DISCUSSION PAPER SERIES

DP16837
(v. 3)

Fear of Hiking? Monetary Policy and Sovereign Risk

Martin Wolf and Leopold Zessner-Spitzenberg

INTERNATIONAL MACROECONOMICS AND FINANCE

MONETARY ECONOMICS AND FLUCTUATIONS

ISSN 0265-8003

Fear of Hiking? Monetary Policy and Sovereign Risk

Martin Wolf and Leopold Zessner-Spitzenberg

Discussion Paper DP16837
First Published 26 December 2021
This Revision 29 July 2022

Centre for Economic Policy Research
33 Great Sutton Street, London EC1V 0DX, UK
Tel: +44 (0)20 7183 8801
www.cepr.org

Fear of Hiking? Monetary Policy and Sovereign Risk

Abstract

What are the implications of a monetary tightening in a currency union for sovereign default risk in a union member? We study this question in a quantitative sovereign default model and obtain two results. First, a monetary tightening reduces default risk in the union member when its debt/GDP ratio is below a critical threshold, driven by increased incentives to reduce the level of debt. Second, the monetary tightening increases default risk when debt/GDP is above the critical threshold. We quantify this "Fear of Hiking" zone and study its policy implications by applying our model to the euro area.

JEL Classification: E52, F34, F41

Keywords: monetary policy, Sovereign debt, sovereign default, Spreads, Currency Union, monetary fiscal interaction

Martin Wolf - martin.wolf@unisg.ch
University of St. Gallen and CEPR

Leopold Zessner-Spitzenberg - leopold.zessner-spitzenberg@hu-berlin.de
Humboldt University Berlin

Acknowledgements
We thank Mark Aguiar, Javier Bianchi, Guido Cozzi, Winfried Koeniger, Paul Pichler, Giorgio Primiceri and Joost Röttger for very useful comments. We also thank conference participants at the Barcelona 2022 Summer Forum, the D-A-CH 2021 workshop "Towards a micro-funded theory of monetary policy", and seminar participants at Bundesbank, Humboldt University of Berlin, University of St. Gallen, University of Konstanz and University of Vienna for useful comments. Leopold Zessner-Spitzenberg gratefully acknowledges fi nancial support by the Austrian Science Fund (FWF) Grant Nr. I 3840-G27..

Fear of Hiking?
Monetary Policy and Sovereign Risk

Martin Wolf and Leopold Zessner-Spitzenberg*

This draft: July 2022
First draft: December 2021

Abstract

What are the implications of a monetary tightening in a currency union for sovereign default risk in a union member? We study this question in a quantitative sovereign default model and obtain two results. First, a monetary tightening reduces default risk in the union member when its debt/GDP ratio is below a critical threshold, driven by increased incentives to reduce the level of debt. Second, the monetary tightening increases default risk when debt/GDP is above the critical threshold. We quantify this "Fear of Hiking" zone and study its policy implications by applying our model to the euro area.

JEL Codes: E52, F34, F41

Keywords: monetary policy, sovereign debt, sovereign default, monetary-fiscal interaction, monetary union

*Martin Wolf: University of St. Gallen and CEPR; martin.wolf@unisg.ch. Leopold Zessner-Spitzenberg: Humboldt University of Berlin; leopold.zessner-spitzenberg@hu-berlin.de. We thank Mark Aguiar, Javier Bianchi, Guido Cozzi, Winfried Koeniger, Paul Pichler, Giorgio Primiceri and Joost Röttger for very useful comments. We also thank conference participants at the Barcelona 2022 Summer Forum, the D-A-CH 2021 workshop "Towards a micro-funded theory of monetary policy", and seminar participants at Bundesbank, Humboldt University of Berlin, University of St. Gallen, University of Konstanz and University of Vienna for useful comments. Leopold Zessner-Spitzenberg gratefully acknowledges financial support by the Austrian Science Fund (FWF) Grant Nr. I 3840-G27.

"*As former central bankers and as European citizens, we are witnessing the ECB's ongoing crisis mode with growing concern. [...] In contrast, the suspicion that behind this measure lies an intent to protect heavily indebted governments from a rise in interest rates is becoming increasingly well founded.*" Open letter to the ECB in October 2019—see footnote 1.

1 Introduction

The aftermath of the Great Recession has been characterized by two developments in the euro area. First, public debt levels have been exceptionally high in several euro area countries. Second, the European Central Bank (ECB) has kept its main interest rates at very low levels. The joint observation of high public debt and low interest rates has sparked a debate whether the ECB keeps its interest rates low to reduce the risk of a sovereign default. Indeed, some observers claim the ECB has a fear of "hiking" - a fear of raising interest rates as this might intensify the debt problems in high-debt euro area countries (see the above quote).[1] But some crucial questions are open. Does a rate hike by the central bank necessarily lead to a higher risk of a sovereign default? And if not, what are the conditions under which interest rates can be raised without increasing the risk of a sovereign default?

In this article, we tackle these question by providing an Eaton and Gersovitz (1981) style model of a small member of a monetary union. The domestic country takes developments in the rest of the union, particularly the policy rate of the union-wide central bank, as given. The main insight from our analysis is that, when the union-wide central bank tightens, the domestic country may respond in one of two different ways. The first is a "benign" way, where the rate hike incentivizes the country to start repaying its debt, *reducing* the probability of a sovereign default. However, the country may also respond in a second, "not-so-benign" way. Indeed, the rate hike may incentivize the country to borrow even more, which *increases* the probability of a sovereign default. We show that the country responds by borrowing more when its debt/GDP ratio is sufficiently high to begin with, which we dub the "Fear of Hiking" zone of the model. We quantify the Fear of Hiking zone in a calibration of the model to Italy, and we also study some policy implications of our framework, particularly which policies might be used to make the Fear of Hiking zone smaller.

The Eaton-Gersovitz framework is a natural starting point for our analysis, because it is the workhorse framework for studying sovereign default risk in the literature (Arellano, 2008; Aguiar and Amador, 2014). The model we use is a quantitative version of the traditional Eaton-Gersovitz framework, with two main ingredients. First, the local government issues defaultable (long-term) debt to investors outside the domestic country but inside the monetary union, implying the policy rate by the union-wide central bank is the safe rate in the price of debt. Second, as in Bianchi and

[1] In October 2019, several former senior European central bankers formulated an open letter to the ECB, criticizing then-President Mario Draghi for the ECB's ultra-loose monetary stance. The open letter can be found here: https://www.ft.com/content/71f90f42-e68f-11e9-b112-9624ec9edc59. Sovereign default risk is still an important topic in 2022. On June 15, 2022, the Governing Council of the ECB held an ad hoc emergency meeting, discussing how to contain the rise in spreads in several euro area countries. Arguably, the rise in spreads was caused by expectations that the ECB may soon start to increase its interest rate in the face of surging inflation.

Mondragon (2021), we assume the presence of nominal wage rigidities, implying that monetary policy has real effects. We treat the central bank as exogenous, i.e. we do not take a stance why the central bank raises interest rates. Rather, we focus our attention on the response of the domestic country following the rise in interest rates.

We start by showing that the effects of a rate hike on borrowing decisions by the government can be framed in terms of two familiar forces which push the incentives to take on debt in opposite directions: a substitution and an income effect. The substitution effect is that borrowing becomes more expensive, creating incentives for the government to increase its primary budget balance in order to reduce the level of debt taken to the next period. The substitution effect therefore implies that debt levels - and by implication sovereign default risk - *decline* following a rise in interest rates. In contrast, the income effect is that refinancing of (maturing) debt occurs at a higher interest rate, implying that debt levels climb to the next period unless the government increases its primary budget balance to offset the higher interest expense. The income effect therefore implies that debt levels *rise* following a rise in interest rates.

We next show that the current level of debt/GDP governs which of the two effects dominates. When debt/GDP is below a critical threshold level, the substitution effect dominates whereas the income effect dominates when debt/GDP is above the critical threshold level. The intuition for the existence of a critical threshold is that the substitution effect is a marginal effect whereas the income effect becomes larger, the larger is the stock of debt (i.e., the income effect scales with the level of debt). We derive an analytical expression for the threshold, which depends in intuitive ways on model parameters. For instance, a lower elasticity of intertemporal substitution or a lower debt maturity both reduce the threshold, as the former implies a weaker substitution effect while the latter implies a stronger income effect. We also argue that the threshold is reduced, when frictions such as distortionary taxation limit the government's ability to adjust its primary budget balance following a rise in interest rates.

In light of these results, a natural question to ask is how large is the threshold in a reasonable calibration of the model, and what are its business cycle characteristics. To answer this question, we calibrate the model to Italy during the post-Great Recession period. We find that the threshold is 51% debt/GDP in the mean of the stationary distribution of the model. When interpreting this number, it is important to bear in mind that in the Eaton-Gersovitz framework, all public debt is external, i.e. held by investors outside the domestic country.[2] In turn, 51% debt/GDP is close to the *actual* amount of external debt issued by Italy in our calibration period. We next identify the set of states which imply that debt/GDP lies above the critical threshold in the equilibrium of the model - the "Fear of Hiking" zone of the model. We depict the Fear of Hiking zone graphically, and, by using simulations, we find that the model finds itself in the Fear of Hiking zone 71% of the time in our calibration to Italy.

[2]The key difference between external and internal debt is that the latter has no income effect as interest payments on public debt remain in the domestic country. The same holds (partly) true for public debt held by official creditors such as the ECB, as interest payments are partly redeemed to the borrowing country through the ECB's capital key. A sovereign default model for the euro area with both internal and external public debt is provided by Bocola et al. (2019).

In a last part of the paper, we turn to the policy implications of our framework.[3] Motivated by the fact that interest rates have been very low since 2008 in the euro area, we first study the implications of a long phase of low interest rates. A low interest rate environment is commonly seen as raising sovereign debt sustainability. In our model, this force shifts the stationary distribution of debt/GDP to the right, thus inducing the country to accumulate more debt. As we show, the stationary distribution of the critical threshold also shifts rightward, but by strictly less than the one of debt/GDP. By implication, following a long phase of low interest rates, the economy is now more likely to be in the Fear of Hiking zone. A recent literature has emphasized that monetary policy normalization becomes more difficult following a long period of low interest rates, due to for instance an accumulation of private debt which reduces aggregate demand or due to an increase in financial stability risk (Mian et al., 2021; Boissay et al., 2021). Our results are similar in spirit. In our model, raising interest rates becomes more likely to increase the risk of a sovereign default following a long period of low interest rates.

Assume now the economy is currently in the Fear of Hiking zone, implying the central bank cannot raise its interest rate without triggering more debt and a higher risk of a sovereign default. Which policies may be used to shift the critical threshold upward? We first discuss two possibilities: debt maturity extension and counter-cyclical fiscal transfers from the rest of the monetary union. We then show that forward guidance, i.e. announcements about *future* monetary policy, can also be used to increase the critical threshold. This makes forward guidance a complement for current monetary policy, as it enables the central bank to raise its current interest rate without increasing the risk of a sovereign default. To the best of our knowledge, the idea that forward guidance may be a *complement* (rather than a *substitute*) for current monetary policy is new. In our model, this is possible because the effects of monetary policy are highly state dependent, and because forward guidance changes the economy's current state. An appropriate mix between a current rate hike and expectation management by the central bank may thus be required for monetary policy normalization in a context of high public debt and the risk of a sovereign default.

Related literature. A classic literature has studied the impact of fluctuations in global safe real interest rates on developing economies (Neumeyer and Perri, 2005; Uribe and Yue, 2006), focusing in particular on spillover effects of changes in U.S. monetary policy (Kalemli-Özcan, 2019). A smaller subset of this literature has looked explicitly at the implications of changes in global rates on the risk of a sovereign default. Using Eaton-Gersovitz frameworks as we do, Johri et al. (2020) study the implications of changes in the level and volatility of global rates on default risk in emerging markets, and Centorrino et al. (2022) study the effects of *expected* movements in global rates on default risk in emerging markets. Motivated by recent events in the euro area, we study the implications of changes in the safe rate on default risk in a monetary union context; however, as we stress in the paper, our model nests as a special case the conventional sovereign default

[3] Also in this part of the paper, we treat monetary policy as exogenous, studying its effects on the domestic economy without taking a stance on the reasons for the change in monetary policy. All results we derive are therefore positive. The natural next step would be to study the implications of our findings for *optimal* monetary policy, which we consider to a be a fruitful avenue for future research and which we plan to address in a follow-up paper.

framework where a stand-alone small open economy faces variation in the global safe real interest rate (for instance, the canonical models by Arellano (2008) and Chatterjee and Eyigungor (2012) are nested as special cases in our framework). As a result, all our findings - in particular the existence of a threshold for debt/GDP above which the effects of changes in the safe interest rate flip - apply to this literature as well. To the best of our knowledge, we are the first to highlight the strong state dependence of the transmission of safe-rate shocks in the Eaton-Gersovitz model class, and to explore the implications of this finding for policy.[4]

Our paper also adds to the literature which studies policy challenges in the euro area. Closest to us are contributions with an explicit focus on the interaction between monetary policy and sovereign default risk in euro area members.[5,6] Corsetti and Dedola (2016) show that unconventional monetary policy (swaps of government debt for central bank reserves) can reduce the risk of self-fulfilling rollover crises. Bianchi and Mondragon (2021), Bocola and Dovis (2019) and Aguiar et al. (2015) also study rollover crises in the euro zone context. In our analysis, we only study conventional monetary policy and we focus our attention on default due to bad fundamentals. de Ferra and Romei (2020) build a model of a monetary union and study the interaction between monetary policy and sovereign default risk in some member countries. In their framework, the central bank affects the economy by changing the relative price of non-tradables, but it has no effect on the safe (real) interest rate which enters the price of debt. Our analysis is therefore complementary, because in our model the central bank affects the economy exclusively through the price of debt. Nuño et al. (2022a) argue that the ECB's asset purchases move the term structure of interest rates in euro area members by affecting the term premium, but also through changes in sovereign default risk. Na et al. (2018) study sovereign default risk in a euro area member using an Eaton-Gersovitz framework as we do, but without studying the effects of changes in monetary policy. Kriwoluzky et al. (2019) show that the possibility of a "Grexit" reinforced the 2012 sovereign debt crisis in Greece.

Finally, in terms of results, we also add to a third literature which stresses the presence of non-linearities when the level of public debt exceeds a critical threshold. In models with self-fulfilling default, the "Crisis zone" is typically reached when debt levels are high enough, whereas self-fulfilling default may not occur for low debt levels (Cole and Kehoe, 2000; Bianchi and Mondragon, 2021; Conesa and Kehoe, 2017). The literature on "fiscal limits" highlights that when debt levels are high enough, countries are no longer able to stabilize their stock of debt through higher taxes which may trigger expectations of inflation (Davig et al., 2011). We contribute to this literature by introducing a "Fear of Hiking" zone - in our analysis, a high level of debt changes how monetary policy influences borrowing decisions and sovereign risk, because it implies that the income effect

[4] Johri et al. (2020) also stress that there is a strong state dependence of the transmission of safe-rate shocks in the Eaton-Gersovitz framework. However, in their analysis, a higher safe rate still always implies that debt levels decline going forward, i.e. the substitution effect is always dominant. This happens because, in their model calibration, the threshold for debt/GDP above which the income effect dominates is too high to be attained in equilibrium.

[5] Another pressing policy challenge for the euro area is the possibility of self-fulfilling capital flights due to the high degree of capital mobility between euro area members (Fornaro, 2021).

[6] See also Arellano et al. (2020) and Nuño et al. (2022b), who use Eaton-Gersovitz frameworks to study the links between monetary policy and sovereign default risk in stand-alone countries.

of bond price changes becomes dominant.

The rest of the paper is structured as follows. In Section 2 we present our baseline model. In Section 3 we derive analytical results, notably the existence of a threshold for debt/GDP above which the effects of monetary policy flip. In Section 4 we quantify the threshold and introduce the Fear of Hiking zone. Section 5 discusses policy implications. Section 6 concludes. In the Appendix we collect the proofs of all propositions as well as additional materials.

2 Baseline model

In this section, we develop our baseline model of a small open economy inside a monetary union. The economy is small, because domestic developments have no repercussions on the rest of the monetary union. The economy issues defaultable debt which is held by investors in the rest of the union. A union-wide central bank sets short-term nominal interest rates. We assume the existence of nominal rigidities which implies that monetary policy has real effects. Time is discrete and indexed by $t \in \{0, 1, 2, ...\}$.

2.1 Households

The domestic economy is populated by a large number of identical households. There is no disutility from working and each household supplies one unit of labor inelastically. However, due to the presence of nominal wage rigidities to be described below, households may be able to sell less than their endowment on the labor market. For future reference, we say that the economy operates below potential / in a slack labor market whenever equilibrium employment is below households' unit-endowment. Households' preferences over consumption are given by

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{C_t^{1-\sigma}}{1-\sigma}, \qquad 0 < \beta < 1, \quad \sigma > 0, \tag{1}$$

where C_t is a composite of goods produced domestically $C_{h,t}$ and goods produced in the rest of the monetary union $C_{f,t}$

$$C_t = \zeta C_{h,t}^{1-\gamma} C_{f,t}^{\gamma}, \qquad 0 < \gamma < 1, \tag{2}$$

where $\zeta \equiv (1-\gamma)^{-(1-\gamma)}\gamma^{-\gamma}$. The parameter β captures impatience, $1/\sigma$ measures the elasticity of intertemporal substitution, and γ captures the economy's openness to trade.[7]

Each households' budget constraint is given by

$$P_{h,t}C_{h,t} + P_{f,t}C_{f,t} = W_t L_t - P_{h,t}T_t, \tag{3}$$

where $W_t L_t$ is labor income and where T_t is a lump-sum tax payment to the government (transfer if negative). In the budget constraint, the prices of domestic and foreign consumption goods are

[7] As in Galí and Monacelli (2005), Itskhoki and Mukhin (2021) and many others, we subsume in the trade openness parameter γ a combination of home bias in preferences, trade costs and non-tradable goods. Modeling non-tradable goods separately as done in many open economy models would not materially affect our results.

denoted $P_{h,t}$ and $P_{f,t}$, respectively. Cost minimization then leads to the demand functions for domestically produced and foreign goods

$$C_{h,t} = (1-\gamma)\frac{P_t}{P_{h,t}}C_t \tag{4}$$

$$C_{f,t} = \gamma\frac{P_t}{P_{f,t}}C_t, \tag{5}$$

with the cost-minimizing price index (the consumer price index) $P_t = P_{h,t}^{1-\gamma}P_{f,t}^{\gamma}$.

