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Abstract

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JEL Classification: G21, G28, L13

Keywords: information-based bank runs

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Learning in Bank Runs*

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Introduction

Studies have indicated that bank runs are caused by signals about fundamental insolvency risk (e.g., Chari and Jagannathan (1988); Gorton (1988)) and coordination problems among investors (e.g., Diamond and Dybvig (1983)), or both (e.g., Goldstein and Pauzner (2005)). But not all bank runs are marked by panicked depositors lined up to withdraw their money. Bank runs also have a more subtle time dimension that involves learning.

Learning has been found to be important for early withdrawals in runs. Individual depositor behavior is hard to observe, but Iyer *et al.* (2016) discovered an increase in withdrawals by bank staff and uninsured depositors at a bank just after the Indian Central Bank found troubling information in an audit, but before that information was released to the public.[1] Blickle *et al.* (2019) show that in Germany in 1931, bank funding dried up first in the interbank market, followed by the wholesale market, and then finally in the retail market. Gráda and White (2003) detail that the panic of 1857 in the U.S. began as a run by more wealthy and experienced[2] depositors. Michaelides (2014) documents a "slow but steady deposit run" in Cyprus, where MFIs (monetary and financial institutions such as foreign banks) pulled their money out of the Cypriot banks while all other depositors did nothing.[3] For U.S. money market mutual funds, Schmidt *et al.* (2016) find that investors with lower expense ratios and higher minimum investments, which they term "sophisticated" investors, withdrew earlier in response to the Lehman crisis.[4]

This paper analyzes the dynamics of depositors' withdrawal behavior when there is the possibility of learning fundamental information about the value of a bank's assets. There are two types of learning in the model. Informed depositors observe the information initially for free (we later extend the model to allow for costly

[1] It is not surprising that bank staff (who likely have low information acquisition costs and high incentives due to lack of diversification) and uninsured depositors (who have high incentives) ran.

[2] These measures of sophistication are taken from the professions of the depositors who withdrew and their time living in the U.S.

[3] Artavanis *et al.* (2019) provide insights into the slow bank run in Greece in 2014-2015.

[4] There are runs in which depositor/debtholder data are unavailable and that appear to depend on information. He and Manela (2016) argue that information acquisition played a major role in the run on the U.S. commercial bank Washington Mutual. U.S. money market mutual funds (MMFs) suffered outflows of about 11% over three months in 2011, just after a Moody's review for downgrades of BNP Paribas, Credit Agricole, and Societe Generale, due to fear about the MMF's eurozone holdings (Chernenko and Sunderam (2014)).

information acquisition). Uninformed depositors can learn by waiting and seeing whether informed depositors withdrew.

In the model, informed depositors have two advantages: (i) a timing advantage that helps them react early whenever the bank has low-quality assets; and (ii) a screening advantage that allows them to avoid losses from liquidating high-quality assets.

Uninformed depositors face a trade-off between an information and a timing disadvantage. First, they can withdraw immediately, but only based on incomplete knowledge about the asset quality. Second, they can decide to wait, which allows them to free-ride on the information of the informed depositors. Waiting avoids losses from inefficiently liquidating high-quality assets. This, however, comes at the cost that their claims become junior to the claims of the informed depositors.

The model is solved using the global games methodology (as in Goldstein and Pauzner (2005)). All agents are risk-neutral. Uninformed depositors receive a noisy signal about the quality of the asset, while informed depositors learn the quality perfectly. There are two opportunities (periods) where depositors can withdraw before their investment and the asset mature. Early withdrawals force partial costly liquidation of the asset.

In the model, learning by both types of depositors increases the probability of preemptive runs on the bank in two ways.

First, introducing the opportunity for uninformed depositors to gain additional information by waiting creates a real option problem. One might expect that the option to decide later with more information may delay runs on the bank. However, we show that the opposite is true because of the coordination problem among depositors and the fact that liquidation is costly. When uninformed depositors finally learn the true asset quality, they run when the asset is low-quality. This forces an inefficient liquidation of the low-quality asset and, hence, decreases the payoff of waiting, leading to a greater likelihood of a preemptive run. This seems in line with reality: Artavanis *et al.* (2019) document strategic withdrawals before the resolution of a 2015 Greek election that had the potential to affect the value of deposits.[5]

Second, when there are more informed depositors, uninformed depositors become increasingly worried about the possibility that the informed depositors will

[5]The election could have affected deposits through the possibility of "Greece leaving the Euro zone and the conversion of deposits from Euros to a new Greek currency, [and] the nationalization of the banking sector" (Artavanis *et al.*, 2019, p.4) .

find out that the bank's assets are low-quality and will grab the entire value for themselves. Therefore, uninformed depositors have a fear of missing out and are more likely to preemptively run on otherwise solvent banks when there are more informed depositors.

Moreover, we demonstrate that having a fraction of depositors be informed may be worse in terms of surplus than having all depositors informed *or* having them all uninformed.

We extend the model to allow for endogenous information acquisition by depositors. We find that depositors overinvest in information relative to the surplus-maximizing choice. This is the case because they (i) benefit from information giving them a first-mover advantage and (ii) do not take into account that their information acquisition will increase the likelihood of preemptive runs.

We also show that when banks' assets have a higher liquidation value, the likelihood of runs may increase or decrease depending on the proportion of informed depositors. The likelihood of runs increases with the liquidation value when a higher fraction of depositors are informed. The reason is that the amount of a low-quality asset remaining in the future is small, making immediate withdrawal more attractive. This may incentivize banks to invest in more illiquid assets and regulators to not intervene with injections of liquidity.

If we were to replace informed depositors with a fully informative public signal in the second period, the real option effect, with its inefficiency, would still be present. Stress tests are prime examples of regulators revealing fundamental information about banks; this result points out that stress tests may amplify runs *before* the tests occur. This is in contrast to the finance literature, which examines stress test design to avoid runs *after* the test.

Withdrawals by informed depositors are fundamental-based runs. Preemptive withdrawals by uninformed depositors are panic-based runs brought about by coordination failure when economic fundamentals are poor. Our model demonstrates that fundamental-based runs and panic-based runs may interact in a subtle way. The threat of a fundamental-based run can trigger a panic-based run.

In an economic environment with good fundamentals, runs on banks may be slow-motion bank runs, where informed depositors withdraw early and uninformed depositors withdraw late. Uninformed depositors wait because their private signal tells them the bank assets are likely to be high-quality, but this information turns out to be incorrect. They realize this when they observe the withdrawals of the informed depositors and then choose to withdraw as well.

We now summarize the related theoretical literature. In Section 2, we present the model, and in Section 3, we solve the model. In Section 4, we examine surplus in the model and illustrate its key mechanisms. In Section 5, we discuss the model in the context of stress tests. In Section 6, we extend the model to make the initial information acquisition endogenous. In Section 7, we conclude. All proofs are in the Appendix. The Internet Appendix studies the optimal deposit contract in our model when we add some features from Calomiris and Kahn (1991).

Theoretical Literature

Our paper is related to the literature on asymmetric information among depositors in run-type situations.

In Chari and Jagannathan (1988), some depositors receive information about the performance of the bank's assets. Other depositors observe the number of withdrawals but not their cause, which could be a negative signal or a liquidity shock, making withdrawals a noisy signal. In our paper, agents correctly deduce information from the behavior of others, which, nevertheless, decreases surplus. We use the global games methodology to pin down a unique equilibrium and are able to examine comparative statics.

He and Manela (2016) study the impact of rumors and noisy information acquisition on the survival time of a bank. We also have what they call a "fear of bad-signal agents," which can lead to panic runs. In contrast to their model, our core result is that the opportunity to wait and obtain better information in the future increases preemptive uninformed runs. Moreover, we capture the interplay of fundamental runs and preemptive runs in the absence of any negative rumors. We show that even in situations without any adverse shock on the fundamentals, increased hard information can reduce surplus.

Ahnert and Kakhbod (2017) analyze how investors' endogenous information choice can amplify financial crises. In their model, the higher frequency of crises is driven by the fundamental, while we allow for preemptive runs that are inefficient. Calomiris and Kahn (1991) have endogenous information acquisition by investors for the purpose of monitoring a bank. They do not allow for spillover effects from this information. Chen (1999) models a negative payoff externality among depositors that makes uninformed investors more sensitive to noisy signals about solvency (coming from the failure of other banks) and more likely to run. In contrast to Chen's paper, uninformed agents in our model already deduce information from withdrawal behavior before the actual failure of institutions. Moreover, while we focus on panic-driven preemptive runs, his model considers only fundamental and, therefore, efficient runs.

Schotter and Yorulmazer (2009) present an experimental study of the dynamics of bank runs. In their experiment, the presence of a small fraction of insiders who know the quality of the bank's assets mitigates the severity of runs, as uninformed agents delay their withdrawal decision to learn from the actions of the insiders. Aside from technical points, the key difference with respect to our model is that Schotter and Yorulmazer (2009) assume that there is no cost/inefficiency of liquidating assets. This cost drives our main results.

A growing literature studies the effect of information acquisition in global games. Angeletos *et al.* (2006) show that endogenous signaling by a policy maker can lead to multiple equilibria. In contrast to their model, the public information in our model is not a decision by one agent, but an outcome of the choices of many infinitesimal agents (the informed investors). Dasgupta (2007) introduces the option to delay an investment decision in a global game. In contrast to our results, the option to delay reduces the incidence of coordination failure in equilibrium relative to the standard case. This is because, unlike in our model, the reduction of payoffs due to the delay is exogenous and independent of the behavior of the other agents. Angeletos *et al.* (2007) analyze a global game with multiple rounds, with updating in each round due to the knowledge that the regime survived past attacks and newly arriving information. Szkup and Trevino (2015) study endogenous information acquisition in a global game and show that strategic complementarities in actions do not always translate into strategic complementarities in information choices. Ahnert and Bertsch (2015) investigate the contagion of a crisis from one region to another based on agents' endogenous decisions to gather more information about the nature of the shock.

1 The Model

We consider an economy in which there exists one good that is used for consumption and investment. We consider two types of risk-neutral agents: a bank and depositors. There is no deposit insurance in the model. Therefore, deposits resemble short-term debt.

We begin by writing down the basic timing of the model, and subsequently discuss each aspect in detail. There are five periods $t \in \{-1, 0, 1, 2, 3\}$. For simplicity, we do not allow for discounting between periods.

t= -1 Nature draws the fundamental θ, which determines the ex ante fraction $p(\theta)$ $(1 - p(\theta))$ of assets that have quality H (L).

t= 0 The bank attacts deposits and invests in an asset drawn randomly from the pool of assets.

Informed depositors observe the asset quality $Q \in \{L, H\}$ directly.

Uninformed depositors observe a private noisy signal of θ (denoted for depositor i by θ_i).

t= 1 All depositors decide to withdraw or keep their money in the bank (wait). The number of withdrawals are observable.
The bank liquidates a fraction of the asset to serve any withdrawals.

t= 2 The remaining depositors decide to withdraw or keep their money in the bank (wait).
The bank liquidates a fraction of the asset to serve withdrawals.

t= 3 The remaining fraction of the asset matures.
The bank repays remaining deposits if solvent.

1.1 Actions and Payoffs

1.1.1 The bank and the asset

The bank has no funds of its own but has access to an asset with a positive net expected value when it matures ($t = 3$). We assume that depositors cannot directly invest in the long-term asset but can invest in the bank's short-term debt. This maturity transformation is the main role of the financial intermediary in our model.[6]

[6] We show in the Internet Appendix that intermediation by a short-term deposit-funded bank is optimal when the banker may abscond with funds, as in Calomiris and Kahn (1991). The

There is a pool of assets that the bank has access to. The pool has a fraction $p(\theta)$ of risky assets that are high-quality ($Q = H$) and have a return of R_H at $t = 3$. The remaining fraction $1 - p(\theta)$ are low-quality ($Q = L$) and have a return of $R_L < R_H$. The bank draws an asset from this pool at random.

We assume that $p(\theta)$ is continuous and strictly increasing in the fundamental θ, which realizes at $t = -1$ and reflects the economic state. We assume that the prior over θ is uniformly distributed in the interval $[0, 1]$ and define $p := E[p(\theta)]$ as the ex ante probability that the bank has a high-quality asset.

There are two types of depositors. Uninformed depositors receive a signal (detailed below) about the quality of the asset pool. Given that they receive a signal, these depositors are not fully "uninformed," but are uninformed in a relative sense. Informed depositors perfectly learn the quality of the asset the bank has drawn.

Portions of the asset can be prematurely sold (at $t = 1, 2$), but in that case, the value of the amount sold shrinks to a fraction $\lambda < 1$ of the final return.[7] This embeds two assumptions. First, we assume that, like the informed depositors, those who invest in the firm's liquidated assets can distinguish between the high and low-quality asset.[8] Second, we assume that the liquidation costs result from the general illiquidity of the bank's financial assets. This liquidity discount may arise from cash-in-the-market pricing, banks being the most efficient holders of the assets (due to monitoring or risk management), and/or outside investors being unwilling to park their money in long-term assets. The assumption that the early liquidation of assets reduces their value has an important implication for surplus: when the asset quality is low, there are fundamental runs at $t = 2$ that decrease the expected return and make preemptive runs more likely at $t = 1$.

At $t = 0$, the bank issues short-term debt that is uniformly distributed among a measure-one continuum of depositors. We assume a sequential service constraint,

banking literature offers several alternative explanations of why households invest in short-term liquid debt rather than directly investing in long-term risky assets such as anticipated liquidity shocks (Diamond and Dybvig (1983)), asymmetric information (Diamond (1984) and Gorton and Pennacchi (1990)), or a maturity rat race (Brunnermeier and Oehmke (2013)).

[7]For simplicity, we assume that λ does not vary with the asset quality. However, allowing the liquidation value parameter λ to depend on the asset quality - i.e., $\lambda_L \neq \lambda_H$ - does not affect our results as long as our general assumptions on the parameters hold.

[8]As we will demonstrate in the text, observing the withdrawals from $t = 1$ at $t = 2$ provides full information about the asset quality in equilibrium. Therefore, this assumption is necessary only for liquidations that occur at $t = 1$.

which creates a coordination problem among depositors. The bank promises to pay ρ at $t = 3$. If the value of the bank's asset at $t = 3$ is lower than the promised repayment - i.e., if the assets have low-quality and $\rho > R_L$ - the bank is declared insolvent and the residual value is allocated evenly to the remaining depositors.[9] We assume that depositors, if indifferent between current and future expected payoffs, prefer immediate consumption.

We assume that depositors can withdraw at any time before the asset matures, receiving a short-term payment $D < \rho$. This is meant to represent uninsured[10] short-term liabilities such as wholesale debt. In the Internet Appendix, we endogenize the liability structure using the logic of Calomiris and Kahn (1991); short-term debt/withdrawable deposits are optimal, as runs prevent the bank manager from absconding. In that specification, we demonstrate that $\rho = R_H$ and that parameters can be found for which D can take any value in the set $(\lambda R_L, R_H)$.[11] Henceforth, to save on notation, we will set $\rho = R_H$.

To further simplify the analysis, we will restrict the value of D such that:

Assumption 1: $R_L < D \leq \lambda R_H$.

This assumption implies that the bank can always make the promised repayment in each period if the bank asset quality is high. However, a low-quality asset can not cover all deposits.

We assume that without any information on the asset quality, not withdrawing deposits until the asset matures ($t = 3$) is optimal, as the bank's asset has positive expected net value to depositors in every period: $pR_H + (1-p)R_L > D$. As we will show later, in equilibrium, the low-quality asset is always completely liquidated before it matures. Therefore, we impose a slightly stricter assumption on the expected value of the asset - i.e., that the asset has a positive expected net value when the low-quality asset is fully liquidated:

Assumption 2: $pR_H + (1-p)\lambda R_L > D$.

[9] Any asset value not paid out to depositors will go to the bank's owners.

[10] He and Manela (2016) argue that even insured depositors may withdraw their deposits prematurely if they fear the illiquidity or insolvency of their bank. Such insured withdrawals could be driven by the fear that the supervisor will temporarily freeze their accounts, or that the deposit insurance simply cannot cover the liabilities of a large bank. Furthermore, in reality, large depositors will only have a fraction of their deposits insured.

[11] As we discuss in the Internet Appendix , Assumption 2 must also be satisfied, which restricts the upper bound of D further.

This assumption implies that depositors who anticipate the liquidation of the low-quality asset find it worth investing in the bank in the first place. This requires that, ex-ante, the probability that the asset is high-quality must be sufficiently large - i.e., $p > \frac{D-\lambda R_L}{R_H-\lambda R_L}$.

We make an additional assumption to guarantee a unique equilibrium:

Assumption 3: $pR_H < pD + (1-p)\lambda R_L$.

This assumption implies that without any additional information, a depositor prefers to withdraw if he expects everyone else to withdraw as well. The right-hand side is the value of withdrawing if everyone else withdraws. If the asset is high-quality, the depositor gets a payment of D because of Assumption 1. If the asset is low-quality, the asset is fully liquidated, and we assume that the liquidation value is evenly allocated among depositors. The left-hand side is the highest value that a depositor can obtain from waiting if everyone else withdraws. If the asset is high-quality, the depositor gets, at most, R_H in expectation. If the asset is low-quality, the entire asset is liquidated immediately, leaving the waiting depositor with nothing.

Assumptions 1 to 3 guarantee that strategic complementarities exist among the uninformed depositors. That is, we show below that the expected payoffs of uninformed depositors from waiting compared to withdrawing immediately increases the more other agents also wait.

The three assumptions combined together imply that $\frac{\lambda R_L}{D} > \frac{1}{2}$. In other words, we assume that early liquidation is costly, but the liquidated low-quality asset still has considerable value. We use this implication explicitly in the proof of Lemma 7.

We now formalize depositors' payoffs and how they depend on the other depositors' actions.

In period $t = 1$, there is a mass 1 of risk-neutral depositors who could be one of two types $z \in \{U, I\}$: a proportion π are informed depositors ($z = I$), and the remaining proportion $1 - \pi$ are uninformed depositors ($z = U$). In periods $t = 1$ and $t = 2$, depositors can choose to withdraw or keep their deposit at the bank.

Whenever a proportion of depositors withdraws the promised amount D in period t, they force the bank to liquidate some fraction of the bank's asset, causing a liquidation loss. Define the number of withdrawals of type $z \in \{U, I\}$ depositors in period t given the (not necessarily known) asset quality $Q \in \{L, H\}$ as:

$$n_t^z(Q)$$

Moreover, denote the total number of withdrawals at time t as:

$$N_t(Q) = n_t^I(Q) + n_t^U(Q).$$

As depositors are risk-neutral, their objective is to maximize their expected consumption by withdrawing at the time when the return is highest. Thus, the amount each agent is able to consume in each period depends on the realized asset quality of the bank and, if the asset is low-quality, on the actions of the other depositors.

At $t = 3$, all asset cash flows are allocated to residual depositors. If the asset quality is high, depositors receive $\rho = R_H$. If the asset quality is low, given that $\rho = R_H > R_L$, the remaining value is allocated evenly to the depositors.

The payoff structure is summarized in Table 1. Consider first a bank with a high-quality asset. A depositor withdrawing at date $t = 1$ or $t = 2$ receives D. The bank has to liquidate a fraction $\frac{N_t(H)D}{\lambda R_H}$ of the asset to serve the withdrawn liabilities $N_t(H)D$. Given Assumption 1, the liquidation value of the high-quality asset is larger than or equal to the liability claim D, such that the bank is always able to cover all early deposits.[12] The fraction of the asset that has not been liquidated produces an amount when it matures that is strictly greater than the outstanding deposits at that time.[13] Depositors that did not withdraw earlier, therefore, receive R_H at $t = 3$ as contracted. Consequently, no negative externality exists between depositors if the bank's asset quality is high; the high-quality bank will not be forced into insolvency by early withdrawals. If the asset quality was known to be high, there would be no self-fulfilling runs.

In contrast, if the asset quality turns out to be low, the bank may not be able to pay all deposit withdrawals. Note that $\frac{\lambda R_L}{D}$ is the default threshold of the bank - i.e., the number of withdrawals that forces the low asset quality bank to liquidate the entire asset.[14]

If only a fraction of depositors withdraw D at dates $t = 1$ and $t = 2$ - i.e.,

[12] The value of bank equity actually strictly increases with early withdrawals when $D < \lambda R_H$. If $D = \lambda R_H$, the bank passes through all cash flows, and the value of bank equity is not affected by early withdrawals.

[13] This is easy to see: $(\frac{\lambda R_H - (N_1(H)+N_2(H))D}{\lambda R_H})R_H > (1 - (N_1(H) + N_2(H)))R_H \Leftrightarrow D < \lambda R_H$.

[14] Note that Assumption 1 implies that a bank with a low quality asset is not able to repay its liability holders, even at asset maturity ($t = 3$). A bank with a low quality asset can, therefore, not only become illiquid once the amount of withdrawals exceeds the liquidation value of the asset, but will also be insolvent at $t = 3$, as the promised repayment $\rho = R_H$ exceeds the value of assets at maturity (R_L) independent of any early liquidation.

Table 1: Asset-quality-dependent expected payoffs at time t

Quality	Aggregate Withdrawals	t=1	t=2	t=3
H		D	D	R_H
L	(1) $N_1(L) + N_2(L) \leq \frac{\lambda R_L}{D}$	D	D	$\frac{R_L-(N_1(L)+N_2(L))\frac{D}{\lambda}}{(1-N_1(L)-N_2(L))}$
	(2) $N_1(L) \leq \frac{\lambda R_L}{D} < N_1(L) + N_2(L)$	D	$\frac{\lambda R_L - N_1(L)D}{N_2(L)}$	0
	(3) $\frac{\lambda R_L}{D} < N_1(L)$	$\frac{\lambda R_L}{N_1(L)}$	0	0

$N_1(L)+N_2(L) \leq \frac{\lambda R_L}{D}$- the liquidation value of the asset suffices to cover the promised payment D to all withdrawing depositors, which is described in Case (1) in Table 1. To satisfy withdrawals, the bank had to liquidate a fraction $\frac{(N_1(L)+N_2(L))D}{\lambda R_L}$ of the low-quality asset. The remaining asset returns $(1-\frac{(N_1(L)+N_2(L))D}{\lambda R_L})R_L$, once the asset matures. This residual asset value is strictly lower than the long-term return R_H promised to the $1-(N_1(L)+N_2(L))$ remaining depositors. Therefore, the bank is declared insolvent at $t=3$, and the residual value is allocated evenly among the remaining depositors, who receive $\frac{R_L-(N_1(L)+N_2(L))\frac{D}{\lambda}}{1-(N_1(L)+N_2(L))} < D$.[15]

Whenever more than $\frac{\lambda R_L}{D}$ depositors withdraw before maturity, the bank becomes illiquid and the bank's entire assets are liquidated. This can happen at $t = 2$ (Case (2)) or at $t = 1$ (Case (3)) in Table 1. Due to the sequential service constraint, the depositor's (random) position in the waiting line of withdrawing depositors determines his payoff. If the bank is liquidated in a period, the first depositors approaching the bank receive D , while the later depositors receive nothing. The ex ante probability of being among the first in the line is $\frac{\lambda R_L - N_1(L)D}{N_2(L)D}$ in case (2) and $\frac{\lambda R_L}{N_1(L)D}$ in case 3. This creates a coordination problem among depositors and strategic complementarity between the withdrawal decisions of depositors. A low-asset-quality bank is vulnerable to a bank run in each period.

Depositors who know the realized asset quality would always want to wait until the high-quality asset matures and withdraw immediately from a bank with a low-quality asset. We will show formally that this is, indeed, the dominant strategy of an informed depositor, independent of the strategies of all other depositors.

Uninformed depositors, however, receive only a private noisy signal about the

[15]Insolvency will occur at $t = 3$ for any number of previous withdrawals as the low-asset-quality repayment in $t = 3$ is decreasing in the number of previous withdrawals due to the liquidation cost. Even if no depositor withdrew in $t = 1$ or $t = 2$, the left-hand side is R_L, which, by assumption, is smaller than D.

asset's quality, which we discuss in the next subsection in more detail. Based on that signal, depositors can choose to either withdraw immediately or to wait and observe the informed depositors' withdrawal behavior from which they will be able to learn about the asset quality. Postponing the withdrawal decision has the advantage that uninformed depositors free-ride on the information of informed depositors and withdraw only if the asset is of low-quality. However, waiting comes at a cost: other depositors may have moved first and forced the partial or full liquidation of the bank's asset. Postponing the withdrawal decision, thus, makes the depositor's claim junior to depositors who withdraw earlier, leaving a small or zero payoff when the asset is low-quality. This cost may outweigh the gain of learning the asset quality. As a result, uninformed depositors may withdraw in the first period, based on their noisy signal only.

