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Abstract

This paper conducts general equilibrium (GE) estimation to evaluate the empirical contributions of macroeconomic shocks in explaining the exchange rate disconnect, excess volatility, and the uncovered interest parity (UIP) puzzles. We embed stochastic volatilities and limits-to-international arbitrage in a two-country New Keynesian model and estimate the GE system for the US and Euro area using higher-order approximation and full-information Bayesian methods. Assessing the roles of level vs. volatility shocks and linear vs. higher-order approximations, we find that shocks to macroeconomic fundamentals together with their uncertainties can account for a sizable portion—over 40%—of the observed exchange rate variations. Using the GE estimates, we then evaluate whether the fundamental shocks in our model can deliver the UIP relationship observed in the data, and more importantly, whether the results may differ conditionally vs. unconditionally. In line with findings in previous literature, several fundamental shocks individually can indeed generate patterns consistent with data. However, their contributions unconditionally in the GE setting are quantitatively insufficient to resolve the UIP puzzle. The presence of multiple shocks, their potential interactions, and the need for estimators to fit empirical dynamics of all observables beyond just the exchange rate are all likely reasons behind this "General Equilibrium Puzzle," which underscores the importance of GE estimation beyond simulations or partial-equilibrium analyses.

JEL Classification: E52, F31, F41

Keywords: Exchange rate, Risk premium, International Risk Sharing, stochastic volatility, nonlinear estimation

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1 Introduction

The nominal exchange rate is an important driver of aggregate fluctuations and a key link in the global goods and asset markets. Theoretical models, whether in general equilibrium (GE) or reduced-form, commonly rely on exchange rate adjustments to transmit policy impacts and macroeconomic shocks across country borders. On the empirical front, however, evidence for the proposed macroeconomic transmission mechanisms through the exchange rate channel remains thin to non-existent. For example, estimations of GE models typically find fluctuations in nominal exchange rates to be unrelated to macroeconomic forces, a pattern that reflects the general *exchange rate disconnect* documented in the literature.[1]

This empirical disconnect between the exchange rate and its standard macroeconomic determinants has manifested in numerous puzzles and research agendas. In this paper, we re-evaluate the exchange rate-macro connection from the GE perspective by estimating a full-fledged two-country New Keynesian dynamic stochastic general equilibrium (DSGE) model using quarterly US and Euro area data between 1987 to 2008.[2] In addition to the level or first-moment macroeconomic shocks considered in the literature (shocks to technology, aggregate demand, and monetary policy), our model incorporates recursive preferences *à la* Epstein and Zin (1989) and two additional sources of exchange rate fluctuations: (i) macroeconomic volatility shocks that can generate an endogenous time-varying currency risk premium; and (ii) a direct shock to the exchange rate or the international risking-sharing condition. Using a full-information Bayesian approach and the central-difference Kalman filter, we estimate the nonlinear DSGE system approximated up to the 3rd-order, and evaluate how the two sources of shocks—macro vs. direct—contribute to explaining the excess exchange rate volatility and the uncovered interest rate parity (UIP) puzzle observed in the data. In particular, our setup allows us to assess systematically the roles of linear vs. higher order model approximations, level vs. stochastic volatility shocks to the macroeconomic environment, as well as how these shocks, conditionally and unconditionally, contribute to explaining the patterns observed in the data.

By juxtaposing macro volatility shocks with a direct shock to the exchange rate, our aim is to assess empirically the extent to which exchange rate may be connected *nonlinearly* to the macroeconomy.[3] While both types of shocks have been emphasized in previous work, we bring their evaluations to the GE setting and let the data distinguish their relative contributions directly. The GE approach also delivers estimates for key structural

[1] See, for example, Lubik and Schorfheide (2006). A notable exception is Adolfson et al. (2007) for the small open economy with rather ad-hoc adjustment costs to capture risks in exchange rates.

[2] By connecting the exchange rate to the macroeconomy, we do not mean generating counterfactual patterns such as direct comovements between the exchange rate and the macro fundamentals, or resolving the Meese and Rogoff (1983) forecast puzzle. We mean finding empirical evidence that the exchange rate responds to shocks to its macroeconomic determinants or fundamentals, as predicted by theory.

[3] We note that the shocks we consider, while standard, are by no means exhaustive. Other macroeconomic channels, such as through news shocks or endogenous financial frictions, may also influence exchange rate behavior. Alvarez et al. (2009), for example, incorporate limited participation and costs of entry to *connect* monetary policy with the currency risk premium.

parameters, allowing us to assess whether their relative magnitudes are consistent with theory. Last but not least, our GE evaluations fill in a crucial step commonly skipped in the literature: we test whether structural mechanisms that demonstrate explanatory power *conditionally* in partial equilibrium settings retain their quantitative relevance *unconditionally* under GE analyses. Since multiple shocks are present and allowed to interact in GE, and the estimators need to fit a broader set of data, these additional and more realistic dimensions may alter the relevance of certain shocks in resolving, for example, the UIP puzzle. In this endeavor, we uncover what we term the "General Equilibrium Puzzle" in the exchange rate behavior, which we elaborate below.[4]

In reconnecting the exchange rate with the macroeconomy, we focus on explaining two salient empirical regularities in the exchange rate literature. The UIP condition is one of the fundamental building blocks of modern open economy models. It states that in the absence of risk considerations, countries with relatively high interest rates should expect subsequent currency depreciations to ensure zero expected excess returns, or no arbitrage, from cross-border financial investments. However, as is well-known since Fama (1984), empirical evidence consistently rejects this view, giving rise to the UIP puzzle, along with, for example, the positive returns from carry-trade strategies that invest in high interest rate currencies with funding from low-interest rate ones. There have been numerous attempts to resolve the UIP puzzle, though as pointed out in Itskhoki and Mukhin (2017), any proposed solutions must also account for the high volatilities present in the exchange rates, but absent in other macroeconomic variables. This is a second feature we aim to replicate in our model.

We compare the relative contributions of a direct exchange rate shock and other fundamental shocks in explaining the above exchange rate patterns, following arguments put forth in recent studies. The direct shock aims to capture underlying frictions in financial transactions that hinder cross-border arbitrage through the exchange rate.[5] We model this wedge in the international arbitrage condition as an exogenous shock, following Itskhoki and Mukhin (2017).[6]

In evaluating the roles of fundamental shocks, on the other hand, we consider their nonlinear propagation mechanisms, motivated by the view that the empirical exchange rate puzzles may be the result of first-order approximations of the open economy DSGE models. As endogenous risk premium may arise from the covariance between the stochastic discount factor and returns to international financial investments, their omission in

[4]These issues that surface in our GE analyses are not new conceptually or unique to exchange rate modeling; they nevertheless underscore the need for GE evaluations in this literature.

[5]See, for example, Adolfson et al. (2007), Alvarez et al. (2009), Bacchetta and van Wincoop (2010), Gabaix and Maggiori (2015) and Itskhoki and Mukhin (2017), who point out the importance of financial frictions in accounting for aggregate fluctuations in open economies.

[6]We choose this simple form of risk-sharing shock to facilitate assessment of the extent of exchange rate disconnect from the macroeconomy. As we augment the standard linearized GE system with higher order approximations and macro volatility shocks, we test how they help reconnect the exchange rate by lowering the contribution of the direct shock. Itskhoki and Mukhin (2017) also offer the micro-foundations for such a direct shock.

the linearized equations may result in unexplained exchange rate volatility and biased coefficient estimates in the Fama regressions.[7] As previous attempts to generate endogenous currency risk premium through first-moment shocks show little success, we incorporate second-moment or uncertainty shocks, *i.e.*, shocks to the volatilities of fundamental shocks, as well. If exchange rate fluctuations systematically reflect such risks, one would then infer that the exchange rate is *not* disconnected from macro fundamentals.

To endogenize exchange rate risks, our setup follows closely with Benigno et al. (2011) (hereafter BBN). Through simulations, they show that a rise in the volatility of nominal shocks at home enhances the hedging properties of its currency, thereby inducing endogenously a risk premium for foreign currency-holding.[8] BBN also show that in line with the findings in McCallum (1994) and Backus et al. (2010), when the monetary policy inertia is high and the price rigidity low, volatility shocks can induce a negative correlation between the expected exchange rate depreciation and interest rate differentials, potentially resolving the UIP puzzle. Notice that McCallum (1994), Backus et al. (2010) and BBN conduct simulation analyses and have not estimated fully specified models. In this regard, BBN state that "the estimation of the model is really needed to evaluate its fit. To this purpose, an appropriate methodology should be elaborated to handle the features of our general second-order approximated solutions."[9]

Our paper evaluates these mechanisms using GE system estimation instead of single-equation estimation or simulations with calibrated parameters. To identify various shocks to macroeconomic fundamentals, we estimate the two-country DSGE model using a full information Bayesian approach. The model is solved by perturbation methods up to a third-order approximation in order to accommodate stochastic volatilities in the fundamental shocks.[10] Due to the non-linearity in the system, the standard Kalman filter cannot be applied to evaluate the likelihood function. In the closed-economy setting, Fernández-Villaverde et al. (2011), for example, employ a particle filter to estimate real business cycle models with stochastic volatilities approximated up to the third order. However, for our two-country model, estimating with particle filters would be too time-consuming and practically infeasible, given the rich dynamic structures we emphasize. Instead, we approximate the likelihood function using the central difference Kalman filter proposed by

[7] For example, a structural or macroeconomic fundamental shock—especially to their volatilities—can simultaneously raises interest rates and appreciates the nominal exchange rates. Backus et al. (2001). Duarte and Stockman (2005), Verdelhan (2010), Colacito and Croce (2011), Bansal and Shaliastovich (2012), Benigno et al. (2011), Backus et al. (2010), Gourio et al. (2013) and Engel (2016) are examples in recent literature that aim to solve the UIP puzzle through risk corrections.

[8] A rise in home nominal volatility tends to reduce domestic output and increase inflation, while the domestic nominal interest rate declines relative to the foreign one. Contrary to findings in the previous literature, *e.g.* Engel and West (2005) and Bacchetta and Wincoop (2006), that only a fraction of the exchange rate volatilities can be accounted for by observed economic fundamentals, BBN show the potential of the uncertainty shocks as a key driver behind empirical exchange rate dynamics.

[9] In his discussion to BBN, Uribe (2011) points out that "there remain a number of issues to be addressed. Among the most important ones are the identification of uncertainty shocks. Ideally, this issue will be tackled by a direct estimation of the proposed theoretical DSGE model."

[10] To gauge the impact of stochastic volatilities, BBN employ the efficient solution method with second-order approximation proposed by Benigno et al. (2013), which can account for any distinct and direct effects of volatility shocks, provided that shocks are conditionally linear.

Andreasen (2013). This filter is much faster than particle filters, and its quasi maximum likelihood estimators can be consistent and asymptotically normal for models solved up to the third order.

In terms of parameter estimates, our GE estimation results are broadly consistent with the empirical regularities and mechanisms discussed in previous studies. As in BBN, we find an increase in the volatility of the productivity shock depreciates the currency, while an increase in the volatility of the monetary policy shock appreciates it. Higher monetary policy volatilities also generate excess foreign currency returns and therefore deviations from the UIP condition. Given our estimated parameters, we confirm that *conditionally*, several volatility shocks—to monetary policy, aggregate demand, and direct risk-sharing—can induce currency risk premia endogenously and deliver the negative correlation observed empirically between interest differentials and exchange rate depreciations.[11]

Using the posterior mean estimates from the full-model (baseline) specification, we then assess the relative contributions of various fundamental shocks under different degrees of model approximation. We find that while the relevance of macroeconomic shocks increases marginally between linear and second-order model approximations, stochastic macro volatility shocks under the third-order approximation contribute sizably to exchange rate volatilities. Impulse response functions to various macro shocks show visible dynamic responses of excess currency returns. Variance decompositions further confirm that under the third-order model approximation, the set of macro shocks, together with shocks to their volatilities, explains 43% of the variance of nominal exchange rate changes. We take these as evidence that the exchange rate is not disconnected from the rest of the macroeconomy, once we move beyond linearization assumptions. Our results also show that the direct exchange rate shock remains the main driver (57%) behind the variations in the nominal exchange rate. Conditionally, the direct shock to international risk-sharing can also replicate the negative UIP correlations observed in the data. These findings point to the potential value in exploring additional nonlinearities such as disaster risks, and in incorporating explicit structural channels of information or financial frictions behind the limit-to-arbitrage.

Despite the positive results above in support of our two proposed transmission mechanisms, our GE estimation illustrates the limitations of partial or conditional analyses in providing full resolutions to the empirical puzzles. Even though several shocks can individually generate the observed Fama coefficient (close to or below zero), simulated data based on the estimated model with the full set of shocks all together no longer reproduce this desired data pattern; instead, they show a positive Fama coefficient closer to 1. In other words, the UIP puzzle remains in GE. Putting this *general equilibrium exchange rate puzzle* in a positive light, we view the conditional results as insightful in illustrating transmission mechanisms and their *qualitative* relevance; indeed, our conditional analyses

[11] While not the focus of our analyses, we also find the relative magnitudes of our structural parameter estimates can support the conditions required for the transmission mechanism discussed in Backus et al. (2010) and BBN to resolve the UIP puzzle. See Appendix A.

based on the GE estimates provide support for both of the channels we emphasize. Their ultimate *quantitative* relevance in resolving the unconditional empirical puzzles, however, is to be assessed unconditionally in GE. On that front, our full model incorporating both of these mechanisms fails to replicate the observed empirical pattern.[12]

The remainder of this paper is organized as follows. Section 2 discusses the relevant literature concerning the degrees of model approximation and shows why the exchange rate disconnect may be mitigated with recursive preferences and uncertainty shocks. Section 3 presents our open economy model with recursive preferences, stochastic volatilities, and shocks to international risk-sharing. Section 4 shows how we estimate the model in a nonlinear setting using a full-information Bayesian approach. Section 5 provides our main results and discussions. Finally, Section 6 concludes.

