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Abstract

The choice of an auction mechanism influences which object characteristics bidders learn about and whether the object is allocated efficiently. Some object characteristics are valued equally by all bidders and thus are inconsequential for the efficient allocation. Others matter only to certain bidders, and thus determine the bidder with the highest object value. I show that the efficient auction is the second-price auction: it induces bidders to learn exclusively about object characteristics which matter only to them. An independent private value framework arises endogenously.

JEL Classification: D44, D83

Keywords: Information Choice, endogeneous interdependence, multidimensional auctions, interdependent values, accuracy

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Information Choice in Auctions*

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December 2021

The choice of an auction mechanism influences which object characteristics bidders learn about and whether the object is allocated efficiently. Some object characteristics are valued equally by all bidders and thus are inconsequential for the efficient allocation. Others matter only to certain bidders, and thus determine the bidder with the highest object value. I show that the efficient auction is the second-price auction: it induces bidders to learn exclusively about object characteristics which matter only to them. An independent private value framework arises endogenously.

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1 Introduction

Preparing how to bid in an auction usually involves evaluating multiple characteristics of the object. This paper explores which object characteristics bidders gather information about in cases in which they can learn about their object value before bidding. These issues are relevant to, for example, corporate takeovers, in which acquiring firms have access to a variety of information about a target company. This information includes the company's R&D activities and its stand-alone value. A reasonable assumption is that firms cannot perfectly process or uncover all existing information, and are thus driven to select elements to focus on before the bidding takes place. Should an acquiring firm conduct research on characteristics that are specific to them, such as their R&D

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synergies with the target? Or should they focus on factors that also matter to other acquiring firms, such as the stand-alone value?

Another example is resource rights auctions for oil fields or timber. Each bidder has access to the same uncertain volume of oil or timber when winning the auction. Bidders may incur different costs in extracting the resources from a site because of the use of different drilling or logging technologies. Does a bidder prefer to perform exploratory drilling to learn about oil volume, or to learn about the costs of extracting the resource through estimating the drilling costs specific to him, or some combination?

Allocative efficiency is of central importance in auction theory. Some characteristics are relevant only to individual bidders (henceforth, referred to as a *private component*), and they determine the efficient allocation. Others are relevant to all bidders (henceforth, a *common component*) but irrelevant for the efficient allocation. Different auction mechanisms might give rise to different incentives to bidders regarding which component to acquire information about. In this paper, I find the auction mechanism which gives rise to *efficient* information acquisition.

As a simple example, consider two bidders who compete for one object. Each bidder's valuation is the sum of a common component S and his private component T_i, all drawn independently and uniformly on [0,1]. Assume that (i) bidders can perfectly learn either their own private or the common component, but not both, and (ii) the bidder with the highest expected value wins the auction. If there exists an auction that incentivizes the bidders to learn about their private component, then the object is allocated efficiently with probability one. If an auction incentivizes bidders to learn only about their common component, the object is allocated to the lower-value bidder with probability one-half, achieving the same allocative efficiency as a random lottery.

The contribution of this paper is to find an auction format that leads to efficient information acquisition. This format turns out to be the second-price auction (SPA). The novel contribution lies in its illumination of *which* component of the object bidders seek to learn. This is in contrast to much of the literature on auctions and mechanism design, where the focus is on *how much* costly information bidders seek to acquire about one given component. By restricting the ability of bidders to learn about several components at once, I study which value setting arises endogenously: an independent private values (IPV) framework if bidders learn exclusively about their independent private components, a pure common value (CV) framework if bidders learn exclusively about the common component, or some other interdependent value framework.[1]

The technical contribution of this paper is to develop a framework for comparing experiments with varying degrees of informativeness about a multidimensional random variable, consisting of the components and the total value of an object. To do this, I use the statistical concept of *accuracy* from Lehmann (1988) to rank signals in terms of their accuracy about several components. In addition, I derive a general first-order-stochastic-dominance result for first- and second-order statistics of two random variables as I vary their correlation, that could prove useful in a variety of other settings.

Consider two bidders who compete for a single indivisible object. The valuation of each bidder is increasing and additive in two components: a common component (e.g., the stand-alone value of a firm) and an independent private value component (e.g., match-specific R&D synergies). As a benchmark, each bidder can costlessly learn either about the common or his private component, but not both.[2] Information choice is simultaneous and covert.

Information plays a dual role. Beyond containing information about the object, it is also informative about the signal of the opponent and hence, his bid. A rational bidder conditions his estimate of the object value not only on his own information but also on what he learns from the event of winning. In my model, the extent of the winner's curse and the interdependence between the bids are both endogenous and depend on the information choice. The signals of bidders become more interdependent if they learn about the common component with a higher probability; the winner's curse is exacerbated. If a bidder learns only about his private component, then his information is independent of the other bidder's value—there is no winner's curse.

As a first benchmark, both the common and private-component experiments are equally accurate about a bidder's total value. As I show, this implies that they both have the same marginal distribution (up to a relabeling) and lead to the same posterior distribution of object value. This observation allows me to focus on the strategic effect of learning about the two components, and not the intertwined effect when bidders favor the more informative signal.[3]

[1] The IPV setting and the interdependent values setting lead to different theoretical predictions and vary significantly in their implications for auction design and policy. The literature on auctions typically assumes either one or the other at the outset of the analysis.

[2] This assumption is purely for expositional clarity. The main results still hold if experiments are informative about both components simultaneously, as I show in section 6.

[3] In a posted price mechanism, a single bidder is indifferent between the signals if they are equally accurate about the object's value.

The first main result is that in an SPA, there exists a unique symmetric equilibrium in which bidders learn only about their private component. No resources are wasted by learning about the common component which is irrelevant for efficiency, and the object is allocated to the bidder with the highest estimate of his private component. The intuition is that for any candidate equilibrium in which both bidders learn about the common component, there exists a simple profitable deviation which decreases interdependence (by learning about the private component). This deviation uses the fact that both signals are equally accurate and hence have the same marginal distribution: instead of learning about the common component, a bidder deviates to learning about his private component but continues to use the same bidding function as in the candidate equilibrium (up to relabeling). This deviation strategy does not change the marginal distribution of bids, but makes the bid distributions independent.

A bidder's overall winning probability is not affected by this deviation; the probability that one of two identically and independently distributed random variables (bids) is higher than the other is one half — the same as in any symmetric candidate equilibrium. How does the expected payment change with this deviation strategy? Conditional on winning in the SPA with the same marginal bidding functions, a bidder pays the second-order statistic of the two bids. How does the distribution of the second-order statistic vary with more or less correlation? At the extreme, under perfect correlation, the winning and losing bids coincide and payment is the same as the winning bid. It is intuitive that as correlation decreases, the losing bid becomes more detached from the winning bid, and thus, expected payment decreases. I formalize this intuition as a general statistical property: the distribution of the second-order statistic of two correlated random variables (the distribution of the losing bid in the candidate equilibrium) dominates the distribution of the second-order statistic of two independent random variables (the distribution of the losing bid with the deviation) in terms of first-order stochastic dominance.[4] By decreasing interdependence in the deviation, the distribution of the second-order statistic puts more weight on lower bids, and expected payment decreases.

In addition, the expected value of the object conditional on winning coincides in the candidate equilibrium and the deviation. In contrast to the candidate equilibrium, in the deviation the bidder is more likely to win when his private component is high, and less likely to win when the common component is high. However, due to signals being equally accurate and bidders using the same marginal bidding functions, the deviating

[4]This holds as long as these random variables have the same marginal distributions. The opposite is true for the distribution of the first-order statistic.

bidder's gain in value from his private component can be shown to exactly equal his loss from winning with a lower common component. Hence, the deviation does not change a bidder's expected object value when winning, but only its component composition.

The above argument relies on both bidders' experiments about their two components having the same accuracy about the object's value. One initial guess might be that if bidders already prefer the private-component signal under equal accuracy, then bidders should continue to prefer it if it is even more accurate about the total value. However, I find two drawbacks to using this accuracy notion: a signal that is more accurate about the private component need not be more accurate about the total value too. This gives rise to unintuitive properties. For example, no signal might exist which is more accurate about the value than a fully uninformative signal, as I show in example 3. In addition, a signal with higher accuracy about the value overall need not be better for a bidder. This is because even if the bidder knows his value perfectly, his signal contains additional information about the opponent's bid. Therefore, the two-dimensional learning problem in this paper cannot be reduced to a mathematical problem for which Lehmann (1988) showed that a decision maker is better off with higher accuracy signal.

I circumvent these novel difficulties which arise in a two-dimensional component setting by developing a two-step procedure that relies on some intermediate experiment. Specifically, in any symmetric equilibrium of the SPA, bidders learn only about the private component if (i) the private-component signal is more accurate *about the private component* than some intermediate private-component signal, and (ii) this intermediate signal is as accurate *about the total value* as the common-component signal. The intermediate experiment establishes a point of reference, so I can apply the main result discussed above. Crucially, I compare the intermediate and the private-component experiment in terms of their accuracy about the private component, rather than about the overall object value. As Lehmann (1988) showed, many experiments can be ranked by this, since accuracy is a more complete order than the Blackwell order.

Not every auction format has the same efficient information-choice incentives as the SPA. I allow bidders to also learn about each other's private component and show that if there exists an IPV equilibrium in the first-price auction (FPA), then there also exists an IPV equilibrium in the SPA, but not vice versa. Furthermore, I allow bidders to choose both the component they wish to learn about and the accuracy of their component. Under some additional assumptions on the costs of information, I show that any equilibrium in the SPA is more efficient than any equilibrium in the FPA. In the FPA, bidders waste more resources on the common component and never learn more accurately about the private component than they do in the SPA.

Related literature. In the classic literature in auction theory, the distribution of bidders' private information is exogenous and does not depend on the auction format. The literature on information acquisition in auctions[5] endogenizes the private information of bidders, by asking *how much* costly information they seek to acquire about a one-dimensional, payoff-relevant variable.

In an IPV framework, Hausch and Li (1991) and Stegeman (1996) show that bidders' incentives to acquire information coincide in an FPA and an SPA. For a pure CV framework, Matthews (1977) finds a particular condition on private signals when bidders acquire the same amount of information about their common value in the SPA and the FPA.[6] The closest paper to mine is Persico (2000). In his the affiliated-value model, bidders choose the accuracy (Lehmann, 1988) of their signal about a one-dimensional random variable. Persico (2000) shows that bidders acquire more information in the FPA than in the SPA. In contrast, bidders in my model choose which component to learn about, and the accuracy is fixed. My framework provides an absolute prediction that can be ranked in terms of allocative efficiency: *which* component do bidders learn about? In section 6, I look at the combined problem of how much to learn (the focus in Persico (2000)) and which component to learn about (my focus).

One other related paper is Gleyze and Pernoud (2021). Each agent's valuation is private, and agents acquire costly information about their own and the others' values. They find that generically, an IPV framework does not arise, and bidders learn about others' values unless the mechanism is dictatorial (which the SPA is not). It might seem as if this result and my result (that an SPA leads to IPV) provide opposing predictions. However, this is due to a crucial modelling difference: in Gleyze and Pernoud (2021), how much a bidder learns about his own value can depend on what he learns about the others' value, and learning is costly. Consider an SPA with private values, as in Gleyze and Pernoud (2021), and let all but one bidder learn only about their own values. Then, for the remaining bidder, learning about others has no impact on the best-response bid but might save on learning costs. In particular, if the other bidders' valuations are so high that the bidder never wins, then he would not want to engage in costly learning.

[5]Endogenous information acquisition has been analyzed in other areas of economics. See Bergemann and Välimäki (2002), Crémer et al. (2009), Shi (2012) and Bikhchandani and Obara (2017) in mechanism design, Martinelli (2006) and Gerardi and Yariv (2007) in voting, Crémer and Khalil (1992) and Szalay (2009) in principal-agent-settings, and Rösler and Szentes (2017) in bilateral trade.

[6]This condition requires the reverse hazard rate of an experiment to be independent of the valuation.

It is not the strategic interaction in the auction, but the optimal spending on learning costs which drives the difference between Gleyze and Pernoud (2021) and my model.

In Bergemann et al. (2009), the value of an object is a weighted sum of every bidder's type. Bidders either learn perfectly their type, or they learn nothing. Learning cannot introduce any dependence among the signals of bidders since all types are independent (although they do matter to other bidders). With positive interdependence, Bergemann and Välimäki (2002) show that in a generalized Vickrey-Clarke-Groves mechanism bidders acquire more information than would have been socially efficient.

The above literature on auctions considers *covert* information acquisition where learning decisions are not observed by other bidders. Another strand of the literature analyzes *overt* information acquisition. Hausch and Li (1991) show that the SPA and the FPA induce different incentives to acquire information when information acquisition is overt, and revenue equivalence fails. Compte and Jehiel (2007) show in an IPV setup that an ascending dynamic auction induces more overt information acquisition and higher revenues than a sealed-bid auction. Hernando-Veciana (2009) compares the incentives to overtly acquire information in the English auction and the SPA, when bidders can learn about either a common component or a private component. In contrast to my model, it is exogenous which component the information acquisition is about.

This paper is part of a broader agenda on information acquisition about common or idiosyncratic aspects in other games and decision problems. Bobkova and Mass (2021) ask how agents in a social learning framework split their learning budget between a common-component experiment and an idiosyncratic-component experiment. Perego and Yuksel (2021) analyze the incentives of media outlets to disclose information about issues of common interest versus issues for which readers' preferences are heterogeneous. In a sender-receiver communication framework, Deimen and Szalay (2019) ask whether a sender learns about her own or the receiver's optimal action.

My paper also relates to the literature on information choice in games with strategic complementarities, e.g., Hellwig and Veldkamp (2009) and Myatt and Wallace (2012).[7] My model differs from those in two major ways. First, in my model bidding functions do not exhibit strategic complementarities (see, e.g., Athey, 2002), which leads to a fundamentally different strategic problem. Second, in the models just cited, all information is about the same one-dimensional state of the world. In my model, however, bidders choose which component of the multidimensional state to learn about.

