

No. 1676

**EXTRACTING INFORMATION FROM
ASSET PRICES: THE METHODOLOGY
OF EMU CALCULATORS**

Carlo A Favero, Francesco Giavazzi,
Fabrizio Lacone and Guido Tabellini

**INTERNATIONAL
MACROECONOMICS**



Centre for Economic Policy Research

EXTRACTING INFORMATION FROM ASSET PRICES: THE METHODOLOGY OF EMU CALCULATORS

**Carlo A Favero, Francesco Giavazzi, Fabrizio Iacone and
Guido Tabellini**

Discussion Paper No. 1676
July 1997

Centre for Economic Policy Research
25–28 Old Burlington Street
London W1X 1LB
Tel: (44 171) 878 2900
Fax: (44 171) 878 2999
Email: cepr@cepr.org

This Discussion Paper is issued under the auspices of the Centre's research programme in **International Macroeconomics**. Any opinions expressed here are those of the authors and not those of the Centre for Economic Policy Research. Research disseminated by CEPR may include views on policy, but the Centre itself takes no institutional policy positions.

The Centre for Economic Policy Research was established in 1983 as a private educational charity, to promote independent analysis and public discussion of open economies and the relations among them. It is pluralist and non-partisan, bringing economic research to bear on the analysis of medium- and long-run policy questions. Institutional (core) finance for the Centre has been provided through major grants from the Economic and Social Research Council, under which an ESRC Resource Centre operates within CEPR; the Esmée Fairbairn Charitable Trust; and the Bank of England. These organizations do not give prior review to the Centre's publications, nor do they necessarily endorse the views expressed therein.

These Discussion Papers often represent preliminary or incomplete work, circulated to encourage discussion and comment. Citation and use of such a paper should take account of its provisional character.

Copyright: Carlo A Favero, Francesco Giavazzi, Fabrizio Iacone and Guido Tabellini

July 1997

ABSTRACT

Extracting Information from Asset Prices: The Methodology of EMU Calculators*

This paper develops a particular technique for extracting market expectations from asset prices. We use the term structure of interest rates to estimate the probability the market attaches to a country, Italy, joining the European Monetary Union at a given date. The extraction of such a probability is based on the presumption that the term structure contains valuable information regarding the markets' assessment of a country's chances of joining EMU. The case of Italy is interesting because in the survey regularly conducted by Reuters the probability that Italy joins EMU in 1999 fluctuated, in the first months of 1997, between 0.07 and 0.15 while during the same period the measures computed by financial houses – which are based on the term structure of interest rates – ranged between 0.5 and 0.8. The paper proposes a new method for computing these probabilities and shows that the discrepancies between survey and market-based measures are not the result of market inefficiencies, but of incorrect use of the term structure to compute probabilities. The technique proposed in the paper can also be used to distinguish between convergence of probabilities and convergence of fundamentals, that is to find out whether an observed reduction in interest rate spreads signals a higher probability of joining EMU at a given time, or simply reflects improved fundamentals. It could also be applied, more generally, to extract information on imminent changes in an exchange rate regime from asset prices.

JEL Classification: E43, E52

Keywords: term structure of interest rates, expectational model, probabilities of entering EMU

Carlo A Favero, Francesco Giavazzi, Fabrizio Iacone and Guido Tabellini
IGIER

Università Bocconi

Via Salasco 5

20136 Milano

ITALY

Tel: (39 2) 5836 3306/3304/5327/3301

Fax: (39 2) 5836 3302

Email: carlo.favero@uni-bocconi.it

francesco.giavazzi@uni-bocconi.it

fabrizio.iacone@uni-bocconi.it

guido.tabellini@uni-bocconi.it

Submitted 4 July 1997

NON-TECHNICAL SUMMARY

Extracting market expectations from asset prices is a question which has recently attracted a great deal of interest, both among market participants and central bankers (for a recent review see Söderlind and Svensson (1997)). This paper looks at one specific example: how the term structure can be used to estimate the probability the market attaches to a particular country – Italy in our example – joining the European Monetary Union at a given date.

We have been drawn towards this example because of the striking difference between the surveys regularly conducted among market participants (such as the Reuters Survey), and the probabilities estimated by financial houses (such as JP Morgan and Credito Italiano) using the term structure. For instance, in the early months of 1997 the probability that Italy would join EMU on 1 January 1999 ranged between 7% and 17% according to the Reuters Survey. During the same period, however, the JP Morgan 'EMU Calculator', regularly published in the *Financial Times*, was assigning Italy probabilities ranging between 51–70%.

This paper investigates the sources of these large discrepancies. They could be related to the way in which the 'calculators' are constructed; alternatively they could be the result of market inefficiencies, or of risk-premia terms.

We reach two main results:

- We show that probabilities computed by currently available 'EMU Calculators' are upward biased because they are based on *average* rather than *instantaneous* forward rates.
- We compute the *OUT* spread between Italian and German short-term interest rates, that is the spread that would arise at some date if, at that date, Italy did not belong to EMU, by estimating a reaction function for the Bank of Italy along the lines of a 'Taylor rule'. We are thus able to identify, separately, the component of the total spread between Italian and German interest rates that is due to different fundamentals, from the component due to the markets' assessment of the probability that the country will join EMU in 1999. For example, out of the 214 basis points reduction in the spread between Italian and German forward rates with maturity in 1999 (observed from March 1996 to March 1997), 150 basis points are attributable to the convergence of fundamentals, while only 65 basis points are attributable to

a change in markets' assessment of the probability of Italy joining EMU in 1999.

The 'EMU Calculators' that are currently published compute the probability that a country joins EMU in 1999 by mapping average forward rates into probabilities, making the following assumptions:

- 1) That currently observed forward interest rates with maturity in 1999, and settlement three to ten years later, are a weighted average of a) the spot rate differential between a country and Germany in the case of that country joining, and b) the spot rate differential in the case of that country not joining, where the weights are the probability of joining, and its complement to one;
- 2) The spot rate differential in the case a country joins is zero;
- 3) If the country does not join, the spot rate differential attains a specific value.

Probabilities are then derived computing the ratio of the observed average forward rate differential, to the spot rate differential assumed for the case in which a country is left out. Leaving aside market inefficiencies, assumptions 2) and 3) introduce two potential sources of bias. The first is related to the need to specify a value of the spot rate differential in the case the country does not join EMU. The second is related to the use of average rather than instantaneous forward rates.

We identify and compute these two sources of bias. The first step involves estimating the interest rate differential in the case of no entry in 1999 – the OUT spread. The technique we use to estimate this differential (which is based on the estimation of a reaction function for the Bank of Italy linking the interest rate spread between Italy and Germany to macroeconomic fundamentals) is different from that used by JP Morgan, but essentially confirms the results obtained by that bank. This result rules out the empirical relevance of the first source of bias.

We then construct an EMU calculator based on instantaneous forward rates. The use of instantaneous forward rates is crucial because it provides an easy way to describe the complementary events 'Italy belongs to EMU' and 'Italy is out of EMU' at a specific future point in time. With *m-period* average (rather than instantaneous) forward rates, one estimates the average of the probabilities of being inside EMU at all points in time during the interval of time of size *m*. Since the probability of being inside EMU at a future date *T* increases in *T*, this average of probabilities is greater than the probability of

being inside EMU exactly at the start of the interval considered. To illustrate our point, we consider the situation in March 1997. On that month the probability of Italy joining in 1999, computed using instantaneous forward rates, was 0.24; the same probability computed using average two-year forward rates was 0.41. The corresponding probability according to the Reuter Survey was 0.12.

To understand the relevance of the argument consider the following 'limit case' example. Markets are one-hundred percent sure that Italy will join in 2001; therefore, the probability that Italy will join in 1999 is zero. In this case the Italian instantaneous forward rate for 1999 will lie on the OUT curve, and a computation based on this rate will deliver the correct estimate of the probability of Italy joining in 1999 – namely zero. The three-year average forward rate will be lower than the OUT forward rate, however, because it is an average of (i) two years of OUT Italian instantaneous forward rates, and (ii) of one year of German instantaneous forward rates. As a consequence, the probability of Italy entering EMU in 1999, computed using average forward rates (as one minus the ratio of the average forward-rate differential between Italy and Germany to the differential between the OUT Italian rate and the German rate) will be positive, and thus upward biased.

1. Introduction

Extracting market expectations from asset prices is a question which has recently attracted a great deal of interest both among market operators and central banks (for a recent review see Söderlind and Svensson, 1997.) This paper looks at one specific example: how can the term structure be used to estimate the probability the market attaches to the event that a country, Italy, joins the European Monetary Union at a given date. We have been drawn towards this example observing the striking difference that exists between the surveys that are regularly conducted among market participants, and the probabilities estimated extracting information from the term structure. Table 1 reports five observations (over the interval January to May 1997) on the probability that Italy joins EMU on 1.1.1999. The surveys conducted by Reuters show a remarkably stable assessment of Italy's chances – ranging from a minimum of 0.07 in February, to a maximum of 0.17 the previous month.¹ In the same table we report the probability computed accordingly to the *J.P. Morgan EMU Calculator*, regularly published in the Financial Times, and the *Credito Italiano EMU Calculator*, published in the Italian daily Corriere della Sera. The probabilities computed using these two techniques are very similar, but quite distant from the results of the survey. In particular, the survey reached a minimum in February, which does not coincide with the month in which the “calculators” show a minimum; both the survey and the “calculators” show a maximum in January, but the probability computed using the “calculator” is four times larger than that of the survey.