2 Related Literature

The literature on the general exchange rate disconnect phenomenon is vast and dates back many decades. While it has branched out and manifested itself as various well-known empirical puzzles, a simple summary is that macroeconomic fundamentals—the theoretical determinants of the exchange rate—have essentially no explanatory power for actual exchange rate behavior.[13] The failure of the UIP condition is at the heart of this problem. In this section, we briefly discuss the disconnect problem and puts a particular emphasis on the solutions to the UIP puzzle. Then, we explain why our model with recursive preferences can generate an endogenous time-varying currency risk premium, potentially replicating the empirical deviations from the UIP condition. In Appendix A, we discuss key parameters for generating the negative UIP correlations observed in the data, as emphasized in Backus et al. (2010) and BBN, that hinges on the relative magnitudes between two parameter values: monetary policy smoothing and the persistence of nominal volatility shocks.[14] Our GE estimation can evaluate this condition directly from the data.

Ignoring risk and systematic expectation errors, the absence of arbitrage in the international asset markets implies the UIP condition, which can be expressed in a log-linear form as

$$\mathbf{E}_t \log(e_{t+1}) - \log(e_t) = \log(R_t) - \log(R_t^*) \tag{1}$$

[12]The GE puzzle also serves as a reminder that in GE estimation, there are multiple data dynamics to fit. As we incorporate additional elements into a model to explain one targeted empirical pattern (*e.g.* the exchange rate), GE estimation helps ensure they do not come at a cost of deteriorating fit in other parts of the system.

[13]See, for example, Frankel and Rose (1995), Engel and Rogers (1996), and Obstfeld and Rogoff (2001) for a summary of the earlier literature. Evans (2011), Engel (2014), and Burnside (2019) provide additional surveys of recent developments.

[14]McCallum (1994) emphasizes the importance of monetary policy rules in resolving the UIP puzzle. He shows that if monetary authorities manage interest rate differentials to avoid rapid changes in exchange rates, the slope coefficient in the Fama regression can be negative.

where R and R_t^* respectively denote domestic and foreign (gross) nominal interest rates and e_t is the nominal exchange rate (the price of foreign currency in terms of domestic currency). The UIP implies that countries with higher relative interest rates should see their currencies depreciate subsequently on average. Under the assumption of rational expectations, this implication is commonly tested in what is referred to as the Fama (1984) or UIP regression:[15]

$$\log(e_t) - \log(e_{t-1}) = \alpha_0 + \alpha_1 \left[\log(R_{t-1}) - \log(R^*_{t-1})\right] + u_t, \tag{2}$$

with the null hypothesis $H_0 : a_0 = 0$ and $\alpha_1 = 1$. In a wide range of international data, the literature has consistently found the estimated slope coefficient α_1 to be significantly below one, and often negative, contrary to the theoretical prediction. Structural VARs, such as in Eichenbaum and Evans (1995), also confirm the empirical observation that a country's currency tends to *appreciate* after a positive monetary policy shock.[16] In terms of financial trading, Lustig and Verdelhan (2007) and Burnside et al. (2008), for example, find sizable gains from the carry trade strategy of investing in high interest rate currencies with funding from low interest rate currencies, confirming the robustness of the UIP puzzle in the data.

Besides producing the wrong signs, empirical tests of the UIP generally result in extremely poor fits, with the estimated R-squares near zero. This is indicative of the broader exchange rate disconnect phenomenon, as interest rates and the UIP condition are a key channel through which other macro fundamentals interact with the exchange rate. Engel and West (2004) and Bacchetta and Wincoop (2006), for example, show that the exchange rate volatility is hardly explained by macroeconomic fundamentals. In a GE context, Lubik and Schorfheide (2006) demonstrate the exchange rate disconnect by estimating a two-country DSGE model. Approximating the GE system up to the first order, they find that most of the fluctuations in the nominal exchange are stemming from a direct shock to the exchange rate itself.

The above results lead to two natural approaches for connecting the exchange rate to the rest of the macroeconomy.[17] The first is to augment the UIP condition with a time-varying risk premium through which macro fundamentals can influence the exchange rate. If this risk premium is correlated negatively with the interest rate differentials (or monetary policy shocks) and of sufficient magnitudes, one could attribute the UIP puzzle—the negative slope coefficient in the Fama regression in equation (2)—to omitted variable biases. The second approach introduces financial wedge to the UIP condition, representing limits to arbitrage that can arise from financial or informational frictions.

[15] The Fama regression can also be expressed in terms of the forward premium, $\log(f_t) - \log(e_t)$, where f_t is the (gross) forward rate. Under covered interest parity, $\log(f_t) - \log(e_t) = \log(R_t) - \log(R_t^*)$, which has strong empirical support until recently.

[16] Eichenbaum and Evans (1995), as well as Scholl and Uhlig (2008) and Bjørnland (2009) further explore the pattern of *delayed overshooting* in exchange rate dynamics.

[17] The two approaches are by no means exhaustive. For example, an important branch of the literature focuses on solving the UIP puzzle with deviations from the rational expectations, such as over-confidence, learning dynamics, or ambiguity aversion. These include Gourinchas and Tornell (2004), Chakraborty and Evans (2008), Burnside et al. (2011b), Ilut (2012), and Candian and Leo (2021).

The literature has emphasized various promising channels to motivate a time-varying risk premium, such as through disaster risks, liquidity premium, or uncertainty shocks.[18] The underlying mechanism relies on the covariance of the stochastic discount factor and the expected payoff to generate an endogenous risk premium that moves in the opposite direction with the relative interest rate, so as to induce the overall exchange rate response consistent with the empirical negative Fama coefficient. Note that to obtain the negative slope coefficient, movements in the risk premium must be large enough to overcome the changes in relative interest rates. As is well known in the macro-asset pricing literature, consumption dynamics are very smooth and therefore, the stochastic discount factor as well. In order to generate the requisite variations in the stochastic discount factor, non-time-separable utility functions are often adopted; for example, Verdelhan (2010) considers external habit formation while Backus et al. (2001), Colacito and Croce (2011), and Bansal and Shaliastovich (2012) employ the recursive preference *à la* Epstein and Zin (1989) and Weil (1989). They show that the negative correlations between exchange rate depreciations and interest rate differentials can be replicated under certain parameter conditions.[19] As for the source of the risk premium, we follow Menkhoff et al. (2012) and BBN to focus on the role of stochastic volatilities in explaining exchange rate behavior. While Menkhoff et al. (2012) shows that global foreign exchange volatility risk can explain excess returns from carry trades, BBN consider both real and nominal macro volatility shocks. By simulating a two-country DSGE model with recursive preferences, they find that a rise in the volatility of nominal shocks at home enhance the hedging properties of its currency, thereby inducing endogenously a risk premium for foreign currency-holding. To re-evaluate the exchange rate disconnect, we focus on such uncertainty shocks regarding the economic fundamentals and assess their relevance, using GE estimation, in explaining the UIP puzzle and the excess exchange rate volatility.

Limits of arbitrage à la Shleifer and Vishny (1997) naturally lead to the failure of the UIP, as they put a direct wedge in the UIP condition. The possible mechanisms again vary, and portfolio adjustment costs, limited participation, and regulatory constraints have all been proposed as the possible *micro-foundation* behind the wedge or friction. For instance, Adolfson et al. (2007) assume portfolio adjustment costs based on the lagged real net foreign asset positions, and show that the lagged nominal exchange rate can appear in the UIP and provide better model fits. Alvarez et al. (2009) incorporate limited participation to affect the risk premium. As inflation stemming from monetary easing lowers the cost of entry and increases the fraction of the agents in the asset market, monetary policy affects the marginal utility of market participants and reduces the currency risk premium. Bacchetta and van Wincoop (2010) resolve the UIP puzzle by employing the infrequent portfolio decisions, which leads to the *delayed overshooting* and therefore gradually appreciates the high interest rate currency. Gabaix and Maggiori (2015) extend the model to incorporate large players (financiers) with limited risk-bearing capacities and financial

[18] Engel (2016) and Valchev (2017) emphasize the importance of liquidity premium stemming from the imperfect substitutability between money and bonds for the liquidity service. Rogoff (1977)'s *peso problem* and disaster risk are another important branch of literature, more recently explored in Gourio et al. (2013) and Burnside et al. (2011a).

[19] Appendix A discusses the conditions regarding monetary policy and volatility shocks.

market imperfection. An adverse shock to the financial system can then lead to positive ex ante returns from the carry trade, since financiers cannot fully engage in international arbitrage. Our paper follows Itskhoki and Mukhin (2017) and examines a direct exogenous shock which hinders the perfect international financial transactions.

We adopt this direct shock approach instead of specifying any explicit forms of financial friction in order to gauge how the standard settings in modern macro finance, namely Epstein and Zin (1989) preferences and uncertainty shocks, can help *reconnect* the exchange rate. Any remainder, *e.g.* variations attributed to the direct shock, we view as capturing the empirical relevance of the above-mentioned mechanisms.

3 The Model

The model estimated in this paper is a two-country extension of the standard New Keynesian model but incorporates recursive preferences *à la* Epstein and Zin (1989) and Weil (1989) together with stochastic volatilities in various fundamental shocks. The world economy consists of the US (the domestic or home country) and the Euro area (the foreign country), which are assumed to be of the same size.[20] In each country, the representative household gains utility from aggregate consumption composed of home and foreign goods, and trades state contingent assets in both domestic and international asset markets. Monopolistically competitive firms produce differentiated goods, and are subject to Calvo (1983)-type staggered price-setting. Monetary authorities adjust the nominal interest rates in response to inflation and output growth. While we assume symmetric households preferences, the two regions differ in price-setting, monetary policy, and the stochastic processes of fundamental shocks. The assumptions with regard to preferences, technology and complete financial markets give us a highly tractable framework for the open economy.

3.1 Household

A representative household in the domestic country maximizes the recursive utility:

$$V_t = \left[u\left(C_t, N_t\right)^{1-\sigma} + \beta \left(\mathbf{E}_t V_{t+1}^{1-\varepsilon} \right)^{\frac{1-\sigma}{1-\varepsilon}} \right]^{\frac{1}{1-\sigma}},$$

where σ measures the inverse of the intertemporal elasticity of substitution, and ε is the coefficient of relative risk aversion. N_t denotes labor supply. Aggregate consumption C_t is a composite of home- and foreign-produced goods, $C_{H,t}$ and $C_{F,t}$, given by

$$C_t = \left[(1-\alpha)^{\frac{1}{\eta}} C_{H,t}^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} C_{F,t}^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}},$$

[20]This assumption follows from Lubik and Schorfheide (2006). Indeed, the two regions are roughly the same size and have similar per capita income.

with

$$C_{H,t} = \left[\int_0^1 C_{H,t}(j)^{1-\frac{1}{\mu}} \, \mathrm{d}j \right]^{\frac{\mu}{\mu-1}},$$
$$C_{F,t} = \left[\int_0^1 C_{F,t}(j^*)^{1-\frac{1}{\mu}} \, \mathrm{d}j^* \right]^{\frac{\mu}{\mu-1}},$$

where $C_{H,t}(j)$ and $C_{F,t}(j^*)$ are differentiated consumption goods produced by domestic and foreign firms, each of which are indexed by j and j^* respectively. The parameters α, η, and μ are the steady state share of the domestically produced goods consumption in the aggregate consumption, the elasticity of substitution between domestically produced and imported goods, the elasticity of substitution among differentiated products in each country. Following BBN, we specify the instantaneous utility as

$$u(C_t, N_t) := C_t^{\psi} (1 - N_t)^{1-\psi}.$$

The household's utility maximization is subject to the budget constraint:

$$P_t C_t + B_t + \mathbf{E}_t[\frac{m_{t,t+1}}{\pi_{t+1}} D_{t+1}] = R_{t-1} B_{t-1} + D_t + W_t N_t + T_t,$$

where P_t is the consumer price index, B_t is the holding of the domestic bond, $m_{t,t+1}$ is the real stochastic discount factor, $\pi_t := P_t / P_{t-1}$ is CPI inflation, D_t is the state-contingent payoff, R_t is the nominal interest rate, W_t is nominal wage, and T_t is the net transfer from firms and the government.

The optimality conditions for the home household lead to

$$C_{H,t} = (1 - \alpha) \, p_{H,t}^{-\eta} C_t,$$

$$C_{F,t} = \alpha \, (p_{F,t})^{-\eta} \, C_t,$$

$$C_t = \frac{\psi}{1 - \psi} (1 - N_t) \, w_t,$$

$$1 = \mathbf{E}_t m_{t,t+1} \frac{R_t}{\pi_{t+1}},$$

$$m_{t,t+1} = \beta \left(\mathbf{E}_t V_{t+1}^{1-\varepsilon} \right)^{\frac{\varepsilon-\sigma}{1-\varepsilon}} V_{t+1}^{\sigma-\varepsilon} \frac{C_{t+1}^{\psi(1-\sigma)-1} (1 - N_{t+1})^{(1-\psi)(1-\sigma)}}{C_t^{\psi(1-\sigma)-1} (1 - N_t)^{(1-\psi)(1-\sigma)}},$$

where $w_t := W_t / P_t$. $p_{H,t} := P_{H,t} / P_t$ and $p_{F,t} := P_{F,t} / P_t$, where $P_{H,t}$ and $P_{F,t}$ denote denote the prices of domestically produced and foreign-produced goods, respectively, in the domestic currency unit.

A representative household in the foreign country faces a symmetric utility maximization problem to the one in the home country.