As is standard in the sovereign default literature, we assume households have no direct access to international financial markets. Rather, the decision to borrow or save is taken by the government, and households receive the proceeds from these transactions in the form of transfer payments $-T_t$. However, this assumption is not important for our results. In Appendix A.1, we show that granting a subset of households access to financial markets leaves our main findings unchanged, and we also discuss this model extension in Sections 4.3 and 5.2.

2.2 Firms

The economy is populated by a large number of identical firms which are owned by the households. Firms produce the domestic consumption good by using the technology $Y_t = L_t$, where L_t is labor demand. Firms' profits are $P_{h,t}Y_t - W_tL_t$. The first order condition is

$$P_{h,t} = W_t. \tag{6}$$

Firms make zero profits in equilibrium.

2.3 Nominal rigidities

We assume the presence of nominal wage rigidities in the domestic economy.[8] The presence of wage rigidities plays two roles in the model. First, it creates the possibility of involuntary unemployment. Second, it implies that monetary policy has real effects. Indeed, as we will see, prices inherit the wage stickiness, so that the central bank can affect the real interest rate of the domestic country through movements in the nominal interest rate.

As in recent sovereign default frameworks (Na et al. 2018, Bianchi et al. 2019 and Bianchi and Mondragon 2021), we assume the presence of downward nominal wage rigidity

$$W_t \geq \bar{W}. \tag{7}$$

By combining (7) with labor demand (6), this implies that $P_{h,t} \geq \bar{W}$, that is, the price of domes-

[8] A growing body of evidence emphasizes how nominal wage rigidities represent an important transmission channel through which monetary policy affects the real economy. For instance, this conclusion is reached by Olivei and Tenreyro (2007), who show that monetary policy shocks in the US have a bigger impact on output in the aftermath of the season in which wages are adjusted. Micro-level evidence on the importance of nominal wage rigidities is provided by Fehr and Goette (2005), Gottschalk (2005), Barattieri et al. (2014) and Fabiani et al. (2010).

tically produced goods is downward sticky as well. As it is standard, we close the labor market with the following complementary slackness condition

$$(W_t - \bar{W})(L_t - 1) = 0. \tag{8}$$

This condition implies that when the wage rigidity is slack, households supply their labor endowment; and conversely, that when households supply less than their endowment the wage rigidity binds.

For the rest of the union, in turn, we assume the price level is constant at all times, and we normalize the price level to one, $P_{f,t} = 1$. Because we consider aggregate shocks (changes in the central bank's policy rate), this amounts to assuming some form of nominal rigidity in the rest of the union as well. For simplicity, we are not making the underlying nominal rigidity in the rest of the union explicit, but restrict our attention to modeling in detail the domestic country.

Last, we normalize $\bar{W} = 1$. This implies that, in states when the wage rigidity binds, all prices in the monetary union are equal to one.

2.4 Domestic government

The government collects taxes $P_{h,t}T_t$ from households and issues nominal, defaultable long-term debt B_t, which is held by foreign investors. Because we are not modeling government consumption to keep the analysis parsimonious, the tax corresponds at the same time to the government's *primary budget balance*, an interpretation which we will adopt frequently below.[9] To model long-term debt, we assume that debt pays geometrically declining coupons (Hatchondo and Martinez, 2009; Chatterjee and Eyigungor, 2012). In particular, a bond issued in period t promises to pay $\tilde{\mu}(1-\mu)^{j-1}$ in period $t+j$ for all $j \geq 1$, where $\tilde{\mu} \equiv \mu + \iota$ and $\iota > 0$ is the nominal interest rate in steady state. The maturity of debt is therefore given by $1/\mu$, where $0 < \mu < 1$. The normalization of the debt service payment of the bond to $\mu + \iota$ implies that the default-free bond price in steady state equals 1. This allows us to interpret B_{t-1}/Y_t as debt/GDP, which we do in the rest of the paper.

At the start of each period, the government can choose to default on its debt. Conditional on repayment, the budget constraint of the government is

$$\tilde{\mu}B_{t-1} = P_{h,t}T_t + q_t(B_t - (1-\mu)B_{t-1}), \tag{9}$$

where we denote q_t the price of debt. The left-hand side captures the government's expenditure, consisting of coupon payments, while the right-hand side captures the government's income, given by the primary balance plus the net issuance of new debt.

When the government defaults, the domestic country is excluded from financial markets for a random number of periods. Moreover in this case, a utility loss $\kappa(Y_t, \xi_t)$ realizes, which depends

[9] See Bianchi et al. (2019) for an analysis of optimally chosen government consumption in a similar economic environment.

non-negatively on output Y_t and is subject to exogenous shifts through the variable ξ_t. The utility loss captures various unmodeled costs of default, such as a reputation loss or turmoil in financial markets. As is well known, without the utility loss from default, plausible levels of debt can not be generated in equilibrium.[10] The dependence of the utility loss on Y_t implies that the government' incentives to default are higher in a recession. Finally, the fact that κ can fluctuate through the variable ξ_t implies that the default decision becomes somewhat detached from the business cycle. The fact that default cannot be perfectly explained by fundamentals is a key feature in emerging market data (Aguiar et al., 2016) and it helps us to calibrate the model to observed spread levels in the quantitative application of the model in Section 4.

2.5 Foreign lenders

Debt issued by the domestic government is held by foreign investors. Foreign investors understand the governments' incentives to default, inducing them the charge an ex-ante lower price (a spread over the risk free rate). Assuming risk neutral foreign lenders, the price of debt is given by

$$q_t = \frac{\mathbb{E}_t(1-\delta_{t+1})(\tilde{\mu} + (1-\mu)q_{t+1})}{1+i_t}, \tag{10}$$

where δ_{t+1} is an indicator variable that takes the value of 1 if the government defaults in period $t+1$.

The opportunity cost of funds for foreign investors is the nominal interest rate i_t. It is the nominal interest rate set by the union-wide monetary authority. The fact that investors' opportunity cost is i_t reflects that investors are also members of the monetary union. Indeed, we assume that all debt issued by the domestic government is held inside the monetary union, and that the monetary union as a whole is a closed economy.[11]

2.6 Market clearing

Domestic goods market clearing is given by

$$Y_t = (1-\gamma)P_{h,t}^{-\gamma}C_t + P_{h,t}^{-\gamma}X_t. \tag{11}$$

It implies that output equals domestic plus foreign demand for domestic goods. Domestic demand is governed by equation (4), and is decreasing in the price of domestic goods. In turn, foreign demand is given by X_t, and is treated as an exogenous variable. We assume that also foreign demand for domestic goods is decreasing in the domestic price level, with the same elasticity as

[10] In the original work of Eaton and Gersovitz (1981) and Arellano (2008), rather than assuming a utility loss, the authors assumed an output loss that materializes in the event of default. Both versions of the model have been extensively used and produce very similar results. We follow Bianchi and Mondragon (2021) and assume that the loss is in terms of utility.

[11] Without this assumption, investors from outside the monetary union would rush to buy assets inside the monetary union in case their opportunity cost of funds lies below the rate set by the union-wide monetary authority. As stressed by Fornaro (2021), euro area countries receive the majority of their capital inflows from within the union.

domestic demand, given by γ.

When the wage rigidity is slack, households supply their full labor endowment implying that $Y_t = 1$. In this case, equation (11) can be seen as the equation determining $P_{h,t}$. When aggregate demand is too high, prices increase to the point where demand is again compatible with households' labor endowment (i.e., the domestic real exchange rate appreciates). In contrast, when the wage rigidity binds this implies $P_{h,t} = 1$, and equation (11) can be seen as the equation determining Y_t. Output becomes an endogenous variable, and is determined by fluctuations in domestic and foreign demand. Firms satisfy this demand by hiring the appropriate number of hours, which households supply as long as labor demand is below their labor endowment.

The second equilibrium condition is the domestic resource constraint. Combining equations (3) and (9), using that $W_t L_t = P_{h,t} Y_t$, and using equations (4)-(5), we obtain

$$P_{h,t}^{1-\gamma} C_t = P_{h,t} Y_t - \tilde{\mu} B_{t-1} + q_t (B_t - (1-\mu) B_{t-1}), \tag{12}$$

which holds in periods when the government chooses to repay. In turn, conditional on default the resource constraint is simply $P_{h,t}^{1-\gamma} C_t = P_{h,t} Y_t$.

2.7 Recursive government problem and Markov equilibrium

We consider a benevolent government that chooses sequentially without commitment. At the start of each period, the government decides whether to default on its debt. In case of repayment, the government decides how much debt it issues to the next period. The government takes as given its own action in the following period, while internalizing that its current action influences its action in the future through endogenous state variables. In a rational expectations equilibrium, the government's expectations and its own actions are compatible. Formally, we will thus study a Markov perfect equilibrium.

As is standard in the literature, we state the government's problem using recursive notation, that is, we henceforth omit time subscript t. Denote by $s = (X, \xi)$ the exogenous state. The other state variables are B, the current level of debt, and i, the nominal interest rate set by the central bank. When the government has access to financial markets, at the start of the period, it compares the value of repayment and the value of default, denoted by respectively $V^r(B, i, s)$ and $V^\delta(i, s)$

$$V(B, i, s) = \max_{\delta \in \{0,1\}} \left\{ (1-\delta) V^r(B, i, s) + \delta V^\delta(i, s) \right\}. \tag{13}$$

Let $U(C)$ denote the utility flow. The value of repayment is

$$V^r(B, i, s) = \max_{B', C} \left\{ U(C) + \beta \mathbb{E} V(B', i', s') \right\} \tag{14}$$

subject to the set of constraints

$$
\begin{aligned}
&i) &\quad P_h^{1-\gamma}C = P_hY - \tilde{\mu}B + q(B', i, s)(B' - (1-\mu)B)\\
&ii) &\quad Y = (1-\gamma)P_h^{-\gamma}C + P_h^{-\gamma}X\\
&iii) &\quad P_h \geq 1\\
&iv) &\quad Y \leq 1\\
&v) &\quad (P_h - 1)(Y - 1) = 0.
\end{aligned}
$$

In the set of constraints, the price of debt q is taken as given by the government, but the government understands that q is a function of its borrowing decision B'. Constraints iii) and iv) capture that prices cannot fall below one due to downward nominal wage rigidity and that labor demand cannot be larger than households' unit endowment. Finally, constraint v) is the complementary slackness condition (8).

The value of default is

$$V^{\delta}(i,s) = U\left(\frac{X}{\gamma}\right) - \kappa\left(\min\left\{1, \frac{X}{\gamma}\right\}, \xi\right) + \beta\mathbb{E}\left(pV(0,i',s') + (1-p)V^{\delta}(i',s')\right), \qquad (15)$$

where we denote p the probability of re-entering credit markets in the next period. Here we have used that $C = X/\gamma$ and $Y = \min\{1, X/\gamma\}$ under financial autarky, which follows from combining goods market clearing (11) and the resource constraint under financial autarky.

Definition 1 (Markov-perfect equilibrium) *For a given law of motion governing i and s, a Markov-perfect equilibrium is a set of value functions $\{V(B,i,s), V^r(B,i,s), V^{\delta}(i,s)\}$, a set of policy functions $\{\delta(B,i,s), B'(B,i,s), C(B,i,s)\}$ and a pricing function $q(B',i,s)$ such that*

1. *Given the bond price schedule, the value and policy functions solve equations* (13)-(15)
2. *The bond price schedule satisfies*

$$q(B',i,s) = \frac{\mathbb{E}(1-\delta(B',i',s'))(\tilde{\mu} + (1-\mu)q(B'',i',s'))}{1+i},$$

where $B'' = B'(B',i',s')$.

2.8 Discussion of economic environment

In the analysis that follows, we inspect how changes in the interest rate set by the union-wide monetary authority affect debt accumulation and default incentives in the small union member. We study this issue in light of recent events in the euro area. Our main period of interest is the time after the 2008 Global Financial Crisis, when interest rates have been low, debt levels have been elevated and spreads have been volatile in several euro area countries (notably in Spain, Italy, Portugal and Greece). To be able to speak to developments in these countries, we have made two assumptions to try to capture these countries' experiences, which we add to an otherwise standard model of sovereign default.

The first assumption is that inflation is low and stable due to nominal rigidities. While these countries have experienced high inflation rates before 2008, inflation rates have been low and stable after 2008, in spite of high unemployment. This finding has triggered interests in the macro effects of downward nominal wage rigidity during the European crisis (for instance Schmitt-Grohé and Uribe, 2016; Na et al., 2018; Wolf, 2020). We follow this literature by assuming that nominal wages are rigid downwards. At the same time, as we explain below, in our model surges of inflation are possible in "run-up" phases of the business cycle, when debt levels are currently low but are quickly expanding.

The second assumption is that the economy has a tendency to run external imbalances. Before 2008, the countries of interest have experienced large amounts of capital inflows, which have subsided or turned into outflows after 2008 (Schmitt-Grohé and Uribe, 2016). To see how we capture this in the model, assume the wage rigidity currently binds implying that $P_h = 1$. Combine equations (9), (11) and (12), to obtain

$$Y_t = \frac{1}{\gamma} X_t - \frac{1-\gamma}{\gamma} T_t. \tag{16}$$

As this equation shows, in periods when the primary budget is balanced ($T_t = 0$), output equals foreign demand divided by the openness coefficient, X_t/γ. In our quantitative analysis in Section 4 below, we will assume that foreign demand is typically quite weak ($X_t < \gamma$ in a typical period), implying that $Y_t < 1$ when the primary balance is zero. Intuitively, the economy suffers a structural demand shortage, in the sense that demand is not strong enough to maintain the economy at full employment. How can the government respond to the demand shortage? As equation (16) shows, the government can raise output by running a primary deficit ($T_t < 0$). Intuitively, a tax cut/transfer payment raises households' consumption, which - because households' consumption falls partly on domestic goods when the economy is not fully open ($\gamma < 1$) - also raises output due to a Keynesian-cross type of effect. But of course, the primary deficit is financed by an issuance of debt, that is, by capital inflows. This feature of the model therefore implies a tendency by the government to run external imbalances. Note that, when the primary deficit is large enough, this may even trigger inflation. As shown in equation (16), a large enough deficit implies that $Y_t > 1$. Hence the wage rigidity ceases to bind, and the economy experiences a surge of inflation.

In sum, by calibrating foreign demand X_t to be relatively weak, we capture the following dynamics. When debt levels are currently low (for instance in the periods after a sovereign default), the economy runs primary deficits, operates at full employment and experiences inflation. As debt levels climb, so does the risk of a sovereign default. This entails a rise in spreads, which stabilizes debt levels and implies a well-defined stationary distribution. In the stationary distribution, the primary balance fluctuates around zero or is even slightly positive, hence the wage rigidity mostly binds implying low inflation and unemployment.[12] We believe that this environment provides a useful backdrop to ask about the effects of monetary tightenings on public debt flows and default risk in euro area countries during the post-2008 period.

2.9 Traditional frameworks as a special case

While we have tried to tailor our framework to the euro area, we stress that it also nests the traditional frameworks in the sovereign default literature where a stand-alone country faces variation in the global safe real interest rate - and where Keynesian aggregate demand effects are absent and output is given by an exogenous and stochastic endowment - as a special case. Specifically, the framework by Chatterjee and Eyigungor (2012) emerges in our model once $\gamma = 1$ (the domestic economy is fully open), whereas Arellano (2008)'s model appears once we assume $\gamma = \mu = 1$ (the economy is fully open and debt maturity is one period).

To understand this role of the openness coefficient γ, combine the goods market clearing condition (11) and the resource constraint conditional on repayment (12), and impose that $\gamma = 1$

$$C_t = X_t - \tilde{\mu} B_{t-1} + q_t(B_t - (1-\mu)B_{t-1}).$$

Because foreign demand X_t is an exogenous and stochastic variable, this resource constraint is mathematically identical as the one studied by Chatterjee and Eyigungor (2012). Intuitively, when the economy is fully open, then all of households' consumption falls on foreign goods, implying domestic output becomes insulated from changes in domestic demand. More details can be found in Appendix A.2, where we provide an overview of the model's equilibrium conditions in the case $\gamma = 1$.

The fact that traditional frameworks are nested in our model has one important implication. It implies that all our results that follow - particularly the existence of a threshold for debt/GDP above which the effects of changes in the safe interest rate flip - apply to these traditional framework as well. This implies our analysis can also inform studies of stand-alone countries, where an important driver of the domestic business cycle is exogenous variation in the safe interest rate (see the citations in the related literature part in the introduction).

