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COORDINATION WITH COGNITIVE NOISE

Abstract

We experimentally study how cognitive noise affects behavior in coordination games. When players face small computational errors in valuation, equilibrium play becomes more predictable owing to the disappearance of multiple equilibria. Our experimental data provide novel evidence for this prediction: the frequency of coordination depends systematically on (i) public information and (ii) the distribution from which public information is drawn. We estimate that cognitive noise constitutes roughly half of the observed noise in strategic behavior. The errors that we model are distinct from those in previous behavioral game theory models and give rise to novel predictions that our data support.

JEL Classification: C72, C92, D91, E71

Keywords: Complexity, Context-Dependence, Stochastic Choice, Cognitive Imprecision

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Coordination with Cognitive Noise*

Cary Frydman† Salvatore Nunnari‡

March 16, 2023

Abstract

We experimentally study how *cognitive noise* affects behavior in coordination games. When players face small computational errors in valuation, equilibrium play becomes *more* predictable owing to the disappearance of multiple equilibria. Our experimental data provide novel evidence for this prediction: the frequency of coordination depends systematically on (i) public information and (ii) the distribution from which public information is drawn. We estimate that cognitive noise constitutes roughly half of the observed noise in strategic behavior. The errors that we model are distinct from those in previous behavioral game theory models and give rise to novel predictions that our data support.

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	Not Invest	Invest
Not Invest	θ, θ	θ, a
Invest	a, θ	b, b

Figure 1: **The Game**

1 Introduction

Consider two players who would like to coordinate on an investment opportunity, described by the payoff matrix in Figure 1. When $a \leq \theta \leq b$, there are multiple equilibria. If the row player believes the column player will invest, then she prefers to also invest. If the row player believes the column player will not invest, then she prefers to not invest. How can we predict which action the players will select? In this paper, we argue that *cognitive noise* corrupts a player's ability to compute the value of each action. Cognitive noise, thus, provides a new source of uncertainty about the opponent's action, as each player will be uncertain about the noisy computation implemented by her opponent. This uncertainty eliminates multiple equilibria and gives rise to a unique prediction about game play. Our model of cognitive noise generates additional testable predictions that distinguish it from leading behavioral models of strategic behavior, including Quantal Response Equilibrium (QRE; McKelvey and Palfrey 1995, 1998) and Level-k Thinking (Nagel, 1995; Camerer, Ho and Chong, 2004).

In a pair of pre-registered experiments, we demonstrate that cognitive noise is inherent in strategic play and that it systematically affects the probability of coordination. We experimentally implement the game shown in Figure 1 and we find three main results. First, behavior is consistent with the unique equilibrium that arises in the presence of cognitive noise, whereby the probability of investing declines continuously in θ. Second, we manipulate the level of cognitive noise and find that it causally affects both the frequency of coordination and the sensitivity of choice to game payoffs. Third, we conduct a decomposition analysis which reveals that cognitive noise represents a substantial source of the noise observed in strategic behavior; we estimate that roughly 50% of noise in behavior stems from an imprecise representation of payoffs and the subsequent cognitive processing involved in valuing each action.

Our analyses highlight that the particular manner in which noise is modeled has important implications for equilibrium. Thus, we are careful to ground our assumptions about the source of noise in a recent empirical literature in economics that has begun investigating imprecision in valuation and choice (Woodford, 2020). In particular, a series of recent individual decision-making experiments has shown that noise arising in the subjective valuation process exhibits clear parallels with noise in basic perceptual decisions (Polania, Woodford and Ruff, 2019; Khaw, Li and Woodford, 2021, 2022; Enke and Graeber, 2023; Enke, Graeber and Oprea, 2023; Frydman and Jin, 2022). This conceptual link between perceptual and economic decisions motivates our modeling approach: we assume each player holds a prior about the fundamental parameter θ and then observes only a noisy signal of θ — even after θ is clearly "presented" to the player. The noisy signal is meant to capture errors involved with encoding, retrieving, and further cognitive processing of θ, when computing the value of an action.

Our particular model of cognitive noise generates a sharp and testable prediction about strategic behavior. In the model, while each player faces unavoidable cognitive noise, we assume that each player can optimally choose the distribution of noise. This assumption of *efficient coding* generates the following prediction: as a player's prior about θ becomes more concentrated, she encodes information about θ with greater precision (Barlow, 1961; Laughlin, 1981). The intuition is that a player will be more precise in processing information about those particular values of θ that she expects to face more frequently. This extra precision in processing information about common values of θ will affect the player's valuation of investing, and hence, coordination. We, therefore, test whether cognitive noise plays an important role in coordination games by experimentally manipulating the prior and testing for the impact on game outcomes.

We present the details of our two experiments in the main body of the paper, but here we preview the key aspects of the design. In our first experiment, subjects are randomly matched on each of three hundred rounds, and play the game outlined in Figure 1. We set the values of $a = 47$ and $b = 63$, and the only object that varies across rounds is θ. On each round, we assume that a subject's prior is governed by the distribution of θ that she has experienced during the experiment. Thus, to manipulate the prior, we implement a between-subjects treatment where half of the subjects observe values of θ drawn from a high volatility distribution, and the other half observe values of θ drawn from a low volatility distribution. The key prediction is that the prior affects the manner in which players process information about θ, and this in turn, affects the subjective valuation of investing and not investing.

In our main test, we compare the frequency that a player invests — conditional on θ — across the two experimental treatments. Consistent with our theoretical model, we find that for a given value of θ, the probability of investing depends on the prior to which the player is adapted. In both treatments, behavior is consistent with subjects playing a noisy version of the unique equilibrium threshold strategy. The smoking gun evidence for cognitive noise, however, is that behavior exhibits significantly more randomness in the high volatility treatment, where our model predicts that information about θ will be processed with more noise. This result is consistent with previous work from individual decision-making experiments (Frydman and Jin, 2022); however, a crucial difference is that here, cognitive noise is key to endogenously producing the equilibrium threshold strategy. Put differently, the data we produce are consistent with a cognitive noise mechanism that endogenously generates *both* the equilibrium threshold strategy and the greater degree of randomness that subjects exhibit when implementing this strategy in the high volatility treatment. Overall, our experimental data indicate that coordination (both players investing or neither player investing) is more likely when players are adapted to the low volatility distribution and face a lower amount of cognitive noise.

We emphasize that other models of noisy strategic behavior, such as QRE, do not predict that strategic behavior depends on the agent's prior. The intuition for this difference in predictions is as follows. Our model of cognitive noise assumes that the agent is unable to precisely compute the value of an action. Thus, because the prior is informative about the value of an action, any shift in the prior will affect the subjective valuation of the action. In contrast, QRE assumes that each agent has no problem with precisely computing the value of each action, conditional on θ. The noise in QRE arises only during the process of action selection, where the agent "trembles". In this case, the prior has no bearing on behavior, as the agent is already fully confident about the precise value of each action.

Our results highlight cognitive noise as a novel and important source of *strategic uncertainty* — which refers to uncertainty about an opponent's behavior. Typically, strategic uncertainty is attributed to uncertainty about an opponent's preferences, information, degree of rationality, or which of multiple equilibria the opponent will select. Because cognitive noise corrupts a player's valuation of an action, it necessarily leads the player to be uncertain about an opponent's valuation and selection of an action. An important question, then, is how much of the noise in behavior that we observe is actually driven by cognitive noise, rather than alternative sources of strategic uncertainty?

To address this question, we conduct a second experiment that enables us to decompose the observed noise in behavior into structural uncertainty (arising from cognitive noise) and strategic uncertainty (arising from sources other than cognitive noise). The main innovation in this second experiment is that we incentivize subjects to play the same series of games as in our first experiment, except the opponent is now a computer. Crucially, we inform subjects that the computer plays a known and deterministic strategy. This design feature purges any strategic uncertainty that arises from sources other than cognitive noise. We find that, even when playing against a computer, subjects still make errors that have signature features of cognitive noise. More importantly, we estimate that roughly half of the noise in behavior from our first experiment is driven by cognitive noise. We attribute the remaining noise in behavior to alternative sources of strategic uncertainty.

Taken together, the results from our two experiments emphasize the importance of accounting for cognitive noise in a strategic setting. We show that the specific stage at which noise enters the decision process will fundamentally affect equilibrium predictions. By drawing on the burgeoning economics literature on cognitive noise, we take the stand that noise arises "early" in the decision process when agents are computing the value of each action. This modeling technique is reminiscent of the approach taken in the literature on global games (Carlsson and Van Damme, 1993; Morris and Shin, 2003; Angeletos and Lian, 2016). There is, however, a critical difference in interpretation between the two models. In global games, it is often presumed that the noise is attributed to private information or some *external* source of uncertainty, which is disconnected from any errors in cognitive processing. In contrast, our model supposes that noise arises *internally* within the agent's mind due to unavoidable information processing constraints.

The source of noise — whether it is cognitively or externally generated — turns out to have important implications for equilibrium (Woodford, 2020). One particular and common interpretation of private signals in global games is that they capture asymmetric information between the players.[1] This asymmetric information generates strategic uncertainty, which in turn, produces a unique equilibrium. But under the interpretation that noise arises cognitively, the prediction of a unique equilibrium should hold in an even broader set of applications. To see this, note that even if a public signal becomes available – which can restore multiple equilibria under global games – cognitive noise will prevent each player from processing the public signal with perfect precision. Thus, cognitive noise will act as a private signal generator that maintains the strategic uncertainty needed to support a unique equilibrium. Thus, interpreting noisy signals as arising (at least in part) from unavoidable errors in value computation leads to a clear prediction: the provision of explicit private signals should have little impact on behavior. Indeed, this is exactly what Heinemann et al. (2004) find when exogenously manipulating the provision of private information, though they do not provide an underlying mechanism. In Section 6.1, we compare our model's predictions with those of global games in more detail.

Our experimental results have important implications for the modeling of incomplete information games. Specifically, our results suggest that the class of games in which it is appropriate to assume agents have incomplete information is likely broader than previously thought. Even in situations where there is no explicit private information, cognitive noise

[1] Indeed, a majority of experimental tests of global games involve explicitly endowing subjects with noisy signals of the fundamental value (Heinemann, Nagel and Ockenfels, 2004; Cabrales, Nagel and Armenter, 2007; Van Huyck, Viriyavipart and Brown, 2018; Avoyan, 2019; Szkup and Trevino, 2020)

will break common knowledge about the valuation of each player's action. In addition to providing guidance for appropriate modeling assumptions, the idea that cognitive noise arises near universally also has implications for experimental design. To see this, consider a recent experiment by Goryunov and Rigos (2022) who use a clever design to explicitly inject noise into the perception of a state variable. Subjects in their experiment observe a visual "dot" that represents the state, and the authors rely on the inherent difficulty of visually perceiving the exact location of the dot to generate private noise. Our results suggest that noise in valuation arises in a much broader class of games, owing to the imprecision involved with higher-level cognitive processing of value.[2] As we demonstrate with our experiments, even when information about the state variable is clearly communicated to subjects through symbolic numerals, we find evidence that cognitive noise is present in value computations. We also note that our measurements of cognitive noise are likely to represent a *lower* bound relative to more complex strategic applications outside the lab.

Our results build on a set of papers that have begun testing whether principles of cognitive noise are active in individual economic decision-making (Polania, Woodford and Ruff, 2019; Gershman and Bhui, 2020; Khaw, Li and Woodford, 2021, 2022; Enke and Graeber, 2023; Enke, Graeber and Oprea, 2023; Frydman and Jin, 2022). In addition to testing whether similar mechanisms extend into strategic environments, our setting of a coordination game enables a novel test of the hypothesis that noise arises early in the decision process during the valuation stage. Sharp tests of this hypothesis are important because the distinction between early and late noise can also shed light on choice biases in individual decision-making (Woodford, 2020). Of course, one additional factor that is present in strategic environments is the need for subjects to form beliefs about opponents' behavior. In our setting, it is important for equilibrium that subjects are aware that (or at least believe that) their opponent faces cognitive noise. In the Online Appendix, we provide evidence from an additional experiment which helps to validate such an assumption. We find that subjects report beliefs that their opponent exhibits more errors in a discrimination task as the distance between states gets smaller. In related work, Enke, Graeber and Oprea (2022) demonstrate that meta-cognition of errors is important for understanding how these errors

[2]Goryunov and Rigos (2022) implement an interesting treatment, whereby they provide some subjects with the ability to observe whether the dot falls on either side of a line. In this treatment, which is meant to enable discontinuous stochastic choice, and thus the existence of multiple equilibria, observed behavior is quite similar to the treatment in which subjects do not have the ability to implement discontinuous stochastic choice. Thus, while theory predicts that behavior will vary considerably across treatments, those subjects who have access to the more precise information acquisition technology "do not seem to be following multiple equilibria, at least not more so than their [no line treatment] counterparts." One explanation for this lack of a treatment effect could stem from the fact that even in the treatment where it is easy to detect on which side of the boundary the state is located, players exhibit errors in processing value – in line with the model we propose – which would eliminate the multiple equilibria.

aggregate at the level of institutions.

The model we propose is also closely related to a set of recent theoretical papers that investigate endogenous information acquisition in coordination games. Yang (2015) shows that the uniqueness result from the global games literature breaks down when players endogenously acquire information about the fundamental using a mutual information cost function. Morris and Yang (2022), instead, show that when the cost function satisfies "infeasible perfect discrimination" — so that signal probabilities vary continuously with the fundamental — then uniqueness is restored. Hébert and Woodford (2021) propose a set of "neighborhood-based" cost functions for rational inattention problems, which are motivated in part by evidence from perceptual experiments. These cost functions satisfy the infeasible perfect discrimination property and, thus, lead to a unique equilibrium in a coordination game. Our model of cognitive noise also gives rise to an endogenous information structure that satisfies infeasible perfect discrimination and leads to a unique equilibrium. Importantly, our experimental data provide novel support for infeasible perfect discrimination in a setting where all information is represented numerically.[3]

2 Model

In this section, we present a model in which cognitive noise corrupts the ability of each player to precisely compute the value of actions. We illustrate the strategic implications of noisy valuation in the setting of a 2×2 simultaneous move game. We focus our analysis on those parameter values that generate the essential features of a coordination game.

Consider the game in Figure 1, where $b > a$. In what follows, we assume that a and b are encoded and processed without any noise by both players, and we are interested in how cognitive noise affects the encoding, retrieval, and subsequent cognitive processing of θ.[4] We further assume that each player has linear utility. As a benchmark, we first consider the predictions of a model in which agents have the ability to compute the precise value of actions conditional on θ; we then derive implications under our main assumption of cognitive noise.

[3] For a given fundamental value, we also find intriguing evidence that response times are significantly longer in the high volatility condition. This finding suggests that the implementation of strategies may be more complex (in the spirit of Oprea 2020) in the high volatility condition. The assumption of infeasible perfect discrimination is also consistent with evidence from an additional experiment, described in Online Appendix D, which investigates subjects' awareness of their and others' imprecision in a perceptual task.

[4] Our assumption that a and b are processed without noise can be justified, for example, through a learning mechanism. In our experiment, we keep a and b constant across all rounds, so the amount of noise in processing a and b is arguably minimal.

2.1 Benchmark: No Cognitive Noise

Without any cognitive noise, the game is one of complete information and its Nash equilibria depend on the true value of θ, as outlined below:

- If $\theta > b$, then Invest is a strictly dominated action for each player, and (Not Invest, Not Invest) is the unique Nash (and dominant strategy) equilibrium.
- If $\theta < a$, then Not Invest is a strictly dominated action for each player, and (Invest, Invest) is the unique Nash (and dominant strategy) equilibrium.
- If $a \leq \theta \leq b$, then there are two Nash equilibria in pure strategies: (Not Invest, Not Invest) and (Invest, Invest). There also exists one Nash equilibrium in mixed strategies.

Thus, when θ takes on values in the intermediate range $[a, b]$, there are multiple pure strategy Nash equilibria. This prediction relies on each player's ability to precisely observe and process information about θ, which generates common knowledge about θ. The common knowledge, in turn, enables coordination and gives rise to multiple self-fulfilling equilibria. The predictions change dramatically, however, when we relax the assumption that players can precisely compute the value of investing and not investing, conditional on θ.

2.2 Information Processing Constraint: Cognitive Noise

Suppose now that players compute the subjective value of each action with noise. In order to minimally depart from the rational benchmark, we assume cognitive noise only corrupts the processing of θ. Our assumption that players exhibit noise in computing the value of investing and not investing is based on a recent literature in economics that has uncovered evidence of such noise in individual decision-making experiments (Polania, Woodford and Ruff, 2019; Gershman and Bhui, 2020; Khaw, Li and Woodford, 2021, 2022; Enke and Graeber, 2023; Enke, Graeber and Oprea, 2023; Frydman and Jin, 2022). These studies demonstrate that even when information is displayed in the form of symbolic numbers – so that there is little room for early-stage perceptual errors – behavior exhibits randomness that reflects errors in the computational process of valuation.

For example, (Enke and Graeber, 2023) show that valuation of risky lotteries becomes less sensitive to state probabilities as cognitive noise is increased. Even in less cognitively demanding tasks, there is evidence that humans exhibit cognitive noise when simply comparing which of two symbolically presented numerical quantities is greater (Dehaene, 2011). Given that the game we analyze is arguably at least as complex as the individual decision-making tasks cited above, we hypothesize that cognitive noise should also operate in the strategic setting we consider here. Building on this previous literature, we model cognitive noise through the assumption that each player has access only to a noisy signal of θ.

Assumption 1 (Cognitive Noise) *Each player i, $i = \{1, 2\}$, has a common prior belief that θ is distributed normally, $\theta \sim \mathcal{N}(\mu_\theta, \sigma_\theta^2)$. Conditional on the realized value of θ, each player observes a noisy internal representation, $S_i = m(\theta) + \epsilon_i$, where each ϵ_i is independently and normally distributed: $\epsilon_i \sim \mathcal{N}(0, \sigma_S^2)$ with $\sigma_S^2 > 0$.*

The prior belief about θ, which we denote by $f(\theta)$, can represent public information or past experience in a similar environment that is common to both players. Assumption 1 reflects noise in the computation of the subjective values of each action. This noise could arise from errors in encoding θ, retrieving θ from short-term memory, or integrating θ with other parameters in the game. To be clear, our assumption does not imply that if a subject were asked to repeat the value of θ back to the experimenter (verbally or through written form), that she would exhibit errors. Our framework is consistent with the view that subjects can have conscious access to the presented value of θ, but that there is noise in judgments and computations that take θ as an input.

It is worth highlighting how Assumption 1 introduces uncertainty into various aspects of the decision process. To illustrate, we derive the condition under which each player chooses to invest. Player i will invest if and only if:

$$\begin{aligned}
\text{EU[Not Invest } |S_i] &< \text{EU[Invest } |S_i] \\
\text{E}[\theta|S_i] &< a + [b-a]\text{E}[p(a,b,\theta)|S_i] \\
\int \theta f(\theta|S_i) d\theta &< a + [b-a] \int p(a,b,\theta) f(\theta|S_i) d\theta, \qquad (1)
\end{aligned}$$

where $f(\theta|S_i)$ is player i's posterior belief about the distribution of θ after observing signal S_i. The function, $p(a, b, \theta)$, maps the game payoffs into a belief about the probability that the opponent invests. When we derive equilibria of the game, p will be pinned down endogenously by rational expectations but, for now, it is instructive to consider p as exogenous.

In inequality (1), the noisy internal representation, S_i, appears on both sides of the expression. On the left-hand side, S_i induces uncertainty about player i's own payoff from not investing, which is referred to as *structural uncertainty*. In our setting, structural uncertainty can arise from noisy encoding of θ or noise in computing the utility of θ. On the right-hand side, S_i induces uncertainty about the opponent's probability of investing, which is referred to as *strategic uncertainty*. If, for example, player i believes the opponent uses a cutoff rule, then her belief about the opponent investing depends on her belief about the opponent's signal. Since S_i and S_{-i} are drawn conditional on θ, player $i's$ belief about her opponent's perception of θ will depends on S_i. Both sources of uncertainty will be important for our theoretical results: strategic uncertainty will be responsible for generating a unique equilibrium, whereas structural uncertainty will generate a continuous relationship between θ and the probability of investing.

