DISCUSSION PAPER SERIES

DP16926

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LABOUR ECONOMICS

ISSN 0265-8003

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Discussion Paper DP16926
Published 18 January 2022
Submitted 18 January 2022

Centre for Economic Policy Research
33 Great Sutton Street, London EC1V 0DX, UK
Tel: +44 (0)20 7183 8801
www.cepr.org

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Most of the key amenities of our today jobs did not emerge in private contracts; instead, they appeared in collective agreements and regulations. I argue that understanding this observation can guide the provision of future amenities. I show that markets underprovide an amenity if workers who value it more have a lower average unobserved productivity. Universal mandate of such amenities improves social welfare when taste-productivity correlation is high. Policies that leverage heterogeneity in the taste-productivity correlation by observable characteristics, e.g., quota and tagging, dominate mandate in the presence of a mild adverse selection.

JEL Classification: N/A

Keywords: N/A

Arash Nekoei - arash.nekoei@iies.su.se
IIES and CEPR

Will markets provide humane jobs? A hypothesis

Arash Nekoei*

January 14, 2022

Abstract

Most of the key amenities of our today jobs did not emerge in private contracts; instead, they appeared in collective agreements and regulations. I argue that understanding this observation can guide the provision of future amenities. I show that markets underprovide an amenity if workers who value it more have a lower average unobserved productivity. Universal mandate of such amenities improves social welfare when taste-productivity correlation is high. Policies that leverage heterogeneity in the taste-productivity correlation by observable characteristics, e.g., quota and tagging, dominate mandate in the presence of a mild adverse selection.

Work conditions have improved over the path of development. New amenities – e.g., flexible hours, four-day workweek, and remote work – have recently been on the agenda.[1] The question is whether market forces by themselves will produce jobs with such amenities.

Today many amenities – e.g., paid sick leave, paid holidays/vacations, limited working hours, workers' compensation – are proclaimed by collective agreements, national legislation, or international laws. An example is the Universal Declaration of Human Rights: "Everyone has the right to rest and leisure, including reasonable limitation of working hours and periodic holidays with pay."[2] The steady decline of working hours, as famously predicted by Keynes (1930), is well-trodden ground for economists.[3] Our understanding is that workers prefer to work less since the income effect of rising labor productivity dwarfs the substitution effect. But the concurrent reduction of standard hours stated in labor laws, which I will document, has not been given much attention.[4] Why do laws enforce these preferences, and not the market? What does our answer teach us about the future of amenities?

I hypothesize a failure in the market of amenities, namely adverse selection. The market underprovides an amenity if those who value it more have a lower average unobserved productivity. Employers hesitate to provide such amenities that attract low-quality workers and reduce production.

I demonstrate the theoretical merit of this hypothesis using a framework that captures the information asymmetry about workers' two-dimensional type. One dimension is relevant for

*IIES at Stockholm University, CEPR, CESifo. I thank Konrad Burchardi, Mitchell Downey, Nathaniel Hendren, Alexandre Kohlhas, Hannes Malmberg, Thomas Mikaelsen, Miika Päällysaho, Pepe Montiel Olea, Ali Shourideh, Jósef Sigurðsson, Johannes Spinnewijn, and Alireza Tahbaz-Salehi for useful comments. Bolun Cong provided excellent research assistance.

[1] See issue 4 of Connecticut Law Review in 2010 and Pinsker (2021) on the four-day week, Hegewisch (2009), Goldin (2014), and OECD (2016) on flextime, and Mas and Pallais (2017) on flextime and remote work.

[2] Article 24 of U.N. (1948). Similar articles appear in International Covenant on Economic, Social and Cultural Rights, European Social Charter, and most national legislation.

[3] E.g. Bell and Freeman (2001), Prescott (2004), Huberman and Minns (2007), Boppart and Krusell (2020).

[4] A few exceptions include Hunt (1998), Alesina et al. (2005), and Saez (2021).

their utility (the taste for an amenity), and the other is relevant for production (their unobserved productivity). I assume that the average productivity of workers with a high amenity taste is lower than that of workers with a low amenity taste. I refer to this new notion of dependence as a negative correlation in expectation. I first prove its main properties required for economic analysis and its relations with well-studied dependence concepts.

I establish that under the negative correlation in expectation between taste and productivity, the market equilibrium with endogenous wages always under-provides the amenity. Inefficiency is the price for motivating the amenity provision and compensating employers for the loss in expected worker quality by lowering the compensating wage differential (amenity price). Inefficiency thus increases with the taste-productivity correlation and worker productivity dispersion. The market can even unravel and provide no amenity in equilibrium.

One remedy to this problem is a mandate - a universal provision of the amenity. Assuming uniform distributions of taste and productivities, I show that a mandate increases welfare according to Hicks-Kaldor Criterion if the taste-productivity correlation or productivity dispersion is high. In such cases, the mandate creates both winners and losers, but winners can compensate losers so that everyone is better off.

Alternatively, policies may leverage heterogeneity in the productivity-taste correlation by observable characteristics. For example, employers can offer flexible hours only to guardians of children. I study two such policies, quotas and tagging, whereby private contracts may vary by observables. Quotas additionally proclaim a fair representation of workers across jobs. Both policies increase aggregate welfare relative to laissez-faire or mandate in the presence of a mild adverse selection, but the mandate dominates all three with a severe adverse selection. The ranking of policies differs for sub-populations with low correlation who prefer quotas and tagging over the mandate.

Looking ahead, this hypothesis necessitates an intervention for amenities with a negative taste-productivity correlation. Emanuel and Harrington (2021) provide the first evidence for such a relation in the case of remote work (Figure 1). The question is whether the magnitude of such a correlation justifies mandates. The answer to the question is ultimately empirical and needs further investigation.

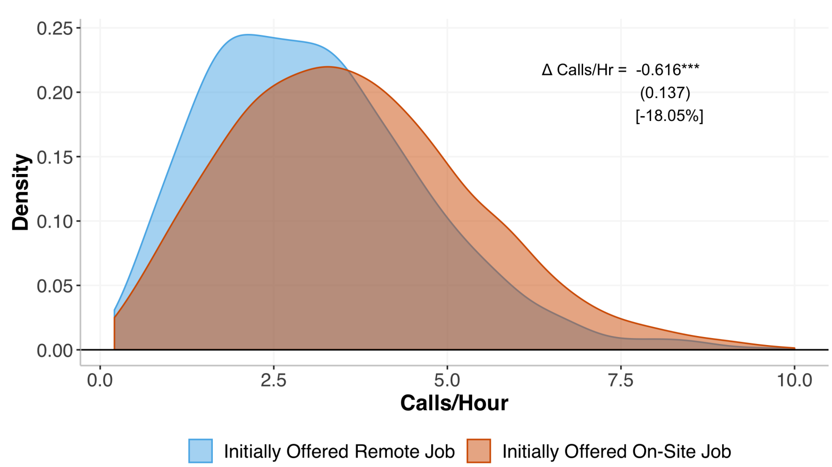

Figure 1: **The Distributions of Productivity for Workers Initially Offered Remote and On-Site Jobs**
Note: This figure compares the distributions of calls handled per hour for workers who were initially offered remote jobs (in blue) and workers who were initially offered on-site jobs (in orange) when all workers were working remotely due to COVID-19's lockdown. The x-axis represents the average calls taken per hour per day. Source: Emanuel and Harrington (2021).

This paper is related to several strands of literature studying the provision of amenities. As a starting point, Summers (1989) points to two rationales behind the mandate of amenity provision: positive externalities of amenities and adverse selection.[5] Hendren (2017) demonstrates that adverse selection leads to market unraveling in the case of unemployment insurance.The micro-foundation and thus the consequences of adverse selection in these papers differs from mine, given my focus on the two-dimensional heterogeneity of employees and heterogeneous employers competing with exclusive contracts.[6]

For limited hours, an alternative explanation for the existence of regulations is that laws are a coordination device in the presence of production or social multipliers. Laws coordinate the working hours by defining weekdays or a daily hours schedule, if productivity increases when workers work simultaneously or the utility of leisure increases when they are off work during the same time period (Glaeser et al. 2003 and Alesina et al. 2005). We also want to limit the hours when they are inefficiently high, e.g., due to their signaling value (Landers et al. 1996 and Anger 2008).

In principle, one possible explanation for the presence of amenities in social contracts instead of individual contracts is collective bargaining. Bargaining over wages is more effective as the wage rate is easier to regulate and monitor. But once a minimum wage is set, mandating amenities helps prevent firms from undercutting the wage regulation. There are two problems with this explanation. First, historically, amenity regulations were introduced earlier than wage regulation. For example, the minimum wage was introduced in 1941 in France, as compared to numerous hours regulations that had been enacted before then (Section 1). Second, in most countries, mandated amenities are more prevalent than wage regulations.

The structure of this paper is as follows. Section 1 overviews the historical development of several key amenities. Section 2 provides the conceptual framework and the main analysis and findings. The mathematical tools used in this section and the proofs are in the Appendix. Section 3 investigates the potential gains from mandating, quotas, and tagging. The last section concludes with a few remarks on the potentials of other policies.

1 Historical Development

This section provides a brief history of some key amenities, focusing on the role of collective action and regulations. To start, Figure 2 illustrates the rise and fall of various amenities in the public debate.

Limited Working Hours The reduction in standard working hours per day, days per week, and weeks per year has appeared in collective regulations as a result of social struggles, instead of appearing in private contracts resulting from individual negotiation.[7] As the American labor leader,

[5] Discussing the role of traditional adverse selection in employers' provision of health insurance, Summers (1989) remarks en passant that "Even leaving aside the consideration that for productivity reasons, firms might not prefer a personnel policy that was most likely to attract unhealthy workers, ...". This aside consideration is the main mechanism in our paper.

[6] Given my goal of presenting an applied idea, I use a realistic and straightforward model with a specific multidimensional heterogeneity. See Azevedo and Gottlieb (2017) and references therein for a comprehensive theoretical treatment.

[7] Chapter 10 of volume 1 of Capital of Marx, "The Working-Day", is entirely devoted to this issue (Marx (1867)). See also Bosch et al. (1994), Alesina et al. (2005), Negrey (2012).

George Meany, summarized: "In effect, the progress towards a shorter work-day and a shorter work-week is a history of the labor movement itself".[8]

Figure 3 illustrates the simultaneous fall of working hours and standard hours in France over the last two centuries. Going further back in time, the history of the U.S. witnessed many collective attempts to enact laws that limit working hours. During the early 17th century, laws in the Virginia and Massachusetts colonies limited working hours, although their bearing on actual practice was limited (Roediger and Foner 1989). The majority of first recorded labor strikes advocated shorter hours, e.g., the unsuccessful strike of Philadelphia Carpenters in May 1791. Beginning in 1845, Sarah Bagley and the New England Female Labor Reform Association petitioned the state legislature to intervene in the determination of hours. Two decades later, Grand Eight Hours Leagues sprang up around the country (Whaples 2001). The haymarket riots of 1886 constituted the next step in the Eight-Hour Movement. Between 1848 and 1921 all but four states passed legislation limiting the working hours for women. Several states declared a general hours legislation unconstitutional, followed by the Supreme Court in 1905 (Goldin 1988). Three decades later, one of the most significant pieces of the New Deal legislation, the Fair Labor Standards Act, sets the 40-hour week.[9]

There has been a parallel movement across the Atlantic. The First International in Geneva in 1866 states that: "We propose 8 hours work as the legal limit of the working day. This limitation being generally claimed by the workmen of the United States of America, the vote of the Congress will raise it to the common platform of the working classes all over the world."

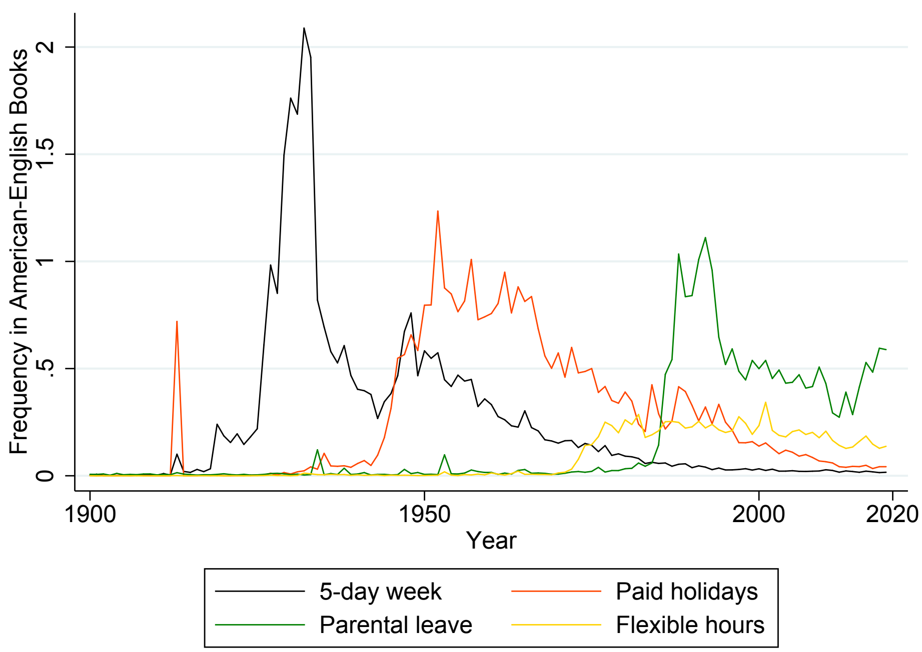

Figure 2: **Amenities in All Books Published in the U.S.**

Note: This figure presents the frequency of appearance of amenities in all American English books published in each year. The y-axis magnitude is the average number of appearances among one million ngrams. Data source: Google Ngram.

