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JEL Classification: D33, L4, O15

Keywords: Double marginalization, input-output linkages, Labor Share, Markups, Outsourcing, structural change

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Labor Share, Markups, and Input-Output Linkages – Evidence from the National Accounts*

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April 20, 2022

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The literature has suggested many possible reasons for the recent decrease in the U.S. labor share. We build a multi-sector model with input-output linkages that allows us to identify the key driving forces. We find that the decrease in the U.S. labor share reflects both sectoral forces, which can be identified with micro or NIPA data, and aggregation effects, which can be identified only with NIPA data. Specifically, we find that the main force was an increase in sectoral markups, which input-output linkages importantly amplified.

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1 Introduction

It is well recognized that the share of national income paid to labor (labor share for short) has decreased in the U.S. The decrease in the labor share is of interest because it tilts the distribution of national income towards the owners of capital, which may contribute to the rising income equality. The decrease in the labor share is also is of interest because it violates one of Kaldor's growth facts that the growth model was constructed to be consistent with.[1] Although a large literature has emerged, there is no consensus yet as to what the main forces are behind the decrease in the labor share. In their recent review article, "The Elusive Explanation for the Declining Labor Share", Grossman and Oberfield (2021) observed that by now we have enough candidates to "explain" a multiple of the decrease in the labor share.[2] They concluded that progress requires a unifying framework that captures the causal and the offsetting forces and accounts for general equilibrium effects. In this paper, we set out to develop such a unifying framework and connect it to the National Income and Product Accounts of the postwar U.S. (NIPA henceforth).

Our unifying framework has multiple sectors and input-output linkages. Disaggregating into different sectors is crucial because the labor share both differs across sectors [Valentinyi and Herrendorf (2008)] and evolves differently across sectors [Elsby et al. (2013)]. Therefore, it is potentially important to model forces that operate at the sectoral level and to capture compositional effects. Modeling input-output linkages is crucial because they can importantly amplify the forces that operate at the sectoral level [Rotemberg and Woodford (1995) and Basu and Fernald (2002)]. As a tractable starting point, we disaggregate into a goods sector and a services sector and consider two types of sectoral forces: those that affect the sectoral output elasticity of labor ("labor intensity") and those that affect the sectoral markup. An example for the former is capital deepening because of automation and an example for the latter is a change in the market structure that leads to more monopoly power. Decomposing the decrease in the U.S. labor share into the different components, we find that the main forces behind it were similar increases in sectoral markups that were amplified through input-output linkages. In contrast, forces that decreased the output elasticities of labor were concentrated in the goods sector. Since the goods sector shrank considerably over time, they contributed little to the decrease in the labor share.

A well-known challenge to implementing our kind of analysis is to separately identify the output elasticity of labor and the markup; see Bond et al. (2020) for further discussion. The underlying problem is that, while it is straightforward to calculate the payments to labor and

[1] See Herrendorf et al. (2019) for a recent update of the Kaldor growth facts.

[2] Important examples include: the increased costs of housing [Rognlie (2015)]; the inclusion of IPP capital in NIPA [Koh et al. (2021)]; increased market power as measured by markups [Farhi and Gourio (2018) and Barkai (2020)]; capital deepening resulting from a declining relative price of capital [Karabarbounis and Neiman (2014)] or from automation [Acemoglu and Restrepo (2018)]; increased competition from globalization [Elsby et al. (2013)]. See Grossman and Oberfield (2021) for a more complete list of the existing work on the labor share.

for intermediate inputs, one needs to split the remaining payments between capital and profits. The payments to capital are the product of the capital stock, which is reported in NIPA, and the user costs of capital, which is not reported in NIPA. We calculate the user costs of capital at the sectoral level since the 1950s by adopting the approach of Farhi and Gourio (2018), who calibrated a growth model to match observable macro trends and then used it to calculate the user costs of capital at the aggregate level since the 1980s. Their approach has three advantages in our contexts: it leads to a measure of the user costs of capital that naturally includes unobservable premia for risk; it can be applied at the sectoral level; it can be extended to the entire postwar period because the required data are available since the 1950s.

Our two-sector model has the following key features. Sectoral gross output can be delivered to the final uses consumption and investment or it can be used as an intermediate input in either sector. Gross outputs in the goods and the services sectors are Dixit-Stiglitz aggregators of given sets of goods and services varieties, respectively. Each variety is produced by a monopolist who charges a sector-specific markup over marginal costs and who is small relative to the rest of the economy (monopolistic competition). The production functions have constant returns and are Cobb-Douglas in capital, labor, and intermediate inputs from both sectors. The output elasticities of the different factors of production are sector specific. In the equilibrium of our two-sector model, the labor share equals the weighted sum of the sectoral labor shares, which in turn equal the ratios of the sectoral output elasticities of labor over the sectoral markups. The weights are the Domar weights, which capture the effects of the sectoral composition of final output as well as the input-output linkages between the sectors.[3]

We connect our two-sector model to the private U.S. economy during the postwar period. To the extent possible, we use NIPA data to restrict the model parameters. Using NIPA data, instead of micro data, has the advantage that it covers all market activity, captures the input-output linkages, and respects the standard adding up constraints. Following Karabarbounis and Neiman (2018) and Barkai (2020), we exclude real estate from our notion of the private economy. The justification is that a large part of the real-estate value added is imputed because it comes from owner-occupied housing that does not lead to observable market transactions. Moreover, NIPA does not include land as part of the capital stock because land is not reproducible. Since land is particularly important in the production of real estate, its omission from the accounts tends to inflate the markups in real estate more than on average [Rognlie (2015)].

We start with the first-order effects on the labor share of changes in the sectoral output elasticities of labor and in the sectoral markups that result when we hold the Domar weights constant. We find that together the first-order effects "explain" a multiple of the decrease in the labor share, just as Grossman and Oberfield (2021) observed. Interestingly, changes in the

[3]We note that our analysis could be generalized to finer disaggregations with more than two sectors. However, very fine disaggregations are problematic because of changes in the industry classification during the postwar period.

sectoral output elasticities of labor have more than twice as large a first-order effect on the labor share than changes in sectoral markups. One might be tempted to conclude that forces which reduce the importance of the input factor labor in production (e.g., automation) must have been a key reason for the decrease in the aggregate labor share. The picture changes dramatically though when we takes into account how the Domar weights evolve: now the main force behind the decrease in the postwar U.S. labor share is that both sectors' markups increased by similar amounts which was importantly amplified by the input-output linkages. The amplification through the input-output linkages arises because of double marginalization, that is, intermediate inputs are marked up when they are produced and when they are used; see Rotemberg and Woodford (1995) and Basu and Fernald (2002) for more discussion. Since the intermediate-input shares are sizeable in both sectors, double marginalization had a large quantitative effect and the aggregate markups turns out to be around twice as large as the sectoral markups.

We also find that forces that decrease the sectoral gross output elasticity of labor were not the main reason for the decrease in the postwar U.S. labor share. These forces fall into two categories: those that increase the value-added elasticity of capital ("capital deepening") and those that increase the gross-output elasticity of intermediate inputs ("outsourcing"). First, while there was considerable capital deepening, it happened exclusively in the goods sector. Since there was also structural change, labor was reallocated from the goods to the services sector, implying that the Domar weight of the goods sector decreased strongly, which largely offset the first-order effect of capital deepening. Second, while there was outsourcing of intermediate inputs in both sectors, it was not strong enough to have a quantitatively strong effect on the labor share.

Our findings imply that the forces behind the labor-share decrease at the aggregate level and in the goods sector are different. The results from the goods sector must therefore not be taken as representative for the aggregate economy. This statement applies particularly to papers that study the labor share decrease in the manufacturing sector, which is the dominant subsector of the goods sector and is not representative for the aggregate economy either.

The rest of the paper is organized as follows. Section 2 lays out the model and Section 3 characterizes the equilibrium. Section 4 describes the calibration strategy and the data we use. Section 5 connects the model to the data and reports our results. Sections 6 compares our results with the those in the literature. Section 7 concludes. An Appendix contains longer derivations and a detailed data description.

2 Model

In this section, we develop a multi-sector version of the model of Farhi and Gourio (2018). Our multi-sector model captures that the sectors produce gross output and that intermediate goods are marked up when they are produced and when they are used. Taking into account the resulting double marginalization is crucial for correctly measuring how sectoral markups affect the labor share.

2.1 Environment

There is a measure one of identical households, implying that all variables are interpreted as per capita variables. Households have the utility function of Epstein and Zin (1989) that separates the degree of risk aversion from the intertemporal elasticity of substitution:

$$U_t = \left((1-\beta)C_t^{1-\sigma} + \beta\left[E_t\left(U_{t+1}^{1-\theta}\right)\right]^{\frac{1-\sigma}{1-\theta}}\right)^{\frac{1}{1-\sigma}}. \tag{1}$$

$\beta \in (0,1)$ is the discount factor; $C_t \geq 0$ is household consumption in period t; $\sigma \geq 0$ is the inverse of the intertemporal elasticity of substitution in the deterministic case; $\theta \geq 0$ is the coefficient of relative risk aversion.

Final-output can be used for consumption and investment. Final output is a Cobb-Douglas aggregator of final goods and services:

$$Y_t = Y_{gt}^{\phi_g} Y_{st}^{\phi_s}. \tag{2}$$

Y denotes final output; Y_g and Y_s denote final goods and services; $\phi_g, \phi_s \geq 0$ are the output elasticities of goods and services; there are constant returns, $\phi_g + \phi_s = 1$.

The production functions of final goods and services are Dixit-Stiglitz aggregators of different varieties:

$$Y_{jt} = \left(\int_0^1 (Y_{j_it})^{\frac{\varepsilon_j - 1}{\varepsilon_j}} di\right)^{\frac{\varepsilon_j}{\varepsilon_j - 1}} \qquad (j \in \{g, s\}), \tag{3}$$

where Y_{j_it} is the quantity of variety $i \in [0,1]$ used in sector $j \in \{g, s\}$; ε_j is the sector-specific elasticity of substitution. Since ε_j is sector specific, markups will be sector-specific too.

The production functions of gross output of the different varieties of goods and services are Cobb-Douglas in capital, labor, and intermediate inputs from both sectors:

$$G_{j_it} = Z_{jt}\left(K_{j_it}\right)^{\alpha_{K_j}}\left(A_{jt}L_{j_it}\right)^{\alpha_{L_j}}\left(M_{jj_it}\right)^{\alpha_{M_{jj}}}\left(M_{j'j_it}\right)^{\alpha_{M_{j'j}}} \qquad (j \neq j' \in \{g, s\},\ i \in [0,1]). \tag{4}$$

G_{j_it} is gross output; K_{j_it} and L_{j_it} are capital and labor; M_{jj_it}, $M_{j'j_it}$ are intermediate inputs produced in sectors j, j' and used in the production of variety j_i; the α's are the sector-specific output elasticities. There are constant returns: $\alpha_{K_j} + \alpha_{L_j} + \alpha_{M_{jj}} + \alpha_{M_{j'j}} = 1$; Z_{jt} is sector-specific, deterministic TFP; A_{jt} is sector-specific technical change, which evolves according to a random walk:

$$A_{j,t+1} = A_{jt} \exp(\chi_{j,t+1}) \qquad (j \in \{g, s\}). \tag{5}$$

$\chi_{j,t+1}$ is an i.i.d. shock with mean zero that shifts productivity permanently and is common across all firms in sector j. Farhi and Gourio (2018) use $\chi_{j,t+1}$ to model rare disaster risk, which implies a risk premium although it materializes only occasionally. Note that to be comparable with Farhi and Gourio (2018), we have followed them and written the production function such that Z_{jt} is TFP whereas A_{jt} is labor-augmenting technical change. Given the Cobb-Douglas functional form, of course, it could be rewritten so that Z_{jt} adds to sectoral labor-augmenting technical change or A_{jt} adds to sectoral TFP.

Gross output of variety j_i is used as final output or as an intermediate input in one of the two sectors:

$$G_{j_it} = Y_{j_it} + M_{j_ijt} + M_{j_ij't} \qquad (j \in \{g, s\}). \tag{6}$$

M_{j_ijt} and $M_{j_ij't}$ are intermediate inputs of variety j_i used by sectors j and j'. To avoid confusion, it is worth mentioning that value added is different from gross output and is given by:

$$p_{V_{j_it}} V_{j_it} = p_{G_{j_it}} G_{j_it} - p_{G_{jt}} M_{jj_it} - p_{G_{j't}} M_{j'j_it} \qquad (j \neq j' \in \{g, s\}), \tag{7}$$

where $p_{V_{j_it}}$ is the price of value added and $p_{G_{j_it}}$ is the price of gross output of sector $j \in \{g, s\}$.