[7]See also Yang (2015) for flexible information acquisition in investment games and Denti (2017) for an unrestricted information acquisition technology.

2 Environment

2.1 Model

Two risk-neutral bidders, indexed by $i \in \{1, 2\}$, compete for one indivisible object. The reservation value of the auctioneer and the outside options of the bidders are zero.

The valuation for the object of bidder i, denoted by V_i, depends on two attributes: a common component S, that is identical for both bidders and admits a density $h_S(.)$ on its support $\mathcal{S} \subset \mathbb{R}$; and a private component T_i that admits a density $h_T(.)$ on its support $\mathcal{T} \subset \mathbb{R}$.[8] The common component and the two private ones $\{S, T_1, T_2\}$ are mutually independent, and T_1 and T_2 are drawn identically. The valuation for the object of bidder i is additive in the common and the private component:[9]

Assumption 1. *The value of bidder i is $V_i = u(S) + w(T_i)$. For every S, T_i, the value is nonnegative and strictly increasing in both components.*

The bidders do not observe the realizations of the random variables S, T_1, T_2. Instead, they choose between experiments about their components. If bidder i learns about the common component and if its realization is $S = s$, then he observes a random variable X_i^S with full support $\mathcal{X}^S$ and density $f^S(.|s)$. If bidder i learns about his private component and if its realization is $T_i = t$, then he observes X_i^T with full support $\mathcal{X}^T$ and density $f^T(.|t)$.

The *information choice* of bidder i is the strategy σ_i which denotes the probability of learning X_i^S. With the remaining probability $1 - \sigma_i$, bidder i learns X_i^T about his private component.[10] The following assumption imposes more structure on these experiments and the correlation structure such that the private signals of bidders are correlated only through learning about the common component.

Assumption 2.

(i) $X_1^T \perp\!\!\!\perp X_2^T$ *and* $X_i^S \perp\!\!\!\perp X_j^T$ *for* $i, j \in \{1, 2\}$*;*

(ii) $X_1^S \perp\!\!\!\perp X_2^S \mid S$*;*

(iii) for all $\ell' > \ell$ *and* $L \in \{S, T\}$, $\frac{f^L(x|\ell')}{f^L(x|\ell)}$ *is strictly increasing in* $x \in \mathcal{X}^L$.

[8] The results can be easily extended to binary components.

[9] This additivity assumption can be relaxed under additional symmetry assumptions. See a previous version of this paper: Bobkova (2019).

[10] For clarity of the presentation, each bidder observes one experiment about only one of his components. However, I also fully solve the case when signals contain information about two components simultaneously in section 6.2. In addition, in section 6.1, I allow bidders to also learn about the payoff-irrelevant private component of their opponents.

By (i), signals are independent if they contain information about different components. By (ii), the two experiments about the common component are independent conditional on S. Furthermore, by (iii), experiments satisfy a strong monotone likelihood ratio property (MLRP) such that higher signal realizations are more indicative of higher realizations of a component. The game consists of two stages:

1. *Information-choice stage.* After an auction format is announced, bidders simultaneously select which component to learn about by choosing σ_1 and σ_2.
2. *Auction stage.* Bidders privately observe their signal X_i^S or X_i^T and bid in the auction.

Bidders choose their information knowing the auction format. Information choice is covert: bidders do not observe which experiment their opponent has learned.

2.2 Efficiency and the equilibrium concept

The model nests two well-known frameworks. If $\sigma_1 = \sigma_2 = 1$, then the environment is a pure CV framework. In that case, both bidders learn only about the common component; no bidder possesses any information about T_1 or T_2. If $\sigma_1 = \sigma_2 = 0$, then this is an IPV framework. In that case, both bidders have no information about the other bidder's value, and their signals are independent.

The IPV case plays a special role in the following analysis: it is the learning outcome in any efficient equilibrium. Both bidders share the same common component, and the values are nonnegative by assumption 1. Thus, allocative efficiency requires the object to go to the bidder with the highest private component, based on all available signals. Efficiency in this environment requires that bidders learn about their private components and the bidder with the highest private signal X_i^T wins the object. From the perspective of efficiency, bidders should not be wasting their learning resources on the common component.

My focus is on symmetric, monotonic Bayesian Nash equilibria $\{\sigma^*, \beta_S^*, \beta_T^*\}$ in which bidders have the same information choice σ^*, and use the same increasing bidding function: β_S^* if learning their common component signals X_i^S, and β_T^* if learning X_i^T.

2.3 Discussion

Any information choice σ_i in my model is costless. This is in contrast to the papers on information acquisition in which agents learn about a one-dimensional variable, and the monetary costs of learning serve as the opportunity costs of learning more. Here, the *opportunity costs* of learning about one component are *not learning about the other component*. Hence, introducing monetary costs into this environment dilutes the pure effects of learning about one or the other component as each signal would then have monetary and nonmonetary opportunity costs. I introduce monetary learning costs in section 6. In my framework, agents would choose to learn one experiment instead of nothing because it is costless and unobserved by others, and can be ignored.

Two of my assumptions might seem restrictive, but are made only for reasons of expositional clarity and relaxed in later parts of the paper. First, I assume that bidders can learn only about their own private component or the common component, but not about the private component of the other bidder. This greatly simplifies the notation, and is relaxed in section 6.

Second, I assume that an experiment is informative about either the common or the private component, but not both. This provides a reasonable and clean benchmark in many contexts where information is discrete. For example, drilling an exploratory well only halfway to the required depth, or performing only the first half of a laboratory procedure does not provide any useful information. In other settings, allowing bidders to learn some continuous combination about both components might be a more realistic assumption. I generalize my results in section 6 to nest experiments which are informative about both components simultaneously. The assumption that signals are one-dimensional —bidders cannot learn the outcomes of two separate experiments about two components— is made for tractability. Even without information choice, establishing existence in the SPA is often not possible for two-dimensional private information, and there are cases in which an equilibrium does not exist (Jackson, 2009).[11]

3 Preliminaries

First, I introduce the notion of *accuracy* of information, which is used repeatedly in what follows to rank experiments.[12] Then, I establish how it relates to the payoff.

Definition 1 (Accuracy)**.** *Fix a random variable Z and two signals X^a and X^b with corresponding distributions $F_a(.|z)$ and $F_b(.|z)$. X^a is* <u>*more accurate*</u> *than X^b about Z*

[11]See Pesendorfer and Swinkels (2000) for a discussion of two-dimensional private information. They show existence in an epsilon equilibrium since they could not establish the existence of an equilibrium.

[12]Lehmann (1988) introduced this concept into the statistical literature as effectiveness. In economics, it is known mainly as *accuracy*. See Persico (2000).

(write $X^a \succ_Z X^b$) if for every x, the function

$$M(x|z) \coloneqq F_a^{-1}(F_b(x|z)|z)$$

is nondecreasing in z. X^a and X^b are equally accurate about Z (write $X^a \sim_Z X^b$) if $X^a \succ_Z X^b$ and $X^b \succ_Z X^a$.

The above definition varies slightly from the standard definitions (e.g., those in Lehmann (1988)) since it defines equal accuracy $\sim_Z$ and includes the variable Z explicitly in the binary relation $\succ_Z$. This captures that in my framework, Z can be several random variables. For example, two signals X^a and X^b can vary in their accuracy about the value $Z = V_i$, but also in their accuracy about the private component $Z = T_i$.[13] The transformation $M(x|z)$ maps a signal realization x from the less accurate experiment to a signal realization for the more accurate experiment.

Let an agent face the following decision problem: he chooses an action $a \in \mathcal{A}$ that, jointly with an unknown state Z, determines the payoff $u(a; z) : \mathcal{Z} \times \mathcal{A} \to \mathbb{R}$. The payoff satisfies a standard single-crossing property:

Definition 2. *$u(a; z)$ satisfies a single-crossing property (SCP) in $(a; z)$ if, for $a' > a$ and $z' > z$, $u(a', z) - u(a; z) > 0 \quad \Rightarrow \quad u(a', z') - u(a; z') \geq 0$.*

The agent does not observe Z, but instead has access to a signal in $X_i \in \{X^a, X^b\}$, based on which he chooses his optimal action. When the concept of accuracy is applied to MLRP signals and payoffs satisfy the single-crossing property, then higher accuracy corresponds to a higher payoff. This is because the more accurate signal is more likely to produce lower signals when Z is low, and high signals when Z is high. Essentially, X^a varies more strongly with Z than X^b.

Theorem 1 (Lehmann (1988))**.** *Let X^a and X^b both satisfy the MLRP, and $X^a \succ_Z X^b$. Then, the expected payoff with X^a is higher than with X^b for any payoff $u(a; z)$ which satisfies SCP in $(a; z)$.*

4 Equal accuracy and SPA

This section distills the strategic value of information about either component. For this reason, I abstract away from signals with varying informativeness, and instead

[13] In a one-dimensional framework with a single payoff-relevant variable, this additional notation is unnecessary: two signals which are equally accurate about a single variable Z are essentially the same.

focus on this stylized question: if either of the two component experiments are equally informative about the total value V_i, which experiment do bidders learn about? The following assumption captures the notion of "equal informativeness".

Assumption 3 (Equal Accuracy)**.** *For $i \in \{1, 2\}$, $X_i^S \sim_{V_i} X_i^T$.*

Let $F^S(X_i^S|v_i)$ be the distribution of X_i^S if $V_i = v_i$, and let $F^T(X_i^T|v_i)$ be the distribution of X_i^T given $V_i = v_i$.[14] Using the definition of equal accuracy, this requires that both ${F^T}^{-1}(F^S(x|v_i)|v_i)$ and ${F^S}^{-1}(F^T(y|v_i)|v_i)$ are nondecreasing in v_i for every $x \in \mathcal{X}^S$ and every $y \in \mathcal{X}^T$. This can hold only if both expressions are constant in v_i. This is summarized in the following lemma which translates equal accuracy into statements about marginal distributions and the value update.

Lemma 1 (Equal Accuracy)**.** *Let $X_i^S \sim_{V_i} X_i^{T_i}$. There exists a bijection $M : \mathcal{X}^S \to \mathcal{X}^T$ such that for every $x \in \mathcal{X}^S$,*

1. *(value-independent transformation) $F^S(x|v_i) = F^T(M(x)|v_i)$ for all v_i;*
2. *(equal marginal distribution) $F^S(x) = F^T(M(x))$;*
3. *(equal posterior value) $\mathbb{E}[V_i|X_i^S = x] = \mathbb{E}[V_i|X_i^T = M(x)]$.*

For any object value v_i, X_i^T and $M(X_i^S)$ are distributed identically. Hence, equal accuracy pins down the same experiment about the total value v_i, up to a relabeling via $M(.)$. If all components are distributed identically, and the valuation is symmetric in the components (e.g., $V_i = S + T_i$), then any fixed signal technology where $F^S(x|s) = F^T(M(x)|t)$ for $s = t_i$ satisfies assumption 3. As the following two examples show, there are several other nonsymmetric frameworks that satisfy assumption 3.

Example 1. *Let $S \in \{0, 1\}$ with equal probability, $T_i \sim \mathcal{U}[1, 2]$, and $V_i = S + T_i$. Then, the following two signal distributions satisfy equal accuracy about V_i: the private-component signal is fully revealing, i.e., $X_i^T = T_i$, and the distribution function of the common-component signal is $f^S(x|S = 0) = 2-2x$ and $f^S(x|S = 1) = 2x$ for $x \in [0, 1]$. In this case, the state-independent transformation is $M(x) = x + 1$.*

Example 2. *Let $S, T_i \in \{0, 1\}$ with equal probability, and $V_i = S+2T_i$. Then, assumption 3 is satisfied by (i) a common-component experiment with $f^S(x|S = 0) = 2-2x$ and $f^S(x|S = 0) = 2x$, and (ii) a private-component experiment with $f^T(x|T_i = 0) = \frac{3}{2} - x$ and $f^T(x|T_i = 0) = \frac{3}{2} + x$ with $x \in \mathcal{X}^S = \mathcal{X}^T = [0, 1]$. In this case, $M(x) = x$.*

[14]This is a slight abuse of notation as $F^S(.|s)$ is the distribution of X_i^S given the common component $S = s$. However, this avoids additional notation and in what follows there should be no confusion since I always include v_i or s to clarify which variable the distribution refers to.

If assumption 3 holds and a bidder is offered the object for a posted price, then he is indifferent between learning X_i^S or X_i^T: both lead to an identical posterior distribution of v_i, and the component composition beyond v_i is payoff-irrelevant. However, in a strategic environment, learning X_i^S might give a bidder a better idea of his opponent's bid. Thus, it need not be the case that a bidder is indifferent between both signals. This is due to the strategic effects of bidders choosing more or less correlated signals, and not due to one signal containing more information about the object's value.

The following main result shows that the SPA is the *efficient* auction format if bidders choose between signals with equal accuracy about their value.

Theorem 2 (SPA induces IPV). *Let $X_i^S \sim_{V_i} X_i^T$ for $i = 1, 2$. In any symmetric equilibrium of the SPA, $\sigma^* = 0$. There exists an equilibrium with $\sigma^* = 0$ and $\beta_T^*(X_i^T) = \mathbb{E}[V_i | X_i^T = x]$.*

The bidder with the highest signal realization about his private component wins, and bidders do not waste their learning resources on the common component, which is irrelevant for the optimal allocation. The remainder of this section is devoted to providing intuition for why only an IPV framework can arise endogenously in the SPA.