1.2 Incomplete Information

We assume that the fundamental θ realizes at $t = -1$ and determines the fraction $p(\theta)$ of the asset pool that are high-quality. This may be thought of as fundamental information about the economy or the financial system. However, uninformed depositors observe that information only in $t = 0$ and with noise. More formally, in period $t = 0$, every uninformed depositor i obtains a private noisy signal about θ:

$$\theta_i = \theta + \varepsilon_i,$$

where the ε_i are arbitrarily small error terms that are independently and uniformly distributed over $[-\varepsilon, \varepsilon]$. The private signal influences a depositor's decision to keep their money in the bank in two ways. First, a higher signal increases the posterior belief of the depositor that R_H has realized, implying a larger incentive to not withdraw. Second, a higher signal provides (imperfect) information about the information of the other depositors and their respective actions. Observing a high signal makes the depositor believe that other uninformed depositors have seen high signals as well, which makes it less attractive to withdraw and more likely that other depositors will not withdraw either.

2 Analysis

We consider perfect Bayesian equilibria. A perfect Bayesian equilibrium is defined by a strategy profile such that each depositor chooses the best action (withdraw or wait) given their information and the strategies of the other uninformed and informed depositors.

2.1 Period 2 decisions

At $t = 2$, given that the bank is still liquid (i.e., $N_1(L)D < \lambda R_L$ if the asset value is low), all remaining depositors have to decide whether or not to withdraw. The withdrawal decision at $t = 2$ clearly depends on the number of withdrawals at $t = 1$ and the information inferred from those withdrawals.

We begin by placing some structure on the number of withdrawals and the information transmitted. We show in the following lemmas that informed depositors have dominant strategies at $t = 1$ that are independent of the proportion $n_1^U(Q) \in [0, 1 - \pi]$ of uninformed depositors withdrawing.

Lemma 1 *If the asset is high-quality, all informed depositors do not withdraw until the asset matures ($t = 3$), such that $n_1^I(H) = n_2^I(H) = 0$ and $n_3^I(H) = \pi$.*

When the asset quality is high, the bank will not fail due to a panic run. If a depositor knows that the bank's asset is high-quality, it can be inferred that the bank is able to serve all liabilities. Keeping their money in the bank until maturity, when they can collect the full promised repayment of R_H, becomes the dominant strategy for informed depositors.

Now consider the optimal decision of a depositor who receives information that the quality of the asset is low.

Lemma 2 *If the asset is low-quality, $n_1^I(L) = \pi$ and $n_2^I(L) = n_3^I(L) = 0$, all informed depositors withdraw at $t = 1$.*

If the asset is low-quality, the short-term payment D is larger than the value of the asset per depositor and the value of the liquidated asset per depositor, so the informed depositors withdraw immediately.

Note that informed depositors do not benefit from hiding their information. As a result, these depositors always withdraw if they know that the asset is low-quality and do not withdraw if they know that the bank's asset is high-quality. Therefore, if uninformed depositors at $t = 2$ were able to directly observe the actions of the informed depositors, they would perfectly infer the quality of the asset.

Nevertheless, we assume that uninformed depositors observe only the aggregate amount of withdrawals at $t = 2$. Given that uninformed depositors may have also withdrawn at $t = 1$, observing the aggregate amount of withdrawals at $t = 2$ may be a noisy signal. We now examine the information that may be inferred from these withdrawals.

In order to do so, we posit that there is a unique threshold fundamental θ' at $t = 1$ that determines the $t = 1$ decision of uninformed depositors: for depositor i, if $\theta_i \leq \theta'$, the depositor withdraws, and if $\theta_i > \theta'$, the depositor waits. We will later prove that there is indeed a unique threshold.

Given the threshold, and acknowledging that ε is assumed to be arbitrarily small, we compute the proportion of depositors $n_1(\theta, \theta', L)$ that withdraw at $t = 1$ given: that the fundamental is θ; that there is a threshold for uninformed withdrawals θ'; and that the asset quality is L:

$$n_1(\theta, \theta', L) = \begin{cases} 1 & \text{if } \theta < \theta' - \varepsilon \\ \pi + (1 - \pi)\left(\frac{1}{2} + \frac{\theta' - \theta}{2\varepsilon}\right) & \text{if } \theta' - \varepsilon < \theta < \theta' + \varepsilon \\ \pi & \text{if } \theta' + \varepsilon < \theta \end{cases} \tag{1}$$

Similarly, the proportion of depositors $n_1(\theta, \theta', H)$ that withdraw at $t = 1$, given: that the fundamental is θ; that there is a threshold for uninformed withdrawals θ'; and that the asset quality is H:

$$n_1(\theta, \theta', H) = \begin{cases} 1 - \pi & \text{if } \theta < \theta' - \varepsilon \\ (1 - \pi)\left(\frac{1}{2} + \frac{\theta' - \theta}{2\varepsilon}\right) & \text{if } \theta' - \varepsilon < \theta < \theta' + \varepsilon \\ 0 & \text{if } \theta' + \varepsilon < \theta \end{cases} \tag{2}$$

We now examine the decision of all depositors that remain at $t = 2$. We make the following assumption (where we define $P(\theta) = \frac{1}{2}p(\theta - 2\varepsilon\frac{\pi}{1-\pi}) + \frac{1}{2}p(\theta))$, which we explain in detail below:

Assumption 4: $P(p^{-1}\left(\frac{D}{R_H}\right) + \varepsilon\frac{2\pi}{1-\pi})R_H + (1 - P(p^{-1}\left(\frac{D}{R_H}\right) + \varepsilon\frac{2\pi}{1-\pi}))\frac{\lambda R_L - \frac{1}{2}D}{\frac{1}{2}} < D.$

The following Lemma demonstrates how all depositors act at $t = 2$.

Lemma 3 *At $t = 2$,*

1. *All remaining depositors do not withdraw if they know the high asset quality has realized.*

2. *All remaining depositors withdraw if they know the low asset quality has realized.*

3. *All remaining depositors who are uncertain about the quality of the asset withdraw, given that Assumption 4 holds.*

The uninformed depositors who remain at $t = 2$ observe the number of withdrawals at $t = 1$ and their own personal signal θ_i about the likelihood that the asset quality is H. Combining this information allows some to perfectly infer the asset quality. Similar to Lemmas 1 and 2, those who infer the asset quality will have a dominant strategy: withdraw if the asset quality is L, and wait if it is H. Given the withdrawals at $t = 1$ and projected withdrawals at $t = 2$ from those who infer the asset quality, the Lemma proves that if the asset quality is low, a minimum of one half of all depositors have already withdrawn/will withdraw by the end of $t = 2$. Assumption 4 states that, given that one half of all depositors will have withdrawn when the asset is low-quality, an uninformed depositor who has not inferred the asset quality will prefer to withdraw. This pins down a unique equilibrium at $t = 2$.[16] This permits us to write down the payoffs at $t = 1$ and solve for the unique equilibrium in that period.

Nonetheless, Assumption 4 is not necessary for our main results to hold. At the end of the proof of Lemma 3, we demonstrate that if we assumed, instead, that all remaining depositors who were uncertain about the quality of the asset wait until $t = 3$, our main results would *still* hold. The utility of Assumption 4 is to be able to write down the payoffs; once we write down the payoffs, the number of uninformed depositors who do not infer the asset quality at $t = 2$ will go to zero when we take the noise ε to zero; that is, when there is no noise, all uninformed depositors will perfectly infer the quality of the asset at $t = 2$, and Assumption 4 will not be relevant.[17]

2.2 Period 1: The decision of uninformed depositors

At $t = 1$, uninformed depositors observe only their private noisy signals about asset quality and have to decide on their action at the same time as the informed depositors. Uninformed depositors correctly anticipate that all informed depositors

[16] Note that Dasgupta (2007) introduces a second source of noise to pin down a unique equilibrium in the second period.

[17] We also note in the proof of Lemma 3 that when $\pi > \frac{1}{2}$, for any amount of noise, all uninformed depositors will infer the quality of the asset perfectly.

will not withdraw when the asset is high-quality and withdraw when the asset is low-quality. The uninformed depositors compare the expected payoffs from withdrawing immediately with the payoff from proceeding to the next period, based on their private signal and the higher-order expectations on the behavior of other uninformed depositors derived from this signal.

The ex-post payoff of uninformed depositors depends on the fundamental θ, the proportion of informed depositors π and the proportion of uninformed preemptive withdrawals n_1^U. We now examine this payoff.

Payoff for uninformed depositors from withdrawing immediately: Note that the consumption of the uninformed depositor who withdraws depends only on the number of withdrawals when the asset quality is low (L); when it is high (H), withdrawals and the resulting asset liquidation does not affect payoffs. The expected consumption of this depositor at $t = 1$ is:

$$E[p(\theta_i)]D + (1 - E[p(\theta_i)]) \min \left[D, \frac{\lambda R_L}{N_1(L)} \right]. \quad (3)$$

Early withdrawal secures an equal share of the residual value for the uninformed depositor if the bank is liquidated. The expected consumption from equation (3) is weakly decreasing in $N_1(L)$.

Payoff for uninformed depositors who wait until t=2: Uninformed depositors can also form expectations about their payoff from waiting. In order to write out this payoff, we define $q(\theta_i, \theta', \varepsilon) \equiv \Pr(\theta' - \varepsilon < \theta < \theta' + \varepsilon | \theta_i)$.

$$E[p(\theta_i)][q(\theta_i, \theta', \varepsilon)D + (1 - q(\theta_i, \theta', \varepsilon))R_H] + (1 - E[p(\theta_i)]) \max \left[\frac{\lambda R_L - N_1(L)D}{1 - N_1(L)}, 0 \right]. \quad (4)$$

We prove that this is the correct payoff at the end of the proof of Lemma 3. The first term is the benefit of waiting if the asset quality is high. In this case, with probability $q(\theta_i, \theta', \varepsilon)$, the depositor is uncertain about the quality of the asset and, by Assumption 4, will decide to withdraw. Otherwise, with probability $1 - q(\theta_i, \theta', \varepsilon)$, the depositor will learn at $t = 2$ that the asset quality is high and decide to wait (with a payoff of R_H). The second term is the payoff when the asset quality is low. In this case, from Lemma 3 and Assumption 4, we know that all remaining depositors at $t = 2$ will withdraw. The depositors split the liquidated asset evenly, if there is any residual asset value to split.

The expected payoff of waiting in equation (4) is also weakly decreasing in $N_1(L)$.

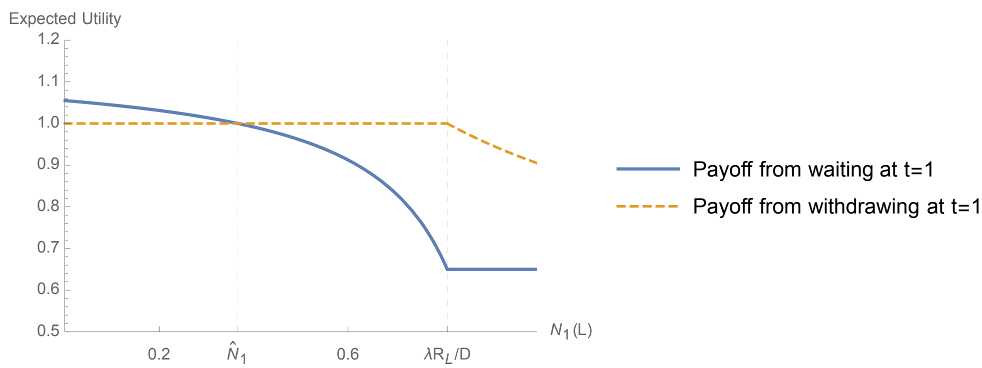


Figure 1: Expected utility from early and late withdrawal.
The parameters used are $D = 1, R_H = 1.3, R_L = 0.9, \lambda = 0.9$, and $p = 0.5$.

2.3 Benchmark: One-period game in which uninformed depositors do not receive private signals

We now define a benchmark where there is no period $t = 2$, and uninformed depositors do not receive private signals θ_i (we set $\theta_i = \theta$). This gives the depositors one period in which to withdraw early. All depositors have symmetric information.

In this setting, we demonstrate that multiple equilibria exist. This provides intuition for the value of the global games setting in refining the set of equilibria.

Lemma 4 *There exists a unique value $\hat{N}_1$ for which an uninformed depositor prefers to withdraw immediately if $N_1(L) > \hat{N}_1$ and wait otherwise.*

Figure 1 illustrates the expected utility from withdrawing immediately and from waiting. The expected utility from withdrawing immediately is constant as long as the bank is expected to be still liquid. The expected utility from waiting is higher than early withdrawal if no other agent withdraws but is decreasing in the withdrawals of other agents when the quality of the asset is low. There exists strategic complementarity among uninformed agents if $\pi \in (0, \hat{N}_1)$ - i.e., agents' actions reinforce each other. Uninformed depositors prefer to wait if they expect other depositors to wait. If an uninformed depositor expects all other depositors to withdraw, choosing to withdraw maximizes the uninformed depositor's expected payoff. This leads to multiple equilibria, which are summarized in the following lemma.

Lemma 5 *When uninformed depositors do not receive private signals, there are multiple equilibria if $\pi < \hat{N}_1$:*

1. *An equilibrium in which uninformed depositors do not withdraw at $t = 1$, with $n_1^U(H) = n_1^U(L) = 0$ such that $N_1(H) = 0$, $N_2(H) = 0$, $N_3(H) = 1$ and $N_1(L) = \pi$, $N_2(L) = 1 - \pi$.*

2. *An equilibrium in which uninformed depositors withdraw at $t = 1$, with $n_1^U(H) = n_1^U(L) = 1 - \pi$ such that $N_1(H) = 1 - \pi$, $N_2(H) = 0$, $N_3(H) = \pi$ and $N_1(L) = 1$, $N_2(L) = 0$.*

If $\pi \geq \hat{N}_1$, (2) would be the unique equilibrium.

Two equilibria exist: one in which none of the uninformed depositors withdraws at $t = 1$, and another in which all uninformed depositors withdraw at $t = 1$. We note that mixed-strategy equilibria are ruled out by our assumption that a depositor withdraws when indifferent between withdrawing in two periods.

2.4 Uninformed depositors receive private signals

In our setup, in which uninformed depositors receive private signals, the realization of θ determines whether uninformed depositors withdraw their investment. Since withdrawing early is not a dominant strategy for moderate realizations of θ, uninformed depositors may withdraw their investment if they believe that the asset quality is low and that others will withdraw as well. We will show that in the unique equilibrium, at $t = 1$, uninformed agents wait if fundamentals are high and withdraw otherwise.

Taking the proportion of informed depositors as given, we first define the extreme realizations of the fundamentals for which uninformed depositors have a dominant strategy. A very low fundamental defines the lower dominance region $\theta < \underline{\theta}(\pi)$, for which an uninformed agent has a dominant strategy to withdraw no matter what other depositors do. The lower dominance region increases in the proportion of informed depositors. We also define the upper dominance region $\theta > \overline{\theta}$, for which an uninformed agent has a dominant strategy to wait no matter what other depositors do. This region does not depend on the fraction of informed depositors.

Lemma 6 *There exists:*

1. *A lower dominance region $\theta \leq \underline{\theta}(\pi)$ where uninformed depositors have a dominant strategy to withdraw, and*

2. *An upper dominance region $\theta \geq \overline{\theta}$ where uninformed depositors have a dominant strategy to wait.*

For realizations of θ between the lower dominance region and the upper dominance region, the expected payoff of an individual depositor depends on the actions of the other depositors; there exists strategic complementarity among uninformed depositors' actions if not too many depositors are informed.

If, however, too many depositors become informed, uninformed depositors prefer to withdraw instead of waiting, no matter what the other uninformed depositors do. The strategic complementarity among uninformed depositors decreases the more depositors become informed. Uninformed depositors correctly anticipate that the large amount of informed depositors will withdraw if the low asset quality has realized, thereby reducing the expected return from waiting. In other words, if the fraction of informed depositors is sufficiently large, the strategic complementarity among uninformed depositors vanishes, such that the depositors' optimal decision of whether or not to withdraw becomes independent of the other depositors' actions. We will formally define the amount of informed depositors for which strategic complementarity vanishes as $\hat{\pi}$ in Lemma 7 (later in the text).

We can make the following statement for $\pi \in (0, \hat{\pi})$: The uninformed depositor's optimal action is uniquely determined by the signal: the uninformed depositor withdraws at $t = 1$ if and only if the signal is below a unique threshold. We define $\theta^*(\pi)$ as the critical fundamental realization that makes uninformed depositors indifferent between waiting or withdrawing immediately for a given amount of informed depositors.

Proposition 1 *At $t = 1$, uninformed depositors have a unique equilibrium strategy to preemptively withdraw if they observe a signal below threshold $\theta^*(\pi)$ and wait if they observe a signal above the threshold.*

The proof is in Appendix A and follows Goldstein and Pauzner (2005). Our setup is similar to theirs, with the main differences being that in our model, (1) the low realization yields strictly positive returns, and (2) there are no impatient depositors but, rather, informed depositors with dominant strategies to withdraw if R_L has realized. We now elaborate on some key elements of this result.

The uninformed depositor's utility differential between waiting and immediate withdrawal is a function of the number of withdrawals $n_1(\theta, \theta^{'}, L)$ as defined in

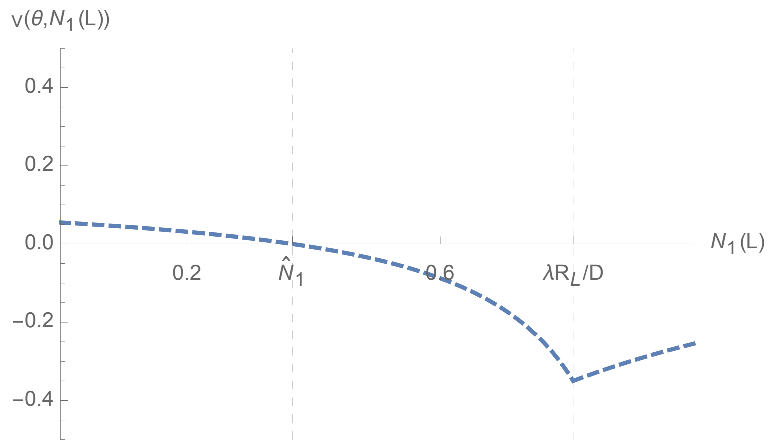


Figure 2: The payoff of waiting minus the payoff of immediate withdrawal. The parameters used are $D = 1, R_H = 1.3, R_L = 0.9, \lambda = 0.9$, and $p = 0.5$.

Equation 1. We obtain:

$$\nu(\theta, n_1(\theta, \theta', L)) = \begin{cases} p(\theta)[(1 - q(\theta_i, \theta', \varepsilon))R_H + q(\theta_i, \theta', \varepsilon)D] & \text{if } \frac{\lambda R_L}{D} > n_1(\theta, \theta', L) \geq \pi \\ +(1 - p(\theta))\frac{\lambda R_L - n_1(\theta,\theta',L)D}{(1-n_1(\theta,\theta',L))} - D & \\ p(\theta)(1 - q(\theta_i, \theta', \varepsilon)(R_H - D) & \text{if } 1 \geq n_1(\theta, \theta', L) \geq \frac{\lambda R_L}{D} \\ -(1 - p(\theta))\frac{\lambda R_L}{n_1(\theta,\theta',L)} & \end{cases} \tag{5}$$

Global strategic complementarity would require that ν always decreases in $n_1(\theta, \theta', L)$. This does not hold in our setting. Figure 2 illustrates the "net incentives" of uninformed agents: the payoff of waiting minus the payoff of immediate withdrawal for a given θ. The withdrawal decision of uninformed agents has one-sided strategic complementarities in the sense of Goldstein and Pauzner (2005): whenever the net incentives are positive, they are monotonically decreasing in the number of other agents.

Intuitively, there is strategic complementarity until the bank is illiquid: The expected gain from waiting decreases in $n_1(\theta, \theta', L)$ until the entire asset is liquidated and then is constant after that point. The expected payoff of withdrawing is constant in $n_1(\theta, \theta', L)$ until the entire asset is liquidated, but it decreases afterwards. Therefore, the net utility from waiting minus withdrawing is decreasing until the entire asset is liquidated but increasing afterwards. In other words, once the asset is liquidated, the incentives to further run on the bank decrease as more depositors withdraw since the share of the liquidated payoff shrinks. As pointed out above, we still have one-sided strategic complementaries as long as Assumption 3 holds - since $\nu(\theta, n_1(\theta, \theta', L))$ is monotonically decreasing in $n_1(\theta, \theta', L)$ as long as $\nu(\theta, n_1(\theta, \theta', L))$ is positive. This is enough to obtain the same uniqueness result as Goldstein and Pauzner (2005).

We can now compute the threshold signal $p(\theta^*(\pi))$. An uninformed depositor that receives the signal $\theta^*(\pi)$ must be indifferent between waiting and withdrawing. That depositor's posterior distribution of θ is uniform over the interval $[\theta^*(\pi) - \varepsilon, \theta^*(\pi) + \varepsilon]$. As we assume that the error term is uniformly distributed as well, the indifferent depositor's posterior distribution of $n_1(\theta, \theta', L)$ is uniform over $[\pi, 1]$. At the limit, as $\varepsilon \to 0$, we can determine the threshold implicitly defined by the condition where the uninformed depositor is indifferent between waiting and withdrawing immediately:[18]

$$\int_{\pi}^{\frac{\lambda R_L}{D}} \{p(\theta^*)R_H + (1 - p(\theta^*))\frac{\lambda R_L - nD}{(1-n)} - D\}dn \\ + \int_{\frac{\lambda R_L}{D}}^{1} \{p(\theta^*)R_H - p(\theta^*)D - (1 - p(\theta^*))\frac{\lambda R_L}{n}\}dn = 0. \tag{6}$$

We can integrate as long as $\pi < \frac{\lambda R_L}{D}$: the number of informed depositors must be low enough such that uninformed depositors still receive a positive amount from the low-quality asset. Integration yields:

$$p(\theta^*)(1-\pi)\left(\frac{R_H}{D} - 1\right) + (1-p(\theta^*))\frac{\lambda R_L}{D}\ln\left(\frac{\lambda R_L}{D}\right) + (1-p(\theta^*))(1-\frac{\lambda R_L}{D})\ln\left(\frac{1-\frac{\lambda R_L}{D}}{1-\pi}\right) = 0.$$

This implicit condition is correct as long as the resulting $\theta^* \in [\underline{\theta}(\pi), \overline{\theta}]$, which holds for $\pi \in [0, \hat{\pi}]$, where $\hat{\pi}$ is defined in Lemma 7 just below. Solving for $p(\theta^*)$ defines the critical realization as a function of payoffs and the proportion of informed depositors:

$$p(\theta^*)_{|\varepsilon \to 0} = \frac{\phi(\pi)}{(1-\pi)(\frac{R_H}{D} - 1) + \phi(\pi)} \tag{7}$$

with $\phi(\pi) := -\frac{\lambda R_L}{D}\ln\left(\frac{\lambda R_L}{D}\right) - (1 - \frac{\lambda R_L}{D})\left(\ln\left(1 - \frac{\lambda R_L}{D}\right) - \ln\left(1-\pi\right)\right)$, which is strictly positive for all $0 \leq \pi < \frac{\lambda R_L}{D} < 1$. This also implies that $p(\theta^*) \in (0, 1)$.

[18] Note that in order to derive this indifference condition, we performed a change of variables (changing θ for $n_1(\theta, \theta', L)$) for the integral over θ, and used the limit $\varepsilon \to 0$ to eliminate the second integral over ε.

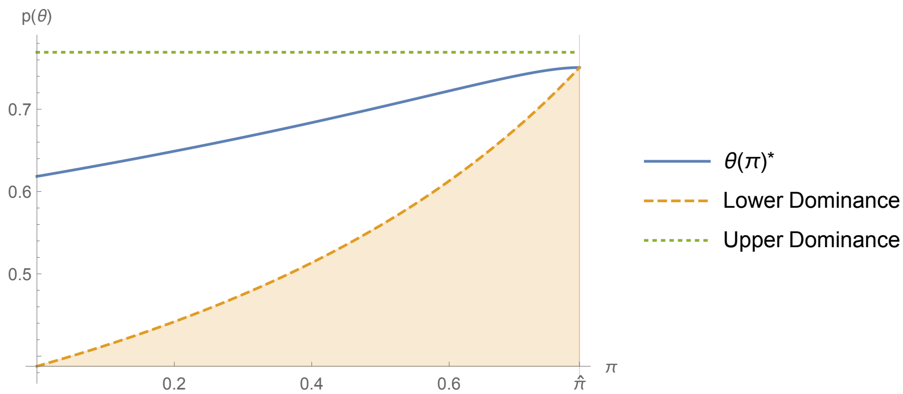


Figure 3: Critical probability threshold that triggers a preemptive run as a function of informed investors. The parameters used are $D = 1, R_H = 1.3, R_L = 0.9$, and $\lambda = 0.9$.

Lemma 7 *Strategic complementarity among uninformed depositors exists only if* $\pi \in (0, \hat{\pi})$*, where* $\hat{\pi} < \frac{\lambda R_L}{D}$ *is implicitly defined by* $p(\theta^*(\hat{\pi})) = p(\underline{\theta}(\hat{\pi}))$*.*

When more informed depositors make use of their first mover advantage when the asset is low-quality, the uninformed depositors benefit less from waiting. As the share of informed depositors increases, the role of strategic complementarity between the agents becomes less important in determining the incidence of preemptive runs, even though the incidence is increasing:

Proposition 2 *The signal threshold* $p(\theta^*)$ *is increasing in* π *for* $\theta^*(\pi) \in (\underline{\theta}(\pi), \overline{\theta})$*.*

The intuition for Proposition 2 is the following. When more depositors are informed, a larger fraction of the low-quality asset is liquidated at $t = 1$. This decreases the residual value of the bank with the low-quality asset at $t = 2$. This can be seen in the indifference condition (Eq. (6)) in the first integral, which has a positive value and shrinks as π increases. This makes a run at $t = 1$ more likely.

