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Tiago Cavalcanti, Angelo Mendes and Pierluca Pannella

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Centre for Economic Policy Research
33 Great Sutton Street, London EC1V 0DX, UK
Tel: +44 (0)20 7183 8801
www.cepr.org

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JEL Classification: E23, L26, O11, O41

Keywords: Distortions, Firm entry, Production Network, Aggregate Misallocation

Tiago Cavalcanti - tvdvc2@cam.ac.uk
University of Cambridge, Sao Paulo School of Economics and CEPR

Angelo Mendes - angeloahm@gmail.com
University of Minnesota

Pierluca Pannella - pierluca.pannella@fgv.br
Sao Paulo School of Economics

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Tiago Cavalcanti † Angelo Mendes ‡ Pierluca Pannella §

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Abstract

This paper investigates how sectoral linkages amplify or diminish misallocation at the intensive and extensive margins. Our analysis is based on a multisector general equilibrium model with input-output linkages, heterogeneous entrepreneurial abilities, and endogenous occupational choice. Distortions misallocate the intensive use of production inputs, but they also affect productivity through two additional wedges: a "labor-entrepreneurship" wedge, which misallocates agents between entrepreneurship and the labor force; and a "between-sector" wedge, which misallocates entrepreneurs among the different sectors. When the most distorted sectors are upstream (downstream), input-output linkages amplify (diminish) the loss from the misallocation of entrepreneurs. We calibrate the model to the US and quantify the output losses from distortions, decomposing the role of networks and the extensive margin decisions. We study an entry subsidy program, showing that it should target sectors with large profit losses, even if they are not necessarily the most distorted.

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†University of Cambridge, CEPR and Sao Paulo School of Economics-FGV

‡University of Minnesota

§Sao Paulo School of Economics-FGV

1. Introduction

It is well known that sectoral distortions reduce aggregate productivity (e.g., Hsieh and Klenow, 2009) and linkages between sectors can amplify the effects of such distortions (e.g., Jones, 2011; Bigio and La'o, 2020; Baqaee and Farhi, 2020b) on economic efficiency. Most of the papers in the literature on production networks and misallocation consider economic environments with a fixed number of firms and no endogenous entry. Existing distortions, however, not only affect the optimal scale of firms, but they also impact entry decisions. This paper investigates how sectoral distortions affect aggregate output in a framework with endogenous occupational choice, entrepreneurship and input-output linkages. The environment is suitable to evaluate the design of an entry subsidy program.

We build a static multisector general equilibrium environment in which sectoral output can be consumed or used by other sectors as input, similarly to Bigio and La'o (2020) and Carvalho and Tahbaz-Salehi (2019). As in Hsieh and Klenow (2009) and Restuccia and Rogerson (2008), distortions are represented by exogenous sectoral wedges between marginal revenues and costs. However, in our economy, returns to scale are decreasing and occupational choice is endogenous, in the same spirit of Lucas Jr (1978). Individuals with heterogeneous managerial productivity can choose to be a worker or an entrepreneur in one of the sectors. Therefore, the mass of workers and the mass of firms by sector and in the aggregate are endogenous objects.

Our framework is tractable enough to allow for the analytical derivation of the aggregate output. Total production depends on the allocation of intermediate goods and workers at the intensive margins, and the allocation of individuals between paid jobs and entrepreneurship at the extensive margin. Specifically, distortions at the extensive margin can be described by two wedges: a "labor-entrepreneurship" wedge, which misallocates individuals between entrepreneurship and the labor force; and a "between-sector" wedge, which misallocates entrepreneurs among the different sectors of economic activity.

The labor-entrepreneurship wedge resembles the labor wedge described by Chari et al. (2007), which appears also in Bigio and La'o (2020) who investigate the effects of sectoral intensive margin distortions (in their case, financial shocks) on the macroeconomy. In their framework, labor supply is elastic, and the mass of firms is constant. In our model, labor supply of each worker is fixed, but the mass of workers and entrepreneurs are endogenous. We show that this labor-entrepreneurship wedge is represented by the ratio of the aggregate share of labor income of the distorted and the undistorted economies. The between-sector wedges affect instead the relative mass of entrepreneurs in each one of the sectors. They are given by the ratio of the sectoral profit shares of the distorted and the undistorted economies.

In the limiting case with no heterogeneous entrepreneurial productivity, we can derive analytical results about the effect of sectoral shocks to the aggregate economy. The Hulten's Theorem (cf., Hulten, 1978) applies to our economy, as the first-order effects of sectoral productivity shocks on aggregate TFP are represented by the efficient-economy Domar weights. Shocks to sectoral distortions induce a reallocation of individuals between the labor force and entrepreneurship. In particular, a negative distortion shock - a rise in distortion - in a sector i reduces (increases) the number of firms in the same sector if the sector has low (high) labor intensity. In addition, the mass of firms in other sectors is also reduced if those sectors are direct or indirect supplier to sector i and they have low labor intensity. Our findings provide alternative insights to explain the decline in business dynamism in Western countries (e.g., Messer et al., 2016; Akicigit and Ates, 2019).[1]

In our model, the output loss from distortions can be approximated, up to second order, by the sum of variance components. In particular, the misallocation of individuals between labor force and entrepreneurship is represented by the variance of our labor-entrepreneurship and

[1]Some papers relate this decline with a process of structural transformation and, in particular, with the rise of the Information and Communications Technology (ICT) sector (e.g., Fernald, 2015; Syverson, 2017). While the ICT sector is relatively more productive, it is also characterized by high market concentration and markups. In particular, the use of intangible capital might lead to higher markups and constraint the creation of new firms (e.g., Caggese and Perez-Orive, 2017; De Ridder, 2019).

between-sector wedges. We analytically compare the output losses in a production network economy to the one suffered by an equivalent horizontal economy.[2] We show how input-output linkages diminish (amplify) the loss from distortions if they directly hit more downstream (upstream) sectors. Intuitively, sectors that are direct or indirect supplier of downstream sectors are indirectly damaged by a lower demand. This may reallocate and rebalance the outflows of entrepreneurs from the originally distorted sector.

We complement our analytical analysis with a quantitative analysis. We calibrated model parameters to match sectoral moments of the United States economy and we then run a series of counterfactual exercises. Using independent estimates of distortions based on markup estimations of De Loecker et al. (2020), we compute the contribution of intensive and extensive margin misallocation on aggregate output loss. We show how the endogenous entry of firms is quantitatively important in amplifying distortions in network economies, especially in an augmented version of our model with fixed technological entry costs.

Finally, we also analyze the effects of sectoral subsidies on output. We use our baseline economy with distortions to identify which sectors should be subsidized to obtain the highest output gain. We compare the effects of a subsidy program which targets one sector at the time but requiring the same total level of expenditure. This exercise is related to the one investigated by Liu (2019). The difference is that we consider subsidies in an environment with endogenous occupational choice. We show that the size of direct distortions are a good statistics to rank the return from subsidies in the equivalent horizontal economy, while they are not necessarily a good measure in production network economies. In the presence of input-output linkages, sectors should be ranked by their between-sector wedge, which is their total loss in profits relative to the undistorted benchmark. The knowledge of the production network structure is needed in designing this entry subsidy program.

The paper is organized as follows. Section 2 presents the related literature and places our

[2]The equivalent horizontal of a production network economy is defined as the economy with no input-output linkages but the same allocation of individuals across sectors at efficiency.

contribution. Section 3 contains our main environment with heterogeneous entrepreneurial productivity and characterizes the equilibrium. Section 4 considers the limiting case with homogeneous managerial skills. We then derive theoretical results about the effects of sectoral distortions on output and firm creation. We also derive analytical formulas for welfare losses from distortions and identify the amplification/reduction role of Input-Output linkages. In Section 5, we present our model calibration and quantitatively compute the losses from distortions. We decompose these losses from misallocation at the intensive and extensive margins. Section 6 analyzes a targeted subsidy program. Section 7 concludes.

2. Related Literature

Since the paper by Long Jr and Plosser (1983), there is a growing literature in macroeconomics studying multisector models to understand the importance of sectoral shocks and their transmission mechanism through input-output linkages (e.g., Acemoglu et al., 2012). Baqaee and Farhi (2018, 2019) are general theoretical references for the production network literature. The first paper characterizes a class of models with heterogeneous agents and input-output linkages, showing that propagation patterns are constrained by the assumption of representative-agent models. The second paper extends the results from Hulten (1978), deriving a decomposition of the first-order effects at and away from efficiency.[3]

Our paper is closely related to a subset of this literature which investigates how misallocation can be amplified through the input-output structure of the economy. Jones (2011) studies the same issue in a standard growth model with neoclassical production functions. Bigio and La'o (2020) also study the effect of wedges between prices and marginal costs and use their model to analyze the role of financial frictions in business cycle fluctuations. Their framework includes elastic labor supply but abstracts from endogenous entrepreneurship and entry of firms.

[3] Carvalho (2014) and Carvalho and Tahbaz-Salehi (2019) provide an extensive review on this literature.

The paper by Liu (2019) is also related to one of the quantitative exercises that we run. He investigates industrial policies in a constant returns to scale, nonparametric production network with market distortions and subsidies. He studies the aggregate effects of sectoral subsidies targeting specific sectors. He proposes a measure of "distortion centrality" to identify which sectors should be subsidized. Our paper considers a parametric production structure but adds endogenous occupational choice. Our subsidy program is slightly different. We subsidize entry while his analysis is based on a production subsidy proportional to input expenditures. In our model, we show that for an entry subsidy program, sectors should be ranked by their total loss in profits relative to the undistorted economy.

Baqaee and Farhi (2020b) investigate the effects of misallocation in economies with production networks and non-parametric input-output structure. They do not consider the role of endogenous firm entry. Baqaee and Farhi (2020a) study the effects of distortions in a framework with endogenous entry from a separate set of potential entrants. They decompose changes in aggregate productivity into changes in technical and allocative efficiency, showing the importance of endogenous entry.[4] Although the focus of this last paper is close to ours, we model entry differently, as an occupational choice decision: an increase in the mass of firms mechanically reduces the labor force; and a higher mass of entrepreneurs in one sector might lead to a lower mass in other sectors. Variations of our model have also been used by researchers to study different macro development questions, such as those related to the implication of regulations and taxes on informal entrepreneurship (e.g., Antunes and Cavalcanti, 2007; Rauch, 1991) or the impact of credit market imperfections on development (e.g., Antunes et al., 2008; Buera et al., 2011). Our environment with Input-Output linkages could be adapted to investigate these and related issues.

[4] Baqaee (2018) considers a model with firm entry/exit with production networks, but the focus of his analysis is on the amplification of productivity shocks.

3. Model

The economy is static. There are N sectors producing intermediate goods indexed by $i \in S = \{1, \ldots, N\}$. Each intermediate good is used as a production input for a final consumption good and other intermediate goods. There is also a continuum of individuals of measure 1. The utility function of each individual is strictly increasing and strictly concave on the consumption of the final good.

Individuals are endowed with one unit of time that can be supplied to firms or used to manage a business. Each individual can open only a firm in one of the N sectors. Entrepreneurial productivity is heterogeneous. Specifically, each individual draws a vector $\mathbf{v}$ of managerial skills from a mutually independent Pareto distributions $\mu(v_i)$, with scale parameter 1 and shape parameter ξ. The production function of an entrepreneur in sector i with productivity v_i is given by

$$y_i(v_i) = a_i v_i l_i^{\theta_i} \prod_{j \in S} x_{ij}^{\sigma_{ij}}, \quad \text{with} \quad \theta_i \geq 0, \quad \sigma_{ij} \geq 0 \qquad \text{and} \quad \eta_i \equiv \theta_i + \sigma_i = \theta_i + \sum_{j \in S} \sigma_{ij} < 1,$$

where y_i denotes the output in sector i, l_i is the labor input, x_{ij} is the quantity of good j used for production of good i, and a_i is a Hicks-neutral productivity factor common to all firms in sector i.

