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JEL Classification: H55, J26

Keywords: option value model, pension reform, structural evaluation

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October 21, 2021

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1 Introduction

In many developed countries, the combination of two trends, namely the aging of the population and the decrease in labor participation of older workers (see, for instance, Gruber and Wise, 1998), has undermined the solvency of retirement systems.[2] To cope with this challenge, much academic work has been undertaken to examine how financial schemes shape individual retirement decision. A first generation of empirical studies tried to explain and to reproduce the sharp increase in labor force exits observed at age 62 and 65 in the U.S. (Gustman and Steinmeier, 1986; Stock and Wise, 1990; Blau, 1994; Lumsdaine, Stock and Wise, 1996). Later, a research stream initiated by Gruber and Wise (2004) broadened the analytical framework by studying how social security wealth may influence retirement decisions in twelve different countries.

From an econometric point of view, we may distinguish two types of approaches. On the one hand, dynamic programming (hereafter DP) aims at modelling the retirement decision as the solution of a finite-horizon stochastic problem. Relaxing progressively simplifying assumptions, models became more complex. For instance, Rust (1989) and Rust and Phelan (1997) incorporate environment uncertainty by introducing into their models multiple state variables (such as health, employment and marital status). Berkovec and Stern (1991) take into account individual heterogeneity by allowing for job-specific effects, whereas French (2005) develop the very first model which allows for savings but not for borrowing.

On the other hand, reduced-form models have progressively been refined, from standard OLS to dynamic panel data analysis. For instance, Asch *et al.* (2005) and Belloni and Alessie (2009) use peak value and accrual (namely, the variation in social security benefits) as the main drivers of exit behavior. Fitzpatrick (2013) uses a differences-in-differences strategy to assess whether introducing health insurance for public school employees affects their labor supply.

Within the class of structural retirement models, the option value model developed by Stock and Wise (1990) has been often considered as a sub-optimal solution of the DP rule set forth by Rust (1989) and Phelan and Rust (1997). In fact, when future incomes are uncertain, the option value model may undervalue future options, and consequently over-predict retirement (Lumsdaine, Stock, and Wise, 1992). However, various empirical applications show that the option value and DP models

[2] As Börsch-Supan *et al.* (2014) notice, in particular, France, Germany, and Italy "have labor markets characterized by low participation rates of young women and individuals aged 55 and over. In spite of these structural problems, France, Germany, and Italy have been remarkably resistant to labor market and pension reforms".

yield similar results (see Lumsdaine, Stock, and Wise, 1996; Burkhauser, Butler, and Gumus 2004). At first considered as a suboptimal solution of the DP rule, the option value model rapidly gained a great popularity among economists to the point where it is often used as a regressor in reduced-form models (see, for instance, Chan and Stevens, 2004). More flexible than DP models, it relies on less ad hoc assumptions. In our application, older teachers, who are civil servants close to retirement, know with a quasi-certainty their future salaries and the amount of their pension, which is communicated to them by the administration. In that case, the option value and the DP models yield the same analytical solution (see Appendix A). Many studies provide evidence that the option value model is at least as good as the dynamic programming model in terms of predictive validity (see, for instance, studies by Daula and Moffit, 1995; Ausink and Wise, 1996; and Burkhauser, Butler and Gumus, 2004). As Belloni (2008) points out, "these works provided evidence that the trade-off between computational complexity and predictive validity did not exist, and that the option value model was at the same time more computationally feasible and more powerful than dynamic programming in approximating actual retirement". Unlike our study, which is one of the very few papers estimating the conditional multiple-years model proposed by Stock and Wise (1990), several studies estimate reduced form (probit-type) models in which the retirement probability depends on the expected gain of postponing retirement, which is there computed exogenously by assuming specific values for the parameters of the utility function (see, for instance, Harris, 2001; Blundell, Meghir and Smith, 2002; Hurd, Loughran, and Panis, 2003; Gruber and Wise, 2004; Börsch-Supan *et alii*, 2004). Most of these studies find unsatisfactory results and have a poor quality of fit.
Generally, the option value model and the dynamic programming model (e.g., Stock and Wise, 1990, and Rust and Phelan, 1997) do not allow for consumption smoothing. Thus, in each period, the level of consumption and the income earned coincide, implying that savings and borrowing are excluded from the analysis, or assumed to be equal to zero (often because of data limitations). However, the role of private wealth in financing retirement is taken into account in the more recent literature on structural models. For instance, French (2005) develops a model of retirement behavior with savings in which future health status and wages are uncertain, which is not our case here since teachers'wages are certain in each period and rather steady (at least in the short term). In French's model (2005), workers can save to insure themselves against health and wage shocks (and for their old age) but cannot borrow from social security, pensions and future wages, in response to adverse shocks. In

our opinion, this appaoch is less relevant for French teachers employed in the public sector, because in general their health expenses (both before and after their retirement) are almost all covered by the public Social Security and supplementary mutual insurances, whose contributions are generally low and which supplement the reimbursement of their health expenses. This last remark limits the objection to having recourse to an option model without savings in our case.

In the option value model developed by Stock and Wise (1990), retirement is seen as an absorbing state and transitions from full-time to part-time work before retirement are excluded. These two restrictions are not binding in our case. First, in the vast majority of cases, retired teachers have no additional salaried activity. Moreover, the proportion of part-time teachers at the date of retirement is very low (4.2 percent). Consequently, we have decided to exclude them of our study sample. In addition, none of the teachers observed in our sample becomes unemployed before retiring (this case is anyway extremely rare in the French public sector at ages close to retirement). It is also well known that the family context and the work choices of other members of the family can affect an individual retirement decision (see, for instance, Blau, 1998). Unfortunately, our sample does not contain all the information concerning the family environment. However, we observe the spouse's age and the presence of at least one child less than 20 in the teacher's household which are two variables likely to affect the retirement decision.

To summarize, we think that we are in a situation which is suitable to be modelled within the option value framework. More precisely, we use the option value model in order to evaluate the impact of the 2003 French national pension reform on high-school teachers' retirement decisions.[3] Due to the complexity of this reform, which modified progressively several parameters of the pension system, we cannot use a reduced-form methodology, such as a differences-in-differences strategy or a regression discontinuity method.

Unlike reduced-form models, the option value model allows to identify several structural parameters of interest, such as the degree of risk aversion and the preference for leisure. In our preferred specification, the estimated quarterly discount factor is equal to 0.97 and the risk aversion coefficient is approximately 0.5: these values are standard in the econometric literature.

The model fits well our data: estimated retirement rates are close to the observed ones. Some alter-

[3]Except estimates contained in the seminal article by Stock and Wise (1990), our paper is, at our best knowledge, the second empirical study that provides a full estimation of the conditional multiple-years model introduced by Stock and Wise (1990), the first being the article published by Belloni and Alessie (2013). However, our study is the first to use this model for estimating the effects of a complex pension system reform on individual retirement decisions.

native scenarios show that financial incentives to continue working impact the retirement age more than financial disincentives associated with early exits (even if this conclusion must be qualified). Finally, we run a partial cost-benefit analysis that is based on the impacted sample observations and that permits some ex-ante limited evaluation of the effectiveness of the reform. The design of the reform is such that parameters of the pension system are modified each year until 2020. We then predict the evolution of the average retirement age as well as that of the partial public cost (defined as the sum of wages and pensions paid by the State to teachers similar to those observed in our sample) for each setup of the reform.

Our study contributes to the literature on pension reforms in several ways. First of all, no structural estimation of a pension reform in France has been undertaken so far. For instance, Bozio (2008) uses a difference-in-differences approach for estimating the effects of the 1993 reform which concerned workers in the private sector only. Depending on their birth year and their length of contribution at age 60, workers were differently affected by this reform ; this feature of the 1993 reform allows Bozio (2008) to define control and treatment groups. He estimates a quarter elasticity of 0.54, meaning that requiring an additional contribution quarter in order to get the full pension rate, leads to an increase of 0.54 contributed quarter. Aubert (2009) pursues this approach with better data and estimates elasticities of 0.7 for men and 0.6 for women.

Benallah (2011) focuses on a specific feature of the 2003 reform, namely the "*surcote*", which consists in a premium paid for each extra contributed quarter. She compares two cohorts (those born in 1938 and 1944), the former being the control group, the latter the treatment one. Using a propensity score matching method, she estimates that this premium increased the average retirement age by 2 months and the probability to stay in activity after age 60 by 12%. Baraton *et al.* (2021) consider younger cohorts, but cannot estimate elasticities, because some workers are still employed at the end of their observation period. Nonetheless, they quantify the impact of the reform by using a regression discontinuity design. They find that the probability to retire between 60 and 61 years old decreased by 9 percentage points between 2004 and 2007. They also document the heterogeneity of this effect: for teachers with a few missing quarters of contribution, the reform has barely affected their retirement age.

Second of all, previous studies about the 2003 reform were limited by the fact that the first targeted cohorts were not fully retired when these studies were conducted. To circumvent this issue, Benallah

(2011) reduced her treatment group to the 1944 cohort, whereas Baraton *et al.* (2021) weighted some observations. By contrast, our sample is composed of fully retired cohorts. Moreover, simulations based on our structural estimates offer an outlook of alternative scenarios. Our results illustrate what would have happened with a different reform setting.

Next sections depict the French institutional framework (Section 2) and our database along with our identification strategy (Section 3). Section 4 is dedicated to the presentation of the Stock and Wise model adapted to the framework of the 2003 French pension reform. Estimates and simulations are then commented in Section 5. Finally, we conclude our study by evoking some research perspectives (Section 6).

2 The institutional framework

2.1 The French context

The French pension system is known to be highly complex. During the last three decades, three major reforms of the French pension system have been implemented, the first one in 1993, under the government of the then Prime Minister Edouard Balladur, and the second one in 2003 under the government of the then Prime Minister Jean-Pierre Raffarin. According to Blanchet (2005), none of these two reforms was “sufficient to fully ensure equilibrium for the pension system, but both of them have had, or should have, very significant impacts”. The third reform was implemented in 2010, under the government of the then Prime Minister François Fillon.

The 1993 reform concerned only the private sector. It increases the number of years required for getting full pension. This number was previously equal to 37.5 years. After the reform, it increased by one quarter each year until 2003, in order to reach 40 years in 2003. Moreover, the 1993 reform changed the number of years on which individual past wages are averaged in order to compute the reference wage used for calculating the pension rate. From 1993 on, this number has increased from 10 to 25 years. Finally, before 1993, past wages and pensions were discounted according to GDP growth, while since the 1993 reform, they are only indexed on inflation. Once again, according to Blanchet (2005), “this new rule considerably strengthens the impact of having shifted from an average of wages over the 10 best years to the average over the 25 best years of one’s career”.

The 2003 reform is presented in detail in the following subsection.