Table 1: Probability that Italy joins EMU from the start, in 1999					
	15th Jan 97	15th Feb 97	15th Mar 97	15th Apr 97	15th May 97
Reuters EMU Survey	0.17	0.07	0.12	0.15	0.14
Credito Ital. EMU Calculator	0.78	0.70	0.54	0.55	0.58
J.P. Morgan EMU Calculator	0.70	0.61	0.51	0.56	0.58

The aim of this paper is to investigate the sources of these discrepancies. They could be related to the way in which the “calculators” are constructed; alternatively they could be the result of market inefficiencies, or of risk premia terms. In the second and third sections of the paper we discuss the construction of a “calculator”, spelling out the assumptions that are needed in order to arrive at an estimate of probabilities. The technique we develop also provides a simple way to investigate a different, although related, question: whether the convergence between Italian and German interest rates, observed since the third quarter of 1996, is the result of converging fundamentals, or of a change in the assessment of Italy's chances to join EMU, related to market

¹ Reuters polls each month 43 experts at banks, research houses, think tanks, universities and employers' associations across Europe. The list of panelist is available from the Reuters code <EMUPOLL37>. Poll details are on Reuters pages <EMUPOLL30> to <EMUPOLL37>.

sentiment but not to fundamentals. Finally, in the last section of the paper, we appraise the “EMU Calculators” currently in use.

These issues will remain relevant even after the start of EMU, with reference to late entrants. The techniques developed in this paper could also be applied more generally, to extract information on imminent changes in an exchange rate regime from asset prices.²

2. The construction of an EMU calculator

The premise of this exercise is that the German yield curve is to be taken as the benchmark for the Euro curve after EMU has started. Hence, we start by estimating the term structure of spot rates for Italy and Germany. From it we then extract the term structure of instantaneous *forward* rates for each country. We interpret this forward curve as the sequence of overnight rates expected to prevail at any date in the future. Forward rates are interest rates on investments made at a future date, the *settlement date*, and expiring at a date further into the future, the *maturity date*. Instantaneous forward interest rates are the limit as the maturity date and the settlement date approach one another. The relationship between a “yield-to-maturity” and the instantaneous forward rate at that maturity is thus analogous to the relationship between marginal and average cost. The curve of instantaneous forward rates thus lies above the curve of spot rates, when this is positively sloped, and below the curve of spot rates, when this is negatively sloped.³

If the pure expectational model is valid and there is no term premium, then instantaneous forward rates at future dates can be interpreted as the overnight spot rates expected to prevail at those future dates. If we think of the overnight rate as the rate controlled by the central bank, then the curve of instantaneous forward rates can be interpreted as an indicator of expected future monetary policy. Instantaneous forward rates are of particular interest in judging the likelihood of a country joining EMU, since in the monetary union the overnight rate will be the same for all participating countries.

At any future date T after December 31, 1998, one of the following two complementary events must be true: either at date T Italy belongs to EMU, or else at date T Italy does not belong to EMU -- we treat Italy being excluded from EMU, or the fact that there is no EMU at all, as the same event here. In the first event, the date T Italian instantaneous forward rates coincides with that of Germany. The future German forward rate is observed and can be extracted from the German yield curve. In the second event, the Italian instantaneous forward rate will be determined by Italian monetary policy; hence it will reflect Italian and international fundamentals, and it will not necessarily coincide with the German instantaneous forward rate. Estimating this hypothetical Italian instantaneous forward curve for date T , if Italy is out of EMU at that date, is the main problem we face. This problem is addressed in the next section.

By risk neutrality, the observed Italian instantaneous forward rate for date T is a weighed average of these two instantaneous forward rates, the German one and that of Italy if out of EMU. The weights are the probability that at date T Italy belongs to EMU, or is out of EMU, respectively. In symbols:

$$ITFW(T)_t = \pi(T)_t BFW(T)_t + (1-\pi(T)_t) OUT(T)_t \quad (1)$$

where :

- $ITFW(T)_t$ is the Italian instantaneous forward rate for date T , observed at date t ;

² Although the problem we study is similar to that investigated by Flood and Marion (1983), and Flood and Garber (1983) -- namely how asset prices incorporate the expectation of a regime change -- those papers do not exploit the information contained in the term structure.

³ See Svensson (1994).

- $BFW(T)_t$ is the German instantaneous forward rate for date T , observed at date t ;
- $OUT(T)_t$ is the Italian instantaneous forward rate for date T , if at that future date Italy is does not belong to EMU, estimated on the basis of information available at date t .
- $\pi(T)_t$ is the probability that at time T Italy belongs to EMU, evaluated on the basis of information available at time t .

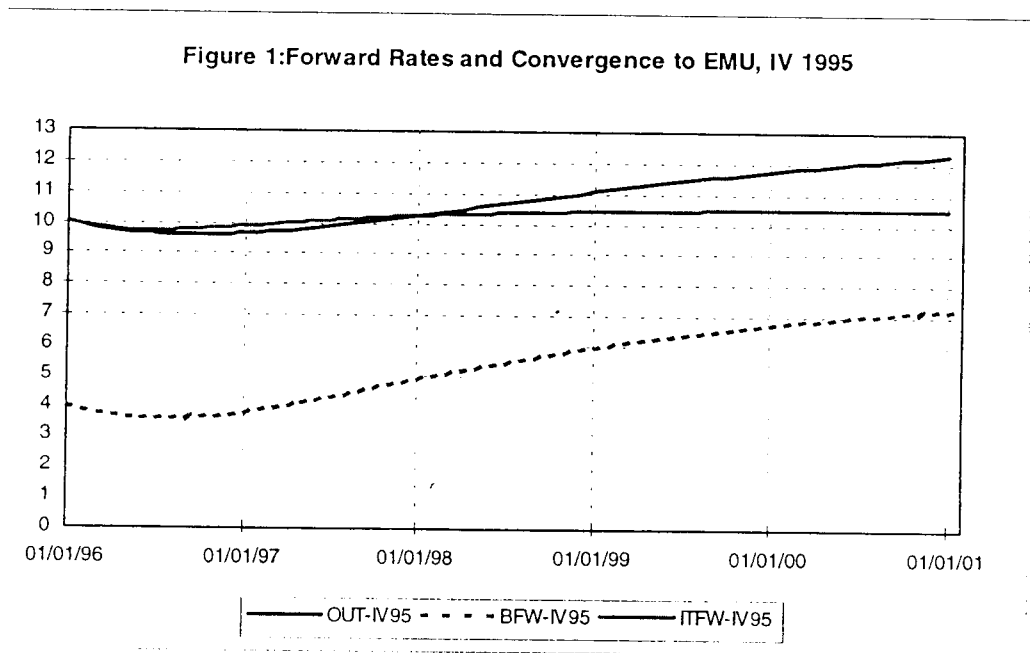
Let $SP(T)_t = ITFW(T)_t - BFW(T)_t$ be the date T forward differential between the instantaneous rates of Italy and Germany observed at date t , and $SPOUT(T)_t = OUT(T)_t - BFW(T)_t$ be the date T differential between the estimated Italian instantaneous forward rate if Italy is out of EMU, and the German forward rate. Then, (1) can be rewritten as:

$$SP(T)_t = (1 - \pi(T)_t) SPOUT(T)_t \quad (2)$$

which in turn immediately implies that the estimated probability that at date T Italy belongs to EMU is simply:

$$\hat{\pi}(T)_t = 1 - \frac{SP(T)_t}{SPOUT(T)_t} \quad (3)$$

Thus, given an estimate of $OUT(T)_t$ for any future date T , and given the observed Italian and German instantaneous forward rates, it is easy to compute the probability assigned by financial markets to the event that Italy belongs to EMU by that date. Figure 1 illustrates the idea graphically.