A representative firm aggregates the sectoral goods into a single final consumption good according to

$$Q = \prod_{i \in S} c_i^{\psi_i}, \quad \text{with} \quad \psi_i \geq 0 \quad \text{and} \quad \sum_{i \in S} \psi_i = 1.$$

The price of this final good is normalized to 1.

Individuals who choose to be workers earn the equilibrium wage w. Entrepreneurs run a business in one of the sectors and make profits. The input choice is distorted by sectoral wedges. An entrepreneur in sector i pays a variable cost $(1 - \phi_i)$ per unit of revenue. This cost creates a wedge between the marginal productivity of each input used in the production of sector i and its rental price. Parameter ϕ_i affects directly the optimal scale of firms in sector i and distorts the optimal occupational choice.

An entrepreneur with productivity v_i in sector i takes prices as given and chooses l_i and x_{ij}, for $j \in S$, to maximize profits:

$$\pi_i \equiv \max \phi_i p_i a_i v_i l_i^{\theta_i} \prod_{j \in S} x_{ij}^{\sigma_{ij}} - w l_i - \sum_{j \in S} p_j x_{ij}. \tag{1}$$

Given the optimal input decisions in each sector i, an individual chooses to be an entrepreneur in sector i if and only if

$$\pi_i \equiv (1 - \eta_i) \phi_i p_i y_i(v_i) \geq \max_{j \neq i} \{w, \pi_j\}.$$

We assume that entrepreneurs are a small fraction of the population, and given the mutually independent Pareto distributions for entrepreneurial ability, the probability of a high entrepreneurial productivity in more than one sector is negligible. Therefore, the previous condition implies a unique productivity cutoff $\hat{v}_i$ for each sector.[5]

Revenues from distortions are equally rebated back to individuals. Therefore, the market clearing condition for good i is

$$Y_i = c_i + \sum_{j \in S} X_{ji}, \tag{2}$$

where Y_i and X_{ji} are respectively the aggregate output in sector i and the aggregate demand of good i from sector j. Finally, the labor market equilibrium condition requires

$$\sum_{i \in S} M_i + L = 1,$$

where M_i is the equilibrium share of entrepreneurs in sector i and L is the equilibrium total share of workers.

Under the assumptions that the entrepreneurs are a small fraction of the population and

[5]See the Appendix for details.

$\xi(1-\eta_i) > 1$ for any i, aggregate output in sector i can be approximated by

$$Y_i \approx A_i \left(L_i^{\theta_i} \prod_j X_{ij}^{\sigma_{ij}} \right)^{\frac{\xi}{1+\xi\eta_i}},$$

with

$$A_i = \left[a_i \left(\frac{(1-\eta_i)\phi_i p_i}{w} \right)^{\frac{\xi(1-\eta_i)-1}{\xi}} \left(\frac{\xi(1-\eta_i)}{\xi(1-\eta_i)-1} \right)^{\frac{1}{\xi}} \right]^{\frac{\xi}{1+\xi\eta_i}}.$$

The complete derivation is presented in the Appendix.

3.1. Equilibrium

Let the Domar weight of sector i be the industry's sales as a fraction of GDP:

$$\lambda(\phi)_i \equiv \frac{p_i Y_i}{Q}.$$

From the market clearing condition of all goods, it is possible to derive the vector of equilibrium Domar weights as a function of model primitives:[6]

$$\lambda(\phi) = \left(\mathbb{I}_N - \Sigma' \circ (1\phi') \right)^{-1} \psi. \tag{3}$$

The vector of Domar weights describes the centrality of each sector in the production network. The weights depend on the vector of final shares, ψ, and the linkages between sectors described by the matrix Σ. Economic distortions affect the Domar weights through these linkages. In particular, distortions in sector i reduce the sales of those other sectors supplying intermediate goods to i.

In the remaining of the paper, we will express our solutions in terms of total share of labor, $s_L(\phi) \equiv \frac{wL}{Q}$, shares of profit of each sector, $s_\Pi(\phi)_i \equiv \frac{\Pi_i}{Q}$, and total share of income, $s_T(\phi)$. Those objects are after tax shares and before rebate. The total share of income is the sum of $s_L(\phi)$ and

[6] See the Appendix for the full derivation.

$s_\Pi(\phi)_i$ across all sectors. We can write these shares as:

$$s_L(\phi) = \sum_j \theta_j \phi_j \lambda(\phi)_j,$$

$$s_\Pi(\phi)_i = \frac{\xi(1-\eta_i)-1}{\xi}\phi_i \lambda(\phi)_i \ \ \forall i \in S, \ \text{and}$$

$$s_T(\phi) = \sum_j \left[\theta_j + \frac{\xi(1-\eta_j)-1}{\xi}\right]\phi_j \lambda(\phi)_j.$$

We can observe that distortions affect theses shares directly and through the Domar weights. Therefore, we also define the following wedges, which present the labor and profit shares relative to an economy without distortions:

$$\tau_L(\phi) \equiv \frac{s_L(\phi)}{s_L(1)},$$

$$\tau_\Pi(\phi)_i \equiv \frac{s_\Pi(\phi)_i}{s_\Pi(1)_i}.$$

We can now characterize the equilibrium.

Proposition 3.1 *The equilibrium in the economy can be described by an aggregate production function*

$$\log Q = \sum_j \psi_j \log \psi_j + \lambda(1)' \log A(\phi) + \lambda(1)' \left\{ \frac{\xi(1-\eta)-1}{\xi} \circ \log M + \theta \log L \right\} \tag{4}$$

with

$$A(\phi)_i = a_i \phi_i \left(\frac{(1-\eta_i)}{s_\Pi(\phi)_i}\right)^{1-\eta_i} \left(\frac{\theta_i}{s_L(\phi)}\right)^{\theta_i} \prod_j \left(\sigma_{ji}\right)^{\sigma_{ji}}, \tag{5}$$

the equilibrium shares of entrepreneurs in each sector

$$M_i = \tau_\Pi(\phi)_i \frac{s_\Pi(1)_i Q}{w} = \frac{s_\Pi(\phi)_i}{s_T(\phi)}, \tag{6}$$

and the equilibrium share of workers

$$L = \tau_L(\phi) \frac{s_L(1) Q}{w} = \frac{s_L(\phi)}{s_T(\phi)}. \tag{7}$$

Proposition 3.1 is proved in the Appendix. Equation (4) describes the aggregate production as a function of two components.[7] The first one, $\sum_i \lambda(1)_i \log A(\phi)_i$, describes the allocation of workers and intermediate goods across sectors.[8] From now on, we will refer to it as the aggregate TFP component. The second component, $\sum_i \lambda(1)_i \left\{\frac{\xi(1-\eta_i)-1}{\xi} \circ \log M_i + \theta_i \log L\right\}$, represents the allocation of individuals between paid jobs and entrepreneurship. Given that the sum of all workers and entrepreneurs is fixed, this component describes misallocation at the extensive margin. In order to distinguish from the previous one, we will refer to the later as the occupational component.

The mass of firms in each sector is an endogenous object. Equation (6) describes the selection of entrepreneurs into sector i: in an undistorted economy the marginal opportunity cost from opening an additional firm, w, must approximately equalize the average profits, $\frac{s_\Pi(1)_i Q}{M_i}$.[9] The term $\tau_{\Pi,i}$ is therefore the deviation in the profit share of sector i relative to an undistorted economy. It represents a "between-sector" wedge distorting the allocation of entrepreneurs into sector i. Equation (7) describes the aggregate selection into the labor force: in an undistorted economy, the marginal cost of labor w must approximately equalize its marginal productivity, $\frac{s_L(1)Q}{L}$. The term τ_L is the deviation in total labor share and represents a wedge in the allocation of individuals between entrepreneurship and paid work. In the next sections, we will generally refer to the $N+1$ wedges τ (N "between-sector" wedges plus the "labor-entrepreneurship" wedge) as occupational wedges.

[7]There is also the component $\sum_j \psi_j \log \psi_j$, but this component is invariant to distortions and equilibrium objects.

[8]When $\phi_i = 1$, then TFP of sector i is only a function of primitives, i.e., $A(1)_i = a_i \left(\frac{(1-\eta_i)}{s_\Pi(1)_i}\right)^{1-\eta_i} \left(\frac{\theta_i}{s_L(1)}\right)^{\theta_i} \prod_j \left(\sigma_{ji}\right)^{\sigma_{ji}}$.

[9]This is an approximated solution, given our assumptions about the small size of entrepreneurs and $\xi(1-\eta_i) > 1$.

4. The economy with homogeneous entrepreneurs ($\xi \to \infty$)

For analytical purposes, we consider the limiting case in which the entrepreneurial ability distribution is degenerated at $v_i = 1$ or $\xi \to \infty$ and individuals have the same entrepreneurial productivity ($v_i = 1$). This is similar to the case of Bigio and La'o (2020) but in our environment there is also endogenous entry. We will also solve the full environment numerically.

We first derive the following Hulten's Theorem result:

Theorem 4.1 *The first-order effect of a sectoral productivity shock on aggregate TFP and total output is equal to the efficient-economy Domar weight of the sector:*

$$\frac{d\sum_j \lambda(1)_j \log A(\phi)_j}{d\log a_i} = \frac{d\log Q}{d\log a_i} = \lambda(1)_i. \tag{8}$$

The shock does not induce any change in the mass of firms.

The Theorem is proved in the Appendix. At efficiency, the effect of a sectoral productivity shock on TFP can be summarized by the Domar weight of the sector. In a Cobb-Douglas economy, once we depart from efficiency, the effect of the shock is still equal to the efficient-economy Domar weight. However, the actual industry's sales shares are modified by distortions.[10]

The Theorem also states that a productivity shock does not alter the allocation of workers and entrepreneurs. This is because the shock in sector i does not change the marginal conditions between paid work and entrepreneurship.

In order to analyze distortion shocks, we start by characterizing the effects of distortions on the equilibrium wage. In the Appendix, we formally prove that the efficient-economy Domar

[10] In our model with Cobb-Douglas production functions and no fixed costs, second-order effects from productivity shocks are irrelevant. This is no more the case when we introduce fixed technological costs in our numerical exercises. See Baqaee and Farhi (2019) for an analysis of second-order productivity shocks in a more general class of models.

weights are once more a sufficient statistic for the first-order effects of both sectoral productivities and distortions on the equilibrium wage, i.e.:

$$\frac{d\log w}{d\log a_i} = \frac{d\log w}{d\log \phi_i} = \lambda(1)_i\,. \tag{9}$$

The first equality follows from the fact that the equilibrium wage does not include rebated taxes, so it is identically affected by a change in a_i or ϕ_i.[11] The second equality is explained by the fact that, for given distortions, the equilibrium wage is a constant fraction of total output.

The total first-order effect of distortions on aggregate output is also a function of the efficient-economy Domar weights, but it is also adjusted by a negative term, capturing the change in the size of rebates:

$$\frac{d\log Q}{d\log \phi_i} = \lambda(1)_i - \frac{d\log s_T(\phi)}{d\log \phi_i}, \tag{10}$$

with

$$\frac{d\log s_T(\phi)}{d\log \phi_i} = \frac{(1-\sigma_i)\lambda(\phi)_i + \sum_j (1-\sigma_j)\phi_j \frac{d\lambda(\phi)_j}{d\phi_i}}{\sum_j (1-\sigma_j)\phi_j \lambda(\phi)_j}\phi_i. \tag{11}$$

From Equation (7), we also derive the effects of distortions ϕ_i on labor supply:

$$\frac{d\log L}{d\log \phi_i} = \frac{d\log s_L(\phi)}{d\log \phi_i} - \frac{d\log s_T(\phi)}{d\log \phi_i}. \tag{12}$$

The change in the mass of workers is given by the difference between the changes in the labor and total shares.