3.2 Firms

In the home country, each firm, indexed by j, produces one kind of differentiated goods $Y_t(j)$ by choosing a cost-minimizing labor input $N_t(j)$, given the real wage w_t, subject to the production function:

$$Y_t(j) = A_{W_t} A_t N_t(j),$$

where A_t is a stationary and country-specific technology shock, and $A_{W,t}$ is a non-stationary worldwide technology component that grows at a constant rate γ, *i.e.*,

$$\frac{A_{W,t}}{A_{W,t-1}} = \gamma.$$

Firms set prices of their products on a staggered basis *à la* Calvo (1983). In each period, a fraction $1-\theta \in (0,1)$ of firms reoptimizes prices, while the remaining fraction θ indexes prices to a weighted average of the past inflation rate for the domestically produced goods $\pi_{H,t-1} := P_{H,t-1}/P_{H,t-2}$ and the steady-state inflation rate π. Then, firms that reoptimize prices in the current period maximize their expected profit

Each firm sets its price in a monopolistically competitive market to maximize the present discounted value of their profits:

$$\mathbf{E}_t \sum_{n=0}^{\infty} \theta^n m_{t,t+n} \left[\frac{P_{H,t}(j)}{P_{t+n}} \prod_{i=1}^{n} \left(\pi^{1-\iota} \pi_{H,t+i-1}^{\iota} \right) - \frac{w_{t+n}}{A_{W_{t+n}} A_{t+n}} \right] Y_{t+n}(j),$$

subject to the firm-level resource constraint

$$Y_t(j) = C_{H,t}(j) + G_{H,t}(j) + C_{H,t}^*(j),$$

and the downward sloping demand curves, which are obtained from the household's optimization problem in each country,

$$C_{H,t}(j) = \left[\frac{P_{H,t}(j)}{P_{H,t}} \right]^{-\mu} (C_{H,t} + G_t),$$
$$C_{H,t}^*(j) = \left[\frac{P_{H,t}^*(j)}{P_{H,t}^*} \right]^{-\mu} C_{H,t}^*,$$

where $\iota \in [0,1]$ denotes the weight of price indexation to past inflation relative to steady-state inflation, $C_{H,t}^*$ is export of the domestically produced goods, $P_{H,t}^*$ is the export price of the domestically produced goods in the foreign currency, G_t—the aggregate of $G_{H,t}(j)$—is an exogenous aggregate demand component other than consumption.[21]

We assume the law of one price:

$$P_{H,t}(j) = e_t P_{H,t}^*(j),$$

[21] We assume that only domestically produced goods are used for the exogenous demand.

where e_t denotes the nominal exchange rate (the price of foreign currency in terms of domestic currency). Then, $\pi_{H,t} := P_{H,t}/P_{H,t-1}$ can be expressed as

$$\pi_{H,t} = \frac{p_{H,t}\pi_t}{p_{H,t-1}}.$$

With the auxiliary variables F_t and K_t, the optimal pricing decision can be written in the recursive form:

$$F_t = \frac{1}{2} p_{H,t} \left(C_{H,t} + G_t + C^*_{H,t} \right) + \theta \mathbf{E}_t m_{t,t+1} \left(\frac{\pi^{1-\iota}\pi^{\iota}_{H,t}}{\pi_{H,t+1}} \right)^{1-\mu} F_{t+1},$$

$$K_t = \frac{1}{2} \frac{\mu}{\mu-1} \frac{w_t}{A_t} \left(C_{H,t} + G_t + C^*_{H,t} \right) + \theta \mathbf{E}_t m_{t,t+1} \left(\frac{\bar{\pi}^{1-\iota}\pi^{\iota}_{H,t}}{\pi_{H,t+1}} \right)^{-\mu} K_{t+1}.$$

Under the present price-setting rule, the inflation rate for the domestically produced goods $\pi_{H,t}$ can be related to these auxiliary variables by

$$\left[\frac{1 - \theta \left(\frac{\pi^{1-\iota}\pi^{\iota}_{H,t-1}}{\pi_{H,t}} \right)^{1-\mu}}{1-\theta} \right]^{\frac{1}{1-\mu}} F_t = K_t.$$

Aggregating the firm-level resource constraint leads to

$$Y_t = \Delta_t \left(C_{H,t} + G_t + C^*_{H,t} \right),$$

where $\Delta_t := \int_0^1 \left[P_{H,t}(j) / P_{H,t} \right]^{-\mu} dj$ represents price dispersion across firms. The price dispersion term evolves according to

$$\Delta_t = (1-\theta) \left[\frac{1 - \theta \left(\frac{\pi^{1-\iota}\pi^{\iota}_{H,t-1}}{\pi_{H,t}} \right)^{1-\mu}}{1-\theta} \right]^{\frac{\mu}{\mu-1}} + \theta \left(\frac{\pi_{H,t}}{\pi^{1-\iota}\pi^{\iota}_{H,t-1}} \right)^{\mu} \Delta_{t-1}.$$

To specify measurement equations in the subsequent section, we define the output growth rate YGR_t:

$$YGR_t := \frac{Y_t}{Y_{t-1}}.$$

Foreign firms' profit maximization problems are symmetric to those presented above.

3.3 Monetary policy

The monetary authority in the home country adjusts the nominal interest in response to deviations of inflation and output growth from their steady state values.

$$\log\left(\frac{R_t}{R}\right) = \phi_r \log\left(\frac{R_{t-1}}{R}\right) + (1-\phi_r)\left[\phi_\pi \log\left(\frac{\pi_t}{\pi}\right) + \phi_y \log\left(\frac{Y_t}{\gamma Y_{t-1}}\right)\right] + \log(\varepsilon_{R,t}).$$

where $\phi_r \in [0,1)$ is the degree of interest rate smoothing, and $\phi_\pi, \phi_y \geq 0$ are the degrees of monetary policy responses to inflation and output growth. $\varepsilon_{R,t}$ is an exogenous shock interpreted as an unsystematic component of monetary policy.

The monetary authority in the foreign country also controls the nominal interest rate following the same type of the monetary policy rule.

3.4 Exchange rate and international linkage

Recall that the law of one price holds for prices of domestically produced goods:

$$P_{H,t} = e_t P^*_{H,t},$$

where e_t denotes the nominal exchange rate (the price of foreign currency in terms of domestic currency). Note that the asterisk (*) indicates variables in the foreign currency. The real exchange rate s_t is defined as

$$s_t = \frac{e_t P^*_t}{P_t},$$

where P^*_t is the foreign aggregate price in the foreign currency. Let $p_{H,t} = s_t p^*_{H,t}$. Then, we have

$$p_{H,t} = s_t p^*_{H,t}.$$

Similarly, we can obtain

$$p_{F,t} = s_t p^*_{F,t}.$$

From the definition of the real exchange rate, we have a expression for the nominal exchange rate depreciation d_t:

$$d_t := \frac{e_t}{e_{t-1}} = \frac{s_t \pi_t}{s_{t-1}\pi^*_t}.$$

Regarding the international asset market, the value of the asset in the foreign currency is given by

$$\mathbf{E}_t[\frac{m_{t,t+1}}{\pi_{t+1}} D^*_{t+1} e_{t+1}]/e_t.$$

Thus, under the perfect risk sharing, the stochastic discount factor in the foreign currency $m^*_{t,t+1}$ must satisfy

$$\frac{m^*_{t,t+1}}{\pi^*_{t+1}} = \frac{m_{t,t+1}}{\pi_{t+1}} \frac{e_{t+1}}{e_t}.$$

Substituting the optimality conditions for the home and foreign households to this equation, we have

$$\left(\frac{\left(V_{t+1}^{*}\right)^{1-\varepsilon}\mathbf{E}_{t}V_{t+1}^{1-\varepsilon}}{V_{t+1}^{1-\varepsilon}\mathbf{E}_{t}\left(V_{t+1}^{*}\right)^{1-\varepsilon}}\right)^{\frac{\varepsilon-\sigma}{1-\varepsilon}}\left[\frac{C_{t+1}^{\psi}\left(1-N_{t+1}\right)^{(1-\psi)}}{\left(C_{t+1}^{*}\right)^{\psi}\left(1-N_{t+1}^{*}\right)^{(1-\psi)}}\right]^{1-\sigma}\frac{C_{t+1}^{*}}{C_{t+1}}s_{t+1}$$
$$=\left[\frac{C_{t}^{\psi}\left(1-N_{t}\right)^{(1-\psi)}}{\left(C_{t}^{*}\right)^{\psi}\left(1-N_{t}^{*}\right)^{(1-\psi)}}\right]^{1-\sigma}\frac{C_{t}^{*}}{C_{t}}s_{t}. \tag{3}$$

Let us denote

$$Q_{t}:=\left[\frac{C_{t}^{\psi}\left(1-N_{t}\right)^{(1-\psi)}}{\left(C_{t}^{*}\right)^{\psi}\left(1-N_{t}^{*}\right)^{(1-\psi)}}\right]^{1-\sigma}\frac{C_{t}^{*}}{C_{t}}s_{t}. \tag{4}$$

Then, equation (3) can be written as

$$Q_{t+1}=Q_{t}\left(\frac{\left(V_{t+1}^{*}\right)^{1-\varepsilon}\mathbf{E}_{t}\left(V_{t+1}^{1-\varepsilon}\right)}{V_{t+1}^{1-\varepsilon}\mathbf{E}_{t}\left[\left(V_{t+1}^{*}\right)^{1-\varepsilon}\right]}\right)^{\frac{\sigma-\varepsilon}{1-\varepsilon}}, \tag{5}$$

where we assume that $Q_0 = 1$, implying that the initial state-contingent wealth equalizes the marginal utilities across countries. If the preferences were non-recursive, *i.e.*, $\sigma = \varepsilon$, then $Q_t = 1$ for all t, and hence the risk-sharing condition would be reduced to the one characterized by equation (4) with $Q_t = 1$. Thus, we regard equation (4) as the international risk-sharing condition and introduce a shock Ω_t to this condition as follows:

$$\Omega_{t}Q_{t}=\left[\frac{C_{t}^{\psi}\left(1-N_{t}\right)^{(1-\psi)}}{\left(C_{t}^{*}\right)^{\psi}\left(1-N_{t}^{*}\right)^{(1-\psi)}}\right]^{1-\sigma}\frac{C_{t}^{*}}{C_{t}}s_{t}. \tag{6}$$

As Itskhoki and Mukhin (2017) indicate, Ω_t captures time-varying financial frictions and works as a direct shock to the exchange rate.

3.5 Fundamental shocks

The model contains the following fundamental shocks: country-specific technology A_t, aggregate demand g_t, and monetary policy $\varepsilon_{R,t}$ shocks in the home country, the corresponding foreign shocks A_t^*, g_t^*, $\varepsilon_{R,t}^*$, and the risk-sharing shock Ω_t. The stochastic processes for these shocks are given by

$$\begin{aligned}
\log\left(A_t\right) &= \rho_A \log\left(A_{t-1}\right) + \sigma_{A,t} u_{A,t}, \\
\log\left(g_t\right) &= \left(1-\rho_g\right) \log \bar{g} + \rho_g \log\left(g_{t-1}\right) + \sigma_{g,t} u_{g,t}, \\
\log(\varepsilon_{R,t}) &= \sigma_{\varepsilon_R,t} u_{\varepsilon_R,t}, \\
\log\left(A_t^*\right) &= \rho_A^* \log\left(A_{t-1}^*\right) + \sigma_{A,t}^* u_{A,t}^*, \\
\log\left(g_t^*\right) &= \left(1-\rho_g^*\right) \log \bar{g} + \rho_g^* \log\left(g_{t-1}^*\right) + \sigma_{g,t}^* u_{g,t}^*, \\
\log(\varepsilon_{R,t}^*) &= \sigma_{\varepsilon_R,t}^* u_{\varepsilon_R,t}^*, \\
\log\left(\Omega_t\right) &= \rho_\Omega \log\left(\Omega_{t-1}\right) + \sigma_{\Omega,t} u_{\Omega,t},
\end{aligned}$$

where $\rho_A, \rho_g, \rho_A^*, \rho_g^*, \rho_\Omega \in [0,1)$ are the autoregressive parameters, $u_{A,t}, u_{g,t}, u_{\varepsilon_R,t}, u_{A,t}^*, u_{g,t}^*, u_{\varepsilon_R,t}^*, u_{\Omega,t} \sim$ i.i.d. $N(0,1)$ are the disturbance terms, and $\sigma_{A,t}, \sigma_{g,t}, \sigma_{\epsilon_R,t}, \sigma_{A,t}^*, \sigma_{g,t}^*, \sigma_{\epsilon_R,t}^*, \sigma_{\Omega,t}$ are their time-varying standard deviations.

In addition to these level shocks, we incorporate shocks to their volatilities. The stochastic processes for the volatility shocks are given by

$$\begin{aligned}
\log\left(\sigma_{A,t}\right) &= \left(1-\rho_{\sigma_A}\right) \log\left(\sigma_A\right) + \rho_{\sigma_A} \log\left(\sigma_{A,t-1}\right) + \tau_A z_{\sigma_A,t}, \\
\log\left(\sigma_{g,t}\right) &= \left(1-\rho_{\sigma_g}\right) \log\left(\sigma_g\right) + \rho_{\sigma_g} \log\left(\sigma_{g,t-1}\right) + \tau_g z_{\sigma_g,t}, \\
\log\left(\sigma_{\varepsilon_R,t}\right) &= \left(1-\rho_{\sigma_{\varepsilon_R}}\right) \log\left(\sigma_{\varepsilon_R}\right) + \rho_{\sigma_{\varepsilon_R}} \log\left(\sigma_{\varepsilon_R,t-1}\right) + \tau_{\varepsilon_R} z_{\sigma_{\varepsilon_R},t}, \\
\log\left(\sigma_{A,t}^*\right) &= \left(1-\rho_{\sigma_A}^*\right) \log\left(\sigma_A^*\right) + \rho_{\sigma_A}^* \log\left(\sigma_{A,t-1}^*\right) + \tau_A^* z_{\tau_A,t}^*, \\
\log\left(\sigma_{g,t}^*\right) &= \left(1-\rho_{\sigma_g}^*\right) \log\left(\sigma_g^*\right) + \rho_{\sigma_g}^* \log\left(\sigma_{g,t-1}^*\right) + \tau_g^* z_{\sigma_g,t}^*, \\
\log\left(\sigma_{\varepsilon_R,t}^*\right) &= \left(1-\rho_{\sigma_{\varepsilon_R}}^*\right) \log\left(\sigma_{\varepsilon_R}^*\right) + \rho_{\sigma_{\varepsilon_R}}^* \log\left(\sigma_{\varepsilon_R,t-1}^*\right) + \tau_{\varepsilon_R}^* z_{\sigma_{\varepsilon_R},t}^*, \\
\log\left(\sigma_{\Omega,t}\right) &= \left(1-\rho_{\sigma_\Omega}\right) \log\left(\sigma_\Omega\right) + \rho_{\sigma_\Omega} \log\left(\sigma_{\Omega,t-1}\right) + \tau_\Omega z_{\sigma_\Omega,t}.
\end{aligned}$$

where $\rho_{\sigma_A}, \rho_{\sigma_g}, \rho_{\sigma_{\varepsilon_R}}, \rho_{\sigma_A}^*, \rho_{\sigma_g}^*, \rho_{\sigma_{\varepsilon_R}}^*, \rho_{\sigma_\Omega} \in [0,1)$ are the autoregressive parameters, $z_{\sigma_A}, z_{\sigma_g}, z_{\sigma_{\varepsilon_R}}, z_{\sigma_A}^*, z_{\sigma_g}^*, z_{\sigma_{\varepsilon_R}}^*, z_{\sigma_\Omega} \sim$ i.i.d. $N(0,1)$ are the innovation to the stochastic volatilities, and $\tau_A, \tau_g, \tau_{\epsilon_R}, \tau_A^*, \tau_g^*, \tau_{\epsilon_R}^*, \tau_\Omega$ are their respective standard deviations.

