly) outside the roots of the quadratic equation $h(\alpha_i)=0$. The lower root of $h(\alpha_i)$ is given by $\alpha_j-(1-\alpha_j)\left(\frac{c_H\frac{v_L}{v_H}-c_L}{v_H-c_H}\right)\in(\breve{\alpha}_{i,L}(\alpha_j),\alpha_j)$, while the upper root is given by $1-\alpha_j$. To sum up, (18) holds if and only if $\alpha_i\leq\alpha_j-(1-\alpha_j)\left(\frac{c_H\frac{v_L}{v_H}-c_L}{v_H-c_H}\right)$, since $\alpha_i\geq 1-\alpha_j$ falls outside the permissible parameter space. Thus, region (iv) can be split into two further subregions as follows : First, if $\alpha_i\in\left[\breve{\alpha}_{i,L}(\alpha_j),\alpha_j-(1-\alpha_j)\left(\frac{c_H\frac{v_L}{v_H}-c_L}{v_H-c_H}\right)\right]$, where $\alpha_i\leq\alpha_j-(1-\alpha_j)\left(\frac{c_H\frac{v_L}{v_H}-c_L}{v_H-c_H}\right)$ is equivalent to $\alpha_j\geq\frac{\alpha_i+\frac{c_H\frac{v_L}{v_H}-c_L}{v_H-c_H}}{1+\frac{c_H\frac{v_L}{v_H}-c_L}{v_H-c_H}}=\alpha_j^{SC}(\alpha_i)$, then for firm $i$, product quality is a strategic complement; second, if instead $\alpha_i\in[\alpha_j-(1-\alpha_j)\left(\frac{c_H\frac{v_L}{v_H}-c_L}{v_H-c_H}\right),\alpha_j)$, product quality is a strategic substitute.

Combining cases (A)-(D), we have that from firm $i$'s perspective product quality is a (weak) strategic complement (substitute) if and only if $\alpha_j \geq \alpha_j^{SC}(\alpha_i)$ ($\alpha_j \leq \alpha_j^{SC}(\alpha_i)$). ■

**Proof of Proposition 3.** Note first that $v_1 = v_2 = v_L$ can never be an equilibrium as then firm 1's profits, $(v_L - c_L)\alpha_1$ would fall short of its min-max profit of $(v_H - c_H)\alpha_1$. Take next a candidate equilibrium with choices $v_1 = v_2 = v_H$. Then, firm 1 makes an expected profit of $\pi_{1,HH} = (v_H - c_H)\alpha_1$, while firm 2 is more aggressive and makes an expected profit of $\pi_{2,HH} = (v_H - c_H)\frac{\alpha_1(1-\alpha_1)}{1-\alpha_2}$. If firm 2 deviates to $v_L$, it remains more aggressive and makes an expected profit of

$$\begin{aligned}\pi_{2,LH} &= (v_L - c_L)\alpha_2 + (1-\alpha_1-\alpha_2)(v_H - c_H)\frac{v_L}{v_H}\left(\frac{1-\frac{c_L}{v_L}}{1-\frac{c_H}{v_H}} - \frac{1-\alpha_1}{1-\alpha_2}\right)\\ &= (v_L - c_L)(1-\alpha_1) - (1-\alpha_1-\alpha_2)\frac{1-\alpha_1}{1-\alpha_2}\left(v_L - c_H\frac{v_L}{v_H}\right),\end{aligned}$$

so that $\pi_{2,HH} \geq \pi_{2,LH}$ if and only if

$$\alpha_2 \geq 1 - \alpha_1\left(\frac{(v_H - v_L)(v_H - c_H)}{c_H v_L - c_L v_H}\right) = \hat{\alpha}_{2,H}(\alpha_1).$$

Turning to firm 1, a deviation to $v_L$ is profitable only if

$$\pi_{1,LH} = (v_L - c_L)(1-\alpha_2) - (1-\alpha_1-\alpha_2)\frac{1-\alpha_2}{1-\alpha_1}\left(v_L - c_H\frac{v_L}{v_H}\right) > (v_H - c_H)\alpha_1,$$

which, after multiplying both sides with $\frac{1-\alpha_1}{1-\alpha_2}$, is equivalent to

$$(v_H - c_H)\frac{\alpha_1(1-\alpha_1)}{1-\alpha_2} < (v_L - c_L)(1-\alpha_1) - (1-\alpha_1-\alpha_2)\left(v_L - c_H\frac{v_L}{v_H}\right).$$

