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## Shuttle Diplomacy

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# Shuttle Diplomacy

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# Shuttle Diplomacy


## Abstract

This paper studies the role of a mediator in helping two parties to evaluate the merits of their positions in economic disputes and to facilitate settlement. In practice, mediation often operates through shuttle diplomacy; the mediator goes back and forth between the parties meeting them in a private caucus. We model shuttle diplomacy as a dynamic procedure where the mediator helps each party to gradually discover her costs and benefits of settlement, as well as to re-assess her bargaining position, while also proposing the terms of the deal and arranging temporary claim exchanges. The way the procedure progresses depends on the feedback the mediator receives from the parties during meetings with the parties. We show that shuttle diplomacy allows parties to always achieve an ex-post efficient and equitable (perfectly fair in the case of symmetric value distributions) final settlement. In contrast, this is not possible with a static mediation procedure.



JEL Classification: C72, D47, D82, D86

Keywords: Bargaining, information design, mechanism design, mediation, Persuasion



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In practice, mediation often operates through shuttle diplomacy: the mediator goes back and forth between parties, meeting them in a private caucus. We model shuttle diplomacy as a dynamic procedure where the mediator helps each party to gradually discover her costs and benefits of settlement, as well as to re-assess her bargaining position, while also proposing the terms of the deal and arranging temporary claim exchanges. We show that shuttle diplomacy always allows parties to achieve an ex-post efficient and equitable (perfectly fair with symmetric value distributions) final settlement. In contrast, this is not possible with a static mediation procedure.

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# 1 Introduction

In April of 2019, U.S. District Court Judge Vince Chhabria appointed Ken Feinberg to facilitate a settlement between Bayer and over 13,000 plaintiffs who had alleged that the weed killer Roundup causes non-Hodgkin's lymphoma due to its active ingredient, glyphosate. Feinberg is an expert mediator who had previously mediated many disputes, including settlement of the 9/11 victims fund and BP Deepwater Horizon disaster. Unlike in arbitration, mediators are not allowed to impose a judgment on the parties. In seeking to bring about a mutually-agreed upon resolution, mediators provide two services. First, in communicating with parties they acquire information that is privately known to one party and selectively transmit it to the other party. Second, the mediator (usually a retired judge or well-experienced lawyer) brings her own expertise to bear on the question of case value. The mediation literature often labels these two roles as facilitative versus evaluative mediation styles.[1]

As pointed out by Carbone (2019): "The best mediators will use an approach that draws upon both styles as the needs of the case require." As an evaluator, the mediator helps parties to assess the merits of the case and provides them with reality checks; she engages in "shuttle diplomacy," meeting with each side in a private caucus. Shuttle diplomacy also allows the mediator to act as a facilitator, collecting information from a party and then leaking some of it back to the other side. The dynamic process of mediation has been described as follows:

> "Mediations are in the main assisted negotiations. The mediator goes back and forth with demands and offers (and counter demands and counter offers), while, at the same time, the mediator asks questions and makes comments about the dispute, so that each side can more objectively and realistically consider the facts and think about what might happen if the case were to go to a verdict."[2]

In lawsuit mediation the mediator's expertise and role as an evaluator is especially important if the case involves issues of first impression, like the Roundup litigation; that

[1] See for example Smartsettle (2017): "The mediator who evaluates assumes that the participants want and need her to provide some guidance as to the appropriate grounds for settlement based on law, industry practice or technology – and that she is qualified to give such guidance by virtue of her training, experience, and objectivity .... the facilitative mediator assumes that her principal mission is to clarify and to enhance communication between the parties in order to help them decide what to do."

[2] National Arbitration and Mediation (2019).

is, new legal issues or interpretations brought before a court that have not been addressed before by that court or that court's jurisdiction. There, as part of the settlement, the plaintiffs want Bayer to label Roundup as including a cancer causing ingredient. Yet the EPA has repeatedly found glyphosate to be safe; the conflict between the proposed settlement and the regulator's explicit opposition sits in unchartered legal territory. In issues of first impression, the mediator often has access to information that parties don't have. How so? The mediator has been involved in other mediations, many of which are not in the public domain. She can thus help to determine costs and benefits for the plaintiff's claim – costs and benefits that the parties themselves can't identify without his help. In the Roundup litigation, for example, these costs and benefit relate to the likely outcomes of the litigation process.

In this paper we examine the mediator's role in helping parties to evaluate the merits of the different options and then in facilitating settlement. We consider two parties, who must reach an agreement about an ongoing dispute regarding an economic transaction. For concreteness, we take the dispute to be about the possible transfer of control of an asset from one party to the other. The parties use a mediator to help them discover their costs and benefits of holding the asset (i.e., of the alternative ways of settling the dispute) and to set the compensation, or price, that must be paid if control of the asset changes hands. In short, our mediator is both a mechanism designer and an information designer. We show that common features of how mediation operates in practice – e.g., shuttle diplomacy – allow to minimize the efficiency loss associated with dispute resolution and may also guarantee the fairness of the final outcome.

The starting point of our analysis is that parties have some beliefs, but dot not fully know their values for the ways of settling the dispute. A novelty of our approach to mediation is that the mediator acts as an expert evaluator, who can transmit each party information about the case that she does not have, thus providing her with reality checks. We maintain however the assumption that whatever a party learns about her own value remains her private information. One interpretation is that the mediator privately and independently runs experiments that inform each party about different features of the asset and each party updates her valuation depending on how much she values the different features; any information about the true values of the parties stays private. An alternative interpretation is that each party directly acquires information about her value, but the mediator monitors and enforces the parties' commitment to discover the prescribed – limited – amount of information.

As in the Bayesian persuasion literature (Kamenica and Gentzkow, 2011), we give extended leeway to the mediator about how much the parties discover about their values; we allow revelation of any information that is Bayes consistent with the prior. A task of the mediator is then to set up an information discovery policy: what to say about the case, to whom in a private session and at what point in the negotiations? But, given the parties' private information, the mediator also plays a facilitative role, by designing procedures to extract information from parties regarding what they learnt and proposing them possible trading deals.

By the revelation principle, if the information available to parties over their valuations is exogenous, then static, direct, mechanisms can induce any achievable outcome; there is no scope for using dynamic mechanisms. This is not the case in the environment we consider. We show that restricting attention to static procedures is *with* loss of generality. Ex-post efficiency (and hence maximal ex-ante gains from trade) can always be achieved with a dynamic procedure, characterized by a sequence of information discoveries by parties and of trading prices. In contrast, this is not possible with static procedures. We also show that while a simple, two-stage policy – single-ride mechanism – implements an ex-post efficient outcome, it does so by giving all the gains from trade to one party. It thus fails any elementary fairness criterion. On the other hand, a dynamic procedure allowing for as many back and forth rides between the parties by the mediator both implements an ex-post efficient outcome and also a more equitable division of the gains from trade, given by a fair, exactly equal division when the parties' prior distributions are symmetric. Thus, our findings provide a clear, strong, rationale for shuttle diplomacy.

The value of shuttle diplomacy is that it allows to condition the information that is released to each party, as well as the price posted at any point in time, on the history of feedbacks received from parties about the information they learnt. It also allows parties to exchange temporary claims on the asset. We show that it is not necessary to change the terms at which trade occurs along the process to achieve ex-post efficiency; the mediator may set a posted price at the outset and keep it constant through the whole process. It is sufficient to provide parties with a small amount of information in each diplomacy round and to let them exchange ownership claims on the asset back and forth.

Limiting the information available to parties and letting them exchange ownership claims improves agents' incentives and facilitates trade. At the same time, the presence of many rounds allows to progressively provide parties with a sufficient amount of information to assess whether or not a settlement featuring the transfer of the asset from the seller to the buyer is ex-post efficient, while retaining the symmetry of the information provided to the two parties. The latter feature in turn helps to sustain the fairness of the outcome. The fact that the price at which trade occurs is kept constant through the whole procedure also adds a robustness property to the mechanism and allows to extend the validity of our results to the case where the two parties have some private information regarding their value at the outset of the procedure.

We also characterize the optimal static mechanism, where information is provided simultaneously to each party, together with the terms at which trade may occur. While ex-post efficiency cannot be achieved in this case, we show the optimality of limiting the information provided to the parties, to an even greater extent than in the optimal dynamic mechanism. Some amount of obfuscation allows to increase trade. Even if some ex-post inefficient trades are then completed, some ex-post efficient transactions occur that would not take place under full information and the latter effect prevails.

Lawsuit mediation is one application of the model, but there are many others. Consider for example the informal role of the US in the Camp David resolution of the Egypt/Israel conflict. What should President Carter have revealed to each side and at what point in the negotiation process? President Carter had access to information unknown to both parties (about the US's position, and the views of the broader international community). Informal mediation has a long tradition across different cultures; in China it goes back to Confucianism and recently "top political-legal authorities of the Chinese Communist Party have been promoting mediation as the key to resolving all disputes" (Pissler, 2013). Disputes which formal or informal mediation is commonly used to resolve include: conflict between members of a family or business relationship, conflict in the workplace, conflict arising out of a commercial transaction including cross border trade, real estate matters and personal property. In some of these cases transfer payments, which play an important role in our analysis, are not commonly used, but in many cases they are.

The paper is organized as follows. Section 2 introduces the setting. Section 3 shows that a single-ride mechanism can implement the ex-post efficient outcome, but it requires that all the gains from trade go to one of the two parties. Section 4 considers sequential discoveries and the role of shuttle diplomacy, establishing that it not only allows to achieve an efficient outcome but also, under suitable circumstances, a fair division of the gains from trade. Section 5 examines static procedures and shows that they cannot implement the ex-post efficient outcome. Section 6 discusses the related literature and Section 7 concludes. All the proofs are in the appendix.

# 2 The Setting

We consider two parties who must reach an agreement about an ongoing economic dispute, the possible completion of an outstanding transaction, or an additional service that one of them, the seller, could provide to the other, the buyer. For concreteness, we refer to the dispute regarding a possible transfer of ownership of an asset. Hence, we describe the settlement options to be that the seller retains control, or ownership, of the asset, or that the buyer acquires the asset in exchange for some compensation to the seller. The parties use a mediator to help them discover the costs and benefits of holding the asset (i.e., of the alternative ways of settling the dispute) and to set the compensation, or price, the buyer must pay if the asset is transferred to her. If the asset transfer takes place, we say that the parties trade.

Let $v_B \in [0, 1]$ and $v_S \in [0, 1]$ be the buyer's and seller's value for holding the asset. They are independently and privately drawn from distributions $F_B(v) = \Pr(v_B \leq v)$ and $F_S(v) = \Pr(v_S \leq v)$. We assume that $F_B$ and $F_S$ have no atoms and admit densities $f_B(v) \geq 0$ and $f_S(v) \geq 0$ almost everywhere; the expectations according to these distributions are $\mathbb{E}_{F_B}^{[0,1]}[v_B] = \int_0^1 v dF_B(v)$ and $\mathbb{E}_{F_S}^{[0,1]}[v_S] = \int_0^1 v dF_S(v)$. We will also use the notation $\mathbb{E}_{F_B}^{[\alpha,\beta]}[v_B]$, $\mathbb{E}_{F_S}^{[\alpha,\beta]}[v_S]$ to denote the expectation of values conditional on the values being in an interval $[\alpha, \beta] \subseteq [0, 1]$, and $\mathbb{E}_{F_B,F_S}^{[\alpha,\beta]}[v_B \mid v_B > v_S]$, $\mathbb{E}_{F_B,F_S}^{[\alpha,\beta]}[v_B \mid v_B < v_S]$, $\mathbb{E}_{F_B,F_S}^{[\alpha,\beta]}[v_S \mid v_S > v_B]$, $\mathbb{E}_{F_B,F_S}^{[\alpha,\beta]}[v_S \mid v_S < v_B]$ for the expectation taken over both values conditional on one value being larger than the other.

Prior to the experiments designed by the mediator, neither the buyer, the seller or the mediator have any information regarding the draws from $F_B$ and $F_S$.[3] The mediator

[3] As we argue later, our results extend to the case where values are correlated and, for the shuttle diplomacy mechanism, also to the case where parties have some private information from the outset.

chooses both the asset allocation procedure and the information discovery policy subject to a privacy constraint; the buyer's discovery experiments provide her with information only about her value, and similarly the seller's experiments yield only information about his value. Otherwise, we allow the mediator to use the most general signaling technology; that is, as in the literature on Bayesian persuasion (e.g., see Kamenica and Gentzkow, 2011, and Bergemann and Morris, 2019) the mediator is free to choose any discovery policy that is consistent with the prior distribution. Thus, our results should be viewed as providing an upper bound on the worth of a mediator in dispute resolution.[4]

We can think of the discovery policy and the privacy constraint as describing a situation where the mediator privately and independently informs each of the two parties about features of the environment that allow them to learn about their values from settlement. The way in which each party updates her valuation depends on how much she values the different features and this is the party's private information. As we pointed out in the introduction, an alternative interpretation is that the information is directly acquired by the parties and the mediator monitors and enforces their commitment to obtain the prescribed, typically incomplete, amount of information.

The mediator is benevolent and his goal is to maximize the gains from trade, or ex-post efficiency; that is, ideally the mediator would want control of the asset to be transferred to the buyer if $v_B > v_S$ and the asset to remain in the seller's control if $v_B < v_S$. The mediator is also impartial and ideally would want any reallocation of the asset to take place at a price that equally splits the gains for reallocating its control.[5]

As there will be several rounds in the shuttle diplomacy mechanism we consider, a strong fairness criterion is that at each round the price proposed by the mediator equally splits the reallocation gains as evaluated by the parties at that point, on the basis of the information they have discovered so far (i.e., at the interim stage). More precisely, this criterion requires that in each round $t$ the *fair price* at which the asset can be reallocated is $p_t^F = \frac{V_t^B + V_t^S}{2}$, where $V_t^B$ and $V_t^S$ are the expected values of buyer and seller for holding

[4] In future research, it would be valuable to explore the consequences of the presence of constraints on the information discovery policies available to a mediator.