The latest reform was introduced in 2010. It increased gradually the minimum statutory retirement age (i.e., the age at which pension benefits become available, albeit not in full) from 60 to 62 years old for those born after 1951. More precisely, the minimum statutory retirement age was set to 60 years and 4 months for employees born in 1951, to 60 years and 9 months for those born in 1952, and to 61 years and 2 months for persons born in 1953 and after. Moreover, the maximum retirement age was maintained at 65 for employees born before 1951 and increased to 67 for those born after. Using a detailed social security administrative database, Rabaté and Rochut (2020) find that, despite a sizable effect on the employment rate of older workers in the private sector, it also strongly increased their unemployment and disability rates. Besides, using a panel data sample of 38,652 high-school teachers, d'Albis *et al.* (2020) find that teachers affected by the 2010 reform had an increased probability to take short sick leaves before retirement. This is mainly due to teachers who decide to retire at the minimum legal retirement age, while those who continue to work above the minimum retirement age do not increase the frequency of their short sick leaves before retirement.

2.2 The 2003 reform

The 2003 pension reform is an extension of the 1993 reform to the French public sector. At that time, three main arguments were highlighted to justify the new reform. The first one was related to fairness: the objective was to bridge the gap between private and public sectors in terms of required years of contribution. Oddly, by offering to civil servants the indexation of pensions on prices, rather than on wages as previously, the reform helped to maintain their purchasing power over time since public wages have been frozen for several years during the previous decade.

The second argument was the demographic concern. Since 1970, life expectancy in France is increasing approximately by one quarter each year.[4] Moreover, generations born after World War 2 were expected to massively retire around 2005. It implies that active workers have to contribute more intensively, and for a longer period, to pension funding.

The third justification was the fiscal requirement triggered off by the demographic issue. Without any reform, the system would have faced a deficit of 43 billions of Euros in 2020, endangering even more the solvency of public accounts. The 2003 reform could bring up to 42% (18 billions of Euros)

[4]Source : Insee, statistiques de l'état civil et estimations de population, http://www.insee.fr/fr/ffc/figure/NATnon02229.xls

of the forecasted 2020 deficit.

2.3 Main features of the 2003 reform

As noticed by Blanchet (2005), the 2003 pension reform had three main features:

- the first objective was to ensure a convergence of conditions for access to a full pension rate in the private and public sectors. Consequently, in the public sector, the number of required years of contributions has been raised from 37.5 to 40 years between 2004 and 2008;
- after 2008, the number of required years has increased by one more year by 2012 in the two sectors;
- in compensation, the penalty for early exits has been reduced, and a financial incentive to postpone retirement has been introduced; it consists of a 3% bonus for each supplementary year.

A key concept in the computation of the pension is the so-called "year for opening rights". It corresponds to the year in which a teacher can legally retire. For cohorts considered in our study (those born between 1940 and 1947), the year for opening rights is the year in which a teacher reaches the age of 60. For instance, a teacher born in 1942 (i.e., who turns 60 in 2002) gets a pension calculated on the basis of the pre-reform scheme, whereas a teacher born in 1947 must comply with the 2007 setup (i.e., when she turns 60).

It must be noted that for the considered cohorts, the year for opening rights always occurs at age 60. This means that teachers can still retire at this age. If they delay retirement, they are eligible to the bonus (see above). We therefore want to estimate the sensitivity of teachers' retirement decisions to these mechanisms. To sum things up, for the cohorts we consider, the reform has introduced four main changes:

1. a gradual increase in the number of quarters required to get the full pension (this number is hereafter denoted *Length FR*),
2. the introduction of two mechanisms, namely a penalty (*"decote"*) for those who retire before getting the required quarters of contribution, and a premium fo those who postpone retirement (*"surcote"*):

- the penalty was implemented from 2006 onwards; it increased since then; for instance, it reduced the pension by 0.125 pp. per missing quarter in 2006, and by 1.250 pp. in 2015;

- the premium was implemented from 2004 onwards; it also increased since then; for instance, the pension was increased by 0.75 pp. per extra quarter in 2004, and by 1.250 pp. in 2015;

3. an increase in the age at which the penalty rate is cancelled (the so-called "age limit")

4. a change in the formula for the pension calculation:

$$P = \tau \times Ind \times Val \times \min \left\{ 1; \frac{\text{Length PS}}{\text{Length FR}} \right\}$$

   where

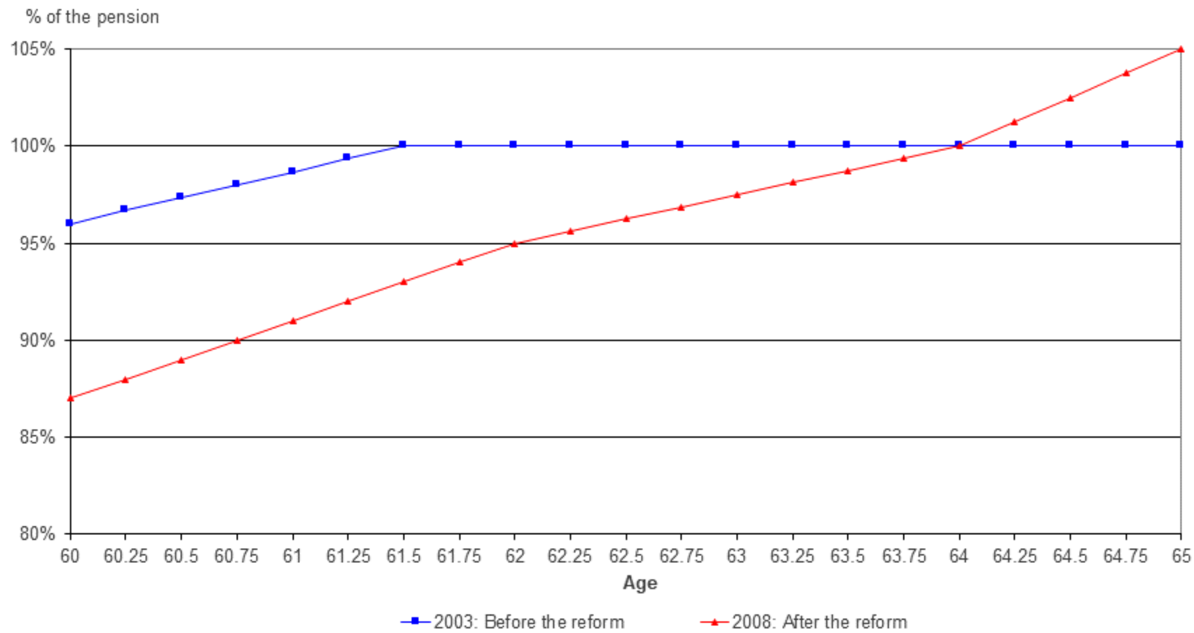
   - τ is the pension rate,
   - Ind is the index of the wage that is reached at least 6 months before the retirement date,
   - Val is the value (in Euros) of one point for the wage index,
   - *Length PS* is the number of quarters spent in the public sector,
   - *Length FR* is the number of quarters required to get the full pension rate.

Before 2003, τ was equal to 0.75 while *Length FR* was equal to 150. After 2003, τ became equal to $0.75 \times (1 - \text{penalty} + \text{premium})$, while *Length FR* increased.

These measures have introduced financial incentives in order to maintain teachers in activity after the legal retirement date. Table 1 sums up the schedule of the evolution of coefficients and Figure 1 illustrates how the reform works.

On Figure 1, one can see that a teacher who contributed for 144 quarters at age 60 in 2003 almost gets the full pension rate at that age. If she contributes a few more quarters (6 supplementary quarters in fact), she gets 100% of her pension at age 61.5 because *Length PS* is now equal to *Length FR*. Beyond that age, she has no incentives to work any more, since she cannot increase her pension.

Figure 1: Pension for a teacher who contributed for 144 quarters at age 60



After the reform, the pension profile of a teacher who is similar to the previous one, except she was born in 1948, undergoes a shift downwards because of the increase in the number of quarters required to get the full pension rate and because of the penalty of 0.375% imposed for each missing quarter. The kink at age 62 corresponds to the "age limit", when the penalty does not apply any more. The pension rate is only reduced because $Length\ PS$ is lower than $Length\ FR$. By contrast, the teacher can now expect to increase her pension rate beyond 100%, thanks to the premium (i.e., bonus) mechanism.

Table 1 shows that the reform has been progressively implemented since 2003. For instance, each extra quarter contributed between 2004 and 2009 increases the pension by 0.750 pp., while after 2009, this premium doubles. The penalty rate is also increasing over the period. In fact, for the youngest cohorts, the cost of each missing quarter is higher, but each additional quarter yields more money.

Table 1 illustrates why a structural model of retirement behavior is preferred to a reduced-form model. One can see that each year, 3 or 4 parameters are varying together, which prevents us to identify precisely the effect of one specific feature of the reform by using either a difference-in-differences procedure or a regression discontinuity method. This is why, using a regression discon-

Table 1: The 2003 reform's schedule: quarters of contribution and coefficients

Year of opening rights	Number of quarters required	penalty rate per missing quarter %	Age limit[a]	Premium rate per extra quarter % [b]
before 2004	150	0.000	60	0.000
2004	152	0.000	60	0.750
2005	154	0.000	60	0.750
2006	156	0.125	61	0.750
2007	158	0.250	61.5	0.750
2008	160	0.375	62	0.750
2009	161	0.500	62.25	1.250
2010	162	0.625	62.5	1.250
2011	163	0.750	62.75	1.250
2012	164	0.875	63	1.250
2013	164	1.000	63.25	1.250
2014	165	1.125	63.5	1.250
2015	166	1.250	63.75	1.250
2016	166	1.250	64	1.250
2017	166	1.250	64.25	1.250
2018	166	1.250	64.5	1.250
2019	167	1.250	64.75	1.250
2020	167	1.250	65	1.250

[a]The age at which the penalty rate is cancelled
[b]This rate does not depend on the year of opening rights but on the year in which the quarter is completed

tinuity method, Baraton *et al.* (2021) can only estimate the overall effect of the reform on the probability to delay retirement between 2004 and 2007. A solution would be to compare cohorts 1944 and 1945, since there is only one parameter that changes in 2005, namely the number of quarters required to get the full pension rate (see Table 1). However, we would not be able to estimate the premium or the penalty effects on teachers' retirement decisions in year 2005.

3 Data

The administrative datasets we use are hosted by the *"Département de l'Évaluation de la Prospective et de la Performance"*, which is the Department for Statistics and Evaluation of the French Ministry of Education. In order to observe wage and pension profiles, we use two files. The first one (*"Fichier des Dossiers d'Examen des Droits à Pensions"*, 1940-1955) gathers information about teachers during the year their rights are opened (i.e., when they are around 58 years old). The second one (*"Fichiers des bénéficiaires d'une pension civile du Ministère de l'Éducation Nationale"*, 1998-2012) collects information concerning teachers who are already retired. These two files allow us to observe wages and pension benefits from age 60 to age 63, in each quarter and for any teacher, but they contain no information about teachers' consumption and savings. Finally, we use mortality tables for professional and intellectual occupations built by the National Institute for Demographic Studies (*"Institut National des Études Démographiques"*") in order to calculate the expected flows of pensions, whatever the retirement age.

3.1 Descriptive statistics

Descriptive statistics are presented in Table 2. Our sample consists in 12,463 teachers, from seven birth cohorts (1940-1947). The 1943 cohort was removed because we do not know whether these teachers were aware of the forthcoming reform or not. They aged 60 in 2003. The first year they could legally retire is also the year the reform was adopted by the Parliament. As we do not really know if option values they calculated were based on the former scheme or on the new one, we withdraw them as a precaution.

