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Keywords: Dynamic moral hazard, productivity, Relational Contracts, Persistence, Limited Commitment

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SELF-ENFORCING CONTRACTS WITH PERSISTENCE*

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Abstract

We show theoretically that, in the presence of persistent productivity shocks, the reliance on self-enforcing contracts due to limited legal enforcement may provide a possible rationale why countries with the worse rule of law might exhibit: (i) higher aggregate TFP volatilities, (ii) larger dispersion of firm-level productivity, and (iii) greater wage inequality. We also provide suggestive empirical evidence consistent with the model's aggregate implications. Finally, we relate the model's firm-level implications to existing empirical findings.

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1. Introduction

Many, if not most, economic interactions are carried out with very incomplete or no formal contracts at all. In their absence, to overcome the possible hold-up problems, economic agents often rely on the repeated nature of their interactions to establish relational or self-enforcing contracts. In this paper, we show how limited external enforceability coupled with persistent productivity shocks can generate several interesting empirical predictions. We show these predictions are consistent with both micro and macro data, and further establish a new link between a country's legal environment and its aggregate-level TFP volatility as well as its wage inequality.

Concretely, we build on the model of relational contracts à la Levin (2003) by allowing for persistence in productivity and show the enforceability constraint, which captures the temptation of the principal to renege on the promised bonus, generates a multiplier effect amplifying the exogenous productivity shocks. This has three implications for economic environments where enforceability constraints are more binding:

(a) aggregate shocks are amplified, resulting in higher TFP volatilities;

(b) idiosyncratic shocks induce a larger dispersion of measured firm productivity levels;

(c) wages would be less evenly distributed.

If common persistent shocks impact all firms in economies, our model predicts that the aggregate volatility of economies with poorer legal enforcement would be greater. Consistent with our predictions, for a large sample of countries, we find a negative relationship between the dispersion of aggregate TFP growth rates and quality of law enforcement. We further address potential endogeneity concerns by exploiting exogenous variations in early settler mortality rates (Acemoglu et al., 2001) to instrument for the quality of law enforcement. The identification strategy suggests that the relationship between the legal environment and aggregate volatility is causal. Our baseline regression without any covariate suggests that one standard deviation increase in the Average Rule of Law is associated with a 0.847 percentage point decrease in the volatility of TFP growth rates.

Alternatively, if persistent idiosyncratic shocks affect firm-level productivity, our model predicts that the spread in firm-level TFP in economies with poorer legal enforcement would be wider. In line with our predictions, we document a negative relationship between the spread in firm-level TFP within a given country and the quality of the legal system in that country. These results also relate to Syverson (2011) who empirically shows large productivity differences within a given industry. Particularly, he highlights that these differences vary dramatically across countries.[4] While in the US a plant in the

[4] See references therein in particular Syverson (2004) and Hsieh and Klenow (2009).

90th percentile within a given industry can be twice as productive as one in the 10th percentile, for India and China differences are fivefold.

There are many possible reasons as to why these cross country differences may arise; our explanation is that it is partly driven by the fact that the US has an effective legal system that facilitates the enforceability of contracts while these institutions are much weaker in developing countries.[5] Thus, a larger fraction of business relationships in India and China are governed by relational contracts. As a result, when two firms in the US and two firms in India experience a pair of low and high exogenous productivity shocks, the spread in measured productivity in India would be larger than in the US. To see why this is the case, suppose the productivity shock does not affect the optimal level of effort. With complete contracts, there are no differences in worker effort between the two firms. As a result, the measured productivity differences in the US firms would be just given by the spread of the exogenous shocks. Instead, with imperfect enforceability, the optimal effort that can be elicited from the workers depends on the shock realization. When the shock is high, the firm can credibly promise high bonuses and, as a result, can demand high effort and become more productive. The opposite effect takes place when the shock is negative. This implies there is an extra wedge in the measured productivity of Indian firms given by the fact that the equilibrium effort also varies with the exogenous shock. Consistent with this prediction, our empirical analysis suggests that India (China) might decrease its TFP dispersion by roughly 55% (65%) if the country could improve the quality of its legal system to that of the United States.[6]

This highlights the role of persistence in productivity during bad times. When the current state is bad, the future value from maintaining the relationship is low, and the principal cannot credibly promise to make large payments. Thus, in bad times the firm cannot motivate the agent to exert high effort. This "low morale effect" is consistent with the evidence that labor productivity levels are pro-cyclical (Baily et al., 2001). We can alternatively recast the model as in Fuchs (2015) where every period the agent receives a stochastic outside option and must decide whether to leave the firm and take the outside option. In such a reformulation, when the future of the firm looks bleak, the agent would be willing to accept lower outside options. This is consistent with the findings presented by Baghai et al. (2016) which show that Swedish firms tend to lose their key talents when their financial health deteriorates. Similarly, using an online search platform, Brown and Matsa (2016) show that job applicants avoid companies with poor financial conditions.

[5]See Syverson (2011) and references therein for a rich discussion of other possible factors influencing firm productivity. Collard-Wexler et al. (2011), for example, argue that these differences arise due to non-convex adjustment costs and persistence of shocks to factors of production.

[6]Since the ORBIS dataset does not contain China. India, and the US in the sample, we compute these values with the following formula:
$1-exp$(Regression Coefficient in Table 4.$\times$The difference in the Average Rule of Law Between the US and the country).

In his review on the determinants of productivity Syverson (2011) highlights:

> "... robust finding in the literature ... is that higher productivity producers are more likely to survive than their less efficient industry competitors. Productivity is quite literally a matter of survival for businesses."

Our model's predictions are consistent with this finding, and further suggest that causality can go both ways. In particular, if a firm is more likely to survive, then it will be able to become more productive.

Our model also has implications for wage inequality in countries with limited legal enforcement. Under imperfect enforcement, the bonus payment that the principal can credibly commit to make to the agent depends on the future value of the relationship, which in turn is shaped by productivity. Persistence in productivity therefore implies that the agent's compensation becomes more sensitive to changes in productivity relative to a contractual environment with better enforcement. Hence, our model predicts that wages would be more evenly distributed within a country as the quality of legal enforcement improves. We use the Theil index in the UTIP-UNIDO database (Galbraith et al., 2014) to proxy for wage inequality and use Acemoglu et al. (2001)'s instrumental variable. Our baseline IV estimate suggests that one standard deviation increase in the average rule of law is predicted to decrease the Theil index by 0.49 in the logarithmic scale.

A related implication is that contracts would exhibit "rewards for luck." We say luck is rewarded since the bonuses are dependent on the realization of the shocks. Yet, the exogenous productivity shocks are perfectly observable and independent of the agent's effort. This stands in contrast to the "Informativeness Principle" (see Hölmstrom (1979)), according to which a performance measure should only affect compensation if it provides information about the agent's hidden effort. The empirical literature has documented several cases in which contracts seem to be rewarding luck. Bertrand and Mullainathan (2001) document that in the oil industry of the US, the compensation of executives in major companies positively correlates with the price of oil, an element arguably outside the executive's control.[7] Finally, DeVaro et al. (2018) (which we discuss further below) show empirically that persistence of the state is important for "reward for luck." They also provide evidence against the capture of boards by CEOs being the main driver of the observed compensation patterns.

There is certainly additional heterogeneity and richness that is not captured by our simple model. Yet, we believe our model provides a very useful lens through which we can start to uncover how seemingly unrelated findings at the firm, industry and aggregate economy level can actually be manifestations

[7] For more recent works, see also Garvey and Milbourn (2006) and Ma (2019).

of a natural friction economic agents must constantly deal with: namely, how to realize the gains from trade without falling prey to the hold-up problem.

Literature Review

Our model belongs to the rich literature on relational incentive contracts that builds on Bull (1987), MacLeod and Malcomson (1989), and Levin (2003). Most of the papers in this literature have focused on implications at the firm level. There has been less work looking at aggregate implications.[8] Our main contribution relative to the literature is to show that a very parsimonious model can be used to simultaneously explain both micro and macro phenomena.

From a technical perspective the most closely related work to ours within this literature are Kwon (2016) and DeVaro et al. (2018). We complement Kwon (2016) and go beyond her analysis by characterizing the optimal relational contracts and analyze comparative statics, which our empirical analysis takes to the data. Similar to ours, DeVaro et al. (2018) offers a theoretical rationale for rewarding luck. In addition, as mentioned earlier, they complement their theoretical analysis with some corroborating evidence on CEO compensation. Although our main argument for rewarding luck is similar in spirit, our model uses a different timing of shocks vis-a-vis theirs. In our model we could have reward for luck without it implying the presence of the morale effect, while the morale effect and the reward for luck effect must always coexist in DeVaro et al. (2018) and Kwon (2016)'s settings.[9] While this distinction is of independent theoretical interest, it is also relevant for policy debates on bonus caps. In particular, under their setting, caps on executive bonuses always decrease efficiency when reward for luck is present. By contrast, in our model, the existence of reward for luck does not necessarily imply inefficiencies when caps are introduced. Thus, the introduction of caps on bonuses might be less detrimental than what prior literature suggests. In addition to this, our main contribution relative to their work is on highlighting the "morale effect" and the related amplification of shocks that is induced by the lack of enforceability.

Three other different theoretical explanations have been put forward for the well-documented "reward for luck" phenomenon. First, Oyer (2004) argues that the shocks identified in some of the empirical studies might affect the agent's options and thus compensation must rise in good states. The second explanation was put forward by Hoffmann and Pfeil (2010) and is also present in DeMarzo et al. (2012). Their models have no enforceability constraints. Instead, the principal needs to finance up-front an agent protected by limited liability. When productivity is high, the potential cash flows are large. Thus, the principal has stronger incentives to avoid reducing the size of the firm or triggering termination (which is the main way of providing incentives within their model). Hence, the principal must give more rents to the agent, and the "reward for luck" effect arises. A third explanation, put

[8] Some exceptions are Powell (2017) and Board and Meyer-ter Vehn (2014) which we discuss below.

[9] For more details, see Remark 1 in section 4.2.

forward by DeMarzo and Kaniel (2017), hinges on agents having a very strong catching up with the Joneses component in their preferences. In such a case, optimal compensation must adjust to favorable shocks to others in a way that looks like luck. None of these models allow for a "morale effect" nor can they be used to study the correlation between enforceability and output volatility.

When considering the "morale effect," Barron et al. (2018) is closely related. They study the problem of an entrepreneur which needs both: (i) to take an initial loan to finance its venture and (ii) to hire an agent to work on it. The contract between the entrepreneur and the worker is not enforceable. Thus the continuation value of the entrepreneur plays an important role in determining the extent to which the enforceability constraint binds. A highly levered entrepreneur is similar to the principal in our model that experiences a low productivity shock. In both cases, their equity value is low and thus there is a limit on how large a bonus they can credibly promise to the agent. Thus, when the firm is in poor financial shape, it cannot induce the worker to work hard. This is similar to the "morale effect." Their paper also provides empirical evidence supporting the mechanism in their model.

Our results on enforceability and dispersion of firm's productivity within industries are related to Powell (2017).[10] His paper focuses solely on this fact. He shows theoretically that limited enforcement combined with pecuniary effects amplify underlying permanent productivity differences and provides empirical evidence of the positive and meaningful correlation between productivity dispersion and contract enforceability. Our model, although similarly reliant on limited enforcement, allows us to look at several other implications of the theory, such as aggregate volatility and wage inequality. Also, we conduct empirical tests on these additional implications and supplement his empirical analysis with an alternative measure of productivity dispersion based on Ackerberg et al. (2015)'s estimation method. This technique overcomes functional dependence problems raised in the literature on productivity function estimation and allows for a more precise estimation of TFP's.

Our results about macroeconomic volatility in productivity are related to Ramey and Watson (1997), Den Haan et al. (2000) and Cooley et al. (2004). Ramey and Watson (1997) develop a theory of labor contracting in which negative productivity shocks lead to costly job loss due to limited enforceability. Their model is used in Den Haan et al. (2000)'s dynamic general equilibrium analysis to illustrate a propagation effect through which cyclical fluctuations in the job-destruction rate magnify the shocks to output and make them persistent. In a dynamic general equilibrium framework, Cooley et al. (2004) characterizes the optimal long-term contracts offered by a competitive financial intermediary to an entrepreneur for investments in innovation. In this model with one-sided commitment, the entrepreneur can repudiate on the contract and this form of limited enforceability amplifies in an endogenous manner the impact of technological innovations on aggregate output. In contrast, we use

[10] Board and Meyer-ter Vehn (2014) also argue that limited enforcement in a model with on-the-job search can explain the emergence of productivity dispersion across ex-ante identical firms.

a model in which neither the principal nor the agent is able to credibly commit to long-term contracts. Furthermore, an amplification mechanism obtains in our framework without any general equilibrium effects, which plays a key role in the amplification mechanism in Cooley et al. (2004)'s model.

Finally, our wage inequality result is also related to Gradstein (2007). Similar to our paper, his model studies the relationship between income inequality and various governance measures (e.g., rule of law, the protection of property rights). The main difference is that his paper mostly focuses on 'appropriation by the state' in the tradition of Acemoglu and Robinson (2000), while our paper studies 'appropriation' between private entities. He also documents an empirical relationship between income inequality and rule of law, whereas we provide a correlation between wage inequality and rule of law.

We see our main contribution relative to the literature, not in being the first to point out a particular effect, but rather providing a unified model capturing multiple relevant phenomena. The existing literature so far provides distinct explanations for these facts. Instead, we offer a parsimonious rationale for all: time-varying limits to contract enforceability.

2. Setup

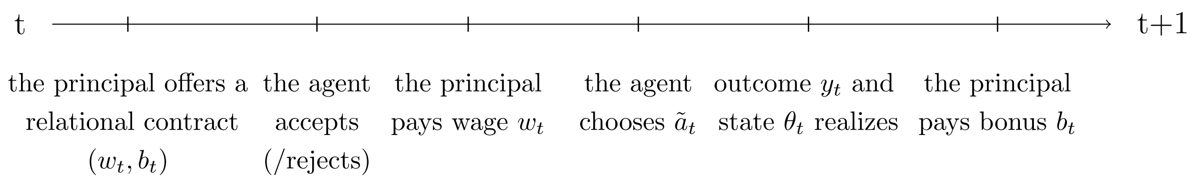
A principal ("she") and an agent ("he") interact repeatedly over time. Time is discrete and indexed via $t \in \{1, 2, ...\}$. At the beginning of every period, the principal makes the agent an offer consisting of an enforceable wage payment w_t, a schedule of non-enforceable bonus payments $\{b_t\}$ and recommends the agent take an action a_t. If the agent rejects, the game ends and both receive their outside options, which we normalize to be zero. If the agent accepts, the agent privately chooses his action $\tilde{a}_t \in [0, 1]$ incurring a cost $c(\cdot)$. We assume $c(\cdot)$ is continuously differentiable, strictly increasing and strictly convex, with $c(0) = c'(0) = 0$ and $1 < c'(1) \leq \infty$.

The firm's publicly observable output is composed of two additively separable components $\pi_t = y_t + \theta_t$. The first component takes on binary values $y \in \{0, 1\}$ and is partially under the control of the agent. Specifically, the agent's unobservable action a_t determines the probability that $y_t = 1$. On the other hand, the distribution over publicly observable state $\theta_t \in \{L, H\}$ is independent of the agent's actions. We assume $H > L > 0$ and it is worth noting that one can show that the value to the firm is weakly decreasing on the spread of the shock $H - L$. Thus, to the extent possible, firms would want to hedge this shock, our underlying assumption is that hedging cannot be done perfectly.[11] The process for θ_t follows a first-order Markov chain with a symmetric persistence parameter $\lambda := \mathsf{Prob}\,(\theta_t = \theta_{t-1} | \theta_{t-1}) >$

[11] Even if θ_t is contractible, firms cannot fully hedge θ_t as long as the court (or any outside third party) cannot verify whether one party has reneged. Assume that the principal and agent write a long-term contract based on θ_t. The long-term contract would always be in effect regardless of whether a player has reneged or not. Hence, the long-term contract does not affect any party's decision to renege.

1/2 for both $\theta_{t-1} = H, L$. The model could be extended to richer stochastic processes. For instance, the additive separability assumption on output can be relaxed. Yet, the additive structure provides a very clear benchmark since it implies the first best action is independent of the state.[12]

Figure 1: Timeline



See Figure 1 for the order of play. We assume that there is a sufficiently high utility cost of continuing the relationship with the party that has reneged before,[13] and therefore the relationship terminates the first time a party reneges on payments, or the first-time the agent rejects a contract. Let τ denote the time when the contracting relation terminates. After termination, both players receive the value of outside option, normalized to 0.

To account for heterogeneity in contract enforcement in our empirical analysis we introduce in our model a parameter α that measures the strength of contract enforcement institutions. In particular, we assume that whenever the principal reneges on her promised payments, she can walk away with only a fraction $(1-\alpha)$ of the bonus.

Both players are risk-neutral, have a common discount factor δ, and have deep pockets. The principal maximizes the present value of the expected discounted output streams minus her payments to the agent:[14]

$$\Pi_t = (1-\delta)\,\mathbb{E}_t \sum_{s=t}^{\tau} \delta^{s-t} \left(\pi_s - w_s - b_s\right).$$

The agent maximizes the expected discounted payments received minus his cost of effort:

$$v_t = (1-\delta)\,\mathbb{E}_t \sum_{s=t}^{\tau} \delta^{s-t} \left(w_s - c(\tilde{a}_s) + b_s\right).$$

Given any public history $h^t = \{\theta_0, \theta_1, ...\theta_{t-1}, y_1, ...y_{t-1}, b_1, ...b_{t-1}, w_1, ...w_{t-1}\} \in \mathcal{H}^t$, a contract specifies

[12]If the output is additively separable, the first-best effort level a^{FB} satisfies: $\frac{\partial}{\partial \tilde{a}_t}\Big(\mathbb{E}(\pi_t|\tilde{a}_t, \theta_{t-1}) - c(\tilde{a}_t)\Big)\Big|_{\tilde{a}_t = a^{FB}} = 1 - c'(a^{FB}) = 0$.

[13]Alternatively, we may assume that outside options are higher conditional than when the agent exerts zero effort. However, the current formulation is more convenient for analytic purposes.

[14]More precisely, the players' payoffs should also reflect the disutility from staying in the relationship with the party that has reneged before. Yet, since the disutility is only incurred off paths, we shall drop the disutility from the payoffs.

the compensation mix the principal offers; whether or not the agent accepts it; and the effort and discretionary bonus payment decisions. The compensation is given by the functions of the following form: $w_t : \mathcal{H}^t \to \mathbb{R}$, $b_t : \mathcal{H}^t \times (\{L, H\} \times \{0, 1\}) \to \mathbb{R}^+$.[15]

A contract is self-enforcing if it is a perfect public equilibrium of the repeated game. There are a large number of such equilibria, but we are interested in characterizing the efficient self-enforcing arrangements that would govern this relationship. As a first step, applying the methods used by Spear and Srivastava (1987) and Abreu et al. (1990) we can conveniently summarize a given public history of the game h^t, into a pair of a continuation value for the agent v and a past state θ_{t-1}. The principal chooses the optimal current actions (a, w, b, v') based on the public history (v, θ_{t-1}). This method allows us to set up the principal's problem recursively as follows:

$$
\begin{aligned}
\Pi(v, \theta_{-1}; \alpha) &= \max_{a,w,b,v'} \mathbb{E}\left[(1-\delta)\left(\pi - w(\theta_{-1}, v; \alpha) - b\left(y, \theta_{-1}, \theta, v; \alpha\right)\right) + \delta \Pi\left(v'\left(y, \theta_{-1}, \theta, v; \alpha\right), \theta; \alpha\right) | a, \theta_{-1}\right] \\
& \quad s.t. \\
v &= \mathbb{E}\left[(1-\delta)\left(w + b - c\left(a\right)\right) + \delta v' | a, \theta_{-1}\right]; && \text{[PK]} \\
a &\in \arg\max_{\tilde{a} \in [0,1]} \mathbb{E}\left[(1-\delta)\left(w + b - c\left(\tilde{a}\right)\right) + \delta v' | \tilde{a}, \theta_{-1}\right]; && \text{[IC]} \\
\delta \Pi(v', \theta; \alpha) &\geq (1-\delta)(1-\alpha) b\left(y, \theta_{-1}, \theta, v; \alpha\right); && \text{[DEP]} \\
\Pi(v', \theta; \alpha) &\geq 0; && \text{[PCP]} \\
v' &\geq 0, && \text{[PCA]}
\end{aligned}
$$

where [PK] is the promise keeping constraint, [IC] the incentive compatibility constraint for the agent to follow the recommended action a, [DEP] the dynamic enforceability constraint guaranteeing that the principal prefers to pay the promised bonus rather than to renege and effectively terminate the relationship, and [PC]'s the participation constraints that each party needs to be at least as well off as the outside option every period. Dependence of the solution on the strength of contract enforcement α in the economic environment is noted.

3. Basic Properties of Optimal Markovian Contracts

We start with formally defining Markovian contracts in this environment, and turn to show that it is without loss of generality to focus on this class of contracts. This definition is similar to Kwon (2016)'s history-independent contract, but the two definitions are not equivalent due to differences in timing of the shocks.

[15] Equivalently, we could instead impose a dynamic enforceability constraint on the agent's part to guarantee that the agent prefers to pay the promised bonus rather than reneging and effectively terminating the relationship. This alternative formulation is more consistent with Levin (2003), but our formulation is also standard in the literature as well (see e.g., Fuchs (2007)).

Definition 1 (Markovian Relational Contracts)**.** A **Markovian relational contract** consists of the non-output-contingent wage $w_t = w(\theta_{t-1}; \alpha)$, the output-contingent bonus $b_t = b(y, \theta_{t-1}, \theta_t; \alpha)$, and recommended action $a_t = a(\theta_{t-1}; \alpha)$.

According to this definition, in a Markovian relational contract at any given period t the bonus scheme upon receiving the output y depends on the current state θ_{t-1} and the state θ_t in the next period but beyond this it does not depend on the period. As noted previously, the contract in this setting depends on the strength of contract enforcement α.

The general problem can be significantly simplified by restricting our analysis to Markovian contracts. This is analogous to the main contribution in Levin (2003) showing that it is without loss to focus on stationary contracts when shocks are i.i.d. over time, and its extension to Markovian productivity states in Kwon (2016). As we show next, this implies the Pareto frontier is linear. For notational convenience, we proceed as if the initial value of the agent is 0. Yet, if the initial value of the agent is greater than 0, it can always be delivered with a transfer in the initial period. Furthermore, we give a characterization of the optimal contract.

Proposition 1 (Optimality of Markovian Relational Contracts)**.** There exists a Markovian relational contract such that

(R-1) it attains maximum surplus state-by-state and it is without loss to set $v'^* = 0$; and

(R-2) it is without loss to set $b^* (y = 0, \theta_{-1}, \theta) = 0$.

This result holds for any given strength α of contract enforcement. It says that for any contract that maximizes the expected total surplus we can find a Markovian relational contract that does the job. The surplus-maximizing Markovian contract also maximizes the principal's profit, because the principal can pay base wages to deliver the promised utility to the agent without affecting the total surplus. Without loss of generality, the offered base wages and bonuses are only functions of the last and current realization θ. Importantly, since beyond participation, continuation values don't play a role, we can simply focus on maximizing the current total surplus generated by the pair to characterize the optimal arrangements.

Since we have established that bonuses are only paid after success, we simplify our notation and denote the bonus payment upon success simply as $b(\theta_{-1}, \theta; \alpha)$. The only role for the base wage is to ensure that the contract delivers the right promised value to the agent. We can thus solve the problem of figuring out the effort and bonus choices that maximize surplus, which we denote by $\Pi(\theta_{-1}; \alpha)$, first and then divide it between the principal and the agent by adjusting the base wages. Also note that we can drop the participation constraints since a contract that implements zero effort is always available and generates strictly positive surplus. Thus, the problem simplifies to:

$$\Pi(\theta_{-1};\alpha) = \max_{a(\cdot),b(\cdot)} (1-\delta)\left(a(\theta_{t-1};\alpha) + \mathbb{E}(\theta_t|\theta_{t-1}) - c(a(\theta_{t-1};\alpha))\right) + \delta\mathbb{E}(\Pi(\theta_t;\alpha)|\theta_{t-1}) \tag{1}$$

subject to:

$$[\text{IC}] \quad a(\theta_{t-1};\alpha) = \operatorname*{argmax}_{\tilde{a}} \mathbb{E}(b(\theta_{t-1},\theta_t)|\theta_{t-1})\tilde{a} - c(\tilde{a});$$

$$[\text{DEP}]' \quad \delta\Pi(\theta_t;\alpha) \geq (1-\delta)(1-\alpha)b(\theta_{t-1},\theta_t;\alpha).$$

Denote the optimal effort and bonus payment by $a^*(\theta_{-1};\alpha)$ and $b^*(\theta_{-1},\theta;\alpha)$, respectively, and resulting profit $\Pi^*(\theta_{-1};\alpha)$. The optimal bonus scheme $b^*(\theta_{-1},\theta;\alpha)$ and the optimal effort $a^*(\theta_{-1};\alpha)$ that solves the reduced problem (1) are related as follows:

Lemma 1. In any optimal Markovian relational contract, the optimal effort satisfies the following properties:

(E-1) (Monotonicity in effort) $a^*(L;\alpha) \leq a^*(H;\alpha) \leq (c')^{-1}(1) = a^{FB}$, where a^{FB} denotes the first-best level of effort; and

(E-2) If the optimal effort $a^*(\theta_{-1};\alpha)$ is strictly less than the first best level a^{FB}, the dynamic enforceability constraints bind for both $\theta = H, L$: $b^*(\theta_{-1},\theta;\alpha) = \frac{\delta}{1-\delta}\frac{\Pi^*(\theta;\alpha)}{1-\alpha}$.

