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Abstract

Many goods are allocated via resale networks, reaching their final buyer through a sequence of exchanges. We study a model where a single good is traded by a potentially infinite number of traders who have private valuations for the good and are connected in a random network that determines resale possibilities. Whoever holds the good has bargaining power. We show that large resale networks allocate efficiently in the no-discounting limit, even if resale opportunities are locally-limited. When the network is a stationary random tree, the limiting equilibrium is inefficient if and only if the network is a chain of monopolists.

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On the Efficiency of Large Resale Networks*

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Abstract

Many goods are allocated via *resale networks*, reaching their final buyer through a sequence of exchanges. We study a model where a single good is traded by a potentially infinite number of traders who have private valuations for the good and are connected in a random network that determines resale possibilities. Whoever holds the good has bargaining power. We show that large resale networks allocate efficiently in the no-discounting limit, even if resale opportunities are locally-limited. When the network is a stationary random tree, the limiting equilibrium is inefficient if and only if the network is a chain of monopolists.

KEYWORDS: decentralized markets, resale networks, asymmetric information, efficiency.

JEL CLASSIFICATION: C73; C78; D44; D82.

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1 Introduction

The ideal-type of a centralized market, where all buyers and sellers meet to exchange, plays a prominent role in economic theory. However, many goods and assets, including those whose value is not subject to fluctuation, change hands several times before reaching their final buyers. Often, especially when goods are unique and in short supply, a centralized marketplace, be it physical or virtual, is simply inexistent. Instead, goods are exchanged locally, over-the-counter, and reach their final buyers via *resale networks*. Examples range from the trading of artworks to over-the-counter (OTC) financial markets, such as the market for bonds and complex derivatives.[1]

In this paper we study the dynamic trading of a single good. We assume traders have private valuations over consuming the good, which are given once-and-for-all. The good is exchanged multiple times before consumption because resale opportunities of individual traders are limited. A salient feature of this economy is that the willingness to pay for the good does not depend on the own consumption value alone, as it would be normally the case in a large centralized market, but also on the structure of and behavior in the entire network of resale opportunities.[2] Our main insight is that, for large resale networks where owning the scarce good confers market power, a first-best allocation will be attained in the limit as discounting vanishes, as long as bottlenecks are rare, that is everyone has, frequently enough, a chance of finding multiple potential buyers.

Economic theory suggests at least two reasons why frictions could persist in a resale network, even in the limit as there's no delay-cost from exchanging the good. First, if negotiations take place between privately informed strategic agents, the opportunity of these agents to obtain a rent from their private information typically results in less trade than it would be optimal (see Myerson and Satterthwaite (1983)). While it is known that large centralized markets may resolve informational problems, there's no such general theory for decentralized ones.[3] Second, in a resale network, strategic traders will attempt to extract a rent from restricting access to other members of the network. In small networks, as we outline later in the literature review, this is known to generate inefficiencies, with and without asymmetric information.

Our theory on the efficiency of large resale networks is developed within the following model. A seller owns a single indivisible good and is connected to a, possibly random, number of ex-ante identical trading partners. Each of these partners is a potential buyer and has a monetary valuation for consuming the good, which is *private information*. The seller has *market power* and selects a mechanism to sell the good. If there is no sale, the seller consumes. If one of the buyers acquires the good, then another identical period ensues, where this buyer is now the seller. And so on until con-

[1]In OTC markets, transactions are mostly bilateral, traders often take securities in their inventory, and inter-dealer resale is frequent. For instance, exploiting a dataset of US municipal bonds transactions, Li and Schürhoff (2019) show that more than 20% of trades involve two or more dealers and that their relationships are long-lived, but not permanent.

[2]For instance, suppose a trader participates in an auction for a good knowing that she might resell it to someone who in turn can resell it to others, an so on. How much she is willing to bid depends on the own value from consuming *and* her beliefs about the network structure and the values of all buyers downstream to her. Even if consumption values are private, the presence of resale opportunities generates a common value component in the valuation.

[3]A number of papers demonstrate the efficiency of centralized markets, even when patronized by strategic agents, under a quite permissive set of informational assumptions. For the case of private-values, Rustichini et al. (1994), Cripps and Swinkels (2006) and Fudenberg et al. (2007) show that in one-shot double-auctions (i.e., since Chatterjee and Samuelson (1983) the paradigm for modeling a centralized market with strategic agents) the inefficiency due to private information quickly disappears as the number of buyers and sellers becomes large.

sumption takes place. The number of potential buyers that each seller has is determined, identically in every period, by the degree distribution of the resale network (i.e., the network is the realization of a Galton-Watson random tree) and everyone only observe their local connections. We assume that every trader has always at least one buyer and that the degree distribution has finite expectation. Since the number of potential buyers is, for sure, infinite and the average degree is constant, the *network is large but sparse*, in a formal graph-theoretic sense. All traders discount the future and maximize expected net monetary surplus. We focus on the unique limit equilibrium of this game, as discounting vanishes.

Assume all traders are ex-ante identical and let v_H be the maximum in the support of the buyers' distribution of values. It is easy to see that, if traders do *not* discount the future, then in the game above there is an (almost surely) *ex-post efficient* equilibrium, where agents keep exchanging the object at price v_H and when an agent with such value is eventually found, consumption takes place.[4] In light of this, our research question can be rephrased as follows. Are equilibria with positive discounting converging to this efficient equilibrium in the limit as agents no longer discount the future? How the answer to this question depends on the network structure?

To illustrate our results, let's begin by considering the simplest possible large resale network, an infinite *chain of monopolists*. Each trader has a single buyer and who owns the good makes a take-it-or-leave-it offer and consumes if this offer is refused.[5] In this case we show that the unique equilibrium with positive discounting does *not* converge to the efficient one described above. Even though, when discounting vanishes, it becomes common knowledge that a buyer with value of nearly v_H exists and can be reached almost surely in a finite number of rounds, the absence of downstream buyers competition generates a sizable inefficiency. In fact, the limit equilibrium is not much more efficient than if we were to let just a single seller and a single buyer negotiate under complete information. The opportunities for improving the allocation that are offered by the whole resale network, with its unbounded number of potential buyers, are wasted.

While the above indicates that an efficient outcome won't be attained *always*, our main result is that the chain of monopolists is the *only* case where it won't, under the stated assumptions. In fact, we show that an efficient outcome attains in the presence of even a tiny probability that each seller faces at least two buyers, rather than just one. The idea is that, as discounting disappears, each seller can now insist on asking a price of nearly v_H to *every* buyer, because it is common knowledge that there will be enough opportunities to find buyers to whom it is possible to ask a price of exactly v_H without risking breakdown of trade. For this, it is sufficient that, anytime a seller finds itself with two buyers, it asks a price of v_H to one of them and otherwise sells to the second one at a price that it will be accepted for sure. The (optimal version of this) mechanism now becomes incentive compatible for the seller even with discounting, as even the continuation value of the lowest-value buyer approaches v_H.

We study a stylized model. In Section 5 we argue that, in addition to sufficient downstream competition, two assumptions are really necessary for efficiency: unboundedness of the network and market power of sellers (i.e., bargaining power and ability to price discriminate). Otherwise, convergence to efficiency holds even if we dispense with a number of other assumptions we make mainly

[4]The path of the good is not important as the continuation network departing from *any* node is infinite.

[5]Following Riley and Zeckhauser (1983)), a take-it-or-leave-it offer is an optimal mechanism here.

for simplicity of exposition, including the network-stationarity assumption and that of symmetric information about the network.

This paper contributes to the large literature investigating decentralized markets. One of the central questions is whether or not, as decentralized markets become more and more "competitive", outcomes converge to efficient Walrasian ones. The canonical approach is to assume that a large pool of agents is, at each point in time, randomly matched in small groups to bargain over the terms of trade. If they fail to agree, then they experience a delay before being matched again. Seminal contributions include Rubinstein and Wolinsky (1985, 1990) and Gale (1986, 1987); a sample of more recent work in this tradition includes Lauermann (2012, 2013), Atakan and Ekmekci (2013), Golosov et al. (2014). For the special case where *sellers have bargaining power*, as in our model, it appears that the unambiguous conclusion is one of convergence towards Walrasian outcomes. Further work confirmed this insight for the case of one-sided (Wolinsky (1988), De Fraja and Sákovics (2001)) and and two-sided asymmetric information (Satterthwaite and Shneyerov (2007, 2008)).