3 Monetary policy, sovereign borrowing and sovereign risk

In this section, we study analytically how the government's incentives to borrow and default are shaped by the union-wide monetary policy, proceeding in four subsections. In Section 3.1, we show that the sovereign's incentives to borrow are shaped by two competing effects, an income and a substitution effect. In Section 3.2, we show that a threshold for debt/GDP exists above which the

[12] Our model shares many similarities with the model by Bianchi et al. (2019), who focus explicitly on the experience of Spain. In their model, the government faces a trade-off between stabilizing domestic output through higher government consumption and incurring higher spreads. The trade-off arises, because government consumption is financed with foreign debt. In our model, the government can raise private households' consumption demand and domestic output by issuing government debt, but it also faces the trade-off that this implies higher spreads.

income effect dominates, implying higher interest rates entail more sovereign debt. In Section 3.3, we illustrate our main result by using a graphical approach. Finally, in Section 3.4, we discuss the implications of our results for sovereign default risk and spreads.

Throughout this section, we focus on rate hikes which leave expectations about the future monetary stance unchanged. In Section 5.2 we will add to the picture rate hikes that are expected to occur in the future (i.e., forward guidance).

3.1 Income and substitution effect

To illustrate the forces shaping debt accumulation, we focus on the government's optimality condition in a period when it has access to financial markets. Moreover, as this will be the case most of the time in the stationary distribution of the model, we consider a period when the wage rigidity binds (see Section 2.8). By combining constraints i) and ii) and inserting them in the value function (14), we then obtain

$$V^r(B,i,s) = \max_{B'} \left\{ U\left(\frac{1}{\gamma}\left(X - \tilde{\mu}B + \frac{\tilde{q}(B',s)}{1+i}(B' - (1-\mu)B)\right)\right) + \beta \mathbb{E} V(B',i',s') \right\}.$$

In the last equation, we have defined the bond price exclusive of the safe interest rate, $\tilde{q}(B',s) = q(B',i,s)(1+i)$, for reasons that become apparent shortly. Assume now that all functions in the government's problem are differentiable. Then the first order condition is[13]

$$U'\left(\frac{1}{\gamma}\left(X - \tilde{\mu}B + \frac{\tilde{q}(B',s)}{1+i}(B' - (1-\mu)B)\right)\right) \times \frac{1}{\gamma}\left(\tilde{q}(B',s) + \frac{\partial \tilde{q}(B',s)}{\partial B'}(B' - (1-\mu)B)\right) + \beta(1+i)\frac{\partial}{\partial B'}\mathbb{E}V(B',i',s') = 0. \quad (17)$$

As we can see, the government borrows according to a standard Euler equation, but subject to an endogenous borrowing limit as the price of debt is endogenous. Specifically, a large (negative) derivative $\partial \tilde{q}(B',s)/\partial B'$ indicates a steep price curve which, everything else equal, discourages further borrowing.

The Euler equation can be seen as the equation determining the equilibrium amount of borrowing B'. Because the safe interest rate i also enters this equation, moreover, this equation defines B' as a function of i implicitly. As we can see, the safe interest rate enters in two places. First, it multiplies the time preference rate β (substitution effect). Second, it enters the current marginal utility of consumption (income effect). A key insight of our analysis is that the substitution effect enters borrowing negatively (i.e., a higher interest rate reduces B'), whereas the income effect enters positively. The intuition for this is as follows.

Substitution effect. A higher interest rate implies a lower attractiveness of borrowing, hence the government has an incentive to reduce B' in order to reduce its exposure to the bond market.

[13] We are not making this assumption in Section 4 where we solve the model numerically using value function iteration. This being said, we verify in the numerical solution of the model that all optimality conditions derived in this section describe well the equilibrium of the model.

Formally, this follows because a higher interest rate puts more weight on the continuation value in the Euler equation (17), which is a declining function of B'.

Income effect. The income effect is that, when the interest rate is higher, the same amount of borrowing B' raises less income, implying a drop in consumption in the current period (see again the Euler equation). This gives incentives for the government to increase its borrowing B', in order to cushion the drop in consumption in the current period.

As we illustrate in the following proposition, a clean characterization of the conditions under which each of the two effects dominates is possible.

Proposition 1 *Assume the government takes borrowing decisions according to the Euler equation (17). Consider a marginal increase in the safe interest rate i. Then*

$$\frac{\partial B'}{\partial i} > 0 \iff \frac{\gamma}{\sigma} < \frac{q(B', i, s)(B' - (1-\mu)B)}{C}. \tag{18}$$

Proof. In Appendix A.3. ■

3.2 A threshold for debt/GDP

The interesting aspect of equation (18) is that it implies a threshold for debt/GDP above which the effects of monetary policy flip. Specifically, the substitution effect dominates on one side of the threshold, whereas the income effect dominates on the other side. In the following we explore this result in depth.

Corollary 1 *Assume the domestic government runs a zero primary balance, $T = 0$. Then the threshold for debt/GDP above which the effects of monetary policy flip is*

$$\mathcal{T} = \frac{\gamma}{\tilde{\mu}\sigma}. \tag{19}$$

When $B/Y < \mathcal{T}$, a rate hike reduces B' (the substitution effect dominates). When $B/Y > \mathcal{T}$, a rate hike increases B' (the income effect dominates).

We start with a special case which is particularly instructive. Assume the domestic government runs a zero primary balance, that is, the level of taxes is zero $T = 0$. From the government's budget constraint (9), this is the case when new debt issuance exactly covers the coupon payment, $q(B' - (1-\mu)B) = \tilde{\mu}B$. Using this in the resource constraint (12) reveals that $C = Y$ when the primary balance is zero. Inserting in equation (18), we thus obtain $\gamma/\sigma < \tilde{\mu}B/Y$. This yields the threshold (19) from Corollary 1.

The threshold (19) is governed by three parameters. We explain the influence of each of these parameters in turn.

The elasticity of intertemporal substitution. The first parameter is the EIS, given by $1/\sigma$. The influence of the EIS is easy to understand. The higher the EIS, the more the government is willing to shift consumption across periods. This parameter therefore determines the strength of the substitution effect. The higher the EIS, the stronger the substitution effect, making it more likely that the substitution effect dominates the income effect. This implies that the threshold $\mathcal{T}$ increases in the EIS.

Debt maturity. The second parameter is the coupon payment $\tilde{\mu}$, determined primarily by debt maturity (recall that $\tilde{\mu} = \mu + \iota$, where ι is the steady state nominal interest rate and $1/\mu$ is the average maturity of debt). When debt maturity is large, a small part of the debt must be refinanced every period. Hence the rate hike only affects this small part of the debt, making the income effect weaker. It follows that a low $\tilde{\mu}$ increases the threshold $\mathcal{T}$ above which the income effect dominates. As pointed out by Chatterjee and Eyigungor (2012), the fact that debt becomes insulated from changes in the bond price q is what enables models with long-term debt to explain large equilibrium amounts of debt. For the threshold that we derived, what matters is that long-term debt insulates the government from changes in the safe interest rate i.

Trade openness. The last parameter is the trade openness γ, which governs the degree to which changes in domestic consumption change domestic incomes. Recall from equation (11) that the lower γ (the more closed is the economy), the more consumption matters for domestic incomes due to a Keynesian-cross type of effect. This feedback enters the threshold $\mathcal{T}$ as strengthening the income effect (hence reducing the threshold). Intuitively, when consumption feeds back into domestic incomes, then it becomes more costly for the government to reduce borrowing following a rise in interest rates, as this also reduces domestic incomes.

The role of trade openness turns out to be quantitatively important for the threshold. Recall from Section 2.9 that standard Eaton-Gersovitz frameworks such as Arellano (2008) and Chatterjee and Eyigungor (2012) are nested in our model once the economy is fully open ($\gamma = 1$). As revealed in equation (19), in this case the threshold is $1/\tilde{\mu}\sigma$, which is about 200% for plausible parameter values (see our calibration in Section 4.1). In these models, therefore, the substitution effect is practically always dominant. Intuitively, what partial closedness adds in our model is to effectively increase the desire of the domestic government to smooth consumption (as consumption smoothing also implies smoothing of domestic incomes). This comes out naturally in our model, because nominal rigidities (which are needed anyway to study monetary policy) imply that incomes become demand-determined. However, this insight reveals that other channels which effectively imply a stronger motive for consumption smoothing would equally entail a reduction of the threshold. We will come back to this point in Section 4.3, where we will discuss various model extensions which imply that the threshold is lower.

Corollary 2 *For a general primary balance, the threshold for debt/GDP above which the effects of monetary policy flip is*

$$\mathcal{T} = \frac{\gamma}{\tilde{\mu}\sigma}\left(1 + \left(\frac{\sigma}{\gamma} - 1\right)\frac{T}{Y}\right) \tag{20}$$

When $B/Y < \mathcal{T}$, a rate hike reduces B' (the substitution effect dominates). When $B/Y > \mathcal{T}$, a rate hike increases B' (the income effect dominates).

We next generalize the threshold to the case of an arbitrary primary T balance in the current period. Again using the government's budget constraint (9) and the economy's resource constraint (12), we can see that condition (18) becomes $\gamma/\sigma < (\tilde{\mu}B - T)/(Y - T)$. Rearranging terms yields the threshold in equation (20).

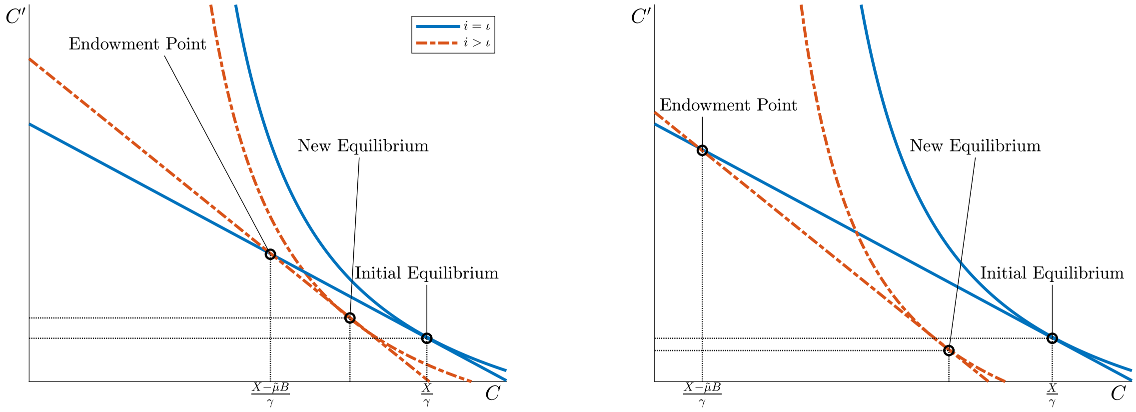
Equation (20) shows that in the general case, the threshold is a function of the state of the economy. In particular, the threshold depends on the primary balance/GDP ratio (positively so in the plausible case $\sigma \geq 1$, that is, assuming an EIS smaller than 1). The intuition is the following. When the government cuts taxes, the shortfall in revenue is compensated by taking on additional debt, that is, by capital inflows (in excess of those flows that refinance the coupon payment). Hence the government is exposed more strongly to changes in interest rates in such a period, making the income effect stronger. This explains that the threshold becomes *smaller* when the primary balance is *in deficit*.

Because the level of T is endogenous in the equilibrium of the model, it is hard to characterize the properties of the threshold further from equation (20). However, we will explore the implications of the dependence of the threshold on the business cycle in depth in our quantitative analysis in Section 4.

3.3 Illustration by using a diagram

The fact that interest rate changes have contrasting effects on borrowing should not come as a surprise. Indeed this result squares with undergraduate teaching of the effects of interest rate changes in the two-period consumption model. In this model, the substitution and income effects are typically explained in terms of consumption. The substitution effect of higher interest rates means that first-period consumption becomes more expensive relative to second-period consumption. Hence first-period consumption falls, whereas second-period consumption rises. In turn, for a *borrowing* agent, the income effect implies that the agent becomes poorer, which reduces consumption in both periods. The overall effect on *second-period* consumption is therefore ambiguous. Because borrowing and second-period consumption are directly related (second-period consumption equals second-period income net of debt repayment), the ambiguous response of second-period consumption maps into an ambiguous response of debt repayment. In addition, the income effect scales with the level of debt: the higher the debt stock, the stronger the income effect. In contrast, the substitution effect is inherently a marginal effect, and operates even when the debt stock is zero. This makes it clear that the income effect may start to dominate the substitution effect when the stock of debt is sufficiently high.

Perhaps the best way to make this link to the two-period consumption model is by using a diagram, which we do in Figure 1. To make the analogy as clear as possible, in the figure we make assumptions that reduce our model effectively to a two-period setup. We do so by assuming that all stochastic variables are constant, by ruling out default risk, and by assuming that the interest rate equals the steady state value $\iota = 1/\beta - 1$ from next period onward. By implication, next-period consumption is $C' = Y' - \iota B' = (X' - \iota B')/\gamma$, where we have used equation (11) to replace Y'. In the current period, instead, consumption is given by $C = (X - \tilde{\mu}B + q(B' - (1 - \mu)B))/\gamma$, where



(a) Low B: Substitution effect dominates. **(b) High B: Income effect dominates.**

Figure 1: Illustration of the effects of interest rate changes. Shown are the effects on current and future consumption of a rise in current-period interest rates i. We distinguish a case of low incoming debt (left panel), and high incoming debt (right panel). In Appendix A.4, we add to the figure a decomposition of the overall response into compensated effect and the effect due to a lower present value of income, plus we provide a formal Slutsky decomposition.

$q = (1+\iota)/(1+i)$. Combining both, we obtain the usual budget line in (C, C') space, connecting different levels of B'. Last, by using an indifference curve, we also plot the consumption bundle in the initial equilibrium. In the figure we focus on an initial equilibrium in which the primary balance is zero (see Corollary 1), implying $C = X/\gamma$.

Now turn to the effects of a rise in current-period interest rates i. As it is standard, the budget line tilts around the endowment point. In our model, however, the endowment point is a function of the model's endogenous state variable B. Precisely, it is given by $C = (X - \tilde{\mu}B)/\gamma$ (as this implies that $q(B' - (1-\mu)B) = 0$). The endowment point is necessarily to the left of the initial equilibrium, and the more so, the higher the level of B. We thus distinguish between a low level of B (left panel) and a high level of B (right panel). When interest rates rise, from the perspective of the initial equilibrium, the budget line not only tilts (substitution effect), but it also shifts (income effect). Moreover, the shift is larger, the larger is the level of B. This implies that, depending on the level of B, the new equilibrium can be characterized either by a higher level of C' (the tilt dominates, left panel), or by a lower level of C' (the shift dominates, right panel). Last, recalling that $C' = (X' - \iota B')/\gamma$, the ambiguous response of C' translates into an ambiguous response of B' in the new equilibrium. The figure thus makes it clear that a rise in interest rates *reduces* B' when current debt B is low, whereas it *raises* B' when current debt B is high.

In sum, there is a clear link between our findings and the conventional two-period consumption model, allowing us to illustrate our main result by using the well-known (C, C') diagram. However, there is also a key difference. In our model, the endowment point around which the budget line tilts is *endogenous*, as it depends on the endogenous state variable B. This implies that - depending on the current debt stock - either the substitution effect or the income effect of bond price changes may be dominant.

3.4 Implications for default risk and spreads

When thinking about the implications of a rise in interest rates for the risk of a sovereign default, we would first like to mention that a rate hike can trigger an *immediate* default in our model. This happens because the value to repay is reduced when the interest rate is higher, implying the government has a stronger incentive to default. However, this happens only when the economy is very close to the default threshold in the first place, which is a very small part of the state space in the numerical solution of the model (see Section 4). We thus proceed under the assumption that the government chooses to repay in the current period. In this case, we have shown that a tighter monetary policy has ambiguous effects on borrowing B'. What are the implications of this result for the probability of a future default and for spreads?

Define the "default set" as the set of exogenous state variables which imply that a default is chosen by the government at the start of the period. It is well known that, in this class of models, a larger amount of debt implies that the default set increases (Aguiar and Amador, 2014). Ex-ante, this implies that more debt is associated with a higher default probability. From this follows that the response of default risk to a rate hike is also shaped by the threshold $\mathcal{T}$. When a rate hike increases borrowing, which happens once debt/GDP is higher than $\mathcal{T}$, then a rate hike increases the probability of default. In contrast, when debt/GDP is lower than $\mathcal{T}$, then a rate hike reduces the probability of default.[14]

However, this does not necessarily imply that a rate hike reduces *spreads* in case debt/GDP is lower than $\mathcal{T}$. In fact, a rate hike may entail a rise in spreads even if it implies a decline in the default probability. The response of spreads is thus a poor guide of whether a rate hike increases or reduces the default probability.

To see this point, recall that the spread is defined as the yield to maturity of the bond with price q relative to a bond of equivalent maturity that does not carry a risk of default (e.g., Hatchondo and Martinez, 2009). In our model, this implies[15]

$$spread = (\mu + \iota)\left(\frac{1}{q} - \frac{1}{q^f}\right), \tag{21}$$

where q^f is the price of a risk-free bond, the solution of $q^f = (\mu + \iota + (1-\mu)q^{f\prime})/(1+i)$. We can simplify the algebra by assuming that debt maturity is one period ($\mu = 1$). In this case, the spread

[14] Because we assume the rate hike leaves expectations about future monetary policy unchanged, the continuation value is not *directly* affected by monetary policy implying that only the response of the endogenous state variable B' matters for the default probability. This will be different in Section 5.2, where we will study the implications of changes in expectations about *future* monetary policy.