The number of observations is larger for later cohorts, due to a sharp increase in recruitment after the World War II. We focus here on two groups of teachers, the first consisting of certified teachers (who hold the *CAPES*[5] certificate), the second consisting of physical education teachers. Women are more numerous, just as certified teachers. Certified teachers represent the majority of teachers in secondary schools. Physical education teachers share exactly the same characteristics, so we decided to include them as well. Unlike more qualified teachers, these two categories of teachers usually do not teach in post-secondary education institutions, which grants they are not cumulating two positions at the end of their career. Our administrative data are exhaustive and individual wage profiles are reliable.

At the date of retirement, wage indices are similar across cohorts.[6] In order to avoid difficulties associated with inflation, we use wage indices instead of nominal wages. For instance, for a teacher born in 1947, a wage index equal to 783 in 2007 corresponds to a monthly gross nominal wage of around 3,550 Euros. In our sample, the median number of years of contribution is approximately equal to 37 years (see Table 2).

The proportion of part-time teachers at the date of retirement is very low (see Table 2). Because

[5] *Certificat d'Aptitude au Professorat de l'Enseignement du Second degré*

[6] Remember that the index is the basis on which the wage is calculated

the decision to be part-time employed is difficult to formalize, we have excluded part-time teachers from our study sample. There is a specific rule for female teachers with three children or more. After the 2003 reform, they were still eligible to the pre-reform pension scheme (which was more generous) as long as they were employed for fifteen years as a civil servant and in addition their third child was born before 2003. Their pension rate was then equal to the one in effect the year their third child was born. Due to the absence of information on that year, we were unable to calculate pension rates of these women. For this reason, they are excluded from our sample, and the maximum number of children is two.

Table 2: Descriptive statistics

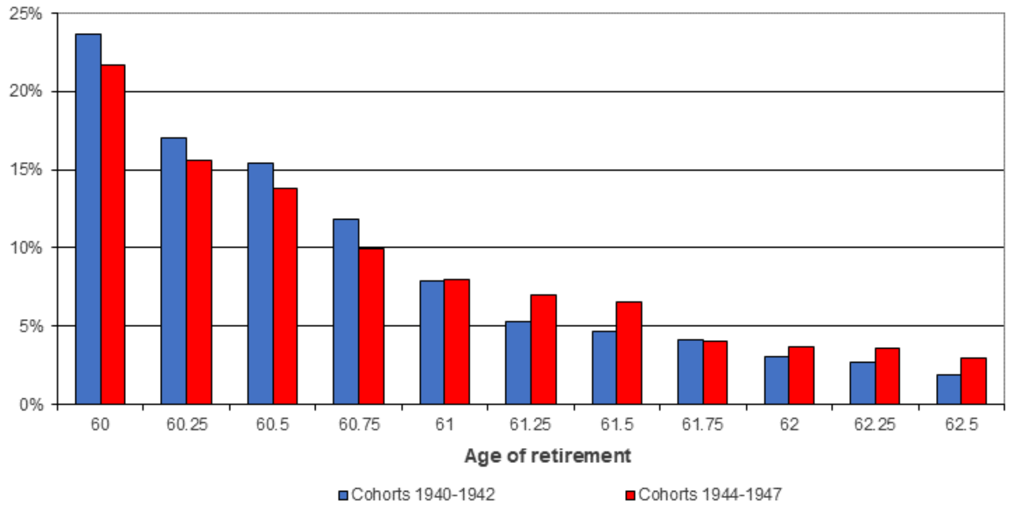
birth cohorts	1940	1941	1942	1944	1945	1946	1947	Total
number of observations	629	773	1,055	1,495	1,817	2,987	3,707	12,463
proportion of women	57.39	58.73	57.54	56.12	58.28	57.21	60.16	58.24
proportion of men	42.61	41.27	42.46	43.88	41.72	42.79	39.84	41.76
proportion of certified teachers	95.07	92.5	92.23	91.17	90.59	91.43	91.42	91.59
proportion of P.E. teachers	4.93	7.5	7.77	8.83	9.41	8.57	8.58	8.41
median wage index	783	741	741	741	783	783	783	772
average wage index	737	734	732	729	730	734	737	734
median number of years of contribution	36.5	36.5	36	36	37	37	37	36.5
median age of entry in the P.S.	27.9	27.3	27.3	27.3	27.6	27.1	26.5	27.1
proportion of part-time teachers at the retirement date	3.5	3.6	2.9	2.9	3.9	4.9	5.7	4.2
average number of children	1.31	1.31	1.35	1.33	1.36	1.35	1.39	1.36

Abbreviations: P.E. for Physical Education, P.S. for Public Sector

3.2 Identification strategy

The 2003 reform introduced an exogenous variation in the way pensions are calculated. Since the reform, entitlement to the full pension rate depends on the number of contributed quarters and on the teacher's birth year. Figure 2 illustrates the fact that younger cohorts, which are directly affected by the reform, are less likely to retire at age 60.[7] By contrast, we observe a shift towards age 61, as shown by Baraton *et al.* (2021).

Figure 2: Distribution of the retirement age according to the birth cohort



We consider that 1940-1942 cohorts constitute the control group whereas the 1944-1947 ones are the treated group. Table 3 shows that the probability to retire between 60 and 61 is reduced by 6.6 percentage points if the teacher belongs to a cohort affected by the reform.

Table 3: Distribution of retirement ages for treated and control groups (in percentage)

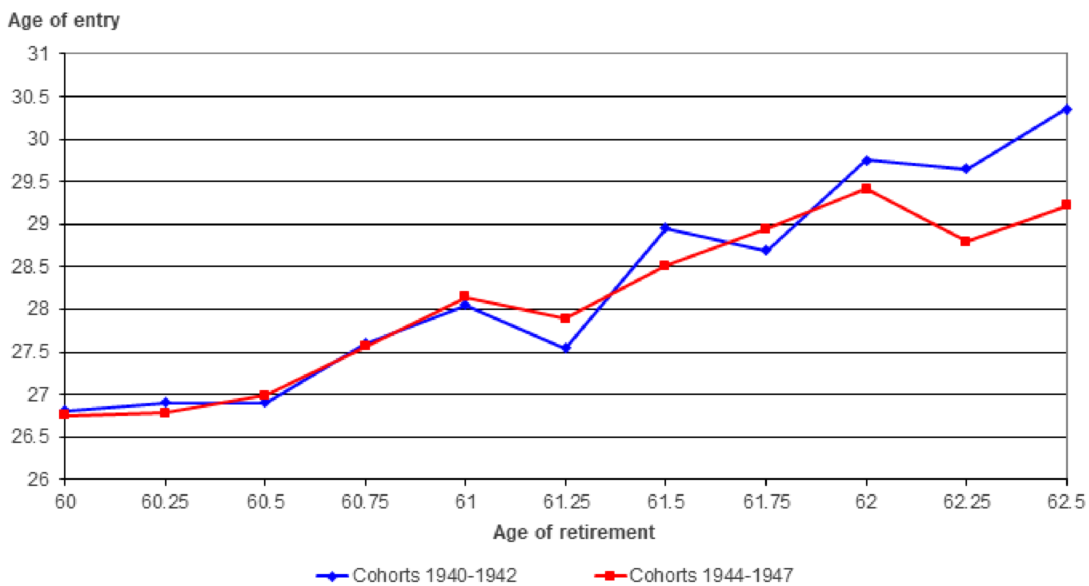
retirement age	cohorts 1940-1942	cohorts 1944-1947
[60; 61[	69.6	63.0
[61; 62[	22.4	26.4
[62; 62.5]	8.0	10.6
Total	100	100

Could this increase in the age of retirement be a consequence of an increase in the age of entry into the public sector? Figure 3 shows that the retirement age increases with the age of entry into the public sector, but this relationship is the same for pre- and post-reform cohorts. Figure 4 shows that the 2003 reform, which increased the number of quarters required to get the full pension rate, induced teachers to declare more frequently their past employment spells in the private sector in

[7] The proportion of teachers who retire beyond 62 years and 6 months is negligible. For this reason, these teachers are not taken into account in simulations carried out in section 5: estimates obtained for them are too imprecise.

order to increase the length of their contribution period.[8]. The upward shift of the baseline curve shows the average difference between our two groups. For instance, for teachers retiring at age 60, there is a non negligible difference of 5 quarters of contribution.

Figure 3: Mean age of entry into the public service, according to the retirement age



After the 2003 reform, declaring these quarters of employment in the private sector has two types of advantages. First, for those teachers who have the required number of quarters of employment in the public sector and who can thus get the full pension rate, these supplementary quarters give entitlement to the premium rate (see Table 1). Second, those for whom the number of employment quarters in the public sector is insufficient, may benefit from such supplementary quarters, which help them to get the full pension rate more rapidly.

Finally, one could argue that the time span between the first and the last cohorts is too large, and that the decision to retire is not only affected by the reform, but also by macroeconomic conditions. For instance, the extreme cohorts (1940 and 1947) can retire from 2000 and from 2007, respectively. Meanwhile, the situation has dramatically evolved, especially because of the 2002 recession. How to be sure that changes in behavior are not linked to changes in the macroeconomic environment, in which case our identification strategy could be blurred? A solution would be to limit the sample to only two close cohorts, say 1942 and 1944. Nevertheless it is not necessary, since we assume that

[8]This fact was already noticed by Bozio (2008)

teachers are facing a certain and secure environment (i.e., a deterministic wage progression, the quasi-impossibility to be dismissed) that makes them less likely to be affected by macroeconomic shocks.

Figure 4: Quarters contributed outside the public service, depending on the age of leave

To sum things up, the 2003 reform (implemented from 2004) modified the calculation of pensions. Cohorts born between 1940 and 1942 were only affected by one feature of the reform, and in a negligible way. Namely, teachers of these cohorts who have not retired in 2004 could claim the premium for their extra quarters of contribution. In our sample, since nobody retires after age 63, this implies that the maximum number of quarters teachers born between 1940 and 1942 could claim is 4 (i.e., a teacher born in 1942 is 62 in 2004 and can get a maximum of 4 quarters when she retires at 63 years old). This deadweight effect is therefore minor, because it only concerns the teachers born in 1942 who are still working at age 62. By contrast, cohorts born between 1944 and 1947 faced at an earlier age the financial incentives introduced by the reform; in addition, they were potentially affected by a penalty per missing quarter and by an increase in the number of quarters required to get the full pension rate.

We think that the exogenous modification of pension calculation is the cause behind the distortion of the retirement age as depicted by Figure 2. Since we reject the hypothesis stating that younger cohorts entered the public sector later (Figure 3), we interpret their delayed retirement as a distortion of their option value profiles.

4 The model

The option value model, which has been introduced by Stock and Wise (1990), has been used several times in the empirical literature. In the so-called reduced-form option value model, economists use the option value as a crucial regressor in the retirement decision equation. For instance, Samwick (1998), Coile and Gruber (2000), and Baker *et al.* (2003) calculate the option value from parameters estimated by Stock and Wise (1990) and then plug it into their regressions. Such a method substantially reduces the computational burden since it avoids estimating directly parameters of the individual utility function.

