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Falko Fecht, Roman Inderst and Sebastian Pfeil

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Centre for Economic Policy Research
33 Great Sutton Street, London EC1V 0DX, UK
Tel: +44 (0)20 7183 8801
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JEL Classification: G21, F36, L25

Keywords: Interbank lending, risk shifting, Debt overhang, integration, deposit insurance

Falko Fecht - f.fecht@fs.de
Frankfurt School of Finance

Roman Inderst - inderst@finance.uni-frankfurt.de
Goethe University Frankfurt and CEPR

Sebastian Pfeil - pfeil@ese.eur.nl
Erasmus School of Economics

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Falko Fecht† Roman Inderst‡ Sebastian Pfeil§

September 2017

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We offer a theory of the "boundary of the firm" tailored to banks as it builds on a single risk-shifting inefficiency and takes into account interbank lending, as an alternative to integration, and insured deposit financing. It explains why deeper economic integration should cause also greater, though still incomplete, financial integration, through both bank mergers and interbank lending, and why economic disintegration, as currently witnessed in the European Union, should cause less interbank exposure. Recent policy measures such as the preferential treatment of retail deposits, the extension of deposit insurance, or penalties on "connectedness" could reduce welfare.

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†Frankfurt School of Finance.

‡Goethe-University Frankfurt and Imperial College London.

§Erasmus School of Economics.

1 Introduction

We offer a theory of the "boundary of the firm" that is tailored to banks as it recognizes the relevance of both (insured) deposit financing and that of interbank lending as a possible substitute for integration. Our theory relies on a single inefficiency that has been at the core of banking theory: risk-shifting incentives in the interest of banks' shareholders. Still, our model is capable of delivering (i) a number of, mostly new, empirical predictions, (ii) a theory of the limits to financial integration, both through interbank lending and the reallocation of funds within a merged bank, as well as (iii) normative implications closely related to the current financial crisis and the respective proposed or already implemented policy measures, such as the preferential treatment of retail deposits and penalties for "interconnectedness".

In our baseline model local banks have specific skills in collecting funds and making loans, so that when banks remain non-integrated, a reallocation of funds across geographically segmented markets relies on interbank lending. Inside an integrated bank, by contrast, funds can be reallocated through an "internal capital market".[1] The extent to which financial integration is achieved through these two channels depends on, first, whether funding relies on insured deposits and, second, on how well the two markets are already integrated economically, as expressed by the correlation in their lending markets. These two parameters determine also whether banks will fully integrate in equilibrium or stay separate.

The key mechanisms at work are the following: (i) More reallocation of funds across markets through interbank lending generates co-insurance benefits for depositors of the lending bank. An inefficiently low level of reallocation is, thus, an expression of risk-shifting behavior to shareholders' benefits; (ii) Integration generates immediate co-insurance for depositors in both markets and, thus, alters the "status-quo" for depositors and shareholders. We obtain from (i) that there will generally be too little interbank lending and from (ii) that there will be too little integration. Combining (i) and (ii) gives rise to a theory of endogenous integration, as either integration or non-integration can, under different circumstances, lead to lower ex-post risk-shifting incentives and, thus, serve as a commitment to (more) efficient resource allocation.

Between non-integrated banks, we find that interbank lending is larger when markets are

[1] Our model thus puts at the forefront the role of the financial system to reallocate resources across otherwise geographically segmented markets, as also Merton and Bodie (1995) or Allen and Gale (2001).

already more closely aligned, as expressed by the correlation between local lending markets. This holds as then the co-insurance externalities from a reallocation of funds across markets are smaller.[2] It is also smaller when a failure to repay the interbank loan has a contagious effect on the creditor bank. We show that there should, thus, be a tendency towards either relatively low or relatively high and contagious levels of interbank exposure between individual banks.

Changes in the correlation of lending markets can derive from an increase or a decrease in economic integration. Our model would thus predict that greater economic integration, such as within in the European Union before the crisis, should itself trigger also more interbank lending (as well as mergers in the banking industry, as we see shortly), while disintegration (or the "de-synchronization" of economic activity) should reduce interbank exposure at the expense of allocative efficiency. The latter observation clearly throws a somewhat different light on the current financial disintegration in the European Union, notably between banks at its core and its periphery.

More generally, our theory thus contributes to a better understanding of the patterns and limits of global financial integration. Such greater financial integration yields potentially large welfare benefits given cross-regional differences in net savings, in productivity, and in exposures to output shocks. This holds both on a global scale but also within relatively homogenous areas such as the Euro zone and the U.S.[3] Various researchers have, however, noted that the extent to which such financial integration has been achieved is still limited. Surprisingly, this observation seems to apply not only to global financial integration, which is restrained by regulation, but also to the financial integration in the Euro area, where de jure obstacles to financial integration have been largely removed.[4] To understand this puzzle, it is important to understand the incentives of banks as they play a key role both in collecting funds from households and in investing, in smaller and medium-sized companies where local

[2] Importantly, the positive relationship between interconnectedness and correlation is thus not a consequence of banks' prospects to be bailed out together (Acharya and Yorulmazer (2006, 2007); cf. also Wagner (2010) for a similar logic).

[3] For evidence and measurements see, for instance, ECB (2013, p. 96-107), Kalemli-Ozcan et al. (2003), or Bonfiglioli (2008).

[4] For a discussion of financial globalization see Stulz (2005). Lane (2009) and more recently van Beers et al. (2014) focus on the Euro area. Claessens and van Horen (2014) report that, while the number of foreign banks has considerably increased since the 1990s (in particular in eastern Europe) most financial intermediation (about 80%) in OECD countries remains done by domestic banks.

proximity is (still) of major importance. We find that incentives for financial integration are typically inefficiently low. This holds also for incentives to merge.[5]

Our model predicts a high degree of fragmentation among banks that rely mainly on insured deposits, such as savings and loan banks or cooperative banks with a strong local retail presence. The "boundary of the bank" also depends on the economic integration of the respective markets: Economic integration that increases the correlation between lending markets makes a bank merger more likely. Notably, as in our model there are no exogenously assumed advantages or disadvantages to the allocation of funds either through interbank lending or within an integrated firm, a bank's boundary is determined solely by the following force: The choice between integration or non-integration generates commitment vis-à-vis the providers of uninsured funding, in terms of the subsequent reallocation of funds and the thereby achieved co-insurance benefit. Interestingly, though our theory builds on a single inefficiency, that is risk-shifting, the trade-off between integration and non-integration is resolved differently, depending on the correlation between the respective lending markets: For low correlation an integrated bank would achieve a less efficient allocation of funds than non-integrated banks relying on interbank lending alone, while for high correlation the allocation is more efficient in the integrated bank.

Rather than excessive interconnectedness or excessive incentives to form "too-big-too-fail" banks, our parsimonious model of banking highlights a different channel pointing in the opposite direction: Too little exposure through interbank lending and too little financial integration through mergers and acquisitions among banks.[6] This is why, in our model, for instance a "tax" or other penalties on size or interconnectedness, may have negative

[5] The role of banks for financial integration, both through cross-border asset holdings and interbank lending as well as through cross-border mergers, has indeed been largely documented in the empirical literature. Globally, Milesi-Ferretti and Tille (2011) argue that the cross-border activity of banks plays a dominant role for financial integration (cf. also Figure 1 in Fecht et al. (2012) for the role of interbank lending). Even within the Euro area the pre-crisis growth in cross-border asset holdings and financial integration was predominantly driven by the internationalization of European banks (cf. van Beers et al. 2014) and interbank lending (Sapir and Wolff 2013). There is also a large literature showing that the deregulation of cross-regional banking improved diversification and capital allocation even though other financial markets were already de facto integrated before. See, for instance, Black and Strahan (2002), Acharya et al. (2006), and Acharya et al. (2010).

[6] Clearly, "too-big-to-fail" as well as "too-connected-to-fail" could generate additional moral hazard problems, from which we abstract. An important insight of our analysis is, however, that there may also be strong disincentives working the opposite way, and those effects need to be considered when determining the optimal degree of regulation.

first-order effects on allocative efficiency and thus welfare. This should throw a new light on several policy initiatives that strive to discourage interbank lending and aim at either directly limiting bank size or imposing additional levies on larger banks.[7]

Yet another policy implication relates to the extension of deposit insurance in the wake of the financial crisis. In our model, this would reduce the commitment role of a bank merger vis-à-vis providers of uninsured funding, so that the extension of deposit insurance can reduce financial integration and welfare. On the other side, as implicit and explicit insurance of bank debt holders is a subtle disincentive to bank mergers in our model, this suggests that the new EU Bank Recovery and Resolution Directive, which increases the bail-in of bank debt holders, could increase Euro area banks' incentives to merge - possibly counteracting the objective of preventing banks from becoming "too-big-too-fail". In yet another twist, current regulatory initiatives that encourage banks' reliance on insured retail deposits, such as their preferential treatment in liquidity coverage ratios and stress tests, would again have the opposite effect of reducing incentives for greater financial integration.

Our paper is embedded in a large banking theory literature, as surveyed for instance in Freixas and Rochet (2008). We share with this literature the following key features of our model: (i) The importance of deposit financing, both insured and uninsured; (ii) banks' role as local and "skilled" collectors of funds and providers of loans; and (iii) risk-shifting as the important inefficiency and friction. Much fewer papers have considered more than one bank and allowed for interbank lending. While our model focuses on the improvement of allocative efficiency through interbank lending, papers such as Bhattacharya and Gale (1987), Allen and Gale (2000), and Freixas et al. (2000) stress the role of the (short-term) interbank market in liquidity risk sharing. None of these papers poses the question of the "boundary of the firm", which is of course addressed in a large separate body of literature.[8]

While a number of empirical papers on multinational banks draw largely on this theoretical literature, regarding both the operation of an internal capital market and the benefits

[7] According to BIS (2011) banks considered as global systemically important financial institutions (G-SIFIs) will be required to hold up to 3.5% additional equity against their risk based assets. Whether a bank is considered a G-SIFI depends among other things on its wholesale funding ratio. On limiting the size of banks see also the respective provisions in the Dodd Frank Act, Section 622.

[8] An exception is Kerl and Niepmann (2014), who study the composition of banks' lending activities, allowing for international interbank lending, intrabank lending, and direct lending to foreign firms. Also Krasa and Villamil (1992) derive the optimal bank size. Key element in their model is the trade-off between banks' lenders' monitoring costs that increase in bank size and better diversification.

of integration, this literature does however not consider the specificities of the banking sector.[9] As noted above, this concerns the reliance on often insured deposits as well as the use of interbank lending.[10] We also focus exclusively on risk shifting as the sole inefficiency,[11] following much of the banking literature, and thereby do not assume other frictions that could provide an (exogenous) disadvantage for integration, such as limits to managerial control, greater conflicts of interest and scope for "rent seeking" in larger organizations,[12] or an (exogenous) advantage, such as asymmetric information or limited contractibility across the boundaries of firms.[13]

The rest of this paper is organized as follows. In Section 2 we introduce our model of segmented funding and lending markets. The analysis with non-integrated banks is contained in Section 3. Section 4 considers the allocation of funds within an integrated bank and compares this with interbank lending. This comparison is then used in Section 5 to endogenize the decision whether to integrate or not. Section 6 collects the key positive and normative implications of our analysis and Section 7 concludes. The Appendix contains all proofs. Additional supportive material is collected in a separate Online Appendix.[14]

[9] See, for instance, Campello (2006) and Cetorelli and Goldberg (2012).

[10] As discussed below, our specification of deposit financing, which we share with the banking literature, allows us to abstract from the endogenization of leverage (cf. Lewellen (1971), Leland (2007), Banal-Estañol et al. (2013)) or, more generally, the financial claims issued by integrated and non-integrated firms (Inderst and Müller (2003)).

[11] Dewatripont and Mitchel (2005) show that financial conglomerates are prone to excessive risk-taking by choosing too (positively) correlated projects. In Freixas et al. (2007) conglomerates with an integrated balance sheet have excessive risk-taking incentives due to deposit insurance while conglomerates with a holding structure practice regulatory arbitrage. In such a setting with subsidiary vs. branch structures, Harr and Rønde (2004) and Lóránth and Morrison (2007) solve for optimal capital requirements and Calzolari and Lóránth (2011) analyze disciplinary actions. Dell'Ariccia and Marquez (2010) highlight the trade-off between a branch structure's ability to shield its capital from expropriation in the host country with a subsidiary's individual limited liability protection.

[12] These have been addressed, for instance, in Rajan et al. (2000), Scharfstein and Stein (2000), or Inderst and Klein (2007). Notably, Stein (2002) considers the interaction of an internal capital market and internal agency problems within a banking context.

[13] With respect to the role of non-contractibility, of course, the seminal approach in Hart and Moore (1990), which focuses on incentives and hold-up, should be noted.

[14] The Online Appendix can be found under www.sebastianpfeil.de/working-papers.

2 The Model

Based on the motivation provided in the Introduction, we consider a stylized model of segmented lending and funding markets. We also build into our model a role for banks in both collecting savings from households and making informed investment decisions through loans. The various assumptions that we thereby make follow closely the large extant literature on banking,[15] which is why the following presentation of our model focuses on those ingredients that are more novel and decisive for our subsequent results.

