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Abstract

We solve a real business cycle model with rational inattention (an RI-RBC model). In the standard model, news about future productivity fails to cause business cycle comovement. In response to a positive news shock, consumption rises but employment and investment fall. Introducing rational inattention helps produce comovement. Agents choose an optimal signal about the state of the economy. The optimal signal turns out to confound current with expected future productivity. Labor and investment demand rise after a news shock, causing an output expansion. Rational inattention also improves the propagation of a standard productivity shock, by inducing persistence.

JEL Classification: D83, E32, E71

Keywords: Information Choice, rational inattention, Real business cycle model, news shock, Productivity Shocks

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Rational Inattention and the Business Cycle Effects of Productivity and News Shocks*

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Abstract

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1 Introduction

The basic challenge for any business cycle model is to specify an impulse and a propagation mechanism that produce business cycle comovement. This challenge is difficult, as Barro and King (1984) first explained.[1] A key insight from the real business cycle model is that fluctuations in productivity generate comovement in the standard neoclassical economy. Employment, investment, output, and consumption move together after a productivity shock, as they do in the data in a business cycle expansion or contraction.[2]

However, this insight is sensitive to the timing of information in the model. In the real world, information about changes in productivity may become available some time before they occur. In the model, it matters if agents can learn in advance about changes in productivity. If agents can learn in advance, variables respond in ways inconsistent with a business cycle. Suppose that productivity will rise in the future (while current productivity is unchanged). The news causes a wealth effect. Firms have no incentive to increase labor demand before productivity improves, while households reduce labor supply due to the wealth effect. As a result, hours worked fall. With capital predetermined and current productivity unchanged, output contracts. Lower saving due to the wealth effect causes a reduction in the capital stock over time. Investment declines while consumption rises. The model fails to produce comovement in response to news about future productivity. It predicts an output contraction after news that productivity will improve.[3]

It is convenient to model the arrival of news about future productivity as "news shocks about productivity" ("news shocks" for short). A shock drawn by nature in quarter t affects productivity in quarter $t + h$, where h is a strictly positive integer. The question is how the economy responds to a news shock before quarter $t + h$. In the standard neoclassical model, labor input, investment, and output fall while consumption rises. Labor input, investment, and output increase only once productivity improves. In the New Keynesian model, each firm commits to supply output at a

[1] Much more recently, Jaimovich and Rebelo (2009), p.1097, write that "the ability to generate comovement is a natural litmus test for macroeconomic models. It is a test that most models fail."

[2] Kydland and Prescott (1982), Hansen (1985), Prescott (1986), and King, Plosser, and Rebelo (1988) are classic references on the RBC model.

[3] With a high elasticity of intertemporal substitution, the model predicts a rise in employment and investment and a fall in consumption. The substitution effect due to an increase in the real interest rate dominates the wealth effect in this case, pushing consumption down and labor supply up.

fixed price, and therefore a rise in consumption exerts upward pressure on the demand for labor and investment. The response of the economy to a news shock depends on monetary policy. With optimal monetary policy the response is identical to the flexible-price neoclassical benchmark. To obtain comovement after a news shock, one can make particular assumptions about production and preferences in the neoclassical model.[4] Alternatively, one can combine imperfect competition and nominal stickiness with suboptimal monetary policy.[5]

This paper takes a different approach, based on a single new assumption. We think that how economic agents respond to news is fundamentally a question about expectation formation. Information about changes in productivity may be available in advance, but people may fail to absorb it completely. We introduce rational inattention to an otherwise standard RBC economy and ask how this single friction changes the propagation of a news shock. Rational inattention is the idea that decision-makers cannot process all available information and they allocate attention optimally (Sims, 2003). In a rational inattention model, an agent chooses an optimal signal about the state of the economy, recognizing that a more informative signal requires more attention, which is costly. The agent takes actions based on the optimal signal, rather than based on perfect information or some exogenous incomplete information set. How does a news shock propagate when people have a limited ability to process information and can choose what information to absorb?

We consider a baseline RBC model. Neoclassical firms produce homogeneous output with capital and labor. There are no adjustment costs. Households have standard preferences for consumption and leisure. The perfect information equilibrium is familiar. We focus on the equilibrium when firms are subject to rational inattention and households have perfect information.[6]

The main qualitative insight from the paper is that rational inattention induces an increase in the firms' demand for labor and investment on impact of a positive news shock. The reason is that the optimal signal confounds current with future productivity.[7] Thus, firms react on impact

[4] See Jaimovich and Rebelo (2009) and Beaudry and Portier (2004, 2007), which we summarize below.

[5] With suboptimal monetary policy a standard New Keynesian model (Smets and Wouters, 2007) produces comovement after news about future productivity. The same is true in a heterogeneous agent version of the model (we thank Christian Wolf for these observations). For a review of the literature on news shocks, see Lorenzoni (2011), Beaudry and Portier (2014), and Jaimovich (2017).

[6] We add rational inattention on the side of households in a later section of the paper.

[7] In Lucas (1972) firms are assumed to observe a one-dimensional signal about nominal aggregate and relative demand. In the rational inattention RBC model with news shocks, firms choose to observe a one-dimensional signal

of a news shock as if productivity has already changed with some probability. The intuition for the optimality of a confounding signal is that, under rational inattention, it is optimal to get low-dimensional signals and it is optimal to get shocks into beliefs early. A signal with a dimension strictly smaller than the dimension of the state of the economy saves on attention. As an example, if the optimal action under perfect information depends on productivity which follows an ARMA(1,1) process $a_t = \rho a_{t-1} + \sigma \varepsilon_{t-1}$ (a simple news shock specification), then a one-dimensional signal saves on attention compared with a two-dimensional signal on both elements of the state vector, $(a_t, \varepsilon_t)'$. Next, compare the action based on a signal on current productivity with the action based on a signal that confounds current with future productivity (a signal on a linear combination of a_t and ε_t in this example). In the former case, the action does not react on impact of the news shock and underreacts once productivity changes, due to the noise in the signal. In the latter case, the action *overreacts* on impact of the news shock and underreacts *by less* once productivity changes. With a quadratic loss from suboptimal actions, the combination of overreaction on impact and less underreaction later on is preferable: the initial overreaction introduces a small additional loss while the subsequent smaller underreaction causes large gains.

The main quantitative insight from the paper is that the rational inattention effect on labor and investment demand is strong enough to change the responses of employment and output on impact of a news shock from negative to positive, and the response of investment from negative to zero. The rational inattention effect on labor demand more than offsets the wealth effect on labor supply. Thus, employment and output increase on impact of a news shock. The rational inattention effect on investment demand offsets the wealth effect on saving supply. As a result, the response of investment on impact of a news shock equals zero, as opposed to a sizable negative number in the standard model. To arrive at these quantitative results, we solve a dynamic stochastic general equilibrium model with rational inattention, which is a non-trivial task.

Hence, the single assumption of rational inattention by firms makes the model predict an output expansion after news that productivity will improve. By assuming that households have perfect information, we stack the deck against us, because in this case the wealth effect that reduces the supply of labor and saving is fully operating. We also solve a version of the model with both firms and households subject to rational inattention. We find that comovement strengthens.

about current and future productivity.

Once we have introduced rational inattention to the neoclassical economy, we can also ask if the new RI-RBC model improves the propagation of a standard productivity shock (a shock that affects productivity in the same period in which the shock is drawn). In the literature it has been standard to compare unconditional second moments in the RBC model and in the data. It has been a challenge for the model to reproduce the persistence in the data. The first-order autocorrelations of employment, investment, and output growth are positive in the data but zero or negative in the baseline model.[8] We find that when firms are subject to rational inattention, the impulse responses of employment, investment, and output to a productivity shock become hump-shaped. Since the optimal signal contains noise, the firms' beliefs are anchored on the steady state and evolve slowly. As a result, employment, investment, and output respond with delay to a productivity shock. The first-order autocorrelations of employment, investment, and output growth in the model become positive and are approximately in line with the data. This finding holds true even though rational inattention is the only source of inertia and the marginal cost of attention is small.

The literature has explored different ways to obtain a model that predicts comovement in response to news about future productivity. Jaimovich and Rebelo (2009) modify the baseline RBC model by adding three assumptions: investment adjustment costs, variable capital utilization, and a new class of preferences. Investment adjustment costs and variable capital utilization produce a change in input demand in response to a news shock, whereas the new preferences control the wealth effect on input supply.[9] Beaudry and Portier (2004, 2007) move to a multi-sector neoclassical setting. They introduce a complementarity so that higher output in one sector makes production more efficient in other sectors, leading to a rise in input demand. Another approach has been to combine imperfect competition and nominal stickiness with suboptimal monetary policy. Lorenzoni (2009) analyzes a New Keynesian economy with a Taylor rule where noise in a public signal about productivity causes comovement.[10] By contrast, we explore how a single new assumption, rational inattention, changes the propagation of a news shock in the baseline RBC model. The assumption

[8] This shortcoming of the RBC model was first noted by Cogley and Nason (1995) and Rotemberg and Woodford (1996).

[9] Schmitt-Grohé and Uribe (2012) estimate a related augmented RBC model.

[10] Angeletos and La'O (2010) study a neoclassical model with strategic complementarity and dispersed information in which a similar noise shock causes comovement. On news and noise see also Blanchard, L'Huillier, and Lorenzoni (2013) and Chahrour and Jurado (2018).

of rational inattention seems well suited to apply to the question if people have an incentive to be perfectly aware of the timing of productivity changes.

A vast empirical literature finds that a sizable fraction of movements in total factor productivity is forecastable.[11] Authors make different assumptions to identify a shock that moves TFP a lot in the future and little, or not at all, on impact. In an influential paper, Beaudry and Portier (2006) show that two alternative identification assumptions in a vector autoregression both yield the result that news shocks cause business cycle comovement. The subsequent research pursues three additional approaches to identification. Papers that use patent data, either as a variable in a VAR or as an external instrument, find that news shocks produce comovement.[12] Papers that identify news shocks using the max-share method of Francis et al. (2014) reach, to some extent, conflicting conclusions. The results depend on the sample period. Barsky and Sims (2011) and Kurmann and Sims (2021) do not find comovement after a news shock in the U.S. data since 1960, while Görtz, Tsoukalas, and Zanetti (2020) who focus on the data since the onset of the Great Moderation do find comovement.[13] Finally, Chahrour and Jurado (2021) identify a fundamental shock to TFP and report that macroeconomic variables exhibit business cycle comovement in anticipation of that shock.

Our model suggests that empirical researchers who study different sample periods can be expected to reach conflicting conclusions regarding comovement. Whether a news shock produces comovement in the model depends on macroeconomic volatility. In a low volatility environment (think of the Great Moderation) agents pay little attention to the macroeconomy, and news shocks cause positive comovement of consumption and labor input because the aforementioned rational inattention effects are strong. In a high volatility environment (think of the period before the Great Moderation) agents pay more attention, and news shocks cause negative comovement because the

[11] Beaudry and Portier (2006), Barsky and Sims (2011), Barsky, Basu, and Lee (2015), Görtz, Tsoukalas, and Zanetti (2020), Miranda-Agrippino, Hacioglu-Hoke, and Bluwstein (2020), Chahrour and Jurado (2021), Kurmann and Sims (2021), Cascaldi-Garcia and Vukotić (2022), and others.

[12] Miranda-Agrippino, Hacioglu-Hoke, and Bluwstein (2020), Cascaldi-Garcia and Vukotić (2022).

[13] The details of the identification assumptions are different in Barsky and Sims (2011), Kurmann and Sims (2021), and Görtz, Tsoukalas, and Zanetti (2020). Görtz, Tsoukalas, and Zanetti (2020), Appendix A.2, show that when they use the identification assumptions of Barsky and Sims (2011) or Kurmann and Sims (2021) and focus on the data since the onset of the Great Moderation, they find that news shocks produce comovement. See also Görtz, Gunn, and Lubik (2022).

equilibrium is closer to the perfect information equilibrium. We illustrate this prediction of the model in an experiment in which we change the volatility of the productivity process. As we discuss, data from the U.S. Survey of Professional Forecasters support the view that agents pay less attention to the macroeconomy since the onset of the Great Moderation than before.