[8] Roediger and Foner (1989). For similar conclusion, see also Hunnicutt (1988).

[9] This was the last legislation in the U.S. shortening working hours, a unique feature of American history (compare for example with France case illustrated in Figure 3). Since the 1930s, no major party had proposed shorter working hours, and the labor movement focused on other demands, e.g., wages and fringe benefits (Hunnicutt, 1988).

The international Eight-Hour Movement with the slogan "eight hours for work, eight hours for rest, eight hours for what we will" is the leading example. In the 19th century, the so-called "factory act" regulated the working hours in manufacturing, initially for children and women and later for everyone.[10] The demand for an international treaty rose in the mid-19th century (Bauer, 1921). A series of reforms reducing the working time has been implemented ever since, such as the Hours of Work Convention of 1919 that stipulates the principle of "eight hours a day and 48 hours a week" and the recent French 35-hour law in 2000 (Hunnicutt 1988 and Cross et al. 1989).[11]

Unemployment Insurance had been instituted in union contracts in the late 19th century in Belgium, the Netherlands and Switzerland (Nelson 1969). These out-of-work benefits were subsidized by the municipal governments, with the famous example of Ghent, Belgium, from which the current Ghent systems borrowed their name. The idea was then implemented at the national level, appearing in labor law and collective bargaining contracts around the world, as illustrated in Figure 4.

Paid sick leave and disability As early as 1673, the French government had a program to protect sailors in old age, sickness, and disability (Rimlinger 1989). 16 out of 182 countries studied provide paid sick leave for workers with more than one year of tenure (WPAC, 2020). It is often part of the legislation and supplemented with collective bargaining agreements, like in Sweden.

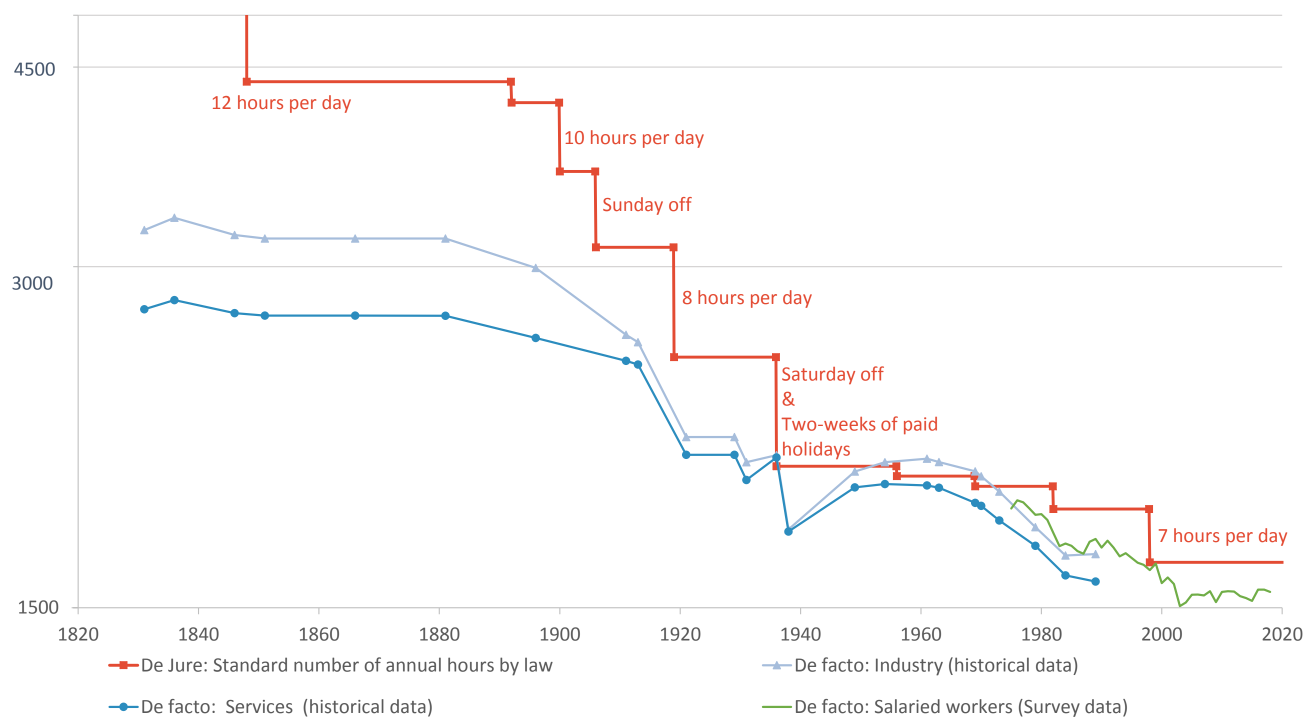

Figure 3: **Annual Hours Worked in France: De Facto vs. De Jure**

Note: De jure, the standard hours by the labor law (Fridenson and Reynaud (2004)). These numbers are just a proxy since the laws did not always immediately apply to all workers. An example is the 35-hour workweek limit initiated in 1998 with fiscal incentives for firms adopting the rule; it became obligatory in 2000 for all firms with more than 20 employees before becoming universal to all in 2002. **De facto**: For the actual hours worked, the historical estiamtes are from Marchand and Thélot (1997) (agriculture excluded); survey data are from INSEE (non-salaried workers excluded).

[10] E.g., the Factory Act of 1847 in the UK ("Ten Hours Act") and the Swiss Factory Act of 1877 (66 hours per week).

[11] It is important to emphasize that the connection between the above-mentioned social movements and the reforms has not been settled (Costa, 2000). For our purpose, the co-existence of the social movements and the laws regulating work conditions is essential, regardless of their causal connection.

Paid maternity leave The pioneering Swiss Factory Act of 1877 was followed by similar acts in ten European countries by 1900, starting with the German Health Insurance Act of 1883, followed up by Holland in 1889, Great Britain in 1891, and the Norwegian Factory Act of 1892 (Peterson 2018). In 1979, The U.N. Convention on the Elimination of All Forms of Discrimination against Women stipulates "maternity leave with pay or with comparable social benefits without loss of former employment, seniority or social allowances" (U.N. (1979)). The Maternity Protection Convention adopted in 2000 by the International Labour Organization proclaims a minimum of 14 weeks of paid maternity leave (ILO (2000)). Today only 4 out of 168 countries studied by Heymann et al. (2006) do not provide paid maternity leave in their labor legislation.

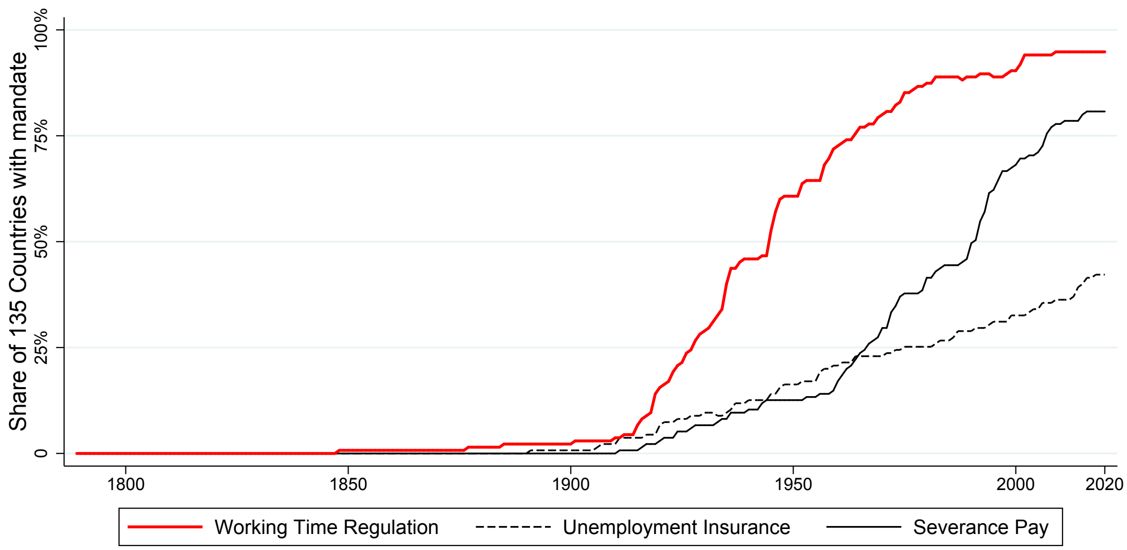

Figure 4: **Mandating Amenities: A Historical and International Perspective**

Note: This figure plots the share of countries with working time regulation and where the law mandates job displacement programs, either in the form of unemployment insurance or severance pay. It is based on a balanced panel of 135 countries that appear in both sources: Gerard and Naritomi (2021) for UI and severance and Rasmussen (2021) for hours regulation. See Online Appendix Section A1 for details on the construction of the underlying data.

2 Economic Model

There is a measure one of both workers and jobs. Workers differ in their productivity and their taste for amenity, (θ, α), both unobservable by employers. The worker utility when working at a job with wage w and with or without amenity $a \in \{0,1\}$ is:

$$u(w, a) = w + \alpha a \tag{1}$$

Jobs differ in their productivity, denoted by y. The productivity of workers and jobs constitute complements, so that the match between job y and worker (θ, α) produces $y\theta$ units of the only consumption good in the economy.[12] Such a match creates the profit

$$\pi(w, a) = y\theta - w - \psi a, \tag{2}$$

[12] The functional form assumption of production can be relaxed, as long as there is some degree of complementarity between the two productivity levels.

that is the production minus wage and minus the cost of the amenity, ψ.

The key difference as compared to the traditional adverse selection model is that our agents vary across two dimensions. One dimension is their preference for the amenity that determines the effect of the amenity on their utility. The other dimension is their productivity type that determines their impact on the firm's profit. In contrast to the traditional adverse selection model, the cost of providing amenities to all workers is the same, and does not depend on the agent's preferences.

Notation 1. I denote the CDF of the random variable X by $F_X(.)$, its survival function by $\overline{F}_X(.)$, its mean by μ_X, and support with support $[x_0, x_1]$.

In the first best, the efficient allocation of amenities would be to provide the amenity to all workers with a taste for the amenity which is higher than the cost of the amenity, i.e. $\alpha > \psi$, independently of the productivity levels of the worker or her job, θ and y. The efficient quantity of the amenity is $q^e = \overline{F}_\alpha(\psi)$.

2.1 Correlation in Expectation

Definition 1. (θ, α) are negatively correlated in expectation when

$$\rho(x) \equiv E(\theta|\alpha < x) - E(\theta|\alpha \geqslant x) \tag{3}$$

is positive for all $x \in [\alpha_0, \alpha_1)$. In addition, the correlation in expectation is higher when $\rho(x)$ is higher everywhere.

Assumption 1. *The worker's productivity* θ *and her taste for amenity* α *are negatively correlated in expectation.*

This assumption states that the average productivity of workers with a high taste is lower than the average productivity of workers with a low taste.

The key challenge of working with this notion of dependence stems from the fact that $\rho(x)$ is not necessarily monotonic. In fact, I show that it is increasing(decreasing) if and only if the expected productivity is convex(concave) in rank of taste, i.e., $E(\theta|F_\alpha(.))$ is convex(concave).

It is useful to re-write $\rho(x)$ using the marginals and the joint CDF as:

$$\rho(x) = -\frac{1}{\overline{F}_\alpha(x)}\frac{1}{F_\alpha(x)}\int F_{\theta,\alpha}(y,x) - F_\alpha(x)F_\theta(y)\,dy \tag{4}$$

This equation reveals that Assumption 1 is implied by negative correlation rank but implies negative Pearson correlation.[13] The former is implied directly by equation 4, but to see the latter I use the fact that the covariance of any two random variables can be written in terms of marginal and joint CDFs (Lehmann (1966)), that is,

$$cov(X,Y) = \int\int F_{Y,X}(y,x) - F_Y(y)F_X(x)\ dydx, \tag{5}$$

[13] For a bivariate normal distribution, a negative correlation in expectation is equivalent to a negative Pearson correlation. However, Gaussian distribution is not a realistic or useful distributional assumption for our economic model (see Section A3.2.2).

Now the equations (4) and (5) together imply that

$$\operatorname{cov}(\alpha,\theta) = -\int F_\alpha(x)\,\overline{F}_\alpha(x)\,\rho(x)\,dx \tag{6}$$

Equation (4) and Sklar's Representation Theorem – a decomposition of the join distribution of (θ,α) into two marginal CDFs, $F_\alpha(.)$ and $F_\theta(.)$, and a bivariate copula, $C(.,.)$ (Nelsen (2007)) – inform the following Lemma used in the next section.

Lemma 1. *A negative correlation in expectation between (α,θ) is strengthened as a result of (i) An increase in the correlation order, that is an increase in the copula keeping the marginal distributions constant; (ii) A mean-preserving change of $F_\theta(.)$ – an increased θ dispersion – keeping the copula unchanged.*

2.2 Market equilibrium

I first characterize the two extreme pooling equilibrium, where all or none of the jobs provide the amenity, before investigating the case where jobs with and without the amenity exist in a separating equilibrium. The focus is on the Rothschild-Stiglitz equilibria.