Having shown how the noisy internal representation can lead to multiple types of uncertainty, we now draw on principles from psychology to put further structure on the distribution of S_i. Following Khaw, Li and Woodford (2021), we constrain the encoding of information so that the mean signal, $m(\theta)$, is a linear function of θ and has a bounded variance:[5]

Assumption 2 (Encoding Function) *The encoding function is linear: $m(\theta) = \xi + \psi\theta$. In addition, there is a power constraint, $E[m^2] \leq \Omega^2 < \infty$.*

The power constraint captures the idea that the brain cannot encode an arbitrarily large set of values. Without the power constraint, the player could choose the noisy internal representation, $S_i = m(\theta) + \epsilon_i$, to be arbitrarily precise by making the variance of $m(\theta)$ as large as needed. By introducing the power constraint, it becomes harder for a player to discriminate between two fundamental values as they become closer together. Specifically, for any two fundamental values $\theta_1 < \theta_2$, it is more difficult for the player to discriminate between the two values as $|\theta_1 - \theta_2|$ approaches zero. This assumption is in the spirit of the cost functions proposed by Hébert and Woodford (2021) and Morris and Yang (2022).

Given the cognitive constraints summarized by Assumptions 1 and 2, we allow the player to choose the encoding function parameters, (ξ, ψ). In this manner, the player can *efficiently code* information about the fundamental to achieve a performance objective. Thus, the conditional distribution of noisy internal representations can vary across environments, depending on the player's prior belief about the fundamental in that environment. Our assumption of efficient coding is built on substantial empirical evidence from the literature on sensory perception, which finds that the distribution of noisy internal representations is *optimally* adapted to the statistical regularities of the environment.[6] In addition to the evidence from sensory perception, recent work has empirically documented effects of efficient coding in economic choices (Polania, Woodford and Ruff, 2019; Frydman and Jin, 2022).

To close the efficient coding model, we need to specify the performance objective which drives the players' optimal choice of the encoding function parameters.

Assumption 3 (Performance Objective) *Players choose the encoding function which minimizes the mean squared error between θ and its conditional mean, $E[\theta|S_i]$.*

[5]Khaw, Li and Woodford (2021) assume a slightly different specification of the encoding function, which is linear in the logarithm of a payoff value. See their Appendix C for details.

[6]For example, see work by (Girshick, Landy and Simoncelli, 2011; Wei and Stocker, 2015; Payzan-LeNestour and Woodford, 2022)

With the player's performance function in hand, we can now derive the efficient coding function that each player optimally chooses, given her cognitive constraints.[7]

Proposition 1 (Efficient Coding) *Given Assumptions 1-3, the optimal encoding function features* $\xi^\star = -\frac{\Omega}{\sigma_\theta}\mu_\theta$ *and* $\psi^\star = \frac{\Omega}{\sigma_\theta}$. *Consider the transformed internal representation,* $Z_i \equiv (S_i - \xi^\star)/\psi^\star$. *The conditional distribution of* Z_i *is* $N(\theta, \omega^2\sigma_\theta^2)$, *where* $\omega = \sigma_S/\Omega$. *The variance of* Z_i *is proportional to the variance of* θ.

Proposition 1 says that the player chooses the slope of the encoding function, $\psi^\star$, such that it becomes steeper as the variance of the prior shrinks. Intuitively, for a given change in θ, a good encoding function is one that exhibits a large change in signal. As the variance of the prior shrinks, signals can become more sensitive to a change in θ while still satisfying the power constraint. Indeed, the important implication of Proposition 1 for our purposes is that the noisy signal distribution is normalized by the prior variance. While this "normalization" result is derived from our three specific assumptions, it is a robust implication of efficient coding that arises in a more general class of models (Polania, Woodford and Ruff, 2019; Khaw, Li and Woodford, 2021; Frydman and Jin, 2022; Payzan-LeNestour and Woodford, 2022).[8]

Given the optimal encoding function in Proposition 1, we can now solve for the equilibria of the game. We restrict our analyses to monotone equilibria of the incomplete information game, that is, equilibria in which actions are monotonic in the transformed internal representation, Z_i. In such a monotone equilibrium, a player's mutual best response is to choose Invest if and only if her transformed internal representation is below a threshold $k^\star$. To derive the equilibrium, we adapt results from the global games literature (Carlsson and Van Damme, 1993; Morris and Shin, 2003; Morris, 2010) to the game in Figure 1, with the further assumption that $\mu_\theta = (a+b)/2$ (as in the experiment described in the next section). We can then establish there exists a monotone equilibrium such that player i invests if and only if $Z_i \leq \mu_\theta$, for any value of σ_θ, σ_S and Ω. Furthermore, if the noise in the transformed internal representation is sufficiently small, this is the unique monotone equilibrium.[9]

Proposition 2 (Equilibrium Existence and Uniqueness) *Suppose Assumptions 1-3 and* $\mu_\theta = (a+b)/2$. *There exists an equilibrium of the game where each player invests if and*

[7] In Section 6.4 and Online Appendix B, we show that our theoretical predictions are robust to different assumptions about the players' performance objective.

[8] The differences across efficient coding models stem from alternative specifications of the encoding constraint and the performance objective (Ma and Woodford, 2020). While model predictions will differ as a function of higher moments of the prior, the prediction of higher precision for lower variance priors is shared among the majority of efficient coding models.

[9] In deriving the equilibrium, we assume common knowledge of the distribution of internal representations. In Section 6.5, we discuss how the equilibrium can arise under weaker assumptions about common knowledge.

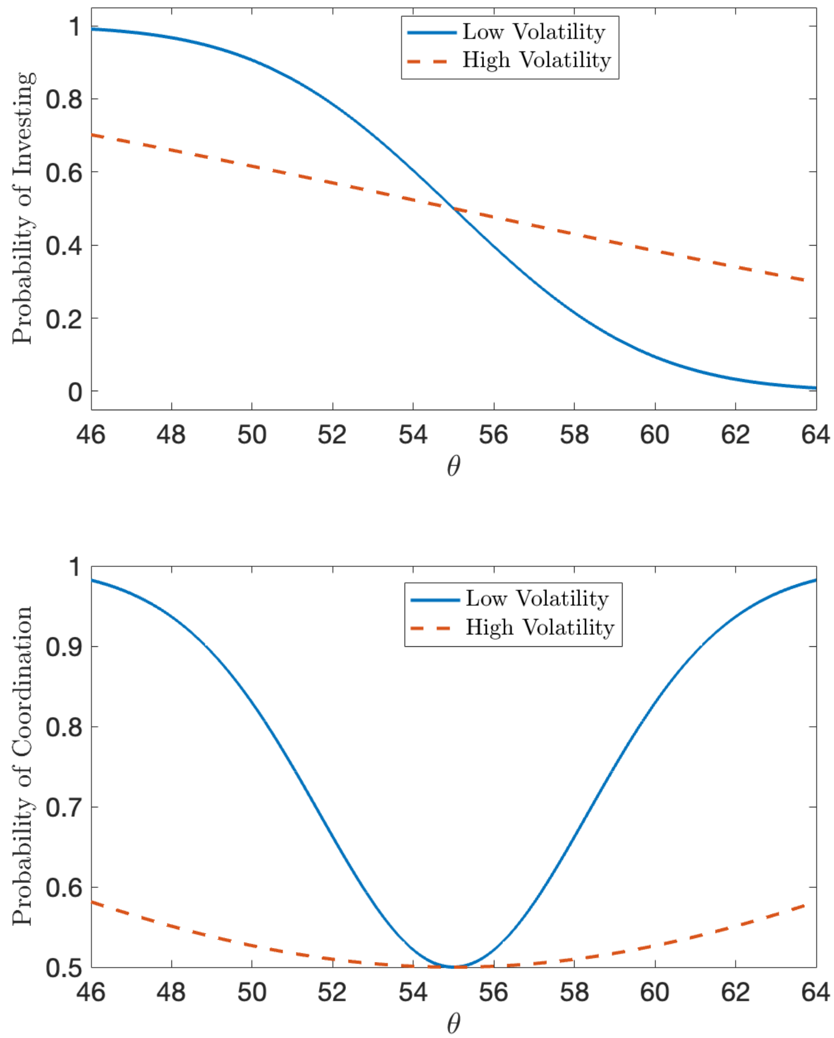


Figure 2: **Probability of Investing and Coordination as a Function of θ.** Note: The upper panel displays the predicted probability of investing. The bottom panel displays the predicted probability of coordination – where coordination is defined as both players investing or both players not investing. In both panels, the solid line denotes the prediction for a low volatility distribution with $\theta \sim N(55, 20)$; the dashed line denotes the prediction for a high volatility distribution with $\theta \sim N(55, 400)$; we set the following parameter values: $a = 47$, $b = 63$, and $\omega = 0.85$.

only if $Z_i \leq \mu_\theta$ (or, equivalently, $E[\theta|Z_i] \leq \mu_\theta$). Moreover, if $\frac{\omega\sqrt{1+\omega^2}}{\sqrt{2+\omega^2}} < \frac{\sqrt{2\pi}}{(b-a)}\sigma_\theta$, this is the unique monotone equilibrium of the game.

Proposition 2 implies a rich set of comparative statics with respect to θ. The probability of investing is pinned down by the distribution of the transformed internal representation:

$Pr[\text{Invest}|\theta] = Pr\left[Z_i \le \mu_\theta|\theta\right] = \Phi\left(\frac{\mu_\theta-\theta}{\omega\sigma_\theta}\right)$, where $\Phi(\cdot)$ is the cumulative density function of the standard normal. This result indicates that, in the unique monotone equilibrium, the probability of investing is continuous and monotonically decreasing in θ. If we operationalize coordination as both players investing or both players not investing, then it follows that coordination will also be systematically related to θ. In particular, the model predicts that the probability of coordination is a continuous and U-shaped function of θ, which has its minimum at $\theta = 55$. We emphasize that the prediction of a systematic relationship between θ and the probability of coordination does not arise in the complete information version of the game.

We can make an even starker prediction about equilibrium outcomes by exploiting the malleability of the encoding function. The probability of investing depends not only on θ, but also on the prior distribution from which θ is drawn. Specifically, σ_θ modulates the optimal encoding function and, therefore, the precision with which a player detects whether a fundamental crosses the equilibrium threshold. It follows that, when ω is sufficiently small (so that a unique equilibrium obtains regardless of σ_θ), the probability of investing declines more rapidly in θ as the prior volatility decreases. This prediction is summarized in the following proposition.

Proposition 3 (Comparative Statics) *Suppose Assumptions 1-3, $\mu_\theta = (a+b)/2$, and $\frac{\omega\sqrt{1+\omega^2}}{\sqrt{2+\omega^2}} < \frac{\sqrt{2\pi}}{(b-a)}\sigma_\theta$. In the unique monotone equilibrium of the game, the probability that each player invests for a given value of θ is $Pr[Invest|\theta] = \Phi\left(\frac{\mu_\theta-\theta}{\omega\sigma_\theta}\right)$. Decreasing the variance of θ will increase the sensitivity of choices to θ (that is, the rate at which $Pr[Invest|\theta]$ decreases with θ) for values of θ close to μ_θ.*

In Figure 2, we illustrate the implications of Proposition 3 for investment behavior and coordination. The upper panel shows that, in both conditions, the probability of investing declines continuously in θ. The negative relationship is a consequence of the unique monotone equilibrium where each player invests if and only if $Z_i \le 55$. We also see that the prior distribution of θ strongly affects the rate at which the probability of investing declines in θ. This dependence of equilibrium behavior on the prior distribution of θ motivates our experimental design. The bottom panel shows that the probability of coordination is a function of both θ and the distribution from which θ is drawn.

3 Experimental Design of Coordination Game

We test the cognitive noise model by incentivizing subjects to play a simultaneous move game, and we manipulate the distribution that generates the fundamental payoff, θ. We pre-register the experiment and recruit 300 subjects from the online data collection platform, Prolific.[10] We restrict our sample to subjects who, at the time of data collection, (i) were UK nationals and residents, (ii) did not have any previous "rejected" submissions on Prolific, and (iii) answered all comprehension quiz questions correctly. Subjects are paid 2 GBP ($\sim$ 2.8 USD) for completing the experiment, and they have the opportunity to receive additional earnings based on their choices and the choices of other participants.

The experiment consists of 300 rounds, and each subject participates in all rounds. In each round, a subject is randomly matched with another subject and, together, they play the simultaneous move game in Figure 1. We hold constant the payoff parameters $a = 47$ and $b = 63$ across all rounds. The only feature of the game that varies across rounds is the value of θ, which is drawn from the condition-specific distribution $f(\theta)$. In each round, both subjects observe the same realization of θ. In order to shut down learning about other participants' behavior, we choose not to provide subjects with feedback about their earnings or their opponent's choice in a given round. At the end of the experiment, one round is selected at random, and subjects are paid according to the number of points they earned in that round, which in turn, depends on their action, their opponent's action, and the (round-specific) value of θ. Points are converted to GBPs using the rate 20:1. The average duration of the experiment was $\sim$ 25 minutes and average earnings, including the participation fee, were $\sim$ 5.5 GBP ($\sim$ 7.7 USD).

Subjects are randomized into one of two experimental conditions: a *high volatility* condition or a *low volatility* condition, which differ only based on the distribution of θ. In the high volatility condition, $f(\theta)$ is normally distributed with mean 55 and variance 400. In the low volatility condition, $f(\theta)$ is normally distributed with mean 55 and variance 20. In both conditions, after drawing θ from its respective distribution, we round θ to the nearest integer, and we re-draw θ if the rounded value is less than 11 or greater than 99. We implement these modifications to the normal distribution to control complexity and ensure that θ is a two-digit number on each round. We do not give subjects any explicit information about $f(\theta)$ in the instructions, as our intention is to test whether a subject can adapt to the statistical properties of the environment without explicit top-down information. Moreover, we believe that such a design is more natural than explicitly telling subjects the distribution of parameters they will experience, as this could artificially direct their attention to the distribution. Each condition contains an identical set of instructions and comprehension quiz.[11] As outlined in our pre-registration, we exclude the first 30 rounds from our analyses, in order to allow subjects time to adapt to the distribution of θ.

[10]The pre-registration document is available at `https://aspredicted.org/IHU_KCE`.

[11]The experimental instructions are available in Online Appendix E.

Option A	**Option B**
45	**47** if other participant chooses A **63** if other participant chooses B

Figure 3: **Sample Screenshot Shown to Participants in Experiment 1.** Note: In this round, the realized value of θ is 45, which is clearly and explicitly displayed to both subjects. Subjects choose "Option A" or "Option B" by pressing one of two keys on the keyboard.

Recall that, in the complete information version of the game, there are multiple equilibria when θ is in the range $[47, 63]$. We therefore focus our analyses on games for which θ lies in this range. We pre-register that our main analyses are restricted to those rounds for which $\theta \in [47, 63]$ and we call these "common rounds." This is a crucial feature of our design, because it allows us to compare behavior across conditions using the exact same set of games and varying only the context, that is, the distribution of past games.

In choosing the parameters for our design (a, b and the two condition-specific values of σ_θ), we strike a balance among three competing objectives: (i) generating a substantial number of common rounds to analyze, (ii) creating a large predicted treatment effect, and (iii) guaranteeing the empirical distributions of θ approximate the distributions that we assume in the theory. There is a tension between the first objective and each of the latter two. First, a natural way to create a large predicted treatment effect is to set a large value of σ_θ in the high volatility condition. However, if this parameter is too large, there will be relatively few draws for which $\theta \in [47, 63]$ and, thus, few common rounds to analyze in this condition. Second, theory requires us to choose an $[a, b]$ range which is not too large. Specifically, equilibrium uniqueness requires that, in both conditions, subjects believe there is some chance of observing games with dominant strategies, that is, games with $\theta < a$ and game with $\theta > b$. At the same time, reducing the distance between a and b — e.g., choosing $a = 50$ and $b = 60$ — would reduce the number of common rounds to analyze.

Figure 3 provides a screenshot of a single round shown to subjects. In order to avoid framing effects, we label the two options "Option A" and "Option B", and the left-right location of each option is randomized across rounds. The number "45" is the realized value of θ on the specific round shown in Figure 3. We emphasize that — while the number is clearly displayed to all subjects and, thus, would traditionally be interpreted as public information — here we rely on cognitive noise to transform the fundamental value into private information. In other words, we assume that cognitive constraints prevent each player from processing the fundamental value and combining it with other information from the game in a precise manner.

Finally, we intentionally choose the visual display of the experiment to be as simple as possible, so that we only present the values of a, b, and θ once on each experimental screen.[12] An alternative approach would be to display the game in matrix form, similar to the display in Figure 1. While the matrix approach is more standard in experimental economics, it may also be interpreted by subjects as more complex compared to our design in Figure 3. Importantly, the complexity of how information is presented has recently been shown to affect the level of cognitive noise (Enke and Graeber, 2023). Thus, we do not believe one display strictly dominates another. On the contrary, differences in display may systematically affect cognitive noise which could motivate modifications of our design to assess the impact on coordination.

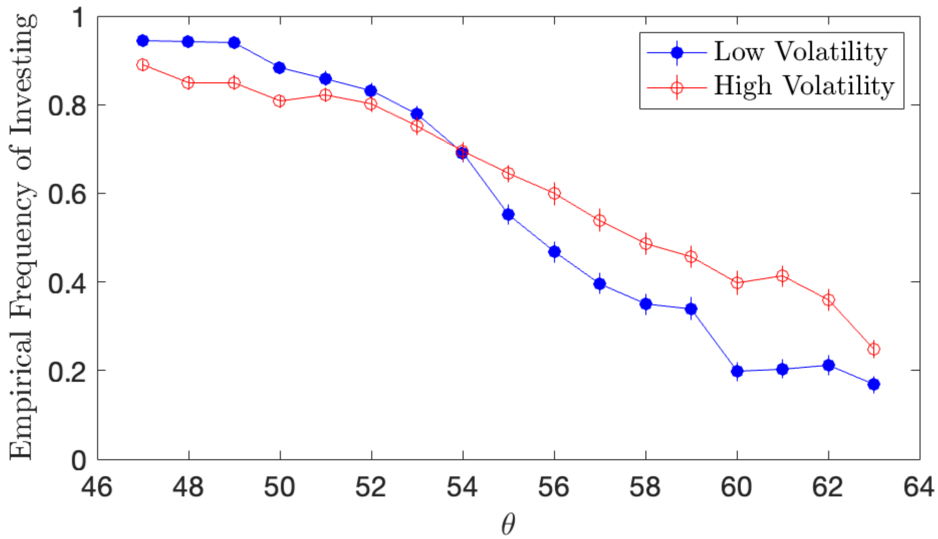
4 Experimental Results from Coordination Game

4.1 Choice Behavior

Following our pre-registration, we restrict our analysis to common rounds in which subjects execute a decision with a response time greater than 0.5 seconds, which generates a sample of 50,129 decisions. Across both conditions, subjects choose to invest on 58.9% of rounds.

In the upper panel of Figure 4, we plot the probability of investing as a function of the fundamental, separately for the two experimental conditions. One can see that, in both conditions, the aggregate data are consistent with the prediction from cognitive noise that subjects implement strategies that are continuous and monotone in θ. The data are therefore consistent with the predicted relationship between θ and the probability of investing from Proposition 3. Importantly, the smooth decreasing relationship between θ and the probability of investing obtains even without introducing any explicit private signals about θ, which are typically implemented in global games experiments. Our interpretation is that subjects generate their own "homegrown" private signals about θ, because cognitive

[12]Heinemann, Nagel and Ockenfels (2009) use a similar visual display of a coordination game (see their Figure 1). Note also that our experimental instructions emphasize that the subject's opponent views the same screen as she does, and our comprehension quiz tests subjects' understanding of how choices translate into earnings for both subjects in the game.