[15] The yield to maturity is defined as the interest rate y_t which solves:

$$q_t = \sum_{s=1}^{\infty}(1+y_t)^{-s}(1+\mu)^{s-1}(\mu+\iota).$$

Inserting q_t and q_t^f, where q_t is defined in equation (10) and where q_t^f is defined in the text, we obtain the spread by using the equation $spread_t = y_t - y_t^f$.

becomes

$$spread = (1+\iota)\left(\frac{1+i}{\mathbb{E}(1-\delta')(1+\iota)} - \frac{1+i}{1+\iota}\right) = (1+i)\left(\frac{1}{\mathbb{E}(1-\delta')} - 1\right).$$

As this equation shows, when the default probability is strictly positive, the spread is increasing in the safe interest rate $1+i$. This implies a monetary tightening may increase the spread, even if it entails a reduction in the default probability $\mathbb{E}\delta'$. This effect is well known in the literature (e.g., Arora and Cerisola, 2001). Intuitively, when there is a risk of default, investors receive a payment only in states of the world in which default does not actually occur. To compensate, when the safe rate rises, the rate on the risky bond must increase more than one-by-one, which shows up as a rise in the spread. Therefore, to assess the implications of a rise in interest rates on the risk of default, it may not be a good idea to inspect the response of sovereign spreads.

4 Quantitative analysis

We have shown that the effects of monetary policy on sovereign borrowing and default risk in the small union member flip once debt/GDP in the small union member exceeds a critical threshold. This begs the question how large is the threshold in a reasonable calibration, and what are its business cycle characteristics. We take up this issue in this section. We calibrate our model to Italy in Section 4.1. In Section 4.2, we study the properties of the threshold in the equilibrium of the model and introduce the Fear of Hiking zone. In Section 4.3, we study the robustness of our results by studying two model extensions.

4.1 Calibration

We first calibrate the parameters which shape the critical threshold $\mathcal{T}$. From equation (20), the three parameters entering the threshold are the openness coefficient γ, the coupon payment $\tilde{\mu}$ and the EIS $1/\sigma$. Using an annual calibration, we assume that the nominal interest rate is 2% in steady state, implying $\iota = 0.02$. Recall that we abstract from trend inflation. In turn, a two-percent yearly real rate is a standard target in many studies. The average maturity of debt $1/\mu$ is observable in euro area countries. It is 7 years in Italy, implying that $\mu = 0.14$. This implies $\tilde{\mu} = \mu + \iota = 0.16$ for the coupon payment. The next parameter is the EIS. The evidence suggests that this parameter is quite close to zero.[16] For instance, Best et al. (2019) estimate the EIS to be 0.1. Havránek (2015) conducts a meta analysis and finds that typical estimates of this parameter lie in the range from 0.3 to 0.4. We proceed by placing ourselves in the middle of these two papers, by using $1/\sigma = 0.25$. The last parameter is the openness coefficient γ. To obtain this parameter we use data on the Italian balance of payments. In our model, gross exports are $P_{h,t}^{1-\gamma} X_t$ and gross imports are $C_{f,t}$. Combining households' demand for imports (5) and goods market clearing (11), we can see that

$$\gamma = \left(1 - \frac{P_{h,t}^{1-\gamma} X_t - C_{f,t}}{P_{h,t} Y_t}\right)^{-1} \left(\frac{P_{h,t}^{1-\gamma} X_t}{P_{h,t} Y_t} - \frac{P_{h,t}^{1-\gamma} X_t - C_{f,t}}{P_{h,t} Y_t}\right).$$

[16] See Hall (1988) for early evidence that the EIS is not much different from zero.

This equation establishes a link between γ and gross trade flows/GDP in our economy. We can measure their evolution in Italian data, and take an average over time in order to obtain a plausible value for γ. This yields $\gamma = 0.27$ implying a relatively small degree of openness, as about three quarters of the consumption basket are domestic goods. A home bias in spending of this magnitude is in line with the literature.

We now calibrate the remaining parameters. Following Bianchi et al. (2019), we set the annual probability of reentry after default $p = 0.18$. All other parameters are set to match key moments in the data through simulation. Details on the construction of moments are provided in Appendix B.2. We calibrate the discount factor β to match the average level of external debt/GDP in Italy between 2000 and 2019, given by 49.4%. We only include debt held by non-residents, as defined in the updated Bruegel database of sovereign bond holdings developed in Merler and Pisani-Ferry (2012). That is, we exclude domestic holdings of debt as well as holdings by official creditors, such as the ECB. The logarithm of foreign demand follows an AR(1) process

$$\log(X_t/\mu_X) = \rho_X \log(X_{t-1}/\mu_X) + \sigma_X \varepsilon_t$$

with $\varepsilon_t \sim_{i.i.d.} \mathcal{N}(0,1)$. The parameters μ_X, ρ_X and σ_X are set to match mean unemployment (our measure for the output gap) as well as the standard deviation and autocorrelation of detrended GDP. For the utility loss in default, we follow Bianchi et al. (2019) and use

$$\kappa(Y_t, \xi_t) = \max(0, L_0 + L_1 \log(Y_t)) + \xi_t.$$

As explained in Section 2, we assume that the cost of default is subject to shocks through the variable ξ_t. This shock plays a dual role in our analysis. First, it partly delinks the default decision from the economy's current state, reducing the sensitivity of the spread to economic fundamentals. This reduces the disciplining effect of spreads on borrowing which means higher spread levels are reached in equilibrium.[17] Second, as is well known, this shock improves the convergence properties of the solution algorithm in models with long-term debt.[18] Following this literature, we assume ξ_t is iid and follows a logistic distribution with location parameter 0 and scale parameter s_ξ. We set s_ξ to match the mean Italian spread over German bonds during 2006-2021. We calibrate the parameters L_0 and L_1 to match the standard deviation of the spread as well as the correlation of the primary balance with GDP. To understand the last point, note that the emergence of spreads tends to make the primary balance less pro-cyclical, a well-known feature of sovereign default models. Hence the loss function κ which matters for spreads can be used to discipline the cyclicality of the primary balance. Matching this cyclicality is important for us, because it maps directly into the cyclicality of the threshold, from equation (20).

The calibrated parameters and the fit of the model are summarized in Table 1. This calibration

[17] As discussed in Aguiar et al. (2016), the spread tends to react very steeply to borrowing in the absence of this shock. As a result, optimal debt levels tend to lie just below this steep part, at a low spread level.

[18] See Chatterjee and Eyigungor (2012) for an explanation. See Gordon (2019), Dvorkin et al. (2021) and Arellano et al. (2020) for recent applications of this method to sovereign default models.

Parameter	Value	Target	Data	Model
ι	0.02	Risk free rate	-	-
$1/\sigma$	0.25	EIS	-	-
γ	0.27	Openness coefficient	-	-
μ	0.14	Debt maturity	-	-
p	0.18	Exclusion period	-	-
β	0.945	mean(B_t/Y_t)	0.499	0.529
μ_X	0.257	mean($1-L_t$)	0.094	0.090
σ_X	0.022	std(Y_t)	0.023	0.021
ρ_X	0.65	corr(Y_t, Y_{t-1})	0.640	0.600
s_ξ	0.66	mean($spread_t$)	0.014	0.014
L_0	2.262	corr(T_t, Y_t)	0.080	0.070
L_1	20	std($spread_t$)	0.011	0.006

Table 1: Parameters and model fit. Details on the calibration are in Appendix B.

implies - in line with our explanations in Section 2.8 - that the wage rigidity binds most of the time in the stationary distribution of the model. We hence characterize an economy that is plagued by a high level of public debt and by weak aggregate demand (see also Figure 3b in Section 4.2).

To solve the model numerically, we discretize the state space for B and X. As it is standard, we approximate the process of X by using the Tauchen (1986) algorithm. We do not require a grid for ξ because this shock does not matter for the solution of the model conditional on repayment in the current period (as this shock is iid). We solve the model only at the steady state for the interest rate, $i = \iota$. To study monetary interventions, we use "MIT shocks", shocks that are zero-probability events. To study such shocks, it is not necessary to solve for policy functions outside of the steady state value for i. This implies that in all policy functions that follow, we suppress the two arguments $i = \iota$ and ξ for better readability. More details on the numerical algorithm can be found in Appendix B.3.

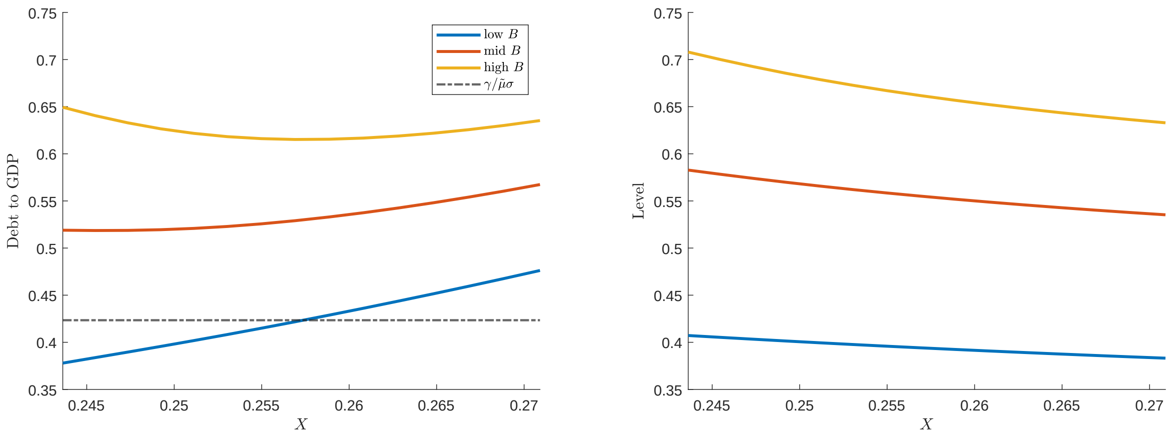
4.2 The Fear of Hiking zone

In the equilibrium of the model, the threshold is a function of the state variables and hence there exists a policy function $\mathcal{T}(B, X)$. To obtain this policy function, we evaluate equation (20) at the policy function of the primary balance/GDP ratio. We start inspecting the properties of $\mathcal{T}(B, X)$ by looking at unconditional statistics. The first row in Table 2 shows the mean of the stationary distribution of the threshold. The mean threshold is 51% debt/GDP, and is therefore in the same ballpark as the actual amount of external debt/GDP issued by Italy during our calibration period (49% debt/GDP, see Table 1).

Because debt/GDP is endogenous in the equilibrium of the model, from inspecting $\mathcal{T}(B, X)$ alone one cannot determine whether debt/GDP is above or below the threshold in the equilibrium

Statistic	Value
mean($\mathcal{T}$)	0.5116
mean($\mathbb{1}_{\mathcal{I}}$)	0.7056
corr($\mathbb{1}_{\mathcal{I}}, Y$)	-0.6781

Table 2: Unconditional statistics. Shown are unconditional statistics of the threshold (20) and the indicator variable (22). The indicator variable equals 1 in states where the income effect dominates, i.e., where the economy is in the Fear of Hiking zone.



(a) Threshold. **(b) Debt/GDP.**

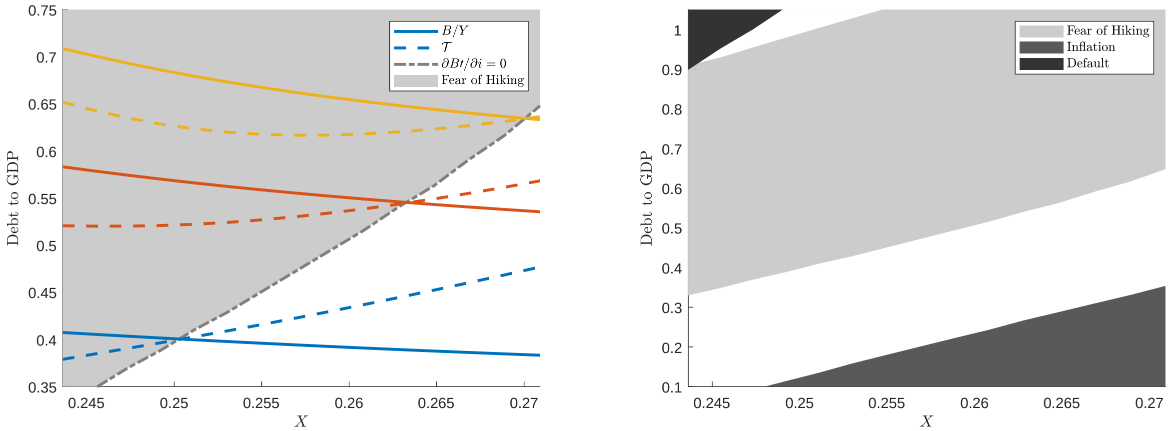
Figure 2: Policy functions for threshold and debt/GDP. Shown are policy functions $\mathcal{T}(B, X)$ (see equation (20)) and $B/Y(B, X)$, both drawn against X for three different values of B.

of the model. We thus additionally inspect the policy function $B/Y(B, X)$. We define the following indicator variable

$$\mathbb{1}_{\mathcal{I}}(B, X) \equiv \mathbb{1}\left\{\frac{B}{Y(B, X)} > \mathcal{T}(B, X)\right\}. \tag{22}$$

This indicator captures what below we will dub the Fear of Hiking zone: the set of state variables which imply debt/GDP is above the threshold in the equilibrium of the model. The second row in Table 2 reports the mean of $\mathbb{1}_{\mathcal{I}}$ in the stationary distribution, which is 0.71. This implies that 71% of the time, the economy finds itself in the Fear of Hiking zone. Finally, the last row of Table 2 shows the correlation between $\mathbb{1}_{\mathcal{I}}$ and output. It is -0.68, implying that visiting the Fear of Hiking zone is more likely when the economy is in a recession. Taken together, the unconditional statistics suggest that the Fear of Hiking zone is a quantitatively important part of the equilibrium of the model.

We next visualize the Fear of Hiking zone by using policy functions. Figure 2 shows the policy functions of the threshold (left panel) and debt/GDP (right panel). We draw the policy functions against the foreign demand variable X, by contrasting three different levels of current indebtedness B. In the case of the threshold, we also add a line at $\gamma/\tilde{\mu}\sigma$ - the threshold in states of the world where the primary balance is zero (Corollary 1). In what follows we briefly explain the shape of



(a) The Fear of Hiking zone. **(b) Adding default zone and inflation zone.**

Figure 3: Equilibrium segmentation of state space in calibrated model. The left panel shows how the Fear of Hiking zone is constructed, using the policy functions $\mathcal{T}(B, X)$ and $B/Y(B, X)$. The right panel adds the zones where the economy defaults in the current period, and where the wage rigidity ceases to bind (implying a surge of inflation).

the policy functions.

Focus first on left panel. We can see that the threshold $\mathcal{T}$ may depart substantially from the value $\gamma/\tilde{\mu}\sigma$, reflecting a high sensitivity of $\mathcal{T}$ with respect to the primary balance (equation (20)). Regarding the comparative statics we find that, for a given level of foreign demand X, a higher level of debt B implies a larger threshold. This reflects that the country starts to repay when its current stock of debt is high, implying a more positive primary balance. The threshold also tends to rise when foreign demand X increases (keeping fixed B). This is true except when debt levels are very high and foreign demand is very low, in which case the primary balance rises when X *declines* due to a rise in the spread - a well known feature of sovereign default models (Arellano, 2008). Turning to the behavior of debt/GDP (right panel), as it is intuitive, debt/GDP is larger when the level of debt B is larger. In turn, a larger X implies that domestic output increases, which explains that debt/GDP is downward sloping in X.

We are now ready to visualize the Fear of Hiking zone of the model. This is done in Figure 3. In the left panel, we draw the policy functions $\mathcal{T}(B, X)$ (dashed lines) and $B/Y(B, X)$ (solid lines) on top of each other. By construction, their intersections indicate the set of state variables where substitution and income effects of interest rate changes exactly balance, hence where $\partial B'/\partial i = 0$. We connect these intersections by using a gray dashed-dotted line. As we can see, moving to the left of this line, debt/GDP starts to rise above the threshold, hence the income effect starts to be dominant. The Fear of Hiking zone thus lies in the upper left part of the figure, and is visited when either debt levels are high or when foreign demand is low.[19]

When deriving the threshold in Section 3, we had made two assumptions. First, we had assumed

[19] The figure shows the Fear of Hiking zone for a marginal increase in the interest rate i. We verified numerically that the zone is very similar (it is slightly smaller) once i is increased by 1 percentage point. Hence, there is no strong non-linearity of the model regarding the size of the interest rate hike.

that the government finds it optimal to repay in the current period (no default). Second, we had assumed that the wage rigidity currently binds (no inflation). In the right panel of Figure 3, we check in which situations both assumptions are justified in the equilibrium of the calibrated model. Specifically, we add to the picture the "default zone", the set of state variables which imply that the government defaults immediately, and we add the "inflation zone", the set of state variables which imply the wage rigidity is slack, implying a surge of inflation. Note that we zoom out relative to the left panel of Figure 3 to make the two zones visible. We find that default occurs for low foreign demand and for very high debt levels - much higher than those that are needed to push the economy in the Fear of Hiking zone.[20] In turn, in line with our explanations in Section 2.8, the wage rigidity ceases to bind when foreign demand is strong and when debt levels are low. Overall, we see that the Fear of Hiking zone covers a relevant part of the state space.

We conclude this section with a remark on how the Fear of Hiking zone can be found in the numerical solution of the model. As we explained above, we obtained this zone by looking for the intersections between the policy functions for the threshold and for debt/GDP. An alternative - and more direct - way of obtaining this zone is by using numerical differentiation, computing the derivative $\partial B'/\partial i$ in the solution of the model and checking if the derivative is positive. We verified that the set of points where the derivative is positive equals exactly the zone that we obtained using policy functions. This testifies to the accuracy of our numerical algorithm, and it also confirms that the threshold in equation (20) provides an accurate description of the equilibrium of the model. This also reveals an alternative strategy to obtain the Fear of Hiking zone, in versions of the model where a closed form expression for the threshold is not readily available. We will make use of this insight in the next section, where we will study the robustness of our results once we extend our baseline model in various directions.