We can get further insights focusing on the efficient economy. From Equations (10) and (12) and considering the case in which the ϕ_is are close to one, we can state the following Proposition:

Proposition 4.2 *Starting from the efficient equilibrium, the first-order effect of a sectoral distortion shock on total output is 0. The shock changes the mass of firms according to:*

$$\left.\frac{d\log\left(\sum M_i\right)}{d\log \phi_i}\right|_{\phi=1} = \left(\frac{d\log s_T(\phi)}{d\log \phi_i} - \frac{d\log s_L(\phi)}{d\log \phi_i}\right)_{\phi=1} = \lambda(1)_i - \frac{\theta_i \lambda(1)_i}{\sum_j \theta_j \lambda(1)_j} - \frac{\sum_j \theta_j \frac{d\lambda(\phi)_j}{d\phi_i}}{\sum_j \theta_j \lambda(1)_j}. \tag{13}$$

[11] Bigio and La'o (2020) analyze the difference between rebated and wasted distortions.

The Proposition is proved in the Appendix. The first result of this Proposition is expected since $\phi_i = 1, \forall i \in S$, corresponds to the point in which $\log Q$ attains its maximum and the derivative of $\log Q$ with respect to each ϕ_i must all be equal to 0. Similar result is also shown in Baqaee and Farhi (2020b).

Equation (13) contains one of the main analytical contributions of this paper. The first term, $\lambda(1)_i$, represents the positive change in the profit share of entrepreneurs when distortions are reduced (higher ϕ). This positive effect is counteracted by the second and third terms, which capture the increase in the labor share. Lower distortions in sector i (a higher ϕ_i) reduces the number of entrepreneurs through the higher demand of workers by the sector. This direct effect depends on the labor intensity θ_i. Intuitively, if sector i is intensive in the use of labor, then distortions in this sector will reduce its labor demand and increase the number of entrepreneurs in the economy. A similar effect occurs through the other sectors in the production network. If the positive shock to sector i increases the Domar weights of labor intensive sectors, then the number of entrepreneurs is reduced. The opposite occurs if the shock reduces the size of these labor intensive sectors.

4.1. The welfare cost of distortions

Next, we analytically derive the output loss from distortions, which also corresponds to the welfare loss from such distortions. The log difference between total output with and without distortions is given by:

$$\log Q(\phi) - \log Q(1) = \sum_i \lambda(1)_i (\log \phi_i) - \log s_T(\phi). \tag{14}$$

We can easily characterize the direct effect of distortions in the first term of Equation (14). The second term, however, is highly nonlinear. We proceed by taking a second-order approximation of Equation (14) around the efficient equilibrium.

Proposition 4.3 *The second-order approximation around efficiency of the output loss from distortions is given by*

$$\log Q(\phi) - \log Q(1) \approx -\frac{1}{2}\left[Var(\log \tau_{\Pi}(\phi)) + \sum_i \sigma_i \lambda(1)_i (\log \tau_{\Pi,i})^2 - \sum_i \lambda(1)_i \left(\log \frac{\lambda(\phi)_i}{\lambda(1)_i}\right)^2\right] \quad (15)$$

with

$$Var(\log \tau_{\Pi}(\phi)) = \sum_i (1-\sigma_i)\lambda(1)_i (\log \tau_{\Pi,i})^2 - \left(\sum_i (1-\sigma_i)\lambda(1)_i \log \tau_{\Pi,i}\right)^2. \quad (16)$$

The derivation is reported in the Appendix. The total loss is given by two misallocation components. The first one, represented by $Var(\log \tau_{\Pi}(\phi))$, refers to the total allocation of individuals, both as entrepreneurs and workers in different sectors. It reminds the measurement of misallocation by Hsieh and Klenow (2009). The loss from distorting the optimal allocation of firms is given by the dispersion of the log wedges of each sector. Remember that those wedges represent how profit shares deviate from the efficient equilibrium. The second component, $\sum_i \sigma_i \lambda(1)_i (\log \tau_{\Pi,i})^2 - \sum_i \lambda(1)_i \left(\log \frac{\lambda(\phi)_i}{\lambda(1)_i}\right)^2$, refers to the propagation of such distortions by the production network and it is zero when $\sigma_i = 0$ for all i.

Next, we want to decompose the total output loss into the aggregate TFP loss and the loss generated by the occupational component. First, we characterize the TFP loss of distortions.

Proposition 4.4 *The second-order approximation around efficiency of the TFP loss from distortions is given by*

$$\log A(\phi) - \log A(1) \approx -\frac{1}{2}\left[s_L(1) Var_L(\log \tau_{\Pi}(\phi)) + \sum_i \sigma_i \lambda(1)_i (\log \tau_{\Pi,i})^2 - \sum_i \lambda(1)_i \left(\log \frac{\lambda(\phi)_i}{\lambda(1)_i}\right)^2\right] \quad (17)$$

with

$$Var_L(\log \tau_{\Pi}(\phi)) = \sum_i \frac{\theta_i \lambda(1)_i}{s_L(1)} (\log \tau_{\Pi,i})^2 - \left(\sum_i \frac{\theta_i \lambda(1)_i}{s_L(1)} \log \tau_{\Pi,i}\right)^2. \quad (18)$$

The steps to obtain the solution are reported in the Appendix. The variance component here is related to the allocation of workers - depends on θ_i and the labor share. The second part referring to the allocation of intermediate inputs is identical to the one presented in Proposition 4.3.

Finally, by subtracting Equation (17) from Equation (15) we can identify the loss associated to the misallocation of individuals between labor and entrepreneurship in the different sectors.

Proposition 4.5 *The occupational loss can be represented by the variance of the log occupational wedges, $\tau_{\Pi,i}$ and τ_L:*

$$[\log Q(\phi) - \log Q(1)] - [\log A(\phi) - \log A(1)] \approx -\frac{1}{2} Var_{Occ}(\log \tau(\phi)), \tag{19}$$

with

$$Var_{Occ}(\log \tau(\phi)) = \sum_i s_\Pi(1)_i (\log \tau_{\Pi,i})^2 + s_L(1)(\log \tau_L)^2 - \left(\sum_i s_\Pi(1)_i \log \tau_{\Pi,i} + s_L(1) \log \tau_L \right)^2. \tag{20}$$

Note that the object $Var_{Occ}(\log \tau_\Pi(\phi))$ includes not just the sectoral entrepreneurial wedges $\tau_{\Pi,i}$ - how profit shares deviates relative to the efficient economy, but also the labor wedge τ_L - how the labor share deviates from the economy without distortions.

4.2. Network linkages and misallocation

In order to evaluate the role of network linkages in propagating distortions for a given network structure Σ, we define an equivalent horizontal one.

Definition 4.6 *The equivalent horizontal economy of an economy with a given production network structure Σ is represented by the following characteristics:*

1. *no input-output linkages, i.e. $\sigma^H_{i,j} = 0 \;\forall\; i, j$;*
2. *same profit shares at efficiency: $(1-\eta_i)\lambda(1)_i = (1-\theta^H_i)\psi^H_i \;\forall\; i$; and*

3. *same labor income shares at efficiency:* $\theta_i \lambda(1)_i = \theta_i^H \psi_i^H \ \forall\ i$.

Given conditions 2 and 3, then the allocation of workers and entrepreneurs is identical at efficiency in the network and the equivalent horizontal economies. The three conditions also imply:

$$(1-\sigma_i)\lambda(1)_i = \psi_i^H,$$

for any i. Note that, for a given horizontal structure identified by θ^H and ψ^H, there exist infinite combinations of matrices Σ, shares θ, and ψ respecting the three conditions above.

Having defined the equivalent horizontal structure of a network, we can measure how distortions are amplified through the network.

Proposition 4.7 *The TFP and occupational loss from distortions in the equivalent horizontal structure of a given network economy Σ are summarized by*

$$\frac{1}{2} Var_L(\log\phi) \quad \textit{and} \quad \frac{1}{2} Var_{Occ}(\log\phi), \ \ \textit{respectively.}$$

With

$$\frac{1}{2} Var_L(\log\phi) = \sum_i \frac{\theta_i \lambda(1)_i}{s_L(1)} (\log\phi_i)^2 - \left(\sum_i \frac{\theta_i \lambda(1)_i}{s_L(1)} \log\phi_i\right)^2$$

and

$$\frac{1}{2} Var_{Occ}(\log\phi) = \sum_i s_\Pi(1)_i (\log\phi_i)^2 + s_L(1)\left(\sum_i \frac{\theta_i \lambda(1)_i}{s_L(1)} \log\phi_i\right)^2 - \left(\sum_i (1-\sigma_i)\lambda(1)_i \log\phi_i\right)^2.$$

In a horizontal economy, the dispersion of original distortions is a sufficient object to describe the TFP loss. The ϕ_is directly distort the optimal allocation of workers and entrepreneurs through the reduction in firm revenues. The presence of input-output linkages alters this result through two channels. First, TFP losses are obviously amplified through the additional misallocation of intermediate inputs. This is captured by the component $\sum_i \sigma_i \lambda(1)_i (\log\tau_{\Pi,i})^2 - \sum_i \lambda(1)_i \left(\log\frac{\lambda(\phi)_i}{\lambda(1)_i}\right)^2$ appearing in (15) and (17). Second, in a network economy, the allocation of workers and entrepreneurs is indirectly influenced by the variation in relative centrality of a sector. This effect is captured by the change in the Domar weights. Specifically, the occupational loss in (19) can be expressed as:

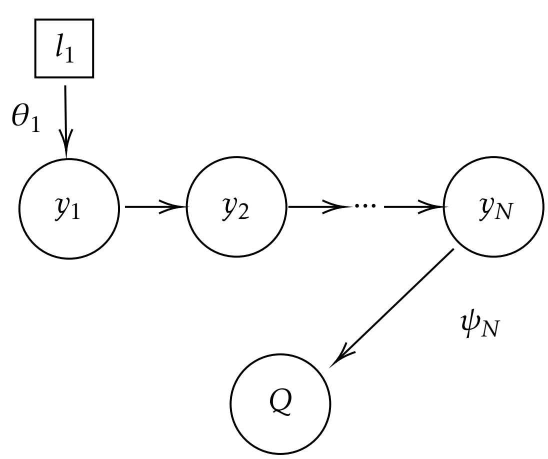
$$\frac{1}{2}\left[Var_{Occ}(\log\phi) + Var_{Occ}\left(\log\frac{\lambda(\phi)}{\lambda(1)}\right) + 2Cov_{Occ}\left(\log\phi, \log\frac{\lambda(\phi)}{\lambda(1)}\right)\right].$$

The term $Var_{Occ}\left(\log\frac{\lambda(\phi)}{\lambda(1)}\right) + 2Cov_{Occ}\left(\log\phi, \log\frac{\lambda(\phi)}{\lambda(1)}\right)$ may be positive or negative, amplifying or diminishing the direct effect of distortions. In particular, a direct effect of distortion ϕ_i to a sector i may be counteracted by the reduction in sales of the main suppliers of i.

In order to get some additional intuitions about the role of linkages in amplifying or diminishing the effect of distortions, in the next subsections we analyze two simple network structures. As it will be clear, network linkages amplify losses when distortions hit more upstream sectors.