[5] The Model Standards of Conduct for Mediators (2005) was prepared by the American Arbitration Association, the American Bar Association's Section of Dispute Resolution, and the Association for Conflict Resolution. Impartiality is Standard II: "A. A mediator shall decline a mediation if the mediator cannot conduct it in an impartial manner. Impartiality means freedom from favoritism, bias or prejudice. B. A mediator shall conduct a mediation in an impartial manner and avoid conduct that gives the appearance of partiality."

the asset, conditional on the information available to them at $t$ and on the asset's control being eventually transferred to the buyer.[6]

Such a fairness criterion not only allows to sustain the impartiality of the mediator along the process, which is desirable per se, as argued above, but another advantage is that it facilitates and justifies the parties' commitment to the discovery and allocation mechanism. That is, it helps to ensure that the parties are not tempted to renege their commitment to the mediation mechanism as the procedure unfolds, to try to achieve a more favorable share of the surplus.

# 3 Single-Ride Mechanism

As we shall demonstrate, by allowing information discoveries to flow sequentially, with current discoveries of a party depending on past discoveries of the other party, shuttle diplomacy plays two important roles in mediation. It facilitates communication and the achievement of efficient trading. It also helps reaching an equitable information discovery and division of the gains from trade.

In this section we focus on the first role, by presenting a simple information discovery and pricing mechanism in which the agents make a single discovery sequentially. We show that a single ride between the parties by the mediator is sufficient to guarantee the achievement of an ex-post efficient outcome, but this comes at the cost of giving all the gains from trade to a single agent. True shuttle diplomacy is needed to achieve a more equitable division of the surplus.

In a *single-ride mechanism* the mediator lets one agent observe his value fully in the first round and asks him to report it. In the second round, the other agent's discovery depends on this report; she only observes whether her value is above or below the value reported by the agent informed in the first round. Finally, the mediator posts a price $p$ and trade takes place if both buyer and seller accept to trade at the posted price. In what follows, we consider the case in which the agent informed in the first round is the seller; the analysis when the buyer is the agent informed is analogous.

Formally, let $\widehat{v}_S$ be the value reported by the seller, and $p(\widehat{v}_S)$ the price chosen by

[6] To be precise, this specification of the fair price $p_t^F$ ensures an exactly equal division of the gains from trade among the two parties, evaluated at $t$, when any transaction occurring in rounds later than $t$ also takes place at the price $p_t^F$.

the mediator as a function of the seller's report. It is clear that if the seller sends a truthful report about his value and the price satisfies the inequalities $\widehat{v}_S \leq p(\widehat{v}_S) \leq \mathbb{E}_{F_B}^{[0,1]}[v_B \mid v_B \geq \widehat{v}_S]$, this mechanism implements an ex-post efficient outcome, as both agents find it optimal to trade if and only if the buyer's value is above the seller's value.

The next proposition shows that there exists a unique price function that implements an ex-post efficient outcome. The price function gives all the surplus from trade to the seller, the agent that is fully informed in the first round. Intuitively, it is clearly a strictly dominant continuation strategy for the seller to accept to trade at the posted price, while the buyer is also willing to trade when her value is above the seller's as in that case she obtains the same payoff (equal to zero) both when trade occurs and when it does not occur. Given this, the seller appropriates all the gains from the trade and we then show that it is always optimal for the seller to report sincerely.

**Proposition 1** *Under the single-ride mechanism, with the seller discovering his value first and the buyer discovering whether her value is above or below the value $\widehat{v}^S$ reported by the seller, $p^S\left(\widehat{v}^S\right) = \mathbb{E}_{F_B}^{[0,1]}\left[v^B \mid v^B \geq \widehat{v}^S\right]$ is the unique price function for which it is a perfect Bayesian equilibrium for the seller to report sincerely his value and for buyer and seller to accept to trade at the posted price if and only if it is ex-post efficient to trade.*

Two features of the single-ride mechanism allow the implementation of an ex-post efficient outcome. The first is that it limits the information that is made available to parties, and by so doing it reduces their misreporting opportunities, slackening the incentive constraints. When trade occurs, the party making a discovery as second does not learn her true value, but only if her value is above or below the reported value of the other party. The second feature is that, by linking the value reported by the first party to the discovery made by the second party, the single-ride mechanism correlates the information that is made available to the parties, without violating their privacy constraints, and this also makes it easier to satisfy the incentive constraints. We must stress that this endogenous correlation between the information available to the two parties, which is exploited by shuttle diplomacy to achieve first best efficiency, is quite different to the exogenous correlation among buyers' values exploited by Crémer and McLean (1988) to construct side-betting, Bayesian, trading mechanisms allowing to obtain full surplus extraction by a seller.

The outcome of the single-ride mechanism, while allowing to extract all gains from trade, is extremely lopsided in their distribution. One simple adjustment of the procedure to ensure a more equitable outcome would be to flip a coin to determine which of the two agents is the one to discover her value fully and thus to extract all surplus from trade. While this procedure would guarantee an equal division of the surplus ex-ante, it exposes the agents to maximum uncertainty about the trading price and would not allow to improve the fairness of the mechanism according to the criterion described in the previous section.

In the single-ride mechanism the price is in fact set at maximum distance from the fair price $p^F$. Suppose, for example, a coin has been flipped and the seller has been selected as the party obtaining all the gains from trade: when the seller's value is $v_S$, the trading price is $p = \mathbb{E}^{[0,1]}_{F_B}\left[v^B | v^B \geq v^S\right]$ while the fair trading price would be $p^F = \frac{v_S + \mathbb{E}^{[0,1]}_{F_B}\left[v^B | v^B \geq v^S\right]}{2}$. While trade is voluntary and each party retains the option to refuse to trade, given the information received, the buyer, having nothing to gain, might refuse to stick to her commitment to mediation. Also, with a markedly unequal distribution of the ex-post gains from trade, the interpretation of the evaluative role of the mediator as allowing to enforce the commitment of the parties to acquire limited information is problematic. This is likely to be an important reason why in practice we rarely see mediators using procedures in which only one party stands to gain ex-post.

It is also important to point out another weakness of the single-ride mechanism: the result that it implements an efficient outcome is not robust to the introduction of a small amount of private information that parties may have about their own value, before the start of the mediation procedure. We will discuss this in greater detail at the end of Section 4 when we compare the situation with shuttle diplomacy.

# 4 The Shuttle Diplomacy Mechanism

In this section, we introduce a shuttle diplomacy mechanism where a mediator approaches the parties sequentially and repeatedly, making them discover the outcome of a suitably designed experiment and asking them to report what they learned and whether they are willing to trade at the proposed price. The key difference from the previous section is the word 'repeatedly': each party is faced with a potentially long sequence of discoveries. We show that by so doing the mediator can still implement the ex-post efficient outcome and, in addition, a more equitable division of the interim gains from trade that, under some conditions, coincides with the fair division.

The first main idea behind the construction of the mechanism is that parties should make asynchronous, but symmetric, discoveries in each round. There are several rounds, and in going from round to round the interval of value uncertainty, that is the set of values of the asset that each party continues to view as possible,[7] shrinks from $v_B, v_S \in [0,1]$ at the beginning of round 1, to $v_B, v_S \in [\alpha_{t-1}, \beta_{t-1}]$ at the beginning of round $t$, for $t = 1, 2, ..T$. In every round, each party learns whether or not her value lies near the boundaries of the value uncertainty interval. In step 1 of round $t$, first the buyer learns if her value is near the lower bound $\alpha_{t-1}$, then the seller learns whether his value is near the upper bound $\beta_{t-1}$. In step 2 of round $t$ roles are reversed; first the seller learns if his value is near $\alpha_{t-1}$, then the buyer learns whether her value is near $\beta_{t-1}$. Thus, discoveries are symmetric across the two parties and the interval of uncertainty evolves in the same way for buyer and seller; this is required to guarantee the efficiency of the outcome. The distribution of values over the interval of uncertainty may however be different for the two parties, when the value distributions $F_B$ and $F_S$ are different, which introduces difficulties in achieving a fair outcome.

The second main idea is that the mediator posts a price $p$ and, after discoveries are made at the end of each step in every round, each party must choose whether or not to trade an ownership claim on the asset at this price. If both parties agree to trade, the claim on the asset changes hands at the posted price $p$ and the procedure continues moving to the next step, or round. If instead one party does not agree to trade, the process stops and the final outcome is that the asset remains with the party holding the ownership claim on the asset at the beginning of the round. The price is kept fixed along the process; hence if parties agree they exchange claims on the asset back and forth at the same price $p$ in each round.

The posted price $p$ and the discovery process are such that in all rounds the interval of value uncertainty contains $p$ and, in the final round, this interval coincides with $p$. Setting a constant price at which the claim can be exchanged in each round adds a robustness

[7] As the densities $f_B, f_S$ could be zero in some interval, the distributions $F_B$ and $F_S$ may have different supports. Hence the interval of value uncertainty should be understood as including all - but possibly not only - the values which a party views as having positive density, given the available information.

feature to the mechanism,[8] since the price does not depend on the exact beliefs entertained by parties along the process, in contrast with the single ride mechanism. It may then allow for some private information of parties, as discussed below. Furthermore, it simplifies the mechanism and lowers the computational burden of agents. As we will see, it also facilitates truthful communication after discoveries and helps to ensure the price is equal, or as close as permitted by incentive compatibility, to the fair price.[9]

In the mechanism we consider, holding or not holding the claim on the asset entitles parties with different rights and hence different benefits and costs. The party with the claim can decide to keep the asset by not agreeing to further exchange and hence stopping the discovery procedure. In contrast, the party without the claim can stop the discovery process, walking away without the asset. As the posted price remains constant at each round, when parties exchange the ownership claim on the asset back and forth during the mediation process all that matters to them is the final allocation arising when the process terminates. The claim on the asset during the process of back and forth exchange can be interpreted as an agreed, temporary, asset allocation.

### The Mechanism

The mechanism is composed by Round 0, when the mediator selects the mechanism parameters, and then a number of rounds of discovery and claim exchange.

ROUND 0. The mediator selects:

- The maximum number $T$ of round of value discovery and claim exchange of the procedure.
- A posted price $p$ at which claim exchange may take place in each round $t$, $1 \leq t \leq T$.
- A collection of lower value discovery intervals: $\{[\alpha_{t-1}, \alpha_t)\}_{t=1}^{T}$ with $\alpha_0 = 0$, $\alpha_t > \alpha_{t-1}$ and $\alpha_T = p$.
- A collection of upper value discovery intervals: $\{(\beta_t, \beta_{t-1}]\}_{t=1}^{T}$ with $\beta_0 = 1$, $\beta_t < \beta_{t-1}$, and $\beta_T = p$.

[8] On robust mechanism design, see Bergemann and Morris (2005).

[9] Recall, in addition, that only with a constant price the specification of the fair price in Section 2 induces an exactly equal division of gains from trades among parties.

Round $t$, Step 1. At the beginning of this step the seller has a claim on the asset.

- The buyer discovers whether her value is in the interval $[\alpha_{t-1}, \alpha_t)$ and decides next whether or not she is willing to acquire the claim on the asset at price $p$.
- If the buyer decides not to acquire the claim, then the procedure stops and the seller retains the asset.
- If the buyer is willing to acquire the claim, then the seller discovers whether his value is in the interval $(\beta_t, \beta_{t-1}]$ and subsequently decides whether or not he is also willing to transfer the claim on the asset to the buyer at price $p$.
- If the seller decides not to transfer the claim, then the procedure stops and the seller retains the asset.
- If the seller is willing to transfer the claim on the asset, then the transfer takes place at price $p$.

Round $t$, Step 2. At the beginning of this step the buyer has a claim on the asset.

- The seller discovers whether his value is in the interval $[\alpha_{t-1}, \alpha_t)$ and subsequently decides whether or not he is willing to buy back the claim on the asset at price $p$.
- If the seller decides not to buy back the claim, the procedure stops and the transfer of the asset to the buyer is finalized.
- If the seller wishes to acquire the claim, the buyer discovers whether her value lies in the interval $(\beta_t, \beta_{t-1}]$ and decides next whether she is willing to transfer the claim on the asset to the seller at price $p$.
- If the buyer decides not to transfer back the claim, the procedure stops and the buyer ends up with the asset.
- If the buyer is willing to transfer back the claim on the asset, then the transfer takes place at price $p$ and the procedure moves to round $t + 1$.

Note that the discovery process is such that, if at the beginning of each round $t$ no party has yet discovered her value, then the interval of value uncertainty is the same for the two parties and equal to $[\alpha_{t-1}, \beta_{t-1}]$. Define $d\alpha_t = \alpha_t - \alpha_{t-1} > 0$ and $d\beta_t = \beta_t - \beta_{t-1} < 0$, as the size of the information discovery intervals at round $t$. We will focus on the case in which $d\alpha_t$ and $d\beta_t$ are infinitesimally "small" and the number of rounds $T$ is thus infinitely "large".

**Stop-at-value Strategies**

In the shuttle diplomacy mechanism we have described, each party must choose a strategy specifying her claim exchange decisions at each step of every round, or equivalently the decision to stop or continue the mechanism.

We introduce next a strategy under which a party stops the mechanism only if she has discovered her value. More specifically, at each step of every round, the agent holding a claim on the asset elects not to exchange the claim (hence stopping the mediation procedure) if and only if she discovers that her value lies in the discovery interval $(\beta_t, \beta_{t-1}]$. Similarly, the agent who does not have a claim on the asset chooses not to exchange the claim (and so to stop the mediation procedure) if and only if she discovers that her value is in the discovery interval $[\alpha_{t-1}, \alpha_t)$.

We call this strategy the *stop-at-value strategy*, because the agent stops the procedure and refuses to exchange the claim at round $t$ when, and only when, she has just "approximately" (i.e., with maximum errors $d\alpha_t$ and $d\beta_t$) discovered her value. If both agents follow the stop-at-value strategy and none of them has stopped the mechanism, then at the beginning of round $t$ they both know that their values lie in the interval $[\alpha_{t-1}, \beta_{t-1}]$. Thus, as the interval $[\alpha_t, \beta_t]$ converges to $[\alpha_T, \beta_T] = [p, p]$, if claim exchange never stops until the last round, agents perfectly learn that both their values are equal to $p$.

Efficiency under stop-at-value strategies

If both agents use the stop-at-value strategies, then the outcome of the shuttle diplomacy mechanism is approximately ex-post efficient. An inefficient outcome can only arise in two circumstances, both occurring in step 1 of each round $t$. First, the buyer stops the claim exchange, as she discovers that her value is in the interval $[\alpha_{t-1}, \alpha_t)$; hence the seller retains the asset, but his value is also in that interval and is lower. The maximal size of the efficiency loss is then equal to the interval size $d\alpha_t$, and, given that $F_B$ and $F_S$ are atomless, its probability is $[F_B(\alpha_{t-1} + d\alpha_t) - F_B(\alpha_{t-1})][F_S(\alpha_{t-1} + d\alpha_t) - F_S(\alpha_{t-1})]$, which for small $d\alpha_t$ is approximately equal to $f_B(\alpha_{t-1})f_S(\alpha_{t-1})(d\alpha_t)^2$.