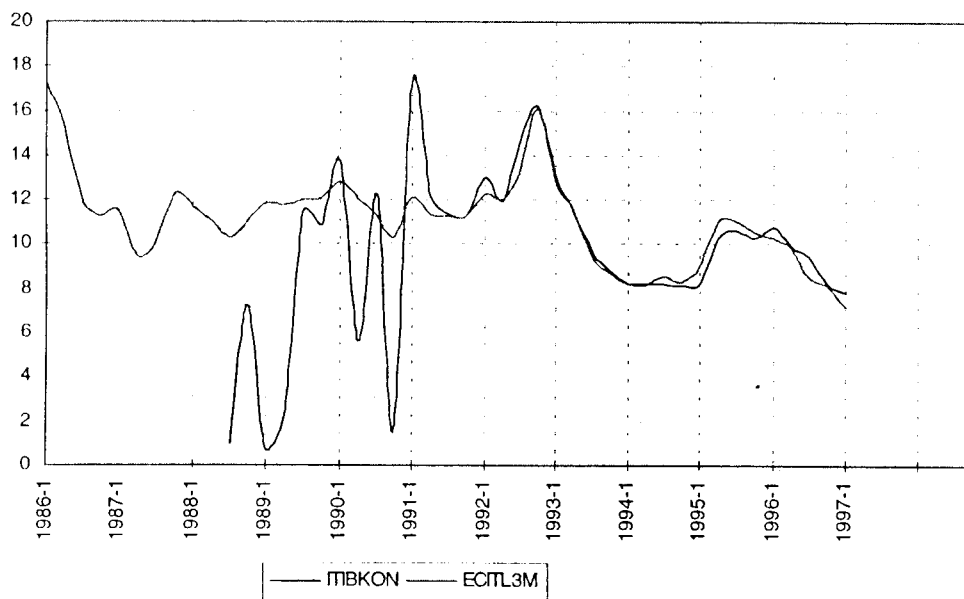
The solid curve is the German instantaneous forward rate observed on December 31, 1995. The grey curve is the Italian instantaneous forward rate observed on December 31, 1995. The dotted curve is our estimate of $OUT(T)$, namely of the Italian instantaneous forward rate if out of EMU. Any point on the grey line after December 31, 1998, is a weighted average of the solid and the dotted curves, with weights that vary over time. The weight on the German curve is the probability that Italy will be inside EMU at that date. As expected, this probability increases over time. The next section explains how these three curves can be estimated.

Note that on January 1, 1999, but only at that date, the event “Italy is inside EMU” coincides with the event “Italy enters EMU”. At all future dates, these two events differ. Throughout the paper, $\pi(T)_i$ refers to the event “Italy is inside EMU at date T ”. The probability of *entering* exactly at date T , for $T > 1/1 1999$, is the time derivative of $\pi(T)_i$ with respect to T . Thus, $1-\pi(T)_i$ is similar to a survival function, while the probability of entering is similar to a hazard rate. Similar but not identical, because $\pi(T)_i$ does not necessarily converge to 1 asymptotically as T goes to ∞ , neither in theory nor in the data (see Figure 6).

3. Forward Curves and probabilities of convergence

In the previous section we have shown that in order to identify the probabilities we must estimate three instantaneous forward curves: the Italian instantaneous forward curve, the German instantaneous forward curve, and the Italian forward curve if out of EMU. We estimate the first two curves using the Nelson-Siegel interpolant discussed in Svensson (1994) -- details of this estimation are provided in the Appendix. Our estimate of the OUT curve is based on a reaction function which explains the Italian short-term rate with macroeconomic fundamentals only. The obvious dependent variable should be the overnight rate, which is the observable equivalent of the instantaneous forward rate. However, as shown in Figure 2, the Italian overnight rate is extremely volatile⁴ -- a result of the reserve requirement liquidity factors in the market for bank reserves prior to the reform of October 1990; in fact from then on the overnight rate moves very close to other short-term rates, such as the 3-months Euro rate.

Figure 2: The Italian overnight rate and the 3m-eurolira rate (quarterly data, end of period)



We have thus specified our reaction function on 3-months Euro rates, taking 1987 as the initial date, and ending the estimation period in 1996:2— a date which allows us to simulate the model to obtain an OUT level of short-term interest rates for at least one-year. Our specification of

⁴ Up to October 1990 the Bank of Italy forced Italian banks to meet the mandatory reserve requirement daily, rather than on average over the maintenance period.

the equation for short-term interest rates is a rule very much in the spirit of Taylor (1995), although adapted to an open economy. The estimated rule is reported on Table 2.

TABLE 2: A rule for short-term Italian interest rates

Modelling IT3MCC by OLS The sample is: 1987 (1) to 1996 (2)						
Variable	Coefficient	Std.Error	t-value	t-prob	PartR ²	
Constant	0.0255049	0.0125307	2.035	0.0510	0.1250	
IT3MCC_1	0.599078	0.126913	4.720	0.0001	0.4345	
INFLGAP	0.314109	0.140926	2.229	0.0337	0.1463	
GRDGAP	0.264385	0.107711	2.455	0.0203	0.1720	
REUGR	-0.184274	0.0914312	-2.015	0.0532	0.1229	
LUSDDM	-0.0793958	0.0231922	-3.423	0.0019	0.2878	
LUSDM_1	0.0469158	0.0222621	2.107	0.0438	0.1328	
BD3MCC	0.905901	0.281672	3.216	0.0032	0.2629	
BD3MCC_1	-0.487597	0.329971	-1.478	0.1503	0.0700	
R ² = 0.77975 F(8, 29) = 12.834 [0.0000] σ = 0.007667534 DW = 1.74 RSS = 0.0017049411422 for 9 variables and 38 observations						
Diagnostic Tests⁵						
AR 1- 3F(3, 26) = 0.318958 [0.8116] ARCH 3 F(3, 23) = 0.145167 [0.9317]						
Normality Chi ² (2) = 10.613 [0.0050] ** Xi ^y F(16, 12) = 1.4256 [0.2703]						
RESET F(1, 28) = 0.563669 [0.4590]						
Solved Static Long Run equation						
IT3MCC = +0.063616 +1.043 BD3MCC -0.45963 REUGR						
(SE) (0.029368) (0.30382) (0.26185)						
-0.081013 LUSDM +0.78347 INFLGAP +0.65944 GRDGAP						
(0.045129) (0.30131) (0.28175)						
EQ(6) Modelling IT3MCC by OLS						
Variable	Coefficient	Std.Error	t-value	t-prob	PartR ²	
Constant	0.0309317	0.00944132	3.276	0.0028	0.2771	
IT3MCC_1	0.576555	0.0950693	6.065	0.0000	0.5678	
INFLGAP	0.228647	0.106887	2.139	0.0413	0.1405	
GRDGAP	0.234366	0.0808244	2.900	0.0072	0.2309	
REUGR	-0.190575	0.0684216	-2.785	0.0095	0.2170	
LUSDM	-0.0659308	0.0175706	-3.752	0.0008	0.3346	
LUSDM_1	0.0452027	0.0166604	2.713	0.0113	0.2082	
BD3MCC	1.0064	0.211752	4.753	0.0001	0.4465	
BD3MCC_1	-0.717288	0.251335	-2.854	0.0080	0.2253	
i1992p4	0.0315236	0.00646128	4.879	0.0000	0.4595	
R ² = 0.880953 F(9, 28) = 23.022 [0.0000] σ = 0.005736896 DW = 1.86 RSS = 0.00092153534533 for 10 variables and 38 observations						
Diagnostic Tests						
AR 1- 3F(3, 25) = 1.1308 [0.3556] ARCH 3 F(3, 22) = 0.248704 [0.8614]						
Normality Chi ² (2) = 0.634888 [0.7280] Xi ^y F(17, 10) = 0.29833 [0.9862]						
RESET F(1, 27) = 0.175522 [0.6786]						
Solved Static Long Run equation						
IT3MCC = +0.073048 +0.68266 BD3MCC -0.45006 REUGR						
(SE) (0.021502) (0.22988) (0.18445)						
-0.048951 LUSDM +0.53997 INFLGAP +0.55347 GRDGAP						
(0.029612) (0.21516) (0.19165)						
+0.074445 i1992p4						
(0.022078)						

⁵ AR is an LM test of the null of absence of autocorrelation of residuals, ARCH and xy are tests of the null of absence of heteroscedasticity, RESET is a test of misspecification due to incorrect functional form, and Normality is a test of normality of residuals. For a complete description of all statistics see Hendry(1995)

The Italian short-term interest rate depends on its own lag, on the current and lagged level of the German short-term rate, and on three weakly exogenous variables: the inflation gap between Italy and Germany, defined as the difference between the headline annual CPI inflation (log of price in quarter t minus log of price in quarter $t-4$) between the two countries; the output gap between Italy and Germany, defined as the difference in annual GDP growth in the two countries; and the current and lagged level of the log of the US dollar-Deutschemark exchange rate. Because of the potential effect of German reunification on our estimated coefficients we have interacted both the inflation gap and the output gap with a reunification dummy, which takes a value of 1 in 1991, and zero anywhere else. The reunification effect is significant when interacted with the output gap, but not significant when interacted with the inflation gap: in the final specification we have thus kept only the product of the output gap and the reunification dummy (REUGER.)