All jobs provide amenity, $q^* = 1$, when the net value of the amenity to the the person that appreciates it the least is larger than the production gain for the most productive job. This is equivalent to the amenity cost being low, $\psi < \psi^F$, where

$$\psi^F \equiv \alpha_0 - y_1\rho(\alpha_0) \tag{7}$$

This condition guarantees that when all jobs come with the amenity, no employer can provide a job without the amenity and increase profit. I must ensure that even the most productive job – the job benefiting the most from higher productivity workers – matched with the most productive workers – the ones with the lowest taste α_0 – do not prefer to deviate from the equilibrium.

This equilibrium is inefficient when $\psi \in (\psi^F, \alpha_0)$, as there are jobs without an amenity when the universal provision is efficient, $q^e = 1 > q^*$ (see Figure 5). There is always a range of costs that leads to inefficient provision since $\psi^F < \alpha_0$. This is more likely, the lower is ψ^F, that occurs when taste-productivity correlation in expectation is higher or the productivity of the most productive job is higher.

Markets unravel and no amenity is provided in equilibrium, $q^* = 0$, when the $\psi^U < \psi$, where

$$\psi^U \equiv \max_{x\in[\alpha_0,\alpha_1]} x - y_0 F_\alpha(x)\,\rho(x)\,. \tag{8}$$

Here I need to guarantee the incentive compatibility for the least productive job, the job least affected by workers' productivity. Unlike the first pooling equilibrium, the most beneficial compensating wage is not a corner solution.[14]

[14] This is because lowering wage by x for a new job with amenity reduces worker productivity, i.e., $E(\theta|\alpha > x)$ is decreasing in x. In contrast to the first pooling equilibrium, where raising wage by x for a new job without amenity reduces worker productivity, i.e., $E(\theta|\alpha < x)$ is also decreasing in x.

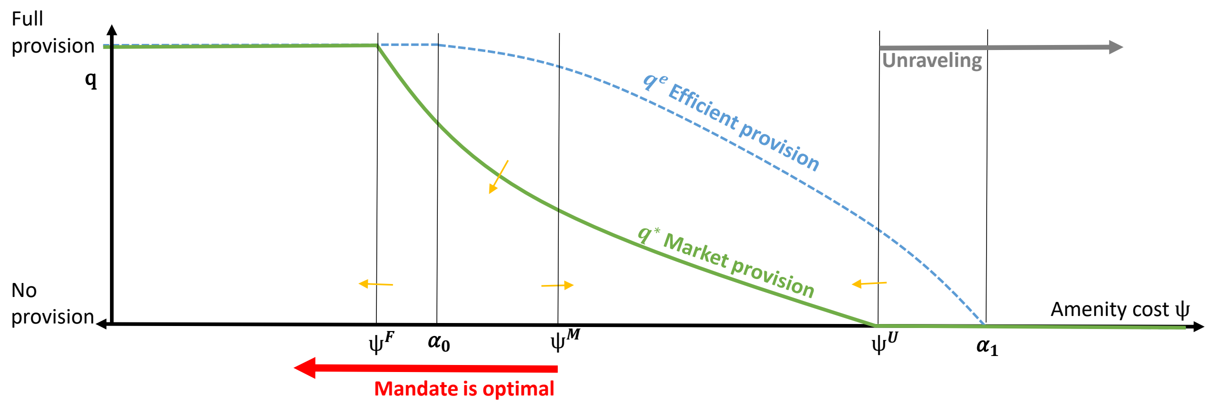

Figure 5: **Efficient vs. the market allocation of the amenity, and the role of mandating**
Note: Quantity of amenity in equilibrium is on the y-axis and the cost of amenity on the x-axis. The small yellow arrows show the effect of a rise in ρ either due to a rise in the negative dependence between taste and productivity, or a higher dispersion of productivity. Here I assume that $\psi^M < \psi^U$, mandating is not optimal when the market unravels, but this is not always the case (see Appendix Figure A1).

This is inefficient when $\psi \in (\psi^U, \alpha_1)$, leading to no market for the amenity where the efficient allocation of the amenity should be positive, $q^e > q^* = 0$. The more negative is the relation between (α, θ), the more likely is this inefficient unraveling.

When the market equilibrium leads to an interior solution, $q^* \in (0, 1)$, two types of contracts are provided: one with a low wage and with the amenity $(w, a = 1)$, and one with a high wage for a job without the amenity $(w + p, a = 0)$, where $p \geqslant 0$ denotes the compensating wage differential for the amenity (The implicit price of amenity).

The contract space is limited – a job is with or without amenity–, similar to Akerlof (1970). So the amenity price (the compensating wage differential) clears the market by equalizing the supply and demand of the amenity that has the flavor similar to the equilibrium in the Akerlof setting.[15]

The demand for the amenity is $\overline{F_\alpha}(p)$, as agents with a higher taste than the wage differential, $\alpha \geqslant p$, choose the low paying jobs with the amenity. Employers choose the contract with the amenity if and only if it leads to a higher profit:

$$yE(\theta|\alpha \geqslant p) - \psi - w > yE(\theta|\alpha < p) - (w + p). \tag{9}$$

Under Assumption 1, only low productivity jobs provide the amenity, since they face a lower productivity loss of low workers. So the supply condition is $\overline{y} > y$, where:

$$\overline{y} \equiv \frac{p - \psi}{\rho(p)} \tag{10}$$

If the equilibrium exists, then it is determined by equalizing the supply and demand of the amenity, that is:

$$q^* = F_y(\overline{y}) = \overline{F_\alpha}(p) \tag{11}$$

[15]See Azevedo and Gottlieb (2017) for a comprehensive state-of-the-art discussion of various definitions of equilibria.

that could be re-written in the form of the following fixed-point problem:

$$p = \psi + \rho(p) F_y^{-1} \circ \overline{F_\alpha}(p) \tag{12}$$

Negative correlation in expectation of taste and productivity implies that the amenity price (the compensating wage differential) is larger than the cost of the amenity, $p > \psi$. The difference between price and cost implies that in equilibrium the amenity is inefficiently under-provided, $q^e > q^*$.[16] Lemma 1 provides us with the following proposition.

Proposition 1. *Under Assumption 1, if a market equilibrium exists, the amenity is always inefficiently under-provided, $q^* < q^e$. The amount of the amenity is lower (the inefficiency is higher), when the taste and the productivity are more correlated in expectation. This occurs with a higher level of the copula or a higher dispersion of worker productivity.*

3 Policy

To compare the market equilibrium with or without an intervention, I use a Utilitarian social welfare function. Given the quasi-linear utility assumption, I do not need to specify the allocation of profits. This is equivalent to using Pareto efficiency allowing for ex-post compensation among employers and employees (Hicks-Kaldor criterion). In addition to be able to rank different policies, I assume uniform distributions for taste, worker and job productivities. For the details of proofs see Online Appendix Section A3.1.

3.1 When is mandating an amenity optimal?

This section will consider the effect of mandating the provision of an amenity. The amenity can either be provided by the employer or by the government. In the latter case, a payroll tax equal to the amenity cost finances the program.

If all jobs provide the amenity, then the total welfare is

$$V^M = \mu_\alpha - \psi + \mu_y \mu_\theta \tag{13}$$

which is better than forbidding the amenity if and only if the average benefit of the amenity is higher than its cost, $\mu_\alpha > \psi$.

Now I consider the market equilibrium. I start with the case where the market does not unravel. Total production in equilibrium is the sum of the production from jobs with and without the amenity:

$$Y^* = F_y(\overline{y}) \underbrace{E(\theta|\alpha > p) E(y|y < \overline{y})}_{E(y|a=1)} + \overline{F}_y(\overline{y}) \underbrace{E(\theta|\alpha < p) E(y|y > \overline{y})}_{E(y|a=0)}$$

[16]The price condition 12 determines the bounds for amenity cost guaranteeing the existence of separating equilibrium that does not necessarily coincide with the ones specified by Rothschild-Stiglitz approach, (ψ^F, ψ^U). The extra layer of complication is beside our main point and is discussed in the Appendix.

I use the market clearing condition, equation (A18), to benchmark the total production with respect to mandated production as

$$Y^* - Y^M = \bar{F}_y(\bar{y})\{E(y|y > \bar{y}) - \mu_y\}\frac{p - \psi}{\bar{y}} \tag{14}$$

The mandate reduces total production. In fact, there is a gain from sorting by productivity through amenity provision in equilibrium but it is lost due to the mandate.

This production loss due to mandating must be traded off against the potential gain in amenity allocation. On the amenity side, similarly, the bench-marking leads to:

$$A^* - A^M = -F_\alpha(p)\, E(\alpha - \psi|\alpha < p) \tag{15}$$

The mandate increases the efficiency of amenity allocation if and only if the average worker in jobs with no amenity in equilibrium appreciates the amenity more than its cost.

Total welfare is the sum of the net benefit from amenities plus the production surplus that is affected by the allocation of workers,

$$V^* - V^M = A^* - A^M + Y^* - Y^M$$

So mandating is welfare improving, $V < V_M$, if and only if the production loss is dominated by the positive amenity gain or using equations (14) and (15),

$$\frac{E(y|y > \bar{y}) - \mu_y}{\bar{y}} < \frac{E(\alpha|\alpha < p) - \psi}{p - \psi} \tag{16}$$

If the market unravels, $\psi^U < \psi$, then then mandate is optimal when there is severe unraveling, $\psi < \mu_\alpha$, that is the cost is below the average taste and still the market unravels.

In the case of a uniform distribution, the above condition can be simplified using the expression for the equilibrium amenity price, which leads to the following result (Appendix Section A3.2.1).

Proposition 2. *In case of uniformly distributed taste and productivity, mandating dominate a la Hicks-Kaldor the market allocation when the taste-productivity correlation in expectation is high,* $\rho > \rho^M$ *where* $\rho^M = \frac{1}{y_0}\frac{\psi - \alpha_0}{\sigma_\alpha}$. *Expressed in terms of amenity cost, mandate is optimal when the amenity cost is low,* $\psi < \psi^M$. *With a separating equilibrium,* $\psi^M = \left(1 - \frac{\rho y_0}{2}\right)\alpha_0 + \frac{\rho y_0}{2}\alpha_1$, *and when markets unravel,* $\psi^M = \mu_\alpha$.

Winners and Losers

I can characterize the winners and losers of the mandate without any distributional assumption. To achieve this goal, I need to specify the mechanism pinning down the wage levels. I assume an infinitely elastic supply of capital necessary for the production in each job.

A mandate, when optimal and necessary, $\psi^F < \psi < \psi^M$, creates both winners and losers. But winners can compensate losers so that all are better off (Hicks-Kaldor Criterion). Workers with a high taste for the amenity who would buy it from the market gain from the mandate through a higher equilibrium compensating wage differential (lower price). Workers who would not receive the amenity from the market experience a wage drop. They are partitioned into winner and losers depending on whether their utility from the amenity dwarfs the wage drop.

3.2 Leveraging Observable Characteristics

The correlation between productivity and taste may vary by observable characteristics. In the case of flexible hours, for example, one might expect the taste-productivity correlation for guardians of young children to be lower than the average worker.

Policies can leverage such observable heterogeneity to deliver tailored solutions instead of the uniform mandate. To examine this issue, I assume that the negative taste-productivity correlation only holds among a sub-population (men), whereas others (women) exhibit zero taste-productivity correlation.

The first policy I consider is a gender quota, whereby both genders must be equally represented in expectation in each job. Quotas would have no effect if distributions of marginal taste were gender-neutral, despite gender differences in correlation. In fact, the market equilibrium without any intervention satisfies the quota in this case.

One way to operationalize the quota system is by allowing gender-specific contracts. In this case, in equilibrium, all jobs offer women a menu including two options: a low wage with the amenity or a high wage without the amenity, making women indifferent among jobs. For men, a separating equilibrium prevails where high productivity jobs provide no amenity with a high wage, and low productivity jobs come with the amenity but pay a low wage.

The quota system with gender-specific contracts dominates market allocations, but mandates dominate both allocations when the taste-productivity correlation is high. For women, such a quota system always increases welfare because quotas achieve the first-best allocation for women. In contrast, the average negative correlation in the entire population (due to correlation among men) determines their market allocation. For men, the quota system leads to an equilibrium allocation based on their own correlation that mandates can dominate in the same fashion as it dominates any market equilibrium under severe adverse selection.

Tagging is an alternative policy that leverages worker characteristics. Each job can be tailored toward either gender, unlike under the quota system where all jobs should be accessible for both sexes. In equilibrium under tagging, middle-range productivity jobs are offered exclusively to women, while the top and bottom jobs are retained for men who can be sorted by their productivity exploiting their taste-productivity correlation. The most productive jobs come without the amenity targeting more productive men and bottom jobs with the amenity targeting the remaining men. In this way, tagging increases total welfare, even for women.

In sum, tagging allows segmentation of the market by the level of taste-productivity correlation: exploiting the negative correlation among some and, at the same time, achieving the efficient allocation of amenities where there is no gain from sorting. Quota is less successful since it requires the accessibility of all jobs for all. It still beats the market as it allows gender-tailored contracts.

The uniform mandates dominate both tailored policies (quota and tagging) and laissez-faire in the presence of a high average taste-productivity correlation due to the adverse selection. But from the perspective of the sub-populations with low correlation, the ranking of policies is different: quota and tagging dominate the uniform mandate.

