DISCUSSION PAPER SERIES

DP16625

HALT: Heterogeneous-Agent Liquidity Traps

Florin Ovidiu Bilbiie

MONETARY ECONOMICS AND FLUCTUATIONS

CEPR

ISSN 0265-8003

HALT: Heterogeneous-Agent Liquidity Traps

Florin Ovidiu Bilbiie

Discussion Paper DP16625
Published 09 October 2021
Submitted 08 October 2021

Centre for Economic Policy Research
33 Great Sutton Street, London EC1V 0DX, UK
Tel: +44 (0)20 7183 8801
www.cepr.org

HALT: Heterogeneous-Agent Liquidity Traps

Abstract

In a tractable heterogeneous-agent New-Keynesian model, I study analytically liquidity traps. Heterogeneity determines whether liquidity traps are confidence-driven or fundamental, excess-saving driven, where the latter can be triggered by shocks to inequality or income risk. Heterogeneity amplifies liquidity-trap recessions (without relying on deep deflations), fiscal multipliers, and forward-guidance power when income inequality and risk are countercyclical. Dampening occurs instead when inequality and risk are procyclical, ruling out confidence-driven traps, neo-Fisherian effects, and the forward guidance puzzle. Optimal monetary policy implies that forward-guidance duration is optimally shortened by the same inequality motives that amplify its power.

JEL Classification: E21, E31, E40, E44, E50, E52, E58, E60, E62

Keywords: Heterogeneity, Inequality, tractable HANK, liquidity traps, neo-Fisher, multipliers, forward guidance, Optimal monetary policy

Florin Ovidiu Bilbiie - florin.bilbiie@gmail.com
University of Lausanne and CEPR

Acknowledgements
This paper supersedes the Aug. 2017 CEPR DP 12231 "The Puzzle, the Power, and the Dark Side: Forward Guidance Redux", then incorporated into "A Catch-22 for HANK Models: No Puzzles, No Amplification" and its subsequent incarnations, the corresponding sections of which (dealing with Liquidity Traps) it also supersedes. I am grateful for comments in particular to Adrien Auclert, Edouard Challe, Keshav Dogra, Gauti Eggertsson, Marcus Hagedorn, Karel Mertens, and to Sushant Acharya, Christian Bayer, Davide Debortoli, Marco Del Negro, Axelle Ferrière, Jordi Galí, Marc Giannoni, Keith Kuester, Eric Leeper, Olivier Loisel, Christina Patterson, Xavier Ragot, Vincent Sterk, Ludwig Straub, Paolo Surico, Andrea Tambalotti, Mirko Wiederholt, Michael Woodford, and a great many other colleagues and seminar and conference participants. I gratefully acknowledge without implicating the support of Banque de France Chair at PSE, and of Institut Universitaire de France, as well as the hospitality of New York University and CREI.

HALT: Heterogeneous-Agent Liquidity Traps[I]

Florin O. Bilbiie[II]

October 2021 (First draft August 2017)[III]

Abstract

In a tractable heterogeneous-agent New-Keynesian model, I study analytically liquidity traps. Heterogeneity determines whether liquidity traps are confidence-driven or fundamental, excess-saving driven, where the latter can be triggered by shocks to inequality or income risk. Heterogeneity amplifies liquidity-trap recessions (without relying on deep deflations), fiscal multipliers, and forward-guidance power when income inequality and risk are countercyclical. Dampening occurs instead when inequality and risk are procyclical, ruling out confidence-driven traps, neo-Fisherian effects, and the forward guidance puzzle. Optimal monetary policy implies that forward-guidance duration is optimally shortened by the same inequality motives that amplify its power.

JEL Codes: E21, E31, E40, E44, E50, E52, E58, E60, E62

Keywords: heterogeneity; inequality; tractable HANK; liquidity traps; neo-Fisher; multipliers; forward guidance; optimal monetary policy.

[I] I am grateful for comments in particular to Adrien Auclert, Edouard Challe, Keshav Dogra, Gauti Eggertsson, Marcus Hagedorn, Karel Mertens, and to Sushant Acharya, Christian Bayer, Davide Debortoli, Marco Del Negro, Axelle Ferrière, Jordi Galí, Marc Giannoni, Keith Kuester, Eric Leeper, Olivier Loisel, Christina Patterson, Xavier Ragot, Vincent Sterk, Ludwig Straub, Paolo Surico, Andrea Tambalotti, Mirko Wiederholt, Michael Woodford, and a great many other colleagues and seminar and conference participants. I gratefully acknowledge without implicating the support of Banque de France Chair at PSE, and of Institut Universitaire de France, as well as the hospitality of New York University and CREI.

[II] University of Lausanne and CEPR; florin.bilbiie@gmail.com. http://florin.bilbiie.googlepages.com.

[III] This paper supersedes the Aug. 2017 "The Puzzle, the Power, and the Dark Side: Forward Guidance Redux", then incorporated into "A Catch-22 for HANK Models: No Puzzles, No Amplification" and its subsequent incarnations, the corresponding sections of which (dealing with Liquidity Traps) it also supersedes.

1 Introduction

Two of the most important and policy-relevant developments in macroeconomic research of the last decades, in particular monetary and fiscal policy, are the study of liquidity traps, and heterogeneity/inequality. This paper is an analytical investigation of the former, in a model that belongs to the latter class. Studying liquidity traps and policy options therein in a model that features distributional, inequality channels is not only interesting in its own right, it is a topical endeavour especially in the current post-COVID world where inequality concerns are front and center in policy discussions, and where the spectre of liquidity traps seems to haunt back most developed economies.

Thus in this paper, I analyze the occurrence of liquidity traps triggered by changes in either confidence or fundamentals in a tractable HANK model, *THANK,* that I developed fully in the companion paper Bilbiie (2018). Then, I analyze the determinants of the size of liquidity-trap recessions, and monetary and fiscal policy solutions therein—including the optimal design of monetary policy as forward guidance.

I analyze all these issues in a three-equation THANK model isomorphic to the textbook representative-agent (RANK) model, which it nests. As argued at length in the companion paper developing it, THANK captures several key dimensions that the recent quantitative-HANK literature finds important for studying macro fluctuations with micro heterogeneity. The key channels of interest for our aggregate implications here are the *cyclicalities of income inequality* (i.e. the constrained agents' income elasticity to aggregate income) and of income risk. As discussed in text following a large segment of the literature, both of these channels can lead to aggregate-demand amplification when they are countercyclical, and to dampening when procyclical. Intertemporally, they translate into *compounding* in the aggregate Euler equation in the former case, and *discounting* in the latter.

I characterize the occurrence of neo-Fisherian effects in the presence of heterogeneity, finding that they are more likely when inequality and risk are countercyclical and less likely when procyclical. Indeed, neo-Fisherian effects are ruled out altogether when inequality and risk are procyclical enough to rule out the forward guidance puzzle and restore determinacy under a peg. The same condition that makes neo-Fisherian effects more likely also increases the likelihood of confidence-driven liquidity traps.

Then, focusing on liquidity traps driven by changes in fundamentals, I show that under heterogeneity liquidity traps can occur in response to exogenous changes in income inequality or risk that induce excess (desired) saving for precautionary, self-insurance purposes. Regardless of the source of the fundamental trap, the *size* of the liquidity-trap recession is (potentially much) larger with *countercyclical* income inequality or risk—the more plausible scenario empirically. Furthermore, it is not necessarily related to or stemming from deflationary forces: indeed, a large LT-recession can occur even with fixed prices, thus alleviating what is known as the "missing deflation puzzle" (Hall, 2011) in RANK models. The same mechanism, however, also magnifies the "paradox of flexibility" (Eggertsson and Krugman, 2012): it amplifies recessionary forces due to deflation, too. Conversely, procyclical inequality and risk mitigate this paradox. Likewise, for LT fiscal multipliers: unlike in the RANK analyses of Eggertsson (2010) and Christiano et al (2011), large multipliers here can occur even with fixed prices—but only under countercyclical inequality; multipliers are instead dampened with procyclical inequality.

Furthermore, I show analytically that in a HALT, forward guidance (keeping interest rates low after the LT-triggering shock is over) also becomes more powerful with countercyclical inequality and risk and less with procyclical. Indeed, the former case aggravates the "FG puzzle" and the latter can cure it. These are merely LT applications to existing results pertaining to FG in the analytical HANK literature, e.g. McKay et al (2016), Bilbiie (2018, 2020), and Acharya and Dogra (2020). The novel contribution in this respect is that in a HALT, *optimal* policy still amounts to forward guidance, but its duration is eventually decreasing with the degree of heterogeneity, even in the "amplification", countercyclical-inequality case. The reason is that amplification also applies to the welfare cost of forward guidance and not only to its benefit, generating inefficient inequality volatility.

Related Literature—Some of the earliest **quantitative** HANK models focused on liquidity traps, e.g. Guerrieri and Lorenzoni (2017) and Heathcote and Perri (2018). Quantitative HANK models have been used to address a wide variety of macro issues.[1] This paper belongs to a literature developing *analytical* representations of the richer-heterogeneity models in order to gain insights into their mechanisms. Bilbiie (2008, 2018, 2020), Acharya and Dogra (2020), Ravn and Sterk (2020), Werning (2015), Broer, Hansen, Krusell, and Oberg (2020), Debortoli and Gali (2018), Hagedorn (2020), Bilbiie, Känzig and Surico (2019). Fiscal multipliers under heterogeneity have been analyzed in several quantitative HANK models cited above and in TANK for spending (Galí et al (2007)), transfers (e.g. Bilbiie, Monacelli and Perotti (2013)) or both, in liquidity traps (Eggertsson and Krugman (2012)). While Eggertsson (2010) and Christiano, Eichenbaum, and Rebelo (2011)'s seminal papers showed that LT *spending multipliers* are large.[2]

The chief insight pertaining to optimal monetary policy in an LT in RANK is that is implies "forward guidance" FG (keeping rates low beyond the end of the trap)—starting with Eggertsson and Woodford (2003), and extended by Jung, Teranishi and Watanabe (2005), Adam and Billi (2006), and Nakov (2008). Refinements focused on uncertainty, institutional arrangements, and "sustainable" policies, see e.g. Nakata (2018) and Nakata and Schmidt (2016). Bilbiie (2019) models FG as a state, calculating its closed-form optimal duration, a proxy to fully-optimal Ramsey policy. We revisit this issue analytically with heterogeneous agents, drawing on result derived for optimal policy analytically in Bilbiie (2018), abstracting from the ZLB; other analytical studies using different models include Challe (2020) and Bilbiie, Monacelli, and Perotti (2020). See the literature review therein, including how this compares to optimal policy in rich-heterogeneity in quantitative HANK such as Bhandari, Evans, Golosov, and Sargent (2020).

Lastly, in a subsequent and complementary analysis, Fernandez-Villaverde et al (2021) solve a quantitative rich-heterogeneity HANK model with an occasionally binding ZLB using global methods, and confirm some of this paper's analytical findings in that much richer environment; that is a clear illustration of the complementarity of these two different approaches, and I do hope that other analytical results derived here can inform further quantitative work on this important topic.