Second, the seller stops the procedure and retains the asset as she discovers that his value is in the interval $[\beta_t, \beta_{t-1})$, but the yet undiscovered buyer's value is also in that interval and is higher. The maximal size of the efficiency loss in this second case is again given by the size of the interval, $-d\beta_t$, and its probability by $[F_S(\beta_{t-1}) - F_S(\beta_{t-1} + d\beta_t)][F_B(\beta_{t-1}) - F_B(\beta_{t-1} + d\beta_t)]$, which for small $d\beta_t$ is approximately equal to $f_B(\beta_{t-1})f_S(\beta_{t-1})(d\beta_t)^2$. It is then immediate that the efficiency loss when agents use stop-at-value strategies converges to zero as $d\alpha_t$ and $d\beta_t$ converge to zero for all $t$.

Given the (approximate) ex-post efficiency of the shuttle diplomacy mechanism under the stop-at-value strategies, and hence the fact that trade of the asset to the buyer is finalized in round $t$ or in subsequent rounds if and only if it is ex-post efficient, i.e., $v_B > v_S$, the value of the fair price at each round $t$ is:

$$p_t^F = \frac{1}{2} \cdot \mathbb{E}_{F_B,F_S}^{[\alpha_{t-1},\beta_{t-1}]}[v_B \mid v_B > v_S] + \frac{1}{2} \cdot \mathbb{E}_{F_B,F_S}^{[\alpha_{t-1},\beta_{t-1}]}[v_S \mid v_B > v_S], \tag{1}$$

where the expectations are taken over both $v_B$ and $v_S$ conditional on $v_B > v_S$, and the parties' expected gains from trade are evaluated with the information available to them at the beginning of round $t$.[10]

Incentive compatibility under stop-at-value strategies

We show first that when an agent "approximately" discovers her value (i.e., she discovers that her value is in the small interval discovered in step 1 or 2 of some round $t$), stopping the mechanism is a weakly dominant behavioural strategy. Consider the party that does not have a claim on the asset (the buyer in step 1, the seller in step 2); if she discovers her value, she discover that it is below the fixed price $p$. Stopping implies that she ends up not owning the asset and not paying anything for it; if she does not stop and acquires a claim for the asset, then she might end up owning the asset having paid a price $p$, which is higher than her value. Similarly for the party that has a claim on the asset; if she discovers her value, she discover that it is above the fixed price $p$. Stopping implies that she ends up owning the asset; if she does not stop and trades away the claim, then she might end up without the asset, having sold it at a price $p$ below her value.

Thus, to prove that adopting the stop-at-value strategies constitutes an equilibrium

[10]Since discoveries in round $t$ are small, the expected gains from trade are approximately the same when evaluated at the beginning and the end of round $t$.

of the mechanism, it only remains to show that for no agent there exists a round $t$ and a step $j \in \{1, 2\}$, at which she prefers to stop and end the procedure even though she has not discovered her value, assuming the other agent follows the stop-at-value strategy.

Consider first the buyer in step 1 of round $t$. Suppose she has not yet discovered her value. Since she does not have a claim on the asset, if she deviates from the stop-at-value strategy and stops, then her continuation payoff is zero. If she follows the stop-at-value strategy, then, as established in Lemma 1 in the appendix, for sufficiently small discovery intervals $\{d\alpha_t, d\beta_t\}_{t=1}^{T}$ her continuation payoff is approximately equal to the expected benefit of gaining the asset at price $p$ whenever trade is ex-post efficient. Letting $\Pr{}_{F_B,F_S}^{[\alpha_{t-1},\beta_{t-1}]}(v_B > v_S)$ be the probability that the buyer's value is greater than the seller's value, conditional on both values being in the interval $[\alpha_{t-1}, \beta_{t-1}]$, this continuation value is equal to:

$$\left(\mathbb{E}_{F_B,F_S}^{[\alpha_{t-1},\beta_{t-1}]}\big[v_B \,|\, v_B > v_S\big] - p\right) \cdot \Pr{}_{F_B,F_S}^{[\alpha_{t-1},\beta_{t-1}]}(v_B > v_S) \tag{2}$$

It follows that for the buyer in step 1 of round $t$ to want to follow the stop-at-value strategy it is necessary and sufficient that the following incentive constraint holds:

$$p < \mathbb{E}_{F_B,F_S}^{[\alpha_{t-1},\beta_{t-1}]}\big[v_B \,|\, v_B > v_S\big]. \tag{3}$$

The seller in step 2 of round $t$ is in a similar position of the buyer in step 1, as he does not have a claim on the asset. We can thus conclude that for the seller in step 2 of round $t$, the incentive constraint for the stop-at-value strategy to be a best response is:

$$p < \mathbb{E}_{F_B,F_S}^{[\alpha_{t-1},\beta_{t-1}]}\big[v_S \,|\, v_S > v_B\big] \tag{4}$$

Now consider the seller in step 1 of round $t$. Suppose he has not discovered his value. Since he has a claim on the asset, if he deviates from the stop-at-value strategy and stops, then his continuation payoff is

$$\begin{aligned} \mathbb{E}_{F_S}^{[\alpha_{t-1},\beta_{t-1}]}\big[v_S\big] \;&=\; \mathbb{E}_{F_B,F_S}^{[\alpha_{t-1},\beta_{t-1}]}\big[v_S \,|\, v_S > v_B\big] \cdot \Pr(v_S > v_B) \\ &+\; \mathbb{E}_{F_B,F_S}^{[\alpha_{t-1},\beta_{t-1}]}\big[v_S \,|\, v_S < v_B\big] \cdot \big[1 - \Pr(v_S > v_B)\big] \end{aligned}$$

If instead he follows the stop-at-value strategy, then, as we also show in Lemma 1 in the appendix, for small discovery intervals $\{d\alpha_t, d\beta_t\}_{t=1}^{T}$ his continuation payoff is approximately:

$$\mathbb{E}_{F_B,F_S}^{[\alpha_{t-1},\beta_{t-1}]}\big[v_S \,|\, v_S > v_B\big] \cdot \Pr(v_S > v_B) + p \cdot \big[1 - \Pr(v_S > v_B)\big] \tag{5}$$

Hence in step 1 of round $t$ the seller wants to follow the stop-at-value strategy if and only if the following condition holds:

$$p > \mathbb{E}_{F_B,F_S}^{[\alpha_{t-1},\beta_{t-1}]}\big[v_S \,|\, v_S < v_B\big] \tag{6}$$

The buyer in step 2 of round $t$ is in a similar position as the seller in step 1, since she does have a claim on the asset. Hence, for the buyer in step 2 of round $t$ to want to follow the stop-at-value strategy it is necessary and sufficient that the following constraint holds:

$$p > \mathbb{E}_{F_B,F_S}^{[\alpha_{t-1},\beta_{t-1}]}\big[v_B \,|\, v_B < v_S\big] \tag{7}$$

The incentive constraints (3), (4), (6), and (7) can be rewritten more compactly as:

$$\mathbb{E}_{F_B,F_S}^{[\alpha_{t-1},\beta_{t-1}]}\left[v_B \,|\, v_B < v_S\right] < p < \mathbb{E}_{F_B,F_S}^{[\alpha_{t-1},\beta_{t-1}]}\left[v_B \,|\, v_B > v_S\right] \tag{8}$$

$$\mathbb{E}_{F_B,F_S}^{[\alpha_{t-1},\beta_{t-1}]}\left[v_S \,|\, v_S < v_B\right] < p < \mathbb{E}_{F_B,F_S}^{[\alpha_{t-1},\beta_{t-1}]}\left[v_S \,|\, v_S > v_B\right] \tag{9}$$

and must hold for all $t \in \{1, ..., T\}$. It thus follows from our previous findings that if the mediator can find a mechanism characterized by sufficiently small discovery intervals and a price $p$ satisfying (8), (9), then the mechanism allows to attain an ex-post efficient allocation. If in addition $p$ is equal to $p_t^F$, specified in (1), for all $t$, then we say that trade occurs at a fair price.

**An Example: Uniform Prior Distributions**

To see the shuttle diplomacy mechanism in action in the clearest way, we now investigate the special case in which the prior value distributions of both parties are uniform. If $F_B$ and $F_S$ are uniform, then (8) and (9) reduce to the following, single condition:

$$\int_{\alpha_{t-1}}^{\beta_{t-1}} \frac{v(\beta_{t-1} - v)}{\int_{\alpha_{t-1}}^{\beta_{t-1}} (\beta_{t-1} - v)\, dv} dv < p < \int_{\alpha_{t-1}}^{\beta_{t-1}} \frac{v(v - \alpha_{t-1})}{\int_{\alpha_{t-1}}^{\beta_{t-1}} (v - \alpha_{t-1})\, dv} dv, \tag{10}$$

for all $t \in \{1, ..., T\}$.

Proposition 2 establishes that equal sized discovery intervals satisfy the incentive constraints (10) for an interval of possible values of the price $p$. Moreover, this interval includes the fair price at all $t$, thus at each round interim gains from trade can be equally divided between the two parties.

**Proposition 2** *Suppose the prior value distributions $F_B$ and $F_S$ are uniform. Set $\alpha_t = \frac{t}{T}p$ and $\beta_t = 1 - \frac{t}{T}(1-p)$. Then, as $T \to \infty$ buyer and seller following a stop-at-value strategy is a perfect Bayesian equilibrium of the shuttle diplomacy mechanism if and only if the posted price $p$ satisfies:*

$$\frac{1}{3} < p < \frac{2}{3}.$$

*Furthermore, $p = 1/2$ is the fair price in all rounds $t$.*

As we shall see, we cannot guarantee that fairness (i.e., the existence of a price that is fair in all rounds) is satisfied for all value distributions $F_B$ and $F_S$. A sufficient condition for a fair price to exist is that parties have the same distribution (i.e., $F_B = F_S$).

### General Prior Distributions

We are now ready to analyze the case of general distributions $F_B, F_S$. We begin by establishing some preliminary results about the conditional expectations that appear in the incentive constraints (8) and (9). It is convenient here to use $i, j \in \{B, S\}$, with $j \neq i$, to denote the two agents. Thus, the value of agent $i \in \{B, S\}$ is $v_i$, and her distributions in $[0, 1]$ is $F_i$, with density $f_i$.

The first claim is that, conditional on $v_j < v_i$ and both values belonging to the same interval $[\alpha, \beta]$, the expectation of $v_i$ is greater than the expectation of $v_j$.

**Claim 1** *For all $[\alpha, \beta] \subseteq [0, 1]$ and $i, j \in \{B, S\}$, $i \neq j$,*

$$\mathbb{E}^{[\alpha,\beta]}_{F_B,F_S}[v_j \,|\, v_j < v_i] < \mathbb{E}^{[\alpha,\beta]}_{F_B,F_S}[v_i \,|\, v_j < v_i]$$

The second claim states that the expectation of $v_j$ conditional on $v_j < v_i$ is smaller than the expectation of $v_j$ conditional on $v_j > v_i$.

**Claim 2** *For all $[\alpha, \beta] \subseteq [0, 1]$ and $i, j \in \{B, S\}$, $i \neq j$,*

$$\mathbb{E}^{[\alpha,\beta]}_{F_B,F_S}[v_j \,|\, v_j < v_i] < \mathbb{E}^{[\alpha,\beta]}_{F_B,F_S}[v_j \,|\, v_j > v_i]$$

The next claim is an important step towards the proof that the parameters of the shuttle diplomacy mechanism – discovery intervals and trading price – can be selected so that the perfect Bayesian equilibrium outcome of the mechanism is ex-post efficient. The claim states that for any given interval $[\alpha, \beta]$ we can find some prices (which may depend on the interval), which satisfy the incentive constraints (8) and (9) for $[\alpha_{t-1}, \beta_{t-1}] = [\alpha, \beta]$. Claim 3 follows directly from Claims 1 and 2, as they imply that each expectation appearing in the left hand side of the inequality in Claim 3 is smaller than each expectation on the right hand side.

**Claim 3** *For all* $[\alpha, \beta] \subseteq [0, 1]$,

$$\max\left\{\mathbb{E}^{[\alpha,\beta]}_{F_B,F_S}[v_B \mid v_B < v_S], \mathbb{E}^{[\alpha,\beta]}_{F_B,F_S}[v_S \mid v_S < v_B]\right\} < \min\left\{\mathbb{E}^{[\alpha,\beta]}_{F_B,F_S}[v_B \mid v_B > v_S], \mathbb{E}^{[\alpha,\beta]}_{F_B,F_S}[v_S \mid v_S > v_B]\right\}$$

We are now ready to show that there exist a posted price and infinitesimal discovery intervals under which the shuttle diplomacy mechanism satisfies the incentive compatibility constraints in all rounds, thus ensuring that the equilibrium outcome is ex-post efficient. By Claim 3, to satisfy incentive compatibility at the beginning of the shuttle diplomacy mechanism, when the interval of uncertainty is $[0, 1]$, the posted price $p$ must satisfy the following condition:

$$\max\left\{\mathbb{E}^{[0,1]}_{F_B,F_S}[v_B|v_B < v_S], \mathbb{E}^{[0,1]}_{F_B,F_S}[v_S|v_S < v_B]\right\} < p < \min\left\{\mathbb{E}^{[0,1]}_{F_B,F_S}[v_B|v_B > v_S], \mathbb{E}^{[0,1]}_{F_B,F_S}[v_S|v_S > v_B]\right\} \tag{11}$$

Proposition 3 below then shows that once the mediator has selected one such a posted price, she can also select a sequence of intervals of uncertainty $[\alpha_t, \beta_t]_{t=1}^{\infty}$, of decreasing size, with $\alpha_0 = 0$, $\beta_0 = 1$, $\alpha_\infty = \beta_\infty = p$, (i.e., a discovery policy) such that the posted price $p$ satisfies the incentive compatibility inequalities (8) and (9) for all intervals of uncertainty.[11]

**Proposition 3** *For all posted prices satisfying (11) there exist an increasing sequence* $\{\alpha_t\}_{t=1}^{\infty}$*, with* $\alpha_0 = 0$ *and* $\alpha_\infty = p$*, and a decreasing sequence* $\{\beta_t\}_{t=1}^{\infty}$*, with* $\beta_0 = 1$ *and* $\beta_\infty = p$*, defining a sequence of infinitesimal discovery intervals and associated intervals of remaining uncertainty, such that buyer and seller following the stop-at-value strategy*

[11]Note that this ex-post efficiency result, as well as the one for the single ride mechanism, holds under no condition on the support of the value distributions, which could then be different for buyer and seller.