4 Conclusion

Good jobs are identified not only by their wages but also by their amenities. The latter include many aspects. This paper studies the inefficiency of the market provision of amenities that are preferred more by less productive workers. Which amenities fall into this category is an empirical question worthy of further investigation.

This paper examines a few remedies for the resulting market failure: the universal mandate and policies leveraging observable characteristics, i.e., quota/tagging. More creative policies need further study. One such option is requiring all vacancies to accept team applications. Two workers with a high taste for flexible hours (remote work) can apply together to a vacancy requiring rigid hours (in office work).

The main goal of this paper has been to put the spotlight on issues related to the supply of new amenities. These concerns are worthy of further theoretical and empirical investigation since the demand for new amenities has always been and will always be present as long as workers' productivity rises (Figure 2). As Orwell (1937) wrote: "To the ordinary working man, the sort you would meet in any pub on Saturday night, Socialism does not mean much more than better wages and shorter hours and nobody bossing you about."

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Part

Online Appendix

A1 Data construction

Figure 4 is constructed using two sources: Gerard and Naritomi (2021) for unemployment insurance (UI) and severance pay (SP) and Rasmussen (2021) for hours regulation. The final dataset is a balanced panel that covers 142 countries and their legislation status regarding working hours, unemployment insurance, and severance payment for the period 1789-2020.

Data of working hours law are obtained from Rasmussen (2021). It contains a binary variable indicating working hours regulation for 202 countries from 1789 to 2020.

Data on UI and SP from Gerard and Naritomi (2021) contains information for 168 countries from 1905 to 2020. UI and SP are also binary variables that equals to one if a country implemented corresponding policy at the given year.

To merge the two datasets and create a balanced panel, I followed the following procedure.

1. For countries that disappeared, their policy dates will be used for their replacing countries. For instance: Toscana, Papal States, and Two Sicilies are historical parts of Italy. The policy start date for Italy will be set to the earliest date of these countries.

2. Countries that came into existence within the time span of our study (e.g., North and South Korea) are excluded if there is no information about their parent country (e.g., Korea). If the information is available, the countries will be merged into one, with the policy's start date set to the earliest date.

I have 142 countries that can be found in both data sets as our sample, that is the 168 countries from Gerard and Naritomi (2021) excluding the 22 new countries and the following four countries that do not exist in Rasmussen (2021): Andorra, Brunei, Liechtenstein and Puerto Rico.

3. I extrapolate missing values to make the final data set a balanced panel in the following instances:

3a. For each country, if the missing value is at the beginning, the value will be set to 0. In the case of Rasmussen (2021), I do this even if the first year is one, since I assume all the policies have been documented in the original data set.

3b. For Rasmussen (2021) , when data is missing in late years, the missing values will be replaced by the most recent available value.

3c. For Rasmussen (2021), there are three countries: Austria, Burkina Faso, and Germany, the values of which are missing in the middle. I found no evidence of change in those periods. Hence, they are replaced by 1 if both the next and the previous available values are 1. Missing values are replaced by 0 if the next and previous available values are not equal or are both equal to 0.

The final dataset also includes new variables to indicate whether the policy status of a country in a given year is extrapolated or taken from the original data sets.

A2 Correlation in expectation

In this section, I define a notion of dependence that is used in the economic model of the next sections.

Notation 1. I denote the CDF of random variable X by $F_X(.)$, its survival function by $\overline{F}_X(.)$, its mean by μ_X, and support with support $[x_0, x_1]$.

Definition 2. Y is positively correlated in expectation with X if and only if $\beta_{Y,X}(x) > 0$ for all x, where

$$\beta_{Y,X}(x) \equiv E(Y|X \geqslant x) - E(Y|X < x).$$

This is equivalent to $E(Y|X \geqslant x) > \mu_Y$ for all x, since $\beta_{Y,X}(x)$ can be rewritten as:

$$\beta_{Y,X}(x) = \frac{\mu_Y - E(Y|X < x)}{\overline{F}_X(x)} = \frac{E(Y|X \geqslant x) - \mu_Y}{F_X(x)}. \tag{A1}$$

Definition 3. (X_2, Y_2) are more correlated in expectation than (X_1, Y_1) if and only if $\beta_{Y_2,X_2}(x) > \beta_{Y_1,X_1}(x)$ for all x.

In particular,Y_2 is more than Y_1 correlated in expectation with X if and only if $\beta_{Y_2,X}(x) > \beta_{Y_1,X}(x)$ for all x.

Now, I show the connection between this notion of dependence and others, starting with the positively quadrant dependent that is $F_{Y,X}(y,x) \geqslant F_X(x) F_Y(y)$ for all x and y (Lehmann (1966)).

Claim 1. Positively quadrant dependent implies positively correlated in expectation.

Proof. Given that $\mu_X = \int \overline{F}_X(x)\, dx$ for any positive support r.v. X, I use the equation (A1), in the following:

$$\begin{aligned}
\overline{F}_X(x) F_X(x) \beta_{Y,X}(x) &= F_X(x)\left[\mu_Y - E(Y|X < x)\right] \\
&= \int F_X(x) \overline{F}_Y(y) - \left(F_X(x) - F_{Y,X}(y,x)\right) dy \\
&= \int F_{Y,X}(y,x) - F_X(x) F_Y(y)\; dy
\end{aligned}$$

So I can write $\beta_{Y,X}(.)$ as a function of the marginal CDFs and the joint CDF.:

$$\beta_{Y,X}(x) = \frac{1}{\overline{F}_X(x)} \frac{1}{F_X(x)} \int F_{Y,X}(y,x) - F_X(x) F_Y(y)\, dy \tag{A2}$$

□

The correlation order ranks couples of random variables with the same marginal distribution, as (X_2, Y_2) is ranked above (X_1, Y_1) if the copula of the latter is standing above the former (Property 6.2.13 Denuit et al. (2006)). The term "concordance order" is sometimes used instead of the "correlation order" (Definition 2.8.1 Nelsen (2007)).

Claim 2. The correlation order implies correlation in expectation.

Proof. Using Sklar's Representation Theorem and Claim 1, I have:

$$\beta_{Y,X}(x) = \frac{1}{1-F_\alpha(x)} \int \frac{C(y, F_\alpha(x))}{F_\alpha(x)} - y \, dF_\theta^{-1}(y) \tag{A3}$$

An increase in the correlation order, i.e. an increase in the level of the copula everywhere keeping the marginal CDFs constant, leads to an increase in $\beta_{Y,X}(x)$ for all x. □

Claim 3. A positive correlation in expectation implies a positive Pearson correlation since the covariance of (X, Y) can be written as a weighted sum of $\beta_{Y,X}(x)$:

$$\int F_X(x) \bar{F}_X(x) \beta_{Y,X}(x) \, dx = \mathrm{cov}(X, Y) \tag{A4}$$

Proof. Using A2 to replace the value of β, it boils down to showing that

$$\mathrm{cov}(X, Y) = \int\int F_{Y,X}(y, x) - F_Y(y) F_X(x) \; dydx, \tag{A5}$$

that is proved in Lehmann (1966). I repeat his elegant proof here for completeness. Assume that (X, Y) and (X', Y') are i.i.d.. And define $I(y, x) = 1$ if $y \leqslant x$, and zero otherwise. Then

$$\begin{aligned}
2\mathrm{cov}(X, Y) &= E\left((X - X')(Y - Y')\right) \\
&= E\left(\int\int (I(x, X) - I(x, X'))\,(I(y, Y) - I(y, Y')) \; dydx\right) \\
&= 2\int\int E\left(I(x, X) I(y, Y) - I(x, X) I(y, Y')\right) \; dydx \\
&= 2\int\int F_{Y,X}(y, x) - F_Y(y) F_X(x) \; dydx
\end{aligned}$$

□

Corollary 1. *If Y_2 is more correlated in expectation with X than Y_1 then it has a higher covariance.*

Another sufficient condition for positive correlation in expectation is that $E(Y|X)$ is an increasing function. The latter is true if Y is stochastically increasing in X, that is $P(Y > y|X)$ is increasing for all y (Balakrishnan and Lai 2009).

Symmetry. If (Y, X) are positively correlated in expectation, then claim 3 implies that (X, Y) cannot be negatively correlated in exp. But the latter can be neither positive nor negative correlated in expectation. In fact, an increasing $E(Y|X)$ does not imply an increasing $E(X|Y)$.

A random variable is more correlated in expectation with itself than with any other random variable with the same marginal distribution.

Claim 4. If X and Y have the same marginal distributions, then X is more correlated in expectation with itself than with Y.

Proof. Using A1, I need to show that $E(Y|X \geqslant x) > E(X|X \geqslant x)$ for all x. This is equivalent to $E(YI(x, X)) < E(XI(x, X))$. Keeping x constant, I denote $I(x, Y)$ with I_Y. Now I appeal to Proposition

1 in Samuels (1991). I repeat the proof here for completeness.

$$E(YI_X) = E(XI_X) + E(Y(I_X - I_Y))$$

Now I need to show that $E(Y(I_X - I_Y)) < 0$. The fact that $Y < x$ when $I_Y - I_X = 1$ and non-degeneracy condition imply that

$$E(Y(I_X - I_Y)) < xP(I_X - I_Y = 1) - xP(I_X - I_Y = -1) = xE(I_X - I_Y) = 0$$

$\square$

Corollary 2. *If X and Y have the same marginal distributions, then being negatively correlated in expectation is equivalent to exhibiting "reversion toward the mean" a la Samuels (1991), that is for all x*

$$E(Y|X > x) - E(X|X > x) < 0 < E(Y|X < x) - E(X|X < x) \tag{A6}$$

and

$$E(Y|X < x) < \mu < E(Y|x < X) \tag{A7}$$

Proof. Definition 2 and Claim 4 together imply that the expected value of Y while X is larger than x is closer to the mean than the expected value of X, that is $E(X|X \geqslant x) > E(Y|X \geqslant x) > \mu$. $\square$

Claim 5. $\beta_{Y,X}(x)$ is an increasing function if and only if $E(Y|F_X(X))$ is convex.

Proof. $\beta_{Y,X}(x)$ is increasing if and only if $\beta_{Y,F_X(X)}(x)$ is increasing. So I have to prove that $\beta_{Y,X}(x)$ is an increasing function if and only if $E(Y|X)$ is convex and X is uniformly distributed.

Lemma 3 shows that the derivative of β can be written as:

$$\frac{\partial}{\partial x}\beta_{Y,X}(x) = \frac{f_X(x)}{F_X(x)\bar{F}_X(x)}\left\{\int E(Y|X = z)\, g(z,x)\, dz - E(Y|X = x)\right\} \tag{A8}$$

where

$$g(z,x) \equiv f_X(z)\left\{\frac{\bar{F}_X(x)}{F_X(x)} I(z,x) + \frac{F_X(x)}{\bar{F}_X(x)}\left[1 - I(z,x)\right]\right\}$$

and $\int g(z,x)\, dz = 1$. Given the convexity of $E(Y|X)$, Jensen's inequality states that:

$$\int E(Y|X = z)\, g(z,x)\, dz > E\left(Y|X = \int z g(z,x)\, dz\right)$$

If X is uniformly distributed then $\int z g(z,x)\, dz = x$, implying that $\frac{\partial}{\partial x}\beta_{Y,X}(x) \geqslant 0$. $\square$

Claim 6. $\beta_{X,X}(x)$ is constant if and only if X is uniformly distributed.

Proof. Claim 5 implies that $\beta_{X,X}(x)$ is constant if and only if $E(X|F_X(X))$ is linear, that is equivalent to a linear CDF.

An alternative direct proof uses equation (A8), that implies $\frac{\partial}{\partial x}\beta_{X,X}(x) = 0$ if and only if

$$\frac{\overline{F}_X(x)}{F_X(x)}\int^x \alpha f_X(\alpha)\, d\alpha + \frac{F_X(x)}{\overline{F}_X(x)}\int_x \alpha f_X(\alpha)\, d\alpha = x. \tag{A9}$$

Differentiating both sides, I get:

$$\frac{-f_X(x)}{F_X(x)^2}\int^x \alpha f_X(\alpha)\, d\alpha + \frac{f_X(x)}{\overline{F}_X(x)^2}\int_x \alpha f_X(\alpha)\, d\alpha + \left(\frac{\overline{F}_X(x)}{F_X(x)} - \frac{F_X(x)}{\overline{F}_X(x)}\right) x f_X(x) = 1.$$

After using equation (A9), I can rewrite this condition as:

$$\int^x \alpha f_X(\alpha)\, d\alpha = F_X(x)\left\{x - \frac{1}{2}\frac{F_X(x)}{f_X(x)}\right\}.$$

Now differentiating both sides, I get $\frac{\partial}{\partial x} f_X(x) = 0$. □

Corollary 3. $\beta_{Y,X}(x)$ *is constant if and only if* $E(Y|X) = \beta F_X(X)$.

I use Sklar's Representation Theorem (Nelsen (2007)), that decomposes the joint distribution of (X, Y) into two marginal CDFs, $F_X(.)$ and $F_Y(.)$, and a bivariate copula, $C(.,.)$. This will help us to prove the following lemma.

Lemma 2. *If* (X, Y) *are negatively correlated in expectation then keeping the copula unchanged, (i) a mean-preserving change of* $F_Y(.)$ *to increase the dispersion of* Y *decreases* $\beta(x)$*; (ii) a shift of* $F_\theta(.)$ *– replacing it with* $F_\theta(x+\delta)$ *– has no effect on* $\beta(x)$.