The first part of Lemma 1 establishes that if there is a distortion in the effort level with respect to first best it is due only to a possible under-provision of effort. This arises due to the principal's dynamic enforceability constraint, i.e., the temptation of the principal to default on the promised bonus. As shown in the second part of the Lemma 1, when the effort is distorted the enforceability constraint must bind. Hence the principal must fully pledge the principal's future surplus (which is the maximum amount the principal can credibly commit to pay) as the bonus.[16] Since there is more surplus in high states than in low states the equilibrium effort must always be weakly lower in the low state.

Denote by $\Pi^{FB}(\theta)$ the maximum surplus that can be obtained without the dynamic enforceability for the state θ. As illustrated in Lemma 1 the dynamic enforceability constraint is the only friction precluding the first-best outcome, and the following monotonicity result obtains:

Lemma 2. (Monotonicity in profits) In any optimal Markovian relational contract, the principal's profits satisfy $\Pi^*(L;\alpha) < \Pi^*(H;\alpha)$ and $\Pi^*(\theta;\alpha) \leq \Pi^{FB}(\theta)$ for both $\theta = L, H$.

[16] To see this, suppose that $b^*(\theta_{-1},\theta;\alpha) < \frac{\delta}{1-\delta}\Pi^*(\theta;\alpha)$ for some θ. The principal could then increase $b(\theta_{-1},\theta;\alpha)$ and simultaneously request that the agent exert more effort. Since there is under-provision of effort that would improve efficiency.

4. Firm Level Implications

We start with presenting below in the subsections 4.1 and 4.2, the firm level implications of our model (the reward-for-luck effect and morale effect) and connect them to existing strands of empirical literature. In the subsequent section 5, we will then show aggregate implications.

4.1. Rewarding Luck

There are numerous papers (see for example Bertrand and Mullainathan (2001), Garvey and Milbourn (2006), DeVaro et al. (2018), and Ma (2019)) documenting empirically that variables outside of the agent's control influence compensation. Consistent with this evidence, our next result shows that when designing the optimal structure of bonus payments *an observable and contemporaneous* shock would not be filtered out, even when the realization of the shock contains no information about possible deviations by the agent in his choice of effort. Hence, it would appear as if employees are partially rewarded for luck.

Proposition 2 (Rewarding Luck)**.** For any contract enforcement level α there exists a threshold discount factor $\bar{\delta}(\alpha) := \frac{1-\alpha}{1-\alpha+a^{FB}-c(a^{FB})+H}$ such that, for all $\delta < \bar{\delta}(\alpha)$, the optimal bonus increases in θ: $b^*(\theta_{-1}, L; \alpha) < b^*(\theta_{-1}, H; \alpha)$ for all $\theta_{-1} \in \Theta$.

The principal can credibly promise to pay out larger amounts when the current shock is revealed to be favorable. This is the case because shocks are persistent. After a good realization the future prospects of the firm are better, hence the enforceability constraint of the firm is relaxed and thus the firm can credibly promise to pay more in such a state. Importantly, even though the size of the bonus varies with the persistent state θ, an element outside the agent's control or influence and with no information content, incentives are only really provided by the expected bonus. Additionally, the bonus is still only paid if the agent delivers on his part of the output. Thus, although luck determines the size of the reward these rewards are still fully driven by incentive motives. Furthermore, in economic environments with poorer contract enforcement, higher importance of future values, as measured by higher discount rates, are needed to maintain contracting relationship.

4.2. Morale Effect

Our next main result shows that the dynamic enforcement constraint is more binding in the low state.

Proposition 3 (Morale Effect)**.** For any contract enforcement level α there exists a threshold discount factor $\bar{\delta}(\alpha) \in (0, 1)$ as given in Proposition 2 such that for all $0 < \delta < \bar{\delta}(\alpha)$, the optimal contract satisfies

(i) $\mathbb{E}(b^*(L, \theta; \alpha) | \theta_{-1} = L) < \mathbb{E}(b^*(H, \theta; \alpha) | \theta_{-1} = H)$;

(ii) $a^*(L;\alpha) < a^*(H;\alpha)$.

There is an intuitive explanation for this result that highlights the effect of persistent productivity during bad times. When the current state is bad, the future value of the relationship is low and the principal can no longer credibly promise to make large payments. Thus, she can no longer motivate the agent to exert very high effort.

Our model thus rationalizes the "low morale effect" during firm distress, once the profitability is interpreted as a driver of financial distress. In our model, firms reduce the expected bonus payments during difficult times, to which workers respond by exerting less effort. Thus, firms' productivity is further depressed due to de-moralization of the workforce.

This mechanism resonates well with multiple labor concession episodes in the domestic steel industry during 1980s (DeAngelo and DeAngelo, 1991). DeAngelo and DeAngelo (1991) illustrates this effect with the labor problems experienced by the US steel plants in the 1980s. This was a challenging time for US steel producers as they faced stiff competition from more productive Japanese counterparts coupled with a decline in global demand.

The paper by Barron et al. (2018) has a similar mechanism at play. Although the shocks in their model are transient, their firms have different levels of leverage which generates persistence. An increase in leverage in the previous period in their model is similar to having experienced a negative persistent shock in our model and leads to a lower labor productivity. They use data on a large sample of European firms to show that indeed, an increase in leverage in year $t-1$ is followed by a decrease in year t productivity. They report that one standard deviation increase in contemporary leverage is correlated with a decrease in TFP-R equal to 9-15% of the median within firm standard deviation.

Furthermore, we can recast our model as in Fuchs (2015) to rationalize why distressed firms are more likely to lose, or less likely to attract workers. This is consistent with the empirical findings in Baghai et al. (2016), and Brown and Matsa (2016). When a negative productivity shock hits the firm, workers with high outside values would leave the company, because firms cannot promise sufficiently high bonuses to retain them.

Remark 1. Importantly, unlike related papers (Kwon, 2016; DeVaro et al., 2018), the reward-for-luck in our setup is distinct from the morale effect. In our setup, the morale effect necessarily leads to the reward-for-luck effect, but not vice versa.

To understand this observation, it is important to recall that the enforceability constraint will always be more binding when $\theta_t = L$ than when $\theta_t = H$ (see Lemma 1). When the enforceability constraint first becomes binding it implies that the principal must promise a lower bonus when $\theta_t = L$ is realized. Absent any other adjustments to the compensation, the expected bonus would decrease and thus effort would inefficiently decrease. To try to avoid this inefficiency, the principal can raise the bonus conditional on $\theta_t = H$ being realized. If the enforceability constraint does not bind in the high state, that means that the principal can fully offset the decrease in expected payments and thus keep the agent at the efficient effort level regardless of the state θ_{t-1}. This is a situation in which we have reward for luck but no morale effect. The morale effect only arises when the enforceability constraint also starts binding for $\theta_t = H$ and, as a result, the expected bonus is no longer sufficient to induce high effort. Note, that in particular, this would imply lower effort when the last period shock was low. This follows because, due to persistence, the low state is more likely to be realized again and thus the expected bonuses must be lower.

5. Amplification and Aggregate Implications

We are now ready to focus on the main contributions of the paper which is to show that this simple yet natural environment also has numerous implications that match well several empirical findings at the country level.

We provide the main theoretical results first. Weaker contractual enforcement implies:

(a) aggregate shocks result in higher TFP volatilities;

(b) idiosyncratic shocks induce a larger dispersion of measured firm; productivity levels within a country or industry.

(c) wages would be less evenly distributed within a country or industry.

We then empirically study each of these implications in turn.

5.1. Theory and Main Predictions

In this section, we study an economy populated by a unit mass of firms indexed by $i \in [0,1]$. Firm i is subject to some persistent shock $\theta_{i,t}$. Let us introduce some auxiliary random variables $\omega_{i,t} \overset{iid}{\sim} U[0,1]$ for each $i \in [0,1]$ and $t \in \{0,1,2,...\}$ and consider two possible stochastic structures regarding the evolution of $\theta_{i,t}$:

Assumption 1 (Pure aggregate volatility without idiosyncratic volatility)**.**

1. $\forall t$, $\theta_{i,t} = \theta_{j,t} = \theta_{1,t}$;

2. $\forall t \geq 1$, $\theta_{1,t} = (H-L) \times [1(\{\omega_{1,t} \leq \lambda, \theta_{1,t-1} = H\}) + 1(\{\omega_{1,t} \leq 1-\lambda, \theta_{1,t-1} = L\})] + L$;

3. $\theta_{1,0} = (H-L)1(\{\omega_{1,0} \leq \frac{1}{2}\}) + L$.

Assumption 2 (Pure idiosyncratic volatility without aggregate volatility)**.**

1. $\forall t \geq 1$, $\theta_{i,t} = (H - L) \times [1(\{\omega_{i,t} \leq \lambda, \theta_{i,t-1} = H\}) + 1(\{\omega_{i,t} \leq 1 - \lambda, \theta_{i,t-1} = L\})] + L$;

2. $\theta_{i,0} = (H - L)1(\{\omega_{1,0} \leq \frac{1}{2}\}) + L$.

For tractability, we specialize on the quadratic cost of effort function $c(x) = \frac{1}{2}cx^2$.

We use the optimal relational contracting problem in (1) for each firm i and determine the optimal effort and the bonus scheme $a^*(\theta_{i,t-1};\alpha)$ and $b^*(\theta_{i,t-1}, \theta_{i,t};\alpha)$. respectively. Using the optimal contract we denote by $I^*(\theta_{i,t-1};\alpha)$ the agent's expected compensation. By an argument similar to the one used in Proposition 3, the agent breaks even every period. Hence, the agent's expected compensation $I^*(\theta_{i,t-1};\alpha)$ is equal to the cost of effort $c(a^*(\theta_{i,t-1};\alpha))$.

Theorem 1 is the main result of our paper. It shows that weak contractual enforcement amplifies the volatility of outputs and the degree of wage inequality, mainly because weak institutions constrain the value promises the principal credibly makes.

Theorem 1.

(a) Under Assumption 1, aggregate volatility $\frac{\mathbb{E}(\int_0^1 y_{i,t}di|\int_0^1 \theta_{i,t-1}di=H)}{\mathbb{E}(\int_0^1 y_{i,t}di|\int_0^1 \theta_{i,t-1}di=L)}$ decreases in α.

(b) Under Assumption 2, dispersion in firm-level productivity $\frac{\mathbb{E}(y_{i,t}|\theta_{i,t-1}=H)}{\mathbb{E}(y_{j,t}|\theta_{j,t-1}=L)}$ decreases in α.

(c) Under Assumption 2, wage inequality $\frac{I^*(\theta_{i,t-1}=H;\alpha)}{I^*(\theta_{j,t-1}=L;\alpha)}$ decreases in α.

To illustrate the intuition behind the parts (a) and (b) of Theorem 1, for simplicity consider two extremes: (1) $\alpha = 1$, which corresponds to formal contracting which would arguably be as in the US and (2) $\alpha = 0$, which corresponds to self-enforcing contracting arguably in India or China. Since dynamic enforceability conditions do not bind in an environment with formal contracting, the optimal effort recommendation in such an environment is always constant at the first-best level $a^*(L;1) = a^*(H;1) = a^{FB}$. Hence, the volatility of US aggregate outputs is given by that of the exogenous component in aggregate productivity shocks. Furthermore, the productivity gap between two US firms is equal to the difference in idiosyncratic productivity shocks. By contrast, when business relationships are governed via self-enforcing contracts, the effort level increases in the past profitability shock due to the morale effect (Proposition 3 and the part (E-1) of Lemma 1). As a result, the dynamic enforceability generates a multiplier effect and makes the aggregate output $a^*(\theta_{-1};\alpha) + \mathbb{E}(\theta|\theta_{-1})$ more volatile. Also, in India or China, the morale effect would further amplify the idiosyncratic productivity shocks, which leads to a wider dispersion in TFP's between two firms in the same country.

The intuition behind part (c) of Theorem 1 is also similar. The agent's expected compensation

$I^*(\theta_{i,t-1};\alpha)$ is equal to his cost of effort $c(a^*(\theta_{i,t-1};\alpha))$. The effort, and hence expected compensations, stays the same when $\alpha = 1$, whereas it increases in the past profitability shock $\theta_{i,t-1}$ when $\alpha = 0$. Hence, the expected compensation becomes more dispersed when $\alpha = 0$. Hence, wages in India and China would be less evenly distributed to those in more developed countries.

Remark 2. An alternative way to obtain the amplification at the aggregate level is to assume that a proportion $\alpha \in [0,1]$ of firms rely on the formal court of law and thus can fully enforce their commitments, while the rest rely on relational contracts with no recovery. Then, as above, the volatility of outputs as measured by $\frac{\mathbb{E}(\int_0^1 y_{i,t}di|\int_0^1 \theta_{i,t-1}di=H)}{\mathbb{E}(\int_0^1 y_{i,t}di|\int_0^1 \theta_{i,t-1}di=L)}$ decreases in α.

5.2. Empirical Evidence

To show that the predictions from Theorem 1 are consistent with the data, we need to proxy for the quality of legal enforcement α. There are several well-established indicators to measure the strength of contracting institutions at the country level. In particular, we use the 'Rule of Law' indicators from the Worldwide Governance Index (Kaufmann et al., 2011) in the results presented below. The Rule of Law variable measures the agents' confidence in contract enforceability, property rights, police, courts and the protection from crime and violence and has been used in the literature on incomplete contracting (Antràs and Yeaple, 2014; Nunn, 2007) as a proxy variable for the quality of contractual enforcement at the country level.

A potential concern is that the Rule of Law measure also includes confidence in police and protection from crime and violence, which do not have a clear connection with our theoretical model.[17] We address this issue in two ways. First, we obtain the intentional homicide rate per 100,000 people in a country from the World Bank's World Development Indicators (WDI) database in order to proxy for its overall crime rate and use it as a co-variate in many of our regressions. In addition, we also conduct robustness checks with an alternative "Rule of Law" variable from IHS Markit World Economic Service, which is available from 1996 to 2013. Despite similarity in names, this variable is only based on the level of risk associated with expropriation and contract alteration by the state and failure to enforce contracts between private entities. Hence, it measures the quality of contractual enforcement in both private and public sectors. Since the principal in our model may also be interpreted as the state and the agent as a private entity (e.g., a government procurement contractor), we believe that this measure captures the spirit of our model well. As illustrated in later sections, our results remain robust even when the crime rate is controlled for, and also when the alternative measure is used.

While, as we show below, our empirical analyses are consistent with our model, we are aware that there may be other factors at play. Unfortunately, it is difficult to entirely rule out alternative explanations.

[17]We thank the referee for bringing up this point.

Indeed, literature discusses other channels through which rule of law affects economic outcomes at the macroeconomic level (Haggard and Tiede, 2011). While we address some of these endogeneity concerns in subsections 5.2.1 and 5.2.3 by adopting the instrumental variable pioneered by Acemoglu et al. (2001), we were not able to use a similar identification strategy in subsection 5.2.2 because there is no overlap between Acemoglu et al. (2001)'s and databases. We are hopeful that future research can utilize more granular datasets to overcome these challenges.[18]

Furthermore, since we consider stationary environments in our model, one can be naturally concerned about the set of countries used for the empirical analysis, including non-stationary countries. To address this concern, we re-conduct our empirical analyses after excluding low-income countries (as defined by the World Bank) from the sample. Our results are qualitatively similar in the restricted sample.

5.2.1. Aggregate TFP Volatility and Rule of Law

The first part of our main theoretical result, the part (a) of Theorem 1, implies that aggregate productivity shocks can be further amplified in poor legal environments, resulting in higher aggregate TFP volatility. To test this implication, we use the latest version of the Penn World Table (PWT 10.0) to obtain aggregate TFP growth rates and merge it with WGI, WDI, and Acemoglu et al. (2001)'s databases. After these procedures, we obtain a sample consisting of 108 countries from 1996 to 2018. Based on this sample, we measure a country's TFP volatility as the standard deviation of its annual TFP growth rates.

Table 1 reports our summary statistics. The "world" column is for the sample obtained above, whereas the "base" column is for the base sample à la Acemoglu et al. (2001), which contains 44 former European colonies with non-missing values of the European settler mortality rate, homicide rate, and TFP volatility. We use the "world" sample in our OLS regressions, and the "base" sample in our IV regressions. Despite differences in data construction procedures, these statistics are quite comparable to those in Acemoglu et al. (2001)'s Table 1. In Table 7 of the Internet Appendix, we also show that Acemoglu et al. (2001)'s results hold in our samples as well.

We first estimate:

$$VOL_c = \alpha + \beta_{OLS} AVG\_RL_c + Controls + \epsilon_c, \tag{2}$$

where VOL_c denotes country c's TFP volatility and AVG_RL_c country c's Average Rule of Law variable. Our co-variates include the absolute values of latitudes (divided by 100), continent fixed effects,[19] and average homicide rates over the sample period.

[18] Ponticelli and Alencar (2016) show a promising avenue toward this direction. They use Brazilian census-level data, together with court congestion data, to establish the effect of improved legal enforcement on economic outcomes.

[19] In continent fixed effect specifications, Acemoglu et al. (2001) also use the "Other continent dummy" variable,

Table 1: Summary Statistics for Volatility Sample

Variable	World	Base	By Quartiles of Mortality			
			(1)	(2)	(3)	(4)
SD of TFP Growth Rates (in Percentage Points)	3.058	2.772	1.896	2.533	2.64	4.315
	(2.474)	(1.909)				
Log GDP Per Capita (PPP) in 1995	8.335	8.092	8.826	8.435	7.847	7.254
	(1.105)	(1.046)				
Average Rule of Law (WGI)	-.032	-.255	.653	-.437	-.446	-.747
	(.982)	(.871)				
Average Rule of Law (IHS)	.582	.521	.708	.482	.476	.430
	(.217)	(.192)				
Average Protection Against Expropriation Risk	7.155	6.594	7.829	6.451	6.127	5.997
	(1.758)	(1.42)				
Average Homicide Rate	7.873	12.494	6.209	21.232	12.158	6.987
	(11.59)	(15.637)				
Constraints On Executives in 1900	1.918	2.293	3.923	3.059	1.154	1
	(1.872)	(2.136)				
Constraints On Executives in 1990	3.573	3.431	4.846	2.529	3.385	3.267
	(2.409)	(2.414)				
Democracy in 1900	1.222	1.702	4.154	2.5	.231	0
	(2.655)	(3.041)				
European Settlers 1900	31.355	16.58	29.464	27.059	9.35	.5
	(42.56)	(25.847)				
Log European Settler Mortality 1900	n.a.	4.63	3.09	4.287	4.824	6.271
		(1.269)				
Number of Observations	108	44				

The "world" column is for the sample used in the OLS regressions. The "base" column is for the sample used in the IV regressions, which consists of former European colonies with non-missing values of European settler mortality rates. Data sources are described at the beginning of 5.2 and 5.2.1. Standard deviations are in parentheses.

We report the OLS results in Panel A of Table 2 and graphically illustrate the baseline regression result in Panel (a) of Figure 2 as well. As predicted by our model, the more a country relies on formal contracts (i.e., the better the legal environment is) the lower the TFP volatility it has experienced over the sample period. In particular, the baseline regression without any control variable shows that one standard deviation increase in the Average Rule of Law is associated with a 1.166 percentage point decrease in TFP volatility. Given that the volatility of TFP growth rates averages around 3.058 percentage points in our world sample, we believe that the quality of contractual enforcement has a quantitatively significant effect on TFP volatility.

which takes the value of 1 when the country is either Australia, Fiji, New Zealand, or Malta, and takes the value of 0 otherwise. Since we believe that this particular set of countries is somewhat arbitrary, we deviate from their approach and exclude this dummy variable in our fixed effect specifications.

Table 2: Volatility Results

Panel A: OLS Regressions						
	(1)	(2)	(3)	(4)	(5)	(6)
Average Rule of Law	-1.188***	-1.284***	-1.125***	-1.191***	-1.395***	-1.397***
	(0.300)	(0.342)	(0.319)	(0.354)	(0.409)	(0.420)
Average Homicide		-0.021		-0.011		-0.001
		(0.019)		(0.019)		(0.018)
Latitude					3.012*	3.004*
					(1.562)	(1.560)
Continent FE	No	No	Yes	Yes	Yes	Yes
Observations	108	108	108	108	108	108
R^2	0.232	0.239	0.252	0.254	0.285	0.285

Panel B: IV Regressions						
	(1)	(2)	(3)	(4)	(5)	(6)
Average Rule of Law	-0.973**	-1.023**	-1.131***	-1.232**	-1.450**	-1.490**
	(0.469)	(0.517)	(0.385)	(0.484)	(0.655)	(0.681)
Average Homicide		-0.016		-0.017		-0.018
		(0.027)		(0.026)		(0.026)
Latitude					2.399	1.897
					(2.875)	(2.484)
Continent FE	No	No	Yes	Yes	Yes	Yes
Observations	44	44	44	44	44	44
Anderson-Rubin statistics	4.20**	3.59*	7.57**	5.47**	5.04**	4.41**

Panel C: First-Stage Regressions						
	(1)	(2)	(3)	(4)	(5)	(6)
Logged Settler Mortality	-0.637***	-0.601***	-0.633***	-0.557***	-0.429***	-0.414***
	(0.095)	(0.096)	(0.114)	(0.101)	(0.130)	(0.133)
Average Homicide		-0.018***		-0.0200***		-0.016**
		(0.006)		(0.007)		(0.006)
Latitude					2.417***	1.888**
					(0.794)	(0.857)
Continent FE	No	No	Yes	Yes	Yes	Yes
Observations	44	44	44	44	44	44
R^2	0.562	0.638	0.568	0.645	0.643	0.687
F-statistics	45.17***	39.07***	30.85***	30.45***	10.85***	9.66***

Panel A reports OLS coefficients of equation (2). Panel B and Panel C report IV and first-stage estimates (respectively) of equations (3). Data sources are described in section 5.2 and 5.2.1. Standard deviations are multiplied by 100, so that coefficients can be interpreted in percentage terms. Robust Standard errors are in parentheses. Asterisks *, **, and *** denote statistical significance at 10%, 5%, and 1%, respectively.

This, of course, is just a correlation. It is well documented in existing literature (Aguiar and Gopinath, 2007; Koren and Tenreyro, 2007) that developing countries' outputs are more volatile. Since contracts are more difficult to enforce in poor countries, even when controlling for continent fixed effects, climate, and crime rates, the OLS approach in Panel A of Table 2 cannot tease out the effects of contractual enforcement from other determinants of a country's TFP volatility.

In order to mitigate the endogeneity concern, we exploit exogenous variations in early European settler's (log) mortality rates S_c from Acemoglu et al. (2001). Specifically, we estimate the following instrumental variable and first-stage regressions in our base sample:

$$
\begin{aligned}
VOL_c &= \alpha + \beta_{IV} AVG\_RL_c + Controls + \epsilon_c, \\
AVG\_RL_c &= \mu + \gamma_{FS} S_c + Controls + \eta_c,
\end{aligned}
\tag{3}
$$

where we use the same set of control variables as in the OLS analogues (2).

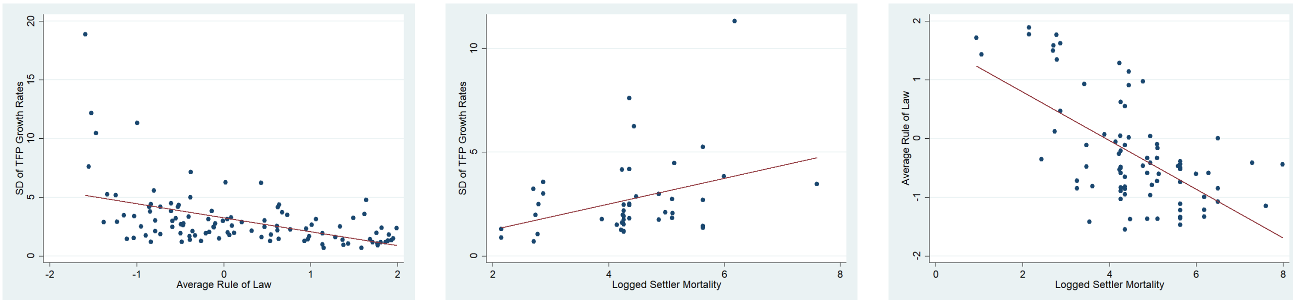
Panel B of Table 2 shows our IV regression results. In particular, our baseline IV regression without any control variable shows that one standard deviation increase in the Average Rule of Law measure is associated with roughly a .847 percentage point decrease in TFP volatility. Other IV estimates are also quite comparable to the results from OLS regressions. In untabulated regressions, we also find that Durbin-Wu-Hausman F-tests fail to reject the null hypotheses that the IV estimates are significantly different from the OLS estimates. These results suggest that endogeneity bias may not be as severe as it seems at first.