4.3. The case of a pure vertical economy

Figure 1: A pure vertical network



Let us consider the example of a pure vertical economy depicted in Figure 1. Labor is used as an input only by the first sector ($\theta_1 > 0$ and $\theta_j = 0$ for $j > 1$). All remaining sectors are chained in a sequence, until a last intermediate sector that supplies inputs to the final consumption good firms ($\psi_N = 1$). In such a network structure, there cannot be any misallocation of intermediate goods and workers across sectors: the variance $Var_L(\log \tau_{\Pi}(\phi))$ and the component $\sum_i \sigma_i \lambda(1)_i (\log \tau_{\Pi,i})^2 - \sum_i \lambda(1)_i \left(\log \frac{\lambda(\phi)_i}{\lambda(1)_i}\right)^2$ are always equal to 0. The reason is that only one sector uses labor and each sector uses the inputs produces by only one sector. Therefore, the difference in welfare loss between the network economy and its equivalent horizontal only depends on the variances of occupational wedges.

For simplicity, let us suppose we only distort one sector at the time. In such a simple structure, it is easy to show that the Domar weights of the economy are unaffected if we distort the first (more upstream) sector ($\phi_1 < 1$ and $\phi_j = 1$ for $j > 1$).[12] Since this sector does not purchase inputs from any other sector, there is no change in the relative industry sales. In this scenario, the welfare loss in the network economy and the equivalent horizontal economy are identical and equal to $\frac{1}{2}Var(\log \phi)$.

Results are different if we distort downstream sectors. In the extreme case of distortions only in the last sector ($\phi_N < 1$ and $\phi_j = 1$ for $j < N$), only the Domar weights of upstream sectors would be affected, so that $\frac{1}{2}Var(\log \tau_{\Pi}(\phi)) = 0$. Intuitively, by distorting the most down-

[12]We can use the Neumann series to analytically solve for

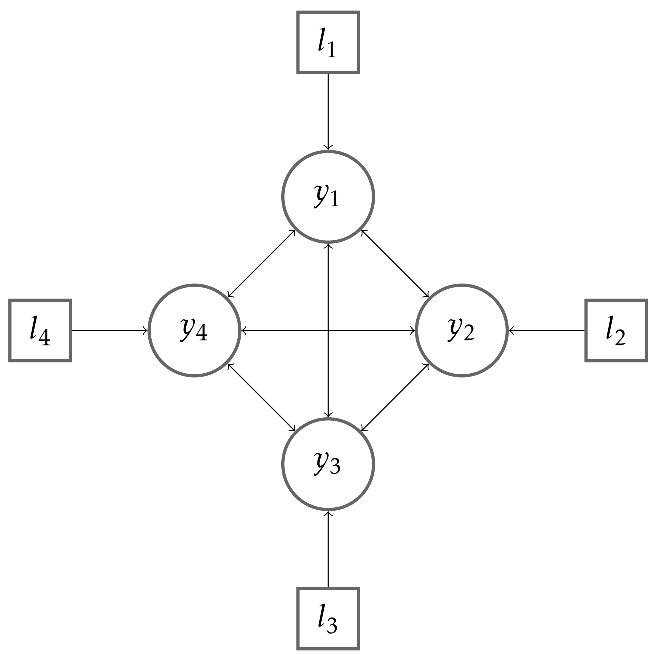
$$\left(\mathbb{I}_N - \Sigma' \circ (1\phi')\right)^{-1}\psi = \left(\mathbb{I}_N - \begin{bmatrix} 0 & 0 & \dots & 0 \\ \phi_2\sigma_{21} & 0 & \dots & 0 \\ 0 & \phi_3\sigma_{32} & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & \dots & \phi_N\sigma_{N(N-1)} & 0 \end{bmatrix}'\right)^{-1} \begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix} = \begin{bmatrix} \prod_{j>1}\phi_j \prod_{j>1}\sigma_{j(j-1)} \\ \prod_{j>2}\phi_j \prod_{j>2}\sigma_{j(j-1)} \\ \prod_{j>3}\phi_j \prod_{j>3}\sigma_{j(j-1)} \\ \vdots \\ 1 \end{bmatrix}.$$

The solution does not depend on ϕ_1, so the weights do not change if we only distort the first sector. If instead we only distort the last sector, the weights are $\begin{bmatrix} \phi_N \prod_{j>1}\sigma_{j(j-1)} \\ \phi_N \prod_{j>2}\sigma_{j(j-1)} \\ \phi_N \prod_{j>3}\sigma_{j(j-1)} \\ \vdots \\ 1 \end{bmatrix}$, so $\tau_{\Pi}(\phi) = \phi \circ \frac{\lambda(\phi)}{\lambda(1)} = \phi_N \begin{bmatrix} 1 \\ 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix}$.

stream sector we indirectly reduce the sales of the previous sectors and offset the outflow of entrepreneurs from sector N. In this scenario, the loss in the network economy would be lower than in the equivalent horizontal one (which is still equal to $\frac{1}{2}Var(\log\phi)$).

4.4. The case of a symmetric economy

Figure 2: A symmetric economy



Another simple production network economy, depicted in Figure 2, is one in which all sectors are identically connected and have the same weights in the undistorted economy. Specifically, let us consider the case in which $\sigma_{ij} = \sigma$ for any i and j, and $\psi_i = \frac{1}{N}$ for any i. In this economy, distorting any one of the sectors will induce exactly the same change in all Domar weights. Therefore, the dispersion in wedges τ_Π is equal to the dispersion of original distortions ϕ, which implies that the occupational loss is the same in the network and equivalent horizontal economies. However, the total output loss is still larger because of the misallocation of intermediate inputs captured by the terms $\sum_i \sigma_i \lambda(1)_i (\log \tau_{\Pi,i})^2 - \sum_i \lambda(1)_i \left(\log \frac{\lambda(\phi)_i}{\lambda(1)_i}\right)^2$.

5. Quantitative analysis

In this section, we calibrate our model using US industry data. We use the environment with heterogeneous abilities presented in Section 3 as our main reference. In order to better match the number of firms from the data, we also consider a modified version in which we add fixed technological entry costs. Specifically, we assume that an individual who wants to open a business in sector i must pay a cost f_i in units of sector i's output. The derivation of the equilibrium for this second model is presented in the Appendix.

We consider a seven-sectors economy. The sectors are: 1) Agriculture, Utilities and Mining (AMU); 2) Construction; 3) Manufacturing; 4) Trade; 5) Transportation; 6) Finance, Insurance, and Real Estate (FIRE)[13]; and 7) Other[14]. We normalize the productivity parameters a_i to unity. The remaining parameters to be calibrated are: (i) intermediate input shares, σ_{ij} (49 parameters); (ii) profit shares , $(1-\eta_i)$, and given σ_{ij} we can identify θ_i (7 parameters); (iii) final good shares ψ_i (7 parameters); (iv) the shape parameter of the entrepreneurial ability distribution, ξ (1 parameter); and (v) sectoral distortions, ϕ_i (7 parameters). Therefore, there are 71 parameters to be set.[15]

We calibrate intermediate input shares σ_{ij} using data from the input-output tables of the Bureau of Economic Analysis (BEA).[16] We calibrate $(1-\eta_i)$ – and therefore θ_i – using the share of Gross Operating Surplus as a fraction of total industry output. Similarly we calibrate final good shares ψ_is, using the share of final use of industry outputs. In order to calibrate ξ, we

[13]Since we do not explicit modeled financial intermediaries we choose to include financial services in the list of calibrated sectors.

[14]Information, Business services, Education, and Entertainment.

[15]For the economy with fixed technological costs, there are 7 additional parameters to be calibrated.

[16]All data refers to the year 2019.

target the share of workers hired by the 10% largest firms. Following Buera et al. (2011), the target is number 69% and our calibrated value are $\xi = 12.38$, for the main model, and $\xi = 8.87$, for the model with fixed costs. Finally, for our model with fixed costs, we estimate those costs denoted by f_is to match the measure of entrepreneurs (or establishment) in each sector out of the total population. These data are from the Bureau of Labor Statistics (BLS).

For distortions ϕ_is, we use independent estimates computed by De Loecker et al. (2020). We assume markups are the only wedges in the economy. The authors use a "production function approach" to compute markups at the industry level. The distortions are: $\phi_1 = 0.31$, $\phi_2 = 0.45$, $\phi_3 = 0.54$, $\phi_4 = 0.43$, $\phi_5 = 0.44$, $\phi_6 = 0.65$, $\phi_7 = 0.55$. Additional details about how they derive those distortions are reported in the Data and Calibration Appendix.

The values from our calibration of the main model are reported in Table 2 in the Data and Calibration Appendix. This table also reports the computed parameters of the equivalent horizontal economy. See also this appendix for the estimated input-output matrix and the calibrated values of the model with fixed costs. Given parameter values, we can compute the loss from distortions.

Table 1 reports the total GDP, TFP, and occupational losses in the main model and the model with fixed technological costs. In both cases, we compare the losses to those of the equivalent horizontal economy. Network linkages always amplify the effect of distortions and this amplification is larger for the TFP component.[17] The cost from misallocating entrepreneurs is only slightly larger in the network economy of our main model than in the equivalent horizontal economy. Through the reduction in demand of intermediate inputs, the direct effect of lowering profits in one sectors is partially offset by the profit decrease in other sectors. This re-balancing effect is less strong in the model with fixed costs. Here the weight of a sector also depends on the production of goods used for establishing a firm. This naturally amplifies the

[17]Observe that, comparing to Baqaee and Farhi (2020b), our TFP loss from distortions is about 2.3 times their reported value in a Cobb-Douglass case. The environments are, however, different. We have heterogeneity within sectors and an endogenous mass of firms in each sector.

direct effect of distorting one sector and the extensive margin appears to be a much stronger mechanism.

Table 1: Economic losses from distortions (baseline) relative to the efficient (undistorted) economy

	Network economy	Equivalent horizontal
Main Model		
GDP	-25.7%	-2.5%
TFP	-23.9%	-1%
Occupational	-1.8%	-1.5%
Model with fixed costs		
GDP	-50.1%	-8.3%
TFP	-24.8%	-1%
Occupational	-25.3%	-7.3%

The main takeaways from Table 1 are: (i) the network structure is quantitatively important to propagate sectoral distortions - this is a common result in the network production literature (e.g, Baqaee and Farhi (2020b) and Bigio and La'o (2020)); and (ii) the occupational choice (extensive margin) or the endogenous entry of firms is also an important mechanism to amplify distortions in an economy with Input-Output linkages, specially in the presence of fixed production costs.

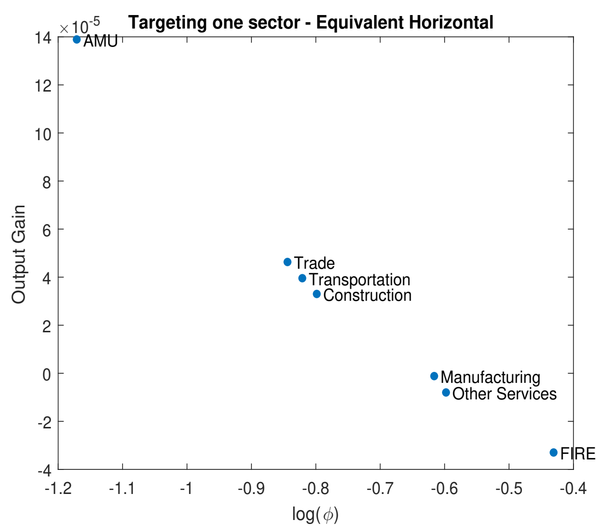
6. The aggregate output effects from entry subsidies

We now study the effect of entry subsidies in our main calibrated model. We run few exercises trying to address the following questions: what is the output gain/loss of an entry subsidy program (involving the same total transfer) targeting one sector at the time? What is the best statistics to identify which sectors should be targeted?