Markets and Technologies. There are two locally segmented markets, $n = A, B$. Each market is populated by a mass one of households. In market A, each household has funds of size M_A. As there is a mass one of households, this also represents the measure of the total funding potential when funds are raised solely in market A. In market B, each household has funds of size M_B. We assume without loss of generality that $M_A \geq M_B \geq 0$. The interesting case will be that where the local funding potential differs across markets. To derive for this a convenient measure, we denote total available funding by $M_A + M_B = 2M$ and write $M_A = M + z$ and $M_B = M - z$ with $z > 0$. When analyzing the role of banks to allocate funding *across* markets, we will conduct a comparative statics analysis in z.

To streamline the model, we abstract from modelling consumption and saving decisions of households and thus take as given that households set aside the respective funds $2M$ for later consumption. Next to a storage technology, which simply preserves the value of funds, we introduce a risky investment technology in each of the two markets. For this we suppose that in each market there is one penniless entrepreneur who has access to a real investment opportunity, as specified next, and we suppose that there is at the same time a large number of fraudulent entrepreneurs who will abscond with any funds that they receive. By specifying that only one locally active bank has the necessary (soft) information to screen out fraudulent entrepreneurs,[16] we grant each local bank monopoly power in the lending market and also preclude any forms of non-intermediated financing. In the case of an integrated bank, AB, we suppose that, by acquiring the respective technology, the integrated bank inherits this

[15] See, for instance, Freixas and Rochet (2008).

[16] In practice, this should hold notably for smaller and medium-sized companies where local proximity is (still) of major importance. See, for instance, Petersen and Rajan (1994) and more recently Degryse and Ongena (2005).

knowledge across both markets.[17]

The project of the (non-fraudulent) entrepreneur, on whom we can focus, is risky as it only succeeds with probability p. In case of success, when having received funds of size F, the project pays back $L(F)$, while it pays back zero when it fails. The (production) function $L(\cdot)$ satisfies $L' > 0$ and $L'' < 0$. As we stipulate that the monopolistic local bank can make a take-it-or-leave-it offer to the local entrepreneur, the function $L(F)$ also represents the local lending (or loan-making) potential. By assuming that it is symmetric across markets, we can focus our analysis on banks' role to bridge funding differences across markets. A crucial parameter in our analysis, however, will be the extent to which the performance of loans in the two markets is correlated. We denote the respective correlation coefficient by ρ and allow for values $0 \leq \rho \leq 1$. As is immediate, the likelihood with which loans in both markets perform is then given by $p^2 + \rho p(1-p)$, which becomes p^2 when projects are fully independent ($\rho = 0$) and equal to p when projects are perfectly (positively) correlated ($\rho = 1$).[18] Next to z, which captures the difference in the local funding base, ρ will be our main comparative variable in what follows.

We further want to focus our analysis on the case where local funding is never in excess, so that we assume throughout that

$$pL'(M+z) > 1. \tag{1}$$

Further, to create scope for default and contagion when interbank loans are not repaid, we suppose that

$$L(M) < 2M. \tag{2}$$

In words, when only *half* of all available funding, M, is invested in one market, then in case of success the resulting payoff is insufficient to pay back *all* available funding, $2M$.

[17] Hence, we abstract from any agency related inefficiencies that larger (merged) banks could have in generating and processing the necessary local information (cf. Stein 1997).

[18] Note at this point that our specification of a single loan opportunity in each market can also be interpreted as a perfect positive correlation for loans in a local market. What is essential for our following arguments is that, in this case, loans in the bank's own lending market are more correlated than loans across banks' local lending markets. Incorporating additional flexibility to allow for more general correlations for a given local loan portfolio has, however, proved to make the analysis much less transparent and at points intractable.

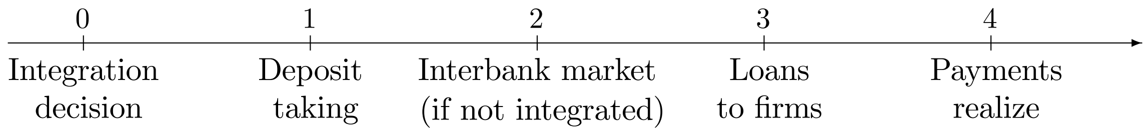

Figure 1: The timing of events

Strategies and Timing. A key part of the analysis in this paper is an endogenization of banks' integration decision and the derivation of its key determinants. We thus start the game at $t = 0$ with banks' decision whether to integrate or not. This, as well as all further decisions, are made in the interest of banks' shareholders. The subsequent game then unfolds depending on whether integration took place or not. We first take the case where banks remain separate.

In $t = 1$ funding can be collected from households. Given our preceding discussion, households will either invest in the storage technology or invest in risky projects through one of the two banks. In our baseline analysis, we further stipulate that households in market n can only invest through bank n, albeit we can extend results to the case where banks compete for funding across markets.[19] Our key assumption is that households' claims on banks' assets will be senior to those of shareholders. We comment shortly on this assumption. We will refer to these claims as *deposits*, so that in our baseline model at $t = 1$ bank n offers a deposit rate r_n in its local market and attracts deposits of total size $R_n \leq M_n$. It should be noted, however, that when these deposits are non-insured, what matters in our model is only that these claims have priority to those of shareholders but that lending decisions are made in the interest of shareholders alone.[20] The assumption of such (debt) deposit financing is shared with a large literature in banking.[21]

Non-integrated banks can arrange interbank lending in $t = 2$, which prescribes a transfer of funds W_n from bank n' to bank n in exchange for a promised repayment w_n. To make our baseline analysis as transparent as possible, we stipulate that the (lending) bank with

[19] In the respective analysis in Part 1 of the Online Appendix we still endow the local bank with an advantage: Households who invest in a non-local bank will incur switching costs.

[20] It is inessential, however, whether or not they are senior to the claims arising from interbank lending.

[21] Though it is there often assumed exogenously as well, seniority of "outside claims" can be given various microfoundations (cf. Diamond and Rajan (2001), albeit there also other aspects of deposit financing, such as a "first-come-first-serve" feature, arise prominently). We wish to abstain from enriching our model in such ways, thereby focusing on what is novel in our analysis compared to the extant literature.

a higher funding base can make a take-it-or-leave-it offer.[22] In $t = 3$ banks extend loans in their local market. Payoffs are realized in $t = 4$. When banks have chosen to integrate in $t = 0$, the subsequent game simplifies as there is no need to arrange interbank lending in $t = 2$. All parties are risk neutral and we abstract from discounting.

3 Non-Integrated Banks

In this section we consider the case of non-integrated banks. Taken this as given for now, we solve the remaining game backwards. We first consider the determination of interbank lending in $t = 2$, taking as given the retail deposit funds R_n that each bank n has already attracted through promising an interest rate r_n. For this stage of the analysis it will not be important whether deposits are insured or not. The main result will be a characterization of optimal interbank lending and its key determinants. We then turn to $t = 1$, where each non-integrated bank secures deposit finance. Taken together, we obtain a full characterization of the funding and lending decisions of non-integrated banks in equilibrium.

3.1 Shortfall of Interbank Lending

As is intuitive (and formally derived in the proof of Lemma 1), in equilibrium there will be at most one interbank loan, i.e., in our model there is no scope for both a loan of bank A to bank B and vice versa. As the purpose of interbank lending is to better align banks' funding with their loan-banking opportunities, it is equally intuitive that an interbank loan will be made, if at all, by the bank with higher initial funding R_n to that with lower funding. We presently suppose that this is bank A, so that $R_A \geq R_B$. Denote thus by $W_B = W \geq 0$ the interbank loan that bank A makes to bank B and by $w_B = w \geq 0$ the respective agreed repayment.

Banks are managed in shareholders' interest. Take first bank A. For given (remaining)

[22] However, we show in Section 2 of the Online Appendix that the key results for interbank lending are unchanged when we stipulate instead a game of Nash bargaining with a more symmetric distribution of bargaining power. Note also that a fully competitive (fragmented) market would seem at odds with the arrangement of interbank lending, while an analysis of a network of interbank lending is beyond the scope of this paper.

funding,

$$F_A = R_A - W,$$

provided that this is then used to make a loan of the same size, the expected profits of shareholders are

$$\begin{aligned} \pi_A &= \left[p^2 + \rho p(1-p)\right]\left[L(F_A) + w - R_A(1+r_A)\right] \\ &\quad + p(1-p)(1-\rho)\left[\max\{0, L(F_A) - R_A(1+r_A)\} + \max\{0, w - R_A(1+r_A)\}\right]. \end{aligned} \tag{3}$$

The first line in (3) accounts for the state where all loans are successful. That is, with the respective probability, $p^2 + \rho p(1-p)$, both lending markets, A and B, perform. This also enables bank B to repay w to bank A.[23] Note that we implicitly assume that the total repayment to bank A, arising from both its own (corporate) loan and the loan made to bank B, is sufficient to cover the repayment that bank A promised to its depositors, $R_A(1+r_A)$. This will always be the case in equilibrium. The second line in (3) accounts jointly for two states that are equally likely: that where only the loans of bank A perform (captured by the first part) and that where only the loans of bank B perform (captured by the second part). When both lending markets do not perform, then clearly shareholders of bank A realize zero profits.

Expression (3) for the payoff of bank A's shareholders thus contains various cases, depending on whether the repayment of the bank's own loans, the repayment of its loan to bank B, or only both together are sufficient to cover claims to its own depositors, $R_A(1+r_A)$. When $L(F_A) > R_A(1+r_A)$, then there is a positive payout to the shareholders of bank A even when bank B cannot repay its interbank loan. This case applies if bank A's funds are mostly invested locally, i.e., F_A remains large, since the size of the interbank loan W and consequently also the respective promised repayment w are small. The other subcase is that where a failure of repayment from bank B causes default of bank A, i.e., interbank lending has a contagious effect. While then the proceeds from its own loans, $L(F_A)$, allow bank A to make some repayment to depositors, when its loan to bank B is not paid back, $L(F_A)$ is no longer sufficient to allow for a payout to shareholders as well. Finally, the case where w is sufficient to fully repay bank A's depositors' claims, $R_A(1+r_A)$ even when its own loans

[23]For instance, when lending markets are independent, so that $\rho = 0$, the respective probability becomes simply p^2.

do not perform, will never arise in equilibrium. Hence, there are only two cases to consider, namely $L(F_A) \geq R_A(1+r_A)$, and $L(F_A) < R_A(1+r_A)$.

We next state the profits of shareholders of bank B,

$$\pi_B = p\left[L(F_B) - w - R_B(1+r_B)\right], \tag{4}$$

where $F_B = R_B + W$. Shareholders of bank B only receive a positive payout when the bank's own loans perform. That profits are positive in this case will naturally arise in equilibrium, so that we can safely restrict consideration to this case. Given the presently assumed take-it-or-leave-it offer by bank A, we have that

$$w = L(F_B) - L(R_B). \tag{5}$$

Hence, in case there is a loan of size W from bank A to bank B, the respective repayment w, as specified in (5), ensures that bank B's profits are just equal to the "standalone payoff" $p\left[L(R_B) - R_B(1+r_B)\right]$.

Lemma 1 *Consider stage $t = 2$, where banks can arrange for an interbank loan W from bank A, which has more retail funding as $R_A \geq R_B$, to bank B. There are two cases to consider.*

- *In Case 1, the loan size W and the repayment w are chosen sufficiently small so that a failure of repayment does not cause the insolvency of the creditor bank A, as $L(R_A - W) \geq R_A(1+r_A)$. Then, there exists a threshold ρ_0, such that $W = W_1^*$ uniquely solves*

$$pL'(R_A - W_1^*) = [p^2 + \rho p(1-p)]L'(R_B + W_1^*) \tag{6}$$

  for $\rho > \rho_0$ and the corner solution $W_1^* = 0$ applies for $\rho \leq \rho_0$.

- *In Case 2, W and w are sufficiently large so that from $L(R_A - W) < R_A(1+r_A)$ a failure of repayment causes insolvency also of the creditor bank A. Then, $W = W_2^*$ uniquely solves*

$$L'(R_A - W_2^*) = L'(R_B + W_2^*). \tag{7}$$

For a discussion, note first that an efficient reallocation of funds through an interbank loan would require that $W = W^{**}$ with W^{**} solving $L'(R_A - W^{**}) = L'(R_B + W^{**})$ - or, expressed differently, $W^{**} = (R_A - R_B)/2$, so that the same amount of funding is allocated to either market. This holds in Case 2 of Lemma 1 (expression (7)), where $W = W_2^* = W^{**}$, but not in expression (6), where $W = W_1^* < W^{**}$, and also not if $W = 0$. In Case 1, unless banks' lending markets are perfectly correlated, so that $\rho = 1$, the interbank loan W_1^* thus remains inefficiently low. As a consequence, more of the total available funding, $R_A + R_B$, is allocated to loans in market A than to loans in market B.

For an illustration, suppose that loan performance across the two banks is independent ($\rho = 0$). Then, the negotiated interbank loan, if positive at all, is such that at this level the *non-risk-adjusted* return from loans of the creditor bank A is equal to the *risk-adjusted* return from loans of the debtor bank B: $L'(R_A - W_1^*) = pL'(R_B + W_1^*)$.