Also, we report our first-stage results in Panel C of Table 2. Consistent with prior findings from the literature on weak instruments (Chernozhukov and Hansen, 2005, 2008), we also find that the first-stage F-statistics based on Acemoglu et al. (2001)'s IV sometimes fall short of Stock and Yogo (2005)'s suggested threshold of 10 and the strength of our instrumental variable depends on the specification. To address the concern that the instrument may be too weak, we follow the recommendations from this literature and report the Anderson-Rubin statistics (Anderson and Rubin, 1949) in Panel B of Table 2. The evidence suggests that the effect of contractual enforcement remains significant (albeit to a statistically lesser extent) even when the strength of the instrument is taken into consideration.

We also graphically illustrate the reduced-form and first-stage relationships in Panel (b) and (c) of Figure 2. From these figures, the reader may be concerned that our results may be driven by a few outliers. In untabulated regressions, we also conduct empirical analyses without outlier countries whose TFP volatility exceeds 10 and find qualitatively similar results.

In the Internet Appendix, we also conduct additional robustness checks to further validate our results. First, we use the alternative measure from IHS and report the results in Table 10. Second, we drop low-income countries from the sample and re-conduct the empirical analyses in Table 11 and 12.

Figure 2: OLS, IV, First-Stage Relationships for TFP Volatility



(a) (b) (c)

Panel (a) illustrates the OLS relationship between TFP volatility (on the y-axis) and the Average Rule of Law measure (on the x-axis). Panel (b) shows the reduced-form relationship between TFP volatility (on the y-axis) and the settler mortality rates (on the x-axis). Panel (c) shows the first-stage relationship between the Average Rule of Law measure (on the y-axis) and settler mortality rates (on the x-axis). In each panel, we also show the line of best fit in red.

These robustness checks yield qualitatively similar results.

5.2.2. TFP Dispersion and Rule of Law

In the presence of idiosyncratic shocks, the same mechanism has implications for the cross-sectional distribution for firm-level TFP's. Namely, the amplification effect implies that firms in countries with poor contract enforcement have a larger dispersion of TFP's at any given point in time as well. The comparison between the findings by Syverson (2004) for the US and Hsieh and Klenow (2009) for India and China are clearly in line with our predictions. While, for the US, there is a twofold difference in productivity between the 10th and 90th percentile firm, in China and India there is a fivefold difference.

To obtain firm-level TFP estimates, we download firm-level financial statements from the ORBIS database through the WRDS platform. We closely follow Gopinath et al. (2017)'s data construction procedure and use the STATA code *prodest* (Manjón and Mañez, 2016) to apply Ackerberg et al. (2015)'s methodology to estimate firm-level TFPs. We refer the interested readers to our data appendix for a more detailed account.

Based on these procedures, we obtain 4,786,856 firm-level TFP's. For each country c and year t, we compute $TFP9010_{c,t}$ as the TFP difference (in the logarithmic scale) between the firm with the top 10% productivity and the bottom 10% firm in a country c and retain only a single observation per one country-year. Then, we merge the data with the WGI and WDI databases, and require a country-year to have non-missing values of homicide rates and the rule of law measure, so that regression coefficients can be interpreted as between-estimates. Our final sample consists of 26 countries and spans from 1996 to 2016. We report the summary statistics for this sample in Table 3. In Table 8 of the Internet Appendix, we also find that Acemoglu et al. (2001)'s original OLS results hold in this sample.

Table 3: Summary Statistics for TFP Dispersion Sample

Variable	Mean	S.D.
Average TFP Dispersion	1.194	.463
Log GDP Per Capita (PPP) in 1995	9.294	.75
Average Rule of Law (WGI)	.973	.774
Average Rule of Law (IHS)	.793	.162
Average Protection Against Expropriation Risk	9.38	.607
Average Homicide Rate	1.793	1.58
Log European Settler Mortality 1900	92	27.689
Number of Observations	26	

Average TFP Dispersion is a country's average level of TFP dispersion.

We are interested in estimating:

$$TFP9010_c = \alpha + \beta_{OLS} AVG\_RL_c + Control_c + \epsilon_c, \tag{4}$$

where $TFP9010_c$ denotes the average level of the (logged) TFP difference $TFP9010_{c,t}$, and AVG_RL_c the average rule of law of country c. Since countries in our sample belong either to Asia or Europe, we only use a dummy variable "Europe" instead of continent-fixed effects. Other co-variates include the average level of a country's homicide rates and the absolute value of its latitude (divided by 100).

We report our regression coefficients in Table 4. The basic OLS relationship without any control variable is also graphically illustrated in Figure 3. Our results show that the relationship implied by the part (b) of Theorem 1 is indeed consistent with what can be observed in the data. In particular, we find that one standard deviation increase in the Average Rule of Law would decrease the productivity gap between the top 10% most productive firm and the bottom 10% firm by 0.407 in a logarithm scale. This translates into a reduction in the productivity gap by 33.4%.

As in the previous analysis on TFP volatility, we conduct various robustness checks in Table 13. First, we report the results based on the IHS measure in Panel A. Second, in Panel B and C, we exclude low-income countries from the sample and repeat the empirical analyses. These robustness checks yield qualitatively similar outcomes.

We could not conduct IV regressions in this subsection because there are zero overlaps between countries in the ORBIS database and those in Acemoglu et al. (2001)'s data set. In the ORBIS universe, only European countries and a small group of Asian countries contain financial statement information required to estimate firm-level TFP's, but none of these countries are former European colonies. We are hopeful that future research can utilize higher-quality datasets to overcome this issue.

Table 4: TFP Dispersion OLS Results

	(1)	(2)	(3)	(4)	(5)	(6)
Average Rule of Law	-0.527***	-0.511***	-0.534***	-0.508***	-0.583***	-0.577***
	(0.0530)	(0.0510)	(0.0542)	(0.0552)	(0.0543)	(0.0727)
Average Homicide		0.018		0.0303		0.004
		(0.026)		(0.023)		(0.030)
Europe			-0.288	-0.321	-0.442	-0.438
			(0.307)	(0.316)	(0.331)	(0.339)
Latitude					1.001**	0.948
					(0.421)	(0.565)
Constant	1.706***	1.660***	1.979***	1.930***	1.630***	1.642***
	(0.0714)	(0.105)	(0.311)	(0.320)	(0.357)	(0.373)
Observations	26	26	26	26	26	26
R^2	0.776	0.779	0.804	0.812	0.823	0.823

In all regressions, the outcome variable is a country's average level of TFP dispersion. Robust Standard errors are in parentheses. Asterisks *, **, and *** denote statistical significance at 10%, 5%, and 1%, respectively.

Figure 3: Rule of Law and Average TFP Dispersion

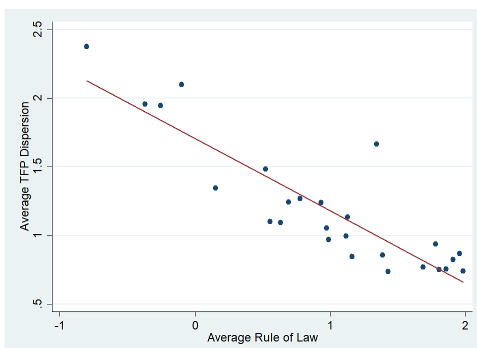


Figure 3 illustrates the OLS relationship between log-TFP differences between the firm with the top 10% productivity and the bottom 10% firm (on the y-axis) and the Average Rule of Law measure (on the x-axis) for 26 sample countries. Above the plot is the line of best fit in red.

5.2.3. *Wage Inequality and Rule of Law*

In the last part of our empirical analyses, we relate the amplification mechanism to pay inequality. To proxy for wage inequality, we use the (logged) Theil index for industrial wage-inequality from the UTIP-UNIDO database (Galbraith et al., 2014).[20] After merging the database with WDI, WGI,

[20] To the best of our knowledge, other popular inequality indices (e.g., Gini indices) are only available for *total income*, but not for *labor income*. Since we believe our model is more relevant for labor income, we use Theil indices from the UTIP-UNIDO database.

and Acemoglu et al. (2001)'s databases, we only retain observations with non-missing values of the Theil indices, Rule of Law, and homicide rates so that we can use between-estimators in our empirical analyses. The sample period is between 1996 and 2015. The summary statistics for this sample are reported in Table 5. In Table 9 of the Internet Appendix, we also replicate Acemoglu et al. (2001)'s main results in this sample and find similar results.

Table 5: Summary Statistics (Inequality Sample)

Variable	World	Base	By Quartiles of Mortality			
			(1)	(2)	(3)	(4)
Average Pay Inequality	-3.126	-2.921	-3.581	-2.749	-2.749	-2.605
	(.896)	(.732)				
Log GDP Per Capita (PPP) in 1995	8.645	8.467	9.696	8.197	8.464	7.658
	(1.051)	(1.051)				
Average Rule of Law (WGI)	.247	.041	1.344	-.395	-.222	-.466
	(.982)	(.871)				
Average Rule of Law (IHS)	.648	.595	.841	.528	.529	.446
	(.982)	(.871)				
Average Protection Against Expropriation Risk	7.765	7.097	8.697	6.635	6.568	6.449
	(1.529)	(1.338)				
Average Homicide Rate	7.324	12.791	5.859	12.964	34.109	9.979
	(13.584)	(20.109)				
Constraints On Executives in 1900	2.333	2.765	4.714	2.714	3.5	1
	(2.216)	(2.4)				
Constraints On Executives in 1990	3.778	3.588	6.429	2.286	2	4.111
	(2.619)	(2.583)				
Democracy in 1900	1.841	2.583	6	1.786	4	0
	(3.256)	(3.596)				
European Settlers 1900	42.119	23.097	50.938	19.857	15.25	6.878
	(46.17)	(30.866)				
Log European Settler Mortality 1900	n.a.	4.027	2.64	4.102	4.358	5.152
		(.963)				
Number of Observations	99	36				

The "world" column is for the sample used in the OLS regressions. The "base" column is for the sample used in the IV regressions, which consists of former European colonies with non-missing values of European settler mortality rates. "Average Pay inequality" is a country's average level of the (logged) Theil indices. Other data sources are described at the beginning of 5.2 and 5.2.1. Standard deviations are in parentheses.

We first estimate:

$$AVG\_Log\_Theil_c = \alpha + \beta_{OLS} AVG\_RL_c + Control_c + \epsilon_c,$$

where $AVG\_Log\_Theil_c$ denotes the average level of our wage inequality measure (i.e., logged Theil index) for country c. We also use the same set of control variables as previous analyses.

Table 6: Wage Inequality Results

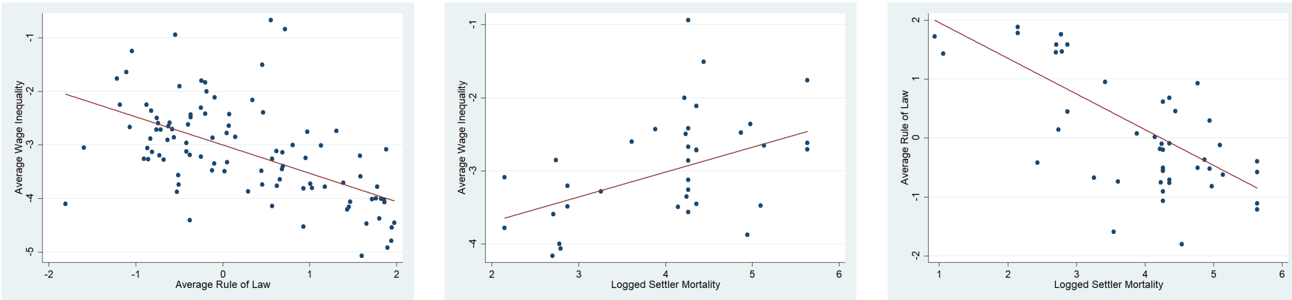
Panel A: OLS Regressions						
	(1)	(2)	(3)	(4)	(5)	(6)
Average Rule of Law	-0.499***	-0.493***	-0.391***	-0.359***	-0.311***	-0.299***
	(0.0770)	(0.0829)	(0.0864)	(0.0975)	(0.0924)	(0.0988)
Average Homicide		0.001		0.005		0.003
		(0.004)		(0.004)		(0.004)
Latitude					-1.032**	-0.989**
					(0.468)	(0.485)
Continent FE	No	No	Yes	Yes	Yes	Yes
Observations	99	99	99	99	99	99
R^2	0.299	0.300	0.369	0.375	0.403	0.404

Panel B: IV Regressions						
	(1)	(2)	(3)	(4)	(5)	(6)
Average Rule of Law	-0.504***	-0.521***	-0.465***	-0.495***	-0.490**	-0.499**
	(0.113)	(0.122)	(0.129)	(0.145)	(0.194)	(0.201)
Average Homicide		-0.004		-0.006		-0.006
		(0.00375)		(0.005)		(0.005)
Latitude					0.188	0.0286
					(0.937)	(0.918)
Continent FE	No	No	Yes	Yes	Yes	Yes
Observations	36	36	36	36	36	36
Anderson-Rubin statistics	16.44***	15.36***	12.08***	11.00***	8.33***	8.30***

Panel C: First-Stage Regressions						
	(1)	(2)	(3)	(4)	(5)	(6)
Logged Settler Mortality	-0.727***	-0.696***	-0.707***	-0.657***	-0.608***	-0.596***
	(0.100)	(0.0904)	(0.111)	(0.0930)	(0.145)	(0.133)
Average Homicide		-0.011***		-0.014***		-0.013**
		(0.004)		(0.005)		(0.005)
Latitude					1.056	0.688
					(0.888)	(0.907)
Continent FE	No	No	Yes	Yes	Yes	Yes
Observations	36	36	36	36	36	36
R^2	0.584	0.638	0.591	0.666	0.612	0.675
F-statistics	52.62***	59.20***	40.75***	49.91***	17.62***	20.13***

Panel A reports OLS coefficients of equation (2). Panel B and Panel C report IV and first-stage estimates (respectively) of the sytem of equations (3). The outcome variable in Panel A and Panel B is "Average Pay inequality," which can be obtained from the UT-UNIDO database. Other data sources are described in section 5.2 and 5.2.1. Robust Standard errors are in parentheses. Asterisks *, **, and *** denote statistical significance at 10%, 5%, and 1%, respectively.

Figure 4: OLS, Reduced-Form, and First-Stage Relationships for Wage Inequality



(a) (b) (c)

Panel (a) shows the OLS relationship between wage inequality (on the y-axis) and Average Rule of Law (on the x-axis). Panel (b) shows the reduced-form relationship between the standard deviations of country-level TFP growth rates (on the y-axis) and the logged morality rates of early European settlers (on the x-axis). Panel (c) shows the first-stage relationship between the Average Rule of Law measure (on the y-axis) and the mortality rates (on the x-axis). In each panel, we also show the line of best fit in red.

We report the OLS estimates in Panel A of Table 6 and also graphically illustrate the OLS relationship in Panel (a) of Figure 4. We find that the relationship implied by the part (c) of Theorem 1 is indeed consistent with the data: countries with better contract enforcement have, on average, less dispersed wages than those with weaker contract enforcement. One standard deviation increase in the average rule of law is predicted to decrease the average level of the wage inequality measure by 0.49. We believe that this is quantitatively large, given that the standard deviation of the wage inequality measure is about 0.896. Furthermore, the R-squared value suggests that variations in the average rule of law would account for 29.9% of variations in country-level dispersion in wage inequality.

As in subsection 5.2.1, we adopt Acemoglu et al. (2001)'s instrumental variable approach and estimate:

$$\begin{aligned} VOL_c &= \alpha + \beta_{IV} AVG\_RL_c + Controls + \epsilon_c, \\ AVG\_RL_c &= \mu + \gamma_{FS} S_c + Controls + \eta_c, \end{aligned} \tag{5}$$

where we use the same set of control variables as in previous analyses.

We report the IV estimates in Panel B of Table 6 and the first-stage estimates in Panel C of Table 6. Also, we show the reduced-form and first-stage relationships graphically in Figure 4. The IV estimates are quite comparable to the OLS estimates, and the first-stage statistics suggest that the inclusion restriction is likely to be satisfied.

As in previous empirical analyses, we report the results of various robustness checks in the Internet Appendix. First, we use the IHS measure and repeat the empirical analyses in Table 14. Second, we report the results after excluding low-income countries in Table 15 and Table 16. These robustness checks yield qualitatively similar outcomes.

6. Conclusion

We have presented a parsimonious model combining two very natural premises: limited enforcement and persistent productivity. Despite its simplicity, time-varying limits to contract enforceability generate a rich set of testable implications both at the micro and macro levels. The empirical findings (both by others and our own) are consistent with our model. In particular, we provided a new set of empirical results showing that cross country differences in contract enforceability can have important implications for aggregate TFP volatility, cross-sectional dispersion in firm-level TFP's and wage inequality.

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Internet Appendix

Appendix for Proofs

Proof of Proposition 1. Throughout we fix a given level of contract enforcement α. We prove this proposition in four steps.

Step 1: In the contracting game, the set of value in the Pareto frontier of Perfect Public Equilibrium (PPE) satisfies: any pair of values v_1, v_2 promised to the agent satisfy $v_1+\Pi^*(v_1,\theta_{-1}) = v_2+\Pi^*(v_2,\theta_{-1})$ for each $\theta_{-1} \in \Theta$.

Suppose, to the contrary, that there exists a pair v_1, v_2 and a state θ_{-1} such that $v_1 + \Pi^*(v_1,\theta_{-1}) < v_2 + \Pi^*(v_2,\theta_{-1})$. Let (a^*, w^*, b^*, v'^*) be an optimal contract that yields value v_1 to the agent. We will construct a contract $(\widetilde{a}, \widetilde{w}, \widetilde{b}, \widetilde{v}')$ when $v = v_1$ that strictly improves upon the contract (a^*, w^*, b^*, v'^*), contradicting the optimality of the latter.

We start with defining a contract $(\widetilde{a}, \widetilde{w}, \widetilde{b}, \widetilde{v}')$ when $v = v_1$ by:

$$\begin{aligned}\widetilde{a}(v_1,\theta_{-1}) = a^*(v_2,\theta_{-1}),\ \widetilde{w}(v_1,\theta_{-1}) := w^*(v_2,\theta_{-1}) + \frac{v_1 - v_2}{1-\delta},\\ \widetilde{b}(y,\theta,\theta_{-1},v_1) := b^*(y,\theta,\theta_{-1},v_2),\ \widetilde{v}'(y,\theta,\theta_{-1},v_1) := v'^*(y,\theta,\theta_{-1},v_2).\end{aligned} \tag{6}$$

The contract defined here replicates the allocation in the contract (a^*, w^*, b^*, v'^*) when $v = v_2$ while it ensures to the agent a value at least $v = v_1$. This follows because shifting the compensation scheme by a constant simply redistributes the values between the parties. To see this rigorously:

$$\begin{aligned}&\mathbb{E}\Big[(1-\delta)\big(\underbrace{\widetilde{w}(v_1,\theta_{-1})}_{=w(v_2,\theta_{-1})+\frac{v_1-v_2}{1-\delta}} + \widetilde{b}(y,\theta,\theta_{-1},v_1) - c\left(\widetilde{a}(v_1,\theta_{-1})\right)\big) + \delta\widetilde{v}'(y,\theta,\theta_{-1},v_1)|\underbrace{\widetilde{a}(v_1,\theta_{-1})}_{=a^*(v_2,\theta_{-1})},\theta_{-1}\Big]\\ &= (v_1 - v_2) + \underbrace{\mathbb{E}\Big[(1-\delta)\left(w^*(v_2,\theta_{-1}) + b^*(y,\theta,\theta_{-1},v_2) - c\left(a^*(v_2,\theta_{-1})\right)\right) + \delta v'^*(y,\theta,\theta_{-1},v_2)|a^*(v_2,\theta_{-1}),\theta_{-1}\Big]}_{=v_2} = v_1.\end{aligned}$$

Moreover, by the appropriate shift in the fixed wage so chosen, the principal's continuation profits $\widetilde{\Pi}(\widetilde{v}'(y,\theta,\theta_{-1},v_1),\theta)$ from the contract $(\widetilde{a}, \widetilde{w}, \widetilde{b}, \widetilde{v}')$ are equal to $\Pi^*(v'^*(y,\theta,\theta_{-1},v_2),\theta)$ for all y, θ.

Now, the contract $(\widetilde{a}, \widetilde{w}, \widetilde{b}, \widetilde{v}')$ satisfies [DEP]', because substituting b^* with $\widetilde{b}$ and Π^* with $\widetilde{\Pi}$ implies:

$$\delta\widetilde{\Pi}(\widetilde{v}'(y,\theta_{-1},\theta,v_1),\theta) = \underbrace{\delta\Pi^*(v'^*(y,\theta_{-1},\theta,v_2),\theta) \geq (1-\delta)(1-\alpha)b^*(y,\theta_{-1},\theta,v_2)}_{[DEP]' \text{ when } v=v_2} = (1-\delta)(1-\alpha)\widetilde{b}(y,\theta_{-1},\theta,v_1).$$

Similarly, it is easy to check that [PCP], and [PCA] are also satisfied. [IC] is also satisfied, because the change only in the base wage from $\widetilde{w}(v_1,\theta_{-1})$ to $w^*(v_2,\theta_{-1})$ does not affect the agent's effort choice.

Hence, the contract $(\widetilde{a},\widetilde{w},\widetilde{b},\widetilde{v}')$ satisfies all constraints and yields the following profit for the principal:

$$\begin{aligned} &\mathbb{E}\left[(1-\delta)\left(\pi-\widetilde{w}(v_1,\theta_{-1})-\widetilde{b}\left(y,\theta_{-1},\theta,v_1\right)\right)+\delta\widetilde{\Pi}\left(\widetilde{v}'\left(y,\theta_{-1},\theta,v_1\right),\theta\right)|\widetilde{a}(v_1,\theta_{-1}),\theta_{-1}\right] \\ =\ &(v_2-v_1)+\underbrace{\mathbb{E}\left[(1-\delta)\left(\pi-w^*(v_2,\theta_{-1})-b^*\left(y,\theta_{-1},\theta,v_2\right)\right)+\delta\Pi^*\left(v'^*\left(y,\theta_{-1},\theta,v_2\right),\theta\right)|a^*(v_2,\theta_{-1}),\theta_{-1}\right]}_{=\Pi^*(v_2,\theta_{-1})}. \end{aligned}$$

Here the equality follows by the construction of $(\widetilde{a},\widetilde{w},\widetilde{b},\widetilde{v}')$ in (6), which implements $\widetilde{a}(v_1,\theta_{-1})=a^*(v_2,\theta_{-1})$. The last equation implies that the alternative contract $(\widetilde{a},\widetilde{w},\widetilde{b},\widetilde{v}')$ yields $v_2-v_1+\Pi^*(v_2,\theta_{-1})$ for the principal. By the initial hypothesis that $v_1+\Pi^*(v_1,\theta_{-1})<v_2+\Pi^*(v_2,\theta_{-1})$, the alternative contract (which may not be optimal) strictly improves upon the principal's profit $\Pi^*(v_1,\theta_{-1})$ under the optimal contract (a^*,w^*,b^*,v'^*) that ensures to the agent payoff at least v_1. This contradicts the optimality of the latter contract.

Step 2: It is without loss of generality to consider $v'^*=0$ in any optimal contract.

Given any optimal contract that ensures a non-negative continuation value to the agent, we construct an alternative contract that delivers zero continuation value to the agent and yields the principal the same payoff. For any quadruple (y,θ,θ_{-1},v), define the compensation scheme $(\widehat{w},\widehat{b},\widehat{v}')$ from an optimal contract yielding value v to the agent as follows:

$$\widehat{w}(v,\theta_{-1}):=w^*(v,\theta_{-1}),\ \widehat{b}(y,\theta,\theta_{-1},v):=b^*(y,\theta,\theta_{-1},v)+\frac{\delta}{1-\delta}v'^*(y,\theta,\theta_{-1},v),\ \widehat{v}'=0.$$

Since $v'^*,b^*\in\mathbb{R}^+$, $\hat{b}=b^*+\frac{\delta}{1-\delta}v'^*\in\mathbb{R}^+$ as well. Moreover, the compensation scheme $(\widehat{w},\widehat{b},\widehat{v}')$ yields the continuation profit of $\Pi^*(0,\theta)$ for the principal.

Here the equality $\widehat{v}'=0$ follows because the difference is absorbed (after appropriately accounting for discounting) in bonus payments, so it does not affect incentives. To see this, $(1-\delta)\widehat{b}+\delta\widehat{v}'=(1-\delta)b^*+\delta v^*$ by definition. Hence, the maximand in [IC] is:

$$\mathbb{E}\left[(1-\delta)\left(\widehat{w}+\widehat{b}-c\left(\widetilde{a}\right)\right)+\delta\widehat{v}'|\widetilde{a},\theta_{-1}\right]=\mathbb{E}\left[(1-\delta)\left(w^*+b^*-c\left(\widetilde{a}\right)\right)+\delta v'^*|\widetilde{a},\theta_{-1}\right].$$

Since the agent faces the same optimization problem under both $(\widehat{w},\widehat{b},\widehat{v})$ and (w^*,b^*,v^*), the effort recommendation $\widehat{a}$ under $(\widehat{w},\widehat{b},\widehat{v})$ would coincide with the effort recommendation a^* under (w^*,b^*,v^*).