4.1 Existence

The existence of an IPV equilibrium in theorem 2 follows from the equal accuracy assumption. If bidder 2 learns X_2^T, then there is no winner's curse for bidder 1. If it is optimal for bidder 1 to bid b following $X_1^S = x$, then bidding b is also optimal when $X_1^T = M(x)$. This is because both signals lead to the same expectation of V_i (lemma 1), and both are independent of the other bidder's bid. Finally, both signals X_1^T and $M(X_1^S)$ are distributed identically, so a bidder is indifferent between learning either since they both lead to the same strategic problem.

4.2 Uniqueness

In what follows, I show by contradiction that there cannot exist a CV equilibrium in which both bidders learn about the common component.[15] Let the candidate equilibrium have $\sigma = 1$ and some increasing bidding function β_S. The following *deviation* constitutes a strictly profitable deviation for bidder 1:

[15]The focus on $\sigma = 1$ is for expositional clarity. The proof in appendix B holds for any $\sigma \in (0, 1]$.

1. *information choice:* $\hat{\sigma}_1 = 0$;
2. *bidding function:* $\hat{\beta}_T(M(x)) = \beta_S(x)$.

I show that this deviation leads to (i) the same winning probability; (ii) a strictly lower expected payment; (iii) the same expected value when winning. Hence, a bidder is better off using the same marginal bid distribution as in the candidate equilibrium on the private-component signal, while avoiding any interdependence with his opponent.

Equal winning probability. In the candidate equilibrium, due to symmetry, both bidders win with probability $\frac{1}{2}$. If bidder 1 deviates, then the bidders' signals are independent. Recall that $M(X_1^S)$ and X_1^T are distributed identically, and both lead to the same bid distribution. Hence, both bidders have the same marginal bid distribution, and win with equal probability; formally, we have:

$$\Pr(X_1^T \geq M(X_2^S)) = \int_{\mathcal{X}^S} f^S(x) \left[1 - F^T(M(x))\right] dx = 1 - \int_{\mathcal{X}^S} f^S(x) F^S(x) dx = \frac{1}{2}. \tag{1}$$

Thus, the deviation has no effect on the winning probability.

Proposition 1. *Let $\sigma = 1$ and β_S be a candidate equilibrium in the SPA. Bidder 1 wins with probability $\frac{1}{2}$ if (i) he plays the candidate equilibrium, or (ii) he deviates to $\hat{\sigma}_1 = 0$ and bidding function $\hat{\beta}_T(M(x)) = \beta_S(x)$.*

Lower expected payment. Let $G_{(2)}(x) \coloneqq \Pr(X_2^S \leq x | X_1^S \geq X_2^S)$ be the distribution of the signal of bidder 2 in the candidate equilibrium, conditional on bidder 1 winning. This is the distribution of the second-order statistic of the two correlated, identically distributed private signals,[16] which is as follows:

$$G_{(2)}(x) = 2F^S(x) - \int_{\mathcal{S}} [F^S(x|s)]^2 h(s) ds. \tag{2}$$

For bidder 1's deviation, let $\hat{G}_{(2)}(x) \coloneqq \Pr(X_2^S \leq x | M^{-1}(X_1^T) \geq X_2^S)$ be the distribution of bidder 2's signal conditional on bidder 1 winning. Recall that signals X_2^S and $M^{-1}(X_1^T)$ are distributed identically and independently. Hence, this distribution equals the well-known distribution of the second-order statistic of two i.i.d. variables:

$$\hat{G}_{(2)}(x) = 2F^S(x) - \left[F^S(x|s)\right]^2. \tag{3}$$

[16] See (8) and proposition 9 in the appendix for further details.

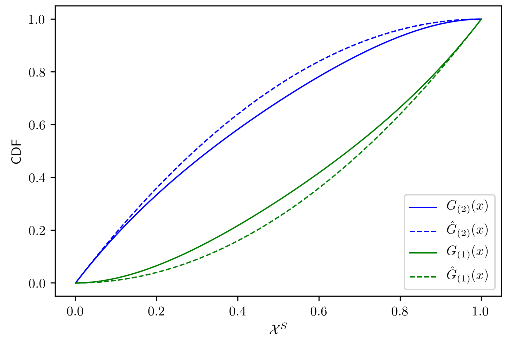

Figure 1: Distributions of first- and second-order statistics of two identically distributed random variables.

When is bidder 1's payment higher? In both the candidate equilibrium and the deviation, bidder 1 pays the bid of bidder 2, who uses the bidding function β_S. If bidder 1 wins, he pays $\int_{\mathcal{X}^S} \beta_S(x)dG_{(2)}(x)$ in the candidate equilibrium, and $\int_{\mathcal{X}^S} \beta_S(x)d\hat{G}_{(2)}(x)$ in the deviation. As the following result shows, $G_{(2)}$ and $\hat{G}_{(2)}$ can be ranked.[17]

Proposition 2 (Lower expected payment.)**.** *Let $\sigma = 1$ and β_S be a candidate equilibrium in the SPA. $G_{(2)}$ first-order stochastically dominates $\hat{G}_{(2)}$. The expected payment for a winning bidder is strictly lower in the deviation than in the candidate equilibrium.*

In the deviation, bidder 2's signal realizations, and hence, his bids, are more likely to be distributed lower when bidder 1 wins. As the bidding function β_S is increasing, it follows that the expected payment is also lower.[18] Consider the extreme case of perfectly correlated signals in the candidate equilibrium. Then, both bidders bid the same, the first- and second-order statistics of the bids coincide, and bidder 1 pays his own bid in the SPA when winning. However, in the deviation, bidder 1 pays less conditional on winning because the two bids no longer coincide: there is a gap between the distribution of the second-order statistic of the bids (which bidder 1 pays when winning) and the first-order statistics of the bids (which bidder 1 bids when winning). The deviation increases the gap between the first- and the second-order statistics distributions. This increase takes the form of first-order stochastic dominance, as depicted in figure 1.

Equal value when winning. In the candidate equilibrium, there is no information about the private components. Conditional on winning, the expected private-

[17]See lemma 2 in the appendix for a general ranking of the distributions of first- and second-order statistics of two variables with the same marginal distribution and varying correlation.

[18]This argument holds for any nondecreasing bidding function β_S, and does not require the functional form of equilibrium bidding functions in the SPA as in Milgrom and Weber (1982).

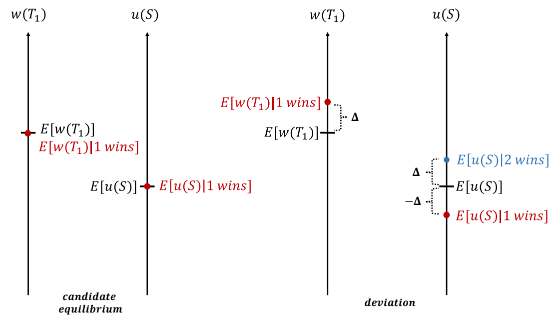

Figure 2: Expected value from components in the candidate equilibrium (left two axes) and deviation (right two axes).

component payoff is therefore $\mathbb{E}[w(T_1)|X_1^S \geq X_2^S] = \mathbb{E}[w(T_1)]$, depicted by the red dot on the left axis in figure 2. Bidder 1 wins at every S with probability $\frac{1}{2}$. Hence, the common-component payoff conditional on winning is $\mathbb{E}[u(S)|X_1^S \geq X_2^S] = \mathbb{E}[u(S)]$, shown by the red dot on the second-from-the-left axis in figure 2.

When deviating, bidder 1 no longer wins with probability 1/2 at every realization of S. Instead, he is more likely to win with a high X_1^T and thus a high T_1.[19] Hence, conditional on winning, the private-component payoff is higher in the deviation than in the candidate equilibrium. Figure 2 shows this increase, denoted by Δ, in the second-from-the-right axis.

Analogously, if bidder 1 deviates, bidder 2 is more likely to win when X_2^S is high. Hence, conditional on winning, bidder 2's expected value of S is higher than in the candidate equilibrium. This is depicted by the blue dot on the far right axis of figure 2. Due to the equal accuracy assumption and symmetry in bid distributions, this increase is exactly Δ, bidder 1's increase from T_1.[20] The total surplus from the common component is $\mathbb{E}[u(S)]$. As the object is always sold (i.e., no surplus is destroyed), and both bidders are equally likely to win when bidder 1 deviates (proposition 1), the sum of the expected values equal the total surplus, as shown here:

$$\mathbb{E}[u(S)] = \frac{1}{2}\underbrace{\mathbb{E}[u(S)|M^{-1}(X_1^T) < X_2^S]}_{\mathbb{E}[u(S)]+\Delta} + \frac{1}{2}\mathbb{E}[u(S)|M^{-1}(X_1^T) \geq X_2^S] \tag{4}$$

This pins down the expected value of the common component for the deviating

[19] Recall that signals satisfy the MLRP (assumption 2).

[20] For further details, see the proof of proposition 11 in the appendix.

bidder 1: (4) can only be satisfied if $\mathbb{E}[u(S)|M(X_1^T) \geq X_2^S] = \mathbb{E}[u(S)]-\Delta$, i.e., bidder 1's common-component payoff decreases by the same amount by which bidder 2's common-component payoff increases. Hence, when deviating and winning, bidder 1's expected object value increases by Δ from the private component, but this is exactly cancelled out by a decrease of Δ in the common component.

Proposition 3 (Equal expected value.)**.** *Let $\sigma = 1$ and β_S be a candidate equilibrium in the SPA. Bidder 1's expected object value when winning is identical in the candidate equilibrium and in the deviation.*

These insights carry over for any candidate equilibrium with $\sigma > 0$. A bidder has a strictly profitable deviation to learn X_i^T instead of X_i^S to avoid any interdependence with his opponent's bid, and to use the same marginal bidding function as in the candidate equilibrium. This strictly decreases the expected payment via a lower distributed second-order statistic of the now independent bids, while keeping the overall winner's curse unaffected: the total value of the object conditional on winning does not change (although its component composition does), and neither does the winning probability.

5 Higher accuracy in the SPA

In the previous section, signals about the common or the private component were equally informative about the bidders' total value. While this isolated the strategic value of information about either component, it is a knife-edge case. In what follows, I relax the equal accuracy assumption and show how theorem 2 can be extended.

An initial naive guess might be that if the private-component signal is more accurate about the value than the common-component signal, i.e., $X_i^T \succ_{V_i} X_i^S$, then bidders learn only about their private component in any symmetric equilibrium. After all, this is true for $X_i^T \sim_{V_i} X_i^S$ by theorem 2, so why shouldn't bidders prefer the private-component signal if it contains even more accurate information about their value V_i? Yet, there are two caveats which make the accuracy order $X_i^T \succ_{V_i} X_i^S$ a suboptimal choice. I discuss both below, before introducing the correct accuracy order to extend theorem 2.

The first caveat is that only few signals might satisfy $X_i^T \succ_{V_i} X_i^S$. A more accurate signal about T_i need *not* be more accurate about V_i. This is due to the two dimensions which influence the value,[21] and gives rise to counterintuitive properties. For the ex-

[21] For the particular value function in Example 3, X_i^T cannot be simultaneously affiliated with T_i and affiliated with V_i, unless it is fully uninformative.

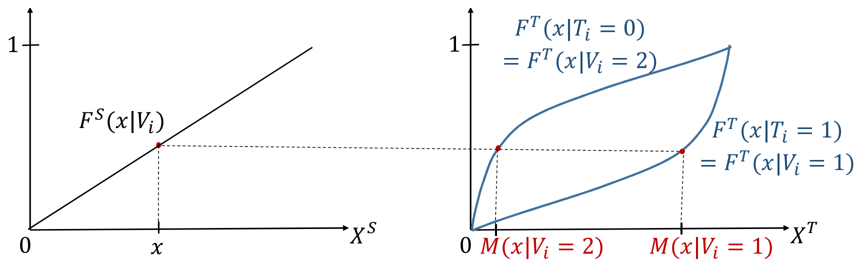

Figure 3: Sketch for Example 3. On the left: X_i^S fully uninformative. On the right: an informative X_i^T, with $M(x|V_i)$ decreasing from $V_i = 1$ to $V_i = 2$.

ample below, there exists no X_i^T which is strictly more accurate about V_i than a *fully uninformative* X_i^S. Yet many signals exist which are more accurate about T_i than a fully uninformative signal, and which are not comparable in terms of $\succ_{V_i}$.

Example 3. *Let $S, T_i \in \{0,1\}$, and $V_i = 2S + T_i$. Note the nonmonotonicity in T_i: $V_i = 1$ only if $T_i = 1$, and $V_i = 2$ only if $T_i = 0$. By assumption 2, X_i^T satisfies the MLRP with respect to T_i,*[22] *and hence,*

$$F^T(x|V_i = 2) = F^T(x|T_i = 0) \geq F^T(x|T_i = 1) = F^T(x|V_i = 1). \tag{5}$$

Then, no informative signal about the private component can have a higher accuracy about V_i than a fully uninformative signal $X_i^S \sim \mathcal{U}[0,1]$. This is because $M(.|v_i) : X_i^S \to X_i^T$ is decreasing from $V_i = 1$ to $V_i = 2$ due to (5), which is sketched in figure 3.