Turning to standard productivity shocks, the literature has explored the idea that moving away from full information rational expectations can improve the propagation mechanism relative to the baseline RBC model. Eusepi and Preston (2011) abandon rational expectations, replacing it by adaptive learning. They find that the first-order autocorrelations of employment, investment, and output growth in the model become positive. We add rational inattention, a form of incomplete information rational expectations, to the baseline RBC model. Surprisingly, the single assumption of rational inattention turns out to be sufficient to bring the first-order autocorrelations of employment, investment, and output growth in the model approximately into line with the data.[14]

Solving a DSGE model with rational inattention is challenging. One needs to solve attention problems (signal choice problems) of individual agents in a dynamic model. Furthermore, one needs to find a fixed point of an economy in which the optimal signal of an agent depends on the signals chosen by other agents. Several papers make progress solving attention problems of individual agents in a dynamic environment (Sims, 2003, Maćkowiak and Wiederholt, 2009, Woodford, 2009, Sims, 2010, Steiner, Stewart, and Matějka, 2017, Maćkowiak, Matějka and Wiederholt, 2018, Stevens, 2020, Afrouzi and Yang, 2021, Jurado, 2021, Miao, Wu, and Young, 2021).[15] Maćkowiak and Wiederholt (2015) solve a DSGE model with rational inattention where the physical environment is similar to a simple New Keynesian model (there is no capital and one side of the market sets the price while the other side of the market chooses the quantity).[16] By contrast, here the physical environment is a standard neoclassical business cycle model (there is capital and prices, which all agents take as given, adjust to guarantee market clearing). We adopt a guess-and-verify

[14] Business cycle models face the challenge of matching the persistence in the macro data more generally, not only conditional on a productivity shock. See Sims (1998) for a general discussion, Christiano, Eichenbaum, and Evans (2005) and Smets and Wouters (2007) in the context of New Keynesian models, and Auclert, Rognlie, and Straub (2020) in the context of a heterogeneous agent New Keynesian model. Our finding may therefore be helpful also for model builders who allow for sources of fluctuations other than productivity.

[15] See the survey of rational inattention by Maćkowiak, Matějka, and Wiederholt (2021) for a summary of these papers.

[16] See also Afrouzi and Yang (2021).

method to find the fixed point, at each iteration using the results of Maćkowiak, Matějka, and Wiederholt (2018) to solve agents' attention problems. This is the first time in the literature that an RBC model with rational inattention is solved.

The next section defines the physical environment. Section 3 introduces rational inattention. Section 4 develops intuition for the effects of rational inattention, by considering special cases of the model. Section 5 studies the effects of productivity shocks and news about future productivity in the complete model. Section 6 considers a version of the model in which all agents, firms and households, are subject to rational inattention. Section 7 concludes and outlines further research. There is an online Appendix with supplementary material.

2 Model – physical environment

We consider a baseline RBC model that allows for an additional factor of production ("an entrepreneurial input") in fixed supply. The production function is Cobb-Douglas and exhibits decreasing returns to scale in the variable factors, capital and labor. We introduce a third factor in fixed supply because to formulate the attention problem of a firm we need the firm's choice of capital and labor under perfect information, not only the capital-labor ratio, to be determinate.

Time is discrete. There is a continuum of firms indexed by $i \in [0,1]$. All firms produce the same good using an identical technology represented by the production function

$$Y_{it} = e^{a_t} K_{it-1}^{\alpha} L_{it}^{\phi} N_i^{1-\alpha-\phi}$$

where Y_{it} is output of firm i in period t, K_{it-1} is capital input, L_{it} is labor input, and e^{a_t} is total factor productivity, common to all firms. N_i is an entrepreneurial input, specific to firm i, in fixed supply. The parameters α and ϕ satisfy $\alpha \geq 0$, $\phi \geq 0$, and $\alpha + \phi < 1$. In the quantitative analysis (Sections 5-6), we will set $\alpha + \phi$ close to 1 so that the third factor plays essentially no role.

The capital stock of firm i evolves according to the law of motion

$$K_{it} - K_{it-1} = I_{it} - \delta K_{it-1}$$

where $\delta \in (0,1]$ is the depreciation rate. The firm maximizes the expected discounted sum of profits or dividends. The dividend of firm i in period t, D_{it}, is given by

$$D_{it} = Y_{it} - W_t L_{it} - I_{it}$$

where W_t is the wage rate. The dividends of all firms flow to a mutual fund. Households own and trade shares in the mutual fund.[17]

Total factor productivity is determined according to the law of motion

$$a_t = \rho a_{t-1} + \sigma \varepsilon_{t-h} \tag{1}$$

where ε_t follows a Gaussian white noise process with unit variance, $\rho \in (0,1)$, $\sigma > 0$, and $h \geq 0$. A shock drawn by nature in period t affects productivity in period $t+h$. We solve the model either with $h = 0$ (a standard productivity shock) or with $h \geq 1$ (a news shock).[18]

There is a continuum of households indexed by $j \in [0,1]$. Each household j maximizes the expected discounted sum of utility. The discount factor is $\beta \in (0,1)$. The utility function is

$$U(C_{jt}, L_{jt}) = \frac{C_{jt}^{1-\gamma} - 1}{1-\gamma} - \frac{L_{jt}^{1+\eta}}{1+\eta}$$

where C_{jt} is consumption by household j in period t, L_{jt} is hours worked, $\gamma > 0$ is the inverse of the elasticity of intertemporal substitution, and $\eta \geq 0$ is the inverse of the Frisch elasticity of labor supply. Typically, we will set $\gamma = 1$ and $\eta = 0$. The budget constraint in period t is

$$V_t Q_{jt} - V_t Q_{jt-1} = W_t L_{jt} + D_t Q_{jt-1} - C_{jt}$$

where V_t is the price of a share in the mutual fund in period t, Q_{jt} is household j's share in the mutual fund, and $D_t \equiv \int_0^1 D_{it} di$ is the dividend from the mutual fund.

Aggregate output is $Y_t \equiv \int_0^1 Y_{it} di$. Aggregate capital and investment are defined analogously. Aggregate consumption is $C_t \equiv \int_0^1 C_{jt} dj$.

In equilibrium in every period the wage adjusts so that labor demand equals labor supply, $\int_0^1 L_{it} di = \int_0^1 L_{jt} dj$, and the price of a share in the mutual fund adjusts so that asset demand equals asset supply normalized to one, $\int_0^1 Q_{jt} dj = 1$.

The non-stochastic steady state of this economy is described in Appendix A. To solve the model when firms and households have perfect information, we log-linearize their first-order conditions and the other equilibrium conditions at the non-stochastic steady state. This yields the completely

[17] When firm i was sold to the mutual fund, the entrepreneurial input was paid the present value of its future marginal products and in return committed to supply its service without additional payments.

[18] We also consider the case when productivity is driven by two orthogonal shocks, a standard productivity shock and a news shock. See the end of Section 4.1. For ease of exposition, we abstract from long-run growth.

standard log-linear equilibrium conditions stated in Appendix B. We refer to the solution as the perfect information equilibrium.

3 Model – rational inattention by firms

A rationally inattentive individual cannot process all available information but can decide what information to focus on. The decision-maker in firm i chooses an optimal signal about the state of the economy. He or she maximizes the expected discounted sum of profits, recognizing that a more informative signal requires more attention, which is costly.[19] This section begins by deriving the agent's objective. We then state the agent's attention problem. Finally, we define the equilibrium in the economy in which firms are subject to rational inattention and households have perfect information.

3.1 Loss in profit from suboptimal actions

We derive an expression for the expected discounted sum of losses in profit when actions of firm i deviate from the profit-maximizing actions – the actions the firm would take if it had perfect information in every period. To obtain this expression, we compute the log-quadratic approximation to the expected discounted sum of profits at the non-stochastic steady state.

Recall that the profit of firm i in period t is given by $Y_{it} - W_t L_{it} + (1-\delta) K_{it-1} - K_{it}$. We assume that the mutual fund instructs each firm to value profits according to the marginal utility of consumption.[20] The profit function can be written in terms of log-deviations from the non-stochastic steady state:

$$C^{-\gamma} e^{-\gamma c_t} Y \left\{ e^{a_t + \alpha k_{it-1} + \phi l_{it}} - \phi e^{w_t + l_{it}} + \left(\frac{\alpha}{\beta^{-1} - 1 + \delta} \right) \left[(1-\delta) e^{k_{it-1}} - e^{k_{it}} \right] \right\}$$

where an upper-case letter without a time subscript denotes the value of a variable in the non-stochastic steady state, and a lower-case letter denotes the log-deviation of a variable from its value in the non-stochastic steady state. The term $C^{-\gamma} e^{-\gamma c_t}$ is the marginal utility of consumption.

Taking the quadratic approximation to the expected discounted sum of profits, we obtain the

[19] The optimal signal may follow a multivariate stochastic process.

[20] All households have the same consumption level so long as households have perfect information.

following expression for the expected discounted sum of losses in profit from suboptimal actions:

$$\sum_{t=0}^{\infty} \beta^t E_{i,-1} \left[\frac{1}{2} (x_t - x_t^*)' \Theta_0 (x_t - x_t^*) + (x_t - x_t^*)' \Theta_1 \left(x_{t+1} - x_{t+1}^* \right) \right] \tag{2}$$

where $x_t \equiv (k_{it}, l_{it})'$, $x_t^* \equiv (k_{it}^*, l_{it}^*)'$, the matrices Θ_0 and Θ_1 are given by

$$\Theta_0 = -C^{-\gamma} Y \begin{bmatrix} \beta\alpha (1-\alpha) & 0 \\ 0 & \phi (1-\phi) \end{bmatrix}$$

$$\Theta_1 = C^{-\gamma} Y \begin{bmatrix} 0 & \beta\alpha\phi \\ 0 & 0 \end{bmatrix}$$

and the stochastic process x_t^* satisfies the equations

$$E_t a_{t+1} - (1-\alpha) k_{it}^* + \phi E_t l_{it+1}^* = \frac{\gamma E_t (c_{t+1} - c_t)}{1 - \beta (1-\delta)} \tag{3}$$

$$a_t + \alpha k_{it-1}^* - (1-\phi) l_{it}^* = w_t \tag{4}$$

and the initial condition $k_{i,-1}^* = k_{i,-1}$. See Appendix C. The vector x_t^* is the *profit-maximizing* input choice when the decision-maker in the firm has perfect information in every period. Equations (3)-(4) are the usual optimality conditions for capital and labor, where E_t denotes the expectation operator conditioned on the entire history up to and including period t. Equation (3) states that the profit-maximizing capital input equates the expected marginal product of capital to the cost of capital, where the latter is proportional to the expected consumption growth rate. Equation (4) states that the profit-maximizing labor input equates the marginal product of labor to the wage. The vector x_t is an *alternative* input choice. Expression (2) gives the expected discounted sum of losses in profit when the stochastic process for the firm's actions, x_t, differs – for whatever reason – from the stochastic process for the profit-maximizing actions, x_t^*. After the quadratic approximation this loss is quadratic in $x_t - x_t^*$. The interaction term $(x_t - x_t^*)' \Theta_1 \left(x_{t+1} - x_{t+1}^* \right)$ appears because bringing too much capital into a period raises the optimal labor input in that period.

Maćkowiak, Matějka, and Wiederholt (2018) derive analytical results for a class of dynamic rational inattention problems known as linear quadratic Gaussian pure tracking problems. In those problems, the period t payoff is a quadratic form in the contemporaneous deviation of the action vector from some target vector, where the target vector follows a Gaussian stochastic process and does not depend on the decision-maker's own past actions. It turns out that objective (2) can be written as the objective of a pure tracking problem by redefining the vectors x_t and x_t^*.

We show in Appendix C that expression (2) is equivalent to

$$\sum_{t=0}^{\infty} \beta^t E_{i,-1} \left[\frac{1}{2} (x_t - x_t^*)' \Theta (x_t - x_t^*) \right] \tag{5}$$

where $x_t \equiv (k_{it}, l_{it} - \frac{\alpha}{1-\phi} k_{it-1})'$, $x_t^* \equiv (k_{it}^*, l_{it}^* - \frac{\alpha}{1-\phi} k_{it-1}^*)'$, the matrix Θ is given by

$$\Theta = -C^{-\gamma} Y \begin{bmatrix} \beta\alpha \left(1 - \alpha - \frac{\alpha\phi}{1-\phi}\right) & 0 \\ 0 & \phi(1-\phi) \end{bmatrix}$$

and the stochastic process x_t^* satisfies

$$x_t^* = \begin{pmatrix} \frac{1}{1-\alpha-\phi} \left[E_t a_{t+1} - \phi E_t w_{t+1} - (1-\phi) \frac{\gamma E_t (c_{t+1} - c_t)}{1-\beta(1-\delta)} \right] \\ \frac{1}{1-\phi} (a_t - w_t) \end{pmatrix}. \tag{6}$$

The first entry of the vector x_t is still the capital stock to be carried into period $t+1$, k_{it}. The second entry of the vector x_t is now the labor input *for a given capital stock*, $l_{it} - [\alpha/(1-\phi)] k_{it-1}$. The target vector x_t^* is given by equation (6). Its first entry – the profit-maximizing capital stock to be carried into period $t+1$ – is proportional to the difference between expected productivity and a weighted average of the expected wage and the cost of capital, where the expectation is conditioned on the entire history up to and including period t. Its second entry – the profit-maximizing labor input for a given capital stock – is proportional to the difference between productivity and the wage. Since the matrix Θ in objective (5) is diagonal, the best response of firm i in period t given any information set $\mathcal{I}_{it}$ is the conditional expectation of x_t^*, $x_t = E(x_t^* | \mathcal{I}_{it})$. Moreover, assuming that the firm chooses (k_{it}, l_{it}) is equivalent to assuming that the firm chooses $(k_{it}, l_{it} - \frac{\alpha}{1-\phi} k_{it-1})$ so long as the firm knows its own past action k_{it-1}, which is the case if $\mathcal{I}_{it-1} \subset \mathcal{I}_{it}$.

