DISCUSSION PAPER SERIES

DP16764

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MONETARY ECONOMICS AND FLUCTUATIONS

ISSN 0265-8003

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Discussion Paper DP16764
Published 01 December 2021
Submitted 29 November 2021

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www.cepr.org

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JEL Classification: N/A

Keywords: N/A

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Acknowledgements
We are grateful for helpful comments from Kjetil Storesletten and seminar participants. Financial support from Handelsbanken's Research Foundations is gratefully acknowledged. All errors are our own.

Macroeconomic Dynamics with Rigid Wage Contracts*

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November 28, 2021

Abstract

We adapt the wage contracting structure in Chari (1983) to a dynamic, balanced-growth setting with re-contracting à la Calvo (1983). The resulting wage-rigidity framework delivers a model very similar to that in Jaimovich and Rebelo (2009), with their habit parameter replaced by our probability of wage-contract resetting. That is, if wage contracts can be reset very frequently, labor supply behaves in accordance with King et al. (1988) preferences, whereas if they are sticky for a long time, we obtain the setting in Greenwood et al. (1988), thus allowing significant responses of hours to wage changes.

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1 Introduction

A widely held view is that there is significant short-run wage rigidity and that this rigidity is an important element of the transmission mechanism of macroeconomic shocks.[1] There is so far no consensus, however, about the modeling of such wage rigidity. The applied quantitative-theory literature following Erceg et al. (2000) (EHL, see also Chapter 6 in Galí (2015)), for example, proceeds in analogy to the modeling of price rigidity in new-Keynesian settings: workers monopolistically choose an hourly wage at which they must supply whatever firms demand of their unique labor service. While both elegant and useful, the monopoly assumption appears quite strong for most individual workers.[2] Moreover, the restriction to a constant nominal hourly wage appears hard to square with actual work practices, in particular when a substantial rise in firms' demand moves workers far away from their supply curve. In this paper, we propose an alternative framework for studying wage rigidity. In particular, we follow Chari (1983) in describing the ex-ante wage-setting stage, in advance of observing macroeconomic shocks, as one of choosing a *wage contract*: a schedule of hours-wage pairs from which the firm, as shocks hit ex post, can choose one. Ex-ante, moreover, there is perfect competition, with the result that firms offer the contract that maximizes workers' ex-ante utility. With this setting, hours worked are demand-determined ex post – firms have the "right to manage". Yet workers are not asked to work harder without extra compensation; in the Chari setup this compensation is in line with their optimal labor-leisure tradeoff.

We depart from Chari's setup in some distinct ways. First, because we are mainly interested in the fluctuation of hours, including on the individual level, we allow hours to be a continuous variable; in contrast, Chari's analysis was motivated by the implicit-contracts literature on employment vs. non-/un-employment.[3] Second, we adopt a preference specification that is consistent with balanced growth – from King et al. (1988) (KPR) – to maintain consistency with the applied macroeconomic literature. Third, we assume that workers are fully insured; again, Chari's interest lay in implicit insurance aspects whereas we focus on the choice of wage-hours pairs and a comparison with the standard representative-agent models. Fourth, we embed the Chari formulation in a dynamic model and assume re-contracting à la Calvo (1983).

We find that the resulting setting, while building on preferences with strong income effects, delivers high short-run intertemporal substitutability of labor. In fact, our model is very similar, in a reduced-form sense, to the setting proposed by Jaimovich and Rebelo (2009), who use a KPR setting with habits, delivering zero income effects in the short run but significant income effects in the long run, when habits have had time to

[1] Christiano et al. (2005) argues that rigid wages is the key friction needed for quantitative new-Keynesian model to match empirical impulse responses to monetary policy shocks. Olivei and Tenreyro (2010) show that wage rigidity plays an important role for the transmission of monetary policy. Wage rigidity also have stark distributional consequences, which greatly affects the dynamics of HANK models, see Broer et al. (2020) and Auclert et al. (2021).

[2] Monopolistic competition in wage setting may be interpreted as stemming from different occupations being organized in small unions, see Galí (2020). A wide body of recent empirical studies have, in contrast to the assumption in the EHL model, documented that labor markets are increasingly characterized by monopsonistic rather than monopolistic competition, see, e.g., Berger et al. (2019) and the literature discussed therein.

[3] See Rosen (1985) for a survey of the implicit-contracts literature.

adjust. The degree of habits in their setting is replaced, in our model, with the probability of re-contracting (the Calvo parameter). The equivalent of their habit term in our model, moreover, is forward- rather than backward-looking, but the main features of the two specifications are quite similar. Thus, if wage contracts can be reset very frequently, labor supply behaves in accordance with KPR, whereas if they are sticky for a long time, we obtain the setting in Greenwood et al. (1988), thus allowing significant responses in hours to wages.[4] In fact, we show that with our contractual setup, depending on the steady-state level of the labor share, the same response of hours to an underlying shock may be consistent with a procyclical, acyclical or countercyclical response of the average real hourly wage.

The details of the wage contract work as follows: the *marginal wage* is chosen so to make the contract ex-post efficient: the equilibrium amount of hours worked maximizes the joint surplus of the firm-worker pair. Thus, it equates the marginal rate of transformation with the marginal rate of substitution. In particular, it takes into account the increasing marginal disutility of hours worked. The *base wage* is chosen to be high enough to make workers agree to the contract. Because the contract is non-contingent on shocks, however, it only takes into account how the equilibrium level of consumption utility responds to shocks on average. With aggregate shocks small in relation to firm-level shocks, consumption does not respond much on average. So the equilibrium shift in income in response to aggregate shocks does not shift the labor supply curve.

With *nominally* rigid wages, the implications of our model turn out to be very similar to EHL. Like EHL, rigidity in our contractual setup means that in the short run, the nominal labor supply curve cannot shift in response to shocks. Because of this feature, our model generates a wage Phillips curve that closely resembles that in EHL. In contrast to the EHL fixed-wage assumption, implying that in the short run, the labor supply curve is flat, the wage-hours schedule implied by our contracting approach is, however, consistent with the upward-sloping labor supply curve implied by worker preferences. Our analysis thus highlights that the sufficient ingredients for a wage Phillips curve are that contracts are rigid in the sense that they are not renegotiable and not indexed to the price level, and that firms rather than workers have the right to manage ex post.

2 The contracting problem

Consider a firm-worker pair that interacts for two periods. In the second period, the firm operates the production function $\mathsf{AF}(\mathsf{N})$, where N is labor input by the worker, and A is a stochastic productivity term. In the first period, before productivity A is known, the firm offers the worker a contract with the aim of maximizing expected profits $\mathsf{AF}(\mathsf{N}) - W^s$ where W^s is the total wage payment. A contract specifies a relation between wage payments W^s and labor input N and can be accepted or rejected by the worker.

[4] Both Jaimovich and Rebelo (2009) preferences and Greenwood et al. (1988) preferences have been extensively used in the quantitative macroeconomic literature, see., e.g., Kaplan et al. (2018), Nakamura et al. (2014), Winberry (2021) and McKay and Wieland (2021).