*is a perfect Bayesian equilibrium of the shuttle diplomacy mechanism and implements an ex-post efficient outcome.*

Note the result requires no condition on the distributions of values for seller and buyer, besides the assumed property that there are no atoms. Hence they may also feature different support.

We now provide some further intuitive explanation for why a shuttle diplomacy mechanism implements an efficient outcome. We can interpret stopping the procedure by an agent as reporting to have discovered her value. The stop-at-value strategy can thus be seen as truthful reporting. The shuttle diplomacy mechanism shares two similarities with the single-ride mechanism, that help implementing an ex-post efficient outcome. It limits the information that parties obtain and, by stopping the discovery of an agent when the other agent reports to have discovered her value, it correlates the information that is made available to them.[12] These features reduce parties' misreporting opportunities and slacken their incentive constraints.

Two important, distinguishing features of the shuttle diplomacy mechanism are the slow information discovery over time and the repeated exchange of the claim on the asset. These features also influence parties' incentives to truthfully report their private information, as they further limit the available lies and affect the payoff from terminating mediation. Consider for example the buyer in step 1 of any round $t$. She discovers whether or not her value $v_B$ is in the interval $[\alpha_{t-1}, \alpha_t]$. If it is, she can only "falsely report" a higher value by not stopping the mechanism, but this is clearly a weakly dominated strategy as the posted price is higher than her value. If instead she does not discover her value, she can only falsely report a value lower than her true $v_B$ by stopping the mechanism. Most importantly, slow information discovery and repeated exchange of the claim on the asset play a key role in facilitating trading at a fair (or fairer) price, as they permit discoveries to be symmetric across parties and put the parties in a symmetric market position, with each of them alternating between having and not having a claim on the asset.

Fair price

The remaining question is, can the posted price $p$ be always set equal to the fair price specified in (1)? For general value distributions, the answer is no. There are two reasons.

[12] In general, the procedure terminates before reaching the final round $T$ with only one party discovering her value and the interval of uncertainty of the other party lying above - or below - that value.

First, the fairness condition at the beginning of the mechanism requires the posted price to be equal to

$$p_1^F = \frac{1}{2} \cdot \mathbb{E}_{F_B,F_S}^{[0,1]}[v_B \,|\, v_B > v_S] + \frac{1}{2} \cdot \mathbb{E}_{F_B,F_S}^{[0,1]}[v_S \,|\, v_S < v_B],$$

but for some distributions $F_B, F_S$ this value of $p_1^F$ lies outside the interval defined by (11) and thus does not satisfy the incentive constraints for the stop-at-value strategies to be an equilibrium. To illustrate this possibility, consider the case where $F_B$ has almost all mass concentrated in a small interval around $v_B = \frac{1}{2}$ and $F_S$ has almost all mass equally concentrated in two small intervals around $v_S = 0$ and $v_S = 1$. Then:

$$\mathbb{E}_{F_B,F_S}^{[0,1]}[v_B \,|\, v_B < v_S] \approx \mathbb{E}_{F_B,F_S}^{[0,1]}[v_B \,|\, v_B > v_S] \approx \frac{1}{2}\,;$$

$$\mathbb{E}_{F_B,F_S}^{[0,1]}[v_S \,|\, v_S < v_B] \approx 0 \qquad \text{and} \qquad \mathbb{E}_{F_B,F_S}^{[0,1]}[v_S \,|\, v_S > v_B] \approx 1.$$

It follows that the fair price is $p_1^F \approx \frac{1}{4}$ and violates condition (11), which in this case becomes $p \approx \frac{1}{2}$. Also, if we swap distributions, so that $F_S$ has almost all mass around $v_S = \frac{1}{2}$ and $F_B$ has almost all mass around $v_B = 0$ and $v_B = 1$, the fair price is $p_1^F = \frac{3}{4}$, while (11) requires $p \approx \frac{1}{2}$.

Second, if $p_1^F$ belongs to the interval defined by (11), by Proposition 3 we can construct discovery intervals such that $p_1^F$ satisfies the incentive compatibility conditions in all rounds, inducing buyer and seller to follow the stop-at-value strategy. However, we cannot be sure that the intervals of uncertainty associated with these discovery intervals are such that $p_1^F$ remains a fair price at all rounds; fairness may be violated at some round $t > 1$.

An important special case where we can set the posted price $p$ equal to the fair price $p_1^F$ and make sure that it is also a fair price in all subsequent rounds $t > 1$ is the symmetric environment, where the prior distributions of valuations are the same for the two agents; that is, when $F_B = F_S = F$. This is because with identical distributions the two expectations on the left hand side and the two on the right hand side of inequality (11) coincide; that is, $\mathbb{E}_{F,F}^{[0,1]}[v_S \,|\, v_S < v_B] = \mathbb{E}_{F,F}^{[0,1]}[v_B \,|\, v_B < v_S]$ and $\mathbb{E}_{F,F}^{[0,1]}[v_S \,|\, v_S > v_B] = \mathbb{E}_{F,F}^{[0,1]}[v_B \,|\, v_B > v_S]$. Furthermore, we have:

$$\begin{aligned} p_1^F &= \frac{1}{2} \cdot \mathbb{E}_{F,F}^{[0,1]}[v_B \,|\, v_B > v_S] + \frac{1}{2} \cdot \mathbb{E}_{F,F}^{[0,1]}[v_S \,|\, v_B > v_S] \\ &= \mathbb{E}_{F,F}^{[0,1]}[v_B \,|\, v_B > v_S] \cdot \Pr\{v_B > v_S\} + \mathbb{E}_{F,F}^{[0,1]}[v_B \,|\, v_S > v_B] \cdot \Pr\{v_S > v_B\} \\ &= \mathbb{E}_{F}^{[0,1]}[v]. \end{aligned}$$

Since: $\mathbb{E}_{F,F}^{[0,1]}[v_B \,|\, v_B < v_S] < \mathbb{E}_{F}^{[0,1]}[v_B] < \mathbb{E}_{F,F}^{[0,1]}[v_B \,|\, v_B > v_S]$, by setting $p = p_1^F$ condition (11) is satisfied. Moreover, the intervals of uncertainty $[\alpha_t, \beta_t]$ can be chosen so that, for all rounds $t$, we have $\mathbb{E}_{F}^{[\alpha_t,\beta_t]}[v] = \mathbb{E}_{F}^{[0,1]}[v]$.

Generally speaking, what makes it impossible to set the posted price equal to the fair price is the presence of large asymmetries in the prior distributions of buyer and seller. We formalize the result for symmetric prior uncertainty in the following proposition.

**Proposition 4** *With symmetric prior uncertainty, i.e., $F_B = F_S = F$, there always exists a sequence of intervals of remaining uncertainty, defined by an increasing sequence $\{\alpha_t\}_{t=1}^{\infty}$, with $\alpha_0 = 0$, $\alpha_\infty = p_1^F$, and a decreasing sequence $\{\beta_t\}_{t=1}^{\infty}$, with $\beta_0 = 1$, $\beta_\infty = p_1^F$, such that: (i) the posted price $p_1^F = \mathbb{E}_{F}^{[0,1]}[v]$ is fair, and (ii) it is a perfect Bayesian equilibrium of the shuttle diplomacy mechanism for buyer and seller to follow the stop-at-value strategy.*

Correlation of values

We have assumed that the buyer and seller's values are independently drawn, but in several applications it seems reasonable to allow them to be correlated. For example, when applying the model to litigation, part of the plaintiff's cost of selling a legal claim (i.e., settling) and part of the defendant's value of buying the legal claim could be correlated, as both the defendant's and the plaintiff's benefit of reaching an agreement relative to going to court depend on the settlement decision that the court will impose if mediation fails. The court's decision in turn depends on characteristics of the case which are relevant for both parties and over which an expert mediator may shed light.

One could then assume that the values of buyer and seller are drawn from an atomless joint distribution $F(v_B, v_S)$ on $[0,1]^2$ with atomless conditional distributions $F(v_B|v_S = v)$ and $F(v_S|v_B = v)$ with support $[0,1]$ and non negative conditional densities, for all $v \in [0,1]$. It is then possible to show that Claims 1–3 still hold for this joint distribution $F(v_B, v_S)$ and then Proposition 3 holds as well. Thus, our results extend to the case of correlated values and, in particular, the ex-post efficient outcome can be implemented without resorting to the side-betting mechanisms of Crémer and McLean (1988).

Private information before mediation

It is well known since Myerson and Satterthwaite (1983) that, if parties had full private information about their values, no procedure could implement the ex-post efficient outcome. Indeed, the procedure which we have studied, with a single posted price $p$, would end up with the asset being traded to the buyer if and only if $v_S < p < v_B$. We have assumed so far that the two parties have no private information before the start of the mediation procedure, but in some practical disputes it is likely they have partial information. The important question is thus whether the shuttle diplomacy mechanism is robust to some amount of private information. The answer is Yes.

To see this, assume that the buyer privately observes a signal $\theta_B \in [0,1]$ about her own value and the seller a signal $\theta_S \in [0,1]$ about his value, before the start of the procedure. Then let $F_B(\cdot \,|\, \theta_B)$ be the distribution of the buyer's value $v_B$ conditional on the buyer having received private signal $\theta_B$. Similarly, let $F_S(\cdot \,|\, \theta_S)$ be the distribution of the seller's value $v_S$ conditional on the seller having received private signal $\theta_S$. A special case is the situation studied so far, where the distributions of values are independent from the signals $\theta_B, \theta_S$. At the other extreme is the case where signals are fully informative, with the distributions putting all mass on $v_i$ being equal to $\theta_i$, $i \in \{B, S\}$.

Following the same approach as the one followed above for the case of no private information, we can derive the incentive constraints that guarantee that the stop-at-value strategies are an equilibrium when parties have private information. They are analogous to constraints (8) and (9) obtained for the case of no private information. Let $\mathbb{E}^{[\alpha,\beta]}_{F_B(\cdot\,|\,\theta_B),F_S}\left[v_B \,|\, v_B < v_S\,;\theta_B\right]$ be the expectation of $v_B$ taken over the values $v_B, v_S$, conditional on the values being in the interval $[\alpha, \beta]$, on $v_B < v_S$, and on the signal realization of the buyer being $\theta_B$. Similarly define $\mathbb{E}^{[\alpha,\beta]}_{F_B,F_S(\cdot\,|\,\theta_s)}\left[v_S \,|\, v_S > v_B\,;\theta_S\right]$ and the analogous expectations conditional on $v_B > v_S$. The incentive constraints can then be rewritten as:

$$\mathbb{E}^{[\alpha_{t-1},\beta_{t-1}]}_{F_B(\cdot\,|\,\theta_B),F_S}\left[v_B \,|\, v_B < v_S\,;\theta_B\right] < p < \mathbb{E}^{[\alpha_{t-1},\beta_{t-1}]}_{F_B(\cdot\,|\,\theta_B),F_S}\left[v_B \,|\, v_B > v_S\,;\theta_B\right] \tag{12}$$

$$\mathbb{E}^{[\alpha_{t-1},\beta_{t-1}]}_{F_B,F_S(\cdot\,|\,\theta_S)}\left[v_S \,|\, v_S < v_B\,;\theta_S\right] < p < \mathbb{E}^{[\alpha_{t-1},\beta_{t-1}]}_{F_B,F_S(\cdot\,|\,\theta_S)}\left[v_S \,|\, v_S > v_B\,;\theta_S\right] \tag{13}$$

and must hold for all $t \in \{1, ..., T\}$ and all $\theta_B, \theta_S \in [0,1]$.

It is immediate to see that Claim 2 generalizes to this environment in the sense that, for all $\theta_j$, $i,j \in \{B,S\}$ it is the case that:

$$\mathbb{E}^{[\alpha,\beta]}_{F_j(\cdot\,|\,\theta_j),F_i}[v_j \,|\, v_j < v_i \,;\, \theta_j] < \mathbb{E}^{[\alpha,\beta]}_{F_j(\cdot\,|\,\theta_j),F_i}[v_j \,|\, v_j > v_i \,;\, \theta_j].$$

Claim 1 (and hence Claim 3), on the other hand, need not generalize; that is, it needs not be the case that for all $\theta_j, \theta_i$, $i,j \in \{B,S\}$:

$$\mathbb{E}^{[\alpha,\beta]}_{F_j(\cdot\,|\,\theta_j),F_i}[v_j \,|\, v_j < v_i \,;\, \theta_j] < \mathbb{E}^{[\alpha,\beta]}_{F_j,F_i(\cdot\,|\,\theta_i)}[v_i \,|\, v_j < v_i \,;\, \theta_i].$$

The reason is that the conditional distributions used to evaluate the expectation on the right and the left are different. In general, Claim 1 fails to generalize if the influence of private information is "strong". For example, in the extreme case where the signals received by each party are fully informative about their values (i.e., $v_i = \theta_i$, $i \in \{B,S\}$, the inequality above becomes $\theta_j < \theta_i$, which cannot hold for all $\theta_i, \theta_j \in [0,1]$. More generally, when the conditional distributions are quite sensitive to the signal realizations the incentive constraints cannot be satisfied by some price $p$ for all $\theta_B, \theta_S$. On the contrary, if the influence of private information is "not too strong", then the conditional expectations do not vary much with $\theta_B$ and $\theta_S$ and we can find a value of $p$ such that the incentive constraints are satisfied for all $\theta_B$ and $\theta_S$. In such a case the result in Proposition 3 extends to this environment with private information; it is possible to find a discovery policy (i.e., intervals of uncertainty) under which the shuttle diplomacy mechanism implements an efficient outcome.