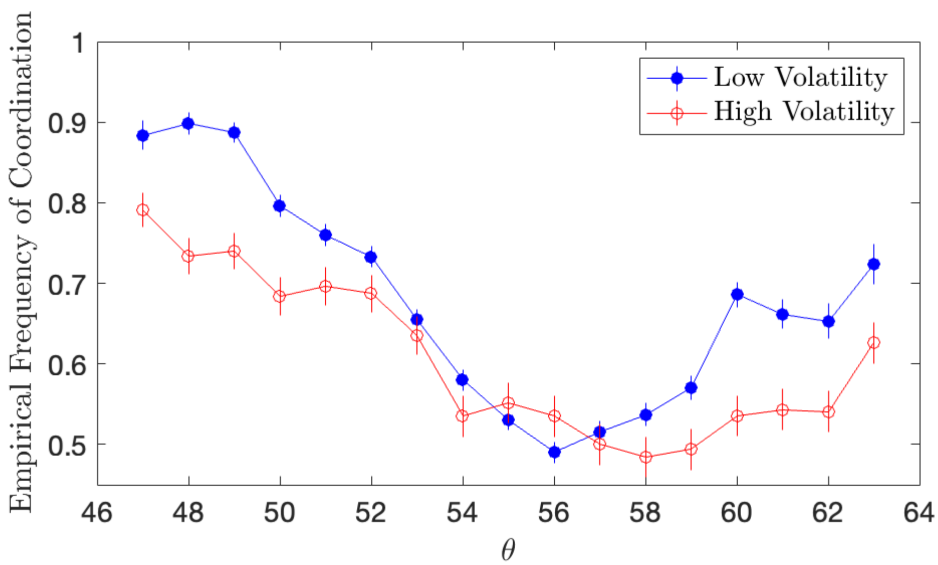


Figure 4: **Empirical Frequency of Investing and Coordination as a Function θ.** Note: In the Upper Panel, for each value of θ between 47 and 63, we plot the proportion of rounds on which a subject chooses to invest, separately for each of the two conditions. Data are pooled across subjects and are shown for rounds 31-300, after an initial 30-round adaptation period. Vertical bars inside each data point denote two standard errors of the mean. Standard errors are clustered by subject. In the Lower Panel, we plot the proportion of games for which the pair of subjects coordinate (both subjects invest or neither subject invests). Data are shown for rounds 31-300. Vertical bars inside each data point denote two standard errors of the mean. Standard errors are clustered by subject pair.

constraints prevent them from precisely processing θ. In the bottom panel of Figure 4, we plot the frequency of coordination outcomes as a function of θ. In both conditions, we observe a systematic relationship between the likelihood of coordination and θ: coordination is more likely as θ becomes farther from 55.

In order to provide a more targeted test of cognitive noise, we focus on the second prediction from Proposition 3, which implies that the distribution of noisy signals should vary systematically across our two experimental conditions. Specifically, efficient coding predicts context-dependent behavior, where subjects in the low volatility condition can more precisely detect whether the fundamental crosses the unique equilibrium threshold. The upper panel of Figure 4 provides evidence consistent with this prediction: the frequency of investing is more sensitive to the fundamental in the low volatility condition. The differential slopes shown in the upper panel of Figure 4 represent our main experimental result, which separates cognitive noise from a broad class of game-theoretic models that do not predict context-dependence.

To formally test the difference in slope, we estimate a series of mixed effects linear regressions which account for the fact that each subject contributes more than one observation to the dataset. Column (1) of Table 1 confirms our main result: the coefficient on the interaction term $(\theta - 55)$ x *Low* is significantly negative ($p < 0.001$), indicating that the probability of investing decreases in the fundamental more rapidly when a subject is adapted to the low volatility condition. Columns (2) and (3) show that this result holds in both early (first 50 trials after adaptation) and late (last 50 rounds of the session) subsamples (both with $p < 0.001$). Column (4) indicates that the treatment effect becomes moderately stronger over the course of the experiment, as the coefficient on the triple interaction is negative ($p = 0.024$). The strengthening of the treatment effect over the course of the experiment suggests that subjects have not fully adapted to the distribution by round 80 and that additional rounds of play provide the opportunity for further adaptation.

The bottom panel of Figure 4 shows that coordination also exhibits a strong degree of context-dependence. Subjects in the low volatility condition are significantly more likely to coordinate their behavior than subjects in the high volatility condition (63.8% vs. 60.5%; $p < 0.001$ for a difference in means). Moreover, this difference in coordination frequency is more pronounced for games where θ is farther from 55, consistent with the theoretical prediction shown in the bottom panel of Figure 2. The difference in coordination frequency across conditions also holds (and becomes moderately stronger) when we control for θ. In sum, our main results in Figure 4 demonstrate that (i) coordination frequency depends systematically on θ and that (ii) increasing the precision with which subjects process information about θ increases the likelihood of coordination.

While subjects do not receive feedback after each round, it is still possible that they learn

Dependent Variable: Pr(Invest)	(1)	(2)	(3)	(4)
$(\theta - 55)$	-0.040***	-0.042***	-0.040***	-0.042***
	(0.002)	(0.003)	(0.003)	(0.003)
$(\theta - 55)$ x Low	-0.022***	-0.018***	-0.024***	-0.018***
	(0.004)	(0.004)	(0.004)	(0.004)
Low	-0.040	-0.032	-0.018	-0.028
	(0.028)	(0.030)	(0.030)	(0.028)
Late				0.001
				(0.008)
$(\theta - 55)$ x Late				0.003
				(0.002)
Low x Late				0.092
				(0.154)
Low x $(\theta - 55)$ x Late				-0.007**
				(0.003)
Constant	0.624***	0.618***	0.607***	0.614***
	(0.018)	(0.020)	(0.021)	(0.019)
Observations	50,129	9,425	9,201	18,626
Rounds	31-300	31-80	251-300	(31-80)
				& (251-300)

Table 1: **Treatment Effect Estimates.** Note: Table displays results from mixed effects linear regressions. Observations are at the subject-round level. The dependent variable takes the value 1 if the subject chooses to Invest and 0 otherwise. The variable *Low* takes the value 1 if the round belongs to the low volatility condition and 0 otherwise. The variable *Late* takes the value 1 if the round number is 251 or greater and 0 otherwise. Only data from rounds where $46 < \theta < 64$ are included in the regressions. There are random effects on $(\theta - 55)$ and the intercept. Standard errors of the fixed effect estimates are clustered at the subject level and shown in parentheses. ***, **, * denote statistical significance at the 1%, 5%, and 10% levels, respectively.

about the strategic environment through repeated exposure to the game, as in Weber (2003) and Rick and Weber (2010). Moreover, our experimental design implies that subjects in different conditions will experience the same game, characterized by θ, a different number of times (e.g., games characterized by a value of θ close to 55 will occur more frequently in the low volatility condition). This raises the potential concern that our observed treatment effect is due to the differential ability to learn, rather than to cognitive noise.[13]

To investigate the learning explanation, Table 2 presents subsample results where we restrict to rounds for which subjects have identical experience with a given game in both conditions. In particular, the first column restricts to those rounds on which subjects in the low and high volatility conditions have previously observed 3 games with the same value of θ as in the current round. Columns (2) – (4) further restrict the data based on more and more experience with a given game. The regression results indicate that our treatment effect obtains among each of the different subsamples (at the 1% significance level). Thus, learning cannot explain the entire treatment effect we observe. Moreover, while learning could potentially modulate the strength of the relationship between θ and the probability of investing, we emphasize that another theory is still needed to explain why there is a continuous and monotonic relationship in the first place. Cognitive noise generates both the monotonicity and the context-dependence.

It is important to point out that the results in Figure 4 are aggregated across subjects. Therefore, while the data are consistent with the prediction that, at the individual subject level, signals are drawn from a noisier distribution in the high volatility condition, there is another potential explanation based on aggregation. Specifically, suppose that subjects perceive θ perfectly and that they use a potentially non-equilibrium cutoff strategy. Further suppose that there is heterogeneity with respect to the cutoff that each subject adopts. If some subjects use low cutoffs, while others use high cutoffs, then this heterogeneity would give rise to the decreasing relationship observed in both aggregate curves in Figure 4. In addition, if the variance in cutoff strategies across subjects is larger in the high volatility condition, then this alternative hypothesis could explain the weaker relationship between θ and the probability of investing in the high volatility condition. To investigate this alternative hypothesis, based on heterogeneity of cutoff strategies, we structurally estimate the model to obtain subject-specific cutoffs and measures of cognitive noise.

[13]For related theoretical work on learning about payoffs in coordination games and the resulting contagion, see Steiner and Stewart (2008).

Dependent Variable: Pr(Invest)	(1)	(2)	(3)	(4)
$(\theta - 55)$	-0.039***	-0.040***	-0.042***	-0.044***
	(0.003)	(0.003)	(0.003)	(0.004)
$(\theta - 55)$ x Low	-0.017***	-0.018***	-0.017***	-0.019***
	(0.004)	(0.004)	(0.004)	(0.005)
Low	-0.042	-0.038	-0.045	-0.056
	(0.026)	(0.028)	(0.028)	(0.030)
Constant	0.625***	0.614***	0.624***	0.629***
	(0.019)	(0.021)	(0.020)	(0.022)
Observations	4249	4001	3582	3065
Rounds of Experience with Game (θ)	3	4	5	6

Table 2: **Controlling for Experience with θ.** Note: Table displays results from mixed effects linear regressions. Observations are at the subject-round level. The dependent variable takes value 1 if the subject chooses to Invest and 0 otherwise. The variable *Low* takes value 1 if the round belongs to the low volatility condition and 0 otherwise. Only data from rounds where $46 < \theta < 64$ are included in the regressions. There are random effects on $(\theta - 55)$ and the intercept. Standard errors of the fixed effect estimates are clustered at the subject level and shown in parentheses. ***, **, * denote statistical significance at the 1%, 5%, and 10% levels, respectively.

4.2 Structural Estimation

According to the model described in Section 2, subject i chooses the parameters of the encoding rule, $m_i(\theta) = \xi_i + \psi_i \theta$. She then observes a noisy internal representation, $S_i = m_i(\theta) + \epsilon_i$. If we define a transformed version of the noisy internal representation as $Z_i = (S_i - \xi_i)/\psi_i$, then, for a cutoff $Z_i^\star$, our model predicts that she invests if and only if $Z_i \leq Z_i^\star$. In the unique monotone equilibrium of the game with cognitive noise, all subjects in the same treatment choose the same $(\xi_i, \psi_i, Z_i^\star)$. Here, we allow subjects to make heterogeneous (non-equilibrium) choices and we structurally estimate these parameters using behavior observed in the experiment.

Consider subject i who adopts a cutoff value of $Z_i^\star$ and, in round t, receives a noisy internal representation $S_{it} = \xi_i + \psi_i \theta_t + \epsilon_{it}$. The probability that subject i invests in round t is the probability that her transformed noisy internal representation is below her cutoff:

$$\mathbb{P}(\text{Invest} | \theta_t, \sigma_S, \psi_i, Z_i^\star) = \Phi\left(\frac{Z_i^\star - \theta_t}{\sigma_S / \psi_i}\right) \tag{2}$$

We structurally estimate the model using maximum likelihood estimation. In particular, for each subject, we estimate the standard deviation of the transformed noisy internal representation, $\sigma_i = \sigma_S/\psi_i$, and the cutoff $Z_i^{\star}$.[14] We maximize the following log-likelihood function over $(\sigma_i, Z_i^{\star})$, using behavior in rounds 31 – 300:

$$LL\left(\sigma_i, Z_i^{\star}, \mathbf{y}_i\right) = \sum_{t=31}^{300} y_{it} \cdot \log\left(\mathbb{P}(\text{Invest}|\theta_t, \sigma_i, Z_i^{\star})\right) + (1 - y_{it}) \cdot \log(1 - \mathbb{P}(\text{Invest}|\theta_t, \sigma_i, Z_i^{\star})), \tag{3}$$

where $\mathbf{y}_i \equiv \{y_{it}\}_{t=31}^{300}$ and y_{it} denotes the subject's choice in round t, with $y_{it} = 1$ if the subject chooses to invest and $y_{it} = 0$ if the subject chooses not to invest. We maximize the log-likelihood function in (3) by searching over grid values of $[\sigma_i, Z_i^{\star}] \in [0.1, 50.1] \times [11, 99]$, in increments of 0.5 along each dimension.

Figure 5 plots the distribution of estimated parameters for the 300 subjects (150 in each condition). Beginning with the upper panel, we see that, for most subjects, the estimated cutoff lies between 50 and 60. The mean cutoff in the high volatility condition is 58.5 and the mean cutoff in the low volatility condition is 57.2. These means are not significantly different from one another ($p = 0.15$). The average cutoff in each condition is, however, significantly greater than 55. As can be seen from the figure, this difference relative to 55 is driven mainly by the right tail of the distribution, which captures a small fraction of subjects who almost always choose to invest.

More importantly, we find that the standard deviation of estimated cutoffs is not significantly different across conditions (8.4 in high volatility vs. 7.5 in low volatility, $p = 0.43$ Levene's test). This suggests that heterogeneity in cutoffs is *not* driving the treatment effect. If it were, we would have observed a more concentrated distribution of cutoffs in the low volatility condition and, thus, a significantly lower standard deviation of estimated cutoffs in the low volatility condition.

Instead, the lower panel of Figure 5 reveals that the difference in behavior across conditions stems from the standard deviation of the noisy internal representations. The mean estimated value of σ_i is significantly higher in the high volatility condition (14.4 vs. 5.9, $p < 0.001$). One can easily see from the figure that this effect holds not only on average, but across the whole distribution. In summary, while the aggregate data in Figure 4 are consistent with subjects in the high volatility condition exhibiting (i) a wider dispersion of cutoffs or (ii) a higher amount of noise in the internal representation of the fundamental, our structural estimation indicates that the effect is coming only through the second channel, as

[14] We cannot separately identify σ_S and ψ_i since these two parameters are perfect substitutes in the conditional density of Z_i. At the same time, while ψ_i is an endogenous variable, we interpret σ_S as an exogenous parameter, capturing the degree of a subject's cognitive capacity. In Section 2, we assume σ_S is homogeneous. Even if we allow for heterogeneity across subjects, the randomization into experimental conditions guarantees a similar distribution of σ_S in the two sub-populations. For this reason, we attribute any difference in the distribution of the estimated σ_i's across conditions to differences in ψ_i.

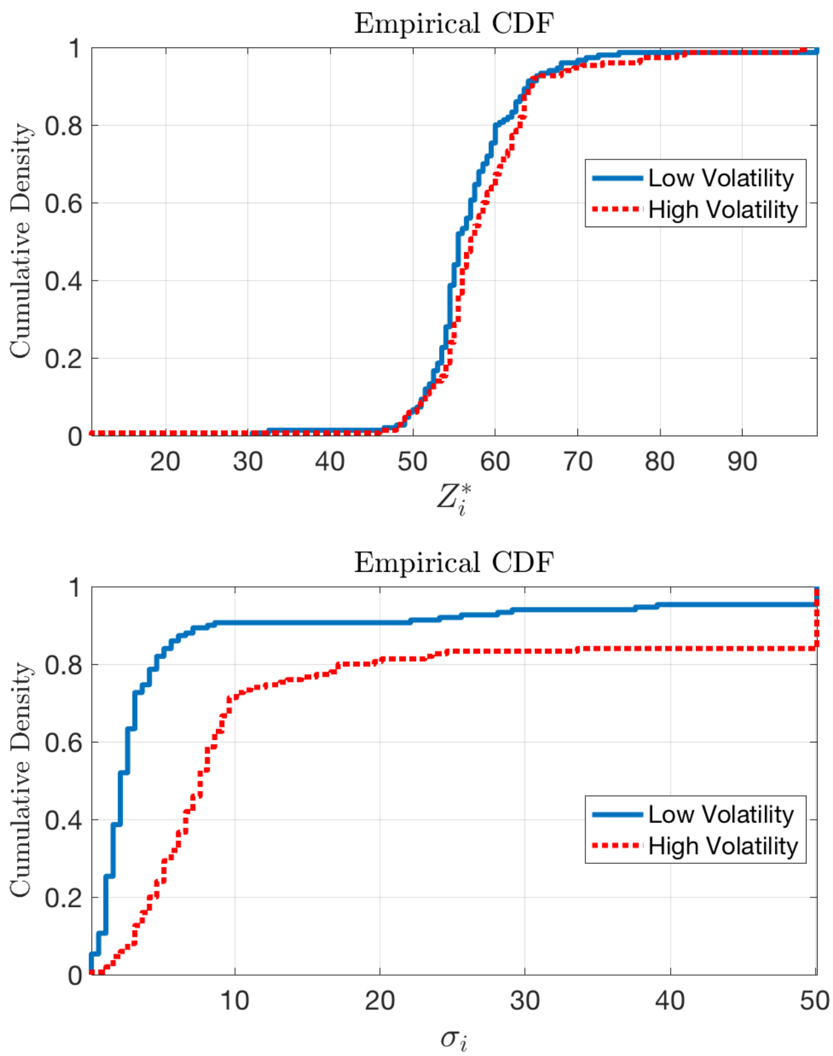


Figure 5: **Empirical CDFs of Subject-Level Structural Estimates.** Note: Upper panel is the empirical CDF of estimated cutoffs. Lower panel is the empirical CDF of estimated standard deviations of noisy internal representations.

predicted by the theory developed in Section 2.

4.3 Response Times

Here we analyze the distribution of response times in both conditions. The response time variable is defined at the round level, and measures how long it takes a player to execute a decision after the game is presented on the screen. As outlined in our pre-registration, we test two hypotheses regarding the distribution of response times. First, response times should peak at the unique equilibrium cutoff level of 55. Second, conditional on θ, response

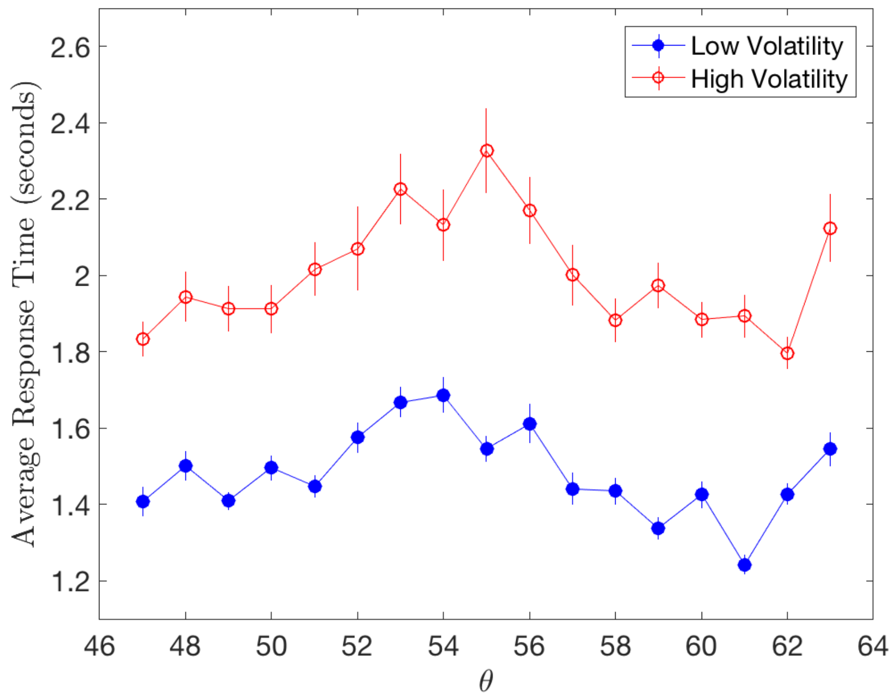


Figure 6: **Average Response Time as a Function of θ.** Note: Response times are averaged across subjects and across rounds. Vertical bars denote two standard errors of the mean. Standard errors are clustered by subject.

times should be longer in the high volatility condition. Our hypotheses are motivated by the literature on sequential sampling models (Ratcliff, 1978; Bogacz, Brown, Moehlis, Holmes and Cohen, 2006), which robustly predict that response times become longer as the values of two items under comparison become closer together. Thus, the tests we present in this section are joint tests of cognitive noise, which predicts that subjects use a unique threshold strategy, and sequential sampling models, which predicts how long it takes to implement the threshold strategy on each round.