We add to our model a fixed (positive) subsidy to any individual who open a business in a targeted sector i. The subsidy is financed with lump-sum taxes. Therefore, in the absence of distortions ϕ, it would definitely misallocate resources and reduce output. However, as distortions ϕ may create a barrier for entrepreneurship in specific sectors, the subsidy might relax this barrier.

We calibrate the size of the aggregate subsidy so that the total equilibrium tax transfer is always equal to 0.01% of the initial aggregate GDP. Therefore, the aggregate size of the entry subsidy is the same relative to the baseline GDP. This allows comparison of the policy of targeting different sectors.

Figure 3: Output gain/loss from subsidizing entry in the equivalent baseline horizontal. The horizontal axis is the log of distortions by economy sectors. The vertical axis displays the percentage deviation of output of the economy with subsidy relative to the baseline output of the horizontal economy. Each dot in the graph corresponds to the change in aggregate output of subsidizing entry only in the respective sector of production. The cost of the policy always amounts to 0.01% of the baseline GDP.



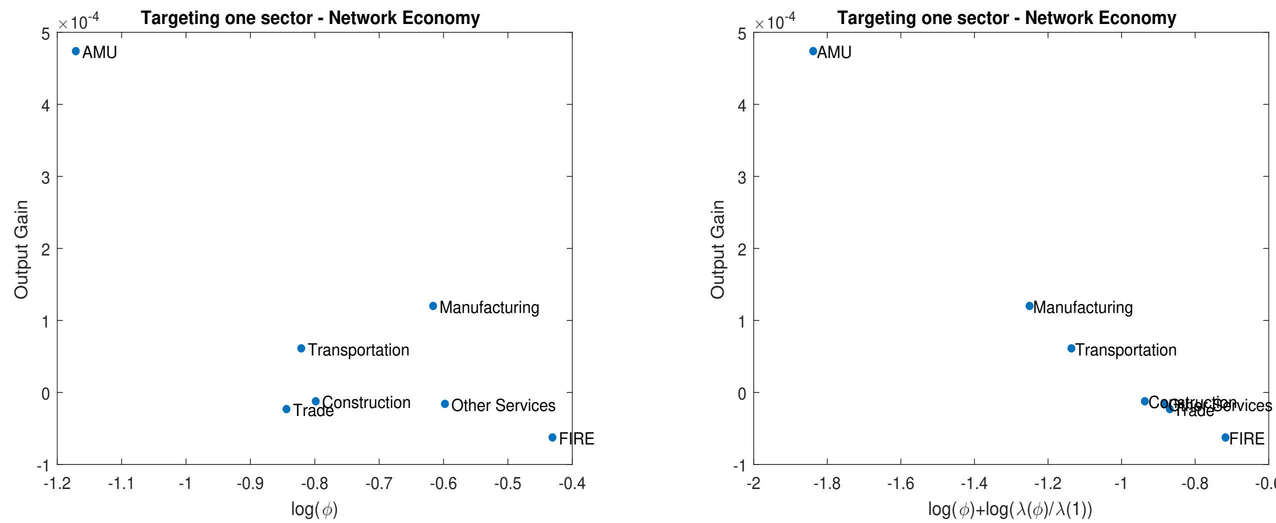
We start considering the effects of entry subsidy on aggregate output in the equivalent horizontal economy of our main calibrated model or baseline model. Figure 3 shows the percentage deviation in output relative to the baseline calibrated economy from subsidizing one sector at the time in the economy without Input-Output linkages. The sectors are ranked in the x-axis from the most distorted to the least distorted. Subsidies increase output only when they target the four most distorted sectors. In particular, distortions ϕ are a sufficient statistic to rank the sectors in terms of gains from entry subsidies.

Results are qualitatively and quantitatively different once we consider our original network economy. Figure 4(a) presents the relation between distortions and output gains from entry subsidies when the observed production network is taken into account. While AMU once more is rightly identified as the sector which generates the highest output rise, distortions are not anymore a good measure to rank the remaining sectors. In particular, while Trade is the second most distorted sector, a targeted entry subsidy program for this sector would actually increase misallocation and reduce aggregate output. This is different when entry in manufacturing is subsidized, which is the fifth most distorted sector. In this case, aggreagate output would rise. Interestingly, in the horizontal economy, subsidizing entry in manufacturing would decrease aggregate output.

The measure of occupational loss presented in Equation (19) helps us to understand the difference of the results of subsidizing entry in the horizontal economy and in the production network economy. The dispersion of wedges τ summarizes this loss. An entry subsidy program should target sectors with the largest profit share reduction. In Figure 4(b), we represent the same output loss/gain of an entry subsidy program against $\log \tau_{\Pi,i} = \log \phi_i + \log\left(\frac{\lambda(\phi)}{\lambda(1)}\right)$. The gains are now monotonically ranked. Different from the horizontal economy, Manufacturing is now identified as the second best sector for an entry targeted subsidy.

Consequently, in a network economy, measuring direct distortions may not be sufficient to design a subsidy program for firms' entry. The profit losses by sector are a superior measure to identify which industries should be targeted. A correct computation of such measure requires not only knowledge of distortions but also information on the production network structure.

Figure 4: Output gain/loss from subsidizing entry in in the baseline network economy. The horizontal axis in graph (a) is the log of distortions by economy sectors, $\log(\phi_i)$; The horizontal axis in graph (b) is the log of profit share by sector relative to the efficient economy, $\log(\tau_{\Pi,i}) = \log(\phi_i) + \log\left(\frac{\lambda(\phi)}{\lambda(1)}\right)$. The vertical axis displays the percentage deviation of output of the economy with subsidy relative to the baseline output of the network economy. Each dot in the graph corresponds to the change in aggregate output of subsidizing entry only in the respective sector of production. The cost of the policy always amounts to 0.01% of the baseline GDP.



(a) x-axis: $\log(\phi_i)$

(b) x-axis: $\log(\tau_{\Pi,i})$

7. Conclusions

We studied the effect of distortions in a multisector general equilibrium model with production network and endogenous occupational choice. Individuals can be workers or they can run a business in one of the production sectors. The environment is an extension of Lucas span of control model (c.f., Lucas Jr, 1978), which has been used to study different issues and questions in the macro economic development literature. At the aggregate level, distortions reduce TFP by misallocating labor and intermediate inputs. In addition, they also misallocate the occupational decision of individuals manifested into two additional wedges: a "labor-entrepreneurship" wedge and a "between-sector" wedge.

We showed that shocks to sectoral distortions induce a reallocation of individuals between the labor force and entrepreneurship in a non-trivial manner. A raise in distortions in a sector i reduces (increases) the mass of firms in the same sector if the sector has low (high) labor intensity. In addition, the mass of firms in other sectors is also reduced if they supply inputs to sector i and they have low labor intensity.

We derived the output loss from distortions, identifying the role of sectoral linkages and endogenous firm entry. Network linkages amplify (diminish) losses if distortions hit more upstream (downstream) sectors. We show that endogenous entry is an important mechanism in evaluating the output loss of distortions. This is particularly relevant when fixed technological costs are present. We also studied the effects of entry subsidies in a calibrated version of our model. We found that subsidies should target sectors suffering large loss in profits from distortions and not necessarily the most distorted sectors. Sectoral distortions are usually not a good measure to represent total profit losses, as they do not include the indirect effect of reduction in intermediate goods demand, which depends on the production network.

We believe that our framework could be extended to investigate specific distortions in models with Input-Output linkages and entrepreneurial decisions, such as credit market imperfections, entry regulations, and taxes.

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A. Mathematical Appendix

A.1. Aggregate Sectoral Output

We derive the equilibrium sectoral outputs in the case with technological fixed costs f_i. By setting $f_i = 0$, we obtain the results presented in Section 2. The first order conditions from the firm's problem (1) are:

$$l_i = \theta_i \phi_i \frac{p_i y_i(v_i)}{w} \tag{21}$$

and

$$x_{ij} = \sigma_{ij} \phi_i \frac{p_i y_i(v_i)}{p_j}, \tag{22}$$

which implies:

$$p_i y_i(v_i) = \left[p_i a_i v_i \left(\frac{\theta_i \phi_i}{w} \right)^{\theta_i} \prod_j \left(\frac{\sigma_{ij} \phi_i}{p_j} \right)^{\sigma_{ij}} \right]^{\frac{1}{1-\eta_i}}.$$

The aggregate output in sector i is

$$Y_i \equiv \int_{\hat{v}_i}^{\infty} \int_1^{\hat{v}_{-i}(v_i)} a_i v_i \left(l_i(v_i) \right)^{\theta_i} \prod_j \left(x_{ij}(v_i) \right)^{\sigma_{ij}} \mu(dv) - f_i \int_{\hat{v}_i}^{\infty} \int_1^{\hat{v}_{-i}(v_i)} \mu(dv).$$

We define:

$$L_i \equiv \int_{\hat{v}_i}^{\infty} \int_1^{\hat{v}_{-i}(v_i)} l_i \mu(dv),$$

$$X_{ij} \equiv \int_{\hat{v}_i}^{\infty} \int_1^{\hat{v}_{-i}(v_i)} x_{ij} \mu(dv),$$

and

$$V_i \equiv \left[\int_{\hat{v}_i}^{\infty} \int_1^{\hat{v}_{-i}(v_i)} v_i^{\frac{1}{1-\eta_i}} \mu(dv) \right]^{1-\eta_i}.$$

Therefore, we can re-express the aggregate output as:

$$Y_i = a_i L_i^{\theta} \prod_j X_{ij}^{\sigma_{ij}} V_i - f_i \int_{\hat{v}_i}^{\infty} \int_1^{\hat{v}_{-i}(v_i)} \mu(dv). \tag{23}$$

The entrepreneurial productivity $\hat{v}_i$ of the marginal entrepreneur in sector i is such that:

$$w = (1-\eta_i)\,\phi_i \hat{v}_i^{\frac{1}{1-\eta_i}} \left[p_i a_i \left(\frac{\theta_i \phi_i}{w}\right)^{\theta_i} \prod_j \left(\frac{\sigma_{ij}\phi_i}{p_j}\right)^{\sigma_{ij}} \right]^{\frac{1}{1-\eta_i}} - p_i f_i. \quad \forall i \in S.$$

In order to derive our analytical results, it is convenient to define the following object:

$$\kappa(\phi)_i \equiv \frac{w}{w + p_i f_i} \tag{24}$$

By assuming that the entrepreneurs are a small measure of the population, we obtain

$$\hat{v}_i = \left[\frac{w}{(1-\eta_i)\phi_i \kappa(\phi)_i p_i a_i} \frac{\left(\frac{\xi(1-\eta_i)}{\xi(1-\eta_i)-1}\right)^{\eta_i}}{L_i^{\theta_i} \prod_j X_{ij}^{\sigma_{ij}}} \right]^{\frac{1}{1+\xi\eta_i}},$$

where we used[18]

$$V_i \approx \left[\int_{\hat{v}_i}^{\infty} v_i^{\frac{1}{1-\eta_i}} \mu(dv) \right]^{1-\eta_i} = \left[\frac{\xi(1-\eta_i)}{\xi(1-\eta_i)-1} \hat{v}_i^{\frac{1}{1-\eta_i}-\xi} \right]^{1-\eta_i}.$$

Finally, by plugging V_i into (23) and using $\int_{\hat{v}_i}^{\infty} \int_1^{\hat{v}_{-i}(v_i)} \mu(dv) \approx \hat{v}_i{}^{-\xi}$, we obtain:

$$Y_i \approx A_i \left(L_i^{\theta_i} \prod_j X_{ij}^{\sigma_{ij}} \right)^{\frac{\xi}{1+\xi\eta_i}}, \tag{25}$$

with

$$A_i = \left(\frac{\xi(1-\eta_i)}{\xi(1-\eta_i)-1} \frac{w}{(1-\eta_i)\phi_i\kappa_i p_i} - f_i \right) \left[a_i \frac{(1-\eta_i)\phi_i\kappa_i p_i}{w} \left(\frac{\xi(1-\eta_i)-1}{\xi(1-\eta_i)} \right)^{\eta_i} \right]^{\frac{\xi}{1+\xi\eta_i}}.$$

A.2. Computing Domar Weights

We derive the solution for Domar Weights in the general model with fixed costs $f_i > 0$. The Domar weight of sector i corresponds to the industry's sales as a fraction of GDP, i.e.,

$$\frac{p_i Y_i}{Q}.$$

[18] We also assume $\xi(1-\eta_i) > 1$ in order to have a finite integral.