Sometimes, all parameters of the utility function are set equal to 1, transforming the utility function into a simple function depending linearly on financial components. This is the solution proposed, e.g., by Asch and Warner (1999) who try to identify determinants of the retirement decision. Coile and Gruber (2001) use the same simplifying assumption to estimate the peak value variable.

Here we intend to estimate the full structural option value model developed by Stock and Wise (1990). Such an estimation is not an easy task, as mentioned for instance by Samwick (1998).[9] At our best knowledge, our study is the second paper providing a full estimation of the conditional multiple-years model introduced by Stock and Wise (1990), the first being the article published by Belloni and Alessie (2013). In the next subsections, we summarize the Stock and Wise model and we explain how we adapt it to our framework.

4.1 The utility function and the decision rule

Let us consider a teacher working at age t:[10] her future wage at age s, which is assumed to be perfectly known,[11] is denoted Y_s. If she retires at age r, her pension is equal to $B_s(r)$. Her indirect utility stems either from her wage, or from her pension.[12] In the first case, her utility is denoted

[9]Samwick (1998) did not succeed in estimating simultaneously the parameter representing the preference for leisure and the discount rate.

[10]In our application, age is measured in quarters.

[11]This assumption is not restrictive for teachers. As civil servants, they cannot be laid off, except in the case of a serious professional misconduct, and they can forecast with certainty their future wages which essentially depend on their seniority.

[12]The model does not take into account individual savings and financial wealth, see subsection 4.5.

U_w, in the latter it is denoted U_r. The teacher chooses her retirement age by maximizing the mathematical expectation of the following value function with respect to r:

$$V_t(r) = \sum_{s=t}^{r-1} \beta^{s-t} U_w(Y_s) + \sum_{s=r}^{S} \beta^{s-t} U_r(B_s(r)) \tag{1}$$

This value function depends on future earnings and retirement pensions, and, above all, on the retirement age r. In this expression, β denotes the discount factor and S is the maximum age of death, which is assumed to be the same for all individuals (in our application, $S = 100$). The earlier the retirement age, the higher the weight assigned to U_r.

At each age t, the teacher is supposed to solve this maximization problem by comparing the expected value of retiring at age t with the greatest among expected values of retiring at any future age r. Then the optimal retirement age r^* is the one which maximizes the function:

$$G_t(r) = E_t V_t(r) - E_t V_t(t)$$

namely,

$$r^* = \underset{\{t+1,t+2,\ldots,S\}}{\text{argmax}} \; E_t V_t(r)$$

Consequently a teacher stops working when there is no older age r such as $G_t(r)$ is positive. In other words, the decision to retire is taken as soon as $G_t(r^*) \leq 0$. This means that the teacher continues working as long as:

$$G_t(r^*) = E_t V_t(r^*) - E_t V_t(t) > 0 \tag{2}$$

It is worth noting that this process is incremental: at each age r, a new r^* is calculated, which can differ from the one computed in the previous period $r - 1$. Unlike a more "sophisticated" rule that requires a one-shot resolution by backward induction, based on a Bellman equation (see Rust, 1987, for instance), the Stock and Wise model is solved one step at a time. However, as claimed by Stock and Wise (1990, p. 1166), if all future revenues are known with certainty, which is the case for secondary school teachers at the end of their career in the public sector, the two rules are equivalent (see Appendix A for a formal proof).

4.2 Functional forms

Under the assumption of a constant relative risk aversion (CRRA), the above utility functions become:

$$U_w(Y_s) = \frac{Y_s^{1-\gamma}}{1-\gamma} + \omega_s \qquad\qquad U_r(B_s) = \frac{(kB_s(r))^{1-\gamma}}{1-\gamma} + \xi_s$$

where γ measures the degree of relative risk aversion ($1/\gamma$ is the elasticity of inter-temporal substitution) and k is a coefficient representing the preference for leisure.

Errors are assumed to follow a first-order autoregressive process, with an individual random effect whose persistence is represented by the parameter ρ ($0 < \rho < 1$):

$$\omega_s = \rho\omega_{s-1} + \varepsilon_{\omega s} \qquad\qquad \xi_s = \rho\xi_{s-1} + \varepsilon_{\xi s}$$

where $E_{s-1}(\varepsilon_{\omega s}) = E_{s-1}(\varepsilon_{\xi s}) = 0 \quad \forall s = t+1, \cdots, S$, and $\varepsilon_{\omega s}$ and $\xi_{\omega s}$ are i.i.d. $\forall s = t, \cdots, S$.

These specifications of utility functions and of errors are then plugged into equation (1). Thus $G_t(r)$ becomes:

$$\begin{aligned} G_t(r) = {} & E_t \sum_{s=t}^{r-1} \beta^{s-t} \frac{Y_s^{1-\gamma}}{1-\gamma} + E_t \sum_{s=r}^{S} \beta^{s-t} \frac{(kB_s(r))^{1-\gamma}}{1-\gamma} - E_t \sum_{s=t}^{S} \beta^{s-t} \frac{(kB_s(t))^{1-\gamma}}{1-\gamma} \\ & + E_t \sum_{s=t}^{r-1} \beta^{s-t} (\omega_s - \xi_s) \end{aligned}$$

which can be written under a more compact form as:

$$G_t(r) = g_t(r) + \phi_t(r)$$

where

$$g_t(r) = E_t \sum_{s=t}^{r-1} \beta^{s-t} \frac{Y_s^{1-\gamma}}{1-\gamma} + E_t \sum_{s=r}^{S} \beta^{s-t} \frac{(kB_s(r))^{1-\gamma}}{1-\gamma} - E_t \sum_{s=t}^{S} \beta^{s-t} \frac{(kB_s(t))^{1-\gamma}}{1-\gamma}$$

and

$$\phi_t(r) = E_t \sum_{s=t}^{r-1} \beta^{s-t} (\omega_s - \xi_s)$$

If we assume that the conditional survival probability at age s given that the teacher is alive at age t ($s > t$), which is denoted $\pi(s|t)$, depends neither on the stream of revenues, nor on the individual disturbance, we can rewrite these two last functions as:

$$\begin{aligned} g_t(r) = & \sum_{s=t}^{r-1} \beta^{s-t}\pi(s|t)E_t\left(\frac{Y_s^{1-\gamma}}{1-\gamma}\right) + \sum_{s=r}^{S} \beta^{s-t}\pi(s|t)E_t\left(\frac{(kB_s(r))^{1-\gamma}}{1-\gamma}\right) \\ & - \sum_{s=t}^{S} \beta^{s-t}\pi(s|t)E_t\left(\frac{(kB_s(t))^{1-\gamma}}{1-\gamma}\right) \end{aligned}$$

and

$$\phi_t(r) = \sum_{s=t}^{r-1} \beta^{s-t}\pi(s|t)E_t(\omega_s - \xi_s)$$

Assuming that errors are AR(1) results in:

$$\phi_t(r) = \sum_{s=t}^{r-1} \beta^{s-t}\pi(s|t)E_t(\omega_s - \xi_s) = \sum_{s=t}^{r-1} (\beta\rho)^{s-t}\pi(s|t)(\omega_t - \xi_t) = K_t(r)\nu_t$$

where

$$K_t(r) = \sum_{s=t}^{r-1} (\beta\rho)^{s-t}\pi(s|t) \qquad \text{and} \qquad \nu_t = \omega_t - \xi_t$$

Finally, we get:

$$G_t(r) = g_t(r) + K_t(r)\nu_t \tag{3}$$

The gain function breaks down into two parts, namely a deterministic component $g_t(r)$ which only depends on future revenues, and a stochastic term ν_t. In equation 3, $K_t(r)$ is a global deflator which evaluates at time t the future value of the random components. When the retirement age r increases, $K_t(r)$ increases too, and a greater weight is put on the random part of the gain function.

4.3 The likelihood function

Let us first consider a single period (quarter). Recall from equation 2 that a teacher continues to work if $G_t(r^*) = g_t(r^*) + K_t(r^*)\nu_t > 0$.

Assuming that $\nu_t \sim \mathcal{N}(0; \sigma_\nu^2)$, the probability to retire at time t is given by:

$$Pr(R = t) = Pr\left(\nu_t \leq -\frac{g_t(r^*)}{K_t(r^*)}\right) = 1 - \Phi\left(\frac{1}{\sigma_\nu}\frac{g_t(r^*)}{K_t(r^*)}\right)$$

where Φ is the c.d.f. of the standard normal distribution $\mathcal{N}(0; 1)$. The individual contribution to the likelihood function is therefore:

$$\mathcal{L}_i = Pr[R = t]^{\mathbb{1}(R=t)} \times [1 - Pr[R = t]]^{\mathbb{1}(R>t)}$$

$$= \left[1 - \Phi\left(\frac{1}{\sigma_\nu}\frac{g_t(r^*)}{K_t(r^*)}\right)\right]^{\mathbb{1}(R=t)} \times \Phi\left(\frac{1}{\sigma_\nu}\frac{g_t(r^*)}{K_t(r^*)}\right)^{\mathbb{1}(R>t)} \quad (4)$$

Now consider multiple periods (quarters). The probability of retirement at age τ is now:

$$Pr[R = \tau] = Pr\left[\nu_t > -\frac{g_t(r_t^*)}{K_t(r_t^*)}\ ;\ ...\ ;\nu_{\tau-1} > -\frac{g_{\tau-1}(r_{\tau-1}^*)}{K_{\tau-1}(r_{\tau-1}^*)}\ ;\ \nu_\tau \leq -\frac{g_\tau(r_\tau^*)}{K_\tau(r_\tau^*)}\right]$$

Considering in addition that random terms ν_s are not independent since $\nu_s = \rho\nu_{s-1} + \varepsilon_s$ (see Appendix B for further details), this probability does not break down into a product of simple components. The individual contribution to the likelihood function is therefore given by:

$$\mathcal{L}_i = \int_{-\frac{g_t(r_t^*)}{K_t(r_t^*)}}^{+\infty} \cdots \int_{-\frac{g_{\tau-1}(r_{\tau-1}^*)}{K_{\tau-1}(r_{\tau-1}^*)}}^{+\infty} \int_{-\infty}^{-\frac{g_\tau(r_\tau^*)}{K_\tau(r_\tau^*)}} f(\nu_t, \cdots, \nu_{\tau-1}, \nu_\tau)\ \mathrm{d}_t \cdots \mathrm{d}_{\tau-1}\mathrm{d}_\tau \quad (5)$$

where f is the p.d.f. of a normal multivariate distribution with a mean of 0 and covariance matrix Σ of dimension τ. At this point, we depart a little bit from Stock and Wise model, because we add an assumption of second-order stationarity, so that the variance does not tend to infinity when t

increases. Thus the covariance matrix has the following convenient form:

$$\Sigma = \frac{\sigma_\varepsilon^2}{1-\rho^2}\begin{bmatrix} 1 & \rho & \rho^2 & \cdots & \rho^{\tau-1} \\ \rho & 1 & \rho & \cdots & \rho^{\tau-2} \\ \vdots & \vdots & & & \vdots \\ \rho^{\tau-2} & & \cdots & 1 & \rho \\ \rho^{\tau-1} & \rho^{\tau-2} & \cdots & \rho & 1 \end{bmatrix}$$

The second-order stationarity assumption reduces the number of parameters to be estimated. Since $Var(\nu_s) = \sigma^2_{\nu s} = \sigma^2_\nu \; \forall s = t+1, \cdots, S$ and because this variance depends only on ρ and σ_ε, we do not estimate directly σ^2_ν. Instead we estimate separately ρ and σ_ε.