By Step 1, $\Pi^*(\widehat{v}',\theta)+\widehat{v}'=\Pi^*(v'^*,\theta)+v'^*$. Together with $(1-\delta)\widehat{b}+\delta\widehat{v}'=(1-\delta)b^*+\delta v^*$, we can check whether $(\widehat{a},\widehat{w},\widehat{b},\widehat{v})$ satisfies [DEP]' using this fact:

$$\delta\Pi^*(\widehat{v}',\theta)=\delta(v'^*-\widehat{v}')+\underbrace{\delta\Pi(v'^*,\theta)\geq(1-\delta)\,(1-\alpha)b^*\,(y,\theta_{-1},\theta,v)}_{[DEP]'\text{ for }(w^*,b^*,v^*)}+\underbrace{\delta(v'^*-\widehat{v}')=(1-\delta)\,(1-\alpha)\widehat{b}\,(y,\theta_{-1},\theta,v)}_{(1-\delta)\widehat{b}+\delta\widehat{v}'=(1-\delta)b^*+\delta v^*}.$$

Other constraints can be verified in a similar manner. Moreover, the principal's profit under $(\widehat{a},\widehat{w},\widehat{b},\widehat{v})$ is:

$$\begin{aligned}&\mathbb{E}\Big[(1-\delta)\,(\pi-\widehat{w}(v,\theta_{-1})-\underbrace{\widehat{b}\,(y,\theta_{-1},\theta,v)}_{(1-\delta)\widehat{b}=(1-\delta)b^*+\delta v^*})+\delta\Pi^*(\underbrace{\widehat{v}'\,(y,\theta_{-1},\theta,v)}_{=0},\theta)|\underbrace{\widehat{a}}_{=a^*},\theta_{-1}\Big]\\=&\mathbb{E}\Big[(1-\delta)\,(\pi-w^*(v,\theta_{-1})-b^*\,(y,\theta_{-1},\theta,v))\underbrace{-\delta v'^*\,(y,\theta_{-1},\theta,v)+\delta\Pi^*\,(0,\theta)}_{=\delta\Pi^*(v'^*(y,\theta_{-1},\theta,v),\theta)}|a^*,\theta_{-1}\Big]\\=&\mathbb{E}\Big[(1-\delta)\,(\pi-w^*(v,\theta_{-1})-b^*\,(y,\theta_{-1},\theta,v))+\delta\Pi^*\,(v'^*\,(y,\theta_{-1},\theta,v)\,,\theta)\,|a^*,\theta_{-1}\Big].\end{aligned}$$

Thus, the principal earns the same profit under the contract $(\widehat{a},\widehat{w},\widehat{b},\widehat{v})$ as that under (a^*,w^*,b^*,v^*).

Step 3: It is without loss of generality to consider $b^*(0,\theta_{-1},\theta,v)=0$ in an optimal contract.

Given any optimal contract with a non-negative bonus to the agent upon failure, we construct an alternative contract that delivers zero bonuses to the agent upon failure and yields the principal the same payoff. By Step 2, we may consider any optimal contract (w^*,b^*,v'^*) such that $v'^*=0$ without loss of generality. For any quadruple (y,θ,θ_{-1},v), define an alternative compensation scheme $(\widehat{w},\widehat{b},\widehat{v}')$ from the optimal contract as follows:

$$\widehat{w}(v,\theta_{-1}):=w^*(v,\theta_{-1})+\mathbb{E}\left[b^*(0,\theta_{-1},\theta,v)|\theta_{-1}\right],\;\widehat{b}(1,\theta,\theta_{-1},v):=b^*(1,\theta,\theta_{-1},v)-b^*(0,\theta,\theta_{-1},v),$$
$$\widehat{b}'(0,\theta,\theta_{-1},v)=0,\;\widehat{v}'=0.$$

One can show that $\widehat{b}(1,\theta,\theta_{-1},v)\in\mathbb{R}^+$.[21] Moreover, the compensation scheme $(\widehat{w},\widehat{b},\widehat{v}')$ yields the continuation profit of $\Pi^*(0,\theta)$ for the principal.

We show that the effort recommendation $\widehat{a}$ under $(\widehat{w},\widehat{b},\widehat{v})$ would coincide with the effort recommendation a^* under (w^*,b^*,v^*). To see this, the agent's payoff under $(\widehat{w},\widehat{b},\widehat{v})$ becomes:

$$\begin{aligned}&\mathbb{E}\left[\left(\widehat{w}+\widehat{b}-c\,(\widetilde{a})\right)|\widetilde{a},\theta_{-1}\right]=\widetilde{a}\left[\widehat{b}\,(1,\theta,\theta_{-1},v)-\widehat{b}\,(0,\theta,\theta_{-1},v)\right]+\widehat{w}\,(v,\theta_{-1})-c\,(\widetilde{a})\\=&\widetilde{a}b^*\,(1,\theta,\theta_{-1},v)+(1-\widetilde{a})b^*\,(0,\theta,\theta_{-1},v)+w^*\,(v,\theta_{-1})-c\,(\widetilde{a})=\mathbb{E}\left[(w^*+b^*-c\,(\widetilde{a}))\,|\widetilde{a},\theta_{-1}\right].\end{aligned}$$

[21] We provide a sketch here (details are available upon request). Assume to the contrary that $b^*(1,\theta,\theta_{-1},v)<b^*(0,\theta,\theta_{-1},v)$ for some (θ,θ_{-1},v). For this (θ,θ_{-1},v), the agent would exert zero efforts $a^*(v,\theta_{-1})=0$. However, the principal can strictly do better by proposing an alternative contract that recommends $\widehat{a}(v,\theta_{-1})=\epsilon>0$ and delivers v to the agent, which is a contradiction.

Since the agent faces the same optimization problem under both $(\widehat{w}, \widehat{b}, \widehat{v})$ and (w^*, b^*, v^*), the effort recommendation $\widehat{a}$ under $(\widehat{w}, \widehat{b}, \widehat{v})$ would coincide with the effort recommendation a^* under (w^*, b^*, v^*).

It remains to verify that the contract $(\widehat{a}, \widehat{w}, \widehat{b}, \widehat{v})$ satisfies the dynamic enforceability constraint [DEP]. Notice that for any triplet (θ_{-1}, θ, v):

$$\delta\Pi^*(\underbrace{\widehat{v}'}_{=0}, \theta) = \delta\Pi^*(\underbrace{v'^*}_{=0}, \theta) \underbrace{\geq}_{\text{[DEP] for }(w^*,b^*,v^*)} (1-\delta)(1-\alpha)b^*(1, \theta_{-1}, \theta, v) \underbrace{\geq}_{b^*(0,\theta_{-1},\theta,v)\geq 0} (1-\delta)(1-\alpha)\widehat{b}(1, \theta_{-1}, \theta, v).$$

Since $\widehat{b}(0, \theta_{-1}, \theta, v) = 0$, $(\widehat{a}, \widehat{w}, \widehat{b}, \widehat{v})$ satisfies [DEP]. Other constraints can be verified in a similar manner. Moreover, the principal's profit under $(\widehat{a}, \widehat{w}, \widehat{b}, \widehat{v})$ is:

$$\begin{aligned}
&\mathbb{E}\Big[(1-\delta)\left(\pi - \widehat{w}(v, \theta_{-1}) - \widehat{b}\left(y, \theta_{-1}, \theta, v\right)\right) + \delta\Pi^*(\underbrace{\widehat{v}'\left(y, \theta_{-1}, \theta, v\right)}_{=0}, \theta) | \underbrace{\widehat{a}}_{=a^*}, \theta_{-1}\Big] \\
=&\mathbb{E}\Big[(1-\delta)\left(\pi \underbrace{- w^*(v, \theta_{-1}) - b^*\left(y, \theta_{-1}, \theta, v\right)}_{\forall \tilde{a},\ \mathbb{E}\left[\widehat{w}+\widehat{b}|\tilde{a},\theta_{-1}\right]=\mathbb{E}[w^*+b^*|\tilde{a},\theta_{-1}]}\right) + \delta\Pi^*(\underbrace{v'^*\left(y, \theta_{-1}, \theta, v\right)}_{=0}, \theta) | a^*, \theta_{-1}\Big].
\end{aligned}$$

Thus, the principal earns the same profit under the contract $(\widehat{a}, \widehat{w}, \widehat{b}, \widehat{v})$ as that under (a^*, w^*, b^*, v^*).

Step 4: An optimal contract such that $b^*(0, \theta_{-1}, \theta, v) = 0$ and $v' = 0$ is a Markovian contract. Consider any optimal contract (a^*, w^*, b^*, v^*) such that $b^*(0, \theta_{-1}, \theta, v) = 0$ and $v'^* = 0$. To show that this is a Markovian contract, we first show that the effort recommendation $a^*(v, \theta_{-1})$ is independent of v. Fix any $\theta_{-1} \in \Theta$, and also fix any value to the agent v on the Pareto frontier.

$$\begin{aligned}
\Pi^*(v, \theta_{-1}) =&\mathbb{E}\Big[(1-\delta)\left(\pi \underbrace{- w^*(v, \theta_{-1}) - b^*\left(y, \theta_{-1}, \theta, v\right)}_{v=(1-\delta)(\mathbb{E}[w^*+b^*|a^*,\theta_{-1}]-c(a^*))\text{ by [PK]}}\right) + \delta\Pi^*(\underbrace{v'^*\left(y, \theta_{-1}, \theta, v\right)}_{=0}, \theta) | a^*, \theta_{-1}\Big] \\
=&\mathbb{E}\Big[(1-\delta)\left(\pi - c(a^*(v, \theta_{-1}))\right) + \delta\Pi^*(0, \theta) | a^*, \theta_{-1}\Big] - v.
\end{aligned}$$

Moving v and $\delta\mathbb{E}\Big[\Pi^*(0, \theta) | a^*, \theta_{-1}\Big]$ to the left hand side, we have:

$$\begin{aligned}
&\underbrace{\Pi^*(v, \theta_{-1}) + v}_{=\Pi^*(0,\theta_{-1})\text{ by Step 1}} - \delta\mathbb{E}\Big[\Pi^*(0, \theta) | a^*, \theta_{-1}\Big] = \mathbb{E}\Big[(1-\delta)\left(\pi - c(a^*(v, \theta_{-1}))\right) | a^*, \theta_{-1}\Big] \\
\iff &\Pi^*(0, \theta_{-1}) - \delta\mathbb{E}\Big[\Pi^*(0, \theta) | a^*, \theta_{-1}\Big] = (1-\delta)\left(a^*(v, \theta_{-1}) - c(a^*(v, \theta_{-1}) + \mathbb{E}\Big[\theta | \theta_{-1}\Big]\right).
\end{aligned}$$

Since the left hand side is independent of v, the surplus from the agent's effort $a^*(v, \theta_{-1}) - c(a^*(v, \theta_{-1}))$

must be also independent of v. From this, $a^*(v, \theta_{-1})$ is independent of v as well.[22]

Since $a^*(v, \theta_{-1})$ is independent of v, the bonus upon success $b^*(1, \theta_{-1}, \theta, v)$ must also be independent of v, because the marginal cost of effort must be equal to the spread in the bonus $b^*(1, \theta_{-1}, \theta, v) - b^*(0, \theta_{-1}, \theta, v)$ using [IC]. The latter in turn is equal to $b^*(1, \theta_{-1}, \theta, v)$ as $b^*(0, \theta_{-1}, \theta, v) = 0$. This show that the bonus scheme $b^*(0, \theta_{-1}, \theta, v) = 0$ is also independent of v, so the optimal contract is a Markovian contract. This establishes the Proposition 1. □

Proof of Lemma 1. As in the proof of Proposition 1], throughout this proof we fix a given level of contract enforcement α. To establish the part (E-1) of Lemma 1, we first show that $\max\{a^*(H; \alpha), a^*(L; \alpha)\} \leq a^{FB}$ under the optimal Markovian contract $\{b^*, a^*, \Pi^*\}$. Suppose to the contrary that there exists a state θ_0 with $a^*(\theta_0; \alpha) > a^{FB}$. Now, consider an alternative bonus system $\hat{b}(\theta_{-1}, \theta; \alpha) = b^*(\theta_{-1}, \theta; \alpha) - \epsilon \times 1(\theta_{-1} = \theta_0)$ for an arbitrarily small $\epsilon > 0$. The IC condition $a(\theta_{-1}; \alpha) = g[\mathbb{E}(b(\theta, \theta_{-1}; \alpha)|\theta_{-1})]$ implies that the bonus scheme $\hat{b}$ induces an effort $\hat{a}(\theta_0; \alpha)$ that is higher than a^{FB} but strictly less than $a^*(\theta_0; \alpha)$. From the system of Bellman equations, the following relationship holds:

$$\begin{bmatrix} \Pi(H; \alpha) \\ \Pi(L; \alpha) \end{bmatrix} = \begin{bmatrix} \gamma & 1-\gamma \\ 1-\gamma & \gamma \end{bmatrix} \begin{bmatrix} a(H; \alpha) - c(a(H; \alpha)) + \hat{H} \\ a(L; \alpha) - c(a(L; \alpha)) + \hat{L} \end{bmatrix}, \tag{7}$$

where $\gamma := \frac{\delta(1-\lambda)+\lambda(1-\delta)}{1+\delta-\delta\lambda} \in (\frac{1}{2}, 1)$. Since the function $x - c(x)$ is concave, by (7), a strict increase in $a - c(a)$ strictly increases all of principal's surpluses Π. We simplify the notation and denote by $\hat{\Pi}$ the principal's payoff from the bonus scheme $\hat{b}$.

Notice that the bonus scheme $\hat{b}$ is weakly smaller than the bonus scheme under the original contract, and hence

$$(1-\delta)(1-\alpha)\hat{b}(\theta_{-1}, \theta; \alpha) \leq (1-\delta)(1-\alpha)b^*(\theta_{-1}, \theta; \alpha) \underbrace{\leq}_{\text{DEP}} \delta\Pi^*(\theta; \alpha) < \delta\hat{\Pi}(\theta; \alpha).$$

Yet, this new contract obtains strictly higher joint payoffs in both states compared to initial optimal contract without violating dynamic enforceability conditions, which contradicts the optimality of the initial contract. This therefore shows that $\max\{a^*\} \leq a^{FB}$.

Next, we show that $a^*(H; \alpha) \geq a^*(L; \alpha)$ under the optimal contract $\{b^*, a^*, \Pi^*\}$. Suppose to the contrary that $a^*(H; \alpha) < a^*(L; \alpha)$. Moreover, as we have showed in the previous part of the current proof that $\max\{a^*\} \leq a^{FB}$, it must be the case that $a^*(H; \alpha) < a^*(L; \alpha) \leq a^{FB}$.

[22]One can show that (1) the surplus $a - c(a)$ strictly increases on $[0, a^{FB}]$ where $a^{FB} = (c')^{-1}(1)$ denotes the first best effort level and (2) $a^*(v, \theta_{-1}) \in [0, a^{FB}]$ (the proof for (2) is almost identical to our Lemma (E-1)). Hence, there is a one-to-one and onto mapping between the surplus $a - c(a)$ and a in the domain of interest.

By the incentive-compatibility condition, the expected bonus payments is equal to the marginal cost of effort. Thus,

$$\mathbb{E}(b^*(L,\theta;\alpha)|\theta_{-1}=L) = c'(a^*(L;\alpha)) > c'(a^*(H;\alpha)) = \mathbb{E}(b^*(H,\theta;\alpha)|\theta_{-1}=H).$$

Here the inequality holds by the assumption that $a^*(L;\alpha) > a^*(H;\alpha)$ and $c'' > 0$.

Now, we construct a new payment scheme $\hat{b}$ as follows:

$$\begin{aligned}
\hat{b}(H,H;\alpha) &= b^*(L,L;\alpha) - \frac{1-\lambda}{\lambda}\left(\frac{\delta}{1-\delta}\Pi^*(L;\alpha) - b^*(L,H;\alpha)\right) \\
\hat{b}(H,L;\alpha) &= \frac{\delta}{1-\delta}\Pi^*(L;\alpha) \\
\hat{b}(L,\theta;\alpha) &= b^*(L,\theta;\alpha) \quad \forall\theta\in\Theta.
\end{aligned}$$

Notice that $\mathbb{E}(\hat{b}(\theta,\theta_{-1};\alpha)|\theta_{-1}) = \mathbb{E}(b^*(L,\theta;\alpha)|\theta_{-1} = L)$ for $\forall\theta_{-1}\in\Theta$. Denote by $\hat{a}$ the action recommendation under the bonus scheme $\hat{b}$ as $\hat{a}$ and by $\hat{\Pi}$ the principal's payoff the $(\hat{b},\hat{a})$. Recall that $x - c(x)$ is concave and attains the maximum value at a^{FB}. As the effort recommendation satisfies $\hat{a}(H;\alpha) = a^*(L;\alpha) \le a^{FB}$, the contract $(\hat{b},\hat{a})$ incentivizes a higher effort $a(H;\alpha)$ in the high state than the original contract. By (7), this in turn implies that the principal obtains a higher payoff from the bonus scheme $\hat{b}$ than the original scheme one b. Summarizing, for any given α the following relationships hold:

$$\hat{\Pi}(H;\alpha) > \hat{\Pi}(L;\alpha) > \Pi^*(L;\alpha),\ \hat{\Pi}(H;\alpha) > \Pi^*(H;\alpha).$$

Now, it remains to verify that dynamic enforceability constraints are satisfied under the new contract. To see this, notice that

$$\begin{aligned}
\delta\hat{\Pi}(H;\alpha) &> \delta\Pi^*(H;\alpha) \ge (1-\delta)(1-\alpha)b^*(L,H;\alpha) = (1-\delta)(1-\alpha)\hat{b}(L,H;\alpha) \\
\delta\hat{\Pi}(L;\alpha) &> \delta\Pi^*(L;\alpha) \ge (1-\delta)(1-\alpha)b^*(L,L;\alpha) = (1-\delta)(1-\alpha)\hat{b}(L,L;\alpha) \\
\delta\hat{\Pi}(L;\alpha) &> \delta\Pi^*(L;\alpha) = (1-\delta)(1-\alpha)\hat{b}(H,L;\alpha),
\end{aligned}$$

which follow from the definition of $\hat{b}$, the system of equations at the beginning of the proof, or DEP of the initial optimal contract. Now,

$$\begin{aligned}
\delta\hat{\Pi}(H;\alpha) &> (2-\frac{1}{\lambda})\delta\Pi^*(L;\alpha) + (\frac{1}{\lambda}-1)\delta\Pi^*(H;\alpha) \\
&\geq (2-\frac{1}{\lambda})\delta\Pi^*(L) + (\frac{1}{\lambda}-1)(1-\delta)(1-\alpha)b^*(L,H;\alpha) \\
&= \delta\Pi^*(L;\alpha) + (1-\frac{1}{\lambda})\delta\Pi^*(L;\alpha) + (\frac{1}{\lambda}-1)(1-\delta)(1-\alpha)b^*(L,H;\alpha) \\
&\geq (1-\delta)(1-\alpha)b^*(L,L;\alpha) + (1-\frac{1}{\lambda})\delta\Pi^*(L;\alpha) + (\frac{1}{\lambda}-1)(1-\delta)(1-\alpha)b^*(L,H;\alpha) \\
&= (1-\delta)(1-\alpha)\hat{b}(H,H;\alpha),
\end{aligned}$$

where the first inequality follows from the observation $\hat{\Pi}(H;\alpha) > \max\{\Pi^*\}$ and $2 > \frac{1}{\lambda}$, the second and the third inequality from the dynamic enforceability constraints of the original contract. Finally, the last equality follows from the definition of $\hat{b}(H,H;\alpha)$. As in the first part of the proof $\max\{a^*\} \leq a^{FB}$, and hence the existence of such a contract $\{\hat{b},\hat{a},\hat{\Pi}\}$ contradicts the optimality of the original contract. These two contradictions together establish the part (E-1) of Lemma 1.

We next turn to establish the part (E-2) of Lemma 1 and start with denoting $g := (c')^{-1}$ the inverse of the marginal cost and $\hat{\theta} = \mathbb{E}(\theta_{+1}|\theta)$ for convenience. Suppose to the contrary. Denote $\{b^*, a^*, \Pi^*\}$ an optimal Markovian contract in which $a^*(\theta') < g(1) = a^{FB}$ with $(1-\delta)b^*(\theta',\theta'') < \delta\Pi^*(\theta'')$ for some $(\theta',\theta'') \in \Theta^2$. Fix such θ',θ'' and let $\epsilon := \frac{1}{2}\min\{\frac{\delta}{1-\delta}\Pi^*(\theta'') - b^*(\theta',\theta''), \frac{1-\mathbb{E}(b^*(\theta',\theta|\theta_{-1}=\theta'))}{\lambda}\} > 0$. Consider the alternative bonus system by $\hat{b}(\theta_{-1},\theta) = b^*(\theta_{-1},\theta) + \epsilon \times 1((\theta_{-1},\theta) = (\theta',\theta''))$. Under this new bonus system $\{\hat{b}\}$:

$$\begin{aligned}
&\mathbb{E}(\hat{b}(\theta',\theta;\alpha)|\theta_{-1}=\theta') \leq \mathbb{E}(b^*(\theta',\theta;\alpha)|\theta_{-1}=\theta') + \lambda\epsilon < 1 \\
&\mathbb{E}(\hat{b}(\theta',\theta;\alpha)|\theta_{-1}=\theta') \geq \mathbb{E}(b^*(\theta',\theta;\alpha)|\theta_{-1}=\theta') + (1-\lambda)\epsilon > \mathbb{E}(b^*(\theta',\theta;\alpha)|\theta_{-1}=\theta').
\end{aligned}$$

To establish the part (E-2) of Lemma 1, denote the principal's surpluses induced from $\hat{b}$ as $\hat{\Pi}$, and the new recommended effort levels $\hat{a}$. From the representation in (7), $\hat{\Pi}(\theta;\alpha) > \Pi^*(\theta;\alpha)\ \forall\theta$, because $\hat{a}(\theta';\alpha) - c(\hat{a}(\theta';\alpha)) > a^*(\theta';\alpha) - c(a^*(\theta';\alpha))$ from $\hat{a}^*(\theta';\alpha) \in (a^*(\theta';\alpha), a^{FB})$ and $\hat{a}(\theta;\alpha) \geq a^*(\theta;\alpha)\ \forall\theta$. Now, we arrive at a contradiction if the new contract $\{\hat{b},\hat{a},\hat{\Pi}\}$ satisfies the dynamic enforceability condition, because the new contract strictly does better than the optimal contract. For the states such that $(\theta_{-1},\theta) \neq (\theta',\theta'')$,

$$(1-\delta)(1-\alpha)\hat{b}(\theta_{-1},\theta;\alpha) = (1-\delta)(1-\alpha)b^*(\theta_{-1},\theta;\alpha) \leq \delta\Pi^*(\theta;\alpha) < \delta\hat{\Pi}(\theta;\alpha).$$

Moreover, for the state $(\theta_{-1}, \theta) = (\theta', \theta'')$,

$$(1-\delta)(1-\alpha)\hat{b}(\theta', \theta''; \alpha) = (1-\delta)(1-\alpha)[b^*(\theta', \theta''; \alpha) + \epsilon] \underbrace{\leq}_{\text{Def'n of } \epsilon} \frac{1}{2}[\delta\Pi^*(\theta''; \alpha) + (1-\delta)(1-\alpha)b^*(\theta', \theta''; \alpha)]$$
$$\leq \delta\Pi^*(\theta''; \alpha) < \delta\hat{\Pi}(\theta''; \alpha).$$

This therefore shows that the bonus scheme $\hat{b}$ satisfies the dynamic enforceability conditions and hence completes the proof. □

Proof of Lemma 2. Applying the part (E-1) of Lemma 1 yields $a^*(H;\alpha) - c(a^*(H;\alpha)) + \hat{H} > a^*(L;\alpha) - c(a^*(L;\alpha)) + \hat{L}$. By the representation (7), this in turn implies that under the high state H both the principal and the agent obtain higher payoffs and hence establishes Lemma 2. □

Proof of Proposition 2. We start with denoting by $\bar{\delta}(\alpha) = \frac{1-\alpha}{1-\alpha+\Pi^{FB}(H)}$ so that $\frac{\bar{\delta}\pi^{FB}(H)}{(1-\bar{\delta})(1-\alpha)} = 1$. We next fix any $\delta < \tilde{\delta}$. By incentive compatibility condition,

$$c'(a^*(H)) = \mathbb{E}(b^*(H, \theta)|\theta_{-1} = H) \leq \frac{\delta}{1-\delta}\frac{\Pi^{FB}(H)}{1-\alpha} < 1,$$

where the second inequality holds by the dynamic enforceability condition, and the last strict inequality by $\delta < \tilde{\delta}$.

Now, due to weak monotonicity in the effort level implied by the part (E-1) of Lemma 1 and the observation above, the optimal efforts in both states are strictly smaller the first-best effort $(c')^{-1}(1)$. This in turn implies that the bonus scheme satisfies $b^*(\theta_{-1}, \theta) = \frac{\delta}{1-\delta}\frac{\Pi^*(\theta;\alpha)}{1-\alpha}$ $\forall\ \theta_{-1}, \theta \in \Theta$ by Lemma 1.