Proof. I use the formulation of β from equation (A1). 1. Keeping the copula unchanged, a mean-preserving spread of Y so that Y is replaced by $(1+\varepsilon)Y - \varepsilon\mu_Y$ for a positive ε, increases $E(Y|X < x)$. To see this, note that:

$$E((1+\varepsilon)Y - \varepsilon\mu_Y|X < x) - E(Y|X < x) = \varepsilon(E(Y|X < x) - E(Y)) > 0.$$

2. Keeping the copula unchanged, a shift of $F_Y(.)$ shifts μ_Y and $E(Y|X < x)$ both with δ. □

Some simple examples are also useful to discuss here.

First, consider the case where $Y = \kappa + \nu X$. In this case, $\beta_{Y,X}(x) = \nu\beta_{X,X}(x)$. Then (X, Y) are positively correlated in expectation if and only if $\nu > 0$. And $Y_1 = \kappa_1 + \nu_1 X$ is more correlated in expectation with X than $Y_2 = \kappa_2 + \nu_2 X$ if and only if $\nu_1 > \nu_2$. In addition if X is uniformly distributed over $[x_0, x_1]$, then $\beta_{X,X}(x) = \frac{x_1 - x_0}{2}$. This leads to the following simple joint uniformly distributed distribution that I will use in section A3.2.1:

$$\beta_{Y,X}(x) = \nu\frac{x_1 - x_0}{2}$$

The second example concerns when (X, Y) are joint normal, then I have

$$\beta_{Y,X}(x) \equiv -\nu\sigma_Y \frac{1}{\overline{\Phi}_X(x)}\frac{\phi_X(x)}{\Phi_X(x)} \tag{A10}$$

where ν denotes the Pearson correlation between (X, Y) and I used

$$E(Y|X < x) = \mu_y - \nu\sigma_y \frac{\phi_X(x)}{\Phi_X(x)}.$$

The latter can be obtained from integrating over

$$E(Y|X = x) = \mu_y + \nu\sigma_y \frac{x - \mu_x}{\sigma_x}.$$

Lemma 3. *The derivative of* β *can be written as:*

$$\frac{\partial}{\partial x}\beta_{Y,X}(x) = \frac{f_X(x)}{F_X(x)\,\overline{F}_X(x)}\left\{\int E(Y|X = z)\, g(z, x)\; dz - E(Y|X = x)\right\}$$

where

$$g(z, x) \equiv f_X(z)\left\{\frac{\overline{F}_X(x)}{F_X(x)} I(z, x) + \frac{F_X(x)}{\overline{F}_X(x)}[1 - I(z, x)]\right\}$$

Proof. Using A1,

$$\frac{\partial}{\partial x}\beta_{Y,X}(x) = -\frac{f_X(x)}{F_X(x)\,\overline{F}_X(x)}\{E(Y|X = x) - E(Y|X \leqslant x)\} + \frac{f_X(x)}{\overline{F}_X(x)}\beta_{Y,X}(x)$$

Using A1 again and the Law of total expectation, I can rewrite this as

$$\frac{\partial}{\partial x}\beta_{Y,X}(x) = \frac{f_X(x)}{\overline{F}_X(x)} E(Y|X > x) + \frac{f_X(x)}{F_X(x)} E(Y|X \leqslant x) - \frac{f_X(x)}{F_X(x)\,\overline{F}_X(x)} E(Y|X = x)$$

Re-arranging the terms, I get

$$\begin{aligned}
\frac{\partial}{\partial x}\beta_{Y,X}(x) &= \frac{f_X(x)}{F_X(x)\,\overline{F}_X(x)}\left\{F_X(x)\, E(Y|X > x) + \overline{F}_X(x)\, E(Y|X \leqslant x) - E(Y|X = x)\right\} \\
&= \frac{f_X(x)}{F_X(x)\,\overline{F}_X(x)}\left\{\frac{F_X(x)}{\overline{F}_X(x)}\int_x E(Y|X = z)\; dF_X(z) + \frac{\overline{F}_X(x)}{F_X(x)}\int^x E(Y|X = z)\; dF_X(z) - E(Y|X = x)\right\} \\
&= \frac{f_X(x)}{F_X(x)\,\overline{F}_X(x)}\left\{\int E(Y|X = z)\, g(z, x)\; dz - E(Y|X = x)\right\}
\end{aligned}$$

□

A3 Application to the Economic Model

The productivity and preference for an amenity, (θ, α) are jointly distributed over bounded and positive supports, that I denote by $[\theta_0, \theta_1] \times [\alpha_0, \alpha_1]$, where $\theta_0 > 0$ and $\alpha_0 > 0$.

Assumption 2. (θ, α) *are negatively correlated in expectation.*

Following the notation in the paper, I define the function $\rho(.)$ as follows.

Notation 2. In this subsection, I will drop the subscript and take the inverse $\rho(x) = |\beta_{\theta,\alpha}(x)|$.

This notation implies that $\rho(x) = E(\theta|\alpha \geqslant x) - E(\theta|\alpha < x)$.

I first study two corner cases when all or none of the jobs provide the amenity in equilibrium (pooling equilibria). The focus is on the Rothschild-Stiglitz equilibria.

I. All jobs provide the amenity, $q^* = 1$, when only contracts $(w^*, 1)$ are provided in the equilibrium, and no employer can provide a job without amenity and make more profit, that is

$$yE(\theta|\alpha < x) - (w^* + x) < y\mu_\theta - \psi - w^*$$

for all y and x. Using A1, I can rewrite this condition as

$$\underbrace{x - \psi}_{\text{Amenity value to the marginal type}} > \underbrace{y\bar{F}_\alpha(x)\,\rho(x)}_{\text{Production gain}} \tag{A11}$$

This condition is equivalent to the most productive employer cannot deviate by providing a job without the amenity, that is, when $\psi^F > \psi$ where

$$\psi^F \equiv \alpha_0 - y_1\rho(\alpha_0) \tag{A12}$$

The net value of the amenity to the individual that appreciates it the least (the lowest α−type) is larger than the production gain to the most productive job.

In the presence of a negative correlation between taste and productivity, there are jobs without an amenity when the universal provision is efficient, $\psi^F < \alpha_0$. This is more likely, ψ^F is lower, with higher correlation, or higher productivity of the most productive job.

If the efficient provision of the amenity is not universal, $q^e < 1$ or $\alpha_0 < \psi$, i.e. there are people for whom the amenity is not worth the cost, then the market would never provide the amenity to all jobs.

II. Unraveling, $q^* = 0$, when only the contract $(w^*, 0)$ is provided in equilibrium, and no employer can increase the profit by providing a job with the amenity and attracting the higher taste workers, that is

$$yE(\theta|\alpha \geqslant x) - \psi - (w^* - x) < y\mu_\theta - w^*$$

for all y and x. This is equivalent to even the least productive job not being able to break the equilibrium and offer jobs with the amenity, or the amenity cost is below a certain level, $\psi^U < \psi$, where

$$\psi^U \equiv \max_{x\in[\alpha_0,\alpha_1]} x - y_0F_\alpha(x)\,\rho(x)\,. \tag{A13}$$

This implies that at least one person should appreciate the amenity more than its cost, $\alpha_0 < \psi$. It also implies that $\alpha_1 - y_0\rho(\alpha_1) < \psi$.

If it is efficient to provide amenity, $q^e > 0$ or $\alpha_1 > \psi$, then an inefficient unraveling may occur if and only if $\psi \in (\psi^U, \alpha_1)$. The more negative is the relation between (α, θ), the more likely is this inefficient unraveling. If $y_0 = 0$, then inefficient unraveling never occurs. The existence of close-to-zero productivity jobs makes unraveling less likely. Such employers do not bear any cost of working with low productivity workers.

If no amenity is efficient, $q^e = 0$ or $\alpha_1 < \psi$, then the market always unravels given the negative correlation.

In order not to have a pooling equilibrium, the amenity cost must satisfies

$$\max_{x \in [\alpha_0, \alpha_1]} x - y_0 F_\alpha(x) \rho(x) > \psi > \alpha_0 - y_1 \rho(\alpha_0) \tag{A14}$$

So, for example, it could be that:

$$\alpha_1 - y_0 \rho(\alpha_1) > \psi > \alpha_0 - y_1 \rho(\alpha_0) \tag{A15}$$

III. Separating the equilibrium is characterized by an anterior solution, $q^* \in (0,1)$ and a compensating wage differential, $p \in [\alpha_0, \alpha_1)$. The provider of amenities is the employer who gets a higher profit providing the amenity, paying the cost, but lower wages,

$$y E(\theta | \alpha \geqslant p) - \psi - w \geqslant y E(\theta | \alpha < p) - (w + p). \tag{A16}$$

This implies that employers with a lower productivity provide amenity, $\overline{y} \geqslant y$ where

$$\overline{y} \equiv \frac{p - \psi}{\rho(p)}. \tag{A17}$$

Due to adverse selection, the share of jobs providing an amenity is not necessarily increasing with the wage differential. This is the case if $\rho(.)$ is constant or a decreasing function.

The market-clearing condition that pins down the compensating wage is

$$q^* = F_y(\overline{y}) = \overline{F}_\alpha(p). \tag{A18}$$

Whether the equilibrium exists or whether it is unique depends on the number of solutions for the following fixed-point equations:

$$p = \psi + \rho(p) F_y^{-1}\left(\overline{F}_\alpha(p)\right) \tag{A19}$$

The first implication of market-clearing condition (A18) is that if the amenity is provided at all $q^* > 0$, then we should have $p > \psi$. This means that the compensating wage differential is larger than the cost of the amenity, an inefficiency $q^e > q^*$. The inefficiency is the price for motivating some jobs to provide the amenity and compensating them for the loss in expected worker quality.

The second implication is that the more negatively they are correlated within the copula, higher ρ, the higher is the inefficiency: lower q^* but q^e is the same as it does not depend on the cupola.

The third implication is that the higher dispersion of θ, the higher σ_θ, the higher the correlation $\rho(x)$, the higher the inefficiency, i.e. the lower q^*.

Proposition 3. *The amenity is always under-provided under the assumption 1.*

We could use the fixed-point condition A19, which implicitly defines the market equilibrium, to also determine the bounds of the amenity cost that ensure the existence of a separating equilibrium. A price exists that satisfies the condition A19, if and only if $\psi \in \left(\psi_a^F, \psi_a^U\right)$, where

$$\psi_a^F \equiv \min_x x - F_y^{-1} \circ \overline{F_\alpha} (x) \rho (x) \tag{A20}$$

and

$$\psi_a^U \equiv \max_x x - F_y^{-1} \circ \overline{F_\alpha} (x) \rho (x) \tag{A21}$$

It is rather straightforward to see that $\psi_a^F \leqslant \psi^F$ and $\psi_a^U \leqslant \psi^U$. The former is true since ψ^F can be achieved by $\min_x x - F_y^{-1} \circ \overline{F_\alpha} (x) \rho (x)$ when $x = \alpha_0$, and the latter since the $F_y^{-1} \circ \overline{F_\alpha} (x) \geqslant y_0 F_\alpha (x)$. There is unique equilibria except when $\psi \in \left(\psi_a^F, \psi^F\right)$ – then there is one pooling with universal coverage and one separating – or when $\psi \in \left(\psi_a^U, \psi^U\right)$ and then there is no equilibria at all.

If $\rho(.)$ is a non-increasing function, which is equivalent of assuming the convexity of expected productivity conditional on rank taste, $E(\theta|F_\alpha(\alpha))$ (Claim 5), then $\psi_a^F = \psi^F$ as the minimum in A20 is achieved at α_0. In this case, we can also pin down $\psi_a^U = \alpha_1 - y_0 \rho(\alpha_1)$. We will encounter such a case when we consider the case of uniform distributions.

A3.1 Policy

A3.1.1 Mandate

I start with the case where the market does not unravel.

Total production in equilibrium is the sum of production from jobs with and without the amenity:

$$Y^* = F_y(\overline{y}) \underbrace{E(\theta|\alpha > p) E(y|y < \overline{y})}_{E(y|a=1)} + \overline{F}_y(\overline{y}) \underbrace{E(\theta|\alpha < p) E(y|y > \overline{y})}_{E(y|a=0)}$$

Now I use the market clearing condition, equation (A18) to rewrite the total production by benchmarking it with respect to mandated production as

$$Y^* - Y^M = \overline{F}_y(\overline{y}) \{E(y|y > \overline{y}) - \mu_y\} \rho(p) \tag{A22}$$

To see this, we use

$$\rho(p) = \frac{E(\theta|\alpha < p) - \mu_\theta}{\overline{F}_\alpha(p)} = \frac{\mu_\theta - E(\theta|\alpha \geqslant p)}{F_\alpha(p)}.$$

to rewrite the total market production as

$$\begin{aligned} Y^* &= F_y(\overline{y}) \left[\mu_\theta - \overline{F}_y(\overline{y}) \rho(p)\right] E(y|y < \overline{y}) + \overline{F}_y(\overline{y}) [\mu_\theta + F_y(\overline{y}) \rho(p)] E(y|y > \overline{y}) \\ &= \mu_\theta \mu_y + F_y(\overline{y}) \overline{F}_y(\overline{y}) \rho(p) [E(y|y > \overline{y}) - E(y|y < \overline{y})] \\ &= Y_M + \overline{F}_y(\overline{y}) \{E(y|y > \overline{y}) - \mu_y\} \rho(p) \end{aligned}$$

The positive sign of the right hand side of equation A22 shows that the production is higher as set by the market than under the mandate. This is because of the gain from assortative sorting by productivity that occurs in equilibrium through firm's choice of amenity provision.