3.2 The attention problem of a firm

In period $t = -1$, the decision-maker in firm i chooses the stochastic process for the signal to maximize the expected discounted sum of profits, (5), net of the cost of attention. In every period $t = 0, 1, 2, ...$, the decision-maker observes a realization of the optimal signal and takes actions – chooses capital and labor.

The statement of the attention problem can be simplified, without loss of generality, based on Maćkowiak, Matějka, and Wiederholt (2018). Let x^*_{1t} denote the first element and x^*_{2t} the second element of x^*_t, $x^*_{1t} = k^*_{it}$ and $x^*_{2t} = l^*_{it} - [\alpha/(1-\phi)]\, k^*_{it-1}$. Suppose that x^*_{1t} and x^*_{2t} each follows a finite-order ARMA process. The vector x^*_t has a first-order VAR representation

$$\xi_{t+1} = F\xi_t + v_{t+1}$$

where v_t is a Gaussian vector white noise process, F is a square matrix, and ξ_t is a vector containing x^*_{1t} and x^*_{2t} and, if appropriate, lags of x^*_{1t} and x^*_{2t} and current and lagged ε_t. The state vector ξ_t contains all information available in period t about the current and future profit-maximizing actions. The analytical results of Maćkowiak, Matějka, and Wiederholt (2018) imply that the optimal signal is a signal about the state vector ξ_t; furthermore, without loss of generality, one can restrict attention to signals that are at most two-dimensional and Gaussian.[21]

The decision-maker in firm i solves:

$$\max_{G,\Sigma_\psi} \sum_{t=0}^{\infty} \beta^t \left\{ E_{i,-1}\left[\frac{1}{2}\left(x_t - x^*_t\right)' \Theta \left(x_t - x^*_t\right)\right] - \lambda I\left(\xi_t; S_{it}|\mathcal{I}_{it-1}\right)\right\} \tag{7}$$

subject to

$$\xi_{t+1} = F\xi_t + v_{t+1} \tag{8}$$

$$x_t = E\left(x^*_t|\mathcal{I}_{it}\right) \tag{9}$$

$$\mathcal{I}_{it} = \mathcal{I}_{i,-1} \cup \{S_{i0}, \ldots, S_{it}\} \tag{10}$$

$$S_{it} = G'\xi_t + \psi_{it} \tag{11}$$

where

$$I\left(\xi_t; S_{it}|\mathcal{I}_{it-1}\right) = H\left(\xi_t|\mathcal{I}_{it-1}\right) - H\left(\xi_t|\mathcal{I}_{it}\right). \tag{12}$$

Expression (7) states that the decision-maker maximizes the expected discounted sum of profits net of the cost of attention. The cost of attention in any period t is proportional to mutual

[21] In Maćkowiak, Matějka, and Wiederholt (2018) the optimal action x^*_t is a scalar while in this model the optimal action x^*_t is a vector, but the proof of Proposition 1 in Maćkowiak, Matějka, and Wiederholt (2018), which states that the optimal signal is a signal about the state vector, extends in a straightforward way from the case of a scalar x^*_t to the case of a vector x^*_t. To show that the optimum can be attained with a two-dimensional signal, one can follow the steps in the proof of Proposition 2 in Maćkowiak, Matějka, and Wiederholt (2018). Finally, Proposition 1 in Jurado (2021) implies that an optimal signal must be Gaussian.

information $I(\xi_t; S_{it}|\mathcal{I}_{it-1})$, where $\lambda > 0$ is the marginal cost of attention. Mutual information is defined below. The decision-maker takes as given the law of motion for the state vector (equation (8)). The agent's actions are equal to the conditional expectation of the profit-maximizing actions given the period t information set (equation (9)). The period t information set $\mathcal{I}_{it}$ consists of the sequence of signal realizations $S_{i0}, \ldots, S_{it}$ and initial information $\mathcal{I}_{i,-1}$ (equation (10)). The optimal signal is a signal about the state vector ξ_t (equation (11)), where the noise ψ_{it} follows a Gaussian vector white noise process with variance-covariance matrix Σ_ψ. The noise ψ_{it} is assumed to be independently distributed across firms.[22] The decision-maker chooses the signal weights G (the number of signals and what each signal is about) and the variance-covariance matrix of the noise Σ_ψ. Equation (12) states that mutual information (between the signal and the state vector) equals the difference between prior uncertainty and posterior uncertainty about the state vector in a given period. $H(\xi_t|\mathcal{I}_{i\tau})$ denotes the entropy of ξ_t conditional on $\mathcal{I}_{i\tau}$, $\tau = t-1, t$. $H(\xi_t|\mathcal{I}_{it-1})$ is the prior uncertainty, before receiving the period t signal, and $H(\xi_t|\mathcal{I}_{it})$ is the posterior uncertainty.

Both the expected discounted sum of profits and the cost of attention in expression (7) depend on conditional second moments.[23] The conditional second moments can in principle vary over time, because the decision-maker conditions on more signal realizations as time passes. To abstract from transitional dynamics in the conditional second moments, we assume that after choosing the signal process in period -1, the agent receives a sequence of signals in period -1 such that the conditional second moments are independent of time. The conditional second moments can then be computed using the steady-state Kalman filter, with state equation (8) and observation equation (11), and problem (7)-(12) can be solved numerically in a straightforward way.[24]

3.3 Equilibrium

We focus on the equilibrium when decision-makers in firms are subject to rational inattention and households have perfect information. For simplicity, until Section 6 we refer to this equilibrium as

[22] Woodford (2003) and Maćkowiak and Wiederholt (2015) make the same assumption. This assumption implies that information is dispersed: in every period, each firm i has a different conditional expectation $E(x_t^*|\mathcal{I}_{it})$.

[23] There is a well-known, closed-form expression for mutual information in the Gaussian case. See Maćkowiak, Matějka, and Wiederholt (2018).

[24] We relax this assumption at the end of Section 4.1. Maćkowiak, Matějka, and Wiederholt (2018) make the same assumption. Woodford (2003) also uses the steady-state Kalman filter to compute conditional second moments in a model in which agents observe exogenously given signals.

the rational inattention equilibrium.[25]

The rational inattention equilibrium can be defined as follows. In period -1, each firm solves problem (7)-(12). In every period $0, 1, 2, ...$, firms and households maximize given their information sets, and markets clear: the wage w_t adjusts so that labor demand equals labor supply, $\int_0^1 l_{it} di = \int_0^1 l_{jt} dj$, and the price of a mutual fund share v_t adjusts so that asset demand equals asset supply, $\int_0^1 q_{jt} dj = 0$.

Some details about firms' and households' maximization are helpful. Equations (9)-(11) together with the choice of G and Σ_ψ and the law of motion of the state (8) yield the input choices of firm i, k_{it} and l_{it}. Firm-level output, investment, and profit follow from $y_{it} = a_t + \alpha k_{it-1} + \phi l_{it}$, $\delta i_{it} = k_{it} - (1-\delta) k_{it-1}$, and $(D/Y) d_{it} = y_{it} - (WL/Y)(w_t + l_{it}) - (I/Y) i_{it}$, whereas aggregate variables from $y_t = \int_0^1 y_{it} di$, $k_t = \int_0^1 k_{it} di$, $i_t = \int_0^1 i_{it} di$, and $d_t = \int_0^1 d_{it} di$.[26] Since households have perfect information, they satisfy the usual first-order conditions

$$\gamma E_t (c_{t+1} - c_t) = \beta E_t v_{t+1} - v_t + (1-\beta) E_t d_{t+1} \tag{13}$$

and

$$w_t - \gamma c_t = \eta l_t. \tag{14}$$

Households are identical, implying that $c_{jt} = c_t$ and $l_{jt} = l_t$ for each j. Finally, the resource constraint reads[27]

$$y_t = (C/Y) c_t + (I/Y) i_t. \tag{15}$$

4 Developing intuition

How does rational inattention affect the propagation of productivity shocks and news about future productivity? To develop intuition this section studies special cases of the model. In the first special case, labor is the only variable input. In the second special case, capital is the only variable input. Section 5 analyzes the rational inattention equilibrium of the complete model.

[25] In Section 6 we add rational inattention on the side of households.

[26] These equations result from log-linearization of the production function, the law of motion of capital, the definition of profit, and the definitions of the aggregate variables. All relevant steady-state ratios appear in Appendix A.

[27] To obtain the resource constraint, we log-linearize the flow budget constraint of household j and we aggregate, imposing market clearing and plugging in the equation for the dividend from the mutual fund.

4.1 The case with labor only

Suppose that labor is the only variable input, $\alpha = 0$. The attention problem of a firm simplifies. The firm's action (labor input choice) is one-dimensional with $x_t = l_{it}$, $x_t^* = l_{it}^* = [1/(1-\phi)](a_t - w_t)$, and $\Theta = -C^{-\gamma}Y\phi(1-\phi)$. Labor supply is governed by equation (14). Households live hand-to-mouth because there is no capital and all households are identical.

The perfect information equilibrium can be solved for analytically. Equating labor demand, $\int_0^1 l_{it}di = [1/(1-\phi)](a_t - w_t)$, and labor supply, which follows from equations (14) and $c_t = y_t = a_t + \phi l_t$, yields the solution for aggregate labor input

$$l_t = \left(\frac{1-\gamma}{1-\phi+\gamma\phi+\eta}\right)a_t. \tag{16}$$

Labor input is proportional to productivity. The impulse response of labor input to a news shock is zero until productivity changes. Firms have no incentive to change labor demand until productivity changes. Similarly, households have no incentive to change labor supply in this special case of the model. The wealth effect on labor supply vanishes, because hand-to-mouth households cannot vary saving and consumption in response to a news shock.

Consider the rational inattention equilibrium. To find the fixed point where all firms are subject to rational inattention and hold correct beliefs about the law of motion of the state, we use a guess-and-verify method. We guess that in equilibrium the profit-maximizing labor input l_{it}^* follows a finite-order ARMA process. This yields the law of motion of the state (8). We solve the attention problem of firm i, (7)-(12) with $x_t = l_{it}$, $x_t^* = l_{it}^*$, and $\Theta = -C^{-\gamma}Y\phi(1-\phi)$. Equations (9)-(11) together with the choice of G and Σ_ψ and the law of motion of the state (8) yield the firm's labor input choice, l_{it}. We verify the guess for the profit-maximizing labor input l_{it}^* from the optimality condition $l_{it}^* = [1/(1-\phi)](a_t - w_t)$ where the market-clearing wage w_t follows from equations (14), $c_t = y_t = a_t + \phi l_t$, and $l_t = \int_0^1 l_{it}di$. One period in the model equals one quarter. As an example, we assume $\gamma = 0.5$, $\eta = 0$, $\phi = 0.6$, $\beta = 0.99$, $\rho = 0.9$, $\sigma = 0.01$, and $\lambda = (4/100{,}000)C^{-\gamma}Y$, which means that the per period marginal cost of attention is equal to $4/100{,}000$ of steady-state output.[28]

The upper-left panel in Figure 1 shows the impulse response of aggregate labor input l_t to a productivity shock ($h = 0$).[29] In the perfect information equilibrium, labor input is proportional to

[28] Section 5 discusses the choice of the value for the marginal cost of attention λ.

[29] In all figures, an impulse response of 1 is a 1 percent deviation from the non-stochastic steady state.

productivity and thus the impulse response peaks on impact and declines monotonically (line with points). The impulse response is weaker and hump-shaped in the rational inattention equilibrium (line with circles). This is the usual result that rational inattention produces dampening and delay due to noise in signals about the state of the economy. For a similar figure, see for instance Figure 1 in Sims (2003).

The upper-right panel in Figure 1 shows the impulse response of l_t to a news shock ($h = 4$). The shock is drawn in period 0 while productivity changes in period $h = 4$. In the perfect information equilibrium, labor input is proportional to productivity (equation (16)) and thus the impulse response is zero until productivity changes (line with points). Under rational inattention labor demand rises on impact of a news shock. The reason is that the optimal signal of firms confounds current with expected future productivity. The increase in labor demand puts upward pressure on the wage. Labor supply is still governed by equations (14) and $c_t = y_t = a_t + \phi l_t$. We find that in equilibrium labor input is positive on impact of a news shock (line with circles) and keeps rising thereafter.