[1] The effects of transfers (Oh and Reis, 2012); job-uncertainty-driven recessions (Ravn and Sterk, 2017; den Haan, Rendahl, and Riegler, 2018); monetary transmission (Gornemann, Kuester, and Nakajima, 2016; Auclert, 2018; Debortoli and Gali, 2018; Auclert and Rognlie, 2017); portfolio composition (Bayer et al, 2016 and Luetticke, 2018); fiscal policy (Ferrière and Navarro, 2018, Hagedorn, Manovskii, and Mitman, 2018; Auclert, Rognlie, and Straub, 2018; McKay and Reis, 2016; Cantore and Freund, 2019); the FG puzzle (McKay et al, 2016; Hagedorn, Luo, Manovskii, and Mitman, 2019).

[2] This is overturned in a confidence-driven liquidity trap, as shown by Mertens and Ravn (2014) and Boneva et al (2018). See Bilbiie (2021) for further analytical implications in RANK.

2 HALT: Liquidity Traps in An Analytical HANK Model

This section outlines the loglinearized THANK model, an analytical HANK model that captures several key channels of complex HANK models: cyclical inequality, self-insurance in face of idiosyncratic uncertainty, and a distinction between liquid and illiquid assets. For a full exposition of the underlying model, I refer the reader to Bilbiie (2018, 2020). There are two states of the world, constrained hand-to-mouth H and unconstrained "savers" S, between which agents switch *exogenously* (idiosyncratic uncertainty). Second, in face of this risk there is *full insurance within* type, after idiosyncratic uncertainty is revealed, but *limited insurance across* types. Third, different assets have different *liquidity:* only one of the two assets can be used to self-insure, i.e. is *liquid*. Specifically, bonds are liquid: they *can* be used to self-insure, before idiosyncratic uncertainty is revealed; while stocks are illiquid, they cannot be used to self-insure. In this paper, I focus on the zero-liquidity equilibrium, assuming that there is no equilibrium bond trading.[3]

The exogenous change of state follows a Markov chain: the probability to *stay* type S is s, and to stay type H is h, with transition probabilities $1-s$ and $1-h$ respectively; the probability s can be a function of aggregate activity. I focus on stationary equilibria whereby the mass of H is the *unconditional* probability:

$$\lambda = \frac{1-s}{2-s-h}.$$

TANK is nested for permanent idiosyncratic shocks ($s = h = 1$) and λ fixed at its initial free-parameter value.

The key equation is the self-insurance Euler equation for liquid bonds:

$$\left(C_t^S\right)^{-\frac{1}{\sigma}} = \beta E_t \left\{ \frac{1+i_t}{1+\pi_{t+1}} \left[s\left(Y_{t+1}\right) \left(C_{t+1}^S\right)^{-\frac{1}{\sigma}} + \left(1-s\left(Y_{t+1}\right)\right) \left(C_{t+1}^H\right)^{-\frac{1}{\sigma}} \right] \right\}, \tag{1}$$

where the transition probability depends on aggregate demand (tomorrow). This equation generates a demand for liquid bonds for precautionary saving by unconstrained agents who seek to self-insure in face of the risk of becoming constrained; the risk is therefore related both to income inequality between the two states and to changes in the transition probability.[4]

We derive an *aggregate* Euler equation, or IS curve for this economy starting from this individual Euler equation and replacing individual consumptions using the rest of the model to express them as a function of aggregate consumption and income. The following key relationships, written in loglinearized form directly, would result from a model of the income distribution:

$$\begin{aligned} c_t^H &= y_t^H = \chi y_t, \chi \lesseqgtr 1, \\ c_t^S &= \frac{1-\lambda\chi}{1-\lambda} y_t, \end{aligned} \tag{2}$$

[3] This follows Krusell, Mukoyama and Smith (2011); other zero-liquidity HANK include Ravn and Sterk (2017), Werning (2015), McKay and Reis (2017), Broer et al (2018), etc. See Bilbiie (2021) for the case with liquidity.

[4] A similar equation holds in the underlying model for H agents who contemplate transitioning to the S state, but we assume that it always holds with strict inequality (H are constrained in the equilibrium we focus on). Furthermore, the Euler equation for *illiquid* assets (e.g. share in firms giving claims to monopolistic profits) is different in that, because the asset is illiquid, it is priced only by agents in the S state, i.e. using only their stochastic discount factor. I refer the reader to Bilbiie (2018) for a complete exposition of the underlying model and a discussion of these issues, including of the within-type perfect insurance scheme necessary to deliver this simple quasi-aggregate representation.

H's consumption comoves one-to-one with *their* income, but *not necessarily* with *aggregate* income. The parameter χ is key and cyclical distributional effects make it different from 1. When $\chi < 1$ S's income elasticity to aggregate income is *larger* than one, and vice versa. Equilibrium *income inequality* $y_t^S - y_t^H = (1-\chi)\frac{y_t}{1-\lambda}$ is *procyclical* iff $\chi < 1$ and *countercyclical* iff $\chi > 1$. Many possible theories of the income distribution can lead to such reduced-form relationships and particular expressions for χ as a function of structural parameters. The TANK model in Bilbiie (2008, 2020) is one such example with χ depending on labor elasticity and fiscal redistribution of profits.[5] Bilbiie, Känzig, and Surico (2019) show how investment in physical capital can also be interpreted as delivering a (different) reduced-form χ.

Replacing the individual consumptions (2) in the loglinearized version of (1) around a steady-state with *inequality* $\Gamma \equiv Y^S/Y^H > 1$, we obtain the *aggregate Euler-IS*:

$$c_t = \delta E_t c_{t+1} - \sigma \frac{1-\lambda}{1-\lambda\chi}\left(i_t - E_t \pi_{t+1} - \rho_t\right), \tag{3}$$

$$\text{with } \delta = 1 + \frac{(\chi - 1)(1-\tilde{s})}{1-\lambda\chi} + \frac{s_Y Y}{1-s}\left(1-\Gamma^{-1/\sigma}\right)(1-\tilde{s})\,\sigma\frac{1-\lambda}{1-\lambda\chi},$$

where $1-\tilde{s} = \frac{(1-s)\Gamma^{1/\sigma}}{s+(1-s)\Gamma^{1/\sigma}} > 1-s$ is the inequality-weighted transition probability measure of risk. $E_t\pi_{t+1}$ is expected inflation, the nominal interest rate i_t is in *levels* (to allow dealing with the zero lower bound transparently later) and ρ_t an exogenous shock that is standard in the liquidity-trap RANK literature (Eggertsson and Woodford, 2003) capturing impatience, or the urgency to consume in the present (its steady-state value is the discount rate $\rho = \beta^{-1} - 1$): when it increases, S households try to bring consumption into the present and "dis-save", and vice versa when it decreases. In a HANK model, however, this reduced-form shock can capture other fundamental changes such as shocks to income inequality or income risk, as spelled out below.

The aggregate Euler equation differs from its RANK counterpart in several ways. The contemporaneous AD elasticity to interest rates is the TANK one, $\sigma\frac{1-\lambda}{1-\lambda\chi}$, reflecting a New Keynesian Cross logic described above. Even though the "direct effect" of a change in interest rates is scaled down by $(1-\lambda)$ (λ agents do not respond directly), the "indirect effect", which amounts to the aggregate-MPC slope of the planned-expenditure curve, is increasing with λ. The rate at which it does so depends on χ, and with $\chi > 1$ the latter effect dominates the former, delivering amplification relative to RANK (while for $\chi < 1$ the reverse holds).

Furthermore, as discussed in detail in Bilbiie (2018, 2020) the Aggregate Euler-IS equation of THANK is characterized by **compounding** ($\delta > 1$) with countercyclical inequality ($\chi > 1$) or risk ($-s_Y < 0$) and by **discounting** ($\delta < 1$) when procyclical ($\chi < 1$ or $-s_Y > 0$); the case isolating the cyclicality of income risk has been studied in isolation, using different models of risk, by Acharya and Dogra (2020) and Ravn and Sterk (2020).[6]

Compounding occurs with countercyclical inequality or risk. Good aggregate income news boost today's demand because they imply less need for self-insurance. Since future consumption in states where the constraint binds over-reacts to good aggregate news when $\chi > 1$, households

[5] Different income distribution models have been advanced in the subsequent literature, e.g. assuming *sticky wages*, see Colciago (2011) in TANK and Broer et al (2018), Hagedorn et al (2018), and Auclert et al (2018) in HANK.

[6] See Werning (2015) for a more general version of aggregation of nonlinear Euler equations with income risk that contains both channels, and Holm (2020) for a different example.

demand less "saving". Likewise, with $-s_Y < 0$, an aggregate expansion reduces the risk of moving to the bad state and mitigates the need for self-insurance, amplifying the initial expansion. But savings still need to be zero in equilibrium, so households consume more than one-to-one and income increases more than without risk.

Discounting occurs instead with procyclical inequality or risk: When good news about future aggregate income arrive, households recognize that in some states of the world they will be constrained and, because $\chi < 1$, not benefit fully from it, so they self-insure and increase consumption less than if they were alone in the economy. With $-s_Y > 0$, good news generate an expansion today to start with, which increases the probability of moving to the bad state and triggers precautionary saving, containing the expansion. Like in RANK and TANK, this (now, self-insurance) increase in saving demand cannot be accommodated as there is no asset, so the household consumes less today and income adjusts accordingly.

Finally, firms' "supply side" is standard, described the loglinearized Phillips curve:

$$\pi_t = \beta E_t \pi_{t+1} + \kappa c_t. \tag{4}$$

2.1 Neo-Fisherian effects and heterogeneity

Assume that the central bank sets i_t as an exogenous process, in levels $i_t = \rho + i_t^*$, with intercept the constant ρ (i.e. ignore natural-rate shocks, $\rho_t = \rho$) and persistence μ, $i_t^* = \mu i_{t-1}^* + \varepsilon_t$ and thus $E_t i_{t+1}^* = \mu i_t^*$. The purely forward-looking model (4)-(3) is then potentially indeterminate, but we choose *one* equilibrium: the "minimum state variable MSV" solution by McCallum (1998) with $E_t \pi_{t+1} = \mu \pi_t$. Under this solution, (4) becomes:

$$\pi_t = \frac{\kappa}{1 - \beta\mu} c_t, \tag{5}$$

Replacing in (3) we obtain:

$$c_t = \nu(\mu) E_t c_{t+1} - \sigma \frac{1-\lambda}{1-\lambda\chi} i_t^*, \tag{6}$$
$$\text{where } \nu(\mu) \equiv \delta + \sigma \frac{1-\lambda}{1-\lambda\chi} \frac{\kappa}{1-\beta\mu}$$

is a key parameter capturing the elasticity to future "news" shocks on aggregate demand-income $E_t c_{t+1}$. With either $\mu = 0$ or $\beta = 0$ this is $\nu(\mu) = \nu = \delta + \kappa\sigma \frac{1-\lambda}{1-\lambda\chi}$: future income means future demand, future expected inflation, lower real rates, and intertemporal substitution towards today. When the shock is persistent and the PC is forward-looking $\beta > 0$, this is further amplified as the whole future path of income matters for current inflation through (5). Replacing the MSV conjecture $E_t c_{t+1} = \mu c_t$ in (6) we obtain Proposition 1.