Now applying the Lemma 1 yields:

$$b^*(\theta_{-1}, L; \alpha) = \frac{\delta}{1-\delta}\frac{\Pi^*(L;\alpha)}{1-\alpha} < \frac{\delta}{1-\delta}\frac{\Pi^*(H;\alpha)}{1-\alpha} = b^*(\theta_{-1}, H; \alpha).$$

Notice that the threshold discount rate $\bar{\delta}(\alpha) := \frac{1-\alpha}{1-\alpha+a^{FB}-c(a^{FB})+H}$ must be decreasing in α. □

Proof of Proposition 3. For any given level α of contract enforcement, the existence of the threshold $\bar{\delta}(\alpha)$ follows using the arguments analogous to those used in the proof of Proposition 2. Moreover, by the incentive compatibility condition,

$$c'(a^*(H;\alpha)) = \mathbb{E}(b^*(H, \theta; \alpha)|H) = \frac{\delta}{1-\delta}\frac{\mathbb{E}(\Pi^*(\theta;\alpha)|H)}{1-\alpha}$$
$$> \frac{\delta}{1-\delta}\frac{\mathbb{E}(\Pi^*(\theta;\alpha)|L)}{1-\alpha} = \mathbb{E}(b^*(L, \theta; \alpha)|L) = c'(a^*(L;\alpha)).$$

where the strict inequality in the middle follows from the strict monotonicity in payoffs, establishing the part (i) of Proposition 3. Finally, convexity of effort cost function $c'' > 0$ implies that $a^*(H;\alpha) > a^*(L;\alpha)$, establishing the part (ii) of Proposition 3. □

Proof of Theorem 1

We first prove together the parts (a) and (b) of Theorem 1 and refer to them together as 'Amplification Effect' and then turn to prove the part (c) of the theorem referred to as 'Inequality Effect.'

Proofs of the parts (a) and (b) of Theorem 1

The proofs of the parts (a) and (b) of Theorem 1 are almost identical because: (1) the parties essentially face the same maximization problem under two stochastic structures (i.e., Assumption 1, 2) and (2) both statements in the part (a) and in the part (b) of Theorem 1 pertain to the dispersion measure $\frac{a^*(H;\alpha)+\mathbb{E}(\theta_{i,t}|\theta_{i,t-1}=H)}{a^*(L;\alpha)+\mathbb{E}(\theta_{i,t}|\theta_{i,t-1}=L)}$. Henceforth, we suppress indices i,t and write $y_{i,t}$ as $Y(\theta_{-1};\alpha) = a^*(\theta_{-1};\alpha) + \mathbb{E}(\theta|\theta_{-1})$. Moreover, we only use the fact that the quadratic function $c(x) = \frac{1}{2}cx^2$ satisfies log-concavity and $c'''(x) \geq 0$.

Outline of the proof of the parts (a) and (b) of Theorem 1

We first sketch the outline of the proof strategy. We provide in Figure 5 a graphical illustration of the proof strategy. In Step 1, we can divide $[0,1]$ into two distinct segments: $I_1 = \{\alpha \in [0,1] : a^*(H;\alpha) < a^{FB}\}$, and $I_2 = \{\alpha \in [0,1] : a^*(H;\alpha) = a^{FB}\}$. Next, we show that $I_1 = [0,\alpha_0)$ and $I_2 = [\alpha_0, 1]$ for some α_0. In step 2, we show that $\frac{Y(H;\alpha)}{Y(L;\alpha)}$ is decreasing in α in the closed interval I_2. According to one of our auxiliary claims presented below, $a^*(\theta;\alpha)$ is weakly increasing in α. Hence, the logic is that $Y(H;\alpha) = a^*(H;\alpha) + \mathbb{E}(\theta|\theta_{-1} = H) = a^{FB} + \mathbb{E}(\theta|\theta_{-1} = H)$ has no more room for further increase because it already reached the maximal possible level, whereas the denominator $Y(L;\alpha)$ can potentially increase when α increases. Step 3 is a preliminary step before implicit differentiation in Step 4, to check whether we can indeed implicitly differentiate $\frac{Y(H;\alpha)}{Y(L;\alpha)}$ w.r.t. α's in I_1. It turns out that we can implicitly differentiate everywhere on I_1, except at most one $\alpha \in I_1$. We shall denote such point as $\hat{\alpha}_0$, the region on the left side of $\hat{\alpha}_0$ as $I_1^L = [0,\hat{\alpha}_0)$, the region on the right side of $\hat{\alpha}_0$ as $I_1^R = (\hat{\alpha}_0, \alpha_0)$. So even when $\hat{\alpha}_0$ does exist, we can still use implicit differentiation to analyze $\frac{Y(H;\alpha)}{Y(L;\alpha)}$ in $I_1^L \cup I_1^R$. In Step 4, we implicitly differentiate the ratio $\frac{Y(H;\alpha)}{Y(L;\alpha)}$ in the region $I_1^L \cup I_1^R$ (or I_1 if $\hat{\alpha}_0$ does not exist), and show that the ratio $\frac{\partial}{\partial\alpha}\frac{Y(H;\alpha)}{Y(L;\alpha)} \leq 0$ whenever implicit differentiation is possible. After we establish Step 4, we know that $\frac{Y(H;\alpha)}{Y(L;\alpha)}$ is decreasing in each of region I_1^L, I_1^R (or I_1 if $\hat{\alpha}$ does not exist) and I_2, but we still do not know what might happen around α_0 and $\hat{\alpha}$ due to jumps. Step 5, the last step, turns to analyze local behaviors around α_0 and $\hat{\alpha}_0$ with "pasting" argument. Thanks to the continuity of $\frac{Y(H;\alpha)}{Y(L;\alpha)}$, irregular jumps are ruled out. Hence, we finish the proof by showing that $\frac{Y(H;\alpha)}{Y(L;\alpha)}$ is decreasing in for all $\alpha \in [0,1]$.

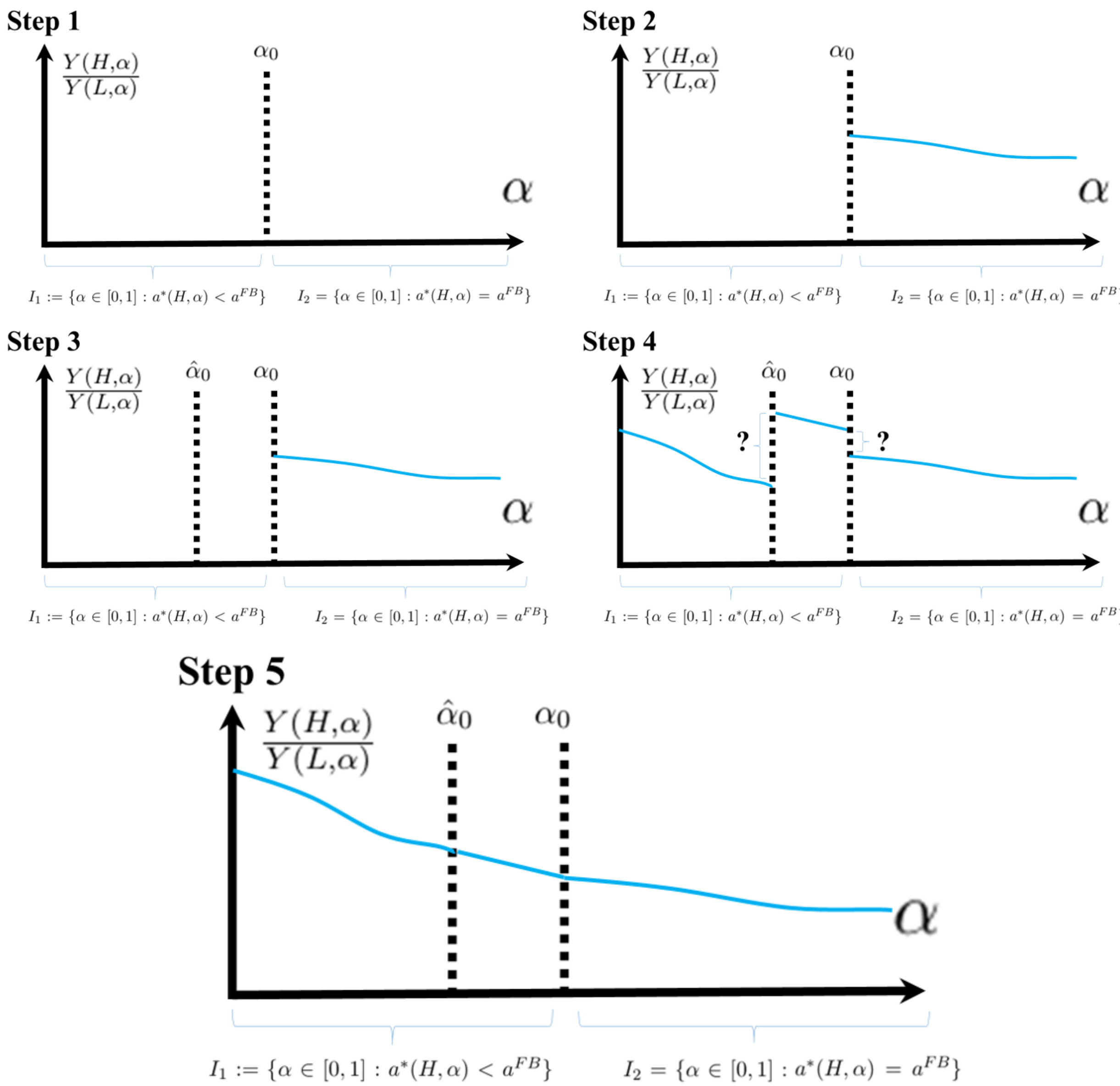

Figure 5: Illustration of the proof strategy to establish the parts (a) and (b) of Thm. 1

Auxiliary Results

To streamline the proof of Theorem 1, in this subsection we begin with listing the auxiliary results that are used in the proof of the theorem. We will then prove the theorem using these auxiliary results. After that, we will turn and provide the proofs of the auxiliary results.

The auxiliary results that we use to prove Theorem 1 are the following three lemmas.

Lemma 3 (Monotonicity and Continuity in α)**.** In any optimal Markovian relational contract, the optimal action and the profit functions satisfy

(I-1) $a^*(H;\alpha)$ and $a^*(L;\alpha)$ are weakly increasing and continuous with respect to α everywhere in

$\alpha \in [0, 1]$.

(I-2) $\Pi^*(H;\alpha)$ and $\Pi^*(L;\alpha)$ are weakly increasing and continuous with respect to α everywhere in $\alpha \in [0, 1]$.

Lemma 4. In any optimal Markovian relational contract with $a^*(H;\alpha) < a^{FB}$, $\frac{(1-\delta)}{\delta}(1-\alpha)c''(a^*(\theta)) - \hat{\gamma}(1 - c'(a^*(\theta))) \geq 0$ for both $\theta = L, H$.

Lemma 5. In any optimal Markovian relational contract with $a^*(H;\alpha) < a^{FB}$, $\frac{c'(a^*(H;\alpha))}{a^*(H;\alpha)+\hat{H}} \leq \frac{c'(a^*(L;\alpha))}{a^*(L;\alpha)+\hat{L}}$.

The proof strategy to establish the parts (a) and (b) of Theorem 1 (Amplification Effect) consists of the following five steps.

Step 1) Consider the two sets defined by $I_1 := \{\alpha \in [0, 1] : a^*(H;\alpha) < a^{FB}\}$, and $I_2 := \{\alpha \in [0, 1] : a^*(H;\alpha) = a^{FB}\}$. Then, there exists a unique $\alpha_0 \geq 0$ such that $I_1 = [0, \alpha_0)$ and $I_2 = [\alpha_0, 1]$, where the set I_1 can be empty if $\alpha_0 = 0$.

We first show that $1 \in I_2$, and hence I_2 is non-empty. Notice that if $\alpha = 1$, DEP does not bind at the optimal relational contract. It therefore implements the first-best allocation and hence $1 \in I_2$.

By continuity of $a^*(H;\alpha)$ from the part (I-1) of Lemma 3, $I_2 = [\alpha \in [0, 1] : a^*(H;\alpha) = 1]$ is a closed set. This implies that $\inf I_2 \in I_2$. Let $\alpha_0 := \inf I_2$. Now, fix any $\alpha \in [\alpha_0, 1]$. Since $a^*(H;\alpha)$ is weakly increasing in α, $a^{FB} = a^*(H, \alpha_0) \leq a^*(H;\alpha)$. Now, applying the part (E-1) of Lemma 1, $a^*(H;\alpha) \leq a^{FB}$. Combining two conclusions, we have that $a^{FB} \leq a^*(H, \alpha_0) \leq a^{FB}$, or $a^*(H, \alpha_0) = a^{FB}$. This implies that $\alpha \in I_2$. Thus, $[\alpha_0, 1] \subset I_2$. By definition of infinimum and supremum, $I_2 \subset [\inf I_2, \sup I_2] \subset [\alpha_0, 1]$. Hence, $I_2 = [\alpha_0, 1]$. Moreover, by monotonicity shown in the part (E-1) of Lemma 1, $a^*(H;\alpha) > a^{FB}$ is impossible. Hence, from the definition, $I_1 = [0, 1] \setminus I_2$. Thus, $I_2 = [0, \alpha_0)$.

Step 2) The ratio $\frac{Y(H;\alpha)}{Y(L;\alpha)}$ is weakly decreasing function of α in I_2.

By definition of the set $I_2 := \{\alpha \in [0, 1] : a^*(H;\alpha) = a^{FB}\}$, $Y(H) = a^{FB} + \hat{H}$ is independent of α in I_2. In contrast, by the part (I-1) of Lemma 3, $Y(L) = a^*(L;\alpha) + \hat{L}$ is a weakly increasing function of α. Moreover, $a^*(L;\alpha) + \hat{L} > 0$, so that $\frac{1}{a^*(L;\alpha)+\hat{L}}$ is always weakly decreasing in α. Hence, when $\alpha \in I_2$, $\frac{Y(H;\alpha)}{Y(L;\alpha)} = \frac{a^{FB}+\hat{H}}{a^*(L;\alpha)+\hat{L}}$ is a weakly decreasing function. This completes Step 2.

Step 3) The optimal effort $a^*(\theta;\alpha)$ is differentiable everywhere in $(0, \alpha_0)$ except at most one point.

By the monotonicity result in the part (E-1) of Lemma 1, $a^*(L;\alpha) \leq a^*(H;\alpha)$. Moreover, by assumption $\alpha \in (0, \alpha_0)$, $a^*(H;\alpha) < a^{FB}$. Hence, the recommended effort level for both states H, L are strictly less than the first best effort level a^{FB}. This implies that dynamic enforceability conditions for all possible past and contemporaneous shocks bind by the part (E-2) of Lemma 1:

$$\frac{(1-\delta)}{\delta}(1-\alpha)b^*(H,H;\alpha) = \Pi^*(H;\alpha), \quad \frac{(1-\delta)}{\delta}(1-\alpha)b^*(H,L;\alpha) = \Pi^*(L;\alpha),$$
$$\frac{(1-\delta)}{\delta}(1-\alpha)b^*(L,H;\alpha) = \Pi^*(H;\alpha), \quad \frac{(1-\delta)}{\delta}(1-\alpha)b^*(L,L;\alpha) = \Pi^*(L;\alpha).$$

Notice that $\mathbb{E}(b^*(H,\theta;\alpha)|\theta_{-1} = H) = \lambda b^*(H,H;\alpha) + (1-\lambda)b^*(H,L;\alpha)$ and $\mathbb{E}(b^*(L,\theta;\alpha)|\theta_{-1} = L) = \lambda b^*(L,L;\alpha) + (1-\lambda)b^*(L,H;\alpha)$. Thus, using the last set of equation and taking the convex combinations of the optimal bonus scheme b^* state by state now imply

$$\frac{(1-\delta)}{\delta}(1-\alpha)[\lambda b^*(H,H;\alpha) + (1-\lambda)b^*(H,L;\alpha)] = [\lambda\Pi^*(H;\alpha) + (1-\lambda)\Pi^*(L;\alpha)]$$
$$\frac{(1-\delta)}{\delta}(1-\alpha)[\lambda b^*(H,H;\alpha) + (1-\lambda)b^*(H,L;\alpha)] = [\lambda\Pi^*(L;\alpha) + (1-\lambda)\Pi^*(H;\alpha)].$$

Using the incentive compatibility condition $c'(a^*(\theta_{-1};\alpha)) = \mathbb{E}(b^*(\theta_{-1},\theta;\alpha)|\theta_{-1})$ we re-write the last two equations above:

$$\begin{bmatrix} \frac{(1-\delta)}{\delta}(1-\alpha)c'(a^*(H;\alpha)) \\ \frac{(1-\delta)}{\delta}(1-\alpha)c'(a^*(L;\alpha)) \end{bmatrix} = \begin{bmatrix} \lambda & 1-\lambda \\ 1-\lambda & \lambda \end{bmatrix} \begin{bmatrix} \Pi^*(H;\alpha) \\ \Pi^*(L;\alpha) \end{bmatrix}.$$

Moreover, using the equation (7) in the proof of Lemma 1 for any Markovian relational contract yields

$$\begin{bmatrix} \Pi^*(H;\alpha) \\ \Pi^*(L;\alpha) \end{bmatrix} = \begin{bmatrix} \gamma & 1-\gamma \\ 1-\gamma & \gamma \end{bmatrix} \begin{bmatrix} a^*(H;\alpha) - c(a^*(H;\alpha)) + \hat{H} \\ a^*(L;\alpha) - c(a^*(L;\alpha)) + \hat{L} \end{bmatrix}.$$

Let $\hat{\gamma} := \gamma\lambda + (1-\gamma)(1-\lambda) \in (\frac{1}{2}, 1)$.[23] The two systems of equations above can be combined as follows:

$$\begin{bmatrix} \frac{(1-\delta)}{\delta}(1-\alpha)c'(a^*(H;\alpha)) \\ \frac{(1-\delta)}{\delta}(1-\alpha)c'(a^*(L;\alpha)) \end{bmatrix} = \begin{bmatrix} \hat{\gamma} & 1-\hat{\gamma} \\ 1-\hat{\gamma} & \hat{\gamma} \end{bmatrix} \begin{bmatrix} a^*(H;\alpha) - c(a^*(H;\alpha)) + \hat{H} \\ a^*(L;\alpha) - c(a^*(L;\alpha)) + \hat{L} \end{bmatrix}. \quad \text{(DE2)}$$

[23] This can be established using the following algebraic manipulations:

$$\hat{\gamma} = \gamma\lambda + (1-\gamma)(1-\lambda) \underbrace{<}_{\lambda > \frac{1}{2}} \gamma\lambda + (1-\gamma)\lambda = \lambda < 1$$
$$\hat{\gamma} - (1-\hat{\gamma}) \underbrace{=}_{1-\hat{\gamma} = (1-\gamma)\lambda + \lambda(1-\gamma)} \gamma\lambda + (1-\gamma)(1-\lambda) - [(1-\gamma)\lambda + \gamma(1-\lambda)] = (2\lambda-1)(2\gamma-1) \underbrace{>}_{\gamma > \frac{1}{2}, \lambda > \frac{1}{2}} 0$$

This system of equations forms a necessary condition for the optimal effort choices $a^*(H;\alpha), a^*(L;\alpha)$. Now, we totally differentiate the system of equations above with respect to α, while treating $a^*(H;\alpha)$ and $a^*(L;\alpha)$ as endogenous variables.

$$\begin{aligned}&\begin{bmatrix}\frac{(1-\delta)}{\delta}(1-\alpha)c''(a^*(H;\alpha))\frac{\partial}{\partial\alpha}a^*(H;\alpha)\\ \frac{(1-\delta)}{\delta}(1-\alpha)c''(a^*(L;\alpha))\frac{\partial}{\partial\alpha}a^*(L;\alpha)\end{bmatrix}-\begin{bmatrix}\frac{(1-\delta)}{\delta}c'(a^*(H;\alpha))\\ \frac{(1-\delta)}{\delta}c'(a^*(L;\alpha))\end{bmatrix}\\ &=\begin{bmatrix}\hat{\gamma} & 1-\hat{\gamma}\\ 1-\hat{\gamma} & \hat{\gamma}\end{bmatrix}\begin{bmatrix}(1-c'(a^*(H;\alpha)))\frac{\partial}{\partial\alpha}a^*(H;\alpha)\\ (1-c'(a^*(L;\alpha)))\frac{\partial}{\partial\alpha}a^*(L;\alpha)\end{bmatrix}.\end{aligned}$$

Rearranging the last equation yields

$$\begin{aligned}-\begin{bmatrix}\frac{(1-\delta)}{\delta}c'(a^*(H;\alpha))\\ \frac{(1-\delta)}{\delta}c'(a^*(L;\alpha))\end{bmatrix}=&\begin{bmatrix}\hat{\gamma} & 1-\hat{\gamma}\\ 1-\hat{\gamma} & \hat{\gamma}\end{bmatrix}\begin{bmatrix}(1-c'(a^*(H;\alpha)))\frac{\partial}{\partial\alpha}a^*(H;\alpha)\\ (1-c'(a^*(L;\alpha)))\frac{\partial}{\partial\alpha}a^*(L;\alpha)\end{bmatrix}\\ &-\begin{bmatrix}\frac{(1-\delta)}{\delta}(1-\alpha)c''(a^*(H;\alpha))\frac{\partial}{\partial\alpha}a^*(H;\alpha)\\ \frac{(1-\delta)}{\delta}(1-\alpha)c''(a^*(L;\alpha))\frac{\partial}{\partial\alpha}a^*(L;\alpha)\end{bmatrix}.\end{aligned}\tag{8}$$

Now, the first row of the right-hand side of the equation (8) can be re-written as:

$$\begin{aligned}&\hat{\gamma}(1-c'(a^*(H;\alpha)))\frac{\partial}{\partial\alpha}a^*(H;\alpha)+(1-\hat{\gamma})(1-c'(a^*(L;\alpha)))\frac{\partial}{\partial\alpha}a^*(L;\alpha)-\frac{(1-\delta)}{\delta}(1-\alpha)c''(a^*(H;\alpha))\frac{\partial}{\partial\alpha}a^*(H;\alpha)\\ =&[\hat{\gamma}(1-c'(a^*(H;\alpha)))-\frac{(1-\delta)}{\delta}(1-\alpha)c''(a^*(H;\alpha))]\frac{\partial}{\partial\alpha}a^*(H;\alpha)+(1-\hat{\gamma})(1-c'(a^*(L;\alpha)))\frac{\partial}{\partial\alpha}a^*(L;\alpha)\\ =&\begin{bmatrix}\hat{\gamma}(1-c'(a^*(H;\alpha)))-\frac{(1-\delta)}{\delta}(1-\alpha)c''(a^*(H;\alpha)) & (1-\hat{\gamma})(1-c'(a^*(L;\alpha)))\end{bmatrix}\begin{bmatrix}\frac{\partial}{\partial\alpha}a^*(H;\alpha)\\ \frac{\partial}{\partial\alpha}a^*(L;\alpha)\end{bmatrix}.\end{aligned}$$

Similarly, the second row of the right-hand side the equation (8) is equal to:

$$\begin{bmatrix}(1-\hat{\gamma})(1-c'(a^*(H;\alpha))) & \hat{\gamma}(1-c'(a^*(L;\alpha)))-\frac{(1-\delta)}{\delta}(1-\alpha)c''(a^*(L;\alpha))\end{bmatrix}\begin{bmatrix}\frac{\partial}{\partial\alpha}a^*(H;\alpha)\\ \frac{\partial}{\partial\alpha}a^*(L;\alpha)\end{bmatrix}.$$

Thus, the system of equations (8) is equivalent to:

$$-\begin{bmatrix}\frac{(1-\delta)}{\delta}c'(a^*(H;\alpha)) \\ \frac{(1-\delta)}{\delta}c'(a^*(L;\alpha))\end{bmatrix}$$

$$=\overbrace{\begin{bmatrix}\hat{\gamma}(1-c'(a^*(H;\alpha)))-\frac{(1-\delta)}{\delta}(1-\alpha)c''(a^*(H;\alpha)) & (1-\hat{\gamma})(1-c'(a^*(L;\alpha))) \\ (1-\hat{\gamma})(1-c'(a^*(H;\alpha))) & \hat{\gamma}(1-c'(a^*(L;\alpha)))-\frac{(1-\delta)}{\delta}(1-\alpha)c''(a^*(L;\alpha))\end{bmatrix}}^{A(\alpha):=}\begin{bmatrix}\frac{\partial}{\partial\alpha}a^*(H;\alpha) \\ \frac{\partial}{\partial\alpha}a^*(L;\alpha)\end{bmatrix} \tag{IFT1}$$

Given this representation, to show the optimal effort $a^*(\theta;\alpha)$ is differentiable, by the Implicit Function Theorem (see, for instance, Sundaram (1996, Thm. 1.77)) it is sufficient to establish that the determinant of the matrix $A(\alpha)$ in the above equation is invertible. The latter is equivalent to showing that $\det(A(\alpha))\neq 0$ for at most one point in $(0,\alpha_0)$ (see, for instance, (Sundaram, 1996, Thm. 1.47)). Computing the determinant yields

$$\begin{aligned}\det(A(\alpha)) = &\left[\hat{\gamma}(1-c'(a^*(H;\alpha)))-\frac{(1-\delta)}{\delta}(1-\alpha)c''(a^*(H;\alpha))\right]\left[\hat{\gamma}(1-c'(a^*(L;\alpha)))-\frac{(1-\delta)}{\delta}(1-\alpha)c''(a^*(L;\right. \\ &-(1-\hat{\gamma})(1-c'(a^*(L;\alpha)))\times(1-\hat{\gamma})(1-c'(a^*(H;\alpha))) \\ &=(1-\hat{\gamma})^2(1-c'(a^*(L;\alpha)))(1-c'(a^*(H;\alpha))) \\ &\times\left[\left(\frac{\hat{\gamma}}{1-\hat{\gamma}}-\frac{(1-\delta)}{\delta}\frac{1-\alpha}{1-\hat{\gamma}}\frac{c''(a^*(H;\alpha))}{(1-c'(a^*(H;\alpha)))}\right)\left(\frac{\hat{\gamma}}{1-\hat{\gamma}}-\frac{(1-\delta)}{\delta}\frac{1-\alpha}{1-\hat{\gamma}}\frac{c''(a^*(L;\alpha))}{(1-c'(a^*(L;\alpha)))}\right)-1\right].\end{aligned}$$