This requirement is at least as strict as that for firm 2 because the term on the LHS is the same as $\pi_{2,HH}$, while the term on the RHS is not larger than $\pi_{2,LH}$. Hence, $v_1 = v_2 = v_H$ constitutes an equilibrium if and only if $\alpha_2 \geq \hat{\alpha}_{2,H}(\alpha_1)$.

Take next the candidate equilibrium with $v_1 = v_H$ and $v_2 = v_L$. Since we assumed (without loss of generality) that $\alpha_1 \geq \alpha_2$, it is immediate that firm 1 can not profitably deviate to $v_L$. Likewise, from the above existence proof of the equilibrium with $v_1 = v_2 = v_H$, we know that for $v_1 = v_H$, firm 2 finds it profitable to choose $v_L$ if and only if $\alpha_2 \leq \hat{\alpha}_{2,H}(\alpha_1)$. Hence, the equilibrium exists if and only if $\alpha_2 \leq \hat{\alpha}_{2,H}(\alpha_1)$.

Finally, take $v_1 = v_L$ and $v_2 = v_H$. Applying Lemma 2 for $i = 1$ and $j = 2$, thus noting that $\alpha_i \geq \alpha_j$, we know that firm 1 finds it profitable to choose $v_L$ in response to $v_2 = v_H$ if and only if $\alpha_1 \leq \tilde{\alpha}_{1,H}(\alpha_2)$ and $\alpha_2 \leq \underline{\alpha}_{2,H}$. And applying Lemma 1 for $i = 2$ and $j = 1$, now with $\alpha_i \leq \alpha_j$, we know that firm 2 finds it profitable to choose $v_H$ in response to $v_1 = v_L$ if and only if $\alpha_2 \geq \tilde{\alpha}_{2,L}(\alpha_1)$ and $\alpha_1 \in (0, \underline{\alpha}_{1,L}]$, or $\alpha_2 \geq \hat{\alpha}_{2,L}(\alpha_1)$ and $\alpha_1 \in [\underline{\alpha}_{1,L}, \overline{\alpha}_{1,L})$, or $\alpha_1 \geq \overline{\alpha}_{1,L}$ (for which automatically $\alpha_2 \geq \hat{\alpha}_{2,L}(\alpha_1)$). Now the conditions $\alpha_1 \leq \tilde{\alpha}_{1,H}(\alpha_2)$ and $\alpha_1 \geq \alpha_2$ (which can only hold jointly if $\alpha_2 \leq \underline{\alpha}_{2,H}$) imply $\alpha_2 \leq \hat{\alpha}_{2,H}(\alpha_1)$,[29] which in turn implies $\alpha_2 < \hat{\alpha}_{2,L}(\alpha_1)$, as is straightforward to prove. Hence, given the required $\alpha_1 \leq \tilde{\alpha}_{1,H}(\alpha_2)$, the condition $\alpha_2 \geq \hat{\alpha}_{2,L}(\alpha_1)$ cannot be satisfied and thus the considered equilibrium exists if and only if $\alpha_1 \leq \tilde{\alpha}_{1,H}(\alpha_2)$ and $\alpha_2 \geq \tilde{\alpha}_{2,L}(\alpha_1)$. ∎

[29]To see this, note that $\alpha_2 \leq \hat{\alpha}_{2,H}(\alpha_1)$ can be rewritten as $\alpha_1 \leq \breve{\alpha}_1(\alpha_2) \equiv \frac{(1-\alpha_2)(c_H v_L - c_L v_H)}{(v_H - v_L)(v_H - c_H)}$, with $\breve{\alpha}_1(0) > \tilde{\alpha}_{1,H}(0)$ and $\breve{\alpha}_1(\alpha_2) = \tilde{\alpha}_{1,H}(\alpha_2)$ if and only if $\alpha_2 = \underline{\alpha}_{2,H} \in (0, 1)$ or $\alpha_2 = 1$. Hence $\alpha_1 \leq \tilde{\alpha}_{1,H}(\alpha_2)$ implies $\alpha_1 \leq \breve{\alpha}_1(\alpha_2)$ – that is, it implies $\alpha_2 \leq \hat{\alpha}_{2,H}(\alpha_1)$ – for $\alpha_2 \leq \underline{\alpha}_{2,H}$.