Consider for example the case in which $v_i = (1-\lambda)\omega_i + \lambda\theta_i$, with $\lambda \in [0,1]$, $\omega_i$ and $\theta_i$ uniformly distributed in $[0,1]$, with $\omega_i$ initially unknown and $\theta_i$ known by agent $i \in \{B,S\}$. Letting $U_i$ be the (uniform) distribution of $\omega_i$ and $F_i$ be the distribution of $v_i = (1-\lambda)\omega_i + \theta_i$, the incentive constraints (12) and (13) reduce to the following condition, which must hold for any $i \neq j \in \{B,S\}$ and for all $\theta_i, \theta_j$:

$$\lambda\theta_i + (1-\lambda)\mathbb{E}^{[\alpha_{t-1},\beta_{t-1}]}_{U_i,F_j}\left[\omega_i \,|\, \lambda\theta_i + (1-\lambda)\omega_i < v_j\right] < p < \lambda\theta_j + (1-\lambda)\mathbb{E}^{[\alpha_{t-1},\beta_{t-1}]}_{U_j,F_i}\left[\omega_j \,|\, \lambda\theta_j + (1-\lambda)\omega_j > v_i\right]$$

Taking $\theta_i = 1$ and $\theta_j = 0$, the condition reduces to

$$\lambda + (1-\lambda)\mathbb{E}^{[\alpha_{t-1},\beta_{t-1}]}_{U_i,F_j}\left[\omega_i \,|\, \lambda + (1-\lambda)\omega_i < v_j\right] < p < (1-\lambda)\mathbb{E}^{[\alpha_{t-1},\beta_{t-1}]}_{U_j,F_i}\left[\omega_j \,|\, (1-\lambda)\omega_j > v_i\right], \tag{14}$$

which can be satisfied if and only if $\lambda$ is below some upper bound $\lambda^*$.

Contrast this finding with the case of the single-ride mechanism. The need to induce the seller to report his true value $v_S$ after he has discovered it uniquely pins down the trading price in this kind of mechanism which allows to achieve ex-post efficiency; it must be set so that the seller extracts all expected surplus from trade. That is, it must be $p(v_S) = \mathbb{E}^{[0,1]}_{F_B}[v_B \,|\, v_B > v_S]$. But a buyer with a signal realization $\theta_B$ such that $\mathbb{E}^{[0,1]}_{F_B(\cdot\,|\,\theta_B)}[v_B \,|\, v_B > v_S\,; \theta_B] < \mathbb{E}^{[0,1]}_{F_B}[v_B \,|\, v_B > v_S]$ will not want to trade at $p(v_S)$; hence the ex-post efficient outcome can no longer be implemented by a single-ride mechanism. Contrary to the case of shuttle diplomacy, the implementation of an efficient outcome under single-ride diplomacy is not robust to the introduction of even a small amount of private information prior to the start of mediation.

# 5 Static Mechanisms

In this section, we examine the case in which the information discovery and exchange mechanism is static: the mediator simultaneously chooses the distribution of the signals received by buyer and seller, as well as the price $p$. Buyer and seller, after observing the realization of their own signal, decide whether or not they wish to trade the asset at price $p$.

As we shall see, an ex-post efficient outcome cannot be implemented by the static mechanisms outlined above. We will also argue that the impossibility result extends to general, static, Bayesian mechanisms. Hence, in the present environment considering only static mechanisms is restrictive. To achieve efficiency the mediator must use dynamic mechanisms, featuring sequences of information discoveries and subsequent trade opportunities, like the ones we have analyzed in Section 4. The analysis of this section also shows that the optimal static mechanism limits the amount of information discovered by the parties, as in the case of the shuttle diplomacy mechanism, but to an even greater extent.

Since ex-post efficiency is not attainable, contrary to the dynamic case, to find the optimal static mechanism we must consider all possible discovery policies. They can be summarized by the induced posterior probability distributions of each agent's expected value. In other words, to keep track of the information discovered by agent $i \in \{B, S\}$, it is sufficient to consider the induced posterior distribution of $i$'s expected value. The situation in which $i$ does not discover any information corresponds to the case in which, from the point of view of the mediator (and the other party), the posterior distribution of agent $i$'s expected value has an atom of mass one on $\mathbb{E}_{F_i}[v_i]$, while the situation in which $i$ has fully discovered her value corresponds to the case in which the posterior distribution of the expected value is $F_i$, the true distribution from which the value is drawn. Intermediate discovery policies must lead to distributions of the expected value $\widetilde{F}$ that are mean preserving spreads of the distribution with unit mass on $\mathbb{E}_{F_i}[v_i]$ and such that the true value distribution $F_i$ is a mean preserving spread of $\widetilde{F}$. Thus, for agent $i \in \{B, S\}$, the family of signal distributions over her expected value that can be feasibly induced by the mediator is:[13]

$$\mathcal{F}_i = \left\{ \widetilde{F} : \int_0^1 v d\widetilde{F}(v) = \mathbb{E}_{F_i}^{[0,1]}[v_i] \text{ and } \int_0^z \widetilde{F}(v)dv \leq \int_0^z F_i(v)dv \text{ for all } z \in [0,1] \right\}$$

Because of the possibility of atoms, when it comes to the buyer it is sometimes convenient to work with the reliability function $R_{\widetilde{F}}(v) = \Pr(v_B \geq v)$ instead of the distribution of signals $\widetilde{F}(v)$; if $R_{\widetilde{F}}$ is the reliability associated with the distribution $\widetilde{F}$, then $R_{\widetilde{F}}(v) = 1 - \widetilde{F}(v) + \mathbf{1}_{\widetilde{F}}(v)$, where $\mathbf{1}_{\widetilde{F}}(v)$ is the probability mass on $v_B = v$.

We can now state formally the mediator's problem in the present environment. The mediator chooses a *static discovery and trading mechanism* $\left\langle \widetilde{F}_B, \widetilde{F}_S, p \right\rangle$; that is, signal distributions $\widetilde{F}_B \in \mathcal{F}_B$ and $\widetilde{F}_S \in \mathcal{F}_S$ for buyer and seller, together with a trading price $p \in \mathbb{R}_+$. After the buyer and seller receive their signals from $\widetilde{F}_B$ and $\widetilde{F}_S$, respectively, each of them decides whether or not to trade at $p$. It is immediate that it is a dominant strategy for the buyer to accept to trade if and only if the signal is an expected value $v_B \geq p$ and for the seller to do so if and only if the signal received is $v_S \leq p$. Thus, since the mediator's goal is to maximize the gains from trade, the optimal static discovery and trading mechanism is obtained as a solution of the following problem:

$$\max_{p \in [0,1],\, \widetilde{F}_B \in \mathcal{F}_B,\, \widetilde{F}_S \in \mathcal{F}_S} \int_p^1 \int_0^p (v^B - v^S) d\widetilde{F}_S(v^S) d\widetilde{F}_B(v^B)$$

or, equivalently:

[13] As the signal distributions could be continuous or discrete, all the integrals in the paper should be understood as Stieltjes integrals.

$$\max_{p\in[0,1],\,\widetilde{F}_B\in\mathcal{F}_B,\,\widetilde{F}_S\in\mathcal{F}_S} \left(\mathbb{E}^{[0,1]}_{\widetilde{F}_B}[v_B \,|\, v_B \geq p] - \mathbb{E}^{[0,1]}_{\widetilde{F}_S}[v_S \,|\, v_S \leq p]\right) R_{\widetilde{F}_B}(p)\widetilde{F}_S(p) \tag{15}$$

Lemma 2 in the appendix shows that it is sufficient for the mediator to select a discovery policy that only has two realizations, a high and a low signal. This is because each party faces a binary decision, to accept or reject trade at the price posted by the mediator. Building on this, we can show that the solution of the mediator problem has a simple, binary and monotone, partition structure.[14] A threshold is chosen for each trader, who then discovers whether her value is above or below the threshold. Buyer types above the buyer's threshold and seller types below the seller's threshold will want to trade, while the other types will refuse to trade. The optimal binary partitions that constitute the signals for seller and buyer are characterized in the next proposition.

**Proposition 5** *In the static information discovery and trading mechanism that maximizes the gains from trade, the buyer observes whether her value is strictly below some threshold $x$ and the seller observes whether her value is strictly above some other threshold $y$, with $x, y$ satisfying:*

$$\mathbb{E}^{[0,1]}_{F_S}[v_S \,|\, v_S \leq y] = x \qquad \textit{and} \qquad \mathbb{E}^{[0,1]}_{F_B}[v_B \,|\, v_B \geq x] = y. \tag{16}$$

*The trading price can be any $p \in \left[\mathbb{E}^{[0,1]}_{F_S}[v_S \,|\, v_S \leq y], \mathbb{E}^{[0,1]}_{F_B}[v_B \,|\, v_B \geq x]\right] = [x, y]$. Thus, in particular, the mediator can choose the fair price $p^F = \; = \frac{x+y}{2}$.*

It is immediate to verify that system (16) always admits a solution for $x, y$ and that $x < y$.[15] At the optimal mechanism, trade occurs when the buyer's valuation is above $x$ and the seller's valuation below $y$. Since $x < y$, trade may occur when the buyer's value is below the seller's value. This is an important difference relative to the case of a static mechanism providing full information to the parties, or also to the optimal shuttle

[14] This is reminiscent of the result in Bergemann and Pesendorfer (2007), that monotone partitions are the optimal static information structures for a revenue maximizing seller in an auction setting.

[15] Since $F_B, F_S$ have no atoms, $\mathbb{E}^{[0,1]}_{F_S}[v_S \,|\, v_S \leq y] < y$ for all $y > 0$ and is continuous and strictly increasing in $y$, while $\mathbb{E}^{[0,1]}_{F_B}[v_B \,|\, v_B \geq x] > x$ for all $x < 1$ and is continuous and strictly increasing in $x$. Hence if a solution exists, we have $x < y$. To show existence define the following function of $x$ with domain and range $[0, 1]$: $\mathbb{E}^{[0,1]}_{F_S}\left[v_S \,|\, v_S \leq \mathbb{E}^{[0,1]}_{F_B}[v_B \,|\, v_B \geq x]\right]$. Since it is the composite function of two continuous functions, it is continuous. By Brouwer's fixed point theorem it has a fixed point $x^*$ and thus $x^*$ and $y^* = \mathbb{E}^{[0,1]}_{F_B}[v_B \,|\, v_B \geq x^*]$ is a solution of (16). Note that multiple solutions may exist, in which case one of them is the optimum.

diplomacy mechanism we have characterized in Section 4. Under the latter, trade is ex-post efficient. Under full information, only buyers with a value above the posted price $p$ and sellers with a value below $p$ trade.

The reason why a full information discovery policy is not optimal is that it does not generate enough trade: any efficient trade with either $(i)$ $p > v_B > v_S$, or $(ii)$ $v_B > v_S > p$ is lost. By limiting the information available to parties, the optimal static information discovery and trading mechanism guarantees completion of a higher volume of trades. Some of the most valuable trades which are lost under full information – those with $(i)$ $v_B = p - \varepsilon_B$ and $v_S = \varepsilon_S$ and those with $(ii)$ $v_B = 1 - \varepsilon_B$ and $v_S = p + \varepsilon_S$ (for $\varepsilon_B, \varepsilon_S$ small) – take place under the optimal mechanism.[16] Inducing completion of valuable trades in the optimal mechanism comes at a cost: some inefficient trades are also completed (this never happens when parties are fully informed), but those are the ones that have smaller losses; trades with $v_S = y - \varepsilon_S > v_B = x + \varepsilon_B$. Moreover, there are also some less valuable, but still efficient, trades that are not completed in the optimal mechanism (e.g., those for which $x > v_B > v_S$ or $v_B > v_S > y$).

The above result and discussion show, also in the case of a static mechanism, the benefits of limiting the information available to parties in order to increase their willingness to trade. At the same time, with a binary partition structure the information provided to parties is too coarse to ensure that all gains from trade are realized; that is, to ensure that trade occurs only and in all the situations where it is ex-post efficient.

Note that the freedom the mediator has in the choice of the price allows her to pursue the additional goal of a fair division of the gains from trade. A 50-50 split of the interim expected surplus between buyer and seller can now always be obtained by posting the fair price $p = \frac{x+y}{2}$. The difference with respect to the case of the optimal dynamic, shuttle diplomacy mechanism should be pointed out. With the latter, the mediator can ensure that the posted price equally splits the interim gains from trade every time the agents trade only for some (e.g., under symmetry, $F_B = F_S$), but not all distributions of values $F_B$ and $F_S$. The fairness requirement is clearly more stringent in a dynamic mechanism, as we ask that it be satisfied in all rounds where trade may occur.

When $F_B$ and $F_S$ are uniform, the solution of the optimal discovery policy we obtain from (16) is $x = 1/3$ and $y = 2/3$; the buyer observes whether her value is above or

[16] This can be seen most clearly when $x < p < y$.

below 1/3, while the seller observes whether her cost is above or below 2/3. Any price $p \in [1/3, 2/3]$ is then an optimal trading price. The expected gains from trade that are realized in the optimal static discovery and trading mechanism are $\left(\frac{2}{3} - \frac{1}{3}\right)\frac{2}{3}\frac{2}{3} = \frac{4}{27}$ or 89% of the ex-post efficient, or first best, level achieved by the shuttle diplomacy mechanism, which is $\frac{1}{6}$. As an additional comparison, when agents are fully informed about their own values, the optimal posted price is $p = \frac{1}{2}$, which yields expected gains from trade of $\left(\frac{3}{4} - \frac{1}{4}\right)\frac{1}{2}\frac{1}{2} = \frac{1}{8}$, or 75% of the first best level. The increase in the gains from trade of the optimal static mechanism relative to full information is due to an increase in the volume of trade. Trade occurs whenever the buyer has a value greater than 1/3 and the seller smaller than 2/3, instead of when the two are, respectively, greater and smaller than 1/2.

In the optimal static information discovery and exchange mechanism, trade occurs whenever its expected benefit exceeds the cost; incentive constraints do not bind. If we retain the static feature of the information discovery of the two parties, but allow for Bayesian mechanisms, where the terms of trade may depend on agents' reporting on the information they have discovered, we can allow for richer patterns of trade. In that case incentives clearly constrain the trading mechanism, which then depends on the information disclosed to the parties. One may wonder whether ex-post efficiency may be achieved by suitably designing the information made available to the parties, while retaining the static property of the information discovery process, if we allow for Bayesian trading mechanisms. The answer is no. The reason is that, when the parties' signal distributions are chosen simultaneously, to ensure that trade occurs if and only if it is efficient the parties must essentially be fully informed, in which case, as is well known (Myerson and Satterthwaite, 1983), ex-post efficiency is not attainable.[17,18]

[17]In the case of $F_B$ and $F_S$ uniform discussed in the previous paragraph, at the optimal Bayesian mechanism when traders are fully informed about their own value trade takes place whenever $v_B \geq v_S + 1/4$ and expected gains from trade are 9/64, or 84% of the first best level (see Chatterjee and Samuelson, 1983, and Myerson and Satterthwaite, 1983). Note this value is also lower than the one we found at the optimal static discovery and exchange mechanism.