The lower dominance cutoff $\underline{\theta}(\pi)$ is also increasing in the fraction of depositors that are informed π. The more depositors that are informed, the smaller is the impact of the behavior of other uninformed depositors on the expected payoff from waiting of an individual uninformed depositor, weakening the strategic complementarity between the uninformed depositors' choices. We depict the threshold value as well as the dominance regions as functions of π in Figure 3. The upper dominance region is independent of π. For fundamentals that produce signals in between the two regions, depositors follow their unique threshold strategy: whenever they observe a signal above the critical threshold, they wait and, otherwise, they withdraw. As the threshold value is increasing in π, the probability that a fundamental value produces signals resulting in a run increases as well.

We can now summarize the main insight from the global game setup for our multi-period game.

Corollary 1 *In the limit, where $\varepsilon \to 0$ and $\pi < \hat{\pi}$, the probability of all uninformed depositors withdrawing is $P[\theta < \theta^*(\pi)]$, which increases in π. With probability $1 - P[\theta < \theta^*(\pi)]$, all uninformed depositors wait until uncertainty is resolved at $t = 2$.*

In the limit, as $\varepsilon \to 0$, preemptive runs occur when the fundamental is low - i.e., $\theta < \theta^*(\pi)$. We assume that depositors at $t = 2$ observe the aggregate withdrawals of all depositors at $t = 1$. The number of withdrawals in the low-quality-asset case (equation (1)) becomes:

$$\lim_{\varepsilon \to 0} n_1(\theta, \theta^*(\pi), L) = \begin{cases} 1 & \text{if } \theta < \theta^*(\pi) \\ \pi & \text{if } \theta > \theta^*(\pi). \end{cases} \tag{8}$$

Similarly, in the limit the number of depositors withdrawing from a bank with a high-quality asset (equation (2)) becomes:

$$\lim_{\varepsilon \to 0} n_1(\theta, \theta^*(\pi), H) = \begin{cases} 1 - \pi & \text{if } \theta < \theta^*(\pi) \\ 0 & \text{if } \theta > \theta^*(\pi). \end{cases} \tag{9}$$

As π is known, remaining uninformed depositors at $t = 2$ are able to perfectly deduce the actual asset quality in $t = 2$ by observing the aggregate number of withdrawals. The aggregate number of withdrawals reveals the actions of informed agents even though depositors cannot observe the identities of the depositors that withdraw.

2.5 Comparative Static on λ

We now examine how the solution is affected by a change in the liquidation value of the asset λ.

Proposition 3 *The equilibrium threshold value for a preemptive run decreases in λ ($\frac{\partial p(\theta^*)}{\partial \lambda} < 0$) if $(1 - \pi) > \frac{D - \lambda R_L}{\lambda R_L}$, and increases otherwise.*

To understand the intuition, we consider the threshold probability of a pre-emptive run that implicitly solves the indifference condition in equation (6), which equates the expected return from waiting for another period and withdrawing immediately. We adapt the indifference condition from equation (6) by separating the utility of waiting (first line) and the utility from withdrawing immediately (second line).

$$\int_{\pi}^{\lambda \frac{R_L}{D}} (p(\theta)R_H + (1-p(\theta))\frac{\lambda R_L - n\,D}{(1-n)})dn + \int_{\lambda \frac{R_L}{D}}^{1} p(\theta)\frac{R_H}{D}\,dn \tag{10}$$
$$= \int_{\pi}^{\lambda \frac{R_L}{D}} D\,dn + \int_{\lambda \frac{R_L}{D}}^{1} (p(\theta)\,D + (1-p(\theta))\frac{\lambda R_L}{n})dn.$$

The first line depicts the expected utility of an uninformed depositor from waiting for another period. This expected utility is clearly increasing in λ, as a higher liquidation value increases the payoff from waiting in both cases.[19] The second line captures the expected utility from immediate withdrawal. It is also increasing in λ, since (i) there are more states in which the depositor receives D; and (ii) there is a larger liquidation value if the bank becomes illiquid.

If the proportion of informed depositors is small, an increase in λ decreases the net incentives to withdraw at $t = 1$, as the remaining fraction of the asset to be liquidated in the second period is high, and the higher future liquidation value outweighs the benefits of immediate withdrawal. However, when there are many informed depositors, the fraction of the low-quality asset remaining for future liquidation is small, and immediate withdrawal becomes more attractive. Therefore, the threshold value increases in λ if π is large.

While we do not model how λ is determined, this result suggests that the incentives to change λ are important. The liquidation discount λ could be affected by the bank when it chooses whether it wants to invest in liquid or illiquid assets. It could also be affected by a regulator, who may try to boost liquidity/reduce fire sale costs by buying illiquid assets such as asset-backed securities. Both the bank and the regulator face an unexpected trade-off; if there are many informed depositors, increasing liquidity can amplify the probability of a run. For the bank, this may

[19] This mechanism is similar to the effect of asset encumbrance, as discussed in Ahnert *et al.* (2018). Not pledging assets to secured debt claims reduces the risk of the unsecured debt claims and, thus, decreases the incidence of an unsecured debt run.

increase its incentives to invest in illiquid assets. For the regulator, this may increase its hesitation to intervene.

3 Surplus

In this section, we study surplus in the model.[20] We look at a simple version of surplus: (i) unlike, for example, Diamond and Dybvig (1983), there is no consumption benefit from early withdrawal in this model; (ii) we look at the surplus from our main model and do not include the benefits from information in disciplining the bank manager (which we model in the Internet Appendix).

In the absence of any early withdrawal, the highest surplus level that can be reached is the first best (FB):

$$S^{FB} = pR_H + (1-p)R_L.$$

However, this is not achievable due to the strategic complementarity of depositors' withdrawal decisions and information externalities.

Now consider the case of full information (FI), where the quality of the bank's asset is known to all depositors, and depositors make their own decisions on when to withdraw. In this case, when the asset quality is high, all depositors will wait until the asset matures ($t = 3$). For the low-quality asset, all depositors will have a dominant strategy to withdraw immediately. This yields a surplus of:

$$S^{FI} = pR_H + (1-p)\lambda R_L.$$

This is below the first best due to the liquidation of the low-quality asset.

3.1 No information

We now analyze the surplus in a static benchmark where (i) there are no informed depositors, and (ii) depositors must choose whether to withdraw or wait without having the opportunity to decide later based on observing better information. This

[20]For ease of reading, we do not take into account the investment 1 of depositors needed to produce the asset's returns.

eliminates period $t = 2$ from the timeline.[21] We call this the no information (NI) benchmark.

The unique equilibrium is again defined as a threshold strategy, in which all depositors withdraw whenever they receive a signal below the threshold and wait until the asset matures otherwise. The threshold is implicitly defined by the depositor who is indifferent between withdrawing and waiting based on the received signal. The indifference condition is similar to Equation 6, with two differences: First, the minimum number of withdrawals is zero, as there are no informed depositors with dominant strategies to withdraw (when the asset quality is low). Therefore, the uninformed depositors have no fear of missing out. Second, the expected return from waiting if the bank is not liquidated is higher. This is because there is no learning that the asset quality is low and, hence, no run. There is no real option of learning, eliminating panic runs and forced liquidation, which increases the surplus. For an arbitrarily small noise term, the new indifference condition implicitly defines the benchmark threshold value $p(\theta^{NI})$:

$$\int_0^{\frac{\lambda R_L}{D}} \{p(\theta^{NI})R_H + (1-p(\theta^{NI}))\frac{R_L - n\frac{D}{\lambda}}{(1-n)} - D\}dn + \int_{\frac{\lambda R_L}{D}}^1 \{p(\theta^{NI})R_H - p(\theta^{NI})D - (1-p(\theta^{NI}))\frac{R_L\lambda}{n}\}dn = 0. \tag{11}$$

We can explicitly solve for $p(\theta^{NI})$:

$$p(\theta^{NI})_{|\varepsilon\to 0} = \frac{\phi^{NI}}{\phi^{NI} + \lambda(\frac{R_H}{D} - 1)} \tag{12}$$

with $\phi^{NI} := -\lambda\frac{\lambda R_L}{D}\ln\left(\frac{\lambda R_L}{D}\right) - (1-\frac{\lambda R_L}{D})\ln\left(1-\frac{\lambda R_L}{D}\right) - (1-\lambda)\frac{\lambda R_L}{D}$.

The surplus created in the no information benchmark, therefore, consists of two parts. First, when a preemptive run occurs - i.e., if the fundamental realization is low ($\theta \leq \theta^{NI}$) - all depositors withdraw and force the bank to liquidate its asset. If the actual asset quality is high, the depositors receive an aggregate amount of D and the bank has to liquidate a fraction $\frac{D}{\lambda R_H}$ of the asset to serve the liabilities.

[21] Note that allowing for period $t = 2$ here complicates matters even if there is no new information in period $t = 2$. This is because there could be a self-fulfilling run in the second period (in addition to the possibility of the run in the first period), and there will not be a unique equilibrium in the second period without further modifications. Thus, the benchmark we examine is more comparable to the main model.

The bank remains liquid, and the remaining part of the asset produces $(1-\frac{D}{\lambda R_H})R_H$ when the asset matures. If, however, the asset quality is low, the bank cannot pay back the withdrawn deposits in full, and the bank is forced into full liquidation. The depositors receive λR_L. The surplus in case of a low fundamental realization can be summarized as:

$$\int_0^{\theta^{NI}} \{p(\theta)\left(R_H(1-\frac{D}{\lambda R_H})+D\right)+(1-p(\theta))\lambda R_L\}d\theta. \tag{13}$$

Second, when the fundamental realization is high ($\theta > \theta^{NI}$), no preemptive run occurs, and the bank does not have to liquidate any fraction of the asset prematurely. The surplus depends only on the realization of the asset quality:

$$\int_{\theta^{NI}}^1 \{p(\theta)R_H+(1-p(\theta))R_L\}d\theta. \tag{14}$$

The expected surplus of the no information benchmark can, therefore, be summarized as the sum of equations 13 and 14:

$$S^{NI}=\underbrace{pR_H+(1-p)\lambda R_L}_{S^{FI}}+\underbrace{(1-\lambda)R_L\int_{\theta^{NI}}^1(1-p(\theta))\,d\theta}_{\text{Gain from no run on } R_L}-\underbrace{(1-\lambda)\frac{D}{\lambda}\int_0^{\theta^{NI}}p(\theta)\,d\theta}_{\text{Loss from run on } R_H}.$$

The expected surplus consists of three parts: the full information surplus; a gain compared to the full information surplus, as the low-quality asset is not liquidated if the fundamentals indicate a good economic environment ($\theta > \theta^{NI}$); and a loss compared to the full information surplus from the liquidation of the high-quality asset if the fundamental indicates a relatively bad economic environment ($\theta < \theta^{NI}$).

3.2 Main model: A proportion π of informed depositors

We now consider the surplus in our main model.

We begin by demonstrating that the threshold in the main model is higher (a run is more likely) than when there is no information.

Proposition 4 *The likelihood of a preemptive run on the bank in $t=1$ in the main model is strictly larger than the likelihood in the no information case: $p(\theta^*(\pi)) > p(\theta^{NI})$ for all π.*

The run threshold of the no information game is strictly smaller than the run threshold in the main model. We prove this by first demonstrating that as $\pi \to 0$,

$p(\theta^*(0)) > p(\theta^{NI})$. Given that $\theta^*(\cdot)$ is increasing in π, the proposition is complete. Note that in the first step, $p(\theta^*(0)) > p(\theta^{NI})$, neither model has any informed depositors withdrawing early. The option to wait for better information, even in the absence of informed depositors, increases the probability of inefficient preemptive runs. By allowing depositors to delay their withdrawal decision, the depositors can learn that the asset has a low-quality; there is then a strict incentive to withdraw at $t = 2$, liquidating the asset inefficiently. This destroys aggregate surplus from waiting in expectation, making preemptive runs more likely at $t = 1$.

We now derive the surplus.

If the fundamental is low ($\theta < \theta^*(\pi)$), a preemptive run forces the liquidation of a proportion $\frac{(1-\pi)D}{\lambda R_H}$ of the high-quality asset to serve the uninformed depositors' withdrawals. The low-quality asset is liquidated entirely since both uninformed and informed depositors withdraw. The ex ante surplus created by low fundamental values can, therefore, be summarized by:

$$\int_0^{\theta^*(\pi)} \{p(\theta)\left(R_H(1 - \frac{(1-\pi)D}{\lambda R_H}) + (1-\pi)D\right) + (1-p(\theta))\lambda R_L\}d\theta. \tag{15}$$

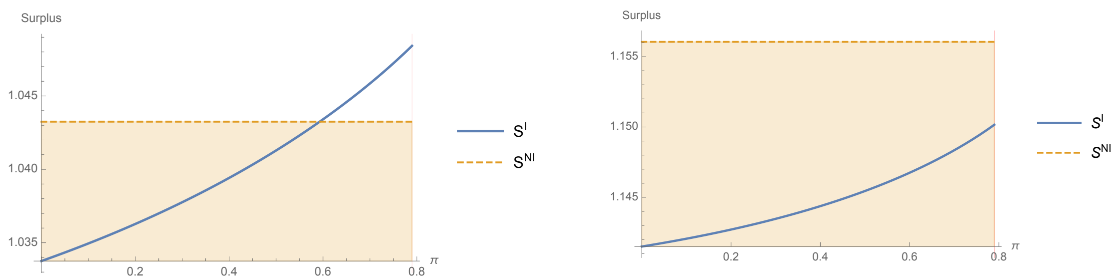
Similarly, we can derive the surplus created for high fundamental values $\theta > \theta^*(\pi)$. For high fundamentals, no preemptive runs by uninformed depositors occur. If the bank has a high-quality asset, no liquidation of the asset is forced. If the bank's asset is low-quality, informed depositors force a partial asset liquidation at $t = 1$, and, subsequently, uninformed depositors deduce that the quality of the asset is low and force the full liquidation of any remaining fraction of the asset. The ex ante value created by high fundamental values can, therefore, be summarized by:

$$\int_{\theta^*(\pi)}^1 \{p(\theta)R_H + (1-p(\theta))\lambda R_L\}d\theta. \tag{16}$$

The surplus is the sum of Equation 15 and 16. Simplifying yields:

$$S^I = \underbrace{pR_H + (1-p)\lambda R_L}_{S^{FI}} \underbrace{-(1-\lambda)\frac{D}{\lambda}\int_0^{\theta^*(\pi)} p(\theta)\,d\theta}_{\text{Loss from run on } R_H} \underbrace{+\pi(1-\lambda)\frac{D}{\lambda}\int_0^{\theta^*(\pi)} p(\theta)\,d\theta}_{\text{Fewer uninformed withdrawals}}. \tag{17}$$

Compared to the full-information benchmark, the surplus is reduced by the cost of the preemptive liquidation of the high-quality asset, forced by the uninformed depositors' withdrawals of $(1-\pi)D$ whenever the economic environment is bad



(a) Ambiguous effect. The parameters used are $D = 1, R_H = 1.3, R_L = 0.9$, and $\lambda = 0.9$.

(b) Learning harms surplus. The parameters used are $D = 1, R_H = 1.5, R_L = 0.9$, and $\lambda = 0.9$.

Figure 4: The Surplus Effect of Information

$(\theta < \theta^*(\pi))$: for every unit withdrawn, one unit of the future value is destroyed, creating only $\lambda < 1$ units of value. This effect is mechanically smaller when there are fewer uninformed depositors; the third term in S^I reflects this.

Compared to the no information benchmark, the total effect of information is ambiguous:

$$S^I - S^{NI} = \tag{18}$$
$$\underbrace{-(1-\lambda)R_L \int_{\theta^{NI}}^{1} (1-p(\theta))\, d\theta}_{\text{Loss from run on } R_L} \underbrace{- (1-\lambda)\frac{D}{\lambda}\int_{\theta^{NI}}^{\theta^*(\pi)} p(\theta)\, d\theta}_{\text{More preemptive runs}} + \underbrace{\pi(1-\lambda)\frac{D}{\lambda}\int_{0}^{\theta^*(\pi)} p(\theta)\, d\theta.}_{\text{Fewer uninformed withdrawals}}$$

Information creates a loss due to liquidation of the low-quality asset and an increase in the probability of inefficient preemptive runs. However, as informed agents do not liquidate the high-quality asset, the size of an inefficient preemptive run decreases as more depositors become informed.

We can decompose the surplus effect into the option to learn about the return at $t = 2$ and the fear-of-missing-out effect.

$$S^I - S^{NI} = \underbrace{\pi(1-\lambda)\frac{D}{\lambda}\int_{0}^{\theta^*(\pi)} p(\theta)\, d\theta}_{\text{Fewer uninformed withdrawals}} \tag{19}$$
$$\underbrace{-(1-\lambda)R_L \int_{\theta^{NI}}^{1} (1-p(\theta))\, d\theta - (1-\lambda)\frac{D}{\lambda}\int_{\theta^{NI}}^{\theta^*(0)} p(\theta)\, d\theta}_{\text{Option to learn}} \underbrace{- (1-\lambda)\frac{D}{\lambda}\int_{\theta^*(0)}^{\theta^*(\pi)} p(\theta)\, d\theta.}_{\text{Amplification: Fear of missing out}}$$

The option to learn unambiguously decreases surplus; it (i) creates inefficient (though fundamental) runs on the low-quality asset and (ii) increases the incidence of preemptive runs and, thus, partial liquidation of the high-quality asset. This effect may have been present in the slow bank run in Greece - Artavanis *et al.* (2019) document strategic withdrawals before the resolution of a 2015 Greek election that had the potential to affect the value of deposits. Furthermore, it tells us that empirical studies need to carefully consider timing and causality in bank runs; here, runs precede the revelation of information, although they do so exactly because it is known that information will be revealed.[22]

The preemptive runs are amplified by the fear of uninformed agents that they will miss out on their chance to withdraw cash before it has run out.

There is a positive side to having more informed depositors. In the first term, we can see that the liquidation of the high-quality asset in a preemptive panic run is less likely.

An increase in the proportion of informed depositors has two effects on the surplus in our model. First, more informed depositors increase the incidence of inefficient preemptive runs. This is a negative effect. Second, the more depositors that are informed, the fewer uninformed depositors there are to preemptively liquidate their high-quality asset if fundamentals are low ($\theta < \theta^*$). This is a positive effect.

The ambiguous effect of learning on surplus is illustrated in Figure 4a. For low π, there is higher surplus when no depositors are informed, while for high π, the reverse is true. Figure 4b illustrates a parameter space in which learning always decreases surplus. If only a few depositors are informed ($\pi \to 0$), the possibility to learn from the withdrawals of the informed unambiguously decreases surplus relative to the no information benchmark. As π becomes large, this gap diminishes. In Figure 4b, the gap does not diminish enough, making learning harmful, while in Figure 4a, learning is superior for larger values of π.

4 Application to Stress Tests

The case in which $\pi \to 0$ and only the real option effect is present is equivalent to a model in which a fully informative public signal is released at $t = 2$ and there

[22] Chen *et al.* (2020) document a significant positive relation between the informativeness of bank earnings and the sensitivity of uninsured deposit flows to bank performance. They do not look at causality, but from our results, any such attempt would need to carefully address the timing and strategic reactions to information.

are no informed depositors. The results for this model are of interest to the debate over the effect of information released by regulators, as it resembles stress testing . There is a recent theoretical literature (Bouvard *et al.* (2015), Goldstein and Leitner (2018), Williams (2017), Orlov *et al.* (2020), and Faria-e Castro *et al.* (2016)) on the information design of stress tests that broadly have the stress test first and depositor reaction to the test second. While we clearly do not examine the optimal way to release information, we point out here that there will be an adverse *ex ante* reaction to the presence of a stress test - runs may occur in anticipation of the test. This is potentially an explanation of why the results of the microprudential version of stress tests, banking supervision exams, are kept private.

Moreover, this implies that there should be a market reaction to the announcement that a stress test will be conducted.[23] In the U.S., this could be examined by looking at the announcements of the first U.S. stress test (the SCAP) and the subsequent stress testing regimes (CCAR and DFAST). However, the announcement may also contain information about future regulatory approaches towards banks.

5 Endogenous Information Acquisition

We now allow depositors to decide ex-ante, before they receive the noisy signal, if they want to become fully informed at a cost. The cost to become informed is assumed to be heterogeneous; we think of this as different levels of resources required to acquire and process information or different levels of financial sophistication. Some sophisticated depositors can become informed at negligible costs, while others have to exert a prohibitively costly effort to gather all necessary information. We assume that depositors draw an information cost that is uniformly distributed over a unit interval $c_i \sim U[0,1]$.[24]

Every depositor i initially has to decide if he/she wants to become informed ($z_i = I$) at an individual cost c_i or remain uninformed ($z_i = U$). The individual

[23] The empirical literature (e.g., Morgan *et al.* (2014)) focuses mainly on event studies examining the reaction to the stress test itself.

[24] Note that we have a discontinuity in the payoff function at the point where nobody becomes informed: if nobody is informed, there would be no information to infer, and the gain from waiting would be the expected return of the asset and not the conditional return. However, there is always an information benefit, such that the investor with zero cost will always acquire information; this results in a strictly positive proportion of informed investors ($\pi > 0$) in equilibrium.

choices aggregate to the proportion of informed depositors

$$\pi^* = \int_0^1 \mathbb{1}\{z_i^* = I\} di$$

that is consistent with the optimal individual information choice of each depositor i:

$$z_i^* = \arg \max_{z_i \in \{I,U\}} \mathbb{1}\{z_i = I\}[EU^I(\pi^*) - c_i] + \mathbb{1}\{z_i = U\} EU^U(\pi^*),$$

where $EU^I(\pi^*)$ and $EU^U(\pi^*)$ are the expected utilities of the depositors who choose to become informed or stay uninformed (before they receive their noisy signal θ_i about the fundamental) at period 0, respectively.[25]

The ex-ante expected utility of a depositor that chooses to become informed when $\pi^* \leq \hat{\pi}$ is:[26]

$$EU^I(\pi) = \int_0^{\theta^*(\pi)} (p(\theta)R_H + (1 - p(\theta))\lambda R_l\,) d\theta + \int_{\theta^*(\pi)}^1 (p(\theta)R_H + (1 - p(\theta))D)\, d\theta.$$

The perfectly informed depositor has a dominant strategy of waiting until the asset matures when observing the high asset quality and to withdraw otherwise. This strategy is independent of the strategy of the uninformed depositors. However, the depositor's payoff *is* affected by the strategy of uninformed depositors: with probability $P[\theta < \theta^*(\pi)]$, the bank is illiquid such that the entire asset is liquidated if the asset is low-quality, and the liquidation value is shared among all depositors. As a result, the expected payoff of an informed depositor is lower the more likely a preemptive run becomes - i.e., the more depositors become informed.

Similarly, the ex-ante expected utility of a depositor that chooses not to invest c_i to become informed when $\pi^* \leq \hat{\pi}$ is:

$$EU^U(\pi) = \int_0^{\theta^*(\pi)} (p(\theta)D + (1 - p(\theta))\lambda R_L\,) d\,\theta + \int_{\theta^*(\pi)}^1 (p(\theta)R_H + (1 - p(\theta))\frac{\lambda R_L - \pi D}{1 - \pi})\, d\,\theta.$$

[25] For the characterization of the endogenous information choice, we use notation similar to that of Ahnert and Kakhbod (2017).

[26] If $\pi^* \in (\hat{\pi}, \frac{\lambda R_L}{D})$, instead of a unique equilibrium with switching strategies, multiple equilibria exist. If $\pi^* \geq \frac{\lambda R_L}{D}$, the bank is forced to liquidate the entire low-quality asset at $t = 1$ because so many informed investors withdraw. Given Assumption 3, withdrawing at $t = 1$ is a dominant strategy for uninformed investors in this case.

The value of becoming informed is defined by the difference in expected utilities; we denote this as the gain from information $\Delta(\pi) = EU^I(\pi) - EU^U(\pi)$, which can be summarized as:

$$\Delta(\pi) = \underbrace{\int_0^{\theta^*(\pi)} p(\theta)(R_H - D)\, d\theta}_{\text{Gain from remaining invested in } H \text{ despite run}} + \underbrace{\int_{\theta^*(\pi)}^1 (1-p(\theta))\left(\frac{D-\lambda R_L}{1-\pi}\right) d\theta.}_{\text{Gain from withdrawing when } L \text{ and no run}} \tag{20}$$

The information gain consists of two parts. The first part reflects the fact that an informed depositor does not withdraw from a bank that has a high-quality asset, even though there is a run. The second part reflects the fact that an informed depositor withdraws when the bank has a low-quality asset and there is no preemptive run. The information benefit is to withdraw before uninformed depositors move and, therefore, gain the difference between D and the liquidation value of the low-quality asset.

