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JEL Classification: D21, G38, M41, M48

Keywords: Mark to market, financial crises, redemption gates, Government Guarantees

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Abstract

We examine the effects of fair-value accounting (FVA) and historical-cost accounting (HCA) regimes on the ex-ante financing of projects by external investors. We formulate a model highlighting the relative merits and demerits of each accounting regime, in particular the sub-optimal continuations under the HCA regime and contagion-induced sub-optimal liquidations under the FVA regime. We show that under homogeneous beliefs about future cash-flows, FVA regime is superior with greater ex-ante financing, even during periods of high market illiquidity, when the failure of some banks leads to adverse spillovers on surviving financial institutions. However, if disclosures under the FVA regime leads to distorted beliefs about future success probabilities, ex-ante financing may suffer under the FVA regime, and it may no longer be superior relative to the HCA regime. In this setting we also analyze the impact on ex-ante financing of (i) ex-post redemption gates, which restrict the extent of investor liquidation under stress, and (ii) government guarantees, which limit the spillovers ex-post but may incentivize excessive risk-taking ex-ante.

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1. Introduction

The 2007-08 global financial crisis (GFC) put the alleged role played by fair value accounting (FVA) in accentuating market instability under intense scrutiny. In its pure form, the FVA regime involves recording the assets and liabilities on an institution's balance sheet at fair value.[1] Within the class of rules that govern the estimation of fair value, the use of market prices is preferred over internal model-based valuations. This measurement framework, known as mark-to-market (MTM) accounting, rests on the economic principle that forward looking market prices provide the best estimate of an asset's fundamental value, and are free from manipulations. However, when markets are stressed and liquidity constrained, trading prices may deviate from the fundamental value depending on the relative demand and supply of assets. Critics of the FVA and MTM accounting regime argue that under such cash-in-the-market pricing scenarios, as in Shleifer and Vishny (1992) and Allen and Gale (1994,1998), FVA regulations may induce artificial volatility in the balance sheet of the institutions holding these assets, given capital requirements that depend on the accounting value. This, in turn, may create a vicious cycle where falling asset prices force institutions to fire-sell assets to meet regulatory restrictions and/or investor redemptions, further depressing market prices and inducing additional forced sales.[2] Such forced selling can affect institutional stability in connected markets as it can lead to contagion-induced failures of otherwise solvent banks.[3]

[1]The International Accounting Standards Board (IASB) defines fair value as the price that would be received when selling an asset or paid to transfer a liability in an orderly-transaction between market participants at the measurement date. Also see FAS 157 and ASC 820 issued by the Financial Accounting Standards Board (FASB) for a framework of determining an asset's fair value under Generally Accepted Accounting Principles, or GAAP. FAS 157 outlines a hierarchy of inputs to estimate the fair value of an asset or liability. Level 1 inputs are transaction or quoted prices of identical assets if such a market exists and is well functioning. However, if Level 1 inputs are not available, the fair value of a security is determined by using a set of observable inputs (Level 2), including prices of similar assets. Finally, in Level 3, also known as mark-to-model, internal model-based valuation measures are used, if observable inputs from Levels 1 and 2 are not available.

[2]Although traditional models of contagion focus on the liability side of the financial institutions, studies have also investigated overlaps between assets held by different financial institutions as a source of systemic risk. See, among others, Allen et al. (2012), Blei and Ergashev (2014), Morris and Shin (2004), Wagner (2008, 2010).

[3]This view has been quite a popular one among market participants. All over the media were calls of suspending the MTM accounting until the market functions returned to normal levels. In an article "Mark it and Weep" published on March 6, 2008, The Economist also shared some of the concerns expressed by the financial institutions: "Regulators worry that mark-to-market may create a liquidity black hole. Nerves jangle at every fire-sale, for fear that this will become the new benchmark for sticky assets. The fear is that value-at-risk systems force investment banks and banks to offload securities, leading to price falls and further sales. The temptation is to sell now, before the next lurch down. The

The main alternative to the FVA regime is the historical cost accounting (HCA), where assets are recorded at historical cost, generally the original purchase price, and updated sporadically subject to impairments and amortization. However, the accounting values of assets and liabilities recorded under the HCA regime are not adjusted for market movements. Critics of the FVA argue that market-based accounting regimes played an important role in undermining financial stability by amplifying asset prices away from the fundamental values.[4] They also contend that HCA may be preferable to FVA, especially during economic downturns, by avoiding fire sales and contagion effects. Contrarily, proponents of FVA, including the International Accounting Standards Board (IASB), and its American counterpart the Financial Accounting Standards Board (FASB), argue that marking assets to market helps incorporate pertinent and timely information into asset prices and thereby reduces asymmetry between originators and holders of many asset classes, such as asset-backed securities. Also, assets recorded at historical prices may incentivize investors to engage in selective gains-trading, selling winners and holding losers in their portfolio, subject to impairments.[5]

The extant literature primarily focuses on the effect of accounting regimes on investor welfare and financial stability in adverse states of the world, ex post. However, there has been little research on the effect of different accounting regimes and the information structures embedded therein on ex ante financing and originations. In this paper, we study the relative merits of the accounting regimes from an ex ante perspective. There are two novel aspects to our study. First, we suppress the interplay between accounting regimes and the banks' internal agency frictions.[6] This allows us to focus on the pure informational aspects of the accounting regimes, and analyze how the information structure impacts ex-ante originations. Second, our setting allows us to analyze how these accounting regimes interact with policies intended to curb excessive liquidations: i) redemption restrictions, which force investors to internalize the costs of demanding liquidity, and ii)

result will be excessive write-downs "as the stable value of assets is above today's distressed level."

[4]Allen and Carletti (2008) show, using a stylized model, that in markets with limited participation, prices may not reflect fundamentals but simply move according to the total liquidity available to the potential asset holders. In such cash-in-the-market pricing settings, MTM may induce fire sale contagion in the markets. Thus, conditional on the adverse state of nature, MTM accounting may be welfare decreasing. Also see Plantin et al. (2008) and Sapra (2008).

[5]See Ellul et al. (2015). Also see Bleck and Liu (2007) and Laux and Leuz (2009) for arguments in favor of fair value regime.

[6]See Li (2017), Lu et al. (2019), Bertomeu et al. (2020) for models where accounting standards interact with principal-agent relationships within an individual bank.

explicit or implicit government guarantees where governments can bailout banks. This analysis is not only appealing from an academic perspective, and also important from a practical policy design angle.

We formulate a stylized two-period model, where a continuum of loans (or projects) becomes available to banks at an initial date. Banks are liquidity constrained, and must raise the initial financing cost from external investors who write short-term debt contracts with these banks. The inherent quality of the projects, which in turn affect their subsequent success rate, is common knowledge, and as such there is no information asymmetry during the initial period. At the intermediate date, aggregate and idiosyncratic shocks are realized, leaving projects in either one of two possible idiosyncratic states: high (H) or low (L). The exact realization of these interim states is private and known only to the managers. Whether this information is subsequently disclosed, or not, depends on the underlying accounting regime. FVA reflects one end of the spectrum with full revelation, while HCA represents the other extreme with complete opacity and no disclosure. Depending on new information, all agents update their beliefs about the unknown aggregate state, and decide on whether to run on banks or not.

Banks that experience a run in the intermediate period are forced to sell the collateral backing these projects to repay withdrawing creditors. We introduce a secondary market for trading collateral, and endogenously derive the equilibrium prices as a function of the available liquidity, agent posteriors, and the quantity of collateral being sold. We show that in presence of both aggregate and idiosyncratic shocks, the agent posteriors, and in turn equilibrium prices, depend on the number of banks reporting an ex-post low type. It has two important effects. First, collateral prices may move away from their fundamental values, creating excessive volatility in the secondary asset markets. Second, it may induce creditors to run on ex-post good banks and liquidate good projects in the intermediate period, creating contagion-induced failures of otherwise solvent banks.

Our result provides insight into the debate regarding the relative merits of the two accounting regimes. When financial institutions operate under a set of common priors about expected future cash flows under different states of nature, FVA regime, despite all the inefficiencies stemming from secondary price distortions and contagion threats, still dominates the HCA regime in terms of ex-ante originations. Critics of FVA argue that use of fair value benchmarks exposes the financial system to additional stress and contagion-induced failures. However, we show that even with downward price spirals and inefficient failures, FVA regime may still dominate Historical Cost regimes from an ex-ante investment perspective. Under FVA with full revelation, investors are able to distinguish between good and bad projects ex-post at the intermediate date and can improve efficiency by terminating poor projects and continuing the good ones. By contrast, under the HCA regime where the intermediate state is revealed only to the projects' operators, external investors may end up inefficiently continuing bad projects.[7] This leads to an ex ante tightening of the funding set as investors demand a higher return to compensate for the risk of such sub-optimal ex post continuations and may thus optimally pass on feasible projects ex ante.

Some authors argue that the FVA regime can jeopardize financial stability by creating dispersion of beliefs among different market participants, and that the valuation of assets at fair value may reduce public information precision and undermine financial system stability by inducing panic-based runs.[8] To investigate this possibility, we allow creditor beliefs about future cash flows to diverge depending on the accounting regime. More specifically, under the FVA regime, liquidations of ex post low-type banks may induce investors to re-evaluate the future success probabilities of intermediate high-type projects . Under such imperfections, the FVA may no longer dominate HCA from an ex-ante perspective. Indeed, under situations of extreme stress and panic, FVA may actually under-perform HCA from an ex-ante perspective.

We next investigate some policy interventions. We examine two specific forms of regulatory policies employed during the 2008 GFC and more recently Brexit: redemption restrictions or gates, and the regulatory bailout of the creditors of failing institutions. Such interventions or restrictions, aimed at reining in downward spirals and cascading financial failures during times of stress, are increasingly adopted by financial institutions, and by central banks globally during periods of financial turmoil.

We first investigate the equilibrium with redemption restrictions or gates where a bank

[7] Since bank managers always gain by continuing projects till maturity, intermediate liquidation decisions are never taken at the bank level, even if they are efficient. This agency conflict becomes particularly relevant under the HCA regime where the intermediate state is revealed only to the bank manager and not to the external investor.

[8] For instance, Magnan et al. (2015) show that the degree of disagreement among analysts is directly related to the presence of fair value assets and liabilities in the bank balance sheets.

or a financial regulator may impose constraints on creditors to prevent intermediate liquidations. The role of such stability policies arises because the private choices of unregulated banks may not in general be socially optimal. We show that the private unregulated equilibrium, though constrained first-best efficient when secondary markets are liquid, start deviating from the social optimum once market liquidity deteriorates and prices deviate from fundamental values. This happens because banks and creditors do not internalize the costs they impose on other parties when choosing their liquidation policies at the intermediate date. Thus, in a private equilibrium, banks take secondary prices as given and never find it optimal to impose redemption restrictions on creditors. Thus, banks may engage in excessive originations and may leave the financial system overly vulnerable to "excessive" liquidations and price declines. By contrast, a social planner, who seeks to maximize total bank profits, and internalizes the dependence between aggregate liquidations and the equilibrium price, optimally sets the redemption restrictions to limit withdrawals for ex-post good banks and curb downward price spirals, while allowing full withdrawals for ex-post bad banks.

Next, we look at the effects of regulatory forbearance on the ex-ante financing and allocation under the two accounting regimes. We consider the case where regulatory authorities may transfer public resources to the creditors facing losses. Public sector bailouts of financial institutions received a lot of attention in the aftermath of the 2007-08 financial crisis and many regulations, including the Consumer Protection Act and Dodd-Frank Wall Street Reform Act were passed to protect the interests of the tax-payers. This, however, can potentially increase ex-ante resource misallocation as the anticipation of future bailouts may distorts incentives of the financial institutions and its investors, exacerbating the fragility of the financial system.[9] In our model, regulatory bailouts can increase ex-ante allocations by partially indemnifying the creditors against failures. Under the HCA regime, up to a certain degree, bailouts can restore first-best equilibrium allocations by insuring the investor against the threats posed by inefficient continuation of bad projects at the intermediate date. Similar improvements are also observed under the FVA regime with belief distortions. In this case, regulatory forbearance

[9]For example, Chari and Kehoe (2013) study an environment where a strict no-bailout policy followed by the government can achieve a constrained-efficient equilibrium. However, they show that the time inconsistencies which typically plague the government commitments worsen ex-ante managerial incentives and reduce overall welfare. Also see Bianchi (2016) for a variant of the above theme.

can improve allocations by reducing contagion-induced liquidations of healthy projects in the intermediate date. However, excessive levels of bailouts reduce welfare by distorting the incentives of the investors who transfer excessive risk to the government by funding negative NPV projects.

We contribute to the strand of literature which highlights the relative merits and demerits of the fair value and historical cost accounting regimes. See, for example, Allen and Carletti (2008), Bleck and Liu (2007), Plantin et al. (2008), Bleck and Gao (2017), Otto and Volpin (2017), Plantin and Tirole (2018). Also see Acharya and Ryan (2016) for an extensive review of the literature which studies the interaction between accounting disclosure regulations and financial stability. A second strand of literature explores the interaction between the accounting regimes and bank capital regulations. See, for example, Heaton et al. (2010), Bertomeu et al. (2020), Lu et al. (2019). We contribute to this literature by analyzing the effects of accounting regimes on ex-ante allocations. Our ex-ante efficiency framework also provides a natural setting to undertake the normative analysis government guarantees or bailouts. Such normative analysis is a novel feature of our paper.

Two papers closest to ours are Plantin et al. (2008) and Bleck and Gao (2017). They show that with liquid asset markets, MTM leads to better securitization of assets and increased efficiency. However, this result may be overturned as market liquidity deteriorates. While these papers focus primarily on loan retention and securitization decisions, we consider the impact of accounting regimes on loan originations.

The remainder of the paper is organized as follows. We introduce the key model in Section 2. Section 3 analyzes the stripped down version of the model, highlighting the key trade-offs between the two accounting regimes. We next analyze the full-scale model featuring a secondary market for assets, endogenous collateral prices, and contagion-induced liquidations under the FVA regime in Section 4. In Section 5, we allow for accounting-regime based belief dispersion, and its effect on ex ante allocations. Next in section 6, we investigate how accounting regimes interact with ex-post policy interventions aimed at curbing excessive liquidations and price distortions, namely redemption restrictions, and regulatory forbearance. Finally, Section 7 concludes.

2. Model

In this section we present a parsimonious model that allows us to characterize how accounting regimes affect ex-ante allocations and efficiency. Time is discrete and extends for three periods, $t = 0, 1, 2$. The economy is populated by a continuum of banks, indexed by $i \in [0, 1]$, each with an opportunity to invest in a project at the initial date $t = 0$. There is also a continuum of investors, indexed by $j \in [0, 1]$, who provide credit to the banks. All agents are assumed to be risk neutral, and have a discount rate of zero.

Project Structure. At $t = 0$, a continuum of projects, indexed by their quality parameter $\theta \in [0, 1]$, becomes available to the banks. Each project yields a payoff $R > 1$ in the final period $t = 2$ when successful, and 0 otherwise. The success probability is a random variable and depends on the underlying project type $s \in \{H, L\}$, which is revealed at date $t = 1$. For projects with type s, the success probability is $q_s \in (0, 1)$, where good quality type-H projects are successful at a higher rate and $q_H > q_L$. All agents know the distribution of types, but the uncertainty concerning an individual project's expected payoff $q_s R$ is only resolved at the intermediate date $t = 1$ when additional information about its type materializes.

In the interim period $t = 1$, the project can be sold in the secondary market at a liquidation value $\bar{\mu}$, which is not directly linked with the expected future payoff. This setup is similar to Eisenbach (2017) and captures the notion that projects are essentially risky loans held by banks. In this case, bank payoffs reflect repayments by the ultimate borrowers, while the liquidation values of projects in the secondary market reflect the fundamental value of the underlying collateral.