In many sequential sampling models (see, e.g., Krajbich, Armel and Rangel 2010), the agent will execute a decision as soon as a stream of incoming signals has reached a pre-defined reliability threshold. Because signals are sampled sequentially, response times increase with the number of signals drawn. While the model we present in Section 2 only allows the agent to draw a single noisy signal, S_i, one could generalize the model to allow a sequence of independent noisy signals. For every additional noisy signal that the player collects, her posterior will become narrower, and, thus, the entire stream of signals provides more reliable evidence about whether θ is less than 55. As signals become more informative about whether θ is below the (equilibrium) threshold, the agent will reach the pre-defined reliability threshold with fewer signals, and thus response times will be shorter.

In our setting, there are two particular ways in which a signal can provide more information about whether θ is less than 55. First, recall that in our model, the mean of S_i varies monotonically with θ. Thus, S_i provides cardinal information about θ, and not just ordinal information about whether θ is below 55. It follows that as $|\theta - 55|$ increases, S_i provides a more informative signal about whether $\theta < 55$. Second, as the precision of S_i increases, this naturally provides more information about whether $\theta < 55$. Taken together, sequential sampling models predict that, when a player is tasked with implementing a cutoff strategy (which is derived as the equilibrium strategy under cognitive noise), response times should decrease as (i) $|\theta - 55|$ increases and (ii) the precision of S_i increases. We can test the first prediction by relying on variation in θ within an experimental condition. We can test the second prediction by relying on the variation in signal precision across conditions, which is endogenously generated by efficient coding.

Figure 6 plots the average response time, conditional on θ, for each of the two experimental conditions. We highlight two features of the figure. First, we see that in the high volatility condition, the peak response time is at $\theta = 55$; in the low volatility condition, the peak is not far away, at $\theta = 54$. Moreover, response times fall almost monotonically as θ moves away from the equilibrium threshold of 55 ($p = 0.001$ in a mixed-effects regression of response time on $|\theta - 55|$ for each of the two conditions). Second, there is a clear separation of the curves across conditions: conditional on θ, response times are longer in the high volatility condition compared to the low volatility condition (unconditionally, the average response time is significantly longer in the high volatility condition, $p < 0.001$). These two features of the data are roughly consistent with the predictions outlined above.

One caveat to our analysis of response times is that the precision of S_i is chosen by the player according to efficient coding, but under the assumption that she can only draw one signal. The predictions may change if one were to endogenize the signal precision and the number of signals to be drawn (or the reliability threshold). That said, the data shown in Figure 6 provide suggestive evidence that subjects are implementing threshold strategies in a manner that is consistent with core predictions of sequential sampling models. In this manner, the response time data help validate our model assumptions about the cognitive constraints that subjects face when playing the game.

5 Experiment on Decomposing Structural Uncertainty and Strategic Uncertainty

In our model, all noise in behavior stems from uncertainty about a game payoff (i.e., *structural uncertainty*) that is induced by cognitive noise. As shown in inequality (1), cognitive noise also generates *strategic uncertainty* — that is, uncertainty about the opponent's valuation and behavior — which is key to pinning down a unique equilibrium. We emphasize that in the model, the only source of strategic uncertainty is cognitive noise, but in reality, there are surely other sources of strategic uncertainty. For example, there may be uncertainty about the opponent's degree of rationality, the opponent's preferences, or which of multiple equilibria (of the complete information version of the game) the opponent is playing. The stochastic behavior we observe in Figure 4 can therefore be a consequence of noise in processing θ or alternative sources of strategic uncertainty. Our objective in this section is to quantitatively assess how much of the observed noise in behavior can be attributed to cognitive noise and how much is driven, instead, by other sources of strategic uncertainty.

To address this question, we conduct a second experiment in which a new sample of subjects plays the same simultaneous move game as in the previous experiment. The only difference is that, here, subjects are told that their opponent is a computer that plays a known and deterministic strategy. In particular, we tell subjects that the computerized opponent chooses to invest if and only if $\theta < 55$. Thus, the computerized opponent's strategy coincides with the unique equilibrium strategy in the game where each player has a small amount of cognitive noise about θ.[15] This treatment should, therefore, eliminate strategic uncertainty — except for the strategic uncertainty that is induced by a subject's own imprecision of θ.

5.1 Experimental Design and Procedures

As in the previous experiment, we incentivize subjects to play the simultaneous move game described in Figure 1. In the previous experiment, we manipulated the distribution from which θ is drawn in each round. Here, we use the distribution from the high volatility condition in the previous experiment, where $\theta \sim N(55, 400)$, but we tell subjects that their

[15] In the game where each human player has cognitive noise, player i is indifferent between investing and not investing when (a) $E[\theta|Z_i] = 55$ and (b) player i believes his human opponent follows the strategy prescribed by the unique equilibrium from Proposition 2. Because our goal here is to completely remove any uncertainty about the opponent's strategy that is not induced by noisy perception of θ, we design the computerized opponent to play a deterministic strategy when $\theta = 55$, namely, not invest with probability 1. As a consequence, in the game where $\theta = 55$, the best response of a human subject who perceives θ without noise is not to invest. This is consistent with the (indeterminate) best response to equilibrium beliefs in the game where each human player has cognitive noise. Because our design choice for the computer strategy when $\theta = 55$ is arbitrary, we show below that our results are robust to removing games for which $\theta = 55$.

opponent is a computer. Subjects play three hundred rounds of the game, where the only difference across games is the random value of θ. Because we tell subjects that the computerized opponent will invest if and only if $\theta < 55$, the subject has a dominant strategy for all θ: invest if and only if her perception of θ is greater than 55.[16]

We pre-register the experiment and recruit 100 subjects from Prolific.[17] We apply the same recruitment restrictions as in the previous experiment. The experimental instructions are in Online Appendix E. Subjects are paid 2 GBPs for completing the experiment and are also paid according to the outcome on one randomly drawn round. Unlike in the previous experiment, here, the outcome depends exclusively on the subject's own decision since the computerized opponent plays a known and deterministic strategy. The median duration of the experiment was around 21 minutes and the average earnings, including the participation fee, were 6.30 GBPs.

5.2 Experimental Results

Following our pre-registration, we restrict our analysis to rounds where $\theta \in [47, 63]$ and where the subject executes a decision with a response time greater than 0.5 seconds. Our focus is on comparing behavior when subjects play against a computerized opponent (Algorithm) with behavior from the high volatility condition from the previous experiment (Human). By fixing the prior distribution across conditions, we control for any efficient coding effects.

Figure 7 plots the data from both the Algorithm and Human conditions. If subjects were precisely implementing the threshold strategy with a threshold of 55, then we should observe a step function around $\theta = 55$. Instead, one can see that there is obviously noise in both conditions. However, behavior appears less noisy in the Algorithm condition compared to the Human condition.

To formally investigate the difference in noise across conditions, we run a linear mixed effects regression where the dependent variable is a dummy that takes on the value 1 if the subject invests and is 0 otherwise. The independent variables are θ, the dummy variable $Human$ which indicates whether the observation is in the Human condition, and the interaction between θ and $Human$. There are random effects on the intercept and on θ. Column (1) of Table 3 shows that the estimated coefficient on θ is significantly negative while the coefficient on the interaction term is significantly positive. These results indicate that the probability of investing declines with θ in both conditions, but also that this probability

[16]For other experiments where a game is reduced to an individual decision problem by using computerized opponents, see Roth and Murnighan (1978), Fehr and Tyran (2001), Esponda and Vespa (2014), and Koch and Penczynski (2018).

[17]The pre-registration document is available at: https://aspredicted.org/339_B5N.

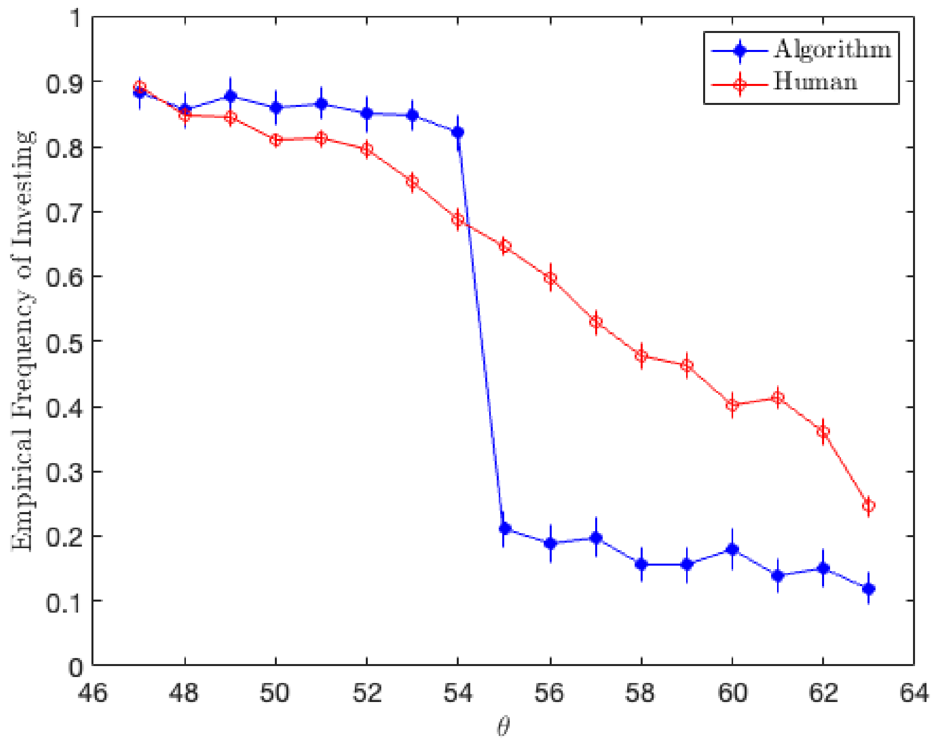


Figure 7: **Empirical Frequency of Investing as a Function of Opponent Type** Note: For each value of θ between 47 and 63, we plot the proportion of rounds on which a subject chooses to invest. The Algorithm condition denotes the data collected in the additional experiment where the subject plays against a computerized opponent. The Human condition denotes the data collected in the high volatility condition from our main experiment. Data are pooled across subjects for all rounds 1-300. Vertical bars inside each data point denote two standard errors of the mean. Standard errors are clustered by subject.

declines more rapidly in the Algorithm condition. This, in turn, suggests that subjects are implementing the threshold strategy with significantly less noise when they play against a computer compared to when they play against a human.

One concern with the previous test about existence of noise in the Algorithm condition is that, even under the null hypothesis of zero noise in the Algorithm condition, the estimated coefficient on θ would be negative (as long as there is some measurement error). This is because the probability of investing drops from 1 to 0 when θ crosses 55. However, continuing under the null hypothesis of zero noise, there should be no variation in behavior when conditioning on values of $\theta > 55$; similarly, there should be no variation in behavior when conditioning on values of $\theta < 55$. In columns (2) and (3) of Table 3, we show that the coefficient on θ remains significantly negative in both subsamples. Therefore, in the Algorithm condition, the probability of investing declines for $\theta \in [47, 54]$ and it also declines for $\theta \in [56, 63]$. This is consistent with the predictions of our model of cognitive noise.

In sum, there are two main takeaways from Table 3: when subjects play a simultaneous move game against a computerized opponent, (i) we continue to detect substantial noise in behavior (and the pattern of noise is consistent with our model) and (ii) the noise is smaller compared to when subjects play against a human opponent. We attribute the reduction of noise to alternative sources of strategic uncertainty that are present in our original experiment and are not driven by imprecision over θ.[18]

To quantitatively assess how much noise in behavior can be attributed to noise in encoding and processing θ compared to other sources of noise, we estimate the amount of noise in each condition non-parametrically. For each subject and each round, we code behavior as "consistent" if and only if the subject chooses the action prescribed by the threshold strategy of "choose invest if and only if $\theta < 55$". If a decision is not coded as consistent, we attribute the decision to noise.[19] We find that, in the Human condition, 31.8% of decisions are driven by noise. In the Algorithm condition, noisy behavior drops significantly to 15.3% of decisions (and the difference is statistically significant at the 0.1% level).[20] Thus, about half of the noise from the Human condition appears to be driven by imprecision in θ while the other half is driven by alternative factors outside our model. Our interpretation is that noise in encoding and processing values of θ drives a substantial portion of observed noise in the Human condition, but that there are clearly other important sources of noise that reflect uncertainty about the human opponent's strategy, preferences, or information. These latter sources of uncertainty are shut down by design in our Algorithm condition.

6 Discussion

6.1 Connection with Global Games

One theme that emerges from both our theoretical and experimental analyses is that the noise in global games models can be interpreted literally as irreducible error stemming from

[18]We find that response times in the Algorithm condition decrease as θ approaches 55 ($p < 0.001$ in a mixed effects linear regression of response time on $|\theta - 55|$). This "distance effect" is similar to the results shown for the Human condition in Figure 6. We do not find any significant difference in average response time across the Human and Algorithm condition. Together with the choice data shown in Figure 7, this indicates that subjects implement the threshold strategy with more precision in the Algorithm condition without taking significantly more time to execute these decisions.

[19]For the remaining analyses in this section, we discard observations for which $\theta = 55$. This restriction is outlined in our pre-registration and is due to the fact that there is no way to unambiguously code behavior in the human condition (because subjects should be indifferent when $\theta = 55$).

[20]When lifting the restriction that $\theta \in [47, 63]$, we find that, in the Human condition, 18.0% of decisions are driven by noise, compared to 11.9% in the Algorithm condition and the difference remains statistically significant ($p = 0.011$).

Dependent Variable: Pr(Invest)	(1) $(46 < \theta < 64)$	(2) $(55 < \theta < 64)$	(3) $(46 < \theta < 55)$
θ	-0.063***	-0.010***	-0.007***
	(0.004)	(0.003)	(0.002)
θ x Human	0.023***		
	(0.004)		
Human	-1.122***		
	(0.243)		
Constant	3.958***	0.765***	1.231***
	(0.206)	(0.162)	(0.108)
Observations	24,966	4,639	4,717

Table 3: **Comparing Behavior Across Human and Algorithm Condition** Note: Table displays results from mixed effects linear regressions. Observations are at the subject-round level. The dependent variable takes value 1 if the subject chooses to Invest and 0 otherwise. The variable *Human* takes value 1 if the round belongs to the Human condition and 0 otherwise. Column (1) includes data from both the Human and Algorithm conditions and results are robust to excluding games where $\theta = 55$. Columns (2) and (3) include data only from the Algorithm condition. There are random effects on θ and the intercept. Standard errors of the fixed effect estimates are clustered at the subject level and shown in parentheses. ***, **, * denote statistical significance at the 1%, 5%, and 10% levels, respectively.

cognitive constraints. This theme is related to the idea from Heinemann, Nagel and Ockenfels (2009) that behavior in a complete information coordination game can be interpreted *as if* players are observing a fundamental parameter with noise. Like us, Heinemann, Nagel and Ockenfels (2009) structurally estimate a global games model and find a sizable standard deviation of private signals. However, Heinemann, Nagel and Ockenfels (2009) argue that the only source of the estimated standard deviation of private signals is strategic uncertainty that does not arise from structural uncertainty.[21] In contrast, we argue that the standard deviation of private signals is driven by cognitive noise. By adopting an "as is" interpretation of noise in private signals, we are able to generate and test novel hypotheses about how the standard deviation of private signals varies across environments.

Another important implication for the literature on global games has to do with the role of public vs. private signals. A series of papers has argued that when an institution like the government or a financial market can generate public signals, then a unique equilibrium may no longer obtain in a global games model (Atkeson, 2000; Angeletos and Werning, 2006; Hellwig, Mukherji and Tsyvinski, 2006). The argument is that a sufficiently precise

[21] Heinemann, Nagel and Ockenfels (2009) argue that "Of course, players know the true payoff. Their uncertainty about others' behavior makes them behave *as if* they are uncertain about payoffs" (p. 203).

public signal can act as a coordination device, and thus restore multiple equilibria. However, our theory and experimental results suggest that there is an important difference between *access* to a public signal and precise processing of a public signal. Specifically, even if all players have access to the public signal, each player may encode the same public signal with noise and thus interpret it slightly differently. This friction, driven by constraints that arise internally in the agent's mind, transforms the public signal into private information and makes it difficult to use the public signal as a coordination device. Our results, therefore, imply that the provision of a public signal is not enough to overturn the classic global games result. The ability to precisely perceive and process public information is also necessary and, as we have shown, this cannot be taken for granted.

6.2 Comparison with Other Behavioral Game Theory Models

Behavioral game theorists have proposed a variety of models which relax the standard assumptions of perfect maximization and rational beliefs. For example, Quantal Response Equilibrium assumes imperfect maximization but retains the rational beliefs assumption; Level-K Thinking relaxes the rational expectations assumption but maintains best responses; M equilibrium relaxes both the rational expectations and perfect maximization assumptions. Below, we derive predictions from these three behavioral game theory models and demonstrate how our model differs in terms of both assumptions and predictions. As we will see, one important conclusion is that none of the theories predict the context-dependent behavior we observe experimentally.

Quantal Response Equilibrium

In our model of cognitive noise, noisy encoding of θ generates stochastic strategic behavior. As such, our model is related to Quantal Response Equilibrium (McKelvey and Palfrey, 1995, 1998; Goeree, Holt and Palfrey, 2016), which is a leading model of stochastic behavior in experimental game theory.[22] For some parameter values, the models of QRE and cognitive noise deliver similar predictions, in that both theories predict that the probability of investing is stochastic and decreases smoothly and monotonically in θ. However, there are fundamental differences in the assumptions of the two theories, which generate distinguishing predictions.

The key difference in assumptions comes from the stage at which noise enters the decision process.[23] In our model, noise arises early in the decision process, before each player has

[22] For other models of strategic interaction with stochastic choice, see Goeree and Holt (2004), Friedman and Mezzetti (2005), and Gonçalves (2022). We discuss Goeree and Louis (2021) later in this sub-section.

[23] We are grateful to Michael Woodford for emphasizing this point in an illuminating discussion of our paper.

computed the expected utility of each action. In contrast, under QRE, noise arises late in the decision process, after each player has perfectly perceived all parameters of the game and precisely computed the expected utility of each action.

In the coordination game we study in this paper, QRE predicts that a player invests if and only if:

$$\begin{aligned} \text{EU[Not Invest]} + \eta_1 &< \text{EU[Invest]} + \eta_2 \\ \theta + \eta_1 &< a + p[b-a] + \eta_2 \\ \theta &< a + p[b-a] - (\eta_1 - \eta_2), \end{aligned} \tag{4}$$

where p is the belief about the probability the opponent invests, and η_1 and η_2 are the late noise perturbations to payoffs. Before making her choice, each player receives a perfectly informative signal about η_1 and η_2 (uncorrelated with the opponent's perturbations to payoffs). If we assume that these perturbations are independently and normally distributed with mean 0 and variance $\sigma_\eta^2 > 0$, we have:

$$\mathbb{P}(\text{Invest}) = \phi(p,\theta) = \Phi\left(\frac{a + p[b-a] - \theta}{\sqrt{2}\sigma_\eta}\right).$$

A quantal response equilibrium then requires that p is a fixed point, conditional on θ; i.e., a QRE is a solution to $p = \phi(p,\theta)$.