Lemma 8 *The gain $\Delta(\pi)$ from learning the actual asset quality at $t = 1$ is positive and increasing in π for all feasible $\pi \in (0, \hat{\pi})$.*

Recalling from Assumption 1 that $R_L < D < \lambda R_H$, it is clear that $\Delta(\cdot)$ must be positive. An increase in π has a direct positive effect on the gain to being informed when the asset is low-quality in a good economic environment (θ high) as the depositor can withdraw early: as more agents are informed, the term in the second integral increases. This direct effect results from the fact that more informed depositors withdrawing before the uninformed depositors withdraw decreases the expected utility of the uninformed but does not directly affect the expected utility of the individual informed depositors. Next, a higher proportion of informed agents increases the critical threshold for preemptive runs. This affects the endpoints of both integrals, but in opposite ways: as a preemptive run becomes more likely, the information gain from not withdrawing from a bank with a high-quality asset, despite a preemptive run, becomes more important than the gain of withdrawing from a bank with a low-quality asset when no preemptive run takes place. We show in the Appendix that this effect is also positive, as the increased gain from not liquidating a high-quality asset outweighs the reduced gain from withdrawing when the asset is low-quality. In other words, the information gain increases when there is a higher likelihood of preemptive runs.

If a unique equilibrium exists, it is defined by a threshold information cost $\bar{c}$ that is implicitly defined by the depositor i that is indifferent between becoming

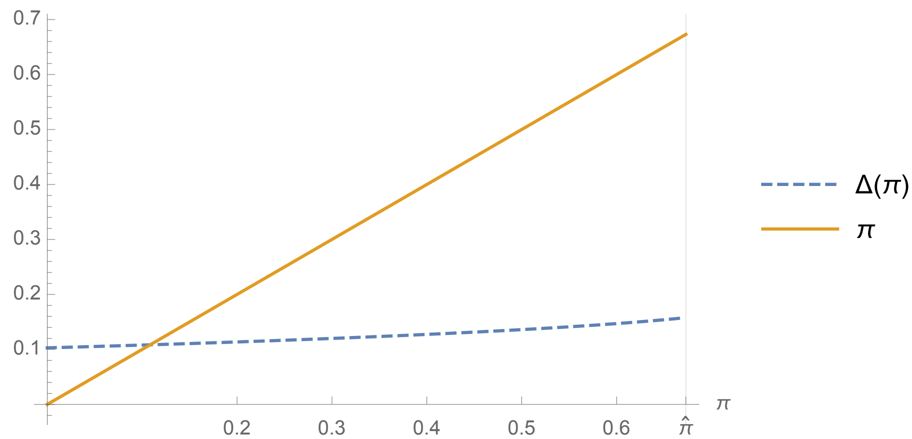


Figure 5: Endogenous Information Choice.
The parameters used are $D = 1, R_H = 1.5, R_L = 0.9$, and $\lambda = 0.8$.

informed or not. All depositors with a lower cost than $\bar{c}$ choose to get informed, and all depositors with a higher cost choose not to acquire information. For uniformly distributed information costs, this threshold value in equilibrium must equal to the proportion of informed depositors: $\pi^* = \bar{c}$. The equilibrium proportion of informed depositors must, therefore, solve: $\Delta(\pi^*) = \pi^*$.

We now demonstrate under what conditions such a fixed point may exist.

Lemma 9 *If $\Delta(\hat{\pi}) < \hat{\pi}$, there exists a fixed point solving $\Delta(\pi^*) = \pi^*$ in $\pi \in (0, \hat{\pi})$.*

The condition for existence $\Delta(\hat{\pi}) < \hat{\pi}$ implies that the expected information gain if many depositors choose to become informed - i.e., $\pi \to \hat{\pi}$ - is lower than the information acquisition cost of the marginal informed depositor. This is depicted in Figure 3. There may be multiple intersections. A condition guaranteeing a unique intersection is $\frac{\partial \Delta(\pi)}{\partial \pi} < 1$ for all π.

The surplus created by a given number π of informed depositors is S^I. A social planner also considers the aggregate cost of becoming informed $\int_0^\pi c\, dc$ and will therefore choose:

$$\pi^S = \arg\max S^I(\pi) - \int_0^\pi c\, dc. \tag{21}$$

If a unique interior solution $\pi^S \in (0, \hat{\pi})$ exists, the surplus-maximizing proportion of informed depositors is implicitly defined by $\frac{dS^I}{d\pi} |_{\pi^S} = \pi^S$.

Proposition 5 *Assuming that there exists a unique interior proportion of informed depositors that invest in information and a unique interior proportion of informed depositors that maximize surplus (π^S), depositors choose to overinvest in information relative to the surplus-maximizing information choice.*

The information benefit for individual depositors is strictly higher than the information benefit for the surplus. The individual benefit has two components: a gain from not liquidating the high-quality asset during a preemptive run and the first-mover advantage of withdrawing from the low-quality asset first. This first-mover advantage does not affect surplus and is, therefore, not taken into account by a surplus maximizer. Moreover, depositors do not consider the effect of their decision to become informed on the decisions of the other depositors to run preemptively on the bank. This negative externality of more preemptive runs reduces surplus.

6 Conclusion

Many bank runs are characterized by heterogeneously informed agents and information transmission. We demonstrate that depositor learning exacerbates panic-based bank runs. The (real) option to learn from previous withdrawals leads to costly liquidation in bad states, which increases the payoff of running ex-ante. And when a fraction of depositors learn the bank's asset quality early, the remaining depositors have a fear of missing out, which also makes preemptive runs more likely. More information may, thus, lead to more panic runs, and surplus may be non-monotonic in the amount of information available.

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A Appendix

Proof of Lemma 1 (Decision of informed depositors when asset is high-quality)

Denote the overall withdrawal before the asset matures by $x \in [0,1]$. Knowing that the high asset quality has realized, an informed depositor would have an incentive to withdraw whenever $D > \min\left[\frac{R_H - x\frac{D}{\lambda}}{(1-x)}, R_H\right]$ or $R_H - D < x\left(\frac{D}{\lambda} - D\right)$. The right-hand side is increasing in x. The highest feasible value is $x = 1$, resulting in the requirement $\lambda\, R_H < D$, which contradicts Assumption 1.

Proof of Lemma 2 (Decision of informed depositors when asset is low-quality)

Denote $E[N_1]$ as a depositor's expectation about how many other depositors withdraw. If an informed depositor expects $E[N_1] > \frac{\lambda R_L}{D}$ (Case (3) in Table 1), it follows that the bank is forced into full liquidation at $t = 1$. Waiting yields a zero return and is, therefore, strictly dominated by immediate withdrawal. If $E[N_1] < \frac{\lambda R_L}{D}$, the informed depositor knows that immediate withdrawal yields D (neglecting the externality on the bank's solvency). From Lemma 3 cases (1) and (2), the depositor forms expectations that waiting yields $\min\left[D, \frac{\lambda R_L - E[N_1]D}{(1-E[N_1])}\right]$ and is, therefore, dominated by immediate withdrawal, as an indifferent depositor prefers immediate consumption. As expectations, in equilibrium, have to match the actual withdrawal behavior, it must hold that $E[N_1] = n_1^U + \pi$, and, hence, $n_1^I = \pi$, which implies that $n_2^I = n_3^I = 0$.

Proof of Lemma 3 (Period 2 decision of uninformed depositors)

We begin by examining the inference problem of the uninformed depositors who waited at $t = 1$ and now must decide to withdraw or wait at $t = 2$.

At $t = 2$, the uninformed depositor has two sources of information: the private signal θ_i from $t = 0$ and the new information from observing the number of withdrawals in $t = 1$. The uninformed depositor knows that at $t = 1$, (1) all informed depositors had the dominant strategy to withdraw when the asset quality was low and to wait when the asset quality was high; and (2) given that we posited the existence of a threshold θ', uninformed depositors withdrew if their private signal was $\theta_i \leq \theta^{'}$ and waited if their signal was $\theta_i > \theta^{'}$.

Therefore, the number of withdrawals depended on the realization of the asset quality, the realization of θ, and the private noise ε. If the asset quality is low, given a threshold strategy of uninformed depositors, the number of withdrawals in $t = 1$ becomes a function of θ as summarized in equation (1). Similarly, the number of withdrawals at $t = 1$ if the asset has high-quality is summarized in equation (2).

First, note that if there had been a realization of θ such that $\theta + \varepsilon < \theta'$, then all uninformed depositors would have withdrawn at $t = 1$, and there would be no uninformed depositors left at $t = 2$ to make a withdrawal decision.

Second, note that if there had been a realization of θ such that $\theta - \varepsilon > \theta'$, then all uninformed depositors would have waited at $t = 1$. In this case, if the asset was low-quality, there would have been π withdrawals ($n_1(\theta, \theta', L) = \pi$), and if the asset was high-quality, there would have been no withdrawals ($n_1(\theta, \theta', H) = 0$). Therefore, the asset quality would be inferred perfectly. As in the case of the informed depositor (Lemmas 1 and 2), the uninformed depositor at $t = 2$ would then withdraw if the asset was low-quality and wait if the asset was high-quality.

An inference problem may arise if $\theta' - \varepsilon < \theta < \theta' + \varepsilon$ because the low-quality asset results in the same amount of withdrawals as the high-quality asset.

In particular, for a given number of withdrawals $\tilde{n}_1$ that satisfy $\theta' - \varepsilon < \theta < \theta' + \varepsilon$, there exist fundamentals $\theta_L \in [\theta' - \varepsilon, \theta' + \varepsilon]$ and $\theta_H \in [\theta' - \varepsilon, \theta' + \varepsilon]$ such that $\tilde{n}_1 = n_1(\theta_L, \theta', L) = n_1(\theta_H, \theta', H)$. Solving for the realization θ_L gives:

$$\theta_H = \theta_L - 2\varepsilon \frac{\pi}{1 - \pi}.$$

The withdrawal amounts for the low-quality asset and high-quality asset are parallel decreasing functions of θ. This is depicted in Figure 6. The distance between the

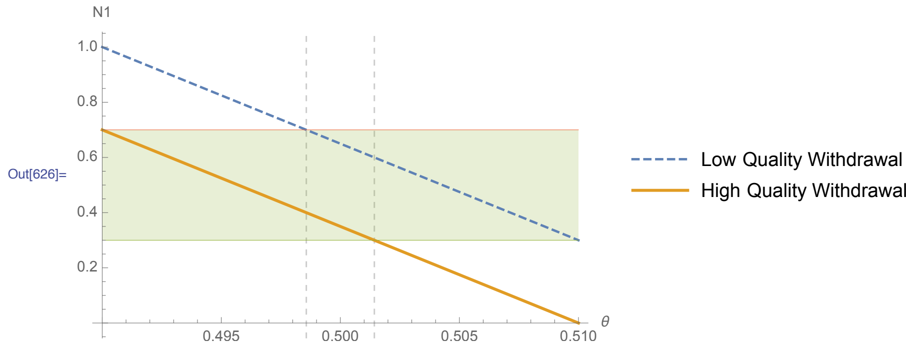


Figure 6: Number of withdrawals for the low asset quality and high asset quality as a function of the fundamental θ with an exogenous threshold $\theta' = 0.5$, $\pi = 0.3$ and $\varepsilon = 0.01$

two realizations that result in the same withdrawal amount is a constant: $2\varepsilon\frac{\pi}{1-\pi}$.

Our first result is that if $\pi \geq 1/2$, the inference problem vanishes entirely. When $\pi > 1/2$, the difference between θ_L and θ_H is $2\varepsilon\frac{\pi}{1-\pi} > 2\varepsilon$. As this is larger than 2ε, it cannot be the case that both θ_L and θ_H are in the interval $[\theta' - \varepsilon, \theta' + \varepsilon]$. The uninformed depositor can then back out the quality of the asset from the number of withdrawals (and will have a dominant strategy of withdrawing if the asset quality is L and waiting if the asset quality is H).

Now consider the case in which $\pi < 1/2$.

The fundamental θ_H must be in the interval $(\theta' - \varepsilon, \theta' + \varepsilon - \frac{2\varepsilon\pi}{1-\pi})$, where the term $\theta' + \varepsilon - \frac{2\varepsilon\pi}{1-\pi}$ is derived by setting $n_1(\theta_H, \theta', H)$ equal to π (see Figure 6 for an illustration). The fundamental θ_L must then lie in the interval $(\theta' - \varepsilon + \frac{2\varepsilon\pi}{1-\pi}, \theta' + \varepsilon)$, where the term $\theta' - \varepsilon + \frac{2\varepsilon\pi}{1-\pi}$ is derived by setting $n_1(\theta_L, \theta', L)$ equal to $1 - \pi$.

Before observing the number of withdrawals at $t = 1$, uninformed depositor i with signal θ_i believed that θ was uniformly distributed over the interval $[\theta_i - \varepsilon, \theta_i + \varepsilon]$. We note that the only uninformed depositors remaining to make a decision at $t = 2$ are depositors with signals $\theta_i > \theta'$, as the others would have withdrawn at $t = 1$. This implies that if $\theta_H + \varepsilon < \theta_i$, there is once again no inference problem, as the depositor will learn that the quality of the asset is L. If $\theta_i < \theta_H + \varepsilon$, then the probability that the fundamental is θ_H is equal to the probability that the fundamental is θ_L. Therefore, depositor i's best estimate of θ is equal to $\frac{1}{2}\theta_H + \frac{1}{2}\theta_L$.

To summarize, when $\theta \in [\theta' - \varepsilon, \theta' + \varepsilon]$, a given number of withdrawals $\tilde{n}_1$ can arise only from two possible draws of θ, θ_H and θ_L such that $\tilde{n}_1 = n_1(\theta_L, \theta', L) =$

$n_1(\theta_H, \theta', H)$. For uninformed depositors with a signal in the interval $(\theta', \theta_H + \varepsilon)$, the best estimate of θ is equal to $\frac{1}{2}\theta_H + \frac{1}{2}\theta_L$. Their best estimate of the probability that the asset is high-quality is, therefore, $P(\theta_L) = \frac{1}{2}p(\theta_L - 2\varepsilon\frac{\pi}{1-\pi}) + \frac{1}{2}p(\theta_L)$.

We denote the probability that uninformed depositors have a signal above $\theta_H + \varepsilon$ (which we will write as $\theta_L + \varepsilon\frac{1-3\pi}{1-\pi}$) as $1 - F(\theta_L + \varepsilon\frac{1-3\pi}{1-\pi})$, where $F(\cdot)$ is a cdf that we will elaborate on below. We denote the probability that uninformed depositors have a signal in the interval $(\theta', \theta_L + \varepsilon\frac{1-3\pi}{1-\pi})$ as $F(\theta_L + \varepsilon\frac{1-3\pi}{1-\pi}) - F(\theta')$.

We make two observations. First, if $\theta_L + \varepsilon\frac{1-3\pi}{1-\pi} < \theta'$, all uninformed depositors have inferred the true asset quality. Second, the only situation in which we will need to use the cdf $F(\cdot)$ is conditioning on the fundamental being L (because that is the only state in which the number of depositors that withdraw matters to payoffs). In that case, the true fundamental is θ_L, and uninformed depositors have signals uniformly distributed over $[\theta_L - \varepsilon, \theta_L + \varepsilon]$ such that we can write $1 - F(\theta_L + \varepsilon\frac{1-3\pi}{1-\pi})$ as $\frac{\theta_L + \varepsilon - (\theta_L + \varepsilon\frac{1-3\pi}{1-\pi})}{2\varepsilon}$, which simplifies to $\frac{\pi}{1-\pi}$. Hence, an uninformed depositor with a signal that does not allow him to infer the true state expects that a proportion of $\frac{\pi}{1-\pi}$ other remaining uninformed depositors receive a signal above $\theta_H + \varepsilon$. Therefore, the uninformed depositor expects that $(1 - n_1(\theta_L, \theta', L))\frac{\pi}{1-\pi}$ withdraw at $t = 2$ if the asset quality is low because they received a signal above $\theta_H + \varepsilon$.

The expected payoff of uninformed depositors with a signal in the interval $(\theta', \theta_L + \varepsilon\frac{1-3\pi}{1-\pi})$ from withdrawing at $t = 2$ is:

$$P(\theta_L)D + (1 - P(\theta_L)) \min \left[D, \frac{\lambda R_L - n_1(\theta_L, \theta', L)D}{(1 - n_1(\theta_L, \theta', L))\frac{\pi}{1-\pi} + n_2} \right],$$

where n_2 indicates the expected number of other uninformed depositors with a signal in the interval $(\theta', \theta_L + \varepsilon\frac{1-3\pi}{1-\pi})$ who will withdraw.

And the expected payoff of waiting is:

$$P(\theta_L)R_H + (1 - P(\theta_L)) \max \left[\frac{\lambda R_L - (n_1(\theta_L, \theta', L) + (1 - n_1(\theta_L, \theta', L))\frac{\pi}{1-\pi} + n_2)D}{1 - (n_1(\theta_L, \theta', L) + (1 - n_1(\theta_L, \theta', L))\frac{\pi}{1-\pi} + n_2)}, 0 \right].$$

The uninformed depositor who has not learned the asset quality and is thinking about waiting must consider (i) the number of withdrawals from $t = 1$ depositors $(n_1(\theta_L, \theta', L))$ and $t = 2$ depositors who have signals that allow them to infer that the asset quality is low $((1 - n_1(\theta_L, \theta', L))\frac{\pi}{1-\pi})$; and (ii) the expected number of other uninformed depositors who have not learned the asset quality that will withdraw. For now, we define the first amount (point (i)):

$$\begin{aligned} & n_1(\theta_L, \theta', L) + (1 - n_1(\theta_L, \theta', L))\frac{\pi}{1-\pi} && \text{(A22)} \\ = \;& \pi + (1-\pi)\left(\frac{1}{2} + \frac{\theta' - \theta_L}{2\varepsilon}\right) + (1 - (\pi + (1-\pi)\left(\frac{1}{2} + \frac{\theta' - \theta_L}{2\varepsilon}\right)))\frac{\pi}{1-\pi} \\ = \;& \frac{1}{2} + \pi + (1 - 2\pi)\left(\frac{\theta' - \theta_L}{2\varepsilon}\right) \end{aligned}$$

The largest possible θ_L sets $\theta_L + \varepsilon\frac{1-3\pi}{1-\pi} = \theta' + \varepsilon$, which, after rewriting, is $\theta_L = \theta' + \varepsilon\frac{2\pi}{1-\pi}$. So, the lowest value that the number of withdrawals can take in equation (A22) is:

$$\frac{1}{2} + \frac{\pi^2}{1-\pi}.$$

So, no matter how many informed depositors there are (as long as there are not zero), the minimum number of withdrawals will be $\frac{1}{2}$ when the asset quality is L, as many of the uninformed depositors will infer the asset quality from the withdrawals at $t = 1$.

Notice, also, that using the largest possible θ_L maximizes $P(\theta_L)$.

If we assume that the expected payoff from waiting is lower than the expected payoff from withdrawing at (i) the largest possible θ_L ($\theta' + \varepsilon\frac{2\pi}{1-\pi}$); (ii) the lowest possible number of $t = 1$ withdrawals plus $t = 2$ withdrawals from uninformed depositors who have inferred the state ($\frac{1}{2}$); and (iii) the lowest possible number of uninformed depositors who have not inferred the state at $t = 2$ withdrawing (zero), we get:

$$P(\theta' + \varepsilon\frac{2\pi}{1-\pi})R_H + (1 - P(\theta' + \varepsilon\frac{2\pi}{1-\pi}))\frac{\lambda R_L - \frac{1}{2}D}{\frac{1}{2}} < D. \qquad \text{(A23)}$$

Lastly, we will demonstrate in Lemma 6 that the threshold θ' must be lower than the cutoff for the upper dominance region $\bar{\theta} = p^{-1}\left(\frac{D}{R_H}\right)$. Therefore, we can modify the condition in Equation A23 to be:

$$P(p^{-1}\left(\frac{D}{R_H}\right) + \varepsilon\frac{2\pi}{1-\pi})R_H + (1 - P(p^{-1}\left(\frac{D}{R_H}\right) + \varepsilon\frac{2\pi}{1-\pi}))\frac{\lambda R_L - \frac{1}{2}D}{\frac{1}{2}} < D.$$

This implies that all remaining uninformed depositors at $t = 2$ who did not infer the asset quality withdraw. We label this condition as Assumption 4 in the text.

Derivation of payoff for uninformed depositors at $t = 1$ waiting until $t = 2$:

Conditional on the asset quality being L, Assumption 4 and the above analysis state that all remaining depositors will withdraw at $t = 2$.

Conditional on the asset quality being H, the above analysis points out several facts for depositor i's decision. If depositor i were to wait at $t = 2$, her payoff would be R_H. If depositor i were to withdraw at $t = 2$, her payoff would be D. Firstly, if $\theta < \theta' - \varepsilon$ or if $\theta > \theta' + \varepsilon$, depositor i would learn the state perfectly at $t = 2$ and wait. Secondly, if $\theta \in [\theta' - \varepsilon, \theta' + \varepsilon]$, then depositor i would be uncertain about the state, and, from Assumption 4, would withdraw.

We denote the probability that $\theta \in [\theta' - \varepsilon, \theta' + \varepsilon]$ given that i's signal is θ_i, as $\Pr(\theta' - \varepsilon < \theta < \theta' + \varepsilon|\theta_i)$. We could write this out explicitly given the assumptions that θ and ε_i are uniformly distributed, but there is no need to since, when we take ε to zero, this probability must be approaching zero as well.

Therefore, an uninformed depositor with signal θ_i can form expectations about her payoff from waiting:

$$E[p(\theta_i)][1 - \Pr(\theta' - \varepsilon < \theta < \theta' + \varepsilon|\theta_i)]R_H + \Pr(\theta' - \varepsilon < \theta < \theta' + \varepsilon|\theta_i)D + (1 - E[p(\theta_i)]) \max\left[\frac{\lambda R_L - N_1(L)D}{1 - N_1(L)}, 0\right]$$

Demonstrating that Assumption 4 does not affect the results:

One might wonder whether the results of the paper depend on Assumption 4. We now demonstrate that they do not.

Consider the opposite assumption (call it Assumption 4'): all uninformed depositors who do not learn the state at $t = 2$ wait until $t = 3$ to withdraw.

Let us derive the payoff of an uninformed depositor at $t = 1$ who decides to wait until $t = 2$.

Now, conditional on the asset quality being H and given Assumption 4', the uninformed depositor will either learn the true state (in which case the depositor will wait) or be uncertain (in which case the depositor will wait).

Conditional on the asset quality being L, if depositor i were to learn the true state, she would withdraw. If depositor i were uncertain about the state, Assumption 4' states that she would wait.

Once again, we denote the probability that $\theta \in [\theta' - \varepsilon, \theta' + \varepsilon]$ given that i's signal is θ_i, as $q(\theta_i, \theta', \varepsilon) = \Pr(\theta' - \varepsilon < \theta < \theta' + \varepsilon|\theta_i)$.

Therefore, an uninformed depositor with signal θ_i can form expectations about her payoff from waiting:

$$\begin{aligned}
& E[p(\theta_i)]R_H + (1-E[p(\theta_i)])\{[1-q(\theta_i,\theta',\varepsilon)]\max\left[\frac{\lambda R_L - N_1(L)D}{[1-q(\theta_i,\theta',\varepsilon)](1-N_1(L))},0\right] \\
& +q(\theta_i,\theta',\varepsilon)\max\left[\frac{\lambda R_L - (N_1(L)+[1-\Pr(\theta'-\varepsilon<\theta<\theta'+\varepsilon|\theta_i)](1-N_1(L)))D}{q(\theta_i,\theta',\varepsilon)(1-N_1(L))},0\right]\} \\
= \; & E[p(\theta_i)]R_H + (1-E[p(\theta_i)])\{\max\left[\frac{\lambda R_L - N_1(L)D}{(1-N_1(L))},0\right] \\
& +\max\left[\frac{\lambda R_L - (N_1(L)+[1-q(\theta_i,\theta',\varepsilon)](1-N_1(L)))D}{(1-N_1(L))},0\right]\}.
\end{aligned}$$

This would replace Equation 4.

We now rewrite the uninformed depositor's utility differential between waiting and immediate withdrawal, where we replace $N_1(L)$ with $n_1(\theta,\theta',L)$:

$$\nu(\theta, n_1(\theta,\theta',L)) =$$
$$\begin{cases}
p(\theta)(R_H-D)+(1-p(\theta))\frac{2(\lambda R_L - N_1(L)D)}{(1-N_1(L))} & \text{if } \frac{\lambda R_L}{q(\theta_i,\theta',\varepsilon)D} - \frac{1-q(\theta_i,\theta',\varepsilon)}{q(\theta_i,\theta',\varepsilon)} > n_1(\theta,\theta',L) \geq \pi \\
-[2-q(\theta_i,\theta',\varepsilon)]D) & \\
p(\theta)(R_H-D) & \text{if } \frac{\lambda R_L}{D} > n_1(\theta,\theta',L) \geq \frac{\lambda R_L}{q(\theta_i,\theta',\varepsilon)D} - \frac{1-q(\theta_i,\theta',\varepsilon)}{q(\theta_i,\theta',\varepsilon)} \\
+(1-p(\theta))(\frac{\lambda R_L - n_1(\theta,\theta',L)D}{(1-n_1(\theta,\theta',L))} - D) & \\
p(\theta)(R_H-D) & \text{if } 1 \geq n_1(\theta,\theta',L) \geq \frac{\lambda R_L}{D} \\
-(1-p(\theta))\frac{\lambda R_L}{n_1(\theta,\theta',L)} &
\end{cases}$$

Notice that the interval $\frac{\lambda R_L}{q(\theta_i,\theta',\varepsilon)D} - \frac{1-q(\theta_i,\theta',\varepsilon)}{q(\theta_i,\theta',\varepsilon)} > n_1(\theta,\theta',L) \geq \pi$ disappears as ε approaches zero. This implies that the key equation that drives our results (equation 6) does not change.