Aggregate Uncertainty. The economy can be in either of two states $\omega \in \{G, B\}$. The state B denotes a crash or recession and the unconditional probability that the economy recedes into a recession at the interim date $t = 1$ is λ.

Aggregate economic uncertainty has important effects on project and collateral valuations. The probability that a project with fundamental quality θ reaches intermediate state $s = H$ is θ when the underlying state is good ($\omega = G$), but it is only $a\theta$ with $a \in \left(\frac{1}{q_H R}, 1\right)$ conditional on an aggregate crash ($\omega = B$). Similarly, a type-s project, $s \in \{H, L\}$, has a success probability q_s under normal times, but only $(1 - \kappa)q_s$ during recessions. The parameter κ captures exposures of individual projects to aggregate economic risk.

We are agnostic about the source of κ, but instead focus on the effect of the underlying accounting regime on κ. In the ensuing analysis, we consider two possible scenarios. Under homogeneous expectations equilibrium, we assume that κ does not depend on the accounting regime, but instead reflects creditor beliefs about cash flows in different states of the world. We also consider a second scenario, where investor expectations are shaped by additional revelations of financial information. Thus, under heterogeneous expectations equilibrium, we set $\kappa = 0$ for the HCA regime with no information revelation, and allow for κ to vary depending on the intermediate state under the FVA regime.

Collateral values are also affected by the underlying state of the economy. Under normal economic conditions, \$1 collateral in the intermediate date $t = 1$ provides a return $\mu_G > 1$ under normal economic conditions, but only $\mu_B < 1$ conditional on an adverse aggregate crash. The notion that collateral values are exposed to systematic risk is examined in Barro (2006, 2009) among others.

Let us consider a simple illustrative example that underlies our modeling choice. We envision an economy where risky projects undertaken by banks reflect the mortgage loans backed by an asset as a collateral. Under normal circumstances, q_H captures the probability that the loan is repaid at the maturity date $t = 2$. However, during the intermediate period, if the bank decides to sell off the loan following an originate to distribute model, it achieves a high price in the secondary market if the collateral backing the loan is expected to appreciate in value ($\mu_G > 1$). However, under adverse conditions, not only do repayment probabilities decline ($q_L < q_H$), but collateral values also go down on average ($\mu_B < 1$).

Financing. To initiate projects, banks must raise the investment cost of one dollar from external investors at $t = 0$. Creditors enter into a bilateral contract with each bank, stipulating a promised repayment D at $t = 2$. While we take debt contracts as given and do not explore other bilateral arrangements, such choices can be justified endogenously as in DeMarzo and Duffie (1999) and Dang et al. (2011).

This debt is information sensitive and exposes banks to rollover risk. That is, creditors may decide to liquidate projects during the intermediate period $t = 1$, and demand payment of the face value D. In such a case, the project is liquidated at a price $P \in (\mu_B, \mu_G)$ and the creditors receive $\min\{D, P\}$ while the residual $\max\{0, D - P\}$ accrues to the bank. In the ensuing analysis, we maintain the assumption (assumption 1):

$$q_L R < \mu_B < 1 < \mu_G < q_H R. \tag{1}$$

The first part of this assumption states that conditional on low project type-L, it is always optimal to liquidate the project in the secondary market even under the worst aggregate economic conditions. On the other hand, if the idiosyncratic type of the project is good and $s = H$, it is always optimal to operate them until maturity under good aggregate conditions. We also assume (assumption 2) that:

$$\lim_{\theta \to 1} \mathbb{E}_{\omega,s}[y_2(\theta)] > 1, \tag{2}$$

where y_2 is the second period cash flow from the project, $y_2 \in \{R, 0\}$, and $\mathbb{E}_{\omega,s}(\cdot)$ is the expectation operator over the underlying state ω and the idiosyncratic type s. Given the unconditional net present value of a project with quality θ:

$$V(\theta) = \mathbb{E}_{\omega,s}[y_2(\theta)] - 1.$$

In the above assumption, it is unconditionally efficient to invest in projects with higher quality ($\theta \to 1$). Also, the net present value $V(\theta)$ is monotonically increasing in the quality θ and satisfies the property $V(0) < 0$ and $V(1) > 1$.[10] Thus, there exists a unique interior quality θ^* such that $V(\theta^*) = 0$. This means that first-best allocation is given by the set $[\theta^*, 1]$ where all positive NPV projects are financed in equilibrium.

Collateral Buyer and Endogenous Secondary Market Prices. We assume that at time $t = 1$, a risk-neutral outside collateral buyer is born with an endowment $e > 0$. He invests this endowment either as capital i in a productive technology yielding an output $r_f \min(i, i^*), r_f > 1$, or in buying collateral from banks in the secondary market. We assume that the output of the production technology is unverifiable, which implies

[10] The net present value of a project with quality parameter θ is

$$V(\theta) = (1 - \lambda)\big[\theta q_H R + (1 - \theta) q_L R\big] + \lambda\big[a\theta q_H R + (1 - a\theta) q_L R\big] - 1$$

It follows directly that $V(0) = q_L R - 1 < 0$ and $V(1) = (1 - \lambda + \lambda a) q_H R - 1 > 1$.

that he cannot borrow against this output from banks and must invest from his own endowment. Also, the functional form implies that the any additional investment beyond the threshold level i^* yields a marginal return of zero. We also make the following assumption about the ordering of payoffs:

$$q_H R - \mu_G \leq r_f \leq \mu_B - q_L R. \tag{3}$$

Accounting Regime. We consider two accounting regimes in the model: the FVA and HCA, as described above. Under the FVA regime, the underlying types $s \in \{H, L\}$ of the projects are revealed publicly in the intermediate period $t = 1$ before the secondary market opens and creditors make rollover decisions. At the other extreme, under the HCA regime, creditors only observe the historical value of the project and no further information is revealed to them. We model this in our setting by assuming that creditors do not observe the underlying type at $t = 1$ and must resort to making optimal rollover decisions based solely on their prior public knowledge.

Timeline. Figure 1 depicts the whole timeline of the economy. Banks secure initial financing and invest in projects with quality θ in the first period $t = 0$. At $t = 1$, the aggregate state of the economy, ω and idiosyncratic project types $s \in \{H, L\}$ are realized. Based on the new information, or lack thereof, creditors decide whether to rollover the debt for another period or to run on the bank. In the event of a creditor run, banks must sell off their project in the secondary market to repay the creditors. Finally, in the last period $t = 2$, continued projects succeed or fail, all private contracts are honored, and all agents consume their payoffs.

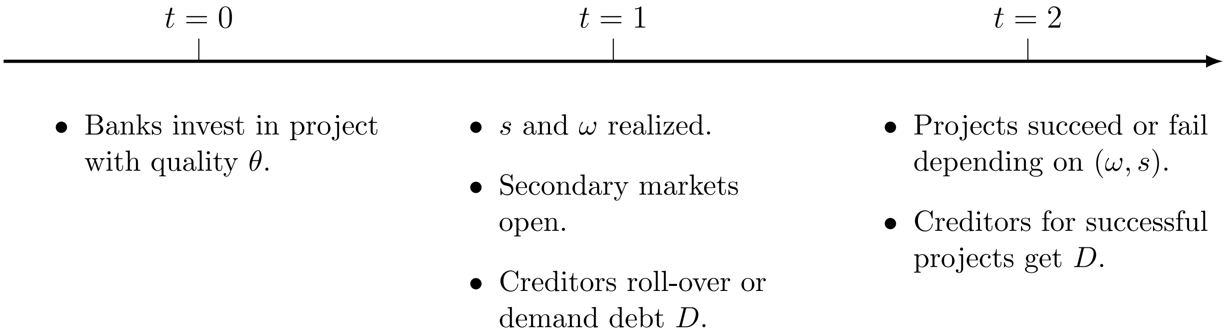


Figure 1: Timeline for the Economy

3. Equilibrium without Aggregate Risk

To clearly delineate the underlying mechanism without imposing additional mathematical complexity, we first consider an equilibrium with a single bank operating a project with fundamental quality θ. We abstract from the case of aggregate uncertainty by setting $\lambda = 0$ and assume that the price of collateral in the secondary market is given by $\bar{\mu} \in [\mu_B, \mu_G]$.

We solve for the creditors' problem by first analyzing the rollover decision in the interim period $t = 1$. Under the FVA regime, each bank releases a signal revealing the idiosyncratic type $s \in \{H, L\}$ of the underlying project. For type-H projects, creditors optimally rollover the debt for one more period for all debt levels D satisfying the incentive compatibility (IC) condition:

$$q_H D \geq \bar{\mu}.$$

On the other hand, type-L projects are liquidated for an immediate payoff since the restriction $q_L R < \bar{\mu}$ is always satisfied under assumption 1.

Given ex-post optimal decisions in the intermediate period, the ex-ante face value of debt D set by the creditors at $t = 0$ must satisfy the following participation constraint

$$\theta q_H \max(D, \frac{\bar{\mu}}{q_H}) + (1 - \theta)\bar{\mu} \geq 1. \tag{4}$$

In equation 4, the first expression gives the unconditional expectation of creditor payoffs from projects that become type-H ex-post at date $t = 1$, and the max term specifies the minimum debt level which ensures optimal continuation for such projects. The second expression provides the unconditional expected payment to the creditors from projects that reveal themselves to be type-L ex-post. The optimal debt level which satisfies both incentive compatibility and creditor rationality restrictions must lie in the region

$$D \in \left[\max\left(\frac{1 - (1 - \theta)\bar{\mu}}{\theta q_H}, \frac{\bar{\mu}}{q_H} \right), R \right].$$

The first term involving the max operator comes directly from the IC condition highlighted above. The upper bound R reflects the limited liability protection enjoyed by bankers, where the total repayment value D promised to the creditors cannot exceed the total cash flow R from successful projects in the terminal period $t = 2$. These restrictions together bound the total set of projects, which acquire financing at $t = 0$, to the set $[\theta_{FVA}, 1]$, where the lower bound of the financing set is given by

$$\theta_{FVA} = \max\left(\frac{1-\bar{\mu}}{q_H R - \bar{\mu}}, 0\right). \tag{5}$$

Under the HCA regime on the other hand, creditors do not receive any updated information about the underlying type $s \in \{H, L\}$ at $t = 1$. In this case, creditors rollover their debt in the intermediate period only if the unconditional continuation value

$$\theta q_H D + (1-\theta) q_L D$$

exceeds that $\bar{\mu}$ obtained from immediate liquidation. From an ex-ante perspective, the promised payment to the creditors, which satisfies both incentive compatibility and individual rationality conditions, must lie in the region

$$D \in \left[\frac{\max(1, \bar{\mu})}{\theta q_H + (1-\theta) q_L}, R\right].$$

The max term in the numerator comes from the incentive compatibility constraint. These restrictions imply that only a subset of projects $[\theta_{HCA}, 1]$ receive financing from creditors at $t = 0$, where the lower bound of the funding set is given by:

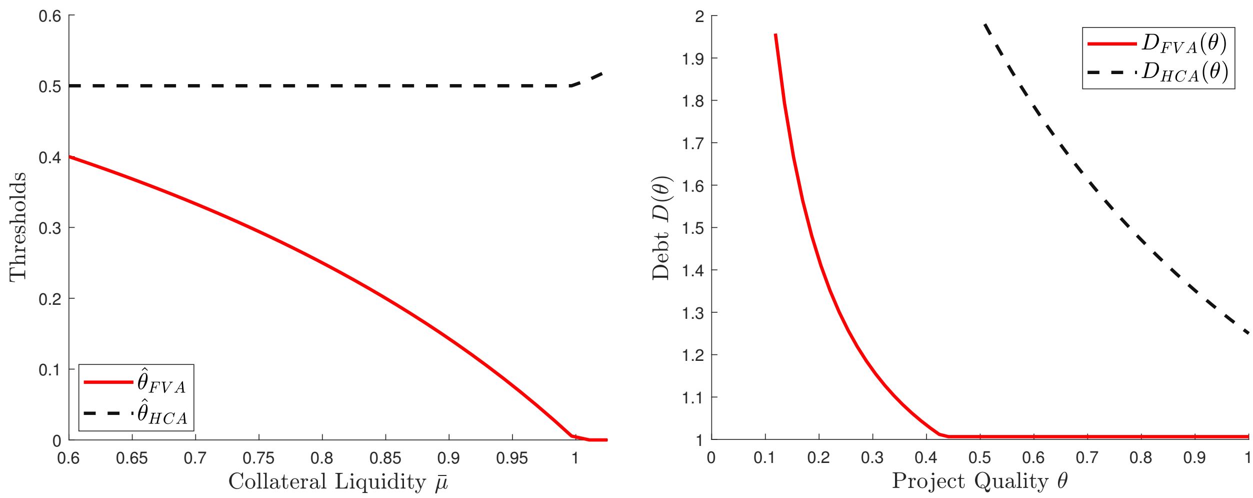
$$\theta_{HCA} = \frac{\max(\bar{\mu}, 1) - q_L R}{(q_H - q_L) R}. \tag{6}$$

Comparison of Regimes. Creditor information sets differ under the two accounting regimes, and, in turn, exposes them to different kinds of risks. For example, creditors under the HCA regime, lacking pertinent information about the underlying state, continue bad type-L projects in equilibrium. This additional risk increases the face value of debt D demanded in equilibrium, and constrains the set of projects which acquire financing ex-ante. With no aggregate risk, FVA on the other hand reflects first-best allocations. Under FVA, all efficient projects are rolled over by creditors in the intermediate period, while bad type-L projects get liquidated. This option value accorded to the investors reduces the credit risk of funding ex-ante, and allows a wider allocation set. The following proposition proved in the appendix formalizes the intuition presented above.

Proposition 1 *The total set of projects funded under FVA is wider than that under the HCA regime; i.e.,*

$$\theta_{FVA} < \theta_{HCA}.$$

Moreover, the FVA regime coincides with the first-best allocation. Finally, the contractual debt $D_{FVA}(\theta)$ under the FVA regime is lower than that under the HCA regime $D_{HCA}(\theta)$.



(a) Thresholds **(b)** Debt Levels, $\bar{\mu} = 0.8$

Figure 2: Funding Sets under FVA and HCA Regime

Figure 2 provides a graphical illustration of proposition 1. Collateral values are drawn from the set $[\mu_B, \mu_G]$, while the standard project parameters are $R = 2, q_H = 0.8, q_L = 0.3$ respectively. The variation of the lower bound of the funding sets under the FVA and HCA regime with the secondary market liquidation value in presented in Panel A; while the optimal face value $D(\theta)$ as a function of project quality is shown in Panel B. The lower bound of the funding set under the FVA regime is decreasing in collateral value $\bar{\mu}$ since

$$\frac{\partial \theta_{FVA}}{\partial \bar{\mu}} = \frac{1 - q_H R}{(q_H R - \bar{\mu})^2} < 0$$

Indeed, the payoff to creditors from bad type-L projects increases with $\bar{\mu}$, relaxing the ex-ante participation constraint 4. On the other hand, liquidation value only affects allocations under the HCA regime through the incentive compatibility condition term

$\max(\bar{\mu}, 1)$ in the equation 6. For $\bar{\mu} \leq 1$, threat of intermediate liquidation is not credible under the HCA regime since it violates their individual rationality constraint. When $\bar{\mu} > 1$ on the other hand, banks must provide enough incentives to creditors to rollover the debt at $t = 1$ in the form of a larger debt level D, constraining the total set of ex-ante allocations. As a result, the lower bound is a flat line up to $\bar{\mu} = 1$ and then it increases linearly in the region $(1, \mu_G]$.

4. Equilibrium with Aggregate Risk

The baseline model introduced in the previous section leaves two important criticisms during the GFC behind. In this section, we extend the baseline model to include fire sale pricing of assets and contagion possibilities, and investigate how they interact with accounting regulations. As Allen and Carletti (2008) and Allen et al. (2012) argue, FVA disclosure requirements led to artificial volatility in asset prices and induced cash-in-the-market pricing, especially when the secondary markets for assets became illiquid.