3.6 Detrending

To make the model stationary and obtain the steady state, real variables in the home country are detrended by non-stationary worldwide technology component $A_{W,t}$ so that $v_t := V_t/A_{W,t}^\psi$, $y_t := Y_t/A_{W,t}$, $c_{H,t} := C_{H,t}/A_{W,t}$, $c_{F,t} := C_{F,t}/A_{W,t}$, $\tilde{w}_t := w_t/A_{W,t}$, and $g_t := G_t/A_{W,t}$. Foreign variables are also detrended in the same manner.

The Steady-state conditions in terms of detrended variables are presented in Appendix B, whereas the detrended system of equations are shown in Appendix C.

4 Solution and Estimation Methods

The model is solved using perturbation methods up to the third-order approximation in order to take account of the stochastic volatilities in the fundamental shocks. To ensure stability, we employ the pruning method developed by Andreasen et al. (2018).

We estimate the model using a full-information Bayesian approach. Because the model is no longer linear, the standard Kalman filter is not applicable to evaluate the likelihood function. Instead, we approximate the likelihood function using the central difference Kalman filter proposed by Andreasen (2013).[22]

To approximate the posterior distribution, this paper exploits the generic Sequential Monte Carlo (SMC) algorithm with likelihood tempering described in Herbst and Schorfheide (2014, 2015).[23] In the algorithm, a sequence of tempered posteriors $\varpi_n(\boldsymbol{\theta})$ are defined as

$$\varpi_n(\boldsymbol{\theta}) = \frac{[p(\mathbf{Y}^T|\boldsymbol{\theta})]^{\tau_n} p(\boldsymbol{\theta})}{\int [p(\mathbf{Y}^T|\boldsymbol{\theta})]^{\tau_n} p(\boldsymbol{\theta}) d\boldsymbol{\theta}}, \;\; n = 0, ..., N_\tau,$$

where $p(\mathbf{Y}^T|\boldsymbol{\theta})$ denotes the likelihood function for observations $\mathbf{Y}^T := [\mathbf{Y}_1, ..., \mathbf{Y}_T]'$ given a draw of a parameter vector $\boldsymbol{\theta}$, $p(\boldsymbol{\theta})$ is a prior density, and N_τ is the number of tempering stages. The tempering schedule $\{\tau_n\}_{n=0}^{N_\tau}$ is determined by $\tau_n = (n/N_\tau)^\chi$, where χ is a parameter that controls the shape of the tempering schedule. The SMC algorithm generates parameter draws $\boldsymbol{\theta}_n^{(i)}$ and associated importance weights $w_n^{(i)}$—which are called particles—from the sequence of posteriors $\{\varpi_n\}_{n=1}^{N_\tau}$; that is, at each stage, $\varpi_n(\boldsymbol{\theta})$ is represented by a swarm of particles $\{\boldsymbol{\theta}_n^{(i)}, w_n^{(i)}\}_{i=1}^N$, where N denotes the number of particles. For $n = 0, ..., N_\tau$, the algorithm sequentially updates the swarm of particles $\{\boldsymbol{\theta}_n^{(i)}, w_n^{(i)}\}_{i=1}^N$ through importance sampling.[24] Posterior inferences about parameters to be estimated are made based on the particles $\{\boldsymbol{\theta}_{N_\tau}^{(i)}, w_{N_\tau}^{(i)}\}_{i=1}^N$ from the final importance sampling. The SMC-based approximation of the marginal data density is given by

$$p(\mathbf{Y}^T) = \prod_{n=1}^{N_\tau} \left(\frac{1}{N} \sum_{i=1}^{N} \tilde{w}_n^{(i)} w_{n-1}^{(i)} \right),$$

where $\tilde{w}_n^{(i)}$ is the incremental weight defined as $\tilde{w}_n^{(i)} = [p(\mathbf{Y}^T|\boldsymbol{\theta}_{n-1}^{(i)})]^{\tau_n - \tau_{n-1}}$. In the subsequent empirical analysis, the SMC algorithm uses $N = 2,000$ particles and $N_\tau = 50$ stages. The parameter that controls the tempering schedule is set at $\chi = 2$ following Herbst and Schorfheide (2014, 2015).

Seven quarterly time series ranging from 1987Q1 to 2008Q4 are used for estimation: the per-capita real GDP growth rate ($100\Delta \log GDP_t, 100\Delta \log GDP_t^*$), the inflation rate of the GDP implicit price deflator ($100\Delta \log PGDP_t, 100\Delta \log PGDP_t^*$), and the three-month

[22] Andreasen (2013) argue that quasi maximum likelihood estimators based on the central difference Kalman filter can be consistent and asymptotically normal for DSGE models solved up to the third order.

[23] Creal (2007) is the first that applied the SMC methods to the estimation of DSGE models.

[24] This process includes one step of a single-block RWMH algorithm.

nominal interest rate(INT_t, INT_t^*), in the US and the Euro Area, and the nominal exchange rate depreciation of the US dollar against the Euro ($100\Delta \log EXR_t$). The construction of the data basically follows from Lubik and Schorfheide (2006): the US data are extracted from the FRED database maintained by the Federal Reserve Bank of St. Louis, whereas the Euro Area data and the exchange rate series are taken from the Area-Wide Model (AWM) database of the European Central Bank.[25] The observation equations that relate the data to model variables are given by

$$\begin{bmatrix} 100\Delta \log GDP_t \\ 100\Delta \log PGDP_t \\ INT_t \\ 100\Delta \log GDP_t^* \\ 100\Delta \log PGDP_t^* \\ INT_t^* \\ 100\Delta \log EXR_t \end{bmatrix} = \begin{bmatrix} \bar{\gamma} \\ \bar{\pi} \\ \bar{r} \\ \bar{\gamma} \\ \bar{\pi} \\ \bar{r} \\ 0 \end{bmatrix} + \begin{bmatrix} 100Y\hat{G}R_t \\ 100\hat{\pi}_t \\ 100\hat{r}_t \\ 100Y\hat{G}R_t \\ 100\hat{\pi}_t \\ 100\hat{r}_t \\ 100\hat{d}_t \end{bmatrix},$$

where $\bar{\gamma} = 100(\gamma - 1)$, $\bar{\pi} = 100(\pi - 1)$, $\bar{r} = 100(R - 1)$, and the circumflex (ˆ) denotes log deviation from the steady-state value.

Before estimation, we fix some parameters to avoid an identification issue: the share of foreign goods, the elasticity of substitution between home and foreign goods, the elasticity of substitution across the goods within each country, the share of aggregate demand, the steady-state growth, inflation, and interest rates, and relative risk aversion are fixed at $\bar{\gamma} = 0.346$, $\bar{\pi} = 0.639$, $\bar{r} = 1.274$, $\bar{g}/\bar{y} = 0.18$, $\alpha = 0.13$, $\eta = 1.5$, $\mu = 6$, $\psi = 0.333$, $\epsilon = \epsilon^* = 5$, respectively. The values for $\bar{\gamma}$, $\bar{\pi}$, $\bar{r}$, and $\bar{g}/\bar{y}$ are set at the sample mean of the corresponding data across the two countries so that the ergodic mean of the model-implied observables tend to be close to the sample mean. The other parameter values are chosen based on the calibration in BBN. All other parameters are estimated; their prior distributions are shown in the first four columns of Table 1. The priors are set according to those used in Smets and Wouters (2007) and the calibrated values in BBN. For the standard deviations of the stochastic volatilities ($\tau_A, \tau_g, \tau_{\epsilon_R}, \tau_A^*, \tau_g^*, \tau_{\epsilon_R}^*, \tau_\Omega$), the prior mean is set in line with the upper bound of the estimated standard deviation of the stochastic volatility for a technology shock reported in Fernández-Villaverde and Rubio-Ramírez (2007).

5 Results

This section first reports the estimation results and then show that the dynamic properties of the estimated model are broadly consistent with those obtained in the previous literature. We finally discuss how our model can account for the aggregate fluctuations observed in the US-Euro area data, specifically the exchange rate dynamics. We focus on

[25] For the nominal exchange rate series for the period prior to the introduction of the Euro in 1999, the USD-ECU (Euroepan Currency Unit) exchange rate is used.

Table 1: Prior and posterior distributions of parameters

	Prior			Posterior	
Parameter	Distribution	Mean	S.D.	Mean	90% interval
ε	Gamma	5.000	0.500	4.331	[3.993, 4.669]
σ	Gamma	2.000	0.250	1.879	[1.682, 2.118]
θ	Beta	0.667	0.100	0.525	[0.473, 0.575]
ι	Beta	0.500	0.150	0.340	[0.212, 0.442]
θ^*	Beta	0.667	0.100	0.766	[0.713, 0.827]
ι^*	Beta	0.500	0.150	0.389	[0.248, 0.548]
ϕ_r	Beta	0.750	0.100	0.772	[0.703, 0.836]
ϕ_π	Gamma	1.500	0.200	2.103	[1.893, 2.348]
ϕ_y	Gamma	0.125	0.050	0.196	[0.164, 0.232]
ϕ_r^*	Beta	0.750	0.100	0.794	[0.733, 0.866]
ϕ_π^*	Gamma	1.500	0.200	1.651	[1.462, 1.819]
ϕ_y^*	Gamma	0.125	0.050	0.151	[0.099, 0.204]
ρ_A	Beta	0.500	0.150	0.481	[0.363, 0.590]
ρ_g	Beta	0.500	0.150	0.862	[0.757, 0.972]
ρ_A^*	Beta	0.500	0.150	0.822	[0.733, 0.928]
ρ_g^*	Beta	0.500	0.150	0.390	[0.245, 0.507]
ρ_Ω	Beta	0.500	0.150	0.955	[0.927, 0.990]
ρ_{σ_A}	Beta	0.500	0.150	0.683	[0.588, 0.780]
ρ_{σ_g}	Beta	0.500	0.150	0.513	[0.373, 0.692]
$\rho_{\sigma_{\epsilon_R}}$	Beta	0.500	0.150	0.739	[0.612, 0.882]
$\rho_{\sigma_A}^*$	Beta	0.500	0.150	0.567	[0.454, 0.710]
$\rho_{\sigma_g}^*$	Beta	0.500	0.150	0.337	[0.193, 0.461]
$\rho_{\sigma_{\epsilon_R}}^*$	Beta	0.500	0.150	0.362	[0.189, 0.528]
ρ_{σ_Ω}	Beta	0.500	0.150	0.389	[0.262, 0.498]
$100\sigma_A$	Inverse Gamma	5.000	2.590	2.048	[1.452, 2.528]
$100\sigma_g$	Inverse Gamma	5.000	2.590	9.235	[8.100, 10.928]
$100\sigma_{\epsilon_R}$	Inverse Gamma	0.500	0.260	0.144	[0.106, 0.186]
$100\sigma_A^*$	Inverse Gamma	5.000	2.590	5.293	[4.034, 6.461]
$100\sigma_g^*$	Inverse Gamma	5.000	2.590	7.734	[6.522, 8.799]
$100\sigma_{\epsilon_R}^*$	Inverse Gamma	0.500	0.260	0.168	[0.107, 0.223]
$100\sigma_\Omega$	Inverse Gamma	5.000	2.590	4.652	[3.833, 5.407]
τ_A	Inverse Gamma	1.000	0.517	0.538	[0.408, 0.674]
τ_g	Inverse Gamma	1.000	0.517	0.862	[0.545, 1.115]
τ_{ϵ_R}	Inverse Gamma	1.000	0.517	1.339	[1.016, 1.686]
τ_A^*	Inverse Gamma	1.000	0.517	0.720	[0.582, 0.877]
τ_g^*	Inverse Gamma	1.000	0.517	1.162	[0.972, 1.338]
$\tau_{\epsilon_R}^*$	Inverse Gamma	1.000	0.517	1.287	[1.032, 1.553]
τ_Ω	Inverse Gamma	1.000	0.517	0.635	[0.486, 0.774]

Notes: This table summarizes the prior and posterior distributions of parameters. The posterior mean and 90 percent highest posterior density intervals are calculated from 2,000 particles from the final importance sampling in the SMC algorithm.

the sources of exchange rate variations, and also present the *general equilibrium puzzle* in the context of the UIP slope coefficient.

5.1 Parameter estimates

The last two columns in Table 1 reports the posterior mean and 90% highest posterior density intervals for the estimated parameters.[26] While the posterior mean estimates of the parameters for the household's preferences ε and σ do not differ from the prior mean, those on the firms' price setting are notably different across the two countries as well as from the priors.[27] The price stickiness parameter θ is smaller in the US than θ^* in the Euro area. Regarding the degrees of indexation to the past inflation, ι and ι^*, both are smaller than the prior mean. We also note that the degree of policy response to inflation ϕ_π in the US is substantially larger than ϕ_π^* in the Euro area.

Moreover, compared with priors, remarkable differences arise in the AR(1) coefficients on the aggregate demand shock in the US ρ_A^*, the technology shock in the Euro area ρ_A^*, the risk-sharing shock ρ_Ω. In particular, ρ_Ω is very large, implying very persistent deviation from the risks-sharing condition and the resulting UIP relationship. In this regard, if the model is estimated without stochastic volatilities (irrespective of the degrees of approximation), the posterior mean of ρ_Ω increases to 0.997—almost a unit root—, as shown in Tables 5 and 6 in Appendix D. These findings suggest that the risk-sharing shock is absorbing some key empirical properties of the exchange rate (its persistent or random walk-like behavior), and the introduction of the stochastic volatilities do affect its role somewhat.