4.4 Seasonality of retirement

French teachers are more likely to retire in early September, first because the school year generally ends after summer vacation, second because teachers are paid during summer months, even if they are not present at school. In order to take into account this phenomenon, we add to the threshold of the decision rule (given by equation 2) a shift function indicating that the month of September belongs to the quarter in which the teacher retires.[13] Then a teacher decides to retire at age r^* if:

$$G_t(r^*) \leq 0 + \psi \times \underbrace{\mathbb{1}(sept \in quarter)}_{=z}$$

which implies that:

$$g_t(r^*) + K_t(r^*)\nu_t \leq \psi z$$

or equivalently:

$$\frac{g_t(r^*)}{K_t(r^*)} - \frac{\psi z}{K_t(r^*)} \leq \nu_t \tag{6}$$

[13]Let us recall that in our application, the time unit is the quarter.

Consequently, in the single-period case, the individual contribution to the likelihood function is:

$$\mathcal{L}_i = \left[1 - \Phi\left(\frac{1}{\sigma_\nu\ K_t(r^*)}(g_t(r^*) - \psi z)\right)\right]^{\mathbb{1}(R=t)} \times \Phi\left(\frac{1}{\sigma_\nu\ K_t(r^*)}(g_t(r^*) - \psi z)\right)^{\mathbb{1}(R>t)} \tag{7}$$

while, in the multiple-periods case this contribution becomes:

$$L_i = \int_{\frac{-g_t(r_t^*)+\psi z_t}{K_t(r_t^*)}}^{+\infty} \cdots \int_{\frac{-g_{\tau-1}(r_{\tau-1}^*)+\psi z_{\tau-1}}{K_{\tau-1}(r_{\tau-1}^*)}}^{+\infty} \int_{-\infty}^{\frac{-g_\tau(r_\tau^*)+\psi z_\tau}{K_\tau(r_\tau^*)}} f(\nu_t, \cdots, \nu_{\tau-1}, \nu_\tau)\ \mathrm{d}_t \cdots \mathrm{d}_{\tau-1}\mathrm{d}_\tau \tag{8}$$

4.5 A model without savings

Because of the lack of relevant information in our dataset, our empirical model abstracts from personal savings. Indeed it is likely that individual assets and wealth could affect the retirement decision. One could yet argue that teachers who have almost the same length of service within the public sector and have benefited from a similar wage progression during their career should roughly own the same amount of savings. However this argument is somewhat weak, and several factors, like the spouse's earnings, the household composition, home ownership and family bequests, may invalidate it.

Nonetheless, as noticed by Belloni and Alessie (2013), "earlier reduced-form studies based on UK and USA data (see, e.g., Blundell *et al.*, 2004; Hausman and Wise, 1985) show that financial wealth has at most a small impact on retirement choices once one controls for pension wealth. [...] In particular Blundell *et al.* (2004) report that retirement choices in the UK are not significantly partially correlated with financial wealth."

Moreover, in order to show that omitting savings in the Stock and Wise model has not serious consequences, Belloni and Alessie (2013) re-estimate this model on a subsample of Italian blue-collar workers who hold much less wealth than other workers. In fact they find that "the model for blue-collar workers generates very similar actual and predicted retirement rates to those of the overall model" (Belloni and Alessie, 2013, p. 524). Thus, they conclude that the effects of omitting savings are not so dramatic.

In the light of these previous studies, we can reasonably think that abstracting from personal savings should not significantly modify the accuracy of the model predictions for a relatively homogenous group of workers like the French secondary school teachers. The good fit of our estimates (see next section) strengthens this argument.

5 Results

5.1 Parameter estimates

The adapted Stock and Wise model is estimated using our sample of 12,463 secondary school teachers from cohorts 1940-1942 and 1944-1947. Likelihood functions represented by equations (7) and (8) are maximized in order to get parameter estimates of the static and dynamic models. Our estimation of the static model is based only on the quarter in which a teacher reaches the age of 60. In theory, the dynamic model should take into account all the quarters between 60 and the age of retirement. Considering that all observed retirement ages are mostly comprised between 60 and 62.5, this implies that the full dynamic model would include individual likelihood contributions involving 11-dimensional integrals. Consequently, this model should be estimated by simulated maximum likelihood or other simulation techniques (like simulated moments or MCMC procedures). In order to circumvent the computational burden associated with the estimation of the full dynamic model, we set at 3 the maximum number of successive periods, which was the set-up adopted by Stock and Wise (1990). For those teachers retiring either at 60 or 60.25 (0.25 representing one quarter after the 60th birthday), the individual likelihood contribution still consists in one or two components. Parameters estimates of the two models are reported in Table 4. In each model the preference for leisure is assumed to be a linear function of the spouse's age and of the presence of at least one child less than 20 in the teacher's household. This implies that coefficient k has the following functioning form:

$$k = k_0 + k_1 \mathbb{1}(\text{the teacher's spouse is over 60}) + k_2 \mathbb{1}(\text{the teacher has a child under 20})$$

By doing so, we aim at introducing some observed heterogeneity into the models. Parameter k_1 is expected to be positive: in fact, it is intended to capture the higher utility resulting from the fact that a spouse who has more than 60 years old could also be retired, which should increase the teacher's preference for leisure. When a teacher has at least one child less than 20, she has to cover expenditures associated with this child's care. She has consequently higher incentives to continue working in order to increase the household's income. Thus, parameter k_2 is expected to be negative. Table 4 shows that parameter estimates are all highly significant and have a theoretically credible order of magnitude. We nonetheless focus on estimates of the dynamic model for several reasons. First, the quarterly discount factor is estimated to be equal to 0.92 in the static model, which implies an annual discount factor of 0.78. Although this value is close to the one found by Stock and Wise (1990), it seems somewhat low. We prefer the one obtained with the dynamic model specification which gives a quarterly discount factor equals to 0.97, corresponding to an annual discount rate of 0.91. This latter value is, for instance, close to those found or calibrated by Eckstein and Wolpin (1999) and Keane and Wolpin (2001) with discrete choice dynamic models.

According to the risk-aversion classification proposed by Holt and Laury (2002), teachers of our sample are estimated to be risk averse within the multiple-period model,[14] while they are found to be risk-neutral within the one-period model. From this point of view, the dynamic model is clearly more credible than the static one. The difference between the estimates of relative risk-aversion coefficients may be explained by the fact that, in the one-period model, the teacher chooses only once the optimal date of retirement, which leaves no room for potential substitution across periods, and then could result in a lower β. On the contrary, in the multiple-period model, the decision rule is repeated up to three times: each time a teacher delays the retirement decision, this choice increases her expected stream of revenues (if not, she would have retired).

Apart from this, the AR(1) coefficient ρ, which is estimated to be 0.60 within the multiple-period model, ensures that exogenous shocks may have long lasting effects. We do believe that this error structure is more adapted to our data because it captures the effect of some unobserved variables, such as health for instance.[15] Of course, further research using individual health status (which is missing in our dataset) would be worthwhile.

The coefficient associated with the dummy variable indicating that the month of September belongs to the current quarter is highly significant, which means that seasonality plays a crucial role in the decision to retire. Including this variable improves substantially the fit of the model: omitting it would have biased upwards other parameter estimates.

[14]The estimated relative risk-aversion coefficient, equal to 0.49, implies that in a lottery where a teacher either wins 10,000 euros with probability 0.5 or wins 20,000 euros with the same probability, her certainty equivalent is 14,562 euros, which is lower than her expected gain of 15,000 euros.

[15]Let us remark that in the multiple period model, the standard error of the random term ν is significantly reduced from 32.8 to 21.8. In other terms, this model restricts the relative magnitude of the random component.

The coefficient associated with the preference for leisure, denoted k_0, is estimated to be 1.23 for single teachers or teachers whose spouse is less than 60 years old and who have no child under 20. This value implies that one euro of pension benefit has a higher weight than one euro of wage. This parameter might as well be considered as a measure of work disutility, since its value implies that 0.82 euros of pension provides the same utility level than one euro of wage. Parameters k_1 and k_2 modify this value. A teacher whose spouse is over 60 but who has no child under 20 only needs 0.75 euros of pension benefit to compensate for one euro in wages. This value increases to 0.87 euros if she has a child under 20 and a spouse below 60.

Table 4: Parameter estimates of the Stock and Wise model

Parameters	Definition	One-period model	Multiple-period model
σ_ν	standard error of ν	32.8 (0.47)	21.8 (calculated)
σ_ε	standard error of ε		17.4 (6.8e-5)
ρ	AR(1) coefficient		0.60 (6.8e-7)
γ	relative risk aversion	0.09 (4.2e-9)	0.49 (7.0e-7)
β	discount factor	0.92 (2.8e-9)	0.97 (7.0e-7)
ψ	September	38.6 (0.82)	55.8 (6.9e-5)
k_0	preference for leisure	1.10 (5.9e-9)	1.23 (7.0e-7)
k_1	spouse over 60	0.08 (1.7e-8)	0.11 (6.9e-7)
k_2	child under 20	-0.04 (1.5e-8)	-0.08 (6.8e-7)
Number of observations		12,463	12,463
Log-likelihood value		-5,434	-12,601

Remark: standard errors are presented between brackets

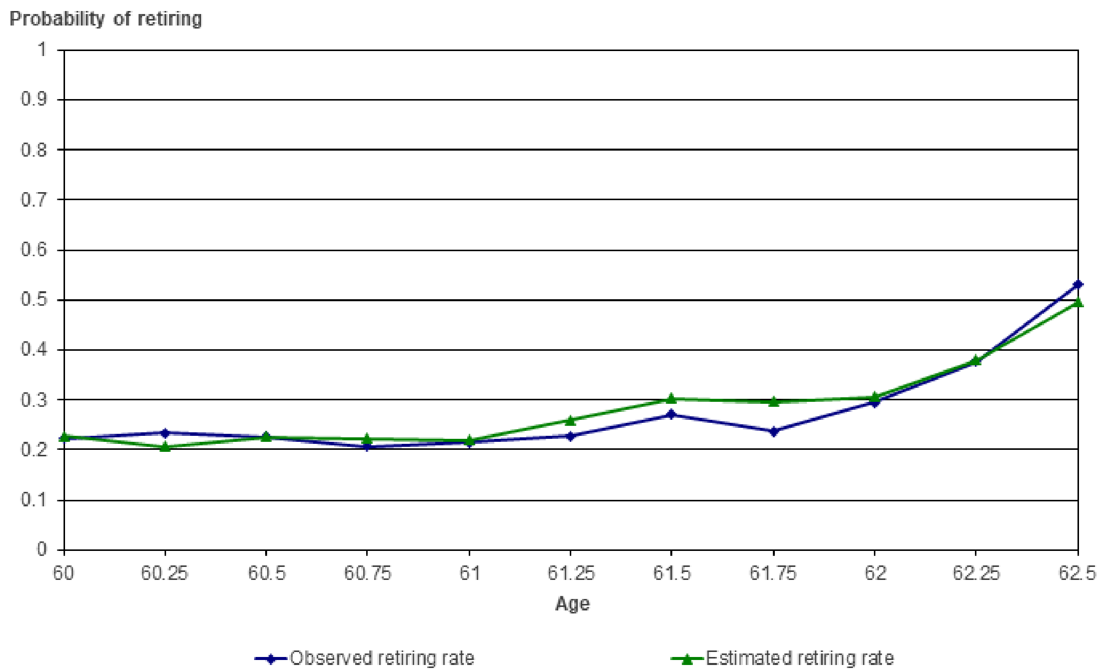
5.2 Simulations

The model fit can be illustrated by plotting simulated retirement rates, compared to the observed ones. This exercise is presented on Figure 5. Using estimates from the multiple-periods model, we simulate retirement rates at each age by using those teachers who are still working at this age.