To investigate the effects of aggregate uncertainty and price volatility on ex-ante allocation efficiency, we extend the baseline model along two dimensions. First, we introduce aggregate uncertainty into the financial system by allowing for the crash risk λ to vary. This has two potent effects on the equilibrium allocations of projects, especially under the FVA regime. First, it directly impacts the distribution of good and bad projects in equilibrium through the parameter a introduced in Section 2. Second, the presence of aggregate uncertainty affects collateral prices in the secondary market. This additional price risk for running creditors affects their participation constraint and creates distortions in the set of projects that get funded ex-ante.

Second, we introduce exposures of projects to the aggregate crash risk through the parameter κ. Under adverse conditions, projects that are rolled over by creditors in the intermediate period become successful with a lower probability $(1-\kappa)q_s$, where $s \in \{H, L\}$ refers to the ex-post idiosyncratic type of projects. We consider two separate equilibrium.

The first equilibrium reflects homogeneous beliefs about future cash flows under the two accounting regimes, and assumes that the parameter κ is same across the two regimes. As hinted before, we are agnostic about the source of κ. It may indeed reflect the fundamental exposures of projects to an aggregate crash risk under rational expectations equilibrium; or it may reflect subjective creditor beliefs about probabilities associated with future cash flows. The only criterion we impose is that these beliefs are invariant across the two regimes. The second equilibrium relaxes this restriction and allows creditors beliefs to vary depending on the intermediate information revealed in the FVA regime. In particular, we assume that κ reflects creditor posteriors in the FVA regime after project types are revealed.[11]

4.1. Beliefs and Secondary Market Prices

Before deriving optimal allocations under the two accounting regimes, we first explore the evolution of beliefs and its effect on the secondary market prices of collateral. Under the FVA regime, all agents receive a vector of signals $\mathbf{S}$ pertaining to the underlying types of each project. With ex-ante identical banks and binomial support over idiosyncratic types $\{H, L\}$, the information content of signal $\mathbf{S}$ can be encoded succinctly using a single parameter k, indicating the number of banks that report a low-type L in the intermediate period. In the absence of any private information, all agents use this signal to update their beliefs about the underlying aggregate state. Let λ_k denote the posterior belief that the underlying state is adverse, conditional on k banks reporting a low type-L. By Bayes' Rule, the updated belief λ_k about an underlying aggregate crash is:

$$\lambda_k = P(\omega = B|k) = \frac{\lambda(a\theta)^{n-k}(1-a\theta)^k}{\lambda(a\theta)^{n-k}(1-a\theta)^k + (1-\lambda)(\theta)^{n-k}(1-\theta)^k}. \tag{7}$$

Intuitively, agent posteriors progressively deteriorate as more banks report low state in the intermediate period. We formalize the intuition in the following lemma and provide the proof in the appendix.

[11] A similar setup is explored in Ottaviani and Sørensen (2015) where investor's subjective beliefs about future events change with successive revelation of new information. For example, the new posteriors follow the odds ratio Bayes' formula

$$\frac{(1-\kappa)q_s}{1-(1-\kappa)q_s} = \frac{q_s}{1-q_s}L$$

where L is the new signal about the underlying project types. We consider a setup where the information content L of the idiosyncratic types becomes progressively worse with number of low types $k \in \{0, 1, 2 \cdots, n\}$ reported in the intermediate period.

Lemma 1 *Consider an economy with n banks of quality θ. The belief λ_k of agents about the underlying state being $\omega = B$, conditional on observing $k \leq n$ failures, is increasing in k.*

Whether or not the signal $\mathbf{S}$ reveals pertinent information about the underlying state ω depends on the exposure parameter a. When $a \to 1$, ex-post bank types do not depend on the aggregate state, but only on project quality θ. In this case, $\lambda_k = \lambda$ for all k, and the signal is completely uninformative about the underlying state ω. Thus in the ensuing analysis we maintain the assumption that $a < 1$.

Next we investigate how these updated posteriors affect secondary market liquidity and equilibrium collateral prices. Recall from our introductory description of the model in Section 2 that a representative collateral buyer is born at date $t = 1$ who allocates his endowment e between investing in a project and buying collateral in the secondary market. Conditional on signal $\mathbf{S}_1$ about the ex-post types of the banks at $t = 1$, the expected terminal payoff at date $t = 2$ from holding collateral is:

$$\mu_k \equiv (1 - \lambda_k)\mu_G + \lambda_k \mu_B \tag{8}$$

which also equals the maximum price he is willing to pay. However, the equilibrium price depends on the total endowment e, as well as the amount ϕ of collateral sold by the banks at $t = 1$.

First, note that the collateral manager only invests an amount $i \leq i^*$ in the safe project. Thus, if $e \geq \bar{e}_k \equiv i^* + \phi\mu_k$, the secondary collateral market clears at fundamental price $P_k(\phi) = \mu_k$. At the other extreme, if the endowment e is such that investing in collateral implies $i < i^*$, no arbitrage condition requires that the rate of return from holding collateral must equal the marginal product of the project r_f, implying

$$r_f = \frac{\mu_k}{P_k(\phi)}.$$

This floor price of $P_k(\phi) = \frac{\mu_k}{r_f}$ obtains when the endowment is smaller than $\underline{e}_k = i^* + \phi\frac{\mu_k}{r_f}$. This floor price sustains until the endowment is not enough to liquidate all collateral supplied at this price, $\phi\frac{\mu_k}{r_f}$. Beyond this level, there is cash-in-the-market pricing and the price merely reflects the available market liquidity $P_k = \frac{e}{\phi}$. Finally, when the endowment is between the thresholds $(\underline{e}_k, \bar{e}_k)$, the collateral price is given by $\frac{e-i^*}{\phi}$. Taken together, the secondary market price of collateral in the intermediate period is

$$P_k(\phi) = \begin{cases} \dfrac{e}{\phi} & \text{if } e \leq \phi \dfrac{\mu_k}{r_f} \\ \dfrac{\mu_k}{r_f} & \text{if } e \in \left[\phi \dfrac{\mu_k}{r_f}, i^* + \phi \dfrac{\mu_k}{r_f}\right] \\ \dfrac{e - i^*}{\phi} & \text{if } e \in \left[i^* + \phi \dfrac{\mu_k}{r_f}, i^* + \phi \mu_k\right] \\ \mu_k & \text{if } e \geq i^* + \phi \mu_k. \end{cases} \tag{9}$$

A few important observations about the secondary market price are in order. First, $P_k(\phi)$ decreases with the number of bank failures k in the intermediate period $t = 1$. This follows directly from Lemma 1 that posteriors are increasing in the number of ex-post bad types, and Equation 8 which links agents' beliefs to collateral payoffs. As shown in Figure 3, price $P_2(2)$, when the signal is $k = 2$ and both banks liquidate their projects, is lower than $P_1(2)$, when $k = 1$, except when the prices are determined by the available cash in the market.

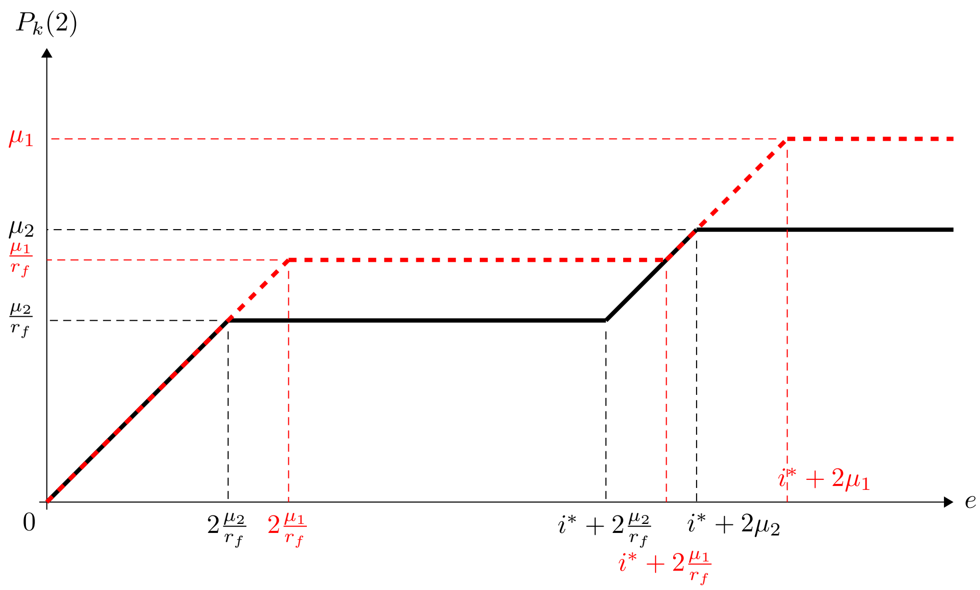


Figure 3: Secondary Market Prices. The black solid line plots the prices for collateral paid by the outside investor when $k = 2$ and both banks report an ex-post low type, while the dashed red line plots the prices when $k = 1$, and only one of the two banks reports a low state, while the other is revealed to be a high type. To facilitate fair comparison, we consider the case where both banks liquidate collateral in the secondary market, $\phi = 2$.

Second, holding the state of the banking system k constant, prices depend on the total amount of collateral sold. As shown in Figure 4, the whole space can be partitioned into two separate price regions. In the first region, the price is determined by the no arbitrage condition and the two prices $P_1(1)$ and $P_1(2)$ coincide. This happens when either there is plentiful liquidity in the secondary market, $e \geq i^* + 2\mu_1$, or when liquidity is scarce, $e \in \left[\frac{2\mu_1}{r_f}, i^* + \frac{\mu_1}{r_f}\right]$, but not enough to shut down outside production $i > 0$. In the second region however, prices are determined by the total available liquidity, and $P_1(2) < P_1(1)$. Finally, notice that in the region $e \in \left[i^* + \mu_1, i^* + 2\mu_1\right]$, banks can improve liquidations values of projects by moving from $\phi = 2$ to $\phi = 1$. In other words, by controlling the creditor's option to liquidate a good project, and thereby the extent of contagion, the banks can improve upon the prices they receive on the collateral by liquidating the ex-post low type project. This provides a rationale for imposing penalties, or direct restrictions, on creditors who choose to run and withdraw liquidity in the intermediate period. We return to this point when we consider the case of redemption gates later.

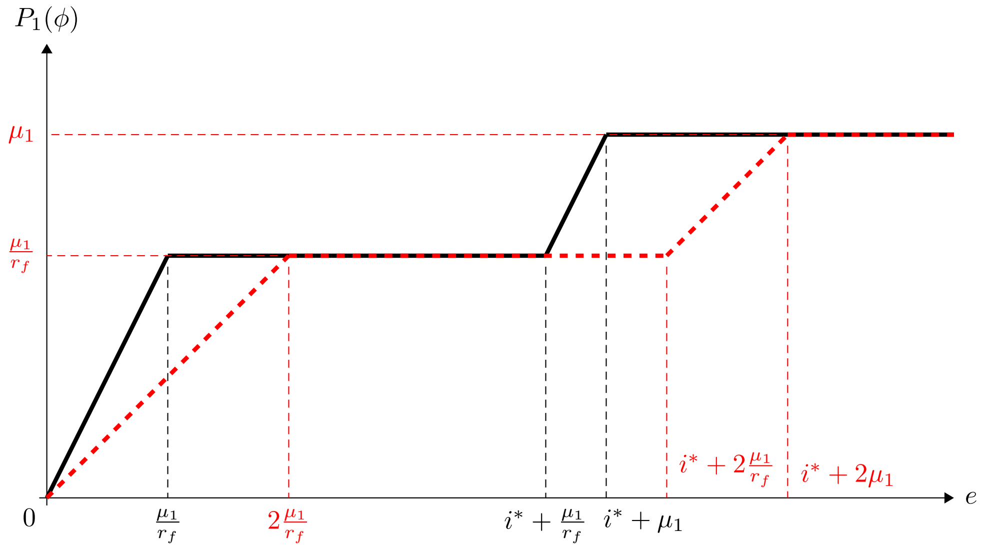


Figure 4: Secondary Market Prices when $k = 1$ and there is one ex-post good (bad) bank in the financial system. The red dashed line plots the prices for collateral paid by the outside investor when both banks sell collateral in the intermediate period,($\phi = 2$ in equation 9) while the black solid plots the prices when only one bank liquidates collateral.

Bank investors take the secondary market liquidity as given, and decide whether to continue the projects or liquidate in the intermediate period. In the next section, we characterize the ex-post equilibrium and show how the possibility of contagion may arise.

4.2. Equilibrium under FVA Regime

We solve for the equilibrium backwards, by first considering the ex-post optimal choices of the investors, and then solving for the threshold project quality that receives funding ex-ante. We consider two levels of collateral supplier's liquidity: first, when the endowment $e > i^* + 2\mu_1$ is large enough to allow all collateral to be liquidated at their fundamental prices, and second when $e \in \left[\frac{\mu_1}{r_f}, i^* + \frac{\mu_1}{r_f}\right]$, and collateral prices are discounted by $r_f > 1$.

4.2.1. Ex-post Equilibrium

Investors use the intermediate signal $\mathbf{S}$ to update their posteriors about the underlying state. This also changes their beliefs about the future success probabilities of the projects to:

$$q_H(k) = (1 - \lambda_k)q_H + \lambda_k(1 - \kappa)q_H$$
$$q_L(k) = (1 - \lambda_k)q_L + \lambda_k(1 - \kappa)q_L.$$

Banks that reveal themselves to be type-L ex-post are always liquidated in the interim period $t = 1$. This follows directly from assumption (1) and the fact that $q_L(k)D < q_L R < \mu_B < P_k(\phi)$. That is, the expected payoff to the investors from type-L projects is smaller than the worst possible price $\frac{\mu_k}{r_f}$ of collateral in the secondary markets. However, the decision of investors of good banks is more involved, and depends on the aggregate state of the financial system in the intermediate period. For the ease of exposition, we focus on the case of a banking duopoly, and take $N = 2$. The incentive compatibility condition of type-H investors dictate that they continue the good projects if and only if that total payoff from continuation, $q_H(k)D$, exceeds that from selling the collateral in the secondary market, $P_k(2)$.[12] Thus, to incentivize the good banks to continue the projects in state k, the ex-ante contract must promise a face value of at least:

$$D_k = \frac{P_k(\phi)}{q_H(k)}.$$

[12]When the intermediate state of the banking system is k, there are k type-L banks and $N - k$ type-H banks. In a symmetric equilibrium, if it is incentive compatible for investors of one type-H bank to run on the bank and liquidate the project, investors of all ex-post good banks reach the same decision. In this case, all banks sell their collateral and the total supply in the secondary market is $\phi = N$. If on the other hand, only ex-post bad banks sell their collateral, then $\phi = k$.