Therefore, Assumption 4 does not affect the analysis, as we study the limiting case. Assumption 4 allows us to write the payoffs clearly so that we can take the limit appropriately.

Proof of Lemma 4 (Threshold when uninformed depositors do not receive private signals)

Here, uninformed depositors do not receive private signals. We define the consumption of an uninformed depositor as $c_t^U(N_t(L),Q)$, where $t\in\{1,2\}$and $Q\in\{H,L\}$.

As in the rest of the text, we define the unconditional ex-ante probability that the asset quality is high by p.

For any realized π, the uninformed depositor has to decide to withdraw immediately or wait. Assumption 2 guarantees that $E[c_2^U(0,Q)] > E[c_1^U(0,Q)]$. If very few depositors withdraw when the asset is low-quality, waiting gives a higher expected payoff than withdrawing early. Both utilities weakly decrease with the number of expected withdrawals. Before the bank becomes illiquid($N_1(L) < \frac{\lambda R_L}{D}$), the utility of waiting $E[c_2^U(\cdot,Q)]$ is continuously decreasing, while $E[c_1^U(\cdot,Q)]$ is a constant until illiquidity. At the point of illiquidity, $E[c_2^U(\frac{\lambda R_L}{D},Q)] = pR_H < E[c_1^U(\frac{\lambda R_L}{D},Q)] = D$, given Assumption 3. There must exist a threshold $N_1(L)$ at which both utilities are equal. This threshold is unique: after the bank is illiquid, $E[c_2^U(\cdot,Q)] = pR_H$ is constant and $E[c_1^U(\cdot,Q)]$ is continuously decreasing until $E[c_1^U(1,Q)] = pD+(1-p)\lambda R_L$. Assumption 3 guarantees that $E[c_1^U(1,Q)] > E[c_2^U(1,Q)]$. There is a unique threshold $\hat{N}_1$ for which an uninformed depositor is indifferent between withdrawing and waiting and the bank is still liquid. This threshold is implicitly defined by:

$$pR_H + (1-p)\left[\frac{\lambda R_L - \hat{N}_1 D}{1-\hat{N}_1}\right] = D.$$

Solving gives:

$$\hat{N}_1 = \frac{p\,R_H + (1-p)\lambda R_l - D}{p(R_H - D)}.$$

Proof of Lemma 5 (Equilibria when uninformed depositors do not receive private signals)

We continue to use the notation from the proof of Lemma 4.

The equilibrium is defined as a number of uninformed withdrawals n_1^U (where we simplify our notation to $n_1^U = n_1^U(H) = n_1^U(L)$ since uninformed depositors do not observe/infer the quality of the asset at any point), given the dominant strategies of π informed depositors, for which it is true that $n_1^U + \pi = N_1(L)$, and depositor i's expected payoff is maximized for all i. We show that two equilibria exist for $\pi < \hat{N}_1$:

(1) Consider an uninformed depositor expecting $n_1^U = 0$. Immediate withdrawal implies that $E[c_1^U(N_1(L),Q)] = D$ given $\pi < \frac{\lambda R_L}{D}$ which always holds for $\pi < \hat{N}_1$. Waiting yields $E[c_2^U(N_1(L),Q)] = pR_H + (1-p)\frac{\lambda R_L - \pi D}{1-\pi}$. Thus, $E[c_1^U(N_1(L),Q)] < E[c_2^U(N_1(L),Q)]$ for $\pi < \hat{N}_1$, implying that $n_1^U = 0$. and, indeed, no uninformed depositor has an incentive to deviate.

(2) Consider an uninformed depositor expecting $n_1^U = 1 - \pi$. Immediate withdrawal implies that $E[c_1^U(N_1(L), Q)] = pD + (1-p)\lambda R_L$. Waiting yields $E[c_2^U(N_1(L), Q)] = pR_H$. Thus, $E[c_1^U(N_1(L), Q)] > E[c_2^U(N_1(L), Q)]$ in the assumed parameter range, implying that $n_1^U = 1 - \pi$, and, indeed, no uninformed depositor has an incentive to deviate.

There is a possible mixed strategy equilibrium in which uninformed depositors withdraw with positive probability. However, given that we have assumed that if a depositor is indifferent between withdrawing in two periods, the depositor withdraws earlier, this is not an equilibrium.

Finally, note that if $\pi > \hat{N}_1$, $E[c_1^U(N_1(L), Q)] > E[c_2^U(N_1(L), Q)]$, and all uninformed depositors withdraw, implying that $n_1^U = 1 - \pi$.

Proof of Lemma 6 (Upper and lower dominance regions)

We first have to show that there are extreme regions of the fundamental for which the uninformed depositor's actions are independent of beliefs about other depositors' behavior. For a given number of informed depositors $\pi \in (0, \frac{\lambda R_L}{D})$, an uninformed depositor is better off by withdrawing immediately, for any beliefs about the other uninformed depositors' actions, if $D > E[p(\theta_i)][1 - q(\theta_i, \theta', \varepsilon)R_H + q(\theta_i, \theta', \varepsilon)D] + (1 - E[p(\theta_i)]\frac{\lambda R_L - \pi D}{1-\pi}$. This condition implicitly defines a lower bound of the fundamental realization at equality $\underline{\theta}(\pi)$. Even if all other uninformed depositors decide to not withdraw, a depositor is better off by withdrawing immediately whenever $\theta_i \leq \underline{\theta}(\pi) - \varepsilon$. The interval $[0, \underline{\theta}(\pi)]$ then defines the lower dominance region. As the difference between the fundamental and an observed signal in our setup is at most ε, an uninformed depositor will withdraw early, whenever observing $\theta_i \leq \underline{\theta}(\pi) - \varepsilon$. To guarantee the existence of the lower dominance region, we assume that ε is sufficiently small - i.e., that $\underline{\theta}(\pi) > 2\varepsilon$. Since we will assume in our analysis that ε is arbitrarily close to 0, it is sufficient to assume that $\underline{\theta}(\pi) > 0$, which holds for all feasible π under our assumptions.[27] For $\lim \varepsilon \to 0$, we can explicitly solve for the lower dominance region:

$$\underline{\theta}(\hat{\pi}(\gamma))_{\varepsilon \to 0} = \frac{D - \lambda R_L}{R_H(1 - \hat{\pi}) + D\hat{\pi} - \lambda R_L}. \qquad \text{(A24)}$$

[27]For $\varepsilon \to 0$ also, the probability of not being able to infer the asset quality in $t = 1$ $q(\theta_i, \theta', \varepsilon) \to 0$. As $\underline{\theta}(\pi)$ is increasing in π, inserting the lowest possible value $\pi \to 0$ shows that a lower dominance region exists as $\lim_{\pi \to 0} \underline{\theta}(\pi) = \frac{D - R_L\lambda}{R_H - R_L\lambda}$, which is strictly greater than zero, given our assumptions.

Similarly, for very high fundamentals, there exists a region in which an uninformed depositor would prefer to wait, independent of other depositors' actions. Waiting is the preferred action if the expected benefit is positive, even though all other uninformed depositors withdraw such that the low-asset-quality bank is liquidated. In this case, the depositor who decides to wait receives a positive return only from the high asset quality, i.e., $D < E[p(\theta_i)][1 - q(\theta_i, \theta', \varepsilon)R_H + q(\theta_i, \theta', \varepsilon)D]$, implicitly defining the $\overline{\theta}$ at equality. Letting $\varepsilon \to 0$, we can also explicitly solve for the upper dominance region:

$$\overline{\theta} = p^{-1}\left(\frac{D}{R_H}\right). \tag{A25}$$

The realizations of the fundamental in the range $[\overline{\theta}, 1]$ define the upper dominance region.

Note that the lower dominance region is increasing in π, while the upper dominance region does not change with π . As $\lim_{\pi \to \frac{\lambda R_L}{D}} \underline{\theta}(\pi) = \overline{\theta}$, both conditions become equal such that the set of signal realizations that result in multiple equilibria becomes empty.

Proof of Proposition 1 (Unique equilibrium threshold)

Assume that all uninformed depositors have the same threshold strategy, characterized by a threshold θ', where uninformed depositors withdraw at all signals below θ' and wait at all signals above θ'. This implies that the aggregate withdrawals also follow the threshold, as summarized in equations (1) and (2). We can then define the expected difference in utility for depositor i, given the signal θ_i and the threshold θ' :

$$\Delta(\theta_i, \theta') = \frac{1}{2\varepsilon}\int_{\theta_i-\varepsilon}^{\theta_i+\varepsilon} \nu(\theta, n_1(\theta, \theta', L))d\theta.$$

Note that $\Delta(\theta_i, \theta')$ is continuous and increasing in θ_i and θ'.

In a threshold equilibrium, an uninformed depositor prefers to withdraw when observing $\theta_i < \theta'$ and to wait if $\theta_i > \theta'$. Given the dominance regions, we know that $\Delta(\theta', \theta')$ is negative for sure if $\theta' \leq \underline{\theta}(\pi) - \varepsilon$ and positive for sure if $\theta' \geq \overline{\theta} + \varepsilon$. As $\Delta(\theta', \theta')$ is continuous and increasing in θ', there must exist a signal θ^* where it holds that $\Delta(\theta^*, \theta^*) = 0$ and this value must be unique. As the depositor that obtains signal $\theta_i = \theta^*$ expects that $\Delta(\theta^*, \theta^*) = 0$, the depositor is, indeed, indifferent between withdrawing and waiting.

We have, therefore, identified a unique candidate for a threshold equilibrium.For the remainder of the proof that any equilibrium must be a threshold equilibrium, we refer the reader to the proof of Goldstein and Pauzner (2005).

Proof of Lemma 7 (Range for which strategic complementarity exists)

Define, as before, $\gamma = \frac{\lambda R_L}{D}$. First, we show that $p(\theta^*(0)) > p(\underline{\theta}(0))$. This can be simplified to $\phi(0) > 1 - \gamma$. A sufficient (though not necessary) condition for this to hold is the parameter restriction $\gamma > \frac{1}{2}$, which is implied by our Assumptions 2 and 3. To see this, remember that the project must have a positive net expected value $pR_H + (1-p)\lambda R_L > D$, which can be written as $D - (1-p)\lambda R_L < pR_H$. The single crossing property requires that $pD + (1-p)\lambda R_L > pR_H$. Combining both conditions requires: $pD + (1-p)\lambda R_L > D - (1-p)\lambda R_L$, which can be simplified to $2\lambda R_L > D$. Hence, for $\pi \to 0$, the threshold value $\theta^*(\pi)$ is above the lower dominance region cutoff . For small π, strategic complementarity among agents exists and leads to self-fulfilling runs. However, as π increases, the strategic complementarity becomes less important. As both functions are continuous in π, we can solve for the intersection $p(\theta^*(\pi)) = p(\underline{\theta}(\pi))$. After solving, this is $\hat{\pi}(\gamma) = 1 - e(1-\gamma)\gamma^{\frac{\gamma}{1-\gamma}}$, where e is Euler's number .

We will now prove that $\hat{\pi}(\gamma) < \gamma \; \forall \gamma \in (1/2, 1)$ in a series of steps.

Step 1: We show that $\hat{\pi}(\gamma)$ is increasing in $\gamma \in (1/2, 1)$. This is directly from $\frac{\partial \hat{\pi}(\gamma)}{\partial \gamma} = -\frac{e\gamma^{\frac{\gamma}{1-\gamma}} \ln(\gamma)}{1-\gamma}$, which is positive for all $\gamma < 1$.

Step 2: We demonstrate that $\frac{\partial^2 \hat{\pi}(\gamma)}{\partial \gamma^2}$is negative for $\gamma \in (1/2, 1)$.The term:

$$\begin{aligned}\frac{\partial^2 \hat{\pi}(\gamma)}{\partial \gamma^2} &= -\frac{e\gamma^{\frac{\gamma}{1-\gamma}-1}}{1-\gamma} - \frac{e\gamma^{\frac{\gamma}{1-\gamma}} \ln(\gamma)}{(1-\gamma)^2} - \frac{e\gamma^{\frac{\gamma}{1-\gamma}} \ln(\gamma)\left(\frac{1}{1-\gamma} + \left(\frac{\gamma}{(1-\gamma)^2} + \frac{1}{1-\gamma}\right)\ln(\gamma)\right)}{1-\gamma} \\ &= \frac{e\gamma^{\frac{1}{1-\gamma}-2}\left((\gamma-1)^2 + \gamma\ln(\gamma)(-2\gamma + \ln(\gamma) + 2)\right)}{(\gamma-1)^3}\end{aligned}$$

Define $f(\gamma) \equiv (1-\gamma)^2 + \gamma \ln(\gamma)(2(1-\gamma) + \ln(\gamma))$. Then, the term $\frac{\partial^2 \hat{\pi}(\gamma)}{\partial \gamma^2}$ is negative if $f(\gamma) > 0$. The slope of $f(\gamma)$ is $\frac{\partial f(\gamma)}{\partial \gamma} = \ln(\gamma)(4(1-\gamma) + \ln(\gamma))$; the term $\ln(\gamma)$ is clearly negative. We now show that the second term is positive. The derivative of the second term is negative: $\frac{\partial(4(1-\gamma)+\ln(\gamma))}{\partial \gamma} = -4 + 1/\gamma < 0 \leftrightarrow \gamma > 1/4$. The second term, evaluated at $\gamma = 1$, is equal to 0. Therefore, the term $4(1-\gamma) + \ln(\gamma)$ is positive, and, thus, the derivative $\frac{\partial f(\gamma)}{\partial \gamma} < 0 \; \forall \gamma \in (1/2, 1)$. Note that $f(1) = 0$, so, indeed, $f(\gamma) > 0 \; \forall \gamma \in (1/2, 1)$. This implies that $\frac{\partial^2 \hat{\pi}(\gamma)}{\partial \gamma^2} < 0 \; \forall \gamma \in (1/2, 1)$.

Step 3: We note the following facts about the endpoints. First, at $\gamma = \frac{1}{2}$, $\hat{\pi}(\gamma) = 1 - \frac{e}{4} < \frac{1}{2}$, so $\hat{\pi}(\frac{1}{2}) < \frac{1}{2}$. Second, at $\gamma = 1$, $\hat{\pi}(\gamma) = 1$, so $\hat{\pi}(1) = 1$. The slope $\frac{\partial \hat{\pi}(\gamma)}{\partial \gamma}$ also approaches 1 as γ approaches 1. To see this,we first apply the product rule to the slope:

$$\lim_{\gamma -> 1} \frac{\partial \hat{\pi}(\gamma)}{\partial \gamma} = \lim_{\gamma -> 1} -\frac{e\gamma^{\frac{\gamma}{1-\gamma}} \ln(\gamma)}{1-\gamma} \tag{A26}$$
$$= e \lim_{\gamma -> 1} \left(\gamma^{\frac{\gamma}{1-\gamma}}\right) \lim_{\gamma -> 1} \left(\frac{\ln(\gamma)}{\gamma - 1}\right).$$

Applying l'Hôpital's rule to the third term:

$$\lim_{\gamma -> 1} \left(\frac{\ln(\gamma)}{\gamma - 1}\right) = \lim_{\gamma -> 1} \frac{\frac{d}{d\gamma}\ln(\gamma)}{\frac{d}{d\gamma}(\gamma - 1)} = \lim_{\gamma -> 1} \frac{\frac{1}{\gamma}}{1} = \lim_{\gamma -> 1} \frac{1}{\gamma} = 1.$$

We can now rewrite Eq (A26) as:

$$\lim_{\gamma -> 1} \frac{\partial \hat{\pi}(\gamma)}{\partial \gamma} = e \lim_{\gamma -> 1} \gamma^{\frac{\gamma}{1-\gamma}}.$$

Rewriting using the exponential of a logarithm:

$$\begin{aligned} &= e \lim_{\gamma -> 1} e^{\ln(\gamma^{\frac{\gamma}{1-\gamma}})} \\ &= e \lim_{\gamma -> 1} e^{\frac{\gamma \ln(\gamma)}{1-\gamma}} \\ &= e * e^{\lim_{\gamma -> 1} \frac{\gamma \ln(\gamma)}{1-\gamma}}. \end{aligned}$$

We now apply l'Hôpital's rule again:

$$\begin{aligned} &= e * e^{\lim_{\gamma -> 1} \frac{\ln(\gamma) + \frac{1}{\gamma}}{-1}} \\ &= 1. \end{aligned}$$

The slope approaches 1 from above. Hence, the term $\hat{\pi}(\gamma) < \gamma$ for all $\gamma \in (1/2, 1)$.

Proof of Proposition 2 (Threshold increasing in π)

Consider the derivative with respect to π. To save on notation, we use the definition $\gamma = \frac{\lambda R_L}{D}$.

$$\frac{\partial p(\theta^*)_{|\varepsilon\to 0}}{\partial \pi} = \frac{(\frac{R_H}{D} - 1)\,(\phi(\pi) - (1-\gamma))}{\big(\phi(\pi) + (1-\pi)(\frac{R_H}{D} - 1)\big)^2} \tag{A27}$$

The sign of the slope is determined by the sign of the numerator - i.e., the sign of $\phi(\pi) - (1-\gamma)$. This term is strictly decreasing in π as: $\frac{\partial \phi(\pi)}{\partial \pi} = -\frac{1-\gamma}{1-\pi} < 0\,\forall\,\pi \in (0,\gamma)$ where $\gamma \leq 1$. Moreover, the numerator is positive for small π - i.e., we can show that for $\pi = 0$, it must hold that $\phi(0) > (1-\gamma)$. In the proof of Lemma 7, we show that this is implied by our single crossing property. Hence, for $\pi = 0$, it must hold that $\phi(0) > (1-\gamma)$, which implies that $\frac{\partial p(\theta)^*_{|\pi\to 0}}{\partial \pi} > 0$ is strictly positive for small π.

As the numerator is a continuous function, we have to identify the critical π for which $\phi(\pi) = (1-\gamma)$. As the right-hand side is a constant and $\phi(\pi)$ is decreasing in π over the feasible parameter space, π has to be unique. Solving $\phi(\pi) = (1-\gamma)$ for π gives $\pi(\gamma) = 1 - e(1-\gamma)\gamma^{\frac{\gamma}{\gamma-1}}$, which is equal to $\hat{\pi}$: the point at which the strategic complementarity among agents vanishes - i.e. where $p(\theta^*(\hat{\pi})) = p(\underline{\theta})$. Intuitively, as long as there is strategic complementarity among agents, the threshold for uninformed depositors to run increases with more informed depositors until $\hat{\pi}$. The slope is zero at the point where no further strategic complementarity among agents exists - i.e., where $p(\theta^*(\hat{\pi})) = p(\underline{\theta})$. Therefore, $\phi(\pi) > (1-\gamma)\,\forall\,\pi \in (0,\hat{\pi}(\gamma)) \Leftrightarrow \frac{\partial p(\theta^*)_{|\varepsilon\to 0}}{\partial \pi} > 0\,\forall\,\pi \in (0,\hat{\pi}(\gamma))$.

Proof of Proposition 3 (Comparative statics)

We first write the critical threshold in terms of λ:

$$p(\theta^*) = \frac{\phi(\pi, \lambda\frac{R_L}{D})}{(1-\pi)(\frac{R_H}{D} - 1) + \phi(\pi, \lambda\frac{R_L}{D})}, \tag{A28}$$

where $\phi(\pi, \lambda\frac{R_L}{D}) = -\lambda\frac{R_L}{D}\ln\left(\lambda\frac{R_L}{D}\right) - (1-\lambda\frac{R_L}{D})\left(\ln\left(1-\lambda\frac{R_L}{D}\right) - \ln(1-\pi)\right) > 0\,\forall\,0 \leq \pi < \lambda\frac{R_L}{D} < 1$.

The derivative with respect to λ is:

$$\frac{\partial p(\theta^*)}{\partial \lambda} = \frac{\frac{R_L}{D}(1-\pi)(\frac{R_H}{D} - 1)\ln\left(\frac{1-\lambda\frac{R_L}{D}}{(1-\pi)\lambda\frac{R_L}{D}}\right)}{\big((1-\pi)(\frac{R_H}{D} - 1) + \phi(\pi)\big)^2}. \tag{A29}$$

The sign of the derivative depends on the sign of the function $\ln\left(\frac{1-\lambda\frac{R_L}{D}}{(1-\pi)\lambda\frac{R_L}{D}}\right)$. The sign is positive, whenever $(1-\lambda\frac{R_L}{D}) > \lambda\frac{R_L}{D}(1-\pi)$ and negative otherwise. Therefore, the threshold value is decreasing in $\lambda\frac{R_L}{D}$ if π is small $(1-\pi > \frac{D-\lambda R_L}{\lambda\frac{R_L}{D}})$ and increasing otherwise. Note that our assumptions imply that $\lambda\frac{R_L}{D} > \frac{1}{2}$, as shown in Proposition 2. Moreover, it holds that $1-\hat{\pi} > \frac{D-\lambda R_L}{\lambda\frac{R_L}{D}}$. Hence, the sign of the derivative switches at $1-\pi = \frac{D-\lambda R_L}{\lambda\frac{R_L}{D}}$.

Proof of Proposition 4 (No information threshold vs. main model)

We have to show that $p(\theta^{NI}) < p(\theta^*(0))$ since both $p()$ is increasing in θ and $\theta^*()$ is increasing in π. Using equations 12 and 7, this can be rewritten as:

$$\frac{\phi^{NI}}{\phi^{NI} + \lambda\left(\frac{R_H}{D} - 1\right)} < \frac{\phi(0)}{\phi(0) + \left(\frac{R_H}{D} - 1\right)}, \tag{A30}$$

with $\phi^{NI} = -\lambda\gamma\ln(\gamma) - (1-\gamma)\ln(1-\gamma) - (1-\lambda)\gamma$ and $\phi(0) = -\gamma\ln(\gamma) - (1-\gamma)\ln(1-\gamma)$ where $\gamma = \frac{\lambda R_L}{D}$. This can be simplified to:

$$\begin{aligned} \frac{\phi^{NI}}{\phi(0)} &< \lambda \\ -\lambda\gamma\ln(\gamma) - (1-\gamma)\ln(1-\gamma) - (1-\lambda)\gamma &< -\lambda\gamma\ln(\gamma) - \lambda(1-\gamma)\ln(1-\gamma) \\ -(1-\lambda)\gamma &< (1-\lambda)(1-\gamma)\ln(1-\gamma) \\ \frac{\gamma}{1-\gamma} &> \ln(1-\gamma). \end{aligned}$$

Defining $g(\gamma) = \frac{\gamma}{1-\gamma} - \ln(1-\gamma)$ it is straightforward to show that $g(\gamma) > 0\ \forall\gamma \in (0,1)$.

Proof of Lemma 8 (Incentive to acquire information increases in π)

As $R_H > D > \lambda R_L$ by Assumption 1, it must hold that $\Delta(\theta^*(0), 0) > 0$. The slope of $\Delta(\pi)$ can be characterized as the following:

$$\begin{aligned} \frac{\partial\Delta(\theta^*(\pi),\pi)}{\partial\pi} &= \int_{\theta^*(\pi)}^{1} (1-p(\theta))\frac{D-\lambda R_L}{(1-\pi)^2}\, d\theta \\ &+ \frac{\partial\theta^*(\pi)}{\partial\pi}\left(p(\theta^*(\pi))(R_H - D) - (1-p(\theta^*(\pi))\frac{D-\lambda R_L}{(1-\pi)}\right) > 0. \end{aligned} \tag{A31}$$

The first term of the slope is always positive. We now show that the second term is positive. First, from Proposition 2, the critical threshold value increases in the proportion of informed depositors $\frac{\partial \theta^*(\pi)}{\partial \pi} > 0$. Second, the term in brackets strictly increases in $p(\theta^*(\pi))$. Third, we know, also from Proposition 2, that $p(\theta^*(\pi)) > p(\underline{\theta}(\pi))$ for any $\pi \in (0, \hat{\pi}(\gamma))$. Inserting $p(\underline{\theta}(\pi))$, defined by equation (A24), into the term in brackets gives zero. Hence, the term in brackets must be strictly positive for all $p(\theta^*(\pi)) > p(\underline{\theta}(\pi))$.

Proof of Lemma 9 (Equilibrium information acquisition)

First, note that $\Delta(0) > 0$. From Lemma 8, we know that $\Delta(\cdot)$ is continuous and increasing for all $\pi \in (0, \hat{\pi}(\gamma))$. This implies, that if a fixed point exists, it is unique if $\frac{\partial \Delta(\pi)}{\partial \pi} < 1$ for all $\pi \in (0, \hat{\pi}(\gamma))$.