[18] Characterizing the optimal static information discovery and Bayesian trading mechanism is a difficult task. Schottmüller (2021) provides a closed form solution for the case in which the prior type distribution is binary. For more general type distributions he is able to show that the optimal information structure is a monotone partition of the type space and that the optimal mechanism is deterministic.

# 6 Related Literature

In the law and economics models that study the role of mediation (e.g., Brown and Ayres, 1994, Doornik, 2014, and Goltsman et al., 2009), each party is assumed to have full information about her own value at the outset of the process. The mediator thus only plays a facilitative role, collecting information from both parties and conveying it strategically via a proposed trading deal, which allows parties to update their view of their bargaining position. Brown and Ayres (1994) argue that mediators reduce adverse selection by committing parties to simple mechanisms; e.g., "(1) by committing parties to break off negotiations when private representations to a mediator indicate that there are no gains from trade; (2) by committing parties to equally divide the gains from trade; and (3) by committing to send noisy translations of information disclosed during private caucuses." Doornik (2014) argues that the role of a mediator is to avoid a costly trial by verifying the private information of an informed party and communicating it to the other party, without disclosing confidential details that would disadvantage the informed party. Goltsman et al. (2009) studies mediation in a cheap talk framework and shows that by adding noise a mediator may relax the incentive compatibility constraint of the informed party and thus facilitate information transmission. While insightful, these models cannot explain either the evaluative role of the mediator, or the benefits of the mediator engaging in shuttle diplomacy.

Two recent papers by Fanning (2021a,b) also focus on the facilitative role of mediation, in the framework of the dynamic, reputational bargaining model of Abreu and Gul (2000). In this model each of the two parties is privately informed about whether she is rational, or a commitment type who demands a fixed share of the surplus and does not accept less. Without a mediator, Abreu and Gul (2000) show that there is a unique equilibrium, similar to the equilibrium in the war of attrition. Fanning (2021a) then shows that an uninformed mediator may help the parties to achieve a better outcome by collecting private messages from them at the beginning of the game, and revealing with positive but less than one probability whether parties have communicated their willingness to compromise. Fanning (2021b) extends this result by taking a mechanism design approach and looking at the optimal mechanism with an uninformed mediator. He shows that noisy communication remains an important feature of the optimal mechanism.

The Bayesian persuasion literature, initiated by Kamenica and Gentzkov (2011) (see also Rayo and Segal, 2010, Kolotilin et al., 2017 and Li and Norman, 2021) and recently reviewed by Bergemann and Morris (2019), studies how the release of appropriately chosen information may allow a principal to incentivize an agent to behave in the desired way. Information design in dynamic settings with a sequence of information releases has then been studied, among others, in Ely et al. (2015), Ely (2017), Ely and Szydlowski (2020), Ball (2019), Orlov et al. (2020) and Zhao, Mezzetti, Renou and Tomala (2021). As the information designer in this literature, our mediator is able to commit to an information structure that maps states of the world (a party's value) into stochastic signals privately disclosed (to each party) and not observed by the mediator. In addition, and also in common with this literature, our mediator knows the prior value distributions and may use all feasible information discovery policies. But in contrast to the information designer of the persuasion literature, who cannot affect the outcomes available to the players, and like the designer in the classic mechanism design literature (e.g., see Myerson and Satterthwaite, 1983), our mediator may also affect outcomes, by setting the price at which parties may trade. Thus, our mediator plays both the role of an information designer in the persuasion literature, and the classical designer in the mechanism design literature (see Mezzetti, 2019, for a brief discussion on this dual role of a designer).

This feature is shared by our paper with the literature on information discovery and surplus extraction by the seller of a single item, where the seller chooses both an information and a sale policy (Bergemann and Pesendorfer, 2007, Esö and Szentes, 2007, Li and Shi, 2017 and Krähmer, 2020).[19] There are several differences, however, apart from the objective of our mediator being to maximize welfare. First, in our paper there is private information on both sides of the market and an information structure must be chosen for both. In contrast, in the papers cited above there is no uncertainty regarding the seller's value but there could be several buyers and the focus is then on the information of each

[19]The combination of information and mechanism design is also present in Roesler and Szentes (2017) and Condorelli and Szentes (2020), but in those papers it is the buyer that selects her own information structure so as to protect herself from the seller's choice of a sale mechanism aiming to maximize his surplus.

of them about their own value.[20] Second, we focus on simple price posting mechanisms,[21] and do not allow the mediator to add privately observed (but payoff irrelevant) signals to the information structure that is provided to the buyer and the seller.[22] Third, and most important, as opposed to the single-round discovery policies of most of the surplus extraction literature, our main contribution is to study a dynamic shuttle diplomacy procedure and show that a sequence of simple discoveries and trade opportunities allow to significantly increase the set of outcomes the mediator may achieve; in particular, it allows implementation of the first best outcome with an equitable division of the surplus.

We are only aware of two important exceptions to single-round discovery in the surplus extraction literature, Bergemann and Wambach (2015) and Heumann (2020). Bergemann and Wambach (2015) must be credited for being the first to show the benefits of slow information release. They introduce an auction with sequential discovery of information in which each bidder learns a progressively higher lower bound $v$ on her value for the item for sale. Each bidder may elect to continue to stay or to stop and drop out of the auction. Once all but a single bidder $i$ have dropped out, bidder $i$ gets the item and is charged her expected value conditional on her value being above the value $v$ at which the last bidder dropped out. Thus, clearly, this auction implements an efficient outcome and extracts all surplus. Our single-ride mechanism also gives all the surplus to one of the parties, but it is different as it does so by revealing all information to one party, and linking what is revealed to the other to the report of the informed party. Our shuttle diplomacy mechanism also reveals information gradually and only allows agents to choose if they want to stop or continue, but there are many differences with Bergemann and

[20] These papers impose, like us, a privacy constraint on the information disclosed to each buyer. In contrast, Bergemann et al. (2015) examine how the division of the gains from trade associated with the optimal pricing policy of a monopolist seller varies with the amount of information the seller has about buyers' private valuations.

[21] It is useful to point out that when agents are fully informed about their values, the only dominant strategy mechanisms that balance the budget at all signal realizations (i.e., such that the law of one price holds) and satisfy agents' participation constraints ex-post are price posting mechanisms in which the mediator posts a single price at which trade may take place (see Hagerty and Rogerson, 1987, Čopič and Ponsati, 2016, and Čopič, 2017; for the robustness of dominant strategy mechanisms, see Bergemann and Morris, 2005).

[22] In contrast, Krähmer (2020) allows the seller to: ($i$) randomize secretely among alternative information structures all of which fully inform the buyer about her own valuation, and ($ii$) condition the terms of trade on the buyer's report of the signal obtained from the realized information structure. This guarantees that a lie is detected with positive probability and can be severely punished. Hence the seller can extract all the surplus from the buyer; that is, the good is allocated efficiently and the buyer can be charged her valuation.

Wambach (2015). Contrary to their model, with shuttle diplomacy: (*i*) stopping might imply that the agent keeps the asset; (*ii*) agents alternate between discovering whether their values are high or low; (*iii*) the main goal is efficiency, which cannot be achieved if the agents are fully informed, contrary to the auction setting; (*iv*) a fair outcome can be achieved with symmetric agents, and finally and importantly (*v*) agents trade the claim on the asset back and forth along the mediation procedure.

Heumann (2020) studies a single-buyer, single-seller, model with variable quantity, in which the type $\theta$ of the buyer affects both her valuation for the quantity $q$ of the good supplied by the seller and the seller's cost of supplying it. As in Bergemann and Wambach (2015), the seller wants to maximize profit. To do so, the seller can choose the information content of a continuous-time signal process observed by the buyer, the quantity to be sold, and the price to be charged, as functions of the entire history of reports by the buyer about the information she has obtained. Heumann (2020) provides a lower bound on the rent that the buyer can attain and demonstrates the optimality of an upward disclosure policy as in Bergemann and Wambach (2015), where the buyer learns a progressively higher lower bound $\theta$, which corresponds to a higher lower bound on her value for any quantity $q$ of the good. As in our paper, slow information release relaxes the incentive constraints; however, in our paper uncertainty and private information concerns both the buyer and seller, the goals are efficiency and fairness, and it is not sufficient only to provide information about a lower boundary on value. Downward disclosures (i.e., the slow release of information on the upper bound of value), must go hand in hand with upward disclosures.

# 7 Conclusions

Psychologists have argued that mediators help parties to overcome psychological barriers to conflict resolution. We have argued that a mediator will also help when parties are rational, strategic negotiators.

The approach adopted in this paper may be described as an information discovery and allocation mechanism design approach. Both the private information disclosed to agents and the claim exchange protocol are chosen by a designer, in our case the mediator. The approach blends the classical mechanism design approach with the information design, or

Bayesian persuasion, approach. In classical mechanism design, agents have full private information and the designer only selects a procedure to determine the allocation as a function of the information reported by the parties. In information design, the allocation mechanism is exogenously fixed and the designer may only select the information discovery policy.

We have shown that shuttle diplomacy, whereby private information is progressively and symmetrically revealed to the parties involved in a dispute, and each time new information arrives parties decide whether to settle the dispute, allows the mediator to achieve an efficient outcome by reducing the opportunities for misrepresentation of value. With symmetric prior value uncertainty, the mediator is also able to set a fair price at which the parties share equally the gains from trade. We believe that the approach considered in this paper could be fruitfully applied to other settings, beyond the case of the a dispute between two parties that we have considered.

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# Appendix

In this appendix we present results and proofs omitted from the main body of the paper.

**Proof of Proposition 1** The seller's payoff when he has value $v^S$ and reports $\widehat{v}^S$ is:

$$u^S\left(\widehat{v}^S; v^S\right) = \max\left\{\left[p^S\left(\widehat{v}^S\right) - v^S\right]\left[1 - F_B\left(\widehat{v}^S\right)\right], 0\right\}$$

At an interior solution, the first order condition of the seller's problem of choosing $\widehat{v}^S$ to maximize $u^S\left(\widehat{v}^S; v^S\right)$ is:

$$\frac{dp^S\left(\widehat{v}^S\right)}{d\widehat{v}^S}\left[1 - F_B\left(\widehat{v}^S\right)\right] - f_B(\widehat{v}^S)\left[p^S\left(\widehat{v}^S\right) - v^S\right] = 0. \tag{17}$$

Denote by $u^S\left(v^S\right) = u^S\left(v^S; v^S\right)$ the seller's indirect utility when reporting sincerely (i.e., $\widehat{v}^S = v^S$) is optimal. By the envelope theorem, treating the true value $v^S$ as a parameter, we have:

$$\frac{du^S\left(v^S\right)}{dv^S} = -\left[1 - F_B\left(v^S\right)\right].$$

Integrating both sides from $v^S$ to 1, using the boundary condition $u^S(1) = 0$, we obtain:

$$\left[p^S\left(v^S\right) - v^S\right]\left[1 - F_B\left(v^S\right)\right] = u^S\left(v^S\right) = \int_{v^S}^{1}\left[1 - F_B\left(\widetilde{v}^S\right)\right] d\widetilde{v}^S.$$

Integrating by parts and rearranging we obtain:

$$p^S\left(v^S\right) = \int_{v^S}^{1} \frac{\widetilde{v}^S}{1 - F_B\left(v^S\right)} dF_B\left(\widetilde{v}^S\right) = \mathbb{E}_{F_B}^{[0,1]}\left[v^B \mid v^B \geq v^S\right]. \tag{18}$$

This shows that $p^S\left(\widehat{v}^S\right) = \mathbb{E}_{F_B}^{[0,1]}\left[v^B | v^B \geq \widehat{v}^S\right]$ is the only price function that satisfies the first order condition for truthful reporting of his value by the seller to be incentive compatible. Replacing (18) into (17), the first order condition can be written as:

$$-\widehat{v}^S f_B\left(\widehat{v}^S\right) + p^S\left(\widehat{v}^S\right) f_B\left(\widehat{v}^S\right) - f_B(\widehat{v}^S)\left[p^S\left(\widehat{v}^S\right) - v^S\right] = f_B(\widehat{v}^S)\left[v^S - \widehat{v}^S\right] = 0.$$

Differentiating it with respect to the report $\widehat{v}^S$ and evaluating it at $\widehat{v}^S = v^S$, shows that the second order condition is satisfied:

$$\left.\left(\frac{df_B(\widehat{v}^S)}{d\widehat{v}^S}[v^S - \widehat{v}^S] - f_B(\widehat{v}^S)\right)\right|_{\widehat{v}^S = v^S} = -f_B(v^S) \leq 0.$$

□

**Lemma 1** *If the seller (resp., buyer) follows the stop-at-value strategy, then the buyer's (resp., seller's) payoff from following the stop-at-value strategy in step 1 of round $t$ converges to the expression in (2) (resp., (5)) as $d\alpha_t$ and $d\beta_t$ converge to zero.*

**Proof** To compute the buyer's and seller's payoffs when they follow the stop-at-value strategy from step 1 of round $t$, observe that if they do, the outcome of the shuttle diplomacy procedure is that the buyer ends up owning the asset when it is efficient – that is, her value $v_B$ is higher than the seller's value $v_S$ – with the exception of two events.

The first event is when in step 1 of some round $\tau \geq t$ the seller stops the mechanism, but the buyer's value is higher. The buyer loses and the seller gains relative to efficient trading as they both value the asset more than the posted price and the asset would end up in the buyer's hands under efficient trading. As argued in the main text when discussing efficiency under stop-at-value-strategies, the probability of this event can be approximated by $f_S(\beta_\tau)d\beta_\tau f_B(\beta_\tau)d\beta_\tau$. An upper bound on the loss to the buyer and on the gain to the seller relative to thepayoff under efficient trading is $\beta_{\tau-1} - p$. Thus, relative to the payoff from trading at price $p$ when it is efficient, the loss in the expected payoff of the buyer and the expected gain for the seller due to this event occurring at $\tau \geq t$ is (approximately) bounded above by:

$$(\beta_{\tau-1} - p)\frac{f_S(\beta_\tau)}{[F_S(\beta_\tau) - F_S(\alpha_\tau)]}\frac{f(\beta_\tau)}{[F_B(\beta_\tau) - F_B(\alpha_\tau)]}(d\beta_\tau)^2.$$

Adding up over all $\tau \geq t$ yields an upper bound on the expected loss in payoff for the buyer and gain for seller with respect to the payoff from efficient trading because of the occurrence of the event described at some date $\tau \geq t$ equal to:

$$\sum_{\tau=t}^{T}(\beta_{\tau-1} - p)\frac{f_S(\beta_\tau)}{[F_S(\beta_\tau) - F_S(\alpha_\tau)]}\frac{f(\beta_\tau)}{[F_B(\beta_\tau) - F_B(\alpha_\tau)]}(d\beta_\tau)^2,$$

which converges to zero as $d\beta_\tau$ goes to zero.