Observe that we can rewrite the market clearing conditions for intermediate good j as:

$$c_j + \sum_{i \in S} X_{ij} = Y_j. \tag{26}$$

Multiplying both sides by p_j:

$$p_j c_j + \sum_{i \in S} p_j X_{ij} = p_j Y_j. \tag{27}$$

By firms' first order condition:

$$p_j c_j + \sum_{i \in S} \phi_i \sigma_{ij} p_i (Y_i + M_i f_i) = p_j Y_j. \tag{28}$$

Using the fact that $p_i = \psi_i Q / c_i$, which follows from final producers' first order condition, we have:

$$\psi_j Q + \sum_{i \in S} \phi_i \sigma_{ij} p_i (Y_i + M_i f_i) = p_j Y_j \quad (\div Q), \tag{29}$$

$$\psi_j + \sum_{i \in S} \phi_i \sigma_{ij} \left(\frac{\psi_i Y_i}{c_i} + \frac{p_i M_i f_i}{Q} \right) = \frac{\psi_j Y_j}{c_j}. \tag{30}$$

Now, define $v_i = \frac{\psi_i Y_i}{c_i}$. Then:

$$\psi + \Sigma' [\phi \circ (v + F)] = v \Longrightarrow v^* = \left(\mathbb{I}_S - \Sigma' \circ (\mathbf{1} \phi') \right)^{-1} \left[\psi + \left(\sum_i \phi_i \sigma_{ij} F(\phi, z)_i \right) \right] =: \lambda(\phi, z), \tag{31}$$

where $\lambda_i = p_i Y_i / Q$ is the Domar weight of sector i, Σ is the firm I-O matrix, and $F(\phi)_i \equiv \frac{p_i M_i f_i}{Q}$.

A.3. Proof of Proposition 3.1

The derivation below applies to the general case with $f_i > 0$. By setting $f_i = 0$, we obtain the results presented in Proposition 3.1. Aggregating both sides of (21) and (22), we obtain:

$$L_i = \theta_i \phi_i \frac{p_i}{w} (Y_i + M_i f_i) = \theta_i \phi_i \left[\lambda(\phi)_i + F(\phi)_i \right] \frac{Q}{w},$$

$$X_{ij} = \sigma_{ij} \phi_i \frac{p_i}{p_j} (Y_i + M_i f_i) = \sigma_{ij} \phi_i \left[\lambda(\phi)_i + F(\phi)_i \right] \frac{Q}{p_j}.$$

The share of entrepreneurs in sector i can be approximated as:

$$M_i = \int_{\hat{v}_i}^{\infty} \int_{1}^{\hat{v}_{-i}(v_i)} \mu(dv) \approx \hat{v}_i^{-\xi} = \frac{Y_i}{\frac{\xi(1-\eta_i)}{\xi(1-\eta_i)-1} \frac{w}{(1-\eta_i)\phi_i \kappa_i p_i} - f_i}$$

$$= \frac{\xi(1-\eta_i)-1}{\xi} \phi_i \kappa(\phi)_i \left[\lambda(\phi)_i + F(\phi)_i\right] \frac{Q}{w}.$$

Using the labor market clearing condition, $\sum_i M_i + \sum_i L_i = 1$, we obtain the equilibrium solution for M_i and L as presented in (6) and (7).

Using the definition of Domar weights and $p_i c_i = \psi_i Q$, we can rewrite (25) as:

$$\frac{c_i}{\psi_i} = a_i \phi_i \left(\frac{(1-\eta_i)\kappa(\phi)_i}{s_\Pi(\phi)_i} \right)^{1-\eta_i} \left(\frac{\theta_i}{s_L(\phi)} \right)^{\theta_i} M_i^{\frac{\xi(1-\eta_i)-1}{\xi}} L^{\theta_i} \prod_j \left(\sigma_{ij} \frac{c_j}{\psi_j} \right)^{\sigma_{ij}}. \tag{32}$$

Taking logs and plugging into

$$\log Q = \sum_i \psi_i \log \psi_i + \sum_i \psi_i \log \left(\frac{c_i}{\psi_i} \right), \tag{33}$$

we obtain the solution for the aggregate production function.

A.4. Computing Prices

Let us plug the inputs' first order conditions

$$x_{ij} = \sigma_{ij} \phi_i \frac{p_i y_i}{p_j} \tag{34}$$

into the production function:

$$p_i y_i = (p_i a_i)^{\frac{1}{1-\sigma_i}} \phi_i^{\frac{\sigma_i}{1-\sigma_i}} \prod_j \left(\frac{\sigma_{ij}}{p_j} \right)^{\frac{\sigma_{ij}}{1-\sigma_i}} l_i^{\frac{\theta_i}{1-\sigma_i}} \tag{35}$$

From labor first order condition

$$l_i = \theta_i \phi_i \frac{p_i y_i}{w} = \theta_i \phi_i \frac{\lambda(\phi)_i}{M_i} \frac{Q}{w} \tag{36}$$

we can write

$$p_i = \frac{1}{\phi_i a_i}\left(\frac{w}{\theta_i}\right)^{1-\sigma_i}\prod_j\left(\frac{p_j}{\sigma_{ij}}\right)^{\sigma_{ij}} l_i^{1-\eta_i} \tag{37}$$

From (36) and

$$M(\phi)_i = \frac{s_\Pi(\phi)_i}{s_T(\phi)}$$

we obtain

$$l_i = \frac{\theta_i}{(1-\eta_i)} \tag{38}$$

We define

$$B(\phi,w)_i := \log\left[\frac{w^{1-\sigma_i}}{\phi_i a_i}\left(\frac{1}{\theta_i}\right)^{\theta_i}\prod_j\left(\frac{1}{\sigma_{ij}}\right)^{\sigma_{ij}}\left(\frac{1}{(1-\eta_i)}\right)^{1-\eta_i}\right] \tag{39}$$

Therefore, the solution for the vector of p is

$$\log p = \left(\mathbb{I}_S - \Sigma\right)^{-1} B(\phi,w) \tag{40}$$

A.5. Proof of Theorem 4.1

Let us plug Equations (6) and (7) into the aggregate production function (4):

$$\left[1 - \sum_j\left(1-\sigma_j\right)\lambda(1)_j\right]\log Q = \sum_j \psi_j \log\psi_j +$$
$$\lambda(1)'\left\{\log A(\phi) + (1-\eta)\circ\log\left[(1-\eta)\phi\lambda(\phi) - \log w\right] + \theta\left[\log\left(\sum_j \theta_j\phi_j\lambda(\phi)_j\right) - \log w\right]\right\}. \tag{41}$$

We now plug (5) and use $\sum_j\left(1-\sigma_j\right)\lambda(1)_j = 1$ to obtain a solution for w:

$$\log w \;=\; \sum_j \psi_j\log\psi_j \;+\; \lambda(1)'\{\log a + \log\phi + (1-\eta)\circ\log(1-\eta) + \theta\log\theta + \Sigma'\circ\log\Sigma'\mathbf{1}_N\}. \tag{42}$$

From the last equation, we observe that $\frac{d\log w}{d\log a_i} = \frac{d\log w}{d\log\phi_i} = \lambda(1)_i$. By plugging 6 and 7 into the population clearing condition, we obtain the relation between Q and w,

$$\log Q = \log w - \log\left[\sum_j (1-\sigma_i)\,\phi_j\lambda(\phi)_j\right]. \tag{43}$$

Therefore, we conclude that $\frac{d\log Q}{d\log a_i} = \lambda(1)_i$.

A.6. Proof of Proposition 4.2

We start by showing that (10) is equal to 0. We already showed that the first component is equal to $\lambda(1)_i$, so we need to prove that the second one is $-\lambda(1)_i$. We solve for $\frac{d\lambda(\phi)_j}{d\phi_i}$ starting from the intermediate goods market clearing (2):

$$\frac{d\lambda(\phi)_j}{d\phi_i} = \sigma_{ij}\lambda(\phi)_i + \sum_n \phi_n \sigma_{nj} \frac{d\lambda(\phi)_n}{d\phi_i}. \tag{44}$$

The solution of the system is:

$$\frac{d\lambda(\phi)}{d\phi_i} = (\mathbb{I}_N - \Sigma)^{-1} \boldsymbol{\sigma_i} \lambda(1)_i. \tag{45}$$

By plugging (45) into the second component of (10), we obtain $-[(1-\sigma_i)\lambda(1)_i + \sigma_i\lambda(1)_i] = -\lambda(1)_i$.

To find equation (13), we start simplifying (12):

$$\frac{d\log L}{d\log\phi_i} = -\frac{[(1-\sigma_i)L(\phi) - \theta_i]\lambda(\phi)_i + \sum_j \left[\left(1-\sigma_j\right)L(\phi) - \theta_j\right]\phi_j \frac{d\lambda(\phi)_j}{d\phi_i}}{L(\phi)\sum_j \left(1-\sigma_j\right)\phi_j\lambda(\phi)_j}\phi_i. \tag{46}$$

By plugging (45), we obtain our final result.

A.7. Proof of Proposition 4.3

By taking the second-order Taylor expansion with respect to all $\log\phi_j$ of Equation (14), we obtain:

$$
\begin{aligned}
&\log Q(\phi) - \log Q(1) \approx \\
&\sum_j \lambda(1)_j (\log\phi_j) - \sum_j \frac{ds_T(\phi)}{d\phi_j}\frac{\phi_j}{s_T(\phi)}\bigg|_{\phi=1} \left(\log\phi_j\right) \\
&- \frac{1}{2}\sum_j \frac{ds_T(\phi)}{d\phi_j}\frac{\phi_j}{s_T(\phi)}\bigg|_{\phi=1} \left(\log\phi_j\right)^2 + \frac{1}{2}\sum_j\sum_i \frac{ds_T(\phi)}{d\phi_j}\frac{ds_T(\phi)}{d\phi_i}\frac{\phi_j\phi_i}{s_T(\phi)^2}\bigg|_{\phi=1} (\log\phi_j)(\log\phi_i) \\
&- \frac{1}{2}\sum_j\sum_i \frac{d^2 s_T(\phi)}{d\phi_j d\phi_i}\bigg|_{\phi=1} (\log\phi_j)(\log\phi_i) = \\
&- \frac{1}{2}\sum_j \frac{d\log s_T(\phi)}{d\log\phi_j}\bigg|_{\phi=1} \left(\log\phi_j\right)^2 + \frac{1}{2}\sum_j\sum_i \frac{d\log s_T(\phi)}{d\log\phi_j}\bigg|_{\phi=1} \frac{d\log s_T(\phi)}{d\log\phi_i}\bigg|_{\phi=1} (\log\phi_j)(\log\phi_i) \\
&- \sum_j\sum_i (1-\sigma_i)\frac{d\lambda(\phi)_i}{d\phi_j}\bigg|_{\phi=1} (\log\phi_j)(\log\phi_i) - \frac{1}{2}\sum_j\sum_i \left[\sum_k (1-\sigma_k)\frac{d^2\lambda(\phi)_k}{d\phi_j d\phi_i}\bigg|_{\phi=1}\right] (\log\phi_j)(\log\phi_i).
\end{aligned}
\tag{47}
$$

Note that, at efficiency, it is $\frac{d\log s_T(\phi)}{d\log\phi_j}\Big|_{\phi=1} = \frac{ds_T(\phi)}{d\phi_j}\frac{\phi_j}{s_T(\phi)}\Big|_{\phi=1} = \lambda(1)_j$. Therefore the total first-order effect is zero.