Capital is sector specific and accumulates according to:

$$K_{j,t+1} = \left[Q_t X_{jt} + (1 - \delta_j) K_{jt}\right] \exp(\chi_{j,t+1}) \qquad (j \in \{g, s\}), \tag{8}$$

where $\delta_j \in [0, 1]$ is the sector-specific depreciation rate; Q_t is the economy-wide marginal rate of transformation between output and investment, which captures that the quality of capital has been improving [Greenwood et al. (1997)]. Note that specification (8) assumes that the capital stock changes immediately after a shock to labor-augmenting technical change. Farhi and Gourio (2018) introduced this feature and interpreted it as a quality shock to existing capital that is in sync with the shock to labor-augmenting technical change. The technical reason for having it is that it shuts down the usual shock propagation through capital accumulation and keeps the economy on a balanced growth path even after a rare disaster shock happened. This feature implies that one can obtain an analytical solution for the equilibrium path without having to solve for transitional dynamics.

(8) implies that sectoral investment is described by:

$$X_{jt} = \exp(-\chi_{j,t+1})\frac{Q_{t+1}}{Q_t}Q_{t+1}^{-1}K_{j,t+1} - (1-\delta_j)\,Q_t^{-1}K_{jt} \qquad (j \in \{g, s\}). \tag{9}$$

Looking ahead to the equilibrium, Q_t^{-1} will be the price of capital relative to final goods implying that $Q_t^{-1}K_{jt}$ is the capital stock in units of the numeraire final good. In the NIPA, $Q_t^{-1}K_{jt}$ is called the capital stock evaluated at current cost or at replacement costs.

Feasibility requires the usual adding up constraints:

$$G_{jt} \equiv \int_0^1 G_{j_it}di \qquad (j \in \{g, s\}), \tag{10}$$

$$Y_{jt} \equiv \int_0^1 Y_{j_it}di \qquad (j \in \{g, s\}), \tag{11}$$

$$M_{j'jt} \equiv \int_0^1 M_{j'j_it}di \qquad (j', j \in \{g, s\}), \tag{12}$$

$$K_{jt} \equiv \int_0^1 K_{j_it}di \qquad (j \in \{g, s\}), \tag{13}$$

$$L_{jt} \equiv \int_0^1 L_{j_it}di \qquad (j \in \{g, s\}), \tag{14}$$

$$1 = \sum_{j=g,s} L_{jt}, \tag{15}$$

$$G_{jt} = Y_{jt} + \sum_{j'\in\{g,s\}} \int_0^1 M_{j'j_it}di \qquad (j' \in \{g, s\}), \tag{16}$$

$$Y_t = C_t + \sum_{j\in\{g,s\}} X_{jt}. \tag{17}$$

The state variables in period t are $\{Q_t, Z_{jt}, A_{jt}, K_{jt}\}_{j\in g,s}$. Q_t and $\{Z_{jt}\}_{j=g,s}$ grow at constant exogenous rates:

$$\begin{aligned} Q_{t+1} &= (1+\gamma_Q)Q_t, \\ Z_{j,t+1} &= (1+\gamma_{Z_j})Z_{jt} \qquad (j \in \{g, s\}). \end{aligned}$$

$\{A_{jt}, K_{jt}\}_{j=g,s}$ follow the laws of motion (5) and (8).

From here on out, we will no longer mention that $j, j' \in \{g, s\}$ when j, j' show up.

2.2 Producer Problems

We start with the production of final-goods. The market for final goods is competitive and the representative firm maximizes profits:

$$\max_{\{Y_{gt},Y_{st}\}} Y_{gt}^{\phi_g} Y_{st}^{\phi_s} - P_{gt}Y_{gt} - P_{st}Y_{st},$$

where the final good is the numeraire. The first-order conditions imply the usual Cobb-Douglas result that the values of goods and services are constant shares of the value of final output:

$$\frac{p_{G_{jt}} Y_{jt}}{Y_t} = \phi_j. \tag{18}$$

We continue with the production of goods and services. The markets for goods and services are also competitive and profit maximization gives:

$$\max_{\{Y_{j_it}\}_{i\in[0,1]}} p_{G_{jt}} \left(\int_0^1 (Y_{j_it})^{\frac{\varepsilon_j-1}{\varepsilon_j}} di \right)^{\frac{\varepsilon_j}{\varepsilon_j-1}} - \int_0^1 p_{G_{j_it}} Y_{j_it} di.$$

The first-order condition implies the standard demand function for each variety:

$$Y_{j_it} = Y_{jt} \left(\frac{p_{G_{j_it}}}{p_{G_{jt}}} \right)^{-\varepsilon_j}. \tag{19}$$

Each variety is produced by a monopolist. The equilibrium concept is monopolistic competition, that is, the monopolist takes aggregate variables as given but takes into account the demand function for the monopolist's variety, (19). Taking r_{jt}, w_t, $p_{G_{jt}}$, $p_{G_{j't}}$ as given, profit maximization gives:

$$\max_{\{G_{j_it},K_{j_it},L_{j_it},M_{jj_it},M_{j'j_it}\}} p_{G_{j_it}} G_{j_it} - r_{jt}K_{j_it} - w_t L_{j_it} - p_{G_{jt}} M_{jj_it} - p_{G_{j't}} M_{j'j_it} \quad \text{s.t.} \quad (4),\ (19).$$

The first-order conditions imply that the monopolist charges a markup over the rental prices. We denote the (gross) markup by μ_j. In equilibrium, it is given as:

$$\mu_j \equiv \frac{\varepsilon_j}{\varepsilon_j - 1}.$$

Imposing that there be symmetry in equilibrium, $j_i = j$, the first-order conditions can be written as:

$$\frac{\alpha_{K_j} p_{G_{jt}} G_{jt}}{K_{jt}} = \mu_j r_{jt}, \tag{20}$$

$$\frac{\alpha_{L_j} p_{G_{jt}} G_{jt}}{L_{jt}} = \mu_j w_t, \tag{21}$$

$$\frac{\alpha_{M_{jj}} p_{G_{jt}} G_{jt}}{M_{jjt}} = \mu_j p_{G_{jt}}, \tag{22}$$

$$\frac{\alpha_{M_{j'j}} p_{G_{jt}} G_{jt}}{M_{j'jt}} = \mu_j p_{G_{j't}}. \tag{23}$$

The first-order conditions reflect the usual result with monopolistic competition that the marginal value product of each factor equals a markup up over its marginal costs. Combining the first-order conditions with the input-output linkages, we obtain the relationship between sectoral gross output and aggregate final output.

Proposition 1 *$p_{jt} G_{jt}$ is proportional to Y_t:*

$$p_{G_{jt}} G_{jt} = \Phi_j Y_t, \tag{24}$$

where:

$$\Phi_j \equiv \frac{\left(1 - \frac{\alpha_{M_{j'j'}}}{\mu_{j'}}\right)\phi_j + \frac{\alpha_{M_{jj'}}}{\mu_{j'}}\phi_{j'}}{\left(1 - \frac{\alpha_{M_{jj}}}{\mu_j}\right)\left(1 - \frac{\alpha_{M_{j'j'}}}{\mu_{j'}}\right) - \frac{\alpha_{M_{jj'}}}{\mu_{j'}}\frac{\alpha_{M_{j'j}}}{\mu_j}}. \tag{25}$$

Proof. See Appendix A.1.

Note that the proposition implies that $\Phi_j = p_{G_{jt}} G_{jt}/Y_t$. In other words, Φ_j is the Domar weight of sector j; see e.g. Carvalho and Tahbaz-Salehi (2019). As usual, the Domar weights do not add up to one. The reason for this is that they do not only reflect the sectoral weights in final output, but also the input-output linkages.

Note too that (18) and (24) imply that, in equilibrium, sectoral gross and final output are proportional to each other as well:

$$\frac{G_{jt}}{Y_{jt}} = \frac{p_{G_{jt}} G_{jt}}{p_{G_{jt}} Y_{jt}} = \frac{\Phi_j Y_t}{\phi_j Y_t} = \frac{\Phi_j}{\phi_j} \qquad \Longrightarrow \qquad G_{jt} = \frac{\Phi_j}{\phi_j} Y_{jt}. \tag{26}$$

Next, we characterize the equilibrium labor allocation. (21) determines the sectoral labor share in gross output:

$$\frac{w_t L_{jt}}{p_{G_{jt}} G_{jt}} = \frac{\alpha_{L_j}}{\mu_j}. \tag{27}$$

Substituting (24) into (27) gives:

$$\frac{w_t L_{jt}}{\Phi_j Y_t} = \frac{\alpha_{L_j}}{\mu_j}. \tag{28}$$

Thus, sectoral labor is proportional to aggregate labor:

$$L_{jt} = \omega_{L_j} L = \omega_{L_j}. \tag{29}$$

Equation (28) also implies the decomposition of the aggregate labor share that we mentioned in the introduction and will use in the empirical part:

$$\frac{w_t L_t}{Y_t} = \Phi_g \frac{w_t L_{gt}}{p_{G_{gt}} G_{gt}} + \Phi_s \frac{w_t L_{st}}{p_{G_{st}} G_{st}} = \Phi_g \frac{\alpha_{L_g}}{\mu_g} + \Phi_s \frac{\alpha_{L_s}}{\mu_s}. \tag{30}$$

2.3 Household Problem

The household problem is:[4]

$$\max_{C_t, \{X_{jt}, K_{jt+1}\}_{j \in \{g,s\}}} \quad U_t = \left((1-\beta) C_t^{1-\sigma} + \beta \left[E_t \left(U_{t+1}^{1-\theta} \right) \right]^{\frac{1-\sigma}{1-\theta}} \right)^{\frac{1}{1-\sigma}}$$

$$\text{s.t.} \qquad (9), \quad C_t + X_{gt} + X_{st} = r_{gt} K_{gt} + r_{st} K_{st} + w_t. \tag{31}$$

We note that the household problem abstracts from taxes, which is as in Farhi and Gourio (2018). Taxes are not of first-order importance for the user cost calculations in our context; see the discussion in Barkai (2020).

Appendix A.2 shows that the first-order conditions imply the familiar Euler equation:

$$1 = E_t \left(D_{t+1} R_{t+1} \right), \tag{32}$$

where D_{t+1} is the stochastic discount factor and R_{t+1} is the stochastic return on capital:

$$D_{t+1} \equiv \beta \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \left[\frac{U_{t+1}}{\left[E_t \left(U_{t+1}^{1-\theta} \right) \right]^{\frac{1}{1-\theta}}} \right]^{\sigma - \theta}, \tag{33}$$

$$R_{jt+1} \equiv \left(1 - \delta_j + r_{jt+1} Q_{t+1} \right) \frac{Q_t}{Q_{t+1}} \exp(\chi_{jt+1}). \tag{34}$$

The stochastic discount factor does not only depend on the usual components β, C_{t+1}/C_t, and σ but also on the underlying risk and the degree of risk aversion θ. In the calibration that fol-

[4]Note that Farhi and Gourio (2018) allow for population growth and a changing employment-to-population ratio. Since these features are not essential for what we do, we assume for simplicity that labor equals the population, which is normalized to one.

lows below, we will not separately identify the different components because it is not required for obtaining the relevant parameters. Nonetheless, the fact that the different components are present will allow the model to match the usual targets and have a realistic risk premia. This is as in Farhi and Gourio (2018).

3 Risky Balanced Growth Path

3.1 Definition

Since there is rare disaster risk, we are looking for a *risky* balanced growth path ("RBGP" henceforth) instead of a standard BGP. In concrete terms, RBGP means that expected variables grow at constant trends including zero but there are unexpected, occasional level shifts of the trend. Put differently, growth along the RBGP comprises both a deterministic trend and a stochastic random walk. If there were no shocks, $\chi_{jt} = 0$, then the model would have a standard BGP along which all variables would grow at constant rates including zero.

Let $p_{K_t} \equiv Q_t^{-1}$ denote the price of capital relative to output and ρ_j the expected discount rate of sector j's profits.

Definition 1 *A RBGP is an equilibrium path along which the following holds:*

- Y_t, K_t, X_t, C_t, w_t, p_{gt}/p_{st}, $\{G_{jt}, Y_{jt}, K_{jt}, X_{jt}, M_{jjt}, M_{jj't}\}_{j,j' \in \{g,s\}}$ *grow at constant expected rates;*
- $\{L_{jt}\}_{j \in \{g,s\}}$, ρ_{jt}, r_{jt}/p_{K_t} *are constant.*

In the rest of this section, we construct a RBGP. While the basic steps are the same as in Farhi and Gourio (2018), the construction is more involved here because one must establish that despite the multi-sector structure, aggregate variables grow at constant expected rates. Crucial in this context is that the production functions for the varieties of each sector and the aggregator for the two sectoral final outputs are all of the Cobb-Douglas form. We will show that this implies that there is an aggregate Cobb-Douglas production function so that the proof of Farhi and Gourio (2018) applies. This is not a foregone conclusion in our model because the output elasticities of the production functions of varieties are sector specific. In contrast, the usual aggregation result with Cobb-Douglas production functions needs the assumption that the sectoral output elasticities are the same; see for example Herrendorf et al. (2014).