However, to show that a fixed point exists, we need to assume only a much weaker condition:

$$\Delta(\hat{\pi}(\gamma)) < \hat{\pi}(\gamma). \tag{A32}$$

We now demonstrate that parameters exist for which Equation A32 holds. If we assume that $p(\theta(\pi)) = \theta(\pi)$, we can write out $\Delta(\pi)$ as:

$$\Delta(\pi) = \frac{1}{2} R_H \theta(\pi)^2 - \frac{1}{2} D\theta(\pi)^2 + \frac{(1-\theta(\pi))^2 (D - \lambda R_L)}{2(1-\pi)}. \tag{A33}$$

We know that for $\pi = \hat{\pi}$, the following holds:

$$p(\theta^*(\hat{\pi}(\gamma))) = p(\underline{\theta}(\hat{\pi}(\gamma))) \Leftrightarrow \theta^*(\hat{\pi}(\gamma)) = \underline{\theta}(\hat{\pi}(\gamma)).$$

where $\underline{\theta}(\hat{\pi}(\gamma))_{\varepsilon \to 0}$ as defined in Equation A24, where $q(\theta_i, \theta', \varepsilon) \to 0$ as $\varepsilon \to 0$. Substituting $\underline{\theta}(\hat{\pi}(\gamma))_{\varepsilon \to 0}$ into Equation A33 and simplifying gives:

$$\begin{aligned} \Delta(\hat{\pi}) &= \frac{R_H - D}{2} \frac{(D - \lambda R_L)^2}{(R_H(1-\hat{\pi}) + D\hat{\pi} - \lambda R_L)^2} + \frac{(D - \lambda R_L)\left(1 - \frac{D - \lambda R_L}{R_H(1-\hat{\pi}) + D\hat{\pi} - \lambda R_L}\right)^2}{2(1-\hat{\pi})} \\ &= \frac{(R_H - D)}{2} \underline{\theta}(\hat{\pi}(\gamma))_{\varepsilon \to 0} \end{aligned} \tag{A34}$$

We can write the sufficient condition for an existence of the fixed point as

$$\frac{\frac{R_H}{D} - 1}{2} \frac{1 - \gamma}{\frac{R_H}{D}(1 - \hat{\pi}(\gamma)) + \hat{\pi}(\gamma) - \gamma} < \hat{\pi}(\gamma).$$

For $\gamma \in (1/2, 1)$, we will now demonstrate that this condition always holds if $\frac{R_H}{D}$ is not too high (which is excluded by our Assumption 3). Recall that $\hat{\pi}(\gamma) = 1 - e(1-\gamma)\gamma^{\frac{\gamma}{1-\gamma}}$.

First, note that the left-hand side of this condition increases in γ: The slope is $\frac{e\gamma^{\frac{\gamma}{\gamma-1}}(\frac{R_H}{D}-1)^2(\gamma-\ln(\gamma)-1)}{2(\gamma-1)^2\left(\gamma^{\frac{\gamma}{\gamma-1}}+e(\frac{R_H}{D}-1)\right)^2}$, which is positive, iff the term $\gamma - \ln(\gamma) - 1 > 0$. The term $\gamma - \ln(\gamma) - 1$ is positive for $\gamma \in (1/2, 1)$, as it is (i) decreasing in γ for $\gamma < 1$, and (ii) equal to zero at $\gamma = 1$. This implies that $\Delta(\hat{\pi}(\gamma))$ is increasing in γ.

The term approaches

$$\lim_{\gamma->1} \Delta(\hat{\pi}(\gamma)) = \frac{\frac{R_H}{D} - 1}{2} \lim_{\gamma->1} \frac{1-\gamma}{\frac{R_H}{D}(1-\hat{\pi}(\gamma)) + \hat{\pi}(\gamma) - \gamma}.$$

Inserting $\hat{\pi}(\gamma)$ and simplifying, we get:

$$\lim_{\gamma->1} \Delta(\hat{\pi}(\gamma)) = \frac{\frac{R_H}{D} - 1}{2} \lim_{\gamma->1} \frac{1}{e\gamma^{\frac{\gamma}{1-\gamma}}(\frac{R_H}{D} - 1) + 1}.$$

Consider only the second term (the one with a limit). We take the exponential of the natural logarithm:

$$\begin{aligned}
&\lim_{\gamma->1} \exp\left(\ln\left(\frac{1}{e\gamma^{\frac{\gamma}{1-\gamma}}(\frac{R_H}{D} - 1) + 1}\right)\right) \\
= &\lim_{\gamma->1} \exp\left(-\ln\left(e\gamma^{\frac{\gamma}{1-\gamma}}(\frac{R_H}{D} - 1) + 1\right)\right) \\
= &\exp\left(-\lim_{\gamma->1} \ln\left(e\gamma^{\frac{\gamma}{1-\gamma}}(\frac{R_H}{D} - 1) + 1\right)\right) \\
= &\exp\left(-\ln\left(e(\frac{R_H}{D} - 1)\lim_{\gamma->1}\left(\gamma^{\frac{\gamma}{1-\gamma}}\right) + 1\right)\right)
\end{aligned}$$

Once again applying an exponential of a logarithm:

$$\begin{aligned}
= &\exp\left(-\ln\left(e(\frac{R_H}{D} - 1)\lim_{\gamma->1}\exp(\ln(\gamma^{\frac{\gamma}{1-\gamma}})) + 1\right)\right) \\
= &\exp\left(-\ln\left(e(\frac{R_H}{D} - 1)\lim_{\gamma->1}\exp\left(\frac{\gamma}{1-\gamma}\ln(\gamma)\right) + 1\right)\right) \\
= &\exp\left(-\ln\left(e(\frac{R_H}{D} - 1)\exp\left(-\lim_{\gamma->1}\frac{\ln(\gamma)}{\gamma - 1}\right) + 1\right)\right)
\end{aligned}$$

Applying l'Hôpital's rule gives us that:

$$\lim_{\gamma->1} \frac{\ln(\gamma)}{\gamma - 1} = \lim_{\gamma->1} \frac{\frac{d}{d\gamma}\ln(\gamma)}{\frac{d}{d\gamma}(\gamma - 1)} = \lim_{\gamma->1} \frac{1}{\gamma} = 1.$$

We now rewrite the expression:

$$\begin{aligned} &= \exp\left(-\ln\left(e(\frac{R_H}{D} - 1)\exp(-1) + 1\right)\right) \\ &= \exp\left(-\ln\left(\frac{e(\frac{R_H}{D} - 1)}{e} + 1\right)\right) \\ &= \frac{1}{\frac{R_H}{D}} \end{aligned}$$

Inserting in the original term, we get:

$$\lim_{\gamma->1} \Delta(\hat{\pi}(\gamma)) = \frac{\frac{R_H}{D} - 1}{2} \frac{1}{\frac{R_H}{D}}$$

This is the highest possible value for the information rent within the parameter space. At the same time $\hat{\pi}$, is also increasing in γ. The lowest possible value that $\hat{\pi}(\gamma)$ can take is $\lim_{\gamma->1/2} \hat{\pi}(\gamma) = 1 - \frac{e}{4} = 0.32043$. This is greater than the highest possible information rent - i.e.,$\lim_{\gamma->1} \Delta(\hat{\pi}(\gamma)) < \lim_{\gamma->1/2} \hat{\pi}(\gamma)$, if $\frac{R_H}{D} < \frac{2}{e-2} = 2.78442$.

Proof of Proposition 5 (Overinvestment in information acquisition)

The endogenous information choice results in π^* implicitly defined by

$$\underbrace{\int_0^{\theta^*(\pi)} p(\theta)(R_H - D)\, d\theta}_{+} + \underbrace{\int_{\theta^*(\pi)}^1 (1 - p(\theta)) \left(\frac{D - \lambda R_L}{1 - \pi}\right) d\theta}_{+} = \pi. \qquad \text{(A35)}$$

Both terms on the left-hand side are positive, and the left-hand side is increasing in π (this was proven in Lemma (8)).

The surplus-optimizing proportion of informed depositors is implicitly defined by $\frac{\partial S^I}{\partial \pi} = \pi$. We can write:

$$D\left(\frac{1-\lambda}{\lambda}\right)\left(\int_0^{\theta^*(\pi)} p(\theta)\, d\theta - (1-\pi)p(\theta^*(\pi))\frac{\partial p(\theta^*(\pi))}{\partial \pi}\right) = \pi \qquad \text{(A36)}$$

$$\underbrace{\int_0^{\theta^*(\pi)} p(\theta) D\left(\frac{1-\lambda}{\lambda}\right) d\theta}_{+} - \underbrace{(1-\pi) D\left(\frac{1-\lambda}{\lambda}\right) p(\theta^*(\pi)) \frac{\partial p(\theta^*(\pi))}{\partial \pi}}_{+} = \pi. \quad \text{(A37)}$$

The first term on the left-hand side is positive and the second positive term is subtracted. Notice that the right-hand sides of the endogenous choice (Eq. A35) and the surplus maximization (Eq. A37) are both equal to π . We will now show that the left-hand side that implicitly defines the surplus maximization is always smaller than the left-hand side of the the endogenous choice for the same π. The first term in the equations can be compared as follows:

$$\int_0^{\theta^*(\pi)} p(\theta)(R_H - D)\, d\,\theta \geq \int_0^{\theta^*(\pi)} p(\theta) D\left(\frac{1-\lambda}{\lambda}\right) d\theta$$

because point-wise:

$$\begin{aligned} (R_H - D) &\geq D\left(\frac{1-\lambda}{\lambda}\right) \\ \lambda R_H &\geq D \end{aligned}$$

This strictly holds under Assumption 1.

As the first term of the endogenous choice equation is strictly greater than the first term of the surplus optimal choice, adding a positive term in the former and subtracting a positive term in the latter reinforces the inequality for any π. As a result, any endogenous choice solution $\pi^* > \pi^S$ results in a strictly higher proportion of depositors than a surplus-optimal choice. The agents overinvest in information from a surplus perspective.

B Internet Appendix

The Optimal Deposit Contract

We now add some features to our model from Calomiris and Kahn (1991) and examine the optimal deposit contract.[28] The optimal deposit contract consists of a short-term rate D at $t = 1$ and $t = 2$ and a long-term payout $\rho > D$.[29] We show that there exist parameter spaces such that the optimal contract satisfies the assumptions of the model. The risk-neutral manager of the bank is compensated by a small fraction of profits η. We assume that η is close to zero to simplify the expressions. The bank has zero costs. At time $t = -1$, the manager must attract deposits. The manager's decision variables are D, the short-term payment for early withdrawal, and ρ, the payout for waiting until $t = 3$ to withdraw. The promised payoff at maturity has to be greater than the short-term payment, since, otherwise, early withdrawal would be the dominant strategy. There is perfect competition among banks. Due to competitive pressure, the manager will maximize the depositors' payoff when proposing the contract.

To simplify this problem, we will assume that all depositors are uninformed and that information about the asset quality $Q \in \{L, H\}$ is revealed perfectly at the end of $t = 1$. This is equivalent to a perfectly informative public signal or the existence of a very small amount of perfectly informed depositors whose $t = 1$ actions were observable.

Depositors have an outside return of $\bar{R}$ when they do not deposit their funds.

Similar to Calomiris and Kahn (1991), in our model, the bank manager can abscond with a fraction of the asset value at $t = 0$.

In particular, we assume that the asset of each quality $Q \in \{L, H\}$ has a payoff at $t = 3$ of $R_Q + \delta$ with probability $\frac{1}{2}$ and a payoff of $R_Q - \delta$ with probability $\frac{1}{2}$. The manager may attempt to abscond with an amount 2δ when the payoff is $R_Q + \delta$.[30] In this case, a low payoff of $R_Q - \delta$ may be due to a poor realization or due to the

[28]Note that we do not directly apply the Calomiris and Kahn (1991) framework to our model. The most important changes (seen below) to the Calomiris and Kahn (1991) setting here are: (i) the bank faces perfect competition ex ante; and (ii) there is an endogenous probability of getting caught and a cost of absconding.

[29]We restrict the penalty rate to be the same at $t = 1$ and $t = 2$ for simplicity.

[30]We continue to assume that if either quality asset is liquidated early, it is worth a fraction of its expected value: λR_Q, where $Q \in \{L, H\}$.

manager absconding.

The manager has to decide whether to abscond at $t = 0$. A regulator may initiate an investigation and impose a penalty of C on the manager. The probability of an investigation is perfectly positively correlated with a bank run. We denote this probability by z. For simplicity, we assume that a bank run always results in a penalty for a manager that decided to divert funds. The way to think about this is that (i) the regulator may go through the books when there is a run; (ii) depositors will start looking into the bank's books when there is a run; and/or (iii) if the money is already gone at $t = 1$, then depositors who do not get their money will demand an investigation. However, this also implies that if there is no run, and the depositors wait until the end, it is too late to catch the banker who absconded.

The manager will decide to abscond if his expected payoff is greater than the expected penalty if he is caught. Ex ante, the gain from absconding is the probability that assets of quality Q have a low payoff $(\frac{1}{2})$ multiplied by the amount with which the manager could abscond (2δ). Given a probability of a run/getting caught z, the manager will not abscond if:

$$z > \overline{z} := \frac{\delta}{\delta + C}.$$

.

Without the manager's moral hazard problem, setting $D < \lambda R_L$ makes withdrawals before $t = 3$ a dominated strategy and, therefore, allows the bank's asset to reach maturity without any liquidations. This is equivalent to offering long-term debt contracts. This, of course, relies on the assumption that waiting until $t = 3$ satisfies the depositors' outside option. A sufficient condition for this to hold is:

$$pR_H + (1 - p)R_L > \bar{R}. \tag{A38}$$

Hence, from a welfare perspective, it is desirable to fund the project. However, with the manager's moral hazard problem, long-term debt funding implies that the manager will choose to abscond whenever the project returns $R_Q + \delta$ for $Q \in \{L, H\}$. This can create a market failure if depositors do not want to invest when they expect the manager to abscond. A sufficient condition for this to hold is:

$$pR_H + (1 - p)R_L - \delta < \bar{R}. \tag{A39}$$

This assumption makes long-term debt contracts unattractive to depositors. Note that the depositors do not need to observe the manager abscond; they will rationally expect that he absconds whenever $R_Q + \delta$.

Moreover, we will assume that a short-term debt contract, with $D \in (\lambda R_L, R_H)$ and $\rho \in [D, R_H]$, is attractive to depositors when it induces a probability of a bank run that is high enough to prevent the banker from absconding. A short-term contract yields the ex ante expected utility of :

$$\int_0^1 (p(\theta)\rho + (1 - p(\theta))\lambda R_L)\, d\theta - (\rho - D) \int_0^{\theta^*(\rho,D)} p(\theta) d\theta. \qquad (A40)$$

We use the definitions (i) $p = \int_0^1 (p(\theta))\, d\theta$, (ii) $(1 - p) = \int_0^1 (1 - p(\theta))\, d\theta$, and note that the probability of a run is $\int_0^{\theta^*(\rho,D)} p(\theta) d\theta$. From above, the minimum ex ante run probability needed to prevent the manager from absconding is $\bar{z} = \frac{\delta}{\delta+C}$. Substituting these expressions, we obtain a sufficient assumption that ensures that the optimal short-term contract is viable, i.e., the expected utility is greater than the outside option:

$$pR_H + (1 - p)\lambda R_L - \frac{\delta}{\delta + C}(R_H - \lambda R_L) > \bar{R} \qquad (A41)$$

In other words, we assume that, ex ante, the net loss from the inefficient liquidation of a high-quality asset $\rho - D$, caused by a disciplining bank run that occurs with probability $\frac{\delta}{\delta+C}$, is not too high. The maximum expected loss is $\frac{\delta}{\delta+C}(R_H - \lambda R_L)$.

Disciplinary bank runs are efficient if, given their probability $\bar{z}$ of occurring, the net loss for depositors when the manager absconds is greater than the net loss from a short-term contract (i.e., combining conditions A39 and A41) :

Assumption A1: $\delta - (1 - p)(1 - \lambda)R_L > \frac{\delta}{\delta+C}(R_H - \lambda R_L)$.

Another way of stating this is that the penalty a manager faces, once detected, is high enough given the benefit from absconding:

$$C > \underline{C}(\delta) \equiv \delta \frac{(R_H - \lambda R_L) - (\delta - (1 - p)(1 - \lambda)R_L)}{(\delta - (1 - p)(1 - \lambda)R_L)}. \qquad (A42)$$

To attract deposit funding, the manager has to offer a short-term debt contract that deters him from absconding. While long-term debt funding is not attractive because of the moral hazard problem, short-term debt induces a positive probability of panic runs, which may help to overcome the moral hazard problem but creates a liquidation cost.

For simplicity in the rest of the derivations, we assume that $p(\theta) = \theta$.

The optimal short-term contract maximizes the expected utility of depositors under the constraint that it successfully prevents the manager from absconding:

$$\max_{\rho,D} E[U(\theta^*(\rho,D),\rho,D)] \tag{A43}$$

s.t.

$$\int_0^{\theta^*(\rho,D)} \theta d\theta \geq \bar{z} \tag{A44}$$

$$\rho \geq D. \tag{A45}$$

The expected utility of depositors is defined as:

$$E[U(\theta^*(\rho,D),\rho,D)] = \int_0^{\theta^*(\rho,D)} \{\theta \min[D, \lambda R_H] + (1-\theta)\lambda R_L\} d\theta$$
$$+ \int_{\theta^*(\rho,D)}^1 \{\theta(a^H \lambda R_H + (1-a^H)\rho) + (1-\theta)(a^L \lambda R_L + (1-a^L)R_L)\} d\theta.$$

Note that in contrast to our main model, there could be multiple equilibria at $t = 2$. In our main model, the assumption $D \in (R_L, \lambda R_H]$ guarantees that depositors have dominant strategies to run or wait in $t = 2$. Here, we allow (i) D to be smaller than R_L, which allows for multiple equilibria at $t = 2$ when the asset is low-quality and (ii) $D > \lambda R_H$, which allows for multiple equilibria at $t = 2$ if the asset is high-quality. To show that a contract $D \in (R_L, \lambda R_H]$ can, indeed, be optimal, we need to compare the expected utility over the entire contract space. To consider the possible multiple equilibria at $t = 2$, we specify that depositors expect with probability a^Q (where $Q \in \{L, H\}$) that all depositors run at $t = 2$, and with probability $1 - a^Q$ that all depositors wait at $t = 2$. This could be due to, for example, a sunspot state, as in Cooper and Ross (1998).[31]

The probability of a second-stage run depends on D, and we can distinguish three intervals for which the probability differs, summarized in Table 2.

In the first interval (Case I), D is low. In the case of a high-quality asset, there would be no runs. In the case of a low-quality asset, there may be runs at $t = 2$. In the second interval (Case II), there are once again no runs at $t = 2$ if the asset is high-quality, but runs with probability 1 if the asset is low-quality. Finally, in the third interval (Case III), there may be runs if the asset is high-

[31] An extrinsic "sunspot" variable that determines the run probability is commonly used in the literature; see, for example, Peck and Shell (2003), Ennis and Keister (2010), and Mitkov (2020).

Table 2: Second-stage Run Probabilities

	Case I	Case II	Case III
	$D \in (\lambda R_L, R_L]$	$D \in (R_L, \lambda R_H]$	$D \in (\lambda R_H, R_H]$
$Q = H$	$a^H = 0$	$a^H = 0$	$a^H \in [0, 1]$
$Q = L$	$a^L \in [0, 1]$	$a^L = 1$	$a^L = 1$

quality, and there will certainly be runs if the asset is low-quality. For brevity, define $A^Q := a^Q + (1 - a^Q)\frac{1}{\lambda} \geq 1$ and note that $\frac{\partial A^Q}{\partial a^Q} = 1 - \frac{1}{\lambda} < 0$.

A unique threshold equilibrium requires the existence of a lower and upper dominance region for θ. In particular, if θ is very high, it is optimal for each agent to wait, no matter what the other agents do. For Cases I and II, the upper dominance region is implied by our assumption $\rho > D$ because, if the probability that a high-quality asset realizes is high enough, the implied return ρ from waiting is higher than what the agent can possibly get from withdrawing, no matter what the other agents do. However, in Case III, the short-term payment is greater than the high-quality asset's liquidation value, implying that a run may occur. The return from waiting when the asset is high-quality may then be lower than the return from withdrawing early (at $t = 1$). Therefore, we make use of the same assumption as (Goldstein and Pauzner, 2005, p.1301) that for sufficiently high state variables, no patient agent demands early withdrawal.[32]

The existence of a lower dominance region is implied by the strictly positive liquidation value of the asset, which guarantees a secure minimum return from early withdrawal. If the state variable is sufficiently low, all agents prefer this secure return over the low expected return from waiting.

Moreover, we assume that the liquidation value is not too low, which will ensure monotonicity of first-stage run probabilities in the second stage's probability of a run:[33]

[32]Note that our model can even be analyzed if an upper dominance region ceases to exist. Goldstein and Pauzner (2005) apply several equilibrium selection criteria to show that the more reasonable equilibrium is the same as the unique equilibrium obtained with an assumed upper dominance region.

[33]Note that $\lambda > 0.5$ is implied by assumptions A2 and A3, such that assumption A4 is just a slightly stricter, technical assumption.

Assumption A2: $\lambda > 1 - \frac{1}{e} \approx 0.632$.

We will now show that the expected utility in Cases I and II is strictly increasing in ρ, while it is ambiguous in D. The short-term payment D has two opposing effects on the expected utility of depositors. First, there is a direct positive effect because depositors can withdraw more from their account as long as the bank is liquid. Second, a higher withdrawal amount increases the probability of an inefficient run, which reduces the expected utility of depositors. We will see that the expected utility is convex in D such that a corner solution of D maximizes the expected utility. Because of varying run probabilities in the second stage, we need to analyze each case individually and provide a full characterization of the optimal contract.

Case I: $\lambda R_L < D \leq R_L$.

We first analyze the case in which D is low. In this case, it is never optimal to run if depositors learn that the asset has high-quality, but if the asset has low-quality, it is optimal to run if other depositors run as well.

We proceed as follows: In a first step, we show that both contract components ρ and D have a direct effect on the expected utility of depositors and an indirect effect through a change in the run probability. In step 2, we solve for the critical state value θ^* that determines the ex ante run probability under the modified conditions that ensure one-sided strategic complementarity among depositors. In step 3, we show, first, that ρ decreases and D increases the run probability. Consequently, the expected utility is unambiguously increasing in ρ, but D has an ambiguous effect on the expected utility. In the fourth step, we show that the expected utility is convex in D in the interval such that the expected utility is maximized by a corner solution. In step 5, we show that this holds for all possible second-stage run probabilities a^L if λ is not too small. We then summarize the results for this interval.

Step 1: Let us start by analyzing the effect of ρ and D on the expected utility. The ex-ante expected utility of a depositor can be written as:

$$U_I = \int_0^{\theta_I^*} \{\theta D + (1-\theta)\lambda R_L\} d\theta + \int_{\theta_I^*}^1 \{\theta\rho + (1-\theta)(A^L \lambda R_L)\} d\theta, \tag{A46}$$

where θ^* is defined as the signal that makes a depositor indifferent between withdrawing and waiting.

Taking the derivative with respect to ρ, we obtain the following:

$$\frac{\partial U_I}{\partial \rho} = \int_{\theta_I^*}^1 \theta \, d\theta - \left[(\rho - D)\theta_I^* + (1-\theta_I^*)(A^L - 1)\lambda R_L\right] \frac{\partial \theta_I^*}{\partial \rho}.$$

A higher contracted payment has a positive direct effect, as it increases expected consumption. However, the contracted payment also affects the incentives to withdraw early, leading to costly liquidation. As the term in brackets is strictly positive, the direction of the indirect effect depends on the effect of the payment ρ on the critical signal θ_I^*.

Taking the derivative with respect to D, we obtain a similar result:

$$\frac{\partial U_I}{\partial D} = \int_0^{\theta_I^*} \theta \, d\theta - \left[(\rho - D)\theta_I^* + (1-\theta_I^*)(A^L - 1)\lambda R_L \,)\right] \frac{\partial \theta_I^*}{\partial D}.$$

Step 2: To pin down the optimal contract, we have to understand how ρ and D affect the probability of a run. Thus, we solve explicitly for the cutoff θ_I^* as the signal that makes a depositor indifferent between withdrawing and waiting at $t = 1$.