After productivity becomes known, the firm has the "right to manage" the labor input: it can choose any combination of wage payments and labor input allowed by the contract.

The worker has separable preferences in wage payments and hours worked. If rejecting the contract, the worker receives reservation utility $\underline{U}$. If accepting the contract, the worker receives labor income W^s in exchange for N hours demanded by the firm after the shock is realized. Denote the disutility of hours worked with $\nu(N)$ and the value of receiving income W^s with $V(W^s)$. In this section, the worker has reduced-form preferences in wage payments; in an equilibrium model, the value of wage payments is derived from preferences over consumption and the financial market structure. In the case of financial autarky, $V(\cdot)$ is simply the consumption utility from directly consuming W^s.

Both the worker and the firm know the distribution of productivity, which has full support on $[0, \infty)$, at the contracting stage. Contracts are "rigid" in the following sense: first, the contract cannot be renegotiated in the second period. Second, the firm cannot offer a contract contingent on productivity.[5] Importantly, there are no other restrictions on the contract meaning that the firm may offer any wage-hours schedule $W^s(N)$ to the worker.

The wage-hours schedule $W^s(N)$ can be specified in terms of a *marginal wage curve* $W(n)$ and a *base wage* W_{min}: $W^s(N) = \int_0^N W(n)dn + W_{min}$. The base wage, which is paid if zero hours of work is demanded, should not be confused with the expected wage $\mathbb{E}[W^s(N)]$.

Second period The firm's problem in the second period, given a wage schedule specified by $W(\cdot)$ and W_{min}, is given by

$$\max_N AF(N) - \int_0^N W(n)dn - W_{min}. \tag{1}$$

For a standard production function F, the optimal labor input chosen by the firm is such that marginal productivity equals to the *marginal* wage, $AF'(N) = W(N)$. Given the marginal wage function $W(\cdot)$, this optimality condition implicitly solves for hours as a function of productivity: $N = \mathcal{N}(A)$.

[5] This non-contractability may be rationalized by the assumption that the firm, but not the worker, observes the level of productivity in the second period, and that there is some cost to renegotiation.

First period Given second-period labor demand $\mathcal{N}(A)$, the firm's problem in the first period is to maximize expected profits subject to providing the worker her reservation utility. The firm's problem is

$$\max_{W(\cdot),W_{min}} \mathbb{E}\left[AF(\mathcal{N}(A)) - \int_0^{\mathcal{N}(A)} W(n)dn - W_{min}\right] \tag{2}$$

$$\text{s.t. } \mathbb{E}\left(V\left(\int_0^{\mathcal{N}(A)} W(n)dw + W_{min}\right) - \nu(\mathcal{N}(A))\right) \geqslant \underline{U}, \tag{3}$$

$$W(\mathcal{N}(A)) = AF'(\mathcal{N}(A)). \tag{4}$$

By means of a substitution of variables, this program can recasted as a standard calculus-of-variations problem. Section 1 of the Online Appendix shows that the solution to this contracting problem is characterized by an ordinary differential equation. In general, this equation has to be solved using numerical methods. Proposition 2.1 characterizes the optimal contract for the case of a linear value function $V(\cdot)$ in wage payments. This case is particularly relevant for a wide class of macroeconomic models as it describes the contracting problem in an environment with idiosyncratic firm shocks where households can fully insure each other against fluctuations in their individual labor incomes.

Proposition 2.1. *With $V'(\cdot) = \frac{1}{\xi}$, where ξ is a constant, the optimal contract sets the marginal wage equal to the marginal rate of substitution, $W(N) = \xi\nu'(N)$ and hours worked are given by $AF'(N) = \xi\nu'(N)$.*

Proof. With linear $V(\cdot)$, utility is transferable between the worker and the firm. Therefore, the first best can be obtained as follows.

Solving for $\mathbb{E}[\int_0^{\mathcal{N}(A)} W(n)dn - W_{min}]$ in the constraint given by (3) and inserting it into the objective gives the objective (ignoring constants)

$$\mathbb{E}\left[AF(\mathcal{N}(A)) - \xi\nu(\mathcal{N}(A))\right].$$

The unconstrained maximum for this objective is obtained when $\mathcal{N}(A)$ satisfies $AF'(\mathcal{N}(A)) = \xi\nu'(\mathcal{N}(A))$. To obtain this unconstrained maximum while satisfying the constraint given by (4), $W(\cdot)$ has to satisfy $W(\mathcal{N}(A)) = \xi\nu'(\mathcal{N}(A))$, i.e., $W(N) = \xi\nu'(N)$. Finally, W_{min} can be adjusted so that the constraint given by (3) holds. □

Proposition 2.1 states that hours worked will be efficient; the role of the contract is to make the firm internalize the worker's disutility of working more hours. Note that this property does not depend on the particular setup of surplus splitting we have assumed here. The same result follows if a union of workers, rather than the firms, offer the contract subject to a reservation profit level. It also follows if workers and firms bargain over the total surplus under a Nash bargaining protocol with given bargaining weights. Such alternative assumptions will naturally affect the level of base pay W_{min}, but the efficiency property follows directly from the assumption that utility is linear, implying that total surplus is maximized when the contract is ex-post efficient.

Whether the contract only specifies a wage-hours schedule or whether it conditions directly on productivity does not matter: in either case, production is efficient. The specific structure of the contract does however matter once we introduce aggregate shocks.

3 Static equilibrium

We now embed the contracting problem in a static general equilibrium environment with many firms and many workers. We study the response of key aggregate variables to changes in aggregate productivity A in the "long-run" (A is known before contracts are written) and the short-run (A is an unexpected shock realized after contracts are written). The results show how rigid wage contracts make business-cycle models consistent with strong fluctuations in hours worked and small average-wage fluctuations in the short run while still maintaining the property of balanced growth in the long run.

There is a continuum of firms indexed by i owned collectively by workers through a diversified mutual fund. Firms match one-to-one with workers. Each firm operates the production function $Y_i = A_i F(N) = A_i N^{1-\alpha}$, with productivity $A_i = A \times \tilde{A}_i$. Firms and workers expect aggregate productivity to be constant, normalized to $A = 1$. Idiosyncratic productivity $\tilde{A}_i$ is equal to 1 on average, but subject to shocks that are identically distributed and independent across firms. The pool of firms is large, and there is free entry to posting contracts, implying that firms make zero profits on average.

There is a continuum of individual workers of unit mass. With slight abuse of notation and in anticipation of the equilibrium allocation, we index workers by the firm i with which they are matched. Worker utility is separable between consumption and hours worked, and consistent with balanced growth following King et al. (1988):

$$\begin{aligned} \mathcal{U}(C, N) &= U(C) - \nu(N), \\ &= \log(C) - \kappa \frac{N^{1+\psi}}{1+\psi}. \end{aligned} \tag{5}$$

Financial markets are complete. In addition to labor income $W^s(N_i)$, worker i receives an exogenous endowment e_i that one may think of as the proceeds from real financial wealth. Without loss of generality we set $e_i = e$ for all households. In addition, the workers also receive profits from their ownership of the firms.