The results of Lemma 1 arise from the incentives of leveraged shareholders to engage in risk-shifting. As long as the correlation between the two lending markets is not perfect, as $\rho < 1$, interbank lending diversifies the overall loan exposure of bank A. That is, when bank A's own (corporate) loans fail, depositors can still be paid back, at least partly, when the loans in market B perform and the interbank loan is paid back. Thus, the diversification that results from interbank lending makes the claims of the depositors of bank A safer.[24] In Case 1 of Lemma 1, this positive externality of diversification generates a wedge between the allocation of funds that would be efficient (through choosing $W = W^{**}$) and the allocation of funds that results as an outcome of optimal interbank lending in shareholders' interest ($W = W_1^* < W^{**}$).

This wedge is intuitively smaller when the two lending markets become more (positively) correlated, in which case depositors of bank A have less to gain from such co-insurance of their deposits through interbank lending.[25] Consequently, the optimally arranged interbank loan W_1^* increases in Case 1 as lending markets become more correlated. The characterization of Case 1 would thus predict a positive correlation between the size of interbank lending and the correlation of local lending markets.

[24] At this point, it is immediate to see that $W < 0$ can not arise in equilibrium. This would be detrimental for shareholders as it both reduces efficiency and generates coinsurance benefits (now of depositors of bank B). Incidentally, such an extreme allocation of resources to one lending market (and thus extreme risk-taking) may however arise when banks are integrated.

[25] Of course, under full deposit insurance these benefits would be reaped rather by the deposit insurance institution than by insured depositors themselves. These considerations will prove important later when we endogenize whether banks are integrated or not.

Corollary 1 *Suppose that Case 1 from Lemma 1 applies. Then, as the correlation between banks' lending markets increases (higher ρ), the size of the interbank loan $W = W_1^*$ increases as well.*

Corollary 1 conducts a comparative analysis only for Case 1. Once we have derived the equilibrium for the full game, we will show that our model predicts a robust positive relationship between interbank lending and the correlation of banks' lending markets. For now, however, we postpone a further discussion of this implication.

The allocation of funding becomes efficient in Case 2 of Lemma 1. The reason is as follows. In this case the exposure of bank A to the risk of bank B is sufficiently large such that failure of repayment of the interbank loan would make bank A insolvent as well, regardless of the performance of its own loan portfolio. Then, $W = W_2^*$ solves (7). Intuitively, once the interbank loan is sufficiently large, so that a failure of repayment has such a "contagious effect", a *marginal* adjustment has no longer the discussed positive externality on depositors of bank A. To now proceed with the analysis, we need to distinguish between the cases where deposits are insured and where deposits are not insured.

3.2 Equilibrium Analysis: The Case with Insured Deposits

We now turn to stage $t = 1$ of our model. Recall that presently we have $R_n = M_n$ and that banks are non-integrated, so that the only way to reallocate funds between the two markets is through interbank lending, as analyzed in the preceding section. With deposit insurance the deposit rate equals $r_A = 0$ and, thus, the equilibrium analysis is simplified by the fact that the funding costs of bank A do not depend on depositors' expectations about the size of its interbank exposure. We then need to compare bank A's shareholders' expected profits in Case 1 which, from (3), is given by

$$\pi_{A1} = p\left[L(R_A - W_1^*) - R_A\right] + \left[p^2 + \rho p(1-p)\right]\left[L\left(R_B + W_1^*\right) - L\left(R_B\right)\right], \tag{8}$$

to their expected profits in Case 2,

$$\pi_{A2} = \left[p^2 + \rho p(1-p)\right]\left[L(R_A - W_2^*) + L(R_B + W_2^*) - L\left(R_B\right) - R_A\right]. \tag{9}$$

This gives rise to the following result.[26]

Proposition 1 *Suppose that deposits are insured and banks non-integrated. We have the following comparative results for the (generically uniquely determined) interbank loan in equilibrium, W^*: There exists a threshold $\widehat{\rho}$, such that*

- *when $\rho < \widehat{\rho}$, it holds that $0 \leq W^* = W_1^* < z$, as determined in Case 1 in Lemma 1,*
- *when $\rho \geq \widehat{\rho}$, it holds that $W^* = W_2^* = z$, as in Case 2 in Lemma 1.*

Overall, W^* is increasing in ρ.

The critical correlation $\widehat{\rho}$ in Proposition 1 is determined such that the expected Case 1 payoffs to bank A's shareholders in (8) equal their expected Case 2 payoffs in (9). Recall that in Case 2, shareholders receive a payment only if loans in both markets perform, which is more likely if the two markets are more closely correlated. Intuitively, their Case 2 payoffs are therefore more sensitive to the correlation between the two loan markets, implying that $\pi_{A2} - \pi_{A1}$ is strictly increasing in ρ. Together with Corollary 1, this immediately implies the comparative result.

For our subsequent discussion of empirical implications, we next state an additional and fairly obvious comparative result:

Proposition 2 *Suppose that deposits are insured and banks non-integrated. When $\rho < 1$, there exists a threshold $0 < \widehat{z} \leq M$ such that*

- *when $z < \widehat{z}$, it holds that $0 \leq W^* = W_1^* < z$, as determined in Case 1 in Lemma 1,*
- *when $z \geq \widehat{z}$, it holds that $W^* = W_2^* = z$, as in Case 2 in Lemma 1.*

As $W^* = W_2^*$ holds always when $\rho = 1$, W^* always increases in z.

[26] Clearly, since the interbank loan in Case 2 is strictly higher than that in Case 1, there exists a set of parameter combinations under which $L(M_A - W_1^*) > R_A > L(M_A - W_2^*)$, such that an interbank loan of size W_2^* is indeed contagious, while an interbank loan of size W_1^* is not.

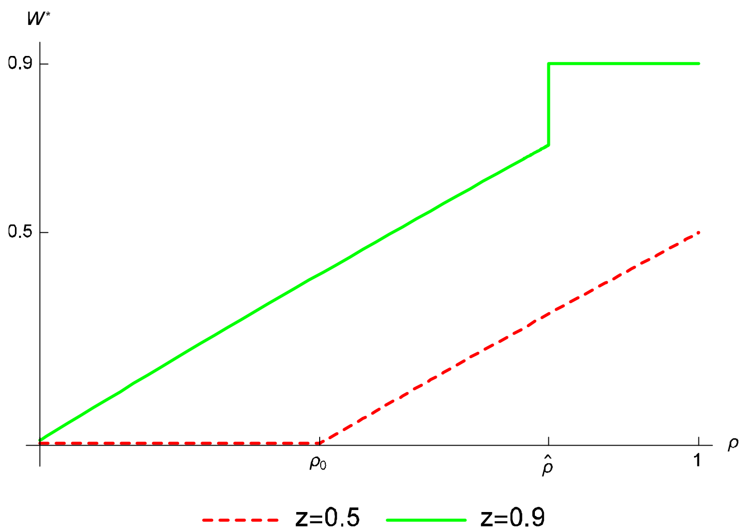

Figure 2: This graph plots the equilibrium size of the interbank loan W^* as a function of ρ for different levels of z. Parameter values are $p = 0.875$, $a = 0.0625$, and $b = 1.8$.

As the difference in the size of the two deposit markets, z, increases, there are two reasons for why the interbank loan increases in size as well, holding now the correlation ρ fixed. First, a larger interbank loan is then needed to reduce the gap between available local funding in the two markets. Second, Case 2 is more likely to apply when z and, thus, the outstanding claims of depositors in market A are larger. The intuition for this result is that in Case 2, depositors' claims $M_A = M + z$ can be fully repaid less often than in Case 1, namely only if loans in both markets are successful, which occurs with probability $p^2 + \rho p(1-p) < p$. Thus, $\pi_{A2} - \pi_{A1}$ is strictly increasing in z.

Illustration. Take a linear-quadratic loan-value function, $L(F) = bF - aF^2$. We normalize the size of funds so that, when there is symmetry, each bank has a potential deposit base of mass one: $M = 1$. For Figure 2 we allow for two different values for the initial funding difference: $z = 0.5$ and $z = 0.9$. The case with contagious interbank lending only arises when the asymmetry of retail deposits is sufficiently large. Note that then, as the correlation increases, W^* jumps upwards (at $\rho = \widehat{\rho}$). Further below we will make use of this feature to derive additional implications on observed exposures through interbank lending.

3.3 Equilibrium Analysis: The Case with Uninsured Financing

The case where debt financing is uninsured[27] is complicated by the fact that now the equilibrium funding rate depends on the bank's riskiness, which in case of bank A depends on its exposure not only to its own loan market but, in case $W > 0$, also to that of bank B. In fact, when depositors of bank A can expect to be co-insured through the repayment from an interbank loan, w, we have $r_A < 1/p - 1$, while when $W = w = 0$, bank A must pay $r_A = 1/p - 1$ to ensure that depositors break even in expectation.

The derivation of an equilibrium is however simplified by the observation that, while the outstanding repayment obligation affects which case of Lemma 1 applies, the optimal choice of W in the respective case is not affected. We denote by $q(r_A)$ the *probability* with which, for a given funding rate r_A, bank A chooses the efficient interbank loan $W_2^* = z$ (Case 2). As r_A determines the claims of bank A's depositors, there exists a threshold $\widehat{r}_A \geq 0$ so that

$$q(r_A) = \begin{cases} 0 & \text{if } r_A < \widehat{r}_A \\ \in [0,1] & \text{if } r_A = \widehat{r}_A \\ 1 & \text{if } r_A > \widehat{r}_A \end{cases} .$$

This reflects the fact that a given level of interbank lending is more likely to be contagious, the higher the repayment required by bank A's depositors (this mirrors the result of Proposition 2).

Having established bank A's optimal response to a given funding rate, r_A, we now determine the funding rate $r_A(q)$ that is required by bank A's depositors for a given *anticipated* strategy q. Recall that when Case 1 applies, bank A's depositors receive the full repayment $R_A(1 + r_A)$ whenever the loans in market A perform, while they receive just the repayment from the interbank loan, $w = L(R_B + W_1^*) - L(R_B)$, when only loans in market B are successful. When Case 2 applies, they receive the full repayment $R_A(1 + r_A)$ if loans in both markets perform, but if only bank A's (corporate) loans perform, they receive just $L(R_A - W_2^*)$. However, when only loans in market B perform, they receive the then higher repayment of the interbank loan, $w = L(R_B + W_2^*) - L(R_B)$. As the co-insurance benefit is, thus, larger in this case, $r_A(q)$ is strictly decreasing in q. Taken together, the break-even

[27] Recall again that for our analysis the precise form of debt that arises not from interbank lending is inconsequential and we uniformly refer to this as (insured or uninsured) deposits.

deposit rate satisfies

$$
\begin{aligned}
R_A = & \Big[q\big(p^2+\rho(1-p)\big)+\big(1-q\big)p\Big]R_A\big(1+r_A(q)\big) \\
& + p(1-p)(1-\rho)\big(1-q\big)\Big[L\left(R_A-W_1^*\right)+L\left(R_B+W_1^*\right)-L\left(R_B\right)\Big] \\
& + p(1-p)(1-\rho)q\Big[L\left(R_B+W_2^*\right)-L\left(R_B\right)\Big].
\end{aligned}
$$

An equilibrium (in possibly mixed strategies) is given by a fixed point for (q^*, r_A^*), at which $q^* = q(r_A^*)$ and $r_A^* = r_A(q^*)$. This is illustrated graphically in Figure 3. The left-hand panel depicts the case of a pure-strategy equilibrium where the equilibrium value $W = W^*$ is characterized by Case 2, so that $W^* = z$ (efficient allocation of resources). Formally, the two graphs $q(r_A)$ and $r_A(q)$ intersect at a combination (q^*, r_A^*) where $q^* = 1$. Consequently, we have $r_A^* = r_A(q = 1)$. The right-hand panel of Figure 3 depicts the case of a mixed-strategy equilibrium. Formally, the two graphs $q(r_A)$ and $r_A(q)$ now intersect at a combination (q^*, r_A^*) where $0 < q^* < 1$ and where $r_A^* = r_A(q^*) = \widehat{r}_A$. That is, the bank obtains funding at interest rate $\widehat{r}_A$ and then chooses with probability q^* a large (and efficient) interbank loan and with probability $1 - q^*$ a small (and inefficient) interbank loan.[28]

Proposition 3 extends the comparative result of Proposition 1 to the case without deposit insurance. Now both the interbank loan $W^* = W_1^* < z$ that solves (6) and the probability that $W^* = W_2^* = z$ is chosen both increase in the correlation between the two markets.

Proposition 3 *Suppose that deposits are uninsured and banks non-integrated. Then, there exists a unique equilibrium, where the size of the equilibrium interbank loan W^* depends on the correlation of banks' local lending markets as follows. There are two thresholds $\widehat{\rho}_l < \widehat{\rho}_h$, such that*

- *when $\rho < \widehat{\rho}_l$, it holds that $0 \le W^* = W_1^* < z$, as in Case 1 in Lemma 1,*
- *when $\rho \ge \widehat{\rho}_h$ it hods that $W^* = W_2^* = z$, as in Case 2 in Lemma 1,*
- *when $\rho \in (\widehat{\rho}_l, \widehat{\rho}_h)$, the bank mixes between the following outcomes: It chooses $W^* = z$, as in Case 2 in Lemma 1, with probability $q^* \in (0,1)$ and with probability $1 - q^*$ it chooses $0 \le W^* < z$, as in Case 1 in Lemma 1.*

[28] Recall that the respective sizes W_2^* and W_1^* are independent of r_A.