In order to capture the salient features of resale networks, our paper departs from the *dynamic search and matching* literature in the way we model a decentralized market. In our work, the market is *not* two-sided, all traders can be both buyers and sellers. Moreover, there is a single good for sale, so, rather than having many local markets in any given period, we have one. Nevertheless, we reach analogous conclusions. On the one hand, as the cost of delaying consumption vanishes, informational asymmetries do not represent a major source of friction. On the other hand, the allocation of bargaining power and the matching protocol (i.e., the network in our case) matter. Sellers' market power and enough instances of local competition among buyers along the network are needed for efficiency.[6]

Recently, a literature has emerged that models markets as *finite* networks. Most of it, starting with Kranton and Minehart (2001), focuses on bipartite buyer-seller networks. A handful of papers consider resale networks, including Gale and Kariv (2007), Gofman (2011), Wright and Wong (2014), Condorelli et al. (2017), Manea (2018), Kotowski and Leister (2019), Blume et al. (2009), Nava (2015), Elliott (2015), Malamud and Rostek (2017), Babus and Kondor (2018); we refer the reader to Condorelli and Galeotti (2016) for a survey on dynamic model of strategic intermediation. A message coming from this line of work is that inefficiencies can persist even when the cost of delay gets small. In particular, traders who provide monopolistic access to parts of the network may not internalize the social value of reselling, which may lead to the good taking a less efficient path or to trade breakdown altogether. We contribute to this literature by showing that this source of inefficiency generally disappears in large resale networks where sellers have market power.

Tangentially related is the study of resale in auctions. A strand of this literature examines how incentives of bidders are affected by the possibility of resale, for instance see Zheng (2002), Haile (2003), Garratt and Tröger (2006), Hafalir and Krishna (2008), Carroll and Segal (2018). Another strand has focused on the effect of information provision following the initial sale, see Calzolari and Pavan (2006a,b) and Dworczak (2020). Partly because they ask different questions than us, these papers model resale as a one-shot phenomenon and therefore are not directly comparable.

[6]Other pioneering papers, such as Corominas-Bosch (2004), Polanski (2007) and Manea (2011) have embedded a network structure into a two-sided matching and bargaining game. These papers do not study intermediation and remain firmly in the tradition of the dynamic matching and bargaining literature.

2 Model

We consider a dynamic model where a single indivisible good may be traded for a possibly unlimited number of periods. At the outset, the good is owned by an *initial seller* who is linked to a finite number of *potential buyers*. In each period, including the first, the current owner of the good resells it to one of its downstream buyers or consumes it. Following consumption, the game ends. If the good is sold, the buyer becomes the new owner and it is matched with a randomly drawn set of new potential buyers. All other traders exit the game and a new period starts. Trading continues until consumption.

In each period, the number of potential buyers is the realization of a random variable taking *strictly* positive values in $\mathbb{N}_+$, with distribution π and finite expectation. The probability of a seller facing $n > 0$ buyers is π_n.[7] All traders (i.e., the initial seller and *all* potential buyers) are *ex-ante* identical, but once they enter the game are endowed with a monetary value for consuming the good. Values are private information and identically and independently drawn from the cumulative distribution F supported in $[0,1]$. We assume that 0 and 1 are in the support.

In each period, the seller publicly commits to a selling mechanism. Without loss of generality, we assume that before the choice of mechanism is made, everyone observes the number of downstream buyers the seller is connected with and the value of the seller.[8] A mechanism is an arbitrary game form, which determines the (possibly random) allocation of the good and the transfers from buyers to the seller, as a function of the history of its play. We assume all transfers are bounded by some constant greater than one.[9] Since participation is voluntary, a mechanism contains a non-participation option for every buyer that delivers a payoff of zero. To avoid specifying the set of feasible mechanisms, we follow Maskin and Tirole (1992) and assume that a seller cannot deviate to a mechanism that has no continuation equilibrium.[10] While not essential, we assume that the outcome of the mechanism is observed by its participants.

All traders discount future payoffs at rate $0 < \delta < 1$ and maximize expected payoff, which is the sum of discounted payments (positive or negative) and value from consumption. For instance, suppose a trader with value v pays p in period t and receives p' in $t+1$. Its payoff is $\delta^t(\delta p' - p)$. If, in addition, it consumes the good, then its payoff is $\delta^t(\delta v + \delta p' - p)$.

[7] We assume $\pi_0 = 0$. This guarantees that the network is infinite with probability one and, therefore, provides us with a natural efficiency benchmark. We discuss relaxing this assumption in Section 5.

[8] The seller can learn about the number of buyers through the design of the mechanism and with risk-neutral buyers it can't exploit this information to his advantage (see McAfee and McMillan (1987)). The second assumption avoids the complications of the informed principal problem. However, following Mylovanov and Tröger (2014) (see Proposition 8), in a linear mechanism design environment with private values and monotonic payoffs, as the one we study, the information buyers have about the seller's value has no influence on the equilibrium outcome.

[9] A bound on the maximum payment eliminates implausible equilibrium bubbles, whereby traders exchange the object forever at growing prices that allow everyone to at least break-even.

[10] As stated in the first paragraph of the section, we maintain that if the good is assigned to the seller, then consumption takes place. A seller cannot draw a new set of buyers in each period. As a result, sellers always prefer to use mechanisms that commit them to consumption whenever the good remains unsold.

3 Equilibrium

The solution concept we adopt is weak Perfect Bayesian equilibrium. To define the extensive-form game more formally, we may begin with a countably infinite set of players and envisage nature generating the resale network (i.e., a stationary random tree) according to the branching process defined by π and then randomly allocating players to nodes.[11]

When trader i enters the game as a buyer he knows his valuation, v_i, observes the mechanism the seller has selected, the seller's valuation, and the number of competing buyers. Trader i has no additional information. In particular, trader i has no information about past trading. Subsequently, when i acts as seller, his only additional information is the number of potential buyers he has and, naturally, the outcome of the mechanism and whatever he has observed during its play. A strategy for trader i is then defined as usual. To ease notation, we do not write down the belief system. We stress that upon entering the game as a buyer, trader i believes that the valuations of all other buyers are identically and independently distributed according to F. Similarly, upon acquiring the good, the seller believes that the valuation of the buyers he has been matched with are identically and independently drawn from F.

We start the analysis with two observations. First, upon acquiring the object, the continuation payoff of a trader, as a seller, only depends on his valuation. Indeed, the buyers the seller is matched with have no information about the history prior to the seller acquiring the good. Hence, their behavior cannot depend on such history. It follows that a seller with given value must expect the same payoff irregardless of previous history. Second, all traders with any given value have the same expected continuation payoff following their purchase. Because buyers cannot condition their behavior on the identity of the seller and all traders are equally likely to take up any position in the network, their continuation payoff, computed before the number of buyers is realized, must be the same. Because payments are bounded, this unique continuation payoff of sellers has to be in $[0,1]$. Since this continuation depends on the valuation of the owner, we let $V : [0,1] \to [0,1]$ be the function mapping valuations to continuation payoffs.

It is a corollary of the above that, at an equilibrium of our dynamic game, the mechanism selected by a seller with value v and the behavior of his n buyers must form an equilibrium of an associated static mechanism design problem. In this static game, a seller with valuation v chooses an individually-rational mechanism to sell to n buyers with identical and independent valuations distributed according $\delta V(\mathbf{v_i})$ with $\mathbf{v_i} \sim_d F$ for $i = 1, \dots, n$.

Let us denote with $R_n(\delta V, v)$ the expected revenue from the seller-optimal static mechanism. If V is measurable, then $R_n(\delta V, v)$ is well defined (see e.g. Page (1998)). We next argue that in any equilibrium of the static game a seller must obtain $R_n(\delta V, v)$. As shown in Duggan (1997), for all $\varepsilon > 0$ there exists a mechanism which guarantees a revenue of $R_n(\delta V, v) - \varepsilon$ in all of its equilibria. Hence, given V, no seller expect a payoff lower than $R_n(\delta V, v)$. It is routine to verify that $v \mapsto R_n(\cdot, v)$ is continuous in v and takes values in $[0,1]$.

We conclude from the previous arguments that, in an equilibrium of our dynamic game with

[11] The process first draws $n_0^1 \in \{1, 2, \dots\}$ nodes with probability $\pi_{n_0^1}$ and connects them to the initial node–node 0. Then, for each new node i, the process draws n_i^2 nodes and connects them to i, and so on.

discounting δ, if V is the continuation payoff upon acquiring the good, then a seller with value v expects payoff $R_n(\delta V, v)$ when facing n bidders. Modulo V, this pins down the choice of mechanism and bidding behavior in the equilibrium of our dynamic game. In particular, the initial seller and all subsequent sellers offer mechanisms achieving the maximal revenue $R_n(\delta V, v)$ when they are n buyers and a seller's valuation is v. Buyers play static equilibria of any mechanism they face, conditional on their continuation value being given by $V(v)$ when their valuation is v.