We also note that the Cobb-Douglas assumption implies that a given parametrization of our model will not be able to match changing sectoral compositions or changing factor intensities at the sector level. To capture these, we will connect the RBGPs for different parametrizations of the model to different subperiods of the postwar period.

3.2 Sectoral and Aggregate Capital

We start with the return on capital, $r_{jt+1}/p_{K_{t+1}}$. Along the RBGP, $r_{jt+1}/p_{K_{t+1}}$ is constant. Thus:

$$1 = E_t\left(D_{t+1}R_{jt+1}\right) = E_t\left(D_{t+1}\exp(\chi_{j,t+1})\right)\frac{1-\delta_j + r_{jt+1}/p_{K_{t+1}}}{1+\gamma_Q}.$$

We define the expected discount rate ρ_j as:

$$\rho_j \equiv \frac{1}{E_t\left(D_{t+1}\exp(\chi_{j,t+1})\right)} - 1.$$

Substituting the definition into the previous equation and rearranging gives an expression for the expected discount rate:

$$1+\rho_j = \frac{1-\delta_j + r_{jt+1}/p_{K_{t+1}}}{1+\gamma_Q}. \tag{35}$$

We continue with capital along RBGP. Substituting (20) into (35), along the RBGP:

$$1+\rho_j = \frac{1-\delta_j}{1+\gamma_Q} + \frac{\alpha_{K_j}}{\mu_j(1+\gamma_Q)}\frac{p_{G_{jt+1}}G_{jt+1}}{p_{K_{t+1}}K_{jt+1}}. \tag{36}$$

Therefore, the sectoral capital-output ratio is given as:

$$\frac{p_{K_{t+1}}K_{jt+1}}{p_{G_{jt+1}}G_{jt+1}} = \frac{\alpha_{K_j}}{\mu_j(\rho_j+\delta_j+\gamma_Q)}. \tag{37}$$

Next, we show that capital is allocated to the sectors in fixed proportions. To this end, substitute (24) into (37) and rearrange:

$$p_{K_{t+1}}K_{jt+1} = \frac{\alpha_{K_j}\Phi_j}{\mu_j(\rho_j+\delta_j+\gamma_Q)}Y_{t+1}. \tag{38}$$

Therefore, $K_{jt}/K_{j't}$ is constant and:

$$K_{jt} = \omega_{K_j}K_t, \tag{39}$$

where $\omega_{K_j} \in [0,1]$ are relative weights that add up to one.

3.3 Capital-Output Ratios

To derive the aggregate capital-output ratio, we construct an aggregate version of the Euler equation. We start by substituting (24) and (39) into (36):

$$1 + \rho_j = \frac{1 - \delta_j}{1 + \gamma_Q} + \frac{\alpha_{K_j}}{\mu_j(1 + \gamma_Q)} \frac{\Phi_j Y_{t+1}}{\omega_{K_j} p_{K_{t+1}} K_{t+1}}. \tag{40}$$

Multiplying this equation with ω_{K_j} and adding up implies that the aggregate capital-final-output ratio is constant too along the RBGP:

$$1 + \rho = \frac{1 - \delta}{1 + \gamma_Q} + \frac{\alpha_K}{\mu(1 + \gamma_Q)} \frac{Y_{t+1}}{p_{K_{t+1}} K_{t+1}}, \tag{41}$$

where:

$$\rho \equiv \sum_{j \in \{g,s\}} \omega_{K_j} \rho_j, \tag{42}$$

$$\delta \equiv \sum_{j \in \{g,s\}} \omega_{K_j} \delta_j, \tag{43}$$

$$\frac{\alpha_K}{\mu} \equiv \frac{\Phi_g \alpha_{K_g}}{\mu_g} + \frac{\Phi_s \alpha_{K_s}}{\mu_s}. \tag{44}$$

(41) implies an aggregate version of (37) that determines the ratio of capital in units of final output to final output:

$$\frac{p_{K_{t+1}} K_{t+1}}{Y_{t+1}} = \frac{\alpha_K}{\mu(\rho + \delta + \gamma_Q)}. \tag{45}$$

Two remarks are in order. First, rewriting (44), we can see that the aggregate markup is a weighted harmonic mean of the sectoral markups:

$$\frac{1}{\mu} = \frac{\Phi_g \alpha_{K_g}}{\alpha_K} \frac{1}{\mu_g} + \frac{\Phi_s \alpha_{K_s}}{\alpha_K} \frac{1}{\mu_s}. \tag{46}$$

Second, (44) determines α_K/μ but not α_K and μ individually. We will solve out for α_K below.

3.4 Trend Growth

To calculate the trend growth rates for output, capital, and intermediate inputs, we first establish that along the RBGP there is an aggregate Cobb-Douglas production function:

Proposition 2 *Along the RBGP, there is an aggregate Cobb-Douglas production function:*

$$Y_t = \Omega Z_t K_t^{\alpha_K} A_t^{\alpha_L}. \tag{47}$$

where Ω *is a constant,* $\alpha_L \equiv 1 - \alpha_K$*, and:*

$$\alpha_K \equiv \frac{\left[(1-\alpha_{M_{ss}})\phi_g + \alpha_{M_{gs}}\phi_s\right]\alpha_{K_g} + \left[(1-\alpha_{M_{gg}})\phi_s + \alpha_{M_{sg}}\phi_g\right]\alpha_{K_s}}{(1-\alpha_{M_{gg}})(1-\alpha_{M_{ss}}) - \alpha_{M_{gs}}\alpha_{M_{sg}}},$$

$$Z_t \equiv \left[Z_{gt}^{\left(1-\alpha_{M_{ss}}\right)\phi_g + \alpha_{M_{gs}}\phi_s} Z_{st}^{\left(1-\alpha_{M_{gg}}\right)\phi_s + \alpha_{M_{sg}}\phi_g}\right]^{\frac{1}{\left(1-\alpha_{M_{gg}}\right)\left(1-\alpha_{M_{ss}}\right)-\alpha_{M_{gs}}\alpha_{M_{sg}}}},$$

$$A_t \equiv \left[A_{gt}^{\alpha_{L_g}\left(1-\alpha_{M_{ss}}+\alpha_{M_{gs}}\right)} A_{st}^{\alpha_{L_s}\left(1-\alpha_{M_{gg}}+\alpha_{M_{sg}}\right)}\right]^{\frac{1}{\left[\left(1-\alpha_{M_{gg}}\right)\left(1-\alpha_{M_{ss}}\right)-\alpha_{M_{gs}}\alpha_{M_{sg}}\right]\alpha_L}}.$$

Proof. See Appendix A.3.

That there is an aggregate Cobb-Douglas production function implies an aggregate first-order condition for labor of the usual form:

$$\frac{\alpha_L Y_t}{L_t} = \mu w_t. \tag{48}$$

Turning now to the growth rates along the RBGP, the aggregate Euler equation (41) implies that $Y_{t+1}/(p_{K_{t+1}} K_{t+1})$ is constant. Using (47), that $\alpha_K + \alpha_L = 1$, and that $p_{K_{t+1}} = Q_{t+1}^{-1}$, the aggregate output-capital ratio can be expressed as:

$$\frac{Y_{t+1}}{p_{K_{t+1}} K_{t+1}} = \Omega \frac{Z_{t+1} K_{t+1}^{\alpha_K} A_{t+1}^{1-\alpha_K}}{p_{K_{t+1}} K_{t+1}} = \Omega \frac{(p_{K_{t+1}} K_{t+1})^{\alpha_K - 1}}{Z_{t+1}^{-1} A_{t+1}^{\alpha_K - 1} Q_{t+1}^{-\alpha_K}} = \Omega \left[\frac{p_{K_{t+1}} K_{t+1}}{Z_{t+1}^{\frac{1}{1-\alpha_K}} A_{t+1} Q_{t+1}^{\frac{\alpha_K}{1-\alpha_K}}}\right]^{\alpha_K - 1}.$$

Since the left-hand side is constant, the numerator and the denominator of the right-hand side grow at the same rate:

$$p_{K_{t+1}} K_{t+1} = T_{t+1} A_{t+1} k^* \qquad \text{where} \qquad T_{t+1} \equiv Z_{t+1}^{\frac{1}{\alpha_L}} Q_{t+1}^{\frac{\alpha_K}{\alpha_L}}, \tag{49}$$

and the constant k^* is such that the aggregate Euler equation holds initially. Note that given we have assumed that Z_{t+1} and Q_{t+1} grow at constant rates, the trend growth rate, γ_T, is constant along the RBGP. Note too that (39) implies that the sectoral capital stocks grow at the same rate as the aggregate capital stock.

Turning now to the growth rate of aggregate final output, the aggregate Euler equation (41) implies that $Y_{t+1}/(p_{K_{t+1}} K_{t+1})$ is constant so that, in expected terms, Y_{t+1} and $p_{K_{t+1}} K_{t+1}$ grow at the same trend growth rates γ_T and:

$$Y_t = T_t A_t y^*, \tag{50}$$
$$y^* \equiv (k^*)^{\alpha_K}, \tag{51}$$

where y^* and k^* are positive constants.

We finish with the remaining growth rates. (18) and (24) imply that

$$\gamma_{p_{G_j}} + \gamma_{G_j} = \gamma_{p_{G_j}} + \gamma_{Y_j} = \gamma_T.$$

Thus, $\gamma_{Y_j} = \gamma_{G_j}$. Moreover, (22) and (23) imply that:

$$M_{jjt} = \frac{\alpha_{M_{jj}}}{\mu_j} G_{jt} \qquad \Longrightarrow \qquad \gamma_{M_{jj}} = \gamma_{G_j}, \tag{52}$$
$$M_{j'jt} = \frac{\alpha_{M_{j'j}}}{\mu_j} \frac{p_{G_{jt}} G_{jt}}{p_{G_{j't}} G_{j't}} G_{j't} = \frac{\alpha_{M_{j'j}}}{\mu_j} \frac{\phi_j}{\phi_{j'}} G_{j't} \qquad \Longrightarrow \qquad \gamma_{M_{j'j}} = \gamma_{G_{j'}}. \tag{53}$$

To obtain $\{\gamma_{G_j}\}_{j\in\{g,s\}}$, we take the growth rates of the production functions while substituting in the previous equations:

$$\gamma_{G_g} = \gamma_{Z_g} + \alpha_{K_g}\gamma_K + \alpha_{M_{gg}}\gamma_{G_g} + \alpha_{M_{sg}}\gamma_{G_s}. \tag{54}$$
$$\gamma_{G_s} = \gamma_{Z_s} + \alpha_{K_s}\gamma_K + \alpha_{M_{ss}}\gamma_{G_s} + \alpha_{M_{gs}}\gamma_{G_g}. \tag{55}$$

These two equations have a unique solution for $\{\gamma_{G_j}\}_{j\in\{g,s\}}$ in terms of exogenous variables and γ_K, which we calculated above already.

3.5 Investment-Capital Ratios

We continue with investment. Along the RBGP, to a first-order approximation, the capital accumulation equation (9) becomes:

$$\frac{X_{jt}}{p_{K_t} K_{jt}} = \gamma_T + \delta_j + \gamma_Q. \tag{56}$$

Since the right-hand side is constant along the RBGP, X_{jt} and $p_{K_t} K_{jt}$ grow at the same rates.

Equation (56) aggregates:

$$\gamma_T + \delta + \gamma_Q = \sum_{j=g,s} \omega_{K_j}(\gamma_T + \delta_j + \gamma_Q) = \sum_{j=g,s} \omega_{K_j} \frac{X_{jt}}{p_{K_t} K_{jt}} = \frac{\sum_{j=g,s} X_{jt}}{p_{K_t} K_t} = \frac{X_t}{p_{K_t} K_t}.$$

Thus, in current prices the aggregate-investment-to-capital ratio is given by the aggregate version of (56):

$$\frac{X_t}{p_{K_t} K_t} = \gamma_T + \delta + \gamma_Q. \tag{57}$$

3.6 Price-Profit Ratios

For the calibration, it is essential to calculate the values of a monopolist in each sector along the RBGP. We assume that all monopolist profits are passed back to the household in the form of dividends. Denoting the dividends of the representative monopolist in sector j by Π_{jt+1}, the standard recursion implies:

$$p_{F_{jt}} = E_t\Big(D_{t+1}\left(\Pi_{jt+1} + p_{F_{jt+1}}\right)\Big).$$

Along the RBGP, the first-order conditions (20)–(23) imply that dividends are given as:

$$\Pi_{jt} = \frac{\mu_j - 1}{\mu_j} p_{G_{jt}} G_{jt}.$$

Moreover, expected dividends are given as:

$$\begin{aligned} E_t \Pi_{jt+i} &= \frac{\mu_j - 1}{\mu_j} E_t\left(p_{G_{jt+i}} G_{jt+i}\right) \\ &= E_t\left(\frac{\mu_j - 1}{\mu_j}(1 + \gamma_{p_{G_{t+i}} G_{t+i}}) \cdot \ldots \cdot (1 + \gamma_{p_{G_{t+1}} G_{t+1}}) p_{G_{jt}} G_{jt}\right) \\ &= (1 + \gamma_T)^i \Pi_{jt}. \end{aligned}$$

Iterating forward while invoking the transversality condition yields a version of the Gordon growth formula:

$$p_{F_{jt}} = \Pi_{jt} \sum_{i=1}^{\infty} \left(\frac{1 + \gamma_T}{1 + \rho_j}\right)^i \qquad \Longrightarrow \qquad \frac{p_{F_{jt}}}{\Pi_{jt}} = \frac{1 + \gamma_T}{\rho_j - \gamma_T}. \tag{58}$$

We will use that relationship to calibrate the expected sectoral discount rate ρ_j.