It is useful to compare the condition for investing under QRE (displayed in inequality (4) above) with the analogous condition for investing under cognitive noise (displayed in inequality (1) in Section 2). Inequality (1) indicates that, with cognitive noise, players remain uncertain about the true value of θ even after θ is realized; the residual uncertainty comes from the fact that players only have access to a noisy representation of θ. As a consequence, player i believes that player j's signal about θ is centered at i's perceived value of θ (which is a function of i's signal about θ). In contrast, the true value of θ appears in inequality (4), which implies that, in QRE, the player has no uncertainty about θ. It follows that, in QRE, player i believes that player j's signal about θ is centered at the true value of θ.

The difference in assumptions about when noise enters the decision process leads to two important distinguishing predictions. The first difference is that, in QRE, each player encodes θ precisely, and thus there is no role for a prior belief over θ. The prior belief does, however, play a key role in our model of cognitive noise. Specifically, our model predicts that the prior belief affects the precision of perceiving θ through efficient coding. Our model therefore endogenizes the noise structure and generates context dependent behavior in equilibrium.[24] The main experimental result in our paper, displayed in Figure 4, clearly shows that the prior distribution has a systematic effect on behavior. The result supports the prediction of cognitive noise and is at odds with the prediction of QRE (unless the researcher is allowed to assume a different and ad hoc distribution of payoff perturbations in the QRE model for each experimental condition).

The second difference between QRE and cognitive noise involves the theoretical conditions that are sufficient to generate a unique equilibrium. As shown in Proposition 3, cognitive noise generates a unique equilibrium when the variance of noise is sufficiently small. One interpretation of this condition is that when players pay sufficient attention to the coordination game, so that the variance of the internal representation Z_i is sufficiently small, then uniqueness obtains under our theory of cognitive noise. In contrast, QRE delivers a unique equilibrium when the variance of the shock to payoffs is sufficiently *large* (Ui, 2006). While our data do not enable us to test between this difference in conditions for uniqueness, one implication is that when players devote a substantial amount of attention to the coordination game, the multiplicity of equilibria is more likely to be eliminated under cognitive noise, compared with QRE.

Level-k Thinking

Our results also relate to another behavioral theory of games called Level-k Thinking (Stahl and Wilson, 1994, 1995; Nagel, 1995; Camerer, Ho and Chong, 2004). In one prominent version of this theory, there are different types of players, and each type best responds to another type who exhibits one less degree of strategic sophistication. For example, a Level-0 type would be characterized by no strategic sophistication and, thus, would exhibit purely random behavior. A Level-1 type would then best respond to a Level-0 player, and a Level-2 player would best respond to a Level-1 player, and so on. What are the predictions of Level-k Thinking for the game in our experiment? Following the analysis in Kneeland (2016) and given that Level-0 players randomize, the expected utility of a Level-1 player from Invest is

$$EU_{L1}(\text{Invest}) = \frac{1}{2}a + \frac{1}{2}b$$

Thus, $EU_{L1}(\text{Invest}) > EU(\text{Not Invest})$ if and only if $\theta < (a+b)/2$. Next, under the assumption that Level-2 players believe they are facing a Level-1 opponent, the expected utility

[24] In QRE, the noise structure is usually taken to be exogenous. Friedman (2020) proposes a model that endogenizes the precision parameter in QRE through the set of payoffs in the current game.

from Invest for a Level-2 player is

$$EU_{L2}(\text{Invest}) = \begin{cases} b \text{ if } \theta < (a+b)/2 \\ a \text{ if } \theta > (a+b)/2 \end{cases}$$

When $\theta < (a+b)/2$, then $EU_{L2}(\text{Invest}) = b > \theta$. Conversely, when $\theta > (a+b)/2$, then $EU_{L2}(\text{Invest}) = a < \theta$. Thus, Level-2 players choose Invest if and only if $\theta < (a+b)/2$. Using the same logic, we obtain the same prediction for all higher levels.

In sum, the fraction of subjects who choose Invest is:

$$\Pr[\text{Invest}] = \begin{cases} \Pr[L_0]\frac{1}{2} + (1 - \Pr[L_0]) & \text{if } \theta < (a+b)/2 \\ \Pr[L_0]\frac{1}{2} & \text{if } \theta > (a+b)/2 \end{cases}$$

where $\Pr[L_0]$ is the fraction of Level-0 players in the population. The theory therefore predicts that, in the aggregate, the probability of investing is monotone in θ and exhibits a sharp decrease at $\theta = (a+b)/2$. We do not observe such a discontinuity in our data. Moreover, Level-k Thinking does not predict any difference across our experimental treatments; thus the theory would need to be augmented with some extra feature in order to explain the clear context-dependence we observe in our data.

M Equilibrium

Finally, we discuss how our theoretical predictions relate to a recent and appealing behavioral game theory model, called "M equilibrium" (Goeree and Louis, 2021). M equilibrium replaces the assumptions underlying Nash equilibrium with two plausible behavioral postulates. First, instead of perfect best response, M equilibrium assumes *monotonicity*. Second, instead of perfectly correct beliefs about others' strategies, M equilibrium assumes *consequential unbiasedness*. Monotonicity allows non-best-response actions to be chosen with positive probability but prescribes that more costly mistakes are less likely to occur (similar to QRE). Consequential unbiasedness allows beliefs to be incorrect (in contrast to QRE), as long as they generate the correct ranking of expected payoffs from actions.

M equilibrium is a a set-valued equilibrium concept and, as such, is characterized by a set of predicted choices and a set of predicted beliefs (where the set of predicted beliefs does not coincide with, but includes, the set of predicted choices). In our game, when $\theta \in [a, b]$, M equilibrium does not make a determinate prediction about how θ affects the probability of investing — or even about which action is most likely to be played. In particular, when $\theta \in [a, (a+b)/2]$, there are two M-choice sets: one where $\Pr[\text{Invest}] \in [1/2, 1]$ and one where

Pr[Invest] $\in [0, (\theta - a)/(b - a)]$.[25] If instead, $\theta \in [(a + b)/2, 1]$, there are also two M-choice sets, one where Pr[Invest] $\in [0, 1/2]$ and one where Pr[Invest] $\in [(\theta - a)/(b - a), 1]$. Besides generating indeterminate predictions about behavior when $\theta \in [a, b]$, M equilibrium does not predict any effect of our experimental manipulation across volatility conditions.

6.3 The Effect of Experience Through the Prior Alone

When presenting our experimental results in Section 4.1, we discussed whether an alternative hypothesis based on learning about the strategic environment could explain the context dependence shown in Figure 4. Holding experience with a particular game constant across conditions, we still found evidence that the probability of investing is more sensitive to fundamentals in the low volatility condition. Here, we discuss whether an alternative specification of learning can generate the observed treatment effect.

The alternative specification we have in mind still allows the player's prior over θ to reflect past experience — as in our model of cognitive noise. Thus, subjects in each condition learn their way to different priors, which reflect the statistical properties of the environment they have experienced. However, here we shut down the efficient coding channel, so that the conditional noisy signal distribution remains fixed across conditions. To illustrate, suppose that in both experimental conditions, we set the distribution of Z_i to an arbitrary distribution. In particular, suppose it is the distribution that arises under efficient coding in the high volatility condition. How does the model prediction of this alternative learning hypothesis compare with the prediction from our model summarized in Figure 2?

It turns out that even when the priors are allowed to differ — for example, based on experience — the predictions for behavior in equilibrium will be identical across conditions. The solid curve in Figure 2 will become flatter and lie directly on top of the dashed curve. The intuition for why the predicted treatment effect vanishes is as follows. In our design, we set the prior mean in both conditions to be the average of the two potential payoffs from investing: $\mu_\theta = (a + b)/2$. Further, in both conditions, the equilibrium threshold in the space of posterior means does not depend on the prior variance and is equal to the prior mean (which, by design, is the same across conditions). Any mental representation Z_i that is below the prior mean leads to a posterior mean that is strictly smaller than the equilibrium threshold. Thus, in our design, it is only the distribution of Z_i that governs behavior when subjects play the equilibrium threshold strategy. When the distribution of Z_i is identical across conditions, there will be no predicted difference in behavior across conditions. In summary, our design gives rise to an environment in which we can cleanly test for the effect

[25] See Online Appendix A for a graphical construction of the M equilibria in our game.

of context dependence generated by efficient coding. A difference in the prior alone is not sufficient to generate our main experimental result in Figure 4.

6.4 Performance Objective for Efficient Coding

Here we revisit the assumption about efficient coding in our model. The specific performance objective that we assume in Section 2 is only one of several plausible specifications (Ma and Woodford, 2020). In particular, there are other possible objective functions that players may have, besides minimizing the mean squared error of the estimate of θ. For example, a prominent alternative efficient coding objective from the literature on sensory perception is to maximize the mutual information between the state and its noisy internal representation. In the proof of Proposition 1, we confirm that the coding rule we use in our model is robust to this alternative objective.

Yet another alternative objective that has been examined in the economics literature is maximization of expected reward. In Online Appendix B, we show that the result in Proposition 1 is robust to using this alternative objective function. Specifically, we maintain the constraints in Assumption 2 and we analyze a two-stage game. In the first stage, each player optimally chooses, simultaneously and independently, the parameters of the encoding function. In the second stage, players choose strategies in the simultaneous move game, conditional on their chosen encoding function from the first stage. We show that the optimal encoding function still takes the form characterized in Proposition 1. Thus, our theoretical predictions are robust to three performance objectives: (i) minimizing mean squared error of the estimate of θ, (ii) maximizing mutual information between the noisy internal representation and θ and (iii) maximizing expected reward.

6.5 Common Knowledge of Internal Representation Distribution

In deriving Proposition 2, we assume common knowledge of the distribution of internal representations. However, precise knowledge of the underlying information structure is not necessary for this equilibrium to arise. As evident from the statement of Proposition 2, the equilibrium exists regardless of the value of σ_θ, σ_S and Ω. It follows that the equilibrium exists even when players have incorrect beliefs about the information structure (maintaining the common knowledge assumption). This is important considering that, while we manipulate σ_θ in the laboratory, we do not control or measure σ_S and Ω.

As we show in Online Appendix C, the equilibrium from Proposition 2 is robust to relaxing the assumption that players have common knowledge of the exact functional forms of the prior and noisy signal distributions. In a model where the coding function is exogenous and equal to $m(\theta) = \theta$, it is enough to assume that (i) $\mu_\theta = (a+b)/2$, (ii) $E[\epsilon_i] = 0$, (iii) the distribution of ϵ_i is symmetric, quasiconcave and independent of the realized value of θ, (iv) the distribution of θ is symmetric and continuous on $\mathbb{R}$, and that there is common knowledge of (i) – (iv). At the same time, the lack of a closed form solution for the posterior distribution of θ under these more general assumptions prevents us from deriving conditions for the equilibrium to be unique (when the variance of noise is finite but positive). More importantly, the lack of a closed form solution prevents us from deriving predictions for the treatment effect. For these reasons, our theoretical analysis is based on a model with a normally distributed prior and normally distributed likelihood function.

It can also be shown that the equilibrium from Proposition 2 is not sensitive to beliefs about one's own or one's opponent's degree of imprecision. Consider the case where $\mu_\theta = (a+b)/2$ (as in the statement of Proposition 2 and in both experimental conditions). If player i (exogenously and possibly incorrectly) believes that the probability player j invests is greater than or equal to 50% for any $\theta \leq \mu_\theta$ and smaller than or equal to 50% for any $\theta \geq \mu_\theta$, then player i's best response is to invest if and only if $E[\theta|Z_i] < \mu_\theta$. In other words, as long as player i believes that player j is noisily implementing a cutoff strategy with cutoff μ_θ, his best response is to use the same cutoff strategy, *independent of his beliefs about his own and his opponent's degree of imprecision.* At the same time, we emphasize that, within our theoretical framework, the differential sensitivity of actions to payoffs across conditions requires that subjects are more precise in detecting whether the fundamental crosses the equilibrium threshold in the low volatility condition.

Apart from the demanding assumption of common knowledge of cognitive noise, here we briefly discuss whether it is plausible to assume that subjects know they are imprecise and that others are imprecise. To investigate the validity of this assumption, we conduct an additional experiment, where subjects are asked to classify whether a two-digit number is greater than a reference level of 55 (which we choose to be the same as the threshold in the unique equilibrium of the game in our main experiment). We incentivize subjects to report their beliefs about (i) the average accuracy of all other subjects in the experiment and (ii) their own accuracy. We find that subjects are aware of their own errors and of others' errors in the classification task. We refer the reader to Online Appendix D for a detailed presentation of the experimental design and results for this additional experiment.

7 Conclusion

In this paper, we have experimentally tested the hypothesis that cognitive noise systematically affects coordination. Cognitive noise plays two important roles in our experiments.

First, a small amount of cognitive noise is sufficient to generate a unique equilibrium whereby each player chooses to invest once the fundamental crosses a threshold. Importantly, the existence of cognitive noise leads to *more* predictable behavior, compared to the case in which there is no cognitive noise (as the strategic uncertainty induced by cognitive noise will eliminate multiple equilibria). The data are consistent with this prediction, as we observe that the frequency of investing predictably and continuously declines in θ. Importantly, our data also demonstrate that the frequency of coordination systematically depends on θ – which is *not* readily predicted under the complete information version of the game. The second role that cognitive noise plays is to modulate the probability of coordination conditional on θ, through an efficient coding mechanism. Our data are also consistent with this prediction, as we observe significantly less randomness in behavior when θ is drawn from a more concentrated distribution. In sum, our experimental results are consistent with a cognitive noise and efficient coding mechanism that produces context-dependent equilibrium behavior.

Our paper pushes forward the broader agenda on cognitive noise in economics by demonstrating that a small amount of cognitive noise can fundamentally alter the information structure and equilibria of a game. The data that we generate provide a proof of principle that such cognitive noise is empirically relevant in a simple coordination game. This result holds even in our treatment where subjects play against a computerized opponent and, thus, any other source of strategic uncertainty is shut down by design.

We believe our analysis paves the way for at least two directions of future work on cognitive noise in games. First, there are additional theory-guided manipulations of cognitive noise which have recently been deployed in individual decision-making experiments, that could be explored in a strategic environment. For example, Polania, Woodford and Ruff (2019) show that cognitive noise can be amplified by imposing time pressure on decisions, and Enke and Graeber (2023) ramp up cognitive noise by increasing the complexity of an action. In our setting, a clear untested prediction is that imposing time pressure should lead the distribution of actions in equilibrium to be compressed towards 50-50, so that the probability of coordination can be modulated by the experimenter. The second direction is along a more theoretical route. Our current framework is confined to a stylized 2×2 coordination game, but we believe there may be much richer implications of cognitive noise in more general strategic environments. In particular, the idea that public signals are universally processed with noise due to cognitive errors is likely to have important implications for strategic behavior in a much broader class of games.

Appendix

Proof of Proposition 1

Here we adapt the theoretical derivation of efficient coding from Khaw, Li and Woodford (2021) to our framework where the distribution of θ is normal rather than lognormal. According to Assumption 1, the internal representation S of θ is drawn from

$$S|\theta \sim N(m(\theta), \sigma_S^2) \tag{5}$$

where the *encoding rule*, $m(\theta)$, is a linear transformation of θ, $m(\theta) = \xi + \psi\theta$, which satisfies the power constraint in Assumption 2. Parameters ξ and ψ are endogenous while the precision parameter σ_S is exogenous. The efficient coding hypothesis requires that the encoding rule $m(\theta)$ is chosen (among all linear functions satisfying the constraint) so as to maximize the system's objective function, for a given prior distribution of θ. As in Khaw, Li and Woodford (2021), we assume that the system produces an estimate of θ on the basis of S, $\tilde{\theta}(S)$, and that the goal of the design problem is to have a system that achieves as low as possible a mean squared error of this estimate. Given a noisy internal representation, the estimate which minimizes the mean squared error is $E[\theta|S]$ for all S. The goal of the design problem is, thus, to minimize the variance of the posterior distribution of θ.

Consider the transformed internal representation, $Z \equiv (S - \xi)/\psi$. The distribution of the transformed internal representation conditional on θ is $Z|\theta \sim N(\theta, \sigma_S^2/\psi^2)$. Thus, the distribution of θ given the transformed internal representation is

$$\theta|Z \sim N\left(\mu_\theta + \frac{\sigma_\theta^2}{\sigma_\theta^2 + (\sigma_S^2/\psi^2)}(Z - \mu_\theta), \frac{\sigma_\theta^2(\sigma_S^2/\psi^2)}{\sigma_\theta^2 + (\sigma_S^2/\psi^2)}\right) \tag{6}$$

The variance of the posterior distribution of θ is strictly increasing in the variance of Z, σ_S^2/ψ^2. Thus, it is desirable to make ψ as large as possible (in order to make the mean squared error of the estimate as small as possible) consistent with the power constraint. When the distribution of θ is normal, we have

$$E[m^2] = \xi^2 + \psi^2 E[\theta^2] + 2\xi\psi E[\theta] = (\xi + \psi\mu_\theta)^2 + \psi^2\sigma_\theta^2 \leq \Omega \tag{7}$$

The largest value of ψ consistent with this constraint is achieved when

$$\xi = -\psi\mu_\theta \ , \ \psi = \frac{\Omega}{\sigma_\theta} \tag{8}$$

Thus, $m^{\star}(\theta) = -\frac{\Omega}{\sigma_\theta}\mu_\theta + \frac{\Omega}{\sigma_\theta}\theta$ and

$$Z|\theta \;\sim\; N\left(\theta, \frac{\sigma_S^2}{\Omega^2}\sigma_\theta^2\right) \tag{9}$$

The same optimal coding rule obtains under an alternative goal of the system. Consider the more conventional hypothesis from sensory perception literature, whereby the encoding rule is assumed to maximize the Shannon mutual information between the objective state θ and its subjective representation S. Denote with ρ_θ the precision of θ and with ρ_S the precision of S. We have $\theta \sim N\left(\mu_x, \frac{1}{\rho_\theta}\right)$, $S|\theta \sim N\left(\xi + \psi\theta, \frac{1}{\rho_S}\right)$, $Z|\theta \sim \left(\theta, \frac{1}{\rho_Z}\right)$, and $\theta|Z \sim N\left(\frac{\rho_\theta\mu_\theta + \rho_Z Z}{\rho_\theta + \rho_Z}, \frac{1}{\rho_\theta + \rho_Z}\right)$, where $Z = \frac{S-\xi}{\psi}$ and $\rho_Z = \psi^2/\sigma_S^2$. The Shannon mutual information between θ and Z is

$$I(\theta, Z) = \frac{1}{2}\log_2\left(\frac{\sigma_\theta^2}{\sigma_{\theta|Z}^2}\right) = \frac{1}{2}\log_2\left(1 + \frac{\rho_Z}{\rho_\theta}\right) \tag{10}$$

which is strictly increasing in ρ_Z and, thus, strictly decreasing in σ_Z^2. This means that, as for the previous goal, it is desirable to make ψ as large as possible (consistent with the power constraint).