The second caveat is that even if two signals satisfy $X_i^T \succ_{V_i} X_i^S$ and both signals are affiliated with V_i, it is not clear why bidders should prefer the more accurate signal about V_i. To see this, consider the expected utility of bidder 1 who receives a signal $X_1^T = x_1$, whose value is v_1 and who places a bid b:

$$\tilde{u}_1^T(b; v_1, x_1) = \underbrace{\sigma_2 \int^{\beta_S^{-1}(b)} [v_1 - \beta_S(x_2)]\, dF_2^S(x_2|v_1, x_1)}_{\text{bidder 2 learns } X_2^S} \tag{6}$$

$$+ \underbrace{(1 - \sigma_2) \int^{\beta_T^{-1}(b)} [v_1 - \beta_T(x_2)]\, dF_2^T(x_2)}_{\text{bidder 2 learns } X_2^T}. \tag{7}$$

Note the dependence of $\tilde{u}^T$ on the signal x_1. Due to the two-dimensional composition of bidder 1's value, the distribution of the opponent's signal $F_2^S(x_2|v_1, x_1)$ depends not

[22] First-order stochastic dominance is a well-known consequence of the MLRP.

only on the realization v_1 but also on the realization of bidder 1's signal. This is because bidder 1's signal conveys information about which of the possible (S, T_1)-combinations is more likely, and hence, which S influences bidder 2's signal X_2^S.[23] This situation does not arise in classic models where the value pins down the distribution of the other bidders (see, e.g., Athey (2001), Persico (2000)). In addition, it is not clear why the function $\tilde{u}^T(b; v_1, x_1)$ should satisfy the SCP. It is well known that (7) (which arises in IPV models) satisfies the SCP in $(b; v_1)$. But even if (6) also satisfies the SCP, it remains the case that, in general, the sum of two functions which satisfy the SCP—as (6) and (7) do in this case—need not itself satisfy the SCP. Thus, theorem 1 cannot be applied because two of its conditions might not hold: the signal is payoff relevant beyond its information about v_1, and the SCP in $(b; v_1)$ might be violated. This means that higher accuracy about V_i might not translate into a higher expected utility.

However, comparing accuracy about a *component* (not about the value) avoids the above problems and allows a comparison among a rich set of experiments:

Theorem 3 (Efficient SPA)**.** *Let bidders choose between X_i^T and X_i^S. If there exists some $\tilde{X}_i^T$ such that $X_i^T \succ_{T_i} \tilde{X}_i^T \sim_{V_i} X_i^S$, then bidders learn X_i^T in any symmetric equilibrium in the SPA.*

Hence, a meaningful accuracy comparison in a two-dimensional valuation framework requires a *two-step approach*: (i) finding an intermediate private-component experiment $\tilde{X}_i^T$ that is as accurate about V_i as X_i^S is, and then (ii) comparing its accuracy about T_i (not about V_i) with the available private-component experiment X_i^T. This preserves the richness of the order $\succ_{T_i}$, since many experiments about the private component can be ranked in this way, regardless of the functional form of $V_i = u(S) + w(T_i)$.

For a sketch of the proof, I show that bidders prefer higher accuracy about the private component, and therefore prefer X_i^T to $\tilde{X}_i^T$. The difficulty of establishing this is similar to the second caveat above for $\succ_{V_i}$: it is not clear whether theorem 1 is applicable. I prove that if $T_i = t_i$ and a bidder places a bid b, his expected payoff (i) satisfies the SCP and (ii) does not depend on the realization of a private-component experiment. This is because the distribution of the opponent's signal is independent of both X_i^T and $\tilde{X}_i^T$ conditional on T_i (but not V_i). In addition, the bidder's inference about S when winning can be integrated out. Hence, theorem 1 can be applied. Finally,

[23] To see this, let $S, T_1 \in \{0, 1\}$ and $V_1 = S + T_1$. If $V_1 = 1$, then either $T_1 = 1$ (and $S = 0$), or $T_1 = 0$ (and $S = 1$). A high signal X_1^T is more indicative of $T_1 = 1$ and $S = 0$, and hence, X_2^S is more likely to be low (as it is affiliated with a low common component). However, a low X_1^T is more indicative of $S = 1$ and $T_1 = 0$. In this case, X_2^S is likely to be higher.

the intermediate experiment $\tilde{X}_i^S$ allows the application of theorem 2.

6 Discussion and extensions

6.1 Robustness of an IPV equilibrium in the SPA

Learning about the opponent's private component. In my model, bidders can learn only about their payoff-relevant components, but not the private component of the other bidder. How relevant is this assumption for the uniqueness of the IPV equilibrium in the SPA? Allowing bidders to learn about the other bidder's private component imposes further restrictions on the IPV equilibrium since it constitutes an additional deviation. In what follows, I show that the IPV outcome remains an equilibrium in the SPA even if bidders can learn about every component in the model.

As in the baseline model, bidder $i \neq j$ has access to the two experiments X_i^T and X_i^S. In addition, each bidder can learn a signal about the private component of the other bidder, Y_i^T, which is informative only about T_j. I do not impose any accuracy ranking about V_j among Y_i^T and X_j^T, and Y_1^T and Y_2^T can have different distributions. The following result shows that the privacy-preserving IPV outcome remains an equilibrium in the SPA.

Proposition 4. *Let bidders choose one signal in $\{X_i^S, X_i^T, Y_i^T\}$ where $X_i^S \sim_{V_i} X_i^T$. Then, there exists an IPV equilibrium in the SPA, in which bidders learn only X_i^T.*

Comparison to an FPA. In addition to allowing bidders to learn about the private components of their opponent, I relax two further assumptions next: signals are now costly,and no accuracy ranking among signals is required anymore. While an absolute prediction as in theorem 2 (i.e., bidders learn only X_i^T in equilibrium) is no longer possible, I can derive a relative prediction between the SPA and the FPA. This shows that not every auction format has the same efficiency properties of the SPA.

Proposition 5. *Let bidders choose among $\{X_i^T, X_i^S, Y_i^T\}$ with corresponding costs $\{c^T, c^S, \tilde{c}^T\}$. If there exists an IPV equilibrium in the FPA, then there exists an IPV equilibrium in the SPA. The reverse is not true.*

The proof relies on revenue equivalence. In an IPV equilibrium, bidders' expected utility coincides in the FPA and the SPA. In addition, I show that the deviation payoff after learning X_i^S also coincides in the FPA and the SPA. This means that if learning about the common component is not a strictly profitable deviation in the FPA, then the same is true for the SPA. As in proposition 4, learning Y_i^T cannot be a strictly profitable deviation in the SPA since it results in a constant bid.

Why does the existence of an IPV equilibrium in the SPA not imply that there also exists an equilibrium in the FPA? Let bidder 2 learn only about T_2. In the FPA, anticipating bidder 2's bid can be useful for bidder 1 who does not want to "leave money on the table" by placing an unnecessarily high winning bid. Learning Y_1^T yields a better estimate of bidder 2's bid. However, learning Y_1^T also comes at the opportunity cost of not learning about V_1. In an FPA, bidders trade off these effects of exploiting higher correlation versus learning more about one's own valuation. Example 4 shows that the former effect can dominate and destroy the existence of an IPV equilibrium in the FPA.

Example 4. *Let* $S, T_1, T_2 \sim \mathcal{U}[0,1]$ *and* $V_i = S + T_i$*, and let signals be perfectly revealing:* $X_1^S = X_2^S = S$ *and* $X_i^T = Y_{j\neq i}^T = T_i$*. Since both signals are equally accurate about* V_i*, it follows that learning* X_i^T *and bidding* $\mathbb{E}[V_i|X_i^T = T_i]$ *constitutes an IPV equilibrium in the SPA.*[24]

Next, I show that there exists no IPV equilibrium in the FPA. Consider a candidate equilibrium in which bidders bid the standard IPV bidding function $\beta(X_i^T = x) = \mathbb{E}\left[V_j|\mathbb{E}[V_j|X_j^T] \leq \mathbb{E}[V_i|X_i^T = x]\right] = \frac{1+x}{2}$*. Bidder i's expected payoff with* $X_i^T = x$ *is* $(\frac{1}{2} + x - \frac{1+x}{2})x = \frac{x^2}{2}$*. Overall, his expected payoff in this candidate equilibrium is* $\frac{1}{6}$*.*

However, the following deviation yields a strictly higher payoff: learn $Y_1^T = T_2$ *and slightly outbid the opponent by bidding* $\frac{1+T_2}{2} + \epsilon$ *for some small* $\epsilon > 0$*. This strategy always wins the object at its prior expected value of* 1*. For* $\epsilon \to 0$*, bidder 1 pays the expected bid of the opponent, which is* $\frac{3}{4}$*. Hence, for* ϵ *sufficiently small, bidder 1's payoff from this deviation approaches* $1 - \frac{3}{4} = \frac{1}{4}$*, making this a strictly profitable deviation.*

6.2 Experiments about both components

So far, bidders could learn about only one component. If possible, would bidders prefer to learn simultaneously about both components in one experiment? To answer this, I allow experiments to vary continuously in their accuracy about the components. To compare the results to theorem 2, each experiment is assumed to have the same accuracy about a bidder's value. I assume that bidders have access to a parametrized set of experiments $\{X^\rho\}_{\rho\in[0,1]}$, where ρ captures the amount of information the experiment

[24] In this case, both bidders are indifferent between learning S and T_i.

contains about S and T_i. Let $f^\rho(.|s,t_i)$ be the positive density of X^ρ with support $\mathcal{X}$.

Assumption 4. *Bidders choose one experiment in the set $\{X^\rho\}_{\rho\in[0,1]}$ such that*

1. *for all $\rho' > \rho$, $X_i^{\rho'} \succ_S X_i^\rho$ (but not $X_i^\rho \succ_S X_i^{\rho'}$), $X_i^\rho \succ_{T_i} X_i^{\rho'}$, and $X_i^\rho \sim_{V_i} X_i^{\rho'}$;*
2. *for all $\rho \in (0,1)$, f^ρ satisfies the strong MLRP with respect to S;*
3. *there exist X_i^T, X_i^S satisfying assumption 2, and $X_i^{\rho=0} = X_i^T$ and $X_i^{\rho=1} = X_i^S$.*

That is, every experiment is equally accurate about the total value V_i. What varies in ρ is how much information the experiment contains about the two payoff-relevant components.[25] A higher ρ is more accurate about the common component, while sacrificing accuracy about the private component T_i. At the extreme, if $\rho \in \{0,1\}$, then the experiment is informative about only one component, as previously.

Bidders choose a signal in the set $\{X^\rho\}_{\rho\in[0,1]}$ by choosing $\rho_i \in [0,1]$. As before, the outcome is efficient if bidders choose $\rho_i = 0$, and do not waste their learning resources on S, which is irrelevant for finding the efficient allocation. And as in section 4, I prove that the SPA is the efficient auction format in this richer informational framework.

Theorem 4. *Let bidders choose one experiment from $\{X^\rho\}_{\rho\in[0,1]}$ which satisfies assumption 4. Then, in any symmetric equilibrium of the SPA, bidders acquire information only about their private components, $\rho^* = 0$. An equilibrium with $\rho^* = 0$ exists.*

Even though bidders can learn about both components simultaneously, they choose not to do so. Thus, IPV arises endogenously again even with a richer signal structure. A crucial assumption in theorem 4 is that every available experiment is equally accurate about the total value. This limits information choice purely to strategic considerations, and not to decisions about how much to learn. In reality, it might be that bidders can more easily learn a bit about both components than a lot about one component. A bidder might then prefer a signal which is simultaneously informative about both components.[26] Hence, in general, a result as strong as theorem 4 will not be possible if bidders consider both the strategic effect and the informativeness of their experiments.

[25] An interior $\rho \in (0,1)$ can be interpreted as a reduced form of a sequential learning procedure, where experiments are conducted on both components based on the outcome of previous experiments, and the bidder receives a one-dimensional summary signal at the end.

[26] As an extreme illustration, consider a (possibly) fully revealing signal $X_i^\rho = \rho_i S + (1-\rho_i)T_i$. While $\rho_i \in \{0,1\}$ perfectly reveals one component, $\rho_i = 1/2$ fully resolves all uncertainty about V_i.

6.3 Choosing accuracy and a component

The preceding analysis focused on which component bidders choose to learn about while fixing the accuracy and abstracting from monetary costs of information. In practice, bidders are likely to face both decisions: how much to learn, and about which component? If the costs of learning vary between the two components, then the information choice of the bidders inevitably depends on the cost functions; a statement as general as theorem 2 that contains an *absolute* information-choice prediction will not always be possible. However, under some regularity conditions on the cost function, I can provide a *relative* prediction of information choice between the SPA and the FPA. I show that the SPA leads to a more efficient outcome than an FPA. To simplify the problem and avoid additional assumptions, I now consider only pure strategy equilibria where bidders learn deterministically about either the common or the private component.[27]

Each bidder chooses one signal from two parametrized sets: $\{X_i^{S,\rho}\}_{\rho\in[0,1]}$ about the common component, and $\{X_i^{T,\tau}\}_{\tau\in[0,1]}$ about the private component. All these signals satisfy assumption 2. A higher ρ or τ is more accurate, respectively, about the common or private component: $X_i^{S,\rho'} \succ_S X_i^{S,\rho}$ for $\rho' > \rho$, and $X_i^{T,\tau'} \succ_{T_i} X_i^{T,\tau}$ for $\tau' > \tau$. To establish a point of reference among experiments about different components, let $X_i^{S,\rho} \sim_{V_i} X_i^{T,\tau}$ for $\rho = \tau$. That is, two experiments with the same parameter are equally accurate about the value. Each experiment comes at a cost $c_S(\rho)$ and $c_T(\tau)$, with $c_T'(.) > 0$ for $\rho, \tau \in (0,1)$, and $c_T(0) = c_S(0) = 0$. For every ρ, the distribution $F^{S,\rho}(x|s)$ is differentiable in ρ, and continuous in s. For every τ, $F^{T,\tau}(x|t)$ is differentiable in τ, and continuous in t.

Finally, I assume that if bidders can learn about only one of the two components, then there exists a unique symmetric equilibrium: If bidders choose only in $\{X_i^{S,\rho}\}_{\rho\in[0,1]}$, then ρ^{SPA} and ρ^{FPA} are the unique information choices in a symmetric equilibrium of the SPA and the FPA. If bidders choose only in $\{X_i^{T,\tau}\}_{\tau\in[0,1]}$, then τ^{SPA} and τ^{FPA} are the unique information choices in a symmetric equilibrium.[28]

Proposition 6 (SPA more efficient than FPA)**.** *Consider symmetric pure strategy equilibria in which bidders choose one signal from* $\{X_i^{S,\rho}\}_{\rho\in[0,1]} \cup \{X_i^{T,\tau}\}_{\tau\in[0,1]}$.