The previous relationships aggregate:

$$\Pi_t \equiv \sum_{j=g,s} \omega_{K_j} \Pi_{jt}, \tag{59}$$

$$\frac{1}{p_{F_t}} \equiv \sum_{j=g,s} \frac{\omega_{K_j} \Pi_{jt}}{\Pi_t} \frac{1}{p_{F_{jt}}}, \tag{60}$$

$$\frac{p_{F_t}}{\Pi_t} = \frac{1 + \gamma_T}{\rho - \gamma_T}. \tag{61}$$

4 Calibration

We have shown that the sectoral shares are constant along a RBGP because the production functions are of the Cobb-Douglas form or are Dixit-Stiglitz aggregators of Cobb-Douglas production functions. Therefore, our model generates structural change only in the sense that the sectoral composition may differ across different model parameterizations. To capture the effects of structural change, we assume that different subperiods represent different model parameterizations. As in Farhi and Gourio (2018), we consider the periods 1984–2000 and 2001–2016. Using the 2000–2001 as the transition years from one to the other period recognizes that around 2000 several trends changed. In addition, we consider 1957–1973 as the initial period. The choice of 1957 and 1973 as the first and last year of the initial period avoids the postwar boom and the decade after the first oil price shock, which cannot be captured well by a balanced growth paradigm.

The advantage of focusing on different RBGPs is that we can solve analytically for the parameter values of interest. The resulting formulas will be straightforward to connect to the data. The disadvantage is that we will only capture the effects of changes in the underlying forces *across* the subperiods, but not *within* the subperiods. If we could measure the annualized effects, they would likely be stronger than the effects across subperiods.

4.1 Calibration Strategy

The calibration proceeds in the following four steps: we calibrate trend growth as average growth in "normal" times; we calibrate the sectoral depreciation from the sectoral investment-capital ratio; we calibrate the sectoral discount factor from the sectoral price-earnings ratio; we calculate markups as revenue over total factor payments; we calculate output elasticities as factor shares in factor payments.

First, we start by calibrating trend growth γ_T. We calculate the trend growth rate as the average growth rate of aggregate final output during the years in which no rare disaster shocks materialize. When implementing this, we will follow Farhi and Gourio (2018) and calibrate γ_T during the third period for just 2001–2006, thereby excluding the Great Recession.

We continue with the calibration at the sectoral level. Second, given γ_T, we calculate $\delta_j + \gamma_Q$ from the investment-capital ratio (56):

$$\delta_j + \gamma_Q = \frac{X_j}{p_K K_j} - \gamma_T.$$

Third, given γ_T, we calculate ρ_j from the Gordon growth formula (58):

$$\rho_j = \gamma_T + \frac{(1+\gamma_T)\Pi_j}{p_{F_j}}. \tag{62}$$

Fourth, given $\rho_j + \delta_j + \gamma_Q$, we calculate μ_j and α_j for each of the two sectors from the first-order conditions (20)–(23) and the capital-gross-output ratio (37):

$$\frac{(\rho_j + \delta_j + \gamma_Q) p_K K_j}{p_{G_j} G_j} = \frac{\alpha_{K_j}}{\mu_j}, \tag{63}$$

$$\frac{w L_j}{p_{G_j} G_j} = \frac{\alpha_{L_j}}{\mu_j}, \tag{64}$$

$$\frac{p_{G_j} M_{jj}}{p_{G_j} G_j} = \frac{\alpha_{M_{jj}}}{\mu_j}, \tag{65}$$

$$\frac{p_{G_{j'}} M_{j'j}}{p_{G_j} G_j} = \frac{\alpha_{M_{j'j}}}{\mu_j}. \tag{66}$$

Imposing that $\alpha_{K_j} + \alpha_{L_j} + \alpha_{M_{jj}} + \alpha_{M_{j'j}} = 1$, we can solve for μ_j:

$$\mu_j = \frac{p_{G_j} G_j}{(\rho_j + \delta_j + \gamma_Q) p_K K_j + w L_j + p_{G_j} M_{jj} + p_{G_{j'}} M_{j'j}}.$$

The markup in sector j equals the value of sectoral output over the factor payments. The factor payments include the imputed user costs of capital, which are consistent with the formula of Hall and Jorgenson (1967). Under perfect competition, the value of output equals the factor payments. Under imperfect competition, the markup increases the value of output above the factor payments. Since we will work with ratios in sectoral gross output, it is worth pointing out that (67) can be restated as:

$$\mu_j = \frac{1}{\frac{(\rho_j+\delta_j+\gamma_Q) p_K K_j}{p_{G_j} G_j} + \frac{w L_j}{p_{G_j} G_j} + \frac{p_{G_j} M_{jj}}{p_{G_j} G_j} + \frac{p_{G_{j'}} M_{j'j}}{p_{G_j} G_j}}. \tag{67}$$

To calculate α_{K_j}, α_{L_j}, $\alpha_{M_{jj}}$, and $\alpha_{M_{j'j}}$, we substitute the expression for μ_j into (63)–(66), which gives:

$$\alpha_{K_j} = \frac{(\rho_j + \delta_j + \gamma_Q)p_K K_j}{(\rho_j + \delta_j + \gamma_Q)p_K K_j + wL_j + p_{G_j}M_{jj} + p_{G_{j'}}M_{j'j}}, \tag{68}$$

$$\alpha_{L_j} = \frac{wL_j}{(\rho_j + \delta_j + \gamma_Q)p_K K_j + wL_j + p_{G_j}M_{jj} + p_{G_{j'}}M_{j'j}}, \tag{69}$$

$$\alpha_{M_{jj}} = \frac{p_{G_j}M_{jj}}{(\rho_j + \delta_j + \gamma_Q)p_K K_j + wL_j + p_{G_j}M_{jj} + p_{G_{j'}}M_{j'j}}, \tag{70}$$

$$\alpha_{M_{j'j}} = \frac{p_{G_{j'}}M_{j'j}}{(\rho_j + \delta_j + \gamma_Q)p_K K_j + wL_j + p_{G_j}M_{jj} + p_{G_{j'}}M_{j'j}}. \tag{71}$$

The output elasticities of the different factors equal the cost shares of the different factors, that is, the individual factor payments divided by the total factor payments. This is a standard result for Cobb-Douglas production functions.

We stress that the individual components of the sum $\rho_j + \delta_j + \gamma_Q$ do not matter for the calibrated parameter values. In particular, we do not have to worry about what part of depreciation is technological, δ_j, and what part is economic, γ_Q. Instead, it is just the sum of the two that enters the calibration. This implies that our assumption that γ_Q is common to both sectors is not restrictive. Note also that:

$$\rho_j + \delta_j + \gamma_Q = \frac{(1+\gamma_T)\Pi_j}{p_{F_j}} + \frac{X_j}{p_K K_j}. \tag{72}$$

The values of Π_j/p_{F_j} and $X_j/(p_K K_j)$ on the right-hand side are of first-order quantitative importance for the sum on the left-hand side and the calibration. In contrast, since γ_T is small, its exact value is not of first-order importance.

Since along the RBGP there is an aggregate Cobb-Douglas production function, the aggregate calibration is essentially as in Farhi and Gourio (2018) and proceeds in the same four steps as the sectoral calibration. In the second step, given γ_T, we calculate the aggregate depreciation rate $\delta + \gamma_Q$ from the aggregate investment-capital ratio (57):

$$\delta + \gamma_Q = \frac{X}{p_K K} - \gamma_T.$$

Given γ_T, in the third step we calculate aggregate ρ from the aggregate Gordon growth formula (61):

$$\rho = \gamma_T + \frac{(1+\gamma_T)\Pi}{p_F},$$

where p_F and Π are calculated according to (59)–(60). Given $\rho + \delta + \gamma_Q$, the fourth step involves solving (45) and (48) for the aggregate markup and output elasticities while using that

$\alpha_K + \alpha_L = 1$:

$$\mu = \frac{Y}{(\rho + \delta + \gamma_Q) p_K K + wL}, \tag{73}$$

$$\alpha_K = \frac{(\rho + \delta + \gamma_Q) p_K K}{(\rho + \delta + \gamma_Q) p_K K + wL}, \tag{74}$$

$$\alpha_L = \frac{wL}{(\rho + \delta + \gamma_Q) p_K K + wL}. \tag{75}$$

We can see that the aggregate results are close relatives of the sectoral results. The aggregate markup equals the value of final output divided by the aggregate factor payments (recall that the price of final output was normalized to one). The aggregate output elasticities of capital and labor equal the cost shares of aggregate capital and labor.

4.2 Implementing the Calibration Strategy

In implementing the four calibration steps, we focus on the private sector without real estate, which is as in Karabarbounis and Neiman (2018), Barkai (2020), and De Loecker et al. (2020a). The rationale for excluding real estate from the analysis is that it looks unusual [Rognlie (2015) and Gutierrez and Philippon (2017)]. Specifically, since owner-occupied housing does not lead to market transactions, real estate is a largely imputed sector. In addition, the NIPA do not include land in its measure of the capital stock although land is an important input into producing real estate services and the scarcity of land contributed to price increases of real estate. Therefore, omitting land from the capital stock inflates markups in real estate more than on average [Rognlie (2015)].

If not mentioned otherwise, the data for the calibration are taken from the U.S. NIPA and the input-output tables.[5] As we pointed out already, the advantage of using NIPA data in our context is that they are constructed with the standard adding up constraints and NIPA identities in mind, implying that they capture how sectoral forces aggregate. Moreover, they include information about the input-output linkages, which micro or industry data lack.

Although our aggregate model is essentially that of Farhi and Gourio (2018), there several important differences between our calibration and theirs. Most importantly, of course, we disaggregate to the sectoral level whereas they only consider the aggregate economy. The portions of the economy that are included in the measurement are also different. As mentioned above, we use the private sector except for real estate whereas they use the entire private sector. Crucially, we exclude residential housing when we calculate labor income and the labor share, because that is consistent with excluding real estate. In contrast, Farhi and Gourio (2018) use the corporate labor share although it represents only part of the private sector.

[5]Appendix B contains a detailed data documentation.

Table 1: Calibration targets

Statistic	1957–1973	1984–2000	2001–2016
wL/Y	0.741	0.708	0.692
$wL_g/(p_{G_g}G_g)$	0.290	0.257	0.229
$wL_s/(p_{G_s}G_s)$	0.477	0.453	0.435
$p_{M_g}M_{gg}/(p_{G_g}G_g)$	0.501	0.514	0.518
$p_{M_s}M_{sg}/(p_{G_g}G_g)$	0.125	0.128	0.130
$p_{M_s}M_{ss}/(p_{G_s}G_s)$	0.299	0.316	0.342
$p_{M_g}M_{gs}/(p_{G_s}G_s)$	0.073	0.079	0.086
$X/(p_K K)$	0.102	0.109	0.105
$X_g/(p_K K_g)$	0.124	0.114	0.111
$X_s/(p_K K_s)$	0.089	0.107	0.103
p_F/Π	28.06	35.46	55.51
p_{F_g}/Π_g	32.35	39.75	54.46
p_{F_s}/Π_s	27.22	34.68	55.71
$p_K K/Y$	1.394	1.476	1.515
$p_K K_g/Y_g$	1.107	1.548	1.884
$p_K K_s/Y_s$	1.589	1.446	1.406

Table 1 summarizes the key data observations for the calibration. While the labor share at the aggregate level shows the usual decrease, the labor share in the goods sector shows a much larger decrease than at the aggregate level.[6] When aggregated, the decrease in the goods sector is mitigated by what happens in the services sector, whose labor share does not decrease as much, and by the increase in the final-output share of services. These observations are consistent with those of Alvarez-Cuadrado et al. (2018). The main difference between their work and ours is that they abstracted from markups and focused on the role of differences in sectoral substitution elasticities between capital and labor and in the sectoral capital biases of technical change.

The intermediate-good shares, $p_{M_j}M_{jj'}/(p_{G_{j'}}G_{j'})$, increased in both sectors. The implied outsourcing of services was considerably stronger in the services sector than in the goods sector.

The investment-capital ratio, $X_j/(p_K K_j)$, did not change much at the aggregate level and the sectoral level. The capital-output ratio $p_K K_j/(p_{G_j}G_j)$ went up considerably in the goods sector and decreased sufficiently in the services sector so as to offset most of the increase in the good sectors so that the aggregate capital-output ratio increases only somewhat. Note that the capital-output ratios are smaller than conventionally found because we have excluded a sizeable part of the capital stock – real estate capital.