In the last expression, if $\alpha\in(0,\alpha_0)$, both $a^*(L;\alpha)\leq a^*(H;\alpha)<a^{FB}=(c')^{-1}(1)$. Hence, $c'(a^*(L;\alpha))<1, c'(a^*(H;\alpha))<1$ by strict monotonicity of c'. Hence, $(1-\hat{\gamma})^2\,(1-c'(a^*(L;\alpha)))\,(1-c'(a^*(H;\alpha)))\neq 0$. The latter in turn implies

$$\det(A(\alpha))=0\leftrightarrow\left[\hat{\gamma}-\frac{(1-\delta)}{\delta}(1-\alpha)\frac{c''(a^*(H;\alpha))}{(1-c'(a^*(H;\alpha)))}\right]\left[\hat{\gamma}-\frac{(1-\delta)}{\delta}(1-\alpha)\frac{c''(a^*(L;\alpha))}{(1-c'(a^*(L;\alpha)))}\right]=(1-\hat{\gamma})^2.$$

Now, we simplify the notation and denote by $h(x):=\frac{(1-\delta)}{\delta}(1-\alpha)\frac{c''(x)}{1-c'(x)}-\hat{\gamma}$. Choose any $\alpha_1,\alpha_2\in(0,\alpha_0)$, with $\det A(\alpha_1)=\det A(\alpha_2)=0$. The latter together with the equivalence relationship in the last equation above imply that $h(a^*(H,\alpha_1))h(a^*(L,\alpha_1))=h(a^*(H,\alpha_2))h(a^*(L,\alpha_2))=(1-\gamma)^2$. Moreover, by Lemma 4, $\frac{(1-\delta)}{\delta}(1-\alpha)c''(a^*(\theta;\alpha))-\hat{\gamma}(1-c'(a^*(\theta;\alpha)))\geq 0$ if $\alpha\in(0,\alpha_0)$. Since $h(x)=[\frac{(1-\delta)}{\delta}(1-\alpha)c''(x)-\hat{\gamma}(1-c'(x))]\times\frac{1}{1-c'(x)}$ and $c'(a(\theta;\alpha))<1$ in the region $(0,\alpha_0)$, all four $h(a^*(H,\alpha_1)),h(a^*(L,\alpha_1))$, $h(a^*(H,\alpha_2))$, $h(a^*(L,\alpha_2))$ are strictly increasing.

$$\begin{aligned}&h(a^*(H,\alpha_1))h(a^*(L,\alpha_1))-h(a^*(H,\alpha_2))h(a^*(L,\alpha_2)) \\ =&[h(a^*(H,\alpha_1))-h(a^*(H,\alpha_2))]h(a^*(L,\alpha_1))+h(a^*(H,\alpha_2))[h(a^*(L,\alpha_1))-h(a^*(L,\alpha_2))].\end{aligned}$$

One can see here that $h(a^*(\theta;\alpha))$ is a strictly increasing function with respect to α in $(0,\alpha_0)$, because both $a^*(\theta,\cdot)$ $h(\cdot)$ are strictly increasing functions in the region $(0,\alpha_0)$. Since $h(a^*(L,\alpha_1)), h(a^*(H,\alpha_2)) > 0$, the expression above is strictly negative or positive if $\alpha_1 \neq \alpha_2$. However, as $h(a^*(H,\alpha_1))h(a^*(L,\alpha_1)) = h(a^*(H,\alpha_2))h(a^*(L,\alpha_2)) = (1-\gamma)^2$, for the case $\det A(\alpha_1) = \det A(\alpha_2) = 0$ to hold, it must be that $\alpha_1 = \alpha_2$. This therefore shows that there can be at most one point α with $\det(A(\alpha)) \neq 0$.

Step 4) If $\alpha \in \{\tilde{\alpha} \in (0,\alpha_0) : \det(A(\tilde{\alpha})) \neq 0\}$, then $\frac{\partial}{\partial\alpha}\frac{Y(H;\alpha)}{Y(L;\alpha)} \leq 0$.

Recall that the system of equations that characterize the $\frac{\partial}{\partial\alpha}a^*(H;\alpha), \frac{\partial}{\partial\alpha}a^*(L;\alpha)$ in I_1 is:

$$
-\begin{bmatrix} \frac{(1-\delta)}{\delta}c'(a^*(H;\alpha)) \\ \frac{(1-\delta)}{\delta}c'(a^*(L;\alpha)) \end{bmatrix}
= \overbrace{\begin{bmatrix} \hat{\gamma}(1-c'(a^*(H;\alpha))) - \frac{(1-\delta)}{\delta}(1-\alpha)c''(a^*(H;\alpha)) & (1-\hat{\gamma})(1-c'(a^*(L;\alpha))) \\ (1-\hat{\gamma})(1-c'(a^*(H;\alpha))) & \hat{\gamma}(1-c'(a^*(L;\alpha))) - \frac{(1-\delta)}{\delta}(1-\alpha)c''(a^*(L;\alpha)) \end{bmatrix}}^{A(\alpha):=}
\begin{bmatrix} \frac{\partial}{\partial\alpha}a^*(H;\alpha) \\ \frac{\partial}{\partial\alpha}a^*(L;\alpha). \end{bmatrix}
$$

If $\alpha \in \{\tilde{\alpha} \in I_1 : \det(A(\tilde{\alpha})) \neq 0\}$, we can post-multiply $\frac{1-\delta}{\delta}$ and the inverse matrix $A(\alpha)^{-1}$ on both sides:

$$
\begin{bmatrix} \frac{\partial}{\partial\alpha}a^*(H;\alpha) \\ \frac{\partial}{\partial\alpha}a^*(L;\alpha) \end{bmatrix} = \frac{(1-\delta)}{\delta}\frac{1}{\det(A(\alpha))}
\begin{bmatrix} \frac{(1-\delta)}{\delta}(1-\alpha)c''(a^*(L;\alpha)) - \hat{\gamma}(1-c'(a^*(L;\alpha))) & (1-\hat{\gamma})(1-c'(a^*(L;\alpha))) \\ (1-\hat{\gamma})(1-c'(a^*(H;\alpha))) & \frac{(1-\delta)}{\delta}(1-\alpha)c''(a^*(H;\alpha)) - \hat{\gamma}(1-c'(a^*(H;\alpha))) \end{bmatrix}
\begin{bmatrix} c'(a^*(H;\alpha)) \\ c'(a^*(L;\alpha)). \end{bmatrix}
$$

By the matrix multiplication on the right-hand side, notice that:

$$\frac{\partial}{\partial\alpha}a^*(H;\alpha) = \frac{1}{\det(A(\alpha))}\underbrace{\frac{(1-\delta)}{\delta}}_{>0}[\underbrace{\frac{(1-\delta)}{\delta}(1-\alpha)c''(a^*(L;\alpha)) - \hat{\gamma}(1-c'(a^*(L;\alpha)))c'(a^*(H;\alpha))}_{\geq 0 \quad \text{by Lemma 4}}$$
$$+\underbrace{(1-\hat{\gamma})(1-c'(a^*(L;\alpha)))}_{>0 \quad \text{by } \alpha\in I_1}\underbrace{c'(a^*(L;\alpha))}_{>0}]$$
$$\frac{\partial}{\partial\alpha}a^*(L;\alpha) = \frac{1}{\det(A(\alpha))}\underbrace{\frac{(1-\delta)}{\delta}}_{>0}[\underbrace{\frac{(1-\delta)}{\delta}(1-\alpha)c''(a^*(H;\alpha)) - \hat{\gamma}(1-c'(a^*(L;\alpha)))c'(a^*(L;\alpha))}_{\geq 0 \quad \text{by Lemma 4}}$$
$$+\underbrace{(1-\hat{\gamma})(1-c'(a^*(H;\alpha)))}_{>0 \quad \text{by } \alpha\in I_1}\underbrace{c'(a^*(H;\alpha))}_{>0}]. \tag{IFT2}$$

Using the part (I-1) of Lemma 3 now implies that the optimal efforts $a^*(H;\alpha)$ and $a^*(L;\alpha)$ are strictly increasing with respect to α in I_1. Hence, the latter implies that $\det(A(\alpha)) > 0$.

Now, $Y(\theta;\alpha) = a^*(\theta;\alpha) + \hat{\theta} > 0$. Thus, we first take the logarithm of the ratio $\frac{Y(H;\alpha)}{Y(L;\alpha)}$ and then differentiate it with respect to α. This yields

$$\frac{\partial}{\partial\alpha}\frac{Y(H;\alpha)}{Y(L;\alpha)} \leq 0 \quad \leftrightarrow \quad \frac{\frac{\partial}{\partial\alpha}Y(H;\alpha)}{Y(H;\alpha)} \leq \frac{\frac{\partial}{\partial\alpha}Y(L;\alpha)}{Y(L;\alpha)}$$
$$\leftrightarrow \quad \left(\frac{\frac{\partial}{\partial\alpha}a^*(H;\alpha)}{a^*(H;\alpha) + \mathbb{E}(\theta|\theta_{-1} = H)}\right) \leq \left(\frac{\frac{\partial}{\partial\alpha}a^*(L;\alpha)}{a^*(L;\alpha) + \mathbb{E}(\theta|\theta_{-1} = L)}\right).$$

We shall plug in the expressions from (IFT2) to show the last inequality above. Since $\frac{1}{\det(A)}\frac{(1-\delta)}{\delta}$ is a common term in (IFT2) for both terms $\frac{\partial}{\partial\alpha}a^*(H;\alpha)$ and $\frac{\partial}{\partial\alpha}a^*(L;\alpha)$ and it is strictly positive, the last inequality is equivalent to:

$$\frac{[(\frac{(1-\delta)}{\delta}(1-\alpha)c''(a^*(L;\alpha)) - \hat{\gamma}(1-c'(a^*(L;\alpha))))c'(a^*(H;\alpha)) + (1-\hat{\gamma})(1-c'(a^*(L;\alpha)))c'(a^*(L;\alpha))]}{a^*(H;\alpha)+\hat{H}}$$
$$\leq \frac{[(\frac{(1-\delta)}{\delta}(1-\alpha)c''(a^*(H;\alpha)) - \hat{\gamma}(1-c'(a^*(H;\alpha))))c'(a^*(L;\alpha)) + (1-\hat{\gamma})(1-c'(a^*(H;\alpha)))c'(a^*(H;\alpha))]}{a^*(L;\alpha)+\hat{L}}. \tag{V1}$$

Similarly, we obtain a weak inequality by collecting the terms with $1 - c'(a^*(L;\alpha))$ inside the brackets

[ ] on the left-hand side and $1 - c'(a^*(H;\alpha))$ inside the brackets on the right-hand side:

$$\frac{[(\frac{(1-\delta)}{\delta}(1-\alpha)c''(a^*(L;\alpha))c'(a^*(H;\alpha)) - (1-c'(a^*(L;\alpha)))(\hat{\gamma}c'(a^*(H;\alpha)) - (1-\hat{\gamma})c'(a^*(L;\alpha)))]}{a^*(H;\alpha)+\hat{H}}$$
$$\leq \frac{[(\frac{(1-\delta)}{\delta}(1-\alpha)c''(a^*(H;\alpha))c'(a^*(L;\alpha)) - (1-c'(a^*(H;\alpha)))(\hat{\gamma}c'(a^*(L;\alpha)) - (1-\hat{\gamma})c'(a^*(H;\alpha)))]}{a^*(L;\alpha)+\hat{L}}. \tag{V2}$$

Now, we will further divide into two possible cases: (A) $\frac{1-c'(a^*(L;\alpha))}{a^*(H;\alpha)+H} < \frac{1-c'(a^*(H;\alpha))}{a^*(L;\alpha)+L}$, and (B) $\frac{1-c'(a^*(L;\alpha))}{a^*(H;\alpha)+H} \geq \frac{1-c'(a^*(H;\alpha))}{a^*(L;\alpha)+L}$. The two inequalities in (V1) and (V2) hold if and only if $\frac{\partial}{\partial\alpha}\frac{Y(H;\alpha)}{Y(L;\alpha)} \leq 0$.

Case A) $\frac{1-c'(a^*(L;\alpha))}{a^*(H;\alpha)+\hat{H}} < \frac{1-c'(a^*(H;\alpha))}{a^*(L;\alpha)+\hat{L}}$.

First, note that:

$$0 \leq \left(\frac{(1-\delta)}{\delta}(1-\alpha)c''(a^*(L;\alpha)) - \hat{\gamma}(1-c'(a^*(L;\alpha)))\right) < \left(\frac{(1-\delta)}{\delta}(1-\alpha)c''(a^*(H;\alpha)) - \hat{\gamma}(1-c'(a^*(H;\alpha)))\right).$$

whereas the first inequality follows from the first part of Lemma 4, and the second step from the convexity of c', c, and monotonicity in the effort. This implies that the left hand side of (V1) is:

$$\frac{[\left(\frac{(1-\delta)}{\delta}(1-\alpha)c''(a^*(L;\alpha)) - \hat{\gamma}(1-c'(a^*(L;\alpha)))\right)c'(a^*(H;\alpha)) + (1-\hat{\gamma})(1-c'(a^*(L;\alpha)))c'(a^*(L;\alpha))]}{a^*(H;\alpha)+\hat{H}}$$
$$= \left[\frac{(1-\delta)}{\delta}(1-\alpha)c''(a^*(L;\alpha)) - \hat{\gamma}(1-c'(a^*(L;\alpha)))\right]\frac{c'(a^*(H;\alpha))}{a^*(H;\alpha)+\hat{H}} + (1-\hat{\gamma})\frac{(1-c'(a^*(L;\alpha)))}{a^*(H;\alpha)+\hat{H}}c'(a^*(L;\alpha))$$
$$< \underbrace{[\left(\frac{(1-\delta)}{\delta}(1-\alpha)c''(a^*(H;\alpha)) - \hat{\gamma}(1-c'(a^*(H;\alpha)))\right)]}_{>\frac{(1-\delta)}{\delta}(1-\alpha)c''(a^*(L;\alpha))-\hat{\gamma}(1-c'(a^*(L;\alpha)))}\underbrace{\frac{c'(a^*(H;\alpha))}{a^*(H;\alpha)+\hat{H}}}_{>0} + (1-\hat{\gamma})\underbrace{\frac{1-c'(a^*(H;\alpha))}{a^*(L;\alpha)+\hat{L}}}_{>\frac{(1-c'(a^*(L;\alpha)))}{a^*(H;\alpha)+\hat{H}};\text{ Case A}}\underbrace{c'(a^*(L;\alpha))}_{>0}$$
$$\leq \underbrace{[\left(\frac{(1-\delta)}{\delta}(1-\alpha)c''(a^*(H;\alpha)) - \hat{\gamma}(1-c'(a^*(H;\alpha)))\right)]}_{>0}\underbrace{\frac{c'(a^*(L;\alpha))}{a^*(L;\alpha)+\hat{L}}}_{\text{by Lemma 5}} + (1-\hat{\gamma})\underbrace{\frac{1-c'(a^*(H;\alpha))}{a^*(L;\alpha)+\hat{L}}}_{>0}\underbrace{c'(a^*(H;\alpha))}_{>c'(a^*(L;\alpha))}.$$

The last expression coincides with the right-hand side of inequality (V1), as required.

Case B) $\frac{1-c'(a^*(L;\alpha))}{a^*(H;\alpha)+H} \geq \frac{1-c'(a^*(H;\alpha))}{a^*(L;\alpha)+L}$.

To see this, we start with showing that

$$\hat{\gamma}c'(a^*(H;\alpha)) - (1-\gamma)c'(a^*(L;\alpha)) \underbrace{>}_{c'(a^*(H;\alpha))>c'(a^*(L;\alpha))} \hat{\gamma}c'(a^*(L;\alpha)) - (1-\hat{\gamma})c'(a^*(H;\alpha)) > 0.$$

The first inequality follows from the part (I-1) of Lemma 3. The second inequality is equivalent to $\frac{\hat{\gamma}}{(1-\hat{\gamma})} > \frac{c'(a^*(H;\alpha))}{c'(a^*(L;\alpha))}$. We now use (DE2) to rewrite $\frac{c'(a^*(H;\alpha))}{c'(a^*(L;\alpha))}$:

$$\frac{c'(a^*(H;\alpha))}{c'(a^*(L;\alpha)))} \underbrace{=}_{\text{DE2}} \frac{\hat{\gamma}\Pi^*(H;\alpha) + (1-\hat{\gamma})\Pi^*(L;\alpha)}{\hat{\gamma}\Pi^*(L;\alpha) + (1-\hat{\gamma})\Pi^*(H;\alpha)} = \frac{\Pi^*(L;\alpha) + \hat{\gamma}(\Pi^*(H;\alpha) - \Pi^*(L;\alpha))}{\Pi^*(L;\alpha) + (1-\hat{\gamma})(\Pi^*(H;\alpha) - \Pi^*(L;\alpha))}$$
$$= \frac{1 + \hat{\gamma}\left(\frac{\Pi^*(H;\alpha)}{\Pi^*(L;\alpha)} - 1\right)}{1 + (1-\hat{\gamma})\left(\frac{\Pi^*(H;\alpha)}{\Pi^*(L;\alpha)} - 1\right)}.$$

Note that $\frac{\hat{\gamma}}{1-\hat{\gamma}} > 1$ as $\hat{\gamma} > \frac{1}{2}$, and $\Pi^*(H;\alpha) > \Pi^*(L;\alpha)$. This in turn implies that $\frac{1+\hat{\gamma}\left(\frac{\Pi^*(H;\alpha)}{\Pi^*(L;\alpha)}-1\right)}{1+(1-\hat{\gamma})\left(\frac{\Pi^*(H;\alpha)}{\Pi^*(L;\alpha)}-1\right)}$ is between 1 and $\frac{\hat{\gamma}}{1-\hat{\gamma}}$, so that $\frac{c'(a^*(H;\alpha))}{c'(a^*(L;\alpha))} < \frac{\hat{\gamma}}{1-\hat{\gamma}}$.

Now, the left-hand side of V2 is:

$$\frac{\frac{(1-\delta)}{\delta}(1-\alpha)c''(a^*(L;\alpha))c'(a^*(H;\alpha)) - (1-c'(a^*(L;\alpha)))(\hat{\gamma}c'(a^*(H;\alpha)) - (1-\hat{\gamma})c'(a^*(L;\alpha)))}{a^*(H;\alpha)+\hat{H}}$$
$$= \frac{(1-\delta)}{\delta}(1-\alpha)c''(a^*(L;\alpha))\frac{c'(a^*(H;\alpha))}{a^*(H;\alpha)+\hat{H}} - \frac{(1-c'(a^*(L;\alpha))}{a^*(H;\alpha)+\hat{H}}(\hat{\gamma}c'(a^*(H;\alpha)) - (1-\hat{\gamma})c'(a^*(L;\alpha)))$$
$$\leq \underbrace{\frac{(1-\delta)}{\delta}(1-\alpha)c''(a^*(L;\alpha))}_{>0} \underbrace{\frac{c'(a^*(L;\alpha))}{a^*(L;\alpha)+\hat{L}}}_{\text{by Lemma 5}} - \underbrace{\frac{(1-c'(a^*(H;\alpha))}{a^*(L;\alpha)+\hat{L}}}_{\leq\frac{1-c'(a^*(L;\alpha))}{a^*(H;\alpha)+\hat{H}};\text{ Case B}} \underbrace{(\hat{\gamma}c'(a^*(H;\alpha)) - (1-\hat{\gamma})c'(a^*(L;\alpha)))}_{>0}$$
$$< \underbrace{\frac{(1-\delta)}{\delta}(1-\alpha)c''(a^*(H;\alpha))}_{\geq c''(a^*(L;\alpha))\text{ from convexity of }c'} \underbrace{\frac{c'(a^*(L;\alpha))}{a^*(L;\alpha)+\hat{L}}}_{>0} - \underbrace{\frac{(1-c'(a^*(H;\alpha))}{a^*(L;\alpha)+\hat{L}}}_{>0} \underbrace{(\hat{\gamma}c'(a^*(L;\alpha)) - (1-\hat{\gamma})c'(a^*(H;\alpha)))}_{<(\hat{\gamma}c'(a^*(H;\alpha))-(1-\hat{\gamma})c'(a^*(L;\alpha)))}.$$

We have therefore showed that the equation (V2) holds under Case B. Hence, $\frac{Y(H;\alpha)}{Y(L;\alpha)}$ is locally decreasing in α when $\alpha \in \{\tilde{\alpha} \in I_1 : \det(A(\tilde{\alpha})) \neq 0\}$.

Step 5: The ratio $\frac{Y(H;\alpha)}{Y(L;\alpha)}$ is decreasing everywhere on $[0,1]$.

$\frac{Y(H;\alpha)}{Y(L;\alpha)} = \frac{a^*(H;\alpha)+\hat{H}}{a^*(L;\alpha)+\hat{L}}$ is continuous in $[0,1]$, because $a^*(\theta;\alpha) \geq 0$ is continuous by the part (I-1) of Lemma 3. Recall that $\alpha_0 = \inf\{\alpha : a^*(H;\alpha) = a^{FB}\}$ and $\hat{\alpha}_0 \in (0,\alpha_0)$ in which $\det A(\alpha) = 0\}$. If $\hat{\alpha}_0$ does not exist, Step 2 and Step 4 in the current proof imply that $\frac{Y(H;\alpha)}{Y(L;\alpha)}$ is piece-wise decreasing in each region $(0,\hat{\alpha}_0), (\hat{\alpha}_0,\alpha_0), (\alpha_0,1]$. If $\hat{\alpha}_0$ exists, Step 2 and Step 4 imply that $\frac{Y(H;\alpha)}{Y(L;\alpha)}$ is piece-wise decreasing in each region $(0, \alpha_0), (\alpha_0, 1]$. Since $\frac{Y(H;\alpha)}{Y(L;\alpha)}$ is everywhere continuous, the piece-wise monotonicity implies the ratio $\frac{Y(H;\alpha)}{Y(L;\alpha)}$ is a decreasing function everywhere. This completes the proof of the parts (a) and (b) of Theorem 1. □

Proof of the part (c) of Theorem 1 (Inequality Effect)

As the ex-ante promised utility to the agent is zero and the agent's continuation utility in each state is zero, the agent's expected per-period utility is also zero at each period t and at each state θ_t. Therefore, in any Markovian relational contract $\{b^*(\theta_{-1}, \theta; \alpha), w^*(\theta_{-1}; \alpha), a^*(\theta_{-1}; \alpha)\}$, the expected per-period compensation is equal to the agent's cost of efforts:

$$\underbrace{w^*(\theta_{-1};\alpha) + \mathbb{E}(b^*(\theta_{-1},\theta;\alpha)|\theta_{-1}) - c(a^*(\theta_{-1};\alpha))}_{\text{Agent's per-period utility given } \theta_{-1}} = 0 \Longleftrightarrow c(a^*(\theta_{-1};\alpha)) = w^*(\theta_{-1};\alpha) + \mathbb{E}(b^*(\theta_{-1},\theta;\alpha)|\theta_{-1}) \tag{9}$$

The rest of the proof of the part (c) of Theorem 1 follows from using the arguments analogous to those used above in the proof of the parts (a) and (b) of the theorem. In particular, by replacing the expected output measure $Y(\theta_{-1}; \alpha)$ in the proofs of these earlier parts with the expected payments to the agent $w^*(\theta_{-1}; \alpha) + \mathbb{E}(b^*(\theta_{-1}, \theta; \alpha)|\theta_{-1})\Big(= c(a^*(\theta_{-1}; \alpha))$ given by the equation (9)$\Big)$, we can establish the statements analogous to Step 1, Step 2, Step 3, and Step 5 given in the previous section. Therefore, the proof is complete if we establish the remaining step, which plays an analogous role to Step 4 above.

Step 4)' If $\alpha \in \{\tilde{\alpha} \in (0, \alpha_0) : \det(A(\tilde{\alpha})) \neq 0\}$, then it is the case that $\frac{\partial}{\partial\alpha}\left(\frac{w^*(H)+\mathbb{E}(b^*(H,\theta)|\theta_{-1}=H)}{w^*(L)+\mathbb{E}(b^*(L,\theta)|\theta_{-1}=L)}\right) \leq 0$.