We are left with characterizing V. Because all traders have the same continuation payoff, if V is an equilibrium continuation payoff of our dynamic game, then we must have

$$V(v) = T_\delta(V)(v) := \sum_n \pi_n R_n(\delta V, v). \tag{1}$$

That is, in any equilibrium, the continuation payoff V must be a fixed point of the operator T_δ, which computes the expected revenue of a seller as a function of her value before the number of buyers is realized. Since $v \mapsto T_\delta(V)(v)$ is bounded and continuous, we can confine our search to the space of bounded and continuous functions $V : [0,1] \to [0,1]$, a complete metric space. In Appendix A we verify that T_δ is a contraction and, therefore, admits a unique fixed point, which we denote as V_δ.

To sum up, we have shown the following result, which asserts existence of a (seller-payoff-unique) equilibrium. Let G_δ be the game defined in Section 2.

Proposition 1 *For all $\delta < 1$, there exists an equilibrium of G_δ. Moreover, in all equilibria, the continuation payoff of each trader upon acquiring the good is V_δ, the unique fixed point of the operator T_δ.*

We have argued that V_δ is continuous and $0 \le V_\delta(v) \le 1$ for all v. It follows from standard mechanism design arguments that V_δ is increasing and convex. We sketch in Figure 1 the main geometrical properties of V_δ. Since δ is the highest-possible willingness to pay of a buyer (corresponding to the buyer being able to obtain 1 in the next round from ownership of the good), sellers with value above δ will consume and so $V_\delta(v) = v$ for $v \ge \delta$.

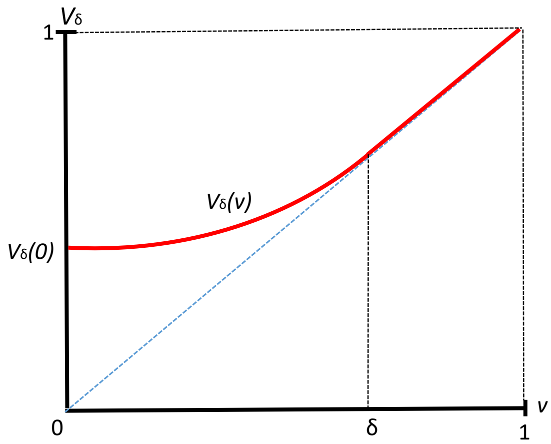

Figure 1: An arbitrary resale value function

4 Ex-post (in)efficiency of the limit equilibrium

Our main contribution is to characterize the efficiency properties of the *limit equilibrium* as $\delta \to 1$ and how they depend on the degree distribution π, which determines the realized resale network. As a preliminary step, we state that a limit equilibrium exists and all limit equilibria have the same continuation payoff function; the proof is in Appendix B.

Proposition 2 *Consider any δ-indexed sequence of the constructed equilibria presented in Proposition 1 with $\delta \to 1$. The limit equilibrium exists, it is an equilibrium of the limit-game G_1 and its continuation payoff is a continuous, increasing and convex function $V_1 = \lim_{\delta \to 1} V_\delta$ (uniformly). In addition, any other limit equilibrium must have continuation payoff equal to V_1.*

We can now state our main result.

Theorem 1 *A limit equilibrium is almost-surely ex-post efficient if, and only if, $\pi_1 < 1$ (i.e., unless the network is a chain of monopolists).*

Asserting that a limit equilibrium is almost surely ex-post efficient is equivalent to claiming that, with probability one, a player with value 1 consumes in equilibrium. Without discounting, only the value of the consuming trader matters for welfare because side-payments are welfare-neutral.[12] That a value 1 trader must consume follows from the fact that, since $\pi_0 = 0$, there are infinite buyers in the network and, almost surely, there is a buyer with the highest consumption value of 1.[13] In turn, in an efficient equilibrium trading must take place until a buyer with value arbitrarily close to 1 is found, and this requires that even *lowest* value buyers must be ready to purchase from *highest* value sellers, to keep the good in the market. Because a seller will always set a reserve price above its own valuation, we conclude that, for efficiency to attain, the willingness to pay of *all* buyers must converge to 1. It is immediate to see that if the willingness to pay for all buyers converge to 1, then consumption will only take place by sellers with values greater than or equal to $\delta V_\delta(0) \to 1$ and so the outcome is efficient. We therefore make the following useful observation; see proof in Appendix B.

Lemma 1 *The limit equilibrium is ex-post efficient if, and only if, $\lim_{\delta \to 1} V_\delta(0) = V_1(0) = 1$.*

Since $V_\delta(1) = 1$ and V_δ is non-decreasing, $\lim_{\delta \to 1} V_\delta(0) = V_1(0) = 1$ implies $V_1(v) = 1$ for all $v \in [0,1]$. With this in hand, we now develop the proof of Theorem 1, covering, in turn, the case of $\pi_1 < 1$ and the case of $\pi_1 = 1$. The following proof for the $\pi_1 < 1$ case delivers compelling intuition and we recommend the reader not to skip it.

[12] Monetary payments are welfare neutral in the sense that the sum of ex-post payoffs of all players must be equal to the discounted consumption value of the trader that consumes. Then, the ex-post sum of payoffs is zero if consumption never takes place. Otherwise, it is equal to $\delta^t v$, where t is the round in which the object is consumed and v the value of the consuming trader. Since when $\delta = 1$ utility between traders is freely transferable through a chain of payments, it follows that an ex-post efficient outcome must maximize the sum of ex-post payoffs.

[13] Denote with M_n the maximum of n i.i.d. random variable with CDF F. Note that $\Pr\{M_n \leq x\} = F^n(x)$ and that for $x < 1$, $F^n(x) < 1$. Then $\lim_{n \to \infty} F^n(x) = 0$. Hence, M_n converges in probability to 1. Since $\{M_n\}_n$ is a non-decreasing sequence, convergence is almost surely.

Proof of Theorem 1 ($\pi_1 < 1$ case) We want to show that $\lim_{\delta\to 1} V_\delta(0) = 1$. To do so, we derive a *lower bound* on $V_\delta(0)$, and show that it can be made arbitrarily close to 1 as δ increases. Consider the *equilibrium* resale-value function, V_δ, and recall that it will be shared by all sellers. Observe that a seller can always use the following mechanism: If the seller faces only one buyer, it makes a take-it-or-leave-it offer at price $\delta V_\delta(0)$; if it has two or more buyers, it makes a take-it-or-leave-it offer at price $\delta - \varepsilon$ to one of them and, if it is refused, asks price $\delta V_\delta(0)$ to another buyer, with $\varepsilon > 0$.

The offers at $\delta V_\delta(0)$ are accepted with probability one. The offers at a price $\delta - \varepsilon$ are accepted by buyers with consumption values in $\{v : \delta V_\delta(v) \geq \delta - \varepsilon\}$. Therefore, such offers are accepted by a positive measure of valuations in $[V_\delta^{-1}(\frac{\delta-\varepsilon}{\delta}), 1]$, where V_δ^{-1} is the generalized inverse of V_δ.[14]

Denote with $\gamma = (1-\pi_1)\left[1 - F\left(V_\delta^{-1}(\frac{\delta-\varepsilon}{\delta})\right)\right]$ the probability that the seller faces at least two buyers *and* that an offer at $\delta - \varepsilon$ is accepted when made. Observe that γ is strictly positive for any $\varepsilon > 0$, $\pi_1 < 1$ and $\delta \in (0,1)$. The revenue generated by this mechanism is

$$(\delta - \varepsilon)\gamma + (1-\gamma)\delta V_\delta(0).$$

Since all sellers can make use of this mechanism, this revenue is a lower bound of $V_\delta(0)$, i.e.,

$$V_\delta(0) \geq (\delta - \varepsilon)\gamma + (1-\gamma)\delta V_\delta(0) \iff V_\delta(0) \geq \frac{\gamma}{1-\delta(1-\gamma)}(\delta - \varepsilon).$$

Taking limits and substituting for γ, we obtain

$$\lim_{\delta\to 1} V_\delta(0) \geq \lim_{\delta\to 1} \frac{\gamma}{1-\delta(1-\gamma)}(\delta-\varepsilon) = \frac{(1-\pi_1)\left[1 - \lim_{\delta\to 1} F\left(V_\delta^{-1}(\frac{\delta-\varepsilon}{\delta})\right)\right]}{(1-\pi_1)\left[1 - \lim_{\delta\to 1} F\left(V_\delta^{-1}(\frac{\delta-\varepsilon}{\delta})\right)\right]}(1-\varepsilon) = 1-\varepsilon,$$

because $\lim_{\delta\to 1} F\left(V_\delta^{-1}(\frac{\delta-\varepsilon}{\delta})\right)$ exists and is strictly less than 1, since 1 is in the support of F and $\varepsilon > 0$. Since ε is arbitrary, it follows that $V_1(0) = \lim_{\delta\to 1} V_\delta(0) = 1$ if $\pi_1 < 1$. $\square$

The intuition that emerges from the proof is the following. Whenever there are at least two downstream buyers, a price of $\delta \cdot 1$ can be demanded to one of the buyers and, upon rejection, the good can be offered at a price $\delta V_\delta(0)$ which is accepted by all buyers. With this (potentially sub-optimal) mechanism, the object flows through the network until it reaches a buyer with a value above δ, who then consumes and to whom at most an information rent of $1-\delta$ is offered. As discounting vanishes, it is just sufficient that at least two buyers appear infinitely often along each path of the network, that is $\pi_1 < 1$, for these information rents to go to zero and so all resale values approach 1.[15] This provides the right incentive to each seller with a valuation lower than 1 to keep the object flowing in the network, which is what is needed to obtain ex-post efficiency when $\delta \to 1$.[16]

[14]That is, $V_\delta^{-1}(v) = \inf\{v' \in [0,1] : \delta V_\delta(v') \geq v\}$

[15]Note that the condition that $\pi_1 < 1$ does not mean that each seller will face two buyers in every period.