When the other bank $-i$ also reports itself to be a type-H,[13] creditors of bank i continue the project if and only if the face value of debt exceeds the threshold D_0. Similarly, if the other bank reports itself as a type-L, the bank has to promise creditors at least D_1 to meet their incentive compatibility condition. How do these thresholds compare? It can be shown with straightforward algebra that $D_0 \geq D_1$ if the crash exposure κ is smaller than $(1 - \frac{\mu_B}{\mu_G})$. Beyond this level, the threshold D_1 exceeds D_0, creating the possibility that creditors roll over a type-H bank if and only if the other bank also reports to be an ex-post type-H, while the good bank is liquidated in the intermediate period if the other bank becomes type-L ex post.[14]

4.2.2. Ex-ante Equilibrium with Liquid Markets

We next compute the lower bounds of the funding set under the FVA regime, separating our analysis into two parts to account for the possibility of contagion in the interim period. Recall that D_0 is the minimum value of debt that must be provided to the creditors to align incentives such that the project is continued at least in the best idiosyncratic state ($\mathbf{S} = (H, H)$). So, $D \geq \max(D_0, D_1)$ implies that good projects are always continued at $t = 1$ irrespective of the type reported by the other bank. To secure ex-ante financing, banks must offer a debt level D that satisfies the creditor's participation constraint:

$$\begin{aligned}(1-\lambda)\big[\theta^2 q_H D + \theta(1-\theta) q_H D + \theta(1-\theta) P_1(1) + (1-\theta)^2 P_2(2)\big] \\ + \lambda\big[a^2\theta^2(1-\kappa) q_H D + a\theta(1-a\theta)(1-\kappa) q_H D + a\theta(1-a\theta) P_1(1) + (1-a\theta)^2 P_2(2)\big] \geq 1,\end{aligned} \tag{10}$$

as well as ex-post incentive compatibility condition $D \geq \max(D_0, D_1)$ and limited liability restriction $D \leq R$. When the secondary market is liquid, the prices converge to their

[13] This corresponds to the case where there are no bad projects, and $k = 0$.

[14] Whether such contagion possibilities arise as an equilibrium outcome in the intermediate period depends on the aggregate crash intensity λ. Note that as the probability of crash vanishes ($\lambda \to 0$), the threshold D_1 tends to $\dfrac{\mu_G}{q_H} < R$. This condition is not sufficient to induce contagion since investors continue all type-H banks at the interim date $t = 1$ if the optimal debt lies in the region (D_1, R). On the other hand, if aggregate crash becomes imminent ($\lambda \to 1$), the threshold D_1 approaches $\frac{\mu_B}{(1-\kappa)q_H}$ which create a possibility for contagion as the bank exposure κ increases.

fundamental values, and

$$P_1(1) = P_1(2) = \mu_1 = (1-\lambda_1)\mu_G + \lambda_1\mu_B$$
$$P_2(2) = \mu_2 = (1-\lambda_2)\mu_G + \lambda_2\mu_B.$$

These conditions, taken together, limit the set of possible debt levels to

$$D \in \left[\max\left(\frac{1-(1-\lambda)(1-\theta)\mu_G - \lambda(1-a\theta)\mu_B}{(1-\lambda+a\lambda(1-\kappa))\theta q_H}, \max(D_0, D_1)\right), R\right].$$

The set of projects that get funded ex ante under this regime is given by $[\theta_{FVA}, 1]$ where the lower bound is:

$$\theta_{FVA} = \frac{1-(1-\lambda)\mu_G - \lambda\mu_B}{(1-\lambda+a\lambda(1-\kappa))q_H R - (1-\lambda)\mu_G - a\lambda\mu_B}. \tag{11}$$

On the other hand, under the contagion region, good banks are rolled over in the intermediate date if and only if the other bank also reports an ex-post type-H. For a system with two banks, this implies that short-term bank debts are rolled over only in the state (H, H), and liquidated in all the other intermediate states. The individual rationality constraint in this case is:

$$\begin{aligned}(1-\lambda)&\big[\theta^2 q_H D + \theta(1-\theta)P_1(2) + \theta(1-\theta)P_1(2) + (1-\theta)^2 P_2(2)\big]\\ &+ \lambda\big[a^2\theta^2(1-\kappa)q_H D + a\theta(1-a\theta)P_1(2) + a\theta(1-a\theta)P_1(2) + (1-a\theta)^2 P_2(2)\big] \geq 1,\end{aligned} \tag{12}$$

and the optimal debt level under such conditions is given by

$$D = \max\left[\frac{1-(1-\lambda)(1-\theta^2)\mu_G - \lambda(1-a^2\theta^2)\mu_B}{(1-\lambda+\lambda a^2(1-\kappa))q_H\theta^2}, D_0\right]$$

The minimum quality θ_{FVA} under the contagion region which successfully acquires financing in the initial period $t = 0$ is:

$$\theta_{FVA} = \sqrt{\frac{1-(1-\lambda)\mu_G - \lambda\mu_B}{(1-\lambda+a^2\lambda(1-\kappa))q_H R - (1-\lambda)\mu_G - a^2\lambda\mu_B}}. \tag{13}$$

In sum, the total set of projects funded under the FVA regime depends on the aggregate uncertainty. When aggregate crash risk is small, all projects that reveal themselves to be of high type-H are continued until maturity and the total set of projects funded is $[\theta_{FVA}, 1]$, where the lower bound of funding set is given by Equation 11. On the other hand, if aggregate uncertainty is high, there is financial contagion and the decision to continue or liquidate good (type-H) projects depends on the state reported by other banks. The lower bound of the funding set in this case is given by equation 13. The following Lemma 2 records the key results in this section.

Lemma 2 *The total funding set under the FVA regime with liquid markets is given by* $[\theta_{FVA}, 1]$*, where*

$$\theta_{FVA} = \begin{cases} \dfrac{1-(1-\lambda)\mu_G - \lambda\mu_B}{(1-\lambda+a\lambda(1-\kappa))q_H R - (1-\lambda)\mu_G - a\lambda\mu_B} & \text{if No Contagion,} \\ \sqrt{\dfrac{1-(1-\lambda)\mu_G - \lambda\mu_B}{(1-\lambda+a^2\lambda(1-\kappa))q_H R - (1-\lambda)\mu_G - a^2\lambda\mu_B}} & \text{if Contagion .} \end{cases}$$

4.3. Equilibrium under HCA Regime

Under the HCA regime, agents do not receive any updated signal about the idiosyncratic bank types in the intermediate period. As a result, their beliefs about the underlying state remain unchanged at the prior λ. To solve for the equilibrium, we again start with ex-post decisions at the interim period $t = 1$. Immediate liquidation of projects yields an expected value:

$$\bar{\mu} \equiv (1-\lambda)\mu_G + \lambda\mu_B,$$

while continuation until maturity provides an expected payoff to creditors of:

$$(1-\lambda)[\theta q_H D + (1-\theta)q_L D] + \lambda[a\theta(1-\kappa)q_H D + (1-a\theta)(1-\kappa)q_L D]. \tag{14}$$

The above expression includes expectations over both the aggregate state (λ), as well as the individual state of the projects. For lower levels of aggregate uncertainty (specifically when $\lambda \leq \frac{\mu_G - 1}{\mu_G - \mu_B}$), the expected payoff $\bar{\mu}$ from immediate liquidation exceeds 1 and imposes an additional incentive constraint on the banks since they must provide a minimum repayment of $D \geq \bar{\mu}$ to creditors to align their interests with that of the bank. For higher aggregate uncertainty however, creditors cannot threaten intermediate liquidation to extract higher rents from the banks as this violates their individual rationality constraint.

The set of projects that secure ex-ante financing under the HCA regime must satisfy the creditors' individual rationality condition as in Equation 14, incentive compatibility constraints $D \geq \max(\bar{\mu}, 1)$, as well as the banks' limited liability constraint $D \leq R$. The minimum project quality that satisfies all the above conditions is:

$$\theta_{HCA} = \frac{1 - (1 - \lambda\kappa)q_L R}{(1 - \lambda + \lambda a(1 - \kappa))(q_H - q_L)R}. \tag{15}$$

The optimal contract between the banks and creditors specifies a face value:

$$D = \max\left(\bar{\mu}, \frac{1}{(1-\lambda)[\theta q_H + (1-\theta)q_L] + \lambda[a\theta(1-\kappa)q_H + (1-a\theta)(1-\kappa)q_L]}\right).$$

4.4. Comparison of Accounting Regimes

We next compare the aggregate investments under the two accounting regimes. The information structure associated with the HCA regime creates an inefficiency in the liquidation policies where type-L projects are sub-optimally continued by uninformed creditors in the intermediate period. However, the introduction of aggregate risk also creates inefficiencies under the FVA regime. As we demonstrate, information about individual bank health spills over under the FVA regime to other banks, and introduces a possibility where type-H projects may also get liquidated sub-optimally. This threat is particularly potent when the aggregate uncertainty is high ($\lambda \uparrow$), and banks are highly exposed to aggregate risk or when subjective beliefs deviate by a large extent from the rational benchmark ($\kappa \uparrow$).

We first consider the situation where contagion threat does not materialize in the intermediate period. This happens when either of the two conditions are satisfied: (i) the exposure of individual projects to the aggregate risk lies below the threshold $\kappa_* := (1 - \frac{\mu_B}{q_H R})$, in which case the level of aggregate risk does not matter; and (ii) for any $\kappa > \kappa_*$, the level of aggregate risk λ is smaller than a threshold $\lambda_*(\kappa)$. We formalize the conditions described above in the following Lemma 3, and characterize the solution methods for finding the thresholds.

Lemma 3 *The sufficient condition for information contagion in the financial market in*

the intermediate period is characterized by two thresholds (κ_*, λ_*) *such that*
(i) contagion does not occur for any exposure $\kappa \leq \kappa_* \equiv 1 - \frac{\mu_B}{q_H R}$
(ii) contagion does not occur for any exposure $\kappa > \kappa_*$ *if the level of aggregate risk is below a threshold* $\lambda_*(\kappa)$*, characterized endogenously by the solution of the equation*

$$D_1(\kappa, \lambda_*(\kappa)) = R.$$

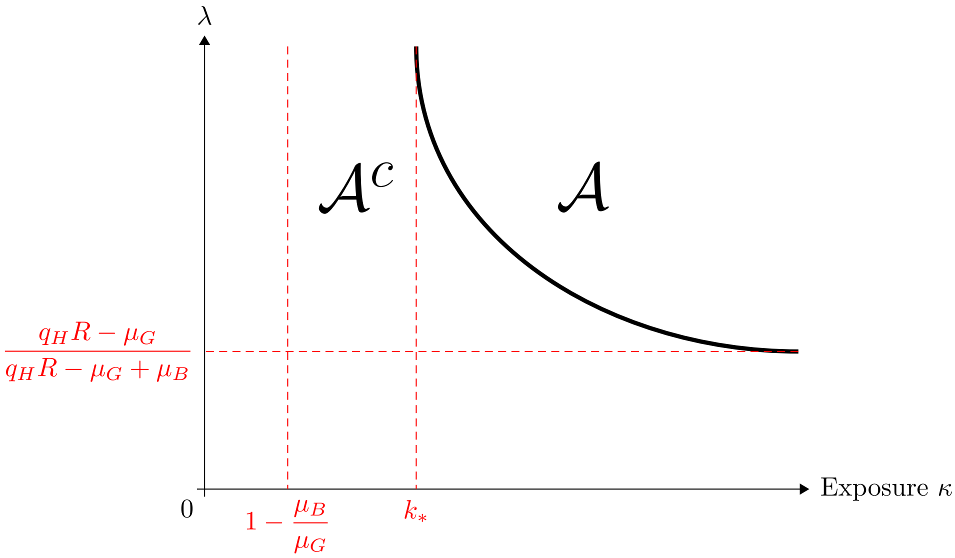
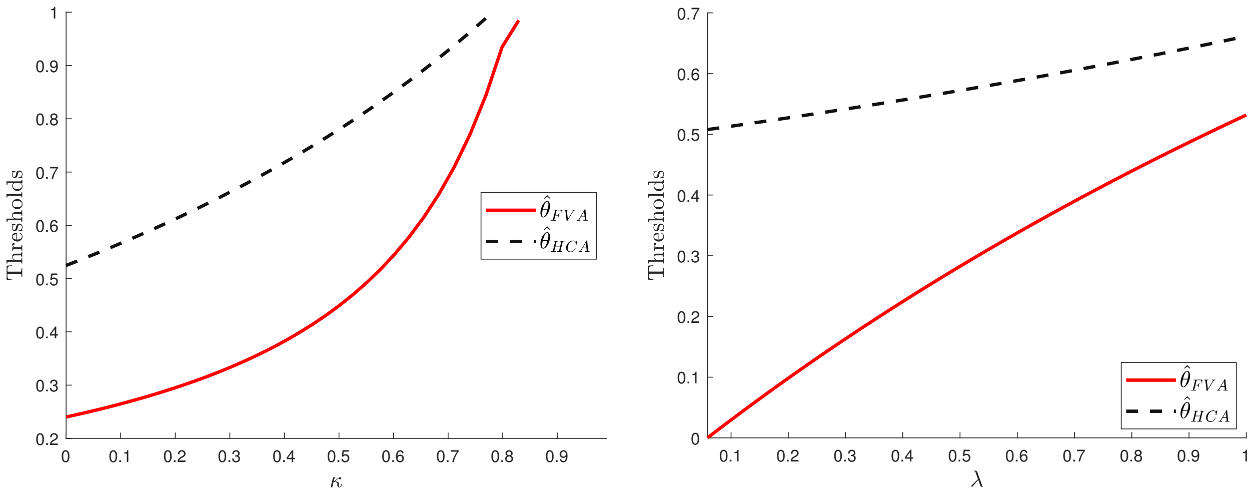


Figure 5: Contagion in the Intermediate Period. The figure plots the contagion region $\mathcal{A} = [\kappa_*, 1] \times [\lambda_*(\kappa), 1]$ and the no-contagion region in the parameter (κ, λ) space. The solid curve represents the threshold $\lambda_*(\kappa)$ beyond which the good projects are liquidated in the intermediate period.

Figure 5 depicts the whole contagion region, that is the set of parameters (κ, λ) for which good projects are liquidated in the intermediate period in the state (H, L). From Lemma 3, this region is characterized by $\mathcal{A} = [\kappa_*, 1] \times [\lambda_*(\kappa), 1]$. The region to the left of the point $\lambda_*(\kappa_*) = 1$ depicts the range of $\kappa \in [0, \kappa_*]$ for which there is no contagion in the intermediate period.

The results presented above show how the incidence of contagion affects the ex-ante allocation of projects under the FVA regime. When the parameters (κ, λ) lie within the contagion region, the FVA suffers from the inefficiency when creditor decisions for good type-H bank depend on the intermediate type reported by the other bank. The HCA regime, on the other hand, suffers from the inefficiencies associated with the continuation of bad type-L projects in equilibrium. To understand how these inefficiencies affect the total set of projects funded in equilibrium, we compare the funding thresholds under both the FVA and HCA regimes. This entails a comparison of thresholds 13 and 15 when the parameters $(\kappa, \lambda) \in \mathcal{A}$ lie in the contagion region, and a comparison of equations 11 and 15 otherwise, when $(\kappa, \lambda) \in \mathcal{A}^c$. This leads to the following Proposition 2. The proof is in the appendix.

Proposition 2 *Under liquid markets, total investment under the FVA regime is always greater than that under HCA.*



(a) $\lambda = 0.09, a = 0.95$ **(b)** $\kappa = 0.50, a = 0.95$

Figure 6: Funding Sets under FVA and HCA Regime

Figure 6 provides a graphical comparison of the lower bounds of the funding sets θ_{FVA} and θ_{HCA} under the FVA and HCA regimes respectively. In the first panel, we hold the aggregate crash intensity λ constant at 9% and plot the variation in the thresholds with project exposure κ. For this range of parameters, there is no contagion in the financial markets and good projects are optimally continued under the FVA regime. On the other hand, inefficiencies stemming from sub-optimal continuation of bad type-L projects constrain the funding set under the HCA regime, as creditors demand a higher debt to compensate for this additional risk. In Panel B, we hold $\kappa = 0.50$ constant. This high exposure level creates a possibility of contagion spillovers in the intermediate period for aggregate risk $\lambda > \lambda_*(\kappa)$.

In the preceding analysis, we focus on the case where secondary market is liquid, and collateral always fetches fundamental values to the seller. We next focus on the case of illiquid markets where asset prices are discounted, and explore the ex-ante effects on allocations.