To see this we start with noticing that by the equation (9), the inequality measure is equal to $\frac{w^*(H)+\mathbb{E}(b^*(H,\theta)|\theta_{-1}=H)}{w^*(L)+\mathbb{E}(b^*(L,\theta)|\theta_{-1}=L)} = \frac{c(a^*(H;\alpha))}{c(a^*(L;\alpha))}$. Differentiating the ratio $\frac{c(a^*(H;\alpha))}{c(a^*(L;\alpha))}$ w.r.t. α yields:

$$\frac{\partial}{\partial\alpha}\Big(\frac{c(a^*(H;\alpha))}{c(a^*(L;\alpha))}\Big) \leq 0 \quad \leftrightarrow \quad \left(\frac{\frac{\partial}{\partial\alpha}a^*(H;\alpha)c'(a^*(H;\alpha))}{c(a^*(H;\alpha))}\right) \leq \left(\frac{\frac{\partial}{\partial\alpha}a^*(L;\alpha)c'(a^*(L;\alpha))}{c(a^*(L;\alpha))}\right).$$

Using (IFT2) in the above expression we show that it holds with inequality. In particular, using $c(x) = \frac{1}{2}cx^2$ and noticing that the expression $\frac{1}{\det(A)}\frac{(1-\delta)}{\delta}$ is a common term in both $\frac{\partial}{\partial\alpha}a^*(H;\alpha), \frac{\partial}{\partial\alpha}a^*(L;\alpha)$ and in (IFT2), which is strictly positive, the last inequality yields:

$$\begin{aligned}&\frac{\Big[(\frac{(1-\delta)}{\delta}(1-\alpha)c - \hat{\gamma}(1-ca^*(L;\alpha)))ca^*(H;\alpha) + (1-\hat{\gamma})(1-ca^*(L;\alpha))ca^*(L;\alpha)\Big]}{\frac{1}{2}ca^*(H;\alpha)^2} \times ca^*(H;\alpha) \\ \leq &\frac{\Big[(\frac{(1-\delta)}{\delta}(1-\alpha)c - \hat{\gamma}(1-ca^*(H;\alpha)))ca^*(L;\alpha) + (1-\hat{\gamma})(1-ca^*(H;\alpha))ca^*(H;\alpha)\Big]}{\frac{1}{2}ca^*(L;\alpha)^2} \times ca^*(L;\alpha).\end{aligned} \tag{V1'}$$

Dividing the both sides of the last expression by 2, the inequality above is equivalent to:

$$\left(\frac{(1-\delta)}{\delta}(1-\alpha)c - \hat{\gamma}(1 - ca^*(L;\alpha))\right)c + (1-\hat{\gamma})\frac{(1-ca^*(L;\alpha))ca^*(L;\alpha)}{a^*(H;\alpha)}$$
$$\leq \left(\frac{(1-\delta)}{\delta}(1-\alpha)c - \hat{\gamma}(1 - ca^*(H;\alpha))\right)c + (1-\hat{\gamma})\frac{(1-ca^*(H;\alpha))ca^*(H;\alpha)}{a^*(L;\alpha)}.$$

Adding to the both sides of the last expression $\left(\frac{(1-\delta)}{\delta}(1-\alpha)c - \hat{\gamma}\right)c$ and dividing by $c > 0$ yield:

$$\hat{\gamma}ca^*(L;\alpha) + (1-\hat{\gamma})\frac{(1-ca^*(L;\alpha))a^*(L;\alpha)}{a^*(H;\alpha)} \leq \hat{\gamma}ca^*(H;\alpha) + (1-\hat{\gamma})\frac{(1-ca^*(H;\alpha))a^*(H;\alpha)}{a^*(L;\alpha)}.$$

Subtracting $ca^*(L;\alpha) + (1-\hat{\gamma})\frac{(1-ca^*(H;\alpha))a^*(H;\alpha)}{a^*(L;\alpha)}$ from the both sides of the inequality above, it takes the following form:

$$(1-\hat{\gamma})\Big(\frac{(1-ca^*(L;\alpha))a^*(L;\alpha)}{a^*(H;\alpha)} - \frac{(1-ca^*(H;\alpha))a^*(H;\alpha)}{a^*(L;\alpha)}\Big) \leq \hat{\gamma}c\Big(a^*(H;\alpha) - a^*(L;\alpha)\Big).$$

Rearranging the left-hand side of this inequality yields:

$$\begin{aligned}
&(1-\hat{\gamma})\Big(\underbrace{(1-ca^*(L;\alpha))}_{\geq 0}\underbrace{\frac{a^*(L;\alpha)}{a^*(H;\alpha)}}_{\leq 1} - \underbrace{(1-ca^*(H;\alpha))}_{\geq 0}\underbrace{\frac{a^*(H;\alpha)}{a^*(L;\alpha)}}_{\geq 1}\Big)\\
&\leq (1-\hat{\gamma})\Big((1-ca^*(L;\alpha)) \times 1 - (1-ca^*(H;\alpha)) \times 1\Big)\\
&\quad = (1-\hat{\gamma})\underbrace{(ca^*(H;\alpha) - ca^*(L;\alpha))}_{\geq 0 \text{ by (E-1)}}\underbrace{\leq}_{\hat{\gamma}>\frac{1}{2}}\hat{\gamma}(ca^*(H;\alpha) - ca^*(L;\alpha)),
\end{aligned}$$

which establishes the required inequality and completes the proof of the part (c) of Theorem 1. □

Proofs of the Auxiliary Results

Proof of the part (I-1) of Lemma 3. To show that the continuity property holds, we start with observing that in the optimization programme (1) the constraint set is linear in α and hence as a correspondence is continuous in α. By the Theorem of Maximum the unique solution for optimal effort α^* is a continuous function of α. We next turn to monotonicity.

Suppose to the contrary, so that there exists a triple $(\theta_0, \alpha_1, \alpha_2)$ with $\alpha_1 < \alpha_2$ such that the optimal effort $a^*(\theta', \alpha_2) < a^*(\theta', \alpha_1)$. Fix α_1, α_2 and let $\hat{\Theta} = \{\theta' \in \Theta : \alpha_1 < \alpha_2,\ a^*(\theta', \alpha_2) < a^*(\theta', \alpha_1)\}$. Now, define a new bonus system under α_2 as following: for all $\theta_{-1} \in \hat{\Theta}$, $\hat{b}(\theta_{-1}, \theta, \alpha_2) := b^*(\theta_{-1}, \theta, \alpha_1)$; for all other $\theta_{-1} \notin \hat{\Theta}$, let $\hat{b}(\theta_{-1}, \theta, \alpha_2) := b^*(\theta_{-1}, \theta, \alpha_2)$.

If $\theta' \in \hat{\Theta}$ so that $a^*(\theta', \alpha_2) < a^*(\theta', \alpha_1)$, the effort $\hat{a}^*(\theta', \alpha_2)$ satisfies:

$$\begin{aligned}\hat{a}^*(\theta', \alpha_2) \underbrace{=}_{IC} &(c')^{-1}(\mathbb{E}(\hat{b}(\theta_{-1}, \theta, \alpha_2)|\theta_{-1} = \theta')) = (c')^{-1}(\mathbb{E}(b^*(\theta_{-1}, \theta, \alpha_1)|\theta_{-1} = \theta')) \\ \underbrace{=}_{IC} &(c')^{-1}(c'(a^*(\theta', \alpha_1))) = a^*(\theta', \alpha_1) = \max_{\alpha_1, \alpha_2} a^*(\theta'; \alpha).\end{aligned}$$

Similarly, if $\theta' \notin \hat{\Theta}$ so that $a^*(\theta', \alpha_2) \geq a^*(\theta', \alpha_1)$, the effort $\hat{a}^*(\theta', \alpha_2)$ satisfies:

$$\begin{aligned}\hat{a}^*(\theta', \alpha_2) \underbrace{=}_{IC} &(c')^{-1}(\mathbb{E}(\hat{b}^*(\theta_{-1}, \theta, \alpha_2)|\theta_{-1} = \theta')) = (c')^{-1}(\mathbb{E}(b(\theta_{-1}, \theta, \alpha_2)|\theta_{-1} = \theta')) \\ \underbrace{=}_{IC} &(c')^{-1}(c'(a^*(\theta', \alpha_2))) = a^*(\theta', \alpha_2) = \max_{\alpha_1, \alpha_2} a^*(\theta'; \alpha).\end{aligned}$$

In other words, the bonus scheme $\{\hat{b}(\cdot, \cdot, \alpha_2)\}$ induces the effort $\max_{\alpha_1, \alpha_2} a^*(\theta; \alpha)$ for each θ. Furthermore, we observe that for all $\theta' \in \hat{\Theta}$, $a^*(\theta', \alpha_1) < a^*(\theta', \alpha_2) \leq a^{FB}$ by monotonicity given in the part (E-1) of Lemma 1. Therefore, $a^*(\theta', \alpha_1) - c(a^*(\theta', \alpha_1)) < \hat{a}^*(\theta', \alpha_2) - c(\hat{a}^*(\theta', \alpha_2))$.

Now, by the system of equations (7) in the proof of Lemma 1, for any stationary contract,

$$\begin{bmatrix}\Pi^*(H; \alpha) \\ \Pi^*(L; \alpha)\end{bmatrix} = \begin{bmatrix}\gamma & 1-\gamma \\ 1-\gamma & \gamma\end{bmatrix}\begin{bmatrix}a^*(H; \alpha) - c(a^*(H; \alpha)) + \hat{H} \\ a^*(L; \alpha) - c(a^*(L; \alpha)) + \hat{L}.\end{bmatrix}$$

Similarly, we now use an analogous representation of the continuation values under the bonus system $\{\hat{b}(\cdot, \cdot, \alpha_2)\}$ for $\hat{\Pi}(\cdot, \alpha_2)$. Since the bonus scheme $\hat{b}$ induces higher efforts $\max_{\alpha_1, \alpha_2} a^*(\theta; \alpha)$, the system of equations above shows that $\hat{\Pi}(\theta, \alpha_2) \geq \max\{\Pi^*(\theta, \alpha_2), \Pi^*(\theta, \alpha_1)\}$. Now, for all $\theta_{-1} \in \hat{\Theta}$,

$$\begin{aligned}&(1-\delta)(1-\alpha_2)\hat{b}(\theta_{-1}, \theta, \alpha_2) = (1-\delta)(1-\alpha_2)b^*(\theta_{-1}, \theta, \alpha_1) \\ <&(1-\delta)(1-\alpha_1)b^*(\theta_{-1}, \theta, \alpha_1) \underbrace{\leq}_{\text{DE-P for } (\theta_{-1}, \theta, \alpha_1)} \delta\Pi^*(\theta_{-1}, \alpha_1) \leq \delta\hat{\Pi}(\theta_{-1}, \alpha_2).\end{aligned}$$

Now, for all $\theta_{-1} \notin \hat{\Theta}$,

$$\begin{aligned}&(1-\delta)(1-\alpha_2)\hat{b}(\theta_{-1}, \theta, \alpha_2) = (1-\delta)(1-\alpha_2)b^*(\theta_{-1}, \theta, \alpha_2) \\ \underbrace{\leq}_{\text{DE-P for } (\theta_{-1}, \theta, \alpha_2)} &\delta\Pi^*(\theta_{-1}, \alpha_2) \leq \delta\hat{\Pi}(\theta_{-1}, \alpha_2).\end{aligned}$$

Since $\hat{\Theta} \neq \emptyset$ and $a^*(\theta', \alpha_2) - c(a^*(\theta', \alpha_2)) < \hat{a}^*(\theta', \alpha_2) - c(\hat{a}^*(\theta', \alpha_2))$, $\Pi^*(\theta, \alpha_2) < \hat{\Pi}(\theta, \alpha_2)$. This contradicts the optimality of the bonus scheme $\{b(\cdot, \cdot, \alpha_2)\}$, because we obtained strictly higher continuation values without violating the dynamic enforceability conditions. This contradiction establishes the part (I-1) of Lemma 3.

□

Proof of the part (I-2)of Lemma 3. We start with showing continuity of the value functions in α. By the argument analogous to that made in the proof of the part (I-1) of Lemma 3, in the optimization the value of the objective function at the optimum is a continuous function of α. We next turn to monotonicity.

Note that $x - c(x)$ is a strictly increasing function on $[0, a^{FB}]$. By monotonicity shown in the part (E-1) of Lemma 1, the optimal efforts satisfy $a^*(L;\alpha) \leq a^*(H;\alpha) \leq a^{FB}$. Hence $a^*(H;\alpha) - c(a^*(H;\alpha))$ and $a^*(L;\alpha) - c(a^*(L;\alpha))$ are both increasing in $\alpha \in [0,1]$ as well. By the equation (7) in the proof of Lemma 1, for any stationary contract

$$\begin{bmatrix} \Pi(H;\alpha) \\ \Pi(L;\alpha) \end{bmatrix} = \begin{bmatrix} \gamma & 1-\gamma \\ 1-\gamma & \gamma \end{bmatrix} \begin{bmatrix} a^*(H;\alpha) - c(a^*(H;\alpha)) + \hat{H} \\ a^*(L;\alpha) - c(a^*(L;\alpha)) + \hat{L} \end{bmatrix}.$$

Hence, $\Pi^*(H;\alpha)$ and $\Pi^*(L;\alpha)$ are both strictly increasing in α. □

Proof of Lemma 4. Suppose to the contrary. We will first assume so that $\frac{(1-\delta)}{\delta}(1-\alpha)c''(a^*(H;\alpha)) - \hat{\gamma}(1 - c'(a^*(H;\alpha))) < 0$ for some $\alpha \in I_1$. Our goal is to show that we can increase the continuation values without violating dynamic enforceability conditions by increasing the bonuses by small amounts, which contradicts the optimality of the bonus scheme and the recommended effort. Denote the bonus system $\{b^*(\cdot, \cdot; \alpha)\}$, the current recommended efforts $\{a^*(\cdot; \alpha)\}$ and continuation values $\{\Pi^*(\cdot; \alpha)\}$ under which $\frac{(1-\delta)}{\delta}(1-\alpha)c''(a^*(H;\alpha)) - \hat{\gamma}(1 - c'(a^*(H;\alpha))) < 0$ is true.

For any $\epsilon \in \left(0, \frac{1-c'(a^*(H;\alpha))}{2}\right)$, we now define a new bonus scheme $\{\hat{b}(\cdot, \cdot, \alpha)\}$, and new recommended effort policies $\{\hat{a}^*(\cdot;\alpha)\}$ as:

$$\hat{b}(H,H;\alpha) = b^*(H,H;\alpha) + \frac{\gamma}{\hat{\gamma}}\epsilon, \quad \hat{b}(H,L;\alpha) = b^*(H,L;\alpha) + \frac{1-\gamma}{\hat{\gamma}}\epsilon, \quad \hat{b}(L,L;\alpha) = b^*(L,L;\alpha), \quad \hat{b}(L,H;\alpha) = b^*(L$$

$$\hat{a}^*(H;\alpha) = (c')^{-1}(\lambda\hat{b}(H,H;\alpha) + (1-\lambda)\hat{b}(H,L;\alpha)) \underbrace{=}_{\hat{\gamma}=\gamma\lambda+(1-\lambda)(1-\gamma)} (c')^{-1}(\underbrace{\lambda b(H,H;\alpha) + (1-\lambda)b(H,L;\alpha)}_{=c'(a^*(H;\alpha))} + \epsilon)$$

$$= (c')^{-1}(c'(a^*(H;\alpha)) + \epsilon)$$

$$\hat{a}^*(L;\alpha) = (c')^{-1}(\underbrace{\lambda\hat{b}(L,L;\alpha) + (1-\lambda)\hat{b}(L,H;\alpha)}_{=c'(a^*(L;\alpha))}) = a^*(L;\alpha).$$

Given the range $\epsilon \in \left(0, \frac{1-c'(a^*(H;\alpha))}{2}\right)$, $c'(\hat{a}^*(H;\alpha)) = c'(a^*(H;\alpha)) + \epsilon < 1$. Since $a^*(H;\alpha) < \hat{a}^*(H;\alpha) < a^{FB}$, $a^*(H;\alpha) - c(a^*(H;\alpha)) < \hat{a}^*(H;\alpha) - c(\hat{a}^*(H;\alpha))$. Using the equation (7) in the proof of Lemma 1, new continuation values $\{\hat{\Pi}(\cdot;\alpha)\}$ are also pinned down. Also, it must be that $\hat{\Pi}(H;\alpha) > \Pi(H;\alpha)$ and $\hat{\Pi}(L;\alpha) > \Pi(L;\alpha)$. Our next task is show that the new bonus system $\{\hat{b}(\cdot,\cdot,\alpha)\}$ satisfies the dynamic enforceability condition.

To simplify the notation we denote the increment in the new recommended level by $\hat{a}^*(H;\alpha)$ relative to the effort level $a^*(H;\alpha)$ by $\eta(\epsilon) := \hat{a}^*(H;\alpha) - a^*(H;\alpha)$. Next, we observe that:

$$
\begin{aligned}
&\frac{\delta}{1-\delta}\hat{\Pi}(H;\alpha) - (1-\alpha)\hat{b}(H,H;\alpha) \\
\underbrace{=}_{\text{by (7)}} &\frac{\delta}{1-\delta}\Big[\gamma(a^*(H;\alpha) - c(a^*(H;\alpha)+\eta(\epsilon)) + \hat{H}) + (1-\gamma)(a^*(L;\alpha) - c(a^*(L;\alpha)) + \hat{L})\Big] - (1-\alpha)(b^*(H,H;\alpha) \\
&+ \frac{\delta}{1-\delta}\gamma\eta(\epsilon) - (1-\alpha)\frac{\gamma}{\hat{\gamma}}\epsilon \\
= &\underbrace{\frac{\delta}{1-\delta}\Big[\gamma(a^*(H;\alpha) - c(a^*(H;\alpha)) + \hat{H}) + (1-\gamma)(a^*(L;\alpha) - c(a^*(L;\alpha)) + \hat{L})\Big] - (1-\alpha)(b^*(H,H;\alpha)}_{=0 \ \because \ \Pi^*(H;\alpha) = \frac{(1-\delta)}{\delta}b^*(H,H;\alpha) \text{ from } a^*(H;\alpha) < a^{FB}} \\
&+ \frac{\delta}{1-\delta}\gamma\left(\eta(\epsilon) - [c(a^*(H;\alpha)+\eta(\epsilon)) - c(a^*(H;\alpha))]\right) - (1-\alpha)\frac{\gamma}{\hat{\gamma}}\epsilon \\
= &\frac{\delta}{1-\delta}\gamma\left(\eta(\epsilon) - [c(a^*(H;\alpha)+\eta(\epsilon)) - c(a^*(H;\alpha))]\right) - (1-\alpha)\frac{\gamma}{\hat{\gamma}}\epsilon =: f(\epsilon).
\end{aligned}
$$

Clearly, $f(0) = 0$. Since $\frac{d\eta(\epsilon)}{d\epsilon}|_{\epsilon=0} = \frac{1}{c''(a^*(H;\alpha))}$,

$$
f'(0) = \gamma\frac{\delta}{1-\delta}\frac{1-c'(a^*(H;\alpha))}{c''(a^*(H;\alpha))} - \frac{\gamma}{\hat{\gamma}}(1-\alpha).
$$

By the current assumption that $\frac{(1-\delta)}{\delta}(1-\alpha)c''(a^*(\theta_{-1})) - \hat{\gamma}(1-c'(a^*(\theta_{-1}))) < 0$, $f'(0) > 0$. This in turn implies that the $f(\epsilon) > 0$ for $\epsilon > 0$ small. This implies that $\frac{\delta}{1-\delta}\hat{\Pi}(H;\alpha) - (1-\alpha)\hat{b}(H,H;\alpha) > 0$ if our choice of $\epsilon > 0$ was small enough. Similarly, an analogous argument yields that $\frac{\delta}{1-\delta}\hat{\Pi}(L;\alpha) - (1-\alpha)\hat{b}(H,L;\alpha) > 0$ if ϵ is small enough. Hence, $\{\hat{b}(\cdot,\cdot,\alpha), \hat{a}^*(\cdot;\alpha), \hat{\Pi}(\cdot;\alpha)\}$ satisfies the dynamic enforceability conditions for small enough $\epsilon > 0$, but attains higher continuation values by the equation (7) in the proof of Lemma 1. However, this contradicts the optimality of the initial contract $\{b^*(\cdot,\cdot;\alpha), a^*(\cdot;\alpha), \Pi^*(\cdot;\alpha)\}$. This contradiction implies that $\frac{(1-\delta)}{\delta}(1-\alpha)c''(a^*(H;\alpha)) - \hat{\gamma}(1 - c'(a^*(H;\alpha))) \geq 0$.

We can derive a similar contradiction if we assume $\frac{(1-\delta)}{\delta}(1-\alpha)c''(a^*(L;\alpha)) - \hat{\gamma}(1-c'(a^*(L;\alpha))) < 0$ by increasing the $b(L,\cdot;\alpha)$ incrementally instead of $b(H,\cdot;\alpha)$. Hence, it must be that $\frac{(1-\delta)}{\delta}(1-$

$\alpha)c''(a^*(\theta;\alpha)) - \hat{\gamma}(1 - c'(a^*(\theta;\alpha))) \geq 0$ for both $\theta = H, L$. □

Proof of Lemma 5. Rearranging the expression in the statement, it suffices to show that

$$\frac{c'(a^*(H;\alpha))}{c'(a^*(L;\alpha))} \leq \frac{a^*(H;\alpha) + H}{a^*(L;\alpha) + L}.$$

By the assumption that the cost function $c(\cdot)$ is log-concave, and hence the derivative $\frac{d}{dx}log[c(x)] = \frac{c'(x)}{c(x)}$ is a decreasing function. Since $a^*(H;\alpha) \geq a^*(L;\alpha)$, the part (E-1) of Lemma 1 implies:

$$\frac{c'(a^*(H;\alpha))}{c(a^*(H;\alpha))} \leq \frac{c'(a^*(L;\alpha))}{c(a^*(L;\alpha))} \leftrightarrow \frac{c'(a^*(H;\alpha))}{c'(a^*(L;\alpha))} \leq \frac{c(a^*(H;\alpha))}{c(a^*(L;\alpha))}.$$

Moreover, using the equation (7) in the proof of Lemma 1 yields

$$\begin{aligned}\frac{c'(a^*(H;\alpha))}{c'(a^*(L;\alpha))} &= \frac{\hat{\gamma}(a^*(H;\alpha) - c(a^*(H;\alpha)) + \hat{H}) + (1-\hat{\gamma})(a^*(L;\alpha) - c(a^*(L;\alpha)) + \hat{L})}{\hat{\gamma}(a^*(L;\alpha) - c(a^*(L;\alpha)) + \hat{L}) + (1-\hat{\gamma})(a^*(H;\alpha) - c(a^*(H;\alpha)) + \hat{H})} \\ &< \frac{a^*(H;\alpha) - c(a^*(H;\alpha)) + \hat{H}}{a^*(L;\alpha) - c(a^*(L;\alpha)) + \hat{L}}.\end{aligned}$$

Since $x - c(x)$ is a strictly increasing function on $[0, (c')^{-1}(1)]$, $a^*(L;\alpha) - c(a^*(L;\alpha)) \leq a^*(H;\alpha) - c(a^*(H;\alpha))$ holds from the assumption that $a^*(H;\alpha) < a^{FB} = (c')^{-1}(1)$ and the monotonicity in effort $a^*(L;\alpha) \leq a^*(H;\alpha)$. Hence, it must be that $a^*(H;\alpha) - c(a^*(H;\alpha)) + \hat{H} \geq a^*(L;\alpha) - c(a^*(L;\alpha)) + \hat{L}$, which shows the inequality above.

Now, since $\frac{a^*(H;\alpha)+\hat{H}}{a^*(L;\alpha)+\hat{L}}$ lies between $\frac{a^*(H;\alpha)-c(a^*(H;\alpha))+\hat{H}}{a^*(L;\alpha)-c(a^*(L;\alpha))+\hat{L}}$ and $\frac{c(a^*(H;\alpha))}{c(a^*(L;\alpha))}$,[24] it follows that:

$$\frac{c'(a^*(H;\alpha))}{c'(a^*(L;\alpha))} \leq \min\left\{\frac{a^*(H;\alpha) - c(a^*(H;\alpha)) + \hat{H}}{a^*(L;\alpha) - c(a^*(L;\alpha)) + \hat{L}}, \frac{c(a^*(H;\alpha))}{c(a^*(L;\alpha))}\right\} \leq \frac{a^*(H;\alpha) + H}{a^*(L;\alpha) + L},$$

which completes the proof. □

[24] This follows from the following fact: assume $a_1, a_2, b_1, b_2 > 0$ and $\frac{a_1}{a_2} \leq \frac{b_1}{b_2}$, then $\frac{a_1}{a_2} \leq \frac{a_1+b_1}{a_2+b_2} \leq \frac{b_1}{b_2}$. These inequalities are strict if $\frac{a_1}{a_2} < \frac{b_1}{b_2}$.

Appendix for Empirics

Data Appendix

Sources and Sample Construction

A. Rule of Law Measure

We obtain this measure directly from the Worldwide Governance Indicators (WGI) project website (Kaufmann et al., 2011). The database covers 216 countries from 1996 to 2020.

B. Country-Level TFP Growth Rates

We use country-level TFP growth rates from the latest version of the Penn World Tables (PWT 10.0). After we merge this database with WGI database, our sample covers 123 countries over 1996-2019. For each country c, the variable VOL_c denotes the volatility of the country c's TFP growth rates from 1996 to 2016.

C. Firm-Level TFP Dispersion

We closely follow data construction procedures in Gopinath et al. (2017). We download all unconsolidated financial statements of all manufacturing firms (2-digit NACE code from C.10-C.33) with from the entire ORBIS universe. Then, we drop the entire history of a firm if the firm has any basic accounting errors in any year (e.g. negative total assets, employment, sales). After removing observations that indicate basic accounting errors, we focus on ones that has non-missing data on total assets, tangible fixed assets, sales, operating revenues, employment, and country information. We also keep a firm-year only if value added, defined as the difference between the operating revenue and material costs, is positive. Also, we drop countries with less than one hundred observations (firm-years). After these procedures, we have 4,786,856 firm-years with 27 countries. Nominal values are deflated via Producer Price Indices (PPI) obtained from the Datastream database.