But we have to compare this production loss due to mandating with the potential gain in amenity allocation. On the amenity side, similarly, I can write:

$$A^* - A^M = \overline{F}_\alpha(p)\, E(\alpha - \psi | \alpha > p) - (\mu_\alpha - \psi) \tag{A23}$$
$$= -F_\alpha(p)\, E(\alpha - \psi | \alpha < p) \tag{A24}$$

This is negative, i.e. the mandate improves amenity allocation, if and only if the average worker without amenity in equilibrium appreciates the amenity more than its cost.

Total welfare is the sum of the net benefit from amenities plus the production surplus that is affected by the allocation of workers,

$$V^* - V^M = A^* - A^M + Y^* - Y^M$$

This can be written, using the equations (A22) and (A23), as

$$V^* - V^M = \overline{F}_y(\overline{y}) \left[\{E(y|y > \overline{y}) - \mu_y\} \rho(p) - E(\alpha - \psi | \alpha < p) \right] \tag{A25}$$

So mandating is welfare improving, $V^M > V^*$, if and only if the production loss is dominated by the positive amenity gain:

$$Y^* - Y^M < A^M - A^*,$$

Replacing each side of the inequality from (A22) and (A23), and using labor market equilibrium condition (A18), the optimal mandating condition can be written as

$$\frac{E(y|y > \overline{y}) - \mu_y}{\overline{y}} < \frac{E(\alpha|\alpha < p) - \psi}{p - \psi} \tag{A26}$$

Both sides of this inequality measure skewness. The right side measures the skewness of the left-hand tail of the taste for the amenity, and the other side the tail of productivity. Loosely speaking, mandating is optimal if the right-hand tail of productivity is smaller than the left-hand tail of amenity taste. The average productivity of most productive firms is lower than the amenity taste of workers who appreciate the amenity the least.

If the market unravels, $\psi > \psi^U$, then mandating is optimal as long as the amenity cost is lower than the average amenity taste, that is $\psi < \psi^M$, where

$$\psi^M = \mu_\alpha. \tag{A27}$$

In this case, mandating creates a positive surplus from the universal provision of the amenity. Therefore, mandating is optimal while the market unravels when $\psi^U < \psi < \mu_\alpha$. This only occurs when there is severe unraveling, that is the cost is below the average taste and still the market unravels. That is possible if and only if

$$\max_{x\in[\alpha_0,\alpha_1]} x - y_0\left(\mu_\theta - E\left(\theta|\alpha \geqslant x\right)\right) < \mu_\alpha$$

Winners and losers

The goal of this subsection is to discover the winners and losers of mandating when it is optimal and necessary, $\psi^F < \psi < \psi^M$.[17] To achieve this goal, the wage setting system shall be specified in order to set the wage levels and not only the compensating wage differentials.

I assume that there is a free entry of entrepreneurs or, equivalently, an infinitely elastic supply of capital that, after opening a vacancy, they match with a random task from the distribution of y. The wage is set to equalize the expected net profit equal to zero.

In the case of pooling equilibrium, $q^* = 1$,

$$E\left(\pi\right) = \mu_y \mu_\theta - \psi - w^*,$$

after normalizing the interest rate equal to zero. This implies a wage level that would be the same level of wage achieved after a mandate, that is,

$$w^M = \mu_y \mu_\theta - \psi.$$

In the case of separating equilibrium, the wage level for jobs with the amenity would be:

$$w^* = F_y\left(\overline{y}\right)\left[E\left(y|\overline{y} \geqslant y\right) E\left(\theta|\alpha \geqslant p\right) - \psi\right] + \overline{F}_y\left(\overline{y}\right)\left[E\left(y|\overline{y} < y\right) E\left(\theta|\alpha < p\right) - p\right]$$

Using equations (A1) and (A17), the change in the wage due to mandating can be written as

$$w^M - w^* = \left[\overline{y} + \mu_y - E\left(y|\overline{y} < y\right)\right] \rho\left(p\right) F_\alpha\left(p\right)$$

or, using the correlation in the expectation definition, as

$$w^M - w^* = \left[\overline{y} - F_y\left(\overline{y}\right) \beta_{y,y}\left(\overline{y}\right)\right] \rho\left(p\right) F_\alpha\left(p\right)$$

This wage change implies that the mandate is optimal when the mandating ensures a small wage loss, that is, the optimal mandate condition (A26) can be thought of as a lower bound for the wage change

$$w^M - w^* > F_\alpha\left(p\right)\left(p - E\left(\alpha|\alpha < p\right)\right).$$

In particular, the optimal mandate leads to a rise (fall) in wage for those who would (not) buy the amenity from the markets:

$$w^* + p > w^M > w^* \tag{A28}$$

Agents' utility rises by the wage gap for those who bought the amenity on the market, $\alpha > p$. For others, $\alpha < p$, the change in utility is $w^M + \alpha - (w^* + p)$. The utility increases as a response to the

[17] If $\psi < \psi^F$, the mandate is unnecessary as it has no effect.

mandate if and only if the agent\s taste is above a certain level,

$$\alpha > \alpha^M = w^* + p - w^M$$

where the cutoff is $p > \alpha^M > 0$.

To summarize, a mandate, when optimal and necessary, $\psi^F < \psi < \psi^M$, creates winners and losers. Winners are all workers with a high taste for the amenity, either those who would buy it from the market or those who would not. The former group gains through higher wages due to the mandate, whereas the latter group experiences a wage drop, but it is dwarfed by the extra utility they receive from the amenity that they would otherwise not have bought. The losers are those workers with a low taste for the amenity who did not buy it from the market and who do not appreciate the mandated amenity given the accompanying wage drop.

A3.1.2 Leveraging Observable Characteristics

Consider the case of gender differences in the correlation, assuming that one subgroup (women) exhibits zero correlation between productivity and taste, whereas the remaining workers (men) have a negative correlation.

$$\rho_m(p) > \rho_f(p) = 0,$$

for all p, where subscripts indicate gender.

Quota system

Consider the case of a quota system that allows for gender-specific contracts. All jobs offer women a menu including two options: a low wage $\mu_y \mu_\theta - \psi$ with amenity or a high wage $\mu_y \mu_\theta$ without amenity. Women are indifferent among jobs, so they will appear uniformly across job distribution satisfying the quota requirement.

Women's production under quotas is the same as the mandate, achieving its first best. But the two systems allocate amenities differently. While the mandate provides all women with the amenity, the quotas take into account the zero-correlation and assign the amenity only to female workers with taste above the cost.

Quotas lead to a level of total welfare for women that when benchmarked against its level under the mandate can be written as

$$V_f^Q - V_f^M = \frac{1}{2} F_\alpha(\psi) \{\psi - E(\alpha | \alpha < \psi)\} \tag{A29}$$

where sub-scripts Q and M indicate quota and mandate systems.

Women are better off under the quota system with gender-specific contracts than under the mandate system. This conclusion is aligned with our previous characterization of the market outcomes when the correlation is zero. More precisely, the quota system creates an isolated market for women, where the absence of taste-productivity correlation within the female sub-population leads to a market equilibrium that dominates the mandate.

The same calculation for men points to the difference:

$$V_m^Q - V_m^M = \frac{1}{2} F_\alpha\left(p_m^Q\right)\left[\left\{E\left(y|y > \overline{y}_m^Q\right) - \mu_y\right\}\rho_m\left(p_m^Q\right) - E\left(\alpha - \psi|\alpha < p_m^Q\right)\right] \tag{A30}$$

which is equivalent to the welfare equation (A25) for the male sub-population. Intuitively, in the absence of a taste-production correlation, $\rho(.) = 0$, the welfare equation (A30) boils down to (A29).

Tagging

Consider the case of separating equilibrium. The level of profit for a job is tailored to women is

$$\pi_f = y\mu_\theta - \psi - w_f$$

independently of whether it is offered low wage with amenity or high wage without amenity. Profit in the market of male workers is equal to

$$\pi_{m,0} = yE\left(\theta|\alpha \geqslant p_m^T\right) - (w_m + \psi)$$

for jobs with amenity, or

$$\pi_{m,1} = yE\left(\theta|\alpha < p_m^T\right) - \left(w_m + p_m^T\right)$$

for jobs coming with no amenity. On the men market, the cutoff of amenity provision would, similar to the equation (A17), be:

$$\overline{y}_m \equiv \frac{p_m^T - \psi}{\rho_m\left(p_m^T\right)} \tag{A31}$$

Now I study the set of jobs be offered to women in equilibrium.

I. When $y < \overline{y}_m$, then $\pi_{m,0} > \pi_{m,1}$. In this case, $\pi_f > \max\left(\pi_{m,0}, \pi_{m,1}\right)$ if and only if $y \in [\widetilde{y}_0, \overline{y}_m]$ where

$$\widetilde{y}_0 \equiv \frac{1}{F_\alpha\left(p_m^T\right)}\frac{w_f - w_m}{\rho\left(p_m^T\right)} \tag{A32}$$

II. When $y > \overline{y}_m$, then $\pi_{m,1} > \pi_{m,0}$. In this case, $\pi_f > \max\left(\pi_{m,0}, \pi_{m,1}\right)$ if and only if $y \in [\overline{y}_m, \widetilde{y}_1]$ where

$$\widetilde{y}_1 \equiv \frac{1}{\overline{F}_\alpha\left(p_m^T\right)}\left[\overline{y}_m - \frac{w_f - w_m}{\rho_m\left(p_m^T\right)}\right] \tag{A33}$$

The equations (A32) and (A33) together imply that jobs in the intermediate interval of $[\widetilde{y}_0, \widetilde{y}_1]$ are allocated to women.[18]

The equilibrium is characterized by

$$F_y\left(\widetilde{y}_1\right) - F_y\left(\widetilde{y}_0\right) = \frac{1}{2} \tag{A34}$$

[18]This is a non-empty set if and only if

$$w_f - w_m < F_\alpha\left(p_m^T\right)\left[p_m^T - \psi\right].$$

and

$$F_y(\widetilde{y}_0) = \frac{1}{2}\overline{F}_\alpha^m(p_m^T) \tag{A35}$$

where the former guarantees gender-balance in labor demand and the latter is the market-clearing condition for amenity for men, equivalent to the equation (A18). These equilibrium conditions will pin down the equilibrium allocation and the gender wage gap. The equation that will set the wage level is the expected zero net profit condition, that is,

$$\frac{1}{2}\pi_f + F_y(\widetilde{y}_0) E(\pi_{m,0}|y < \widetilde{y}_0) + \overline{F}_y(\widetilde{y}_1) E(\pi_{m,1}|y > \widetilde{y}_1) = 0.$$

The higher is the taste-productivity correlation among men, the higher is the under-provision of the amenity (higher p_m^T), leading to a shift downward of the jobs presented to women (lower $\widetilde{y}_0$ and $\widetilde{y}_1$).

Now consider the welfare level at the tagging equilibrium. For women, the total welfare is

$$V_f^T = \frac{1}{2}\mu_\theta E(y|\widetilde{y}_0 < y < \widetilde{y}_1) + \frac{1}{2}\overline{F}_\alpha(\psi)\{E(\alpha|\alpha > \psi) - \psi\}$$

using the gender-balanced equilibrium condition (A34).

Comparing it with welfare under quota, equation (A29), shows that women prefer the quota system if their access to jobs is limited downward by targeting, as

$$V_f^T - V_f^Q = \frac{1}{2}\mu_\theta\{E(y|\widetilde{y}_0 < y < \widetilde{y}_1) - \mu_y\} + \frac{1}{2}\{\mu_\alpha - \psi\} \tag{A36}$$

For men, the total production is

$$Y_m^T = F_y(\widetilde{y}_0) E(\theta|\alpha > p_m^T) E(y|y < \widetilde{y}_0) + \overline{F}_y(\widetilde{y}_1) E(\theta|\alpha < p_m^T) E(y|y > \widetilde{y}_1) \tag{A37}$$

that can be written, the gender-balanced equilibrium condition (A34), as

$$\begin{aligned} Y_m^T &= F_y(\widetilde{y}_0)(\mu_\theta - \rho_m(p_m)\overline{F}_y(\widetilde{y}_1)) E(y|y < \widetilde{y}_0) + \overline{F}_y(\widetilde{y}_1)(\rho_m(p_m) F_y(\widetilde{y}_0) + \mu_\theta) E(y|y > \widetilde{y}_1) \\ &= Y^M - Y_f^T + \rho_m(p_m^T) F_y(\widetilde{y}_0)\overline{F}_y(\widetilde{y}_1)[E(y|y > \widetilde{y}_1) - E(y|y < \widetilde{y}_0)] \end{aligned}$$

This implies that

$$Y^T - Y^M = \rho_m(p_m^T) F_y(\widetilde{y}_0)\overline{F}_y(\widetilde{y}_1)[E(y|y > \widetilde{y}_1) - E(y|y < \widetilde{y}_0)] \tag{A38}$$

production is always higher under tagging than mandate, $Y^T > Y^M$, as tagging is a more flexible type of market equilibrium where gender can be separated.