To see analytically that a confounding signal is optimal, consider the following special case. Suppose that a measure zero of firms are subject to rational inattention. Since a measure one of firms have perfect information, the equilibrium employment is given by equation (16) and the equilibrium wage is $w_t = [(\gamma + \eta)/(1 - \gamma)]l_t$, implying that the profit-maximizing labor input of an individual firm is proportional to productivity: $l_{it}^* = [1/(1-\phi)](a_t - w_t) = [(1-\gamma)/(1-\phi+\gamma\phi+\eta)]a_t$. Suppose that $a_t = \rho a_{t-1} + \sigma\varepsilon_{t-1}$ (i.e., $h = 1$). The profit-maximizing labor input then has a first-order VAR representation with state vector $\xi_t = (l_{it}^*, \varepsilon_t)'$, or equivalently $\xi_t = (a_t, \varepsilon_t)'$. The optimal signal follows from Propositions 1, 2, and 5 in Maćkowiak, Matějka, and Wiederholt (2018). Proposition 1 states that the optimal signal is about the state vector, $S_{it} = G'\xi_t + \psi_{it}$. Proposition 2 states that with a one-dimensional action (here, labor input), the optimal signal is a *one-dimensional* signal about the state vector, $S_{it} = a_t + g\varepsilon_t + \psi_{it}$. Proposition 5 states that $g \neq 0$. Hence the optimal signal confounds current with expected future productivity. It turns out that this result still holds when all firms are subject to rational inattention and $h > 1$.

We offer the following intuition for the optimality of a confounding signal. The intuition consists of two parts. First, it is a well-known result in the rational inattention literature that the dimension of the optimal signal can be strictly smaller than the dimension of the state of the economy.[30] The advantage of a lower-dimensional signal is that it saves on attention, while the disadvantage of a lower-dimensional signal is that it provides a less complete view of the economy. In the special case discussed in the previous paragraph, Proposition 2 in Maćkowiak, Matějka, and Wiederholt (2018) implies that the optimal signal is one-dimensional.

Second, given the optimality of a one-dimensional signal (more generally, given the optimality of a lower-dimensional signal), the question is: What is the optimal signal about? To gain intuition for the answer, consider the lower-left panel of Figure 1. Here we continue to assume that a measure zero of firms are subject to rational inattention (partial equilibrium) and $h = 1$, implying that the optimal labor input of a firm is given by $l_{it}^{*} = [(1-\gamma)/(1-\phi+\gamma\phi+\eta)]a_t$ with $a_t = \rho a_{t-1} + \sigma\varepsilon_{t-1}$. The line with points depicts the impulse response of the optimal labor input to a news shock. The optimal labor input perfectly tracks the evolution of productivity. The line with circles shows the impulse response of the labor input to a news shock under the optimal signal, which has the form $S_{it} = a_t + g\varepsilon_t + \psi_{it}$ with $g \neq 0$. The line with squares shows the impulse response of the labor input to a news shock in the case of a noisy signal on current productivity, $S_{it} = a_t + \psi_{it}$. To make the line with circles and the line with squares comparable, the optimal signal under rational inattention and the noisy signal on current productivity are constrained to have the same signal-to-noise ratio, $var\left(G'\xi_t|\mathcal{I}_{it-1}\right)/var\left(\psi_{it}\right)$, which implies that the two signals are associated with the same information flow, $I\left(\xi_t; S_{it}|\mathcal{I}_{it-1}\right)$. The signal on current productivity generates no overreaction of the labor input on impact of the news shock, but it generates underreaction of the labor input to the increase in productivity in all following periods, due to the noise in the signal. The optimal signal, by contrast, generates overreaction of the labor input on impact of the news shock and less underreaction of the labor input once productivity actually increases, by pushing the entire impulse response function up. With a quadratic objective, the overreaction on impact of the news shock causes only a small profit loss, because it introduces a small mistake, while the reduction in the underreaction in the following periods causes large gains in profit, because it reduces large mistakes. In addition, the overreaction occurs for one period, while the reduction in

[30] For this result in the static case, see Proposition 1 in Köszegi and Matějka (2020). For this result in different dynamic models, see Proposition 2 in Maćkowiak, Matějka, and Wiederholt (2018), Proposition 5 in Miao, Wu, and Young (2021), Theorem 2.2 in Afrouzi and Yang (2021), and Proposition 7 in Jurado (2021).

the unterreaction is present for many periods. As a result, the confounding signal generates smaller profit losses due to suboptimal actions than the signal on current productivity.[31]

Jurado (2021) offers a different intuition for why rational inattention endogenously produces forward-looking behavior, in the sense that, whenever possible, the agent responds too early to future innovations in optimal actions. Formulating a dynamic rational inattention problem in the frequency domain, he shows that the solution can be understood as the outcome of a reverse water-filling strategy in the frequency domain: The agent concentrates the uncertainty reduction on the variation in the most important frequencies. Furthermore, learning about what is happening to productivity at the most important frequencies is different from choosing to have more precise knowledge about whether productivity is changing today versus tomorrow. We view these two intuitions as complementary.

In the same partial equilibrium setting as in the lower-left panel in Figure 1, let us vary the marginal cost of attention λ. The impulse response of the action under rational inattention (here, labor input) on impact of a news shock, in period 0, is non-monotonic in λ. With a λ near zero (Figure 1, lower-right panel, line with asterisks), the solution is close to the perfect information case in which the impulse response on impact is zero. With a high λ (line with diamonds), the solution is close to a "no information" model in which the impulse response in all periods is zero.

Appendix D reports additional numerical results. One result is that the more distant is the change in productivity, the weaker is the response of the action on impact of a news shock. If productivity will change in the near future, a rationally inattentive agent believes that productivity has already changed with a non-trivial probability. The short-run response of the action can then be strong (even though the perfect information response is zero). If productivity will change only in a longer run, the agent is fairly confident that productivity has not yet changed. The short-run response of the action approaches the perfect information response.

Appendix D also considers a version of the model in which productivity is driven by two orthogonal shocks, a standard productivity shock and a news shock. The result that labor input rises on impact of a positive news shock is unchanged (and the impulse response of labor input to a news

[31] The profit loss due to suboptimal actions depends also on the noise in the actions, but it turns out that the response of labor input to a noise shock is very similar under the optimal signal and under the signal on current productivity.

shock is very similar to the one reported in this subsection). In addition, in Appendix D we drop the assumption that, after choosing the signal process in period -1, the agent receives a sequence of signals in period -1 such that the conditional second moments are independent of time. The impulse response of labor input is almost identical with and without this assumption.

4.2 The case with capital only

Suppose that capital is the only variable input, $\phi = 0$. The attention problem of a firm is analogous to Section 4.1. The firm's action (capital input choice) is one-dimensional with $x_t = k_{it}$, $x_t^* = k_{it}^* = \frac{1}{1-\alpha}\left[E_t a_{t+1} - \frac{\gamma E_t(c_{t+1}-c_t)}{1-\beta(1-\delta)}\right]$, and $\Theta = -C^{-\gamma} Y \beta\alpha(1-\alpha)$. Consumption-saving behavior of households is governed by equation (13).

Assume log utility from consumption, $\gamma = 1$, and full capital depreciation, $\delta = 1$. The perfect information equilibrium can be solved for analytically: $k_t = \alpha k_{t-1} + a_t$, $k_t = i_t = y_t = c_t = d_t = v_t$. In this special case, the model can produce some positive autocorrelation in investment and output growth. However, the impulse responses of all variables to a news shock are zero until productivity changes. An increase in expected productivity creates an incentive to invest in the period before productivity improves, but this incentive is completely offset by a rise in the cost of capital.

Consider the rational inattention equilibrium. To find the fixed point, we guess that in equilibrium the profit-maximizing capital input k_{it}^* follows a finite-order ARMA process. This yields the law of motion of the state (8). We solve the attention problem of firm i, (7)-(12) with $x_t = k_{it}$, $x_t^* = k_{it}^*$, and $\Theta = -C^{-\gamma} Y \beta\alpha(1-\alpha)$. Equations (9)-(11) together with the choice of G and Σ_ψ and the law of motion of the state (8) yield the firm's capital input choice, k_{it}. Aggregating across firms produces k_t, i_t, y_t, and d_t, while the budget constraint implies that $c_t = d_t$. We verify the guess for the profit-maximizing capital input k_{it}^* from the optimality condition $k_{it}^* = [1/(1-\alpha)]\left[E_t a_{t+1} - E_t(c_{t+1} - c_t)\right]$. The market-clearing mutual fund share price v_t follows from equation (13) and the solution for c_t. As an example, we assume $\gamma = 1$, $\alpha = 0.33$, $\beta = 0.99$, $\delta = 1$, $\rho = 0.9$, $\sigma = 0.01$, and $\lambda = (4/100{,}000)C^{-\gamma}Y$.

The top panel in Figure 2 displays the impulse response of aggregate investment i_t to a productivity shock ($h = 0$). The rational inattention equilibrium (line with circles) features more first-order autocorrelation in the growth rate of investment compared with the perfect information equilibrium (line with points). The middle panel in Figure 2 shows the impulse response of i_t to a news shock ($h = 4$). In the perfect information equilibrium, the impulse response of investment is zero until productivity changes in period 4 (line with points). In the rational inattention equilibrium, investment is positive in period 0 (line with circles) and keeps rising thereafter. Rational inattention induces an increase in investment demand on impact of a news shock. As a result, investment rises in equilibrium.

Since the attention problem of a firm is analogous to Section 4.1, the intuition for what happens to investment demand is the same as the intuition given there. The forward-looking attention choice leads investment demand to react immediately to a news shock, as if productivity has already changed with some probability. Similarly to Section 4.1, the bottom panel in Figure 2 compares the rational inattention model with the alternative model (the model with the signal on current productivity) with $h = 4$. The alternative model yields no capital input mistakes conditional on a news shock from period 0 through period $h - 1$, followed by larger mistakes than in the rational inattention model. By smoothing the action, the signal in the rational inattention model lowers the overall expected profit loss.[32]

Let us summarize Section 4. The impulse responses to a productivity shock and a news shock change significantly when firms are subject to rational inattention. Employment and investment react with *delay* to a standard productivity shock, because the optimal signal is noisy. Moreover, employment and investment *rise* in response to news that productivity will improve in the future, because the optimal signal confounds current with expected future productivity.

5 Predictions of the model

What does rational inattention imply about the business cycle effects of productivity shocks and news about future productivity? We return to the complete model with variable capital and labor, $\alpha > 0$ and $\phi > 0$, and compare the rational inattention equilibrium with the perfect information

[32] We emphasized that rational inattention makes very different predictions than the alternative model in the case of news shocks ($h \geq 1$). With $h = 0$ actions based on the optimal signal are also different from actions based on the signal on current productivity, except when the optimal action follows an AR(1) process. How much difference there is depends on the details of the model. In this model the difference turns out to be modest. Consider the partial equilibrium analysis with $h = 0$ and the same parameter values. The profit-maximizing capital input follows an AR(2) process. The investment growth rate of rationally inattentive firms has a serial correlation of 0.61. With the alternative signal the serial correlation rises to 0.65.

equilibrium. We focus on $h = 2$ and $h = 4$, following the key papers on news shocks which also focus on changes in productivity a few quarters ahead ($h = 3$ in Beaudry and Portier, 2004, $h = 2$ in Jaimovich and Rebelo, 2009).

We set $\gamma = 1$, $\eta = 0$, $\alpha = 0.33$, $\phi = 0.65$, $\beta = 0.99$, $\delta = 0.025$, $\rho = 0.9$, and $\sigma = 0.008$. Thus, we assume log utility from consumption and linear disutility from work, $\alpha + \phi$ close to 1, a depreciation rate of 2.5 percent per quarter, and a persistent productivity process with an innovation that has a standard deviation of 0.8 percent.[33] We know that the model with perfect information ($\lambda = 0$) does not produce comovement after a news shock, and we conjecture based on Section 4 that the model with rational inattention ($\lambda > 0$) may be able to do so. We set $\lambda = (6/100,000)C^{-\gamma}Y$, which means that the per period marginal cost of attention is equal to $6/100,000$ of steady-state output, because this is the lowest value of λ for which the model produces comovement with $h = 2$. Increasing λ somewhat would strengthen comovement.[34] It turns out, as we show below, that with this value of λ the model is also consistent with survey data on expectations.