We adopt a standard methodology for TFP estimation in the literature. For each country c, we use the methodology from Ackerberg et al. (2015) to separately estimate parameters $\beta_{l,c}$ and $\beta_{k,c}$:

$$log(y_{i,c,t}) = log(Z_{i,c,t}) + \beta_{l,c}log(l_{i,c,t}) + \beta_{k,c}log(k_{i,c,t}) + \epsilon_{i,c,t}$$

$log(y_{i,c,t})$ denotes the nominal value added deflated by PPI for the domestic manufacturing firms, $log(l_{i,c,t})$ the nominal wage bill deflated by the same PPI, and $log(k_{i,c,t})$ the value of the sum of tangible and intangible fixed assets deflated by PPI for the investment good.[25] We use intermediate

[25] We deflate all variables by economy-wide PPI's for following countries, because other PPI's were are not available: Bosnia and Herzegovina, Montenegro, the Republic of Macedonia, Romania, Serbia, and Ukraine.

goods as a control variable. To do this, we use the STATA code prodest from Manjón and Mañez (2016).

For each country c and year t, $LN\_TFP9010_{c,t}$ denotes the TFP difference (in the logarithmic scale) between the firm with the top 10% productivity and the bottom 10% firm in a country c in a year. As samples above, AVG_RL_c denotes the average rule of law of the country c over the corresponding sample period. After merging the ORBIS sample with the WGI and WDI data, we obtain a sample of 26 countries.

Table 7: Replication of Acemoglu et al. (2001)'s Main Results (Volatility Sample)

Panel A: OLS Regressions						
	(1)	(2)	(3)	(4)	(5)	(6)
Average Protection Against Expropriation Risk	0.538***	0.554***	0.434***	0.429***	0.407***	0.407***
	(0.0330)	(0.0374)	(0.0313)	(0.0426)	(0.0562)	(0.0573)
Average Homicide		0.007		-0.001		-0.000
		(0.004)		(0.005)		(0.00566)
Latitude					0.356	0.356
					(0.484)	(0.535)
Continent FE	No	No	Yes	Yes	Yes	Yes
Observations	101	101	101	101	101	101
R^2	0.609	0.614	0.700	0.700	0.702	0.702
Panel B: IV Regressions						
	(1)	(2)	(3)	(4)	(5)	(6)
Average Protection Against Expropriation Risk	0.854***	0.882***	0.817***	0.843***	0.957***	0.963***
	(0.118)	(0.122)	(0.152)	(0.158)	(0.253)	(0.251)
Average Homicide		0.012**		0.005		0.004
		(0.006)		(0.007)		(0.008)
Latitude					-1.283	-1.168
					(1.429)	(1.448)
Continent FE	No	No	Yes	Yes	Yes	Yes
Observations	44	44	44	44	44	44
Anderson-Rubin statistics	14.16	14.91	15.53	10.12	13.29	12.61
Panel C: First-Stage Regressions						
	(1)	(2)	(3)	(4)	(5)	(6)
Logged Settler Mortality	-0.888***	-0.875***	-0.774***	-0.738***	-0.581***	-0.576***
	(0.154)	(0.157)	(0.184)	(0.172)	(0.199)	(0.200)
Average Homicide		-0.007		-0.010		-0.005
		(0.011)		(0.012)		(0.012)
Latitude					2.294*	2.132
					(1.216)	(1.312)
Continent FE	No	No	Yes	Yes	Yes	Yes
Observations	44	44	44	44	44	44
R^2	0.503	0.508	0.529	0.537	0.560	0.562
F-statistics	33.16	31.22	17.71	18.31	8.51	8.27

We replicate Acemoglu et al. (2001)'s main results in the volatility sample. Panel A reports the OLS coefficients of equation (2) in the main text. Panel B and Panel C report IV and first-stage estimates (respectively) of the sytem of equations (3) in the main text. Data sources are described in section 5.2 and 5.2.1 of the main text. Standard deviations are in percentage terms. Robust Standard errors are in parentheses. Asterisks *, **, and *** denote statistical significance at 10%, 5%, and 1%, respectively.

Table 8: Replication of Acemoglu et al. (2012)'s OLS Results (TFP Dispersion Sample)

	(1)	(2)	(3)	(4)	(5)	(6)
Average Protection Against	0.734***	0.687***	0.754***	0.694***	0.735***	0.672***
Expropriation Risk	(0.132)	(0.0984)	(0.148)	(0.109)	(0.135)	(0.0940)
Average Homicide		-0.329*		-0.309		-0.316
		(0.172)		(0.205)		(0.190)
Europe			-0.296**	-0.0671	-0.420*	-0.197
			(0.132)	(0.158)	(0.197)	(0.157)
Latitude					0.867	0.945
					(0.761)	(0.693)
Constant	2.728*	3.579***	2.803*	3.545***	2.626*	3.369***
	(1.289)	(0.960)	(1.361)	(1.035)	(1.299)	(0.906)
Observations	17	17	17	17	17	17
R^2	0.691	0.794	0.725	0.795	0.741	0.815

Standard errors in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 9: Replication of Acemoglu et al. (2012)'s Main Results (Inequality Sample)

Panel A: OLS Regressions						
	(1)	(2)	(3)	(4)	(5)	(6)
Average Protection Against Expropriation Risk	0.519***	0.539***	0.417***	0.417***	0.397***	0.398***
	(0.0493)	(0.0574)	(0.0429)	(0.0605)	(0.0725)	(0.0792)
Average Homicide		0.005		0.000		0.000
		(0.003)		(0.005)		(0.005)
Latitude					0.254	0.258
					(0.525)	(0.543)
Continent FE	No	No	Yes	Yes	Yes	Yes
Observations	76	76	76	76	76	76
R^2	0.561	0.566	0.643	0.643	0.644	0.644

Panel B: IV Regressions						
	(1)	(2)	(3)	(4)	(5)	(6)
Average Protection Against Expropriation Risk	0.775***	0.821***	0.710***	0.743***	0.752***	0.762***
	(0.155)	(0.161)	(0.168)	(0.181)	(0.230)	(0.227)
Average Homicide		0.014***		0.007		0.008
		(0.005)		(0.006)		(0.006)
Latitude					-0.374	-0.166
					(0.977)	(0.900)
Continent FE	No	No	Yes	Yes	Yes	Yes
Observations	36	36	36	36	36	36
Anderson-Rubin statistics	33.88***	33.71***	23.17***	18.65***	7.23***	6.95***

Panel C: First-Stage Regressions						
	(1)	(2)	(3)	(4)	(5)	(6)
Logged Settler Mortality	-0.916***	-0.867***	-0.841***	-0.765***	-0.705***	-0.687***
	(0.185)	(0.175)	(0.194)	(0.172)	(0.224)	(0.210)
Average Homicide		-0.017***		-0.021***		-0.020***
		(0.005)		(0.005)		(0.005)
Latitude					1.450	0.873
					(1.328)	(1.343)
Continent FE	No	No	Yes	Yes	Yes	Yes
Observations	36	36	36	36	36	36
R^2	0.434	0.496	0.464	0.549	0.483	0.556
F-statistics	24.53***	24.58***	18.73***	19.85***	9.95***	10.70***

Standard errors in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 10: Volatility Results Based on IHS Measure

Panel A: OLS Regressions						
	(1)	(2)	(3)	(4)	(5)	(6)
Average Rule of Law	-5.197***	-5.860***	-5.030***	-5.538***	-6.437***	-6.630***
	(1.373)	(1.638)	(1.499)	(1.730)	(1.949)	(2.067)
Average Homicide		-0.0280		-0.0174		-0.00793
		(0.0222)		(0.0211)		(0.0202)
Latitude					3.312**	3.222**
					(1.638)	(1.602)
Continent FE	No	No	Yes	Yes	Yes	Yes
Observations	108	108	108	108	108	108
R^2	0.154	0.186	0.210	0.225	0.238	0.247

Panel B: IV Regressions						
	(1)	(2)	(3)	(4)	(5)	(6)
Average Rule of Law	-4.529**	-4.839**	-5.271***	-5.975**	-7.692**	-8.047**
	(2.161)	(2.406)	(1.816)	(2.322)	(3.637)	(3.706)
Average Homicide		-0.0216		-0.0252		-0.0296
		(0.0302)		(0.0291)		(0.0317)
Latitude					3.903	3.141
					(3.585)	(2.887)
Continent FE	No	No	Yes	Yes	Yes	Yes
Observations	44	44	44	44	44	44
Anderson-Rubin statistics	4.38**	3.75*	7.57***	6.44**	5.36**	4.78**

Panel B: First-Stage Regressions						
	(1)	(2)	(3)	(4)	(5)	(6)
Logged Settler Mortality	-0.137***	-0.127***	-0.136***	-0.115***	-0.0809***	-0.0766***
	(0.0206)	(0.0212)	(0.0265)	(0.0220)	(0.0272)	(0.0262)
Average Homicide		-0.005***		-0.006***		-0.004***
		(0.001)		(0.001)		(0.001)
Latitude					0.651***	0.504***
					(0.141)	(0.150)
Continent FE	No	No	Yes	Yes	Yes	Yes
Observations	44	44	44	44	44	44
R^2	0.556	0.680	0.565	0.691	0.682	0.756
F-statistics	44.23 ***	35.83***	26.28***	27.12***	8.86***	8.59***

We report the empirical results based on IHS Markit's Rule of Law measure. Panel A reports OLS coefficients of equation (2). Panel B and Panel C report IV and first-stage estimates (respectively) of the sytem of equations (3). Data sources are described in section 5.2 and 5.2.1. Standard deviations are in percentage terms. Robust Standard errors are in parentheses. Asterisks *, **, and *** denote statistical significance at 10%, 5%, and 1%, respectively.

Table 11: Volatility Results (Without Low-income Countries)

Panel A: OLS Regressions						
	(1)	(2)	(3)	(4)	(5)	(6)
Average Rule of Law	-1.054***	-1.160***	-1.025***	-1.075***	-1.326***	-1.300***
	(0.334)	(0.402)	(0.337)	(0.388)	(0.446)	(0.461)
Average Homicide		-0.0196		-0.00806		0.00538
		(0.0207)		(0.0193)		(0.0175)
Latitude					3.265*	3.349**
					(1.681)	(1.653)
Continent FE	No	No	Yes	Yes	Yes	Yes
Observations	98	98	98	98	98	98
R^2	0.203	0.211	0.236	0.237	0.282	0.282

Panel B: IV Regressions						
	(1)	(2)	(3)	(4)	(5)	(6)
Average Rule of Law	-0.529**	-0.497**	-0.740***	-0.698***	-0.805**	-0.782**
	(0.208)	(0.251)	(0.187)	(0.212)	(0.350)	(0.384)
Average Homicide		0.00606		0.00632		0.00628
		(0.0214)		(0.0198)		(0.0197)
Latitude					0.521	0.674
					(2.322)	(2.026)
Continent FE	No	No	Yes	Yes	Yes	Yes
Observations	40	40	40	40	40	40
Anderson-Rubin statistics	6.26**	3.58 *	6.77***	9.84***	5.36**	3.76*

Panel C: First-Stage Regressions						
	(1)	(2)	(3)	(4)	(5)	(6)
Logged Settler Mortality	-0.706***	-0.646***	-0.678***	-0.594***	-0.470***	-0.446***
	(0.127)	(0.130)	(0.135)	(0.128)	(0.150)	(0.158)
Average Homicide		-0.0162***		-0.0185***		-0.0140**
		(0.00565)		(0.00662)		(0.00648)
Latitude					2.457***	1.990**
					(0.819)	(0.887)
Continent FE	No	No	Yes	Yes	Yes	Yes
Observations	40	40	40	40	40	40
R^2	0.578	0.638	0.585	0.650	0.663	0.698
F-statistics	30.83***	24.71***	25.43***	21.47***	9.80 ***	7.98 ***

We report the empirical results after dropping low-income countries (as defined by World Bank). Panel A reports OLS coefficients of equation (2). Panel B and Panel C report IV and first-stage estimates (respectively) of the sytem of equations (3). Data sources are described in section 5.2 and 5.2.1. Standard deviations are multiplied by 100, so that coefficients can be interpreted in percentage terms. Robust Standard errors are in parentheses. Asterisks *, **, and *** denote statistical significance at 10%, 5%, and 1%, respectively.

Table 12: Volatility Results Based on IHS Measure (Without Low-income Countries)

Panel A: OLS Regressions

	(1)	(2)	(3)	(4)	(5)	(6)
Average Rule of Law	-4.585***	-5.354***	-4.576***	-5.023**	-6.122***	-6.185***
	(1.559)	(2.010)	(1.614)	(1.969)	(2.162)	(2.340)
Average Homicide		-0.0271		-0.0143		-0.00247
		(0.0250)		(0.0228)		(0.0202)
Latitude					3.513*	3.482**
					(1.785)	(1.711)
Continent FE	No	No	Yes	Yes	Yes	Yes
Observations	98	98	98	98	98	98
R^2	0.185	0.197	0.231	0.234	0.282	0.282

Panel B: IV Regressions

	(1)	(2)	(3)	(4)	(5)	(6)
Average Rule of Law	-2.440**	-2.360**	-3.434***	-3.383***	-4.213**	-4.205**
	(0.951)	(1.192)	(0.875)	(1.041)	(1.855)	(2.132)
Average Homicide		0.00330		0.00165		0.000434
		(0.0231)		(0.0216)		(0.0228)
Latitude					1.341	1.350
					(2.682)	(2.356)
Continent FE	No	No	Yes	Yes	Yes	Yes
Observations	40	40	40	40	40	40
Anderson-Rubin statistics	6.26**	3.58*	14.18***	9.84***	5.36**	3.76 *

Panel C: First-Stage Regressions

	(1)	(2)	(3)	(4)	(5)	(6)
Logged Settler Mortality	-0.153***	-0.136***	-0.146***	-0.123***	-0.0898***	-0.0829**
	(0.0275)	(0.0286)	(0.0303)	(0.0280)	(0.0303)	(0.0314)
Average Homicide		-0.00459***		-0.00520***		-0.00400***
		(0.00103)		(0.00118)		(0.00111)
Latitude					0.664***	0.531***
					(0.146)	(0.157)
Continent FE	No	No	Yes	Yes	Yes	Yes
Observations	40	40	40	40	40	40
R^2	0.580	0.683	0.590	0.701	0.713	0.774
F-statistics	31.06***	22.42***	23.34***	19.31***	8.79***	6.98**

We report the empirical results after dropping low-income countries (as defined by World Bank) and the IHS Markit's Rule of Law measure. Panel A reports OLS coefficients of equation (2) in the main text. Panel B and Panel C report IV and first-stage estimates (respectively) of the sytem of equations (3). Data sources are described in section 5.2 and 5.2.1 of the main text. Standard deviations are multiplied by 100, so that coefficients can be interpreted in percentage terms. Robust Standard errors are in parentheses. Asterisks *, **, and *** denote statistical significance at 10%, 5%, and 1%, respectively.

Table 13: Average TFP Dispersion Robustness Checks

Panel A: Regressions Based on IHS Measure						
	(1)	(2)	(3)	(4)	(5)	(6)
Average Rule of Law	-2.467***	-2.663***	-2.538***	-2.655***	-2.514***	-2.707***
	(0.252)	(0.249)	(0.216)	(0.258)	(0.233)	(0.323)
Average Homicide		-0.0345		-0.0214		-0.0263
		(0.0286)		(0.0269)		(0.0346)
Latitude					-0.143	0.147
					(0.453)	(0.524)
Europe FE	No	No	Yes	Yes	Yes	Yes
Observations	26	26	26	26	26	26
R^2	0.743	0.752	0.787	0.790	0.787	0.790

Panel B: Regressions Without Low-income Countries						
	(1)	(2)	(3)	(4)	(5)	(6)
Average Rule of Law	-0.488***	-0.490***	-0.494***	-0.490***	-0.543***	-0.562***
	(0.0600)	(0.0602)	(0.0621)	(0.0641)	(0.0688)	(0.0787)
Average Homicide		-0.00523		0.0104		-0.0185
		(0.0254)		(0.0167)		(0.0248)
Latitude					0.799*	0.996*
					(0.403)	(0.534)
Europe FE	No	No	Yes	Yes	Yes	Yes
Observations	25	25	25	25	25	25
R^2	0.713	0.714	0.754	0.755	0.769	0.771

Panel C: Regressions Based on IHS Measure, Without Low-income Countries						
	(1)	(2)	(3)	(4)	(5)	(6)
Average Rule of Law	-2.319***	-2.533***	-2.391***	-2.543***	-2.329***	-2.634***
	(0.320)	(0.286)	(0.286)	(0.291)	(0.299)	(0.349)
Average Homicide		-0.0629**		-0.0470***		-0.0567**
		(0.0235)		(0.0134)		(0.0226)
Latitude					-0.276	0.268
					(0.419)	(0.470)
Europe FE	No	No	Yes	Yes	Yes	Yes
Observations	25	25	25	25	25	25
R^2	0.658	0.691	0.718	0.735	0.720	0.736

Panel A reports OLS coefficients of equation (4). Panel B estimates the same equation (4), but instead uses the IHS Markit's Rule of Law measure as the outcome variable. Panel C reports the results after low-income countries (as defined by World Bank) are dropped from the sample. Other data sources are described in section 5.2 and 5.2.1. Robust Standard errors are in parentheses. Asterisks *, **, and *** denote statistical significance at 10%, 5%, and 1%, respectively.

Table 14: Pay Inequality Results Based on IHS Measure

Panel A: OLS Regressions						
	(1)	(2)	(3)	(4)	(5)	(6)
Average Rule of Law	-1.999***	-1.975***	-1.392***	-1.242**	-1.081**	-0.993*
	(0.409)	(0.459)	(0.468)	(0.540)	(0.483)	(0.540)
Average Homicide		0.001		0.008		0.005
		(0.011)		(0.009)		(0.010)
Latitude					-1.034*	-1.003*
					(0.529)	(0.532)
Continent FE	No	No	Yes	Yes	Yes	Yes
Observations	94	94	94	94	94	94
R^2	0.230	0.230	0.323	0.327	0.355	0.356

Panel B: IV Regressions						
	(1)	(2)	(3)	(4)	(5)	(6)
Average Rule of Law	-2.471***	-2.645***	-2.384***	-2.660***	-2.707**	-2.733**
	(0.602)	(0.677)	(0.703)	(0.844)	(1.075)	(1.123)
Average Homicide		-0.015		-0.020		-0.021
		(0.010)		(0.013)		(0.013)
Latitude					0.536	0.117
					(1.141)	(1.008)
Continent FE	No	No	Yes	Yes	Yes	Yes
Observations	32	32	32	32	32	32
Anderson-Rubin statistics	14.57***	13.42***	11.51***	10.82***	8.70***	8.51***

Panel C: First-Stage Regressions						
	(1)	(2)	(3)	(4)	(5)	(6)
Logged Settler Mortality	-0.155***	-0.144***	-0.145***	-0.130***	-0.116***	-0.115***
	(0.0234)	(0.0201)	(0.0242)	(0.0198)	(0.0297)	(0.0257)
Average Homicide		-0.006***		-0.007***		-0.007***
		(0.001)		(0.001)		(0.001)
Latitude					0.334	0.192
					(0.200)	(0.190)
Continent FE	No	No	Yes	Yes	Yes	Yes
Observations	32	32	32	32	32	32
R^2	0.549	0.666	0.588	0.744	0.635	0.759
F-statistics	43.96***	35.74***	35.74***	43.35***	15.19***	19.93***

We report the empirical results based on the IHS Markit's Rule of Law measure. Panel A reports OLS coefficients of equation (2). Panel B and Panel C report IV and first-stage estimates (respectively) of the sytem of equations (3). The outcome variable in Panel A and Panel B is "Average Pay inequality," which can be obtained from the UT-UNIDO database. Other data sources are described in section 5.2 and 5.2.1. Robust Standard errors are in parentheses. Asterisks *, **, and *** denote statistical significance at 10%, 5%, and 1%, respectively.

Table 15: Wage Inequality Results (Without Low-income Countries)

Panel A: OLS Regressions

	(1)	(2)	(3)	(4)	(5)	(6)
Average Rule of Law	-0.539***	-0.538***	-0.445***	-0.419***	-0.371***	-0.364***
	(0.0705)	(0.0762)	(0.0782)	(0.0897)	(0.0816)	(0.0885)
Average Homicide		0.000		0.004		0.001
		(0.004)		(0.004)		(0.004)
Latitude					-0.939**	-0.915*
					(0.461)	(0.486)
Continent FE	No	No	Yes	Yes	Yes	Yes
Observations	90	90	90	90	90	90
R^2	0.329	0.329	0.401	0.404	0.429	0.430

Panel B: IV Regressions

	(1)	(2)	(3)	(4)	(5)	(6)
Average Rule of Law	-0.483***	-0.503***	-0.437***	-0.464***	-0.397***	-0.405***
	(0.120)	(0.132)	(0.116)	(0.129)	(0.147)	(0.151)
Average Homicide		-0.004		-0.005		-0.005
		(0.00402)		(0.005)		(0.005)
Latitude					-0.339	-0.492
					(0.820)	(0.830)
Continent FE	No	No	Yes	Yes	Yes	Yes
Observations	33	33	33	33	33	33
Anderson-Rubin statistics	13.66***	12.20***	12.45***	11.17 ***	7.03***	6.98***

Panel C: First-Stage Regressions

	(1)	(2)	(3)	(4)	(5)	(6)
Logged Settler Mortality	-0.841***	-0.801***	-0.830***	-0.776***	-0.751***	-0.737***
	(0.0914)	(0.0823)	(0.0984)	(0.0785)	(0.133)	(0.114)
Average Homicide		-0.010**		-0.013**		-0.012**
		(0.004)		(0.005)		(0.005)
Latitude					0.794	0.420
					(0.868)	(0.882)
Continent FE	No	No	Yes	Yes	Yes	Yes
Observations	33	33	33	33	33	33
R^2	0.666	0.714	0.672	0.738	0.683	0.741
F-statistics	84.71***	94.73***	71.03***	97.69***	32.04***	41.84***

We report the empirical results after dropping low-income countries (as defined by World Bank). Panel A reports OLS coefficients of equation (2). Panel B and Panel C report IV and first-stage estimates (respectively) of the sytem of equations (3). The outcome variable in Panel A and Panel B is "Average Pay inequality," which can be obtained from the UT-UNIDO database. Other data sources are described in section 5.2 and 5.2.1. Robust Standard errors are in parentheses. Asterisks *, **, and *** denote statistical significance at 10%, 5%, and 1%, respectively.

Table 16: Pay Inequality Results Based on IHS Measure (Without Low-income Countries)

Panel A: OLS Regressions						
	(1)	(2)	(3)	(4)	(5)	(6)
Average Rule of Law	-2.191***	-2.209***	-1.677***	-1.560***	-1.394***	-1.345***
	(0.386)	(0.422)	(0.429)	(0.505)	(0.432)	(0.497)
Average Homicide		-0.000846		0.00519		0.00241
		(0.0105)		(0.00910)		(0.00984)
Latitude					-0.901*	-0.884*
					(0.519)	(0.527)
Continent FE	No	No	Yes	Yes	Yes	Yes
Observations	86	86	86	86	86	86
R^2	0.252	0.252	0.348	0.350	0.373	0.373

Panel B: IV Regressions						
	(1)	(2)	(3)	(4)	(5)	(6)
Average Rule of Law	-2.318***	-2.483***	-2.137***	-2.373***	-2.084***	-2.097***
	(0.595)	(0.670)	(0.621)	(0.704)	(0.784)	(0.805)
Average Homicide		-0.013		-0.018		-0.017
		(0.010)		(0.011)		(0.011)
Latitude					-0.091	-0.469
					(0.942)	(0.867)
Continent FE	No	No	Yes	Yes	Yes	Yes
Observations	30	30	30	30	30	30
Anderson-Rubin statistics	12.60***	11.42***	10.90***	10.32***	6.93***	6.74***

Panel C: First-Stage Regressions						
	(1)	(2)	(3)	(4)	(5)	(6)
Logged Settler Mortality	-0.174***	-0.162***	-0.167***	-0.151***	-0.142***	-0.141***
	(0.0208)	(0.0171)	(0.0215)	(0.0131)	(0.0277)	(0.0198)
Average Homicide		-0.006***		-0.00707***		-0.007***
		(0.001)		(0.001)		(0.001)
Latitude					0.260	0.110
					(0.201)	(0.183)
Continent FE	No	No	Yes	Yes	Yes	Yes
Observations	30	30	30	30	30	30
R^2	0.641	0.755	0.663	0.812	0.690	0.817
F-statistics	70.16***	89.07***	60.09***	133.10***	26.27***	51.12***

We report the empirical results after dropping low-income countries (as defined by World Bank). Panel A reports OLS coefficients of equation (2). Panel B and Panel C report IV and first-stage estimates (respectively) of the sytem of equations (3). The outcome variable in Panel A and Panel B is "Average Pay inequality," which can be obtained from the UT-UNIDO database. Other data sources are described in section 5.2 and 5.2.1. Robust Standard errors are in parentheses. Asterisks *, **, and *** denote statistical significance at 10%, 5%, and 1%, respectively.