Proof of Proposition 2

First, we show that, when the conditions in the statement of the Proposition are satisfied, there exists a unique monotone equilibrium of the game. Remember that $Z_i \sim N(\theta, \sigma_Z^2)$, where $\sigma_Z^2 = \omega^2\sigma_\theta^2 = (\sigma_S^2/\Omega^2)\sigma_\theta^2$. Thus, player 1's posterior distribution of θ given Z_1 is

$$\theta|Z_1 \sim \mathcal{N}\left(\frac{\sigma_Z^2}{\sigma_\theta^2 + \sigma_Z^2}\mu_\theta + \frac{\sigma_\theta^2}{\sigma_\theta^2 + \sigma_Z^2}Z_1, \frac{\sigma_\theta^2\sigma_Z^2}{\sigma_\theta^2 + \sigma_Z^2}\right)$$

Therefore, we have:

$$EU[\text{Not Invest}|Z_1] = E[\theta|Z_1] = \frac{\sigma_Z^2\mu_\theta + \sigma_\theta^2 Z_1}{\sigma_\theta^2 + \sigma_Z^2}$$

On the other hand, player 1's expected utility from investing is

$$EU[\text{Invest}|Z_1] = a + (b-a)\Pr[\text{Opponent Invests}|Z_1]$$

Assume player 1 believes his opponent uses a monotone strategy with threshold k. In this case, player 1's expectation that the opponent invests is $\Pr[Z_2 \leq k|Z_1]$. Player 1's belief about the distribution of Z_2 given Z_1 is:

$$Z_2|Z_1 \sim \mathcal{N}\left(E[\theta|Z_1] = \frac{\sigma_Z^2}{\sigma_\theta^2+\sigma_Z^2}\mu_\theta + \frac{\sigma_\theta^2}{\sigma_\theta^2+\sigma_Z^2}Z_1, \frac{2\sigma_\theta^2\sigma_Z^2+\sigma_Z^4}{\sigma_\theta^2+\sigma_Z^2}\right)$$

Thus, we have:

$$\Pr[Z_2 \leq k|Z_1] = \Phi\left(\frac{\left(\sigma_\theta^2+\sigma_Z^2\right)k - \sigma_Z^2\mu_\theta - \sigma_\theta^2 Z_1}{\sqrt{2\sigma_\theta^2\sigma_Z^2+\sigma_Z^4}\sqrt{\sigma_\theta^2+\sigma_Z^2}}\right)$$

where $\Phi(\cdot)$ is the cumulative distribution of the standard normal.

Player 1's best response is to invest if and only if

$$\frac{\sigma_Z^2\mu_\theta+\sigma_\theta^2 Z_1}{\sigma_\theta^2+\sigma_Z^2} \quad\leq\quad a+(b-a)\Phi\left(\frac{\left(\sigma_\theta^2+\sigma_Z^2\right)k - \sigma_Z^2\mu_\theta - \sigma_\theta^2 Z_1}{\sqrt{2\sigma_\theta^2\sigma_Z^2+\sigma_Z^4}\sqrt{\sigma_\theta^2+\sigma_Z^2}}\right) \tag{11}$$

If we write $\overline{Z}(k)$ for the unique value of Z_1 such that player 1 is indifferent between investing and not investing (this is well defined since player 1's expected payoff from not investing is strictly increasing in Z_1 and player 1's expected payoff from investing is strictly decreasing in Z_1), the best response of player 1 is to follow a monotone strategy with threshold equal to $\overline{Z}(k)$, that is, to invest if and only if $Z_1 \leq \overline{Z}(k)$.

Observe that as $k \to -\infty$ (that is, player 2 never invests), $EU[\text{Invest}|Z_1, k]$ tends to a, so $\overline{Z}(k)$ tends to $\frac{\left(\sigma_\theta^2+\sigma_Z^2\right)a-\sigma_Z^2\mu_\theta}{\sigma_\theta^2}$. As $k \to \infty$ (that is, player 2 always invests), $EU[\text{Invest}|Z_1]$ tends to b, so $\overline{Z}(k)$ tends to $\frac{\left(\sigma_\theta^2+\sigma_Z^2\right)b-\sigma_Z^2\mu_\theta}{\sigma_\theta^2}$. A fixed point of $\overline{Z}(k)$ — that is a value $k^\star$ such that $\overline{Z}(k^\star) = k^\star$ — is a monotone equilibrium of the game where each player invests if and only if his signal is below $k^\star$. Since $\overline{Z}(k)$ is a mapping from $\mathbb{R}$ to itself and is continuous in k, there exists $k \in \left[\frac{\left(\sigma_\theta^2+\sigma_Z^2\right)a-\sigma_Z^2\mu_\theta}{\sigma_\theta^2}, \frac{\left(\sigma_\theta^2+\sigma_Z^2\right)b-\sigma_Z^2\mu_\theta}{\sigma_\theta^2}\right]$, such that $\overline{Z}(k) = k$ and a threshold equilibrium of this game exists.

When is there a unique equilibrium? Define $W(\overline{Z}(k), k)$ as

$$W(\overline{Z}(k), k) = \frac{\sigma_Z^2\mu_\theta+\sigma_\theta^2\overline{Z}(k)}{\sigma_\theta^2+\sigma_Z^2} - a - (b-a)\Phi\left(\frac{\left(\sigma_\theta^2+\sigma_Z^2\right)k - \sigma_Z^2\mu_\theta - \sigma_\theta^2\overline{Z}(k)}{\sqrt{2\sigma_\theta^2\sigma_Z^2+\sigma_Z^4}\sqrt{\sigma_\theta^2+\sigma_Z^2}}\right)$$

At a fixed point, $\overline{Z}(k^\star) = k^\star$. Thus, we have:

$$W(k^\star) = \frac{\sigma_Z^2\mu_\theta+\sigma_\theta^2 k^\star}{\sigma_\theta^2+\sigma_Z^2} - a - (b-a)\Phi\left(\frac{\sigma_Z^2}{\sqrt{2\sigma_\theta^2\sigma_Z^2+\sigma_Z^4}\sqrt{\sigma_\theta^2+\sigma_Z^2}}\left(k^\star-\mu_\theta\right)\right)$$

Then,

$$\frac{\partial W(k^\star)}{\partial k^\star} = \frac{\sigma_\theta^2}{\sigma_\theta^2+\sigma_Z^2} - \phi\left(\frac{\sigma_Z^2}{\sqrt{2\sigma_\theta^2\sigma_Z^2+\sigma_Z^4}\sqrt{\sigma_\theta^2+\sigma_Z^2}}\left(k^\star-\mu_\theta\right)\right)\frac{\sigma_Z^2(b-a)}{\sqrt{2\sigma_\theta^2\sigma_Z^2+\sigma_Z^4}\sqrt{\sigma_\theta^2+\sigma_Z^2}}$$

And there is a unique fixed point if and only if $\frac{\partial W(k^\star)}{\partial k^\star} > 0$ at the fixed point. When $\frac{\partial W(k^\star)}{\partial k^\star} < 0$, there are at least three fixed points. Since $\phi(y) \leq \frac{1}{\sqrt{2\pi}}$, this is a sufficient condition for $\frac{\partial W(k^\star)}{\partial k^\star} > 0$:

$$\begin{aligned}
\frac{\sigma_\theta^2}{\sigma_\theta^2+\sigma_Z^2} &> \frac{1}{\sqrt{2\pi}}\frac{\sigma_Z^2(b-a)}{\sqrt{2\sigma_\theta^2\sigma_Z^2+\sigma_Z^4}\sqrt{\sigma_\theta^2+\sigma_Z^2}} \\
\frac{\sigma_\theta^2\sqrt{2\sigma_\theta^2\sigma_Z^2+\sigma_Z^4}}{(b-a)\sigma_Z^2\sqrt{\sigma_\theta^2+\sigma_Z^2}} &> \frac{1}{\sqrt{2\pi}} \\
\sqrt{2\pi} &> \frac{(b-a)\sigma_Z^2\sqrt{\sigma_\theta^2+\sigma_Z^2}}{\sigma_\theta^2\sqrt{2\sigma_\theta^2\sigma_Z^2+\sigma_Z^4}}
\end{aligned}$$

The condition $\frac{\omega\sqrt{1+\omega^2}}{\sqrt{2+\omega^2}} < \frac{\sqrt{2\pi}}{(b-a)}\sigma_\theta$ is obtained by replacing $\sigma_Z = \omega\sigma_\theta$ in the condition above and re-arranging terms. Thus, this shows that, when the conditions in the statement of the Proposition are satisfied, there exists a unique monotone equilibrium of the game.

Second, we show that, when $\mu_\theta = \frac{(a+b)}{2}$, there exists a monotone equilibrium of the game where $k^\star = \mu_\theta$ for any value of σ_θ, σ_S and ω (or, equivalently, for any value of σ_θ and σ_Z). Assume player 2 uses a threshold strategy where he invests if and only if $Z_2 \leq k = \mu_\theta$. Is this an equilibrium, that is, is $\overline{Z}(\mu_\theta) = \mu_\theta$? $\overline{Z}(\mu_\theta)$ is the value of Z_1 such that the following equation is satisfied with equality:

$$\begin{aligned}
\frac{\sigma_Z^2\mu_\theta+\sigma_\theta^2 Z_1}{\sigma_\theta^2+\sigma_Z^2} &= a+(b-a)\Phi\left(\frac{\left(\sigma_\theta^2+\sigma_Z^2\right)k-\sigma_Z^2\mu_\theta-\sigma_\theta^2 Z_1}{\sqrt{2\sigma_\theta^2\sigma_Z^2+\sigma_Z^4}\sqrt{\sigma_\theta^2+\sigma_Z^2}}\right) \\
\frac{\sigma_Z^2\mu_\theta+\sigma_\theta^2 Z_1}{\sigma_\theta^2+\sigma_Z^2} &= a+(b-a)\Phi\left(\frac{\sigma_\theta^2\mu_\theta-\sigma_\theta^2 Z_1}{\sqrt{2\sigma_\theta^2\sigma_Z^2+\sigma_Z^4}\sqrt{\sigma_\theta^2+\sigma_Z^2}}\right)
\end{aligned}$$

If we set $Z_1 = \mu_\theta$, we get:

$$\begin{aligned}
\mu_\theta &= a+(b-a)\Phi\left(0\right) \\
\mu_\theta &= \frac{(a+b)}{2}
\end{aligned}$$

which is true by one of the assumptions in the statement of the Proposition.

Proof of Proposition 3

From Proposition 2 and the condition in the statement of Proposition 3, we know that there exists a unique monotone equilibrium of the game where each player invests if and only if his transformed internal representation is smaller than μ_θ. In this equilibrium, $Pr[\text{Invest}|\theta] = Pr\left[Z_i \le \mu_\theta|\theta\right] = \Phi\left(\frac{\mu_\theta-\theta}{\omega\sigma_\theta}\right)$ and $\frac{\partial Pr[\text{Invest}|\theta]}{\partial\theta} = -\phi\left(\frac{\mu_\theta-\theta}{\omega\sigma_\theta}\right)\left(\frac{1}{\omega\sigma_\theta}\right)$. Thus, $Pr[\text{Invest}|\theta]$ grows with σ_θ if $\theta < \mu_\theta$ and it decreases with σ_θ is $\theta > \mu_\theta$. Moreover, the sensitivity of choices to θ decreses with σ_θ for values of θ around the cutoff.

Indeed, we have

$$\begin{aligned}
\frac{\partial Pr\left[\text{Invest}|\theta\right]}{\partial\theta\partial\sigma_\theta} &= \phi\left(\frac{\mu_\theta-\theta}{\omega\sigma_\theta}\right)\left(\frac{1}{\omega\sigma_\theta^2}\right) + \phi'\left(\frac{\mu_\theta-\theta}{\omega\sigma_\theta}\right)\left(\frac{\mu_\theta-\theta}{\omega\sigma_\theta^2}\right)\left(\frac{1}{\omega\sigma_\theta}\right) \\
&= \phi\left(\frac{\mu_\theta-\theta}{\omega\sigma_\theta}\right)\left(\frac{1}{\omega\sigma_\theta^2}\right) - \left(\frac{\mu_\theta-\theta}{\omega\sigma_\theta}\right)\phi\left(\frac{\mu_\theta-\theta}{\omega\sigma_\theta}\right)\left(\frac{\mu_\theta-\theta}{\omega\sigma_\theta^2}\right)\left(\frac{1}{\omega\sigma_\theta}\right) \\
&= \phi\left(\frac{\mu_\theta-\theta}{\omega\sigma_\theta}\right)\left(\frac{1}{\omega\sigma_\theta^2}\right) - \phi\left(\frac{\mu_\theta-\theta}{\omega\sigma_\theta}\right)\left(\frac{(\mu_\theta-\theta)^2}{\omega^3\sigma_\theta^4}\right) \\
&= \phi\left(\frac{\mu_\theta-\theta}{\omega\sigma_\theta}\right)\left(\frac{\omega^2\sigma_\theta^2-(\mu_\theta-\theta)^2}{\omega^3\sigma_\theta^4}\right)
\end{aligned}$$

which is positive if and only if $(\mu_\theta-\theta)^2 < \omega^2\sigma_\theta^2$.

(In the second line, we used the fact that $\phi'(x) = -x\phi(x)$.)

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Online Appendix

A Construction of M Equilibria

In this Appendix, we follow the graphical procedure from Goeree and Louis (2021) to construct the set of all M equilibria in our game. We note also that the M-choice sets we characterize through this procedure contain all Quantal Response Equilibria of our game. Let p and q denote the probabilities with which Player 1 and Player 2 choose Invest. Let ν and ω denote Player 1's and Player 2's beliefs that the other player chooses Invest, respectively. Since Not Invest is chosen with complementary probability, the sets of choice and belief profiles can be summarized by unit squares consisting of the pairs (p, q) and (ν, ω) respectively. These unit squares are displayed in the top row of Figures A1 and A2 for, respectively, games with $\theta < (a + b)/2$ and games with $\theta > (a + b)/2$. In the left panel of each figure's top row, the quadrants reflect the four possible orderings of choice probabilities. In the right panel of each figure's top row, π_1 and π_2 denote the expected payoffs for Player 1 and Player 2. The vertical line at $\omega = \frac{\theta - a}{b - a}$ indicates the belief for which Player 2 is indifferent and the horizontal line at $\nu = \frac{\theta - a}{b - a}$ does the same for Player 1.

These "indifference curves" divide the unit square in four rectangles where expected payoffs are strictly ordered. To check for monotonicity, we match each of these four rectangles with a quadrant in the left panel. For instance, consider Figure A1: the largest rectangle on the right for which the expected payoff of Invest exceeds that of Not Invest for both players is matched with the north-east quadrant on the left. Likewise, the smallest rectangle on the right for which the expected payoff of Not Invest exceeds that of Invest for both players is matched with the south-west quadrant on the left. These matchings are such that the choice probabilities on the left are ranked the same way as the payoffs given beliefs on the right. Consequential unbiasedness requires that the ranking of expected payoffs based on beliefs is the same as that based on choices.

Graphically, this equilibrium condition is implemented by superimposing the rectangles of the right panel on the left unit square. The intersection of any quadrant with the matched rectangle determines the M-choice and M-belief sets. For instance, superimposing the rectangle of the right panel where Player 1's expected payoff of Invest exceeds that of Not Invest but the opposite is true for Player 2 on the left panel yields an empty intersection, i.e. there is no M equilibrium in which Player 1 is more likely to choose Invest while Player 2 is more likely to choose Not Invest. In contrast, superimposing the largest rectangle of the right panel on the left panel yields a non-empty intersection indicated by the red M-choice set in the bottom-left panel of Figure A1. The corresponding M-belief set is simply the large rectangle itself, as indicated by the red set in the bottom-right panel. Repeating this procedure for the different payoff rankings yields two full-dimensional M equilibria that are colorable, see the bottom panels of Figures A1 and A2.

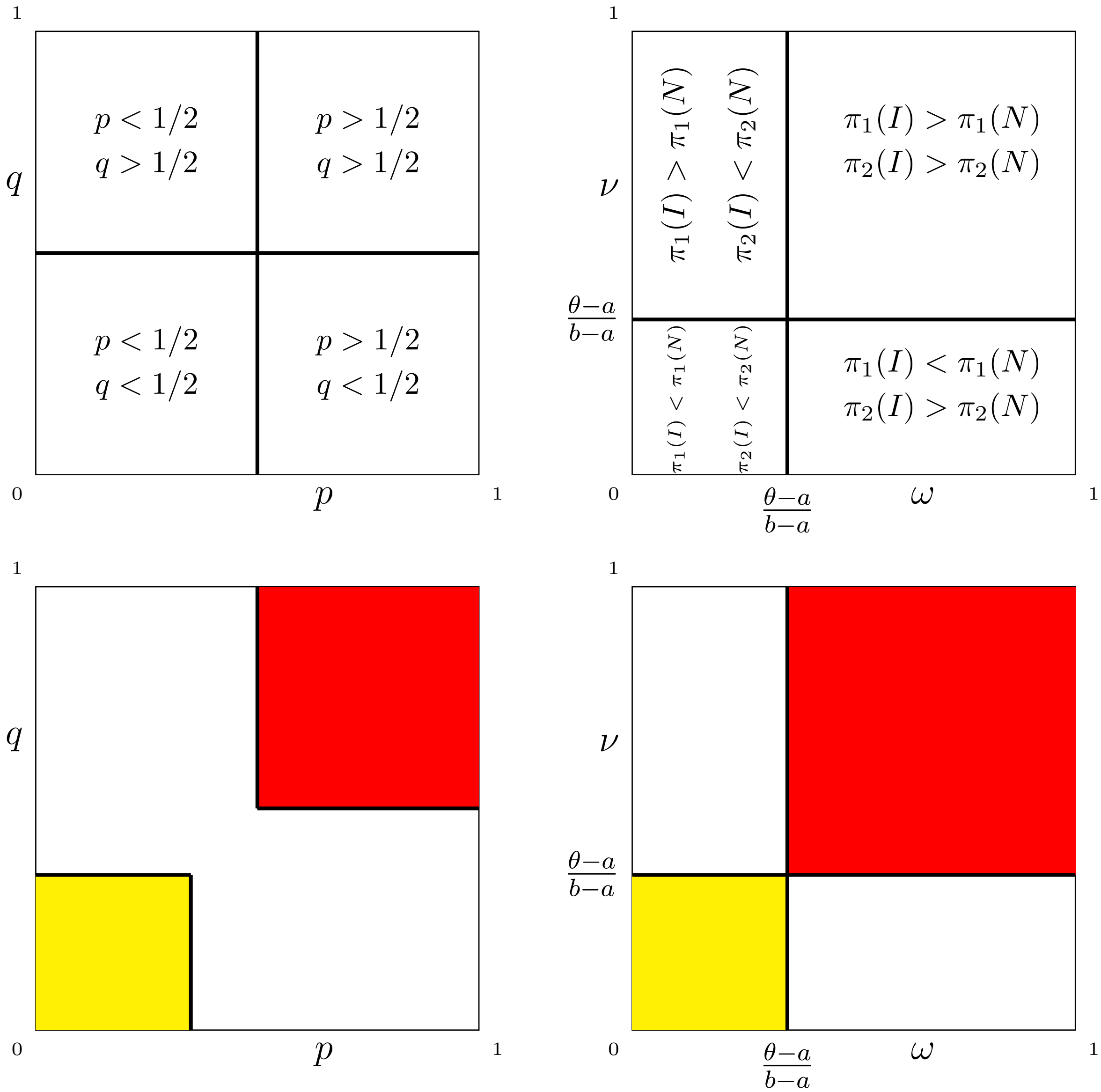

Figure A1: **Construction of M-Choice and M-Belief Sets for Games with $\theta \in [a, (a+b)/2]$.** The top panel show partitions of the unit square based on orderings of choice probabilities (left) and expected payoffs (right). There are two M-choice sets for which these orderings match, see the lower-left panel, which can be labeled or colored by the ordering they represent. The colored sets in the lower-right panel show the beliefs that generate the same ordering of expected payoffs as choices of the same color.