The total amenity utility for men is

$$A_m^T - A_m^M = -\frac{1}{2}F_\alpha(p_m^T) E(\alpha - \psi|\alpha < p_m^T) \tag{A39}$$

that is the same as the market equilibrium if jobs productivity has been truncated, i.e., $y \in [\widetilde{y}_0, \widetilde{y}_1]$. So the total welfare as

$$V^T - V^Q = \rho_m\left(p_m^T\right) F_y\left(\widetilde{y}_0\right) \bar{F}_y\left(\widetilde{y}_1\right)\left[E\left(y|y > \widetilde{y}_1\right) - E\left(y|y < \widetilde{y}_0\right)\right] + \frac{1}{2}\left\{\mu_\alpha - \psi - F_\alpha\left(p_m^T\right) E\left(\alpha - \psi|\alpha < p_m^T\right)\right\}.$$

A3.2 Special Cases: Parametric Distributions

A3.2.1 Uniform Distribution

Assume that (α, θ) are joint uniformly distributed. As discussed in section A2, in this case, $\rho(x) = \rho\sigma_\alpha$, where $\sigma_\alpha \equiv \frac{\alpha_1 - \alpha_0}{2}$. So market equilibrium leads to a universal provision of the amenity, $q^* = 1$, if and only if the amenity is cheap enough, $\psi < \psi^F$, where

$$\psi^F = \alpha_0 - \rho\sigma_\alpha y_1. \tag{A40}$$

Putting differently, $q^* = 1$ if and only if the correlation is low, $\rho < \rho^F$, where

$$\rho^F \equiv \frac{1}{y_1}\frac{\alpha_0 - \psi}{\sigma_\alpha}.$$

Given the negative correlation, this requires that $\alpha_0 > \psi$.

Unraveling, $q^* = 0$, occurs in equilibrium when the amenity is expensive enough, $\psi > \psi^U$, where

$$\psi^U = \max_{x \in [\alpha_0, \alpha_1]} \frac{\rho y_0}{2}\alpha_0 + \left(1 - \frac{\rho y_0}{2}\right) x. \tag{A41}$$

The optimal compensating wage differential for the deviation is

$$x^* = \begin{cases} \alpha_1 & \text{if } \frac{\rho y_0}{2} < 1 \\ \alpha_0 & \text{if } \frac{\rho y_0}{2} \geqslant 1 \end{cases}$$

That leads to the unraveling's cost cutoff of

$$\psi^U = \begin{cases} \alpha_0 \frac{\rho y_0}{2} + \left(1 - \frac{\rho y_0}{2}\right)\alpha_1 & \text{if } \frac{\rho y_0}{2} < 1 \\ \alpha_0 & \text{if } \frac{\rho y_0}{2} \geqslant 1 \end{cases}$$

So $\psi^U \in [\alpha_0, \alpha_1]$, and, as long as $\frac{\rho y_0}{2} < 1$, it is a weighted average, so that the smaller $\frac{\rho y_0}{2}$ is, the larger is the ψ^U. Putting differently, unraveling, $q^* = 0$, occurs in equilibrium when $\psi > \alpha_0$ and $\rho > \rho^U$, where

$$\rho^U = \frac{2}{y_0} + \frac{1}{y_0}\frac{\alpha_0 - \psi}{\sigma_\alpha}$$

Market unravels more often, the larger is the correlation in expectation, ρ. When the correlation is too large, $y_0\rho > 2$, then the market unravels except when the universal provision is efficient ($\psi \leqslant \alpha_0$), i.e., when every worker values amenity more than its cost.

Now considering the interior cases, $\psi \in \left[\psi^F, \psi^U\right]$, where a separating equilibrium prevails. The amenity equilibrium condition is

$$q^* = F_y\left(\frac{p-\psi}{\rho\sigma_\alpha}\right) = \frac{\alpha_1 - p}{2\sigma_\alpha}. \tag{A42}$$

If I further assume that firm productivity y is also uniformly distributed, I can pin down the compensating wage differential as the weighted average of the amenity's cost and a measure of taste:

$$p = (1-\zeta)\left(\mu_\alpha + \sigma_\alpha \frac{\mu_y}{\sigma_y}\right) + \zeta\psi, \tag{A43}$$

where the weight is $\zeta = \frac{1}{1+\rho\sigma_y} \in [0,1]$. In the absence of a correlation between productivity and amenity, $\zeta = 1$, the efficient allocation prevails in equilibrium, $p = \psi$.[19]

The first implication of equation (A43) is that the market allocation is not efficient, since $p > \psi$, in the presence of negative taste-productivity correlation in expectation.

The inefficiency is higher (the further is the wage differential p from the amenity cost ψ),

- the more negative is the correlation between taste and productivity,
- the higher is the mean or standard deviation of taste for amenity,
- the higher is the average and the lower is the standard deviation of job productivity,
- neither the variation nor the average worker's productivity matters.

Mandate

I can write the difference between the market and mandate welfare levels, using the equation (A25) and the market-clearing condition (A18), as:

$$V^* - V^M = \frac{p-\alpha_0}{2\sigma_\alpha}\left(\frac{\alpha_1 - p}{2}\rho\sigma_y - \frac{p+\alpha_0}{2} + \psi\right) \tag{A44}$$

Using expressions (A43), I can rewrite it as

$$V^* - V^M = \frac{\zeta}{4\sigma_\alpha}\left(\psi - \alpha_0 + \rho\sigma_\alpha y_1\right)\left(\psi - \alpha_0 - \rho\sigma_\alpha y_0\right) \tag{A45}$$

To see this, note that

$$V^* - V^M = \frac{\zeta}{4\sigma_\alpha}\frac{p-\alpha_0}{\zeta}\left\{2\left(\psi - \alpha_0 + \rho\sigma_\alpha\sigma_y\right) - \frac{p-\alpha_0}{\zeta}\right\}$$

and $\frac{p-\alpha_0}{\zeta} = \psi - \alpha_0 + \rho\sigma_\alpha y_1$.

The difference between the market and mandate welfare levels, equation A45, implies that mandate is optimal if and only if $\psi^M > \psi > \psi^F$,where

$$\psi^M \equiv \left(1 - \frac{\rho y_0}{2}\right)\alpha_0 + \frac{\rho y_0}{2}\alpha_1$$

[19]In this case, it is easy to verify that we have $\psi_a^F = \psi^F$ and $\psi_a^U \leqslant \psi^U$ as $\psi_a^U = \alpha_0\frac{\rho y_0}{2} + \left(1 - \frac{\rho y_0}{2}\right)\alpha_1$.

So the optimal mandate condition when the market provides a separating equilibrium is that the mandate cost must be below certain level,[20]

$$\psi < \psi^M \tag{A46}$$

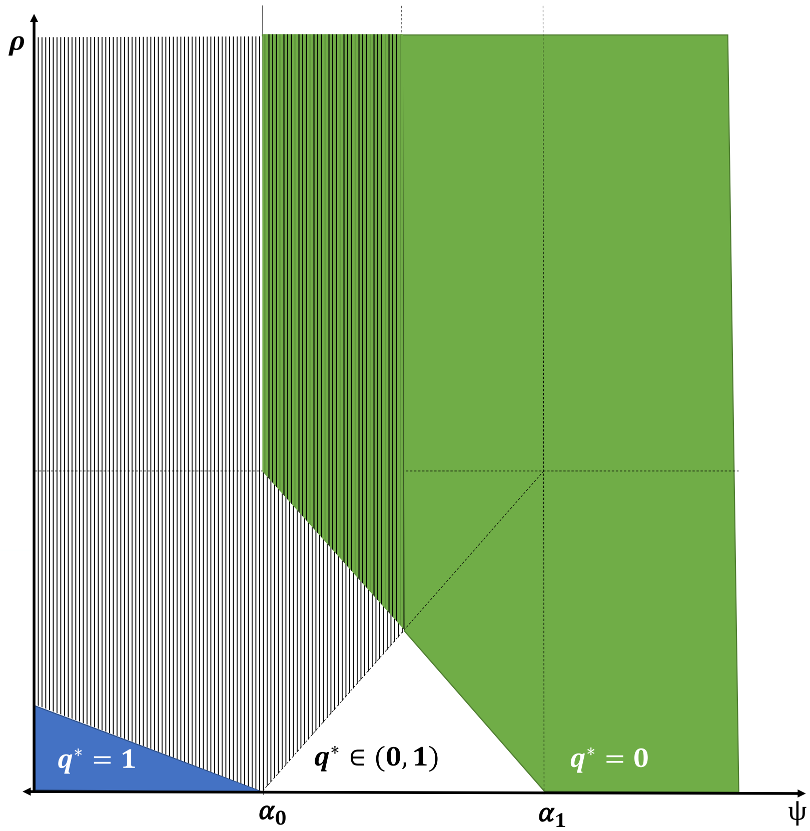

Figure A1: **Market allocation of the amenity, and the role of mandating**
Note: Taste-productivity correlation in expectation on the y-axis, and the amenity cost on the x-axis. Three colors indicate regions with different market equilibrium: green where market unravels, $q^* = 0$, blue where all jobs come with the amenity, $q^* = 1$, and white where some jobs provide the amenity and some do not, $q^* \in (0, 1)$. Stripped region indicates where mandating is welfare improving. Figure 5 in the main text shows a similar graph when fixing ρ.

The comparison with the unraveling cutoff in this case reveals that $\psi^M < \psi^U$ if and only if $\frac{\rho y_0}{2} < \frac{1}{2}$. This implies that there are three cases to consider.

The case I, when $\frac{\rho y_0}{2} < \frac{1}{2}$: In this case, $\alpha_0 < \psi^M < \mu_\alpha < \psi^U < \alpha_1$. The mandate is never optimal if the market unravels. When the market leads to a separating equilibrium, the mandate is optimal if and only if $\psi < \psi^M$. So mandate is only optimal when $\psi \in [\psi^F, \psi^M]$.

The case II, when $\frac{\rho y_0}{2} \in [\frac{1}{2}, 1]$: In this case, $\alpha_0 < \psi^U < \mu_\alpha < \psi^M < \alpha_1$. The mandate is optimal when $\psi \in [\psi^F, \mu_\alpha]$. This can occur when the market unravels $\psi > \psi^U$ or not $\psi \in [\psi^F, \psi^U]$. When the market unravels, $\psi > \psi^U$, the mandate is optimal when $\psi \in [\psi^U, \mu_\alpha]$, using the equation A27. When the market does not unravel, the mandate is always optimal, and necessary when $\psi > \psi^F$. So

[20] Alternatively, we could drive this condition using the condition (A26).

mandate is only optimal when $\psi \in \left[\psi^F, \mu_\alpha\right]$.

The case III, when $\frac{\rho y_0}{2} \geqslant 1$: In this case, $\psi^U < \alpha_0 < \alpha_1 < \psi^M$. The mandate is always optimal if the market does not unravel, and when the market unravels, the mandate can still be optimal if $\psi < \mu_\alpha$. So mandate is only optimal when $\psi \in \left[\psi^F, \mu_\alpha\right]$.

To summarize, when $\rho y_0 < 1$, mandate is only optimal when $\psi \in \left[\psi^F, \psi^M\right]$, otherwise when $\psi \in \left[\psi^F, \mu_\alpha\right]$.

In terms of the correlation in expectation, the mandate is optimal when the correlation is high, $\rho > \rho^M$. When $\psi \in [\alpha_0, \mu_\alpha]$,

$$\rho^M = \frac{1}{y_0}\frac{\psi - \alpha_0}{\sigma_\alpha}.$$

When the amenity is cheap enough, $\psi < \alpha_0$, the mandate never reduces welfare, and it is optimal (strictly welfare increasing) when market does uniformly providing it, $\rho > \rho^F$, i.e., $\rho^M = \rho^F$.

Proposition 4. *In case of uniformly distributed taste and productivity, mandating dominate a la Hicks-Kaldor the market allocation when the taste-productivity correlation in expectation is high, $\rho > \rho^M$ where $\rho^M = \frac{1}{y_0}\frac{\psi-\alpha_0}{\sigma_\alpha}$. Expressed in terms of amenity cost, mandate is optimal when the amenity cost is low, $\psi < \psi^M$. With a separating equilibrium, $\psi^M = \left(1 - \frac{\rho y_0}{2}\right)\alpha_0 + \frac{\rho y_0}{2}\alpha_1$, and when markets unravel, $\psi^M = \mu_\alpha$.*

Quota

Using the equation (A29), I can write the gain from quotas for women as:

$$V_f^Q - V_f^M = \frac{1}{2\sigma_\alpha}\left(\frac{\psi - \alpha_0}{2}\right)^2$$

The same for men, using the equation (A30), would be

$$V_m^Q - V_m^M = \frac{p_m^Q - \alpha_0}{4\sigma_\alpha}\left(\frac{\alpha_1 - p_m^Q}{2}\rho_m\sigma_y - \frac{p_m^Q + \alpha_0}{2} + \psi\right) \tag{A47}$$

that is equivalent to the market vs. mandate comparison in the case of uniform distribution, equation (A44). So in the same way, quotas are preferred to mandate for men if the correlation is low, that is:

$$V_m^Q > V_m^M \Leftrightarrow \rho_m < \rho_m^M$$

Taken together,

$$V^Q - V^M = \frac{1}{2}\frac{1}{4\sigma_\alpha}\left[\frac{1}{1+\rho_m\sigma_y}\left(\psi - \alpha_0 + \rho_m y_1\sigma_\alpha\right)\left(\psi - \alpha_0 - \rho_m y_0\sigma_\alpha\right) + \left(\psi - \alpha_0\right)^2\right]$$

Now comparing the mandate with the market equilibrium, as shown in Lemma 4, total welfare can be written as:

$$V^* - V^M = \frac{1}{4\sigma_\alpha}\left[\frac{1}{1+\frac{\rho_m}{2}\sigma_y}\left(\psi - \alpha_0 + \frac{\rho_m}{2}y_1\sigma_\alpha\right)\left(\psi - \alpha_0 - \frac{\rho_m}{2}y_0\sigma_\alpha\right)\right]$$

Define the function $\Omega\left(.\right)$ as

$$\Psi(x) \equiv \frac{1}{1+x\sigma_y}\left(\psi-\alpha_0+xy_1\sigma_\alpha\right)\left(\psi-\alpha_0-xy_0\sigma_\alpha\right)$$

This is a convex function for positive values of x. Then we have:

$$V^Q - V^M = \frac{1}{4\sigma_\alpha}\left[\frac{1}{2}\Psi(\rho_m)+\frac{1}{2}\Psi(0)\right]$$

and

$$V^* - V^M = \frac{1}{4\sigma_\alpha}\Psi\left(\frac{\rho_m}{2}\right)$$

The Jensen inequality, given the convexity of $\Psi(.)$, implies that quotas always dominate the market equilibrium:

$$V^Q > V^* \Leftrightarrow \frac{1}{2}\Psi(\rho_m)+\frac{1}{2}\Psi(0) > \Psi\left(\frac{\rho_m}{2}\right)$$

Mandate dominate the market only when the correlation is larger than certain cutoff, i.e. the positive solution of $\Psi(x)=0$. We denote this cutoff by ρ^*. Mandate dominates quota when the correlation is larger than a different cutoff, denoted by ρ^{**}. The latter is large than ρ^* since $V^Q > V^*$.