The second event is when in step 1 of some round $\tau \geq t$ the buyer stops, but her value is higher than the seller's value and hence it would be efficient for the buyer to obtain the asset. In this case, the departure from efficiency is beneficial to the buyer and costly for the seller, as they both value the asset less than the posted price. The probability of this event happening is equal to the probability that $v_B$ and $v_S$ are both in $[\alpha_{\tau-1}, \alpha_\tau)$, which is approximately equal to $f_S(\alpha_\tau)d\alpha_\tau f_B(\alpha_\tau)d\alpha_\tau$. An upper bound on the buyer's payoff gain and seller's loss relative to efficient trading is $p - \alpha_{\tau-1}$. Thus, relative to the payoff from trading at price $p$, the buyer's expected gain and seller's expected loss from this event happening at $\tau$ is (approximately) bounded above by

$$(p - \alpha_{\tau-1}) \frac{f_S(\alpha_\tau)}{[F_S(\beta_\tau) - F_S(\alpha_\tau)]} \frac{f_B(\alpha_\tau)}{[F_B(\beta_\tau) - F_B(\alpha_\tau)]} (d\alpha_\tau)^2.$$

It then follows that, as for the first event, adding up over all $\tau > t$ yields again an upper bound on the expected gain of the buyer and loss of the seller with respect to the payoff from efficient trading, due to the second event occurring at any $\tau$, and this upper bound converges to zero as $d\alpha_\tau$ goes to zero.

Hence, as $d\alpha_t$ and $d\beta_t$ converge to zero for all rounds $\tau \geq t$, the buyer and seller's expected payoffs from the stop-at-value strategies, evaluated before step 1 of round $t$, converge, respectively, to:

$$\left(\mathbb{E}_{F_B,F_S}^{[\alpha_{t-1},\beta_{t-1}]}\left[v_B \,|\, v_B > v_S\right] - p\right) \cdot \Pr(v_B > v_S)\,,$$

which is equal to (2), and

$$\mathbb{E}_{F_B,F_S}^{[\alpha_{t-1},\beta_{t-1}]}\left[v_S \,|\, v_S > v_B\right] \cdot \Pr(v_S > v_B) + p \cdot \left[1 - \Pr(v_S > v_B)\right],$$

which is equal to (5). □

**Proof of Proposition 2** Condition (10) can be written as

$$\int_{\alpha_{t-1}}^{\beta_{t-1}} \frac{\beta_{t-1}v - v^2}{\frac{1}{2}(\beta_{t-1} - \alpha_{t-1})^2} dv < p < \int_{\alpha_{t-1}}^{\beta_{t-1}} \frac{v^2 - \alpha_{t-1}v}{\frac{1}{2}(\beta_{t-1} - \alpha_{t-1})^2} dv, \qquad \text{or}$$

$$\frac{1}{2}\beta_{t-1}(\beta_{t-1}^2 - \alpha_{t-1}^2) - \frac{1}{3}(\beta_{t-1}^3 - \alpha_{t-1}^3) < \frac{1}{2}(\beta_{t-1} - \alpha_{t-1})^2 p < \frac{1}{3}(\beta_{t-1}^3 - \alpha_{t-1}^3) - \frac{1}{2}\alpha_{t-1}(\beta_{t-1}^2 - \alpha_{t-1}^2), \quad \text{or}$$

$$\frac{\beta_{t-1}}{3}(\beta_{t-1}^2 - \alpha_{t-1}^2) - \frac{2\alpha_{t-1}^2}{3}(\beta_{t-1} - \alpha_{t-1}) < (\beta_{t-1} - \alpha_{t-1})^2 p < \frac{2\beta_{t-1}^2}{3}(\beta_{t-1} - \alpha_{t-1}) - \frac{\alpha_{t-1}}{3}(\beta_{t-1}^2 - \alpha_{t-1}^2), \quad \text{or}$$

$$\frac{\beta_{t-1}}{3}(\beta_{t-1} + \alpha_{t-1}) - \frac{2\alpha_{t-1}^2}{3} < (\beta_{t-1} - \alpha_{t-1})p < \frac{2\beta_{t-1}^2}{3} - \frac{\alpha_{t-1}}{3}(\beta_{t-1} + \alpha_{t-1}), \quad \text{or}$$

$$\frac{\beta_{t-1}^2 - \alpha_{t-1}^2}{3} + \frac{\alpha_{t-1}(\beta_{t-1} - \alpha_{t-1})}{3} < (\beta_{t-1} - \alpha_{t-1})p < \frac{2(\beta_{t-1}^2 - \alpha_{t-1}^2)}{3} - \frac{\alpha_{t-1}}{3}(\beta_{t-1} - \alpha_{t-1}), \quad \text{or}$$

$$\frac{\beta_{t-1}^2 - \alpha_{t-1}^2}{3} + \frac{\alpha_{t-1}(\beta_{t-1} - \alpha_{t-1})}{3} < (\beta_{t-1} - \alpha_{t-1})p < \frac{2(\beta_{t-1}^2 - \alpha_{t-1}^2)}{3} - \frac{\alpha_{t-1}}{3}(\beta_{t-1} - \alpha_{t-1}), \quad \text{or}$$

$$\frac{1}{3}\beta_t + \frac{2}{3}\alpha_t < p < \frac{2}{3}\beta_t + \frac{1}{3}\alpha_t$$

Since the above inequality must hold at the beginning of the discovery procedure, $t = 0$, when $\alpha_0 = 0$ and $\beta_0 = 1$, it must be $\frac{1}{3} < p < \frac{2}{3}$. To see that this is the only required condition, let $\alpha_t = \frac{t}{T}p$ and $\beta_t = 1 - \frac{t}{T}(1-p)$, so that all discovery intervals on the same side of $p$ are of the same size. Then the above condition, for all $t$, reduces to:

$$\frac{1}{3}\left(1 - \frac{t-1}{T}(1-p)\right) + \frac{2}{3}\frac{t-1}{T}p < p < \frac{2}{3}\left(1 - \frac{t-1}{T}(1-p)\right) + \frac{1}{3}\frac{t-1}{T}p, \qquad \text{or}$$

$$\frac{1}{3}\left(1 - \frac{t-1}{T}\right) + \frac{t-1}{T}p < p < \frac{2}{3}\left(1 - \frac{t-1}{T}\right) + \frac{t-1}{T}p, \qquad \text{or}$$

$$\frac{1}{3} < p < \frac{2}{3}.$$

The fair price $\frac{1}{2} \cdot \mathbb{E}^{[\alpha_{t-1},\beta_{t-1}]}_{F_B,F_S}[v_B \mid v_B > v_S] + \frac{1}{2} \cdot \mathbb{E}^{[\alpha_{t-1},\beta_{t-1}]}_{F_B,F_S}[v_S \mid v_B > v_S]$ is equal to

$$\begin{aligned}
\frac{1}{2}\int_{\alpha_{t-1}}^{\beta_{t-1}} \frac{\beta_{t-1}v - v^2}{\frac{1}{2}(\beta_{t-1}-\alpha_{t-1})^2}dv + \frac{1}{2}\int_{\alpha_{t-1}}^{\beta_{t-1}} \frac{v^2 - \alpha_{t-1}v}{\frac{1}{2}(\beta_{t-1}-\alpha_{t-1})^2}dv &= \int_{\alpha_{t-1}}^{\beta_{t-1}} \frac{(\beta_{t-1}-\alpha_{t-1})v}{(\beta_{t-1}-\alpha_{t-1})^2}dv \\
&= \frac{1}{2}(\beta_{t-1}+\alpha_{t-1}),
\end{aligned}$$

which equals 1/2 when $\alpha_{t-1} = \frac{t-1}{T}\cdot\frac{1}{2}$ and $\beta_{t-1} = 1 - \frac{t-1}{T}\cdot\frac{1}{2}$. □

**Proof of Claim 1** It follows from the fact that:

$$\mathbb{E}^{[\alpha,\beta]}_{F_B,F_S}[v_j \mid v_j < v_i] - \mathbb{E}^{[\alpha,\beta]}_{F_B,F_S}[v_i \mid v_j < v_i] = \mathbb{E}^{[\alpha,\beta]}_{F_B,F_S}[v_j - v_i \mid v_j < v_i] < 0.$$

□

**Proof of Claim 2** For fixed $v_i$ we have :

$$\mathbb{E}^{[\alpha,\beta]}_{F_j}[v_j|v_j < v_i] < v_i < \mathbb{E}^{[\alpha,\beta]}_{Fj}[v_j|v_j > v_i].$$

Integrating over $v_i$ yields the lemma. □

**Proof of Claim 3** Let $j = \text{argmax}_{B,S}\left\{\mathbb{E}^{[\alpha,\beta]}_{F_B,F_S}[v_B \mid v_B < v_S], \mathbb{E}^{[\alpha,\beta]}_{F_B,F_S}[v_S \mid v_S < v_B]\right\}$.

By Claim 1:

$$\mathbb{E}^{[\alpha,\beta]}_{F_B,F_S}[v_j \mid v_j < v_i] < \mathbb{E}^{[\alpha,\beta]}_{F_B,F_S}[v_i \mid v_i > v_j].$$

By Claim 2:

$$\mathbb{E}^{[\alpha,\beta]}_{F_B,F_S}[v_j \mid v_j < v_i] < \mathbb{E}^{[\alpha,\beta]}_{F_B,F_S}[v_j \mid v_j > v_i].$$

The inequality in Claim 3 then follows. □

**Proof of Proposition 3** For $\lambda \in (0,1)$, select the posted price

$$p = \lambda \max\left\{\mathbb{E}^{[0,1]}_{F_B,F_S}[v_B|v_B < v_S], \mathbb{E}^{[0,1]}_{F_B,F_S}[v_S|v_S < v_B]\right\} + (1-\lambda)\min\left\{\mathbb{E}^{[0,1]}_{F_B,F_S}[v_B|v_B > v_S], \mathbb{E}^{[0,1]}_{F_B,F_S}[v_S|v_S > v_B]\right\}. \tag{19}$$

Define $i_t, j_t$ as follows:[23]

$$i_t = \arg\max_{B,S}\left\{\mathbb{E}^{[\alpha_t,\beta_t]}_{F_B,F_S}[v_B|v_B < v_S], \mathbb{E}^{[\alpha_t,\beta_t]}_{F_B,F_S}[v_S|v_S < v_B]\right\}$$

$$j_t = \arg\min_{B,S}\left\{\mathbb{E}^{[\alpha_t,\beta_t]}_{F_B,F_S}[v_B|v_B > v_S], \mathbb{E}^{[\alpha_t,\beta_t]}_{F_B,F_S}[v_S|v_S > v_B]\right\}$$

Letting $h_t, k_t \in \{B, S\}$, $h_t \neq i_t$, $k_t \neq j_t$, to prove the proposition we will show that, for all $t$, there is a choice of $\alpha_t = \alpha_{t-1} + d\alpha_t > \alpha_{t-1}$ and $\beta_t = \beta_{t-1} + d\beta_t < \beta_{t-1}$ such that the price $p$ defined in (19) satisfies the following condition for all $t \in \{0, ..., \infty\}$:

$$\begin{aligned} p &= \lambda \mathbb{E}^{[\alpha_t,\beta_t]}_{F_B,F_S}[v_{i_t} \mid v_{i_t} < v_{h_t}] + (1-\lambda)\mathbb{E}^{[\alpha_t,\beta_t]}_{F_B,F_S}[v_{j_t} \mid v_{j_t} > v_{k_t}] \\ &= \lambda \int_{\alpha_t}^{\beta_t} \frac{v_{i_t}[F_{h_t}(\beta_t) - F_{h_t}(v_{i_t})]}{\int_{\alpha_t}^{\beta_t}[F_{h_t}(\beta_t) - F_{h_t}(v)]dF_{i_t}(v)} dF_{i_t}(v_{i_t}) + (1-\lambda)\int_{\alpha_t}^{\beta_t} \frac{v_{j_t}[F_{k_t}(v_{k_t}) - F_{k_t}(\alpha_t)]}{\int_{\alpha_t}^{\beta_t}[F_{k_t}(v) - F_{k_t}(\alpha_t)]dF_{j_t}(v)} dF_{j_t}(v_{j_t}) \end{aligned} \tag{20}$$

Take an increasing sequence $\{\alpha_t\}_{t=1}^{\infty}$ with $\alpha_0 = 0$, $\alpha_\infty = p$ and use (20) to implicitly define an associated sequence $\{\beta_t\}_{t=1}^{\infty}$. It is clear that it must be $\beta_0 = 1$ and $\beta_\infty = p$. Thus, it only remains to show that the sequence $\{\beta_t\}_{t=1}^{\infty}$ is decreasing. To establish that this is the case, we totally differentiate (20) with respect to $\alpha_t$ and $\beta_t$ to obtain:

[23] Note it could be $i_t = j_t$ or $i_t \neq j_t$.