The price-dividend ratio increased considerably, which the calibration will translate into

[6] We calculate the labor shares, $w_jL_j/(P_{G_j}G_j)$, according to the standard methodology that involves splitting proprietors income between capital and labor income according to the economy-wide proportions; see Gollin (2002) and Valentinyi and Herrendorf (2008) for the details.

a decrease in the expected discount rate. Of course, the price-dividend ratio, p_{Fj}/Π_j, is not in NIPA. As Farhi and Gourio, we obtain it from Kenneth French's stock data library; see Appendix B.2. French reports prices and dividends of all public firms and assigns each of them to the main industry in which it was active, implying that we can do this calibration step by sector. Dividends are the difference between reported returns with and without dividends.

There are two concerns with using the price-dividend ratio in the calibration. First, one might wonder how well firms from the services sector are represented in stock data. Perhaps surprisingly, they are well represented indeed: during the first calibration period 1957–1973, one third of all listed firms are in the services sector and during the two other calibration periods half of them are. Second, the price-dividend ratio is for firms whereas the NIPA data are for establishments. The distinction is relevant in our context because firms in the goods sector also produce some in-house services, which often takes place in their headquarters. Unfortunately, it is impossible to calculate separate price-earnings ratios for in-house services produced in the goods sector. This is not likely to be a serious issue for our calibration because in-house services are a small part of total services.

Table 2: Calibrated parameters for the private sector without real estate

Parameters	1957–1973	1984–2000	2001–2016
γ_T	0.025	0.026	0.017 (2001–2006)
γ_Q	-0.007	-0.019	-0.013
δ	0.084	0.102	0.102
δ_g	0.106	0.107	0.108
δ_s	0.072	0.100	0.099
ρ	0.061	0.054	0.035
ρ_g	0.057	0.052	0.035
ρ_s	0.063	0.055	0.035

Table 2 reports the parameter values that are pinned down by the first three calibration steps. We obtain γ_T from NIPA as the average growth rate of real output per capita. Following Farhi and Gourio (2018), we estimate γ_T only for the first part of the last period, 2001–2006. This is consistent with the view that the Great Recession constituted a realization of a rare disaster shock, χ_t, that changed the level of trend output growth. Given that (72) implies that the value of γ_T is not of first-order importance for the results, this interpretation is not crucial for the results. We find that trend growth is around 2.5 percent in the first two periods and 1.7 percent in the last period.

We obtain γ_Q from NIPA as the inverse of the average growth rate of the price of investment relative to output. Consistent with conventional wisdom, investment-specific technical change is strongest in the 1980s and 1990s; see for example Duernecker et al. (2021). In our context, this manifests itself in the absolute value of γ_Q being twice as large in the middle period than in the other two periods.

The depreciation rates are between 7% and 11%, which is reasonable given that we exclude residential housing, which has a much lower depreciation rate, from the analysis. Note that the depreciation rate increased over time, which is consistent with the observation of Bridgman (2018) that intellectual property product has high depreciation rates and is growing as a share of overall capital.

The expected discount rates ρ_j fell from around 6% to around 3.5%. This is broadly in line with what Farhi and Gourio (2018) found for the aggregate for the last two periods. They argued that the inclusion of a risk premium in ρ implies that its recent values are somewhat higher than those from Aaa interest rates used by Barkai (2020).

Note that in the model ρ_j applies to all firms whereas in the data it applies to publicly listed firms only. This leads to two concerns. First, it is sometimes claimed that the publicly listed firms have lower user costs of capital than the other firms. If that is correct, then our calibrated $\rho_j + \delta_j + \gamma_Q$ is downward biased; compare expression (67). Second, it is often claimed that in recent decades firms increased their retained earnings, which decreased dividends and increased firm prices. If these retained earnings had been paid out, then the price-dividend ratio would have increased by less, ρ_j and ρ would have fallen less. These considerations mean that our calibrated values of μ_j and μ will be an upper bound and our calibrated labor shares will be a lower bound to the actual values. So, if anything, the recent labor shares may be somewhat higher than what we find. Be that as it may, in the discussion of Table 7 below we will see that the potential bias cannot be very large, as our aggregate labor share estimates are in the same ballpark as the usual estimates in the literature.

5 Results

We organize the presentation of our results around four key aggregation relationships that the equilibrium of our model implies: (30), (46), (73), and (75). For convenience, we state them again:

$$\frac{w_t L_t}{Y_t} = \frac{\alpha_L}{\mu} = \Phi_g \frac{w_t L_{gt}}{p_{G_{gt}} G_{gt}} + \Phi_s \frac{w_t L_{st}}{p_{G_{st}} G_{st}} = \Phi_g \frac{\alpha_{L_g}}{\mu_g} + \Phi_s \frac{\alpha_{L_s}}{\mu_s}, \tag{76}$$

$$\frac{1}{\mu} = \frac{\Phi_g \alpha_{K_g}}{\alpha_K} \frac{1}{\mu_g} + \frac{\Phi_s \alpha_{K_s}}{\alpha_K} \frac{1}{\mu_s}, \tag{77}$$

where the Domar weights are given by (25):

$$\Phi_j \equiv \frac{\left(1 - \frac{\alpha_{M_{j'j'}}}{\mu_{j'}}\right)\phi_j + \frac{\alpha_{M_{jj'}}}{\mu_{j'}}\phi_{j'}}{\left(1 - \frac{\alpha_{M_{jj}}}{\mu_j}\right)\left(1 - \frac{\alpha_{M_{j'j'}}}{\mu_{j'}}\right) - \frac{\alpha_{M_{jj'}}}{\mu_{j'}}\frac{\alpha_{M_{j'j}}}{\mu_j}}. \tag{78}$$

(76) says that the aggregate labor share is an arithmetic average of the sectoral labor shares weighted by the Domar weights. The aggregate or sectoral labor shares are the ratios of the relevant output elasticity of labor over the relevant markup. (76) says that the aggregate markup is a harmonic average of the sectoral markups weighted by the product of the Domar weights and the relative output elasticities of capital.

A natural first step towards understanding the decrease in the labor share is to inspect the behavior of the sectoral labor shares and the Domar weights. Both are straightforward to calculate directly from the data. The labor share is the share of the sectoral payments to labor over sectoral gross output, which we already reported in Table 1. The Domar weight is the ratio of sectoral gross output over total GDP; see Equation (24). Table 3 reports the results. The level of the sectoral labor share is considerably larger in the services sector than in the goods sector. Moreover, both sectors' labor shares decreased but the decrease was larger in the goods sector than in the services sector (6.1 versus 4.2 percentage points). The aggregate decrease of 4.9 percentage points comes out only a little larger than the decrease in the services sector. The reason is that the Domar weights changed dramatically. While in the initial period the Domar was somewhat larger in the goods sector, in the final period it was less than half in the goods sector than in the services sector. Therefore, what happens in the goods sector is less and less reflected at the aggregate. While these conclusions may be obtained without a model, it is important to realize that they do not tell us what drives the evolution of the sectoral labor shares or of the Domar weights. To answer that important question, we need our model.

Table 3: Aggregate versus sectoral labor shares

	1957–1973	**1984–2000**	**2001–2016**
wL/Y	0.741	0.708	0.692
$wL_g/(p_{G_g}G_g)$	0.290	0.257	0.229
$wL_s/(p_{G_s}G_s)$	0.477	0.453	0.435
Φ_g	1.018	0.726	0.572
Φ_s	0.933	1.153	1.289

5.1 Determinants of the Labor Share

Table 4 displays the parameters implied by the fourth calibration step. We can see that structural change reduced the final output share of goods by more than half from 0.460 to 0.194. This explains in part the movements of the Domar weights in Table 3.

Turning now to the output elasticity of labor, it decreased by 5.9 percentage points in the goods sector and 2.0 percentage points in the services sector between 1957–1973 and 2001–2016. There are two sets of forces behind the decrease in the labor share: those that increase the value-added elasticity of capital ("capital deepening") and those that increase the gross-output elasticity of intermediate inputs ("outsourcing"). We find that there was considerable capital deepening in the goods sector but not in the services sector or at the aggregate. In particular, $\alpha_{L_g}/(\alpha_{K_g} + \alpha_{L_g})$ *decreased* from 0.807 to 0.702 but $\alpha_{L_s}/(\alpha_{K_s} + \alpha_{L_s})$ *increased* from 0.785 to 0.811. Overall, α_L decreased by a tiny amount from 0.794 to 0.787.

Table 4: Calibrated parameters for the private sector without real estate – continued

	1957–1973	1984–2000	2001–2016
ϕ_g	0.460	0.283	0.194
α_L	0.794	0.777	0.787
α_{L_g}	0.302	0.270	0.247
$\alpha_{L_g}/(\alpha_{K_g} + \alpha_{L_g})$	0.807	0.754	0.702
$\alpha_{K_g} + \alpha_{L_g}$	0.374	0.358	0.352
$\Phi_g\alpha_{K_g}/\alpha_K$	0.380	0.259	0.201
α_{L_s}	0.493	0.475	0.464
$\alpha_{L_s}/(\alpha_{K_s} + \alpha_{L_s})$	0.785	0.785	0.811
$\alpha_{K_s} + \alpha_{L_s}$	0.628	0.605	0.572
$\Phi_s\alpha_{K_s}/\alpha_K$	0.558	0.705	0.743
μ	1.072	1.097	1.137
μ_g	1.040	1.049	1.078
μ_s	1.033	1.050	1.066

There are two underlying forces that may lead to capital deepening in the goods sector: decreases in the relative price of capital [Karabarbounis and Neiman (2014)]; automation [Acemoglu and Restrepo (2018)]. For decreases in the relative price of capital to lead to capital deepening, the elasticity of substitution between capital and labor must be larger than one. Herrendorf et al. (2015) found that, while the elasticity of substitution is indeed larger than one in U.S. agriculture, it is considerably smaller than one in the large remaining part of the goods sector. Oberfield and Raval (2021) found that the elasticity of substitution is less than one in U.S. manufacturing. The evidence on the elasticity of substitution casts doubt on the possibility that the substitution of increasingly cheap capital for labor is behind capital deepening in the goods sector. A more likely story is that automation reduced the output elasticity of labor in the goods sector but not in the services sector. Interestingly, the effects of capital deepening in the goods sector did not translate to the aggregate α_L, which hardly changed. The reason is that structural change reallocated labor from the goods sector to the services sector, which offset the capital deepening in the goods sector.

Outsourcing happened in both sectors, but it was stronger in the services sector. In particular, $\alpha_{K_g} + \alpha_{L_g} = 1 - \alpha_{M_{gg}} + \alpha_{M_{sg}}$ decreased from 0.374 to 0.352 whereas $\alpha_{K_s} + \alpha_{L_s}$ decreased from 0.628 to 0.572. Together, capital deepening and outsourcing decrease the output elasticity of labor in both sectors, with a larger decrease in the goods sector than in the services sector.

We continue with the calibration results for the markups. Between 1957–1973 and 2001–2016, the markups in both sector increased by roughly similar amounts and they roughly doubled. While aggregate markups also roughly doubled, we find that the levels of markups in both sectors are only around half of what they are at the aggregate. The mechanical reason is revealed by applying the fact that the weights in Equation (77) add up to a number smaller than one: $\sum_j \Phi_j \alpha_{K_j} < \alpha_K$. The economic reason for the difference between the levels of aggregate and sectoral markups is double marginalization, that is, the sectoral markups are not only applied to the rental prices of capital and labor but also to the purchase prices of intermediate inputs. As a result, intermediate inputs get marked up when their producers sell them and when their users sell their output. Average sectoral markups must therefore be smaller than aggregate markups [Basu (2019)]. Our analysis shows that the difference is very large: sectoral markups are around half of aggregate markups.[7]

We emphasize that taking the input-output linkages into account is of first-order importance for the aggregation of sectoral to aggregate markups. Simple cost- or revenue-weighted averages miss the important amplification of sectoral markups that results from input-output linkages. One way to see this is to write down a version of our a model without input-output linkages in which each sector produces value added without intermediate inputs. In that case, the sectoral markups would be considerable higher and the markup in the goods sector would actually exceed the aggregate markup considerably. The details including the calibrated parameter values can be found in Appendix C.[8]

[7]Note that our analysis rests on the assumption that most intermediate inputs are not produced within vertically integrated firms that do not charge markups to themselves. While NIPA data do not allow us to assess how restrictive this assumption is, evidence presented by Atalay et al. (2014) for the goods sector suggests that the vast majority of intermediate inputs are traded outside vertically integrated firms.

[8]There is a mounting body of work on the implications of input-output linkages for a variety of macro phenomena other than markups. Valentinyi (2021) reviews the recent literature and discusses its potential implications for the literature on productivity and structural change. An example from the literature on structural change is Herrendorf et al. (2013). An example from the literature on misallocation is Hang et al. (2020). They found a result similar in spirit to ours: there is less sectoral misallocation in a gross-output model than in the corresponding value-added model, because only in the gross-output model sectoral does misallocation get amplified through input-output linkages.