Proposition 1 ***Neo-Fisher Effect****: Under a peg, interest rate increases are expansionary, inflationary, and reduce the real rate $r_t = i_t - E_t \pi_{t+1}$:*

$$\frac{\partial c_t}{\partial i_t^*} = -\frac{\sigma \frac{1-\lambda}{1-\lambda\chi}}{1 - \mu\nu(\mu)}; \frac{\partial \pi_t}{\partial i_t^*} = \frac{\kappa}{1-\beta\mu} \frac{\partial c_t}{\partial i_t^*}; \frac{\partial r_t}{\partial i_t^*} = \frac{1-\mu\delta}{1-\mu\nu(\mu)}. \tag{7}$$

*if and only if they are **persistent enough** and there is enough **news-amplification**, namely:*

$$\mu\nu(\mu) > 1. \tag{8}$$

In THANK there can be no neo-Fisherian effects if

$$\nu(\mu) < 1.$$

Conversely, neo-Fisherian effects are more likely with countercyclical inequality and/or risk $\delta > 1$.

This result is derived for simplicity under a peg but generalizes directly to the case of a Taylor rule, see the Appendix. In particular, the very same condition dictates whether there is positive comovement of the nominal rate and equilibrium inflation/consumption.

The necessary and sufficient condition for ruling out neo-Fisherian effects $\nu < 1$ requires that $1 - \delta > \frac{\kappa}{1-\beta\mu}\sigma\frac{1-\lambda}{1-\lambda\chi}$, i.e. that the HANK-AD discounting dominate the AS-compounding of news (right side) that drives the occurrence of neo-Fisherian effects in RANK.

To gain intuition, focus first on the long-run effect: the (old) Fisher effect is either enhanced or lacking altogether, depending on whether inequality and risk are counter- or pro-cyclical. Consider the steady state of the HANK Euler equation (1):

$$1 = \beta\frac{1+i}{1+\pi}\left[s(Y) + (1 - s(Y))\,\Gamma^{\frac{1}{\sigma}}\right]. \tag{9}$$

The *level* of the long-run natural rate informs us about the possibility of *secular stagnation*:

$$\left(\frac{1+i}{1+\pi}\right)^{*} = \frac{1}{\beta\left[1 + (1 - s(Y))\left(\Gamma^{\frac{1}{\sigma}} - 1\right)\right]} < 1;$$

secular stagnation occurs in particular if the levels of risk and inequality are high enough, namely:

$$(1 - s(Y))\left(\Gamma^{\frac{1}{\sigma}} - 1\right) > \beta^{-1} - 1$$

The Fisher effect is instead about the *derivative*:

$$\frac{d\pi}{di} > 0,$$

where $\frac{1+\pi}{1+i} = \beta\left[1 + (1 - s(Y))\left(\Gamma^{\frac{1}{\sigma}} - 1\right)\right]$. To eliminate the Fisher effect, i.e. have a less than 1-to-1 long-run response, the condition is:

$$d\frac{(1 - s(Y))\left(\Gamma^{\frac{1}{\sigma}} - 1\right)}{d(1+i)} < 0.$$

In other words, this requires procyclical inequality Γ or risk $1 - s$, which instead implies Euler-equation discounting.

This result for permanent shocks $\mu = 1$ is intimately related to the issue of local determinacy under a peg. Indeed, the THANK model (4)-(3) is *determinate* under a peg iff $\nu(1) < 1$, that is:[7]

$$\delta + \sigma \frac{1-\lambda}{1-\lambda\chi} \frac{\kappa}{1-\beta} < 1.$$

This is *the same* as the condition ruling out the "old" Fisher effect, i.e. ruling out neo-Fisherian effects in Proposition 1 for *permanent* shocks $\mu = 1$; naturally, neo-Fisherian effects cannot occur if the Fisher effect is absent, no matter how persistent the shock is. When the Fisher effect *is* present (i.e. there is indeterminacy under a peg and the FG puzzle is *not* ruled out), ruling out neo-Fisher effects requires *in addition* that the shock be transitory enough as formalized by (8), namely:

$$1 - \frac{\sigma \frac{1-\lambda}{1-\lambda\chi}\kappa}{1-\beta} < \delta \leq \frac{1 - \beta\mu - \sigma \frac{1-\lambda}{1-\lambda\chi}\kappa\mu}{\mu(1-\beta\mu)}.$$

Figure 1 illustrates the two cases along with the RANK benchmark. In the "discounting" case $\nu(\mu) < 1$ (but not enough to rule out the "old" Fisher efect) in the left panel, interest rate *increases* are short-run *contractionary* and *deflationary* for a larger region of the shock persistence region. With countercyclical inequality and risk giving rise to compounding $\delta > 1$ in the right panel, neo-Fisherian effects are more likely, i.e. they hapen for lower values of the shock persistence. Consider first the "standard" region (for small enough μ) whereby interest rate increases are contractionary and deflationary. It is clear that the effect of shocks is dampened by heterogeneity in the procyclical inequality/risk case (left panel), and amplified in the countercyclical case. The intuition follows directly from our discussion above, as procyclical inequality and risk induce Euler discounting and smaller AD elasticity to interest rates, while countercyclical inequality and risk trigger compounding of both news and contemporary interest changes. Persistent shocks imply larger responses because they trigger income effects: anticipating high future rates, households increase desired savings and income has to adjust down to keep equilibrium savings fixed; deflation happens in equilibrium to deliver the higher real interest rate consistent with lower income and demand today, and thus imply the "right" substitution effect.

Consider next the "neo-Fisherian" region, whereby interest-rate increases are expansionary, inflationary and decrease real rates; thereby, persistent enough shocks trigger over-compensating income effects that dominate the substitution effects—and this is more likely to occur with countercyclical inequality/risk, and less likely with procyclical. The initial, direct effect is still that households want to save more at given inflation; but with strong enough compounding of news $\nu(\mu)$ this leads to higher income, higher demand, and inflation. It is then the substitution effect that needs to adjust to deliver zero equilibrium savings as the real rate goes down instead of up. This"fallacy of composition" equilibrium—whereby one agent wants to save more, but in general equilibrium the whole economy actually saves less and consumes more—is more likely to appear when inequality and risk are countercyclical, and less likely to appear when they are procyclical. Indeed, if procyclical "enough" in the sense made explicit above they in fact do not occur at all as the (old) Fisher effect is altogether eliminated.

[7] This is also related to the *Forward Guidance FG puzzle* of Del Negro et al (2012), as we discuss below). See the HANK-modified Taylor principle Propositions 1 and 8 in Bilbiie (2018) for determinacy properties with Taylor rules and a discussion of the FG puzzle in that context.

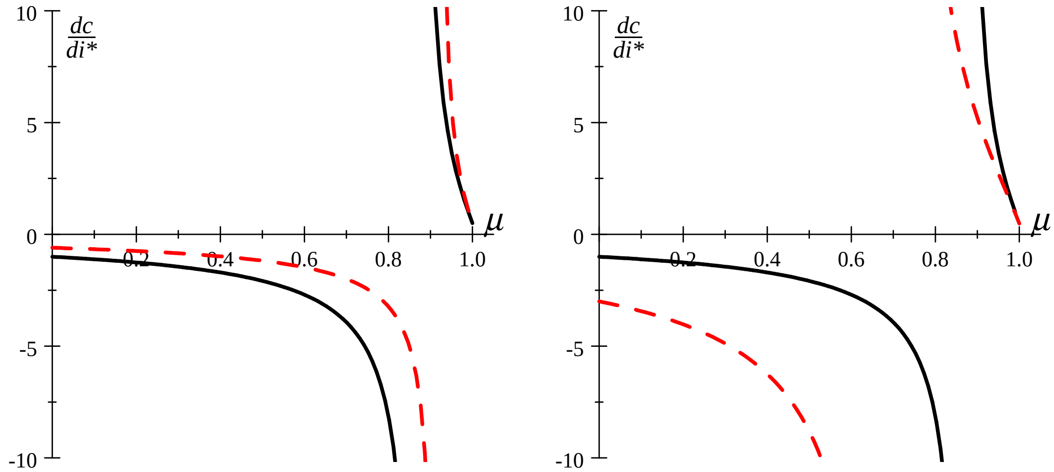


Fig. 1: Neo-Fisherian Effects in THANK vs RANK.
Procyclical (left $\chi = 0 < 1$) vs countercyclical ($\chi = 2$ right) inequality and risk.

2.2 Liquidity Traps and Heterogeneity

In this Section I analyze the two LT varieties under an active policy rule subject to the zero lower bound. Throughout, I assume that uncertainty (be it fundamental or not) evolves according to a Markov chain. Since the model is linear and forward-looking, endogenous variables inherit the persistence of these exogenous forces. In particular, there are two states: the first, "intended" steady state $(i, \pi, c)^I = (\rho, 0, 0)$ is absorbing (once in it, the economy stays there).

Liquidity traps occur when the second state materializes and the model's endogenous forces make the lower bound bind. This state is transitory with persistence probability z_j, and transition probability $1 - z_j$, where j is an index for the shock source: no index when fundamental (z), and s for sunspot-confidence (z_s). The duration of the transitory state is a random variable T with expected value $E(T) = (1 - z_j)^{-1}$. Given this Markov chain structure and the Taylor rule:

$$i_t = \max\left(0, \rho_t + i_t^* + \phi \pi_t\right), \tag{10}$$

with $\phi > 1$, we conjecture and verify that the LT equilibrium is time-invariant, regardless of the source of uncertainty; denote by (c_L, π_L) consumption and inflation in this equilibrium, prevailing for any time t between 0 and T (after date T it is straightforward to show that the system formed by (4), (3) and (12) has a unique equilibrium $(i_t, \pi_t, c_t)^I = (i, \pi, c)^I = (\rho, 0, 0)$). Equation (4) implies $\pi_L = \frac{\kappa}{1-\beta z_j} c_L$ and, with a binding lower bound $i_L = 0$, (3) implies the following, depending on what shock triggers the LT and using again the notation for news-compounding $\nu\left(z_j\right) \equiv \delta + \frac{\kappa}{1-\beta z_j} \sigma \frac{1-\lambda}{1-\lambda \chi}$.

2.3 Confidence-driven HALT

This LT variety has been originally studied by Benhabib Schmitt-Grohe Uribe (2000, 2001) and extended by Mertens and Ravn (2014) to study fiscal multipliers. See Bilbiie (2021) for other analytical implications in RANK. A confidence-driven LT occurs *without* any fundamental shock: $\rho_j = \rho > 0$. Agents recognize a second "unintended" steady state where the ZLB binds $(i, \pi, c)^U = (0, -\rho, -\rho(1-\beta)/\kappa)$ and randomize between it and $(i, \pi, c)^I$: they believe that the former occurs today and will persist tomorrow with probability z_s. The conjectured equilibrium solutions for consumption and inflation during the trap are:

$$c_L = \frac{\sigma \frac{1-\lambda}{1-\lambda\chi}}{1 - z_s \nu\left(z_s\right)}\rho;\ \pi_L = \frac{\kappa}{1-\beta z_s} c_L.$$

A self-fulfilling LT occurs if:

$$z_s \nu\left(z_s\right) > 1 \tag{11}$$

Notice that, as for neo-Fisherian effects, the condition is impossible to satisfy if $\nu < 1$, that is if inequality and/or risk are procyclical enough. Conversely, however, countercyclical inequality and risk make confidence-driven traps more likely because they magnify the news-amplification mechanism that generates confidence-driven traps in the first place; as we will see, relatedly, they also amplify fundamental-driven recessions.