[27] When one bidder mixes between a common- and a private-component signal, it is not straightforward to establish a SCP to guarantee that more accurate information about S is better via theorem 1. While the SCP can be established under additional assumptions on the distributions and value function, this goes beyond the scope of this section.

[28] If bidders learn about only one component, then this is the framework of Persico (2000). See his corollary 1 for assumptions such that a unique symmetric equilibrium exists. By assuming existence and uniqueness, I focus on the additional effect of choosing between two components, instead of one.

1. *Let $c_S(a) \geq c_T(a)$ for all $a \in [0,1]$. In the SPA, $\sigma^{SPA} = 0$ in any symmetric equilibrium of the SPA. The FPA might have a pure CV equilibrium, but also has an IPV equilibrium. In the IPV equilibrium of both auctions, $\tau^{SPA} = \tau^{FPA}$.*
2. *Let $c_S(a) < c_T(a)$ for all $a \in [0,1]$. Then, there exists no IPV equilibrium in the SPA or in the FPA. In any symmetric pure CV equilibrium , $\rho^{SPA} \leq \rho^{FPA}$.*

In both cases, any SPA equilibrium is more efficient than any FPA equilibrium.

If it is cheaper to learn about one's private component than about the common component, then bidders learn only about their private component in the SPA. Ignoring the costs, under equal accuracy, bidders prefer the private-component signal (Theorem 2). Thus, bidders prefer it even more when it is cheaper, making learning about S unsustainable in equilibrium. If it is cheaper to learn about S, then IPV cannot be an equilibrium since bidders can get the same amount of information about their value from the common-component experiment instead of their private-component experiment, still have independent signals, but pay strictly less.

The relative comparison of accuracy between the auctions in the proposition, $\tau^{SPA} = \tau^{FPA}$ and $\rho^{SPA} \leq \rho^{FPA}$, follows as a corollary of Persico (2000). He considers a framework related to mine: bidders choose how accurately to learn about their one component, and bidders' components are affiliated and (possibly) payoff-relevant for all.[29] He shows that bidders acquire more information in the FPA than in the SPA. His framework does not make any statement about the efficiency of information choice. My analysis sheds light on the intricate interaction between higher accuracy and higher efficiency. In my model, the additional information which is acquired in the FPA is about the common component. Hence, learning more does not correspond to a higher allocative efficiency, since it is both costly and useless for finding the efficient allocation.

It is worth noting in which sense the SPA is more efficient than the FPA: the SPA does not necessarily lead to more learning about the private component. In fact, in an IPV equilibrium, both auction formats lead to the same allocative efficiency since they induce bidders to learn about their private components with the same accuracy in the FPA and SPA. However, the SPA has two advantages over the FPA: (1) It is more likely to induce an equilibrium in IPV. (2) It leads to less wasteful information acquisition about the common component. If the mechanism designer also takes the costs of information acquisition into account, then he might want to choose the SPA over

[29] Thus, he considers information acquisition about only one component, but his one-dimensional learning framework is more general than what I allow for each of the two components.

the FPA even both result in a pure CV equilibrium. An exciting follow-up question is whether this finding — that all additional information acquired in the FPA, as compared to the SPA, is wasteful — would survive in a richer model with multidimensional costly signals. This is left for future research as it is outside the scope of this paper.

6.4 Many bidders

So far, I have focused exclusively on the case of two bidders. Does an IPV equilibrium still exist in the SPA for more than two bidders? The answer is an unambiguous yes.

Proposition 7. *Let $N \geq 2$ bidders choose between $\{X_i^T, X_i^S\}$ with $X_i^T \sim_{V_i} X_i^S$. Then, learning X_i^T and bidding $\beta(X_i^T) = \mathbb{E}[V_i|X_i^T]$ are an equilibrium outcome in the SPA.*

As was true previously for two bidders, bidders are indifferent between learning two equally informative signals if both signals are independent of the information of others. With more than two bidders, is the IPV equilibrium also the unique symmetric equilibrium? The answer to this is more complicated. If $N \geq 3$ and a bidder uses the same class of deviation strategies as described previously, then he might be strictly more likely to win when deviating. As a consequence, the deviating bidder's expected payment might be strictly higher than in the candidate equilibrium. However, under additional assumptions, previous results can be extended to any number of bidders. For example, for binary states and a symmetric value function, Proposition 8 shows that a pure CV framework cannot be an equilibrium outcome in the SPA.[30]

Proposition 8. *Let $S, T_i \in \{0, 1\}$ uniformly, $V_i(S = 1, T_i = 0) = V_i(S = 0, T_i = 1)$, and $X_i^T \sim_{V_i} X_i^S$. For any $N \geq 2$, there exists no pure CV equilibrium in the SPA.*

7 Conclusion

This paper explores the impact of the auction mechanism on the type of information bidders choose to learn. Information about the common component simplifies coordination and is informative about the other bidder's bid. However, it is socially wasteful, since it comes at the opportunity cost of learning less socially valuable information about their own private component. In this paper, I found that the efficient auction format is the SPA, as long as all available information is equally informative about bidders' total valuation. In the SPA, an IPV framework arises endogenously. This result

[30]This result can be extended further to rule out candidate equilibria with mixing, $\sigma \in (0, 1)$.

can be extended in several ways – to compare signals with different accuracy, and to show that the SPA is more efficient than the FPA. This shows the endogenous nature of the IPV framework. If the auctioneer switches the auction format, bidders might react by learning more about the common component in the FPA than in the SPA. In the worst case scenario, bidders learn exclusively about the common component, and the FPA has the same chance of achieving the efficient allocation as a random lottery.

In addition to efficiency considerations, revenue also varies with bidders' information choice. Which auction format maximizes revenue? In an IPV framework, the SPA and the FPA yield the same revenue, while in a pure CV framework, the SPA yields higher revenue than the FPA (Milgrom and Weber, 1982). However, when bidders choose which components to learn about, an auctioneer might be facing the choice of an SPA with an IPV framework, or an FPA with a pure CV framework. Finding the revenue-maximizing mechanism remains an open question and is left for future research.

More broadly, agents choose information in a variety of other strategic settings beyond an auction environment, for example, in voting and public good problems. Understanding how a mechanism impacts the type of information that agents seek is essential if agents have multidimensional valuations with varying efficiency implications. Many models in these frameworks have well-known results if agents have common or independent private valuations; and, importantly, policy recommendations vary based on these two frameworks. However, if the change in private information is not taken into account, then policy recommendations might backfire and result in less efficiency: the policies might drive people to learn about aspects which do not matter for an efficient outcome. The question as to for which mechanisms an IPV or interdependent valuation framework arises endogenously in other essential strategic problems provides an exciting new avenue for future research.

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A Auxiliary Results

A.1 Order statistics and stochastic dominance

The following auxiliary lemma establishes a strong stochastic ordering of variables with the same marginal distribution but varying correlation.

Let X_1, $\hat{X}_1$ and X_2 be three continuous random variables, distributed identically with marginal distribution $F(x)$ for $x \in \mathcal{X}$. $\hat{X}_1$ and X_2 are drawn independently. X_1 and X_2 are drawn independently conditional on some random variable S with positive density $h(s)$. X_1 and X_2 satisfy the strong MLRP with respect to S.

Let $Y_{(2)} \coloneqq \min\{X_1, X_2\}$ be the second-order statistic of X_1 and X_2. Denote the respective distribution by $G_{(2)}$. Let $Y_{(1)} \coloneqq \max\{X_1, X_2\}$ be the first-order statistic of X_1 and X_2 with distribution $G_{(1)}$. Similarly, define $\hat{Y}_{(2)} \coloneqq \min\{\hat{X}_1, X_2\}$ be the second-order statistic of $\hat{X}_1$ and X_2, with distribution $\hat{G}_{(2)}$. Let $\hat{Y}_{(1)} \coloneqq \max\{\hat{X}_1, X_2\}$ be the first-order statistic of $\hat{X}_1$ and X_2 with distribution $\hat{G}_{(1)}$.

Lemma 2. *For any $x \in \mathcal{X}$ such that $F(x) \in (0,1)$,*

1. *$\hat{G}_{(1)}$ first-order stochastically dominates $G_{(1)}$, and $\hat{G}_{(1)}(x) < G_{(1)}(x)$,*
2. *$G_{(2)}$ first-order stochastically dominates $\hat{G}_{(2)}$, and $\hat{G}_{(2)}(x) > G_{(2)}(x)$.*

Proof. Proof of part 1. The distribution of the first-order statistic of the two independent random variables, $\hat{Y}_{(1)}$, is $\hat{G}_{(1)}(x) = F(x)^2$. The distribution of the first-order statistic of the two interdependent variables, $Y_{(1)}$, is $G_{(1)}(x) = \int_S F(x|s)^2 h(s)ds$. For all x with $F(x) \in (0,1)$, using the strict Cauchy-Bunyakowski-Schwartz inequality and the strong MLRP,[31]

$$\hat{G}_{(1)}(x) = F(x)^2 = \left[\int_{\mathcal{S}} F(x|s)h(s)ds\right]^2 < \underbrace{\int_{\mathcal{S}} h(s)ds}_{=1} \int_{\mathcal{S}} F(x|s)^2 h(s)ds = G_{(1)}(x).$$

Proof of part 2. The distribution of the second-order statistics of two i.i.d. random variables is $\hat{G}_{(2)}(x) = 2F(x) - F(x)^2$. For a fast way to derive $G_{(2)}$, note that the weighted first-order and second-order statistic distributions have to add up to the marginal distribution, i.e., $F(x) = \frac{1}{2}G_{(1)}(x) + \frac{1}{2}G_{(2)}(x)$. Therefore,

$$G_{(2)}(x) = 2F(x) - G_{(1)}(x) = 2F(x) - \int_{\mathcal{S}} F(x|s)^2 h(s)ds. \tag{8}$$

The same argument as above (the strict Cauchy-Bunyakovski-Schwartz inequality and the strict MLRP property) yields the result. □

A.2 SCP and supermodularity

In this subsection, I present sufficient conditions when a function $u_1 : \mathbb{R}^+ \times \mathcal{T} \to \mathbb{R}$ satisfies SCP in $(b; t_1)$, which will be used in later proofs.

Definition 3. *A function $y : \mathbb{R}^+ \times \mathcal{T} \to \mathbb{R}$ is supermodular (spm) if for every $t_1' > t_1$ and every $b' > b$, it holds that $y(b'; t_1') + y(b; t_1) \geq y(b; t_1') + y(b'; t_1)$.*

The following is a well-known result (see, e.g., Athey (2001)).

Lemma 3. *If $u_1 : \mathbb{R}^+ \times \mathcal{T} \to \mathbb{R}$ is spm, then it is SCP in $(b; t_1)$.*

The following are sufficient conditions for u_1 to be spm.[32]

Lemma 4. *Let $u_1(b; t_1) = f(t_1, b)g(b)$. If (i) $g(b)$ is nondecreasing in b and nonnegative, and (ii) $f(t_1, b)$ is nondecreasing in t_1 and spm, then $u_1(b; t_1)$ is spm.*

[31] The Cauchy-Bunyuakovsky-Schwarty inequality $\left[\int_a^b c(s)d(s)ds\right]^2 \leq \int_a^b c(s)^2 ds \cdot \int_a^b d(s)^2 ds$ is strict unless $c(s) = \alpha \cdot d(s)$ for some constant α (see Hardy et al. (1934), Chapter VI.) In the above argument, $c(s) = \sqrt{h(s)}$, and $d(s) = F(x|s)\sqrt{h(s)}$. Due to the strong MLRP, unless x such that $F(x) \in \{0,1\}$, $F(x|s)$ is not constant in s.

[32] A similar observation has been made in Athey (2001) in the proof of Theorem 7 in the appendix.

Proof. For any $b' > b$ and $t_1' > t_1$,

$$\begin{aligned} f(t_1', b') - f(t_1, b') &\geq f(t_1', b) - f(t_1, b) \\ \Rightarrow \quad g(b')\left[f(t_1', b') - f(t_1, b')\right] &\geq g(b)\left[f(t_1', b) - f(t_1, b)\right] \\ \Rightarrow \quad f(t_1', b')g(b') + f(t_1, b)g(b) &\geq f(t_1', b)g(b) + f(t_1, b')g(b'). \end{aligned}$$

□

Hence, to show that a function $u_1(b; t_1) = f(t_1, b)g(b)$ satisfies SCP in $(b; t_1)$, it is sufficient to establish the above properties (i) and (ii) in lemma 4.

B Omitted Proofs

Proof of Lemma 1. For existence of a value-independent $M(.)$, see the main text before the lemma. As $\forall v_i$, $F^S(x|v_i) = F^T(M(x)|v_i)$, it holds that $F^S(x) = F^T(M(x))$. Then, by Bayes rule, it holds that $F^S(v_i|x) = F^T(v_i|M(x))$. Hence,

$$\mathbb{E}[V_i|X_i^S = x] = \int v_i dF^S(v_i|x) = \int v_i dF^T(v_i|M(x)) = \mathbb{E}[V_i|X_i^T = M(x)]. \quad \square$$

Proof of Theorem 2. Section 4.1 in the main text establishes existence. Next, I show uniqueness. By contradiction, let there exist a candidate equilibrium $\{\sigma^{CE} > 0, \beta_S^{CE}, \beta_T^{CE}\}$. Then, bidder 1 has a profitable deviation: learn X_1^T ($\sigma_1 = 0$) and bid $\beta_T(M(x)) = \beta_S^{CE}(x)$, where a bijection $M : \mathcal{X}_S \rightarrow \mathcal{X}_T$ exists by lemma 1. This deviation yields a strictly higher payoff than learning X_1^S and bidding $\beta_S^{CE}(.)$, as I show next.