The intuition behind our specification is that of a small open-economy Taylor-rule where the central bank has an objective function which includes, along with the usual macroeconomic variables such as inflation and growth, exchange rate stability. The objective of exchange rate stability is implemented by defining the target values of the macroeconomic variables as those assumed by these variables in the reference country -- Germany. The Lira-Deutschemark rate cannot be assumed to be weakly exogenous: it was thus replaced, in the estimation, with the dollar-Deutschemark rate, which we interpret as a weakly exogenous instrument correlated with the Italian Lira-Deutschemark rate (see Giavazzi and Giovannini, 1989.) Exogeneity of the macroeconomic variables is guaranteed if monetary policy takes some time -- at least one quarter -- to affect such variables -- by now a standard assumption in the literature on monetary transmission mechanism which uses structural VAR models (Bernanke-Mihov, 1996, Christiano and Eichenbaum, 1992, Sims and Leeper and Zha, 1996.)⁶

Before commenting on the empirical results, it is worth mentioning that we have experimented with alternative specifications. In particular, we have included, on the right-hand-side of the equation, the Italian GDP gap -- deviation of actual GDP from a Hodrick-Prescott trend -- and a commodity price index. The first variable would be justified if we allow for the possibility that even a small open economy targets growth independently of the reference country; the second variable has an established tradition as a leading indicator for inflation, and a relevant argument in the reaction functions of central banks.⁷ Neither the deviation of GDP from its trend, nor commodity prices turn out to be significant when added to our basic specification. On the basis of these results we concluded that there is no statistical evidence that the Italian central bank targets the deviation of GDP from its trend (as identified by the HP filter), and thus omitted this variable from the estimated equation. We have also omitted commodity prices: as we include the German policy rate, and considering that this rate reacts significantly to commodity prices (Bernanke and Mihov, Clarida and Gertler), we concluded that commodity prices would not play an independent role in our specification. Finally, we also included a measure of fiscal policy: in Italy the exchange rate and more generally exchange rate expectations react to the perceived sustainability of fiscal policy (cf. Sargent and Wallace (1981) for an example of a theoretical argument along these lines). Since exchange rate stability was certainly a policy goal during this period, it is likely that the Bank of Italy would have responded to fiscal policy news to the extent that these were associated with expected depreciation. As a measure of fiscal policy, we used seasonally adjusted quarterly budget

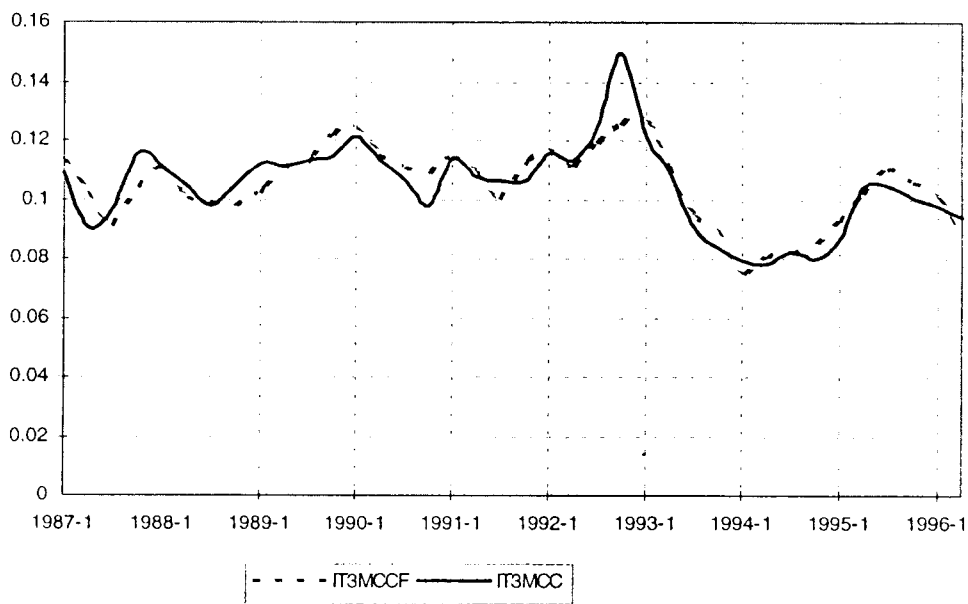
⁶ Note that our single-equation rule is consistent with the rules derived within VAR models, and, under the validity of our identifying assumption, the parameters estimated in a structural VAR would coincide with the ones delivered in our single-equation framework.

⁷ The original Taylor rule does not include commodity prices as an explanatory variable in the determination of short-term interest rates. However, the structural VAR approach has shown that the omission of this variable may lead to misspecification of the reaction function, and to some puzzling impulse responses, *i.e.* prices declining in response to an expansionary monetary policy shock.

deficits as a % of GDP. It never was statistically significant. Hence, we omitted it from the final specification. Probably, the quarterly budget deficit is not a good indicator of the perceived sustainability of the government budget; but we could not find any other observable and exogenous measure of budgetary sustainability.

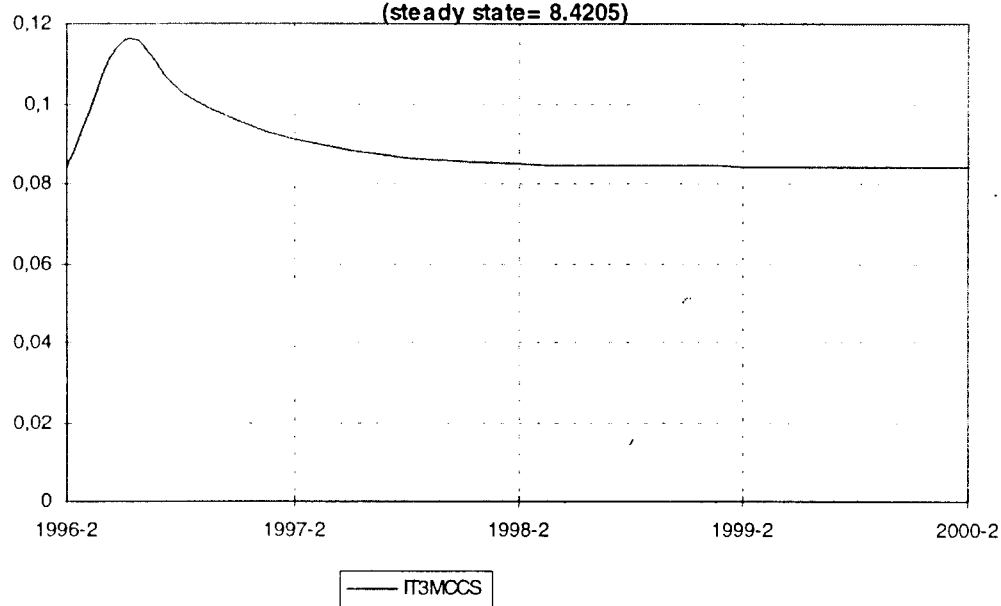
Table 2 reports our results. The coefficient on the lagged dependent variable is 0,6 and that on short-term German interest rates is not significantly different from one. The coefficients on the inflation and output gaps are, respectively, 0,31 and 0,26 in the short-run, 0,78 and 0,66 in the long-run. These long-run coefficients are significantly different from zero, but not from 0,5. Both the short-run and the long-run coefficient on the dollar-Deutschemark exchange rate take a value of -0,08, implying that a one percentage point appreciation of the US dollar against the Deutschemark will be reflected in an eight basis points fall in Italian short-term rates. The estimated equation passes all the diagnostics with the exception of the normality of residuals. As shown in Figure 3, which displays the time series of actual and fitted Italian short-term rates, the absence of normality in the residuals may be due to the presence of an outlier in 1992:4.

**Figure 3: Observed and Fitted Italian short term rate (3m-euolira)
(quarterly data, end of period)**



This is likely to be associated to a shock in the risk premium, or to unusual behavior of financial markets during the EMS crisis, rather than to a structural break of the model. This suspicion is confirmed by the results, also reported in Table 2, obtained including a point dummy for 1992:4. Including the dummy eliminates the non-normality problem, but does not alter the other coefficients. To further investigate the effects of shocks to the risk premium we have simulated a shock of 315 basis points -- the value of the coefficient on the EMS crisis dummy. The inflation and output gaps, the US dollar-Deutschemark rate, and the German three-month rate are all set at their observed values in 1997:1. The model is initialised at the steady state. The simulation, reported in Figure 4, shows that a shock to the risk premium of the dimension observed on the occasion of the Italy's exit from the ERM is re-absorbed within two-years.

Figure 4: Simulating the effect of a 315 basis point shock to the risk premium on the Italian 3m-eurolira rate
(steady state = 8.4205)



Having estimated the rule for Italian short-term rates up to 1996:2, we proceed as follows:

- we map this rule into the *OUT* curve, projecting it forward for two years. We use “Consensus” forecasts for Italian and German inflation and growth, and for the dollar-Deutschemerk exchange rate; the path of short-term German rates is instead derived from the estimated German instantaneous forward curve; this gives us eight observations for *OUT*.
- we fit a Nelson-Siegel forward function through ten points: the current overnight rate, the eight rates obtained projecting the rule as described above, and an asymptot which corresponds to the long-run solution of the estimated rule. The ten points thus obtained are sufficient to identify the four parameters in the Nelson-Siegel forward function.

This procedure allows us to map fundamentals into an *OUT* curve, and thus to associate the level of the *OUT* Italian interest rate to the evolution of fundamentals. Except for the German forward rate, all these fundamentals are obtained from the consensus forecasts formulated every quarter from the last quarter 1995 onwards with a forecasting horizon of the following four. From then on, the *OUT* reflects the asymptotic solution of the Nelson-Siegel forward function. This method of computing the *OUT* curve is further discussed in section 5, and compared to other commonly used procedures.

We now have all the necessary information for the computation of probabilities: we show in Figure 5 the estimated curves for the actual Italian forward rate, for the German forward rate, and for the Italian *OUT* forward rates, at several different dates during 1996 and 1997 (the dates correspond to those of the consensus forecasts). On the basis of these curves, we have computed the probabilities that Italy belongs to EMU at two future dates: January, 1999 and January, 2001. These probabilities are reported in Table 3.