5.2 Decomposition of the Labor Share Decrease

We now decompose the change in the aggregate labor share into changes in the sectoral output elasticities of labor, in sectoral markups, and in the Domar weights. We start by taking the Taylor expansion of Equation (76), which we call the "bottom-up" approach. Note that given the functional form of the right-hand side, the third-order Taylor expansion is exact. Table 5 reports the first-, second-, and third-order effects. The first two lines reveal that measuring the first-order effects implies that changes in the output elasticity of labor reduce it by a full 8 percentage points whereas changes in markups reduce the aggregate labor share by 2.4 percentage points. Since both first-order effects add up to over 10 percentage points, they do exactly what Grossman and Oberfield (2021) observed: together they "explain" more than twice the 4.9 percentage points decrease in the aggregate labor share. The table also shows that a sizeable part of the first-order effects is offset by other effects. For starters, the first-order effects of changes in the Domar weight offset 4 percentage points of the total first-order effect of changes in the output elasticities and the markups. The second- and third-order effects that result from the interaction of the output elasticity of labor with the Domar weights offset another 1.4 percentage points. These results highlight that identifying the forces behind the decrease of the aggregate labor share requires a unifying framework like ours that captures causal and offsetting forces and accounts for aggregation effects.

Table 5: Decomposition of changes in aggregate labor share – bottom up

Data		-0.049
First-order effect of	μ_j	-0.024
	α_{L_j}	-0.080
	Φ_j	0.040
Second-order effect of	α_{L_j}, Φ_j	0.014
	μ_j, α_{Lj}	0.003
	Φ_j, μ_j	-0.001
Third-order effect of	$\alpha_{L_j}, \mu_j, \Phi_j$	-0.001

One complication with interpreting the bottom-up approach is that changes in the Domar weights reflect changes in the final output shares (structural change), changes in the output elasticities of the intermediate inputs (which, given constant returns to scale, are linked to changes in the output elasticities of labor), and changes in the markups; compare Equation (78). It is straightforward to take the first-order Taylor expansion of the Domar weights with respect to the final output weights, ϕ_j. This implies that the aggregate labor share increases by 2.7 percentage points due to changes in ϕ_{jt} that lead to changes in Φ_{jt}. How changes in μ_{jt} and $\alpha_{jj't}$ affect Φ_{jt}, however, is somewhat hard to disentangle, and so the bottom up approach does not yield estimates for the full individual effects of μ_{jt} and of $\alpha_{jj't}$. To measure them, we use instead a “top-down” approach, which we describe next.

Table 6: Decomposition of changes in aggregate labor share – top down

		1957–1973	**1984–2000**	**2001–2016**	**Δ LS 57–73 to 01–16**
Data		0.741	0.708	0.692	-0.049
Constant	μ_j	0.741	0.726	0.735	0.043 smaller decrease
	α_{Lj}	0.741	0.725	0.719	0.027 smaller decrease
	$\alpha_{Mjj}, \alpha_{Mjj'}$	0.741	0.711	0.700	0.008 smaller decrease
	ϕ_j	0.741	0.700	0.664	0.028 larger decrease
	μ_j, ϕ_j	0.741	0.718	0.711	0.019 smaller decrease
	α_{Lj}, ϕ_j	0.741	0.720	0.698	0.006 smaller decrease
	$\alpha_{Mjj}, \alpha_{Mjj'}, \phi_j$	0.741	0.703	0.671	0.021 larger decrease

The top-down approach shuts down one or more forces at a time by fixing the corresponding parameters at the values of the first period. This captures the total effects on the aggregate labor share of the first-, second-, and third-order effects related to the change in the parameter. Table 6 reports the results. Without changes in sectoral markups, the labor share would have decreased by 4.3 percentage points less ($0.735 - 0.692 = 0.043$). Without changes in output elasticity of labor, the labor share would have decreased by 2.7 percentage points less ($0.719 - 0.692 = 0.027$). Without outsourcing, the labor share would have decreased by 0.8 percentage points less ($0.692 - 0.700 = 0.008$).[9] Without structural change, the labor share would have decreased by 2.8 percentage points more ($0.692 - 0.664 = 0.0028$).

Several remarks are at order. First, the top-down approach shows that the total effect of changes in sectoral markups on the aggregate labor is a 4.3 percentage point decrease. Since the bottom-up approach gave a direct effect on the aggregate labor share was a 2.4 percentage point decrease, markups must have an additional indirect effect of 1.9 percentage points through affecting the Domar weights. Second, as with the bottom up approach, the parameter changes that decrease the labor share in the top-down approach add up to more than the 4.9 percentage point decrease of the labor share. In particular, they decrease the labor share by $7.8 = 4.3 + 2.7 + 0.8$ percentage points. The excess decrease of 2.9 percentage points is offset by the effect of structural change, which increases the labor share by 2.8 percentage points. Note that without rounding errors, the two numbers should be exactly the same.

Our results have some important implications. First, double marginalization hugely amplifies the effects of the even changes in sectoral markups, μ_j. Taking into account the input-output linkages is therefore crucial for our results. Instead, NIPA data is required to capture the quan-

[9] Our finding that outsourcing hardly decreased the labor share is consistent with the finding of Giannoni and Mertens (2019) for the subperiod 1996–2016. In particular, they found that while outsourcing may affect industry labor shares, it does not affect the aggregate labor share much because labor that is outsourced in one industry shows up in another unless it is offshored. This insight reiterates the importance of using a unifying framework in which sectoral variables aggregate to economy-wide variables and everything is accounted for.

titatively important roles played by structural change and the input-output linkages. Second, together the individual forces explain a multiple of the labor share decrease because aggregation effects offset some of them. A particular aggregation effect comes from the changes in the sectoral composition that result from structural change. It is important to realize that structural change offsets mostly the forces of capital deepening but not the increases in markups. To see this, we go back to Table 4. The table shows that, the aggregate output elasticity of labor, α_L, remains almost constant whereas both sectoral and aggregate markups roughly double. The reason for the difference is that, given that capital deepening happens only in the goods sector, it is mostly offset by the reallocation of labor to the services sector. In contrast, given that both sectoral markups increase by similar amounts, changes in the sectoral composition do not matter much for how the sectoral changes in markups aggregate. We conclude that to understand the reasons for the decrease in the aggregate labor share, we need an explanation for why markups increased by similar amounts in both sectors.

6 Comparison with Literature

To lend additional credibility to our results, we now establish that our parameter values for the aggregate labor share and the aggregate markups are broadly in line with those in the literature.

6.1 Labor Share

Table 7 compares our aggregate labor share estimates with those of the literature. The second line repeats the first line of Table 1 for comparison. The last line reports what our framework implies if we followed Farhi and Gourio (2018) in using the *corporate* labor share but took all other data from the entire private sector. The results are broadly in line with what other authors found, which is reassuring. The difference between the first two lines reflects the fact that real estate is relatively capital intensive, and so the labor share increases in all three periods when one takes real estate out. However, the labor share still decreases by 4.9 percentage points without real estate, compared to its 6.2 percentage points decrease with real estate. This implies that what happened within real estate is not the main reason for the decrease in the labor share. The difference between the first and last lines is more sizeable. This reflects that the corporate labor share is larger than the labor share in the private sector and that it decreased by less (6.2 versus 3.9 percentage points). We note that it is not ideal to use the corporate labor share when the rest of the model is calibrated to the entire private sector.

Table 7: Labor share for different parts of the private sector

Covered part of economy	1957–1973	1984–2000	2001–2016
Private sector	0.657	0.618	0.595
Private sector without real estate	0.741	0.708	0.692
Private sector without FIRE	0.738	0.706	0.692
Corporate sector without FIRE	0.699	0.697	0.651
Private sector with corporate labor share – Farhi-Gourio type estimates	0.699	0.701	0.660

6.2 Markups

As a second consistency check, Table 8 compares our aggregate markup estimates with those of the literature. The second line repeats the second line of Table 4 for comparison. The third line reports the markups our framework implies if instead of using the calibrated ρ_j, we followed Barkai (2020) and used the Aaa interest rate from Moody's. Note that the Aaa interest rate is not available during the first period, which explains the blank space in the table.[10] The last line reports what our framework implies if we followed Farhi and Gourio (2018) in using the *corporate* labor share but all other data from the entire private sector. Again, the results are broadly in line with what the other authors found. Moreover, in the final period the markup estimates are fairly similar to each other, ranging from 12.8% to 14.5%. In contrast, in the first period, there is somewhat more variation in the markup estimates; our method implies a markup of 7.5% whereas using the private sector with the corporate labor share implies the lower estimate of 3.7%. It is important to realize that this difference is expected because the initial corporate labor share is larger than the labor share of the private sector with real estate. Equation (73) implies that the estimate of aggregate markups decreases as a result:

$$\mu \downarrow = \frac{1}{(\rho + \delta + \gamma_Q)\frac{p_K K}{Y} + \frac{wL}{Y} \uparrow} \tag{79}$$

Table 8 also shows that markups are larger for the private sectors with real estate than without real estate (lines one versus two). One explanation for why including real estate increases markups is that NIPA includes land income in Y but not in K. Hence, $p_{K_j} K_j / Y_j$ is underestimated for real estate, which biases the markups in real estate upwards:

$$\mu_j \uparrow = \frac{1}{(\rho_j + \delta_j + \gamma_Q)\frac{p_{K_j} K_j \downarrow}{Y_j} + \frac{w_j L_j}{Y_j}}$$

[10]Following the methodology of Barkai (2020), Esfahani et al. (2020) calculated disaggregate markups at the industry level for many countries from World KLEMS data on input-output tables. Since these data are available for 1996–2014, their period of investigation is close to the last period of our calibration, that is, 2001–2016. Therefore, their analysis does not speak to what happened since the 1950s.

Table 8: Markups for different parts of the private sector

Covered part of economy	1957–1973	1984–2000	2001–2016
Private sector	1.071	1.121	1.194
Private sector without real estate	1.072	1.097	1.137
Private sector without real estate and with Barkai type user costs		1.101	1.151
Private sector without FIRE	1.070	1.093	1.128
Corporate sector without FIRE	1.088	1.086	1.138
Private sector with corporate labor share – Farhi-Gourio type estimates	1.037	1.080	1.145

We also compare our estimates of markups at the macro level with existing estimates of markups at the micro level. By and large, micro estimates tend to be larger than macro estimates. For example, a common parameter choice in the industrial organization literature is an elasticity of substitution of 4, which leads to markups of $\varepsilon/(\varepsilon - 1) = 1.25$; see for example Broda and Weinstein (2006). Recent studies of markups in Compustat find similar or even larger firm-level markups as the industry studies; see for example Edmond et al. (2018), Traina (2018), and De Loecker et al. (2020b). The reason why the micro studies find larger markups than we do is that they make different assumptions about the returns to scale. In particular, we have assumed that the production functions has constant returns to scale and that all capital and labor are variable inputs, implying that markups cover pure profits only. In contrast, micro studies typically assume that there are fixed costs, implying that the production function does not have constant returns to scale, part of the payments to capital and labor are fixed costs, and markups do not only cover pure profits but also the fixed costs. As a result, markups tend to be larger with fixed costs than without.[11] We emphasize that for our purpose here, assuming constant returns and abstracting from fixed costs is appropriate because we are after the determinants of the share of labor income in GDP, irrespective of whether labor is a fixed or variable input in production. The key object that is required for calculating the labor share is what Karabarbounis and Neiman (2018) called "factorless income", that is, profits that do not accrue to capital or labor. Our macro approach measures factorless income.

6.3 Intellectual Property Products

It is also of interest to study the effect on the aggregate labor share of the recent inclusion in the NIPA of the investment in intellectual property products (IPP). IPP investment comprises the expenditure on software, research and development, and entertainment, literary, and artistic

[11] See De Loecker et al. (2020b) for the analytical details. See Syverson (2019) for further issues regarding the integration of the micro and the macro literature on markups.

originals. The revision of 1999 included software in NIPA and the revision of 2013 included the rest of IPP in NIPA. IPP investment is a sizeable and growing subset of unmeasured investment that importantly affects the aggregate labor share.

Koh et al. (2021) studied what happens to the aggregate labor share when one takes IPP investment and the related factor income out of the NIPA, which captures how the NIPA was constructed before 1999. They found that without IPP investment, the aggregate labor share hardly decreased over the postwar period. We now show that our analysis is consistent with their results. Since they analyzed the aggregate economy, and since it is challenging to apportion IPP investment to the two sectors, we will analyze only the aggregate effects of taking IPP investment out.