Proposition 9 (Equal winning probability). *Let $\{\sigma^{CE} > 0, \beta_S^{CE}, \beta_T^{CE}\}$ be a candidate equilibrium. Bidder 1 wins with the same probability if (i) he plays the candidate equilibrium and learns X_1^S, or (ii) he plays the above deviation.*

Proof. With probability σ^{CE}, bidder 2 learns X_2^S. Then, winning probability in (i) the candidate equilibrium after learning X_1^S and (ii) in the deviation strategy is 1/2, which is proven in the main text in (1).

With probability $1 - \sigma_{CE}$, bidder 2 learns X_2^T. As X_2^T is independent of both X_1^S and X_1^T, and X_1^T and $M(X_1^S)$ are distributed identically (lemma 1), winning probability in (i) and (ii) also coincides,

$$\Pr\left[\beta_T(X_1^T) \geq \beta_T^{CE}(X_2^T)\right] = \Pr\left[\beta_T(M(X_1^S)) \geq \beta_T^{CE}(X_2^T)\right] = \Pr\left[\beta_S^{CE}(X_1^S) \geq \beta_T^{CE}(X_2^T)\right].$$

□

Proposition 10 (Lower expected payment.). *Let $\{\sigma_{CE} > 0, \beta_S^{CE}, \beta_T^{CE}\}$ be a candidate equilibrium. Bidder 1's expected payment is strictly lower in the deviation than in the candidate equilibrium when learning X_1^S.*

Proof. With probability $1 - \sigma_{CE}$, bidder 2 learns X_2^T and bids $\beta_{CE}^T(X_2^T)$, which bidder 1 pays when winning. In both the deviation and in the candidate equilibrium when learning X_1^S, bidder 1 has the same marginal bidding function: he places a bid $\beta_S^{CE}(x) = \beta_T(M(x))$ or below with probability $F^S(x) = F^T(M(x))$. Bids are independent in both cases. Hence, bidder 1's expected payment is the same.

With probability σ_{CE}, bidder 2 learns X_2^S. In the candidate equilibrium after learning X_1^S, bidder 1 wins if $X_1^S \geq X_2^S$. Hence, the expected payment of bidder 1 when winning after learning X_1^S is $\int_{\mathcal{X}_S} \beta_S^{CE}(x)dG_{(2)}(x)$, where $G_{(2)}(x)$ is the second-order statistic distribution of two identically distributed signals X_1^S and X_2^S as derived in (8).

In the deviation strategy, bidder 1 wins if $M^{-1}(X_1^T) \geq X_2^S$. Both random variables X_1^S and $M(X_1^T)$ are i.i.d. with distribution $F^S(.)$. Hence, if bidder 1 wins, he pays the bid of the second-order statistics, $\int_{\mathcal{X}_S} \beta_S^{CE}(x)d\hat{G}_{(2)}(x)$, where $\hat{G}_{(2)} = 2F^S(x) - F^S(x)^2$ is the second-order statistic distribution of two i.i.d. variables with distribution $F^S(.)$.

Lemma 2 in Appendix A establishes that $G_{(2)}$ first-order stochastically dominates $\hat{G}_{(2)}$ (in the notation of the lemma, let $X_1 = X_1^S$, $\hat{X}_1 = M^{-1}(X_1^T)$, and $X_2 = X_2^S$). Thus, because $G_{(2)}(x) < \hat{G}_{(2)}(x)$ for every x for which $F(x) \notin \{0, 1\}$ and as β_S^{CE} is increasing, it follows that $\int_{\mathcal{X}_S} \beta_S^{CE}(x)d\hat{G}_{(2)}(x) < \int_{\mathcal{X}_S} \beta_S^{CE}(x)dG_{(2)}(x)$. If bidder 2 learns X_2^S, then bidder 1's expected payment conditional on winning is strictly lower in the deviation strategy than in the candidate equilibrium after learning X_1^S. Finally, by proposition 9, overall winning probability is $\frac{1}{2}$ in the candidate equilibrium and the deviation if bidder 2 learns X_2^S. Hence, the unconditional expected payment is also strictly lower in the deviation. □

Proposition 11 (Equal expected value.)**.** *Let $\{\sigma_{CE} > 0, \beta_S^{CE}, \beta_T^{CE}\}$ be a candidate equilibrium. In the candidate equilibrium and the deviation strategy, bidder 1's expected value when winning, $\mathbb{E}[v_1|\text{bidder 1 wins}]$, is identical.*

Proof. With probability $1 - \sigma_{CE}$, bidder 2 learns X_2^T. Bidder 1 with signal $X_1^S = x$ in the candidate equilibrium wins with the same probability as with $X_1^T = M(x)$ in the deviation.[33] There is no winners curse for bidder 1. By lemma 1, $\mathbb{E}[V_1|X_1^S = x] = \mathbb{E}[V_1|X_1^T = M(x)]$. Hence, the value of V_1 conditional on winning is the same due to the same marginal distribution of $M(X_1^S)$ and X_1^T.

With probability σ_{CE}, bidder 2 learns X_2^S. Then, in the candidate equilibrium, the expected object value for bidder 1 who learns X_1^S and wins is $\mathbb{E}[u(S)] + \mathbb{E}[w(T_1)]$, as bidders are symmetric and win with probability $\frac{1}{2}$ at every $s \in S$, and do not have any information about their private components.

In the deviation strategy, signals are independent and X_1^T is informative only about S, while X_2^S is informative only about S. As $M^{-1}(X_i^T)$ and X_i^S are distributed i.i.d.,

[33] Bidder 1 places the same bid with $X_1^S = x_1$ and $X_1^T = M(x_1)$, $\beta_S^{CE}(x_1) = \beta_T(M(x_1))$. $M(X_1^S)$, X_1^T, X_2^T are i.i.d., so $\Pr\left[\beta_S^{CE}(x_1) \geq \beta_T^{CE}(X_2^T)\right] = \Pr\left[\beta_T(M(x_1)) \geq \beta_T^{CE}(X_2^T)\right]$.

bidder 1's expected value of the private component when winning can be written as

$$\begin{aligned}\mathbb{E}\left[w(T_1)|\beta_T(X_1^T) \geq \beta_S^{CE}(X_2^S)\right] &= \int_{\mathcal{X}^S} \mathbb{E}\left[w(T_1)|X_1^T = M(x)\right] f^S(x)F^S(x)\, dx \\ &= \int_{\mathcal{X}^S} \mathbb{E}\left[u(S)|X_1^S = x\right] f^S(x)F^S(x)\, dx \\ &\quad + \mathbb{E}\left[w(T_1)\right] - \mathbb{E}\left[u(S)\right], \end{aligned} \tag{9}$$

where the last equality followed because

$$\begin{aligned} &\mathbb{E}\left[V_1 = u(S) + w(T_1)|X_1^S = x\right] = \mathbb{E}\left[V_1 = u(S) + w(T_1)|X_1^T = M(x)\right] \\ \Leftrightarrow \quad &\mathbb{E}\left[u(S)|X_1^S = x\right] + \mathbb{E}[w(T_1)] = \mathbb{E}\left[w(T_1)|X_1^T = M(x)\right] + \mathbb{E}[u(S)]. \end{aligned}$$

Note the similarity of (9) to bidder 2's common component value when winning:

$$\mathbb{E}\left[u(S)|\beta_S^{CE}(X_2^S) > \beta_T(X_1^T)\right] = \int_{\mathcal{X}^S} \mathbb{E}\left[u(S)|X_2^S = x\right] f^S(x)F^S(x)\, dx. \tag{10}$$

The object is always sold, the common component surplus to be divided between the bidders is $\mathbb{E}[u(S)]$, and each bidder wins with probability $\frac{1}{2}$ (see proposition 9) if bidder 2 learns X_2^S. Hence,

$$\mathbb{E}\left[u(S)\right] = \frac{1}{2}\mathbb{E}\left[u(S)|\underbrace{\beta_S^{CE}(X_2^S) \leq \beta_T(X_1^T)}_{\text{bidder 1 wins}}\right] + \frac{1}{2}\mathbb{E}\left[u(S)|\underbrace{\beta_S^{CE}(X_2^S) > \beta_T(X_1^T)}_{\text{bidder 2 wins}}\right].$$

This and equations (9) and (10) pin down bidder 1's value when deviating and winning,

$$\begin{aligned} &\mathbb{E}\left[u(S) + w(T_1)|\beta_T(X_1^T) \geq \beta_S^{CE}(X_2^S)\right] \\ =&2\mathbb{E}\left[u(S)\right] - \mathbb{E}\left[u(S)|\beta_S^{CE}(X_2^S) > \beta_T(X_1^T)\right] + \mathbb{E}\left[w(T_1)|\beta_S^{CE}(X_2^S \leq \beta_T(X_1^T)\right] \\ =&2\mathbb{E}\left[u(S)\right] + \mathbb{E}\left[w(T_1)\right] - \mathbb{E}\left[u(S)\right] = \mathbb{E}\left[u(S)\right] + \mathbb{E}\left[w(T_1)\right], \end{aligned}$$

which is the same expected value conditional on winning as in the candidate equilibrium when both bidders learn X_i^S. □

Together, Propositions 9, 10 and 11 establish that bidder 1 has a strictly profitable deviation in any candidate equilibrium with $\sigma_{CE} > 0$. If bidder 2 learns X_2^S (a positive probability event), then the deviation performs strictly better (lower expected payment and equal value of the object) than learning X_1^S in the candidate equilibrium. If bidder 2 learns X_2^T, the deviation strategy yields the candidate equilibrium payoff. □

Proof of Propositions 1, 2 and 3. These propositions are special cases of propositions 9, 10 and 11 in the proof of theorem 2. □

Proof of Theorem 3. By contradiction, assume there exists a symmetric candidate equilibrium with $\sigma > 0$ and bidding functions β_S and β_T. In what follows, I show that bidder 1 has a strictly profitable deviation. Denote bidder 1's expected utility after learning $X_1 \in \{X_1^T, \tilde{X}_1^T\}$ and bidding optimally as $EU_1^*[X_1]$.

Claim 1. $EU_1^*[X_1^T] \geq EU_1^*[\tilde{X}_1^T]$.

Proof. Let $T_1 = t_1$. Let $\Pr[1 \text{ wins}|b]$ be the probability that bidder 1 wins with a bid b when bidder 2 plays the candidate equilibrium. When winning with b, denote bidder 1's object value as $\mathbb{E}[u(S) + w(t_1)|b, 1 \text{ wins}] := w(t_1) + \mathbb{E}[u(S)|b, 1 \text{ wins}]$, and his expected payment as $\mathbb{E}[\text{pay}|b, 1 \text{ wins}]$. Then, bidder 1's expected utility when he bids b is

$$u_1(t_1, b) = (w(t_1) + \mathbb{E}[u(S)|b, 1 \text{ wins}] - \mathbb{E}[\text{pay}|b, 1 \text{ wins}]) \Pr[1 \text{ wins}|b]. \tag{11}$$

The above expression does not depend on whether bidder 1 learns X_1^T or $\tilde{X}_1^T$ and its realization. This is because with both signals, bidders' information is independent and hence, winning probability and inference about S only depends on the placed bid b.

Next, I show that $u_1(t_1, b)$ satisfies SCP in $(b; t_1)$, a prerequisite of theorem 1. To establish this, in section A.2 I showed that it is sufficient to show that (i) the function $f(t_1, b) := w(t_1) + \mathbb{E}[u(S)|b, 1 \text{ wins}] - \mathbb{E}[\text{pay}|b, 1 \text{ wins}]$ is supermodular (spm)[34] and nondecreasing in t_1, and (ii) $g(b) := \Pr[1 \text{ wins}|b]$ is nondecreasing and nonnegative. $\Pr[1 \text{ wins}|b]$ is nonnegative and nondecreasing as a higher bid is weakly more likely to win in an SPA. Hence, (ii) is satisfied. The function $f(t_1, b)$ is nondecreasing in t_1, as only the first term $w(t_1)$ depends on t_1 and is nondecreasing by assumption. Finally, none of the additive terms in f depend on both t_1 and b; it is straightforward that $f(.,.)$ is spm. Hence, (i) is satisfied. This establishes that $u_1(t_1, b)$ is SCP in $(b; t_1)$. Thus, theorem 1 can be applied: bidder 1 weakly prefers X_1^T to $\tilde{X}_1^T$ as $X_1^T \succ_{T_1} \tilde{X}_1^T$. □

Denote bidder 1's expected utility in the candidate equilibrium when learning X_1^S as $EU_1^{CE}[X_1^S]$.

Claim 2. $EU_1^*[\tilde{X}_1^T] > EU_1^{CE}[X_1^S]$.

Proof. As $X_1^S \sim_{V_1} \tilde{X}_1^T$, by lemma 1, there exists a transformation $M(.)$ which maps the common-component signal into a signal $\tilde{X}_1^T$. As was shown in the proof of theorem 2, instead of learning X_1^S and bidding $\beta_S(X_1^S)$ in the candidate equilibrium, bidder 1 is strictly better off learning $\tilde{X}_1^T$ and bidding $\beta_S(M^{-1}(\tilde{X}_1^T))$. This deviation is strictly profitable whenever bidder 2 learns X_2^S, and results in the same payoff whenever bidder 2 learns X_2^T.[35] Finally, note that the above deviation strategy is a lower bound for $EU_1^*[\tilde{X}_1^T]$, bidder 1's optimal payoff from $\tilde{X}_1^S$. This establishes the claim. □

Hence, bidder 1 is strictly better off learning X_1^T than $\tilde{X}_1^T$, and strictly better off learning $\tilde{X}_1^T$ instead of X_1^S (which he learns with strictly positive probably in the candidate equilibrium). Hence, by transitivity, $\sigma > 0$ cannot arise in equilibrium. □

[34]See Definition 3 in section A.2 in the appendix.