The most important discrepancy between observed and estimated retirement ages is at age 61.75

(i.e., 61 years and 3 quarters). At all other ages, especially between 60 and 61 years old, estimates are quite accurate. The peak at age 61.5 is well accounted for, but the model overpredicts the retirement rate at age 61.75. Let us remark that at this exact age, there is no difference in the proportions of retirements in the two groups of cohorts, namely the ones having 60 before the reform (1940-1942) and those having 60 after the reform (1944-1947).[16]

Figure 5: Conditional probability to retire depending on the teacher's age



Since the model fits the data conveniently, we can now develop counterfactual scenarios, which may be simulated by using the estimated structural parameters. The 2003 French reform was aimed at delaying retirement. This goal was expected to be reached through three mechanisms, namely the increase in the number of quarters required to get the full pension rate, a financial incentive (the "premium") and a financial disincentive (the "penalty").[17]

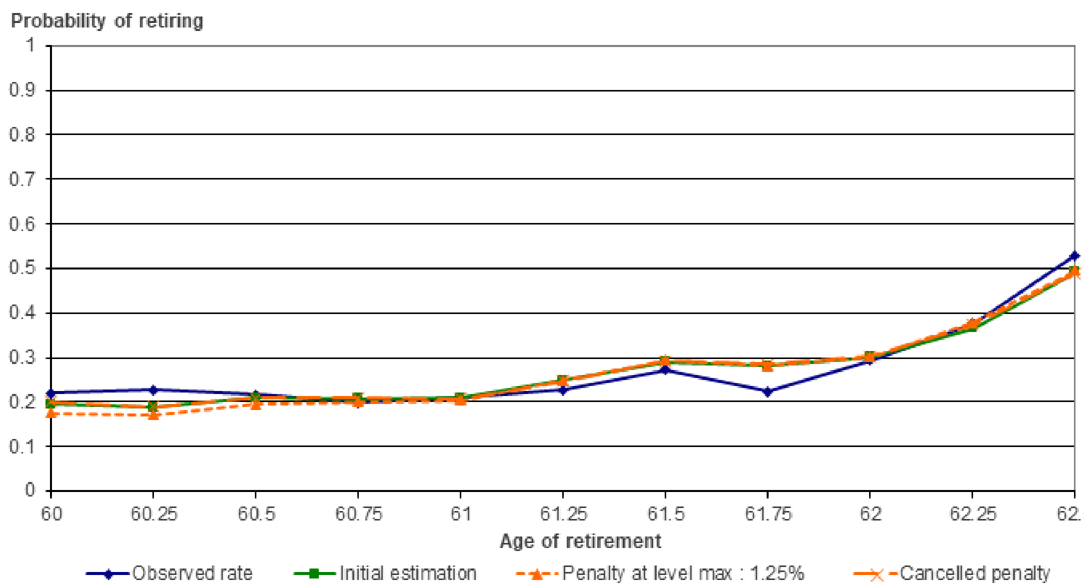
The following simulations focus on these two last mechanisms. In particular, we analyze the sensitivity of retirement rates to changes in coefficients determining the percentage that is applied to any missing or extra quarter of contribution. These simulations are run on the cohorts 1944-1947, since they are those directly impacted by the reform.

[16]See Figure 2.

[17]See section 2.3.

Figure 6 reports what would be (according to the parameter estimates of the structural model) the effects of cancelling the penalty and those associated with a penalty sets at its final (maximal) level, i.e., 1.25% for each missing quarter. These changes in the penalty rate do not affect retirement rates. At first glance, this could be interpreted as an efficient measure allowing to reduce pensions, without having any impact on teachers' retirement behavior. This statement has to be qualified though. In fact, in our sample, only 14.5% of teachers are subject to a small penalty when they effectively retire.[18] Given that they can only retire at 60 and above and that the age at which the penalty is cancelled (the so-called "limit age") is 61, the maximum number of quarters subject to the penalty is 4. In other words, the maximum penalty rate that can be applied to one's pension rate is equal to 1%, which is very small.

Figure 6: Conditional probability to retire depending on the teacher's age and on the level of the penalty rate (cohorts 1944-1947)



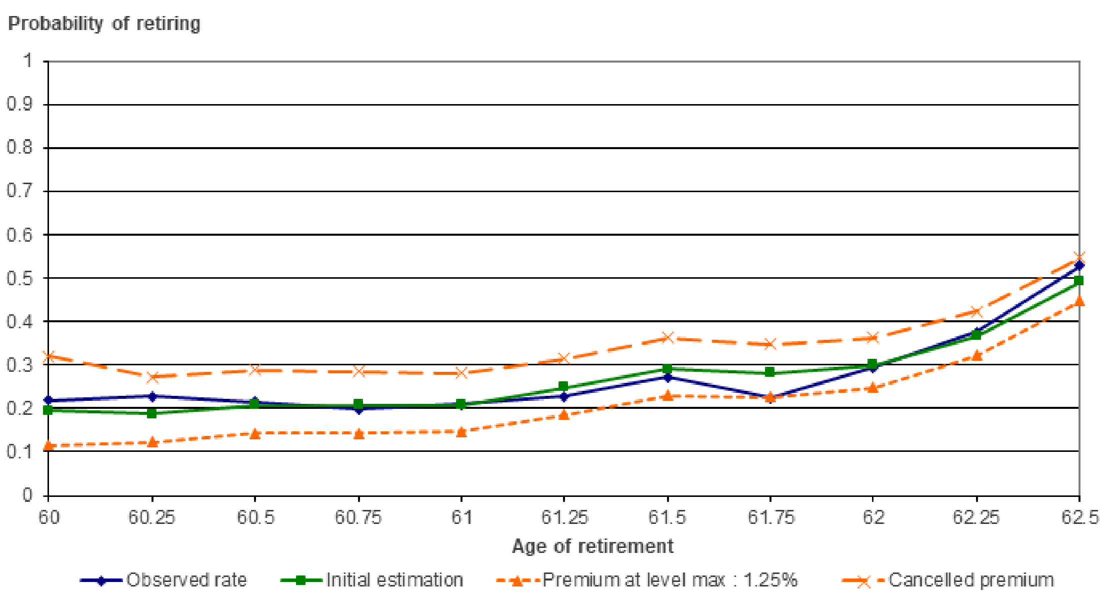
Therefore it is not surprising that this mechanism has not decreased retirement rates. On the one hand, our sample is constituted by teachers that are barely concerned by the penalty. On the other hand, the magnitude of the penalty coefficient is too small to affect teachers' decision. Starting from 0.125% per missing quarter in 2006, the penalty rate reaches the initial level of the premium rate (0.75%) five years later, and its maximum level (1.25%) in 2015 when all the teachers of our

[18]For instance, the penalty rate was equal to 0.125% for a teacher born in 1946 and to 0.25% for one born in 1947 (see Table 1).

sample will be retired. Finally, the existence of a limit age, which is really close to the age at which teachers can legally retire in the very first cohorts affected by the reform, worsens even more the ineffectiveness of the financial disincentive (i.e., the penalty rate) in our sample.

More interesting is the simulation concerning the premium mechanism. Figure 7 shows that doubling the legal premium rate divides approximately by two retirement rates between 60 and 61 years old. Above 61, and especially after 62 years old, the decrease in the retirement rate is much lower. On the opposite, cancelling the premium rate would have increased by more than 50% the retirement rate between 60 and 61 years old. After 61, the increase in the retirement rate would have been less important. Another interesting feature lies in the fact that dashed lines converge towards the initial estimation line, showing that as times goes by, teachers are less and less sensitive to changes in the level of the premium rate. This argument is consistent with a labor disutility (or a preference for leisure) that increases with age, meaning that teachers are less sensitive to financial incentives as they get older.

Figure 7: Conditional probability to retire depending on the teacher's age and on the level of the premium rate (cohorts 1944-1947)



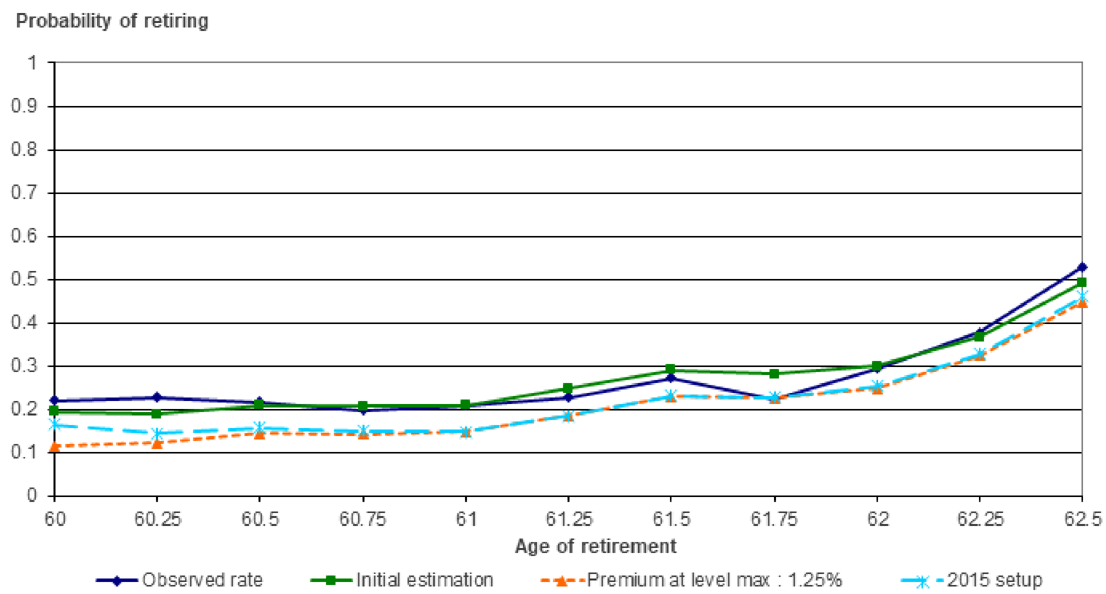
Once again, this result must be qualified. Changing the financial incentive (i.e., the premium rate) affects a wider proportion of the sample, since 57% of teachers were indeed eligible for at least one extra quarter when they retired. In our scenario, changing the premium rate directly affects these teachers, but also those who have contributed for a number of quarters close to the legal requirement and who may decide to continue working to benefit from the premium rate. However, the behavioral responses to the two scenarios, namely cancelling or doubling the premium rate, is asymmetric: cancelling the premium has an higher effect (in absolute value) than doubling it.