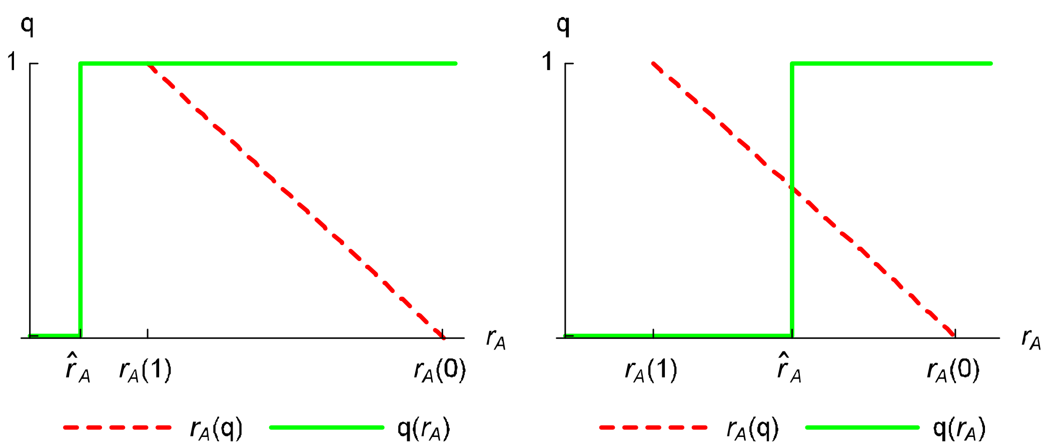

Figure 3: This graph plots bank A's optimal choice, $q(r_A)$, and the required interest rate of its depositors, $r_A(q)$. For different values of ρ, the left hand panel illustrates a pure-strategy equilibrium with $q^* = 1$ and the right hand panel plots the case of a mixed-strategy equilibrium. We resort to the same linear-quadratic loan-value function, $L(F) = bF - aF^2$, as in Figure 2 with parameter values $p = 0.875$, $a = 0.0625$, $b = 1.8$, $z = 0.6$.

The probability q^* increases in ρ (strictly so for $\rho \in (\widehat{\rho}_l, \widehat{\rho}_h)$). As also W^* increases in ρ in Case 1, and it stays constant in Case 2, the expected interbank loan surely increases in ρ.

Finally, we state the analogous comparative result to Proposition 2.

Proposition 4 *Suppose that deposits are uninsured and banks non-integrated. When $\rho < 1$, there are now two thresholds $0 < \widehat{z}_l < \widehat{z}_h \leq M$, such that*

- *when $z < \widehat{z}_l$, it holds that $0 \leq W^* < z$, as in Case 1 in Lemma 1,*
- *when $z > \widehat{z}_h$, it holds that $W^* = z$, as in Case 2 in Lemma 1,*
- *when $z \in (\widehat{z}_l, \widehat{z}_h)$, the bank mixes between the following two outcomes: It chooses $W^* = W_2^* = z$ as in Case 2 in Lemma 1 with probability $q^* \in (0,1)$ and with probability $1 - q^*$ it chooses $0 \leq W^* = W_1^* < z$, as in Case 1 in Lemma 1.*

The probability q^* increases in z (strictly so for $z \in (\widehat{z}_l, \widehat{z}_h)$). As also W^* increases both in Case 1 and Case 2, the expected interbank loan increases in z.

4 Allocation of Funds within an Integrated Bank

We now suppose that a single bank operates across both markets, A and B. We will ask how the resulting allocation of funds differs from that achieved when markets are served by separate banks. While the present analysis will be of interest on its own, as we notably derive conditions for when an integrated bank may either achieve more or less efficiency in its lending than separate banks, it will also form the background for our subsequent analysis of endogenous integration.

When a single bank, AB, operates in both markets, the question of whether retail deposit markets are fully segmented or not becomes superfluous. Also, as the repayment of all deposits is served by all of the bank's assets, in $t = 1$ the integrated bank will now offer the same interest rate r_{AB} to depositors in both markets. As there is no interbank lending, the game then proceeds to $t = 3$, where the bank allocates its aggregate funds over the two segmented lending markets, choosing F_A and F_B. Payoffs are again realized in $t = 4$. The integrated bank's shareholders' profits are given by

$$\begin{aligned} \pi_{AB} \quad = \quad & \left[p^2 + \rho p (1-p)\right] \left[L(F_A) + L(F_B) - R_{AB}(1 + r_{AB})\right] \\ & + p(1-p)(1-\rho) \max\{0, L(F_A) - R_{AB}(1 + r_{AB})\}. \end{aligned} \tag{10}$$

Note first that without loss of generality we restrict attention to cases where weakly more funds are allocated to market A: $F_A \geq F_B$. The first line in (10) accounts for the outcome where loans in both markets are successful.[29] With respect to the second line in (10), note first that the case where the repayment from loan market B alone would already be sufficient for the integrated bank to remain solvent can be ruled out. This follows from condition (2) and the observation that no resources will be wasted due condition (1), such that $F_B = 2M - F_A$ which, from $F_B \leq F_A$ implies that $F_B \leq M$. Hence, the shareholders of the integrated bank can only expect to receive a payout when the loans in market A perform. The case distinction in the second line of (10) is then whether this is indeed sufficient to make depositors whole, i.e., whether $L(F_A) > R_{AB}(1 + r_{AB})$ holds.

[29] Again, as in the case of separate banks, we abbreviate the analysis by stipulating that in this case the bank can indeed fully repay its depositors. This will clearly be the case in equilibrium.

4.1 Integrated Bank with Insured Deposits

As in the case of interbank lending, the optimal allocation of funds across the two markets is driven by two considerations: On the one hand, the maximization of total profits and thus efficiency, which obtains when M is allocated to either market, and, on the other hand, the reduction of a co-insurance effect (or, likewise, the maximization of risk-shifting) to the benefits of shareholders, though it reduces the value of depositors' claims.

Note now that for the following proposition we relabel the threshold for the case distinction with separate banks from Proposition 1 by $\widehat{\rho}^S$. Recall that $\widehat{\rho}^S$ denotes the threshold for the correlation between lending markets so that for $\rho \geq \widehat{\rho}^S$ interbank lending leads to an efficient allocation of funds among the two markets. In Proposition 5 below the corresponding threshold above which an integrated bank achieves the efficient allocation of funds will be denoted by $\widehat{\rho}^I$.

Proposition 5 *Suppose that deposits are insured and banks integrated. There exists a threshold $\widehat{\rho}^I$, such that the (generically unique) equilibrium allocation of funds F_B^* and F_A^* with $F_B^* + F_A^* = 2M$ is uniquely characterized as follows: When $\rho < \widehat{\rho}^I$ it is inefficient with $F_B^* < F_A^*$ as*

$$pL'(F_A^*) = \left[p^2 + \rho p (1-p)\right] L'(F_B^*) \tag{11}$$

or $F_B^* = 0$ holds and when $\rho \geq \widehat{\rho}^I$ it is efficient with $F_A^* = F_B^* = M$. Overall, the allocation of funds thus becomes more efficient as ρ increases ($F_A^* - F_B^* \geq 0$ decreases as both F_B^* increases and F_A^* decreases).

Though the characterization when condition (11) applies is analogous to that when condition (6) applies without integration (Case 1), the efficiency properties of this case with integration and non-integration can be markedly different. We first report the respective comparison before providing an intuition also for the characterization in Proposition 5.

Proposition 6 *Suppose that deposits are insured. When $\rho > \widehat{\rho}^I$ the equilibrium allocation of funds across markets is more efficient in the integrated bank, while for $\rho < \widehat{\rho}^I$ the allocation is less efficient in the integrated bank compared to when banks are non-integrated (and a reallocation of funds is thus achieved through interbank lending).*

To understand the difference between the allocation of funds through the interbank market and that in an integrated bank in Proposition 6, the treatment of depositors is key. When

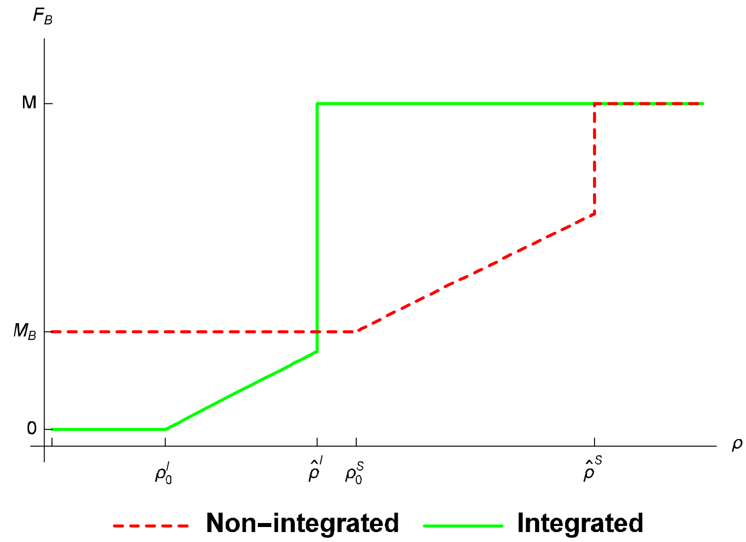

Figure 4: This graph plots the allocation of funds to market B achieved by non-integrated and integrated banks, respectively. For $\rho \geq \hat{\rho}^I$ ($\rho \geq \hat{\rho}^S$) integrated banks (separate banks) achieve an efficient allocation of funds, as $M_B + W^* = M$ ($F_B^* = M$). For $\rho < \rho_0^S$, there is no reallocation to market B via interbank lending ($W^* = 0$) and for $\rho < \rho_0^I$, an integrated bank allocates no funds at all to market B. We resort to the same linear-quadratic loan-value function, $L(F) = bF - aF^2$, as in Figure 2 with parameter values $p = 0.875$, $a = 0.04$, $b = 1.5$, $z = 0.7$.

banks are non-integrated, it is only through an interbank loan from A to B that depositors obtain claims on loans in market A *and* loans in market B. When no interbank loan is made, deposits in bank A and deposits in bank B will only be repaid when the loan in the respective local market performs. Instead, all deposits in the integrated bank represent senior claims, compared to those of shareholders, to the proceeds from loans in both market A and market B. The key difference lies thus in the "status quo" regarding the treatment of deposits, which for separate banks means that each bank's deposits are secured only by the assets of this bank, while for an integrated bank depositors in either market have senior access, compared to shareholders, to repayments of loans made in both markets.

As illustrated in Figure 4, when banks are non-integrated and the co-insurance externality is large, as $\rho < \rho_0^S$, the case with $W = 0$ provides the limit of risk-shifting through a lack of reallocation of funds across markets, as then no such co-insurance externality exists.[30]

[30] Recall from Lemma 1, that $W < 0$ will in fact never be optimal if $M_B < M_A$.

But this is different in an integrated bank, where an allocation of $F_A = M_A$ and $F_B = M_B$, which would correspond to the case with $W = 0$ for non-integrated banks, still involves co-insurance benefits for depositors. If these are sufficiently severe, i.e., for $\rho < \rho_0^I$, the integrated bank allocates all funds to market A. This is the reason why the allocation in the integrated bank is less efficient when $\rho < \widehat{\rho}^I$. We can thus say that in the integrated firm there exist a greater potential for risk-taking (through less diversification), which is indeed exploited when ρ is sufficiently low.

However, the larger repayment obligations of the integrated bank make it more likely that there is "contagion", i.e. that the failure of loans in one market fully erodes the claims of shareholders (cf. Proposition 2). Recall that contagion becomes more likely also when both markets are more likely to perform simultaneously (cf. Proposition 1). Taken together, this implies that the critical correlation above which a further increase in reallocation does not generate an additional co-insurance externality is lower for the integrated bank than in the case with interbank lending, i.e. $\widehat{\rho}^I < \widehat{\rho}^S$. Hence, when $\rho > \widehat{\rho}^I$, the allocation in the integrated bank is (weakly) more efficient compared to when banks are nonintegrated and a reallocation of funds is achieved through interbank lending.

The comparison in Proposition 6 derives clear-cut conditions for when an allocation of funds inside an integrated bank is more efficient than that achieved through interbank lending. To our knowledge, such a comparison has not yet been undertaken. Though our analysis is admittedly highly stylized, the respective simplifications allow to clearly isolate incentives for risk-shifting by leveraged shareholders as the driving force between the difference in allocations. Incentives and the scope for risk shifting, as manifested by a more asymmetric allocation of funds between the two markets, can both be lower and higher in an integrated bank, depending on the correlation between the loan-making opportunities in the two markets, ρ. We return in Section 6 to various normative and positive implications of Proposition 6.

4.2 Integrated Bank with Uninsured Financing

We show next how the basic insights of the comparison with insured deposits extend to the case with uninsured deposits. The key difference between the cases with and without deposit insurance will be uncovered only subsequently when we ask whether and when integration will arise in equilibrium.

For a characterization recall that without integration there was a mixed strategy equilibrium for intermediate values of the correlation coefficient ρ. We now denote the respective boundaries (in Proposition 3) with a superscript S and the analogous boundaries under integration with a superscript I (in Proposition 7). The following result comprises both a characterization and a comparison with the case of non-integration.