A worker's labor income is determined by the outcome of the labor contract to which she has agreed. Because all workers and firms are ex ante identical, and thus write identical contracts, a worker's consumption is therefore independent of her contract and identical for all workers, equal to the sum of output from production Y and the endowment e: $C_i = Y + e$. The assumption of perfect insurance implies that the contracting parties take the marginal value of wealth as given at the contracting stage. The contracting problem in this environment is therefore the same as that considered in Proposition 2.1. However, the reservation utility $\underline{U}$ and the inverse level of marginal utility ξ are determined in equilibrium. In particular, free entry of firms implies that $\underline{U}$ adjusts so that expected profits are zero.

Definition 3.1. *A competitive equilibrium consists of a wage-hours schedule $W^s(N)$, an hours schedule $N(A_i)$, individual consumption C_i, and aggregate production Y such that*

- *given the worker's inverse marginal utility of consumption ξ, $W^s(N)$ solves the contracting problem,*
- *the reservation utility $\underline{U}$ is such that $\mathbb{E}[A_i F(N_i) - W(N_i)] = 0$,*
- *ex-post hours for worker i, $N_i = N(A_i)$, satisfies firm optimality given the contract $W^s(N_i)$ and realized productivity A_i,*
- *the goods market clears: $C_i = C = Y + e$ for all i with $Y = \int_{i=0}^{1} A_i F(N_i) di$,*
- *and the inverse marginal utility of consumption is $\xi = \frac{1}{U'(C)} = C$.*

Computing the equilibrium Proposition 2.1 characterizes the marginal wage schedule $W(N)$ given inverse marginal utility ξ and the response of hours to idiosyncratic productivity shocks: $N_i = N_i(A_i; \xi)$. Given the marginal wage schedule, the zero-profit condition pins down the base wage W_{min}. In equilibrium, inverse marginal utility of consumption is given by $\xi = Y + e$, so we write $N_i = N_i(A_i; Y + e)$. Equilibrium aggregate production Y is solved from the market clearing condition $Y = \int_i A_i F(N_i(A_i; Y + e)) di$.

The response to changes in expected productivity Consider a change in aggregate productivity that is fully anticipated at the contracting stage. On top of increasing the productivity of matches, we also assume that this shock scales households' endowment income: $e' = e \times A'$. Although our economy is static, this experiment corresponds to a balanced-growth path in a dynamic economy, in which all household income grow at the same rate. Proposition 3.1 establishes that in response to such a change, hours worked are not affected. The fact that firms and workers agree to the wage-hours contract ex ante, as opposed to hours being determined in a spot market ex post, does not change the balanced-growth property of the KPR preference specification (5).

Proposition 3.1. *In response to a change in aggregate productivity that is anticipated in the contracting* *period, total hours are unchanged, and output moves one-for-one with productivity,*

$$Y' = A'Y,$$
$$N' = N.$$

Proof. By Proposition 2.1, hours worked in each contract i is given by

$$A_i F'(N_i) = \xi \nu'(N_i)$$

where inverse marginal utility ξ of consumption is given by $\xi = Y + e$ and total output is given by $Y = \int_0^1 A_i F(N_i) di$. Equilibrium hours are thus characterized by

$$A_i F'(N_i) = \left(\int_0^1 A_j F(N_j) dj + e \right) \nu'(N_i).$$

With a shift in aggregate productivity such that $A_i' = A' A_i$, the new equilibrium is characterized by

$$A' A_i F'(N_i) = \left(\int_0^1 A' A_j F(N_j) dj + A' e \right) \nu'(N_i)$$

and it is readily seen that equilibrium hours do not depend on the aggregate productivity level A'. □

Given that output scales with aggregate productivity and hours are unchanged, the zero-profit condition implies that total wage payments W^s and the average wage $\frac{W^s}{N}$ scale with aggregate productivity as well.

The response to changes in unexpected productivity We now consider the response to an unexpected increase in aggregate productivity from $A = 1$ to $A' > A$ (an "MIT" shock).

Proposition 3.2. *In response to an aggregate productivity shock that is unexpected at the contracting stage, total hours and total output respond by*

$$Y' = (A')^{1+(1-\alpha)/(\alpha+\psi)} Y > AY,$$
$$N' = (A')^{1/(\alpha+\psi)} N > N.$$

Proof. At the match level, hours are given by $A' A_i F'(N_i') = \xi \nu'(N_i')$. With $F(N) = N^{1-\alpha}$ and $\nu(N) = \kappa \frac{N^{1+\psi}}{1+\psi}$, we have $(1-\alpha) A' A_i (N_i')^{-\alpha} = \xi \kappa (N_i')^{\psi}$ or rearranging,

$$N_i' = \left(\frac{1-\alpha}{\xi \kappa} A' A_i \right)^{1/(\alpha+\psi)} = (A')^{1/(\alpha+\psi)} N_i.$$

Inserting hours worked into the production function yields $Y'_i = (A')^{1+(1-\alpha)/(\alpha+\psi)} Y_i$ and aggregating over the matches yields the proposition. $\square$

Why does the response of output and hours to unexpected productivity shocks differ from that in proposition 3.1? In Proposition 3.1, the contract was conditioned on the expected increase in productivity, while in Proposition 3.2, it was not. In response to the unexpected shock, firms find it optimal to increase hours worked ex post. This is achieved by raising the marginal wage payment, in line with the contract. The fact that the contract is not conditioned on the shock, however, means that it does not take into account that the equilibrium increase in consumption of the workers also diminishes their marginal utility of consumption. Contracts written before aggregate shocks are realized thus "turn off" the income effect that otherwise offsets the substitution effect in response to anticipated shocks in Proposition 3.1.

We now consider the response of the average hourly wage $\tilde{W}_t = \frac{W^s}{N}$ to the unexpected shock. Proposition 3.3 shows that the cyclicality of the average wage is determined by relationship between the output elasticity of the production function and the labor share.

Proposition 3.3. *In response to an unexpected productivity shock, the equilibrium elasticity of the average wage with respect to hours, $\epsilon_N^{\tilde{W}}$, is given by*

$$\epsilon_N^{\tilde{W}} = \frac{1-\alpha}{\mathcal{L}} - 1$$

where $\mathcal{L} = \frac{W^s}{Y}$ is the steady-state labor share of income.