[35]This holds irrespective of bidder 2's bidding function as long as it is independent of bidder 1's private signal.

Proof of Proposition 4. By the same argument as in theorem 2 and section 4.1, bidders do not have a profitable deviation involving X_i^S. I show that learning Y_i^T cannot be part of a strictly profitable deviation. Bidder i's best-response bid with any payoff-irrelevant signal realization $Y_i^T = y$ is constant: $\beta(Y_i^T = y) = \mathbb{E}[V_i|Y_i^T = y] = \mathbb{E}[V_i]$. Bidder i can obtain the same payoff by learning X_i^T, and bidding $\beta(X_i^T) = \mathbb{E}[V_i]$ for any realization. Hence, his payoff after learning X_i^T and bidding optimally (as in the candidate equilibrium) is weakly higher, and no strictly profitable deviation exists. □

Proof of Proposition 5. Let there exist an equilibrium in the FPA with $\sigma = 0$. Without loss, consider bidder 1. By standard IPV arguments for the FPA, bidder 2 bids the symmetric equilibrium bidding function

$$\beta^{FPA}(X_2^T) = \mathbb{E}\left[\mathbb{E}[V_1|X_1^T]|\mathbb{E}[V_1|X_1^T] \leq \mathbb{E}[V_2|X_2^T]\right].$$

By the same argument, bidder 1's best-response bid when deviating to X_1^S is

$$\tilde{\beta}^{FPA}(X_1^S) = \mathbb{E}\left[\mathbb{E}[V_2|X_2^T]|\mathbb{E}[V_2|X_2^T] \leq \mathbb{E}[V_1|X_1^S]\right].$$

Now consider an IPV candidate equilibrium in the SPA with bidding function $\beta^{SPA}(X_i^T) = \mathbb{E}[V_i|X_i^T]$. By the same logic as in proposition 4, bidders do not have a strictly profitable deviation to learn Y_i^T, as it is payoff irrelevant, and leads to a constant best-response bid that a bidder could as well place after learning X_i^T.

It remains to be shown that learning X_i^S cannot be part of a strictly profitable deviation. Let bidder 1 deviate and learn X_1^S. Then, $\tilde{\beta}^{SPA}(X_1^S) = \mathbb{E}[V_1|X_1^S]$ is an optimal bid by a standard IPV argument. Note that the optimal expected payoff for each realization $X_1^S = x$ is the same in the SPA and the FPA because

1. winning probability is identical,[36]

$$\begin{aligned}\Pr\left[\tilde{\beta}^{FPA}(X_1^S = x) \geq \beta^{FPA}(X_2^T)\right] &= \Pr\left[\mathbb{E}[V_1|X_1^S = x] \geq \mathbb{E}[V_2|X_2^T]]\right] \\ &= \Pr\left[\tilde{\beta}^{SPA}(X_1^S = x) \geq \beta^{SPA}(X_2^T)\right]\end{aligned}$$

2. expected value if winning depends only on $X_1^S = x$, i.e., $E[V_1|X_1^S = x, 1 \text{ wins}] = E[V_1|X_1^S = x]$
3. expected payment when winning is identical,

$$\begin{aligned}\mathbb{E}[\beta^{SPA}(X_2^T)|\beta^{SPA}(X_2^T) \leq \tilde{\beta}^{SPA}(X_1^S = x)] &= \mathbb{E}\left[\mathbb{E}[V_2|X_2^T]|\mathbb{E}[V_2|X_2^T] \leq \mathbb{E}[V_1|X_1^S = x]\right] \\ &= \tilde{\beta}^{FPA}(X_1^S = x).\end{aligned}$$

Hence, the deviation payoff after learning X_1^S and bidding optimally coincides in the FPA and the SPA. By revenue equivalence, bidder 1 obtains the same payoff in the

[36]For $X_1^S = x$ and $X_1^T = y$ such that $\mathbb{E}[V_1|X_1^S = x] = \mathbb{E}[V_1|X_1^T = y]$, bidder 1 places the same bid in the deviation and the candidate equilibrium. By symmetry, $\mathbb{E}[V_1|X_1^T = y] = \mathbb{E}[V_2|X_2^T = y]$. Thus, bidder 1 with $X_1^S = x$ wins if $X_2^T \leq y$, or equivalently, if $\mathbb{E}[V_2|X_2^T] \leq \mathbb{E}[V_2|X_1^T = y] = \mathbb{E}[V_1|X_1^S = x]$.

IPV (candidate) equilibrium of the FPA and the SPA. By assumption, deviating to X_1^S is not a strictly profitable deviation in the FPA. As shown above, this deviation has the same payoff in the SPA, and is thus also not a strictly profitable deviation.

Finally, I show that the existence of an IPV equilibrium in the SPA does not imply the existence of an IPV equilibrium in the FPA. Example 4 in the main text provides a counterexample which relies on perfectly revealing signals, and thus does not satisfy assumption 2. Consider the following variation on this example with noisy signals satisfying the requirements of the model, $X_i^S \sim \mathcal{N}(S, \sigma^2)$ and $X_i^T \sim \mathcal{N}(T_i, \sigma^2)$. For any $\epsilon > 0$, there exists a σ^2 sufficiently small such that after bidder 1 learns Y_1^T, then he knows with probability approaching one that bidder 2's value falls within the ϵ-interval of $E[V_2|Y_1^T]$. Then, bidder 1 is almost sure of bidder 2's bid and can outbid him by bidding $\mathbb{E}[V_2|Y_1^T] + \epsilon$. Thus, as $\sigma^2 \to 0$, bidder 1's deviation is strictly profitable as it approaches the strictly profitable full information benchmark. □

Proof of Theorem 4. Existence follows by the same argument as in section 4.1. The following notation for a bidder's signal distribution with information choice ρ_i will be used in the proof. Let the (full) support of $X_i^{\rho_i}$ be $\mathcal{X}^{\rho_i}$. Let $f^{\rho_i}(x|s,t)$ be the density of bidder i's signal with components $S = s$ and $T_i = t$. Let $f_S^{\rho_i}(x|s) := \int_{\mathcal{T}} f^{\rho_i}(x|s,t)h_t(t)dt$ be the marginal density of $X_i^{\rho_i}$ given S, and $F_S^{\rho_i}(x|s)$ the distribution. Similarly, let $f_T^{\rho_i}(x|t)$ be the marginal density of $X_i^{\rho_i}$ given $T_i = t$, and $F_T^{\rho_i}(x|t)$ the distribution. Let $f^{\rho_i}(x)$ ($F^{\rho_i}(x)$) be the marginal density (distribution).

Assume by contradiction that there exists a candidate equilibrium with $\rho > 0$ and $\beta(X_i^\rho)$. I show that bidder 1 has a strictly profitable deviation: $\rho_1 = 0$ and $\hat{\beta}(X_1^0) = \beta(M^{-1}(X_1^0))$ where M is the invertible bijection $M : X_1^\rho \to X_1^0$. Such a map exists because lemma 1 extends to $X_1^\rho \sim_{V_1} X_1^0$ (by substituting X_1^S with X_1^ρ in the proof). Similarly to the proof of theorem 2, I show that when deviating from the candidate equilibrium (i) the winning probability is the same; (ii) the expected payment is strictly lower; (iii) the expected object value when winning is weakly higher.

Proof of (i). In the symmetric candidate equilibrium, winning probability is $\frac{1}{2}$. In the deviation strategy, bidder 1 wins if $X_1^0 \geq M(X_2^\rho)$. Using the same argument as in lemma 1, these two random variables are i.i.d. Hence, winning probability is $\frac{1}{2}$.

Proof of (ii). This follows from the same arguments as proposition 10. In the candidate equilibrium, both bidders have the same marginal bid distribution which is correlated via the common component. In the deviation strategy, bidders have the same bid distributions but the bids are now independent. The distribution of the second-order statistic under correlation strictly first-order stochastically dominates that under independence. This is a general statistical property which I establish in lemma 2. A winning bidder pays the second-order statistics of the two bids. Hence, expected payment conditional on winning is strictly lower in the deviation.

Proof of (iii). Let bidder 2 play the candidate equilibrium. Then, denote bidder 1's net gain in private-component value when winning from deviating from the candidate equilibrium as $\Delta_1^T := \mathbb{E}[w(T_1)|1 \text{ wins}, \rho_1 = 0] - \mathbb{E}[w(T_1)|1 \text{ wins}, \rho_1 = \rho]$. Analogously, define $\Delta_i^S := \mathbb{E}[u(S)|i \text{ wins}, \rho_1 = 0] - \mathbb{E}[u(S)|i \text{ wins}, \rho_1 = \rho]$ for bidder i for the common component. By the same argument as in proposition 9, the object is always sold, and both bidders win with probability 1/2. Therefore,

$$\begin{aligned}\mathbb{E}[u(S)] &= \frac{1}{2}\mathbb{E}[u(S)|1 \text{ wins}, \rho_1 = \rho] + \frac{1}{2}\mathbb{E}[u(S)|2 \text{ wins}, \rho_1 = \rho] \\ &= \frac{1}{2}\mathbb{E}[u(S)|1 \text{ wins}, \rho_1 = 0] + \frac{1}{2}\mathbb{E}[u(S)|2 \text{ wins}, \rho_1 = 0].\end{aligned}$$

Hence, $\Delta_1^S = -\Delta_2^S$. Using this observation, I prove in the following that bidder 1's deviation is strictly profitable as $\Delta_1^T + \Delta_1^S = \Delta_1^T - \Delta_2^S \geq 0$.

In the candidate equilibrium, due to symmetry, both bidders win at every s with probability 1/2. Hence, $\mathbb{E}[u(S)|i \text{ wins}, \rho_1 = \rho] = \mathbb{E}[u(S)]$ and

$$\begin{aligned}\Delta_1^T - \Delta_2^S =& \mathbb{E}[w(T_1)|1 \text{ wins}, \rho_1 = 0] - \mathbb{E}[w(T_1)|1 \text{ wins}, \rho_1 = \rho] \\ &- \mathbb{E}[u(S)|2 \text{ wins}, \rho_1 = 0] + \mathbb{E}[u(S)].\end{aligned} \tag{12}$$

Let $g_1(x;\rho)$ be the density of the first-order statistic of the two signals X_1^ρ and X_2^ρ. Conditional on winning in the candidate equilibrium, bidder 1's signal when winning is distributed with density g_1. We can write

$$\begin{aligned}\mathbb{E}[w(T_1)|1 \text{ wins}, \rho_1 = \rho] &= \int_{\mathcal{X}^\rho} \mathbb{E}[w(T_1)|X_1^\rho = x] g_1(x;\rho) dx \\ &\leq \int_{\mathcal{X}^\rho} \mathbb{E}[w(T_1)|X_1^\rho = x] 2f^\rho(x)F^\rho(x)dx,\end{aligned} \tag{13}$$

where $2f^\rho(x)F^\rho(x)$ is the density of the first-order statistic of two i.i.d. random variables with marginal distribution $F^\rho(x)$. The inequality in the last step followed from lemma 2: the first-order statistic of two i.i.d. variables first-order stochastically dominates that of correlated variables with the same marginal distribution, and $\mathbb{E}[w(T_1)|X_1^\rho = x]$ is nondecreasing in x. As $F^\rho(x) = F^0(M(x))$, we can write

$$\begin{aligned}\mathbb{E}[w(T_1)|1 \text{ wins}, \rho_1 = 0] &= \int_{\mathcal{X}^\rho} \mathbb{E}[w(T_1)|X_1^0 = M(x)] 2f^\rho(x)F^\rho(x)dx. \\ \mathbb{E}[u(S)|2 \text{ wins}, \rho_1 = 0] &= \int_{\mathcal{X}^\rho} \mathbb{E}[u(S)|X_2^\rho = x] 2f^\rho(x)F^\rho(x)dx. \\ \mathbb{E}[u(S)|2 \text{ wins}, \rho_1 = \rho] =& \mathbb{E}[u(S)] = \int_{\mathcal{X}^\rho} \mathbb{E}[u(S)] 2f^\rho(x)F^\rho(x)dx.\end{aligned}$$

Plugging this and (13) back into (12) yields

$$\begin{aligned}\Delta_1^T - \Delta_2^S \geq \int_{\mathcal{X}^\rho} (&\mathbb{E}[w(T_1)|X_1^0 = M(x)] - \mathbb{E}[w(T_1)|X_1^\rho = x] \\ &- \mathbb{E}[u(S)|X_2^\rho = x] + \mathbb{E}[u(S)]) 2f^\rho(x)F^\rho(x)dx = 0.\end{aligned}$$

The last equality followed by symmetry, $\mathbb{E}[V_1|X_1^\rho = x] = \mathbb{E}[V_2|X_2^\rho = x]$, and by the equal accuracy assumption, as for every $x \in \mathcal{X}^\rho$,

$$\begin{aligned} & \mathbb{E}[V_1|X_1^0 = M(x)] = \mathbb{E}[V_1|X_1^\rho = x] \\ \Leftrightarrow \quad & \mathbb{E}[u(S)] + \mathbb{E}[w(T_1)|X_1^0 = M(x)] = \mathbb{E}[u(S)|X_1^\rho = x] + \mathbb{E}[w(T_1)|X_1^\rho = x]. \qquad \square \end{aligned}$$

Proof of Proposition 6. Part 1. Let $c_s(a) \geq c_T(a)$ for all $a \in [0,1]$. First, by contradiction, let there exist an equilibrium in pure CV with $\sigma^{SPA} = 1$ in which bidders choose $\rho^{SPA} = a$ and learn $X_i^{S,a}$. By the same argument as in theorem 2, and while ignoring the monetary costs of information, a bidder has a strictly profitable deviation to $X_i^{T,a}$. In addition, such information is cheaper, $c_S(a) \geq c_T(a)$, and thus, it is a strictly profitable deviation. Hence, $\sigma^{SPA} = 0$ in any symmetric pure equilibrium.