Proposition 7 *Suppose that deposits are uninsured and banks integrated. Then, there exist two thresholds $\widehat{\rho}_l^I < \widehat{\rho}_h^I$, such that*

- *when $\rho \leq \widehat{\rho}_l^I$, the equilibrium allocation of funds is inefficient as $F_B^* = 0$ or as (11) holds,*
- *when $\rho \geq \widehat{\rho}_h^I$, the equilibrium allocation of funds is efficient with $F_A^* = F_B^* = M$,*
- *when $\widehat{\rho}_l^I < \rho < \widehat{\rho}_h^I$, the bank mixes between the following outcomes: It chooses $F_A^* = F_B^* = M$ with probability $q^I \in (0,1)$ and with probability $1 - q^I$ it chooses F_A^* and F_B^* according to (11), where q^I strictly increases in ρ.*

Furthermore, there exists a unique threshold $\widehat{\rho}_l^I \leq \tilde{\rho} \leq \widehat{\rho}_h^I$ such that when $\rho \geq \tilde{\rho}$, the expected amount of funds allocated to market B is larger in the integrated bank, while for $\rho \leq \tilde{\rho}$ the expected amount of funds allocated to market B is smaller in the integrated bank compared to when banks are non-integrated (and a reallocation of funds is thus achieved through interbank lending).

5 Endogenous Integration

As discussed previously, integration can – at least when correlation between lending markets is not too low – lead to a more efficient reallocation of funds from market A, which has a larger deposit base, to loans made in market B. On the other hand, we showed as well how integration can lead to greater risk shifting when the correlation is low. Integration has, in addition, the immediate effect of providing co-insurance for all deposits, as depositors then have jointly a claim on all assets of A and B, albeit the scope of such co-insurance depends on the ensuing equilibrium allocation of funds across markets. Taking all these observations together, we now ask whether integration arises endogenously in stage $t = 0$ of our model.

For uninsured funding, interest rates positively react to the extent to which claims are co-insured by investments in both markets A and B. When banks are separated, this is only the case for the depositors of bank A and only when subsequently an interbank loan is made. Likewise, in cases where integration leads to greater risk taking, this will be equally anticipated by depositors and lead to higher funding costs. Such a feedback channel between funding costs and the decision to integrate is fully absent with insured deposits. Then only the immediate co-insurance externality remains, so that integration is never beneficial for shareholders.

Proposition 8 *Consider the case where funding is from insured deposits. Then banks will remain separate as integration would reduce shareholders' joint profits.*

A key prediction of Proposition 8 is that banks financed by insured deposits are likely to remain small and to resist mergers.[31] This should thus apply particularly to smaller, traditional savings and loans banks. These banks can reap the benefits from reallocating resources also through interbank loans, to the extent that they wish to do so, but without providing at the same time co-insurance benefits to depositors (or the deposit insurance fund) of the creditor bank B. Such an immediate co-insurance benefit also exists without deposit insurance, but in this case shareholders internalize the benefit through a lower interest rate.

Proposition 9 *Consider the case where funding is uninsured. Then there exists a unique threshold $\widehat{\rho}_l^I \leq \tilde{\rho}^* \leq \widehat{\rho}_h^I$ such that for $\rho \leq \tilde{\rho}^*$ banks will remain separate as integration would reduce shareholders' joint profits, and for $\rho \geq \tilde{\rho}^*$ banks will integrate as this increases shareholders' joint profits.*

Note that for ease of exposition, we have omitted in the statement of Proposition 9 a distinction between whether, in the respective parameter regions, banks strictly or weakly prefer to remain separate or to integrate. This distinction is made precise in the proof. In

[31] Note that one reason why we have in the main text abstracted from possible competition for deposits without integration is that then integration of banks would trivially lead to benefits, namely by lowering funding costs. We conjecture that the non-profitability result survives as long as competition between these two banks is not too intense without integration (or when there is sufficient competition from other institutions even after integration).

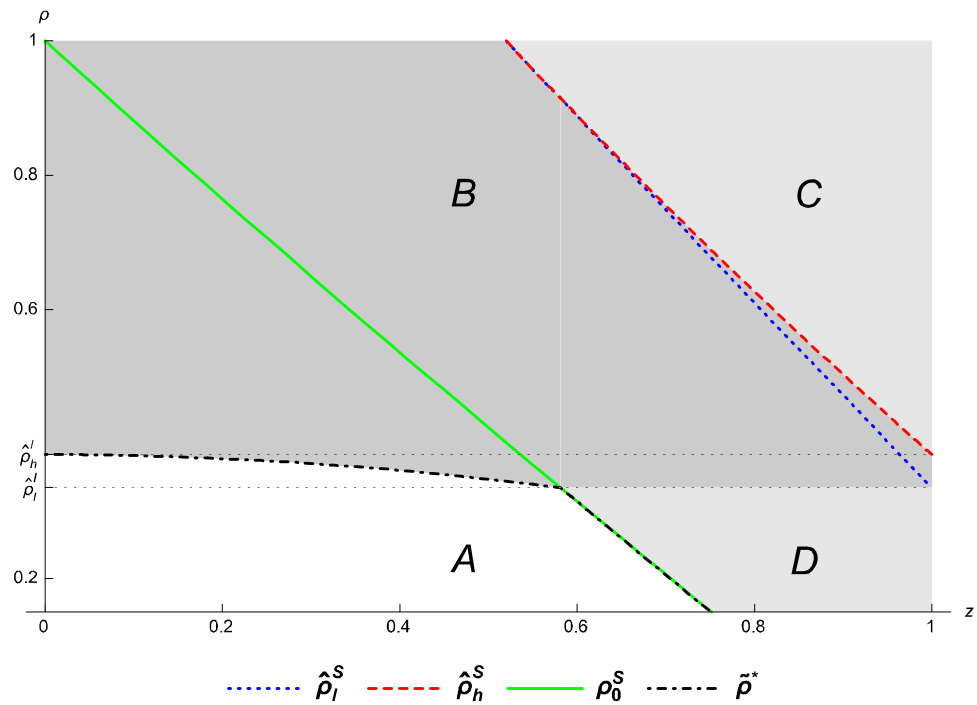

Figure 5: This graph plots the critical thresholds for ρ as a function of z. For $\rho \geq \widetilde{\rho}^*$, integration (weakly) increases banks' joint profits and for $\rho < \widetilde{\rho}^*$, integration would (strictly) decrease joint profits. Parameter values are $p = 0.875$, $a = 0.0625$, and $b = 1.8$.

addition, it is illustrated in Figure 5. There, $\tilde{\rho}^*$ is represented by the black dot-dashed line. Banks will stay separate as integration would strictly reduce their joint profits in region A, where $\rho < \tilde{\rho}^*$. The threshold $\tilde{\rho}^*$ is strictly decreasing in z. Integration would, by contrast, strictly increase banks' joint profits in region B. In region C, where $\rho \geq \widehat{\rho}_h^S \geq \widehat{\rho}_h^I$, integrated and non-integrated banks achieve an efficient allocation of funds, so that banks' joint profits are not affected by integration. A similar result prevails in region D, where integrated and non-integrated banks choose with probability one the same allocation of funds $F_A^* = M_A - W_1^*$ and $F_B^* = M_B + W_1^*$ according to (6) and (11), respectively, as $\rho_0^S \leq \rho \leq \widehat{\rho}_l^I \leq \widehat{\rho}_l^S$.

As we noted, the interest rate for uninsured deposits internalizes the expected co-insurance benefits. We also noted repeatedly that in our model there is no built-in disadvantage of non-integration in terms of additional frictions. Why then does the choice between integration and non-integration make a difference, as predicted by Proposition 9? The "boundaries of the bank" play the role of a commitment device vis-à-vis the providers of uninsured financing, given that once financing is obtained, the choice of the allocation of funding is made in the interest of shareholders alone. As shareholders are the residual claimants, from an ex-ante perspective they choose the "boundaries of the bank" so that the subsequent allocation of funds across loan markets is as efficient as possible. The "boundaries of the bank" are thereby derived from a single inefficiency that, as noted in the Introduction, is also at the heart of the vast majority of contributions to the theory of banking: shareholders' risk-shifting incentives.

6 Collection of Implications

We conclude our analysis by collecting the main implications. We have both testable positive implications and normative implications on the effects of regulation.

Empirical Implications. In our model, as (local) banks have an advantage in making loans, to achieve a more efficient allocation when there are differences in local funding, it is necessary that funds are either reallocated through interbank lending or within an integrated bank that operates across markets.[32] We derive implications both for loans made between banks and for whether and when we should observe integration that could facilitate the reallocation of funds.

Implication 1. *The size of an interbank exposure should increase both with the difference in banks' local funding base and with the correlation between local lending markets.*

As in much of the theoretical literature on banking, recall that our results are driven by a risk-shifting motive of shareholders. In our model this expresses itself in an insufficient realization of efficiency gains from reallocating resources as the ensuing diversification would benefit depositors. This is also the rationale for why interbank lending increases with the correlation between local lending markets. It should also be noted that this result is not driven by banks speculating on a "joint" bail-out and that the increase in interbank lending increases efficiency. This should be born in mind when considering our next implication.

[32] Notably, also retail competition alone is insufficient as long as a local bank still enjoys an advantage also on the funding side, e.g., due to switching costs of depositors; cf. Part 1 of the Online Appendix.

Implication 2. *We expect interbank exposure to be clustered in the following way: Provided that lending markets are only weakly correlated, interbank exposure should be very low but increasing in correlation (or zero if funding bases are of similar size). For sufficiently high levels of correlation, interbank exposure should be very high, unaffected by a further increase in correlation and bear the risk of contagion.*

Recall that the potential "clustering‘" of (empirical) observations (at low or high interconnectedness) follows from the described contagious effect, which decreases the positive externality of higher interbank lending on depositors and which only kicks in when the interbank exposure is sufficiently large.

Implication 3. *An integrated bank that operates in different (funding and lending) markets can have both a more and a less symmetric allocation of funds across the different markets when compared to the operations of a non-integrated bank that rely on interbank lending to reallocate funds across markets. The allocation of the integrated bank is more diversified when the correlation between the loans across markets is relatively high, and it is less diversified when the correlation is relatively low.*

Recall that the key insight that leads to Implication 3, where one compares allocative efficiency and diversification across markets, is the following: In an integrated bank that secures funding from various markets all deposits represent claims to all assets, i.e., to all loans made in different markets, whereas for non-integrated banks the respective deposits are only secured by local loans, unless there is interbank lending as well. It should be noted, however, that Implication 3 does not yet take into account that integration is itself endogenous. Still, as in practice there could be other obstacles to integration, such as regulatory or cultural constraints, but also conducive factors, such as managerial hubris, Implication 3 may also lend itself to the derivation of empirical predictions. The next implications focus, instead, on the equilibrium choice of integration.

Implication 4. *Banks that rely on insured deposits have lower (or even no) incentives to integrate, even when this leads to an inefficiently low reallocation of funds through interbank lending.*

When deposits are insured, shareholders can not benefit through lower funding costs from higher co-insurance of deposits when integration would lead to greater diversification of loan-making. Unless integration leads to other gains, such as reduced competition in the deposit market, it will thus not materialize when banks rely mainly on insured deposits. This could apply, for instance, to cooperative or savings and loan banks that have a strong retail presence and thus typically a large retail deposit funding base. Implication 4 predicts that this segment of the banking industry should remain heavily fragmented. This is different for banks that rely mainly on uninsured funding.

Implication 5. *Integration is more likely between banks with a more correlated lending market.*

The correlation between two lending markets could itself be the outcome of smaller or greater economic integration between the two regional or national economies. When this is taken as given, Implication 5 predicts that also banking mergers between these two already more integrated economies become more likely. Recall that Implication 1 obtains an analogous prediction for interbank lending. Taken together, economic integration through real activity, such as trade, and financial integration through interbank lending or bank merger are thus complementary, rather than one being a (perfect) substitute for the other.

This observation has also some direct normative implications with respect to policy and regulation that we explore further below. Note finally that rather than applying only "cross-sectionally", Implication 5 applies also when other forces, such as increasing trade or joint economic policy as witnessed in the European Union, lead to increasing economic integration between different regions and countries. Then the integration of banks should follow suit, beyond what the removal of legal and regulatory obstacles would suggest. That however the level of integration through the "banking channel" remains still insufficient is stated in the Implication 7 below.

Over time, the economic integration between different regions or countries may also decrease, or there may be other reasons for why lending markets become less correlated. Though this may admittedly be a far shot, given that our model is on purpose as parsimonious as possible, the currently witnessed disintegration (or "de-synchronization") of the European economies, notably the different development of those on its southern periphery, may be a case in place. Our model would predict that this should also reduce interbank lending *beyond* what can be accounted for by a worsening of economic prospects or financial fragility of debtor banks.

Implication 6. *As economic integration between two regions or countries deepens, also financial integration through the "banking channel", that is both through interbank lending and the reallocation of funds through integrated banks, should increase. Instead, when the correlation between two markets decreases, also financial integration through bank mergers and interbank lending should decrease.*

Normative Implications. From an efficiency perspective, the following implication is key.