3 Fundamental HALT: Implications

Fundamental LTs can occur in response to any shocks that generate an increase in desired saving and change the natural, flexible-price interest rate. As discussed above and shown in the Appendix, in addition to the standard change in patience (of savers), in this class of models other such shocks include increases in inequality or income risk that trigger an increase in saving for precautionary, self-insurance reasons. However, algebraically, I show in the Appendix that these are isomorphic (up to a rescaling) to changes in patience.

Following the seminal paper of Eggertsson and Woodford (2003), I analyze fundamental LTs assuming that ρ_t, whatever its "structural" source, follows a Markov chain with two states. The first is the good, "intended" steady state denoted by I, with $\rho_t = \rho$, and is absorbing: once in it, there is a probability of 1 of staying. The other state is transitory and denoted by L: $\rho_t = \rho_L < 0$ with persistence probability z (conditional upon starting in L, the probability that $\rho_t = \rho_L$ is z, while the probability that $\rho_t = \rho$ is $1 - z$). At time t, there is a negative realization of $\rho_t = \rho_L < 0$. The duration of the transitory state is a random variable T with expected value $E\left(T\right) = \left(1 - z\right)^{-1}$.

Given this Markov chain structure and the Taylor rule subject to a zero lower bound:

$$i_t = \max\left(0, \rho_t + i_t^* + \phi\pi_t\right), \tag{12}$$

with $\phi > 1$, the LT equilibrium is found by conjecturing that it is time-invariant, denoting it by $\left(c_L, \pi_L\right)$ which prevails for any time t between 0 and T (thereafter, it is straightforward to show that the model returns to the steady state). Equation (4) and, with a binding lower bound $i_L = 0$, the aggregate IS implies:

$$c_L = \frac{1}{1 - z\nu\left(z\right)} \sigma \frac{1-\lambda}{1-\lambda\chi} \rho_L; \pi_L = \frac{\nu\left(z\right) - 1}{1 - z\nu\left(z\right)} \rho_L \tag{13}$$

As clear from (13), a recession and deflation ($c_L < 0;\ \pi_L < 0$) occur if and only if:

$$z\nu\left(z\right) < 1. \tag{14}$$

Finally, verifying that ZLB binds indeed: $\rho_L + \phi\pi_L < 0$ implies $1 + \phi\frac{\nu-1}{1-z\nu} > 0$, which always holds as long as (14) holds. Of course, condition (14) rules out the occurrence of confidence-driven liquidity traps (see Bilbiie, 2021 for further discussion of this duality in RANK). In the "discounting" case with $\nu < 1$, the restriction is *a fortiori* satisfied no matter how pessimistic agents are (how high the sunspot persistence), since z is a probability $z < 1 < \nu^{-1}$.[8]

Notice that in the discounting case with $\nu(z) < 1$ there is no asymptote/bifurcation: recessions are bounded and multipliers are small. This brings us to the topic of amplification of LT recessions.

3.1 Amplification: Deeper (and Deflationless) Recessions?

The mechanism by which LT-recessions occur is similar to the one familiar from the RANK model; but in THANK, their magnitude and amplification relative to RANK depends on the key parameters λ, χ, $1-s$, through both the within-period demand elasticity to interest rates ($\sigma\frac{1-\lambda}{1-\lambda\chi}$) and through the AD effect of news under a peg parameter ν.

As clear by direct inspection of (13), the amplification mechanisms emphasized above can generate a deep recession in THANK *even for fixed prices* $\kappa = 0$, if and only if income inequality and risk are countercyclical, i.e. $\chi > 1$. This resolves what is sometimes called "the missing deflation puzzle" (Hall, 2011). This amplification mechanism is very general and transcends the simple model used here: indeed, a generalized version of it holds in the very rich globally-solved HANK+ZLB model subsequently analyzed by Fernandez-Villaverde et al (2021).

Amplification, understood as an LT recession deeper than in RANK, obtains if and only if inequality is countercyclical $\chi > 1$, by the same amplification logic emphasized above) Generally, this occurs through three forces. First, the within-the-period, TANK amplification of changes in interest rates through a New Keynesian Cross mechanism ($\frac{1-\lambda}{1-\lambda\chi}$).[9] Second, the HANK, intertemporal extension of that: the *self-insurance* channel yielding *compounding* in the aggregate Euler equation ($\delta > 1$) which amplifies the effect of "news". Insofar as the liquidity trap is expected to persist, bad news about future aggregate income reduce today's demand because they imply *more* need for self-insurance saving. Since future consumption in states where the constraint binds over-reacts to bad "aggregate news" (countercyclical inequality), households internalize this by attempting to self-insure *more*. And since saving needs to be zero in equilibrium, households consume less and income falls to deliver this, thus magnifying the recession even further. Third, the *expected deflation channel*: a shock that is expected to persist triggers self-insurance because of expected deflation ($\kappa\sigma\frac{1-\lambda}{1-\lambda\chi}$), which at the ZLB means an increase in interest rate, so more saving and, since equilibrium saving is zero, less consumption and less income. This last effect operates in the standard representative-agent model too, but here it is amplified proportionally to $\frac{1-\lambda}{1-\lambda\chi}$. Evidently, in the procyclical inequality and risk region, *all* these channels imply *dampening* instead of amplification.[10]

The model has implications for the **paradox of flexibility** (Eggertsson and Krugman, 2012), that

[8]Notice, nevertheless, that a sunspot equilibrium may *always* be constructed, e.g. insofar as prices are flexible enough (or whatever makes $\nu > 1$). In fact, they can always be constructed as long as the ZLB equilibrium is a steady state.

[9]This mechanism is also at play in Eggertsson and Krugman's deleveraging-based LT model, where it compounds a debt-deflation channel. The borrowers whose constraint binds at all times are effectively hand-to-mouth (even though their income comprises financial income that I abstract from, at the core of Eggertsson and Krugman's analysis).

[10]Turning the above logic over its head, in the *dampening* case ($\chi < 1$) the LT-recession is *decreasing* with λ and $1-s$: the more H agents and the more risk, the lower the elasticity to interest rates within the period, and the lower the discount factor of the Euler equation δ—both of which lead to dampening (and increasingly so when taken together).

increasing price flexibility κ makes the ZLB recession worse (c_L more negative), specifically:

$$\partial\left(\frac{\partial c_L}{\partial \rho_L}\right)/\partial\kappa = z\left(\frac{\sigma\frac{1-\lambda}{1-\lambda\chi}}{1-z\nu(z)}\right)^2$$

The paradox is merely *mitigated*, i.e. the derivative above decreases, with λ iff $\chi < 1$ (the proof follows immediately as $\sigma\frac{1-\lambda}{1-\lambda\chi}$ and δ, and hence ν, are decreasing with λ iff $\chi < 1$). Conversely, the paradox is aggravated with λ if $\chi > 1$.

3.2 Amplification: Inflationless Fiscal Multipliers in HALT?

As it is by now well understood from the seminal studies of Eggertsson (2010) and Christiano, Eichenbaum, and Rebelo (2011), government-spending multipliers can be high in LTs (the opposite holds in confidence-driven LTs, see Mertens and Ravn (2014)); however, the mechanism by which this happens relies on expected inflation – and thus requires both a persistent inflationary shock and intertemporal substitution. This is no longer the case under heterogeneity. This is easily analyzed in our model; introduce public spending: the government buys an amount of goods G_t with zero steady-state value ($G = 0$) and taxes all agents uniformly in order to finance it.[11] Straightforward derivation delivers the aggregate Euler-IS, denoting $\zeta \equiv \left(1+(\varphi\sigma)^{-1}\right)^{-1}$:

$$c_t = \delta E_t c_{t+1} - \sigma\frac{1-\lambda}{1-\lambda\chi}\left(i_t - E_t\pi_{t+1}\right) + \zeta\left[\frac{\lambda(\chi-1)}{1-\lambda\chi}\left(g_t - E_t g_{t+1}\right) + (\delta-1)E_t g_{t+1}\right]. \quad (15)$$

Together with an extended Phillips curve $\pi_t = \beta E_t\pi_{t+1} + \kappa c_t + \kappa\zeta g_t$ and using a process for spending with the same Markov structure as the shock, with persistence z, we obtain the *HALT multiplier* (with balanced-budget and uniform taxation $t_t = g_t$):

$$\frac{dc_L}{dg_L} = \frac{\zeta}{1-\nu(z)z}\left[\underbrace{\frac{\lambda(\chi-1)}{1-\lambda\chi}(1-z) + (\delta-1)z}_{\text{TANK + HANK AD}} + \underbrace{\frac{\kappa}{1-\beta z}\sigma\frac{1-\lambda}{1-\lambda\chi}z}_{\text{RANK AS, E}(\pi)}\right].$$

Notice the same condition for obtaining multipliers through heterogeneity even with fixed prices ($\kappa = 0$): either countercyclical inequality $\chi > 1$;[12] or, for persistent shocks, countercyclical risk engendering Euler-compounding $\delta > 1$. The intuition is a manifestation of the more general aggregate-demand amplification logic already covered above: a spending expansion triggers an increase in income that either falls disproportionately on constrained agents, begetting further demand expansion rounds; or, if persistent, future aggregate income expansions and, with countercyclical risk, less of a precautionary saving motive and higher demand today. That is, heterogeneity channels can deliver positive and high consumption multipliers in a LT without relying on expected

[11] The implicit redistribution of the taxation scheme used to finance the spending is of the essence for the multiplier—see Bilbiie (2021) for a discussion.

[12] A version of this of course holds in Eggertsson and Krugman (2012) in deleverage-induced LTs, and away from the ZLB in Gali et al (2007) and Bilbiie and Straub (2004).

inflation—unlike the RANK model. It is worth noticing, however, that the debate as to the size or even sign of fiscal multipliers *in the data* is far from settled, see Ramey (2016) and Ramey and Zubairy (2018) for reviews. The virtue of heterogeneity in this realm is that it provides for a wider possible range of multipliers and several distinct degrees of freedom for their structural determinants.

4 Forward Guidance and Optimal Monetary Policy in HALT

Finally, we can also analyze forward guidance and optimal monetary policy. To obtain maximum tractability and closed forms, I first focus on the simplest special case:

$$\pi_t = \kappa c_t, \tag{16}$$

nested in (4) above with $\beta = 0$ for firms used previously in Bilbiie (2018, 2019); but the results derived here generalize for the more familiar forward-looking (4), see the Appendix.

4.1 Forward Guidance Puzzle and Power in HALT

Forward guidance has been discussed in particular in the context of LTs, as a policy tool that remains available when the standard ones are not, and as a characteristic of optimal policy; see Eggertsson and Woodford (2003) for the original analysis, and Bilbiie (2019) for a more recent treatment and an up-to-date discussion of the literature.