Since our structural model shares many similarities with the one presented in BBN, we compare our parameter estimates with their parameter values calibrated for their simulation analyses. According to Table 2, whereas our posterior mean estimates for parameters on the household's preferences and the price stickiness $(\varepsilon, \sigma, \theta, \theta^*)$ are in line with their calibrated values, those on the monetary policy responses $(\phi_\pi, \phi_y, \phi_\pi^*, \phi_y^*)$ exhibit some differences. McCallum (1994), Backus et al. (2010) and BBN stress the importance of parameters in monetary policy rules in generating the observed negative correlation between exchange rate depreciation and interest rate differentials, through simulation analyses. In what follows, we examine whether their proposed mechanism is empirically valid, based on our GE estimates.

5.2 Impulse responses

This subsection demonstrates that our estimated model can broadly replicate the key observations about volatility shocks described in BBN: (i) an increase in the volatility of the

[26] We also estimate a linearized version of the model and a model *without* stochastic volatilities approximated up to the second and third order to better understand the effect of higher order approximations and the introduction of stochastic volatilities on the parameter estimates. The results are shown in Tables 5 and 6 in Appendix D.

[27] Recall that we assume symmetric households preferences, but the two countries are different in price-setting, monetary policy and fundamental shocks.

Table 2: Comparison of parameters

Parameter	Our estimate	BBN's calibration
ε	4.33	5.00
σ	1.88	2.00
θ	0.53	0.66
θ^*	0.77	0.75
ϕ_r	0.77	0.76
ϕ_π	2.10	1.41
ϕ_y	0.20	0.66
ϕ_{r^*}	0.79	0.84
ϕ_{π^*}	1.65	1.37
ϕ_{y^*}	0.15	1.27

Note: This table compares the posterior mean estimates of a subset of parameters shown in Table 1 with the corresponding parameter values calibrated in BBN.

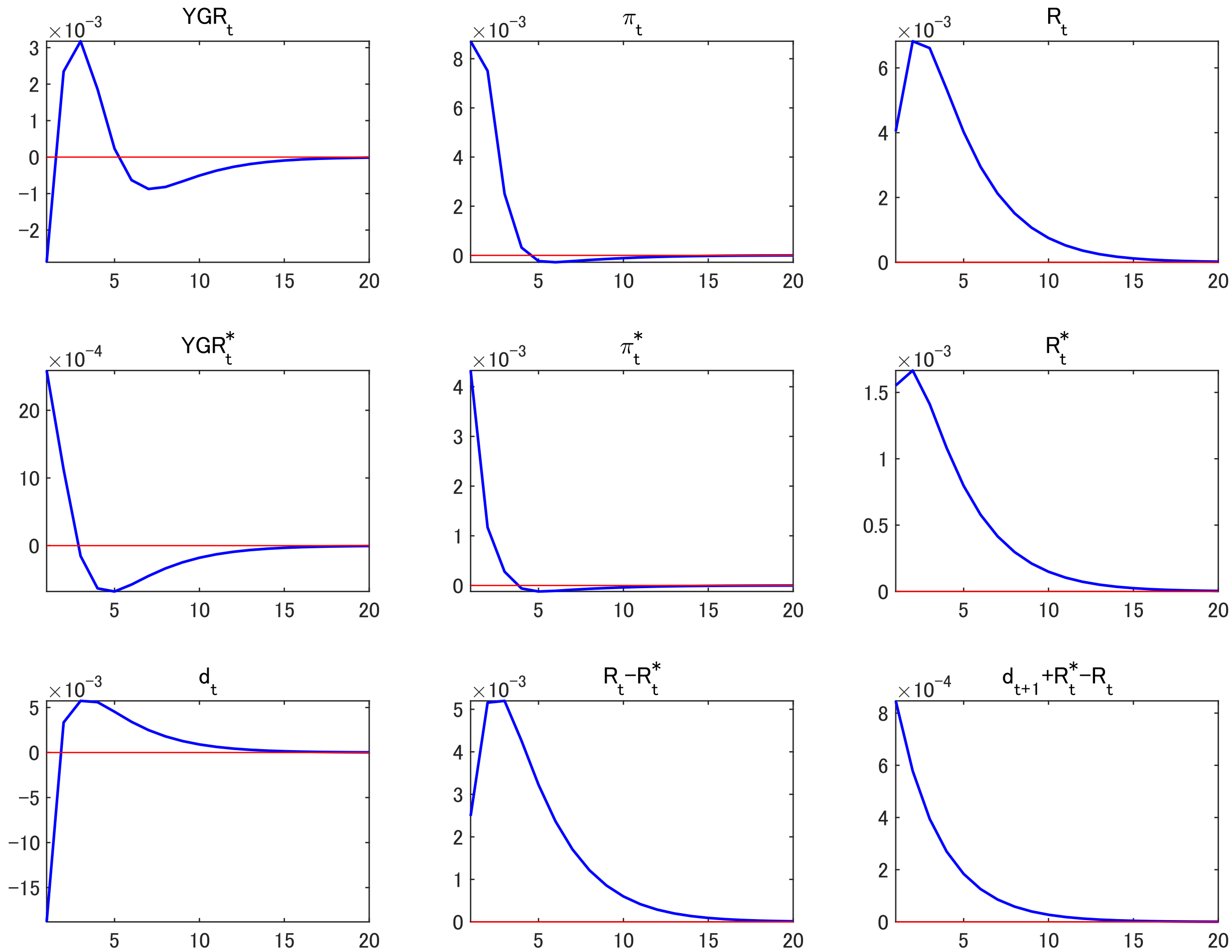
productivity shock induces an exchange rate depreciation; (ii) an increase in the volatility of the monetary policy shock induces an exchange rate appreciation; and (iii) an increase in the volatility of the monetary policy shock also causes deviations from the UIP via increasing the excess return on the foreign currency.

We present the impulse responses for the set of observed macro variables (YGR_t, π_t, R_t, YGR_t^*, π_t^*, R_t^*, d_t), nominal interest rate differential ($R_t - R_t^*$), and the excess return on the foreign currency ($d_{t+1} + R_t^* - R_t$). Figures 1 and 2 respectively consider volatility shocks to home technology and home monetary policy, both at the ergodic mean of the state variables, given the posterior mean estimates of parameters obtained in our baseline estimation. A positive uncertainty shock to home technology causes a currency appreciation upon impact and subsequent depreciation. On the other hand, in response to an increase in the volatility of home monetary policy, the exchange rate appreciates upon impact, and deviation from the UIP materializes as demonstrated in BBN.[28] We, however, note that the quantitative impacts of these volatility shocks are small.

In addition to the macro volatility shocks, Figure 3 presents the impulse responses to the direct (level) risk-sharing shock that we also incorporated into our model. Not surprisingly, the direct shock depreciates the dollar upon impact. We then see a persistent deviation from the UIP as foreign currency excess return declines. This direct shock also affects other macroeconomic variables to a substantial degree, especially compared to the marginal quantitative impacts of the volatility shocks. Since deviation from the UIP is an unconditionally observed phenomenon, it is necessary to check whether this direct shock is also an important driver of the aggregate fluctuations in open economies. This issue will be discussed in Section 5.5.

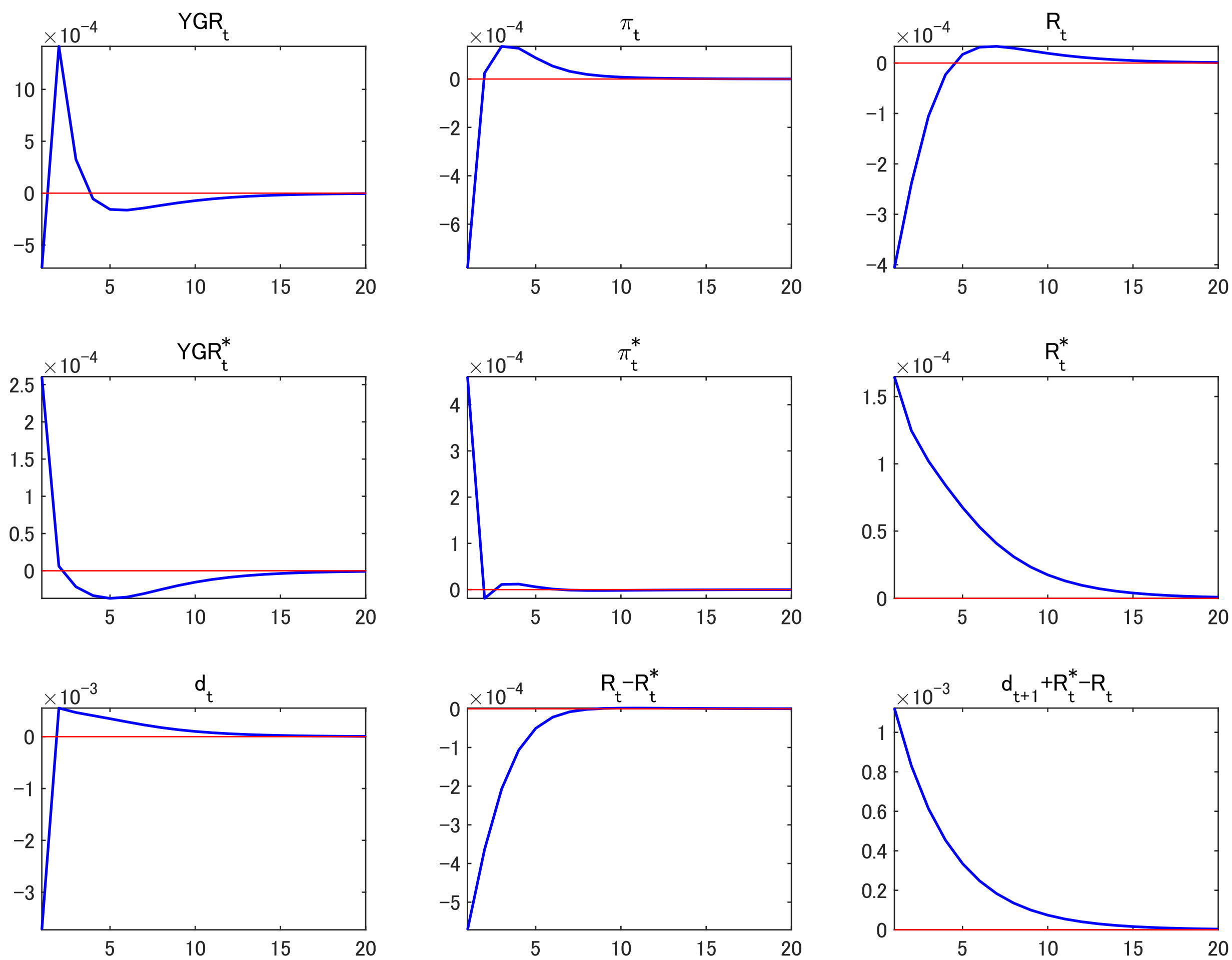
[28] See Appendix A or BBN for more detailed discussions on the explanations and economic intuitions behind these results.

Figure 1: Responses to volatility shock to home technology



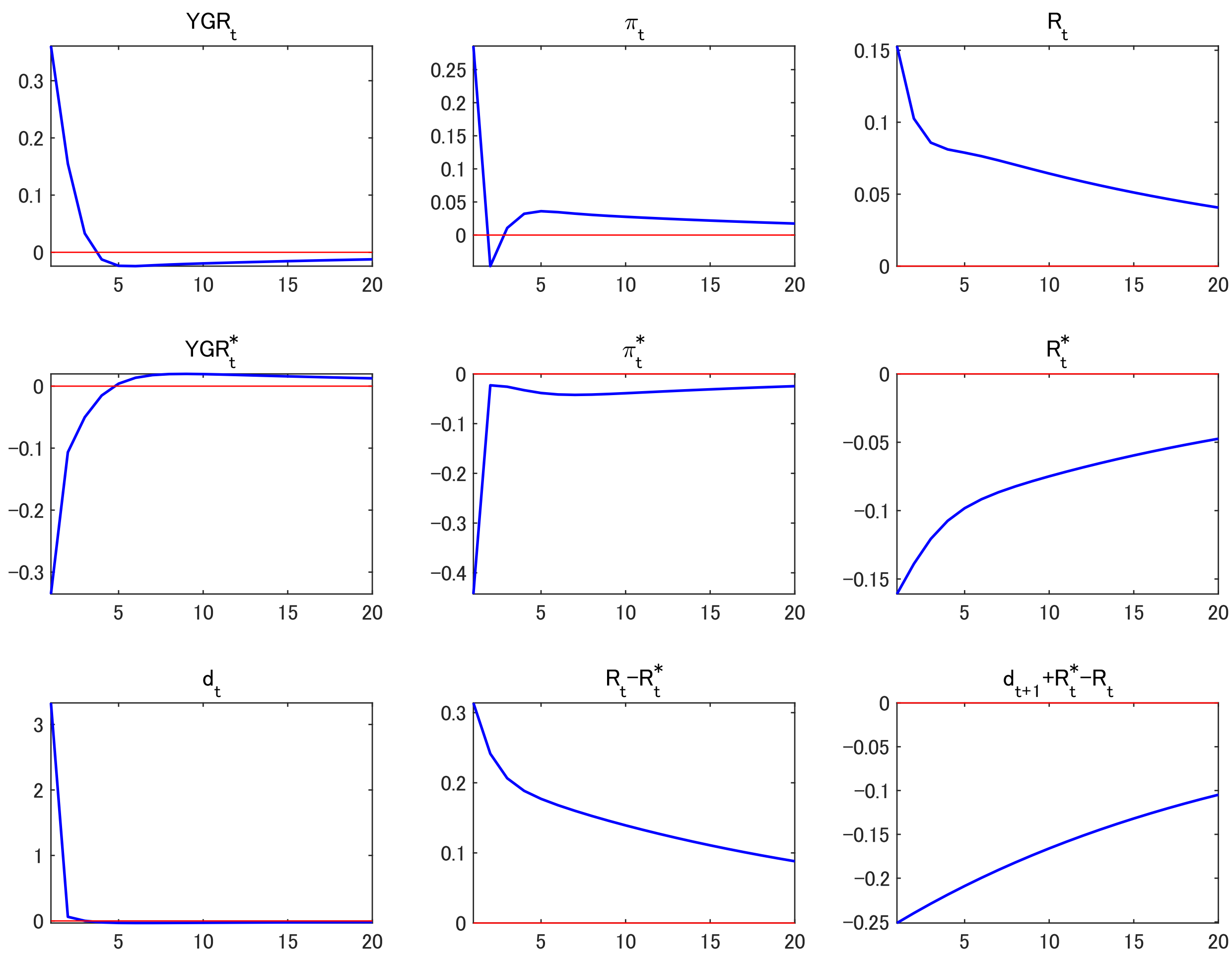
Note: This figure shows the impulse responses of output growth, inflation, the nominal interest rate in both countries, depreciation of the nominal exchange rate, nominal interest rate differential, and deviation from the UIP, in terms of deviations from steady-state values, to a one-standard-deviation volatility shock to home technology, given the posterior mean estimates of parameters in the baseline model.