[16]An ex-post efficient equilibrium always exists in the G_1 game, where $\delta = 1$. For example, G_1 admits an equilibrium where sellers with value 1 consume, while all other sellers set a second-price auction with reserve price of 1 and sell with probability one. This equilibrium is efficient and every trader, except for the initial seller, breaks-even. Since the limit equilibrium must be an equilibrium of G_1 a corollary of the first part of Theorem 1 is that the limit equilibrium when

We now turn to the second part of Theorem 1, where it is assumed that the resale network is a chain of monopolists. In this case a seller either offers the object at a price that even the lowest value buyer is willing to accept ($\delta V_\delta(0)$) *or* will ask a higher price, thereby excluding some buyers from purchasing. Efficiency requires all sellers follow the former strategy. This implies that the resale value of a lowest value seller, $V_\delta(0)$, must be equal to $\delta V_\delta(0)$, which is only possible if $V_\delta(0) = 0$. This, of course, violates incentive compatibility for sellers, who can make positive profit by asking a positive price below δ. Rather than formalizing this argument, the proof that we provide next develops an upper-bound, $B(v)$, to $V_1(v)$ and shows that this upper bound is strictly lower than 1 for type $v = 0$, i.e., $B(0) < 1$. The derivation allows to determine a lower bound to the total welfare loss in the chain of monopolists.

Proof of Theorem 1 ($\pi_1 = 1$ case). Define the function $B : [0,1] \to [0,1]$ by

$$B(v) = 1 - \int_v^1 F(r)dr.$$

We first prove that B is an upper bound to V_1. Note that $B(v) = 1 - \int_v^1 F(r)dr \geq 1 - \int_v^1 1dr = v$ for all v. Moreover, B is convex with derivative $F(v)$ at v for all v. Therefore, for all (v,r),

$$\begin{aligned} B(v) &\geq B(r) + F(r)(v-r) \geq B(r) + F(r)(v-r) + \underbrace{F(r)(r - B(r))}_{\leq 0} \\ &= (1-F(r))B(r) + F(r)v. \end{aligned}$$

Hence,

$$B(v) \geq \max_{r \in [0,1]} (1-F(r))B(r) + F(r)v = T_1(B)(v).$$

It follows that

$$B \geq T_1(B) \geq T_\delta(B) \geq T_\delta(T_\delta(B)) \geq \cdots \geq \lim_{n\to\infty} T_\delta^n(B) = V_\delta,$$

for all δ, where we use the fact that T_δ is increasing and $T_\delta \geq T_{\delta'}$ for all $1 \geq \delta \geq \delta'$. Consequently, we have $\lim_{\delta \to 1} V_\delta = V_1 \leq B$.

Second, it is easy to see that $B(0) = \mathbb{E}[\mathbf{v}] < 1$ and therefore, in light of Lemma 1, this implies that ex-post efficiency will not attain in the limit, i.e., a positive mass of sellers with value greater than $V_1(0)$, with $V_1(0) \leq B(0) < 1$, consume with strictly positive probability in the limit equilibrium. $\square$

We now use the function B to construct a lower bound on the total welfare loss arising in equilibrium with a chain of monopolists. To do so, observe that a buyer with valuation v will never buy at a price higher than $B(v)$, which is an upper bound to the resale value, while a seller will never sell the good at a price below the own value. This implies that a seller with value u will consume whenever the buyer has value v such that $u > B(v)$. The probability that consumption happens when the seller has value u is, therefore, at least $\lim_{x\uparrow u} F(B^{-1}(x))$. Compared to the first best, the welfare loss in the limit equilibrium when a seller with value u consumes is $1-u$. Averaging over the possible valuations

$\pi_1 < 1$ indeed converges to this efficient equilibrium of G_1.

of the seller, we obtain a lower bound on the equilibrium welfare loss which is equal to

$$\int_0^1 (1-y) \lim_{x \uparrow y} F(B^{-1}(x)) dF(y).$$

The lower bound above is conservative. In practice, a seller always has an incentive to raise the price strictly above its value and a buyer's resale value will be above the lower bound B.

To appreciate the extent of the inefficiency let's focus on the the welfare of the first seller, whose payoff is equal to the total surplus when $\pi_1 < 1$. Assume $\mathbf{z} \sim_d F$, and let's rewrite the bound as

$$B(v) = 1 - \int_v^1 F(x)dx = 1 - \int_v^1 \int_0^x dF(z)\, dx = (1-F(v))\mathbb{E}[\mathbf{z} \mid \mathbf{z} > v] + F(v)v,$$

where the second equality follows by exchanging the order of integration. This rewriting indicates that the maximum (undiscounted) resale value of the initial seller with value v cannot exceed the expected *first-best* surplus from a bilateral trade between the initial seller and the next buyer in the chain. Hence, despite there are infinitely many resale opportunities along the chain of monopolists starting from the initial seller, only the value of selling to the direct downstream buyer is internalized by the initial seller. As we illustrate next with our examples, the fact that the initial seller's payoff is limited, does not imply that in a chain of monopolists the outcome remains close to efficient but the surplus is distributed to buyers along the chain. Instead, the seller extracts most of the surplus and inefficiencies are sizable.

Example (Two-value Distribution) Let the two possible valuations be $\{v_L, v_H\}$ with $0 < v_L < v_H = 1$ and assume that the probability of high value 1 is $0 < \mu < 1$. In this case, buyers have two valuations, $V_\delta(v_L)$ and $V_\delta(1)$. Since, the high-value seller always consumes, we have $V_\delta(1) = 1$.

The binary-valuation case is interesting in a number of ways. First, a closed-form solution for $V_\delta(v_L)$ is available. That is

$$V_\delta(v_L) = \begin{cases} \delta(1-\sum_n \pi_n (1-\mu)^n) + v_L \sum_n \pi_n (1-\mu)^n & \text{if } \tau^* < 0 \\ \frac{\delta(1-\sum_n \pi_n (1-\mu)^{n-1})}{1-\delta \sum_n \pi_n (1-\mu)^{n-1}} & \text{if } \tau^* \geq 0 \end{cases}$$

where

$$\tau^* = \delta \frac{\delta - \mu - \delta \sum_n \pi_n (1-\mu)^n}{1-\mu-\delta \sum_n \pi_n (1-\mu)^n} - v_L.$$

Observe that if $\tau^* < 0$ then the seller will use a reserve price equal to 1. Otherwise, the reserve price is $\delta V_\delta(v_L)$ and the outcome is inefficient. Consistently with our main result it can be seen that $\tau^* < 0$ if $\pi_1 = 1$ while $\lim_{\delta \to 1} \tau^* = 1 - v_L > 0$ if and only if $\pi_1 < 1$.

Second, the two-type case clearly highlights the logic behind the inefficiency result in the chain of monopolists. Facing a single buyer, a seller with value v_L either asks price δ, thus obtaining a payoff of $\mu\delta + (1-\mu)v_L$, or asks $\delta V_\delta(v_L)$, selling for sure and obtaining $\delta V_\delta(v_L)$. For the equilibrium to be ex-post efficient, sellers must prefer this second option. However, the fixed-point equation (1) implies in this case that $V_\delta(v_L) = \delta V_\delta(v_L)$, which is only satisfied if $V_\delta(v_L) = 0$. But this is clearly impossible in equilibrium, as $\mu\delta + (1-\mu)v_L > 0$. Hence, we conclude, the initial seller will ask price 1 in equilibrium and trade will never proceed for more than two rounds.

Third, the bound B is tight. In fact $B(v_L) = \mu + (1-\mu)v_L$. All traders beyond the initial seller make no surplus. Moreover, only the initial seller makes a positive payoff and the welfare loss is equal to $(1-v_L)(1-\mu)$. As one would expect, the loss increases when high-valuation traders become more difficult to find.

Example (Uniform Distribution) Suppose valuations are uniformly distributed. We are unable to solve for the limit resale value analytically, as it involves solving a system of two differential equations (i.e. the equation that determines the optimal price as a function of the resale value and equation (1), which depends on the optimal price).