Consider an IPV candidate equilibrium in the SPA in which bidders learn about the private component with precision τ^{SPA}. This is the equilibrium accuracy level if only private-component experiments were available, so by construction no bidder has a profitable deviation to another private-component signal. Furthermore, for any experiment, a bidder's signal is independent of the information and the bid of his opponent. Hence, by the same argument as before, experiment $X_i^{S,a}$ and experiment $X_i^{T,a}$ yields the same payoff when bidding optimally. This is because both result in the same distribution of the posterior and the same optimal bid. Hence, in an SPA, a bidder also has no profitable deviation from learning with any available precision about the common component. The exact same argument for a candidate equilibrium with τ^{FPA} and $\sigma^{FPA} = 0$ establishes existence of an IPV equilibrium in the FPA.

The proof of $\tau^{SPA} = \tau^{FPA}$ is a corollary of a result in Persico (2000): For the IPV case, the variable V_i in Persico (2000) corresponds to variable T_i in my model. No bidder acquires any information about the common component, hence, signals are independent for any precision $\tau \in [0,1]$. The common component is a mere normalization in the expected payoff. Hence, information choice about the private component in my model is a special case of Persico's one-dimensional information choice. Hence, his result (Proposition 2, Fact 2 and Footnote 10 for independent components) applies. In his terminology, the expected payoff in the SPA and the FPA is equally risk sensitive, and hence, equilibrium accuracy is the same.

Part 2. Let $c_s(a) < c_T(a)$ for all $a \in [0,1]$. Assume by contradiction that there is an IPV equilibrium in the SPA in which both bidder learn $X_i^{T,a}$ for some $\sigma = a$ and get independent signals. By assumption, experiment $X_i^{T,a} \sim_{V_i} X_i^{S,a}$. Thus, $X_i^{S,a}$ yields the same information about V_i, leads to the same distribution of best response bids, and yields the same payoff as when learning $X_i^{T,a}$ and bidding optimally. However, $X_i^{S,a}$ is strictly cheaper than $X_i^{T,a}$ and hence, a strictly profitable deviation. The exact same argument rules out an IPV equilibrium in the FPA.

The proof of $\rho^{SPA} \leq \rho^{FPA}$ in any pure CV equilibrium follows from Persico (2000). For the pure CV case, V_i in Persico (2000) corresponds to S in my model. As bidders acquire no information about their T_i in a pure CV equilibrium, the expectation of the private component is just a normalization in the payoff. Hence, once again, information choice just about one component is a special case of the one-dimensional learning problem in Persico (2000). Hence, his result (Proposition 2 and Fact 2) applies to my framework: equilibrium accuracy is weakly higher in the FPA than in the SPA. □

Proof of Proposition 7. Without loss, consider bidder 1. If all other bidders learn $X^T_{i\neq 1}$, then both experiments available to bidder 1 result in independent signals and the same distribution of the posterior value because $X^T_1 \sim_{V_1} X^S_1$. Thus, learning X^S_1 and bidding optimally results in the same payoff as learning X^T_1 and bidding optimally. Thus, X^S_1 cannot be part of a strictly profitable deviation. After learning X^T_i, it is well known that bidding the expected value is a weakly dominant strategy in the SPA. □

Proof of Proposition 8. The following properties are used in the proof and follow from $X^S_i \sim_{V_i} X^T_i$ and lemma 1. For any realization $X^S_1 = x$, there exists $X^T_1 = M(x)$ such that $F^S(x) = F^T(M(x))$. By equally accuracy, $\mathbb{E}[V_i|X^S_i = x] = \mathbb{E}[V_i|X^T_i = M(x) = y]$. For this to hold and as by assumption, $\Pr(S = 1) = \Pr(T_i = 1)$, it holds that $F^S(X^S_i = x|S = 1) = F^T(X^T_i = y|T_i = 1)$ and $F^S(X^S_i = x|S = 0) = F^T(X^T_i = y|T_i = 0)$. Let $\mathbf{v}_0 := \mathbb{E}[V_i|S = 0, T_i = 0]$, $\mathbf{v}_2 := \mathbb{E}[V_i|S = 1, T_i = 1]$, and $\mathbf{v}_1 := \mathbb{E}[V_i|S = 1, T_i = 0] = \mathbb{E}[V_i|S = 0, T_i = 1]$. Finally, due to symmetry, $\mathbb{E}[V_i] = \mathbf{v}_1$.

By contradiction, consider a pure CV candidate equilibrium where each bidder learns X^S_i and bids $\beta(X^S_i)$. I show that bidder 1 has a strictly profitable deviation: learn X^T_1 and bid $\beta(X^T_1) = \beta_S(M^{-1}(X^T_1))$. Let $Y_{(1)} = \max\{X^S_2, ..., X^S_N\}$ be the highest common-component signal of all other bidders. Let $G^S_{(1)}(x|s) := \Pr(Y_{(1)} \leq x|S = s) = F^S(y|S = 1)^{N-1}$ be the distribution of $Y_{(1)}$, and $g_{(1)}$ the corresponding density.

Let $V_1 = \mathbf{v}_0$. Then, $S = 0$ and $T_1 = 0$. In the candidate equilibrium and the deviation, bidder 1 has the same marginal bid distribution $F^S(x|S = 0) = F^T(M(x) = y|T = 0)$, so the deviation has no effect on the payoff which is in both cases

$$\int_{\mathcal{X}^S} [\mathbf{v}_0 - \beta_S(x)]\, g^S_{(1)}(x|0)[1 - F^S(x|0)]dx.$$

Similarly, if $V_1 = \mathbf{v}_2$ ($S = 1$ and $T_1 = 1$), deviating has no effect on the payoff. For the remaining case, if $V_1 = \mathbf{v}_1$, then in the candidate equilibrium the expected payoff is

$$\int_{\mathcal{X}^S} [\mathbf{v}_1 - \beta_S(x)] \left[\frac{1}{2} g^S_{(1)}(x|1)[1 - F^S(x|1)]dx + \frac{1}{2} g^S_{(1)}(x|0)[1 - F^S(x|0)]\right] dx. \tag{14}$$

The expected payoff in the deviation is

$$\begin{aligned}
&\frac{1}{2}\int_{\mathcal{X}^S} [\mathbf{v}_1 - \beta_S(x)]\, g^S_{(1)}(x|1)[1 - F^T(M(x)|1)]dx \\
&\quad + \frac{1}{2}\int_{\mathcal{X}^S} [\mathbf{v}_1 - \beta_S(x)]\, g^S_{(1)}(x|0)[1 - F^T(M(x)|1)]dx \\
&= \int_{\mathcal{X}^S} [\mathbf{v}_1 - \beta_S(x)] \left[\frac{1}{2} g^S_{(1)}(x|1)[1 - F^S(x|0)]dx + \frac{1}{2} g^S_{(1)}(x|0)[1 - F^S(x|1)]\right] dx. \qquad (15)
\end{aligned}$$

Next, I show that the payoff with the deviation is higher with $\mathbf{v}_1$. Note that (15) minus (14) (the net payoff from the deviation) can be written as

$$\frac{1}{2}\int_{\mathcal{X}^S}[\beta_S(x)-\mathbf{v}_1]\left(g^S_{(1)}(x|0)-g^S_{(1)}(x|1)\right)\left(F^S(x|1)-F^S(x|0)\right)dx. \tag{16}$$

Let $\alpha(x) \coloneqq \left(g^S_{(1)}(x|0)-g^S_{(1)}(x|1)\right)\left(F^S(x|1)-F^S(x|0)\right)$. It captures the difference in winning probability between the candidate equilibrium and the deviation for bidder 1. Next, I show that by deviating, bidder 1 is more likely to win.

Lemma 5. $\int_{\mathcal{X}^S}\alpha(x)dx<0$.

Proof. Using integration by parts, we can write

$$\begin{aligned}\int_{\mathcal{X}^S}\alpha(x)dx &= -\int_{\mathcal{X}^S}\left(f^s(x|1)-f^S(x|0)\right)\left(F^S(x|0)^{N-1}-F^S(x|1)^{N-1}\right)dx\\ &= +\frac{2}{N}-\left[\frac{1}{N}+\frac{N-1}{N}\right]\int_{\mathcal{X}^S}\left(f^S(x|1)F^S(x|0)^{N-1}+f^S(x|0)F^S(x|1)^{N-1}\right)dx\end{aligned} \tag{17}$$

Integrating both terms of the middle $\frac{1}{N}$-term by parts yields

$$\begin{aligned}&\frac{1}{N}\int_{\mathcal{X}^S}\left(f^S(x|1)F^S(x|0)^{N-1}+f^S(x|0)F^S(x|1)^{N-1}\right)dx\\ &=\frac{2}{N}-\frac{N-1}{N}\int_{\mathcal{X}^S}f^S(x|1)F^S(x|1)^{N-2}F^S(x|0)+f^S(x|0)F^S(x|0)^{N-2}F^S(x|1)dx.\end{aligned}$$

Plugging this into (17) for the $\frac{1}{N}$-term, the expression simplifies to

$$-\frac{N-1}{N}\int_{\mathcal{X}^S}\left(F^S(x|0)^{N-2}-F^S(x|1)^{N-2}\right)\left(f^S(x|1)F^S(x|0)-f^S(x|0)F^S(x|1)\right)dx$$

By $N\geq 3$ and the strong MLRP, for all interior x, $F^S(x|0)^{N-2}-F^S(x|1)^{N-2}>0$ and $\frac{f^S(x|1)}{F^S(x|1)}>\frac{f^S(x|0)}{F^S(x|0)}$ (reverse-hazard-rate dominance). Hence, $\int_{\mathcal{X}^S}\alpha(x)dx<0$. $\square$

Lemma 6. *There exists $\underline{x},\overline{x}$ with $\underline{x}\leq\overline{x}$ such that*

1. *$\alpha(x)$ crosses zero exactly once from below at some $\overline{x}\leq\hat{x}$ where $f^S(\hat{x}|1)=f^S(\hat{x}|0)$.*
2. *$\int_{\mathcal{X}^S\geq\underline{x}}\alpha(x)dx=0$.*

Proof. By Theorem 2 and 4 in Milgrom and Weber (1982), $Y_{(1)}$ and S are affiliated (due to conditional independence of all X_i^S given S). Hence, $\frac{g_{(1)}(x|1)}{g_{(1)}(x|0)}$ is nondecreasing in x. MLRP implies first-order stochastic dominance, $F^S(x|1)\leq F^S(x|0)$. This establishes the single-crossing property of $\alpha(x)$. As $\int_{\mathcal{X}^S}\alpha(x)dx\leq 0$ by lemma 5, and $\alpha(x)$ crosses zero once, there exists $\underline{x}\leq\overline{x}$ (the crossing point) as in the Lemma.

Because the signal X_i^S has the MLRP, and $\int_{\mathcal{X}^S}f^S(x|s)dx=1$, there exists a 'neutral' signal $\hat{x}$ such that $f^S(\hat{x}|1)=f^S(\hat{x}|0)$. At this signal $\hat{x}$, $\alpha(\hat{x})\geq 0$ because $F^S(\hat{x}|1)\leq$

$F^S(\hat{x}|0)$ and

$$\begin{aligned} g^S_{(1)}(\hat{x}|0) - g^S_{(1)}(\hat{x}|1) =& (N-1)\left[f^S(\hat{x}|1)F^S(\hat{x}|1)^{N-2} - f^S(\hat{x}|0)F^S(\hat{x}|0)^{N-2}\right] \\ \leq & (N-1)F^S(\hat{x}|0))^{N-2}\left[f^S(\hat{x}|1) - f^S(\hat{x}|0)\right] = 0. \end{aligned}$$

Hence, $\hat{x} \geq \overline{x}$: $\alpha(x)$ crosses zero at some realization below $\hat{x}$. □

Let $\gamma(x) \coloneqq \beta_S(x) - \mathbf{v}_1$, which is nondecreasing in x. Using this, (16) can be written as two sums,

$$\frac{1}{2}\int_{\mathcal{X}^S<\underline{x}} \gamma(x)\alpha(x)dx + \frac{1}{2}\int_{\mathcal{X}^S\geq\underline{x}} \gamma(x)\alpha(x)dx.$$

The first summand is strictly positive. This is because by lemma 6, $\alpha(x) < 0$ as $\underline{x} \leq \overline{x}$ where the crossing at zero from below occurs. In addition, the bidding function in an SPA is $\beta_S(x) = \mathbb{E}[V_i|X_i^S = x, Y_{(1)} = x]$ and increasing in x by Milgrom and Weber (1982). Thus, at any $x \leq \underline{x}$, $\beta_S(x) \leq \beta_S(\overline{x}) \leq \beta_S(\hat{x}) = \mathbb{E}[V_i|X_i^S = \hat{x}, Y_{(1)} = \hat{x}] \leq \mathbb{E}[V_i] = \mathbf{v}_1$. The second to last inequality followed because any signal lower than the neutral signal $\hat{x}$ is bad news for the value of the object. Thus, for any $x \leq \underline{x}$, $\gamma(x)\alpha(x) \geq 0$, and for a positive measure of x, $\gamma(x)\alpha(x) > 0$.

The following follows from lemma 1 in Persico (2000), the proof is therefore omitted.

Lemma 7. *Let $\int_{\mathcal{X}} \alpha(x)dx = 0$, cross zero once from below and $\gamma(x)$ be a nondecreasing function. Then, $\int_{\mathcal{X}} \alpha(x)\gamma(x) \geq 0$.*

As I established in lemma 6, all these assumptions are satisfied by $\alpha(.)$ and $\gamma(.)$ for $\mathcal{X}^S \geq \underline{x}$. Hence, the second summand is nonnegative. This establishes that the deviation payoff is positive for $N \geq 3$. The result for $N = 2$ follows by theorem 2. □