Figure 2: Responses to volatility shock to home monetary policy



Note: This figure shows the impulse responses of output growth, inflation, the nominal interest rate in both countries, depreciation of the nominal exchange rate, nominal interest rate differential, and deviation from the UIP, in terms of deviations from steady-state values, to a one-standard-deviation volatility shock to home monetary policy, given the posterior mean estimates of parameters in the baseline model.

Figure 3: Responses to risk sharing shock



Note: This figure shows the impulse responses of output growth, inflation, the nominal interest rate in both countries, depreciation of the nominal exchange rate, nominal interest rate differential, and deviation from the UIP, in terms of deviations from steady-state values, to a one-standard-deviation shock to the risk-sharing condition, given the posterior mean estimates of parameters in the baseline model.

The impulse responses to all the other shocks are presented in the Appendix D.

5.3 Accounting for exchange rate volatility

Under log-linearization, the equilibrium conditions presented in section 3 imply the following modified UIP condition, which directly incorporates the risk-sharing shock Ω_t:

$$\hat{R}_t - \hat{R}_t^* = \mathbf{E}_t \hat{d}_{t+1} + \hat{\Omega}_t - \mathbf{E}_t \hat{\Omega}_{t+1}, \tag{7}$$

where the circumflex (ˆ) denotes log deviation from the steady-state value. Since deviations from the UIP are now directly captured by this risk-sharing shock, all deviations from the UIP in the data will be attributable to this risk-sharing shock, implying its contributions to exchange rate dynamics.

Moving beyond linearization, ignoring the direct risk-sharing shock, the UIP condition up to the second-order approximation is given by

$$\hat{R}_t - \hat{R}_t^* = \mathbf{E}_t \hat{d}_{t+1} - \frac{\mathbf{cov}_t[\hat{M}^*_{t,t+1}, -\hat{d}_{t+1}]}{2} + \frac{\mathbf{cov}_t[\hat{M}_{t,t+1}, \hat{d}_{t+1}]}{2},$$

and that up to the third-order approximation is given by

$$\begin{aligned} \hat{R}_t - \hat{R}_t^* = \mathbf{E}_t \hat{d}_{t+1} &- \frac{\mathbf{cov}_t[\hat{M}^*_{t,t+1}, -\hat{d}_{t+1}]}{2} + \frac{\mathbf{cov}_t[\hat{M}_{t,t+1}, \hat{d}_{t+1}]}{2} \\ &- \frac{\mathbf{E}_t[\hat{M}^2_{t,t+1}\hat{d}_{t+1}]}{2} - \frac{\mathbf{E}_t[\hat{d}^2_{t+1}\hat{M}^*_{t,t+1}]}{2} - \frac{\mathbf{E}_t[(\hat{M}^*_{t,t+1})^2\hat{d}_{t+1}]}{2} + \frac{\mathbf{E}_t[\hat{d}^2_{t+1}\hat{M}_{t,t+1}]}{2}, \end{aligned}$$

where $M_{t,t+1} := m_{t,t+1}/\pi_{t+1}$ denotes the nominal stochastic discount factor. Because the higher order terms are also driven by other fundamental shocks including volatility shocks, the contribution of the risk-sharing shock to overall exchange rate fluctuations is therefore expected to decrease, compared to the linear case.

To assess the relative contributions of the two channels—the direct shock to the risk-sharing condition vs. the higher order terms driven by other fundamental shocks—, we consider different orders of approximation with and without stochastic volatilities: (i) a linearized version of the model, (ii) the model approximated to the second order without stochastic volatilities; (iii) the model approximated to the third order without stochastic volatilities, and (iv) the full (baseline) model approximated to the third order with stochastic volatilities. For these four cases, the parameters are fixed at the posterior mean estimates of the full model shown in Table 1 in computing the model-implied variances of the observed variables. To evaluate the relative contribution of each shock, we calculate the relative variances of the observables by *excluding each single shock from the model*. We adopt this method because standard variance decompositions would underestimate the contributions of each shock, as they cannot capture cross-terms or interactions among shocks that arise in nonlinear settings. The relative variances enable us to measure how its *exclusion* affects the data variations the model can explain. To compute these variances, each model is simulated for 10,100 periods with the first 100 observations discarded.

The numbers reported in each row of Table 3 represent the fraction of fluctuations in the observed column variables the model can explain without the particular fundamental shocks shown in the left column. We consider the same set of level shocks in all specifications: u_A, u_g, u_{ϵ_R}, u_A^*, u_g^*, $u_{\epsilon_R}^*$, and u_Ω, denoting shocks to home technology, home aggregate demand, home monetary policy, their foreign counterparts indicated with * superscripts, and also the international risk-sharing condition. In the bottom part of the table, we add in the volatility shocks to the respective level shocks: z_{σ_A}, z_{σ_g}, $z_{\sigma_{\epsilon_R}}$, $z_{\sigma_A}^*$, $z_{\sigma_g}^*$, $z_{\sigma_{\epsilon_R}}^*$, and z_{σ_Ω}. In what follows, we focus on the last column, which shows the results for nominal exchange rate changes d_t.

We see that excluding the international risk-sharing shock, the linear model with its remaining macroeconomic shocks can explain only 25% of the exchange rate volatility, implying that 75% of the exchange rate fluctuations are driven by the direct risk-sharing shock. The second-order approximation slightly increases the collective contribution from macro shocks to 26%. Without stochastic volatilities, even if the model takes account of nonlinearities up to the third order, the contribution of other fundamental shocks than the risk-sharing shock remains almost the same as in the case of the second-order approximation. By stark contrast, when stochastic volatilities are incorporated in the model approximated up to the third order, the macro shocks including their volatility shocks can explain 43% of the exchange rate fluctuations. This result is consistent with BBN's findings based on simulations, that macroeconomic uncertainties can induce a time-varying exchange rate risk premium that acts as a key source behind exchange rate fluctuations.[29]

Last but not least, our results show that despite significant contributions from the macro side, the direct risk-sharing shock still accounts for more than half (57%) of the exchange rate fluctuations. These findings support the arguments put forth in *e.g.* Itskhoki and Mukhin (2017) on the importance of financial frictions in accounting for the exchange rate dynamics and aggregate fluctuations in the open economy. To confirm this point, we re-estimate the baseline model *excluding* the direct risk-sharing shock, and report the estimates in the last two columns of Table 6 in Appendix D. We see that the price indexation parameters and several AR(1) coefficients become larger, to account for the persistence in the observed variables that was captured by the risk-sharing shock in the baseline specification. The log marginal data density $\log p(\mathbf{Y}^T)$ is substantially lower (-919.4) than that in the baseline estimation (-807.3), indicating a much worse fit of the model without the risk-sharing shock.

We note that the high (estimated) persistence of the risk-sharing shock is likely behind why it plays such a significant role in explaining exchange rate fluctuations. As reported in Tables 5 and 6 in Appendix D, without stochastic volatilities, the mean estimates for its AR(1) coefficient ρ_Ω are all very close to unity (0.997), regardless of approximation

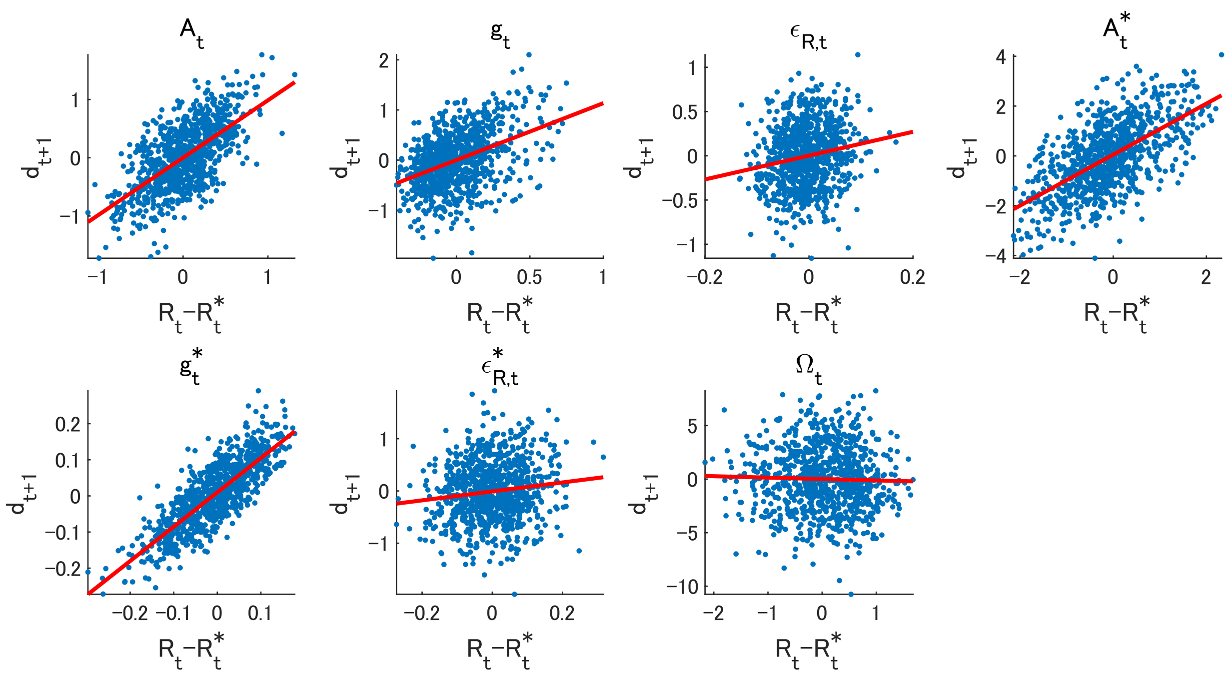
[29] These finding also support the view in Bloom (2009) and Fernandez-Villaverde et al. (2015) that uncertainties can play a major role in explaining aggregate economic fluctuations observed in the data.

Table 3: Relative variances excluding each shock

		$\Delta \log Y_t$	$\log \pi_t$	$\log R_t$	$\Delta \log Y_t^*$	$\log \pi_t^*$	$\log R_t^*$	d_t
Linear Approx. w/o Stochastic Volatilities								
w/o:	u_A	0.733	0.315	0.597	0.995	0.980	0.992	0.971
	u_g	0.331	0.944	0.809	0.999	0.996	1.000	0.972
	u_{ϵ_R}	0.980	0.940	0.991	1.000	0.999	1.000	0.988
	u_A^*	0.988	0.925	0.792	0.584	0.161	0.115	0.840
	u_g^*	0.999	0.999	1.000	0.458	0.996	0.995	1.000
	$u_{\epsilon_R}^*$	1.000	0.991	0.996	0.964	0.975	0.988	0.975
	u_Ω	0.957	0.884	0.799	0.981	0.909	0.938	0.245
2nd-Order Approx. w/o Stochastic Volatilities								
w/o:	u_A	0.738	0.313	0.598	0.995	0.979	0.992	0.970
	u_g	0.330	0.938	0.800	0.999	0.996	1.000	0.970
	u_{ϵ_R}	0.980	0.941	0.991	1.000	0.999	1.000	0.988
	u_A^*	0.991	0.926	0.798	0.575	0.161	0.115	0.834
	u_g^*	0.998	0.999	1.000	0.458	0.997	0.995	1.000
	$u_{\epsilon_R}^*$	1.000	0.992	0.996	0.963	0.975	0.988	0.974
	u_Ω	0.960	0.884	0.800	0.984	0.914	0.942	0.255
3rd-Order Approx. w/o Stochastic Volatilities								
w/o:	u_A	0.742	0.313	0.600	0.995	0.978	0.992	0.970
	u_g	0.328	0.934	0.793	1.000	0.994	1.000	0.969
	u_{ϵ_R}	0.980	0.941	0.991	1.000	0.999	1.000	0.988
	u_A^*	0.996	0.922	0.795	0.585	0.161	0.117	0.835
	u_g^*	0.999	0.999	1.000	0.441	0.997	0.995	1.000
	$u_{\epsilon_R}^*$	1.000	0.992	0.996	0.963	0.975	0.988	0.974
	u_Ω	0.962	0.885	0.801	0.984	0.913	0.942	0.258
3rd-Order Approx. with Stochastic Volatilities								
w/o:	z_{σ_A}	0.905	0.734	0.815	0.998	0.991	0.997	0.984
	z_{σ_g}	0.480	0.952	0.808	1.000	0.996	0.998	0.967
	$z_{\sigma_{\epsilon_R}}$	0.913	0.692	0.957	1.000	0.996	1.000	0.920
	$z_{\sigma_A}^*$	0.996	0.963	0.871	0.823	0.457	0.427	0.877
	$z_{\sigma_g}^*$	0.999	0.999	0.998	0.475	0.994	0.993	0.998
	$z_{\sigma_{\epsilon_R}}^*$	1.000	0.987	0.992	0.946	0.938	0.976	0.933
	z_{σ_Ω}	0.986	0.971	0.940	0.997	0.964	0.959	0.711
	u_A, z_{σ_A}	0.830	0.523	0.664	0.998	0.984	0.993	0.975
	u_g, z_{σ_g}	0.295	0.932	0.722	1.000	0.994	0.994	0.956
	u_{ϵ_R}, $z_{\sigma_{\epsilon_R}}$	0.907	0.673	0.956	1.000	0.996	1.000	0.915
	u_A^*, $z_{\sigma_A}^*$	1.000	0.936	0.777	0.717	0.169	0.108	0.816
	u_g^*, $z_{\sigma_g}^*$	0.999	0.998	0.998	0.338	0.992	0.988	0.997
	$u_{\epsilon_R}^*$, $z_{\sigma_{\epsilon_R}}^*$	1.000	0.985	0.992	0.940	0.927	0.975	0.926
	u_Ω, z_{σ_Ω}	0.976	0.932	0.859	0.995	0.934	0.941	0.425

Note: This table shows the variances of the output growth rate, the inflation rate, the nominal interest rate in the home and foreign countries, and the nominal exchange rate depreciation excluding each shock, relative to those with all the shocks, given the posterior mean estimates of parameters in the baseline model.