Implication 7. *When the reallocation of funding across two (regional or national) funding and lending markets relies crucially on banks and their specific ability to collect funds from households and to invest in local business, then there is a strong tendency for too little financial integration (through both interbank lending as well as bank mergers and the subsequent reallocation within the integrated bank).*

Rather than excessive interconnectedness or excessive integration to form "too-big-too-fail" international banks, our parsimonious model of banking predicts the opposite: Too little exposure through interbank lending and too little financial integration through mergers and acquisitions among banks. As noted in the Introduction, we clearly abstract from other reasons for why banks may want to become "too-big-too-fail" or "too-interconnected-to-fail", namely if the expectation of a bail-out will lower their funding costs. What is however key, in our view, is the prediction that in the *absence* of such additional considerations the outcome will not be first-best efficient, but that it may involve a considerable gap in financial integration through interbank lending and mergers. The first-order effect of regulatory activities that further curb these activities may then be non-negligible and negative by further reducing allocative efficiency.

Our results also point to an unintended and likely ignored consequence of extended deposit insurance. Then, integration does no longer benefit banks through a commitment to more diversified lending, which then leads to lower funding costs. Instead, as we showed only positive co-insurance effect on depositors would remain, making integration unprofitable.

Implication 8. *Suppose through regulatory intervention banks' reliance on insured (retail) deposits becomes larger. Then rather than increasing financial integration, this makes fi-*

nancial integration through mergers and a reallocation of funds within integrated banks less likely, thereby reducing efficiency.

Note finally that banks may also choose to rely more on insured retail deposits when regulation makes funding through other (wholesale) sources more expensive (e.g., through liquidity requirements that are, however, outside our model; cf. the Introduction). Again, less financial integration may then be an unintended and negative consequence, according to Implication 8.

7 Conclusion

Our analysis presents a simple model of segmented funding and lending markets. Interbank lending as well as integration of banks can bridge funding differences and lead to more efficient lending across markets. As we discussed and analyze more formally in the Online Appendix to this paper, these channels prove relevant even when banks can compete for (deposit) funding across markets, as long as competition remains imperfect, e.g., due to the low granularity of deposits and switching costs. What makes our model particularly tractable is the focus on a single difference between markets, that is in the provision of local funding (z), and as we can capture the joint distribution of lending opportunities by a single variable (the correlation coefficient ρ). By varying z and ρ, next to considering both insured and uninsured funding, we derive a range of implications on integration, interbank lending, or on various policy measures. While we see the limitations given by tractability, future work could allow for more flexible specifications, e.g., by considering also imperfect correlation in local lending markets, introducing further asymmetries between markets, or considering and even endogenizing the mix of insured and non-insured funding that each bank receives.

Our theory of the "boundary of the bank" is tailored to the specific industry as it features interbank lending, insured deposit financing, and risk shifting as the primary agency problem. In fact, to our knowledge the extant literature on the "boundary of firms" does not consider the first two features, which is also why our theory allows to derive novel implications that are specific to the banking industry. On the other hand, notably Stein (1997) but also other contributions to the theory of the firm, have considered agency problems that should also be of first-order importance for banks, such as the use of soft information in the allocation of funding in a bank's "internal capital market". Combining the different approaches could also prove a fruitful avenue for future research.

Our model is clearly too simplistic to provide policy advice. In particular, it neglects aspects of systemic risk that are at the heart of the current policy debate (cf. the Introduction), notably as we do not consider an externality on the economy that could increase more-than-proportionally with the number and size of the failing banks. Such an externality could be included in the analysis, and one could then ask when limitations to interbank lending decrease or increase welfare. Presently, our analysis at least provides a reminder to policymakers that the formation of larger (integrated) banks and interbank lending both serve the purpose of facilitating the allocation of resources across otherwise segmented markets. Imposing limits or additional costs on these channels of reallocating resources should have a first-order effect on welfare, albeit this should in turn depend on the overall importance of these channels. Thus, a further avenue for future research would be to include other financial and non-financial institutions that could be active in collecting funds and directing these to investments in other markets, albeit these institutions may rely for this on different (hard instead of soft) information and may have access to only a fraction of household savings.

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Appendix

Proof of Lemma 1. Recall that implicit in expression (3) there are four different cases: Case 1 with $L(F_A) \geq R_A(1+r_A)$ and $w < R_A(1+r_A)$, Case 2 with $L(F_A) < R_A(1+r_A)$ and $w < R_A(1+r_A)$, Case 3 with $L(F_A) \geq R_A(1+r_A)$ and $w \geq R_A(1+r_A)$, and Case 4 with $L(F_A) < R_A(1+r_A)$ and $w \geq R_A(1+r_A)$. We treat these cases in turn and show that only Case 1 and Case 2 will be relevant for our subsequent analysis.

Consider first Case 1 where, after substituting $w = L(R_B + W) - L(R_B)$, it follows from (3) that the profits of bank A's shareholders are given by

$$\pi_{A1} = p\left[L(R_A - W_1^*) - R_A(1+r_A)\right] + \left[p^2 + \rho p(1-p)\right]\left[L(R_B + W_1^*) - L(R_B)\right].$$

Note that the program is strictly concave in this case. From inspection of expression (6) in Proposition 1, note next that W_1^* strictly increases in ρ. Using strict concavity, we can define for given $z > 0$ and p a value ρ_0 so that $W_1^* > 0$ only if $\rho > \rho_0$:

$$\rho_0 := \frac{1}{1-p}\left(\frac{L'(M+z)}{L'(M-z)} - p\right), \tag{12}$$

where further

$$\frac{d\rho_0}{dz} = \frac{1}{1-p} \frac{L''(M+z)L'(M-z) + L'(M+z)L''(M-z)}{L'(M-z)^2} < 0. \tag{13}$$

In Case 2 shareholder profits are from (3) equal to

$$\pi_{A2} = \left[p^2 + \rho p (1-p)\right] \left[L(R_A - W_2^*) - R_A(1+r_A) + L(R_B + W_2^*) - L(R_B)\right],$$

and the first-order condition yields (7). Note that also in this case the program is strictly concave.

Now consider Case 3, where (3) becomes

$$\pi_{A3} = p\left[L(R_A - W_3^*) - R_A(1+r_A) + L(R_B + W_3^*) - L(R_B)\right],$$

and the first order condition would imply that $R_A - W_3^* = R_B + W_3^* = M$. We now argue that if the interbank loan is sufficiently low so that repayment from its own loans is sufficient to repay A's depositors, the repayment from the interbank loan w can not at the same time be sufficiently high to repay depositors of bank A, i.e.,

$$L(R_B + W) - L(R_B) - R_A(1+r_A) < 0. \tag{14}$$

Condition (14) is for $R_B = 0$ implied by assumption (2). It also holds for $R_B \in (0, M)$ since the partial derivative of (14) with respect to R_B is, using $R_A = 2M - R_B$, given by

$$1 + r_A - L'(R_B) \leq \frac{1}{p} - L'(R_B),$$

which is strictly negative for $R_B \in [0, M]$ due to concavity of L and (1). (Recall also that $r_A \leq \frac{1}{p} - 1$.)

Finally, in Case 4 shareholders' profits in (3) are given by

$$\begin{aligned}\pi_{A4} &= p\left[L(R_B + W_4^*) - L(R_B) - R_A(1+r_A)\right] \\ &\quad + \left[p^2 + \rho p(1-p)\right] L(R_A - W_4^*)\end{aligned}$$

with first order condition

$$\left[p^2 + \rho p(1-p)\right] L'(R_A - W_4^*) = pL'(R_B + W_4^*).$$

Now consider two subcases. If $[p^2 + \rho p(1-p)] L'(R_B) > pL'(R_A)$, we have $W_1^* > 0$, implying that Case 4 is always (weakly) inferior to Case 1 as

$$\pi_{A1} - \pi_{A4} = p(1-p)(1-\rho) L(R_B).$$

When instead $[p^2 + \rho p(1-p)] L'(R_B) \leq pL'(R_A)$, then in Case 1 $W = 0$ so that A's profits are given by $\pi_{A0} := p[L(R_A) - R_A(1+r_A)]$ and the difference

$$\begin{aligned} \pi_{A0} - \pi_{A4} &= p[L(R_A) + L(R_B) - L(R_B + W_4^*)] \\ &\quad - \left[p^2 + \rho p(1-p)\right] L(R_A - W_4^*) \end{aligned} \tag{15}$$

is strictly positive as well. To see this note that when $[p^2 + \rho p(1-p)] L'(R_B) = pL'(R_A)$, it holds that $\pi_{A0} = \pi_{A1}$ and thus (15) is equal to $p(1-p)(1-\rho) L(R_B)$. Differentiating (15) with respect to R_B, again using $R_A = 2M - R_B$, yields

$$\frac{d}{dR_B}(\pi_{A0} - \pi_{A4}) = p[L'(R_B) - L'(R_A)] > 0.$$

Finally, note that $W < 0$ is never optimal as long as $R_A \geq R_B$. To see this, note that for $W \leq 0$ bank A's profits are given by

$$p[L(R_A + W) - w - R_A(1+r_A)].$$

Bank B breaks even if

$$w = \frac{p}{p^2 + \rho p(1-p)}[L(R_B) - L(R_B - W)],$$

which leads to first order condition

$$\left[p^2 + \rho p(1-p)\right] L'(R_A + W) = pL'(R_B - W).$$

Clearly, this implies that $W = 0$ unless

$$\left[p^2 + \rho p(1-p)\right] L'(R_A) > pL'(R_B),$$

which is ruled out by $R_A \geq R_B$. **Q.E.D.**

Proof of Proposition 1. Recall first that when a bank is not able to repay, then the deposit insurance agency covers the full repayment obligation. Hence, bank A's depositors break even with $r_A = 0$. Thus, starting from $W = 0$ we have from (3) the following derivative of π_A:

$$\frac{d\pi_A}{dW} = \begin{cases} [p^2 + \rho p(1-p)] L'(R_B + W) - pL'(R_A - W) & \text{if } L(R_A - W) \geq R_A \\ [p^2 + \rho p(1-p)] [L'(R_B + W) - L'(R_A - W)] & \text{if } L(R_A - W) < R_A \end{cases},$$

where $R_A = M + z$ and $R_B = M - z$. Suppose now first that even when $W^* = z$, Case 2 does not apply as

$$L(M) \geq M + z, \tag{16}$$

Clearly, (16) holds when $z = 0$, while it does not hold for $z = M$ due to (2). Hence, there is a cutoff $\tilde{z}$ defined by

$$L(M) = M + \tilde{z}, \tag{17}$$

so that we can altogether rule out Case 2 if and only if $z \leq \tilde{z}$. Suppose now that Case 2 is feasible as $z > \tilde{z}$. Clearly, in Case 2 $W^* = z$ no longer depends on ρ. Also, it holds in Case 1 that $W^* < z$ (unless $\rho = 1$, so that there is perfect positive correlation). The crux is now that the objective function for the maximization problem with respect to W is now altogether no longer quasiconcave as we shift between different cases. We now compare bank A's shareholders' profits across the different cases evaluated at the respective optimal interbank loan. Consider first

$$\frac{d}{d\rho}(\pi_{A2} - \pi_{A0}) = p(1-p)\left[2L(M) - L(M-z) - (M+z)\right],$$

which is strictly positive. This is surely the case for $z = 0$ (cf. the much stronger condition (1)). Next, differentiating the expression with respective to z, it is strictly increasing when $L'(M-z) > 1$, which is also implied by (1). We finally show that also $\pi_{A2} - \pi_{A1}$ is increasing in ρ. Making use of the first-order condition (6), we have

$$\frac{d}{d\rho}(\pi_{A2} - \pi_{A1}) = p(1-p)\left[2L(M) - L(M - z + W^*) - (M+z)\right]. \tag{18}$$

To confirm that this is strictly positive, it is sufficient to do so at the highest value $L(M - z + W^*)$ that is still compatible with Case 1, which in turn is the lowest value at which still $L(M + z - W^*) = M + z$. But then the sign of the derivative is determined by

$$2L(M) - L(M - z + W^*) - L(M + z - W^*) > 0,$$

where we used strict concavity of L. **Q.E.D.**

Proof of Proposition 2. We are now rather brief as the analysis is largely analogous to the comparative analysis in ρ of Proposition 1. Taking first Case 1, note that from (12) we can now define, for given ρ, a cutoff z_0 so that indeed $W^* > 0$ when $z > z_0$, where $z_0 < M$ when

$$\rho > \frac{1}{1-p}\left(\frac{L'(2M)}{L'(0)} - p\right).$$

When $W^* > 0$, which is the case for $z > z_0$, where z_0 satisfies

$$\frac{L'(M + z_0)}{L'(M - z_0)} = p + \rho(1-p),$$

it is also strictly increasing in z. Next, note that $W^* = z$ will arise indeed only if z is sufficiently high, as

$$\begin{aligned} \frac{d}{dz}(\pi_{A2} - \pi_{A0}) &= \left[p^2 + p(1-p)\rho\right] L'(M - z) - pL'(M + z) + p(1-p)(1-\rho) \\ &> p(1-p)(1-\rho), \end{aligned} \tag{19}$$

which follows from $\left[p^2 + p(1-p)\rho\right] L'(M - z) > pL'(M + z)$ for $z > z_0$ and

$$\frac{d}{dz}(\pi_{A2} - \pi_{A1}) = p(1-p)(1-\rho).$$

Denote the critical level where $\pi_{A2} = \pi_{A1}$ by $\widehat{z}$. **Q.E.D.**