5.1 The effects of productivity shocks

To begin, consider the perfect information equilibrium with $h = 0$. Figure 3 shows the impulse responses to a productivity shock (lines with points). Aggregate labor input, investment, output, and consumption move in the same direction, consistent with a business cycle. The impulse responses of labor input, investment, and output peak on impact and decline monotonically. Following common practice in the RBC literature, we compare unconditional second moments in the model and in the data. We want to find out if rational inattention improves the performance of the model in this standard exercise. Table 1 reports selected unconditional moments for the model (column "Perfect

[33] Fernald (2014) constructs a quarterly series on the growth rate of TFP adjusted for capacity utilization. Regressing Fernald's series on its own lag in the sample 1955Q1-2007Q4 yields a point estimate of -0.08, which would imply a coefficient of $1 - 0.08 = 0.92$ in equation (1). The estimated standard deviation of the error term is 0.0083. Rounding off these estimates, we arrive at $\rho = 0.9$ and $\sigma = 0.008$. One can also convert Fernald's series into a series on the log level of TFP and fit an AR(1) to that series after detrending, but the estimated coefficient depends on the detrending method.

[34] Only the ratio λ/σ^2 matters for the equilibrium impulse responses, because the first term in objective (7) is linear in σ^2 and the second term is linear in λ.

information") and for the quarterly post-war data from the United States.[35] The comparison is familiar. Let us focus on the persistence of growth rates. The first-order autocorrelations of employment, investment, and output growth are positive in the data but negative in the model. In the model these variables inherit the autocorrelation of exogenous productivity growth.[36]

Consider the rational inattention equilibrium. Searching for the fixed point is more difficult than in Section 4, because we must consider two inputs, capital and labor, and two factor prices, the cost of capital and the wage. To find the fixed point, we guess that in equilibrium consumption c_t follows a finite-order ARMA process. With $\gamma = 1$ and $\eta = 0$, the optimality condition (14) states that the wage process w_t equals the consumption process c_t. This condition holds because households have perfect information. Therefore, a guess about consumption implies a guess about both factor prices, the cost of capital (the expected consumption growth rate) and the wage. We calculate the implied ARMA representations of the optimal inputs $x_{1t}^* = k_{it}^*$ and $x_{2t}^* = l_{it}^* - [\alpha/(1-\phi)]\, k_{it-1}^*$ from equation (6). This yields the law of motion of the state (8). We solve the attention problem of firm i, (7)-(12). Equations (9)-(11) together with the choice of G and Σ_ψ and the law of motion of the state (8) yield the firm's inputs, $k_{it} = x_{1t}$ and $l_{it} = x_{2t} + [\alpha/(1-\phi]k_{it-1}$. Aggregating across firms produces k_t, i_t, l_t, y_t, and d_t. We verify the guess for the equilibrium consumption process by solving for c_t from the resource constraint (15). The market-clearing mutual fund share price v_t follows from equation (13) and the solution for d_t and c_t.

What are the effects of rational inattention on the propagation of a productivity shock? In the rational inattention equilibrium, the impulse responses of employment, investment, and output become hump-shaped (Figure 3, lines with circles). These impulse responses are hump-shaped even though there are no adjustment costs. The first-order autocorrelations of employment, investment, and output growth become positive (Table 1, column "Rational inattention"). The model

[35] We use the data from Eusepi and Preston (2011). The sample period is 1955Q1-2007Q4. Productivity is defined as real GDP divided by hours worked, measured as in Francis and Ramey (2009). See Data Appendix in Eusepi and Preston (2011). The unconditional moments from the model are computed from the equilibrium MA representation of each variable.

[36] The model matches well the standard deviation of consumption, investment, and productivity relative to output, while underpredicting the volatility of hours worked. The model matches well the correlation of consumption, hours worked, and investment with output, while overstating the correlation of productivity with output. Finally, the model matches well the first-order autocorrelation of consumption growth. It turns out that rational inattention has little effect on these predictions of the model. See Table 1.

matches well the first-order autocorrelation of employment growth in the data, even though rational inattention is the only source of inertia and the marginal cost of attention is small. The model underpredicts somewhat the serial correlation of output and investment growth. In the rational inattention equilibrium, we can compute the expected profit loss of firm i from suboptimal actions. This is equal per period to $3/100,000$ of steady-state output, even smaller than the marginal cost of attention λ.

In Figure 3 note also that consumption declines somewhat when firms become subject to rational inattention. Households consume less because rationally inattentive firms underestimate productivity and produce less than in the perfect information equilibrium.

Section 4 explained the effects of rational inattention one input at a time. In this section the new feature is that rational inattention induces delay in the demand for both inputs, capital and labor, at the same time. Figure 3 shows the impulse response of the conditional expectation of productivity by firms to a productivity shock. The impulse response is hump-shaped, indicating that the firms' beliefs are anchored on the steady state and evolve slowly. The rational inattention effect turns out to be sufficient to bring the first-order autocorrelations of employment, investment, and output growth in the model approximately into line with the data.

The amount of inattention in the model, governed by the parameter λ, can be compared to survey data on expectations. Coibion and Gorodnichenko (2015) show that models with an informational friction predict a regression relationship between the average forecast error and forecast revision in a cross-section of agents. Suppose that firms in this model report their forecasts of output. Let $\hat{y}_{t+\tau|t}$ denote the period t average forecast of output in period $t+\tau$, where τ is a positive integer. The average forecast error, $y_{t+\tau} - \hat{y}_{t+\tau|t}$, is positively related to the average forecast revision, $\hat{y}_{t+\tau|t} - \hat{y}_{t+\tau|t-1}$. The regression coefficient increases in the size of the informational friction, in this model governed by the value of λ. Coibion and Gorodnichenko (2015) and Bordalo, Gennaioli, Ma, and Shleifer (2020) estimate this regression relationship using survey data on forecasts of a number of variables. Typically, these authors report coefficients in the range of 0.3-1.4.[37] We repeat their estimation using quarterly data on median forecasts of output (real GDP) from the U.S. Survey of Professional Forecasters for the period 1968Q4-2019Q4 obtained from the Federal

[37] See in particular Coibion and Gorodnichenko (2015), Table 1 and Figures 1-2, and Bordalo, Gennaioli, Ma, and Shleifer (2020), Table 3.

Reserve Bank of Philadelphia. Focusing on $\tau = 3$, we estimate a regression coefficient of 0.76 with a standard error of 0.30, which is a finding in the typical range from the previous work.[38] Next, we simulate data from our model with the parameter values used in this section, including the value of λ. When we run the same regression on the simulated data, on average across the simulations we obtain a coefficient of 0.96. We conclude that the amount of inattention in the model is consistent with the survey data on expectations. It is remarkable that the first-order autocorrelations of employment, investment, and output growth in the model are approximately in line with the macro data and, at the same time, the model is consistent with the survey data on expectations.

5.2 The effects of news about future productivity

Consider the perfect information equilibrium with $h \geq 1$. Figures 4 and 5 show the impulse responses with $h = 2$ and $h = 4$, respectively (lines with points). The shock is drawn in period 0 while productivity changes in period h. A news shock causes a wealth effect. Consumption and leisure are normal goods, and therefore households want to consume more (save less) and work less after a positive news shock. Firms have no incentive to increase labor demand before productivity improves, while households reduce labor supply due to the wealth effect. As a result, employment falls. With capital predetermined and current productivity unchanged, output contracts. On impact firms have no incentive to increase investment, while the wealth effect reduces desired saving by households. Investment declines while consumption rises. The model fails to produce comovement in response to news about future productivity. It predicts an output contraction after news that productivity will improve. Note also that, after decreasing on impact, employment, investment, and output keep falling between when the news arrives (period 0) and when productivity changes (period h). This is particularly clear in Figure 5 ($h = 4$). An increase in expected productivity creates an incentive to invest in the period before productivity improves (period $h-1$), but this incentive is more than offset by a rise in the cost of capital. Employment, investment, and output increase only once productivity improves.

With a high elasticity of intertemporal substitution, the model predicts a fall in consumption and

[38] Coibion and Gorodnichenko (2015) and Bordalo, Gennaioli, Ma, and Shleifer (2020) also focus on $\tau = 3$. Both papers report results for forecasts of output *growth* but not output *level*. Some observations on forecasts of the level of output cannot be used due to base year changes in the dataset; furthermore, we remove as outliers the top 1 percent of forecast errors and revisions.

a rise in labor input and investment. The substitution effect due to an increase in the real interest rate dominates the wealth effect in this case, pushing consumption down and labor supply up. "However, no combination of parameters can generate a joint increase in consumption, investment, and employment." (Lorenzoni, 2011, p.539.)

Consider the rational inattention equilibrium (Figures 4-5, lines with circles).[39] In both figures employment is positive in period 0 and keeps rising thereafter. The conditional expectation of productivity by firms increases on impact, which pushes up labor demand. In general equilibrium, the desire of households to reduce labor supply is pulling employment down. It turns out that the rational inattention effect on labor demand is strong enough to *more than offset* the wealth effect on labor supply. As a result, employment rises in equilibrium.

Figures 4-5 show the impulse response of investment in general equilibrium ("RI general equilibrium") and the impulse response of investment by rationally inattentive firms of measure zero when other firms have perfect information ("RI partial equilibrium," line with asterisks). In partial equilibrium, investment is positive in period 0 and keeps rising thereafter. The conditional expectation of productivity by rationally inattentive firms increases on impact, which pushes up investment demand. In general equilibrium, the desire of households to reduce saving for a given level of output is pulling investment down. We find that the rational inattention effect on investment demand approximately offsets the wealth effect on saving supply. The response of investment on impact of a news shock is close to zero (whereas it is nearly -3 percent in the perfect information equilibrium). Note also that investment rises between period 0 and period h. This is particularly clear in Figure 5 ($h = 4$).

With capital predetermined and an increase in employment in period 0, the impulse response of output on impact of a news shock is positive. Output increases further between period 0 and period h, as employment and investment rise. The rational inattention effect on input demand induces an output expansion in response to a news shock.

Consider in more detail what affects investment in general equilibrium. Investment rises on impact of a positive news shock relative to the perfect information equilibrium. The cost of capital

[39] In the economy with $h = 2$ we find a per period expected profit loss equal to $4/100,000$ of steady-state output. In the economy with $h = 4$ we set $\lambda = (22/100,000)C^{-\gamma}Y$ (with $h = 4$ the model needs a higher λ to produce comovement after a news shock). The per period expected profit loss in this case turns out to equal $15/100,000$ of steady-state output.

increases (the expected consumption growth rate rises). The profit-maximizing capital input of an individual firm falls. See the first line in equation (6). Capital is a strategic substitute. An individual firm demands less capital when other firms invest more. This general equilibrium feedback effect turns out to be very strong. The coefficient on the expected consumption growth rate in the first line of equation (6) equals -504.[40] The coefficient on the expected consumption growth rate increases in the depreciation rate, δ, and decreases in the elasticity of output with respect to labor, ϕ. In Section 4.2, with full capital depreciation and without labor input ($\delta = 1$, $\phi = 0$), this coefficient decreases in absolute value by more than two orders of magnitude, to -1.5, implying that the strategic substitutability is much weaker. The impulse response of equilibrium investment on impact of a news shock is positive in this case.

To summarize, rational inattention induces an increase in the demand for labor and investment in response to news that productivity will improve. The rational inattention effect on labor demand more than offsets the wealth effect on labor supply. Thus, employment and output rise on impact. The rational inattention effect on investment demand offsets the wealth effect on saving supply. As a result, the response of investment on impact equals zero, as opposed to a sizable negative number in the perfect information equilibrium.

In Figures 4-5 note also that consumption increases somewhat when firms become subject to rational inattention. Households consume more because rationally inattentive firms overestimate productivity and produce more than in the perfect information equilibrium.

What is the optimal signal? In problem (7)-(12) the firm can in principle choose a multi-dimensional signal process, consisting of signals on elements of the state vector ξ_t, signals on linear combinations of the elements of ξ_t, or both. We find that a one-dimensional signal on all elements of the state vector is optimal.[41] A signal on all elements of the state vector confounds current with

[40] Labor is also a strategic substitute. However, the general equilibrium dampening of labor demand due to a higher wage is weak. The coefficient on the wage in the second line of equation (6) equals -2.9.

[41] This finding is a numerical result, but one can show analytically that even with a multi-dimensional state of the economy (here the dimension of ξ_t) and a multi-dimensional action at each point in time (here the dimension of x_t) the dimensionality of the optimal signal does not exceed the number of structural shocks that hit the economy per period. This is a corollary of Proposition 7 of Jurado (2021). Hence, in the model with only news shocks a one-dimensional signal is optimal, while in the model with news shocks and standard productivity shocks, which is studied in Appendix D, a signal with at most two dimensions is optimal. Furthermore, even with two structural shocks the optimal signal is one-dimensional for sufficiently low attention. In Section 4.1, we explain in detail why

expected future productivity. Furthermore, we find that the impulse response of the optimal signal to a news shock is positive on impact (Appendix Figure 1, upper-left panel, $h = 2$). To simplify, the message to firms from a positive signal realization is: "Hire and invest, productivity is either already up or about to rise (and it is not that important precisely when productivity rises)."