TABLE 3: The Probability of Italy Being in EMU in 1999 and in 2001.

date	$\pi_{99,t}$	$\pi_{01,t} - \pi_{99,t}$	$1 - \pi_{01,t}$	ITFW(99)	ITFW(01)	BFW(99)	BFW(01)	OUT(99)	OUT(01)
11/12/95	0,12692	0,22262	0,65046	10,433	10,507	6,017	7,229	11,075	12,268
11/03/96	0,30337	0,10690	0,58973	9,772	10,378	6,496	7,597	11,199	12,312
10/06/96	0,23875	0,23944	0,52181	8,806	9,367	6,052	7,464	9,669	11,111
09/09/96	0,31362	0,20886	0,47748	8,230	9,006	5,394	7,028	9,526	11,171
09/12/96	0,52379	0,17154	0,30467	6,406	7,426	4,584	6,249	8,410	10,113
10/03/97	0,24311	0,24926	0,50763	6,707	7,416	4,440	5,947	7,436	8,842

ITFW(99) is the observed Italian instantaneous forward rate for January 1999, at date t

ITFW(01) is the observed Italian instantaneous forward rate for January 2001, at date t

BFW(99) is the observed German instantaneous forward rate for January 1999, at date t

BFW(01) is the observed German instantaneous forward rate for January 2001, at date t

OUT(99) is the theoretical Italian instantaneous forward rate associated to the event Italy is not in EMU in January 1999, at date t

OUT(01) is the theoretical Italian instantaneous forward rate associated to the event Italy is not in EMU in January 2001, at date t

$\pi_{99,t}$ is the estimated probability at date t of Italy being in EMU in January 1999

$\pi_{01,t}$ is the estimated probability at date t of Italy being in EMU in January 2001

Hence, $(\pi_{01,t} - \pi_{99,t})$ is the probability of entering EMU after January 1999 but not after January 2001, while $(1 - \pi_{01,t})$ is the probability of being still out of EMU on January 2001.

Note that $\pi(T)_t$ is the unconditional probability of being inside EMU at date T . We can also compute the probability of being inside at date T , conditional on being out at some earlier date $S < T$. This conditional probability is simply $(\pi(T)_t - \pi(S)_t)$. It measures the probability of *entering* EMU after date S but not after date T . This conditional probability is also displayed in Table 3, with reference to the dates January 1999 and January 2001.

Figure 6 displays the probability computed on the 10/3/1997 of Italy being in the EMU at any date between January 1999 and April 2002. As expected, the probability increases over time and converges asymptotically to 0.5. Note that the slope is practically flat after January 2001, indicating that the probability of entry after that date is perceived by financial markets to be very small. In other words, in March 1997 Italy was still perceived as a likely late entrant - the probability of Italy being inside EMU is always below 50%. But the delay in Italy entry was perceived to be short, namely two-year at the most.

Naturally, if our assumption of risk neutrality is false, our measure of the probability of being inside EMU could be biased downwards, as Italian interest rates could be kept high by a risk premium rather than by the perceived probability of being left out.

Figure 5.1: The estimated Italian forward rates

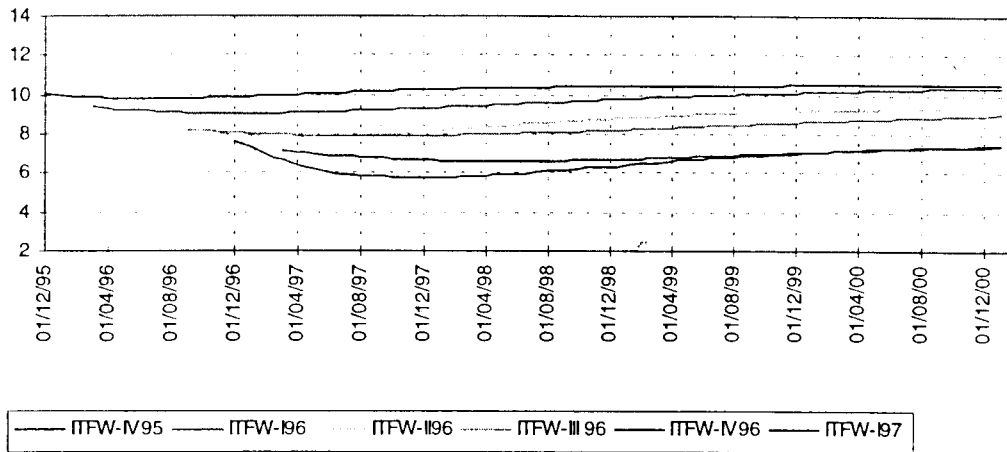


Figure 5.2: The estimated German forward rates

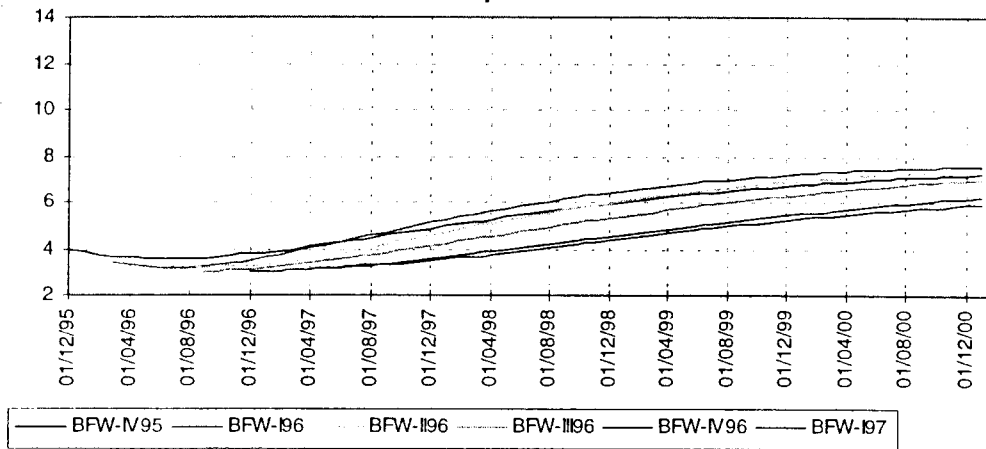
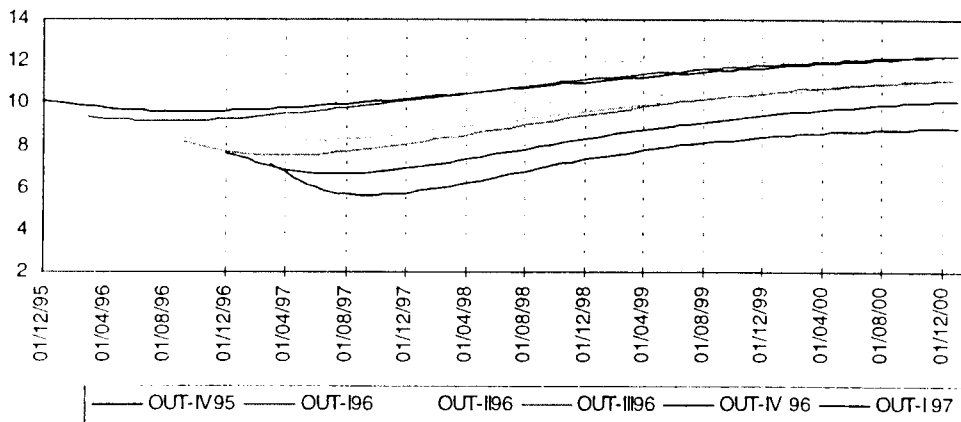
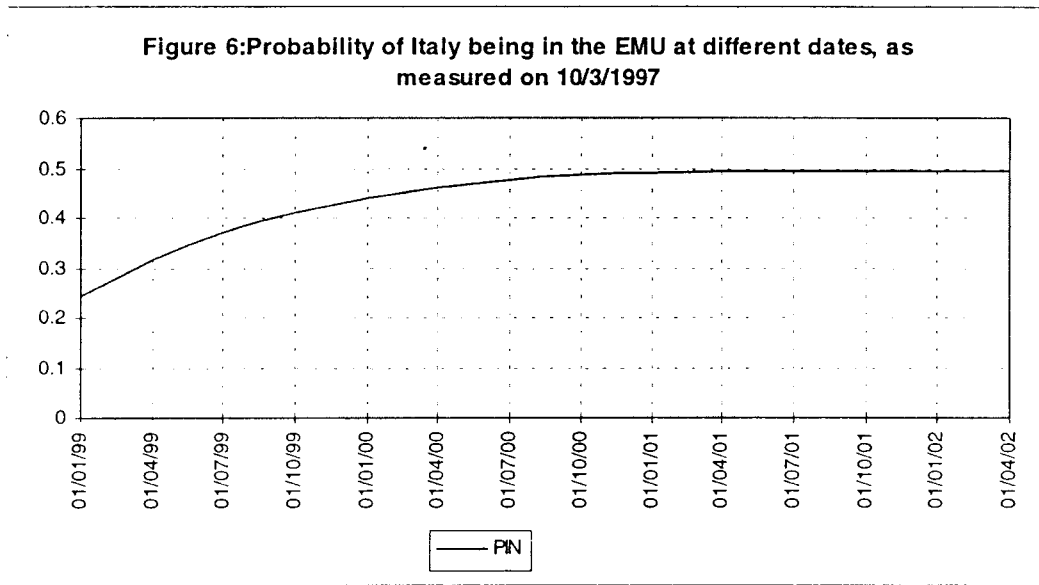


Figure 5.3: The estimated OUT Italian forward rates



Several features of our results are worth commenting. First, the substantial decrease in Italian forward rates from the beginning to the end of our sample are associated more with a decrease in the *OUT* forward rates than with an increase in the probability of Italy being inside EMU. Second, the probabilities of Italy joining in 1999, computed according to our procedure, are uniformly lower than those derived using the EMU calculators presented in Table 1. Third, the probabilities of being inside EMU by 2001 are much closer to the probabilities of convergence in 1999 computed by the EMU calculators. We shall devote the next two sections to a closer investigation of these results.