From the intermediate market clearing (2), we can solve for the second-order effect on Domar weights:

$$\frac{d^2\lambda(\phi)_k}{d\phi_i d\phi_j} = \sigma_{ik}\frac{d\lambda(\phi)_i}{d\phi_j} + \sigma_{jk}\frac{d\lambda(\phi)_j}{d\phi_i} + \sum_n \phi_n \sigma_{nk}\frac{d^2\lambda(\phi)_n}{d\phi_i d\phi_j}.$$

The solutions are:

$$\frac{d^2\lambda(\phi)}{d\phi_i d\phi_j} = (\mathbb{I}_N - \Sigma)^{-1}\left(\boldsymbol{\sigma_i}\frac{d\lambda(\phi)_i}{d\phi_j} + \boldsymbol{\sigma_j}\frac{d\lambda(\phi)_j}{d\phi_j}\right).$$

Given this result, we can re-express the last line of 47:

$$-\sum_j \sum_i (1-\sigma_i) \frac{d\lambda(\phi)_i}{d\phi_j}\bigg|_{\phi=1} (\log\phi_j)(\log\phi_i) - \frac{1}{2}\sum_j \sum_i \left[\sum_k (1-\sigma_k) \frac{d^2\lambda(\phi)_k}{d\phi_j d\phi_i}\bigg|_{\phi=1}\right] (\log\phi_j)(\log\phi_i) =$$

$$-\sum_j \sum_i \frac{d\lambda(\phi)_i}{d\phi_j} (\log\phi_j)(\log\phi_i) = -\sum_i \lambda(\phi)_i \left(\log\frac{\lambda(\phi)_i}{\lambda(1)_i}\right)(\log\phi_i).$$

We can also substitute:

$$-\frac{1}{2}\sum_j \frac{d\log s_T(\phi)}{d\log\phi_j}\bigg|_{\phi=1} \left(\log\phi_j\right)^2 = -\frac{1}{2}\sum_j \lambda(1)_j \left(\log\phi_j\right)^2$$

and

$$\frac{1}{2}\sum_j \sum_i \frac{d\log s_T(\phi)}{d\log\phi_j}\bigg|_{\phi=1} \frac{d\log s_T(\phi)}{d\log\phi_i}\bigg|_{\phi=1} (\log\phi_j)(\log\phi_i) \approx \frac{1}{2}\left[\sum_j (1-\sigma_j)\lambda(1)_j \left(\log\tau_{\Pi,j}\right)\right]^2.$$

Therefore, we obtain:

$$\begin{aligned}
&\log Q(\phi) - \log Q(1) \approx \\
&\qquad -\frac{1}{2}\sum_i (1-\sigma_i)\lambda(1)_i (\log\tau_{\Pi,i})^2 + \frac{1}{2}\left(\sum_i (1-\sigma_i)\lambda(1)_i \log\tau_{\Pi,i}\right)^2 \\
&\qquad\qquad -\frac{1}{2}\sum_i \sigma_i\lambda(1)_i (\log\tau_{\Pi,i})^2 + \frac{1}{2}\sum_i \lambda(1)_i \left(\log\frac{\lambda(\phi)_i}{\lambda(1)_i}\right)^2.
\end{aligned} \tag{48}$$

A.8. Proof of Proposition 4.4

From Equation (5), the aggregate TFP loss is given by

$$\begin{aligned}
&\log A(\phi) - \log A(1) = \\
&\sum_k \lambda(1)_k (\log\phi_k) - \sum_k (1-\eta_k)\lambda(1)_k \left[\log s_\Pi(\phi)_k - \log s_\Pi(1)_k\right] - \sum_k \theta_k \lambda(1)_k [\log s_L(\phi) - \log s_L(1)].
\end{aligned} \tag{49}$$

By taking the second-order Taylor expansion with respect to all $\log\phi_j$, we obtain:

$$
\begin{aligned}
\log A(\phi) - \log A(1) \approx & \sum_j \lambda(1)_j (\log\phi_j) \\
& - \sum_j \sum_k (1-\eta_k)\lambda(1)_k \frac{ds_\Pi(\phi)_k}{d\phi_j} \frac{\phi_j}{s_\Pi(\phi)_k}\Big|_{\phi=1} \left(\log\phi_j\right) - \sum_j \sum_k \theta_k \lambda(1)_k \frac{ds_L(\phi)}{d\phi_j} \frac{\phi_j}{s_L(\phi)}\Big|_{\phi=1} \left(\log\phi_j\right) \\
& - \frac{1}{2} \sum_j \sum_k (1-\eta_k)\lambda(1)_k \frac{ds_\Pi(\phi)_k}{d\phi_j} \frac{\phi_j}{s_\Pi(\phi)_k}\Big|_{\phi=1} \left(\log\phi_j\right)^2 - \frac{1}{2} \sum_j \sum_k \theta_k \lambda(1)_k \frac{ds_L(\phi)}{d\phi_j} \frac{\phi_j}{s_L(\phi)}\Big|_{\phi=1} \left(\log\phi_j\right)^2 \\
& + \frac{1}{2} \sum_j \sum_i \sum_k (1-\eta_k)\lambda(1)_k \frac{ds_\Pi(\phi)_k}{d\phi_j} \frac{ds_\Pi(\phi)_k}{d\phi_i} \frac{\phi_j \phi_i}{s_\Pi(\phi)_k^2}\Big|_{\phi=1} \left(\log\phi_j\right)\left(\log\phi_j\right) \\
& + \frac{1}{2} \sum_j \sum_i \sum_k \theta_k \lambda(1)_k \frac{ds_L(\phi)}{d\phi_j} \frac{ds_L(\phi)}{d\phi_i} \frac{\phi_j \phi_i}{s_L(\phi)^2}\Big|_{\phi=1} \left(\log\phi_j\right)(\log\phi_i) \\
& - \frac{1}{2} \sum_j \sum_i \sum_k (1-\eta_k)\lambda(1)_k \frac{d^2 s_\Pi(\phi)_k}{d\phi_j d\phi_i} \frac{\phi_j}{s_\Pi(\phi)_k}\Big|_{\phi=1} (\log\phi_j)(\log\phi_i) \\
& - \frac{1}{2} \sum_j \sum_i \sum_k \theta_k \lambda(1)_k \frac{d^2 s_L(\phi)}{d\phi_j d\phi_i} \frac{\phi_j}{s_L(\phi)}\Big|_{\phi=1} (\log\phi_j)(\log\phi_i). \quad (50)
\end{aligned}
$$

We can simplify further, using $(1-\eta_k)\lambda(1)_k = s_\Pi(1)_k$ and $\sum_k \theta_k \lambda(1)_k = s_L(1)$:

$$
\begin{aligned}
\log A(\phi) - \log A(1) \approx & \\
& \sum_j \lambda(1)_j (\log\phi_j) - \sum_j \sum_k \frac{ds_\Pi(\phi)_k}{d\phi_j}\Big|_{\phi=1} \left(\log\phi_j\right) - \sum_j \frac{ds_L(\phi)}{d\phi_j}\Big|_{\phi=1} \left(\log\phi_j\right) \\
& - \frac{1}{2} \sum_j \sum_k s_\Pi(1)_k \frac{d\log s_\Pi(\phi)_k}{d\log\phi_j}\Big|_{\phi=1} \left(\log\phi_j\right)^2 - \frac{1}{2} \sum_j s_L(1) \frac{d\log s_L(\phi)}{d\log\phi_j}\Big|_{\phi=1} \left(\log\phi_j\right)^2 \\
& + \frac{1}{2} \sum_j \sum_i \sum_k s_\Pi(1)_k \frac{d\log s_\Pi(\phi)_k}{d\log\phi_j}\Big|_{\phi=1} \frac{d\log s_\Pi(\phi)_k}{d\log\phi_i}\Big|_{\phi=1} \left(\log\phi_j\right)\left(\log\phi_j\right) \\
& + \frac{1}{2} \sum_j \sum_i s_L(1) \frac{d\log s_L(\phi)}{d\log\phi_j}\Big|_{\phi=1} \frac{d\log s_L(\phi)}{d\log\phi_i}\Big|_{\phi=1} \left(\log\phi_j\right)(\log\phi_i) \\
& - \frac{1}{2} \sum_j \sum_i \sum_k \frac{d^2 s_\Pi(\phi)_k}{d\phi_j d\phi_i}\Big|_{\phi=1} (\log\phi_j)(\log\phi_i) - \frac{1}{2} \sum_j \sum_i \frac{d^2 s_L(\phi)}{d\phi_j d\phi_i}\Big|_{\phi=1} (\log\phi_j)(\log\phi_i). \quad (51)
\end{aligned}
$$

Since $\sum_k s_\Pi(\phi,\delta)_k + s_L(\phi) = s_T(\phi,\delta)$, we can rewrite:

$$\log A(\phi) - \log A(1) \approx \sum_j \lambda(1)_j (\log\phi_j) - \sum_j \left.\frac{ds_T(\phi)}{d\phi_j}\right|_{\phi=1} \left(\log\phi_j\right)$$

$$- \frac{1}{2}\sum_j\sum_k s_\Pi(1)_k \left.\frac{d\log s_\Pi(\phi)_k}{d\log\phi_j}\right|_{\phi=1} \left(\log\phi_j\right)^2 - \frac{1}{2}\sum_j s_L(1) \left.\frac{d\log s_L(\phi)}{d\log\phi_j}\right|_{\phi=1} \left(\log\phi_j\right)^2$$

$$+ \frac{1}{2}\sum_j\sum_i\sum_k s_\Pi(1)_k \left.\frac{d\log s_\Pi(\phi)_k}{d\log\phi_j}\right|_{\phi=1} \left.\frac{d\log s_\Pi(\phi)_k}{d\log\phi_i}\right|_{\phi=1} \left(\log\phi_j\right)\left(\log\phi_j\right)$$

$$+ \frac{1}{2}\sum_j\sum_i s_L(1) \left.\frac{d\log s_L(\phi)}{d\log\phi_j}\right|_{\phi=1} \left.\frac{d\log s_L(\phi)}{d\log\phi_i}\right|_{\phi=1} \left(\log\phi_j\right)(\log\phi_i)$$

$$- \frac{1}{2}\sum_j\sum_i \left.\frac{d^2 s_T(\phi)}{d\phi_j d\phi_i}\right|_{\phi=1} (\log\phi_j)(\log\phi_i) =$$

$$- \frac{1}{2}\sum_j\sum_k s_\Pi(1)_k \log\tau_\Pi\left(\phi_j\right)_k \left(\log\phi_j\right) + \frac{1}{2}\sum_j\sum_i\sum_k s_\Pi(1)_k \log\tau_\Pi\left(\phi_j\right)_k \log\tau_\Pi(\phi_i)_k$$

$$- \frac{1}{2}\sum_j s_L(1)\log\tau_L\left(\phi_j\right)\left(\log\phi_j\right) + \frac{1}{2}\sum_j\sum_i s_L(1)\log\tau_L\left(\phi_j\right)\log\tau_L(\phi_i)$$

$$- \sum_j\sum_i (1-\sigma_i)\left.\frac{d\lambda(\phi)_i}{d\phi_j}\right|_{\phi=1} (\log\phi_j)(\log\phi_i) - \frac{1}{2}\sum_j\sum_i \left[\sum_k (1-\sigma_k)\left.\frac{d^2\lambda(\phi)_k}{d\phi_j d\phi_i}\right|_{\phi=1}\right] (\log\phi_j)(\log\phi_i). \tag{52}$$