Proof. With $\tilde{W} = \frac{W^s}{N}$, the first-order response of the average wage is given by (using differentials)

$$\frac{d\tilde{W}}{\tilde{W}} = \frac{dW^s}{W^s} - \frac{dN}{N}.$$

Since wage payments for a match i is given by $W^s(N_i) = \int_0^{N_i} W(n)dn + W_{min}$, we have

$$dW^s(N_i) = W(N_i)dN_i.$$

Firm optimality implies $W(N_i) = A_i F'(N_i)$. With the Cobb-Douglas production function, we thus have $W(N_i)dN_i = \frac{F'(N_i)N_i}{F(N_i)} A_i F(N_i) \frac{dN_i}{N_i} = (1-\alpha)Y_i \frac{dN_i}{N_i}$. From the proof of Proposition 3.2, the hours response to an aggregate productivity shock is equal across matches, $\frac{dN_i}{N_i} = \frac{dN}{N} = \frac{dA}{\alpha+\psi}$. We therefore arrive at $dW^s(N_i) = (1-\alpha)Y_i \frac{dN}{N}$ and, integrating over all matches, we get,

$$dW^s = (1-\alpha)Y\frac{dN}{N}.$$

Finally, this together with $\mathcal{L} = \frac{W^s}{Y}$ gives us

$$\frac{d\tilde{W}}{\tilde{W}} = \left(\frac{1-\alpha}{\mathcal{L}} - 1\right)\frac{dN}{N}.$$

$\square$

The efficiency condition $W(N) = AF'(N)$ restricts the cyclicality of the marginal wage with respect to hours. Given this cyclicality, a higher steady-state labor share $\mathcal{L}$ reduces the comovement between average wages and output. Under our benchmark assumptions, steady state profits are zero, implying $\mathcal{L} = 1$. The elasticity of output to labor inputs equals $\epsilon_N^Y = 1 - \alpha$ and we thus have $\epsilon_N^{\tilde{W}} < 0$: average wages are countercyclical.

More generally, the zero-profit condition implied by free entry of firms is but one way to embed our contracting model in a general-equilibrium environment for macroeconomic analysis. The same contracting problem can be included in, e.g., a competitive model with capital or a frictional labor market. With capital (that is predetermined at the contracting stage) and a Cobb-Douglas production function , we have that $\mathcal{L} = 1 - \alpha$ and the average wage is acyclical. In an alternative version of our environment with a frictional labor market and linear production ($\alpha = 0$), where firm profits in expectation must compensate firm for vacancy posting costs (as in the canonical Diamond-Mortensen-Pissarides model), steady-state firm profits are positive, and the average hourly wage of existing matches is procyclical.

Comparisons Figure 3.1 displays the short-run behavior of hours worked in response to unexpected changes in aggregate productivity in our rigid-contracts model alongside three comparison models.

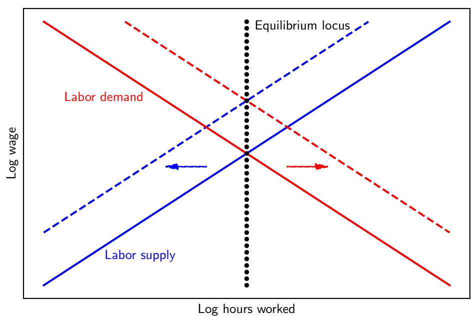
In Subfigure 3.1a, we display the Marshallian cross in a neoclassical spot labor market. The labor demand curve is given by the F.O.C. of the firm's optimality problem, which, when log-linearized, is

$$\log w = a + \alpha \log n. \tag{6}$$

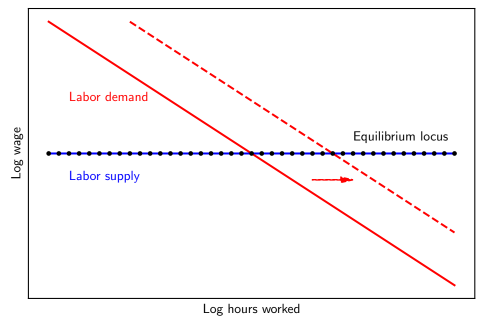
The supply curve is given be the household's intratemporal optimality condition, which with preferences as in Equation (5) is given by

$$\log w = c + \psi \log n. \tag{7}$$

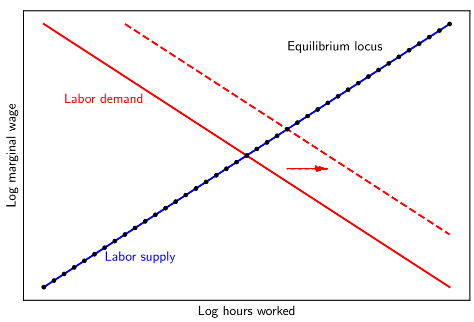
In response to a positive shock to aggregate productivity a, the demand curve shifts out. With an upward-sloping supply curve, hours worked and the wage level increase in partial equilibrium. In general equilibrium, the income effect from the increase in household consumption shifts the supply curve inward. The balanced-growth property of preferences (5) implies that the partial-equilibrium substitution effect and the general-



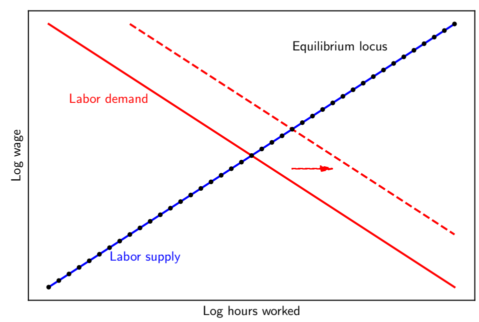
(a) Neoclassical spot market



(b) Rigid wages à la Erceg et al. (2000)



(c) Rigid wage contracts



(d) Neoclassical spot market with GHH preferences

Figure 3.1: The Marshallian cross in four types of labor markets.

equilibrium income effect cancel, and the equilibrium locus in response to an aggregate productivity shock is formed by a vertical line.

In Subfigure 3.1b, we display the Marshallian cross in a labor market with rigid, or predetermined, hourly wages, and where the firm unilaterally determines hours in response to shocks. This is the labor-market setup in the Erceg et al. (2000) (EHL) model, widely used in the new-Keynesian literature (we discuss this model further in Section 4). Here, the labor demand curve is the same, but the short-run supply curve is now a flat line: workers have to accept any amount of hours worked demanded by the firm ex post. In response to a positive shock to aggregate productivity a, the demand curve shifts out, and since the supply curve is flat, there is a larger partial-equilibrium increase in hours worked compared to the neoclassical spot market. Moreover, with the wage contract not being contingent on the shock, there is no income effect in general equilibrium. The supply curve does not shift, and the equilibrium locus coincides with the flat supply curve.

In Subfigure 3.1c, we display the Marshallian cross in our model, with unrestricted wage-hours contracts. The labor demand curve is the same, and the optimal contract prescribes that the supply curve is also given by the worker optimality condition (7). Here, however, w in (7) is the marginal wage rather than the total wage. In the neoclassical spot market and the EHL model, the marginal wage and the average wage are the same; in our setup, they are not. As in the EHL model, the contract is not contingent on aggregate shocks which implies that the supply curve does not respond to the aggregate shock. In effect, the general-equilibrium response to the productivity shock in our model mimics the partial-equilibrium response in the neoclassical spot market, without any feedback from the increase in income on consumption utility.