Proof of Proposition 3. First, recall that for a given interest rate r_A, the interbank loan set by bank A in $t = 2$ is either given by $W^* = z$ or W^* which solves (6), where the latter contains also the boundary solution with $W^* = 0$. We will now construct the optimal choice of bank A for a given interest rate r_A. If Case 2 applies and $W^* = z$, bank A's shareholders' profits are given by

$$\pi_{A2}(r_A) = \left(p^2 + \rho p (1-p)\right)\left[2L(M) - L(M-z) - (M+z)(1+r_A)\right].$$

If Case 1 applies and W^* solves instead (6), profits are given by

$$\begin{aligned} \pi_{A1}(r_A) &= p\left[L(M+z-W^*) - (M+z)(1+r_A)\right] \\ &\quad + \left(p^2 + \rho p(1-p)\right)\left[L(M-z+W^*) - L(M-z)\right]. \end{aligned}$$

Let q denote the probability with which bank A sets $W^* = z$, such that shareholders' expected profits are given by

$$\pi_A(q, r_A) := q\pi_{A2}(r_A) + (1-q)\pi_{A1}(r_A).$$

Note that Case 2 can only arise for $r_A \in [r_{Al}, \overline{r}_A]$, where $\overline{r}_A := [2L(M) - L(M-z)]/(M+z) - 1$ and $r_{Al} := L(M)/(M+z) - 1$. Next, Case 1 where W^* solves (6) can only arise for $r_A < r_{Ah} := L(M+z-W^*)/(M+z) - 1$. Now consider the difference in profits for a given interest rate r_A,

$$\Delta(r_A) := \frac{\partial \pi_A(q, r_A)}{\partial q} = \pi_{A2}(r_A) - \pi_{A1}(r_A), \tag{20}$$

and observe that

$$\begin{aligned} \Delta(r_{Ah}) &= \left(p^2 + \rho p(1-p)\right)\left[2L(M) - L(M-z+W^*) - L(M+z-W^*)\right] \\ &> 0, \end{aligned}$$

which follows immediately from strict concavity of L. Next,

$$\begin{aligned}\Delta(r_{Al}) &= \left(p^2+\rho p(1-p)\right)[L(M)-L(M-z+W^*)]-p[L(M+z-W^*)-L(M)]\\ &< \left[\left(p^2+\rho p(1-p)\right)L'(M-z+W^*)-pL'(M+z-W^*)\right](z-W^*)\\ &= 0,\end{aligned}$$

where the inequality follows from concavity of L and the last equality from (6). Furthermore, since $\partial\Delta/\partial r_A = p(1-p)(1-\rho)(M+z) > 0$, there exists a unique cutoff $\widehat{r}_A \in (r_{Al}, r_{Ah})$ such that $\Delta(\widehat{r}_A) = 0$ and the best response to r_A is given by

$$q(r_A) = \begin{cases} q=0 & \text{for} \quad r_A \in [0,\widehat{r}_A) \\ q\in[0,1] & \text{for} \quad r_A = \widehat{r}_A \\ q=1 & \text{for} \quad r_A \in (\widehat{r}_A, \overline{r}_A] \end{cases}.$$

The interest rate $r_A(q)$ at which depositors of bank A break even, given an anticipated probability q, is determined by

$$\begin{aligned}R(q,r_A) := &\left[q\left(p^2+\rho p(1-p)\right)+(1-q)p\right](M+z)(1+r_A)\\ &+p(1-p)(1-\rho)[q2L(M)+(1-q)L(M-z+W^*)-L(M-z)]-(M+z)\\ = &\ 0,\end{aligned} \tag{21}$$

where

$$r_A(0) = \frac{1-p}{p} - \frac{p(1-p)(1-\rho)}{p}\frac{L(M-z+W^*)-L(M-z)}{M+z} \tag{22}$$

and

$$r_A(1) = \frac{1-\left(p^2+\rho p(1-p)\right)}{p^2+\rho p(1-p)} - \frac{p(1-p)(1-\rho)}{\left(p^2+\rho p(1-p)\right)}\frac{2L(M)-L(M-z)}{M+z}. \tag{23}$$

An equilibrium is therefore given by a fixed point (q^*, r_A^*) of the correspondence (q, r_A) : $[0,\overline{r}_A]\times[0,1] \to [0,1]\times[r_A(0), r_A(1)]$. Note that (q, r_A) is non empty since the sign of $\overline{r}_A - r_A(1)$ is determined by the following expression

$$p(2L(M)-L(M-z)) - \left(p^2+\rho p(1-p)\right)(M+z),$$

which is positive by concavity of L, and $r_A(0) > 0$, so that there must indeed exist a fixed point with $r_A^* = r_A(q^*)$ and $q^* = q(r_A^*)$.

To show uniqueness, it is helpful to consider two cases. First, if there exists a $\tilde{q} < 1$ such that $r_A(\tilde{q}) \leq r_{Al}$, then there will be a unique fixed point with $q^* \in [\tilde{q}, 1]$ as $r_A(q)$ is strictly decreasing in q for $q \geq \tilde{q}$, i.e.

$$\frac{\partial r_A(q)}{\partial q} = -\frac{\partial R(q)/\partial q}{\partial R(q)/\partial r_A} < 0,$$

which follows from

$$\frac{\partial R(q, r_A)}{\partial q} = p(1-p)(1-\rho)\left[2L(M) - L(R_B + W_1^*) - (M+z)(1+r_A)\right]$$

which is strictly positive for $r_A \leq r_{Ah}$ and

$$\begin{aligned}\frac{\partial R(q, r_A)}{\partial r_A} &= \left[q\left(p^2 + \rho p(1-p)\right) + (1-q)p\right](M+z) \\ &> 0.\end{aligned}$$

Second, if such a $\tilde{q}$ does not exist, then $r_A(q) > r_{Ah}$ for all $q \leq 1$, so that there can only be a fixed point with $q^* = 1$, which must therefore be unique.

Comparative Analysis in ρ: Consider the smallest $\widehat{\rho}_h$, such that $q^* = 1$. Then $q^* = 1$ also for $\rho \in [\widehat{\rho}_h, 1]$. This follows as the critical interest rate $\widehat{r}_A$ decreases in ρ as

$$\frac{\partial \widehat{r}_A}{\partial \rho} = -\frac{\partial \Delta/\partial \rho}{\partial \Delta/\partial r_A} < 0, \tag{24}$$

where

$$\frac{\partial \Delta(r_A)}{\partial \rho} = p(1-p)\left[2L(M) - L(M - z + W^*) - (M+z)(1+r_A)\right] > 0, \tag{25}$$

and

$$\frac{\partial \Delta(r_A)}{\partial r_A} = p(1-p)(1-\rho)(M+z) > 0. \tag{26}$$

Furthermore, the interest rate that is required when $q = 1$ increases in ρ which follows from differentiating the break even condition (21):

$$\begin{aligned}\frac{\partial r_A(1)}{\partial \rho} &= -\frac{\partial R(1, r_A)/\partial \rho}{\partial R(1, r_A)/\partial r_A} \\ &> 0,\end{aligned}$$

where

$$\begin{aligned}\frac{\partial R(1, r_A)}{\partial r_A} &= \left(p^2 + \rho p(1-p)\right)(M+z) \\ &> 0\end{aligned} \tag{27}$$

and

$$\begin{aligned}\frac{\partial R(1, r_A)}{\partial \rho} &= -p(1-p)\left[2L(M) - (M+z)(1+r_A) - L(M-z)\right] \\ &< 0.\end{aligned}$$

Since $\partial r_A(q)/\partial q < 0$, the only equilibrium that can be supported for $\rho \in (\widehat{\rho}_h, 1]$ is therefore that where $W^* = z$ is chosen with probability one.

Now consider $\widehat{\rho}_l$, the largest value where $q^* = 0$ can be supported and where thus $\Delta(r(0)) = 0$. Next, note that at $\widehat{\rho}_h$ it holds that $\Delta(r(1)) = 0$. Since $\partial r_A(q)/\partial q < 0$ we must have that $r_A(1) < r_A(0)$. Together with (26) and (25), this implies that $\widehat{\rho}_h > \widehat{\rho}_l$. Hence, for $\rho \in (\widehat{\rho}_l, \widehat{\rho}_h)$ there does not exist a pure strategy equilibrium and we will now show that q^* is strictly increasing in ρ on that interval.

Note first that (24) can be written more explicitly

$$\frac{\partial}{\partial \rho}\left[(M+z)(1+\widehat{r}_A)\right] = -\frac{2L(M) - L(M-z+W^*) - (M+z)(1+\widehat{r}_A)}{1-\rho}. \tag{28}$$

Now consider the break even condition for depositors evaluated at q^* and $\widehat{r}_A$

$$\begin{aligned}&\left[q^*\left(p^2 + \rho p(1-p)\right) + (1-q^*)p\right](M+z)(1+\widehat{r}_A) \\ &+p(1-p)(1-\rho)\left[q^* 2L(M) + (1-q^*)L(M-z+W^*) - L(M-z)\right] - (M+z) = 0.\end{aligned}$$

Differentiation with respect to ρ yields

$$\begin{aligned} & p(1-p)\left[q^*(M+z)(1+\widehat{r}_A)-q^*2L(M)-(1-q^*)L(M-z+W^*)+L(M-z)\right] \\ & +\frac{\partial}{\partial\rho}\left[(M+z)(1+\widehat{r}_A)\right]\left[q^*\left(p^2+\rho p(1-p)\right)+(1-q^*)p\right] \\ & +\frac{\partial q^*}{\partial\rho}p(1-p)(1-\rho)\left[2L(M)-L(M-z+W^*)-(M+z)(1+\widehat{r}_A)\right] \\ & +p(1-p)(1-\rho)(1-q^*)L'(M-z+W^*)\frac{\partial W^*}{\partial\rho}=0. \end{aligned} \tag{29}$$

Substituting (28) gives us an expression for

$$\begin{aligned} \frac{\partial q}{\partial\rho} &= \frac{1}{(1-p)(1-\rho)^2} \\ &+\left(\frac{1}{1-\rho}\right)\frac{L(M-z+W^*)-L(M-z)-(1-\rho)(1-q^*)L'(M-z+W^*)\frac{\partial W^*}{\partial\rho}}{2L(M)-L(M-z+W^*)-(M+z)(1+\widehat{r}_A)}, \end{aligned} \tag{30}$$

which, as we will show now, is positive for $z<z_0$ and increasing in z for $z>z_0$ First, consider $z\in[0,z_0)$,[33] and recall from Proposition 2 that $W^*=\frac{\partial W^*}{\partial\rho}=0$ for $z<z_0$, implying that (30) becomes

$$\begin{aligned} \frac{\partial q^*}{\partial\rho} &= \frac{1}{(1-p)(1-\rho)^2} \\ &> 0. \end{aligned}$$

Next, for $z\in[z_0,M]$, we differentiate (30) with respect to z, noting that $\frac{\partial}{\partial z}L(M-z+W^*)=\frac{\partial}{\partial z}\left[L'(M-z+W^*)\frac{\partial W^*}{\partial\rho}\right]=\frac{\partial}{\partial z}\left[(M+z)(1+\widehat{r}_A)\right]=0$,[34] to get

$$\frac{\partial^2 q^*}{\partial\rho\partial z}=\left(\frac{1}{1-\rho}\right)\frac{L'(M-z)+(1-\rho)L'(M-z+W^*)\frac{\partial W^*}{\partial\rho}\frac{\partial q^*}{\partial z}}{2L(M)-L(M-z+W^*)-(M+z)(1+\widehat{r}_A)}. \tag{31}$$

This is strictly positive since $\frac{\partial W^*}{\partial\rho}>0$ (which is shown in Proposition 1) and $\frac{\partial q^*}{\partial z}>0$ (which is shown in Proposition 4). **Q.E.D.**

[33] Note that $z_0>0$ since $-pL'(M)+\left[p^2+\rho p(1-p)\right]L'(M)<0$.

[34] The first two statements follow immediately from the first order condition for W^* and the second statement from differentiating the indifference condition $\Delta(\widehat{r}_A)=0$.