$$0 = \left( \lambda \left[ \int_{\alpha_t}^{\beta_t} \frac{v_{i_t}}{F_{i_t}(\beta_t) - F_{i_t}(\alpha_t)} dF_{i_t(v_{i_t})} - \mathbb{E}_{F_B,F_S}^{[\alpha_t,\beta_t]}[v_{i_t}|v_{i_t} < v_{h_t}] \right] \frac{[F_{i_t}(\beta_t) - F_{i_t}(\alpha_t)] f_{h_t}(\beta_t)}{\int_{\alpha_t}^{\beta_t} [F_{h_t}(\beta_t) - F_{h_t}(v)] dF_{i_t}(v)} \right.$$

$$\left. + (1-\lambda) \left[ \beta_t - \mathbb{E}_{F_B,F_S}^{[\alpha_t,\beta_t]}[v_{j_t}|v_{j_t} > v_{k_t}] \right] \frac{[F_{k_t}(\beta_t) - F_{k_t}(\alpha_t)] f_{j_t}(\beta_t)}{\int_{\alpha_t}^{\beta_t} [F_{k_t}(\beta_t) - F_{k_t}(v)] dF_{j_t}(v)} \right) d\beta_t$$

$$+\lambda \left( \left[ \mathbb{E}_{F_B,F_S}^{[\alpha_t,\beta_t]}[v_{i_t}|v_{i_t} < v_{h_t}] - \alpha_t \right] \frac{[F_{h_t}(\beta_t) - F_{h_t}(\alpha_t)] f_{i_t}(\alpha_t)}{\int_{\alpha_t}^{\beta_t} [F_{h_t}(\beta_t) - F_{h_t}(v)] dF_{i_t}(v)} \right.$$

$$\left. +(1-\lambda) \left[ \mathbb{E}_{F_B,F_S}^{[\alpha_t,\beta_t]}[v_{j_t}|v_{j_t} > v_{k_t}] - \int_{\alpha_t}^{\beta_t} \frac{v_{j_t}}{F_{j_t}(\beta_t) - F_{j_t}(\alpha_t)} dF_{j_t}(v_{j_t}) \right] \frac{[F_{j_t}(\beta_t) - F_{j_t}(\alpha_t)] f_{k_t}(\alpha_t)}{\int_{\alpha_t}^{\beta_t} [F_{k_t}(\beta_t) - F_{k_t}(v)] dF_{j_t}(v)} \right) d\alpha_t$$

Both the term multiplying $d\beta_t$ and the one multiplying $d\alpha_t$ are positive. Hence, the above expression defines a negative value $d\beta_t$ associated to any positive value $d\alpha_t$, which guarantees that the price $p$ satisfies (20). This concludes the proof. □

**Lemma 2** *Given any solution to the mediator's maximization problem* (15)*, there is a payoff equivalent solution in which the mediator chooses a two-point signal distribution both for the buyer and the seller.*

**Proof** Suppose $p, \widetilde{F}_B, \widetilde{F}_S$ are maximizers of (15). We show in what follows that $p$ and the following two point distributions are also maximizers of (15). For buyers, take the distribution that puts mass $1-R_{\widetilde{F}_B}(p)$ on $\mathbb{E}_{\widetilde{F}_B}[v_B|v_B < p]$ and mass $R_{\widetilde{F}_B}(p)$ on $\mathbb{E}_{\widetilde{F}_B}[v_B|v_B \geq p]$. For sellers, take the distribution that puts mass $\widetilde{F}_S(p)$ on $\mathbb{E}_{\widetilde{F}_S}[v_S|v_S \leq p]$ and mass $1 - \widetilde{F}_S(p)$ on $\mathbb{E}_{\widetilde{F}_S}[v_S|v_S > p]$. It is then immediate to see that the expression in (15), evaluated at the solution $p, \widetilde{F}_B, \widetilde{F}_S$, is equal to:

$$\left( \mathbb{E}_{\widetilde{F}_B}^{[0,1]}[v_B|v_B \geq p] - \mathbb{E}_{\widetilde{F}_S}^{[0,1]}[v_S|v_S \leq p] \right) R_{\widetilde{F}_B}(p) \widetilde{F}_S(p),$$

that is, to the value of (15) at the two-point distributions described above. Note that if $\widetilde{F}_B \in \mathcal{F}_B$, then the described two-point distribution for the buyer also belongs to $\mathcal{F}_B$, and similarly for the seller. This concludes the proof that the mediator may restrict attention to two-point discrete distributions. □

**Proof of Proposition 5** To establish the result, we characterize first the classes of feasible two-point signal distributions. For buyers, let $\{v_B^L, v_B^H\}$ be the set of possible signals with associated probabilities $\widetilde{f}_B^L$, $\widetilde{f}_B^H = 1 - \widetilde{f}_B^L$. The following constraints must hold to guarantee that the true distribution $F_B$ is a mean preserving spread of the two-point signal distribution:

$$v_B^L \widetilde{f}_B^L + v_B^H \left(1 - \widetilde{f}_B^L\right) = \mathbb{E}_{F_B}^{[0,1]}[v_B] \tag{21}$$

$$\left(x - v_B^L\right) \widetilde{f}_B^L \leq \int_0^x F_B(v)dv \quad \text{for } x \in [v_B^L, v_B^H) \tag{22}$$

$$\left(v_B^H - v_B^L\right) \widetilde{f}_B^L + \left(x - v_B^H\right) \leq \int_0^x F_B(v)dv \quad \text{for } x \in \left[v_B^H, 1\right] \tag{23}$$

We can then solve (21) for $v_B^L$ and replace the obtained solution into the other two constraints:

$$(x - v_B^H)\widetilde{f}_B^L - \mathbb{E}_{F_B}^{[0,1]}[v_B] + v_B^H - \int_0^x F_B(v)dv \leq 0 \quad \text{for } x \in [v_B^L, v_B^H) \tag{24}$$

$$x - \mathbb{E}_{F_B}^{[0,1]}[v_B] - \int_0^x F_B(v)dv \leq 0 \quad \text{for } x \in \left[v_B^H, 1\right] \tag{25}$$

It is immediate to see that (25) holds since, integrating by parts,

$$\begin{aligned} x - \mathbb{E}_{F_B}^{[0,1]}[v_B] - \int_0^x F_B(v)dv &= x[1 - F_B(x)] - \mathbb{E}_{F_B}^{[0,1]}[v_B] + \int_0^x vdF_B(v) \\ &= xR_{F_B}(x) - \int_x^1 vdF_B(v) \leq 0. \end{aligned}$$

It is also immediate to see that the left hand side of (24) is a concave function of $x$. Define $x_B$ as the solution to $\widetilde{f}_B^L = F_B(x)$. There are three possible cases.

Case 1: $x_B \in [v_B^L, v_B^H]$. Then the left hand side of (24) is maximized at $x_B$ and condition (24) holds if it is satisfied for $x = x_B$. Hence we can rewrite this condition as:

$$(x - v_B^H)F_B(x) - \mathbb{E}_{F_B}^{[0,1]}[v_B] + v_B^H - xF_B(x) + \int_0^x vdF_B(v) \leq 0 \quad \text{or,}$$

$$v_B^H R_{F_B}(x) - \int_x^1 vdF_B(v) \leq 0 \tag{26}$$

Condition (26) is quite intuitive: it says that the value of the buyer when she receives the high signal, $v_B^H$, cannot exceed the expected value of the buyer, conditional on this value lying above some threshold $x$, evaluated according to the posterior distribution $F_B$.

Case 2: $x_B < v_B^L$, that is the two point signal distribution is such that $F(v_B^L) > \widetilde{f}_B^L$. In this case the left hand side of (24) is maximized at $x = v_B^L$, hence condition (24) can be rewritten as:

$$-\int_0^x F_B(v)dv \leq 0,$$

and is always satisfied.

Case 3: $x_B > v_B^H$, the two point signal distribution is such that $\widetilde{f}_B^L > F(v_B^H)$. The expression on the left hand side of (24) is now maximized at $x = v_B^H$ and we can rewrite (24) as:

$$-\mathbb{E}_{F_B}^{[0,1]}[v_B] + v_B^H - \int_0^{v_B^H} F_B(v)dv \leq 0, \tag{27}$$

which we can also show always to hold.[24]

We may repeat the same argument for the seller, letting $\{v_S^L, v_S^H\}$ be the set of possible signals with probabilities $\widetilde{f}_S^L, \widetilde{f}_S^H$, with $v_S^L \widetilde{f}_S^L + v_S^H \widetilde{f}_S^H = \mathbb{E}_{F_S}^{[0,1]}[v_S]$ and the counterparts of (24) and (25). Using $y$ instead of $x$, we see that the left hand side of the counterpart of (24) is a concave function of $y$. The expression on the left hand side of the constraint reaches then the highest value either at $y$ equal to $v_S^L$ or $v_S^H$, in which case the constraint is always satisfied, or at $y$ such that $\widetilde{f}_S^L = F_S(y)$, in which case the constraint can be rewritten as:

$$\begin{aligned} v_S^H R_{F_S}(y) - \int_y^1 v dF_S(v) \leq 0 \quad \text{or,} \\ \int_0^y v dF_S(v) - v_S^L F_S(y) \leq 0 \end{aligned} \tag{28}$$

We can then use Lemma 2 and the characterization we obtained of two-point signal distributions to rewrite the mediator's problem, (15). As we said, a mediator aiming to

[24] We can in fact rewrite (27) as

$$\begin{aligned} -\mathbb{E}_{F_B}^{[0,1]}[v_B] + v_B^H \left[1 - F_B(v_B^H)\right] + \int_0^{v_B^H} v dF_0(v) \leq 0 \quad \text{or,} \\ v_B^H \left[1 - F_B(v_B^H)\right] - \int_{v_B^H}^1 v dF_B(v) \leq 0, \end{aligned}$$

which always holds.

maximize the gains from trade will choose distributions and a price such that the buyer is willing to trade when he gets the high signal and the seller with the low signal, that is: $v_B^H \geq p \geq v_S^L$. Also, we can show that the situation described in Cases 2 and 3 above never arises at a solution of the mediator's problem,[25] hence neither constraint (26) nor (28) can be ignored, and we can replace the choice variables $\widetilde{f}_B^L$ and $\widetilde{f}_S^L$ with $F_B(x)$ and $F_S(y)$. The mediator problem (15) can then be rewritten as follows:

$$\max_{v_B^H, v_S^L, x, y} \left(v_B^H - v_S^L\right) F_S(y) R_{F_B}(x) \quad \text{s.t.} \tag{29}$$

$$v_B^H R_{F_B}(x) + v_B^L F_B(x) = \mathbb{E}_{F_B}^{[0,1]}[v_B]$$

$$v_B^H R_{F_B}(x) - \int_x^1 v dF_B(v) \leq 0$$

$$v_S^H R_{F_S}(y) + v_S^L F_S(y) = \mathbb{E}_{F_S}^{[0,1]}[v_S]$$

$$\int_0^y v dF_S(v) - v_S^L F_S(y) \leq 0$$

Furthermore, it is immediate to see that both inequality constraints must bind, otherwise the mediator would profit from raising $v_B^H$ or lowering $v_S^L$. Hence, substituting the constraints into the objective function, the problem reduces to:

$$\max_{x,y} \left(\mathbb{E}_{F_B}^{[0,1]}[v_B \mid v_B \geq x] - \mathbb{E}_{F_S}^{[0,1]}[v_S \mid v_S \leq y]\right) F_S(y) R_{F_B}(x) \tag{30}$$

The interpretation of (30) is as follows. The fact that the two constraints in (29) hold as equality means that the mediator lets the buyer observe exactly whether her value is greater than or equal to $x$ and lets the seller observe whether her value is smaller than or equal to $y$. Trade takes place when both events realize, as ensured by posting any price $p \in \left[\mathbb{E}_{F_S}^{[0,1]}[v_S \mid v_S \leq y], \mathbb{E}_{F_B}^{[0,1]}[v_B \mid v_B \geq x]\right]$. The values of $x$ and $y$ are then optimally chosen to maximize expected gains from trade.

[25] Consider the information provided to the buyer. Suppose first the solution of the mediator's problem falls in Case 2. Then $F(v_B^L) > \widetilde{f}_B^L = F(x_B)$ and it is possible to raise the gains from trade by keeping $\widetilde{f}_B^L$ constant, reducing $v_B^L$ and increasing $v_B^H$ while satisfying the only binding constraint $v_B^L \widetilde{f}_B^L + v_B^H \left(1 - \widetilde{f}_B^L\right) = \mathbb{E}_{F_B}^{[0,1]}[v_B]$, a contradiction. Second, suppose the solution falls in Case 3. We have so $F(v_B^H) < \widetilde{f}_B^L = F(x_B)$. If $x_B < 1$ it is again possible to raise the gains from trade by keeping $\widetilde{f}_B^L$ constant, reducing $v_B^L$ and increasing $v_B^H$ while satisfying $v_B^L \widetilde{f}_B^L + v_B^H \left(1 - \widetilde{f}_B^L\right) = \mathbb{E}_{F_B}^{[0,1]}[v_B]$, a contradiction. If instead $x_B = 1$, then $f^L = 1$ and with probability 1 there is no trade; this is also a contradiction as full discovery and any interior posted price $p \in (0, 1)$ would generate positive gains from trade.

The final step of the proof is then to characterize the solutions of program (30). Since the domain of $(x, y)$ is compact (the unit square) and the objective function and constraints are continuous in $x$ and $y$, program (30) has a solution. Setting $x = 1$ or $y = 0$ cannot be optimal, as it yields a zero payoff to the mediator, which is less than the payoff that could be achieved by setting $0 < x = y < 1$. Thus the only possible boundary solutions have $x = 0$ and/or $y = 1$.

Since $F_B$ and $F_S$ have no atoms, the first order conditions of program (30), taking into account the constraints $x \geq 0$ and $1 - y \geq 0$, are:

$$\begin{aligned}
-xf_B(x)F_S(y) + \int_0^y vdF_S(v)f_B(x) &\leq 0 \\
\left(-xf_B(x)F_S(y) + \int_0^y vdF_S(v)f_B(x)\right)x &= 0 \\
\int_x^1 vdF_B(v)f_S(y) - yf_S(y)R_{F_B}(x) &\geq 0 \\
\left(\int_x^1 vdF_V(v)f_S(y) - yf_S(y)R_{F_B}(x)\right)(1-y) &= 0
\end{aligned}$$

Note that if $x = 0$, the first inequality is violated, as the term on the left hand side is strictly positive. Similarly, if $y = 1$ the second inequality is violated, as the expression on the left hand side is strictly negative. Thus there are no boundary solutions, the solution is interior and satisfies the conditions:

$$\begin{aligned}
&-xf_B(x)F_S(y) + \int_0^y vdF_S(v)f_B(x) = 0 \quad \text{and} \\
&\int_x^1 vdF_B(v)f_S(y) - yf_S(y)R_{F_B}(x) = 0
\end{aligned}$$

which can be written as

$$\begin{aligned}
&\mathbb{E}^{[0,1]}_{F_S}[v \,|\, v \leq y] = x \quad \text{and} \\
&\mathbb{E}^{[0,1]}_{F_B}[v \,|\, v \geq x] = y.
\end{aligned}$$

This concludes the proof of the proposition. □