4.3 Robustness

While we view our baseline model as a reasonable benchmark to study the effects of monetary policy on sovereign debt and default risk, our framework is flexible enough to also incorporate various model extensions. These extensions shift the Fear of Hiking zone up or down, but leave our main conclusions otherwise unchanged.

In Appendix A.1, we study a model extension in which a subset of households have access to international bond markets (so called Ricardian households). This extension implies that government borrowing has less powerful effects on the domestic economy. In particular, more borrowing does not raise domestic consumption and output to the same extent as in our baseline model, because the borrowing is partly "offset" by Ricardian households raising their private foreign saving. This also implies that the critical threshold $\mathcal{T}$ in the model extension is now larger. As we show in

[20]Notice that the substitution effect becomes again dominant when the economy is very close to the default zone. In this region, the bond price is extremely steep, hence the government chooses to repay following a rise in interest rates. Formally, the primary balance is very positive in this region due to the high level of spreads, implying that $\mathcal{T}$ climbs again above B/Y. To construct the default zone, we assume the stochastic variable ξ entering the cost of default κ equals its mean value.

Appendix A.1, the threshold is still given by equation (20), but once the openness coefficient γ is replaced with $\lambda\gamma + 1 - \lambda$, where $1 - \lambda$ is the share of Ricardian households. That is, the economy with Ricardian households is effectively "more open", which for the reasons discussed earlier makes the threshold larger (implying the Fear of Hiking zone shrinks).

It may thus appear that in a model version with Ricardian households the Fear of Hiking zone would be quantitatively not important. However, this is not necessarily the case. This is because we have omitted at least two other features in our baseline model which imply that the Fear of Hiking zone is larger.

The first feature is that foreign demand X_t may be directly a function of the interest rate i_t. It is plausible to think that foreign demand depends on the interest rate, for instance, if foreign output contracts following a rate hike implying a reduction of imports of goods produced in the domestic economy. We thus assess the sensitivity of our results once we replace X_t with $X_t(1 + i_t - \iota)^{-\chi}$, where $\chi > 0$ is an interest semi-elasticity.

The second feature is that the government may face costs in the adjustment of the primary balance. Such costs may reflect, for instance, that a certain amount of revenue *must* be raised in every period (subsistence consumption, as in Bocola et al. 2019), or the presence of distortionary taxes.[21] To illustrate the effects of such adjustment frictions, we will simply assume that raising taxes enters negatively households' utility

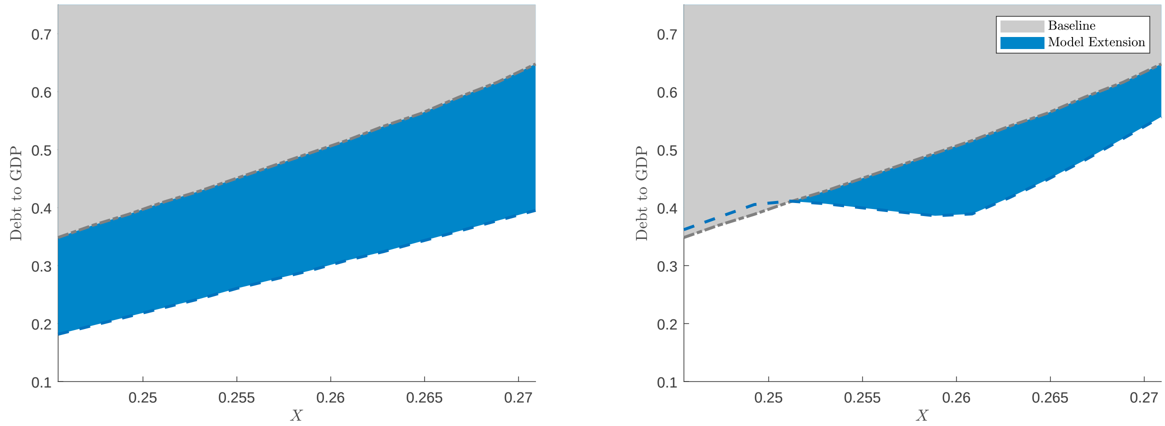
$$\tilde{U}(C_t, T_t) \equiv U(C_t) - \mathbb{1}\{T_t > 0\}\,\zeta T_t^2, \quad \zeta > 0,$$

which we use to replace utility $U(C_t)$ in equation (1).

Figure 4 shows the effects of both model extensions on the Fear of Hiking zone of the model. The left panel shows the case where foreign demand depends on the interest rate, the right panel shows the case where adjusting the primary budget balance is costly. In each case, we obtain the Fear of Hiking zone by using numerical differentiation, as explained at the of Section 4.2. We find that both model extensions make the Fear of Hiking zone *larger*. They imply, in other words, that a rate hike is now more likely to raise the level of debt and default risk by the government. The intuition for the first model extension is easy to understand. When a rate hike reduces foreign demand, this directly adds to the recession in the domestic economy. The government thus finds it optimal to borrow even more (despite the higher cost of borrowing), in order to cushion the recession.

To understand the effects of the second model extension, assume that the government finds it optimal to reduce its level of borrowing in the absence of adjustment costs (the substitution effect dominates), which it does by increasing its primary balance (raise taxes) to repay part of its debt. This may no longer be optimal under the model extension, because we assumed raising taxes is

[21] What about the government following fiscal rules? We view as one benefit of the Eaton-Gersovitz setup that the government is making optimal decisions, even if subject to constraints such as costs in the adjustment of the primary balance. In contrast, in the presence of fiscal rules there are no decisions to be made by the government (except the decision to default), and whether debt levels rise or fall following changes in interest rates is a mechanical outcome of the particular fiscal rule in place.



(a) Foreign demand. **(b) Adjustment costs.**

Figure 4: Fear of Hiking zone under two model extensions. The left panel shows how the Fear of Hiking zone changes when foreign demand is sensitive to the central bank's policy rate. The right panel shows how it changes when adjusting the primary budget balance is costly. The additional parameters used to construct this figure are $\chi = 0.05$ and $\zeta = 5$.

costly. Indeed, the government may now find it optimal to not adjust its primary balance, but to let the maturing part of its debt grow at the higher interest rate. Another way of stating this result is that adjustment costs effectively raise the desire of the government to smooth consumption. As we explained in Section 4.2, more consumption smoothing reduces the threshold $\mathcal{T}$, thus making the Fear of Hiking zone larger.

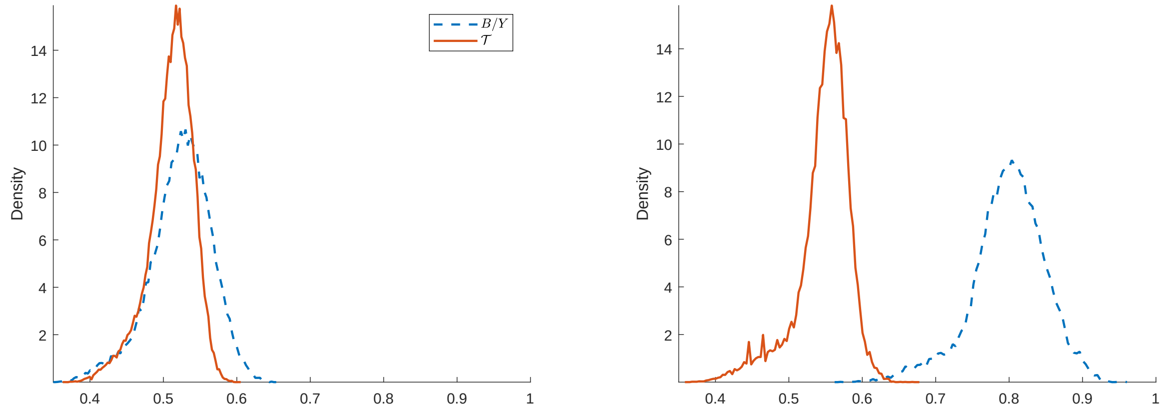
5 Policy implications

In this section we discuss some policy implications of our framework. To be clear, the objective of this section is not to provide a careful quantitative evaluation of actual policy proposals or to replicate any particular historical event. Moreover, we keep discussing our findings from a purely positive standpoint, without touching on normative issues such as optimal monetary policy. While we view such questions as very important and interesting, they are beyond the scope of this paper and hence we reserve them for future research.

In Section 5.1 we study the implications of a long period of low interest rates. In Section 5.2, we ask which policies may be used to shift to critical threshold upward, thus shrinking the Fear of Hiking zone.

5.1 Long period of low interest rates

Safe interest rates have declined steadily across major industrial economies in the last decades, which may have occurred due to a shortage of safe assets (Caballero et al., 2008) or due to other forces emphasized by the secular stagnation literature, such as a decline in potential output growth (Benigno and Fornaro, 2018) or rising inequality and population aging (Mian et al., 2021; Eggertsson et al., 2019). The euro area in particular has been characterized by very low interest rates in



(a) Baseline calibration. **(b) Long period of low interest rates.**

Figure 5: Stationary distributions for debt/GDP and threshold following a long period of low interest rates. Shown are stationary distributions for B/Y and $\mathcal{T}$, baseline calibration in the left panel ($\iota = 0.02$), low interest rates in the right panel ($\iota = 0$).

the aftermath of the Great Recession of 2008.

A recent literature has highlighted that a low-interest environment may pose particular challenges for monetary policy. For instance, Boissay et al. (2021) inspect a New Keynesian model with financial crises and stress that "financial crises may occur after a long period of unexpectedly loose monetary policy as the central bank abruptly reverses course". Mian et al. (2021) stress that a central bank may face "limited ammunition". In their model, a low interest environment reduces the *natural* rate of interest due an accumulation of private (and public) debt, making monetary policy normalization more difficult without sacrificing aggregate demand.[22] Motivated by this literature, we now inspect the implications of a long spell of low interest rates for sovereign default risk through the lens of our model.

We capture a long spell of low interest rates by assuming the steady state interest rate ι is now lower. We illustrate our results with the help of Figure 5. The left panel shows our baseline calibration $\iota = 0.02$, while the right panel considers a significant drop in the steady state interest rate, to $\iota = 0$. The figure shows that low interest rates shift the stationary distribution of debt/GDP to the right (blue-dashed). We are not showing it, but also the stationary distribution of default risk shifts to the right (despite the fact that interest rates are lower, which by itself tends to reduce default risk). Hence default risk is higher in the long term following a long spell of low interest rates, which is brought about by higher average levels of debt.

What is interesting in the context of our analysis is to contrast the shifts in the stationary distribution of debt/GDP with the one of the threshold $\mathcal{T}$ (red solid). We find that the stationary distribution of $\mathcal{T}$ also shifts to the right, but by less than the one of debt/GDP. Intuitively, the rightward shift of the stationary distribution for $\mathcal{T}$ reflects that the primary balance/GDP ratio is on average larger (recall equation (20)). This follows because debt levels are on average higher,

[22]See also Boissay et al. (2016) and Gerke and Röttger (2021).

requiring a larger primary surplus to maintain the stock of debt bounded.

Comparing stationary distributions for debt/GDP and the threshold we thus find that, following a long spell of low interest rates, the economy is now more likely to be in the Fear of Hiking zone. But this implies returning to higher interest rates becomes more difficult without increasing *even more* the level of debt by the government (thus increasing even more the risk of a sovereign default). In this sense, our results are in line with the literature stressing that a long spell of low interest rates increases financial stability risk (Boissay et al., 2021, 2016). Interestingly, low interest rates are often seen as making a high stock of debt more sustainable (indeed, it is this force which is pushing up the stationary distribution of debt/GDP in the first place). As we can see, however, the endogenous debt dynamics which are triggered by low interest rates may make it hard for the central bank to reverse course in the future.

In concluding we would like to stress that, of course, the reverse holds as well. While a long spell of high interest rates may make current debt levels less sustainable, this has the benefit of keeping the economy away from the Fear of Hiking zone. This may enhance the central bank's room the maneuver in the future.

5.2 Shifting up the critical threshold

Assume debt/GDP in the domestic economy is currently above the critical threshold, implying the central bank cannot raise its interest rate without triggering even more debt and a higher risk of a sovereign default. We now discuss policies which may be used to shift the critical threshold upward, focusing on three possibilities: debt maturity extension, the implementation of union-wide fiscal transfers, and expectation management.

5.2.1 Debt maturity extension

Our formula (20) reveals that the threshold depends critically on the debt maturity. Specifically, the threshold rises if the government issues longer maturity debt. As we explained in Section 3.2, this happens because the income effect of interest rate changes becomes weaker.

A downside of this policy, however, might be that a larger maturity not only increases the critical threshold, but it also increases the incentives of the government to accumulate more debt. Indeed as pointed out in Chatterjee and Eyigungor (2012), a large debt maturity (and the associated debt-dilution effect) is what enables Eaton-Gersovitz models to generate large equilibrium amounts of debt in the first place. Depending on which of the two effects is more important - the incentive to accumulate more debt versus the rise in the threshold - such policy may thus turn out to be self-defeating.

A recent literature has started to integrate *endogenous* maturity choice in the Eaton-Gersovitz framework. Arellano and Ramanarayanan (2012) show that governments tend to shorten their debt maturity in recessions (see also Aguiar et al. 2019), as a shorter debt maturity makes the debt-dilution problem less severe and thus makes refinancing cheaper. This channel may thus represent an additional force pushing the economy into the Fear of Hiking zone in a recession (as we argued in Section 4.2, visiting the Fear of Hiking zone is more likely to happen in recessions). Indeed Bocola and Dovis (2019) find that due to this channel, the maturity of Italian debt tended to decline at the height of the sovereign debt crisis. We view exploring the interaction between endogenous maturity choice and the critical threshold as an interesting model extension for future research.

5.2.2 Countercyclical fiscal transfers

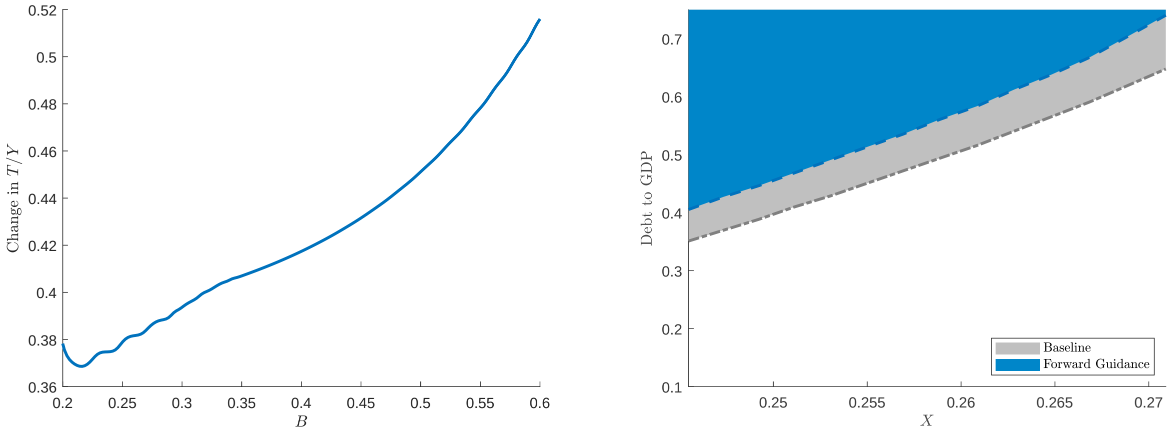
Our model implies that a system of countercyclical (fiscal) transfers from the rest of the monetary union to households in the domestic county shifts upward the critical threshold.

To understand this point, we go back to the model extension in which a subset of households is Ricardian (i.e. has access to financial markets in the monetary union). As we discussed in Section 4.3, this model extension implies the critical threshold is larger than in our baseline model. The mechanism through which this occurs is as follows. In the model extension, Ricardian households receive a transfer payment from the rest of the monetary union in states of the world where the government increases its primary balance following a rise in interest rates. The transfer payment cushions the negative effects of the government's austerity (the drop in consumption and output), thus making it less costly for the government to increase its primary balance and to reduce its level of debt. It is through this force that the critical threshold is increased.

One example of a countercyclical transfer scheme that has been discussed in the context of the euro area is a European unemployment insurance. Through the lens of our model, such a scheme would shield domestic output and the unemployment rate from a rise in taxes by the government. At the margin, this implies the government has stronger incentives to reduce its level of debt following a rise in interest rates. Ignaszak et al. (2020) discuss how such an unemployment scheme could be designed in the euro area, by taking account of moral hazard concerns as member states retain authority over local labor-market policies. In the context of our framework, additional moral hazard concerns would have to be addressed, as member states also have authority over the level of taxes/transfers, which interact with labor market outcomes.

The idea that more fiscal integration may be needed in the euro area has been debated in other contexts. In his seminal article, Kenen (1969) argued that fiscal integration was critical to a well functioning monetary union. Farhi and Werning (2017) provide a model of a monetary union showing that private risk sharing across member countries is constrained inefficient due to an aggregate demand externality. A role for fiscal transfers thus emerges, making the monetary union also a "fiscal union". In the model by Fornaro (2021), a role for a fiscal union emerges due to the possibility of self-fulfilling capital flights across the unions' member countries.

We identify a different role for fiscal transfers in a monetary union, by arguing that they provide incentives for member countries to reduce their public debt stock in the face of rising interest rates, implying default risk declines.