4.5. Equilibrium Under Illiquid Markets

In illiquid markets, collateral sells at a discount relative to their fundamental value. In this section, we consider two separate cases of liquidity decline. In the first case, we assume that the market is semi-liquid in the sense that prices still reflect fundamental values if only one bank liquidates project in the intermediate period. However, if both banks sell collateral, assets trade at a discount. That is, in the first case, we consider the following price menu

$$P_k(1) = \mu_k, P_k(2) = \frac{\mu_k}{r_f}$$

This happens when the collateral buyer's endowment lies in the range $e \in \left[i^* + \mu_1, i^* + \frac{2\mu_2}{r_f}\right]$. The secondary market in the second case is more constrained, and prices are discounted by a factor r_f irrespective of the number of units of collateral sold. That is, in this case, the asset prices are

$$P_k(\phi) = \frac{\mu_k}{r_f} \text{ for all } \phi \in \{1, 2\}$$

First consider the situation when there is contagion in the intermediate period and ex-post good projects are liquidated in the state $k = 1$. Since collateral prices $P_1(2)$ and $P_2(2)$ remain the same in either case, the threshold quality of projects funded under the contagion region θ_{FVA} is obtained by plugging the price expressions in the investor participation constraint 12, and using the limited liability condition $D \leq R$.

$$\theta^C_{FVA} = \sqrt{\frac{1 - \frac{(1-\lambda)\mu_G + \lambda\mu_B}{r_f}}{(1 - \lambda + \lambda a^2(1-\kappa))q_H R - \frac{(1-\lambda)\mu_G + \lambda a^2 \mu_B}{r_f}}} \tag{16}$$

However, differences emerge in the no-contagion region when the good project is continued in the intermediate period in the state $k = 1$. In the first case, the collateral price in the first case is $P_1(1) = \mu_1$, while is only $P_1(1) = \mu_1/r_f$ in the second case. Using these expressions for prices in the participation constraint under the no- contagion region 10 yields the threshold quality of projects funded as

$$\theta_{FVA}^{NC} = \begin{cases} \dfrac{-\xi_1 + \sqrt{\xi_1^2 + 4\zeta_1\left(1 - \frac{(1-\lambda)\mu_G + \lambda\mu_B}{r_f}\right)}}{2\zeta_1} & \text{in semi-liquid markets} \\ \dfrac{1 - \frac{(1-\lambda)\mu_G + \lambda\mu_B}{r_f}}{(1-\lambda+a\lambda(1-\kappa))q_H R - \frac{(1-\lambda)\mu_G + a\lambda\mu_B}{r_f}} & \text{in illiquid markets} \end{cases} \quad (17)$$

where

$$\xi_1 = (1-\lambda+\lambda a(1-\kappa))q_H R + \left(\frac{2}{r_f} - 1\right)\Big[(1-\lambda)\mu_G + \lambda a\mu_B\Big]$$
$$\zeta_1 = \left(\frac{1}{r_f} - 1\right)\Big[(1-\lambda)\mu_G + \lambda a^2\mu_B\Big]$$

Market illiquidity also impacts the total amount of projects funded ex-ante under the FVA regime by reducing the payoffs from early liquidations of projects which become bad ex-post. Differentiating the threshold project quality θ_{FVA} with respect to the discount parameter r_f yields

$$\frac{d}{dr_f}\theta_{FVA} = \Big[(1-\lambda+\lambda a(1-\kappa))q_H R\Big]\frac{(1-\lambda)\mu_G + \lambda\mu_B}{r_f^2} - \frac{(1-\lambda)\mu_G + \lambda a\mu_B}{r_f^2}$$
$$= \Big[(1-\lambda+\lambda a(1-\kappa))q_H R - 1\Big]\frac{(1-\lambda)\mu_G + \lambda\mu_B}{r_f^2} + \lambda\frac{\mu_B(1-a)}{r_f^2}$$

Finally, the constraint that $\theta_{FVA} \leq 1$ also makes the above derivative positive. Intuitively, lower liquidation values in the intermediate period constrains the investor's zero profit condition and reduces the total ex-ante set of projects funded. More interestingly, this raises a further question about the relative efficiencies of the two accounting regimes when markets are illiquid.

Proposition 3 *Under illiquid markets, total investment under the FVA regime is always greater than that under the HCA regime, and converges to the funding set under HCA regime when there is a complete shutdown of secondary asset markets.*

As the secondary market discount r_f increases, all incentive compatibility constraints D_k become slack. As long as the IC condition in the worst possible state $k = 2$,

$$\frac{(1-\lambda_2)\mu_G + \lambda_2\mu_B}{r_f} \geq \big[(1-\lambda_2)q_L + \lambda_2 q_L(1-\kappa)\big]R$$

is valid, proposition 3 shows that the total funding set under FVA dominates that under the HCA regime. Let r_f^* be the threshold level of discount factor for which the IC condition in state (L, L) binds with equality. When $r_f > r_f^*$, the IC in the lowest state is violated and the projects are continued unconditionally in the intermediate period. In this region, the equilibrium under the FVA coincides with that under the HCA regime.

5. Equilibrium with Heterogeneous Beliefs

In this section, we explore the effect of belief distortions on investment allocations under the two accounting regimes. To operationalize this setup, we make two modifications to the two-bank model introduced in the previous section. Firstly, we assume that individual projects are not fundamentally exposed to aggregate crash risk, and set $\kappa = 0$. Secondly, we allow for the possibility of pessimistic beliefs and panic-driven contagion. We assume that conditional on observing the state (H, L), creditors revise their expectations about the future success probability of the good type-H project from q_H to $q_H(1 - \kappa_D)$ under the adverse aggregate state.

There are two ways to justify the above modeling choice. First, empirically, after the failure of Lehman Brothers, the spreads on credit default swaps of other major investment banks including Goldman Sachs, Citibank, and J.P. Morgan Chase showed a large jump. The notion that investors downgrade success probabilities of good projects captures this scenario where investors become more pessimistic about the intermediate healthy projects. Alternately, one can also think of this modeling choice as a reduced form version of an elaborate investor optimization problem. Using a setup where investors hold multiple risky securities in their portfolio, Kyle and Xiong (2001) show that losses on one part of the portfolio can render investors more risk averse with respect to their remaining portfolio, changing the stochastic discount factor for other stocks. This potentially leads to a contagion effect with investors selling stocks with positive returns. The downward bias in the success probabilities captures the distortions of stochastic discount factors originating in a more complex model.

In an equilibrium with pessimistic beliefs, any project that secures financing ex-ante must provide a face value of debt D to creditors that satisfies their participation constraint:

$$
\begin{aligned}
&(1-\lambda)\big[\theta^2 q_H D + \theta(1-\theta)q_H D + \theta(1-\theta)P_1 + (1-\theta)^2 P_2\big] \\
&+ \lambda\big[a^2\theta^2 q_H D + a\theta(1-a\theta)(1-\kappa_D)q_H D + a\theta(1-a\theta)P_1 + (1-a\theta)^2 P_2\big] \geq 1.
\end{aligned}
$$

In the no-contagion region $\mathcal{A}^c$, creditors roll over type-H projects at $t = 1$ if the contractual debt level D exceeds the threshold level:

$$
D_1(\kappa_D) = \frac{P_1}{(1-\lambda_1)q_H + \lambda_1(1-\kappa_D)q_H}.
$$

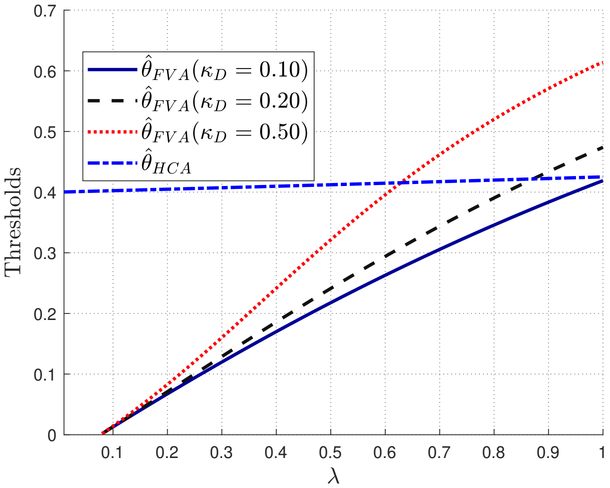
Here, P_1 reflects the immediate payoff to the creditors if they withdraw at $t = 1$, while $\big[(1-\lambda_1)q_H + \lambda_1(1-\kappa_D)q_H\big]D$ is the total creditor payoff from rolling over the debt for one more period. In this region, the lower bound of projects funded under the FVA regime is θ_{FVA} where

$$
\begin{aligned}
\theta_{FVA} &= \frac{-\chi + \sqrt{\chi^2 + 4\lambda a^2\kappa_D q_H R(1-(1-\lambda)\mu_G - \lambda\mu_B)}}{2\lambda a^2 \kappa_D q_H R} \\
\chi &= (1-\lambda+\lambda a)q_H R - \lambda a\kappa_D q_H R - (1-\lambda)\mu_G - \lambda a\mu_B.
\end{aligned}
$$

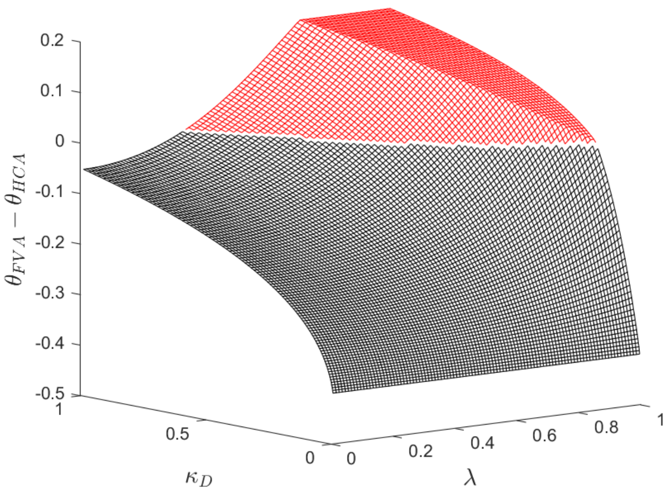
Finally, when the set of parameters (κ_D, λ) lie in the contagion region, the total set of projects funded under heterogeneous beliefs is the same as in Equation 13 where $\kappa = 0$.

Let us compare investments under the FVA and HCA regime under heterogeneous beliefs. When creditors are not pessimistic and $\kappa_D \to 0$, investment under the FVA regime unconditionally dominates that under HCA. On the other hand, as creditors revise beliefs about success probability of type-H projects conditional on the state (H, L) and $\kappa_D \to 1$, the set of projects funded under the FVA regime shrinks while that under HCA regime remains constant. For certain combinations of (κ_D, λ), the set of projects funded under the HCA dominates that under FVA. The following proposition provides a formalization of the intuition presented above, and a graphical representation of the above result is shown in panels A and B of Figure 7.

Proposition 4 *Assume that conditional on aggregate crash, the fundamental exposure of individual projects is $\kappa = 0$, while creditors beliefs about the exposure is $\kappa_D > 0$. In this case, aggregate investment under HCA regime is greater than that under FVA if the* *set of parameters* (κ_D, λ) *lies in the set* $(\kappa_D^*, 1] \times [\lambda_*(\kappa), 1]$.



(a) $\lambda = 0.09, a = 0.95$



(b) $a = 0.95$

Figure 7: Funding Sets under FVA and HCA Regime

6. Policy Analysis

The analysis presented in the previous section highlights the relative inefficiencies associated with the two accounting regime: sub-optimal continuation of ex-post bad banks under the HCA regime, and excessive contagion-induced liquidations of ex-post good banks by investors under the FVA regime. Inefficient continuations of bad projects increases the total amount of risk borne by bank creditors, who demand a higher return as compensation for this additional risk, in turn reducing the total ex-ante originations in the HCA model. Similarly, under the FVA regime, excessive liquidation of projects in the intermediate period, in an illiquid market, imposes an externality on other banks by creating price distortions away from fundamental values.

This creates an opportunity for a social planner to implement policies aimed at improving ex-ante allocation, either by (i) sharing failure risk with creditors, or (ii) by forcing banks to internalize the price externality created by excessive liquidations in the secondary markets. In this section, we investigate two two policy interventions, both of which were implemented during the GFC, and more recently Brexit. We investigate the impact of the interactions between the accounting regimes and the policy interventions on ex-ante allocations, and their efficacy relative to the first-best. The first policy is Suspension of Convertibility or Redemption Gates, where, during periods of market stress, financial institutions can impose restrictions on investor withdrawals temporarily to avoid fire sales and contagion-induced systemic failure. In our setting, the banks impose a hard constraint on the external investors such that at the intermediate date, only up to a fraction $x \in [0,1]$ of the project can be liquidated. The second policy intervention we consider is *regulatory forbearance* where the government commits to bailout the creditors of failed projects with a probability $\beta \in [0,1]$.

6.1. Social Planner Equilibrium

We begin by considering the ex-ante allocations that a constrained social planner, who also takes endowments and prices as given, can achieve. The following proposition shows that ex-ante originations under the first best coincides with that achieved under the fair-value regime with homogeneous beliefs.

The constrained planner chooses an optimal threshold θ_{FB} such that all projects which non-negative net present value are funded in the initial period $t = 0$. The value function of the banking system which the planner seeks to maximize is

$$\begin{aligned} V(\theta) = &\big[(1-\lambda+\lambda a(1-\kappa))\big]\theta q_H R + \big[(1-\lambda)\theta(1-\theta)+\lambda a\theta(1-a\theta)\big]P_1(1) \\ &+ \big[(1-\lambda)(1-\theta)^2+\lambda(1-a\theta)^2\big]P_2(2) - 1 \end{aligned} \tag{18}$$

if there is no contagion, and

$$\begin{aligned} V(\theta) = &\big[(1-\lambda+\lambda a^2(1-\kappa))\big]\theta^2 q_H R + 2\big[(1-\lambda)\theta(1-\theta)+\lambda a\theta(1-a\theta)\big]P_1(2) \\ &+ \big[(1-\lambda)(1-\theta)^2+\lambda(1-a\theta)^2\big]P_2(2) - 1 \end{aligned} \tag{19}$$

if there is. First consider the case where the secondary markets are liquid, and collateral can be sold in the intermediate period at their fundamental value. That is, we assume that

$$P_1(1) = P_1(2) = \mu_1, P_2(2) = \mu_2$$

In this case, simple computations yield the set of projects funded by the constrained planner as $[\theta_{FB}, 1]$, where

$$\theta_{FB} = \begin{cases} \dfrac{1-(1-\lambda)\mu_G - \lambda\mu_B}{(1-\lambda+a\lambda(1-\kappa))q_H R - (1-\lambda)\mu_G - a\lambda\mu_B} & \text{if No Contagion} \\ \sqrt{\dfrac{1-(1-\lambda)\mu_G - \lambda\mu_B}{(1-\lambda+a^2\lambda(1-\kappa))q_H R - (1-\lambda)\mu_G - a^2\lambda\mu_B}} & \text{if Contagion} \end{cases} \tag{20}$$

These thresholds are obtained by solving for the project quality θ for which the net present value of the banking system $V(\theta) = 0$. Notice that the same thresholds are achieved by the decentralized banking system under the FVA regime (see threshold equations 11 and 13). Thus, the FVA regime is first best when market are liquid.

Proposition 5 *When the secondary market for collateral is liquid, the total set of projects funded by a benevolent social planner coincides with that under the FVA regime.*

However, as secondary markets liquidity deteriorates, the social planner allocations may deviate from that achieved under private banking equilibrium. We analyze this problem in the next section, where we also introduce the potential of imposing liquidation costs on withdrawing creditors, and explore its effects on ex-ante loan originations under social planner and private banking equilibrium.