4. Convergence of probabilities or convergence of fundamentals ?

Does the observed convergence of Italian and German interest rates, observed from the second half of 1996 onwards, depend on the convergence of fundamentals (inflation and output gaps), or is it the result of an increase in the probability that Italy joins EMU ? The Bank of Italy has recently proposed a methodology that allows to identify the relative contribution of these factors (see Bollettino Economico, February 1997). It shows that the reduction in the spread between Italian and German interest rates is more related to the convergence of fundamentals than to a change in the market assessment of the probability of Italy joining EMU. Here we adapt and extend the proposed methodology to investigate the same issue within our framework.

Differentiating (2), for any future date T we can decompose the time variation of the forward spread between Italy and Germany in its components:

$$\Delta SP(T)_t = (1 - \pi(T)_t) \Delta SPOUT(T)_t - SPOUT(T)_{t-1} \Delta \pi(T)_t \quad (3)$$

The term $(1 - \pi(T)_t) \Delta SPOUT(T)_t$ captures that part of the change in the spread related to domestic fundamentals, since they affect the spread if out of EMU. The term $(- SPOUT(T)_{t-1} \Delta \pi(T)_t)$ captures instead the change in markets' perception of the probability of being inside EMU, and it is thus not directly related to fundamentals.

Consider again the two dates of January 1999 and January 2001.

TABLE 4: ASSESSING CONVERGENCE							
date	SP(99)	SP(01)	SPOUT(99)	SPOUT(01)	$\pi_{99,t}$	$\pi_{01,t}$	
11/12/95	4.416	3.278	5.058	5.039	0.126917	0.222619	
11/03/96	3.277	2.780	4.704	4.715	0.303365	0.106896	
10/06/96	2.753	1.903	3.617	3.647	0.238746	0.239438	
09/09/96	2.836	1.978	4.132	4.143	0.313622	0.208896	
09/12/96	1.822	1.177	3.826	3.863	0.523787	0.171540	
10/03/97	2.267	1.470	2.996	2.895	0.243106	0.249261	
SP(99)=ITFW(99)-BFW(99) SP(01)=ITFW(01)-BFW(01) SPOUT(99)=OUT(99)-BFW(99) SPOUT(01)=OUT(01)-BFW(01) ITFW(99) is the observed Italian instantaneous forward rate for January 1999, at date t ITFW(01) is the observed Italian instantaneous forward rate for January 2001, at date t BFW(99) is the observed German instantaneous forward rate for January 1999, at date t BFW(01) is the observed German instantaneous forward rate for January 2001, at date t OUT(99) is the theoretical Italian instantaneous forward rate associated to the event Italy is out of EMU on January 1999, at date t OUT(01) is the theoretical Italian instantaneous forward rate associated to the event Italy is out of EMU on January 2001, at date t $\pi_{99,t}$ is the estimated probability at date t of Italy being inside EMU on January 1999 $\pi_{01,t}$ is the estimated probability at date t of Italy being inside EMU on January 2001							
Decomposition of the time variation in the spread between Italian and German forward rates							
date	$\Delta SP(99)$	FUND99	PR99	$\Delta SP(01)$	FUND01	PR0199	PR01
11/03/96	-1.139	-0.24664	-0.89244	-0.497	-0.19146	-0.88915	0.583149
10/06/96	-0.524	-0.82755	0.30395	-0.877	-0.55724	0.304645	-0.62487
09/09/96	0.083	0.353473	-0.2708	0.075	0.23699	-0.27304	0.111376
09/12/96	-1.014	-0.14555	-0.86833	-0.801	-0.08526	-0.8707	0.154762
10/03/97	0.445	-0.62846	1.073887	0.293	-0.49144	1.084305	-0.30024
TOTAL	-2.148	-1.495	-0.654	-1.808	-1.088	-0.644	-0.076
percent	100	69,6	30,4	100	60,2	35,6	4,2
$\Delta SP(99)_t = (1 - \pi_{99,t}) \Delta SPOUT(99)_t - SPOUT(99)_{t-1} \Delta \pi_{99,t} = FUND99 + PR99$ $\Delta SP(01)_t = (1 - \pi_{01,t}) \Delta SPOUT(01)_t - SPOUT(01)_{t-1} \Delta \pi_{01,t} = FUND99 + PR0199 + PR01$							

The top panel of Table 4 shows that the forward spread between Italy and Germany for January 1999 has dropped from 441 basis points at the end of 1995, to 226 basis points at the end of the first quarter of 1997. Similarly, the forward spread for January 2001 has decreased from 328 basis points to 147. Note, however, that *SPOUT*, the spread if out of EMU, also fell -- from 505 to 299 basis points and from 504 to 290 basis points, for 1999 and 2001 respectively. The lower panel of Table 4 shows the decomposition period by period, and over the entire interval. Out of a total reduction of 214 basis points in the forward spread for January 1999, 149 basis points can be attributed to the direct effect of fundamentals, only 65 to the probability effect. Similarly, out of a

total reduction of 181 basis points in the forward spread for January 2001, 109 are accounted for by fundamentals, and only 76 by the probability effect -- 64 are accounted for by a change in the probability of Italy converging in 1999, and 8 by a change in the probability of Italy converging in 2001. These numbers suggest that the observed convergence between Italian and German interest rates is more due to an improvement in Italian fundamentals than to an increase in the probability of being inside EMU -- thus confirming the observation made by the Bank of Italy.

This conclusion must be interpreted with caution, however, as our suggested decomposition could suffer from the following problem. In constructing our estimate of $SPOUT(T)$, we used the consensus forecasts for Italian inflation that refer to future dates up to 1998:1. The last observation of this consensus forecast is extrapolated in the indefinite future to construct the asymptotic solution of $SPOUT$. It can be argued that this forecast, although referring to a date prior to the start of EMU, is affected by the probability of entry, for two reasons.

First, if at date t Italy is perceived as more likely to be out of EMU in the future, the Lira could weaken immediately; this in turn would increase future expected inflation. We doubt that this problem is quantitatively important, however: the reduction in the inflation forecasts is probably largely due to the remarkable and unexpected drop of actual inflation in the period 1995:4 - 1997:1.

Second, and perhaps more important, future monetary policy as well as future expected inflation and hence future wages could also be affected by whether Italy is in or out of EMU. If Italy remains out of EMU, and in particular if the EMU project is scrapped altogether, the incentives of the Bank of Italy and of the government to converge could be weakened. If so, the inflation forecast conditional on Italy being out of EMU could be much higher than the inflation forecast referring to 1998. This suggests that it could be difficult to disentangle the improvement in the consensus forecast in inflation from an increase in the probability of entry. This second problem is likely to be critical in the event that EMU is scrapped altogether, but it does not seem to be a major problem if Italy is simply left out, for in this case the incentives to converge would remain very strong. A similar qualification could apply to the dollar /DM rate, that could also be affected by the existence of an EMU project. Hence, the event "Italy is out of EMU" is best interpreted as referring to the case of a late entry of Italy, but not of a delay or an abandonment of the EMU project altogether.

Concluding, our results indicate that the observed reduction of forward interest rates is largely due to an improvement in domestic fundamentals rather than to an increase in the probability of Italy being inside EMU, but conditional on the EMU project not being scrapped altogether or delayed. What would be the behavior of Italian interest rates if this latter event were to materialize is admittedly very hard to tell.

5. Evaluating EMU Calculators

The probability that Italy joins EMU in 1999, computed according to the technique we have described, is significantly lower than that computed by J.P. Morgan and Credito Italiano, and reported in Table 1. Why is there a difference, and which technique should be used ?