(a) Primary balance/GDP ratio. **(b) Fear of Hiking zone.**

Figure 6: Effect of forward guidance. Shown is the response of T/Y and the Fear of Hiking zone following the announcement of a future rise in the safe interest rate. The response of T/Y is expressed in percentage point deviation to the value taken without forward guidance.

5.2.3 Management of expectations and forward guidance

One key insight of our analysis is that the critical threshold is not just depending on the model's parameters (i.e. the *structure* of the model) but also on the economy's *current state*. As equation (20) shows, the threshold depends on the primary balance/GDP ratio - an equilibrium object. This suggests another way to shift up the critical threshold is to create incentives for the government to increase its primary balance. We next illustrate how this can be achieved via a management of expectations. Specifically, we show that contractionary forward guidance, i.e. announcing a rate hike in the future, can be used to shift the critical threshold upwards.

We provide the following numerical experiment. The central bank announces an interest rate hike by one percentage point that is implemented in the next period. Moreover, agents attach a probability of 90% that interest rates will still be higher by one percentage point in the period after next period, of 81% two periods after next period, and so forth. The result of this announcement is shown in Figure 6. The left panel shows the response of the primary balance/GDP ratio, drawn against different levels of current debt B while keeping foreign demand X at its mean value. The right panel shows the response of the Fear of Hiking zone. As we can see, the announcement has the effect of increasing the primary balance/GDP ratio by about 0.5 percentage points and, as a consequence, to shrink the Fear of Hiking zone.

This experiment thus assigns a different role to forward guidance than is commonly found in the literature. Typically, forward guidance is seen as a *substitute* for monetary policy, for instance when the central bank is constrained by the zero lower bound on its policy rate. By making an expansionary announcement, the central bank may then substitute for a rate cut in the current period (Eggertsson and Woodford, 2003). In contrast, we highlight the role of forward guidance as a *complement* for monetary policy. Namely, by shrinking the Fear of Hiking zone, forward guidance improves the terms of the central bank in the *current period*, in the sense of being (more) able to tighten without adverse effects on the risk of a sovereign default.

But what about the *direct* effects of the announcement on the risk of a sovereign default? There are two forces pushing default risk in opposite directions. On the one hand, the rise in the primary balance reduces debt in the next period, which reduces the risk of a sovereign default. Indeed the announcement reduces the debt going forward *everywhere* in the state space, implying there is no analogous object to a critical threshold for forward guidance.[23] On the other hand, the announcement reduces the value of repayment in future periods, expanding the default set for a given amount of borrowing. This increases the risk of a sovereign default. Quantitatively, we find both effects are roughly offsetting in our calibrated model, as we find default risk hardly responds to the announcement.

In sum, we have illustrated that the Fear of Hiking zone shrinks following contractionary forward guidance. This implies the economy may no longer be in the Fear of Hiking zone in the current period, enabling the central bank to raise its current interest rate without triggering more debt and a higher probability of default. In our model, this role of forward guidance emerges because the effects of monetary policy are highly state dependent, and because forward guidance changes the economy's current state.[24] How these insights translate into an optimal policy mix for the central bank between current rate hike and forward guidance represents a question that we plan to address in future research.

6 Concluding remarks

Recent developments in the euro area have exposed the need to better understand how monetary policy interacts with the risk of a sovereign default in a union member. Motivated by this question, in this paper we have constructed a quantitative sovereign default model of a small member of a monetary union. We have found that a rise in interest rates by the union-wide monetary authority may either raise or reduce sovereign borrowing and the risk of a sovereign default, and that this depends on the current debt/GDP ratio in the small union member. Specifically, a rate hike may reduce default risk going forward, but this will only work if debt/GDP is low enough to begin with, that is as long as the economy is not in the "Fear of Hiking" zone. We have quantified the Fear of Hiking zone and shown that it is sensitive to policy, for instance, it becomes smaller following an announcement by the central bank about a future tight monetary policy.

The paper is only a first step in a broader research agenda, and several question are still open.

[23] To understand this, take the perspective of the economy in the next period where the announcement is implemented. While borrowing B'' in this period responds ambiguously - due to the forces we emphasized in the paper - consumption in the next period C' still unambiguously declines. To smooth consumption, the government responds in the *current* period by reducing its borrowing. There are two other effects driving the response of B' in the current period, which we will mention briefly but which turn out to be swamped by the consumption smoothing effect just explained. First, due to long-term debt the announcement reduces the current bond price, which triggers contrasting income and substitution effects on B'. Second, the continuation value changes because of a change in future default risk. Due to the complexity of the problem, we cannot show the effects of forward guidance on B' analytically, but in our numerical solution, B' declines everywhere in the state space following the announcement.

[24] To be clear, this property of forward guidance should arise in any model in which monetary policy's effect on the economy is strongly state dependent.

First, we have analyzed this issue taking a purely positive perspective. The logical next step is to think about the welfare implications and in particular, to ask about the implications of our results for optimal monetary policy. This analysis will need to take care of central bank credibility and time consistency issues, as announcements by the central bank are likely to be not credible. Ex-post, the central bank may always have an incentive to cut the interest rate, as this makes an *imminent* default less likely.

Another open question is how monetary policy interacts with the possibility of self-fulfilling sovereign debt crises. In our analysis, we have only studied default risk due to bad fundamentals. However, a big issue in the euro area is whether default can be the outcome of self-fulfilling beliefs or animal spirits (De Grauwe and Ji, 2013; Bocola and Dovis, 2019; Bianchi and Mondragon, 2021). It would be interesting to assess if a tighter monetary policy makes the possibility of self-fulfilling default more or less likely.

Appendix (for online publication)

A Appendix analytical derivations

A.1 Model with Ricardian households

A.1.1 Setup

We normalize the total mass of households to 1. A subset λ of households is hand-to-mouth as in our baseline model, and has budget constraint

$$P_t C_t^i = W_t L_t^i - P_{h,t} T_t^i,$$

for households $i \in [0, \lambda]$. In turn, a subset $1-\lambda$ of households has access to international financial markets (so called Ricardian households). Following Bocola et al. (2019), we assume Ricardian households have access to a complete set of state contingent assets. As it will become clear below, this makes our analysis particularly tractable. Their budget constraint is

$$P_t C_t^i = W_t L_t^i - P_{h,t} T_t^i + \mathbb{E}_t d_{t,t+1} \Lambda_{t,t+1}^i - \Lambda_t^i$$

for all household $i \in [\lambda, 1]$. In both budget constraints, we have used that households solve their optimal expenditure problem according to (4)-(5), as in the baseline model.

In the budget constraint of Ricardian households, Λ_t^i denotes foreign assets, priced at the stochastic discount factor d. This implies the optimality condition $U'(C_t^i) = \kappa U'(C^*)$, where C^* is consumption in the rest of the monetary union and κ is a normalization factor (capturing relative initial wealth). Without loss of generality we may thus normalize $C_t^i = 1$ - a constant. Under the alternative assumption that C^* and thus C_t^i depend negatively on the central bank's policy rate, the Fear of Hiking zone would become correspondingly larger, for the same reason that it becomes larger when foreign demand is interest-dependent (see Section 4.3).

We focus on symmetric equilibria where all households face the same rationing in the labor market and the same tax/transfer from the government, $T_i^t = T_t$ and $L_t^i = L_t$ for all $i \in [0, 1]$. For Ricardian households, this implies their foreign assets also coincide in equilibrium $\Lambda_t^i = \Lambda_t$.

The problem of firms and the conditions characterizing the government are the same as before. Hence equations (6)-(10) still hold.

Aggregate consumption is given by $\lambda C_t + (1-\lambda)$, where C_t denotes consumption of the hand-to-mouth households, as in the baseline model. The goods market clearing condition thus becomes

$$Y_t = (1-\gamma) P_{h,t}^{-\gamma} (\lambda C_t + (1-\lambda)) + P_{h,t}^{-\gamma} X_t.$$

Conditional on repayment, the resource constraint of the economy is now given by

$$P_{h,t}^{1-\gamma} (\lambda C_t + 1 - \lambda) = P_{h,t} Y_t - \tilde{\mu} B_{t-1} + q_t (B_t - (1-\mu) B_{t-1}) + (1-\lambda)(\mathbb{E}_t d_{t,t+1} \Lambda_{t,t+1} - \Lambda_t),$$

where we took the sum of the budget constraints of hand-to-mouth and Ricardian households and replaced T_t by using the government's budget constraint. This shows that capital flows are now composed of two parts. There are public flows, given by $\tilde{\mu}B_{t-1} + q_t(B_t - (1-\mu)B_{t-1})$, and there are private flows, given by the term $\mathbb{E}_t d_{t,t+1}\Lambda_{t,t+1} - \Lambda_t$.

Conditional on default, the public flow part disappears, but we assume Ricardian households still have access to state contingent assets implying their consumption is still perfectly insured:

$$P_{h,t}^{1-\gamma}(\lambda C_t + 1 - \lambda) = P_{h,t}Y_t + (1-\lambda)(\mathbb{E}_t d_{t,t+1}\Lambda_{t,t+1} - \Lambda_t).$$

A.1.2 Equilibrium

The government borrows in international financial markets to maximize utility of all households, $\int U(C_t^i)di$. Using that households' consumption of subgroup $[\lambda, 1]$ is a constant, this yields the objective $\lambda U(C_t) + (1-\lambda)U(1)$. Ignoring the additive constant, the government's objective conditional on repayment is thus given by

$$V^r(B,i,s) = \max_{B',C} \left\{ \lambda U(C) + \beta \mathbb{E} V(B',i',s') \right\}$$

subject to the set of constraints

$$\begin{aligned}
&i) && P_h^{1-\gamma}C = P_hY - \tilde{\mu}B + q(B',i,s)(B' - (1-\mu)B) \\
&ii) && Y = (1-\gamma)P_h^{-\gamma}(\lambda C + (1-\lambda)) + P_h^{-\gamma}X \\
&iii) && P_h \geq 1 \\
&iv) && Y \leq 1 \\
&v) && (P_h - 1)(Y-1) = 0.
\end{aligned}$$

The first constraint is obtained by combining hand-to-mouth households' budget constraint with the government budget constraint, and looks the same as in the baseline model. The second constraint is the goods market clearing condition, which is different compared to the baseline model. The remaining constraints are the usual inequality and complementary slackness constraints. The budget constraint of Ricardian households plays no role in the maximization, because it determines the amount of private capital flows $\mathbb{E}_t d_{t,t+1}\Lambda_{t,t+1} - \Lambda_t$ as the residual.

Conditional on default, the value function is

$$V^\delta(i,s) = \lambda U\left(\frac{(1-\gamma)(1-\lambda) + X}{1-(1-\gamma)\lambda}\right) - \kappa\left(\min\left\{1, \frac{(1-\gamma)(1-\lambda)+X}{1-(1-\gamma)\lambda}\right\}, \xi\right) \\ + \beta\mathbb{E}\left(pV(0,i',s') + (1-p)V^\delta(i',s')\right).$$

Finally, the default variable δ is determined by the optimality condition

$$V(B,i,s) = \max_{\delta \in \{0,1\}} \left\{ (1-\delta)V^r(B,i,s) + \delta V^\delta(i,s) \right\}.$$

Notice that, when $\lambda = 1$ (all households are hand-to-mouth), then all value functions collapse to the ones studied in the baseline model.

Definition 2 (Equilibrium with Ricardian households) *For a given law of motion governing i and s, a Markov-perfect equilibrium is a set of value functions $\{V(B,i,s), V^r(B,i,s), V^\delta(i,s)\}$, a set of policy functions $\{\delta(B,i,s), B'(B,i,s), C(B,i,s)\}$ and a pricing function $q(B',i,s)$ such that*

1. *Given the bond price schedule, the value and policy functions solve the equations defined in this subsection*
2. *The bond price schedule satisfies*

$$q(B',i,s) = \frac{\mathbb{E}(1-\delta(B',i',s'))(\tilde{\mu} + (1-\mu)q(B'',i',s'))}{1+i},$$

where $B'' = B'(B',i',s')$.

A.1.3 Threshold

Due to complete markets assumption, the threshold $\mathcal{T}$ of the model with Ricardian households can also be obtained in closed form. When the wage rigidity binds, the value function conditional on repayment can be written as

$$V^r(B,i,s) = \max_{B'} \left\{ \lambda U \left(\frac{1}{1-(1-\gamma)\lambda} \left((1-\gamma)(1-\lambda) + X - \tilde{\mu}B + \frac{\tilde{q}(B',s)}{1+i}(B' - (1-\mu)B) \right) \right) + \beta \mathbb{E} V(B',i',s') \right\}.$$

This reveals three differences relative to our baseline model. First, the utility benefit of borrowing is scaled down by λ, reflecting that a part of the transfers to households is lost as it ends up in the hands of Ricardian households. Rather than consume the transfer, these households send it one-for-one abroad via a state contingent payment. Hence this part of the transfer is lost for consumption in the domestic economy. The second difference is that the autonomous income component in the value function is now given by $(1-\gamma)(1-\lambda) + X$ and not just X, reflecting that also Ricardian households' consumption is independent of domestic developments. The last difference is that the Keynesian-cross effect of consumption into domestic output is now weaker. Precisely, the multiplier of autonomous income into consumption is now given by $1/(1-(1-\gamma)\lambda)$, rather than $1/\gamma$. In the main text, we called γ the openness coefficient. Hence, the economy becomes now effectively *more open* as γ is replaced by

$$1-(1-\gamma)\lambda = \lambda\gamma + (1-\lambda)1,$$

which is necessarily larger than γ. Intuitively, the increased openness arises because Ricardian households' consumption is independent of output (their marginal propensity to consume out of current income is zero). When output increases, their consumption does not rise; rather, the additional output flows abroad via a state contingent payment.

By taking the same steps as in Proposition 1, we obtain the condition

$$\frac{\partial B'}{\partial i} > 0 \iff \frac{1-(1-\gamma)\lambda}{\sigma} < \frac{q(B',i,s)(B'-(1-\mu)B)}{C}.$$

Correspondingly, the threshold (20) is now replaced by the condition

$$\mathcal{T} = \frac{1-(1-\gamma)\lambda}{\tilde{\mu}\sigma}\left(1+\left(\frac{\sigma}{1-(1-\gamma)\lambda}-1\right)\frac{T}{Y}\right).$$

This shows two things. First, a threshold for public debt/GDP above which the effects of changes in the safe interest rate flip also exists in a model with Ricardian households. Second, compared to our baseline model, the threshold is now necessarily *larger*, making the Fear of Hiking zone *smaller*. This happens because the economy is now effectively more open, as γ has been replaced by $1-(1-\gamma)\lambda$. Intuitively, cutting borrowing by the government following a rise in interest rates becomes now more attractive, because domestic consumption and incomes are partly insulated from the cut in borrowing. Precisely, the public capital *outflow* resulting from the cut in borrowing is partly compensated by a private capital *inflow* as Ricardian households draw down their foreign assets, sustaining domestic consumption and domestic incomes.

A.2 Model conditions when $\gamma = 1$

When the economy is fully open ($\gamma = 1$), our model becomes mathematically identical as Chatterjee and Eyigungor (2012)'s setup. This implies our main results - particularly the existence of a threshold for debt/GDP above which the effects of changes in the safe interest rate flip - carry over to the case of stand-alone small open economies (ones that are not part of a monetary union) as well.

A.2.1 Setup

Households. When $\gamma = 1$, households buy only foreign goods, see equations (4)-(5) which now imply $C_{h,t} = 0$ and $C_{f,t} = C_t$. Here we have used that $P_t = P_{f,t} = 1$, where the first equality follows from the definition of the price index and using that $\gamma = 1$, and where the second equality is the normalization for $P_{f,t}$ that we also used in the main text. Using these results, households' budget constraint (3) becomes $C_t = W_t L_t + P_{h,t} T_t$.

Other agents. The problem of all other agents (firms, international investors, and domestic government) is unchanged from before. We also assume the same nominal rigidity as before. Hence the equilibrium conditions (6)-(10) are the same as before.

Market clearing. The goods market clearing condition (11) is now $Y_t = P_{h,t}^{-1} X_t$. Note that domestic output is matched exlusively by foreign demand. This happens because all of domestic demand falls on foreign goods (imports) when $\gamma = 1$.

We use that $W_t L_t = P_{h,t} Y_t$ and combine households' budget constraint (derived above), the government's budget constraint (9), and goods market clearing, to derive the resource constraint conditional on repayment

$$C_t = X_t - \tilde{\mu} B_{t-1} + q_t(B_t - (1-\mu)B_{t-1}.$$

Under financial autarky, this becomes simply $C_t = X_t$.

A.2.2 Equilibrium

At the start of the period, the government decides whether to default

$$V(B,i,s) = \max_{\delta \in \{0,1\}} \left\{ (1-\delta)V^r(B,i,s) + \delta V^\delta(i,s) \right\}.$$

The value of repayment is now

$$V^r(B,i,s) = \max_{B',C} \left\{ U(C) + \beta \mathbb{E} V(B',i',s') \right\}$$

subject to

$$C = X - \tilde{\mu} B + q(B',i,s)(B' - (1-\mu)B),$$

and subject to the bond price schedule (10), which is unchanged from before. In turn, conditional on autarky, the value is

$$V^\delta(i,s) = U(X) - \kappa\left(\min\{1,X\},\xi\right) + \beta \mathbb{E}\left(pV(0,i',s') + (1-p)V^\delta(i',s') \right).$$

We then define equilibrium as follows.