As in Section 5.1, we can compare the amount of inattention in the model to the SPF data. When we run the Coibion-Gorodnichenko regression on data simulated from the economy with $h = 2$ (with $\tau = 3$), on average we obtain a coefficient of 1.17. This amount of inattention is consistent with the survey data on expectations.[42] With $h = 4$ the model needs a higher marginal cost of attention to produce an increase in employment after a positive news shock. When we run the Coibion-Gorodnichenko regression on data simulated from the economy with $h = 4$ (with $\tau = 3$), on average we obtain a coefficient of 2.81. This amount of inattention is somewhat greater than implied by the SPF data.[43]

5.3 Changing macroeconomic volatility

The impulse responses in the model depend on how much attention agents choose to pay, and the optimal attention varies with the environment. In Sections 5.1-5.2, we set the volatility of the productivity process based on the post-war U.S. data. Specifically, we set $\sigma = 0.008$ to match the standard deviation of the quarterly growth rate of TFP adjusted for capacity utilization in the period 1955Q1-2007Q4. The TFP growth rate was less variable in the second half of this sample than in the first half, a part of the decline in macroeconomic volatility known as the Great Moderation. The standard deviation of the TFP growth rate decreased from 0.9 percent (1955Q1-1984Q4) to 0.7 percent (1985Q1-2007Q4). Let us resolve the model with $\sigma = 0.009$ (higher volatility) and again with $\sigma = 0.007$ (lower volatility). The other parameter values remain as in Section 5.2.

We find that in the lower volatility economy ($\sigma = 0.007$) the period 0 impulse response of labor input to a news shock is positive (like in the baseline with $\sigma = 0.008$). A news shock produces positive comovement of labor input and consumption on impact. In the higher volatility economy

this one-dimensional signal is on all elements of the state vector.

[42] Recall that in the SPF data the analogous regression coefficient is 0.76 with a standard error of 0.30.

[43] It seems plausible that in the real world decision-makers in small and medium-sized firms pay less attention to the aggregate economy than professional forecasters.

($\sigma = 0.009$), the period 0 impulse response of labor input is negative. Here a news shock produces negative comovement of labor input and consumption on impact (Appendix Figure 1, upper-right panel, $h = 2$). The reason behind the change in the sign is intuitive. With higher volatility agents pay about 50 percent more attention to the state of the economy, and therefore the response of labor input is closer to the perfect information RBC model, than with lower volatility. This effect is strong enough to change the sign of the impulse response of employment to a news shock. Thus, the model suggests that empirical researchers who study different sample periods can be expected to reach conflicting conclusions regarding comovement.[44] Note also that the imperfect information version of the baseline RBC model in which firms take actions based on signals of the form "current productivity a_t plus noise" or "future productivity a_{t+h} plus noise" does not produce business cycle comovement, regardless of the value of σ or the amount of noise.[45]

The SPF data support the view that agents pay less attention to the macroeconomy since the onset of the Great Moderation than before. Coibion and Gorodnichenko (2015, Section III.A) make this point in detail. In Section 5.1, we reported the result from running the Coibion-Gorodnichenko regression on median forecasts of output from the SPF for the period 1968Q4-2019Q4 (with $\tau = 3$). We estimated a coefficient of 0.76 with a standard error of 0.30. Let's split the sample in half and rerun this regression in the two subsamples.[46] In the first half of the sample the coefficient falls to 0.48 (the standard error is 0.38). In the second half of the sample the coefficient rises to 1.21 (the standard error is 0.48). This finding is in line with the hypothesis of "less attention since the onset of the Great Moderation than before."[47]

[44] Recall from the introduction that Barsky and Sims (2011) and Kurmann and Sims (2021) do not find business cycle comovement after a news shock, while Görtz, Tsoukalas, and Zanetti (2020) who focus on data since the onset of the Great Moderation find comovement. Görtz, Tsoukalas, and Zanetti (2020) also show that when they use the identification assumptions of Barsky and Sims (2011) or Kurmann and Sims (2021) and focus on the data since the onset of the Great Moderation, they do find comovement.

[45] Recall the discussion of the optimal confounding signal in Section 4.1.

[46] The SPF sample starts only in 1968 and therefore it seems reasonable to split the sample in half, rather than divide it into unequal "before" and "after" the onset of the Great Moderation subsamples. That alternative approach, however, happens to yield regression results very similar to the ones reported here.

[47] In Section 5.2 we also ran the same Coibion-Gorodnichenko regression on data simulated from the baseline rational inattention economy, obtaining a coefficient of 1.17 ($h = 2$). Repeating this regression in the model with $\sigma = 0.009$ and $\sigma = 0.007$ yields coefficients of 0.78 and 1.42, respectively.

6 Rational inattention by firms and households

We focused on the equilibrium when decision-makers in firms are subject to rational inattention and households have perfect information. To obtain comovement in response to a news shock, it seems critical to find a mechanism leading to a shift in labor demand and investment demand for a given level of productivity. Rational inattention *by firms* is such a mechanism. To illustrate in the most transparent way the effects of rational inattention by firms, we assumed that households have perfect information. We found that rational inattention by firms also improves the propagation of a standard productivity shock. In this section, we add rational inattention on the side of households. Let us first describe the attention problem of an individual household j and afterwards explain how we solve for the fixed point when all firms and all households are subject to rational inattention.[48]

Each household j chooses a signal about the state of the economy to maximize the expected discounted sum of utility. The household recognizes that a more informative signal requires more attention, which is costly. Proceeding analogously to Section 3, we derive an expression for the expected discounted sum of losses in utility when actions of household j deviate from the utility-maximizing actions – the actions the household would take if it had perfect information in every period. To obtain this expression, we compute the log-quadratic approximation to the expected discounted sum of utility at the non-stochastic steady state, arriving at

$$\sum_{t=0}^{\infty} \beta^t E_{j,-1} \left[\frac{1}{2} \left(x_t - x_t^* \right)' \Theta \left(x_t - x_t^* \right) \right], \tag{17}$$

where

$$x_t = \begin{pmatrix} \omega_V \left(q_{jt} - q_{jt-1} \right) \\ \gamma \left[\omega_V \left(\frac{1}{\beta} q_{jt-1} - q_{jt} \right) + \omega_W l_{jt} \right] + \eta l_{jt} \end{pmatrix} \tag{18}$$

$$\Theta = -C^{1-\gamma}\gamma \begin{bmatrix} \left(1 - \frac{1}{1+\frac{\eta}{\omega_W \gamma}}\right) \frac{1}{\beta} & 0 \\ 0 & \frac{1}{1+\frac{\eta}{\omega_W \gamma}} \frac{1}{\gamma^2} \end{bmatrix} \tag{19}$$

and

$$x_t^* = \begin{pmatrix} z_t - (1-\beta) \sum_{s=t}^{\infty} \beta^{s-t} E_t \left[z_s \right] + \left(1 + \omega_W \frac{\gamma}{\eta} \right) \frac{1}{\gamma} \beta \sum_{s=t}^{\infty} \beta^{s-t} E_t \left[r_{s+1} \right] \\ w_t - \gamma \left(\omega_W w_t + \omega_D d_t \right) \end{pmatrix}. \tag{20}$$

[48] The attention problem of each firm i is essentially unchanged. Households no longer have the same consumption level in this version of the model. We assume that firm i values profits according to the marginal utility of consumption of the representative (average) household.

Here $z_s \equiv \omega_W \left(w_s + \frac{1}{\eta} w_s \right) + \omega_D d_s$ and $r_{s+1} \equiv \beta v_{s+1} - v_s + (1-\beta)\, d_{s+1}$. See Appendix E.[49]

This objective has a simple interpretation. The first element of x_t is the change in asset holdings. The second element of x_t is the component of the marginal rate of substitution between consumption and leisure that is directly controlled by the household through the choice of asset holdings, q_{jt}, and hours worked, l_{jt}. The vector x_t^* is the vector of optimal choices under perfect information in period t. It is optimal to increase asset holdings when income is high relative to permanent income or when the return on saving is high. It is optimal to equate the marginal rate of substitution between consumption and leisure to the wage. When the household deviates from these optimal choices, the household loses an amount of utility determined by the matrix Θ. This matrix is diagonal, because a *sub*optimal marginal rate of substitution between consumption and leisure does not affect the optimal change in asset holdings, and a *sub*optimal change in asset holdings does not affect the optimal marginal rate of substitution between consumption and leisure.[50]

We assume that the household chooses asset holdings, q_{jt}, and hours worked, l_{jt}, in every period t. One can also think of the household as choosing directly the vector x_t in equation (18). These two assumptions are equivalent so long as the household knows its own past action q_{jt-1}, which is the case if $\mathcal{I}_{jt-1} \subset \mathcal{I}_{jt}$, where $\mathcal{I}_{jt}$ denotes the information set of household j in period t. The fact that the matrix Θ is diagonal implies that the best response of household j in period t given any information set $\mathcal{I}_{jt}$ is the conditional expectation of x_t^*, $x_t = E\left(x_t^* | \mathcal{I}_{jt}\right)$.

We also assume that the vector x_t^* given by equation (20) has a first-order VAR representation, $\xi_{t+1} = F\xi_t + v_{t+1}$. The rational inattention problem of a household then has the exact same form as the rational inattention problem of a firm (equations (7)-(12)), and it can be solved using any solution method for linear quadratic Gaussian pure tracking problems (Maćkowiak, Matějka, and Wiederholt, 2018, Afrouzi and Yang, 2021, Jurado, 2021, Miao, Wu, and Young, 2021).

Finding a fixed point of an economy in which all firms and households are subject to rational inattention and hold correct beliefs about the law of motion of the state is more difficult than what we have done so far. Now equilibrium depends on the signals chosen by firms and on the signals chosen by households. To find the fixed point, we guess that in equilibrium consumption c_t and the wage w_t each follows a finite-order ARMA process. We calculate the implied ARMA representation

[49] The coefficients ω_V, ω_W and ω_D denote the steady-state ratios V/C, WL/C and D/C, respectively.

[50] A given change in asset holdings can be financed with different combinations of consumption and hours worked. One of these combinations equates the marginal rate of substitution between consumption and leisure to the wage.

of the optimal inputs of firm i and we solve the firm's attention problem, as in Section 5. From the solution we obtain the firm's inputs, k_{it} and l_{it}, and the aggregate variables k_t, i_t, y_t, and d_t, again as in Section 5. Turning to the attention problem of household j, we note that the optimal choice vector x_t^* depends on the process for w_t, d_t, and v_t (the first element x_{1t}^* depends on w_t, d_t, and v_t, and the second element x_{2t}^* depends on w_t and d_t). See equation (20). The price of a mutual fund share v_t adjusts so that in equilibrium asset demand equals asset supply, $\int_0^1 q_{jt} dj = 0$. To impose this asset market clearing condition, we compute the process for v_t such that x_{1t}^* equals 0 given the guess for w_t and the solution for d_t. We also calculate the ARMA representation of x_{2t}^* implied by the guess for w_t and the solution for d_t. We then solve the household's attention problem. Since $x_{1t}^* = 0$, the perfect tracking of x_{1t}^* requires no attention and the solution to the household's attention problem has the feature that $x_{1t} = 0$, which implies that $q_{jt} = 0$ and $\int_0^1 q_{jt} dj = 0$. The household's optimal signal choice together with the equation for x_{2t} (the second line in (18)) and $q_{jt} = 0$ yield the household's hours worked, l_{jt}. We adjust the guess for consumption c_t and the wage w_t until the resource constraint, equation (15), holds and labor demand equals labor supply, $\int_0^1 l_{it} di = \int_0^1 l_{jt} dj$.

We assume the same parameter values as in Section 5, except that the marginal cost of attention to a household, which we call μ, no longer equals 0 as is implicit there. We set $\mu = (1/100,000)\, C^{1-\gamma}$ in the economy with $h = 0$ and $\mu = (3/100,000)\, C^{1-\gamma}$ in the economy with $h = 2$, which means that the household's marginal cost of attention is equal to $1/100,000$ of steady-state consumption ($3/100,000$, respectively) per period.[51] The derivation of the household's objective assumes that η is a strictly positive number, whereas $\eta = 0$ in Section 5. Therefore we now set η equal to a very small, strictly positive number (so that utility is approximately linear in hours worked). The equilibria studied in Section 5 are essentially identical whether $\eta = 0$ or η equals a very small, strictly positive number.