6.2. Illiquid Markets and Redemption Gates

Under illiquid markets, when collateral price in the secondary market is limited by the available supply of liquidity, a regulator can potentially improve bank profitability, as well as ex-ante allocations, by imposing penalties or explicit restrictions on the withdrawing creditors. As shown in Figure 4, in the regions where collateral prices are determined by the total cash-in-the market, as in $e \in [0, 2\frac{\mu_1}{r_f}] \cup [i^* + \frac{\mu_1}{r_f}, i^* + 2\mu_1)$, the price of collateral, when only one bank accesses the secondary market, $P_1(1)$ is significantly larger than when both banks approach the market, $P_1(2)$. Under such circumstances, financial institutions can impose temporary restrictions on investor withdrawals temporarily to avoid fire sales and contagion-induced systemic failure.

We next explore the effects of such redemption gates on ex-ante allocations and welfare. Specifically, we assume that a benevolent social planner sets state-contingent restrictions

$\mathbf{x} = (x_{HH}, x_{HL}, x_{L*})$ such that the creditors are only allowed to withdraw up to a maximum fraction x_k of the project at $t = 1$ for a total payoff of $x_k P_k$ in the intermediate period, while the residual amount $(D - x_k P_k)$ is paid at the terminal date $t = 2$ upon the successful completion of the project.[15] We assume the following liquidity structure of the secondary markets. If only a single bank liquidates collateral, then prices reflect the fundamental value. But if both banks sell collateral, either there is cash-in-the-market pricing, and prices are given by e/ϕ, or the prices are discounted by a factor r_f. For ease of exposition, we focus on the latter case, but the generalization also yields an identical result.

We first consider the case of $\kappa \leq \kappa_*$, for which there is no contagion in the intermediate period. In this case, the restrictions x_{HH} and x_{HL} does not matter, as the projects are rolled over until the terminal period. We assume that the social planner imposes limits x_{LH} and x_{LL} on the investors in the intermediate period. In this case, the investors withdraw up to the maximum limit x_{L*} [16] yielding a total payoff of

$$\begin{aligned}(1-\lambda)\Bigg[&\theta q_H D + \theta(1-\theta)\Bigg\{x_{LH}P_1(1) + q_L(D - x_{LH}P_1(1))\Bigg\} + (1-\theta)^2\Bigg\{x_{LL}P_2(2) + \\ &q_L(D - x_{LL}P_2(2))\Bigg\}\Bigg] + \lambda\Bigg[a\theta q_H(1-\kappa)D + a\theta(1-a\theta)\Bigg\{x_{LH}P_1(1) \\ &+ q_L(1-\kappa)(D - x_{LH}P_1(1))\Bigg\} + (1-a\theta)^2\Bigg\{x_{LL}P_2(2) + q_L(1-\kappa)(D - x_{LL}P_2(2))\Bigg\}\Bigg]\end{aligned} \tag{21}$$

[15] This clearly identifies the external investor as a creditor as opposed to a preferred equity holder. We can design this alternate case in a similar manner. In this case, the investor receives the intermediate value xP from the liquidated part as before, but on the residual fraction $(1 - x)$ that is continued, the investor receives, upon success, a total value $(1 - x)D$. The residual part $(1 - x)(R - D)$ accrues to the junior claimant - the bank. The results from this altered version are qualitative similar to the ones presented here and are omitted for the sake of brevity.

[16] This follows directly from the incentive compatibility condition of the investors. Assume that they withdraw a fraction $m \leq x$ in the intermediate period for a total payoff mP at date $t = 1$, and an expected payoff of $q_L(k)(D - mP)$ at date $t = 2$. The investor's optimization problem can then be written as

$$\max_{m \leq x} mP + q_L(k)\min(D - mP, (1-m)R)$$

When $D - mP \leq (1-m)R$ the payoff is maximized at $m = x$. Consider the other case where $(1-m)R \leq (D - mP)$. In this case, the total payoff of the investor is $mP + (1-m)q_L(k)R$. From assumption 1, we have $q_L(k)R \leq q_L R \leq P$, and the optimal withdrawal strategy is again $m = x$.

Finally the bank's payoff in the terminal period is

$$\begin{aligned}\Pi_B(x_{LH}, x_{LL}) = (1-\lambda+\lambda a(1-\kappa))\theta q_H(R-D) + \big[(1-\lambda)\theta(1-\theta)+ \\ \lambda a\theta(1-a\theta)(1-\kappa)\big] q_L \Big\{(1-x_{LH})R - D + x_{LH}P_1(1)\Big\} + \\ \big[(1-\lambda)(1-\theta)^2 + \lambda(1-a\theta)^2(1-\kappa)\big] q_L \Big\{(1-x_{LL})R - D + x_{LL}P_2(2)\Big\}\end{aligned} \tag{22}$$

In a rational expectations equilibrium, the regulator anticipates optimal investor response to the imposed limits, and chooses them ex-ante to maximize the second period bank profits Π_B. The following proposition characterizes the optimal bank choice for the states $(L, H), (L, L)$ respectively.

Proposition 6 *The regulator optimally allows full redemption in states where it is revealed to be a low-type ex-post. That is* $x_{L*} = 1$.

Next consider the case where $\kappa > \kappa_*$ and investors also liquidate the ex-post good project in the intermediate period in the state $k = 1$. Since markets are illiquid, we assume that there exists a $x^* \in (0, 1)$ such that when the total collateral supply $\phi \leq 1+x^*$, markets clear at fundamental prices. However, for $\phi > 1+x^*$, there is cash-in-the-market pricing and the price of collateral drops to $P_1(\phi) = e/\phi$. We set $x_{L*} = 1$ directly using proposition 6, and assume $x_{HH} = 0$.[17] Total investor payoffs in this case is

$$\begin{aligned}&(1-\lambda+\lambda a)\theta q_H D + (1-\lambda)\theta(1-\theta)\big[(1+x_{HL}(1-q_H))P_1(\phi)\big] + \\ &\lambda a\theta(1-a\theta)\big[(1+x_{HL}(1-q_H(1-\kappa)))P_1(\phi)\big] + \big[(1-\lambda)(1-\theta)^2 + \lambda(1-a\theta)^2\big]P_2(2)\end{aligned} \tag{23}$$

while total bank profits equal

$$\begin{aligned}\Pi_B = \big[(1-\lambda) + \lambda a(1-\kappa)\big]\theta^2 q_H(R-D) + \big[(1-\lambda)\theta(1-\theta) \\ + \lambda a\theta(1-a\theta)(1-\kappa)\big] q_H((1-x_{HL})R - D + x_{HL}P_1(\phi))\end{aligned} \tag{24}$$

The regulator chooses an optimal x_{HL} to maximize profits 24 subject to the investor break

[17] This assumption is just for the sake of avoiding clutter. Using an identical approach one can easily show that the bank prefers continuation in the state (H, H), and chooses $x_{HH} = 0$ to restrict investors from withdrawing in the best possible intermediate state.

even condition and price process $P_1(\phi)$. The following Proposition 7 then shows that a social planner, who internalizes the effect of total liquidations on the secondary market price of collateral $P_1(\phi)$, sets optimal thresholds ϕ^* so as to minimize liquidation-induced price distortions.

Proposition 7 *Assume that for a given level of collateral liquidity e, there exists an interior $\phi^* = 1 + x^*$ such that the secondary market price of collateral in the state $k = 1$ is*

$$P_1(\phi) = \begin{cases} \mu_1 & \text{for all } \phi \leq \phi^* \\ \frac{e}{\phi} & \text{if } \phi > \phi^* \end{cases}$$

Then in an equilibrium with redemption gates, the regulator optimally sets the maximum redemption level at $x = x^*$ if the bank is high-type ex-post, and the other bank is low-type.

When markets are semi-liquid, the regulator optimally sets redemption restrictions such that secondary collateral markets clear at fundamental values for ex-post type-H banks, while allowing a full run on type-L banks. It is important to note that propositions 6 and 7 also work when markets are completely illiquid and there is cash-in-the-market pricing irrespective of the total quantity of collateral sold. In this case, $\phi^* = 0$ and secondary market prices are given by $P_1(\phi) = P_2(\phi) = \frac{e}{\phi}$. In this case, the regulator halts all withdrawals from good banks completely.

Proposition 8 *Under cash-in-the-market pricing, the regulator halts all redemptions from ex-post good banks. That is, she sets $x_{H*} = 0$. On the other hand, creditors are allowed to run on ex-post bad banks, and $x_{L*} = 1$.*

Finally, we investigate the impact of such restrictions on ex-ante allocations. One the one hand, restricting withdrawals improves secondary market prices of collateral, while on the other, it also leads to a tightening of the investors' participation constraint. Since such gates are only effective in stopping liquidations of good projects in the contagion state, they forcibly subvert incentive compatibility conditions, thereby exposing investors to excess risk, and leading to a reduction of ex-ante funding liquidity. To compare the effect of redemption gates on ex-ante allocations, we consider two scenarios. In the first case, banks impose no constraint, and investors liquidity both projects in the state $k = 1$, for a total outlay of $2 \cdot e/2 = e$ in the intermediate period. The participation constraint of the investor in this case is

$$(1-\lambda)\left[\theta^2 q_H D+\theta(1-\theta)e+(1-\theta)^2\frac{e}{2}\right]+\lambda\left[a^2\theta^2 q_H(1-\kappa)D+a\theta(1-a\theta)e+(1-a\theta)^2\frac{e}{2}\right]\geq 1 \quad (25)$$

Notice that the projects are continued only in the state (H, H), and liquidated otherwise. In the second case, the bank imposes a limit x^* on the investor such that in the state (H, L), only a fraction x^* of the good project is sold, and the price of collateral is $P_1(1) = \mu_1$ and the total payoff to investors in the intermediate period is $(1 + x^*)\mu_1 = e$. The participation constraint in this case is

$$(1-\lambda)\left[\theta^2 q_H D+\theta(1-\theta)e+(1-\theta)^2\frac{e}{2}\right]+\lambda\left[a^2\theta^2 q_H(1-\kappa)D+a\theta(1-a\theta)e+(1-a\theta)^2\frac{e}{2}\right]$$
$$+\underbrace{\left[(1-\lambda)\theta(1-\theta)q_H\kappa+\lambda a\theta(1-a\theta)q_H(1-\kappa)\right](D-\mu_1 x^*)}_{\text{Payment Due at } t=2 \text{ to type-}H \text{ investor when } k=1}\geq 1 \quad (26)$$

A comparison of participation constraints directly leads to the following Proposition 9.

Proposition 9 *Redemption gates expand the ex-ante allocations under the Fair Value regime when there is contagion in the intermediate period.*

6.2.1. Why Banks do not impose Gates?

A natural question is: Why can't banks impose these restrictions themselves? Under our framework of symmetric information under the FVA regime, both banks and creditors possess same information about the ex-post state of the projects. Thus, as argued in Gorton (1985), there is no role for restrictions in a private equilibrium. To see this formally, first note that the banks' privately optimal decision for ex-post bad projects coincide with that of the regulator, and they allow full creditor redemptions in the intermediate period. However, a key point of deviation appears between the social planner decision and the banks' privately optimal choice regarding the restrictions for ex-post good banks in the state (H, L). Assume that a bank wants to privately choose an optimal restriction level x_{HL}. Total bank profits is given by equation 38 in the appendix.

An individual bank takes the secondary market price P_1 as given and chooses and optimal x_{HL} to maximize total profits. If the creditors find it optimal to run on the good bank in the state (H, L), that is there is contagion in the secondary market, the term T_2, especially the one inside the bracket multiplying x_{HL} is positive and banks optimally set $x_{HL} = 1$. The regulator, on the other hand, realizes that $P_1 \cdot (1 + x_{HL}) = e$ in an illiquid market, and chooses $x_{HL} = 0$.

6.3. Regulatory Forbearance

In this section we consider the effect of regulator interventions in private debt contracts in the form of implicit or explicit bailout guarantees. In our model, at the initial date $t = 0$, before private contracts are finalized, the regulator announces forbearance rules (β_H, β_L). Under full commitment equilibrium, creditors now expect type-H (type-L) projects to be bailed out with probability β_H (β_L) at $t = 2$.

The presence of such implicit or explicit bailout guarantees has the potential to expand ex-ante allocation efficiency as the creditors now share failure risk with the regulator. Conditional on a bailout guarantee, creditors now bear bankruptcy cost in period $t = 2$ with only probability

$$(1 - \lambda_k)\big[q_S + (1 - q_S)\beta_S\big] + \lambda_k[q_S(1 - \kappa) + (1 - q_S(1 - \kappa))\beta_S]$$

when the aggregate state in the intermediate period is $\omega = k$ and the type revealed by the individual bank is $S \in \{H, L\}$. However, under indiscriminate bailouts, regulators can also introduce inefficiency into the financial system by incentivizing creditors to transfer excessive failure risk to the taxpayers by inefficiently funding worse quality projects, or sub-optimally continuing them in the intermediate period.

As an illustrative example, consider the incentives of creditors of type-L projects in the state (H, L). While such projects are never continued in a decentralized equilibrium, a high bailout threshold β_L can overturn this result. Specifically, any bailout level $\beta_L > \beta_L^*$ distorts creditor incentives to the extent where type-L projects are also continued in the intermediate period. This threshold level is given by

$$\beta_L^* = \frac{(1-\lambda_1)\mu_G + \lambda_1\mu_B - q_L R(1-\lambda_1\kappa)}{(1-q_L+\lambda_1\kappa q_L)R}$$

Any regulator intervention aimed at creating a Pareto improvement will not allow such inefficiencies associated with continuations of bad projects. Consequently, without loss of generality, we set $\beta_L = 0$ for the FVA regime.

We first consider the full commitment bailout equilibrium under the FVA regime. For brevity of space, the relevant details are relegated to appendix section 7. The total set of projects funded under the FVA regime under the regulatory bailout policy $(\beta_H, 0)$ is given in equation 41. Under HCA regime, a regulator, who does not possess superior insider information about underlying types, chooses a symmetric bailout level β. The total funding set in this case is given by equation 44 in appendix 7.

Two observations are immediate. Government bailout increases total ex-ante allocations in the financial system, since

$$\frac{\partial \theta_{FVA}}{\partial \beta_H} < 0 \qquad\qquad \frac{\partial \theta_{HCA}}{\partial \beta_H} < 0$$

More importantly, as the probability of bailouts increases $\beta_H \to 1$, creditors impose a higher financial burden on taxpayers by funding inefficient projects. This conclusion immediately obtains from the observation that the first best benchmark with no fundamental exposure risk, obtained by setting $\kappa = 0$ in expression 11 is larger than the threshold quality θ_{FVA} funded with $\beta_H = 1$.