The expected consumption of a depositor withdrawing at $t = 1$ is:

$$\theta_i D + (1-\theta_i) \min\left[D, \frac{\lambda R_L}{N_1(L)}\right]. \tag{A47}$$

The expected consumption of a depositor waiting at $t = 1$ is:

$$\theta_i \rho + (1-\theta_i) A^L \max[\frac{\lambda R_L - N_1(L) D}{1 - N_1(L)}, 0]. \tag{A48}$$

The payoff of waiting minus the payoff of running is:

$$\tilde{\nu}(\theta, n_1) = \begin{cases} \theta[\rho - D] + (1-\theta)[\frac{\lambda R_L - n_1 D}{1-n_1} A^L - D] & \text{if } \frac{\lambda R_L}{D} > n_1 \geq 0 \\ p(\theta)[\rho - D] - (1-\theta)\frac{\lambda R_L}{n_1} & \text{if } 1 \geq n_1 \geq \frac{\lambda R_L}{D} \end{cases} \tag{A49}$$

Integrating over the number of withdrawals in $t = 1$ yields:

$$\begin{aligned} 0 = & \theta_I^*(\rho - D) + (1-\theta_I^*)(A^L - 1)\lambda R_L + (1-\theta_I^*)\lambda R_L \ln(\frac{\lambda R_L}{D}) \\ & + (1 - \theta_I^* A^L)(D - \lambda R_L) \ln(1 - \frac{\lambda R_L}{D}). \end{aligned}$$

Rearranging, we obtain:

$$\theta_I^* = \frac{\phi_I}{(\rho - D) + \phi_I} \tag{A50}$$

and

$$1 - \theta_I^* = \frac{(\rho - D)}{(\rho - D) + \phi_I}, \tag{A51}$$

where $\phi_I = -(A^L - 1)\lambda R_L - \lambda R_L \ln\left(\frac{\lambda R_L}{D}\right) - A^L(D - \lambda R_L)\ln\left(1 - \frac{\lambda R_L}{D}\right)$.

Step 3: We can now analyze the impact of ρ and D on the ex ante run- probability. It is straightforward to see that:

$$\frac{\partial \theta_I^*}{\partial \rho} = -\frac{\phi_I}{D((\frac{\rho}{D}-1)+\phi_I)^2} < 0.$$

Increasing the payout ρ increases the expected utility of depositors ex ante as it increases their returns and reduces the incentive to run.

The impact of the contracted short-term payment is, however, less obvious:

$$\frac{d\theta_I^*}{dD} = \frac{\phi_I + (\rho - D)\frac{d\phi_I}{dD}}{((\rho - D)+\phi_I)^2}.$$

While the first term in the numerator is positive, the second term needs further analysis:

$$\frac{\partial \phi_I}{\partial D} = -(A^L - 1)\frac{\lambda R_L}{D} - A^L \ln\left(1 - \frac{\lambda R_L}{D}\right). \tag{A52}$$

The first term is negative and the second term is positive. To see that the sum is, nevertheless, positive for $D \in (\lambda R_L, R_L]$ given any possible $A^L = a_L + (1-a_L)/\lambda \geq 1$ with $a_L \in [0,1]$ and $\lambda \in (0,1)$, we first show that the whole term is decreasing in D:

$$\frac{\partial^2 \phi_I}{\partial D^2} = -\frac{\lambda R_L}{D^2}\frac{D + (A^L-1)\lambda R_L}{D - \lambda R_L} < 0.$$

As the derivative is unambiguously decreasing over the interval $(\lambda R_L, R_L]$, it must hold that $\frac{\partial \phi_I}{\partial D} > 0$ if we can show that $\frac{\partial \phi_I}{\partial D}|_{D=R_L} > 0$, which is true if:

$$-\lambda(1-a_L)(1-\lambda) - (a_L\lambda + (1-a_L))\ln(1-\lambda) > 0. \tag{A53}$$

The LHS of Equation (A53) is increasing in λ with slope equal to:

$$\begin{aligned} &-(1-a_L)(1-\lambda) + (1-a_L)\lambda + \frac{(1-a_L)+a_L\lambda}{1-\lambda} - a_L\ln(1-\lambda) \\ &= \left(2(1-a_L) + \frac{1}{1-\lambda}\right)\lambda - a_L\ln(1-\lambda) > 0 \end{aligned}$$

and approaches zero for $\lim \lambda \to 0$. Hence condition (A53) must hold for any $a_L \in [0,1]$ and $\lambda \in (0,1)$ such that $\frac{\partial \phi_L(A^L)}{\partial D}|_{D=R_L} > 0$. As $\frac{\partial \phi_I}{\partial D}$ decreases in D, but is positive for the highest value in the interval it must be positive in the whole interval. Therefore:

$$\frac{\partial \phi_I}{\partial D} > 0,$$

which implies that:

$$\frac{d\theta^*(A^L)}{dD} > 0.$$

A higher D increases the likelihood of a run, which decreases ex ante expected utility. As the direct effect is positive, the total effect of D on expected utility is ambiguous.

Step 4: We now examine the shape of U_I. Rewriting $\frac{\partial U_I}{\partial D}$, we get:

$$\frac{\partial U_I}{\partial D} = \frac{\theta_I^{*2}}{2} - \frac{d\theta_I^*}{dD}[(\rho - D)\theta_I^* + (1-\theta_I^*)(A^L - 1)\lambda R_L]. \tag{A54}$$

Substituting the derivative $\frac{d\theta_I^*}{dD}$ and using (A50) and (A51), yields:

$$\frac{\partial U_I}{\partial D} = \theta_I^*\{\frac{\theta_I^*}{2} - (\theta_I^* + (1-\theta_I^*)\frac{\partial \phi_I}{\partial D})(1-\theta_I^*)\left(1 + \frac{(A^L-1)\lambda R_L}{\phi_I}\right)\}.$$

Hence, the expected utility increases in D if:

$$\frac{\theta_I^*}{2} - \{\theta_I^* + (1-\theta_I^*)\frac{d\phi(A^L)}{dD}\}(1-\theta_I^*)(1+\Phi)) > 0 \tag{A55}$$

and decreases otherwise, where $\Phi = \frac{(A^L-1)\lambda R_L}{\phi(A^L)} > 0$, which is increasing in A^L with slope $\frac{\lambda R_L\left(\phi_I - (A^L-1)\frac{\partial \phi_I}{\partial A^L}\right)}{\phi_I^2} > 0$ because:

$$\frac{\partial \phi_I}{\partial A^L} = -(D - \lambda R_L)\ln\left(1 - \frac{\lambda R_L}{D}\right) - \lambda R_L < 0.$$

To see this, we divide the derivative by D and denote $\gamma = \frac{\lambda R_L}{D}$ for brevity. We need to show that $-(1-\gamma)\ln 1 - \gamma) < \gamma$. For $\lim_{\gamma \to 0}$ both terms go to zero. The RHS is increasing in γ with slope 1, while the LHS has a slope smaller than 1. Hence, the condition must hold for any $\gamma > 0$.

We now show that U_I is convex in D, and, thus, the optimal D is a corner solution within the interval. To show this, we first consider the case $a^L = 1$, implying that $A^L = 1$ such that $\Phi = 0$, meaning that we can rewrite condition A55 as:

$$\theta^*(0)(2\theta^*(0) - 1) > 2\frac{d\phi(0)}{dD}(1 - \theta^*(0))^2. \tag{A56}$$

The left-hand side is negative for $\theta^*(0) < \frac{1}{2}$ and positive otherwise. In contrast, the right-hand side is always positive. The depositor's expected utility is, therefore, decreasing in D for $\theta^*(0) < 1/2$.

The slope of the left-hand side of Equation (A56) is:

$$\frac{\partial(2\theta^*(0)^2 - \theta^*(0))}{\partial D} = \frac{\partial \theta^*(0)}{\partial D}(4\theta^*(0) - 1).$$

The left-hand side of Equation A56 is decreasing in D for $\theta^*(0) < 1/4$ and increasing in D for $\theta^*(0) > 1/4$. Hence, the left-hand side of Equation A56 is increasing in D whenever it is positive (i.e., when $\theta^*(0) > \frac{1}{2}$).

The right-hand side of Equation A56 is always positive and strictly decreasing in D:

$$\frac{\partial(\frac{d\phi(0)}{dD}(1-\theta^*(0))^2)}{\partial D} = -\frac{\lambda R_L}{D}\frac{(1-\theta^*(0))^2}{D-\lambda R_L} + 2\ln\left(1-\frac{\lambda R_L}{D}\right)(1-\theta^*(0))\frac{\partial\theta^*(0)}{\partial D} < 0.$$

As the left-hand side of Equation A56 is increasing when positive, and the right-hand side of Equation A56 is always positive and decreasing, and we know that $\frac{d\theta^*(0)}{dD}$ is positive, there must be a unique $\hat{D}$ for which the slope $\frac{\partial U_I}{\partial D}$ is zero.

If $\hat{D} > R_L$, the solution within this interval is $\lambda R_L + \varepsilon$, as the expected utility is unambiguously decreasing in D. If $\hat{D} \in (\lambda R_L, R_L)$, the solution for this interval is one of the endpoints. If $\hat{D} < \lambda R_L$, the solution for this interval is R_L, as the expected utility is increasing over the interval.

Step 5: Now we allow $A^L > 1$. That is, we consider the effect of a second-stage run probability $a^L \in (0,1)$ on the impact of D on the expected utility. Dividing the condition (A55) by $(1-\theta)$ and rewriting, we obtain:

$$\frac{1}{2}\frac{\phi_I}{\rho - D} > \{\theta_I^* + \frac{d\phi_I}{dD}(1-\theta_I^*)\}(1+\Phi)).$$

The left-hand side is decreasing in A^L because $\frac{\partial\phi_I}{\partial A^L} < 0$, as we showed above. We now examine how the right-hand side changes with an increase in A^L:

$$\frac{\partial\Phi}{\partial A^L}\{\theta_I^* + \frac{d\phi_I}{dD}(1-\theta_I^*)\} + \frac{\partial\{\theta_I^* + \frac{d\phi(A^L)}{dD}(1-\theta_I^*)\}}{\partial A^L}(1+\Phi)).$$

The first term is clearly positive, the second term can be positive or negative depending on λ and A^L. We can break down the second term into:

$$\frac{\partial\{\theta_I^* + \frac{d\phi(A^L)}{dD}(1-\theta_I^*)\}}{\partial A} = \frac{\partial\theta_I^*}{\partial A^L}\left(1-\frac{\partial\phi_I}{\partial D}\right) + \frac{\partial^2\phi_I}{\partial D\partial A^L}.$$

The second term is positive. The expression $\frac{\partial^2\phi_I}{\partial D\partial A^L} = -\frac{\lambda R_L}{D} - \ln(1-\frac{\lambda R_L}{D}) > 0$ for $\frac{\lambda R_L}{D} \in (0,1)$ given it is increasing in the term $\frac{\lambda R_L}{D}$ and zero for $\frac{\lambda R_L}{D} \to 0$.

The first term is positive if:

$$\frac{\partial\phi_I}{\partial D} > 1$$

because

$$\frac{\partial\theta_I^*}{\partial A^L} = \frac{(\rho-D)\frac{\partial\phi_I}{\partial A^L}}{(\rho-D)+\phi(A^L))^2} < 0.$$

Now we acknowledge that

$$\frac{\partial^2 \phi_I}{\partial D^2} = -\frac{(1-\frac{\lambda R_L}{D}) + A^L \frac{\lambda R_L}{D}}{1-\frac{\lambda R_L}{D}} < 0.$$

Second, the derivative also increases in A^L because

$$\frac{\partial^2 \phi_I}{\partial D \partial A^L} = -\frac{\lambda R_L}{D} - \ln\left(1-\frac{\lambda R_L}{D}\right) > 0,$$

which we showed to be positive above.

Now it must be the case that if $\frac{\partial \phi_I}{\partial D} > 1$ holds for the lowest possible A^L (that is, $A^L = 1$) and the highest D in the interval (that is, $D = R_L$), the expected utility is unambiguously decreasing in A^L. Using equation A52, we can write:

$$\frac{\partial \phi_I}{\partial D}|_{A^L=1, D=R_L} = -\ln(1-\lambda).$$

For this expression to be larger than 1, it must be that $\lambda > 1 - \frac{1}{\mathrm{e}}$, which is our Assumption A2.

The minimum of the expected utility function $\hat{D}_I(A^L)$ is implicitly defined by $\frac{\partial U_I(\hat{D}_I, A^L)}{\partial \hat{D}_I} = 0$. The impact of changes in $A^L > 1$ on the threshold value can be determined by the implicit function theorem:

$$\frac{d\hat{D}_I}{dA^L} = -\frac{\frac{\partial^2 U_I}{\partial \hat{D}_I \partial A^L}}{\frac{\partial^2 U_I}{\partial \hat{D}_I^2}} > 0$$

because $\frac{\partial^2 U_I}{\partial D^2} > 0$ and $\frac{\partial^2 U_I}{\partial D \partial A^L} < 0$, as shown above. The critical threshold value $\hat{D}_I(A^L)$ is increasing in A^L (and, hence, decreasing in a^L). This implies that the critical value for a full second-stage run is strictly smaller than the critical value with a partial run: $\hat{D}_I(1) < \hat{D}_I(a^L)$ if $a^L < 1$.

Finally, it is straightforward to show that the expected utility increases in A^L:

$$\frac{\partial U_I}{\partial A^L} = \int_{\theta_I^*}^{1} (1-\theta)\lambda R_L \, d\theta - ((\rho - D)\theta_I^* + (A-1)\lambda R_L(1-\theta_I^*)) \frac{\partial \theta_I^*}{\partial A^L} > 0$$

because $\frac{\partial \theta_I^*}{\partial A^L} < 0$, as shown above. This implies that the expected utility decreases in the probability of a second-stage run a^L. Defining $U_I(\rho, D, a^L)$, it must, therefore, hold that $U_I(R_H, R_L, 1) \leq U_I(R_H, D_I^*, a^L)$.

To summarize, the optimal (unconstrained) contract parameters in Case I are: $\rho^* = R_H$, and D^* is equal to either $\lambda R_L + \varepsilon$ or R_L.

Case II: $D \in (R_L, \lambda R_H]$. We proceed using the same approach as in Case I. Consider setting D slightly above R_L. This induces withdrawals for the low-quality asset such that none of it remains at $t = 3$. Hence, $a^L = 1$ and $A^L = 1$. This is the major change from Case I. We can, therefore, take the expression for expected utility there (Equation A57) and simplify it to:

$$U_{II} = \int_0^{\theta_I^*} \{\theta D + (1-\theta)\lambda R_L\} d\theta + \int_{\theta_I^*}^1 \{\theta\rho + (1-\theta)\lambda R_L\} d\theta, \tag{A57}$$

where $\theta^*(0)$ is the critical threshold defined in equation (7) of our main model with $\pi = 0$. The effect of the payout ρ is again unambiguously positive because a higher ρ reduces the probability of a run ($\frac{\partial\theta^*(0)}{\partial\rho} < 0$):

$$\frac{\partial U_{II}}{\partial \rho} = \int_{\theta^*(0)}^1 \theta\, d\theta - (\rho - D)\theta^*(0)\frac{\partial\theta^*(0)}{\partial\rho} > 0. \tag{A58}$$

The effect of the short-term payment D is ambiguous, giving us:

$$\frac{\partial U_{II}}{\partial D} = \int_0^{\theta^*(0)} \theta\, d\theta - (\rho - D)\theta^*(0)\frac{\partial\theta^*(0)}{\partial D}. \tag{A59}$$

Our main model gives (using our assumption here that $p(\theta) = \theta$):

$$\theta^*(0) = \frac{\phi(0)}{(\rho - D) + \phi(0)} \tag{A60}$$

and

$$1 - \theta^*(0) = \frac{(\rho - D)}{(\rho - D) + \phi(0)}, \tag{A61}$$

with $\phi(0) := -\lambda R_L \ln\left(\frac{\lambda R_L}{D}\right) - (D - \lambda R_L)\ln\left(1 - \frac{\lambda R_L}{D}\right)$. We note that:

$$\frac{d\phi(0)}{dD} = -\ln\left(1 - \frac{\lambda R_L}{D}\right) > 0.$$

We can now write the expression for $\frac{d\theta^*(0)}{dD}$:

$$\frac{d\theta^*(0)}{dD} = \frac{\phi(0) + (\rho - D)\frac{d\phi(0)}{dD}}{[(\rho - D) + \phi(0)]^2} > 0.$$

Rewriting $\frac{\partial U_{II}}{\partial D}$ from Equation A59, we get:

$$\frac{\partial U_{II}}{\partial D} = \theta^*(0)\{\frac{1}{2}\theta^*(0) - \frac{d\theta^*(0)}{dD}(\rho - D)\}. \tag{A62}$$

After inserting $\frac{d\theta^*(0)}{dD}$ and rearranging we obtain that the expected utility is increasing in D if condition (A56) holds for $D \in (R_L, \lambda R_H]$. We apply the same argumentation as in Case I, but we now consider the interval $D \in (R_L, \lambda R_H]$.

As the left-hand side of Equation A56 is increasing when positive, and the right-hand side of Equation A56 is always positive and decreasing in D, and we know that $\frac{d\theta^*(0)}{dD}$ is positive, there exists a $\hat{D}_{II}$ for which $\frac{\partial U_{II}}{\partial D} = 0$. Because the expected utility in Case II equals the expected utility in Case I if $a^L = 1$, it follows that $\hat{D}_{II} = \hat{D}_I(1)$. Moreover, it immediately follows that $\hat{D}_{II} < \hat{D}_I(a^L)$ for $a^L < 1$.

For $D < \hat{D}_{II}$, the slope $\frac{\partial U_{II}}{\partial D}$ is negative, and for $D > \hat{D}_{II}$, the slope $\frac{\partial U_{II}}{\partial D}$ is positive. This implies that there is no interior solution for the optimal unconstrained D in the interval $(R_L, \lambda R_H]$. If $\hat{D}_{II} \geq \lambda R_H$, the optimal D for this interval is $D = R_L$. If $\hat{D}_{II} \in (R_L, \lambda R_H)$, the optimal D is a corner solution. If $\hat{D} \leq R_L$, the optimal D for this interval is $D = \lambda R_H$. Note that since $\frac{\partial U_{II}}{\partial \rho} > 0$, the optimal $\rho = R_H$.

Therefore, we have proven that the solution for this interval is: $\rho = R_H$, and D is equal to either $R_L + \varepsilon$ or λR_H.

Comparing Case I and Case II, it becomes clear that the run probability resulting from the optimal contract in Case I is always smaller than the run probability induced by the optimal contract for Case II. To see this, first note that the run probability θ_I^* decreases in A^L (increases in a^L). But for the highest value $a^L = 1$, the run probabilities are equal $\theta_I^*|_{a_L=1} = \theta_{II}^*$ for any short-term payment D. The short-term payment $D_I \in (\lambda R_L, R_L]$ that is optimal in Case I, however, must be weakly smaller than the optimal rate $D_{II} \in (R_L, \lambda R_H]$. As θ_{II}^* increases in D, the run probability resulting from the optimal contract in Case II must be strictly greater than the run probability resulting from the optimal contract in Case I.

Moreover, denoting the expected utility in Cases I and II as $U_I(\rho, D)$ and $U_{II}(\rho, D)$, respectively, it is the case that $U_{II}(R_H, R_L) \leq U_I(R_H, D_I^*)$ for any a^L: the expected utility with $D_{II}^* = R_L$ is less than or equal to the expected utility achievable with any optimal contract in Case I.

Case III: $\lambda R_H < D < R_H$

Increasing D above λR_H leads to the possibility of multiple equilibria at $t = 2$ when the asset quality is H. Uninformed depositors then can either all run or all wait. Consider the situation in which depositors expect with probability a^H that all depositors run at $t = 2$, and with probability $1 - a^H$ that all depositors wait at $t = 2$. As discussed earlier, this could be due to a sunspot.

The expected utility in this case can be summarized as:

$$U_{III} = \int_0^{\theta^*_{III}} \{\theta\lambda R_H + (1-\theta)\lambda R_L\} d\theta + \int_{\theta^*_{III}}^1 \{\theta(a^H\lambda R_H + (1-a^H)\rho) + (1-\theta)\lambda R_L\} d\theta. \tag{A63}$$

If $D > \lambda R_H$, the positive direct effect of D on the expected utility vanishes because now the high-quality asset also needs to be entirely liquidated in case of a bank run (because the liquidation value λR_H does not cover the liability to depositors D). Hence, an increase of D affects only the probability of a run on the bank:

$$\frac{\partial U_{III}}{\partial D} = -(1-a^H)(\rho - \lambda R_H)\theta^*_{III}\frac{\partial \theta^*_{III}}{\partial D} \tag{A64}$$

and

$$\frac{\partial U_{III}}{\partial \rho} = (1-a^H)\left(\int_{\theta^*_{III}(\rho,D)}^1 d\theta - (\rho - \lambda R_H)\theta^*_{III}\frac{\partial \theta^*_{III}}{\partial \rho}\right). \tag{A65}$$

To understand the effect of ρ and D on the probability of a run on the bank, we need to solve for the threshold θ^*_{III} in this case. We proceed in two steps. The first step is to evaluate $\theta^*_{III} \mid_{a^H=0}$, which we will denote $\theta^*_{III}(0)$, as the case in which all depositors wait.

Step 1: The expected consumption of a depositor withdrawing at $t = 1$ is:

$$\theta_i \min\left[D, \frac{\lambda R_H}{n_1}\right] + (1-\theta_i)\min\left[D, \frac{\lambda R_L}{n_1}\right]. \tag{A66}$$

The expected consumption of a depositor waiting at $t = 1$ is:

$$\theta_i \min[\rho, \max(\frac{R_H - \frac{n_1 D}{\lambda}}{1-n_1}, 0)] + (1-\theta_i)\max[\frac{\lambda R_L - n_1 D}{1-n_1}, 0]. \tag{A67}$$

This gives three cutoffs for withdrawals in the first stage: if $n_1 = \frac{R_H-\rho}{\frac{D}{\lambda}-\rho}$, the promised payment ρ at $t = 3$ equals the residual value of the high-quality asset after liquidating a fraction $n_1\frac{D}{\lambda R_H}$ at $t = 1$. Note that this value approaches zero if $\rho = R_H$ and equals 1 for $D = \lambda R_H$ given any $\rho > D$. The second cutoff, $n_1 = \frac{\lambda R_H}{D}$, defines the withdrawal amount that forces the full liquidation of the hiqh quality asset. Note that $\frac{R_H-\rho}{\frac{D}{\lambda}-\rho} \leq \frac{\lambda R_H}{D}$ with both being equal to 1 for $D = \lambda R_H$. The final cutoff is the number of withdrawals that force the bank into full liquidation of the low-quality asset, $n_1 = \frac{\lambda R_L}{D}$. We note that $\frac{R_H-\rho}{\frac{D}{\lambda}-\rho}$ can be greater or smaller than $\frac{\lambda R_L}{D}$ depending on ρ and D. We, therefore, distinguish two sets of parameters.

Parameter Set A: $\frac{R_H-\rho}{\frac{D}{\lambda}-\rho} \in \left(0, \frac{\lambda R_L}{D}\right)$ We define the payoff of waiting minus the payoff of running as:

$$\hat{\nu}(\theta, n_1) = \begin{cases} \theta[\rho - D] & \text{if } \frac{R_H-\rho}{\frac{D}{\lambda}-\rho} > n_1 \geq 0 \\ +(1-\theta)[\frac{\lambda R_L - n_1 D}{1-n_1} - D] & \\ \theta[(\frac{\lambda R_H - n_1 D}{1-n_1}) - D] & \text{if } \frac{\lambda R_L}{D} > n_1 \geq \frac{R_H-\rho}{\frac{D}{\lambda}-\rho} \\ +(1-\theta)[\frac{\lambda R_L - n_1 D}{1-n_1} - D] & \\ \theta[(\frac{\lambda R_H - n_1 D}{1-n_1}) - D] & \text{if } \frac{\lambda R_H}{D} \geq n_1 \geq \frac{\lambda R_L}{D} \\ -(1-\theta)\frac{\lambda R_L}{n_1} & \\ -\theta\frac{\lambda R_H}{n_1} & \text{if } 1 \geq n_1 \geq \frac{\lambda R_H}{D} \\ -(1-\theta)\frac{\lambda R_L}{n_1} & \end{cases} . \tag{A68}$$

We can now compute the threshold signal θ^*_{III} for which the uninformed depositor is indifferent between waiting and withdrawing.

$$\int_0^{\frac{R_H-\rho}{\frac{D}{\lambda}-\rho}} \{\theta^*_{III}(\rho - D)\} dn - \int_0^{\frac{\lambda R_L}{D}} \{(1-\theta^*_{III})\frac{D-\lambda R_L}{1-n}\} dn \tag{A69}$$
$$- \int_{\frac{R_H-\rho}{\frac{D}{\lambda}-\rho}}^{\frac{\lambda R_H}{D}} \{\theta^*_{III}(0)\frac{D-\lambda R_H}{1-n}\} dn$$
$$- \int_{\frac{\lambda R_L}{D}}^{1} \{(1-\theta^*_{III}(0))\frac{\lambda R_L}{n}\} dn - \int_{\frac{\lambda R_H}{D}}^{1} \{\theta^*_{III}(0)\frac{\lambda R_H}{n}\} dn = 0.$$

Integration yields:

$$\begin{aligned} 0 \;=\; & \theta^*_{III}(0)(\rho - D)\frac{R_H-\rho}{\frac{D}{\lambda}-\rho} + \theta^*_{III}(0)(D-\lambda R_H)\ln(1-\lambda\frac{\rho}{D}) + \theta^*_{III}(0)\lambda R_H \ln(\frac{\lambda R_H}{D}) \\ & +(1-\theta^*_{III}(0))(D-\lambda R_L)\ln(1-\frac{\lambda R_L}{D}) + (1-\theta^*_{III}(0))\lambda R_L \ln(\frac{\lambda R_L}{D}). \end{aligned}$$

Rearranging yields:

$$\theta^*_{III}(0) = \frac{\phi(0)}{\phi(0) + \xi(\rho, D)}$$

with:

$$\xi(\rho, D) = (\rho - D)\frac{R_H-\rho}{\frac{D}{\lambda}-\rho} + (D-\lambda R_H)\ln(1-\frac{\lambda\rho}{D}) + \lambda R_H \ln(\frac{\lambda R_H}{D}),$$

and $\phi(0)$ as in Equation (A60), which is the same as in our main model for $\pi = 0$.