This appears to be the first time in the literature that a general equilibrium model is solved in which all agents are subject to rational inattention and prices, which the agents take as given,

[51] In equilibrium, the per period expected utility loss from inattention turns out to equal $5/1,000,000$ of steady-state consumption with $h = 0$ and $8/1,000,000$ with $h = 2$. We focus on the same value of λ as in Section 5 and small μ to learn how rational inattention by households changes the equilibrium. In principle, one could choose the values of λ and μ jointly to maximize the model's fit to, e.g., the unconditional moments in the data from Table 1. However, one would need to solve the model many times and each solution is time-consuming.

adjust so that markets clear (here, the wage adjusts to equate labor demand and supply and the price of a mutual fund share adjusts to equate asset demand and supply).[52]

Figure 6 shows the equilibrium with firms and households subject to rational inattention (lines with asterisks). The top row is the case of $h = 0$. The bottom row is the case of $h = 2$. The perfect information equilibrium (lines with points) and the equilibrium from Section 5 with rationally inattentive firms and perfectly informed households (lines with circles) are displayed for comparison.

Begin with a standard productivity shock, $h = 0$. On impact rational inattention by households reduces labor supply for a given wage, because it takes time for households to recognize that working conditions have improved. To restore equilibrium in the labor market the wage rises (the impulse response of the wage is stronger on impact when households are rationally inattentive than when they have perfect information). A higher wage depresses investment demand (the profit-maximizing capital stock is decreasing in the expected wage, see the first line of equation (6)). In equilibrium employment, investment, output, and consumption fall compared with the equilibrium from Section 5.1. Thus, rational inattention by households adds further dampening and delay to the impulse responses of these variables to a productivity shock.

Next, consider a news shock, $h = 2$. Rational inattention by households has two effects in the model. It weakens the wealth effect on labor supply and saving supply, since a news shock is an instantaneous change in the present value of income and rational inattention creates a dampened and delayed reaction of consumption and leisure to this change in permanent income. In addition, under rational inattention the labor supply decision becomes forward-looking, which makes households even more willing to supply labor at a given wage on impact of a positive news shock. The payoffs from future work rise, and the optimal signal of households confounds the payoff from current work with the payoffs from future work. Both effects of households' rational inattention strengthen comovement. To restore equilibrium in the labor market the wage falls (the impulse response of the wage is weaker on impact when households are rationally inattentive than when they have perfect information). A lower wage stimulates investment demand. In equilibrium employment, investment, and output rise on impact of a news shock compared with the equilibrium

[52] In Maćkowiak and Wiederholt (2015), all firms and households are also subject to rational inattention but in each market one side of the market sets the price while the other side chooses the quantity. Moreover, there is no capital in that model.

from Section 5.2. In parallel, saving rises while consumption falls. These effects get reversed once productivity rises (the wage increases, investment falls, and so on, relative to the equilibrium from Section 5.2, as we have seen from the impulse responses to a productivity shock).

We conclude that rational inattention by households *strengthens* comovement after a news shock. With rationally inattentive households employment, investment, and output are *even higher* on impact of a news shock, because these households then supply more labor and save more than perfectly informed households.

7 Conclusions

Very few papers so far have solved a DSGE model with rational inattention. This paper solves a benchmark RBC model with rational inattention (RI-RBC).

The RI-RBC model generates over-reaction to news by decision-makers in firms on impact of a news shock and under-reaction once productivity actually changes, where over- and under-reaction are defined relative to the profit-maximizing actions. It is the anticipation of the under-reaction later on that makes the over-reaction early on desirable. We find that the rational inattention effect on labor demand on impact of a news shock more than offsets the wealth effect on labor supply; thus, employment and output increase. We also find that the rational inattention effect on investment demand on impact of a news shock offsets the wealth effect on saving supply; and that rational inattention by households strengthens comovement.

Comovement after news shocks is usually generated by introducing preferences that weaken the wealth effect on labor supply, investment adjustment costs, and variable capital utilization (Jaimovich and Rebelo, 2009), or complementarities in a multi-sector setting (Beaudry and Portier, 2007). We find it interesting that comovement emerges in the benchmark RI-RBC model. Furthermore, the rational inattention explanation for comovement after news shocks can potentially also rationalize why researchers who study different sample periods can reach conflicting conclusions regarding comovement in the data.

Hump-shaped impulse responses are usually generated by introducing adjustment costs or exogenous imperfect information. We find it interesting that a single friction (costly attention) and the agents' optimal response to that friction (rational inattention) generates both hump-shaped impulse responses and comovement after news shocks.

When we introduced rational inattention on the side of households, we assumed that they choose how much to save and how much to work. In future work, it would be worthwhile to solve the model under the assumption that households choose how much to consume and how much to work. We conjecture that one would obtain very similar results in that alternative setup. To see this, consider the response of consumption and saving to a positive news shock. An inattentive household which chooses consumption under-reacts to the optimal consumption response, and thus consumes less (saves more) than under perfect information. An inattentive household which chooses saving under-reacts to the *negative* optimal saving response, and thus also saves more (consumes less) than under perfect information. In future research, it would also be interesting to study the implications of rational inattention for how the economy responds to fiscal news shocks (news about future government spending, future taxes, or future transfers).

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Table 1: Business cycle statistics

	Data	Model, $h = 0$ Perfect information	Model, $h = 0$ Rational inattention
Relative standard deviation			
σ_c/σ_y	0.55	0.56	0.59
σ_l/σ_y	0.92	0.66	0.58
σ_i/σ_y	2.89	3.05	2.94
σ_a/σ_y	0.52	0.46	0.51
Correlation			
$\rho_{c,y}$	0.79	0.78	0.81
$\rho_{l,y}$	0.86	0.85	0.83
$\rho_{i,y}$	0.90	0.93	0.92
$\rho_{a,y}$	0.40	1.00	0.99
First-order serial correlation			
Δc	0.27	0.23	0.28
Δl	0.41	-0.06	0.44
Δi	0.35	-0.06	0.14
Δy	0.30	-0.05	0.13
Δa	-0.06	-0.05	-0.05

Data: United States, 1955Q1-2007Q4, from Eusepi and Preston (2011).
Model: Unconditional moments computed from the equilibrium MA representation of each variable.