# 7 Appendix B: Equilibrium Characterization for the Benchmark (Absolute Thinking)

In this Appendix we provide a full characterization of the equilibrium when consumers have standard preferences (absolute thinking). For given qualities, we now find that firm $i$ is more aggressive than its rival $j$ if it holds that

$$(v_i - c_i)(1 - \alpha_i) > (v_j - c_j)(1 - \alpha_j), \tag{19}$$

while if the converse holds strictly, firm $j$ is more aggressive. The full characterization of the pricing equilibrium is as follows (where we omit the proof, as it follows standard arguments):

**Proposition 4** *Suppose consumers are absolute thinkers and that, without loss of generality, condition (19) holds weakly. Then there exists a unique pricing equilibrium such that firm $i$ randomizes over prices $p_i \in [\underline{\underline{p}}_j + v_i - v_j, v_i)$, while firm $j$ randomizes over prices $p_j \in [\underline{\underline{p}}_j, v_j)$ and chooses $p_j = v_j$ with probability*

$$\gamma_j^{abs} = 1 - \left(\frac{1-\alpha_j}{1-\alpha_i}\right)\left(\frac{v_j - c_j}{v_i - c_i}\right), \tag{20}$$

*Firm profits are given by $\pi_i^{abs} = (v_i - c_i)[\alpha_i + (1 - \alpha_i - \alpha_j)\gamma_j^{abs}]$ and $\pi_j = (v_j - c_j)\alpha_j$. The CDF over $[\underline{p}_i, \overline{p}_i)$ is given by*

$$F_i^{abs}(p) = 1 - \frac{\frac{\pi_j}{p+v_j-v_i-c_j} - \alpha_j}{1-\alpha_i-\alpha_j}$$

*and that over $[\underline{p}_j, \overline{p}_j)$ by*

$$F_j^{abs}(p) = 1 - \frac{\frac{\pi_i^{abs}}{p+v_i-v_j-c_i} - \alpha_i}{1-\alpha_i-\alpha_j}.$$

*Firm $i$ promotes (weakly) more often ($g_i^{abs} \geq g_j^{abs}$), while the (maximum) depth of promo-*

*tions is the same for both firms* ($d_i^{abs} = d_j^{abs} = (v_j - c_j)\left(1 - \frac{\alpha_j}{1-\alpha_i}\right)$*).*

In the main text we also referred briefly to a comparison of promotional strategies with relative thinking and with standard preferences. The following corollary presents a more complete and formal comparison:

**Corollary 4** *Suppose that firm $i$ has a product that offers consumers an absolutely higher value than that of its rival, $v_i > v_j$. Then comparing the two unique equilibrium outcomes with absolute and with relative thinkers (as characterized in Propositions 4 and 1), the following differences emerge: With relative thinkers,*
*i) the higher-value product $i$ is promoted less often and the lower-value product $j$ more often (precisely, $g_i$ decreases and $g_j$ decreases);*
*ii) the maximum depth of promotion of the higher-value product $i$ increases and that of the lower-value product $j$ decreases (precisely, $d_i$ increases and $d_j$ decreases);*
*iii) the profits of firm $i$ with a higher-value product decrease and those of firm $j$ with a lower-value product increase (precisely, $\pi_i$ decreases and $\pi_j$ increases).*

**Proof.** Recall that $v_i > v_j$. There are three possible cases (as the fourth is mathematically not possible):

Case (A): Conditions (19) and (2) hold, such that $i$ is more aggressive with absolute and relative thinkers.
Case (B): The converse of (19) and (2) hold, such that $j$ is more aggressive with absolute and relative thinkers.
Case (C): Condition (19) holds but not (2), such that $i$ is more aggressive with absolute thinkers and $j$ more aggressive with relative thinkers.