Proof of Proposition 4. Consider the smallest value z_h for which $q^* = 1$ can be supported. Then, only $q^* = 1$ can be supported for $z \in [\widehat{z}_h, 1]$. To see this, note first that the profit difference $\Delta(r_A) = \pi_{A2} - \pi_{A1}$ is strictly increasing in the outstanding repayment obligation, $(1 + r_A)(M + z)$, as

$$\frac{d\Delta(r_A(1))}{dz} = p(1-p)(1-\rho)\left[1 + r_A(1) + (M+z)\frac{\partial r_A(1)}{\partial z}\right], \tag{32}$$

where

$$1 + r_A(1) + (M+z)\frac{\partial r_A(1)}{\partial z} = \frac{1 - p(1-p)(1-\rho)L'(M-z)}{p^2 + \rho p(1-p)}.$$

Note that Case 2 can only arise if $L(M) < (M+z)(1 + r_A(1))$ or, after substituting $r_A(1)$ from (23), if

$$L(M) < \frac{1}{p^2 + \rho p(1-p)}\left[M + z - p(1-p)(1-\rho)(2L(M) - L(M-z))\right]. \tag{33}$$

Note further that (33) is satisfied for $z = M$ due to Assumption (2) and it is violated for $z = 0$ due to Assumption (1). Furthermore Assumption (2) implies that for $z = M$ it also holds that

$$L(2M - W^*) < \frac{1}{p^2 + \rho p(1-p)}\left[2M - p(1-p)(1-\rho)(2L(M) - L(0))\right], \tag{34}$$

that is, the repayment obligation $(1 + r_A(1))(M+z)$ exceeds the repayment from bank A's corporate loans in Case 1 where W^* satisfies (6). Hence, as the right hand side of (33) is strictly concave in z, it must be strictly increasing in z, i.e.,

$$1 > p(1-p)(1-\rho)L'(M-z), \tag{35}$$

for $z \leq \tilde{z}$ which is defined as the smallest value for which (34) holds with equality. As for $z > \tilde{z}$ only $q^* = 1$ can be supported and for $z \leq \tilde{z}$ we get that $d\Delta/dz > 0$, $q^* = 1$ for $z > \widehat{z}_h$.

Finally, since $1 + r_A(0) + (M+z)\frac{\partial r_A(0)}{\partial z} < 1 + r_A(1) + (M+z)\frac{\partial r_A(1)}{\partial z}$, it follows immediately that $\widehat{z}_l < \widehat{z}_h$, where $\widehat{z}_l$ is the highest value for which $q = 0$ can be supported, and bank

A mixes for $z \in (\widehat{z}_l, \widehat{z}_h)$. Differentiating (21) with respect to z yields

$$\begin{aligned} \frac{dR\left(q^*, r_A^* = \widehat{r}_A\right)}{dz} &= \frac{\partial q^*}{\partial z} p\left(1-p\right)\left(1-\rho\right)\left[2L\left(M\right) - L\left(M - z + W^*\right) - \left(M+z\right)\left(1+r_A\right)\right] \\ &\quad + \left[q^*\left(p^2 + \rho p\left(1-p\right)\right) + \left(1-q^*\right)p\right]\left[\left(1+r_A\right) + \left(M+z\right)\frac{\partial \widehat{r}_A}{\partial z}\right] \\ &\quad - \left[1 - p\left(1-p\right)\left(1-\rho\right)L'\left(M-z\right)\right] \\ &= 0. \end{aligned} \tag{36}$$

It then follows from differentiating the indifference condition $\Delta\left(\widehat{r}_A\right) = 0$, that

$$\left(1 + \widehat{r}_A\right) + \left(M+z\right)\frac{\partial \widehat{r}_A}{\partial z} = 0, \tag{37}$$

such that due to (35), $\frac{\partial q^*}{\partial z} > 0$. **Q.E.D.**

Proof of Proposition 5. Note first that the profits of the integrated bank's shareholders (10) are equal to the profits of bank A's shareholders (3) once we substitute (5) and set $R_A = 2M$ and $R_B = 0$ in (3). The required interest rate when deposits are insured is given by $r_{AB} = 0$. Hence, the equilibrium characterization and comparative statics can be inferred from extending the analysis of separate banks in Lemma 1 and Proposition 1 to the case with $z = M$. More explicitly, when $\rho \leq \widehat{\rho}^I$, the equilibrium allocation of funds by an integrated bank, F_n^*, satisfies (11) which mirrors Case 1 in Lemma 1 (the corner solution of Case 1 with $F_B^* = 0$ and $F_A^* = 2M$ applies if $\rho \leq \rho_0^I$, given by (12) for $z = M$). We will refer to the F_n^* that solves (11) – analogously to W_1^* – as F_{n1}^*, for $n = A, B$. When $\rho > \widehat{\rho}^I$, the efficient allocation is achieved with $F_n^* = M$, which mirrors Case 2 in Lemma 1.

From (13) it follows then immediately that $\rho_0^S > \rho_0^I$ and, thus, the integrated bank achieves a strictly less efficient allocation than separate banks, as $F_B^* < R_B + W^*$ for $\rho < \rho_0^S$. Next, from combining (18) and (19) it follows immediately that $\widehat{\rho}^S > \widehat{\rho}^I$. Hence, for $\widehat{\rho}^I < \rho < \widehat{\rho}^S$, we get $R_B + W^* < F_B^* = M$, while $R_B + W^* = F_B^*$ for $\rho_0^S < \rho < \widehat{\rho}^I$. Finally, and $R_B + W^* = F_B^* = M$ for $\rho \geq \widehat{\rho}^S$. **Q.E.D.**

Proof of Proposition 7. As in Proposition 5, the equilibrium and comparative analysis can be inferred from extending the analysis of separate banks in Lemma 1 and Proposition 3 to the case with $z = M$. Note that now the required interest rate r_{AB} is given depositors' break even condition (21) for $z = M$. It then follows immediately from (36) that $\widehat{\rho}_l^I < \widehat{\rho}_l^S$ and $\widehat{\rho}_h^I < \widehat{\rho}_h^S$.

Now consider the expected amount allocated to market B, which, in case of an integrated bank, is given by

$$T_B^I := q^I \left(M - F_{B1}^*\right) + F_{B1}^*, \tag{38}$$

and in case of separate banks, it is given by

$$T_B^S := q^S \left(z - W_1^*\right) + \left(M - z + W_1^*\right). \tag{39}$$

Note first that for $\rho \leq \widehat{\rho}_l^I$,

$$T_B^I - T_B^S = F_{B1}^* - \left(M - z + W_1^*\right) \leq 0$$

with strict inequality for $\rho < \rho_0^S$. (Recall that this threshold denotes the value above which there is a positive interbank loan.) Next, for $\widehat{\rho}_l^I < \rho < \widehat{\rho}_h^I$, we show that $T_B^I - T_B^S$ is strictly increasing in ρ and eventually turns positive. Note that we have to consider various cases, depending on whether one or both thresholds $\rho_0^I < \rho_0^S$ fall into this interval. Differentiating (38) yields

$$\frac{dT_B^I}{d\rho} = \frac{dq^I}{d\rho}\left(M - F_{B1}^*\right) + \left(1 - q^I\right)\frac{dF_{B1}^*}{d\rho},$$

where $\frac{dF_{B1}^*}{d\rho} \geq 0$ (with strict inequality for $\rho > \rho_0^I$). Differentiating (39) yields

$$\frac{dT_B^S}{d\rho} = \frac{dq^S}{d\rho}\left(z - W_1^*\right) + \left(1 - q^S\right)\frac{dW_1^*}{d\rho},$$

where $\frac{dW_1^*}{d\rho} \geq 0$ (with strict inequality for $\rho > \rho_0^S > \rho_0^I$). Recall next that for $\rho < \rho_0^S$, it holds that $M - F_{B1}^* > z - W_1^*$. Furthermore, from (31), it follows that $\frac{dq^I}{d\rho} > \frac{dq^S}{d\rho}$. Taken together, we thus have $\frac{\partial T_B^I}{\partial \rho} \geq \frac{\partial T_B^S}{\partial \rho}$ for $\rho < \rho_0^S$. For $\rho \geq \rho_0^S$, we have $F_{B1}^* = M - z + W_1^*$ and, thus,

$$T_B^I - T_B^S = \left(q^I - q^S\right)\left(M - F_{B1}^*\right) > 0.$$

Finally, for $\rho \geq \widehat{\rho}_h^I$,

$$T_B^I - T_B^S = \left(1 - q^S\right)\left(z - W_1^*\right) \geq 0$$

with strict inequality ($q^S < 1$) for $\widehat{\rho}_h^I \leq \rho < \widehat{\rho}_h^S$. Hence, we have shown that there exists a unique $\widehat{\rho}_l^I \leq \tilde{\rho} \leq \widehat{\rho}_h^I$ such that $T_B^I - T_B^S \geq 0$ for $\rho \geq \tilde{\rho}$ and $T_B^I - T_B^S \leq 0$ for $\rho \leq \tilde{\rho}$. **Q.E.D.**

Proof of Proposition 8. Consider first $\rho \geq \widehat{\rho}^S$, where where we have

$$\pi_{AB2} - (\pi_{A2} + \pi_B) = -p(1-p)(1-\rho)[L(M-z) - (M-z)] < 0.$$

Next, take $\widehat{\rho}^I \leq \rho < \widehat{\rho}^S$, for which $\pi_{A1} > \pi_{A2}$ and, thus, $\pi_{AB2} - (\pi_{A1} + \pi_B) < \pi_{AB2} - (\pi_{A2} + \pi_B)$. Finally, for $\rho < \widehat{\rho}^I$, we have

$$\begin{aligned} \pi_{AB1} - (\pi_{A1} + \pi_B) &= pL(F_{A1}^*) + \left(p^2 + \rho p(1-p)\right) L(F_{B1}^*) \\ &\quad -pL(M + z - W_1^*) - \left(p^2 + \rho p(1-p)\right) L(M - z + W_1^*) \\ &\quad -p(1-p)(1-\rho) L(M-z) \\ &\leq -p(1-p)(1-\rho) L(M-z) \\ &< 0, \end{aligned}$$

where the first inequality follows from concavity of L (it holds strictly for $\rho < \rho_0^S$ where $F_{A1}^* > M + z - W_1^*$ and $F_{B1}^* < M - z + W_1^*$). **Q.E.D.**

Proof of Proposition 9. Note that at $t = 1$ when banks decide whether or not to integrate, expected profits of an integrated bank are given by

$$\pi_{AB} = p\left[q^I 2L(M) + \left(1 - q^I\right)\left(L(F_{A1}^*) + L(F_{B1}^*)\right)\right] - 2M$$

and joint expected profits of bank A and B are given by

$$\pi_A + \pi_B = p\left[q^S 2L(M) + \left(1 - q^S\right)\left(L(M + z - W_1^*) + L(M - z + W_1^*)\right)\right] - 2M.$$

Note first that for $\rho \leq \widehat{\rho}_l^I$,

$$\begin{aligned} \pi_{AB} - (\pi_A + \pi_B) &= p\left[\left(L(F_{A1}^*) + L(F_{B1}^*)\right) - \left(L(M + z - W_1^*) + L(M - z + W_1^*)\right)\right] \\ &\leq 0 \end{aligned}$$

with strict inequality for $\rho < \rho_0^S$.

Next, for $\widehat{\rho}_l^I < \rho < \widehat{\rho}_h^I$, we show that $\pi_{AB} - (\pi_A + \pi_B)$ is strictly increasing in ρ and eventually turns positive. First, for $\rho < \rho_0^I$, where $F_{B1}^* = W_1^* = 0$,

$$\frac{\partial}{\partial \rho}\left[\pi_{AB} - (\pi_A + \pi_B)\right] = p \left[\begin{array}{c} \frac{\partial q^I}{\partial \rho}\left(2L\left(M\right) - L\left(2M\right)\right) \\ -\frac{\partial q^S}{\partial \rho}\left(2L\left(M\right) - L\left(M+z\right) - L\left(M-z\right)\right) \end{array} \right] > 0,$$

which follows from concavity of L and the observation that $\frac{\partial q^I}{\partial \rho} > \frac{\partial q^S}{\partial \rho}$ by (31). By the same arguments (recall also that $F_{B1}^* < F_{A1}^*$ and $\frac{\partial F_{B1}^*}{\partial \rho} > 0$) it follows that for $\rho_0^I \leq \rho < \rho_0^S$,

$$\frac{\partial}{\partial \rho}\left[\pi_{AB} - (\pi_A + \pi_B)\right] = p \left[\begin{array}{c} \frac{\partial q^I}{\partial \rho}\left(2L\left(M\right) - L\left(F_{A1}^*\right) - L\left(F_{B1}^*\right)\right) \\ -\frac{\partial q^S}{\partial \rho}\left(2L\left(M\right) - L\left(M+z\right) - L\left(M-z\right)\right) \\ +\left(1-q^I\right)\left(L'\left(F_{B1}^*\right) - L'\left(F_{A1}^*\right)\right)\frac{\partial F_{B1}^*}{\partial \rho} \end{array} \right] > 0.$$

Next, for $\rho \geq \rho_0^S$, we have $F_{B1}^* = M - z + W_1^*$ and, thus,

$$\pi_{AB} - (\pi_A + \pi_B) = p\left(q^I - q^S\right)\left[2L\left(M\right) - \left(L\left(F_{A1}^*\right) + L\left(F_{B1}^*\right)\right)\right] > 0.$$

Finally, for $\rho \geq \widehat{\rho}_h^I$,

$$\pi_{AB} - (\pi_A + \pi_B) = p\left(1 - q^S\right)\left[2L\left(M\right) - L\left(M + z - W_1^*\right) + L\left(M - z + W_1^*\right)\right] \geq 0,$$

with strict inequality for $\widehat{\rho}_h^I \leq \rho \leq \widehat{\rho}_h^S$. Thus, there exists a unique cut off $\widehat{\rho}_l^I \leq \tilde{\rho}^* \leq \widehat{\rho}_h^I$, such that $\pi_{AB} - (\pi_A + \pi_B) \leq 0$ for $\rho \leq \tilde{\rho}^*$ and $\pi_{AB} - (\pi_A + \pi_B) \geq 0$ for $\rho \geq \tilde{\rho}^*$. **Q.E.D.**