Beside these scenarios, one could wonder what would have occurred if the cohorts 1944-1947 would have been immediately subject to the maximum premium and penalty rates implemented from 2015 onwards. In this scenario, the premium and penalty rates are both equal to 1.25%, the number of required quarters is equal to 166 and the limit age is 63.75 years old. Figure 8 plots the result.

This simulation is close to the doubled premium scenario (which is equivalent to set the premium rate at its maximal level), but conditions to get the full pension rate are now hardened. This means that, with the 2015 setup, the budget cost should be lower than in the doubled premium scenario.

This point addresses the question of budgetary effectiveness of the reform: at similar retirement rates, some reform setups may be fiscally preferable, as will be tackled in the next subsection.

Figure 8: Conditional probability to retire depending on the teacher's age with the 2015 setup (cohorts 1944-1947)



Unfortunately these results do not allow to conclude definitively about the relative efficiency of these two incentive mechanisms, namely the premium (*the carrot*) and the penalty (*the stick*). For the first cohorts affected by the reform (cohorts 1944-1947), the premium coefficient was 6 times larger than the penalty one. In addition, the premium was introduced two years before the penalty, which explains that many more teachers were eligible to the premium within these cohorts.

Thus, our simulations may be depending on the composition of our sample. Their replication using more recent cohorts, whose structure in terms of gender, marital status, number of children and number of contributed quarters is likely to differ, could yield alternative results.

However, even if we have to be prudent when generalizing our results, the previous simulations illustrate the fact that the premium rate played a crucial role for the first cohorts affected by the reform. Offering a bonus to teachers that have contributed for the required minimum number of quarters is a strong incentive for postponing retirement. It is also a political strategy in order to increase the acceptability of the reform, at least in the short term.[19]

5.3 A partial effectiveness analysis

It can also be interesting to assess the effectiveness of the 2003 pension reform. Reforming the pension system should indeed increase teachers' mean retirement age. However, the relative effect of the 2003 reform both on wages and on pensions paid to teachers is *a priori* unclear since the schedule of this reform (see Table 1) is quite complex.

In order to evaluate, at least approximately, the effectiveness of this reform, we run the following exercise. We pool the four cohorts (1944-1947) impacted by the reform and we assume that all these teachers reach age 60 in a given year (namely, 2003, 2004, ..., 2010).[20] First we simulate their individual retirement age according to the pension scheme implemented before the reform. We then calculate both the total wage and pension bills, and the average retirement age. These estimations are our baseline (counterfactual) values for the year 2003. We then repeat this simulation for each annual setup of the reform schedule (see Table 1). In this exercise we assume 1) that the survival probabilities remain unchanged over the period 2003-2010, and 2) that the statutory retirement age is always equal to 60 years old. These two assumptions are probably too strong and they should somewhat restrict the validity of this last exercise. We insist on the fact that our effectiveness analysis is partial. It is only based on the four impacted cohorts of teachers included in our sample. A more general (and probably more exact) analysis should also use cohorts born after year 1947,

[19]This last result is somewhat in opposition with those generally obtained in the policy-reform retirement literature, which generally finds a small role for financial incentives (see, e. g., Burtless, 1986, Krueger and Pischke, 1992, Brown, 2013, or Manoli and Weber, 2016), with the exception of Mastrobuoni (2009) who found somewhat larger effects. Our result could be due to the fact that, in the case of the 2003 French reform, the bonus is granted continuously, for each additional year worked beyond the number of years required to obtain the full rate.

[20]We stop the simulation in 2010, since it is the year of implementation of the pension reform which increased the statutory retirement age to 62 years.

say between 1948 and 1955, whose structure could be different from the structure of the impacted sample cohorts (1944-1947). Moreover, this analysis exclude many elements that matter in a general cost-benefit analysis, for instance the effects of longer working times on teachers' health, income taxes, indexation of pensions, frequency and duration of sick leaves, absenteeism, consequences of these events on health insurance expenses, reversion pensions in the event of the death of one of the spouses, the salary cost resulting from the hiring of new teachers who will have to work longer, etc. Evolutions of the mean retirement age and of the total wage bill are reported on Figure 9. From 2003 to 2010, we observe a steady rise in the mean retirement age, which is mechanically mirrored by the increase in the total wage bill.

Figure 9: Evolutions of the mean retirement age and of the total wage bill

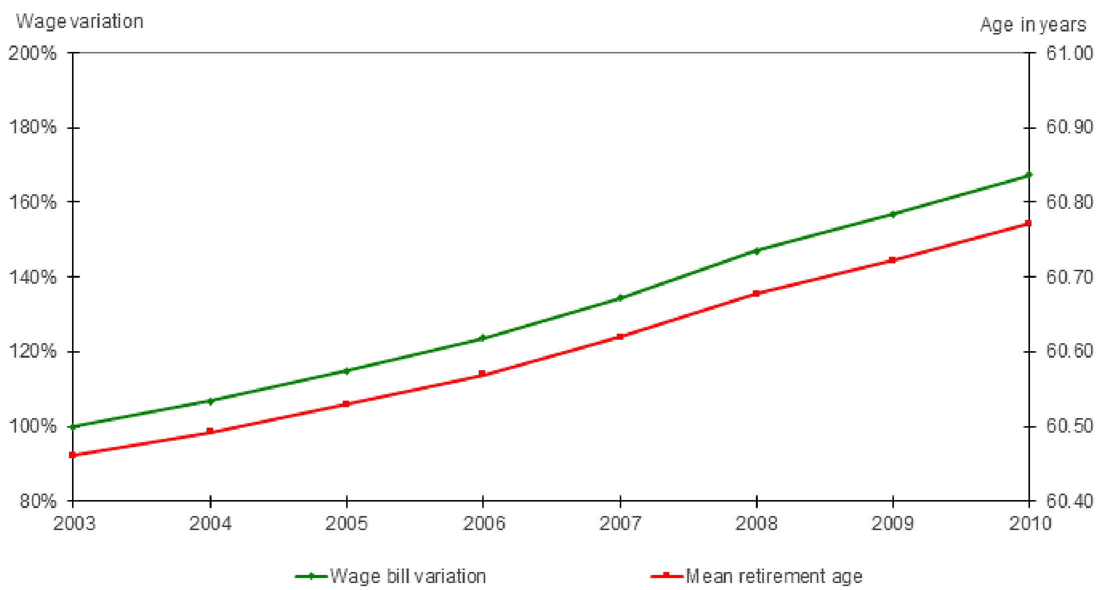
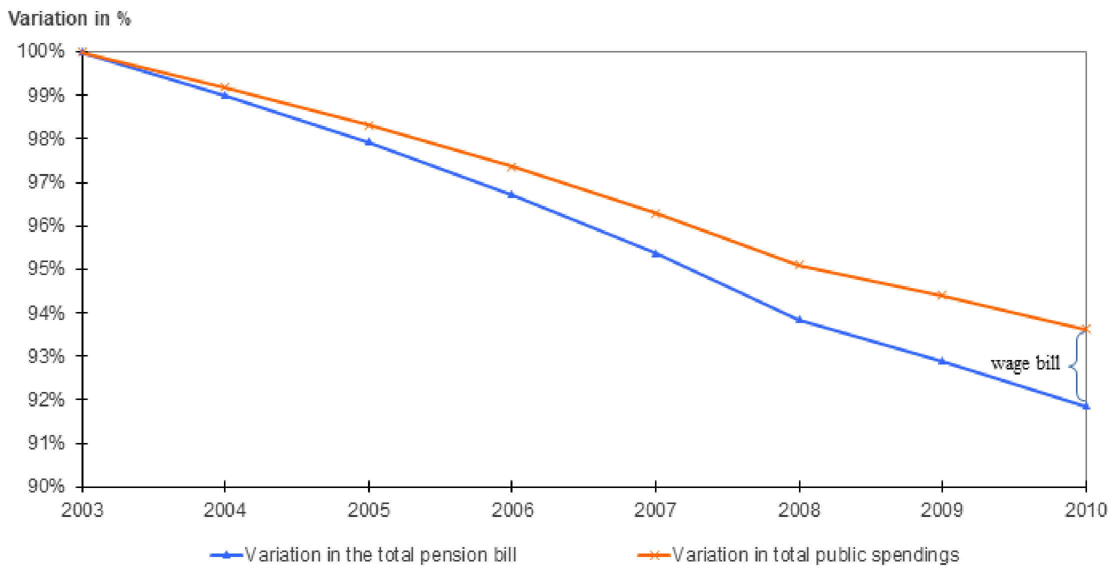


Figure 10 shows how the total pension bill decreases when the reform schedule comes closer to the end of the period of study. Two mechanisms may explain this result. First, the increase in the mean retirement age tends to lower the total pension bill. Second, since conditions to get the full pension rate have been toughened over these years, teachers are more likely to retire before reaching the increasing number of quarters required to get the full pension rate (because of their high disutility of labor; see the estimated value of parameter k_0 in Table 4). Therefore they prefer to incur the penalty, which is relatively small, than continuing to work. This explains why their average retirement age increases by less than one year between 2003 and 2010, while the number of quarters required to get the full pension rate rises by 12 quarters over the same period. Consequently, a continuously decreasing proportion of teachers getting the full pension rate induces a reduction in the total pension bill. Moreover, the premium, which could counterbalance this reduction, is progressively more difficult to obtain.

Figure 10: Evolutions of the total pension bill and of public spendings



However, the improvement of public accounts is not as strong as expected. In fact, under the 2010 setup, the estimated decrease in the total pension bill is around 8% (see Figure 10), but this has to be corrected by the increase in the total wage bill of 67% (see Figure 9). The overall effect is then a 6.5% reduction in public spendings, which means that the reduction in the total pension bill overcomes the increase in the total wage bill.

In the very first years of the reform (2004-2006), the variation in the pension cost corresponds roughly to the variation in public spendings, meaning that the increase in the wage bill is negligible. The positive but small impact on public accounts is mainly driven by the toughening in conditions imposed to get the full pension rate. But as the reform strengthens, the gap between the two curves widens because the mean retirement age does not increase quickly enough. A significant mandatory increase in the statutory retirement age, say up to 63 years old, would have probably reduced the improvement in public spendings because teachers would then have been obliged to work longer without being able to obtain the number of quarters required to get the full pension rate. A way to improve the pension reform effectiveness would be to increase significantly the penalty rate, which should have not much consequences on the retirement rate (see Figure 6), because teachers have a relatively high disutility of labor.

6 Conclusion

Using a unique sample of 12,463 secondary-school teachers, we have evaluated the effect of the 2003 French pension reform on the retirement rate of teachers. To do so, we have used the Stock and Wise option value model (1990).