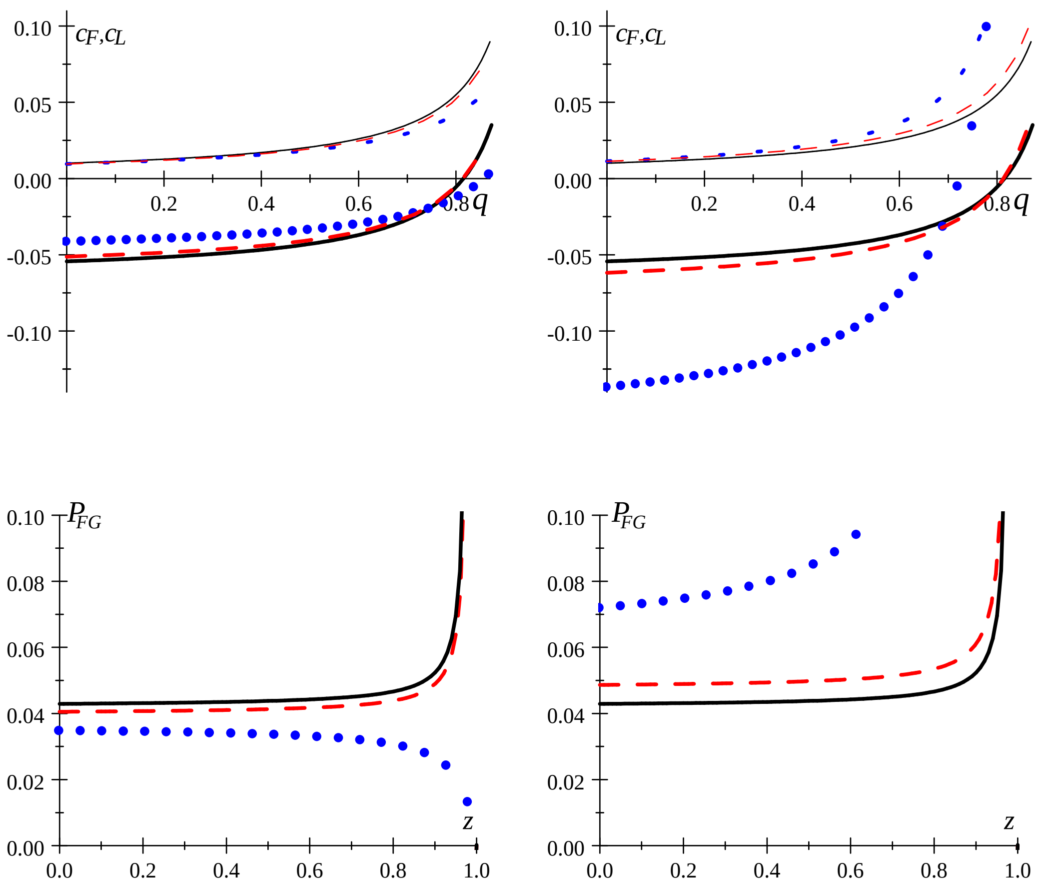
Amplification also applies to *forward guidance*. To discuss it, including optimal policy in the context of LTs, I follow the latter paper to model forward guidance stochastically through a Markov chain as follows. After the trap end-time T_L (with expected value $E(T_L) = (1-z)^{-1}$) the central bank commits to keep the interest rate at 0 while $\rho_t = \rho > 0$, with probability q. Denote this state by F, with expected duration $T_F = (1-q)^{-1}$. The Markov chain has *three* states: liquidity trap L ($i_t = 0$ and $\rho_t = \rho_L$), forward guidance F ($i_t = 0$ and $\rho_t = \rho$) and absorbing steady state ($i_t = \rho_t = \rho$). The probability to transition from L to L is still z, and from L to F it is $(1-z)\,q$. The persistence of F is q, and the probability to move back to steady state from F is hence $1-q$.

Under this stochastic structure, expectations are determined by $E_t c_{t+1} = z c_L + (1-z)\, q c_F$ and similarly for inflation. Evaluating the aggregate Euler-IS (3) and Phillips ($\pi_t = \kappa c_t$) curves during states F and L respectively and solving for the time-invariant equilibria delivers (the solution with NKPC (4) is in Appendix B.2):

$$\begin{aligned} c_F &= \frac{1}{1-q\nu_0}\sigma\frac{1-\lambda}{1-\lambda\chi}\rho; \\ c_L &= \frac{1-z}{1-z\nu_0}\frac{q\nu_0}{1-q\nu_0}\sigma\frac{1-\lambda}{1-\lambda\chi}\rho + \frac{1}{1-z\nu_0}\sigma\frac{1-\lambda}{1-\lambda\chi}\rho_L, \end{aligned} \tag{17}$$

and $\pi_F = \kappa c_F$, $\pi_L = \kappa c_L$, where $\nu_0 \equiv \delta + \kappa\sigma\frac{1-\lambda}{1-\lambda\chi}$. It is immediate that the future expansion c_F is increasing in q regardless of the model, but more so with countercyclical inequality and risk.

The upper row of Figure 2 illustrates these findings. Distinguishing between $\chi < 1$ (left) and $\chi > 1$ (right), it plots in both panels consumption in the liquidity trap (thick) and in the FG state (thin), as a function of the FG probability q. Other than the parameter values used for Figure 1, it uses $z = 0.8$ and a shock of 4 percent per annum ($\rho_L = -0.01$). This delivers a recession of 5 percent and annualized inflation of 1 percent in RANK without FG ($q = 0$). The domain is such that $q < \nu_0^{-1}$. RANK is with solid lines, TANK with red dashed, and the iid limit of THANK model ($1 - s = h = \lambda$) with blue dots. At given q, low future rates have a lower effect on both c_F and c_L in TANK, and an even lower one in HANK, with procyclical inequality/risk. The dampening is magnified by higher risk $1 - s$ (blue dots). Whereas with countercyclical inequality/risk (right panel) low rates have an amplified effect in TANK and even higher in THANK because of compounding. Indeed, even though $\chi = 2$ is a rather conservative number and λ is very small (0.1), making amplification in TANK limited, THANK amplification is substantial: the recession is three times larger than in RANK. This number goes up steeply with the forward-looking (4), or increase λ or χ albeit slightly; indeed, with $\beta = 0.99$ in (4), the recession is *10* (ten) times larger.



$\chi = .5$ (procyclical inequality) $\qquad$ $\chi = 2$ (countercyclical inequality)

Fig. 2: Upper: c_L (thick) and c_F (thin); Lower: FG power $\mathcal{P}_{FG}$. RANK (black solid), TANK (red dashed) and THANK-iid (blue dots).

To illustrate the FG puzzle in a liquidity trap, I define *forward-guidance power* $\mathcal{P}_{FG}$ as:

$$\mathcal{P}_{FG} \equiv \frac{dc_L}{dq} = \left(\frac{1}{1 - q\nu_0}\right)^2 \frac{(1-z)\,\nu_0}{1 - z\nu_0}\sigma\frac{1-\lambda}{1-\lambda\chi}\rho.$$

As apparent from inspecting the top row Figure 2, this is much larger in THANK with countercyclical inequality, following the same logic as for any demand shock.

The *FG puzzle* is then in this context that $\mathcal{P}_{FG}$ increases with the persistence (and thus expected duration) of the trap z:

$$\frac{d\mathcal{P}_{FG}}{dz} \geq 0.$$

When does the model resolve the FG puzzle in a LT?

Proposition 2 *Ruling out neo-Fisherian effects and the confidence-driven LT also rules out the FG puzzle:*

$$\nu_0 < 1 \rightarrow \frac{d\mathcal{P}_{FG}}{dz} < 0.$$

The result follows directly calculating the derivative $d\mathcal{P}_{FG}/dz = \frac{(\nu_0 - 1)\nu_0}{[(1-q\nu_0)(1-z\nu_0)]^2}\sigma\frac{1-\lambda}{1-\lambda\chi}\rho$ and then replacing the expression for ν_0. The bottom row of Figure 2 illustrates this by plotting $\mathcal{P}_{FG}$ as a function of z (fixing $q = 0.5$) for the same cases as before. This shows most clearly that it is the complementarity between procyclical inequality and idiosyncratic risk that resolves the puzzle: the power becomes decreasing in the duration of the trap (blue dots, left). Procyclical inequality *by itself* (red dash, left, TANK) alleviates the puzzle relative to RANK but does not make the power decrease with the horizon z. While idiosyncratic risk by itself added to countercyclical-inequality magnifies power even further, *aggravating* the puzzle (blue dots, right).

4.2 Optimal Monetary Policy in a HALT

Optimal policy in THANK can under some conditions be reduced to a linear-quadratic problem in aggregate output and inflation volatility, with coefficients that depend on the heterogeneity parameters (Bilbiie (2018)). In particular, maximizing aggregate welfare of the two types around an efficient, perfect-insurance flexible-price equilibrium is equivalent to minimizing the loss function:

$$\min_{\{c_t,\pi_t\}} \frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \pi_t^2 + \alpha y_t^2 \right\}, \tag{18}$$

$$\text{where } \alpha \equiv \frac{\sigma^{-1}+\varphi}{\psi}\left(1 + \frac{\lambda}{1-\lambda}\sigma^{-1}\varphi^{-1}(\chi-1)^2\right)$$

captures the inequality motive that amounts in this simple case, with no wedge between inequality and output gap, to an additional motive for stabilization of real activity.

In a liquidity trap, one notion of optimal policy consists of solving for the optimal forward-guidance duration, found by maximizing welfare with respect to q. This is developed in Bilbiie (2019) in RANK and shown to be close to the full Ramsey-optimal policy calculated by Eggertsson and Woodford (2003) and several others since. The aggregate welfare function, given the Markov chain structure, is of the form:

$$\mathbb{W} = \frac{1}{1-\beta z}\frac{1}{2}\left[c_L^2 + \omega(q)\, c_F^2\right],$$

where $\omega(q)$ is the appropriate discount factor for the F state.[13] The central bank chooses forward guidance duration (probability q) by solving the optimization problem $\min_q \mathbb{W}$ taking as constraints the equilibrium values c_F and c_L given in (17) above. The first-order condition of this problem is:

$$c_L \frac{dc_L}{dq} + \omega(q) c_F \frac{dc_F}{dq} + \frac{1}{2}\frac{d\omega(q)}{dq} c_F^2 = 0. \tag{19}$$

This has a clear intuitive interpretation. The first term is the welfare *benefit* of more FG: mitigating the trap-recession and minimizing consumption volatility therein. This is proportional to the *level* of consumption in the trap: the larger the initial recession, the higher the marginal utility of extra consumption, and the larger the scope for a policy delivering it. The last two terms are the *total cost* of FG: the former is the direct cost, a future consumption boom creating inefficient volatility; the latter is the discounting effect discussed above: the longer FG duration, the larger the cost, which is proportional to consumption volatility in the F state.

The basic **analytical insights** can be obtained by focusing first on a simpler case, assuming in addition that the central bank attaches equal weights to future and present: $\omega(q) = 1, \omega'(q) = 0$. This provides an *upper bound* on optimal FG because it ignores the second-order discounting costs.[14] The optimal duration can then be solved in closed-form: (19) becomes $c_L \frac{dc_L}{dq} = -c_F \frac{dc_F}{dq}$, which replacing c_F and c_L from (17) delivers the following.

Proposition 3 *The optimal FG duration is $q = 0$ if $\Delta_L < \frac{(1-z\nu_0)^2}{1-z}$ and $q^* > 0$ otherwise, with:*

$$q^* = \frac{1}{\nu_0} \frac{\Delta_L - \frac{(1-z\nu_0)^2}{1-z}}{1 - z + \Delta_L},$$

where $\Delta_L \equiv -\rho_L/\rho > 0$ is the financial disruption causing the ZLB.