We now discuss the three cases in turn, drawing always on the characterization of the unique equilibria with absolute and with relative thinkers in Propositions 4 and 1.

Case (A): Take the high-quality firm $i$, which thus promotes with probability one with absolute and relative thinkers. Its profits are lower with relative thinkers if $\gamma_j < \gamma_j^{abs}$, which easily follows from $v_i > v_j$. Further, its depth of promotions increases if $d_i > d_i^{abs}$, which is directly implied by $v_i > v_j$. Turning to firm $j$, we have that its profits are unchanged, as is its depth of promotions. Its likelihood of promotions is strictly higher with relative thinkers as $\gamma_j < \gamma_j^{abs}$.

Case (B): Now, starting again with firm $i$, $\pi_i$ and $d_i$ are not affected, while the likelihood of promotions is strictly lower with relative thinkers if $\gamma_i > \gamma_i^{abs}$, which follows from $v_i > v_j$. Turning to firm $j$, which promotes with probability one with absolute and relative thinkers, its expected profit increases with relative thinkers as $\gamma_i > \gamma_i^{abs}$. Its depth of promotions decreases with relative thinkers since $d_j^{abs} = (v_i - c_i)\left(1 - \frac{\alpha_i}{1-\alpha_j}\right) < d_j = \frac{v_j}{v_i}(v_i - c_i)\left(1 - \frac{\alpha_i}{1-\alpha_j}\right)$.

Case (C): As now (19) holds but not (2), it is immediate that the likelihood of promotions is (weakly) higher for the high-quality firm $i$ with absolute thinkers (that is, equal to one) than with relative thinkers (that is, weakly below one), while the opposite holds for the low-quality firm $j$. Likewise, profits exceed the min-max profits for firm $i$ only with absolute thinkers and for firm $j$ only with relative thinkers. For the depth of promotions it holds that $d_i^{abs} = (v_j - c_j)\left(1 - \frac{\alpha_j}{1-\alpha_i}\right) < d_i = (v_i - c_i)\left(1 - \frac{\alpha_i}{1-\alpha_j}\right)$ as follows from (19) and $d_j^{abs} = (v_j - c_j)\left(1 - \frac{\alpha_j}{1-\alpha_i}\right) \geq d_j = \frac{v_j}{v_i}(v_i - c_i)\left(1 - \frac{\alpha_i}{1-\alpha_j}\right)$ as follows from the converse of (2). ■

We next turn to the endogenization of qualities. The equilibrium outcome is as follows:

**Proposition 5** *If consumers are absolute thinkers, in every product-choice equilibrium, both firms select (one of) the product(s) which is "absolutely strongest" in the sense that it maximizes the difference $v - c$.*

**Proof of Proposition 5.** This is obvious when noting that firm $i \in \{1,2\}$'s expected profit can, irrespective of whether (19) holds or not, always be written as

$$\pi_i^{abs}(v_i, v_j) = (v_i - c_i)[\alpha_i + (1 - \alpha_i - \alpha_j)\max\{\gamma_j^{abs}(v_i, v_j), 0\}],$$

where $\gamma_j^{abs}(v_i, v_j)$ is given by (20). Since $\gamma_j^{abs}(v_i, v_j)$ is now strictly increasing in $v_i - c_i$, it is a dominant strategy for either firm to choose (one of) the product(s) which maximizes $v - c$. ■

# 8 Appendix C: Extension of the Characterization of Pricing and Promotions with Relative Thinking to More Firms (and Products)

In this Appendix, we show how the equilibrium characterization (of prices and promotions) extends to more than two firms. We thus take now $I > 2$ firms. Recall next that with two firms only, we had to distinguish between two different cases, depending on which firm was more aggressive (i.e., promoted its product more often, as described by condition (2)). Depending on firms' loyal shares and the absolute as well as relative strength of their products, with more firms the number of possible cases increases substantially. Still, the characterization always follows the same logic, which we now illustrate for a particular case. We choose symmetric shares $\alpha_i = \alpha < 1/I$. Without loss of generality, we now suppose that firms are ordered such that the respective ratio $\frac{v_i}{c_i}$ is increasing in $i$.