The J.P. Morgan and the Credito Italiano “EMU Calculators” both use the following assumption to compute the probability that Italy belongs to EMU in 1999:

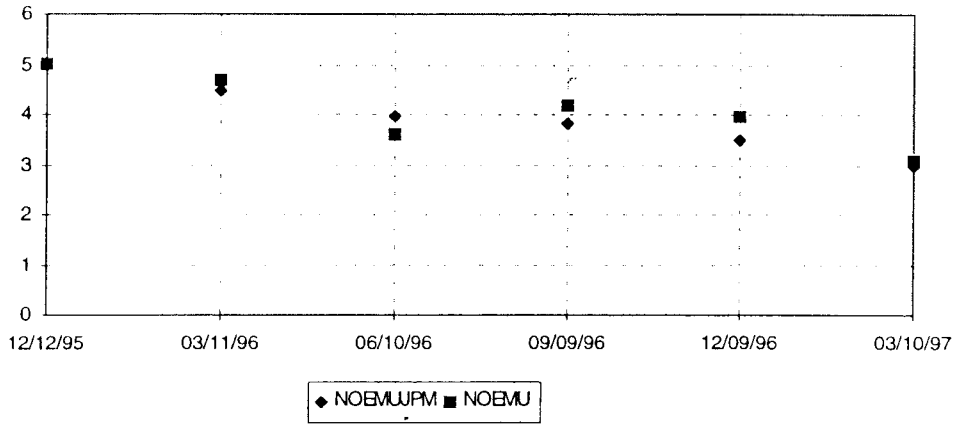
$$SP(99)_{m,t} = (1 - \pi_{99,t}) SPOUT(99)_{m,t} \quad (4)$$

where $SP(99)_{m,t}$ is the observed spread between the Italian and German m -year forward rates with settlement in 1999, and $SPOUT(99)_{m,t}$ is the theoretical spread between the Italian and German m -year forward rates with settlement in 1999 in the event “Italy does belong to EMU in 1999”. m is set to three-years by Credito Italiano, and to five years by J.P. Morgan.

The two institutions also use different assumptions to pin down the value of SPOUT, the differential in the event Italy does not join EMU. Credito Italiano sets this spread equal to its average in 1993 -- on the presumption that in 1993 the start of EMU was not incorporated in market prices. J.P. Morgan, instead, identifies this spread by claiming that there is an international price for the risks of policy, inflation, and volatility, whose aggregation constitutes the total spread. Therefore “if EMU were not around, the spread between Italian and German bonds would be highly correlated with international measures of risks such as US-Australia, US-Canada, Brady-Treasury, the US long bond-2yr bond spread, as well as the order of performance of non-European currencies and market volatilities” [J.P. Morgan, 1997]. J.P. Morgan thus estimates a regression of the observed spread on an average non-European spreads, the level of American market rates, the steepness of the American yield curve, and non-European measures of volatility, using daily data over the sample January 1989-December 1991. On the basis of the estimated coefficient of the regression in a no-convergence period they are able to map current observations on the regressors into a level of the spread if out of EMU. The value of the spread thus computed can be updated daily, thus at a much higher frequency than our measure of SPOUT, which is based on macroeconomic fundamentals updated quarterly.

The choice of a different method of calculating SPOUT is an obvious candidate to explain the divergence between the probabilities we compute, and those computed by the two institutions. We can rule out this explanation, however. The probabilities computed by J.P. Morgan and Credito Italiano move very closely to one another, and the level of SPOUT based on the measure of the price of risk computed by J.P. Morgan is not very different from that estimated in this paper on the basis of Italian macroeconomic fundamentals. Figure 7 plots both J.P. Morgan measure of 2-year forward differential for January 1, 1999, estimated at different future dates t , $SPOUT(99)_{2,t}$ as well as the corresponding value of $SPOUT(99)_{2,t}$ used in this paper. To compute our measure of $SPOUT(99)_{2,t}$, naturally, we have converted the instantaneous forward differential estimated as described in section 3, in a two-year forward differential. The difference between our measure and J.P. Morgan’s never exceeds 30 basis within our sample.

Figure 7: The JP-Morgan NOEMU spread and our NOEMU spread



The reason why the probabilities estimated by the two financial houses differ from our own is that they use forward rates with a maturity date that is different from the settlement date. While in this paper we use *instantaneous* forward rates, that is, in practice, interest rates with settlement some time in the future and maturity a day after, both financial houses use forward rates with settlement some time in the future, and maturity some years after -- three years in the case of Credit, five years in the case of J.P. Morgan. This implies that the probability computed by these institutions refers to a different event. We estimate the probability that Italy belongs to EMU at a specific moment in time, say January 1, 1999. These two institutions instead estimate the *average* of the probabilities that Italy belongs to EMU at all point in time between date t , say January 1, 1999, and date $t+m$. Since the probability of being inside EMU at a future date T increases with T , this measure clearly overestimates the probability of being inside EMU exactly on January 1, 1999.

To see this point more clearly, recall that the m -year forward spread is the average of instantaneous forward spreads over the relevant period. Specifically:

$$SP(T)_{m,t} = \frac{1}{m} \int_T^{T+m} SP(z) dz \quad (5)$$

and similarly for $SPOUT(T)_{m,t}$. J.P. Morgan assumes that the instantaneous forward spread if out of EMU increases linearly with the settlement date T , while Credito Italiano assumes that it is constant overtime. That is, they assume that

$$SPOUT(T)_{m,t} = \lambda SPOUT(T) \quad (6)$$

where $\lambda = 1$ for Credito Italiano, while $\lambda > 1$ for J.P. Morgan. By (2), (5) and (6):

$$SP(T)_{m,t} = \frac{1}{m} \lambda SPOUT(T) \int_T^{T+m} (1 - \pi(z)_t) dz \quad (7)$$

These two financial institutions estimate the probability that Italy belongs to EMU at date T as:

$$\hat{\pi}(T)_t = 1 - \frac{SP(T)_{m,t}}{SPOUT(T)_{m,t}} \quad (8)$$

By (6)-(7), this estimate reduces to:

$$\hat{\pi}(T)_t = \frac{1}{m} \int_T^{T+m} \pi(z)_t dz \quad (9)$$

where the right hand side of (9) is the average probability of being inside EMU between date T and date $T+m$. Since the probability of being inside EMU increases overtime:

$$\pi(T+m)_t > \hat{\pi}(T)_t > \pi(T)_t$$

Thus, the probability computed from the EMU calculators based on m -year forward rates overestimates the true probability of being inside EMU at date T , the more so the larger is the maturity date m . If m is zero, as with instantaneous forward rates, however, and if $SPOUT(T)_t$ is correctly estimated, then the estimate of the probability is unbiased.

Consider, as an example, March 10, 1997, when our methodology delivers an estimate of 0,24 for the probability of being inside EMU on January 1, 1999, and 0,48 for the probability of being inside EMU by January 1, 2001. If we compute the same probabilities using 2-year average forward rates, instead of instantaneous forward rates, without any change in the scenario if out of EMU, the resulting probability of entry in 1999 is 0,41. Thus, there is an upward bias of 0,17 (= 0,41-0,24). A full comparison of the results provided by alternative "EMU Calculators" for March 10, 1997 is shown in Table 5. Clearly, the different methodology is highly relevant from a quantitative point of view.

Table 5: Probabilities (as of March 10, 1997) of Italy joining EMU in 1999					
	Reuters EMU Survey	Credito Ital. EMU Calculator	J.P. Morgan EMU Calculator	Our methodology using instantaneous forward rates	Our methodology using average 2- year forward rates
	0.12	0.46	0.57	0.24	0.41

6. Conclusions

We have proposed a method for extracting, from the term structure, information about the probability that a country joins the EMU at given dates. We have derived two main results:

- We have shown that probabilities computed by currently available "EMU Calculators" are upward biased because they are based on *average* rather than *instantaneous* forward rates.
- We have estimated the *OUT* spread between Italian and German short-term interest rates, that is the spread that would arise at some date t if, at that date, Italy did not belong to EMU, by estimating a reaction function for the Bank of Italy along the lines of a "Taylor rule". We are thus able to identify, separately, that component of the total spread between Italian and German interest rates that is due to different fundamentals, from the component due to the markets' assessment of the probability that the country will join EMU in 1999. For example, out of the

214 basis points reduction in the spread between Italian and German forward rates with maturity in 1999, observed from March '96 to March '97, 150 basis points are to be attributed to the convergence of fundamentals, while only 65 basis points are to be attributed to a change in markets' assessment of the probability of Italy joining EMU in 1999.

We have also assessed the properties of the "EMU Calculators" that are currently used by two financial houses. We have first shown that both are derived from a common set of assumptions, namely that:

1. that currently observed forward interest rates with maturity in 1999, and settlement three to ten years later, are a weighted average of (a) the spot rate differential between a country and Germany in the case that country joins, and (b) the spot rate differential in the case that country does not join, where the weights are the probability of joining, and its complement to one;
2. the spot rate differential in the case a country joins is zero;
3. if the country does not join, the spot rate differential attains a specific value.

Probabilities are then derived computing the ratio of the observed average forward rate differential, to the spot rate differential assumed for the case in which a country is left out. Leaving aside market inefficiencies, assumptions (2) and (3) introduce two potential sources of bias. The first is related to the need of specifying a value of the spot rate differential in the case the country does not join EMU. The second is related to the use of average rather than instantaneous forward rates.

We have identified and computed these two sources of bias. The first step involves estimating the interest rate differential in the case of no entry in 1999 – the OUT spread. The technique we use to estimate this differential (which is based on the estimation of a reaction function for the Bank of Italy linking the interest rate spread between Italy and Germany to macroeconomic fundamentals) is different from that used by J.P. Morgan, but essentially confirms the results obtained by that bank. This result rules out the empirical relevance of the first source of bias. We then construct an EMU calculator based on instantaneous forward rates.