It is optimal to refrain from FG altogether ($q^* = 0$) when there is not enough "news-amplification": when ν_0 is *smaller* than a certain threshold $\tilde{\nu}$.[15] Thus in the amplification case ($\chi > 1$ and $\nu_0 > 1$) the region of λ for which FG is optimal will be ceteris paribus smaller than in the "dampening ($\chi < 1$ and $\nu_0 < 1$) case. Moreover, since in the former case ν_0 is increasing both with λ and with $1-s$, an increase in either restricts the case for optimal FG.[16] The reason is that more amplification also brings about a higher welfare cost of FG. Conversely of course, in the latter (dampening) case the opposite is true: an increase in either λ or $1-s$ pushes up the threshold and enlarges the region for which FG is optimal (ν_0 is decreasing in both parameters).

Optimal FG duration depends on the key heterogeneity parameters through the key composite parameter elasticity-to-news ν_0:

$$\frac{dq^*}{d\nu_0} = \frac{1}{\nu_0^2}\left(\frac{1-(z\nu_0)^2}{1-z} - \Delta_L\right).$$

[13] The equilibrium being time-invariant in each state, the per-period loss is: $\pi_j^2 + \alpha c_j^2 = (\alpha + \kappa^2) c_j^2$, $j = \{L, F\}$. The optimal weight $\omega(q) = \frac{1-\beta z+\beta(1-z)q}{1-\beta q}$ counts the time spent in F, with $\omega'(q) > 0$: the longer time spent in F, the larger the welfare cost. See Bilbiie (2016) for details, including second-order sufficient conditions.

[14] See Bilbiie (2019) for an analysis of the accuracy of this in a RANK model.

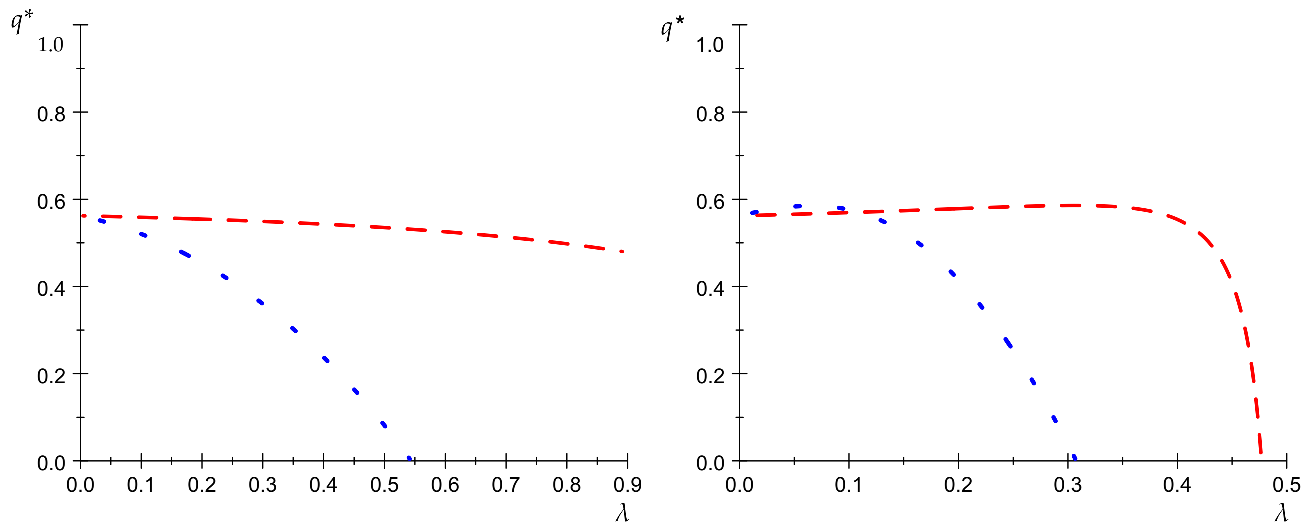
[15] Specifically, $\tilde{\nu} \equiv \left(1 - \sqrt{(1-z)\Delta_L}\right)/z$ which under the baseline calibration is 0.86.

[16] Formally: $\frac{d\nu_0}{d(1-s)} = \frac{\chi-1}{1-\lambda\chi}$; $\frac{d\nu_0}{d\lambda} = (\chi - 1)\frac{\chi(1-s)+\kappa\sigma}{(1-\lambda\chi)^2}$.

When the disruption causing the liquidity trap is lower than a certain threshold $\Delta_L < (1-z)^{-1}$ (the more empirically plausible case),[17] q^* is increasing in ν_0 if $\nu_0 < \bar{\nu} \equiv \sqrt{1-\Delta_L(1-z)}/z$ and decreasing otherwise. Notice that this threshold is larger than the threshold needed for FG to be optimal at all ($q^* > 0$) derived above: $\bar{\nu} > \tilde{\nu}$. We have $dq^*/d\nu_0 > 0$ when $\tilde{\nu} < \nu_0 < \bar{\nu}$ and $dq^*/d\nu_0 < 0$ when $\tilde{\nu} < \bar{\nu} < \nu$. It is useful to again distinguish the two cases depending on χ.

In the dampening case ($\chi < 1$) ν_0 is decreasing in λ and $1-s$; if we start with $\nu_0 > \bar{\nu}$, optimal FG duration first increases, then decreases as ν_0 crosses the threshold. Whereas if we start below the threshold, optimal FG duration decreases uniformly (this is the case shown in the Figure below). The effect is mitigated by idiosyncratic risk which, because it reduces both the power of FG and the scope for it (the LT recession is smaller) implies uniformly lower optimal duration.

With amplification ($\chi > 1$), ν_0 is increasing in both λ and $1-s$; therefore, if we start below the threshold $\bar{\nu}$, optimal FG first increases up to a maximum level (reached at the threshold) and then decreases abruptly. Furthermore, it increases faster and reaches its maximum sooner when there is idiosyncratic risk, because of the complementarity: amplification itself is in that case magnified—by the same token, the welfare cost of FG suffers from the same amplification, so the point where FG ceases to be optimal is reached sooner than without risk $s = 1$.



TANK (red dashed); THANK iid (blue dots)

Figure 3 Optimal FG persistence as a function of λ for $\chi < 1$ (left) and $\chi > 1$ (right)

Figure 3 plots the optimal duration in the general case, the solution of (19), as a function of λ, under our baseline parameterization, distinguishing $\chi < 1$ (left) and $\chi > 1$ (right). With procyclical inequality, optimal FG is *decreasing* with λ, the more so, the higher idiosyncratic risk. Intuitively, all forces work in the same direction: the recession is lower to start with, implying *less* scope for forward guidance, and forward-guidance power is monotonically decreasing in λ.

The amplification case is, in view of our previous results, more surprising: the optimal duration is almost invariant to λ in TANK because of two counterbalancing forces. On the one hand, the benefit is higher: the recession is larger, creating more scope for using forward guidance, whose

[17] If instead $\Delta_L > (1-z)^{-1}$, q^* is uniformly *decreasing* in ν_0: that is, it is decreasing in χ , λ, and $1-s$ in the "amplification" case $\chi > 1$. The reason is that the contractionary effect coming from the steeper recession dominates the expansionary effect of increased FG effectiveness; the opposite is of course true with $\chi < 1$: q^* is increasing in λ and $1-s$.

power is also higher. But on the other hand, the welfare cost is also increasing and at some threshold λ level, it is no longer worth bearing: the implied inefficient volatility during F is so high that the optimal duration drops rapidly to zero. In THANK, these effects are further amplified by the complementarity with risk: an increase in λ makes the recession larger and accelerates the increase in forward-guidance power, making the optimal duration initially increasing; but the same amplification applies to the welfare cost of future volatility, which kicks in at a lower λ making the optimal duration drop abruptly towards zero. This sharp increase in the welfare cost occurs precisely when the power is large: the "dark side" of forward-guidance power.

In both cases, it becomes optimal to do no forward guidance at all beyond a threshold λ. The underlying reason is, however, very different. With dampening, it is because a higher λ implies both *low* power and a weaker scope for forward guidance. With amplification, it is because a high λ implies *high* power, but also a high welfare cost, and the former effect is dwarfed by the latter.[18]

5 Conclusions

How are the occurrence and severity of liquidity traps, and their management by monetary and fiscal policy affected by heterogeneity? This paper provided analytical answers to these questions, using a tractable heterogeneous-agent model that captures several essential channels of richer HANK models. Whether liquidity traps are more likely to be caused by confidence (be "neo-Fisherian") or fundamental shocks is crucially determined by distributional considerations; moreover, in a HA model liquidity traps themselves can be triggered by increases in inequality or income risk leading to changes in desired precautionary savings. Heterogeneity amplifies the magnitude of the liquidity-trap recession endogenously when income inequality and risk are countercyclical—and it does so without relying on deflation, a necessary ingredient instead in the RANK model; the same amplification extends to fiscal multipliers and to the power of forward guidance. When inequality and risk are instead procyclical, heterogeneity can lead to the elimination of confidence-driven traps and neo-Fisherian effects altogether, as well as of the forward guidance puzzle in a liquidity trap. At the same time, it also implies that the severity of fundamental-trap recession and the power of monetary and fiscal policies therein are mitigated. Optimal monetary policy in a HALT, however, implies that even with countercyclical inequality and/or risk the very same amplification that boosts forward-guidance power also magnifies its welfare cost, thus containing its optimal duration.

These analytical results can inform the quantitative HANK literature that only recently overcame the technical challenges of solving globally HA models with occasionally binding constraints, see Fernandez-Villaverde et al (2021). Further work would analyze forward guidance and optimal policies in liquidity traps in such quantitative models. The spectre of liquidity traps in the current post-COVID crisis, whereby distributional channels like the ones emphasized by this whole literature are paramount, speaks directly to the relevance of such results.

[18] In the Appendix, I address a caveat raised by Nakata, Schmidt, and Yoo (2018): shouldn't optimal policy imply doing more (rather than less) FG when it is less effective? The short answer is that keeping fixed the observable recession (rather than the unobservable disturbance) *does not necessarily* imply a stronger case for longer optimal guidance duration. Indeed, in the "amplification" case, whereby FG power is highest, it unambiguously implies an even weaker case.

References

Acharya, S. and K. Dogra, 2018 "Understanding HANK: Insights from a PRANK," Econometrica

Aiyagari, R., 1994, "Uninsured Idiosyncratic Risk and Aggregate Saving," Quarterly Journal of Economics, 109, 659–684.

Andrade, P., G. Gaballo, E. Mengus, and B. Mojon, 2016, "Forward Guidance with Heterogeneous Beliefs," American Economic Journal: Macroeconomics

Angeletos, G.M. and C. Lian, 2017, Forward Guidance without Common Knowledge, American Economic Review

Auclert, A. 2018, "Monetary Policy and the Redistribution Channel," American Economic Review

Auclert, A. and M. Rognlie 2016, "Inequality and Aggregate Demand," Mimeo

Auclert, A., M. Rognlie, and L. Straub 2018, "The Intertemporal Keynesian Cross," Mimeo

Bayer, C., R. Luetticke, L. Pham-Dao, and V. Tjaden, 2018, "Precautionary Savings, Illiquid Assets, and the Aggregate Consequences of Shocks to Household Income Risk," Econometrica

Benhabib, J., Schmitt-Grohé, S., and Uribe, M., 2001a, "The Perils of Taylor Rules," Journal of Economic Theory

Benhabib, J., Schmitt-Grohé, S., and Uribe, M., 2001b, "Monetary Policy and Multiple Equilibria", American Economic Review, 91, 167–186.