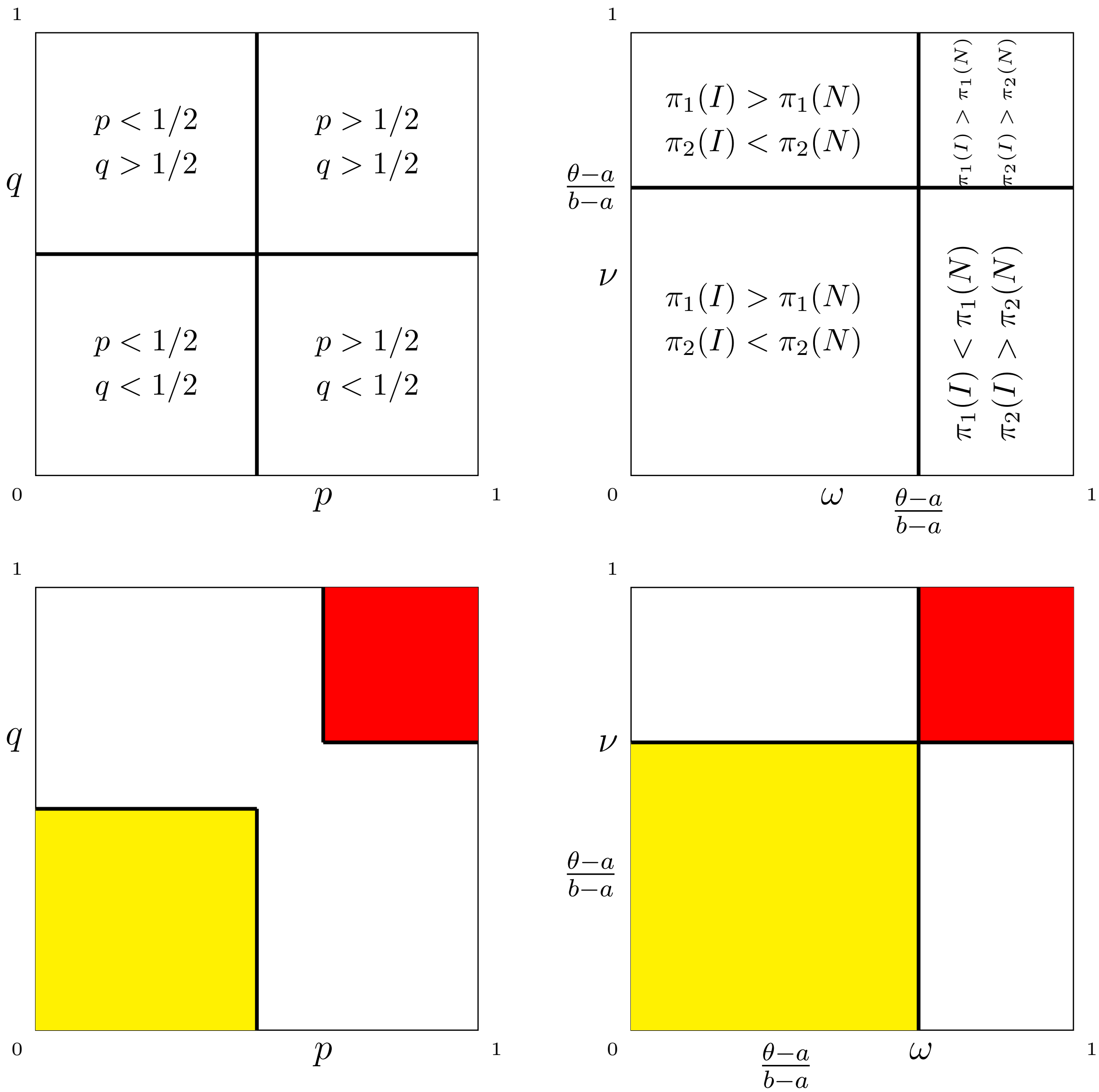

Figure A2: **Construction of M-Choice and M-Belief Sets for Games with $\theta \in [(a+b)/2.b]$.** The top panel show partitions of the unit square based on orderings of choice probabilities (left) and expected payoffs (right). There are two M-choice sets for which these orderings match, see the lower-left panel, which can be labeled or colored by the ordering they represent. The colored sets in the lower-right panel show the beliefs that generate the same ordering of expected payoffs as choices of the same color.

B Alternative Model of Efficient Coding

Assumption 4 (Alternative Performance Objective) *Players choose the encoding function which maximizes their expected reward in the simultaneous move game.*

Consider the following two-stage game: in stage 1, each player $i = \{1, 2\}$ chooses simultaneously and independently the parameters of his encoding function, (ξ_i, ψ_i), to maximize the performance objective in Assumption 4 under the constraints in Assumption 2; in stage 2, players participate to the simultaneous move game endowed with the encoding functions chosen in the previous stage. We solve this game by backward induction.

Stage 2: Simultaneous move Game (with Exogeneous Encoding Functions)

For each player $i = \{1, 2\}$, we have $S_i|\theta \sim N(m_i(\theta), \sigma_S^2)$, where $m_i(\theta) = \xi_i + \psi_i\theta$.

Consider the transformed internal representation $Z_i = (S_i - \xi_i)/\psi_i$. We have:

$$Z_i|\theta \sim N\left(\theta, \beta_i^2\right)$$

where $\beta_i = (\sigma_S/\psi_i)$.

Proposition 4 *Suppose Assumptions 1, 2, 4 and $\mu_\theta = (a + b)/2$. Regardless of σ_θ, σ_S, (ξ_1, ψ_1), and (ξ_2, ψ_2), there exists an equilibrium of the game where each player invests if and only if $Z_i \leq \mu_\theta$. Moreover, if $\frac{\sigma_\theta^2\sqrt{\beta_i^2(2\sigma_\theta^2+\beta_i^2)}}{(b-a)\beta_i^2\sqrt{\sigma_\theta^2+\beta_i^2}} > \frac{1}{\sqrt{2\pi}}$ for all $i = \{1, 2\}$, this is the unique monotone equilibrium of the game.*

Proof. Since the likelihood function of Z_i is conjugate to the prior distribution of θ, we have a closed form solution for the distribution of player i's posterior beliefs over θ. In particular, player 1's posterior distribution of θ given Z_1 is

$$\theta|Z_1 \sim \mathcal{N}\left(\frac{\beta_1^2\mu_\theta + \sigma_\theta^2 Z_1}{\sigma_\theta^2 + \beta_1^2}, \frac{\sigma_\theta^2\beta_1^2}{\sigma_\theta^2 + \beta_1^2}\right)$$

Thus, we have:

$$EU[\text{Not Invest}|Z_1] = E[\theta|Z_1] = \frac{\beta_1^2\mu_\theta + \sigma_\theta^2 Z_1}{\sigma_\theta^2 + \beta_1^2}$$

On the other hand, player 1's expected utility from investing is

$$EU[\text{Invest}|Z_1] = a + (b - a)\Pr[\text{Opponent Invests}|Z_1]$$

Assume player 1 believes his opponent uses a monotone strategy with threshold k_2. In this case, player 1's expectation that the opponent invests is $\Pr[Z_2 \leq k_2|Z_1]$. Player 1's belief over the distribution of Z_2 conditional on Z_1 is:

$$Z_2|Z_1 \sim \mathcal{N}\left(\frac{\beta_1^2\mu_\theta + \sigma_\theta^2 Z_1}{\sigma_\theta^2 + \beta_1^2}, \frac{\sigma_\theta^2\left(\beta_1^2 + \beta_2^2\right) + \beta_1^2\beta_2^2}{\sigma_\theta^2 + \beta_1^2}\right)$$

Thus, we have:

$$\Pr[Z_2 \leq k_2|Z_1] = \Phi\left(\frac{k_2\left(\sigma_\theta^2 + \beta_1^2\right) - \beta_1^2\mu_\theta - \sigma_\theta^2 Z_1}{\sqrt{\sigma_\theta^2 + \beta_1^2}\sqrt{\sigma_\theta^2\left(\beta_1^2 + \beta_2^2\right) + \beta_1^2\beta_2^2}}\right)$$

where $\Phi(\cdot)$ is the cumulative distribution of the standard normal.

Player 1's best response is to invest if and only if

$$\frac{\beta_1^2\mu_\theta + \sigma_\theta^2 Z_1}{\sigma_\theta^2 + \beta_1^2} \leq a + (b-a)\Phi\left(\frac{k_2\left(\sigma_\theta^2 + \beta_1^2\right) - \beta_1^2\mu_\theta - \sigma_\theta^2 Z_1}{\sqrt{\sigma_\theta^2 + \beta_1^2}\sqrt{\sigma_\theta^2\left(\beta_1^2 + \beta_2^2\right) + \beta_1^2\beta_2^2}}\right)$$

Assume $k_2 = \mu_\theta$. We want to show that player's best response is to use the same cutoff. In this case, player 1's best response is to invest if and only if

$$E\frac{\beta_1^2\mu_\theta + \sigma_\theta^2 Z_1}{\sigma_\theta^2 + \beta_1^2} \leq a + (b-a)\Phi\left(\frac{\sigma_\theta^2\left(\mu_\theta - Z_1\right)}{\sqrt{\sigma_\theta^2 + \beta_1^2}\sqrt{\sigma_\theta^2\left(\beta_1^2 + \beta_2^2\right) + \beta_1^2\beta_2^2}}\right)$$

First, note that the LHS is a convex combination of μ_θ and Z_1 and that, thus, it is a) equal to μ_θ when $Z_1 = \mu_\theta$, b) smaller than μ_θ when $Z_1 < \mu_\theta$, and c) larger than μ_θ when $Z_1 > \mu_\theta$. Second, remember that $\mu_\theta = (a+b)/2$ and note that the RHS is a) equal to μ_θ when the argument of $\Phi(\cdot)$ is 0 (that is, when $Z_1 = \mu_\theta$, since the denominator is strictly positive); b) larger than μ_θ when the argument of $\Phi(\cdot)$ is strictly positive (that is, when $Z_1 < \mu_\theta$), and c) smaller than μ_θ when the argument of $\Phi(\cdot)$ is strictly negative (that is, when $Z_1 > \mu_\theta$). This means that, when player 2 invests if and only if $Z_2 \leq k_2 = \mu_\theta$, then player 1's best response is to invest if and only if $Z_1 \leq \mu_\theta$. This proves that there exists an equilibrium where both players use a monotone strategy with cutoff equal to μ_θ for any value of (ξ_1, ψ_1), (ξ_2, ψ_2), σ_S and σ_θ. Finally, to show that, when the condition in the statement of the proposition is satisfied, this is the unique equilibrium of the game, we can use the same steps in the proof of Proposition 2 to show that the best response mapping is a contraction (and that, thus, we can apply the contraction mapping theorem). In particular, it is sufficient to show that the derivative of the best response function of player 1 with respect to k_2 and the derivative of the best response function of player 2 with respect to k_1 have both an absolute value strictly smaller than 1. ■

Stage 1: Encoding Function Choice

When deriving the optimal choice of the encoding function in stage 1, we assume that, in stage 2, players use the cutoff strategy in the (unique) equilibrium from Proposition 4.

Proposition 5 *Suppose Assumptions 1, 2, 4, and* $\mu_\theta = (a+b)/2$. *The optimal encoding function is the same for both players and is given by* $m^\star(\theta) = \xi^\star + \psi^\star\theta = -\frac{\Omega\mu_\theta}{\sigma_\theta} + \frac{\Omega}{\sigma_\theta}\theta$.

Proof. In stage 2, each player $i = \{1, 2\}$ invests if and only if $Z_i \leq \mu_\theta$. Given the conditional distribution of Z_i, the probability player i invests for a given θ and encoding function is

$$\mathbb{P}_i(\text{Invest}|\theta, \psi_i) = \Phi\left(\frac{\mu_\theta - \theta}{\sigma_S/\psi_i}\right)$$

Thus, the expected utility player i gets from the game with a given value of θ is

$$\begin{aligned} EU_i(\theta, \psi_i) &= \mathbb{P}_i(\text{Invest}|\theta, \psi_i)\left(a + \mathbb{P}_{-i}(\text{Invest}|\theta, \psi_{-i})(b-a)\right) + (1 - \mathbb{P}_i(\text{Invest}|\theta, \psi_i))\theta \\ &= \theta + \Phi\left(\frac{\mu_\theta - \theta}{\sigma_S/\psi_i}\right)\left(a + \Phi\left(\frac{\mu_\theta - \theta}{\sigma_S/\psi_{-i}}\right)(b-a) - \theta\right) \end{aligned}$$

where we use $-i$ to denote i's opponent. How does this expected utility change with ψ_i (taking ψ_{-i} as given)?

$$\frac{\partial EU_i(\theta, \psi_i)}{\partial \psi_i} = \phi\left(\frac{\mu_\theta - \theta}{\sigma_S/\psi_i}\right)\left(\frac{\mu_\theta - \theta}{\sigma_S}\right)\left(a + \Phi\left(\frac{\mu_\theta - \theta}{\sigma_S/\psi_{-i}}\right)(b-a) - \theta\right) \tag{12}$$

Since $\phi(\cdot)$ is strictly positive for any argument, the sign of equation (12) is determined by the product of its second and third term. First, note that the second term is a) equal to 0 when $\theta = \mu_\theta$, b) strictly positive when $\theta < \mu_\theta$ and c) strictly negative when $\theta > \mu_\theta$. Second, note that — since $\mathbb{P}_{-i}(\text{Invest}|\theta, \psi_{-i})$ is greater than 1/2 if and only if $\theta < \mu_\theta$ and $\mu_\theta = (a+b)/2)$ — the third term is a) strictly positive when $\theta < \mu_\theta$ and b) strictly negative when $\theta > \mu_\theta$. This means that the product of the second and third term of equation (12) is always positive, with the exception of the case when $\theta = \mu_\theta$, in which case it is 0.

We have shown that the expected payoff in a game with a given θ is strictly increasing in ψ_i for any value of $\theta \neq \mu_\theta$ and it is constant in ψ_i for $\theta = \mu_\theta$. This means that, from an ex-ante perspective (that is, when a player knows the distribution of θ but does not know its actual realization), each player's expected reward from the simultaneous move game — that is, $EU_i(\psi_i) = \int EU_i(\theta, \psi_i) f(\theta) d\theta$ — is strictly increasing in ψ_i. Therefore, it is desirable to make ψ_i as large as possible consistent with the power constraint. When the distribution of θ is normal, we have

$$E[m^2] = \xi^2 + \psi^2 E[\theta^2] + 2\xi\psi E[\theta] = (\xi + \psi\mu_\theta)^2 + \psi^2\sigma_\theta^2 \leq \Omega$$

The largest value of ψ consistent with this constraint in Assumption 2 is achieved when

$$\xi = -\psi\mu_\theta \ , \ \psi = \frac{\Omega}{\sigma_\theta}$$

Thus, $m^\star(\theta) = -\frac{\Omega}{\sigma_\theta}\mu_\theta + \frac{\Omega}{\sigma_\theta}\theta$. ■

C Robustness of Monotone Equilibrium with $k^\star = \mu_\theta$

Let us introduce the following definitions from Chambers and Healy (2012):

Definition 1 *A random variable with cumulative density function F and mean μ is* ***symmetric*** *if, for every $a \geq 0$, $F(\mu + a) = 1 - \lim_{x \to a^-} F(\mu - a)$.*

Definition 2 *A random variable is* ***quasiconcave*** *(or unimodal) if it has a density function f such that for all $x, x' \in \mathbb{R}$ and $\lambda \in (0,1)$, $f(\lambda x + (1-\lambda)x') \geq \min\{f(x), f(x')\}$.*

Definition 3 *The error term ϵ_i satisfies* ***symmetric dependence*** *with respect to the random variable θ if, for each realization of θ, $\epsilon_i|\theta$ has a continuous density function $f_{\epsilon_i|\theta}$ satisfying $f_{\epsilon_i|\theta}(\epsilon_i|\mu_\theta + a) = f_{\epsilon_i|\theta}(\epsilon_i|\mu_\theta - a)$ for almost every ϵ_i and a in $\mathbb{R}$. (Note that error terms that are independent of θ satisfy this definition).*

Consider the following assumptions:

(A1) $S_i = \theta + \epsilon_i$

(A2) $E[\theta] = \mu_\theta < \infty$

(A3) θ is a symmetric random variable and its density is continuous on $\mathbb{R}$

(A4) $E[\epsilon_i|\theta] = 0$ for each θ

(A5) ϵ_i is a symmetric and quasiconcave random variable

(A6) ϵ_i satisfies symmetric dependence with respect to θ

Lemma 1 (Chambers and Healy 2012, Proposition 2) *Assume A1-A6. A Bayesian agent updates his beliefs over θ in the direction of the signal, that is, for almost every $S_i \in \mathbb{R}$, there exists some $\alpha \geq 0$ such that $E[\theta|S_i] = \alpha S_i + (1-\alpha)\mu_\theta$.*

Proposition 6 *Assume common knowledge of both A1-A6 and $\mu_\theta = (a+b)/2$. There exists a monotone equilibrium of the game where $k^\star = \mu_\theta$.*

Proof of Proposition 6 The proof can be carried out with general values for a and b (such that $b > a$). For ease of exposition, we focus on the experimental parameters: $a = 47$, $b = 63$, $\mu_\theta = 55$. Assume that player j uses threshold $k_j = 55$, that is, he invests if and only if $S_j < 55$. We want to show that player i's best response is to use the same threshold, $k_i = 55$. Player i prefers to invest if and only if $EU[\text{Not Invest}|S_i] < EU[\text{Invest}|S_i, k_j]$.

Thus, we want to show that (1) when $S_i = 55$, $EU[\text{Not Invest}|S_i] = EU[\text{Invest}|S_i, k_j = 55]$; (2) when $S_i < 55$, $EU[\text{Not Invest}|S_i] < EU[\text{Invest}|S_i, k_j = 55]$; and (3) when $S_i > 55$, $EU[\text{Not Invest}|S_i] > EU[\text{Invest}|S_i, k_j = 55]$.

By Lemma 1, $EU[\text{Not Invest}|S_i] = E[\theta|S_i] = \alpha S_i + (1-\alpha)\mu_\theta$ where $\alpha \geq 0$. Note also that $EU[\text{Invest}|S_i, k_j = 55] = 47 + (63 - 47)Pr[S_j < k_j = 55|S_i]$. First, we prove (1). Assume $S_i = 55$. We want to show that $EU[\text{Not Invest}|S_i] = EU[\text{Invest}|S_i, k_j = 55]$. By Lemma 1, $EU[\text{Not Invest}|S_i = 55] = \alpha S_i + (1 - \alpha)\mu_\theta = \alpha(55) + (1 - \alpha)(55) = 55$. Thus, the equality we want to show becomes $55 = 47 + (63 - 47)Pr[S_j < k_j = 55|S_i = 55]$. This equality is satisfied if and only if $Pr[S_j < k_j = 55|S_i = 55] = 1/2$. By A1 and A4 (and linearity of expectation), $E[S_j|S_i] = E[\theta|S_i] = 55$. By A5, the density of of $S_j|S_i$ is symmetric. Thus, the probability S_j takes a value below its posterior mean (55) is 1/2. This proves (1).

Second, we prove (2). Assume $S_i < 55$. By Lemma 1, $EU[\text{Not Invest}|S_i] = \alpha S_i + (1 - \alpha)55$. This is smaller than 55 for any positive α. This also means that, by A1 and A4, $E[S_j|S_i] = E[\theta|S_i] < 55$. The probability that the opponent invests is the posterior probability that his signal is below 55 (given S_i). Since the conditional distribution of the opponent's signal is symmetric around its mean (by A5), the median is equal to the mean. This means that the conditional CDF of the opponent signal equals 1/2 at the posterior mean, is greater than 1/2 for values of S_j above the mean and is lower than 1/2 for values of S_j below the mean. Since the posterior mean of the opponent's signal is lower than 55, the probability that player j's signal is lower than 55 (conditional on $S_i < 55$) is greater than 1/2. Thus, $EU[\text{Invest}|S_i, k_j = 55] = 47 + (63 - 47)Pr[S_j < k_j = 55|S_i] > 55$. This proves that $EU[\text{Invest}|S_i, k_j = 55] > 55 > EU[\text{Not Invest}|S_i]$. (3) can be proven analogously.

D Experiment on Awareness of cognitive noise

Here we report results from an additional experiment that is designed to investigate whether subjects are aware of their own imprecision and the imprecision of others. If subjects are not aware of the cognitive noise of others, then this would shut down the channel that generates strategic uncertainty in our model, which is key to generating the unique threshold equilibrium.