Nonetheless, we were able to obtain a solution computationally. Figure 2 reports both the upper bound of the resale value of a buyer with value v, which in this case is $B(v) = \frac{v^2+1}{2}$ and the numerical computation of equilibrium. In the figure, the monopoly line indicates the seller's revenue when it sells under *asymmetric information*, to a single buyer who has no resale possibilities, and the 45-degree line indicates the autarchy outcome, where the seller consumes without selling.

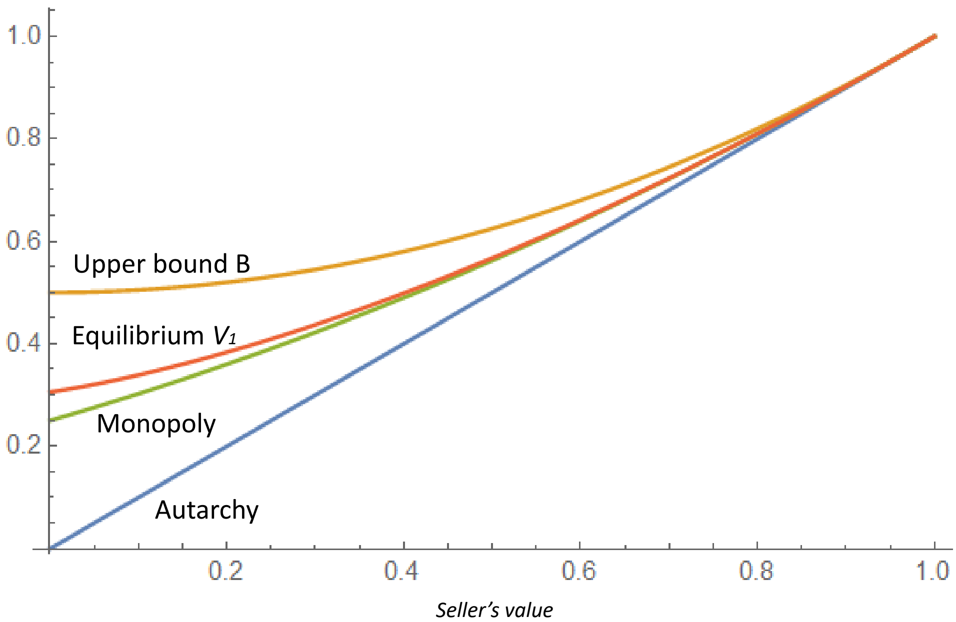

Figure 2: V_1 in the chain of monopolies ($\pi_1 = 1$) with the uniform distribution

5 Robustness

We conclude by investigating the limits and robustness of the efficiency result presented in Theorem 1.

5.1 Bargaining power and mechanism design

In our model, sellers choose a selling mechanism within the set of all feasible mechanisms. We argue that two conditions are necessary to the efficiency result in the $\pi_1 < 1$ case: (a) sellers have *all the bargaining power*, in the sense that they always choose the selling mechanism, and (b) sellers can *price-discriminate across buyers*.

To see why (a) is necessary, suppose that, with some probability, one of the buyers is randomly chosen to propose a mechanism to the seller, while with the remaining probability the seller makes the offer. Otherwise, the model remains the same and consumption by the owner of the good follows from a refused offer. Then, assuming the value of the seller is not known to the buyer, trade will be generically inefficient. This is an immediate consequence of the standard result on the inefficiency of monopoly pricing.

To see why (b) is necessary, contrasts the case in which sellers can offer different prices to different buyers with the case in which they can only post a uniform price. Buyers must decide whether to buy or not at the posted price and the object is randomly allocated among those willing to buy. In contrast with Theorem 1, the following proposition shows that the outcome is ex-post inefficient regardless of the resale network; see Appendix B for the proof.

Proposition 3 *Suppose the only mechanism available to sellers is to post a single price. Consider a sequence of equilibria of this modified game as $\delta \to 1$. The outcome of the limit equilibrium is not ex-post efficient.*

This result has a simple intuition. Consider the situation in which the resale network is so that every seller has n buyers. Because a seller can only post a uniform price, the probability that a buyer buys the object depends on the distribution of the maximum valuation of the n buyers. Hence, this situation is isomorphic to one where there is a *chain of monopolists* but the distribution of the value of the subsequent monopolist is now given by the distribution of maximum of n values drawn from F.

Finally, we argue that (a) and (b) are, in a sense, jointly sufficient for efficiency to attain when $\pi_1 < 1$. That is, the assumption that sellers can select *any* mechanism is not necessary for efficiency, which can be attained as long as sellers have bargaining power and, in addition, can charge different prices to different agents. Indeed, as we demonstrated in the first part of the proof of Theorem 1 efficiency is attained in the limit even if sellers are restricted to a sequence of bilateral negotiations in their local network, via, possibly discriminatory, take-it-or-leave-it offers.

5.2 Finite Networks.

We have assumed that $\pi_0 = 0$. When $\pi_0 > 0$, starting from any node in the network, the continuation network of resale will be finite with some positive probability.[17] This has an important consequence. That is, if we take any two buyers connected to the initial seller, say A and B, there will be a positive probability that the highest value among the traders who can be reached via buyer A will differ from the highest-value among the traders who can be reached via buyer B. However, since buyers have no information about future resale opportunities, there is no guarantee that in equilibrium the good will take the path where the highest-value trader is located. Since equilibrium will sometimes result in consumption by an agent who has not the highest-value in the network of resale, we conclude that when $\pi_0 > 0$, the equilibrium will not be ex-post efficient.

Ex-post efficiency becomes an excessively demanding benchmark when the network can be finite. With that in mind, we confirm next that the efficiency result of Theorem 1 still obtains as the probability that the network becomes finite vanishes, i.e., $\pi_0 \to 0$. To do so, we consider games $G_\delta(\pi)$ with resale network π that satisfies $\pi_0 > 0$ and $\pi_0 + \pi_1 < 1$. We first take a sequence of games where δ goes to 1 and we denote by $G_1(\pi)$ the limiting game. Then we take a sequence of games, $\{G_1(\pi), ..., G_1(\pi^k)...\}$, that starts from $G_1(\pi)$ and, along the sequence, we decrease π_0 in a way that $\lim_{k\to\infty} \pi^k = \hat{\pi}$ has the following properties: $\hat{\pi}_0 = 0$ and $\sum_{l\geq 1} \hat{\pi}_l - \pi_l = \pi_0$. That is, along the sequence we shift mass from π_0 to others realization of resale opportunities (see Appendix B for proof).

Proposition 4 *Suppose that $\pi_0 > 0$ and $\pi_0 + \pi_1 < 1$. The equilibrium outcome of $\lim\limits_{k\to\infty} G_1(\pi^k)$ is almost surely ex-post efficient.*

5.3 Asymmetric information about the resale network.

In our model traders are symmetrically informed about the resale network. In a way, we model an extreme scenario, where everyone knows very little about resale opportunities that will be available to themselves and to others. Instead, suppose each buyer is informed, before buying, of the number of its own downstream buyers, but all other traders treat this as random draw from π. In this case, at the time the selling mechanism is selected and played, there is asymmetric information both about the consumption value and about resale opportunities.

The equilibrium will exhibit differentiated resale value functions $\{V_\delta^1, V_\delta^2, \dots\}$, one for each possible realization of the number of downstream buyers of the given upstream seller. An analogous existence result can be established. Sellers will post mechanisms that maximize profit considering that now the willingness to pay of a buyer is $V_\delta^{\mathbf{n}}(\mathbf{v})$, with $\mathbf{n} \sim_d \pi$ and $\mathbf{v} \sim_d F$. Moreover, efficiency still attains if and only if $V_\delta^n(0) \to 1$ for all n.

We first argue that as long as $\pi_1 + \pi_0 = 0$, then the limit equilibrium will be ex-post efficient. In fact, if all traders have at least two downstream connections we use a simple adaptation of the argument we presented in the proof of the first part of Theorem 1. In particular, consider a seller with value equal to zero and two buyers. The seller proposes a price of $\delta - \varepsilon$ to one buyer and in case of

[17] This is equivalent to asserting that the extinction probability in the Galton-Watson process defined by π is positive, which is true when $\pi_0 > 0$.

rejection offers the good to the other buyer at a price of $\delta V_\delta^2(0)$. Note that $V_\delta^2(0)$ is the lowest resale value because $V_\delta^n(v)$ is going to be increasing in v and n. Hence, the offer $\delta V_\delta^2(0)$ is accepted surely by all buyers. We then have that

$$V_\delta^2(0) \geq (\delta-\varepsilon)\hat{\gamma} + (1-\hat{\gamma})\delta V_\delta^2(0)$$

where $\hat{\gamma} = 1 - F(V_\delta^{2,-1}\left(\frac{\delta-\varepsilon}{\delta}\right))]$. This implies that

$$\lim_{\delta\to 1} V_\delta^2(0) \quad \geq \quad \lim_{\delta\to 1} \frac{\hat{\gamma}}{1-\delta(1-\hat{\gamma})}(\delta-\varepsilon) = 1-\varepsilon.$$

As ε is arbitrary and as $V_\delta^n(v)$ are increasing in v and in n, we have that all resale values converge to 1, which implies ex-post efficiency.