Therefore, the equilibrium response to unexpected productivity shocks in our model is identical to that in an alternative environment with a neoclassical spot market for labor and where preferences feature no income effects. A class of such preferences was proposed Greenwood et al. (1988) ("GHH preferences"):

$$\mathcal{U}(C, N) = U(C - \nu(N)). \tag{8}$$

With GHH preferences, the optimality condition in a neoclassical spot market is $AF'(N) = \nu'(N)$, just as in our contractual setup. We summarize this result in Proposition 3.4.

Proposition 3.4. *The response of output and hours to an unexpected shock to aggregate productivity in our rigid-contracts model is identical to that in an alternative environment where hours worked are determined in a competitive spot market but where worker preferences are given by* (8).

The Marshallian cross with GHH preferences is displayed in Subfigure 3.1d. GHH preferences are popular in business-cycle analysis, as they give rise to larger equilibrium fluctuations in labor inputs in response to short-run shocks when embedded in a standard spot labor market. Because of the absence of income effects on labor supply, they are not, however, consistent with the key stylized fact of balanced growth in the long run. Our contractual setup, in contrast, features both balanced growth (in response to "long-run", or anticipated changes in productivity) and no income effects in the short run.

Remarks We have considered completely unexpected ("MIT") shocks. In general, workers and firms may expect that aggregate productivity is drawn from some ex-ante known distribution. As long as aggregate shocks are small relative to idiosyncratic shocks, our results carry over. In the limit of arbitrarily small aggregate shocks, certainty equivalence holds and the results from this section hold identically. If aggregate shocks are relatively large, then workers will require compensation for the unconditional correlation between consumption and hours, increasing the importance of income effects.

We have assumed that asset markets are complete. Maintaining the rigidity assumption, other assumptions regarding market structure will change the slope of the supply curve in Figure 3.1c, but not the result that the supply curve is unresponsive to the shock. For example, we conjecture that embedding the contracting problem in a Aiyagari-style model of limited consumption insurance, the contract will specify a flatter supply curve, as risk-averse workers will be willing to trade less consumption volatility against lower base pay. This will move the outcome of the contract closer to the complete-markets EHL model, and we might expect a larger response in hours worked to an unexpected productivity shock.

4 Dynamic equilibrium

The previous section presented a static model with fully rigid contracts, which we used to interpret the effects of contract rigidity on macroeconomics dynamics in the short vs. long-run. In this section, we embed our contracting problem in an infinite-horizon model with Calvo (1983)-style rigidity, where each contract can be rewritten with a constant probability every period. We maintain the assumptions that asset markets are complete and that there is no capital. Workers have per-period preferences given by (5) and seek to maximize expected discounted utility with a constant time discount factor β. For simplicity, we set the endowment e and the net supply of assets to zero.

In the previous section we focused on real shocks to aggregate productivity, implicitly setting the price of consumption goods to a constant, normalized to 1. In this section we explicitly introduce a nominal price of consumption goods P, which allows us to also consider shocks to the price level. We stipulate that contracts are such that the wage-hour schedule cannot condition on any future aggregate shocks and that it is specified in nominal terms, i.e., the contract is nominally rigid.

As before, optimal firm behavior prescribes setting the marginal nominal productivity equal to the marginal wage prescribed by the contract, $P_t A_t A_{i,t} F'(N_t) = W(N_t)$ where $W(N_t)$ is now the *nominal* wage-hours schedule. The problem of firms that can change their contract in period 0 is to offer an optimal nominal wage-hours schedule, knowing that the contract will be in force in period $t > 0$ with probability θ^t, where θ is the constant probability of the contract surviving to the next period.

We consider the firm's problem under a perfect-foresight path. Firms take the path of aggregate productivity A_t, the price level P_t, and the discount factor $\beta^t \Lambda_t$ as given. Because firms are owned by the workers through a diversified mutual fund and workers insure each other against income risk, $\beta^t \Lambda_t$ is the discount factor of the representative worker. The problem of a re-contracting firm is thus

$$\max_{W(\cdot), W_{min}} \sum_{t=0}^{\infty} (\beta\theta)^t \Lambda_t \mathbb{E}\left[A_t A_{i,t} F(\mathcal{N}(P_t A_t A_{i,t})) - \frac{\int_0^{\mathcal{N}(P_t A_t A_{i,t})} W(n)dn + W_{min}}{P_t}\right] \tag{9}$$

$$\text{s.t.} \quad \sum_{t=0}^{\infty} (\beta\theta)^t \mathbb{E}\left[\Lambda_t \left(\frac{\int_0^{\mathcal{N}(P_t A_t A_{i,t})} W(n)dn + W_{min}}{P_t}\right) - \nu(\mathcal{N}(P_t A_t A_{i,t}))\right] \geqslant \underline{U} \tag{10}$$

$$P_t A_t A_{i,t} F'(N_{i,t}) = W(N_{i,t}) \tag{11}$$

Given the marginal wage schedule $W(\cdot)$, the constraint (11) implicitly defines $N_{i,t} = \mathcal{N}(\tilde{A}_{i,t})$, where $\tilde{A}_{i,t} = P_t A_t A_{i,t}$. Since the contract takes the discount factor $\beta^t \Lambda_t$ as given, utility is again transferable as before. The solution to the firm's problem amounts to selecting $\mathcal{N}(\cdot)$ such that joint surplus is maximized:

$$\max_{\mathcal{N}(\cdot)} \sum_{t=0}^{\infty} \frac{(\beta\theta)^t \Lambda_t}{P_t} \mathbb{E}\left[\tilde{A}_t F(\mathcal{N}(\tilde{A}_{i,t})) - \frac{P_t}{\Lambda_t} \nu(\mathcal{N}(\tilde{A}_{i,t})\right], \tag{12}$$

and adjusting W_{min} so that the worker accepts the contract. The solution is characterized by a first-order condition that sets average expected marginal productivity equal to the average expected marginal rate of substitution, see the Online Appendix. Log-linearizing this optimality condition, we have that the log deviation of $\mathcal{N}(\tilde{A})$ is given by

$$\hat{n}(\tilde{A}) \quad = \quad -\frac{1}{\alpha + \psi} \underbrace{(1 - \beta\theta)\mathbb{E}_t \sum_{l=0}^{\infty} (\beta\theta)^l \left(\hat{p}_{t+l} - \hat{\lambda}_{t+l}\right)}_{\text{average price of cons. utility}} \tag{13}$$

where $\hat{x}_t$ represents the log deviation of any variable X_t from its value in the absence of aggregate shocks. Write $\hat{\xi}_t = (1 - \beta\theta)\mathbb{E}_t \left[\sum_{k=0}^{\infty} (\beta\theta)^k (\hat{p}_{t+k} - \hat{\lambda}_{t+k})\right]$ for the "expected average price of consumption utility". The corresponding wage-hours schedule is given by

$$w(N_{i,t+k}) = (1 + \hat{\xi}_t)\kappa N_{i,t+k}^{\psi}.$$

The slope of the optimal contract is set such that the contract matches the marginal rate of substitution for the worker on average. As before, the base wage W_{min} can easily be calculated but does not affect the determination of hours worked in response to shocks.