Excluding IPP in the measurement of markups requires two modifications to Equations (73)–(75): first, one must take the user costs paid to the stock of unmeasured capital, r^*K^*, out of factor payments; second, one must expense IPP investment, X^*, thereby taking it out of total GDP.[12] Given that the income and product approaches to measuring GDP must give the same answer, the two modifications must conceptually be equal in value: $r^*K^* = X^*$. Since $Y > rK + wL$, (73) implies that excluding unmeasured capital *increases* aggregate markups:[13]

$$\mu = \frac{Y}{rK + wL} < \frac{Y - X^*}{rK - r^*K^* + wL} = \mu^*.$$

Moreover, (75) implies that excluding unmeasured capital *increases* the aggregate output elasticity of labor:

$$\alpha_L = \frac{wL}{rK + wL} < \frac{wL}{rK - r^*K^* + wL} = \alpha_L^*. \tag{80}$$

Table 9 reports the results. As expected, the aggregate markup and the aggregate output elasticity of labor are both larger without than with IPP capital. Moreover, without IPP capital, both markups and the output elasticity of labor increase. Interestingly, the two increases largely offset each so that the aggregate labor share declines by only one percentage point from 0.743 to 0.732, instead of from 0.741 to 0.692. This is a version of the finding of Koh et al. (2021). Our decomposition adds the insight that with IPP capital the aggregate output elasticity of labor no longer increases but remains almost constant. This is intuitively plausible: including an increasingly important part of the capital – IPP capital – increases the output elasticity of capital and therefore decreases the output elasticity of labor.

[12] The detailed steps are explained in Koh et al. (2021).

[13] Atkeson (2020) arrived at a similar conclusion regarding the effect of unmeasured capital on markups.

Table 9: Aggregate calibration without IPP capital and real estate

Targets	1957–1973	1984–2000	2001–2016
wL/Y^*	0.743	0.732	0.732
α_L^*	0.814	0.815	0.839
μ^*	1.097	1.114	1.146

7 Conclusion

We have developed a unifying framework to study why the labor share decreased in the postwar U.S. Our framework captures key causal forces and important amplifying and offsetting effects. We have found that the main force behind the decrease in the postwar U.S. labor share is increases in sectoral markups that are amplified through input-output linkages. In contrast, forces that lead to capital deepening (e.g., automation) were concentrated in the goods sector and were largely offset by structural change. Our analysis implies that input-output linkages and structural change are essential for estimating the effects of sectoral forces on the aggregate labor share. Micro data alone are not sufficient to capture them, implying that it is crucial that we use NIPA data for our analysis.

Our analysis suggests several directions for future work. First, it would be useful to do the decomposition also for finer industry disaggregations and investigate what patterns arise in the aggregation of more disaggregate force behind the labor share decrease. While we have focused on the two-sector split between goods and services as a useful and tractable first step, one can apply our methodology to finer industry disaggregations as long as they do not get so fine that they change with the changes in the industry classification. Second, it would be valuable to extend our analysis beyond the U.S. to other countries. Since our calibration procedure has limited data requirements, that appears to be feasible. We plan to turn to these tasks next.

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A Proofs and Derivations

A.1 Proof of Proposition 1

$p_{jt}G_{jt}$ are linked to each other Y_{jt} through the feasibility constraint. (16) implies:

$$p_{G_{jt}}G_{jt} = p_{G_{jt}}Y_{jt} + p_{G_{jt}}M_{jjt} + p_{G_{jt}}M_{jj't}.$$

Using (18), (22), and (23) gives:

$$p_{G_{jt}}G_{jt} = \phi_j Y_t + \frac{\alpha_{M_{jj}}}{\mu_j} p_{G_{jt}}G_{jt} + \frac{\alpha_{M_{jj'}}}{\mu_{j'}} p_{G_{j't}}G_{j't}.$$

Denoting column vectors and matrices in boldface,

$$\frac{\boldsymbol{p_{G_t}G_t}}{\boldsymbol{Y_t}} \equiv \begin{bmatrix} \frac{p_{G_{gt}}G_{gt}}{Y_t} \\ \frac{p_{G_{st}}G_{st}}{Y_t} \end{bmatrix}, \quad \boldsymbol{\phi} \equiv \begin{bmatrix} \phi_g \\ \phi_s \end{bmatrix}, \quad \boldsymbol{\Omega} \equiv \begin{bmatrix} \frac{\alpha_{M_{gg}}}{\mu_g} & \frac{\alpha_{M_{gs}}}{\mu_s} \\ \frac{\alpha_{M_{sg}}}{\mu_g} & \frac{\alpha_{M_{ss}}}{\mu_s} \end{bmatrix},$$

this implies:

$$\frac{\boldsymbol{p_{G_t}G_t}}{\boldsymbol{Y_t}} = \boldsymbol{\phi} + \boldsymbol{\Omega}\frac{\boldsymbol{p_{G_t}G_t}}{\boldsymbol{Y_t}}.$$

If $\boldsymbol{\Omega}$ is invertible, then:

$$\frac{\boldsymbol{p_{G_t}G_t}}{\boldsymbol{Y_t}} = [\boldsymbol{I} - \boldsymbol{\Omega}]^{-1}\boldsymbol{\phi}, \tag{A.1}$$

where $[\boldsymbol{I} - \boldsymbol{\Omega}]^{-1}$ is the so called Leontief inverse. Solving for the Leontief inverse gives:

$$\begin{aligned} [\boldsymbol{I} - \boldsymbol{\Omega}]^{-1} &= \begin{bmatrix} 1 - \frac{\alpha_{M_{gg}}}{\mu_g} & -\frac{\alpha_{M_{gs}}}{\mu_s} \\ -\frac{\alpha_{M_{sg}}}{\mu_g} & 1 - \frac{\alpha_{M_{ss}}}{\mu_s} \end{bmatrix}^{-1} \\ &= \frac{1}{\left(1 - \frac{\alpha_{M_{gg}}}{\mu_g}\right)\left(1 - \frac{\alpha_{M_{ss}}}{\mu_s}\right) - \frac{\alpha_{M_{gs}}}{\mu_s}\frac{\alpha_{M_{sg}}}{\mu_g}} \begin{bmatrix} 1 - \frac{\alpha_{M_{ss}}}{\mu_s} & \frac{\alpha_{M_{gs}}}{\mu_s} \\ \frac{\alpha_{M_{sg}}}{\mu_g} & 1 - \frac{\alpha_{M_{gg}}}{\mu_g} \end{bmatrix}. \end{aligned}$$

Plugging the Leontief inverse into (A.1) implies that, in equilibrium, sectoral gross output is proportional to aggregate final output:

$$p_{G_{jt}}G_{jt} = \Phi_j Y_t \qquad \text{where} \qquad \Phi_j \equiv \frac{\left(1 - \frac{\alpha_{M_{j'j'}}}{\mu_{j'}}\right)\phi_j + \frac{\alpha_{M_{jj'}}}{\mu_{j'}}\phi_{j'}}{\left(1 - \frac{\alpha_{M_{jj}}}{\mu_j}\right)\left(1 - \frac{\alpha_{M_{j'j'}}}{\mu_{j'}}\right) - \frac{\alpha_{M_{jj'}}}{\mu_{j'}}\frac{\alpha_{M_{j'j}}}{\mu_j}}.$$

QED

A.2 Derivation of the Euler equation

Substituting out X_{jt} in the household problem (31) by using (9), the problem simplifies to:

$$\max_{C_t,\{K_{jt+1}\}_{j\in\{g,s\}}} \quad U_t = \left((1-\beta)C_t^{1-\sigma} + \beta\left[E_t\left(U_{t+1}^{1-\theta}\right)\right]^{\frac{1-\sigma}{1-\theta}}\right)^{\frac{1}{1-\sigma}}$$

$$\text{s.t.} \quad C_t = \sum_{j\in\{g,s\}}\left[\left(\frac{1-\delta_j}{Q_t} + r_{jt}\right)K_{jt} - \frac{K_{jt+1}}{Q_t\exp(\chi_{jt+1})}\right] + w_t.$$

Substituting the constraint into the life-time utility function gives:

$$\max_{K_{t+1}} U_t = \left(1-\beta)N_t^{\sigma}\left[\left(\frac{1-\delta}{Q_t}+r_t\right)K_t + w_tN_t - \frac{K_{t+1}}{Q_t\exp(\chi_{t+1})}\right]^{1-\sigma} + \beta\left[E_t\left(U_{t+1}^{1-\theta}\right)\right]^{\frac{1-\sigma}{1-\theta}}\right)^{\frac{1}{1-\sigma}}$$

The first-order conditions are:

$$0 = -\frac{U_t^{\sigma}(1-\beta)C_t^{-\sigma}}{Q_t\exp(\chi_{t+1})} + U_t^{\sigma}\beta\left[E_t\left(U_{t+1}^{1-\theta}\right)\right]^{\frac{\theta-\sigma}{1-\theta}} E_t\left[U_{t+1}^{-\theta}\frac{\partial U_{t+1}}{\partial K_{t+1}}\right]$$

$$\frac{\partial U_{t+1}}{\partial K_{t+1}} = U_{t+1}^{\sigma}(1-\beta)C_{t+1}^{-\sigma}\left(\frac{1-\delta}{Q_{t+1}} + r_{t+1}\right)$$

Hence,

$$\frac{C_t^{-\sigma}}{Q_t\exp(\chi_{t+1})} = \beta E_t\left(C_{t+1}^{-\sigma}\left(\frac{1-\delta}{Q_{t+1}}+r_{t+1}\right)\left[\frac{U_{t+1}}{\left[E_t\left(U_{t+1}^{1-\theta}\right)\right]^{\frac{1}{1-\theta}}}\right]^{\sigma-\theta}\right)$$

Rewriting this gives the Euler equation (32) stated in the text:

$$1 = E_t\left(D_{t+1}R_{t+1}\right)$$

where

$$D_{t+1} \equiv \beta\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma}\left[\frac{U_{t+1}}{\left[E_t\left(U_{t+1}^{1-\theta}\right)\right]^{\frac{1}{1-\theta}}}\right]^{\sigma-\theta}$$

$$R_{t+1} \equiv (1-\delta+r_{t+1}Q_{t+1})\,\frac{Q_t}{Q_{t+1}}\,\exp(\chi_{t+1})$$

A.3 Proof of Proposition 2

(26) implies that along the RBGP Y_{jt} is proportional to G_{jt}, which we write as $Y_{jt} \propto G_{jt}$. Thus,

$$Y_t \propto G_{gt}^{\phi_g}G_{st}^{\phi_s}. \tag{A.2}$$

(22) implies that $M_{jjt} \propto G_{jt}$ and (23) implies that:

$$M_{j'jt} = \frac{\alpha_{M_{j'j}}}{\mu_j} \frac{p_{G_{jt}} G_{jt}}{p_{G_{j't}} G_{j't}} G_{j't}.$$

Using (24), the previous equation implies that $M_{j'jt} \propto G_{j't}$. (29) and (39) imply that $L_{jt} \propto 1$ and $K_{jt} \propto K_t$. Substituting all these equilibrium relationships into (4) gives:

$$G_{jt} \propto Z_{jt} K_t^{\alpha_{K_j}} A_{jt}^{\alpha_{L_j}} G_{jt}^{\alpha_{M_{jj}}} G_{j't}^{\alpha_{M_{j'j}}} \qquad (j, j' \in \{g, s\}).$$

We can solve this equation for G_{jt}:

$$G_{jt} \propto Z_{jt}^{\frac{1}{1-\alpha_{M_{jj}}}} K_t^{\frac{\alpha_{K_j}}{1-\alpha_{M_{jj}}}} A_{jt}^{\frac{\alpha_{L_j}}{1-\alpha_{M_{jj}}}} G_{j't}^{\frac{\alpha_{M_{j'j}}}{1-\alpha_{M_{jj}}}} \qquad (j, j' \in \{g, s\}).$$

Substituting the same equation for $G_{j't}$ into the previous equation:

$$G_{jt} \propto Z_{jt}^{\frac{1}{1-\alpha_{M_{jj}}}} K_t^{\frac{\alpha_{K_j}}{1-\alpha_{M_{jj}}}} A_{jt}^{\frac{\alpha_{L_j}}{1-\alpha_{M_{jj}}}} \left[Z_{j't}^{\frac{1}{1-\alpha_{M_{j'j'}}}} K_t^{\frac{\alpha_{K_{j'}}}{1-\alpha_{M_{j'j'}}}} A_{j't}^{\frac{\alpha_{L_{j'}}}{1-\alpha_{M_{j'j'}}}} G_{jt}^{\frac{\alpha_{M_{jj'}}}{1-\alpha_{M_{j'j'}}}} \right]^{\frac{\alpha_{M_{j'j}}}{1-\alpha_{M_{jj}}}}.$$

Solving for G_{jt}, we find:

$$G_{jt} \propto \left[Z_{jt}^{1-\alpha_{M_{j'j'}}} Z_{j't}^{\alpha_{M_{j'j}}} K_t^{\alpha_{K_j}\left(1-\alpha_{M_{j'j'}}\right)+\alpha_{K_{j'}}\alpha_{M_{j'j}}} A_{jt}^{\alpha_{L_j}\left(1-\alpha_{M_{j'j'}}\right)} A_{j't}^{\alpha_{L_{j'}}\alpha_{M_{j'j}}} \right]^{\frac{1}{\left(1-\alpha_{M_{jj}}\right)\left(1-\alpha_{M_{j'j'}}\right)-\alpha_{M_{jj'}}\alpha_{M_{j'j}}}}.$$

Substituting the previous equation for G_{gt} and G_{st} into (A.2) gives us the aggregate Cobb-Douglas production function (47). **QED**

B Data Documentation

B.1 NIPA Data

The full suite of industry data by NAICS industries is not available for the full post WWII period. We project previous shares of the missing components using previous industry classifications (SIC87 and SIC72).