However, this argument fails when $\pi_1 \in (0,1)$, for reasons analogous to those that prevents attainment of an efficient outcome in the case of the chain of monopolists in the benchmark model. To see this, consider a seller with valuation equal to zero and with only one buyer. For the outcome to be efficient, the seller must offer a price that a buyer always accept. This is equal to the lowest resale value across all buyers, $\delta V_\delta^1(0)$. But this implies that the resale of the lowest value seller with one buyer $V_\delta^1(0)$ must be equal to $\delta V_\delta^1(0)$. This is only possible if $V_\delta^1(0) = 0$. Again, for reasons already discussed, this continuation value is not compatible with equilibrium.

5.4 Non-stationary degree distribution.

We assume that the resale network is stationary, i.e., the degree distribution determining the number of buyers is the same in every period. However, one could envisage a different stochastic process $\{n_t \in \mathbb{N}_+, t = 0,1,\ldots\}$, where n_t is the random variable representing the number of buyers in period t. Then, the question arises about whether it is essential that *every* seller has *some* probability of having at least two buyers, or not. We argue, relying on the proof of Theorem 1, that efficiency would be attained quite generally even in this environment, as long as *each* seller expects an *infinite* number of future instances in which the number of available buyers is at least two.

To fix ideas, imagine a case where in odd periods sellers have just a single buyer, while in even periods they face at least two buyers with probability $(1-\pi_1) > 0$. This model intertwines a chain of monopolists with the case with multiple buyers. Then, consider sellers employing the same sequential mechanism we used in the proof of Theorem 1. As we show next, we are able to obtain the same conclusion we obtain in Theorem 1 for the $\pi_1 < 1$ case, that is the outcome is efficient.

Let V_δ^o the continuation of sellers in odd period and V_δ^e the continuation of sellers in even ones. We have

$$\begin{aligned} V_\delta^e &= (\delta-\varepsilon)\gamma + (1-\gamma)\delta V_\delta^o(0); \\ V_\delta^o &= \delta V_\delta^e(0). \end{aligned}$$

Substituting the bottom equation into the top one we have

$$V^e_\delta = (\delta - \varepsilon)\gamma + (1-\gamma)\delta^2 V^e_\delta(0).$$

Solving for $V^e_\delta(0)$ and recalling sellers are not using an optimal mechanism we obtain

$$V_\delta(0) \geq \frac{\gamma}{1-\delta^2(1-\gamma)}(\delta - \varepsilon),$$

which again converges to $1-\varepsilon$ as $\delta \to 1$. As it can be easily seen, the argument above does not rely on there being a positive probability of competition every two period. In fact, it is sufficient that competition is present with some probability every k periods, for some fixed finite k.

Appendix A: Proof that T_δ is a contraction

Throughout, we abuse notation and do not distinguish random variables and their realizations. It shall not create any confusion.

Recall that $R_n(\delta V, v)$ is the optimal revenue a seller with (consumption) value v obtains when he faces n buyers. Buyer i value the good at $\delta V(v_i)$ when his consumption value is v_i. Consumption values are independently and identically distributed with distribution F.

Recall that the mapping T_δ is given by:

$$T_\delta(V)(v) := \sum_n \pi_n R_n(\delta V, v),$$

for all functions $V : [0,1] \to [0,1]$ and v. Let $\mathscr{CV}$ be the space of bounded, continuous and convex functions from $[0,1]$ to $[0,1]$. Note $\mathscr{CV}$ is a closed subspace of the space of bounded and continuous functions from $[0,1]$ to $[0,1]$. Since the space of bounded and continuous functions from $[0,1]$ to $[0,1]$ is a complete metric space (equipped with the sup-norm), so is $\mathscr{CV}$.

We now show that the mapping $T_\delta : \mathscr{CV} \to \mathscr{CV}$ is a contraction.

We first derive important properties of $R_n(\delta V, v)$. From the revelation principle, we can restrict out attention to direct mechanisms. Let (x, p) be a direct mechanism. The mechanism is incentive compatible if:

$$U_i(v_i) := \mathbb{E}[x_i(v_i, v_{-i})\delta V(v_i) - p_i(v_i, v_{-i})|v_i] \geq \mathbb{E}[x_i(v_i', v_{-i})\delta V(v_i) - p_i(v_i', v_{-i})|v_i],$$

for all v_i', for all v_i, for all i. The mechanism is individual rational if

$$\mathbb{E}[x_i(v_i, v_{-i})\delta V(v_i) - p_i(v_i, v_{-i})|v_i] \geq 0,$$

for all v_i, for all i. The maximal expected revenue is therefore:

$$R_n(\delta V, v) = \sup_{(x,t)\text{: IR and IC}} \mathbb{E}\left[\left(1 - \sum_{i=1}^{n} x_i(v_i, v_{-i})\right) v + \sum_{i=1}^{n} p_i(v_i, v_{-i})\right],$$

when the seller faces n buyers and v is the seller's consumption value.

Note that $v \mapsto R_n(\delta V, v)$ is continuous (Berge's maximum theorem), convex (as the supremum of convex functions), bounded (since transfers are bounded) and increasing.

In the sequel, we write $X_i(v_i)$ (resp., $P_i(v_i)$) for $\mathbb{E}[x_i(v_i, v_{-i})|v_i]$ (resp., $\mathbb{E}[p_i(v_i, v_{-i})|v_i]$). From the convexity of V, incentive compatibility and positivity of X_i, we have that

$$\begin{aligned}
\delta X_i(v_i')V'(v_i')(v_i - v_i') &\leq \delta X_i(v_i')\left[V(v_i) - V(v_i')\right] \\
&\leq U_i(v_i) - U_i(v_i') \\
&\leq \delta X_i(v_i)\left[V(v_i) - V(v_i')\right] \leq \delta X_i(v_i)V'(v_i)(v_i - v_i'),
\end{aligned}$$

for all (v_i, v_i'), where V' denotes the left-derivative of V.

Two observations follow immediately. First, if $V(v_i') > V(v_i)$, then $X_i(v_i') \geq X_i(v_i)$. Second, if $V(v_i') = V(v_i)$, then $U_i(v_i) = U_i(v_i')$.

We now argue that at the optimum, we can assume without loss of generality that $X_i(v_i) = X_i(v_i')$ if $V(v_i') = V(v_i)$. It follows that X_i is increasing in v_i at an optimum. So, suppose that $V(v_i') = V(v_i)$ and assume without loss of generality that

$$(\delta V(v_i) - v) X_i(v_i) - U_i(v_i) > \big(\delta V(v_i') - v\big) X_i(v_i') - U_i(v_i').$$

We consider two distinct cases. First, assume that $F(v_i') - F(v_i'-) = 0$, where $F(v_i'-)$ is the left-limit. We can modify the mechanism without affecting the revenue by requiring that $x_j(v_i', v_{-i}) = x_j(v_i, v_{-i})$ and $p_j(v_i', v_{-i}) = p_j(v_i, v_{-i})$ for all v_{-i}. By construction, when buyer i reports v_i', his expected payoff is $U_i(v_i) = U_i(v_i')$, so that we do not affect his incentive to be truthful. Similarly, it does not affect the incentive of others and the revenue as we modify the mechanism on a set of measure zero. Second, assume that $F(v_i') - F(v_i'-) > 0$. Then, we have a contradiction with optimality noting that the expected revenue is:

$$v + \mathbb{E}\left[\sum_{j=1}^{n} \Big(\big(\delta V(v_j) - v\big) X_j(v_j) - U_j(v_j)\Big)\right].$$

It is therefore without loss of optimality to require that

$$(\delta V(v_i) - v) X_i(v_i) - U_i(v_i) = \big(\delta V(v_i') - v\big) X_i(v_i') - U_i(v_i'),$$

i.e., $X_i(v_i) = X_i(v_i')$ for all (v_i, v_i') such that $V(v_i) = V(v_i')$.

We are now ready to state and prove two important lemmas.