Aggregation The Calvo structure of contract duration implies, to a first-order approximation, simple dynamic relationships between labor-market aggregates when aggregating across contract vintages. A vintage t is associated with the slope of its nominal marginal-wage curve, ξ_t. Within a contract cohort t, idiosyncratic productivities $a_{i,t}$ average out, and hours in period $t+k$ are given by

$$\hat{a}_{t+k} + \hat{p}_{t+k} = \hat{\xi}_t + (\psi + \alpha)\hat{n}_{t+k|t}.$$

At time $t+k$, a share $(1-\theta)\theta^k$ of the population has a contract of vintage t so aggregate hours worked are given by

$$\hat{n}_{t+k} = \frac{1}{\psi+\alpha}\hat{a}_{t+k} - \frac{1}{\psi+\alpha}(1-\theta)\sum_{s=0}^{\infty}\theta^s(\hat{\xi}_{t+k-s} - \hat{p}_{t+k}). \tag{14}$$

Define the backward-looking sum of the contract vintages $w^{all}_{t+k} = (1-\theta)\sum_{s=0}^{\infty}\theta^s\xi_{t+k-s}$. The object w^{all}_{t+k}, the across-vintages average of the slopes of the marginal wage schedules, is like an allocative nominal wage: Equation (14) can be interpreted as a demand curve in a Marshallian diagram like those considered in Figure 3.1.

Define the *allocative real wage* as $\omega^{all}_t = w^{all}_t - p_t$. In the Online Appendix, we show that Equation (14) can be rewritten like a forward-looking Phillips curve relating the growth rate in the nominal allocative wage $\pi^{w^{all}}_t$ to the current level of the real allocative wage. Together with the firm-optimality condition that determines hours ex post, we have

$$\pi^{w^{all}}_t = \beta\mathbb{E}_t\pi^{w^{all}}_{t+1} - \gamma(\hat{\omega}^{all}_t + \hat{\lambda}_t), \tag{15}$$

$$\hat{\omega}^{all}_t + \psi\hat{n}_t = \hat{a}_t - \alpha\hat{n}_t, \tag{16}$$

with $\gamma = \frac{(1-\theta)(1-\beta\theta)}{\theta}$. Put together, the growth rate in the nominal allocative wage is related to the deviation of marginal productivity, $\hat{a}_t - \alpha\hat{n}_t$, from the marginal rate of substitution, $-\hat{\lambda}_t + \psi\hat{n}_t$. We arrive at the following proposition:

Proposition 4.1. *Taking goods-price inflation π_t, marginal consumption utility λ_t and the initial real allocative wage $\hat{\omega}_{-1}$ as given, the labor-market equilibrium $\{\hat{n}_t, \hat{\omega}_t\}$ is summarized by a wage-Phillips curve*

$$\pi^{w^{all}}_t = \beta\mathbb{E}_t\pi^{w^{all}}_{t+1} - \gamma(\hat{a}_t + \hat{\lambda}_t - (\alpha+\psi)\hat{n}_t), \tag{17}$$

and an accounting identity

$$\hat{\omega}^{all}_t = \hat{\omega}^{all}_{t-1} + \pi^{w^{all}}_t - \pi_t. \tag{18}$$

Here, we have considered the labor-market equilibrium in isolation, i.e., π_t and λ_t are taken as exogenous. To close the general equilibrium, we need to add equations describing the dynamics of consumption and inflation. For example, integrated in the textbook new-Keynesian model, aggregate consumption and inflation obey the usual IS curve and a Taylor rule for monetary policy.

The dynamics of rigid real contracts is obtained by setting $\pi_t = 0$. Then $\pi^{w^{all}}$ represents the growth rate of the real allocative wage.

Comparisons Our dynamic model of rigid wage contracts implies equilibrium dynamics that are qualitatively similar to two other labor-market models widely used for business-cycle analysis: the spot-market model with habit preferences introduced by Jaimovich and Rebelo (2009) and the monopolistic model of nominal wage rigidity introduced by Erceg et al. (2000).

Jaimovich and Rebelo (2009) (JR) considered a neoclassical spot labor market in which workers have a per-period utility function:

$$V(C_{i,t}, N_{i,t}, X_{i,t}) = \frac{\left(C_{i,t} - \frac{\kappa N_{i,t}^{1+\psi} X_{i,t}}{1+\psi}\right)^{1-\sigma} - 1}{1-\sigma} \tag{19}$$

where $X_{i,t}$ represents a habit. When habits are determined by aggregate consumption, $X_{i,t} = X_t = C_t$, and thus external to individual workers, X_t evolves according to[6]

$$X_t = C_t^{\gamma} X_{t-1}^{1-\gamma}.$$

JR's preference specification nests GHH preferences, discussed in the previous section, with $\gamma = 0$. In contrast to GHH, with $\gamma \in (0, 1]$, optimal household choice is consistent with balanced growth in the long run.

With JR preferences, the marginal rate of substitution between consumption and leisure is given by $MRS_{i,t} = \kappa N_{i,t}^{\psi} X_t$. With a spot market for labor, marginal productivity equals this marginal rate of substitution every period, which, when aggregated across household and log-linearized, provide the equilibrium conditions

$$\hat{x}_t = \gamma \hat{c}_t + (1-\gamma)\hat{x}_{t-1}, \tag{20}$$

$$\hat{a}_t - \alpha \hat{n}_t = \psi \hat{n}_t + \hat{x}_t. \tag{21}$$

Equations (20) and (21) are the equivalent to Equations (15) and (16) in our benchmark model, summarizing

[6] Jaimovich and Rebelo (2009)'s original preference specification had internal rather than external habits. This distinction matters for the welfare properties of the model, but is immaterial for the results we discuss here.

the labor-market equilibrium. Both models thus feature a short-run wedge between the marginal rate of substitution and the marginal rate of transformation between hours and consumption goods as both $\hat{x}_t$ and $\hat{\omega}_t^{all}$ adjust slowly. In the case of JR preferences, this wedge results from a persistent habit that depends on past aggregate consumption $\hat{c}_t$. In the case of our rigid-contracts model, the wedge results from past expectations of the aggregate discount factor $-\hat{\lambda}_t$. In general equilibrium with KPR preferences, we typically have $-\hat{\lambda}_t = \hat{c}_t$. In the long-run, the marginal rate of substitution equals the marginal rate of transformation, consistent with balanced growth in both models. The rigid-contract model thus shares the three essential properties that Jaimovich and Rebelo (2009) sought when introducing the preferences (19): i) there are limited income effects in the short run, ii) the model converges to balanced growth in the long run and iii) there is a parameter that controls the speed of convergence in relation to aggregate consumption/output. With JR preference, the parameter is the degree of habit formation γ in (20); in our model, it is the Calvo contract duration parameter θ.