Figure 4: UIP regressions based on simulated series driven by each level shock



Note: This figure shows the UIP regressions based on the simulated series of the exchange rate depreciation and the nominal interest rate differentials driven by each level shock indicated at the top of each panel.

orders.[30] Even with stochastic volatilities incorporated, the coefficient estimate remains high at 0.96. This dynamic property of the risk-sharing shock appears consistent with the (near) random-walk behavior of exchange rates observed in the data.

Overall, our results show that by allowing higher order approximations and stochastic volatilities, the exchange rate behavior is no longer disconnected from the macroeconomic fundamentals. Yet, our findings also point to the indispensable role of the direct risk-sharing shock in explaining most of variations in the Euro-Dollar exchange rate.

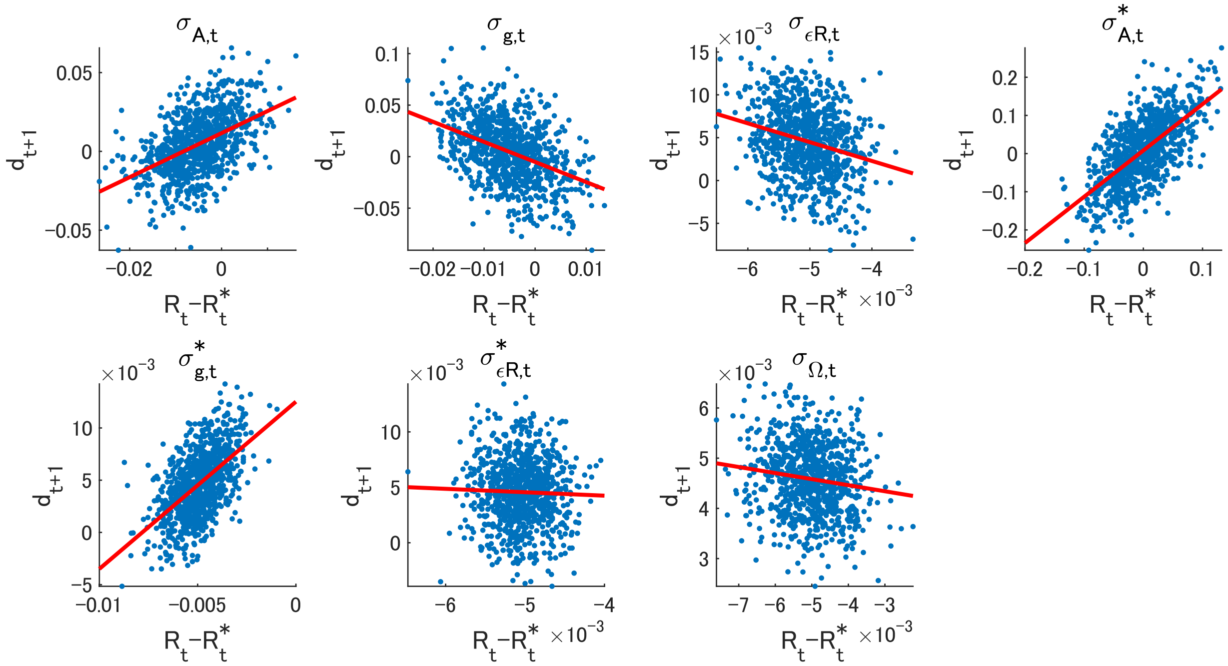
5.4 The UIP puzzle

We now turn to examine how the different sources of shocks may be behind the UIP puzzle, namely the negative Fama coefficient observed in the data. We construct a set of artificial time-series data by shutting down all but a single shock each time, and examine which individual shock can replicate the empirical negative correlation between interest rate differentials and subsequent nominal exchange rate changes, *i.e.*, high relative interest rate currency appreciates subsequently.

Considering the set of level shocks first, Figure 4 presents plots of the UIP regressions

[30] Table 7 in Apprndix D demonstrates that the contribution of the risk-sharing shock to the exchange rate volatility increases to 86% if we use the estimated model under linear approximation.

Figure 5: UIP regressions based on simulated series driven by each volatility shock



Note: This figure shows the UIP regressions based on the simulated series of the exchange rate depreciation and the nominal interest rate differentials driven by each volatility shock indicated at the top of each panel.

using simulated series of the exchange rate depreciation d_{t+1} and the nominal interest rate differentials $R_t - R_t^*$ driven by a single shock, indicated at the top of each panel.[31] *All* of the macro level shocks—to home and foreign technology (A_t and A_t^*), demand (g_t and g_t^*), and monetary policy ($\varepsilon_{R,t}$ and $\varepsilon_{R,t}^*$)—generate *positive* slope coefficients, in line with the theoretical no-arbitrage implication of the UIP, and thus unable to account for the contrary empirical regularities even with nonlinear estimation of the fully specified model. This result aligns with previous attempts to generate endogenous risk premium through first-moment shocks, which show little success. The notable panel is the last one, where the risk-sharing shock Ω_t generates a Fama coefficient in line with the empirics: close to and slightly below zero. This reaffirms our observation based on the variance decomposition analyses above: the direct risk-sharing shock may play a key role in explaining exchange rate dynamics.

We next look at how the volatility shocks perform in replicating the empirical negative- to no-correlation between interest rate differentials and exchange rate changes. Figure 5 depicts UIP regression results using simulated series driven by one individual volatility shock at a time (together with the corresponding level shock). Here we see more promising results: while the volatility shocks to technology ($\sigma_{A,t}$ and $\sigma_{A,t}^*$) and foreign demand ($\sigma_{g,t}^*$) generate positive Fama coefficients, volatility shocks to monetary policy both

[31] Given the posterior mean estimates of parameters in the baseline estimation, the model is simulated for 1,100 periods, and the first 100 observations are discarded.

at home and abroad ($\sigma_{\varepsilon R,t}$ and $\sigma^*_{\varepsilon R,t}$), to home demand, and to the risk-sharing wedge ($\sigma_{\Omega,t}$) all replicate the negative UIP slope coefficients as obtained in the literature.

The results for the monetary policy uncertainty shocks also confirm the mechanism proposed in BBN. As explained in Appendix A, the relative sizes of the persistence parameters between monetary policy and stochastic volatilities are important for generating the target exchange rate dynamics. Backus et al. (2010) and BBN emphasize that the slope of the UIP regression is more likely to be negative when interest rate smoothing is more active and the persistence of the nominal volatility shock is low. Our GE estimates turn out to meet these conditions for replicating the empirical deviations from the UIP condition.

5.5 The general equilibrium puzzle

The above exercises have identified several shocks that can replicate the empirical regularity of a mildly negative Fama coefficient: (i) level and the volatility shocks to the risk-sharing condition, (ii) volatility shocks to home aggregate demand, and (iii) volatility shocks to both home and foreign monetary policy. This set of shocks generally echo findings and mechanisms proposed in previous papers that rely on simulations with calibrated parameters. However, we note that these findings are all based on partial equilibrium or conditional analyses, where all shocks but the proposed one are assumed to be absent. The actual empirical UIP puzzle, on the other hand, is a pattern that manifests unconditionally in GE. As such, the actual empirical relevance of these proposed mechanisms, in the GE context, needs to be further evaluated. Put it differently for our particular context, besides the set of shocks that produced the desired negative Fama coefficient, there is the complement set of shocks that support the theoretical UIP condition instead. Their relative contributions in GE need to be assessed in order to determine whether the proposed models and mechanisms are empirically relevant.

Our paper has the unique advantage that our full model is estimated directly in GE, and thus all parameter values are obtained to fit not just one target variable (*e.g.* the exchange rate) but the full set of relevant open-economy macro dynamics. The importance of this *multiple target* aspect of the GE estimation is well-understood in the broader macroeconomic literature. Models and mechanisms that fit one aspect of the economy, *e.g.* dynamics of the real variables, can fail miserably at explaining another, such as asset returns. As discussed earlier, we choose to incorporate stochastic volatilities and recursive preferences as they have been shown in the close-economy settings to provide reasonable fit of both real variables and asset returns.

Given these GE estimates, we can further examine the aggregate impact of all the proposed shocks, to determine whether the successes in conditional analyses extend quantitatively to GE settings. To this end, we simulate data from our estimated full model and investigate whether, with all the shocks together, we can obtain a Fama coefficient as observed in the actual data.

Table 4: Fama coefficients in actual vs. simulated data

	Fama Coeff. $\widehat{\alpha}_1$	95% C.I.	R^2
Actual data	0.048	[-1.492, 1.587]	0.00
Simulated data with all shocks	0.705	[0.530, 0.880]	0.06
Simulated data without Ω_t	1.084	[0.995, 1.173]	0.36

Note: This table shows the Fama regression coefficients, their 95% confidence intervals, and R^2, based on actual data, simulated data from the baseline model with all shocks, and simulated data from the baseline model excluding the risk-sharing shock.

Table 4 presents the estimated Fama regression coefficient $\widehat{\alpha}_1$ based on three sets of data: (i) the actual data; (ii) simulated data from the baseline model with all shocks; and (iii) simulated data from the model excluding the risk-sharing shock Ω_t. We see that while incorporating the risk-sharing shock does lower the Fama coefficient (from 1.08 to 0.70), the baseline model with the full set of shocks all together still generates the coefficient that is significantly positive.

The finding from the GE analysis leaves us with the *general equilibrium puzzle* of exchange rate dynamics. While our results show that the exchange rate is not disconnected from macro fundamentals, and that the risk-sharing shock can explain a large fraction of the overall exchange rate volatility, their collective impact on actual exchange rate, unconditionally, is not quantitatively large enough to resolve the UIP puzzle. In other words, results derived from partial equilibrium or conditional analyses can offer important qualitative insights about potential transmission mechanisms, but their overall quantitative relevance needs to be evaluated in GE, due to the presence of multiple shocks as well as multiple time-series dynamics to explain simultaneously.

As implied in Table 3, other fundamental shocks such as level technology and monetary policy shocks are still essential drivers of fluctuations in output, inflation, and interest rates, consistent with numerous empirical findings obtained in the previous literature using RBC and New Keynesian models. The importance of these traditional shocks in accounting for aggregate fluctuations in open economies hints that mechanisms to replicate the deviations from the UIP to such shocks needs to be incorporated to resolve the *general equilibrium puzzle*.

6 Conclusion

In this paper, we have estimated a two-country New Keynesian model for the US and Euro area economies with recursive preferences and stochastic volatilities using nonlinear system estimation methods. According to the estimation results, the direct shock to the international risk-sharing condition, which represents the time-varying financial frictions that hinder the international arbitrage, is a major driver for the observed Euro-Dollar exchange rate fluctuations. We also find that macroeconomic shocks, together with shocks to their volatilities, can explain a significant portion of exchange rate dynamics as well. Therefore, by allowing for higher-order terms and volatility shocks, the exchange rate can be reconnected with macroeconomic fundamentals.

Most importantly, our explorations in hope of resolving the UIP puzzle instead point to an important observation that we call the *general equilibrium puzzle.* We show that while certain shocks or mechanisms may work well in partial equilibrium analyses and offer important insights into the possible mechanisms behind certain empirical observations, their ultimate relevance in explaining the actual data would still need to be evaluated in GE, where multiple shocks are present, and multiple observable series are to be fitted simultaneously. This hints at the importance of embedding a mechanism where deviations from the UIP realize to technology and monetary policy shocks, which are major drivers of aggregate fluctuations, to resolve the *general equilibrium puzzle*.

Several considerations are left for future work. First, the exact micro-foundation behind the direct risk-sharing shock warrants additional investigation in GE, so are other mechanisms that can generate endogenous risk premium. Second, additional disturbances such as *news shocks*, whose roles are empirically examined in close-economy models by *e.g.* Fujiwara et al. (2011) and Schmitt-Grohe and Uribe (2012), in particular, anticipated future shocks to both levels and volatilities of fundamentals could be incorporated into our nonlinear GE estimation framework. Finally, from a structural perspective, one could consider a setup with local currency pricing as in Betts and Devereux (2000) and Devereux and Engel (2002), which would result in more disconnected exchange rate behavior.

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Appendix

A Monetary policy and exchange rate dynamics

Backus et al. (2010) and BBN, two papers our modeling approach follows closely, further emphasize that the monetary policy reaction functions play a crucial role in replicating the negative Fama slope coefficient. Specifically, the persistence in the monetary policy uncertainty shock process, and the policy inertia, must meet certain parameter restrictions in order to generate the exchange rate dynamics observed in the data. Since one of the advantages of our GE estimation is that we can obtain direct estimates for these relevant parameters, we will briefly present the conditions described in these papers.