We can now do the following substitutions:

$$- \frac{1}{2}\sum_j\sum_k s_\Pi(1)_k \log\tau_\Pi\left(\phi_j\right)_k \left(\log\phi_j\right) \approx$$

$$- \frac{1}{2}\sum_k s_\Pi(1)_k (\log\phi_k)^2 - \frac{1}{2}\sum_k s_\Pi(1)_k \sum_j \left(\log\phi_j\right)\left(\log\frac{\lambda(\phi_j)_k}{\lambda(1)_k}\right),$$

$$\frac{1}{2}\sum_j\sum_i\sum_k s_\Pi(1)_k \log\tau_\Pi\left(\phi_j\right)_k \log\tau_\Pi(\phi_i)_k \approx$$

$$\frac{1}{2}\sum_k s_\Pi(1)_k \left[(\log\phi_k)^2 + \left(\log\frac{\lambda(\phi)_k}{\lambda(1)_k}\right)^2 + 2(\log\phi_k)\left(\log\frac{\lambda(\phi)_k}{\lambda(1)_k}\right)\right],$$

$$-\frac{1}{2}\sum_j s_L(1)\log\tau_L\left(\phi_j\right)\left(\log\phi_j\right)\approx$$

$$-\frac{1}{2}\sum_k \theta_k\lambda(1)_k(\log\phi_k)^2-\frac{1}{2}s_L(1)\sum_k(\log\phi_k)\sum_i\frac{\theta_i\lambda(1)_i}{s_L(1)}\left(\log\frac{\lambda(\phi_k)_i}{\lambda(1)_i}\right),$$

$$\frac{1}{2}\sum_j\sum_i s_L(1)\log\tau_L\left(\phi_j\right)\log\tau_L(\phi_i)\approx\frac{1}{2}s_L(1)\left[\left(\sum_i\frac{\theta_i\lambda(1)_i}{s_L(1)}\log\phi_i\right)^2+\left(\sum_i\frac{\theta_i\lambda(1)_i}{s_L(1)}\log\frac{\lambda(\phi)_i}{\lambda(1)_i}\right)^2\right]$$

$$+s_L(1)\left(\sum_i\frac{\theta_i\lambda(1)_i}{s_L(1)}\log\phi_i\right)\left(\sum_i\frac{\theta_i\lambda(1)_i}{s_L(1)}\log\frac{\lambda(\phi)_i}{\lambda(1)_i}\right),$$

$$-\sum_j\sum_i(1-\sigma_i)\frac{d\lambda(\phi)_i}{d\phi_j}\bigg|_{\phi=1}(\log\phi_j)(\log\phi_i)-\frac{1}{2}\sum_j\sum_i\left[\sum_k(1-\sigma_k)\frac{d^2\lambda(\phi)_k}{d\phi_j d\phi_i}\bigg|_{\phi=1}\right](\log\phi_j)(\log\phi_i)$$

$$=-\sum_j\sum_i\frac{d\lambda(\phi)_i}{d\phi_j}(\log\phi_j)(\log\phi_i)=-\sum_i\lambda(\phi)_i(\log\phi_i)\left(\log\frac{\lambda(\phi)_i}{\lambda(1)_i}\right).$$

Plugging into 52:

$$\log A(\phi)-\log A(1)\approx-\frac{1}{2}\left[s_L(1)Var_L(\log\tau_\Pi)-\sum_i(1-\sigma_i)\lambda(1)_i\left(\log\frac{\lambda(\phi)_i}{\lambda(1)_i}\right)^2\right]$$

$$-\frac{1}{2}\left[2\sum_i\sigma_i\lambda(1)_i(\log\phi_i)\left(\log\frac{\lambda(\phi)_i}{\lambda(1)_i}\right)+\sum_i\left(\sum_j(1-\sigma_j)\frac{d\lambda(\phi)_j}{d\phi_i}\right)(\log\phi_i)^2\right].$$

Since $\left(\sum_j(1-\sigma_j)\frac{d\lambda(\phi)_j}{d\phi_i}\right)=\sigma_i\lambda(1)_i$, we can substitute and obtain our final result.

B. Data and Calibration Appendix

We analyze a seven-sectors economy. The sectors are: 1) Agriculture, Utilities and Mining (AMU) (sector 1); 2) Construction (sector 2); 3) Manufacturing (sector 3); 4) Trade (sector 4); 5) Transportation (sector 5); 6) Finance, Insurance, and Real Estate (FIRE) (sector 6); and 7) Other (sector 7)[19]. We normalize the productivity parameters a_i to unity.

B.1. Externally calibrated ϕ

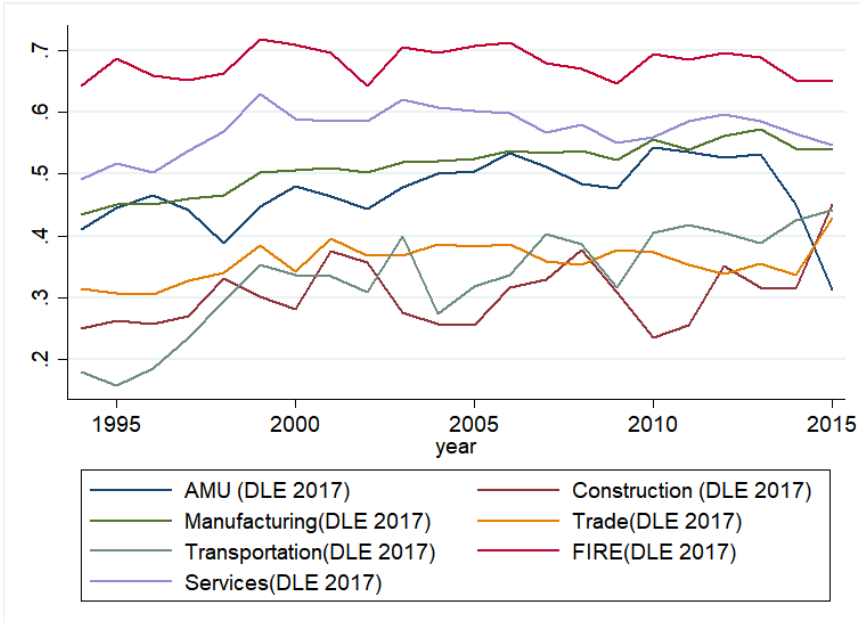
The distortion values we use in our quantitative analysis are obtained from Baqaee and Farhi (2020b) who estimate markups using the "production function approach" proposed by De Loecker et al. (2020). Specifically, given sectoral markups μ_i from Baqaee and Farhi (2020b), we compute the wedges using the following formula:

$$\phi_i := 1 - \frac{1}{\mu_i}.$$

In Figure (5) we plot the estimated ϕ values from 1995 to 2015. In our exercise, we use the most recent ones.

[19]Information, Business services, Education, and Entertainment.

Figure 5: Sectoral wedges (ϕ_i)



B.2. Calibration of technology parameters

We use the input-output tables from the Bureau of Economic Analysis to calibrate the structure of the network. The estimated matrix of intermediate goods shares σ_{ij} is for the seven sectors we are studying is:

$$\Sigma = \begin{bmatrix} 0.1802 & 0.0092 & 0.1624 & 0.0019 & 0.0087 & 0.0647 & 0.0714 \\ 0.0209 & 0.0001 & 0.3635 & 0.0000 & 0.0005 & 0.0322 & 0.0626 \\ 0.1323 & 0.0025 & 0.4000 & 0.0043 & 0.0085 & 0.0167 & 0.0675 \\ 0.0169 & 0.0019 & 0.0556 & 0.0189 & 0.0447 & 0.1194 & 0.1957 \\ 0.0111 & 0.0045 & 0.1267 & 0.0002 & 0.1181 & 0.0994 & 0.1190 \\ 0.0169 & 0.0241 & 0.0187 & 0.0012 & 0.0056 & 0.2103 & 0.1221 \\ 0.0086 & 0.0011 & 0.0772 & 0.0003 & 0.0103 & 0.0866 & 0.2149 \end{bmatrix}.$$

We calibrate $(1-\eta_i)$ – and therefore θ_i – using the share of Gross Operating Surplus as a fraction of total industry output. Similarly we calibrate final good shares ψ_is, using the share of final use of industry outputs. In order to calibrate ξ, we target the share of workers hired by the 10% largest firms. Following Buera et al. (2011), the target is 69% and our calibrated value for the main model, $\xi = 12.38$, generates a perfect fit. The values from our calibration of the main model are reported in Table 2. Table 2 also reports the parameters of the equivalent horizontal economy.

Table 2: Calibrated technology parameters of main model

Network economy													
ψ_1	ψ_2	ψ_3	ψ_4	ψ_5	ψ_6	ψ_7	θ_1	θ_2	θ_3	θ_4	θ_5	θ_6	θ_7
0.05	0.05	0.13	0.13	0.04	0.23	0.37	0.16	0.35	0.18	0.35	0.32	0.15	0.42
Equivalent horizontal													
ψ_1^H	ψ_2^H	ψ_3^H	ψ_4^H	ψ_5^H	ψ_6^H	ψ_7^H	θ_1^H	θ_2^H	θ_3^H	θ_4^H	θ_5^H	θ_6^H	θ_7^H
0.12	0.05	0.21	0.11	0.05	0.42	0.04	0.36	0.76	0.64	0.71	0.69	0.27	0.77

The technology parameters in our network economy with fixed costs are the same as in the first line of table 2 and matrix Σ. To calibrate the fixed costs f_i, we match the average number of establishments in each sector out of the active population between 1987-2015. The moments are obtained from the Bureau of Labor Statistics (BLS). All estimated values for our model with fixed costs are reported in Table 3. Finally, the equivalent horizontal structure of the network model with fixed costs is defined by the following characteristics:

1. no input-output linkages, i.e. $\sigma_{i,j}^H = 0 \ \forall \ i, j$;
2. same profit shares at efficiency:

$$\frac{\xi(1-\eta_i)-1}{\xi}\kappa(1)_i\left[\lambda(1)_i + F(1)_i\right] = \frac{\xi(1-\theta_i^H)-1}{\xi}\kappa^H(1)_i\left[\lambda^H(1)_i + F^H(1)_i\right] \ \forall \ i;$$

3. same labor income shares at efficiency: $\theta_i\left[\lambda(1)_i + F(1)_i\right] = \theta_i^H\left[\lambda^H(1)_i + F^H(1)_i\right] \ \forall \ i$; and
4. same fixed cost shares at efficiency: $F(1)_i = F^H(1)_i \ \forall \ i$.

The last three moments allow us to identify ψ^H, θ^H, and f^H.

Table 3: Calibrated parameters in model with fixed costs

Targeted Moments	US Data	Model	Parameters
Top 10 percentile employment share	69%	73.3%	$\xi = 8.87$
Establishment ratio in AMU	0.04%	0.04%	$f_1 = 0.139$
Establishment ratio in Construction	0.41%	0.41%	$f_2 = 0.004$
Establishment ratio in Manufacturing	0.17%	0.19%	$f_3 = 0.627$
Establishment ratio in Trade	0.84%	0.79%	$f_4 = 0.008$
Establishment ratio in Transportation	0.14%	0.13%	$f_5 = 0.029$
Establishment ratio in FIRE	0.51%	0.48%	$f_6 = 0.776$
Establishment ratio in Other Services	2.4%	2.2%	$f_7 = 0.228$