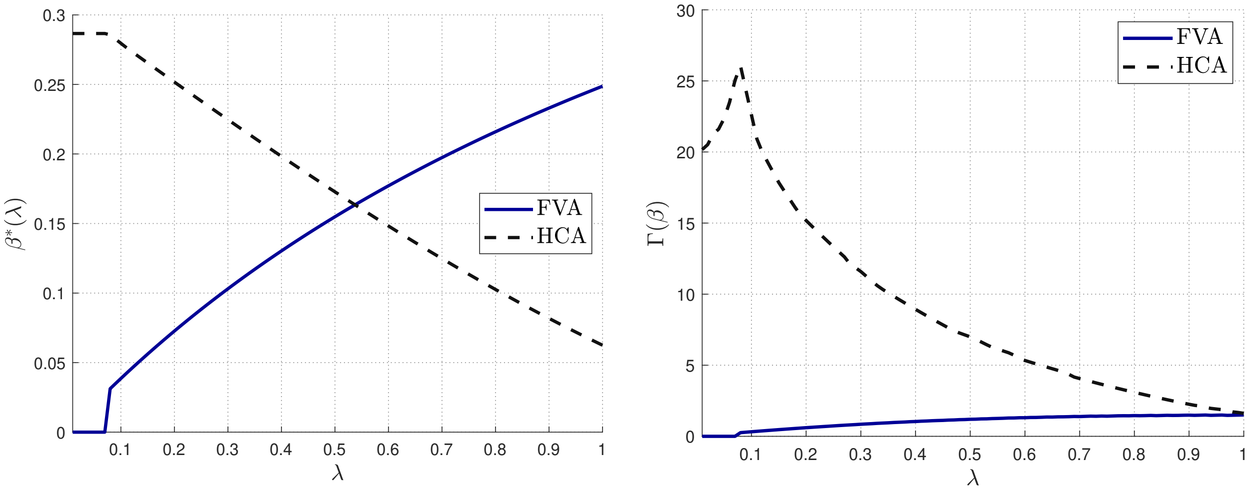
$$\underbrace{\frac{1-\bar{\mu}}{(1-\lambda+\lambda a)q_H R - (1-\lambda)\mu_G - \lambda a \mu_B}}_{FirstBest} > \underbrace{\frac{1-\bar{\mu}}{(1-\lambda+\lambda a)R - (1-\lambda)\mu_G - \lambda a \mu_B}}_{Bailout}$$

On the other hand, as $\beta \to 1$, creditors fund all projects under HCA regime since $\lim_{\beta\to 1}\theta_{HCA} = 0$. The two results jointly imply that there is an intermediate bailout level $\beta^*_{FVA}(\lambda) = (\beta^*_H(\lambda), 0)$ and $\beta^*_{HCA}(\lambda)$ which achieves first best allocation.[18]

The variation of bailout thresholds $\beta^*(\lambda)$ required to achieve first best is depicted

[18]The result immediately follows from the fact that without bailouts, the threshold is larger than the first best and the monotonicity property. A direct application of Intermediate Value Theorem delivers the desired result.

in Panel A of figure 8. The solid line gives the threshold levels for the FVA regime, while the dashed line gives the corresponding level for the HCA regime. As probability of aggregate crash increases, the threshold levels of projects financed by creditors also increase in tandem. As a result, the bailout level required to restore first best allocations also increase under the FVA regime.



(a) $\beta^*(\lambda)$ **(b)** Bailout Cost

Figure 8: Funding Sets under FVA and HCA Regime

On the other hand, the optimal bailout level decreases under HCA regime. The reason behind this apparently counter-intuitive result is as follows. As the probability of aggregate crash increases, creditors under FVA regime are hurt in two ways : (i) the probability of projects reporting low type increases; and (ii) price paid for liquidating collateral in the secondary market drops. While the first reason also plagues creditors under the HCA regime, they are nevertheless shielded from collateral value drop since there is no intermediate liquidation. As a result, the threshold θ_{FB} increases at a much faster rate with λ compared to θ_{HCA} and the level of regulator support required to achieve first best shrinks.

Let us consider the costs of institutionalizing such a bailout. Under the FVA regime, since only type-H projects are rolled over until the terminal date, the total cost to the regulator for bailing out creditors of failed projects is

$$\Gamma_{FVA}(\theta) = (1 - \lambda + \lambda a)\theta(1 - q_H)\beta D \tag{27}$$

On the other hand, under the HCA regime, projects are unconditionally continued till the terminal date. Consequently, regulators end up bailing out not only good, but also low type projects inefficiently continued by the creditors. This in turn increases total bailout costs

$$\Gamma_{HCA}(\theta) = \big[1-\lambda+\lambda a\big]\theta(1-q_H)\beta D + \underbrace{\big[(1-\lambda)(1-\theta)+\lambda(1-a\theta)\big](1-q_L)\beta D}_{\text{Bailout Cost of Low type inefficient projects}} \quad (28)$$

Finally to compute the total bailout costs, we assume that project qualities are distributed uniformly over the support $[0, 1]$. Hence, the total cost facing the regulator who chooses an optimal bailout level $\beta^*(\lambda)$ depending on the ex-ante expectation of an aggregate crash is

$$\Gamma_{FVA} = \int_{\theta_{FB}}^{1} \Gamma_{FVA}(\theta;\beta^*(\lambda))d\theta$$
$$\Gamma_{HCA} = \int_{\theta_{FB}}^{1} \Gamma_{HCA}(\theta;\beta^*(\lambda))d\theta$$

where the optimal bailout level chosen by the regulator aims to restore first best allocations under each regime. Panel B of figure 8 plots the total bailout cost for the regulator under each regime. The high level of bailout that needs to be promised to creditors to restore first best allocations under the HCA regime also implies a considerably higher bailout cost ex-post.

7. Conclusion

We investigate the effects of accounting regimes and their inherent information structures on the ex-ante allocation and bilateral contracts written between external investors and the banks. We formulate a simple stylized two-period model where banks borrow from creditors in the initial period to invest in assets, referred to as projects in the paper, which are subject to both aggregate and idiosyncratic shocks. Under the fair-value accounting (FVA) regime, the intermediate state of these assets are revealed to the creditors, who must then decide to rollover their investment, or demand payment in which case a bank run ensues. In a financial system with multiple banks, we show how information of the underlying assets in the intermediate period may induce creditors to even run on ex-post healthy banks, creating contagion-induced failures. On the other hand, under the historical-cost accounting (HCA) regime, the ex-post state of the bank assets or projects are not revealed only to the manager.

Our analysis provides insight into the debate between the FVA and HCA regimes. We show that even with contagion-induced failures, FVA is more efficient relative to the HCA regime in terms of ex-ante originations. That is creditors fund more projects ex-ante under the FVA. Lack of information about the ex-post states of the banks forces creditors to continue projects inefficiently, even when it is optimal to liquidate them at the intermediate date. This in turn restricts the creditors' participation constraint and reduces the total funding set ex-ante. Our results continue to hold even when secondary markets for assets are illiquid, and prices deviate from their fundamental values under the FVA regime.

We also consider an extension of our model, where we allow for accounting-regime dependent belief dispersions. In particular, we assume that when creditors observe large number of bank failures, their beliefs about future success probabilities of ex-post healthy projects worsen considerably. Such extreme "panic"-driven liquidations can also render the FVA regime inefficient relative to the HCA.

We next investigate how ex-post interventions designed to curb excessive liquidations, like redemption restrictions or implicit government bailout guarantees, interact with the accounting regimes to affect ex-ante allocations. We show that optimally designed interventions like redemption gates, where the regulator places state-dependent restrictions on ex-post good banks, while allowing full withdrawal of the bad banks, can improve ex-ante allocations under the fair-value regime by forcing creditors to partially internalize the liquidation costs and reducing ex-ante probabilities of destabilizing negative price spirals. Similarly, governments and regulatory authorities can increase efficiency by partially indemnifying creditors against losses. We show that ex-ante allocations expand under either regime in an equilibrium with government guarantees. However, such taxpayer-funded bailouts may prove to be excessively costly as banks and investors may anticipate ex-post bailouts and transfer the entire failure risk to the government and taxpayers. Such ex-ante analysis is not only appealing from an academic perspective, but are also important from practical policy design angle.

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Appendix

Proof of Proposition 1

We first show that the total allocation under the FVA regime is greater compared to that under HCA regime. The total set of projects funded under the FVA regime is given by equation 5 while that under the HCA regime is given by 6. A direct comparison of the thresholds yields the following result

$$\theta_{FVA} \leq \theta_{HCA} \iff \max\left[0, \frac{1-\bar{\mu}}{q_H R - \bar{\mu}}\right] \leq \frac{\max(1,\bar{\mu}) - q_L R}{q_H R - q_L R}$$

First consider the case where $\bar{\mu} > 1$. In this case, all projects are funded under FVA regime since $(1-\bar{\mu}) < 0$. The lower bound of the projects funded is strictly greater than 0 given the assumption $\bar{\mu} < q_H R$. Let us consider the case where $\bar{\mu} < 1$. In this case FVA funds more projects compared to HCA if and only if

$$q_L R \leq \bar{\mu}$$

which is always true from assumption about collateral values $\bar{\mu} \geq \mu_B \geq q_L R$.

Second, we compare the net present value of the threshold project financed under the fair value regime. Since only those projects which reveal itself to be high type are rolled over by the creditors in the intermediate period, the total expected payoff which accrues to the bank is

$$\theta q_H (R - D)$$

where $(R-D)$ is the residual payoff to the bank after all creditors are paid, and θq_H gives the unconditional probability of success. Individual rationality condition for creditors also imply

$$\theta q_H D + (1-\theta)\bar{\mu} = 1$$

where all creditors break even in competitive markets. The NPV of the threshold project is then

$$NPV = \theta q_H R + (1-\theta)\bar{\mu} - 1 = 0$$

where we substituted the debt level from the IR expression and the final equality obtains after plugging in the value of the threshold quality θ_{FVA}.

Finally, let us compare the debt levels under the two accounting regimes. This is akin to comparing the participation constraints for creditors under the two regimes for the same quality level θ. The participation constraint for the FVA regime is given above, while that for the HCA regime is

$$(1-\theta)q_H D_{HCA} + \theta q_L D_{HCA} = 1$$

The final result obtains from a direct comparison of the accounting regimes and the fact that $\bar{\mu} > q_L R \geq q_L D$.

■

Proof of Lemma 1

An investor with prior λ about the underlying state observes the number k of banks reporting a low state L and $n-k$ banks reporting a high state H in the intermediate period. Denote the updated prior of the underlying state being bad by λ_k. Since conditional on the good (bad) state, banks report an underlying low state with probability $(1-\theta)$ (respectively $(1-a\theta)$ with $a<1$), the updated belief obtains from Bayes' rule as

$$\lambda_k = \frac{\lambda(a\theta)^{n-k}(1-a\theta)^k}{(1-\lambda)\theta^{n-k}(1-\theta)^k + \lambda(a\theta)^{n-k}(1-a\theta)^k} \tag{29}$$

To show that the posterior is increasing in the number of failures, we consider the x case with $(k+1)$ failures and write

$$\lambda_{k+1} = \frac{1}{1 + \frac{1-\lambda}{\lambda}a^{k-n}\left(\frac{1-\theta}{1-a\theta}\right)^k \left[a\left(\frac{1-\theta}{1-a\theta}\right)\right]}$$

By removing the term in brackets from the denominator, one obtains the expression for λ_k. Note that the term inside brackets is always smaller than one for $a<1$. Thus, the denominator for λ_{k+1} is smaller than that for λ_k. ■

Proof of Lemma 3

The proof proceeds in two steps. First, we show that the threshold exposure limit $\kappa^*(\lambda)$ cannot be smaller than $\kappa_0 = (1-\frac{\mu_B}{\mu_G})$. It can be shown by direct algebra that debt thresholds satisfy the relationship $D_0 \geq D_1$ whenever the exposure is smaller than κ_0. Recall that D_0 is the minimum promised payment required to align creditor incentives with that of the bank, and is given by

$$D_0 \equiv \frac{p_0}{q_H(\kappa)} = \frac{(1-\lambda)\mu_G + \lambda a^2 \mu_B}{(1-\lambda)q_H + \lambda a^2(1-\kappa)q_H}$$

When project exposure is bounded above by κ_0, aggregate uncertainty does not induce contagion in the financial markets. To see this, we take the limit as $\lambda \to 1$

$$\lim_{\lambda \to 1} D_0 = \frac{\mu_B}{(1-\kappa)q_H} \leq \frac{\mu_G}{q_H} < R$$

This implies that banks can simply ensure continuation in the state (H, H) by promising a payment $D \in (D_0, R)$. Since $D_1 < D_0$ in this region, any debt $D \geq D_0$ also immediately ensures continuation of good projects when the other bank reports a type-L.

When $\kappa > \kappa_0$, we have $D_0 < D_1$ and banks must pay higher debt $D > D_1$ to incentive continuation of good projects when other bank report a low type-L. The threshold D_1 is given by

$$D_1 = \frac{(1-\lambda)\theta(1-\theta)\mu_G + \lambda a\theta(1-a\theta)\mu_B}{(1-\lambda)\theta(1-\theta)q_H + \lambda a\theta(1-a\theta)(1-\kappa)q_H}$$

Now,

$$\lim_{\lambda \to 1} D_1 = \frac{\mu_B}{(1-\kappa)q_H}$$

If the exposure of projects to aggregate uncertainty is smaller than $(1 - \frac{\mu_B}{q_H R})$, both thresholds D_0 and D_1 are smaller than R, and any debt $D \in (D_1, R)$ ensures unconditional continuation of good projects in the intermediate period.

When the exposure of projects to the aggregate state is greater than $\left(1 - \frac{\mu_B}{q_H R}\right)$, sufficiency condition for contagion in intermediate period is given by a threshold $\lambda^*(\kappa)$ such that contagion happens if and only if $\lambda > \lambda^*(\kappa)$. This threshold solves the equation

$$D_1(\kappa, \lambda^*(\kappa)) = R \tag{30}$$

As an illustrative example, we consider the case when $\kappa \to 1$. The debt threshold D_1 is

$$D_1(1, \lambda^*(1)) = \frac{\mu_G}{q_H} + \frac{\lambda}{1-\lambda}\frac{a(1-a\theta)}{1-\theta}\frac{\mu_B}{q_H}$$

As aggregate uncertainty vanishes, the threshold limit $D_1 = \frac{\mu_G}{\kappa_H} < R$ and there is no contagion in the financial markets. On the other hand, as $\lambda \to 1$, the threshold limit increases beyond all bounds. Then by intermediate value theorem, there exists a unique threshold $\lambda^*(\kappa)$ such that $D_1(\lambda^*(\kappa)) = R$.

Finally, differentiating equation 30 with respect to κ we obtain

$$\frac{d}{d\kappa}\lambda^*(\kappa) = -\frac{\frac{dD_1}{\kappa}}{\frac{dD_1}{\lambda}}$$

It can be shown simply by direct differentiation that D_1 is increasing in both κ and λ, implying that the threshold uncertainty $\lambda^*(\kappa)$ is decreasing in κ. Intuitively, as exposures of individual projects to aggregate state increases, the unconditional probability required to induce contagion in the financial markets decrease. ■

Proof of Proposition 2

The proof proceeds in two steps. We first compare the allocation set under no-contagion restriction, and then move to the contagion case.

First let us consider the FVA regime. When project exposures to aggregate uncertainty is smaller than the threshold value required to induce contagion in the financial markets, type-H projects are continued in the intermediate period irrespective of the type reported by the other bank; while bad projects are always liquidated. In this case, any debt contract with a promised value

$$D \geq \max\left[D_1, \frac{1-(1-\lambda)(1-\theta)\mu_G - \lambda(1-a\theta)\mu_B}{(1-\lambda+a\lambda(1-\kappa))\theta q_H}\right]$$

satisfies both individual rationality and incentive constraints. The final constraint that the promised debt must satisfy the limited liability of banks, $D \leq R$ yields the inequality

$$\left[(1-\lambda+\lambda a(1-\kappa))q_H R - (1-\lambda)\mu_G - \lambda a \mu_B\right]\theta + (1-\lambda)\mu_G + \lambda\mu_B - 1 \geq 0 \qquad (31)$$

Next, under the HCA regime, the promised debt which satisfies the individual rationality and incentive compatibility constraints of the creditors must lie in the region

$$D \in \left[\max\left(\bar{\mu}, \frac{1}{(1-\lambda)[\theta q_H + (1-\theta)q_L] + \lambda[a\theta(1-\kappa)q_H + (1-a\theta)(1-\kappa)q_L]}\right), R\right]$$

Using an analogous approach as the FVA regime, the key inequality which every bank must satisfy to receive ex-ante financing is

$$(1-\lambda+\lambda a(1-\kappa))(q_H - q_L)R\theta + (1-\lambda\kappa)q_L R - 1 \geq 0 \qquad (32)$$

Since the coefficient multiplying θ is positive, inequality 32 only binds at $\theta = \theta_{HCA}$, and will be strictly positive for all larger values of θ. To compare the two thresholds, we combine inequalities 31 and 32 together to write the left hand side as

$$(1-\lambda+\lambda a(1-\kappa))(q_H - q_L)R\theta + (1-\lambda\kappa)q_L R - 1 + f(\theta)$$
$$f(\theta) = \left[(1-\lambda+\lambda a(1-\kappa))q_L R - (1-\lambda)\mu_G - \lambda a\mu_B\right]\theta + (1-\lambda)\mu_G + \lambda\mu_B - (1-\lambda\kappa)q_L R$$

The first term expresses the first inequality 31 in terms of the second 32 and an additional component $f(\theta)$. To prove that FVA funds more investments compared to HCA when there is no intermediate contagion, it suffices to show that $f(\theta_{HCA}) > 0$. Two observations are enough to prove this claim.

$$f(0) = (1-\lambda)\mu_G + \lambda\mu_B - (1-\lambda\kappa)q_L R > \mu_B - q_L R > 0$$
$$f(1) = \lambda(1-a)[\mu_B - (1-\kappa)q_L R] > 0$$

The function is linearly decreasing in θ but it remains positive at the two bounds $\{0, 1\}$. Thus, it must be positive for all the intermediate values, and θ_{HCA} in particular. This concludes the first part of the proof.