The use of instantaneous forward rates is crucial because it provides an easy way to describe the complementary events "Italy belongs to EMU" and "Italy is out of EMU" at a specific future point in time. With *m-period* average (rather than instantaneous) forward rates, one estimates the average of the probabilities of being inside EMU at all points in time during the interval of time of size *m*. Since the probability of being inside EMU at a future date *T* increases in *T*, this average of probabilities is greater than the probability of being inside EMU exactly at the start of the interval considered.

7. Appendix: Spot Rates, the Expectational Model and Forward Rates

To illustrate our derivation of spot and forward rate functions, let us start by considering a zero coupon bond issued at time t , with a face value of 1, maturity of m years and price PZC_{mt} . The simple yield Y_{mt} is related to the price as follows:

$$(1) \quad PZC_{mt} = \frac{1}{(1 + Y_{mt})^m}$$

Define the spot rate r_{mt} as $\log(1 + Y_{mt})$, which is the continuously compounded yield, and define the discount function D_{mt} as the price at time t of a zero coupon that pays one unit at time $t+m$. We then have :

$$(2) \quad PZC_{mt} = \exp(-mr_{mt}) = D_{mt}$$

Consider now a coupon bond that pays a coupon rate of c percent annually, and pays a face value of 1 at maturity. The price of the bond at trade date is given by the following formula:

$$(3) \quad P_{mt} = \sum_{k=1}^m cD_{kt} + D_{mt}$$

Given the prices of coupon bonds, spot rates on zero-coupon equivalent can be derived by fitting a discount function based on the following specification for the spot rates:

$$(4) \quad r_{kt} = \beta_0 + \beta_1 \frac{1 - \exp\left(-\frac{k}{\tau_1}\right)}{\frac{k}{\tau_1}} + \beta_2 \left(\frac{1 - \exp\left(-\frac{k}{\tau_1}\right)}{\frac{k}{\tau_1}} - \exp\left(-\frac{k}{\tau_1}\right) \right) + \beta_3 \left(\frac{1 - \exp\left(-\frac{k}{\tau_2}\right)}{\frac{k}{\tau_2}} - \exp\left(-\frac{k}{\tau_2}\right) \right)$$

The above specification has been originally introduced by Svensson (1994) as an extension of the parametrization proposed by Nelson and Siegel(1987). Note that our estimated spot rate differs from the yield to maturity often quoted for coupon bonds. In fact the quoted yield to maturity, y_{mt} , is defined by the following relation:

$$(5) \quad P_{mt} = \sum_{k=1}^m c \exp(-ky_{mt}) + \exp(-my_{mt})$$

Yield to maturities are averages of spot rates up to the date of maturity. While in general spot rates defined by (3) vary with the maturity, the yield to maturity defined by (5) is constant. Henceforth, the term structure of interest rates estimated on yields to maturity is only valid when the term structure of spot rates is flat. Moreover, the yield to maturity for a bond with a given maturity depends on the coupon rate, the so-called "coupon effect". Spot rates instead are free from such an effect.

Implied forward rates can be computed from spot rates. A forward rate at time t with trade date $t+t'$, and settlement date $t+T$ can be computed as the return on an investment strategy based on buying zero-coupon bonds at time t maturing at time $t+T$, and selling at time t zero-coupon bonds maturing at time $t+t'$. The forward rate is related to the spot rate according to the following formula:

$$(6) \quad f_{T+t,t'+t,t} = \frac{T r_{T,t} - t' r_{t',t}}{T - t'}$$

The forward rate for a 1-year investment with settlement in 2 years, and maturity in 3 years is thus equal to three times the 3-year spot rate, minus twice the two year spot rate.

The instantaneous forward rate is the rate on a forward contract with an infinitesimal investment after the settlement date:

$$(7) \quad f_{mt} = \lim_{T \rightarrow m} f_{T+t,m+t,t}$$

In practice we identify the instantaneous forward rate with an overnight forward rate, *i.e.* a forward rate with maturity one day after the settlement. The relationship between the instantaneous forward rate and the spot rate is then:

$$(8) \quad r_{mt} = \frac{\int_{t=t}^{t+m} f_{\tau} d\tau}{m} \quad \text{or, equivalently}$$

$$(9) \quad f_{mt} = r_{mt} + m \frac{\partial r_{m,t}}{\partial m}$$

Given specification (4) for the spot rate, the resulting forward function is:

$$(10) \quad f_{kt} = \beta_0 + \beta_1 \exp\left(-\frac{k}{\tau_1}\right) + \beta_2 \frac{k}{\tau_1} \exp\left(-\frac{k}{\tau_1}\right) + \beta_3 \frac{k}{\tau_2} \exp\left(-\frac{k}{\tau_2}\right)$$

Therefore, as k goes to zero, the spot and the forward rate coincide at $\beta_0 + \beta_1$, and as k goes to infinity the spot and the forward rate coincide at β_0 . The forward rate function features a constant, an exponential term decreasing when β_1 is positive, and two “hump shape” terms.

We estimate a term structure of spot rates based on the observation of the overnight rate, the Euro 1-month, 3-month, 6-month and 12-month rates. We then consider the 2-year, 3-year, 5-year, 7-year and 10-year fixed interest rates on swaps. We use Euro-rates as spot rates, because they are zero-coupon bonds. We then consider swap rates as the long-term rates to be associated to Euro rates. Fixed interest rates swaps facilitate international comparisons because they are not affected by different taxation regimes, and by default risk. By fitting the discount function to the data, and minimizing error in the yield space, we then estimate spot and forward rates.

References

- Bernanke B.S. and I. Mihov (1995), "Measuring monetary policy", NBER working paper no. 5145.
- Bernanke B.S. and I. Mihov (1996), "What does the Bundesbank target ?", *European Economic Review*, 41,7.
- Campbell J.Y.(1995), "Some lessons from the yield curve", NBER working paper no. 5031.
- Campbell J.Y., Shiller R. J., and K.L. Shoenholtz (1983), "Forward rates and future policy: interpreting the term structure of interest rates", *Brooking Papers on Economic Activity* 1, 173-217.
- Campbell J.Y., and Shiller R.J. (1987), "Cointegration and Tests of Present Value Models", *Journal of Political Economy*, 5, 1062-1088.
- Clarida R., and M. Gertler (1996) "How the Bundesbank conducts monetary policy", NBER working paper no. 5581.
- Christiano L.J., and M. Eichenbaum (1992), "Liquidity effects and the monetary transmission mechanism", *American Economic Review*, 82, 2, 346-353.
- Christiano L.J., M. Eichenbaum and C.L. Evans (1996a) "The effects of monetary policy shocks: evidence from the flow of funds", *Review of Economics and Statistics*, 78, 16-34
- Christiano L.J., M. Eichenbaum and C.L. Evans (1996b) "Monetary policy shocks and their consequences: theory and evidence", *European Economic Review*, 41, 7.
- De Grauwe P. (1996), "Forward rates as predictors of EMU", CEPR discussion paper no.1556.
- Doornik J., and D.F. Hendry (1996), *Interactive econometric modelling of dynamic systems*. London: International Thompson Publishing.
- Flood, R.P. and P.M. Garber (1983), "A model of stochastic process switching", *Econometrica*, 51, 3, 537-51.
- Flood, R.P. and N. Marion (1983), "Exchange-rate regimes in transition: Italy 1974", *Journal of International Money and Finance*, 2, 279-94.
- Gerlach S., and F. Smets (1995), "The Term structure of euro rates: some evidence in support of the expectations hypothesis", mimeo.
- Giavazzi F. and A.Giovannini (1989), *Limiting Exchange Rate Flexibility: The European Monetary System*. Cambridge (Mass): MIT Press.
- Hendry D.F. (1995) *Dynamic Econometrics*, Oxford University Press, Oxford
- J.P. Morgan (1997), "EMU Calculator Handbook", Foreign Exchange Research: Technical series, London, January 1997. Daily updates available at <http://www.jpmorgan.com>.

King M. (1995), "Credibility and Monetary Policy: Theory and Evidence", *Scottish Journal of Political Economy*, 1-19.

Leeper E.M., C.A. Sims and T. Zha (1996), "What does monetary policy do?", available at <ftp://ftp.econ.yale.edu/pub/sims/mpolicy>.

Mac Callum T. (1995), "Monetary policy and the term structure of interest rates", NBER working paper no. 4938.

Nelson C.R., and A.F. Siegel (1987), "Parsimonious modelling of yield curves", *Journal of Business*, 60, 473-489.

Sims C.A., J. H. Stock, and M.Watson (1990), "Inference in linear time-series models with some unit roots", *Econometrica*, 58, 113-144.

~ Söderlind, P. and L.E.O. Svensson (1997), "New techniques to extract market expectations from financial instruments", CEPR discussion paper no. 1556.

Svensson L.E.O. (1994), "Estimating and interpreting forward interest rates: Sweden 1992-94", CEPR discussion paper no. 1051.

Taylor J.B.(1993), "Discretion versus policy rules in practice", *Carnegie Rochester Conference Series on Public Policy* 39,195-214 .