Definition 3 (Equilibrium $\gamma = 1$) *For a given law of motion governing i and s, a Markov-perfect equilibrium is a set of value functions $\{V(B,i,s), V^r(B,i,s), V^\delta(i,s)\}$, a set of policy functions $\{\delta(B,i,s), B'(B,i,s), C(B,i,s)\}$ and a pricing function $q(B',i,s)$ such that*

1. *Given the bond price schedule, the value and policy functions solve the equations defined in this subsection*

2. *The bond price schedule satisfies*

$$q(B',i,s) = \frac{\mathbb{E}(1-\delta(B',i',s'))(\tilde{\mu} + (1-\mu)q(B'',i',s'))}{1+i},$$

where $B'' = B'(B',i',s')$.

A.2.3 Discussion

Chatterjee and Eyigungor (2012) consider a real model of a small open economy that issues defaultable long term debt to international investors, faces an exogenous output stream and a global safe real interest rate r. As it becomes apparent from the equilibrium definition in the previous subsection, our model is mathematically equivalent to such a model when $\gamma = 1$. Particularly, the role of the exogenous output stream is played by the foreign demand variable X, and the role of the global safe real interest rate r is played by the union-wide safe nominal interest rate i. To be clear, ours is still a model of a small member of a monetary union, even when $\gamma = 1$. It is only that the equilibrium conditions *look the same* compared to a model such as the one in Chatterjee and Eyigungor (2012).

Why does our model look like such a model when $\gamma = 1$? First, domestic output $P_{h,t}Y_t$ is still endogenous in our model, even when $\gamma = 1$. However, by the goods market clearing condition, it is determined only by *foreign* demand, which is an *exogenous* variable. This implies that output becomes effectively exogenous when $\gamma = 1$. Second, because all of domestic consumption falls on foreign goods, the domestic nominal (wage) rigidity becomes effectively irrelevant and it drops out of the equilibrium conditions. In particular, when all spending falls on foreign goods, the tradeable-goods-based real interest rate that matters for spending decisions is $(1 + i_t)P_{f,t}/P_{f,t+1}$. Now recall that we normalized $P_{f,t} = 1$. This implies that the nominal interest rate i becomes effectively a *real* interest rate when $\gamma = 1$, independent of the state of the economy.[25]

A.3 Proof of Proposition 1

Denote $\tilde{q}(B', i, s) \equiv q(B', i, s)(1+i)$ the bond price exclusive of the interest rate. By equation (10), it is given by

$$\tilde{q}(B', i, s) = \mathbb{E}(1 - \delta(B', i', s'))(\tilde{\mu} + (1 - \mu)q(B'(B', i', s'), i', s')).$$

Because i' is by assumption independent of i, this equation makes it clear that $\tilde{q}$ is not a function of i. Hence we can write $\tilde{q}(B', s)$. Furthermore we have

$$\frac{\partial q(B', i, s)}{\partial B'} = \frac{\partial}{\partial B'} \frac{\tilde{q}(B', s)}{1 + i} = \frac{1}{1 + i} \frac{\partial \tilde{q}(B', s)}{\partial B'}.$$

This yields the Euler equation (17)

$$\frac{1}{\gamma} U' \left(\frac{1}{\gamma} \left(X - \tilde{\mu} B + \frac{\tilde{q}(B', s)}{1 + i} (B' - (1 - \mu)B) \right) \right) \left(\tilde{q}(B', s) + \frac{\partial \tilde{q}(B', s)}{\partial B'} (B' - (1 - \mu)B) \right) + (1 + i)\beta \frac{\partial}{\partial B'} \mathbb{E} V(B', i', s') = 0.$$

The Euler equation can be viewed as an equality $\Lambda(B', B, i, i', s, s') \equiv 0$, which must hold, in particular, for all levels of i. It thus implies a mapping of i into B', which is implicitly defined

[25] In the case $\gamma < 1$ studied in the main text, the real interest rate that matters for spending decisions also contains domestic tradable goods, hence it does not move one-for-one with the nominal interest. In fact, it moves one-for-one with the nominal rate only when the domestic wage rigidity binds, whereas it contains a relative price term when the domestic wage rigidity is slack.

$\Lambda(B'(i), B, i, i', s, s') \equiv 0$. We now differentiate Λ with respect to i

$$\frac{\partial \Lambda(B', B, i, i', s, s')}{\partial B'} \frac{\partial B'(i)}{\partial i} + \mathcal{I} + \mathcal{S} = 0,$$

where the first summand captures the *indirect* derivatives (the fact that B' depends on i in equilibrium) and where the last two summands capture the *direct* derivatives, given by $\mathcal{I}$ (income effect) and $\mathcal{S}$ (substitution effect)

$$\mathcal{I} \equiv \frac{U''(C)}{\gamma^2} \left(-\frac{\tilde{q}(B', s)}{(1+i)^2} (B' - (1-\mu)B) \right) \left(\tilde{q}(B', s) + \frac{\partial \tilde{q}(B', s)}{\partial B'} (B' - (1-\mu)B) \right)$$

and

$$\mathcal{S} \equiv -\frac{1}{1+i} \frac{U'(C)}{\gamma} \left(\tilde{q}(B', s) + \frac{\partial \tilde{q}(B', s)}{\partial B'} (B' - (1-\mu)B) \right),$$

respectively. To obtain the substitution effect, we have used that $\mathcal{S} = \beta(\partial/\partial B')\mathbb{E}V(B', i', s')$ and then substituted in the Euler equation. Notice that, in the computation above, we have also used that $\mathbb{E}V(B', i', s')$ is not dependent on i, because we assumed that i' is independent of i.

We first pin down the indirect derivatives. As it turns out, we do not need to compute these derivatives in order to pin down their *sign*. This is because, by assumption, we study the Euler equation at a local maximum, hence the second order condition at the equilibrium point is negative:

$$\frac{\partial}{\partial B'} \Lambda(B', B, i, i', s, s') < 0.$$

But this implies that

$$\frac{\partial B'}{\partial i} > 0 \iff \mathcal{I} + \mathcal{S} > 0.$$

Inserting and factorizing terms,

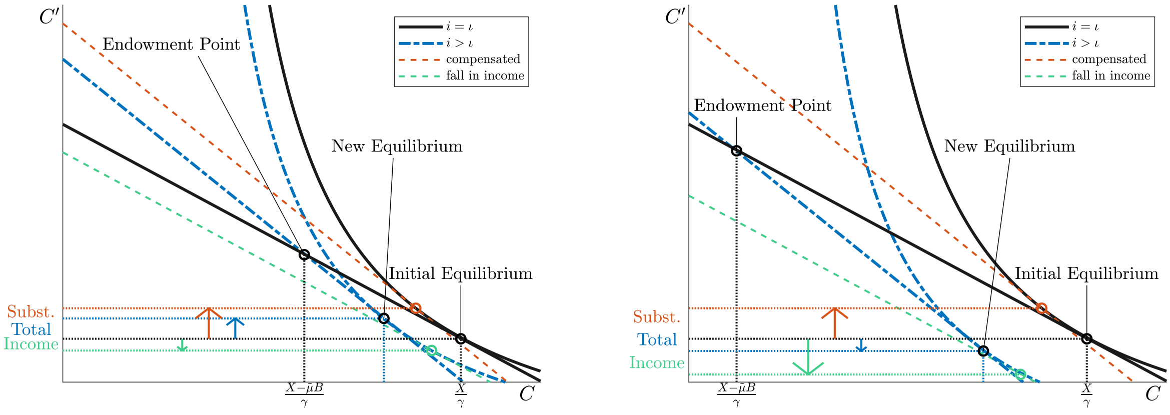
$$\frac{1}{\gamma} U'(C) \left(q(B', i, s) + \frac{\partial q(B', i, s)}{\partial B'} (B' - (1-\mu)B) \right) \left(\frac{\sigma}{\gamma} \frac{q(B', i, s)(B' - (1-\mu)B)}{C} - 1 \right) > 0,$$

where we used that $\sigma = -U''(C)(C/U'(C))$. The first two factors correspond to the left hand side of the Euler equation (17). They are therefore positive, because the continuation value slopes downward in the level of debt (a well known feature of sovereign default models): $\beta(\partial/\partial B')\mathbb{E}V(B', i', s') < 0$. This implies the last factor must be positive for the overall sign to be positive, which concludes the proposition.

A.4 Slutsky decomposition

In this appendix we provide more details on the discussion in Section 3.3. We consider a perfect foresight economy in which foreign demand X is constant and in which interest rates equal $\iota = 1/\beta - 1$, except in the initial period, where we consider an interest rate $i > \iota$. We assume away the possibility of default.

Figure 7 is constructed by following the same steps as outlined in the main text. In the figure,



(a) Low B: Substitution effect dominates.

(b) High B: Income effect dominates.

Figure 7: Illustration of the effects of interest rate changes. Shown are the effects on current and future consumption of a rise in current-period interest rates i. We distinguish a case of low incoming debt (left panel), and high incoming debt (right panel). We add to the figure a decomposition of the overall response into compensated effect and the effect due to a lower present value of income. The corresponding Slutsky decomposition can be found in this appendix.

in addition to plotting the overall effect of the interest rate hike, we also show how the overall effect comes about, as a combination of substitution (compensated) effect (red lines) and an income effect (lower present value of income, green lines).

We now decompose the effect of changes in the interest rate i on future consumption C' by using a Slutsky decomposition. To economize on notation, we write all variables as functions of the current interest rate only. For example, we write $C'(i)$.

Since C is constant for all periods after the current period t, lifetime utility is

$$U(i) = \frac{C(i)^{1-\sigma}}{1-\sigma} + \sum_{t=1}^{\infty} \beta^t \frac{C'(i)^{1-\sigma}}{1-\sigma} = \frac{C(i)^{1-\sigma}}{1-\sigma} + \frac{1}{\iota}\frac{C'(i)^{1-\sigma}}{1-\sigma} \tag{A.1}$$

Using $\beta = \frac{1}{1+\iota}$ the Euler equation becomes

$$C(i) = C'(i)\left(\frac{1+\iota}{1+i}\right)^{\frac{1}{\sigma}}.$$

Substituting for $C(i)$ in equation (A.1) and solving for $C'(i)$, we can write

$$C'(i, U(i)) = \left(\frac{U(i)(1-\sigma)}{\frac{1}{\iota} + \left(\frac{1+i}{1+\iota}\right)^{\frac{\sigma-1}{\sigma}}}\right)^{\frac{1}{1-\sigma}}. \tag{A.2}$$

This expression allows us to decompose the effects of an interest rate change into an income effect (green) and a substitution effect (red)

$$\frac{dC'(i,U)}{di} = \frac{\partial C'(q,U)}{\partial U}\frac{\partial U}{\partial i} + \frac{\partial C'(i,U)}{\partial i}. \tag{A.3}$$

The second term is the *compensated* effect of the interest rate on future consumption, i.e. how future consumption changes with the interest rate holding the level of lifetime utility (the present value of income) constant. Graphically, the income effect corresponds to the shifting of the green line and the substitution effect corresponds to the tilting of the red line in Figure 7. In the following, we determine the values of the derivatives in Equation (A.3).

We start with the compensated (substitution) effect which is given by

$$\frac{\partial C'(i,U)}{\partial i} = \frac{1}{\sigma}C'(i)^{\sigma}\frac{U(i)(1-\sigma)}{\left(\frac{1}{\iota}+\left(\frac{1+i}{1+\iota}\right)^{\frac{\sigma-1}{\sigma}}\right)^2}\left(\frac{1+i}{1+\iota}\right)^{-\frac{1}{\sigma}}\frac{1}{1+\iota}.$$

Using $\frac{U(i)(1-\sigma)}{\frac{1}{\iota}+\left(\frac{1+i}{1+\iota}\right)^{\frac{\sigma-1}{\sigma}}} = C'(i)^{1-\sigma}$ this is

$$\frac{\partial C'(i,U)}{\partial i} = \frac{1}{\sigma}\frac{C'(i)}{\left(\frac{1}{\iota}+\left(\frac{1+i}{1+\iota}\right)^{\frac{\sigma-1}{\sigma}}\right)}\left(\frac{1+i}{1+\iota}\right)^{-\frac{1}{\sigma}}\frac{1}{1+\iota}. \tag{A.4}$$

Next, we derive the partial derivative of C' with respect to U

$$\frac{\partial C'(i,U)}{\partial U} = C'(i)^{\sigma}\left(\frac{1}{\frac{1}{\iota}+\left(\frac{1+\iota}{1+i}\right)^{\frac{\sigma-1}{\sigma}}}\right). \tag{A.5}$$

It remains to determine the derivative of U with respect to i. Because income in every future period is constant, consumption in all future periods is given by permanent income

$$C'(i) = \frac{X-\iota B'(i)}{\gamma}. \tag{A.6}$$

Notice that $q(i) = \frac{1+\iota}{1+i}$ holds and we can write the budget constraint as

$$B'(i) = (1-\mu B) + \frac{1+i}{1+\iota}\left(C(i) + \frac{\tilde{\mu}B - X}{\gamma}\right).$$

This yields future consumption in terms of current consumption and the interest rate

$$C'(i) = \frac{X-\iota(1-\mu)B}{\gamma} - \iota\frac{1+i}{1+\iota}\left(C(i) + \frac{\tilde{\mu}B - X}{\gamma}\right).$$

Lifetime utility then is

$$U(i) = \frac{C(i)^{1-\sigma}}{1-\sigma} + \frac{1}{\iota} \frac{\left(\frac{X-\iota(1-\mu)B}{\gamma} - \iota \frac{1+i}{1+\iota} \left(C(i) + \frac{\tilde{\mu}B-X}{\gamma} \right) \right)^{1-\sigma}}{1-\sigma}.$$

Importantly, $U(i)$ is evaluated at the optimal choice $C(i)$. Therefore the envelope theorem applies and we only have to take the direct derivative with respect to i:

$$\frac{\partial U}{\partial i} = -\frac{1}{1+\iota} C'(i)^{-\sigma} \left(C(i) + \frac{\tilde{\mu}B - X}{\gamma} \right). \tag{A.7}$$

We substitute (A.4), (A.5) and (A.7) into (A.3) to obtain

$$\frac{dC'(i,U)}{di} = -\left(\frac{1}{\frac{1}{\iota} + \left(\frac{1+\iota}{1+i}\right)^{\frac{\sigma-1}{\sigma}}} \right) \frac{1}{1+\iota} \left(C(i) + \frac{\tilde{\mu}B - X}{\gamma} \right) + \frac{1}{\sigma} \frac{C'(i)}{\left(\frac{1}{\iota} + \left(\frac{1+i}{1+\iota}\right)^{\frac{\sigma-1}{\sigma}} \right)} \left(\frac{1+i}{1+\iota} \right)^{-\frac{1}{\sigma}} \frac{1}{1+\iota}$$

Using $C(i) = C'(i) \left(\frac{1+\iota}{1+i}\right)^{\frac{1}{\sigma}}$ and evaluating at the steady state $i = \iota$

$$\frac{dC'(i,U)}{di} = -\frac{1}{\left(\frac{1}{\iota}+1\right)} \frac{1}{1+\iota} \left(C(i) + \frac{\tilde{\mu}B - X}{\gamma} \right) + \frac{1}{\sigma} C(i) \frac{1}{\left(\frac{1}{\iota}+1\right)} \frac{1}{1+\iota}$$

Collecting terms, we find

$$\frac{dC'(i,U)}{di} = \frac{\iota}{(1+\iota)^2} \left(-\left(C(i) + \frac{\tilde{\mu}B - X}{\gamma} \right) + \frac{1}{\sigma} C(i) \right) \tag{A.8}$$

Here we see that the income effect of an interest rate increase on future consumption is negative, while the substitution effect is positive.

The link to current-period borrowing B' is now established from equation (A.6), which is a negative linear relationship between C' and B'. This implies that all terms have to be multiplied by $-\frac{\gamma}{\iota}$ to translate them into effects on B'. From this, it is also straightforward to see that equation (A.8) gives again the threshold $\mathcal{T}$ that we have found in the main text.

B Appendix quantitative analysis

B.1 Data sources

Our main source is the Eurostat database from which we get data on gross trade flows and public debt relative to GDP, as well as unemployment.[26] We complement this with data on debt maturity from the OECD (2021)[27], data on bond yields from the Bundesbank and Banca d'Italia and data on the sectoral decomposition of government debt holdings from the updated Bruegel database of sovereign bond holdings developed in Merler and Pisani-Ferry (2012).[28] We obtain the data for Italy's primary government surplus from the ECB statistical data warehouse.[29] In Table 3 we summarize the details on the data used along with the respective source.

B.2 Calibration details

For the standard deviation and autocorrelation of GDP, as well as the correlation of the trade balance/GDP ratio with GDP, we use the values provided by Uribe and Schmitt-Grohé (2017). They compute these moment on annual Italian data from 1965 to 2010 after removing a log-quadratic trend. We compute the remaining moments by using our own data sources above. For the level of government debt/GDP and government debt holdings by sector we use quarterly observations from Q1 2000 to Q3 2019, the latest observations available. We compute holdings by foreigners as the product of total debt/GDP and the share of government debt held by non-residents, obtained from the Bruegel database. We then compute the mean over the quarterly series. For unemployment we take the mean over the annual observations from 2000-2020. To compute the spread, we use data on 5 year bond yields (which is the closest to average maturity available) for Italy and Germany. We average over monthly observations to aggregate the data to yearly frequency. We then take the difference between the two series to obtain the spread. All data moments can be found in the second to last column of Table 1.

To compute model analogues of these moments, we simulate the model for 100 000 periods. We then exclude the first 1000 periods as well as any periods the economy spends in autarky and the first 30 periods after reentry.[30] We apply log-quadratic detrending to model generated GDP separately for each sub-sample. We report the weighted mean over the sub-samples for the standard deviation and autocorrelation. All model moments can be found in the last column of Table 1.