**Assertion:** *With $I > 2$ firms with symmetric loyalty shares, the following constitutes a pricing equilibrium:*

<u>Case A:</u> *Suppose $\frac{v_I}{c_I} > \frac{v_{I-1}}{c_{I-1}}$ holds strictly. Then i) firms 1 to $I-2$ choose $p_i = v_i$ with probability one, so that they do not promote at all; ii) firm $I$ promotes with probability one and chooses $p \in [\underline{p}_I, v_I)$ according to the CDF*

$$F_I(p) = 1 - \left(\frac{\alpha}{1 - I\alpha}\right)\left(\frac{1 - \frac{p}{v_I}}{\frac{p}{v_I} - \frac{c_{I-1}}{v_{I-1}}}\right);$$

*and iii) firm $I-1$ promotes with probability strictly less than one, as it charges the non-discounted price with probability*

$$\gamma_{I-1} = 1 - \left(\frac{1 - \frac{c_{I-1}}{v_{I-1}}}{1 - \frac{c_I}{v_I}}\right),$$

*and chooses* $p \in [\underline{p}_{I-1}, v_{I-1})$ *according to the CDF*

$$F_{I-1}(p) = 1 - \left(\frac{\alpha}{1-I\alpha}\right)\left(\frac{1-\frac{p}{v_{I-1}}}{\frac{p}{v_{I-1}}-\frac{c_I}{v_I}}\right) - \frac{\frac{c_{I-1}}{v_{I-1}}-\frac{c_I}{v_I}}{\frac{p}{v_{I-1}}-\frac{c_I}{v_I}}.$$

*Case B:* *Suppose that* $\frac{v_I}{c_I} = \frac{v_{I-1}}{c_{I-1}}$. *Then i) firms* $1$ *to* $I-2$ *choose* $p_i = v_i$ *with probability one, so that they do not promote at all and ii) firms* $I$ *and* $I-1$ *promote with probability one and choose promotions as follows: Each* $i \in \{I-1, I\}$ *chooses prices* $p \in [\underline{p}_i, v_i]$ *according to the CDF*

$$F_i(p) = 1 - \left(\frac{\alpha}{1-I\alpha}\right)\left(\frac{v_i-p}{p-c_i}\right).$$

**Proof.** The existence proof extends from the characterization with only two firms as follows. Consider first firms $I-1$ and $I$. Given that firms $1$ to $I-2$ are supposed to choose $v_i$ with probability one and thus do not compete for shoppers, the game between firms $I-1$ and $I$ is essentially that considered with only two firms, albeit there are now only $1 - \sum_{i=1}^{I} \alpha_i = 1 - I\alpha$ shoppers in the market. The characterization for both cases A and B follows from this observation.

We are thus left with assertion i) for firms $1$ to $I-2$, where we need to argue that choosing a strictly lower price is not more profitable. To prove this, note that it is sufficient to show that any firm $k \in \{1, ..., I-2\}$ would not find it profitable to choose a lower price than $v_k$ even if it only needed to beat firm $I$'s price in order to attract the shoppers (ignoring that firm $k$ might still lose vs. firm $I-1$). Then, an upper bound for firm $k$'s deviating profit (for some $p$) is given by

$$(p-c_k)\left[\alpha + (1-I\alpha)\left(1 - F_I\left(p\frac{v_I}{v_k}\right)\right)\right],$$

which, after substituting $F_I(p\frac{v_I}{v_k})$ and simplifying, can be rewritten as

$$\pi_k(p) = \left(\frac{p - c_k}{p\frac{v_{I-1}}{v_k} - c_{I-1}}\right)\alpha(v_{I-1} - c_{I-1}).$$

It is now straightforward to show that the derivative of $\pi_k(p)$ has the same sign as

$$-c_{I-1} + c_k\frac{v_{I-1}}{v_k},$$

which is always non-negative given the way we have ordered firms, i.e., so that the respective ratio $\frac{v_i}{c_i}$ is increasing. Hence, even if firm $k$ only needed to beat firm $I$ in order to attract the shoppers, it would still find it optimal to charge the highest possible price $v_k$. Clearly, this implies that firm $k$ cannot find it profitable to charge a price lower than $v_k$ if it has to compete against both $I$ and $I - 1$, as is the case in the constructed equilibrium.

■