Lemma 4. *When only half of the population exhibit a negative taste-productivity correlation in expectation, the total welfare difference between market equilibrium and under the mandate can be written as*

$$V^* - V^M = \frac{1}{2}Y_M + \overline{F}_y(\overline{y})\left[\left\{E(y|y>\overline{y})-\mu_y\right\}\frac{\rho(p)}{2} - E(\alpha-\psi|\alpha<p)\right]$$

and under uniform distribution,

$$V^* - V^M = \frac{\zeta'}{4\sigma_\alpha}\left(\psi-\alpha_0+\frac{\rho_m}{2}\sigma_\alpha y_1\right)\left(\psi-\alpha_0-\frac{\rho_m}{2}\sigma_\alpha y_0\right)$$

Proof. We start with firms decision, equation 9, which in this case where women exhibit no correlation, would be

$$\frac{1}{2}y\left[E(\theta|\alpha \geqslant p)+\mu_\theta\right]-\psi-w > y\left[\frac{1}{2}E(\theta|\alpha<p)+\mu_\theta\right]-(w+p). \tag{A48}$$

So the supply condition is $\overline{y} > y$, where in this case the cutoff is $\overline{y} \equiv \frac{p-\psi}{\rho(p)/2}$. The price of amenity is pinned down by the market clearing condition $F_y(\overline{y}) = \overline{F_\alpha}(p)$. This is not affected by the hetergenity in correlation because it only dependson marginal distributions.

Total production in equilibrium, equation A22, can be written in this case as

$$\begin{aligned}
Y^* &= F_y(\overline{y})\,E(\theta|\alpha>p)\,E(y|y<\overline{y}) + \overline{F}_y(\overline{y})\,E(\theta|\alpha<p)\,E(y|y>\overline{y}) \\
&= \frac{1}{2}F_y(\overline{y})\left[3\mu_\theta - \overline{F}_y(\overline{y})\rho(p)\right]E(y|y<\overline{y}) + \frac{1}{2}\overline{F}_y(\overline{y})\left[3\mu_\theta + F_y(\overline{y})\rho(p)\right]E(y|y>\overline{y}) \\
&= \frac{3}{2}\mu_\theta\mu_y + F_y(\overline{y})\overline{F}_y(\overline{y})\frac{\rho(p)}{2}\left[E(y|y>\overline{y}) - E(y|y<\overline{y})\right] \\
&= \frac{3}{2}Y_M + \overline{F}_y(\overline{y})\left\{E(y|y>\overline{y})-\mu_y\right\}\frac{\rho(p)}{2}
\end{aligned}$$

The total amenity utility is unchanged, equation A23, as the sorting is the same function of price (although the price is different). This means that the total welfare is

$$V^{*}-V^{M}=\frac{1}{2}Y_{M}+\overline{F}_{y}\left(\overline{y}\right)\left[\left\{E\left(y|y>\overline{y}\right)-\mu_{y}\right\}\frac{\rho\left(p\right)}{2}-E\left(\alpha-\psi|\alpha<p\right)\right]$$

Under the uniform distribution assumption, this can be rewritten as

$$V^{*}-V^{M}=\frac{\zeta'}{4\sigma_{\alpha}}\left(\psi-\alpha_{0}+\frac{\rho_{m}}{2}\sigma_{\alpha}y_{1}\right)\left(\psi-\alpha_{0}-\frac{\rho_{m}}{2}\sigma_{\alpha}y_{0}\right) \tag{A49}$$

since $\frac{p-\alpha_0}{\zeta'}=\psi-\alpha_0+\frac{\rho_m}{2}\sigma_\alpha y_1$ where $\zeta'\equiv\frac{1}{1+\frac{\rho_m}{2}\sigma_y}$. □

Tagging

The systems of equations (A32, A33, and A34) pin down the lowest-productive job allocated to women as

$$\widetilde{y}_{0}=\frac{p_{m}^{T}-\psi}{\rho_{m}\sigma_{\alpha}}-\sigma_{y}\overline{F}_{\alpha}\left(p_{m}^{T}\right)$$

In addition market equilibrium (equation A35) provides a second equation with the same two unknowns,

$$\widetilde{y}_{0}=y_{0}+\sigma_{y}\overline{F}_{\alpha}\left(p_{m}^{T}\right) \tag{A50}$$

So solving for the amenity allocation cutoff for men leads to

$$p_{m}^{T}=\left(1-\zeta^{m}\right)\left(\mu_{\alpha}+\sigma_{\alpha}\frac{\mu_{y}}{\sigma_{y}}\right)+\zeta^{m}\psi$$

Now comparing to the cutoff for men under the quotas – male-specific market equivalent of A43 – indicates that the cutoffs are the same under tagging and quota system, $p_m^T=p_m^Q$. Men's amenity allocation is the same although their job allocation is very different.

Equation A50 pins down the least productive job allocated to women by

$$\widetilde{y}_{0}=y_{0}+\sigma_{y}\times q_{m}^{Q},$$

meaning that the higher is the male taste-productivity correlation, the lower is the efficiency of the quota system or tagging is in allocating amenity to men, and the lower is the job quality allocated to women. The reason for the latter is that the high male correlation motivates a more intense usage of sorting in their market that can be boosted in case of tagging using job differentiation.

Now the welfare effect of tagging for women, using equation (A36), is

$$\begin{aligned}V_{f}^{T}-V_{f}^{Q}&=\frac{1}{2}\mu_{\theta}\left\{\widetilde{y}_{0}-\frac{\sigma_{y}}{2}-y_{0}\right\}+\frac{1}{2}\left\{\mu_{\alpha}-\psi\right\}\\&=\frac{1}{2}\mu_{\theta}\sigma_{y}\left\{q_{m}^{Q}-\frac{1}{2}\right\}+\frac{1}{2}\left\{\mu_{\alpha}-\psi\right\}\end{aligned}$$

This implies that women are better off under tagging with a low correlation among men.

For male workers, the amenity allocation is the same under quotas and tagging, leading to

$A_m^T = A_m^Q$, since $p_m^T = p_m^Q$. The total welfare:

$$V^T - V^Q = Y^T - Y^Q = \rho_m \sigma_\alpha \sigma_y \left(1 - q_m^Q\right) q_m^Q \tag{A51}$$

Tagging is always better than quotas. Although both achieve the same amenity allocation, tagging leads to higher production as tagging exploits gender-specific partition of jobs to sort workers.

More importantly, the higher is the taste-productivity correlation among men, the higher is the gain from tagging relative to quota.

To gain some more intuition, note that using A38 and A47, we can write the production gain of using observable characteristics relative to mandate case as

$$Y^Q - Y^M = \frac{1}{2} \rho_m \sigma_\alpha \sigma_y \left(1 - q_m^Q\right) q_m^Q$$

for the case of quota, and

$$Y^T - Y^M = \frac{3}{2} \rho_m \sigma_\alpha \sigma_y \left(1 - q_m^Q\right) q_m^Q$$

for the case of tagging. Quota system allows separation contracts by gender, thus reaching first best for women, and separating jobs for men by a cutoff. This implies, given the uniform distribution, a difference between averages job quality of more and less productive male workers of $\frac{1}{2}\sigma_y$. Tagging further allows the separation of the jobs by gender, leaving the middle jobs for women that leads to a difference between averages job quality of more and less productive male workers of $\frac{3}{2}\sigma_y$.

A3.2.2 The Case of Normal Distribution

Now assume that (α, θ) are joint normally distributed. Denote their means by (μ_α, μ_θ), standard deviations by $(\sigma_\alpha, \sigma_\theta)$, and the marginal density function by ϕ_α, the cumulative distribution function by Φ_α and the survival function by $\overline{\Phi_\alpha}(x)$. I define

$$\Omega_\alpha(x) \equiv \frac{\phi_\alpha(x)}{\Phi_\alpha(x)\,\overline{\Phi_\alpha}(x)}$$

that is a convex function with the minimum at μ_α. To see this, it is useful to recall that the inverse Mill's ratio $\lambda_\alpha(x) \equiv \frac{\phi_\alpha(x)}{\Phi_\alpha(x)}$ is a decreasing and convex function, whereas the hazard function, $\frac{\phi_\alpha(x)}{\overline{\Phi_\alpha}(x)}$, is increasing and convex, and the two functions are equal at μ_α.[21] I use the same notations for the standard normal distribution by omitting the subscript.

As discussed above, in the case of joint normal, $\rho(x) = \rho\sigma_\theta\Omega_\alpha(x)$, where ρ denotes the negative of a Pearson correlation between (α, θ). So Assumption 1 is equivalent to a negative $\rho > 0$.

In equilibrium, the market always leads to an interior solution, $q^* \in (0, 1)$. Intuitively, this is due to an unbounded distribution of taste, there is always somebody who hates the amenity, $\alpha = -\infty$. Technically speaking, I have $\psi^F = -\infty$ and $\psi^U = \infty$, since

$$\lim_{x \to -\infty} x - \rho y_1 \sigma_\theta \Omega_\alpha(x) = -\infty$$

[21] To show these claims, one needs to use $\frac{\partial}{\partial x}\phi_\alpha(x) = -x\phi_\alpha(x)$.

$$\lim_{x\to\infty} x - \rho y_0 \sigma_\theta \frac{\phi_\alpha(x)}{\overline{\Phi_\alpha}(x)} = \infty$$

The supply condition $\overline{y} > y$, is determined by

$$\overline{y} \equiv \frac{1}{\rho\sigma_\theta} \frac{p-\psi}{\Omega_\alpha(p)}. \tag{A52}$$

If the equilibrium exists, it is determined by an equalization of the supply and demand of the amenity, that is:

$$q^* = F_y(\overline{y}) = \overline{\Phi_\alpha}(p) \tag{A53}$$

Using the equation (A52), the market equilibrium condition can be written as:

$$p = \psi + \rho\sigma_\theta \lambda_\alpha(p) \frac{F_y^{-1}\left(\overline{\Phi_\alpha}(p)\right)}{\overline{\Phi_\alpha}(p)}$$

Now I further assume that the firm's productivity y is also normally distributed. I can rewrite the market equilibrium condition as:

$$p = \psi + \rho\sigma_\theta \left\{\mu_y - \sigma_y \frac{p-\mu_\alpha}{\sigma_\alpha}\right\} \Omega\left(\frac{p-\mu_\alpha}{\sigma_\alpha}\right) \tag{A54}$$

This is a problematic assumption since from the perspective of the economic model, the negative value of y is meaningless. Note that the case of θ is different since it represents the unobserved part of worker productivity. So instead I assume that firms' productivity y is uniformly distributed, then

$$p = \psi + \rho\sigma_\theta \left[\mu_y + \left\{1 - \Phi\left(\frac{p-\mu_\alpha}{\sigma_\alpha}\right)\right\}\sigma_y\right] \Omega\left(\frac{p-\mu_\alpha}{\sigma_\alpha}\right). \tag{A55}$$

In this case, it is easy to see that the equilibrium is unique, using the fixed-point theorem for continuous real functions.

The first implication of condition (A55) is that the market allocation is not efficient, that is $p > \psi$ since $\rho > 0$, implying that $q^* > q$. The comparative statics of the wage differential with respect to different parameters is more complicated in this case.

Mandating is optimal when equation (A26) holds, that is equivalent to

$$E(y|y > \overline{y}) - \mu_y < \frac{\overline{\Phi_\alpha}(p)}{\rho\sigma_\theta} \left\{\frac{\mu_\alpha - \psi}{\lambda_\alpha(p)} - \sigma_\alpha\right\}$$

in case of a normal distribution, after using (A52). Now if I also assume a uniform distribution of y, the mandate optimality condition is

$$p < \mu_\alpha + \sigma_\alpha \lambda^{-1}\left(\frac{\mu_\alpha - \psi}{\sigma_\alpha + \frac{\rho}{2}\sigma_\theta}\right).$$