Next we consider the case where there is contagion under FVA regime in the intermediate period. We again focus on the FVA regime first. If project exposures cross the threshold level, optimal continuation or liquidation decisions pertaining to type-H projects depend on the state reported by the other bank. If the other bank reports a type-H, the good project is continued; while it is liquidated if the other bank reports a type-L. In this case, the optimal debt which respects creditor incentives and banks' limited liability constraints lies in the region

$$\left[\max\left(\frac{1-(1-\lambda)(1-\theta^2)\mu_G - \lambda(1-a^2\theta^2)\mu_B}{(1-\lambda+\lambda a^2(1-\kappa))q_H\theta^2}, D_0\right), R\right]$$

Analogous approach as used in the proof of proposition 2 yields the inequality

$$\left[(1-\lambda+\lambda a^2(1-\kappa))q_H R - (1-\lambda)\mu_G - \lambda a^2\mu_B\right]\theta^2 + (1-\lambda)\mu_G + \lambda\mu_B - 1 \geq 0 \quad (33)$$

Let $\gamma_1 = (1-\lambda+\lambda a^2(1-\kappa))q_H R - (1-\lambda)\mu_G - \lambda a^2\mu_B$ and $\gamma_2 = (1-\lambda+\lambda a(1-\kappa))q_H R - (1-\lambda)\mu_G - \lambda a\mu_B$. When $\kappa > \kappa_*$ and there is contagion in financial markets, direct computations show $\gamma_1 > \gamma_2$. The threshold $D_0 \leq R$ implies that the coefficient $\gamma_1 > 0$. Now, using this condition, we can write

$$\gamma_1\theta^2 + (1-\lambda)\mu_G + \lambda\mu_B - 1 > \gamma_2\theta^2 + (1-\lambda)\mu_G + \lambda\mu_B - 1 \quad (34)$$

Let θ_{FVA} and θ_* respectively be the roots of the expressions on the left and right. It necessarily follows from the inequality 34, that $\theta_{FVA} < \theta_*$. Thus to complete the proof, we simply need to show that $\theta_* < \theta_{HCA}$, which we proceed to do next. To do this, we again follow an approach analogous to the one used in the first part of the proof.

We begin by recognizing that the root θ_* of the right hand side of the inequality 34 is smaller than the root θ_{**} of the following equation

$$\gamma_3\theta^2 + (1-\lambda+\lambda a(1-\kappa))q_L R + \lambda(1-a)\mu_B - 1 \geq 0 \quad (35)$$

where $\gamma_3 = (1-\lambda+\lambda a(1-\kappa))(q_H - q_L)R$. This follows directly from that we can express equation 35 in terms of the right hand side of inequality 34 as

$$\gamma_3\theta^2 + (1-\lambda+\lambda a(1-\kappa))q_L R + \lambda(1-a)\mu_B - 1 = (\gamma_2+\Delta)\theta^2 + (1-\lambda)\mu_G + \lambda\mu_B - \Delta - 1$$

where $\Delta = (1-\lambda)(\mu_G - q_L R) + \lambda a(\mu_B - (1-\kappa)q_L R) > 0$. A direct comparison of the roots proves the claim made above.[19]

The final step requires a comparison of θ_{**} and θ_{HCA}. To do this, we subtract 32 from 35 to obtain

$$\gamma_3\theta^2 - \gamma_3\theta - \lambda(1-a)(\mu_B - (1-\kappa)q_L R)$$

A positive value of the above expression implies $\theta_{**} < \theta_{HCA}$. It is straight-forward to show that the smaller root is negative while the larger root is greater than 1, and in the intermediate region, the expression is positive. Thus for all $\theta \in [0,1]$, the above expression is positive. Thus,

$$\theta_{FVA} < \theta_* < \theta_{**} < \theta_{HCA}$$

The pieces together yield the complete result. ■

Proof of Proposition 3

The proof adopts an approach analogous to the one used in proposition 2. Under illiquid markets, the threshold quality of project which receives funding ex-ante is given by the inequality

$$\left[(1-\lambda+\lambda a(1-\kappa))q_H R - \frac{(1-\lambda)\mu_G + \lambda a\mu_B}{r_f}\right]\theta + \frac{(1-\lambda)\mu_G + \lambda\mu_B}{r_f} - 1 \geq 0 \qquad (36)$$

in the no-contagion region. We re-write this equation as

$$(1-\lambda+\lambda a(1-\kappa))(q_H - q_L)R\theta + (1-\lambda\kappa)q_L R - 1 + f(\theta) \geq 0 \qquad (37)$$

where

$$f(\theta) = (1-\lambda)(1-\theta)\left(\frac{\mu_G}{r_f} - q_L R\right) + \lambda(1-a\theta)\left(\frac{\mu_B}{r_f} - (1-\kappa)q_L R\right)$$

[19] A short heuristic proof is as follows. Assume θ_* and θ_{**} are of the form

$$\theta_* = \sqrt{\frac{p}{q}}, \theta_{**} = \sqrt{\frac{p+\Delta}{q+\Delta}}$$

where p and q are real numbers such that $0 < p < q$. This directly implies $\theta_* < \theta_{**}$ as long as $p < q$ and $\Delta > 0$, which are both true in our case.

Note that when $\theta = \theta_{HCA}$, the left hand side of equation 37 reduces to $f(\theta)$. Thus, to show that the threshold project quality which gets funded under the FVA regime is smaller than that under HCA regime, it suffices to show that $f(\theta_{HCA}) > 0$.

Now consider the investors' incentives when the intermediate state is revealed to be (L, L). In this state, the project is liquidated if and only if the total payoff from liquidation μ_f/r_f exceeds the maximum possible payoff from continuation $(1 - \lambda_2 + \lambda_2(1-\kappa))q_L R$. This leads to the following inequality, after some rearranging

$$(1-\lambda)(1-\theta)^2\left(\frac{\mu_G}{r_f} - q_L R\right) + \lambda(1-a\theta)^2\left(\frac{\mu_B}{r_f} - (1-\kappa)q_L R\right) \geq 0$$

Since $(1-\theta_{HCA}) \leq 1$ and $(1-a\theta_{HCA}) \leq 1$, the above inequality also implies

$$(1-\lambda)(1-\theta_{HCA})\left(\frac{\mu_G}{r_f} - q_L R\right) + \lambda(1-a\theta_{HCA})\left(\frac{\mu_B}{r_f} - (1-\kappa)q_L R\right) \geq 0$$

The left hand side of the above inequality is $f(\theta_{HCA})$, thus completing our proof. The case of contagion follows an analogous approach. ■

Proof of Proposition 4

The lower bound of the funding set under FVA regime and pessimistic creditor beliefs is given in 5 while the lower bound of the funding set under HCA regime is given in equation 15. When the probability of aggregate crash $\lambda \to 0$, the fact that $\theta_{HCA} > \theta_{FVA}$ follows directly from proposition 2. Consider the converse where probability $\lambda \to 1$. In this case, the lower bound of the funding set under HCA regime is

$$\theta_{HCA}(\lambda = 1) = \frac{1 - q_L R}{a(q_H R - q_L R)}$$

while that under the FVA regime is

$$\theta_{FVA}(\lambda = 1) = \sqrt{\frac{1-\mu_B}{a^2(q_H R - \mu_B)}}$$

Direct comparison of the two thresholds yields, after straightforward but tedious algebra, the desired result that $\theta_{FVA} > \theta_{HCA}$. The final step requires the monotonicity of the difference between two thresholds which can be shown by direct differentiation. ■

Proof of Proposition 6

The bank manager chooses the optimal (x_{LH}, x_{LL}) to maximize profit function 22 subject to the constraint that investors break even in equilibrium. Substituting the value of debt from 21 into the bank profit function yields the following optimization problem

$$\max_{x_{LH}\leq 1, x_{LL}\leq 1} \big[(1-\lambda)\theta(1-\theta)(P_1(1)-q_LR)+\lambda a\theta(1-a\theta)(P_1(1)-(1-\kappa)q_LR)\big]x_{LH}$$
$$+\big[\lambda(1-\theta)^2(P_2(2)-q_LR)+\lambda(1-a\theta)^2(P_2(2)-(1-\kappa)q_LR)\big]x_{LL}$$

where we have omitted the terms not involving x for clarity of exposition. Now the term multiplying x_{LL} (respectively x_{LH}) is positive. To see this, consider the incentive compatibility condition for the investors in state (L, L) (respectively (L, H)). Investors have incentives to liquidate the project if and only if the expected payoffs from current liquidation exceeds that from continuation. In other words,

$$P_2(2) \geq (1-\lambda_2)q_LR + \lambda_2 q_L(1-\kappa)R$$

Plugging in the value of λ_2 from equation 7 and rearranging gives exactly the result desired. The linearity of the optimization implies directly that the optimal choices for x_{LH} and x_{LL} bind at the maximum allowable limit of 1. ■

Proof of Proposition 7

Rewrite the bank profit function 24 by plugging in the value of D from equation 23 as

$$\begin{aligned}\Pi_B &= (1-\lambda+\lambda a(1-\kappa))\theta q_HR + \big[(1-\lambda)(1-\theta)^2+\lambda(1-a\theta)^2\big]P_2(2)\\ &+ \underbrace{\big[(1-\lambda)\theta(1-\theta)+\lambda a\theta(1-a\theta)\big]P_1(\phi)}_{T_1} - 1\\ &+ \underbrace{\Big\{\Big[(1-\lambda)\theta(1-\theta)+\lambda a\theta(1-a\theta)\Big]P_1(\phi) - \Big[(1-\lambda)\theta(1-\theta)+\lambda a\theta(1-a\theta)(1-\kappa)\Big]q_HR\Big\}x_{HL}}_{T_2}\end{aligned} \tag{38}$$

The bank chooses x_{HL} to maximize profits subject to the secondary market price process

$$P_1(\phi) = \begin{cases} \mu_1 & \text{if } x_{HL} \leq x^* \\ \frac{e}{\phi} & \text{if } x_{HL} > x^* \end{cases}$$

Consider the two regions separately. We first solve for the optimal redemption limit when $x \leq x^*$, and the collateral market clears at fundamental price $P_1(\phi) = \mu_1$. As long as the investor incentive compatibility condition impel the investors to liquidate the good project in the intermediate date, the term in braces multiplying x_{HL} is positive (see the proof of proposition 6 for a similar argument laid out in greater detail), and the optimal solution binds at the top corner. That is, either the banks set $x_{HL} = x^*$ and the market clears at the fundamental price $P_1(1 + x^*) = \mu_1$.

Next consider the other case where $P_1(\phi) = e/\phi$. Note that in this region there is cash-in-the-market pricing, and $(1 + x_{HL}) \cdot P_1(\phi) = e$. Substituting in the bank profit function yields

$$\begin{aligned}\Pi_B = &(1-\lambda+\lambda a(1-\kappa))\theta q_H R + \left[(1-\lambda)(1-\theta)^2 + \lambda(1-a\theta)^2\right] P_2(2)\\ &+ \left[(1-\lambda)\theta(1-\theta) + \lambda a\theta(1-a\theta)\right] e - 1\\ &- \left[(1-\lambda)\theta(1-\theta) + \lambda a\theta(1-a\theta)(1-\kappa)\right] q_H R \cdot x_{HL}\end{aligned}$$

The solution to the optimization problem for the bank in this region ($x_{HL} > x^*$) is the lower corner $x = x^*$ since the term involving x_{HL} is now negative and the bank chooses the lowest permissible value of x. ■

Equilibrium with Bailouts

Consider a regulatory intervention of the form (β_H, β_L) where type-H (respectively type-L) projects are bailed out with a probability β_H (resp. β_L). Regulatory interventions change the probabilities which creditors to the final payment of debt D in the terminal period $t = 2$. We will define the following quantities

$$\begin{aligned}\tilde{q}_H &= q_H + (1-q_H)\beta_H\\ \hat{q}_H &= (1-\kappa_D)q_H + (1-(1-\kappa_D)q_H)\beta_H\\ \tilde{q}_L &= q_L + (1-q_L)\beta_L\end{aligned}$$

The first quantity reflects the total probability with which creditors expect to get repaid the face value of debt D conditional on the aggregate state being good, while the second expression is the expected payoff probability conditional on an aggregate crash.

Let us consider the FVA regime first. As argued in the main text, any Pareto improving intervention will necessarily set $\beta_L = 0$ to avoid inefficient continuations of type-L projects in the intermediate period. Under this restriction, creditor rationality conditions can be written as

$$\begin{aligned}&(1-\lambda)\left[\theta\tilde{q}_H D + \theta(1-\theta)p_1 + (1-\theta)^2 p_2\right] +\\ &\lambda\left[a^2\theta^2\tilde{q}_H D + a\theta(1-a\theta)\hat{q}_H D + a\theta(1-a\theta)p_1 + (1-a\theta)^2 p_2\right] \geq 1\end{aligned} \tag{39}$$

The optimal debt level which satisfies creditor rationality and banks' limited liability must lie in the interval

$$D \in \left[\frac{1-(1-\theta)(1-\lambda)\mu_G - \lambda(1-a\theta)\mu_B}{(1-\lambda+\lambda a)\theta\tilde{q}_H - \lambda a\theta(1-a\theta)\kappa_D q_H(1-\beta_H)}, R\right] \tag{40}$$

The constraint on the debt level again imposes a restriction on the set of projects which receive financing in equilibrium. Aggregate investment under full commitment equilibrium is given by the threshold quality θ_{FVA} where

$$\theta_{FVA} = \frac{-\chi + \sqrt{\chi^2 + 4(1-\bar{\mu})\lambda a^2 \kappa_D q_H (1-\beta_H) R)}}{2\lambda a^2 \kappa_D q_H (1-\beta_H) R} \tag{41}$$

$$\chi = (1-\lambda+\lambda a)\tilde{q}_H R - \lambda a \kappa_D q_H (1-\beta_H) R - (1-\lambda)\mu_G - \lambda a \mu_B \tag{42}$$

On the other hand, under the HCA regime, creditors expect to recover the face value of debt for good type-H projects with probability $\tilde{q}_H$ and type-L projects with probability $\tilde{q}_L$ respectively. Under competitive markets, they are thus willing to fund projects which provide them with an expected payoff of 1 in the terminal period. Individual rationality under HCA regime constrains the optimal debt level to the region

$$D \in \left[\frac{\max(1, \bar{\mu})}{(1-\lambda+\lambda a)(\tilde{q}_H - \tilde{q}_L)\theta + \tilde{q}_L}, R\right] \tag{43}$$

Under HCA, we will look for a symmetric equilibrium under full commitment. In this case the regulator sets the two bailout probabilities $\beta_H = \beta_L = \beta$. Aggregate funding set under the HCA regime is given by $[\theta_{HCA}, 1]$ where the lower bound is

$$\theta_{HCA} = \frac{1 - \tilde{q}_L R}{(1-\lambda+\lambda a)(1-\beta)(q_H - q_L)R} \tag{44}$$