Lemma 2 *If $W \geq V$, then $R_n(\delta W, v) \geq R_n(\delta V, v)$ for all n, for all v.*

Proof of Lemma 2 Since X_i is increasing, it is Riemann integrable. Similarly, since V is convex, V' is increasing and is, therefore, Riemann integrable. Thus, $X_i V'$ is Riemann integrable. From the above, it follows that

$$U_i(v_i) = U_i(0) + \delta \int_0^{v_i} X_i(s) V'(s) ds.$$

As usual, at the optimum, $U_i(0) = 0$. Therefore,

$$
\begin{aligned}
&R_n(\delta V, v) - R_n(\delta W, v) = \\
&\max_{(X_i):X_i \text{ increasing}} \mathbb{E}\left[\sum_{i=1}^{n}\left((\delta V(v_i) - v)X_i(v_i)) - \delta\int_0^{v_i} X_i(s)V'(s)ds\right)\right] && (2)\\
&\qquad - \max_{(X_i):X_i \text{ increasing}} \mathbb{E}\left[\sum_{i=1}^{n}\left((\delta W(v_i) - v)X_i(v_i) - \delta\int_0^{v_i} X_i(s)W'(s)ds\right)\right]\\
&\leq \max_{(X_i):X_i \text{ increasing}} \mathbb{E}\left[\sum_{i=1}^{n}\left(\delta\left(V(v_i) - W(v_i)\right)X_i(v_i) - \delta\int_0^{v_i} X_i(s)(V'(s) - W'(s))ds\right)\right] && (3)\\
&= \max_{(X_i):X_i \text{ increasing}} \mathbb{E}\left[\sum_{i=1}^{n}\left(\delta\left(V(0) - W(0)\right)X_i(0) + \delta\int_0^{v_i} X_i'(s)(V(s) - W(s))ds\right)\right] && (4)\\
&\leq 0, && (5)
\end{aligned}
$$

where (1) follows from the definition of the revenue and the expression for $U_i(v_i)$, (2) follows from the property that the difference of the maximum of two functions is smaller than the maximum of the difference of the functions, (3) follows from integration by parts and (4) from $W \geq V$ and $X_i'(s) \geq 0$ for all s, for all i since X_i is increasing for all i. □

Lemma 3 *For all $c \geq 0$, $R_n(\delta(V + c), v) \leq R_n(\delta V, v) + \delta c$ for all n, for all v.*

Proof of Lemma 3 Let (x, p) be an optimal mechanism when the value function is $V + c$. Consider the new mechanism (x, p'), where

$$p_i'(v_i, v_{-i}) = p_i(v_i, v_{-i}) - \delta x_i(v_i, v_{-i})c$$

for all (v_i, v_{-i}), for all i. It is immediate to check that this mechanism is incentive compatible and individually rational when the value function is V. Therefore,

$$
\begin{aligned}
R_n(\delta V, v) &\geq \mathbb{E}\left[\left(1 - \sum_{i=1}^{n} x_i(v_i, v_{-i})\right)v + \sum_{i=1}^{n} p_i'(v_i, v_{-i})\right]\\
&= \mathbb{E}\left[\left(1 - \sum_{i=1}^{n} x_i(v_i, v_{-i})\right)v + \sum_{i=1}^{n} p_i(v_i, v_{-i}) - \delta x_i(v_i, v_{-i})c\right]\\
&\geq \mathbb{E}\left[\left(1 - \sum_{i=1}^{n} x_i(v_i, v_{-i})\right)v + \sum_{i=1}^{n} p_i(v_i, v_{-i})\right] - \delta c\\
&= R_n(\delta V + c) - \delta c,
\end{aligned}
$$

the desired result. □

We conclude that the mapping T_δ is a contraction by applying Blackwell's theorem. From Lemma 2, we have that

$$T_\delta(V)(v) = \sum_n \pi_n R_n(\delta V, v) \leq \sum_n \pi_n R_n(\delta W, v) = T_\delta(W)(v)$$

for all v, whenever $V \leq W$. From Lemma 3, we have that

$$T_\delta(V+c)(v) = \sum_n \pi_n R_n(\delta(V+c), v) \leq \sum_n \pi_n R_n(\delta V, v) + \delta c = T_\delta(V)(v) + \delta c,$$

for all v.

Appendix B: Remaining proofs

Proof of Lemma 1 Suppose $V_1(0) < 1$. Then, consider an open interval sellers types, with values $v \in (V_1(0), 1)$. They will never sell with positive probability the object the open set of buyers in $(V_1(0), V_1(0) + \varepsilon)$ for some $\varepsilon > 0$, where this set exists because V_1 is continuous. That the equilibrium is efficient when $V_1(0) = 1$ follows because $V_1(x) = 1$ for all $x \in [0,1]$ and the sum of all traders'expected payoffs, which cannot be lower or exceed $V_1(0) = 1$, can only be equal to 1 if a value 1 trader consumes almost surely. □

Proof of Proposition 2 For each $v \in [0,1]$, the map $\delta \mapsto V_\delta(v)$ is increasing. To see this, observe that T_δ is monotonic, i.e., $T_\delta(V) \geq T_\delta(V')$ if $V \geq V'$, and $T_\delta(V) \geq T_{\delta'}(V)$ if $\delta \geq \delta'$. Therefore,

$$V_{\delta'} = T_{\delta'}(V_{\delta'}) \leq T_\delta(V_{\delta'}) \leq T_\delta(T_\delta(V_{\delta'})) \leq \cdots \leq \lim_{k \to +\infty} T^k_\delta(V_{\delta'}) = V_\delta.$$

Consequently, the sequence $(V_\delta(v))_\delta$ is an increasing sequence in δ, which takes values in the compact set $[0,1]$. Hence, the pointwise $\lim_{\delta \to 1}(V_\delta(v))_\delta$ exists. Moreover, since V_δ is continuous and the sequence is increasing, convergence is uniform by Dini's Theorem. It follows that V_1 is continuous, increasing and convex. Since V_δ is convex, it is also follows that the left-derivative V'_δ converges uniformly to the left-derivative of V'_1.

Recall that the optimal revenue $R_n(\delta V_\delta, v)$ is given by:

$$\max_{(X_i):X_i \text{ increasing}} \mathbb{E}\left[\sum_{i=1}^n \left((\delta V_\delta(v_i) - v)X_i(v_i)) - \delta \int_0^{v_i} X_i(s)V'_\delta(s)ds\right)\right].$$

From the uniform convergence of V_δ and V'_δ, we have that

$$\lim_{\delta \to 1} \max_{(X_i):X_i \text{ increasing}} \mathbb{E}\left[\sum_{i=1}^n \left((\delta V_\delta(v_i) - v)X_i(v_i)) - \delta \int_0^{v_i} X_i(s)V'_\delta(s)ds\right)\right] =$$
$$\max_{(X_i):X_i \text{ increasing}} \mathbb{E}\left[\sum_{i=1}^n \left((\delta V_1(v_i) - v)X_i(v_i)) - \delta \int_0^{v_i} X_i(s)V'_1(s)ds\right)\right].$$

The space of increasing functions $X_i : [0,1] \to [0,1]$ is closed. To see this, we prove that the complementary set is open. Let $f : [0,1] \to [0,1]$ be a non-increasing function, that is, there exist $x > y$ such that $f(x) < f(y)$. Consider the set $\mathscr{O} := \{g : [0,1] \to [0,1] : g(x) < (f(x)+f(y))/2, g(y) > (f(x)+f(y))/2\}$. By construction, $f \in \mathscr{O}$. Moreover, all functions in $\mathscr{O}$ are non-increasing. Thus, we can find an open set of non-increasing functions g around f for every non-increasing f, hence the space of non-increasing functions is closed.

Since the space of increasing functions X_i is a subset of $[0,1]^{[0,1]}$, a compact set by Tychonoff theorem, the space of increasing functions $X_i : [0,1] \to [0,1]$ is compact. Thus, we can extract converging sub-sequences $(X_{i,\delta})_\delta$. In particular, if $X^*_{i,\delta}$ maximizes $R_n(\delta V_\delta, v)$ for all δ, we can extract a converging sub-sequences.