Erceg et al. (2000) considered a model with Calvo-style rigid wage setting and complete asset markets, similar to our model. In their setup, firms do not offer optimal contracts in a competitive market. Rather, workers are monopolistic suppliers of a differentiated labor input and constrained to set a constant nominal hourly wage that remains unchanged across periods with a constant probability. Ex post, workers are committed to providing whatever hours are demanded by the firm at this wage level. Once the wage is allowed to be reset, the worker sets a new nominal wage to minimize the expected future distance between the real average wage and a frictionless optimal markup over her marginal rate of substitution. In their setup, using the preference specification (5), the labor-market equilibrium is summarized by

$$\pi_t^w = \beta \mathbb{E}_t \pi_{t+1}^w - \gamma^{EHL}(\hat{\omega}_t - (\hat{c}_t + \psi \hat{n}_t)), \tag{22}$$

$$\hat{\omega}_t = \hat{\omega}_{t-1} + \pi_t^w - \pi_t, \tag{23}$$

with $\gamma^{EHL} = \frac{(1-\theta)(1-\beta\theta)}{\theta(1+\epsilon_w \psi)}$. Here π_t^w is nominal wage inflation, ω_t is the real wage and ϵ_w is the slope of the CES demand curve. Equations (22) and (23) compare to Equations (17) and (18) in our benchmark model. If assuming, as in our model, that the firms in the EHL model produce final goods and sell them in a competitive market, optimality implies $\hat{\omega}_t = \hat{a}_t - \alpha \hat{n}_t$ in Equation (22). Moreover, in a typical general-equilibrium version of our model, $-\hat{\lambda}_t = \hat{c}_t$ in Equation (17). In consequence, given a path of inflation π_t and aggregate consumption $\hat{c}_t$, and if the slope parameters are equal, $\gamma = \gamma^{EHL}$, the two models are identical in the determination of hours worked. In general, however, the slopes γ and γ^{EHL} of the two Phillips curves differ. In Erceg et al. (2000)'s monopolistic-workers setup, the slope increases with the market power of the workers, whereas in our competitive contractual setup, market power plays no role. In the special case with infinite Frisch elasticity, $\psi = 0$, the two models imply identical dynamics for hours worked. This is not surprising, $\psi = 0$ means that the labor supply curve in the Marshallian cross of our model is horizontal, just as in Subfigure 3.1b.

Erceg et al. (2000)'s model has been criticized for the implication that workers are contractually bound to supply whatever amount of hours the firms demand, without any possibility of exiting the relationship. In a standard calibration, Huo and Ríos-Rull (2020) find that workers are required to work much more than they want for long periods of time. In our model, this critique is less severe. The fact that the contract does not restrict the hours-wage schedule means that workers are compensated by higher wages whenever they provide more hours. They may be asked to work more hours than they would supply at their current consumption and the current marginal wage, but the distance between the hours required and the willingness to work is significantly smaller.

The comparison of our model to the Erceg et al. (2000) model also demonstrates that neither market power nor worker wage setting are essential for generating a Phillips curve. In the EHL model, worker market power is introduced to enable that worker set wages rather than taking them as given. A wide body of recent empirical studies have documented that labor markets are increasingly characterized by monopsonistic rather than monopolistic competition, see, e.g., Berger et al. (2019) and the references therein, and the notion that workers *set* their wages is seemingly in conflict with everyday experience of the labor market. In contrast to the EHL model, our model is competitive and firms, rather than workers, offer wage contracts. The essence for generating a wage Phillips curve is that the contracts cannot be continuously renegotiated and that hours worked are demand-determined in the sense that it is the firms, and not the workers, that chooses hours worked in response to shocks.

5 Conclusion

We have studied macroeconomic dynamics with rigid labor contracts. We defined contracts as agreements between firms and workers on the terms of exchange between hours and wage payments in future periods. The only constraint we imposed on these agreements was that they are rigid: they do not condition on shocks directly and cannot be renegotiated in response to shocks with probability one.

We showed how these assumptions have implications very similar to those of GHH preferences in the short run. In contrast to those preferences, our model is consistent with balanced growth in the long run, and with any cyclical behavior of hourly wages. Its implications are thus similar to those of Jaimovich and Rebelo (2009)'s generalization of GHH, but without assuming consumption habits. We also showed how, when there are shocks to the price level, such contracts generate a wage Phillips curve similar to that in Erceg et al. (2000), but without assuming worker monopoly power, or that workers supply any amount of hours at a rigid nominal hourly wage.

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A Appendix

A.1 Solution to the general contracting problem

Consider the contracting problem (2). The total wage income can be reformulated in terms of the hours-productivity relation $\mathcal{N}$ by a change of variable:

$$\int_0^N W(n)dn + W_{min} = \int_0^A W(\mathcal{N}(a))\mathcal{N}'(a)da + W_{min} = \int_0^A aF'(\mathcal{N}(a))\mathcal{N}'(a)da + W_{min}$$

The firm's problem can thus be expressed in terms of $\mathcal{N}(\cdot)$.

$$\begin{aligned} \max_{\mathcal{N}(\cdot), W_{min}} \quad & \mathbb{E}\left[AF(\mathcal{N}(A)) - \int_0^A aF'(\mathcal{N}(a))\mathcal{N}'(a)da - W_{min}\right] \\ \text{s.t.} \quad & \mathbb{E}\left(V\left(\int_0^A aF'(\mathcal{N}(a))\mathcal{N}'(a)da + W_{min}\right) - \nu(\mathcal{N}(A))\right) = \underline{U} \end{aligned}$$

We have now transformed the problem from choosing a relationship between wage payments and hours (implying a relationship between hours and productivity) to choosing a relationship between hours and productivity (implying a relationship between wage payments and hours).

We reformulate the problem so that we can directly apply Euler's equation from calculus of variations. First, note that $AF(\mathcal{N}(A)) - \int_0^A aF'(\mathcal{N}(a))\mathcal{N}'(a)da = \int_0^A F(\mathcal{N}(a))da$ by integration by parts. Therefore, the problem can be reformulated as

$$\max_{\mathcal{N}(\cdot), W_{min}} \mathbb{E}\left[-W_{min} + \int_0^A F(\mathcal{N}(a))da\right] \quad \text{s.t.} \quad \mathbb{E}\left(V\left(AF(\mathcal{N}(A)) - \int_0^A F(\mathcal{N}(a))da + W_{min}\right) - \nu(\mathcal{N}(A))\right) = \underline{U}.$$

Write $\mathcal{I}(A) = \int_0^A F(\mathcal{N}(a))da$. Note that $\mathcal{I}'(A) = F(\mathcal{N}(A))$. The problem can then be written as

$$\max_{\mathcal{I}(\cdot), W_{min}} \mathbb{E}\left[-W_{min} + \mathcal{I}(A)\right] \qquad \text{s.t.} \qquad \mathbb{E}\left(V\left(A\mathcal{I}'(A) - \mathcal{I}(A) + W_{min}\right) - \nu(F^{-1}(\mathcal{I}'(A)))\right) = \underline{U} \tag{24}$$