[26] See ec.europa.eu/eurostat/en/web/main/data/database.

[27] See www.oecd.org/finance/oecdsovereignborrowingoutlook.htm.

[28] See www.bruegel.org/publications/datasets/sovereign-bond-holdings/.

[29] See https://sdw.ecb.europa.eu/

[30] Since no defaults occur in the data, there is no need to apply a similar procedure.

Name	Unit	Dates	Frequency	Source
Gross domestic product	Chain linked volumes (2010), million euro	2000-2020	annual	Eurostat
Exports of goods and services	Chain linked volumes (2010), million euro	2000-2020	annual	Eurostat
Imports of goods and services	Chain linked volumes (2010), million euro	2000-2020	annual	Eurostat
Unemployment	Percentage of population in the labour force	2000-2020	annual	Eurostat
Government consolidated gross debt	Percentage of gross domestic product	Q1 2000 - Q2 2021	quarterly	Eurostat
Share of government debt held by non residents	Percentage of total government debt	Q1 2000 - Q3 2019	quarterly	Bruegel database
Share of government debt held by central bank	Percentage of total government debt	Q1 2000 - Q3 2019	quarterly	Bruegel database
Average term-to-maturity of outstanding marketable debt	Years	2020	-	OECD
German Bund 5-year notes yield	ppt	9.2006-11.2021	monthly	German Bundesbank
Italy yield of 5 year benchmark BTP	ppt	9.2006-11.2021	monthly	Bank of Italy
Italy Government primary surplus	Percentage of gross domestic product	2000-2020	annual	ECB

Table 3: Data sources

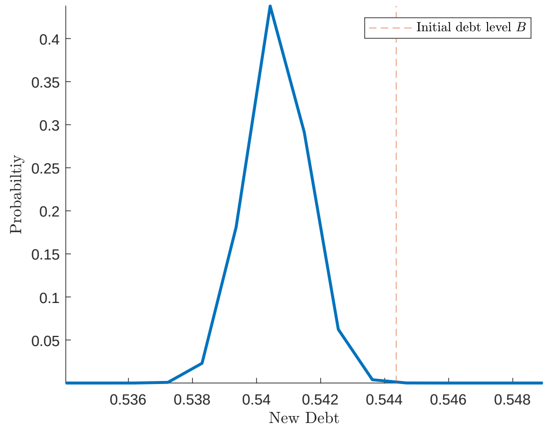


Figure 8: Borrowing policy function. The figure shows the choice probabilities for different values of B' before the realization of the taste shock at the gridpoint $(B, X) = (0.544, 0.255)$.

B.3 Numerical algorithm and accuracy

Our numerical solution relies on discrete choice value function iteration augmented with extreme value taste shocks. As discussed for example in Gordon (2019) and Dvorkin et al. (2021) these shocks smooth out kinks in policy functions and bond price schedules which otherwise lead to convergence problems. Our solution algorithm closely follows Arellano et al. (2020) where a full description can be found.

For borrowing B we choose a discrete grid over the interval [-0.05, 0.8] with 800 linearly spaced points. We denote the set of possible borrowing levels with $\mathcal{B}$. We discretize the autoregressive process for X using the Tauchen (1986) algorithm. We use 15 gridpoints and set the gridwidth to 1.88 standard deviations.

The algorithm involves extending the exogenous state with a taste shock specific to each possible borrowing choice, so $s = (X, \xi, \{\psi_{B'}\}_{B' \in \mathcal{B}})$. The shocks $\psi_{B'}$ follow a Gumble (Extreme Value Type 1) distribution with standard deviation σ_ψ. The value function in repayment is then given by

$$V^r(B, i, s) = \max_{B' \in \mathcal{B}, C} \left\{ U(C) + \beta \mathbb{E} V(B', i', s') + \psi_{B'} \right\}. \tag{B.1}$$

Before taste shocks realize, the policy functions for borrowing B' and default δ are probability distributions over the possible choices at each grid point (B, X).[31] In all figures we report the mean over the borrowing policy function. We set the standard deviation of the borrowing taste shock to a very small value $\sigma_\psi = 10^{-4}$ which ensures that these shocks do not affect our quantitative results while the algorithm still convergences. As an example, Figure 8 shows the borrowing policy function at the gridpoint $(0.544, 0.255)$. It can be seen that nearly all mass lies on the three gridpoints in the narrow interval $[0.539, 0.541]$.

We iterate backward in time on the value function and bond price schedule until updating errors are smaller than 10^{-6}.

[31] See Arellano et al. (2020) for details on how to compute the choice probabilities.

Interest rate changes are modeled as MIT shocks. To do so, we additionally add gridpoints with different interest rates. However, the probability of transitioning from the steady state to any point on the grid outside the steady state is set to zero when computing optimal choices. The grid and transition matrix we use for the interest rate depends on the experiment. For Figure 4, we use a grid of $i \in \{\iota, \iota + 10bps\}$ to compute the numerical derivative of borrowing with respect to the interest rate. For Figure 6, we use a grid of $i \in \{\iota, \iota, \iota + 10bps, \iota + 1ppt\}$. The first point is the steady state and the probability of transitioning to any other state is set to zero. States two and three are the states in which forward guidance has been announced. In these states the probability of transitioning to state four are equal to one. Since the announced interest rate hike is persistent, the probability to remain in state four is $\rho = 0.9$ and the probability to return to steady state is $1 - \rho = 0.1$. Figure 6a is constructed by taking differences between the outcomes in states one and four, isolating the effect of forward guidance. The numerical derivative to find the threshold in Figure 6b is computed as the difference in the policy functions between states two and three.

Our analytical analysis provides a natural test for the accuracy of our numerical solution. As pointed out in the main text, the points in the state space where $\partial B'/\partial i = 0$ can be computed in two ways. First, we can use the analytical threshold in equation (20). Second, we can numerically compute the derivative of the policy function and find the points where it is zero. In our numerical results, we find that these two approaches yield virtually identical results. This gives us confidence in our results in two ways. First, it shows that the Euler equation does in fact describe well the choices made by the sovereign.[32] Second, it shows that our numerical solution is accurate in the sense that it matches the theoretical predictions of the Euler equation.

References

Aguiar, M., S. Chatterjee, H. Cole, and Z. Stangebye (2016) "Chapter 21 - Quantitative Models of Sovereign Debt Crises," Vol. 2 of Handbook of Macroeconomics: Elsevier, pp. 1697–1755.

Aguiar, Mark and Manuel Amador (2014) "Sovereign Debt," Vol. 4: Elsevier, Chap. Chapter 11, pp. 647–687.

Aguiar, Mark, Manuel Amador, Emmanuel Farhi, and Gita Gopinath (2015) "Coordination and crisis in monetary unions," *The Quarterly Journal of Economics*, Vol. 130, No. 4, pp. 1727–1779.

Aguiar, Mark, Manuel Amador, Hugo Hopenhayn, and Iván Werning (2019) "Take the Short Route: Equilibrium Default and Debt Maturity," *Econometrica*, Vol. 87, No. 2, pp. 423–462.

Arellano, Cristina (2008) "Default Risk and Income Fluctuations in Emerging Economies," *American Economic Review*, Vol. 98, No. 3, pp. 690–712.

Arellano, Cristina, Yan Bai, and Gabriel P Mihalache (2020) "Monetary Policy and Sovereign Risk

[32] Due to the complexity of the model and possible non-convexities, this cannot be shown analytically.

in Emerging Economies (NK-Default)," Working Paper 26671, National Bureau of Economic Research.

Arellano, Cristina and Ananth Ramanarayanan (2012) "Default and the Maturity Structure in Sovereign Bonds," *Journal of Political Economy*, Vol. 120, No. 2, pp. 187–232.

Arora, Vivek and Martin Cerisola (2001) "How Does U.S. Monetary Policy Influence Sovereign Spreads in Emerging Markets?" *IMF Staff Papers*, Vol. 48, No. 3, pp. 474–498.

Barattieri, Alessandro, Susanto Basu, and Peter Gottschalk (2014) "Some Evidence on the Importance of Sticky Wages," *American Economic Journal: Macroeconomics*, Vol. 6, No. 1, pp. 70–101.

Benigno, Gianluca and Luca Fornaro (2018) "Stagnation traps," *Review of Economic Studies*, Vol. 85, No. 3, pp. 1425–1470.

Best, Michael Carlos, James S Cloyne, Ethan Ilzetzki, and Henrik J Kleven (2019) "Estimating the Elasticity of Intertemporal Substitution Using Mortgage Notches," *The Review of Economic Studies*.

Bianchi, Javier and Jorge Mondragon (2021) "Monetary Independence and Rollover Crises," *The Quarterly Journal of Economics*, Vol. 137, No. 1, pp. 435–491.

Bianchi, Javier, Pablo Ottonello, and Ignacio Presno (2019) "Fiscal Stimulus under Sovereign Risk," Working Paper 26307, National Bureau of Economic Research.

Bocola, Luigi, Gideon Bornstein, and Alessandro Dovis (2019) "Quantitative sovereign default models and the European debt crisis," *Journal of International Economics*, Vol. 118, pp. 20–30.

Bocola, Luigi and Alessandro Dovis (2019) "Self-Fulfilling Debt Crises: A Quantitative Analysis," *American Economic Review*, Vol. 109, No. 12, pp. 4343–77.

Boissay, Frederic, Fabrice Collard, Jordi Gali, and Cristina Manea (2021) "Monetary Policy and Endogenous Financial Crises," Working Paper 29602, National Bureau of Economic Research.

Boissay, Frederic, Fabrice Collard, and Frank Smets (2016) "Booms and Banking Crises," *Journal of Political Economy*, Vol. 124, No. 2, pp. 489–538.

Caballero, Ricardo J, Emmanuel Farhi, and Pierre-Olivier Gourinchas (2008) "An equilibrium model of global imbalances and low interest rates," *American Economic Review*, Vol. 98, No. 1, pp. 358–93.

Centorrino, Samuele, Lei Li, and Gabriel Mihalache (2022) "Predictable Interest Rate Movements, Implications for Emerging Markets," Working paper.

Chatterjee, Satyajit and Burcu Eyigungor (2012) "Maturity, Indebtedness, and Default Risk," *American Economic Review*, Vol. 102, No. 6, pp. 2674–99.

Cole, Harold L. and Timothy J. Kehoe (2000) "Self-Fulfilling Debt Crises," *The Review of Economic Studies*, Vol. 67, No. 1, pp. 91–116.

Conesa, Juan and Timothy Kehoe (2017) "Gambling for Redemption and Self-Fulfilling Debt Crises," *Economic Theory*, Vol. 64.

Corsetti, Giancarlo and Luca Dedola (2016) "The mystery of the printing press: Monetary policy and self-fulfilling debt crises," *Journal of the European Economic Association*, Vol. 14, No. 6, pp. 1329–1371.

Davig, Troy, Eric M. Leeper, and Todd B. Walker (2011) "Inflation and the fiscal limit," *European Economic Review*, Vol. 55, No. 1, pp. 31–47, Special Issue on Monetary and Fiscal Interactions in Times of Economic Stress.

De Grauwe, Paul and Yuemei Ji (2013) "Self-fulfilling crises in the Eurozone: An empirical test," *Journal of International Money and Finance*, Vol. 34, pp. 15–36, The European Sovereign Debt Crisis: Background and Perspective.

Dvorkin, Maximiliano, Juan M Sanchez, Horacio Sapriza, and Emircan Yurdagul (2021) "Sovereign debt restructurings," *American Economic Journal: Macroeconomics*, Vol. 13, No. 2, pp. 26–77.

Eaton, Jonathan and Mark Gersovitz (1981) "Debt with Potential Repudiation: Theoretical and Empirical Analysis," *The Review of Economic Studies*, Vol. 48, No. 2, pp. 289–309.

Eggertsson, G. and M. Woodford (2003) "The Zero Bound on Interest Rates and Optimal Monetary Policy," *Brookings Papers on Economic Activity*, Vol. 2003, No. 1, pp. 139–211.

Eggertsson, Gauti B., Neil R. Mehrotra, and Jacob A. Robbins (2019) "A Model of Secular Stagnation: Theory and Quantitative Evaluation," *American Economic Journal: Macroeconomics*, Vol. 11, No. 1, pp. 1–48.

Fabiani, S., C. Kwapil, T. Rõõm, K. Galuscak, and A. Lamo (2010) "Wage Rigidities and Labor Market Adjustment in Europe," *Journal of the European Economic Association*, Vol. 8, No. 2-3, pp. 497–505.

Farhi, Emmanuel and Ivan Werning (2017) "Fiscal unions," *American Economic Review*, Vol. 107, No. 12, pp. 3788–3834.

Fehr, E. and L. Goette (2005) "Robustness and Real Consequences of Nominal Wage Rigidity," *Journal of Monetary Economics*, Vol. 52, No. 4, pp. 779–804.

de Ferra, Sergio and Federica Romei (2020) "Sovereign Default in a Monetary Union," Mimeo, Stockholm University.

Fornaro, Luca (2021) "A Theory of Monetary Union and Financial Integration," *The Review of Economic Studies*, rdab057.

Galí, Jordi and Tommaso Monacelli (2005) "Monetary Policy and Exchange Rate Volatility in a Small Open Economy," *The Review of Economic Studies*, Vol. 72, No. 3, pp. 707–734.

Gerke, Rafael and Joost Röttger (2021) "The incentive effects of monetary policy on fiscal policy behaviour," Deutsche Bundesbank Technical Paper.

Gordon, Grey (2019) "Efficient computation with taste shocks."

Gottschalk, P. (2005) "Downward Nominal-Wage Flexibility: Real or Measurement Error?" *Review of Economics and Statistics*, Vol. 87, No. 3, pp. 556–568.

Hall, Robert E (1988) "Intertemporal Substitution in Consumption," *The Journal of Political Economy*, Vol. 96, No. 2, pp. 339–357.

Hatchondo, Juan Carlos and Leonardo Martinez (2009) "Long-duration bonds and sovereign defaults," *Journal of International Economics*, Vol. 79, No. 1, pp. 117–125.

Havránek, Tomáš (2015) "Measuring Intertemporal Substitution: The Importance of Method Choices and Selective Reporting," *Journal of the European Economic Association*, Vol. 13, No. 6, pp. 1180–1204.

Ignaszak, Marek, Philip Jung, and Keith Kuester (2020) "Federal unemployment reinsurance and local labor-market policies," ECONtribute Discussion Papers Series 040, University of Bonn and University of Cologne, Germany.

Itskhoki, Oleg and Dmitry Mukhin (2021) "Exchange Rate Disconnect in General Equilibrium," *Journal of Political Economy*, Vol. 129, No. 8, pp. 2183–2232.

Johri, Alok, Shahed Khan, and César Sosa-Padilla (2020) "Interest Rate Uncertainty and Sovereign Default Risk," Working Paper 27639, National Bureau of Economic Research.

Kalemli-Özcan, Sebnem (2019) "U.S. Monetary Policy and International Risk Spillovers," Working Paper 26297, National Bureau of Economic Research.

Kenen, Peter (1969) "The theory of optimum currency areas: an eclectic view," *Monetary problems of the international economy*, pp. 41–60.

Kriwoluzky, Alexander, Gernot J. Müller, and Martin Wolf (2019) "Exit expectations and debt crises in currency unions," *Journal of International Economics*, Vol. 121, p. 103253.

Merler, Silvia and Jean Pisani-Ferry (2012) "Who's afraid of sovereign bonds?" Bruegel Policy Contribution 2012/02, Brussels.

Mian, Atif, Ludwig Straub, and Amir Sufi (2021) "Indebted Demand*," *The Quarterly Journal of Economics*, Vol. 136, No. 4, pp. 2243–2307.

Na, Seunghoon, Stephanie Schmitt-Grohé, Martín Uribe, and Vivian Yue (2018) “The Twin Ds: Optimal Default and Devaluation,” *American Economic Review*, Vol. 108, No. 7, pp. 1773–1819.

Neumeyer, Pablo A. and Fabrizio Perri (2005) “Business cycles in emerging economies: the role of interest rates,” *Journal of Monetary Economics*, Vol. 52, No. 2, pp. 345–380.

Nuño, Galo, Jim Costain, and Carlos Thomas (2022a) “The Term Structure on Interest Rates in a Heterogeneous Monetary Union,” Working paper.

Nuño, Galo, Samuel Hurtado, and Carlos Thomas (2022b) “Monetary Policy and Sovereign Debt Sustainability,” *Journal of the European Economic Association, forthcoming.*

OECD (2021) *OECD Sovereign Borrowing Outlook 2021*: OECD Publishing, pp.90.

Olivei, G. and S. Tenreyro (2007) “The Timing of Monetary Policy Shocks,” *The American Economic Review*, Vol. 97, No. 3, pp. 636–663.

Schmitt-Grohé, Stephanie and Martín Uribe (2016) “Downward Nominal Wage Rigidity, Currency Pegs, and Involuntary Unemployment,” *Journal of Political Economy*, Vol. 124, No. 5, pp. 1466–1514.

Tauchen, George (1986) “Finite state markov-chain approximations to univariate and vector autoregressions,” *Economics Letters*, Vol. 20, No. 2, pp. 177–181.

Uribe, Martin and Stephanie Schmitt-Grohé (2017) *Open economy macroeconomics*: Princeton University Press.

Uribe, Martín and Vivian Z. Yue (2006) “Country spreads and emerging countries: Who drives whom?” *Journal of International Economics*, Vol. 69, No. 1, pp. 6–36, Emerging Markets.

Wolf, Martin (2020) “Pecuniary externalities in economies with downward wage rigidity,” *Journal of Monetary Economics*, Vol. 116, pp. 219–235.