Figure 1: Impulse responses with α = 0

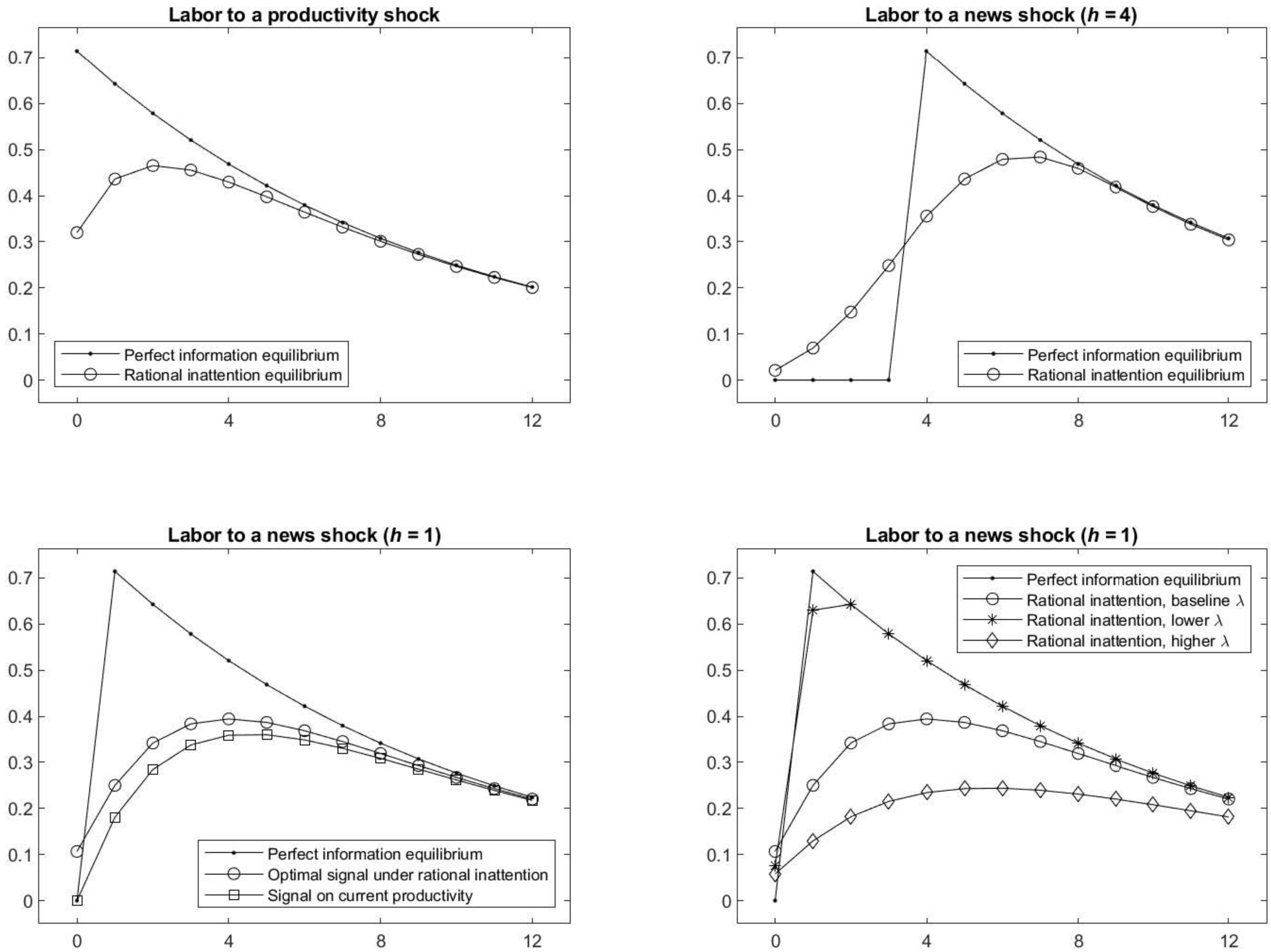

Figure 2: Impulse responses with ϕ = 0

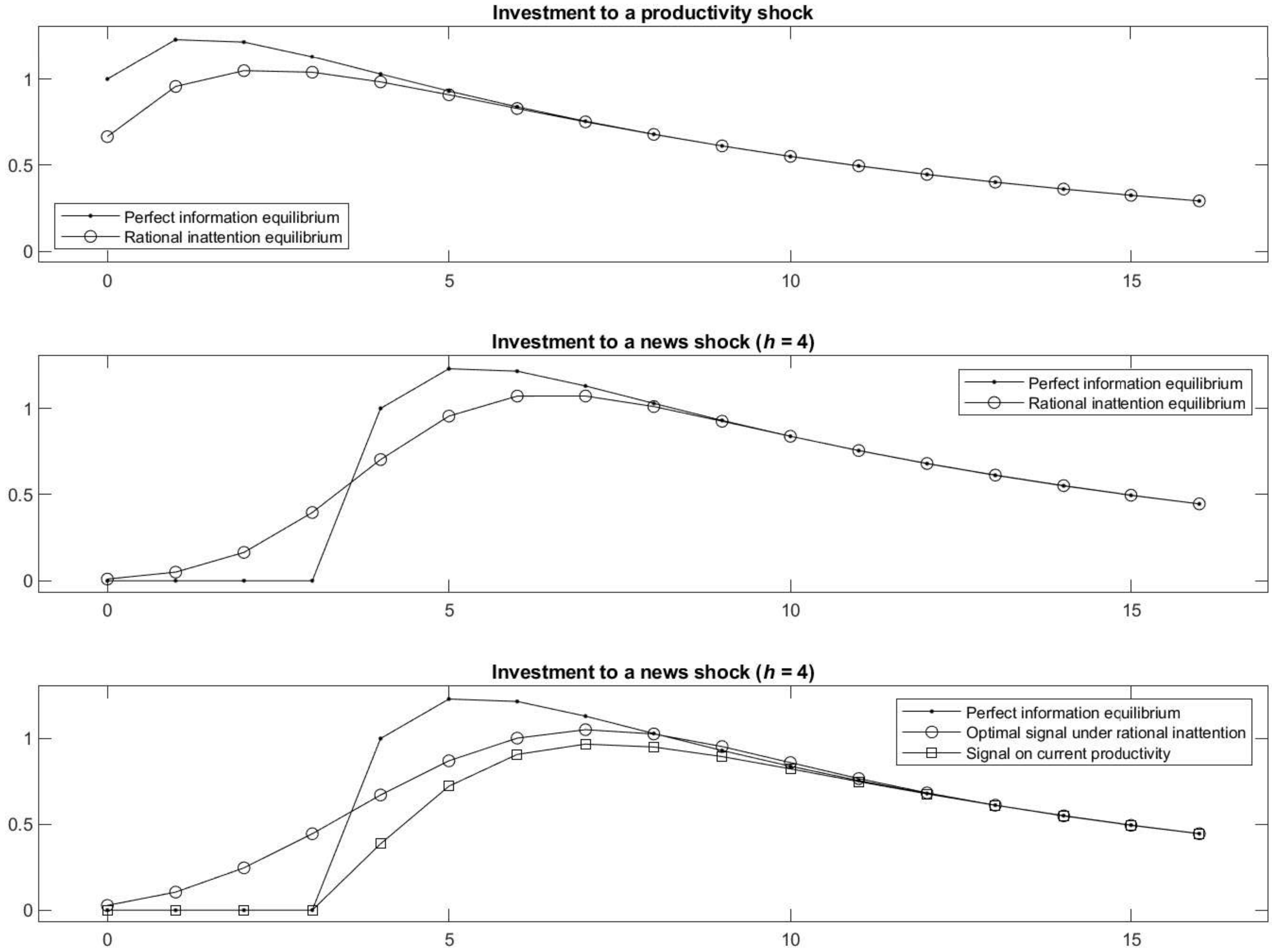

Figure 3: Impulse responses to a productivity shock

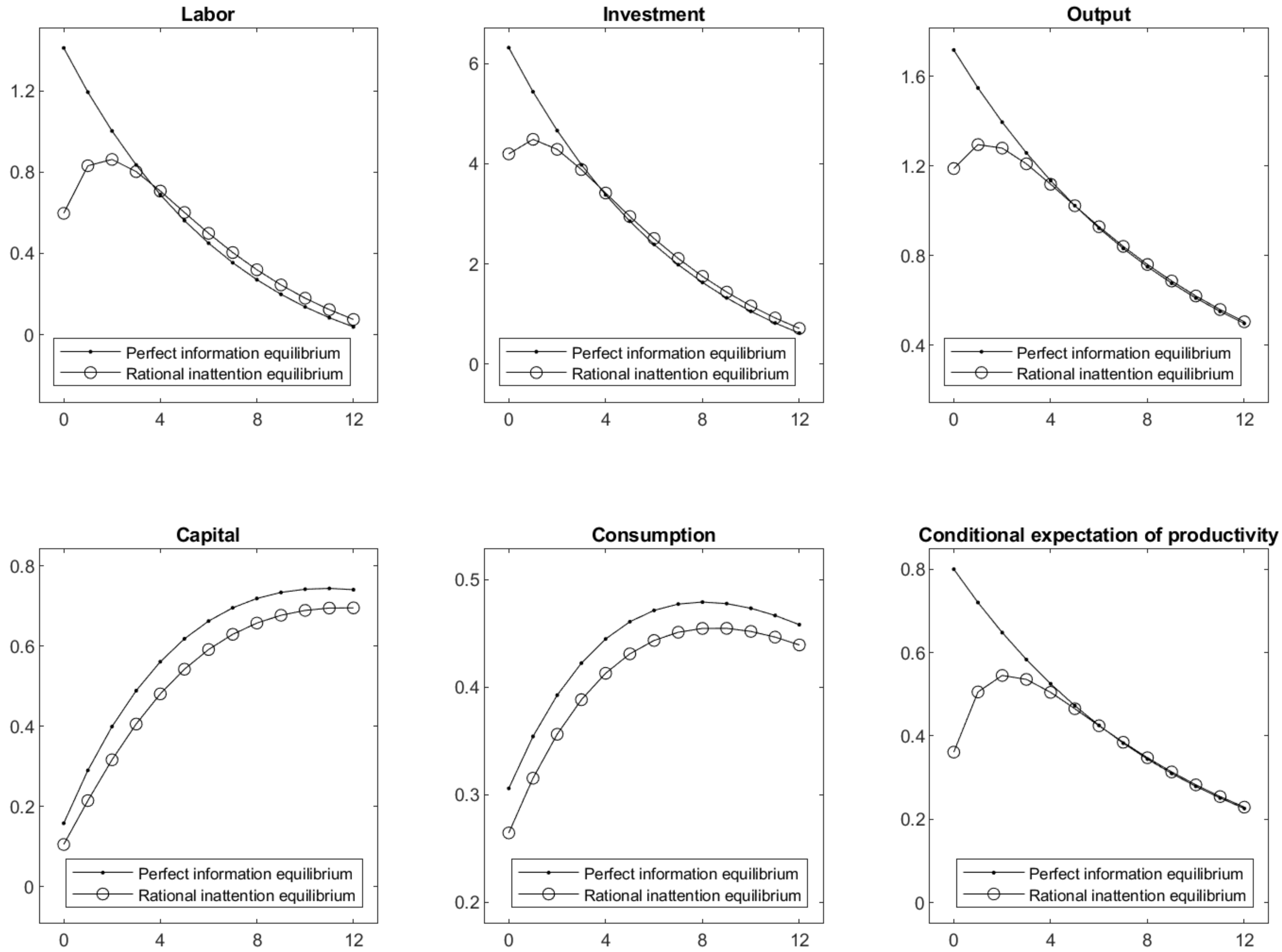

Figure 4: Impulse responses to a news shock (h = 2)

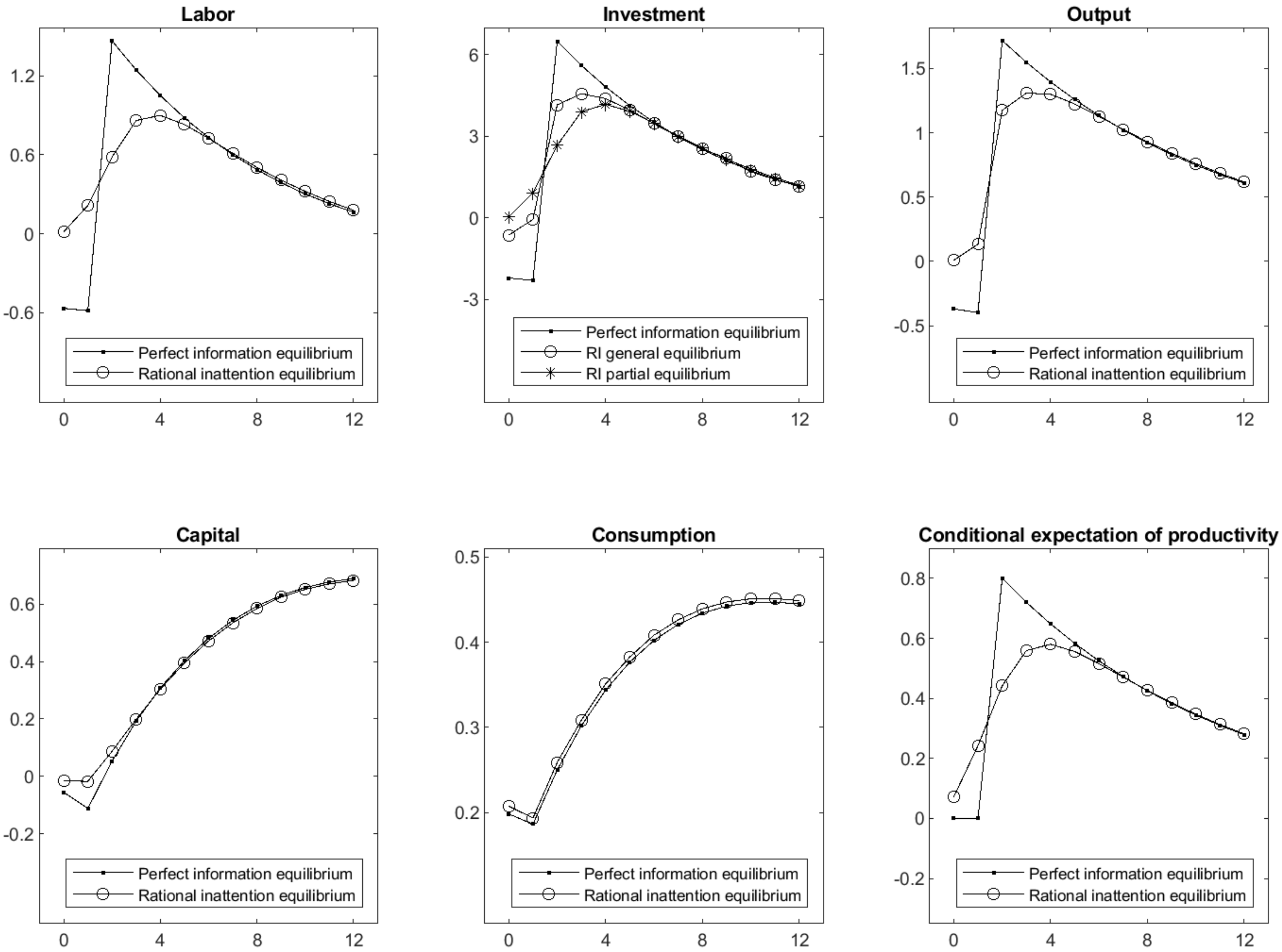

Figure 5: Impulse responses to a news shock (*h* = 4)

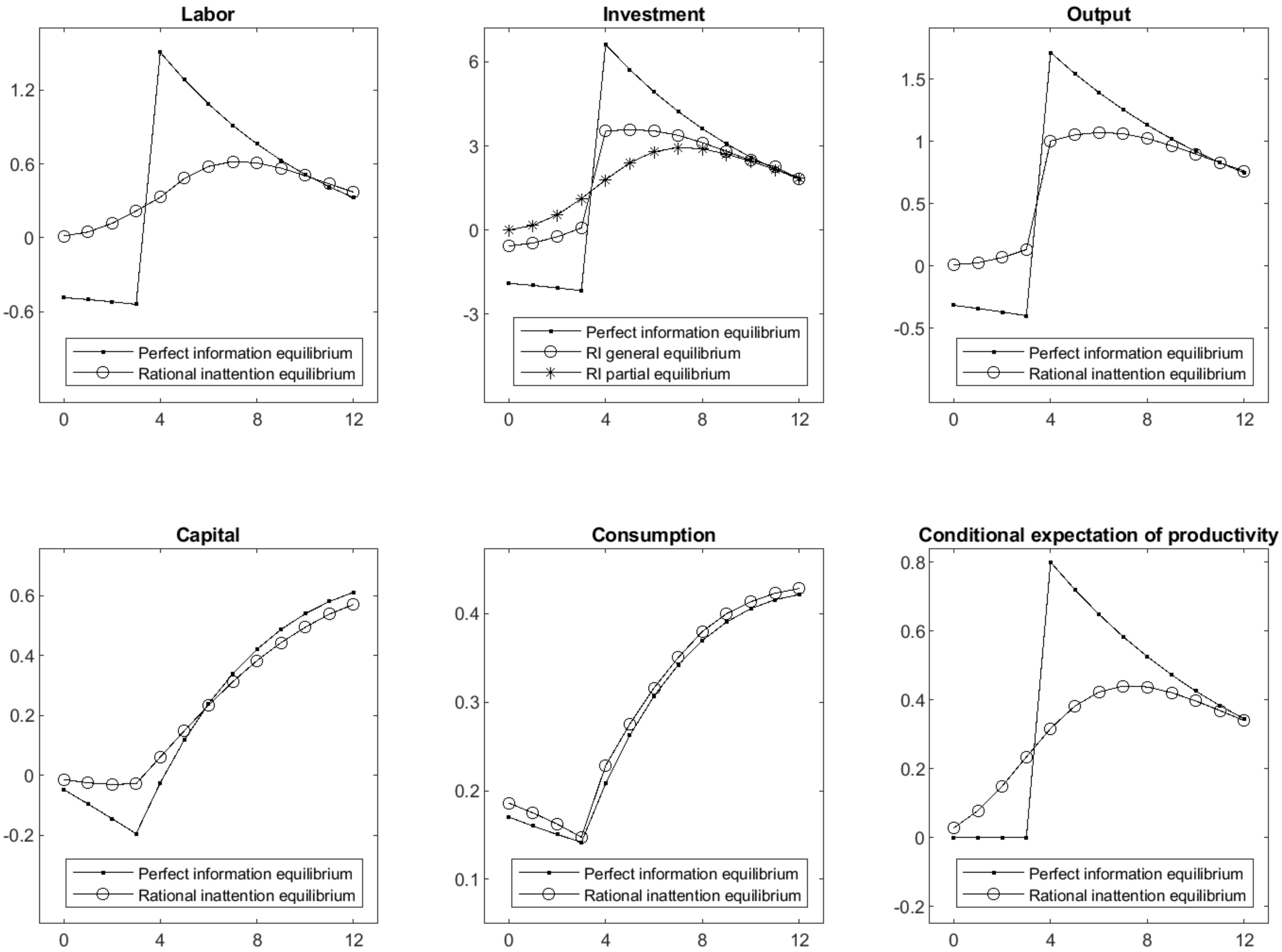

Figure 6: Impulse responses with rational inattention by firms and households