Benhabib, J., Schmitt-Grohé, S., and Uribe, M., 2002, "Avoiding Liquidity Traps", Journal of Political Economy

Bewley, T., 1977, "The Permanent Income Hypothesis: A Theoretical Formulation," Journal of Economic Theory, 16, 252–292.

Bhandari, A., D. Evans, M. Golosov, and T. Sargent, 2017, "Inequality, Business Cycles and Monetary-Fiscal Policy", Mimeo

Bilbiie F. 2008. "Limited Asset Market Participation, Monetary Policy, and (Inverted) Aggregate Demand Logic". Journal of Economic Theory 140, 162-196.

Bilbiie F. 2011. "Non-Separable Preferences, Frisch Labor Supply and the Consumption Multiplier of Government Spending: One Solution to a Fiscal Policy Puzzle," Journal of Money, Credit and Banking, 43(1), 221-251

Bilbiie F. and R. Straub, 2013, "Asset Market Participation, Monetary Policy Rules and the Great Inflation", Review of Economics and Statistics, 95(2), pp 377-392

Bilbiie F., Monacelli T. and R. Perotti 2013, "Public Debt and Redistribution with Borrowing Constraints", The Economic Journal, F64-F98

Bilbiie, F. 2018, "Monetary Policy and Heterogeneity: An Analytical Framework," CEPR DP

Bilbiie, F. 2019, "Optimal Forward Guidance", American Economic Journal: Macroeconomics

Bilbiie, F. and X. Ragot, 2020, "Optimal Monetary Policy and Liquidity with Heterogeneous Households", Review of Economic Dynamics

Bilbiie, F. 2020, "The New Keynesian Cross," Journal of Monetary Economics

Bilbiie, F. 2021, "Neo-Fisherian Policies and Liquidity Traps," American Economic Journal: Macroeconomics

Broer, T., N. J. Hansen, P. Krusell, and E. Oberg, 2020 "The New Keynesian transmission mechanism: a heterogeneous agent perspective," Review of Economic Studies

Broer, T., N. J. Hansen, P. Krusell, and E. Oberg, 2021 "Fiscal Multipliers: a heterogeneous-agent perspective," mimeo

Campbell, J. Y. and Mankiw, N. G. 1989. "Consumption, Income, and Interest Rates: Reinterpreting the Time Series Evidence," NBER Macroeconomics Annual 185-216

Cantore, C. and L. Freund, 2019. "Workers, Capitalists, and the Government: Fiscal Policy and Income (Re)Distribution." Journal of Monetary Economics

Carlstrom, C., T. Fuerst, and M. Paustian, 2015. "Inflation and output in New Keynesian models with a transient interest rate peg," Journal of Monetary Economics, 76(C), 230-243

Challe, E., 2020 "Uninsured Unemployment Risk and Optimal Monetary Policy in a Zero-Liquidity Economy," American Economic Journal

Challe, E, J. Matheron, X. Ragot and J. Rubio-Ramirez, 2017, ""Precautionary Saving and Aggregate Demand", Quantitative Economics

Christiano L., M. Eichenbaum and S. Rebelo, 2011. "When Is the Government Spending Multiplier Large?," Journal of Political Economy, 119(1), pages 78 - 121.

Christiano L., M. Eichenbaum, and B. Johannsen, 2018, "Does the New Keynesian Model Have a Uniqueness Problem," Mimeo

Cloyne, J., C. Ferreira and Surico, P., 2015 "Monetary Policy when Households have Debt: New Evidence on the Transmission Mechanism" Review of Economic Studies

Cochrane, J., 2017 "Michelson-Morley, Fisher and Occam: The Radical Implications of Stable Inflation at the Zero Bound," NBER Macroeconomics Annual.

Colciago, A. 2011, Rule-of-thumb consumers meet sticky wages, Journal of Money Credit and Banking 43(2), 325-353.

Curdia, V. and M. Woodford 2016, "Credit Frictions and Optimal Monetary Policy," Journal of Monetary Economics

Debortoli, D., and J. Galí, 2018, "Monetary Policy with Heterogeneous Agents: Insights from TANK models"; Mimeo

Del Negro, M., M. Giannoni, and C. Patterson 2012: "The Forward Guidance Puzzle," Mimeo

Eggertsson G. and M. Woodford 2003, "The Zero Bound on Interest Rates and Optimal Monetary Policy", Brookings Papers on Economic Activity, No. 1.

Eggertsson, G. 2010, "What Fiscal Policy Is Effective at Zero Interest Rates?" NBER Macroeconomics Annual

Eggertsson G. and P. Krugman 2012, "Debt, Deleveraging, and the Liquidity Trap: A Fisher-Minsky-Koo Approach," Quarterly Journal of Economics, 127(3): 1469-1513.

Fernández-Villaverde, J., J. Marbet, G. Nuno, and O. Rachedi, 2021, "Inequality and the Zero Lower Bound," Mimeo

Gabaix, X., 2019, "A Behavioral New Keynesian Model," American Economic Review

Galí, J. 2008, "Monetary Policy, Inflation, and the Business Cycle: An Introduction to the New Keynesian Framework and Its Applications", Princeton University Press

Galí, J., D. López-Salido, and J. Vallés, 2007. "Understanding the Effects of Government Spending on Consumption". Journal of the European Economic Association, March, vol. 5 (1), 227-270.

Garcia-Schmidt, M. and M. Woodford, 2014. “Are Low Interest Rates Deflationary? A Paradox of Perfect Foresight Analysis,” American Economic Review

Garin, J., R. Lester, and E. Sims 2018, "Raise Rates to Raise Inflation? Neo-Fisherianism in the New Keynesian Model" Journal of Money, Credit and Banking 50(1), 243-259

Gornemann, N., K. Kuester, and M. Nakajima, 2016, "Doves for the Rich, Hawks for the Poor? Distributional Consequences of Monetary Policy," Mimeo

Guerrieri, V. and G. Lorenzoni, 2017, "Credit Crises, Precautionary Savings, and the Liquidity Trap" Quarterly Journal of Economics

Hagedorn, M., 2017, "A Demand Theory of the Price Level", Mimeo

Hagedorn, M., 2018, “Prices and Inflation when Government Bonds are Net Wealth,” mimeo

Hagedorn, M., I. Manovskii, and K. Mitman, 2017, "The Fiscal Multiplier," Mimeo

Hagedorn, M., J. Luo, I. Manovskii, and K. Mitman, 2019, "Forward Guidance," Journal of Monetary Economics

Heathcote, J. and F. Perri, 2018, "Wealth and Volatility," Review of Economic Studies

Huggett, M., 1993, “The Risk-Free Rate in Heterogeneous-Agent Incomplete-Insurance Economies,” Journal of Economic Dynamics and Control, 17, 953–969.

Kaplan, G., B. Moll and G. Violante, 2018, "Monetary Policy According to HANK", American Economic Review

Krusell, P., T. Mukoyama, and A. Smith. 2011. "Asset Prices in a Huggett Economy," Journal of Economic Theory, 146 (3), 812-844

Leeper, E., 1991, "Equilibria Under "Active" and "Passive" Monetary and Fiscal Policies", Journal of Monetary Economics

Lucas R. E. Jr., 1990 "Liquidity and Interest Rates." Journal of Economic Theory 50, 237 - 264.

Luetticke, R., 2018 "Transmission of Monetary Policy with Heterogeneity in Household Portfolios," Mimeo

Mankiw, N. G. 2000. ”The Savers-Spenders Theory of Fiscal Policy,” American Economic Review, 90 (2): 120-125.

McKay, A., E. Nakamura, and J. Steinsson 2015: “The Power of Forward Guidance Revisited,” American Economic Review

Mertens K. and M. Ravn 2013. "Fiscal Policy in an Expectations Driven Liquidity Trap," Review of Economic Studies

Michaillat, P. and E. Saez, 2018, "A New Keynesian Model with Wealth in the Utility Function", Review of Economics and Statistics

Nakata, T., 2018 "Reputation and Liquidity Traps,” Review of Economic Dynamics, 28: 252-268

Nakata, T. and S. Schmidt, 2016 "Conservatism and Liquidity Traps" Journal of Monetary Economics

Nakata, T. and S. Schmidt, 2021 "Expectations-Driven Liquidity Traps: Implications for Monetary and Fiscal Policy," American Economic Journal: Macroeconomics

Nakata, T., S. Schmidt, and Yoo, 2018, "Attenuating the Forward Guidance Puzzle: Implications for Optimal Monetary Policy," Mimeo

Nakov, A., 2008, " Optimal and Simple Monetary Policy Rules with Zero Floor on the Nominal Interest Rate," International Journal of Central Banking

Nistico, S., 2015, "Optimal monetary policy and financial stability in a non-Ricardian economy." Journal of the European Economic Association

Nuño, G. and C. Thomas, 2017, "Optimal Monetary Policy with Heterogeneous Agents" Mimeo

Oh, H. and R. Reis, 2012, "Targeted transfers and the fiscal response to the great recession." Journal of Monetary Economics, 59(S), S50-S64

Patterson, C., 2019, "The Matching Multiplier and the Amplification of Recessions," Mimeo

Ramey, V. and S. Zubairy, 2018, "Government Spending Multipliers in Good Times and in Bad: Evidence from U.S. Historical Data," Journal of Political Economy.

Ramey, V., 2016, "Macroeconomic Shocks and Their Propagation," Handbook of Macroeconomics.

Ravn, M. and V. Sterk, 2017 “Job Uncertainty and Deep Recessions,” Journal of Monetary Economics.

Ravn, M. and V. Sterk, 2018, "Macroeconomic Fluctuations with HANK and SAM: an analytical approach," Journal of the European Economic Association

Schmitt-Grohé, S. and Uribe, M., 2017, "Liquidity Traps And Jobless Recoveries," American Economic Journal: Macroeconomics 9, 165-204

Werning, I., 2015, "Incomplete markets and aggregate demand", Mimeo

Woodford, M., 2003, Interest and prices: foundations of a theory of monetary policy, Princeton University Press

Woodford, M. 2011. "Simple Analytics of the Government Expenditure Multiplier," American Economic Journal: Macroeconomics, 3(1), p 1-35.

Appendix

A Inequality and Risk Shocks as LT Triggers

Consider first a **pure inequality shock** ε_t^{γ} a positive innovation to which reduces income in the H state and increases it in the S state:

$$\begin{aligned} c_t^H &= \chi y_t - \varepsilon_t^{\gamma} \\ c_t^S &= \frac{1-\lambda\chi}{1-\lambda} y_t + \frac{\lambda}{1-\lambda}\varepsilon_t^{\gamma}, \end{aligned} \tag{A.1}$$

Without loss of generality, the aggregate Euler equation approximated around $\Gamma = 1$ (or with $s_Y = 0$) becomes

$$y_t = \delta E_t y_{t+1} - \sigma \frac{1-\lambda}{1-\lambda\chi}\left(i_t - E_t \pi_{t+1} - \rho_t\right) - \frac{\lambda}{1-\lambda\chi}\left(\varepsilon_t^{\gamma} - \left(1 - \frac{1-s}{\lambda}\right) E_t \varepsilon_{t+1}^{\gamma}\right).$$

This shows clearly that an increase in inequality acts as a "demand shock", and is in fact isomorphic to ρ:

$$\sigma\left(1-\lambda\right)\rho_t \sim -\lambda\left(\varepsilon_t^{\gamma} - \left(1 - \frac{1-s}{\lambda}\right) E_t \varepsilon_{t+1}^{\gamma}\right)$$

A fall in the natural rate (negative ρ) is the same as a positive ε_t^{γ}, where the latter has the different interpretation of saving more in order to self-insure, for precautionary reasons. Note that $1 - \frac{1-s}{\lambda}$ is the autocorrelation of idiosyncratic income, so even if the inequality shock is permanent this wedge will generate excess saving (with most saving in the iid case $1 - s = \lambda$).