We will use tools from calculus of variations to solve this problem. First, we rewrite the problem with a Lagrange multiplier on the constraint,

$$\max_{\mathcal{I}(\cdot), W_{min}, \lambda} \mathbb{E}\left[-W_{min} + \mathcal{I}(A)\right] + \lambda(\mathbb{E}\left(V\left(A\mathcal{I}'(A) - \mathcal{I}(A) + W_{min}\right) - \nu(F^{-1}(\mathcal{I}'(A)))\right) - \underline{U})$$

By taking the first-order conditions with respect to W_{min} and λ, we get the optimality conditions

$$\frac{1}{\mathbb{E}(U'(A\mathcal{I}'(A) - \mathcal{I}(A) + W_{min}))} = \lambda,$$
$$\mathbb{E}\left(V\left(A\mathcal{I}'(A) - \mathcal{I}(A) + W_{min}\right) - \nu(F^{-1}(\mathcal{I}'(A)))\right) = \underline{U}.$$

The problem of choosing $\mathcal{I}(\cdot)$ is on a form where Euler's equation for an extremum applies. Let

$$F(A, \mathcal{I}, \mathcal{I}') = \left[-W_{min} + \mathcal{I} + \lambda\left(V(A\mathcal{I}' + W_{min} - \mathcal{I}) - \nu(F^{-1}(\mathcal{I}')) - \underline{U}\right)\right] f_A(A).$$

Euler's equation states that optimality requires $F_{\mathcal{I}} = \frac{d}{dA} F_{\mathcal{I}'}$.

We compute the derivatives in turn. Write $\mathcal{V}(\cdot) = \nu(F^{-1}(\cdot))$. We suppress the argument of $V(\cdot)$. The density function of A is given by f_A.

$$\begin{aligned}
F_{\mathcal{I}} &= [1 - \lambda V'(\ldots)] f_A(A) \\
F_{\mathcal{I}'} &= \lambda(AV'(\ldots) - \mathcal{V}'(\ldots)) f_A(A) \\
\frac{d}{dA} F_{\mathcal{I}'} &= \lambda(V'(\ldots) + A^2 V''(\ldots)\mathcal{I}'' - \mathcal{V}''(\ldots)\mathcal{I}'') f_A + \\
&\quad \lambda(AV'(\ldots) - \mathcal{V}'(\ldots)) f_A'(A)
\end{aligned}$$

Therefore, optimality requires

$$[1 - \lambda V'(\ldots)] f_A(A) = \lambda[V'(\ldots) + A^2 V''(\ldots)\mathcal{I}''(A) - \mathcal{V}''(\ldots)\mathcal{I}''(A)] f_A + \lambda[AV'(\ldots) - \mathcal{V}'(\ldots)] f_A'(A).$$

or

$$\mathcal{I}'' = \frac{\lambda^{-1} - 2V' - \frac{f_A'}{f_A}(AV' - \mathcal{V}')}{A^2 V'' - \mathcal{V}''}. \tag{25}$$

which is an ordinary differential equations.

Wage payments are $W^s = A\mathcal{J}'(A) + \mathcal{J}(A) + W_{min}$. Hours are $N = F^{-1}(\mathcal{J}'(A))$. Therefore, expressing wage payments as a function of hours, we arrive at $W^s = (\mathcal{J}')^{-1}(F(N))F(N) - \mathcal{J}((\mathcal{J}')^{-1}(F(N))) + W_{min}$.

A.2 Missing steps to the proof to Proposition 4.1

Derivation of Equation (13) The solution to (12) is characterized by a first order condition for each level of $\tilde{A}$:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \frac{(\beta\theta)^t \Lambda_t}{P_t} \left(P_t A_t f_A(\tilde{A}/(P_t A_t))\right) \left(\tilde{A}F'(\mathcal{N}(\tilde{A})) - \frac{P_t}{\Lambda_t}\nu'(\mathcal{N}(\tilde{A})\right) = 0$$

where $f(\cdot)$ is the density of the distribution of idiosyncratic productivities $A_{i,t}$. The density $P_t A_t f_A(\tilde{A}/(P_t A_t))$ is the density of nominal productivity $\tilde{A}$ at time t. Since $\tilde{A}$ is given, write $N = \mathcal{N}(\tilde{A})$.

To a first order in aggregate variables, the first-order condition is

$$\mathbb{E}_0 \sum_{t=0}^{\infty} (\beta\theta)^t \Lambda_{ss} f_A(\tilde{A}) \left(\tilde{A}F'(N) - \frac{(1 + p_t - \lambda_t)}{\Lambda_{ss}}\nu'(N)\right) = 0$$

or equivalently to a first order in N (with $N = N_{ss}(1 + n)$),

$$\mathbb{E}_0 \sum_{t=0}^{\infty} (\beta\theta)^t \Lambda_{ss} f_A(\tilde{A}) \left(\tilde{A}F'(N_{ss})(1 - \alpha n) - \frac{(1 + p_t - \lambda_t)}{\Lambda_{ss}}\nu'(N_{ss})(1 + \psi n)\right) = 0.$$

We now remove the outer constants Λ_{ss} and $f_A(\tilde{A})$, and note that $\tilde{A}F'(N_{ss}) = \nu'(Nss)/\Lambda_{ss}$ to arrive at

$$\mathbb{E}_0 \sum_{t=0}^{\infty} (\beta\theta)^t \left(-\alpha n - p_t + \lambda_t - \psi n\right) = 0.$$

or

$$n = -\frac{1}{\alpha + \psi}(1 - \beta\theta)\mathbb{E}_0 \sum_{t=0}^{\infty} (\beta\theta)^t \left(p_t - \lambda_t\right).$$

which is Equation (13).

Derivation of Equation (15) We use the following simple lemma, which lies behind new-Keynesian Phillips curves:

Lemma A.1. *Let* $X_t = (1-\theta)\sum_{k=0}^{\infty}\theta^k x_{t-k}$ *and let* $x_t = (1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k z_{t+k}$. *Then*

$$\Delta X_t = \frac{(1-\theta)(1-\beta\theta)}{\theta}(z_t - X_t) + \beta\Delta X_{t+1}.$$

Proof. We have that $X_t = (1-\theta)x_t + \theta X_{t-1}$. Therefore, $\Delta X_t = (1-\theta)(x_t - X_{t-1})$. Furthermore, $x_t = (1-\beta\theta)z_t + (\beta\theta)x_{t+1}$. Therefore,

$$X_t = (1-\theta)(1-\beta\theta)z_t + (1-\theta)(\beta\theta)x_{t+1} + \theta X_{t-1}.$$

or

$$\theta(X_t - X_{t-1}) = (1-\theta)(1-\beta\theta)(z_t - X_t) + (1-\theta)(\beta\theta)(x_{t+1} - X_t).$$

Finally, noting that $(1-\theta)(x_{t+1} - X_t) = X_{t+1} - X_t$ and rearranging gives the sought after expression. □

Applying the lemma with $X_t = w_t^{all}$, $x_t = \xi_t$, and $z_t = p_t - \lambda_t$ gives Equation (15).