From the extraction of such sub-sequences, it immediately follows that we can construct a sequence of equilibria of G_δ converging to an equilibrium in G_1. As explained in the main text, the equilibria of G_δ we consider consists in sellers offering optimal direct revelation mechanisms and buyers reporting truthfully their valuations. (Off path, buyers play an equilibrium of the static game induced by the mechanism selected by the seller, when the continuation values are given by V_δ. An equilibrium exists by assumption.) The rest of the statements follow from uniqueness of the fixed point V_δ for each $\delta < 0$. □

Proof of Proposition 3 Let F^n be the distribution of the maximum valuation of n draws. Let $B^n(v) = 1 - \int_v^1 F^n(r)dr$ for every n, and let $B(v) = \sum_n \pi_n B^n(v)$. Note that $B^n(v) = 1 - \int_v^1 F^n(r)dr \geq 1 - \int_v^1 1dr = v$ for all v and for all n. We have that for all (v,r,n),

$$\begin{aligned} B^n(v) &\geq B^n(r) + F^n(r)(v-r) \\ &\geq B^n(r) + F^n(r)(v-r) + \underbrace{F^n(r)(r - B^n(r))}_{\leq 0} \\ &= (1 - F^n(r))B^n(r) + F^n(r)v. \end{aligned}$$

Hence,

$$B^n(v) \geq \max_{r\in[0,1]} (1 - F^n(r))B^n(r) + F^n(r)v = \mathbb{E}R_n(\delta V_\delta, \delta),$$

where $\mathbb{E}R_n(\delta V_\delta, \delta)$ is the expected revenue of a seller posting a uniform price to n buyers. It follows that

$$B(v) = \sum_n \pi_n B^n(v) \geq \sum_n \pi_n \mathbb{E}R_n(\delta V_\delta, \delta) := T_\delta(V_\delta)(v).$$

Adapting the proof from Appendix A to the case where sellers can only use a posted uniform price, we obtain that T_δ, defined by $T_\delta(V)(v) := \sum_n \pi_n \mathbb{E}R_n(\delta V, v)$, is a contraction on the complete metric space of bounded, continuous and convex functions $\mathscr{C}\mathscr{V}$. Hence,

$$B \geq T_1(B) \geq T_\delta(B) \geq T_\delta(T_\delta(B)) \geq \cdots \geq \lim_{\ell\to\infty} T_\delta^\ell(B) = V_\delta,$$

for all δ. Consequently, we have that $\lim_{\delta\to 1} V_\delta = V_1 \leq B$. This leads to the following final inequalities:

$$V_1(0) \leq B(0) = \sum_n \pi_n B^n(0) = \sum_n \pi_n \mathbb{E}[\mathbf{v_{n:n}}] < 1,$$

where $\mathbb{E}[\mathbf{v_{n:n}}]$ is the expected value of the maximum of n draws from F, which implies that a strictly positive mass of traders with value strictly less than 1 consumes in equilibrium. □

Proof of Proposition 4 Suppose we start from the case where $\pi_0 + \pi_1 = 1$ Consider the *equilibrium* resale-value function, V_{δ,π_0}, and recall that it will be shared by all sellers. A seller can always make use of the following mechanism, for some fixed small $\varepsilon > 0$: If there is only one buyer, make a take-it-or-leave-it offer at price $\delta V_{\delta,\pi_0}(0)$, if there are more than two buyers, make a take-it-or-leave-it offer at price $\delta - \varepsilon$ to one of them, and if it is refused, then ask price $\delta V_{\delta,\pi_0}(0)$ to another, and if there is no buyer the seller consumes.

On the one hand, offers at $\delta V_{\delta,\pi_0}(0)$ are accepted with probability one. On the other hand, offers at price $\delta - \varepsilon$ are accepted by buyers with consumption values in $\{v : \delta V_{\delta,\pi_0}(v) \geq \delta - \varepsilon\}$. Therefore such offers are accepted by a positive measure of valuations in $[V^{-1}_{\delta,\pi_0}(\frac{\delta-\varepsilon}{\delta}), 1]$, where $V^{-1}_{\delta,\pi_0}(x)$ is the generalized inverse of V (i.e., $V^{-1}_{\delta,\pi_0}(v) = \inf\{v' \in [0,1] : \delta V_{\delta,\pi_0}(v') \geq v\}$).

If a seller with valuation $v = 0$ uses this mechanism, she will generate a revenue of

$$\pi_1 \delta V_{\delta,\pi_0}(0) + (1-\pi_1-\pi_0)\left[(\delta-\varepsilon)\left(1-F\left(V^{-1}_{\delta,\pi_0}\left(\frac{\delta-\varepsilon}{\delta}\right)\right)\right) + \delta V_{\delta,\pi_0}(0) F\left(V^{-1}_{\delta,\pi_0}\left(\frac{\delta-\varepsilon}{\delta}\right)\right)\right]$$

Since the seller is using a potentially suboptimal mechanism is follows that $V_{\delta,\pi_0}(0) \geq$

$$\pi_1 \delta V_{\delta,\pi_0}(0) + (1-\pi_1-\pi_0)\left[(\delta-\varepsilon)\left(1-F\left(V^{-1}_{\delta,\pi_0}\left(\frac{\delta-\varepsilon}{\delta}\right)\right)\right) + \delta V_{\delta,\pi_0}(0) F\left(V^{-1}_{\delta,\pi_0}\left(\frac{\delta-\varepsilon}{\delta}\right)\right)\right]$$

which implies

$$V_{\delta,\pi_0}(0) \geq \frac{(1-\pi_1-\pi_0)\left(1-F\left(V^{-1}_{\delta,\pi_0}\left(\frac{\delta-\varepsilon}{\delta}\right)\right)\right)}{1-\delta\left[\pi_1+(1-\pi_0-\pi_1)F\left(V^{-1}_{\delta,\pi_0}\left(\frac{\delta-\varepsilon}{\delta}\right)\right)\right]}(\delta-\varepsilon).$$

Taking limits on both sides with respect to δ we obtain

$$\begin{aligned}
\lim_{\delta\to 1} V_{\delta,\pi_0}(0) &\geq \lim_{\delta\to 1} \frac{(1-\pi_1-\pi_0)\left(1-F\left(V^{-1}_{\delta,\pi_0}\left(\frac{\delta-\varepsilon}{\delta}\right)\right)\right)}{1-\delta\left[\pi_1+(1-\pi_0-\pi_1)F\left(V^{-1}_{\delta,\pi_0}\left(\frac{\delta-\varepsilon}{\delta}\right)\right)\right]} \times (\delta-\varepsilon) \\
&= \frac{(1-\pi_1-\pi_0)\left(1-\lim_{\delta\to 1} F\left(V^{-1}_{\delta,\pi_0}\left(\frac{\delta-\varepsilon}{\delta}\right)\right)\right)}{1-\left[\pi_1+(1-\pi_0-\pi_1)\lim_{\delta\to 1} F\left(V^{-1}_{\delta,\pi_0}\left(\frac{\delta-\varepsilon}{\delta}\right)\right)\right]} \times (1-\varepsilon) \\
&= \frac{(1-\pi_1-\pi_0)\left(1-F\left(V^{-1}_{1,\pi_0}(1-\varepsilon)\right)\right)}{1-\left[\pi_1+(1-\pi_0-\pi_1)F\left(V^{-1}_{1,\pi_0}(1-\varepsilon)\right)\right]} \times (1-\varepsilon) \\
&= \frac{(1-\pi_1)\left(1-F\left(V^{-1}_{1,\pi_0}(1-\varepsilon)\right)\right)+\pi_0 F\left(V^{-1}_{1,\pi_0}(1-\varepsilon)\right)-\pi_0}{(1-\pi_1)\left(1-F\left(V^{-1}_{1,\pi_0}(1-\varepsilon)\right)\right)+\pi_0 F\left(V^{-1}_{1,\pi_0}(1-\varepsilon)\right)} \times (1-\varepsilon) \\
&= \left[1-\frac{\pi_0}{(1-\pi_1)\left(1-F\left(V^{-1}_{1,\pi_0}(1-\varepsilon)\right)\right)+\pi_0 F\left(V^{-1}_{1,\pi_0}(1-\varepsilon)\right)}\right](1-\varepsilon)
\end{aligned}$$

where we have used the fact that $\lim_{\delta\to 1} F\left(V^{-1}_{\delta,\pi_0}(\frac{\delta-\varepsilon}{\delta})\right)$ exists and is in $(0,1)$, since 1 is in the support of F and $\varepsilon > 0$.

Finally, taking the limit with respect to $\pi_0 \to 0$ we obtain that

$$\lim_{\pi_0 \to 0} \lim_{\delta \to 1} V_{\delta,\pi_0}(0) \geq \lim_{\pi_0 \to 0} \left[1 - \frac{\pi_0}{(1-\pi_1)\left(1 - F\left(V_{1,\pi_0}^{-1}(1-\varepsilon)\right)\right) + \pi_0 F\left(V_{1,\pi_0}^{-1}(1-\varepsilon)\right)} \right] (1-\varepsilon) = 1 - \varepsilon$$

where the equality follows by noticing that $\lim_{\pi_0 \to 0} F\left(V_{1,\pi_0}^{-1}(1-\varepsilon)\right)$ exists and is in $(0,1)$, since 1 is in the support of F and $\varepsilon > 0$, and that, by assumption, since $\pi_0 + \pi_1 < 1$, in the limit game in which $\pi_0 \to 0$ we still have that $\pi_1 < 1$.

The fact that $\lim_{\pi_0 \to 0} \lim_{\delta \to 1} V_{\delta,\pi_0}(0) \geq 1 - \varepsilon$ and that ε is arbitrary concludes the proof that $\lim_{\pi_0 \to 0} \lim_{\delta \to 1} V_{\delta,\pi_0}(0) = 1$ in the equilibrium outcome of the limiting game where traders are perfectly patients and the probability that the network is finite vanishes. $\square$

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