The derivative with respect to D is given by:

$$\frac{\partial \theta^*_{III}(0)}{\partial D} = \frac{\frac{d\phi(0)}{dD}}{\phi(0)+\xi(\rho,D)} - \frac{\phi(0)(\frac{d\phi(0)}{dD}+\xi_D(\rho,D))}{(\phi(0)+\xi(\rho,D))^2}. \tag{A70}$$

We have already shown in Case II that $\frac{d\phi(0)}{dD} = -\ln\left(1-\frac{\lambda R_L}{D}\right) > 0$ is strictly positive for any $D > \lambda R_L$. Therefore, the first term is positive. The second term is positive if $\frac{d\phi(0)}{dD} + \xi_D(\rho,D) < 0$. To show this, first we write out $\xi_D(\rho,D)$:

$$\xi_D(\rho,D) = -\frac{\lambda(R_H-\rho)(\rho+D-2\lambda\rho)}{(D-\lambda\rho)^2} + \ln(1-\frac{\lambda\rho}{D}) < 0.$$

Both terms are negative for $\rho \leq R_H$ and $D > \lambda R_H$. The sum $\frac{d\phi(0)}{dD} + \xi_D(\rho,D) < 0$ is also negative because $\ln(1-\frac{\lambda\rho}{D}) - \ln\left(1-\frac{\lambda R_L}{D}\right) = \ln\left(\frac{D-\lambda\rho}{D-\lambda R_L}\right)$ is strictly negative given that $\frac{D-\lambda\rho}{D-\lambda R_L} < 1$. Therefore, in Case III, the probability of a bank run is also strictly increasing in the contracted short-term payment:

$$\frac{\partial \theta^*_{III}(0)}{\partial D} > 0.$$

Given that in Case III, the short-term payment D only indirectly affects the expected utility via the bank run probability, it immediately follows that:

$$\frac{\partial U_{III}}{\partial D} < 0.$$

The optimal short-term rate is the infimum of the set of possible short-term payments in Case III: $D^*_{III} = \lambda R_H + \varepsilon$.

The derivative with respect to ρ is

$$\frac{\partial \theta^*_{III}(0)}{\partial \rho} = -\frac{\phi(0)\xi_\rho(\rho,D)}{(\phi(0)+\xi(\rho,D))^2}. \tag{A71}$$

Inserting the optimal short-term payment $\lim_{\varepsilon\to 0} D^*_{III} = \lambda R_H$ into $\xi(\rho,\lambda R_H)$ simplifies that expression to $\rho - \lambda R_H > 0$. The derivative with respect to ρ is, therefore, equal to $\xi_\rho(\rho,\lambda R_H) = 1 > 0$. Thus, the run probability decreases in the long-term return ρ, and, thus, the expected return is maximized by setting $\rho^* = R_H$.

We will now demonstrate that the maximum expected utility that can be achieved in Case II is at least as high as the expected utility achieved with the optimal contract for Case III - i.e., $U_{II}(R_H, D^*_{III}) \geq U_{III}(R_H, \lambda R_H + \varepsilon)$. For this, we first demonstrate that $\theta^*_{III}(0) \geq \theta^*(0)$:

$$\frac{\phi(0)}{\xi(\rho,D)+\phi(0)} \geq \frac{\phi(0)}{(\rho-D)+\phi(0))}.$$

This holds since $\xi(\rho, D) < (\rho - D)$; that is:

$$(\frac{\rho}{D} - 1)\frac{R_H - \rho}{\frac{D}{\lambda} - \rho} + (1 - \frac{\lambda R_H}{D})\ln(1 - \frac{\lambda\rho}{D}) + \frac{\lambda R_H}{D}\ln(\frac{\lambda R_H}{D}) < \frac{\rho}{D} - 1.$$

This holds given that $\frac{R_H-\rho}{\frac{D}{\lambda}-\rho} < 1$ and $(1 - \frac{\lambda R_H}{D})\ln(1 - \frac{\lambda\rho}{D}) + \frac{\lambda R_H}{D}\ln(\frac{\lambda R_H}{D}) < 0$ for $D \in (\lambda R_H, R_H)$. Given that expected utility strictly decreases in the run probability, it must, therefore, be true that $U_{II}(R_H, \lambda R_H) \geq U_{III}(R_H, \lambda R_H + \varepsilon)$.

Hence, $U_{II}(R_H, D^*_{II}) \geq U_{II}(R_H, \lambda R_H) \geq U_{III}(R_H, \lambda R_H + \varepsilon)$ when $a^H = 0$; the highest achievable utility in Case III is strictly dominated by the highest achievable utility in Case II. The intuition is straightforward: as liquidations of the high-quality asset are inefficient, setting $D > \lambda R_H$ cannot increase the expected utility since it increases only the incentives to run without increasing the expected utility directly. Any $a^H > 0$ decreases U_{III} such that the achievable utility in Case III is strictly lower than the achievable utility in Case II.

The same remains true for Parameter Set B:

Parameter Set B: $\frac{R_H-\rho}{\frac{D}{\lambda}-\rho} \in \left(\frac{\lambda R_L}{D}, \frac{\lambda R_H}{D}\right)$ We will see that this case yields the same result as Parameter Set A, as the relative size of $\frac{\lambda R_L}{D} < \frac{R_H-\rho}{\frac{D}{\lambda}-\rho}$ does not affect the expected payoff in each case. To see this, we again define the payoff of waiting minus the payoff of running as:

$$\hat{\nu}(\theta, n_1) = \begin{cases} \theta[\rho - D] & \text{if } \frac{\lambda R_L}{D} > n_1 \geq 0 \\ +(1-\theta)[\frac{\lambda R_L - n_1 D}{1-n_1} - D] & \\ p(\theta)[\rho - D] & \text{if } \frac{R_H-\rho}{\frac{D}{\lambda}-\rho} > n_1 \geq \frac{\lambda R_L}{D} \\ -(1-\theta)\frac{\lambda R_L}{n_1} & \\ \theta[(\frac{\lambda R_H - n_1 D}{1-n_1}) - D] & \text{if } \frac{\lambda R_H}{D} > n_1 \geq \frac{R_H-\rho}{\frac{D}{\lambda}-\rho} \\ -(1-\theta)\frac{\lambda R_L}{n_1} & \\ -\theta\frac{\lambda R_H}{n_1} & \text{if } 1 \geq n_1 \geq \frac{\lambda R_H}{D} \\ -(1-\theta)\frac{\lambda R_L}{n_1} & \end{cases} \quad \text{(A72)}$$

The threshold signal θ^*_{IIIB} makes the uninformed depositor indifferent between waiting and withdrawing:

$$\int_0^{\frac{R_H-\rho}{\frac{D}{\lambda}-\rho}} \{\theta^*_{IIIB}(\rho - D)\}dn - \int_0^{\frac{\lambda R_L}{D}} \{(1-\theta^*_{IIIB})\frac{D-\lambda R_L}{1-n}\}dn$$
$$- \int_{\frac{R_H-\rho}{\frac{D}{\lambda}-\rho}}^{\frac{\lambda R_H}{D}} \{\theta^*_{IIIB}\frac{D-\lambda R_H}{1-n}\}dn$$
$$- \int_{\frac{\lambda R_L}{D}}^1 \{(1-\theta^*_{IIIB})\frac{\lambda R_L}{n}\}dn - \int_{\frac{\lambda R_H}{D}}^1 \{\theta^*_{IIIB}\frac{\lambda R_H}{n}\}dn = 0. \quad (A73)$$

This is identical to ($A69$), implying that $\theta^*_{IIIB} = \theta^*_{III}$. The relative size of threshold points $\frac{R_H-\rho}{\frac{D}{\lambda}-\rho}$ and $\frac{\lambda R_L}{D}$ do not matter because the first affects only the payoff of the high-quality asset and the latter only the payoff of the low-quality asset. As both cases result in the same expected payoffs, we do not have to analyze them separately.

We now proceed to the second step.

Step 2: We have already shown that in the "best case scenario," where $a^H = 0$, the expected utility in Case III is strictly lower than the expected utility in Case II. It is straightforward to show that $a^H > 0$ decreases only the expected utility that can be achieved in Case III.

When $a^H > 0$, the expected consumption of a depositor withdrawing at $t = 1$ is the same as in the case where $a^H = 0$ - i.e., Equation A66 still holds.

The expected consumption of a depositor waiting at $t = 1$ is:

$$\theta_i(a^H \max[\frac{\lambda R_H - n_1 D}{1-n_1}, 0] + (1-a^H)\min[\rho, \max(\frac{R_H - \frac{n_1 D}{\lambda}}{1-n_1}, 0)])$$
$$+(1-\theta_i)\max[\frac{\lambda R_L - n_1 D}{1-n_1}, 0]. \quad (A74)$$

The payoff of waiting at $t = 1$ in the equation (A74) is strictly lower than the payoff of waiting at $t = 1$ when $a^H = 0$ (Equation A67). Therefore, the threshold signal that makes the depositor indifferent between running and not running must be larger than the probability of running when all depositors wait at $t = 2$, $\theta^*_{III}(a^H > 0) > \theta^*_{III}$. Furthermore, given that $\theta^*_{III} > \theta^*(0)$, this implies that $\theta^*_{III}(a^H > 0) > \theta^*(0)$. Overall, an increase in a_H has two negative effects on the ex-ante expected utility. First, it increases the run probability. Second, it reduces the expected return if no first-stage run occurs because depositors receive $\lambda R_H < \rho$ with probability a_H instead of ρ. Therefore, the expected utility is strictly decreasing in the probability of second-stage runs $a^H \in [0, 1]$. Formally:

$$\frac{\partial U_{III}}{\partial a^H} = -\left(\int_{\theta^*_{III}(a^H)}^1 (\rho - \lambda R_H)\theta d\theta + (1-a^H)(\rho - \lambda R_H)\theta^*_{III}(a^H)\frac{\partial \theta^*_{III}(a^H)}{\partial a_H}\right) < 0$$

because $\frac{\partial \theta^*_{III}(a^H)}{\partial a_H} > 0$ and $\rho > \lambda R_H$.

The Optimal Contract We now write down the optimal contract. The expected utility in all cases is increasing in ρ, while the optimal short-term payment is pinned down by corner solutions. Therefore, the optimal payout must be $\rho^* = R_H$, the highest feasible rate that can be credibly promised to depositors.

The constrained optimal short-term payment D^* depends on $\hat{D}$ and $\bar{z}$.

First, we define

$$\theta^*(R_H, D) = \begin{cases} \theta^*_I(R_H, D, a^L) & \text{if } D \in (\lambda R_L, R_L] \\ \theta^*_{II}(R_H, D) & \text{if } D \in (R_L, \lambda R_H] \\ \theta^*_{III}(R_H, D, a^H) & \text{if } D \in (\lambda R_H, R_H] \end{cases} . \tag{A75}$$

We have shown that $\theta^*(R_H, D)$ is unambiguously increasing in D. We can now analyze the different parameter spaces. For brevity, we write the utility for each interval as $U_I(D, a^L)$, $U_{II}(D)$, and $U_{III}(D, a^H)$, omitting the argument $\rho^* = R_H$.

1. If $\hat{D}_{II} > \lambda R_H$ the optimal short-term payment is either

$$D^* = \lambda R_L + \varepsilon \text{ if } \int_0^{\theta^*(R_H, \lambda R_L + \varepsilon)} \theta d\theta \geq \bar{z}(\delta, c)$$

or

$$D^* > \lambda R_L + \varepsilon \text{ solving } \int_0^{\theta^*(R_H, D^*)} \theta d\theta = \bar{z}(\delta, c)$$

if a solution exists that prevents absconding. Otherwise, banking is not viable.

If $\hat{D}_{II} > \lambda R_H$, the expected utility is decreasing in D in all three intervals. Given that $\hat{D}_I(a^L) > \hat{D}_{II}$, we showed that $U_I(R_L, a_L) \geq U_{II}(R_L)$ in the discussion at end of Case II. As $U_{II}(D)$ is decreasing in D, it must also hold that $U_{II}(R_L) > U_{II}(\lambda R_H) \geq U_{III}(\lambda R_H, a^H)$. We also proved that $U_{III}(D)$ is decreasing in D. The optimal short-term rate is $D^* = \lambda R_L + \varepsilon$ if the implied run probability deters the banker from absconding. If the run probability is too low to deter absconding, the optimal contract is implicitly defined by the binding constraint.

We run a simulation to make sure that the constrained optimal short-term payment assumed in our main model $D \in (R_L, \lambda R_H]$ can, indeed, be a solution for our modified model. The first case is illustrated in Figures 7a and 7b. All assumptions hold for the parameterization: $R_H = 1.11$; $R_L = 0.95$; $\lambda = 0.86$;

$a^L = 1$; $a^H = 0$; and $\bar{R} = 0.99$. If we set $\delta = 0.5$ and $C = 1.5$, the resulting constrained optimal short-term rate is $\bar{D} \in (R_L, \lambda RH)$, as assumed in our main model.

2. If $\hat{D}_{II} < \lambda R_H$ and $U_I(D_I^*) < U_{II}(D_{II}^*)$, the optimal short-term rate is either

$$D^* = \lambda R_H \text{ if } \int_0^{\theta^*(R_H,\lambda R_H)} \theta d\theta \geq \bar{z}(\delta, c)$$

or

$$\bar{D} > \lambda R_H \text{ which solves } \int_0^{\theta^*(R_H,\bar{D})} \theta d\theta = \bar{z}(\delta, c)$$

if a solution exists that prevents absconding. Otherwise, banking is not viable.

We showed at the end of Case II that $U_I(D_I^*) \geq U_{II}(R_L)$. Therefore, $U_I(D_I^*) < U_{II}(D_{II}^*)$ implies that $D_{II}^* = \lambda R_H$. If the implied run probability that deters the manager from absconding is above the threshold $\bar{z}(\delta, c)$, this constitutes the optimal contract. Otherwise, the short-term payment that just makes the constraint bind constitutes the constrained optimal rate, as the expected utility strictly decreases in $D > \lambda R_H$.

We illustrate this case in Figures 8a and 8b. All assumptions hold for the parameterization: $R_H = 1.01$; $R_L = 0.99$; $\lambda = 0.99$; $a^L = 0.5$; $a^H = 0$; $\bar{R} = 0.99$; and $\delta = 0.5$. The optimal short-term payment is $D^* = \lambda R_H$ if $C > 0.893$, such that the resulting bank run probability that successfully deters the manager from absconding is not too high.

3. If $\hat{D}_{II} < \lambda R_H$ and $U_I(D_I^*) > U_{II}(D_{II}^*)$, we have to distinguish two subcases:

   (a) If $D_{II}^* = R_L$

$$D^* = D_I^* \text{ if } \int_0^{\theta^*(R_H,D_I^*)} \theta d\theta \geq \bar{z}(\delta, c). \tag{A76}$$

   Consider first the case in which $D_I^* = \lambda R_L$. If $\int_0^{\theta^*(R_H,\lambda R_L)} \theta d\theta \geq \bar{z}(\delta, c)$ is violated, then the solution may be:

$$D^* = R_L \text{ if } \int_0^{\theta^*(R_H,R_L)} \theta d\theta \geq \bar{z}(\delta, c) \tag{A77}$$

   or

$$D^* = \bar{D} < R_L, \text{ where } \int_0^{\theta^*(R_H,\bar{D})} \theta d\theta = \bar{z}(\delta, c). \tag{A78}$$

If $\int_0^{\theta^*(R_H,R_L)} \theta d\theta \geq \bar{z}(\delta,c)$ is violated, the short-term payment is defined by the binding constraint:

$$D^* = \bar{D} \in (R_L, \lambda R_H) \text{ where } \int_0^{\theta^*(R_H,\bar{D})} \theta d\theta = \bar{z}(\delta,c) \tag{A79}$$

or

$$D^* = \lambda R_H \text{ if } \int_0^{\theta^*(R_H,\lambda R_H)} \theta d\theta \geq \bar{z}(\delta,c), \tag{A80}$$

depending on whether $U_{II}(\bar{D})$ is larger or smaller than $U_{II}(\lambda R_H)$.

If $\int_0^{\theta^*(R_H,\lambda R_H)} \theta d\theta \geq \bar{z}(\delta,c)$ is violated, the short-term payment is given by:

$$D^* = \bar{D} \text{ where } \int_0^{\theta^*(R_H,\bar{D})} \theta d\theta = \bar{z}(\delta,c) \tag{A81}$$

if a solution exists that prevents absconding. Otherwise, banking is not viable.

Now consider the case in which $D_I^* = R_L$. If $\int_0^{\theta^*(R_H,D_I^*)} \theta d\theta \geq \bar{z}(\delta,c)$ is violated, then the solution may be Condition A79 or Condition A80, depending on whether $U_{II}(\bar{D})$ is larger or smaller than $U_{II}(\lambda R_H)$. If $\int_0^{\theta^*(R_H,\lambda R_H)} \theta d\theta \geq \bar{z}(\delta,c)$ is violated, the short-term payment is given by Condition A81 if a solution exists that prevents absconding. Otherwise, banking is not viable.

We illustrate this parameter space in Figures 9a and 9b. The parameterization is $R_H = 1.14$; $R_L = 0.99$; $\lambda = 0.87$; $a^L = 1$; $a^H = 0$; $\bar{R} = 0.99$; $\delta = 0.5$; and $C = 2$, and all assumptions hold.

(b) If $D_{II}^* = \lambda R_H$ and $U_I(R_L) > U_{II}(\lambda R_H)$, the optimal short-term payment is defined in part (a). If $U_I(R_L) < U_{II}(\lambda R_H)$, the optimal short-term payment is:

$$D^* = \lambda R_L \text{ if } \int_0^{\theta^*(R_H,\lambda R_L)} \theta d\theta \geq \bar{z}(\delta,c). \tag{A82}$$

If $\int_0^{\theta^*(R_H,\lambda R_L)} \theta d\theta \geq \bar{z}(\delta,c)$ is violated, the optimal short-term payment is:

$$D^* = \lambda R_H \text{ if } \int_0^{\theta^*(R_H,\lambda R_H)} \theta d\theta \geq \bar{z}(\delta,c). \tag{A83}$$

If $\int_0^{\theta^*(R_H,\lambda R_H)} \theta d\theta \geq \bar{z}(\delta,c)$ is violated, the optimal short-term payment is:

$$\bar{D} \in (\lambda R_H, R_H) \text{ such that } \int_0^{\theta^*(R_H,\bar{D})} \theta d\theta = \bar{z}(\delta,c). \tag{A84}$$

Otherwise, banking is not viable.

We illustrate this parameter space in Figures 10a and 10b. The parameterization is $R_H = 1.05$; $R_L = 0.95$; $\lambda = 0.91$; $a^L = 0.5$; $a^H = 0$; $\bar{R} = 0.99$; $\delta = 0.5$; and $C = 2$, and all assumptions hold.

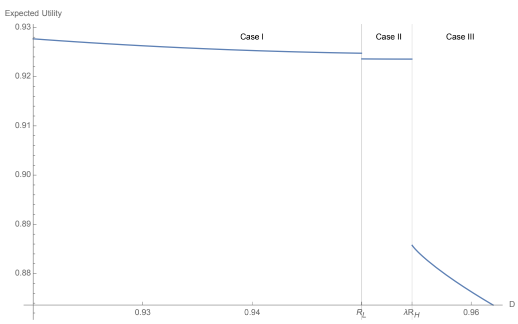
If $\bar{z}(\delta, C)$ is so high that $U_I(\bar{D})$ is lower than $U_{II}(\lambda R_H)$, this case also results in $D^* = \lambda R_H$ as the constrained optimal short-term rate.

Finally, any optimal contract must satisfy Assumption 2 to be ex-ante viable, which gives an upper bound on D. The short-term rate cannot exceed the expected long-term proceeds ex-ante.

Figure 7: Optimal Contract with $\hat{D}_{II} > \lambda R_H$.

The parameters used are $R_H = 1.11$, $R_L = 0.95$, $\lambda = 0.86$, $a^L = 0.9$, $a^H = 0$.

(a) The Expected Utility.



(b) The Run Probability for a given D.

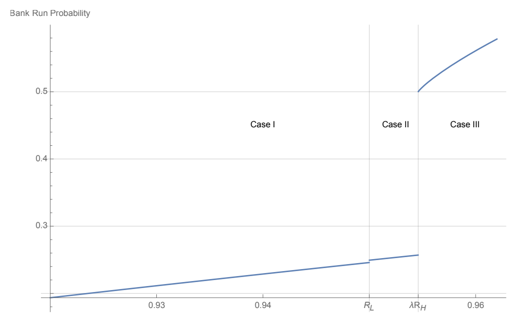
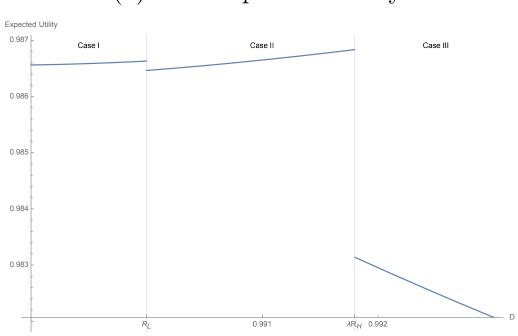


Figure 8: Optimal Contract with $\hat{D}_{II} < \lambda R_H$ and $U_I(D_I^*) < U_{II}(D_{II}^*)$.

The parameters used are $R_H = 1.01$, $R_L = 0.99$, $\lambda = 0.99$, $a^L = 0.5$, $a^H = 0$.

(a) The Expected Utility.



(b) The Run Probability for a given D.

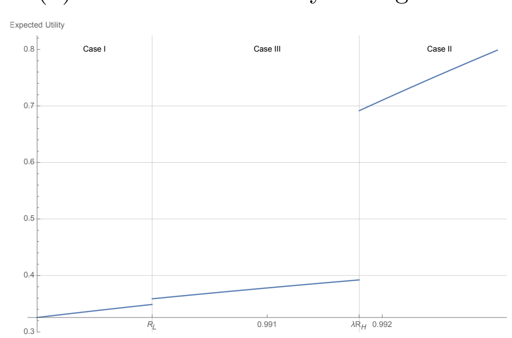
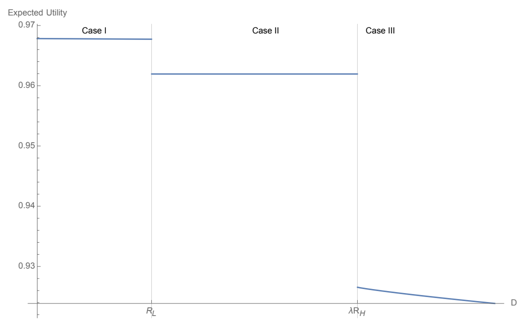


Figure 9: Optimal Contract with $\hat{D}_{II} < \lambda R_H$, $U_I(D_I^*) > U_{II}(D_{II}^*)$ and $D_{II}^* = R_L$. The parameters used are $R_H = 1.14$, $R_L = 0.99$, $\lambda = 0.87$, $a^L = 0.5$, $a^H = 0$.

(a) The Expected Utility.



(b) The Run Probability for a given D.

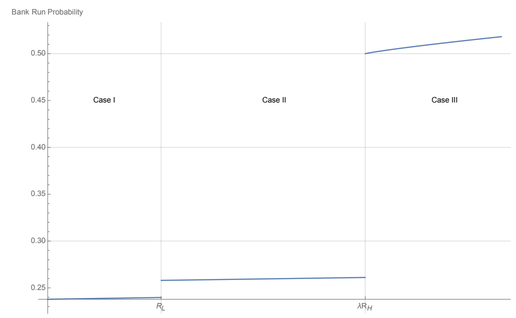
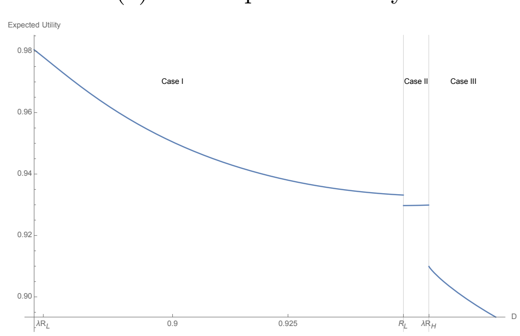


Figure 10: Optimal Contract with $\hat{D}_{II} < \lambda R_H$, $U_I(D_I^*) > U_{II}(D_{II}^*)$ and $D_{II}^* = \lambda R_H$. The parameters used are $R_H = 1.05$, $R_L = 0.95$, $\lambda = 0.91$, $a^L = 0.5$, $a^H = 0$..

(a) The Expected Utility.



(b) The Run Probability for a given D.