Experimental Design

Our method for studying awareness of imprecision is to create a simplified version of the coordination game experiment in the main text, but one that retains the core individual decision-making prediction that subjects play a threshold strategy. We employ a task from the numerical cognition literature where subjects are incentivized to quickly and accurately classify whether a two-digit number is larger or smaller than the number 55. Note that this threshold strategy is identical to the equilibrium strategy in the main experiment; the important difference is that here, we exogenously impose the strategy on subjects without any strategic considerations or equilibrium requirements. We then incentivize subjects to report beliefs about errors in their own classification and in the classification of others. These beliefs are the main object of study in this experiment.

We recruit 300 subjects from Prolific who did not participate in the main experiment. We pay subjects 1 GBP for completing the study, in addition to earnings from three phases of the experiment. In Phase 1, on each of 150 rounds, subjects are incentivized to quickly and accurately classify whether a two-digit Arabic numeral on the experimental display screen is larger or smaller than 55. Subjects earn ($1.5 \times \text{accuracy} - 1 \times \text{speed}$) GBPs, where 'accuracy' is the percentage of trials where the subject classifies the number correctly, and 'speed' is the average response time in seconds.[26] As in the main experiment, there are two conditions, and the only difference across conditions is the distribution from which the two-digit Arabic numeral (which we again denote by θ) is drawn. We use the same two distributions as in the main experiment: in the high volatility condition, $\theta \sim \mathcal{N}(55, 400)$, and in the low volatility condition, $\theta \sim \mathcal{N}(55, 20)$. We then round each value of θ to the nearest integer and re-draw if the rounded integer is less than 11 or greater than 99 (again, to ensure that each number contains exactly two digits).

We note that one difference in incentives compared to the main experiment involves decision speed. Here, we penalize subjects for the time it takes them to respond. The reason we impose the speed incentive comes from the well known "speed-accuracy tradeoff"

[26] The experimental instructions are available in Online Appendix E.

in perceptual decision-making: one can obtain higher accuracy in classification as decision speed slows down. Thus, in order to increase statistical power to detect how accuracy differs for values of θ close and far from the threshold, we jointly reward speed and accuracy.

In Phase 2 of the experiment, we incentivize subjects to report beliefs about others' performance in the task. Furthermore, we collect data on whether subjects believe that others are more imprecise when the number on screen is closer to the reference level of 55, compared to when the number is farther from the reference level. This feature of beliefs is important because the equilibrium predictions from our previous experiment depend on the noise structure in perception. In particular, recent theoretical work has shown that an important property of the noise structure for determining equilibrium is that discriminating between nearby states is harder than discriminating between far away states (Morris and Yang, 2022; Hébert and Woodford, 2021)[27]. We ask subjects to consider the 149 other participants in their experimental condition of the study, who also just completed Phase 1. We then ask subjects the following two questions:

1. Consider only trials where the number on screen was equal to 47. In what percentage of these trials do you think the other participants gave a correct answer, that is, they correctly classified whether the number was smaller or larger than 55?

2. Consider only trials where the number on screen was equal to 54. In what percentage of these trials do you think the other participants gave a correct answer, that is, they correctly classified whether the number was smaller or larger than 55?

For each of the two questions, we pay the subject 0.5 GBP if their forecast is within 1% of the true percentage.[28] Question 1 elicits beliefs about others' imprecision when the distance between the number is far from the threshold (47 vs. 55), whereas Question 2 elicits beliefs about others' imprecision when the distance is close (54 vs. 55). While we could have asked subjects about their beliefs about others' imprecision for a range of numbers — rather than the single numbers 47 and 55 — this would have introduced a confound, since the distribution of numbers is different across conditions.

In Phase 3, we ask subjects about their own performance on the number classification task (that they completed in Phase 1). This question is not trivial because we do not provide subjects with feedback after any round in Phase 1 (nor after the end of Phase 1). Here, we

[27]For example, an alternative model of imperfect perception that does not feature the property that nearby states are harder to distinguish than far away states is proposed in Gul, Pesendorfer and Strzalecki (2017).

[28]Following Hartzmark, Hirshman and Imas (2021), we choose this elicitation procedure as opposed to a more complex mechanism such as the Binarized Scoring Rule (BSR) due to recent evidence showing that the BSR can systematically bias truthful reporting (Danz, Vesterlund and Wilson, 2022).

are also interested in subjects' awareness of their own imprecision for numbers that are close and far from the threshold. Specifically, we ask subjects the following two questions:

1. Consider only trials where the number on screen was between 52 and 58. In what percentage of these trials do you think you correctly classified whether the number was smaller or larger than 55?

2. Consider only trials where the number on screen was less than 52 or greater than 58. In what percentage of these trials do you think you correctly classified whether the number was smaller or larger than 55?

For each of these two questions, we again reward subjects with 0.50 GBP if they provide an answer that is within 1% of their true accuracy. All subjects first go through Phase 1, and the order of Phase 2 and Phase 3 is randomized across subjects. We note that one potential concern with our design, is that when asking subjects about their performance in Phase 1, we are testing memory, not *ex-ante* beliefs. This is a reasonable concern, and an alternative is to have subjects forecast their performance before undertaking the classification task. However, under this alternative design, subjects' classification performance would be endogenous to their beliefs, and would invalidate the incentive compatibility of our belief elicitation procedure. For this reason, we opt to implement Phase 1 first for all subjects.

Experimental Results

The upper panel of Figure A3 replicates the classic result from previous experiments on number discrimination, whereby subjects exhibit errors, and these errors increase as the number on screen approaches the threshold (Dehaene, Dupoux and Mehler, 1990). Moreover, we see that, for numbers between 47 and 63, errors are systematically higher in the high volatility condition (Frydman and Jin, 2022). Similar patterns are reflected in the response times shown in the lower panel of Figure A3: response times increase as the number approaches the threshold of 55, and response times are systematically longer in the high volatility condition.

The purpose of Phase 1 is to create a dataset about performance, over which we can ask subjects about their beliefs in Phases 2 and 3. In the left panel of Figure A4, we see that subjects believe their behavior in the classification task exhibits imprecision (that is, beliefs about accuracy are less than 100%). Moreover, we see that subjects are aware that mistakes are more likely for numbers closer to the threshold (greater than 52 and less than 58) than for numbers farther from the threshold (less than 52 or greater than 58; $p < 0.001$).

The results in the middle panel of Figure A4 help validate a crucial assumption in our model. Specifically, we see that subjects are aware of other subjects' imprecision. Moreover,

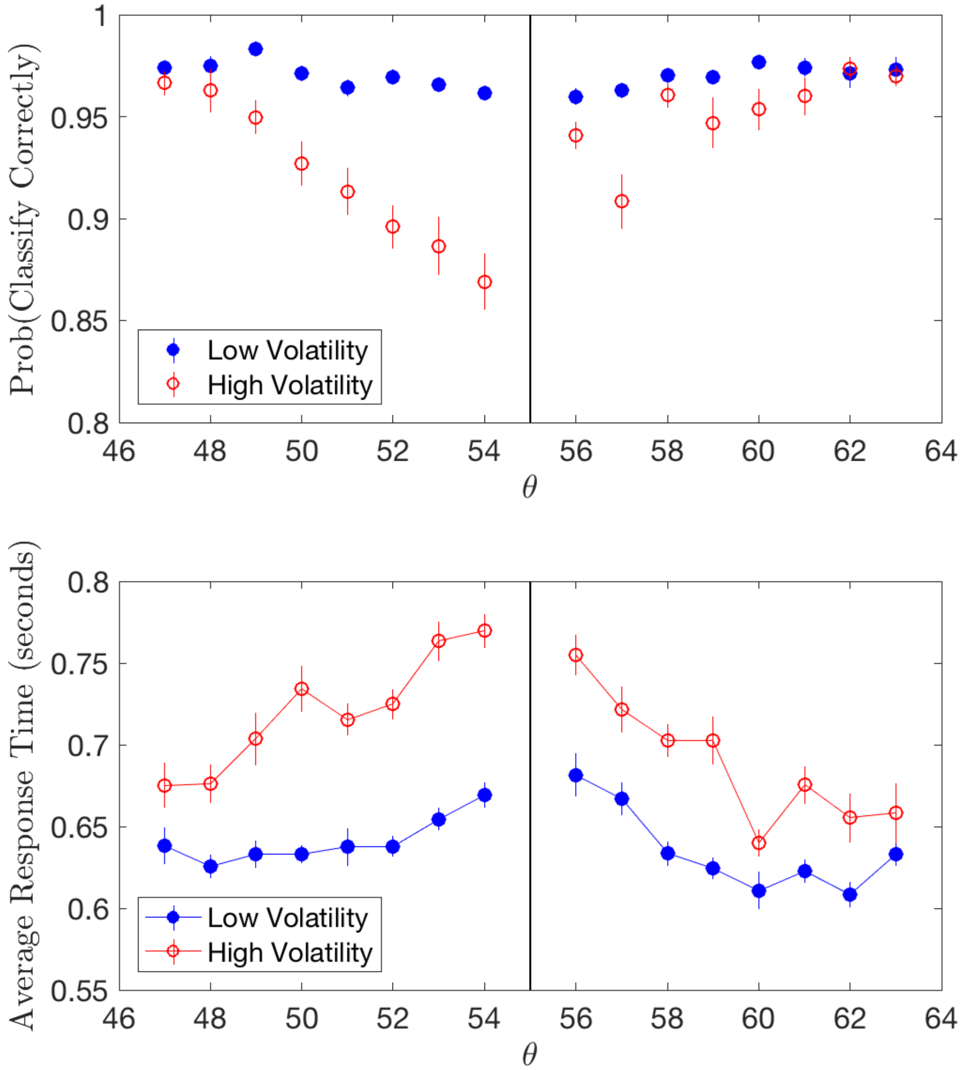

Figure A3: **Accuracy and Response Times in the Classification Task.** Note: Upper panel shows the proportion of rounds on which subjects correctly classify θ as greater than or less than the reference level of 55. Lower panel shows the average response time on rounds where subjects correctly classify θ. In both panels, the vertical bars denote two standard errors of the mean. Standard errors are clustered by subject.

subjects believe that others are less accurate when discriminating 54 vs. 55 compared with discriminating 47 vs. 55 ($p < 0.001$). When embedded in a game, these beliefs are sufficient to generate strategic uncertainty: if player i believes that player j perceives θ with error, then player i is uncertain about player j's perception. The data in the middle panel of Figure A4 therefore provide support for the mechanism that generates strategic uncertainty in our

Figure A4: **Beliefs about Own and Others' Accuracy in the Classification Task.** Note: Left panel shows the average belief about own accuracy for values of θ that are far ($\theta < 52$ or $\theta > 58$) and close ($51 < \theta < 59$) to the threshold 55. Middle panel shows the average belief about others' accuracy for values of θ that are far ($\theta = 47$) and close ($\theta = 54$) to the threshold 55. Right panel shows the average belief about others' accuracy when $\theta = 54$, split by experimental condition. In all panels, vertical bars denote two standard errors of the mean.

model.

Finally, our data also enable us to test one other feature of beliefs about others' imprecision. As outlined in our pre-registration, we test whether beliefs about others' accuracy on rounds when $\theta = 54$ is higher for those subjects who experience the low volatility distribution in Phase 1.[29] Such a test investigates the hypothesis that subjects are aware that others' perception of a given number varies as a function of the experienced distribution. Indeed, the right panel of Figure A4 shows that, for $\theta = 54$, subjects who experience the high volatility distribution in Phase 1 report that others make more errors, compared to those subjects who experience the low volatility distribution in Phase 1 ($p = 0.048$).

[29] Pre-registration document is available at `https://aspredicted.org/OGG_XNK`.

E Experimental Instructions

Main Experiment (Coordination Game)

Welcome!

You will earn £2 for completing this study and will have the opportunity to **earn more money** depending on your decisions during the study.

Specifically, at the end of the study, the computer will randomly select one question. You will receive points from the randomly selected question and the number of points depends on your decision and the decision of another participant. Points will be converted to pounds using the rate 20 points = £1. For example, if you earned 60 points for the selected question, you would then earn 60/20 = £3 (in addition to the completion fee).

All questions are equally likely to be selected so **make all choices carefully**.

The next pages give detailed instructions. Following the instructions, you will take a quiz on them. You will be allowed to continue and will be entitled to payment **only if you answer all questions on the quiz correctly**.

Instructions (1/2)

The study is separated into 6 parts of 50 rounds each.

In each round, you are randomly matched with another participant, who we call your **opponent**.

In each round, both you and your opponent will be asked to choose between **two options**:

"Option A" or "Option B"

Here is how to earn points:

- If you choose Option A, the number of points you receive does not depend on whether your opponent chooses Option A or B. The amount of points you receive for choosing Option A can be different in different rounds and will be displayed on your screen.
- If you choose Option B, the number of points you receive depends on your opponent's decision: if your opponent chooses Option A, you will receive 47 points; if your opponent also chooses Option B you will receive 63 points.

Importantly, your opponent is reading these same exaxt instructions. This means that:

- If your opponent chooses Option A, his/her payoff does not depend on your decision and the number of points he/she earns are those given by Option A.
- If your opponent chooses Option B, the number of points he/she receives depends on your decision: if you choose Option A, your opponent will receive 47 points; if you also choose Option B, your opponent will receive 63 points.

Instructions (2/2)

Below is an example screen from the study:

Option A	Option B
53	**47** if other participant chooses A **63** if other participant chooses B

In this example, Option A is on the LEFT side of the screen and Option B is on the RIGHT.

In each round, you will choose one of the two options by pressing either the "A" key on your keyboard for the LEFT option or the "L" key on your keyboard for the RIGHT option. On some rounds, Option A will be on the LEFT, and in other rounds it will be on the RIGHT.

In the example above:

- Option A pays you 53 points regardless of your opponent's decision, while Option B pays you 47 points if your opponent chooses Option A and 63 points if your opponent chooses Option B.
- Note also that, if your opponent chooses Option A, he/she earns 53 points regardless of your decision. If your opponent, instead, chooses Option B, he/she earns 47 points if you choose Option A and 63 points if you choose Option B.

Experiment 2 (Human vs. Algorithm)

Welcome!

You will earn £2 for completing this study.

You will also have the opportunity to **earn more money** depending on your decisions.

Specifically, at the end of the study, the computer will randomly select one question. You will receive points from the randomly selected question and the number of points depends on your decision and the decision of an opponent. Points will be converted to pounds using the rate 20 points = £1. For example, if you earned 60 points for the selected question, you would then earn 60/20 = £3 (in addition to the completion fee).

All questions are equally likely to be selected so **make all choices carefully**.

The next pages give detailed instructions.

Following the instructions, you will take a quiz on them. You will be allowed to continue and will be entitled to payment **only if you answer all questions on the quiz correctly**.

Instructions (1/2)

The study is separated into 6 parts of 50 rounds each.

In each round, you will play a game against the computer, who we call your **opponent**.

In each round, both you and your opponent will choose between two options:

"Option A" or "Option B"

Importantly, **your computerized opponent chooses according to a rule** that you will learn below.

Here is how to earn points:

- If you choose Option A, the number of points you receive does not depend on whether your opponent chooses Option A or B. The amount of points you receive for choosing Option A can be different in different rounds and will be displayed on your screen.
- If you choose Option B, the number of points you receive depends on your opponent's decision: if your opponent chooses Option A, you will receive 47 points; if your opponent also chooses Option B you will receive 63 points.

Your Opponent's Rule

You will not have to guess how your computerized opponent will behave.

This is because **your opponent is programmed to choose according to the following rule**:

- If Option A delivers 55 points or more, your opponent chooses Option A.
- If Option A delivers less than 55 points, your opponent chooses Option B.

To be clear: your computerized opponent follows the rule above without exceptions. So, you can be certain about how your opponent's choice depends on the number of points that Option A delivers.

Instructions (2/2)

Below is an example screen from the study:

Option A	**Option B**
53	**47** if opponent chooses A **63** if opponent chooses B

In this example, Option A is on the LEFT side of the screen and Option B is on the RIGHT.

In each round, you will choose one of the two options by pressing either the "A" key on your keyboard for the LEFT option or the "L" key on your keyboard for the RIGHT option. On some rounds, Option A will be on the LEFT, and in other rounds it will be on the RIGHT.

In the example above:

- Since Option A pays 53 points, the rule says your opponent chooses Option B.
- Therefore, if you choose Option A, you get 53 points.
- If, instead, you choose Option B, you get 63 points.

Experiment 3 (Awareness of cognitive noise)

Thank you for participating in this study!

Before we begin, please close all other applications on your computer and put away your cell phone. This study will last approximately **10 minutes**. During this time, we ask your complete and undistracted attention. You will earn £1 for completing the study and you will have the opportunity to **earn more money** depending on your answers during the study.

This study consists of **two phases**. The instructions for Phase 1 are given in the next page. After you go through Phase 1, you will be given a new set of instructions for Phase 2.

When you are ready to continue, press ENTER.

In **Phase 1**, you will see a series of numbers and will be asked to classify whether each number is **larger or smaller than 55**. If the number displayed is smaller than 55, press the “A” key on your keyboard. If the number displayed is larger than 55, press the "L" key.

Your bonus payment will depend on the speed and accuracy of your classification. Specifically:

Bonus Payment = £ (1.5 x accuracy – 1 x speed)

where "accuracy" is the percentage of trials where you correctly classified the number as larger or smaller than 55, and "speed" is the average amount of time it takes you to classify the number on all trials throughout the study, in seconds.

Thus, you make the most money by answering as **quickly** and as **accurately** as possible.

For example, if you correctly classified the number on all trials and it took you 0.3 seconds to respond to each question, you would earn £(1.5 x 100% - 10 x 0.3) = £1.20. If instead you only classified 70% of the numbers correctly and took 0.8 seconds to respond to each question, you would earn £(1.5 x 70% - 10 x 0.8) = £0.25.

Phase 1 will be separated into 3 parts of 50 trials each. In between, you can take a short break.

Before starting with the classification task, you will be asked a question to check your understanding of the instructions. You will be allowed to continue **only if you answer this question correctly**.

When you are ready to continue with the comprehension question, press ENTER.

This is **Phase 2** of the study.

Phase 2 consists of four questions, two on this page and two on the next one.

There are 99 other participants in this study.

Consider the task completed by **the other participants** in Phase 1.

Question 1

Consider only trials where the number on the screen was **equal to 47**. In what percentage of these trials do you think **the other participants** gave a correct answer, that is, they correctly classified whether the number was smaller or larger than 55? Give us your forecast on a scale between 0% and 100%, where 0% means you believe no answer in these trials was correct and 100% means you believe all answers in these trials were correct. If your forecast is within plus or minus 1% of the true percentage, you will earn £0.5.

Question 2

Consider only trials where the number on the screen was **equal to 54**. In what percentage of these trials do you think **the other participants** gave a correct answer, that is, they correctly classified whether the number was smaller or larger than 55? Give us your forecast on a scale between 0% and 100%, where 0% means you believe no answer in these trials was correct and 100% means you believe all answers in these trials were correct. If your forecast is within plus or minus 1% of the true percentage, you will earn £0.5.

Press ENTER to confirm your answers.

Consider the task **you** completed in Phase 1.

Question 3

Consider only trials where the number on the screen was **between 52 and 58**. In what percentage of these trials do you think **you** gave a correct answer, that is, you correctly classified whether the number was smaller or larger than 55? Give us your forecast on a scale between 0% and 100% where 0% means you believe no answer in these trials was correct and 100% means you believe all answers in these trials were correct. If your forecast is within plus or minus 1% of your true accuracy, you will earn £0.5.

Question 4

Consider only trials where the number on the screen was **smaller than 52 or larger than 58**. In what percentage of these trials do you think **you** gave a correct answer, that is, you correctly classified whether the number was smaller or larger than 55? Give us your forecast on a scale between 0% and 100% where 0% means you believe no answer in these trials was correct and 100% means you believe all answers in these trials were correct. If your forecast is within plus or minus 1% of your true accuracy, you will earn £0.5.

Press ENTER to confirm your answers.