Estimates show that teachers are risk averse (the relative risk aversion coefficient is approximately 0.5) and have a quarterly discount factor close to unity (equal to 0.97). The coefficient representing their preference for leisure implies that 0.8 Euros of pension generates as much utility as 1 Euro of salary. This preference for leisure increases if the spouse is over 60 years old, and decreases if the household still includes at least one child less than 20 years old.

Simulations based on estimated structural parameters highlight the high sensitivity of teachers' retirement rates to financial incentives (namely, premiums) granted to teachers who continue working beyond the legal duration. This can result from the fact that our simulations are based on the first cohorts affected by the reform. Those cohorts have easily benefited from this premium because the increase in the legal duration of contribution was still limited.

Exploiting successively the annual reform setups, which have continuously changed from 2004 onwards, we have conducted a partial effectiveness analysis with the 1944-1947 cohorts (those cohorts being directly impacted by the reform). That exercise shows how toughening the conditions required for obtaining the full pension rate affects the average retirement age. According to our calculations, in year 2010, this reform should have reduced by 6.5% public spendings directly associated with the payment of wages and pensions of secondary school teachers above 60 years old. This relatively small reduction is essentially driven by three causes, namely *i*) the increase in the number of quarters required to get the full pension rate, *ii*) the teachers' relative high disutility of labor, and *iii*) the penalty imposed to teachers who retire before reaching the legal duration of contribution.

Some important research issues should still be examined. A first interesting extension would be to incorporate the teachers' health status into the analysis. For instance, French (2005) has set forth a model with a utility function that depends on health, which is a more refined framework than ours relying on the AR(1) structure of errors which is supposed to capture the persistence of health shocks. In fact, the individual health status at older ages is an important issue. Inducing teachers who would have otherwise retired, to continue working can generate additional costs for the public accounts: a higher frequency of sick leaves could deteriorate the budget impact of the reform. Last but not least, integrating data on working conditions could be of some interest. For instance, in some disadvantaged areas, financial incentives could be ineffective since teachers would like to leave as early as possible because of a higher disutility of labor.

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Appendix A: Equivalence of decision rules in the dynamic programming and the option value models when future revenues are certain

The decision rule used by Stock and Wise (1990) differs from the dynamic programming decision rule which is deduced from a Bellman equation. Here we show that the two rules are equivalent when future revenues are certain. Let us simplify the problem by setting $\gamma = k = \beta = \pi(s|t) = 1$, and by limiting the time span to three periods. Then the teacher can work during periods t and t_1, but retirement is mandatory in period t_2.

In the option value model, a teacher decides to work in period t if retirement in a later period yields a higher utility. In other terms, the teachers works in period t if:

$$\max[E_t V_t(t+1); E_t V_t(t+2)] > E_t V_t(t)$$

Using the analytical expression of the value function in each state, we get:

$$\begin{aligned} E_t(Y_t + w_t) + \max[E_t(B_{t+1}(t+1) + \xi_{t+1}) \ ; \ E_t(Y_{t+1} + \omega_{t+1})] > {} & E_t(B_t(t) + \xi_t) \\ & + E_t(B_{t+1}(t) + \xi_{t+1}) \end{aligned} \tag{9}$$

Conversely, the dynamic programming rule implies that the teacher works in period t if the expected utility stream induced by postponing retirement is greater than the one associated with current retirement:

$$E_t(Y_t + \omega_t) + E_t \max[V_{t+1}(t+1) \ ; \ V_{t+1}(t+2)] > E_t V_t(t)$$

which is equivalent to:

$$\begin{aligned} E_t(Y_t + \omega_t) + E_t \max[(B_{t+1}(t+1) + \xi_{t+1}) \ ; \ (Y_{t+1} + \omega_{t+1})] > {} & E_t(B_t(t) + \xi_t) \\ & + E_t(B_{t+1}(t) + \xi_{t+1}) \end{aligned} \tag{10}$$

The two rules differ because the expected value of the maximum of two random functions is not the maximum of their expected values. Since the former is usually greater than the latter, it means that the left hand side of equation 10 should be greater than the left hand side of equation 9. Thus the option to delay retirement is underweighted in the option value model. Under the assumption stipulating that survival rates are equal to one in each period, the uncertainty results from the stochastic part of the revenues, namely ξ_{t+1} and ω_{t+1}. If we consider in addition that revenues are perfectly known, the stochastic part disappears and calculating expectations of deterministic revenues becomes null and void. Therefore, equation 9 and equation 10 become equivalent and their common expression is:

$$Y_t + \max[B_{t+1}(t+1)\ ;\ Y_{t+1}] > [B_t(t) + B_{t+1}(t)]$$

A teacher who decided to retire in period t stays out of the labor market in period $t+1$ (i.e., retirement is an absorbing state). By contrast, a teacher who worked in period t has the choice either to work or to retire in period $t+1$. Since retirement is compulsory in period $t+2$, the decision rule in period t is given by:

$$E_{t+1}V_{t+1}(t+2) > E_{t+1}V_{t+1}(t+1)$$

which is equivalent to:

$$E_{t+1}(Y_{t+1} + w_{t+1}) > E_{t+1}(B_{t+1}(t+1) + \xi_{t+1})$$

In this case, there is no need to impose certainty of future revenues: if there is only one period left, the model comes down to a one-shot decision. Option value and dynamic programming decision rules match because there is no maximization any more. The teacher only considers what she would get in the two different states, and choose either to work or to retire accordingly. In period $t+2$, there is no choice left, everyone retires.

Increasing the number of periods would not change the result. The following equations show how incorporating the work option in period t_2, before imposing mandatory retirement in t_3, increases the dimension of integration in order to compute the dynamic programming rule.

More precisely, the decision rule in a four-periods option value model is:

$$E_t(Y_t + w_t) + \max \left\{ \begin{array}{ll} E_t(B_{t+1}(t+1) + \xi_{t+1}) + E_t(B_{t+2}(t+1) + \xi_{t+2}) & ; \\ E_t(Y_{t+1} + w_{t+1}) + E_t(B_{t+2}(t+1) + \xi_{t+2}) & ; \\ E_t(Y_{t+1} + w_{t+1}) + E_t(Y_{t+2} + w_{t+2}) & \end{array} \right\} >$$

$$E_t(B_t(t) + \xi_t) + E_t(B_{t+1}(t) + \xi_{t+1}) + E_t(B_{t+2}(t) + \xi_{t+2})$$

while the decision rule in a four-periods dynamic program model is:

$$E_t(Y_t + \omega_t) + E_t \left[\max \left\{ \begin{array}{ll} B_{t+1}(t+1) + \xi_{t+1} + B_{t+2}(t+1) + \xi_{t+2} & ; \\ Y_{t+1} + \omega_{t+1} + B_{t+2}(t+1) + \xi_{t+2} & ; \\ Y_{t+1} + \omega_{t+1} + Y_{t+2} + \omega_{t+2} & \end{array} \right\} \right] >$$

$$E_t(B_t(t) + \xi_t) + E_t(B_{t+1}(t) + \xi_{t+1}) + E_t(B_{t+2}(t) + \xi_{t+2})$$

Nonetheless, in an environment with certainty, the stochastic part of revenues is left aside, and only the deterministic term remains. The expectation of the maximum and the maximum of the expectations coincide and the two rules are equivalent.

Appendix B: Structure of errors

We assume that $\nu_{t+1} = \rho\nu_t + \varepsilon_{t+1}$ with $\varepsilon_t \sim \mathcal{N}(0, \sigma_\varepsilon)$. Similarly, we assume that the initial distribution of ν_t is a mean-zero normal distribution, i.e., $\nu_t \sim \mathcal{N}(0, \sigma_\nu)$ and that ν_t is independent of ε_s for $s = t+1, ..., S$.

Then we get:

$$
\begin{aligned}
\nu_{t+1} &= \rho\nu_t + \varepsilon_{t+1} \\
\nu_{t+2} &= \rho^2\nu_t + \rho\varepsilon_{t+1} + \varepsilon_{t+2} \\
\nu_{t+3} &= \rho^3\nu_t + \rho^2\varepsilon_{t+1} + \rho\varepsilon_{t+2} + \varepsilon_{t+3} \\
&\vdots \\
\nu_\tau &= \rho^{\tau-t}\nu_t + \sum_{j=0}^{\tau-t-1} \rho^j \varepsilon_{\tau-j} \\
\nu_{\tau+1} &= \rho^{\tau-t+1}\nu_t + \sum_{j=0}^{\tau-t} \rho^j \varepsilon_{\tau+1-j}
\end{aligned}
$$

Because of the stochastic independance between ν_t and ε_s for $s = t+1, ..., S$, the variance of ν_τ is given by:

$$
\begin{aligned}
Var(\nu_\tau) = E(\nu_\tau^2) &= \rho^{2(\tau-t)} E\left(\nu_t^2\right) + \sum_{j=0}^{\tau-t-1} \rho^{2j} E\left(\varepsilon_{\tau-j}^2\right) \\
&= \rho^{2(\tau-t)}\sigma_\nu^2 + \sum_{j=0}^{\tau-t-1} \rho^{2j}\ \sigma_\varepsilon^2 \\
&= \sigma_\nu^2 + (\tau - t)\sigma_\varepsilon^2 \quad \text{if } \rho = 1
\end{aligned}
$$

and the covariance between ν_τ and $\nu_{\tau+1}$ is:

$$\begin{aligned}
Cov(\nu_\tau, \nu_{\tau+1}) = E(\nu_\tau \cdot \nu_{\tau+1}) &= \rho^{2(\tau-t)+1}\sigma_\nu^2 + \rho \sum_{j=0}^{\tau-t-1} \rho^{2j}\ \sigma_\varepsilon^2 + \underbrace{\varepsilon_{\tau+1} \sum_{j=0}^{\tau-t-1} \rho^j \varepsilon_{\tau-j}}_{=0} \\
&= \rho \left[\rho^{2(\tau-t)}\sigma_\nu^2 + \sum_{j=0}^{\tau-t-1} \rho^{2j}\ \sigma_\varepsilon^2 \right] \\
&= \rho\ Var(\nu_\tau)
\end{aligned}$$

For example, in a three-periods setup, the covariance matrix is the following:

$$\Sigma_3 = \begin{bmatrix} \sigma_\nu^2 & \rho\sigma_\nu^2 & \rho^2\sigma_\nu^2 \\ & \rho^2\sigma_\nu^2 + \sigma_\varepsilon^2 & \rho^3\sigma_\nu^2 + \rho\sigma_\varepsilon^2 \\ & & \rho^4\sigma_\nu^2 + (1+\rho^2)\sigma_\varepsilon^2 \end{bmatrix}$$

If the process ν_t is second-order stationary, then the diagonal elements are equal, which implies that the variance of ν_t is constant over time. Under that assumption, the covariance matrix becomes:

$$\Sigma_3 = \frac{\sigma_\varepsilon^2}{1-\rho^2} \begin{bmatrix} 1 & \rho & \rho^2 \\ \rho & 1 & \rho \\ \rho^2 & \rho & 1 \end{bmatrix}$$