B.1.1 Gross Output and Value Added by NAICS

1947–2018: Gross Output by Industry and Components of Value Added by Industry, BEA Industry Accounts, October 29, 2019 release.

B.1.2 Compensation of Employees, Gross Operating Surplus, Indirect Business Taxes

1987–2018: BEA Industry Accounts, October 29, 2019 release.

1947–1986: SIC72 Historical Accounts to calculate the COE and IBT shares of VA by Industry. We multiply these shares by NAICS VA.

B.1.3 Proprietors' Income

1998–2018: Table 6.12D. Nonfarm Proprietors' Income by Industry, July 30, 2019 release plus Farm Proprietor's income from Table 1.12. National Income by Type of Income July 30, 2020 release. 1987–1997: SIC87 Industry Accounts to calculate Proprietor's Income share of GOS and we multiply these shares by NAICS GOS.

1947–1986: SIC72 Historical Accounts to calculate Proprietor's Income share of GOS and we multiply these shares by NAICS GOS.

B.1.4 Real Output

1947–1997: Chain–Type Quantity Indexes for Value Added by Industry (2012 base year), BEA Industry Accounts, October 29, 2019 release.

1997–2018: Real Value Added by Industry (2012 dollars), April 6, 2020 release.

To obtain 1984–2000 sample, we use growth rates from the 1997–2018 source.

B.1.5 Price Indices

Investment price change is investment price growth minus total price growth.

Investment prices from NIPA Table 5.3.4. Price Indexes for Private Fixed Investment by Type, June 25, 2020 release. We use Private Investment (line 1) for private sector and Non-Residential Private Investment (line 2) for Non-Real Estate/Non-FIRE investment.

Total prices change is GDP deflator (line 1), NIPA Table 1.1.4. Price Indexes for Gross Domestic Product, June 25, 2020 release.

B.1.6 Fixed Assets

Capital stock by industry from Fixed Assets Table 3.1ESI. Current-Cost Net Stock of Private Fixed Assets by Industry, August 8, 2019 release.

Investment by industry from Table 3.7ESI. Investment in Private Fixed Assets by Industry, August 8, 2019 release.

B.2 Financial Data

Price-Dividend ratio taken from Kenneth R. French's data library:
https://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html
We use the 38 Industry Portfolio data.

B.3 Population

Non-institutionalized population, ages 16+, Census Bureau. FRED series CNP16OV.

C Value-added versus Gross-output Model

While we have estimated markups in a full-blown gross-output model that takes into account intersectoral input-output linkages, most of the literature on structural change abstracts from them and assumes that each sector produces value added. Such value-added models are popular because they are very tractable; see for example the canonical model in the review article of Herrendorf et al. (2014). In this appendix, we shall establish that using a value-added model would severely bias the estimates.

The value-added version of our model can be obtained as a special case of the gross-output version by setting $\alpha_{M_{jj}} = \alpha_{M_{j'j}} = 0,\ j, j' \in \{g, s\}$. To see the implications, we rewrite the first-order conditions (22)–(23) as:

$$p_{G_{jt}} M_{jjt} = \frac{\alpha_{M_{jj}}}{\mu_j} p_{G_{jt}} G_{jt}, \tag{A.3}$$

$$p_{G_{j't}} M_{j'jt} = \frac{\alpha_{M_{j'j}}}{\mu_j} p_{G_{jt}} G_{jt}. \tag{A.4}$$

Thus, $\alpha_{M_{jj}} = \alpha_{M_{j'j}} = 0$ implies that, in equilibrium, $M_{jjt} = M_{j'jt} = 0$. Imposing symmetric equilibrium, equations (6)–(7) then simplify to:

$$G_{jt} = Y_{jt} = V_{jt}.$$

That is, if $\alpha_{M_{jj}} = \alpha_{M_{j'j}} = 0$, then sectoral gross output equals sectoral final output equals sectoral value added. In this case, the gross-output model reduces to the special case of the value-added model and so it is straightforward to obtain the relevant calibration equations for the value-added model.

The first three calibration steps are the same in the value-added model as in the gross-output model. In contrast, the fourth calibration step changes. Denoting value-added parameters by a tilde, we have at the sectoral level:

$$\tilde{\mu}_j = \frac{p_{V_j} V_j}{(\rho_j + \delta_j + \gamma_Q) p_K K_j + w L_j}, \tag{A.5}$$

$$\tilde{\alpha}_{K_j} = \frac{(\rho_j + \delta_j + \gamma_Q) p_K K_j}{(\rho_j + \delta_j + \gamma_Q) p_K K_j + w L_j}, \tag{A.6}$$

$$\tilde{\alpha}_{L_j} = \frac{w L_j}{(\rho_j + \delta_j + \gamma_Q) p_K K_j + w L_j}. \tag{A.7}$$

At the aggregate level, the calibration of $\tilde{\alpha}_K$ and $\tilde{\alpha}_L$ remains unchanged whereas the calibration of $\tilde{\mu}$ changes to:

$$\tilde{\mu} = \frac{V}{(\rho + \delta + \gamma_Q) p_K K + w L}. \tag{A.8}$$

Comparing the value-added markups with the gross-output markups from before, we obtain the following result:

Proposition 3 *The aggregate markups are the same in the value-added model and the gross-output model: $\tilde{\mu} = \mu$. In contrast, if markups are positive, then the sectoral markups are larger in the value-added model than in the gross-output model: $\tilde{\mu}_j > \mu_j$.*

Proof. We start by showing that the aggregate markups are the same in the gross-output and value-added model. The result follows because, in a closed economy like ours, *aggregate* value added equals *aggregate* final output. The reason, of course, is that total intermediate inputs produced must equal total intermediate inputs used; see (12). Choosing final output as the numeraire, it thus follows:

$$\begin{aligned} V_t &\equiv \sum_{j \in \{g,s\}} p_{V_{jt}} V_{jt} = \sum_{j \neq j' \in \{g,s\}} \left(p_{G_{jt}} G_{jt} - p_{G_{jt}} M_{jjt} - p_{G_{j't}} M_{j'jt} \right) \\ &= \sum_{j \neq j' \in \{g,s\}} p_{G_{jt}} \left(G_{jt} - M_{jjt} - M_{jj't} \right) = \sum_{j \in \{g,s\}} p_{G_{jt}} Y_{jt} \equiv Y_t. \end{aligned}$$

Thus,

$$\tilde{\mu} = \frac{V}{(\rho + \delta + \gamma_Q) p_K K + w L} = \frac{Y}{(\rho + \delta + \gamma_Q) p_K K + w L} = \mu.$$

We now turn to showing that sectoral markups are smaller in the gross-output model than in the value-added model. Starting with the gross-output model, we first substitute the first-order conditions (22)–(23) into (7):

$$p_{V_{jt}} V_{jt} = p_{G_{jt}} G_{jt} \left(1 - \frac{\alpha_{M_{jj}} + \alpha_{M_{j'j}}}{\mu_j} \right). \tag{A.9}$$

Sector j's share of total value added is:

$$\frac{p_{V_{jt}} V_{jt}}{V_t} = \frac{p_{G_{jt}} G_{jt} \left(1 - \frac{\alpha_{M_{jj}} + \alpha_{M_{j'j}}}{\mu_j}\right)}{\sum_{j=g,s} p_{G_{jt}} G_{jt} \left(1 - \frac{\alpha_{M_{jj}} + \alpha_{M_{j'j}}}{\mu_j}\right)}.$$

Since $p_{G_{jt}} G_{jt} = \Phi_j Y_t$, this expression simplifies to:

$$\frac{p_{V_{jt}} V_{jt}}{V_t} = \frac{\Phi_j \left(1 - \frac{\alpha_{M_{jj}} + \alpha_{M_{j'j}}}{\mu_j}\right)}{\sum_{j=g,s} \Phi_j \left(1 - \frac{\alpha_{M_{jj}} + \alpha_{M_{j'j}}}{\mu_j}\right)}.$$

In the value-added model, in contrast, $\tilde{\mu}_j$ is given by (A.5) from above. Using (A.9), we get:

$$\tilde{\mu}_j = \frac{p_{G_{jt}} G_{jt} \left(1 - \frac{\alpha_{M_{jj}} + \alpha_{M_{j'j}}}{\mu_j}\right)}{r_t K_{jt} + w_t L_{jt}}.$$

Using the first-order conditions (20)–(21) gives:

$$\tilde{\mu}_j = \frac{\mu_j}{\alpha_{K_{jt}} + \alpha_{L_{jt}}} \left(1 - \frac{\alpha_{M_{jj}} + \alpha_{M_{j'j}}}{\mu_j}\right).$$

Since $1 = \alpha_{K_{jt}} + \alpha_{L_{jt}} + \alpha_{M_{jj}} + \alpha_{M_{j'j}}$, we end up with:

$$\tilde{\mu}_j - \mu_j = (\tilde{\mu}_j - 1)\left(\alpha_{M_{jj}} + \alpha_{M_{j'j}}\right).$$

Thus, $\tilde{\mu}_j > \mu_j$. **QED**

The result that aggregate markups are the same is intuitive. Since at the aggregate level of a closed economy, intermediate inputs produced equal intermediate inputs used, final output equals total value added, $V = Y$, aggregate markups must be the same in both cases. The result that sectoral markups are larger in the value-added model is intuitive too. With value added, the aggregate markups result from firms marking up the payments to capital and labor once. With gross output, the aggregate markups result from firms marking up the payments to capita and labor once and the payments to intermediate goods twice. Since sectoral gross-output markups are applied twice to intermediate inputs, they must be smaller than value-added markups.

Table C.1 contains the quantitative results for the value-added calibration. The first observation is that, as expected, $\tilde{\alpha}_{L_j} = \alpha_{L_j}/(\alpha_{K_j} + \alpha_{L_j})$; compare Table 4

Reassuringly, regarding markups, the result of the Table are consistent with the implications of Proposition 3. In particular, as in the gross-output model, aggregate markups went up by the 7 percentage points. Moreover, the levels of aggregate and sectoral markups roughly doubled. Different from the gross-output model, however, the levels of the sectoral markups are way

C.1 Calibrated parameters for value-added model of private economy without real estate

Parameters	1957–1973	1984–2000	2001–1916
$\tilde{\phi}_g$	0.460	0.283	0.194
$\tilde{\alpha}_L$	0.794	0.777	0.787
$\tilde{\alpha}_{L_g}$	0.809	0.754	0.702
$\tilde{\alpha}_{L_s}$	0.789	0.789	0.812
$\tilde{\mu}$	1.072	1.097	1.137
$\tilde{\mu}_g$	1.108	1.138	1.222
$\tilde{\mu}_s$	1.046	1.071	1.107

higher in the value-added model. For example, markups in the goods sector now go up all the way to 22% whereas before they went up to only 8%.

Our results show that the value-added model leads to a severe upward bias in the estimates of the sectoral markups, because it erroneously attributes the effect of double marginalization to sectoral markups. Our results have the important implication that the value-added model is fine for estimating aggregate markups, but the gross-output model with input-output linkages is essential for correctly estimating sectoral markups and how they aggregate to the economy-wide markups. This leads to an obvious tension with the literature on structural change, which typically employs versions of the value-added models; see Herrendorf et al. (2014) for a review. Although that is more tractable, our results show that employing value-added models can be very misleading in the presence of distortions (here the monopoly distortion that leads to markups).

We end the discussion by implementing the same decomposition as for gross output. To this end, note that (25) implies that without intermediate inputs, the Domar weights collapse into the final expenditure shares: $\Phi_{jt} = \tilde{\phi}_{jt}$. Substituting this into (30) gives the value added decomposition of the labor share:

$$\frac{w_t L_t}{Y_t} = \tilde{\phi}_g \frac{\tilde{\alpha}_{L_g}}{\tilde{\mu}_g} + \tilde{\phi}_s \frac{\tilde{\alpha}_{L_s}}{\tilde{\mu}_s}. \tag{A.10}$$

Table A1 reports the results:

Table A1: Decomposition of changes in aggregate labor share – top down

		1957–1973	**1984–2000**	**2001–2016**
Data		0.743	0.716	0.702
Constant	$\tilde{\mu}_j$	0.743	0.733	0.749
	$\tilde{\alpha}_{Lj}$	0.743	0.729	0.703
	$\tilde{\phi}_j$	0.743	0.703	0.660