Take next a **pure risk shock**: let ε_t^s be a fall in s so an exogenous increase in risk $1 - s$. Loglinearization delivers:

$$\begin{aligned} -\frac{1}{\sigma} c_t^S \;=\;& \left(i_t - E_t \pi_{t+1}\right) - \frac{1}{\sigma}\frac{s}{s + (1-s)\Gamma^{1/\sigma}} E_t c_{t+1}^S - \frac{1}{\sigma}\frac{(1-s)\Gamma^{1/\sigma}}{s + (1-s)\Gamma^{1/\sigma}} E_t c_{t+1}^H + \frac{s'(Y)\,Y}{1-s(Y)}\frac{(1-s)\left(1-\Gamma^{1/\sigma}\right)}{s+(1-s)\Gamma^{1/\sigma}} E_t c_{t+1} \\ & - \frac{1}{s+(1-s)\Gamma^{1/\sigma}}\varepsilon_t^s + \frac{\Gamma^{\frac{1}{\sigma}}}{s+(1-s)\Gamma^{1/\sigma}}\varepsilon_t^s \end{aligned}$$

which replacing individual consumption levels as function of aggregate becomes:

$$c_t = \delta E_t c_{t+1} - \sigma \frac{1-\lambda}{1-\lambda\chi}\left(i_t - E_t \pi_{t+1} - \rho_t\right) - \sigma \frac{1-\lambda}{1-\lambda\chi}\left(\Gamma^{\frac{1}{\sigma}} - 1\right)\varepsilon_t^s$$

An increase in risk $d\varepsilon_t^s > 0$ triggers precautionary saving (only if inequality $\Gamma > 1$).

B Derivations: with NKPC

This section derives the same results as in text but with the forward-looking NKPC (4).

B.1 Neo-Fisheria effects under Taylor rule

Consder the rule:

$$i_t - \rho = \phi \pi_t + i_t^*. \tag{B.1}$$

Substitute in (4) and (3)

$$c_t = -\frac{\sigma\frac{1-\lambda}{1-\lambda\chi}}{1-\delta\mu+\sigma\frac{1-\lambda}{1-\lambda\chi}\frac{\kappa}{1-\beta\mu}(\phi-\mu)} i_t^* \tag{B.2}$$

$$\pi_t = -\frac{\sigma\frac{1-\lambda}{1-\lambda\chi}\frac{\kappa}{1-\beta\mu}}{1-\delta\mu+\sigma\frac{1-\lambda}{1-\lambda\chi}\frac{\kappa}{1-\beta\mu}(\phi-\mu)} i_t^*, \tag{B.3}$$

$$i_t-\rho = \frac{1-\left(\delta+\sigma\frac{1-\lambda}{1-\lambda\chi}\frac{\kappa}{1-\beta\mu}\right)\mu}{1-\delta\mu+\sigma\frac{1-\lambda}{1-\lambda\chi}\frac{\kappa}{1-\beta\mu}(\phi-\mu)} i_t^*$$

Condition for positive comovement of nominal interest and inflation (and consumption and output) is the same as under a peg, i.e. news-amplification $\mu\nu(\mu) > 1$.

B.2 Liquidity trap and FG

Under the Markov chain structure used in text, we can use the same solution method to obtain the LT equilibrium under forward guidance (which evidently nests the LT equilibrium without FG). Using the notations for news-amplification parameters:

$$\begin{aligned}
\nu(q) &\equiv \delta+\sigma\frac{1-\lambda}{1-\lambda\chi}\frac{\kappa}{1-\beta q} \\
\nu(z,q) &\equiv \delta+\sigma\frac{1-\lambda}{1-\lambda\chi}\frac{\kappa}{(1-\beta q)(1-\beta z)}
\end{aligned}$$

the equilibrium is:

$$\begin{aligned}
c_F &= \frac{1}{1-q\nu(q)}\sigma\frac{1-\lambda}{1-\lambda\chi}\rho; \\
c_L &= \frac{(1-p)\,q\nu(z,q)}{(1-q\nu(q))(1-z\nu(z))}\sigma\frac{1-\lambda}{1-\lambda\chi}\rho+\frac{1}{1-z\nu(z)}\sigma\frac{1-\lambda}{1-\lambda\chi}\rho_L,
\end{aligned} \tag{B.4}$$

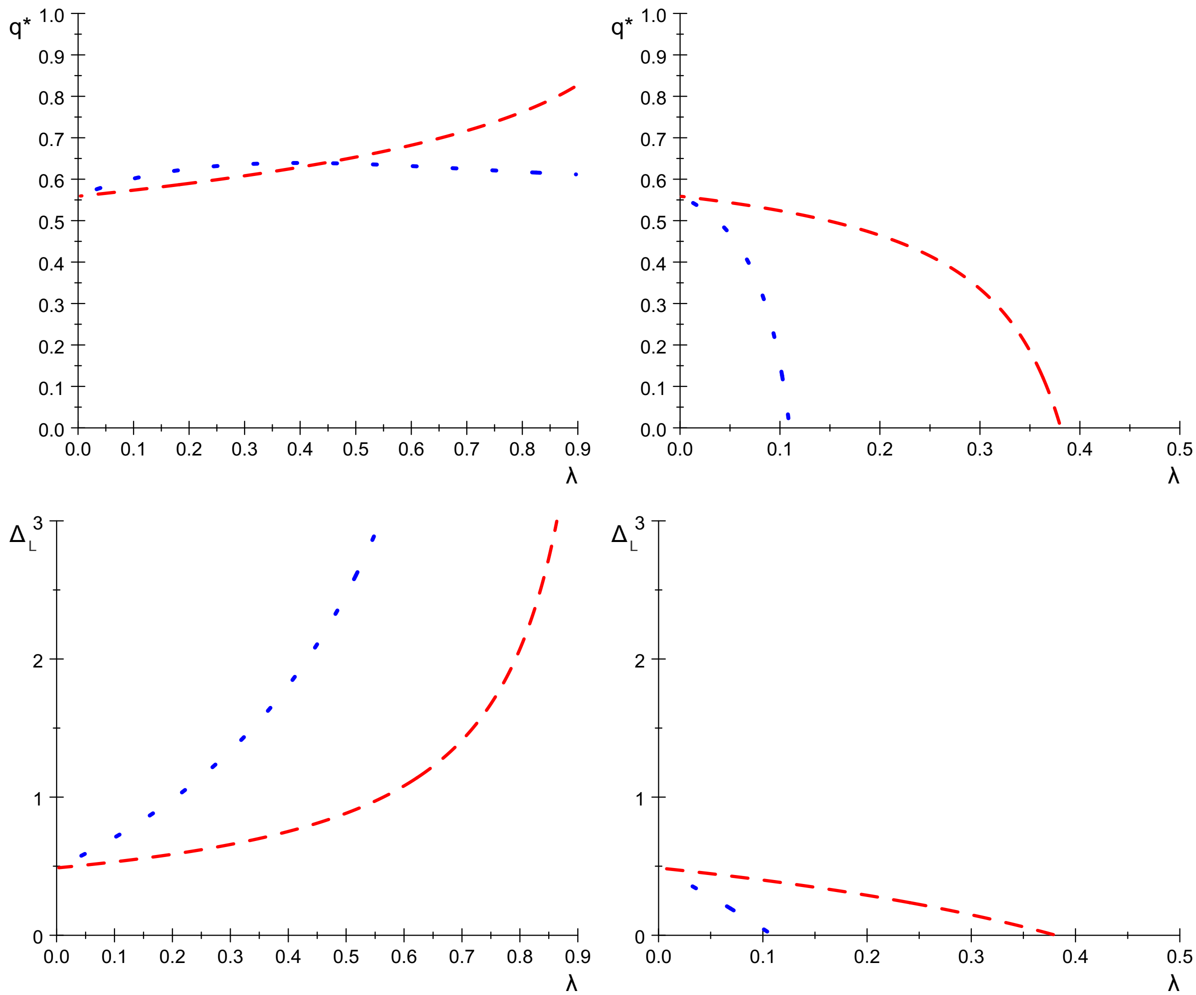
and $\pi_F = \frac{\kappa}{1-\beta q}c_F$, $\pi_L = \beta(1-z)\,q\frac{\kappa}{(1-\beta q)(1-\beta z)}c_F + \frac{\kappa}{1-\beta z}c_L$.

A Caveat is in order: when FG is less effective, shouldn't optimal policy imply doing more (not less) of it? Nakata, Schmidt, and Yoo (2018), in a calibrated model with a discounted Euler equation and FG mitigation, show that, if instead of keeping the size of the disturbance fixed (as this paper does) one fixes the *size of the recession*, itself a function of other structural parameters, one obtains the opposite conclusion to this paper's with $\chi < 1$: the optimal duration of FG becomes *increasing* in the share of constrained households. The reason is that, as λ increases, the shock necessary to generate the given recession gets larger and larger, which adds a force calling for more optimal FG. If this force is strong enough, it can overturn the conclusion obtained above for a given shock.

This also holds in my model with procyclical inequality ($\chi < 1$) and little or no idiosyncratic risk, i.e. TANK (red dash, upper left panel, Figure E1): the optimal duration becomes increasing with λ. There is, however, an important qualification as the level of idiosyncratic risk increases: the blue dotted line in the same panel (corresponding to THANK with the strongest self-insurance motive) is increasing only slightly initially, and decreasing thereafter. The reason is that idiosyncratic risk delivers more dampening overall; so while the shock necessary to deliver a given recession is increasing in λ at a faster rate, FG power also goes down fast. The FG puzzle and having optimal FG increase with λ are two sides of the same coin: in this model, you cannot throw one and keep the other.[19]

[19] Another qualification pertains to the implied shock, plotted in the lower left panel. With so much dampening as implied

Moreover, the same logic that generates increasing FG duration is turned on its head in the amplification, $\chi > 1$ case: as λ gets larger, a smaller shock is needed to generate a given recession (lower right panel). This adds a force calling for *less* optimal FG, so the optimal duration is *lower* (and more rapidly decreasing) than in the "fixed-shock" case. And since amplification is so powerful in THANK, self-insurance makes the optimal duration decrease even faster. The general message is that keeping fixed the observable recession (rather than the unobservable disturbance) is a useful exercise but does not necessarily imply a stronger case for longer optimal guidance duration. Indeed, in some cases such as the "amplification" case whereby FG power is highest (and the puzzle at its most extreme) it unambiguously implies an even weaker case.



TANK (red dashed); THANK iid (blue dots)

Figure E1 q^* as a function of λ: fixed recession, Δ_L adjusts endogenously

by THANK, the shock necessary to replicate an even modest recession (4 percent) becomes very large indeed (several times larger than the normal-times interest rate); while the shock is unobservable, this type of configuration seems unlikely.