For the stochastic volatilities in the model to generate a currency risk premium of the requisite sign and size, Backus et al. (2010) and BBN show that the degree of interest rate smoothing in the Taylor type instrument rule, relative to the degree of persistence in the monetary policy shock, play a key role. Specifically, assuming away any real dynamics for simplicity, Backus et al. (2010) consider the following simple feedback rule for monetary policy:

$$R_t = \gamma_0 + \gamma_1 \pi_t + \gamma_2 R_{t-1} + \varepsilon_t,$$

where ε_t is a monetary policy shock, assumed to be not only persistent but also heteroskedastic:

$$\begin{aligned} \varepsilon_t &= \delta_\varepsilon \varepsilon_{t-1} + \sqrt{v_t} u_t^\varepsilon, \\ v_t &= (1 - \delta_v)\, \bar{v} + \delta_v v_{t-1} + \sigma^v u_t^v. \end{aligned}$$

By assuming symmetry between the two countries, they derive the following result for the slope coefficient in equation (2):

$$\alpha_1 = \frac{\delta_v - \gamma_2}{\gamma_1}. \tag{8}$$

This condition shows that in order for the monetary policy volatility shock to produce the empirically observed negative α_1, the policy inertia parameter γ_2 must be larger than the persistence of the volatility shock, δ_v.[32] Since our GE system estimation with 3rd-order approximation can incorporate both monetary volatility shocks as well as their persistence, we can empirically test whether the above condition is generally satisfied. Moreover, we note that since equation (8) is derived under the stringent assumption of no real dynamics, the presence of addition shocks, including real ones, may affect this condition and the sign of the slope coefficient. Direct estimation of the full system therefore can help reveal the overall balance of these considerations, and reveal (i) whether the exchange rate is connected or not from macroeconomic fundamentals through stochastic volatilities, and

[32] Backus, Gavazzoni, Telmer and Zin (2010) show that without the stochastic volatility, the slope coefficient in the Fama regression for the real variables cannot be negative.

(ii) whether the negative slope coefficient in the UIP regression can be explained by the endogenous risk premium, as shown in Backus et al. (2010).

B Steady state

To ensure stationarity of relative prices, we need to assume

$$\pi = \pi^*$$

at the steady state.

We parameterize g/y instead of g. Thus, g that appears in the subsequent steady-state conditions are given by

$$g = \frac{\frac{\psi(\mu-1)}{\mu-\psi}}{\left(\frac{\bar{g}}{\bar{y}}\right)^{-1} + \frac{\psi(\mu-1)}{\mu-\psi} - 1}.$$

B.1 Domestic

$$p_H = 1,$$

$$\pi_H = \pi,$$

$$R = \frac{\pi}{\beta\gamma^{\psi(1-\sigma)-1}},$$

$$m = \beta\gamma^{\psi(1-\sigma)-1},$$

$$\tilde{w} = \frac{\mu-1}{\mu},$$

$$c = \frac{\psi(\mu-1)}{\mu-\psi}\left(1-g\right),$$

$$N = \frac{\psi(\mu-1)}{\mu-\psi}\left(1-g\right) + g,$$

$$y = \frac{\psi(\mu-1)}{\mu-\psi}\left(1-g\right) + g,$$

$$c_H = \left(1-\alpha\right)c,$$

$$c_F = \alpha c,$$

$$v = \left\{\frac{\left[c^{\psi}\left(1-N\right)^{1-\psi}\right]^{1-\sigma}}{1-\beta\gamma^{\psi}}\right\}^{\frac{1}{1-\sigma}}$$

$$f = \frac{y}{2\left(1-\theta\beta\gamma^{\psi(1-\sigma)}\right)},$$

$$k = \frac{y}{2\left(1 - \theta\beta\gamma^{\psi(1-\sigma)}\right)},$$

$$\Delta = 1,$$

$$YGR = \gamma.$$

B.2 Foreign

$$p_F^* = 1,$$

$$\pi_F^* = \pi^*,$$

$$R^* = \frac{\pi^*}{\beta\gamma^{\psi(1-\sigma)-1}},$$

$$m^* = \beta\gamma^{\psi(1-\sigma)-1},$$

$$\tilde{w}^* = \frac{\mu - 1}{\mu},$$

$$c^* = \frac{\psi(\mu - 1)}{\mu - \psi}\left(1 - g^*\right),$$

$$N^* = \frac{\psi(\mu - 1)}{\mu - \psi}\left(1 - g^*\right) + g^*,$$

$$y^* = \frac{\psi(\mu - 1)}{\mu - \psi}\left(1 - g^*\right) + g^*,$$

$$c_H^* = \alpha c^*,$$

$$c_F^* = \left(1 - \alpha\right)c^*,$$

$$v^* = \left\{\frac{\left[c^{*\psi}\left(1 - N^*\right)^{1-\psi}\right]^{1-\sigma}}{1 - \beta\gamma^{\psi}}\right\}^{\frac{1}{1-\sigma}},$$

$$f^* = \frac{y^*}{2\left(1 - \theta^*\beta\gamma^{\psi(1-\sigma)}\right)},$$

$$k^* = \frac{y^*}{2\left(1 - \theta^*\beta\gamma^{\psi(1-\sigma)}\right)},$$

$$\Delta^* = 1,$$

$$YGR^* = \gamma.$$

B.3 International

$$s = 1,$$

$$Q = 1,$$

$$d = 1.$$

C Detrended system of equations

The detrended system of equations consists of 35 equations as shown below.

C.1 Domestic

$$c_t := \left[(1-\alpha)^{\frac{1}{\eta}} c_{H,t}^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} c_{F,t}^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}},$$

$$v_t^{1-\sigma} = \left[c_t^{\psi} (1-N_t)^{1-\psi} \right]^{1-\sigma} + \beta \gamma^{\psi} \left(\mathbf{E}_t \left[v_{t+1}^{1-\varepsilon} \right] \right)^{\frac{1-\sigma}{1-\varepsilon}},$$

$$\log\left(\frac{R_t}{R}\right) = \phi_r \log\left(\frac{R_{t-1}}{R}\right) + (1-\phi_r) \left[\phi_\pi \log\left(\frac{\pi_t}{\pi}\right) + \phi_y \log\left(\frac{y_t}{y_{t-1}}\right) \right] + \log(\varepsilon_{R,t}),$$

$$c_{H,t} = (1-\alpha)\, p_{H,t}^{-\eta} c_t,$$

$$c_{F,t} = \alpha \left(s_t p_{F,t}^* \right)^{-\eta} c_t,$$

$$c_t = \frac{\psi}{1-\psi} (1-N_t)\, \tilde{w}_t,$$

$$1 = \mathbf{E}_t m_{t,t+1} \frac{R_t}{\pi_{t+1}},$$

$$m_{t,t+1} = \beta \left[\mathbf{E}_t (v_{t+1})^{1-\varepsilon} \right]^{\frac{\varepsilon-\sigma}{1-\varepsilon}} (v_{t+1})^{\sigma-\varepsilon} \gamma^{\psi(1-\sigma)-1} \frac{c_{t+1}^{\psi(1-\sigma)-1} (1-N_{t+1})^{(1-\psi)(1-\sigma)}}{c_t^{\psi(1-\sigma)-1} (1-N_t)^{(1-\psi)(1-\sigma)}},$$

$$\pi_{H,t} = \frac{p_{H,t}\pi_t}{p_{H,t-1}},$$

$$f_t = \frac{1}{2} p_{H,t} \left(c_{H,t} + g_t + c_{H,t}^* \right) + \gamma\theta \mathbf{E}_t m_{t,t+1} \left(\frac{\pi^{1-\iota}\pi_{H,t}^{\iota}}{\pi_{H,t+1}} \right)^{1-\mu} f_{t+1},$$

$$k_t = \frac{1}{2} \frac{\mu}{\mu-1} \frac{\tilde{w}_t}{A_t} \left(c_{H,t} + g_t + c_{H,t}^* \right) + \gamma\theta \mathbf{E}_t m_{t,t+1} \left(\frac{\pi^{1-\iota}\pi_{H,t}^{\iota}}{\pi_{H,t+1}} \right)^{-\mu} k_{t+1},$$

$$y_t = A_t N_t,$$

$$\left[\frac{1-\theta \left(\frac{\pi^{1-\iota}\pi_{H,t-1}^{\iota}}{\pi_{H,t}} \right)^{1-\mu}}{1-\theta} \right]^{\frac{1}{1-\mu}} f_t = k_t,$$

$$y_t = \Delta_t \left(c_{H,t} + g_t + c_{H,t}^* \right),$$

$$\Delta_t = (1-\theta) \left[\frac{1-\theta \left(\frac{\pi^{1-\iota}\pi_{H,t-1}^{\iota}}{\pi_{H,t}} \right)^{1-\mu}}{1-\theta} \right]^{\frac{\mu}{\mu-1}} + \theta \left(\frac{\pi_{H,t}}{\pi^{1-\iota}\pi_{H,t-1}^{\iota}} \right)^{\mu} \Delta_{t-1},$$

$$YGR_t := \gamma \frac{y_t}{y_{t-1}}.$$

C.2 Foreign

$$c_t^* := \left[(\alpha)^{\frac{1}{\eta}}\left(c_{H,t}^*\right)^{\frac{\eta-1}{\eta}} + (1-\alpha)^{\frac{1}{\eta}}\left(c_{F,t}^*\right)^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}},$$

$$(v_t^*)^{1-\sigma} = \left[(c_t^*)^{\psi}\left((1-N_t)^*\right)^{1-\psi}\right]^{1-\sigma} + \beta\gamma^{\psi}\left(\mathbf{E}_t\left[\left(v_{t+1}^*\right)^{1-\varepsilon}\right]\right)^{\frac{1-\sigma}{1-\varepsilon}},$$

$$\log\left(\frac{R_t^*}{R^*}\right) = \phi_r^*\log\left(\frac{R_{t-1}^*}{R^*}\right) + (1-\phi_r^*)\left[\phi_\pi^*\log\left(\frac{\pi_t^*}{\pi^*}\right) + \phi_y^*\log\left(\frac{y_t^*}{y_{t-1}^*}\right)\right] + \log(\varepsilon_{R,t}^*),$$

$$c_{H,t}^* = \alpha\left(\frac{p_{H,t}}{s_t}\right)^{-\eta} c_t^*,$$

$$c_{F,t}^* = (1-\alpha)\left(p_{F,t}^*\right)^{-\eta} c_t^*,$$

$$c_t^* = \frac{\psi}{1-\psi}\left(1-N_t^*\right)\tilde{w}_t^*,$$

$$1 = \mathbf{E}_t m_{t,t+1}^*\frac{R_t^*}{\pi_{t+1}^*},$$

$$m_{t,t+1}^* = \beta\left(\mathbf{E}_t\left(v_{t+1}^*\right)^{1-\varepsilon}\right)^{\frac{\varepsilon-\sigma}{1-\varepsilon}}\left(v_{t+1}^*\right)^{\sigma-\varepsilon}\gamma^{\psi(1-\sigma)-1}\frac{\left(c_{t+1}^*\right)^{\psi(1-\sigma)-1}\left(1-N_{t+1}^*\right)^{(1-\psi)(1-\sigma)}}{(c_t^*)^{\psi(1-\sigma)-1}\left(1-N_t^*\right)^{(1-\psi)(1-\sigma)}},$$

$$\pi_{F,t}^* = \frac{p_{F,t}^*\pi_t^*}{p_{F,t-1}^*},$$

$$f_t^* = \frac{1}{2}p_{F,t}^*\left(c_{F,t} + c_{F,t}^* + g_t^*\right) + \gamma\theta^*\mathbf{E}_t m_{t,t+1}^*\left[\frac{(\pi^*)^{1-\iota}\left(\pi_{F,t}^*\right)^{\iota}}{\pi_{F,t+1}^*}\right]^{1-\mu} f_{t+1}^*,$$

$$k_t^* = \frac{1}{2}\frac{\mu}{\mu-1}\frac{\tilde{w}_t^*}{A_t^*}\left(c_{F,t} + c_{F,t}^* + g_t^*\right) + \gamma\theta^*\mathbf{E}_t m_{t,t+1}^*\left[\frac{(\pi^*)^{1-\iota}\left(\pi_{F,t}^*\right)^{\iota}}{\pi_{F,t+1}^*}\right]^{-\mu} k_{t+1}^*,$$

$$y_t^* = A_t^* N_t^*,$$

$$\left[\frac{1-\theta^*\left(\frac{(\pi^*)^{1-\iota}\left(\pi_{F,t-1}^*\right)^{\iota}}{\pi_{F,t}^*}\right)^{1-\mu}}{1-\theta^*}\right]^{\frac{1}{1-\mu}} f_t^* = k_t^*,$$

$$y_t^* = \Delta_t^*\left(c_{F,t} + c_{F,t}^* + g_t^*\right),$$

$$\Delta_t^* = (1-\theta^*)\left[\frac{1-\theta^*\left[\frac{(\pi^*)^{1-\iota}\left(\pi_{F,t-1}^*\right)^{\iota}}{\pi_{F,t}}\right]^{1-\mu}}{1-\theta^*}\right]^{\frac{\mu}{\mu-1}} + \theta^*\left[\frac{\pi_{F,t}^*}{(\pi^*)^{1-\iota}\left(\pi_{F,t-1}^*\right)^{\iota}}\right]^{\mu}\Delta_{t-1}^*,$$

$$YGR_t^* := \gamma\frac{y_t^*}{y_{t-1}^*}.$$

C.3 International

$$c_t^{\psi(1-\sigma)-1}\left(1-N_t\right)^{(1-\psi)(1-\sigma)} s_t = \Omega_t Q_t \left(c_t^*\right)^{\psi(1-\sigma)-1}\left(1-N_t^*\right)^{(1-\psi)(1-\sigma)},$$

$$Q_{t+1} = Q_t \left(\frac{\left(v_{t+1}^*\right)^{1-\varepsilon} \mathbf{E}_t \left(v_{t+1}\right)^{1-\varepsilon}}{\left(v_{t+1}\right)^{1-\varepsilon} \mathbf{E}_t \left(v_{t+1}^*\right)^{1-\varepsilon}}\right)^{\frac{\sigma-\varepsilon}{1-\varepsilon}},$$

$$d_t = \frac{s_t \pi_t}{s_{t-1}\pi_t^*}.$$

D Additional results

Tables 5 and 6 show parameter estimation results for the models approximated up to the first, second, and third order without volatility shocks and the full model but excluding the risk-sharing shock. We note the similarity in the parameter estimates under different model estimates, compared to our baseline estimates presented in Table 1.

Table 7 shows the relative variances of observed variables excluding each single shock using parameter estimates for a linearized version of our model. We note from the last row and column that the fundamental shocks other than the risk-sharing shock explain only 14% of the exchange rate fluctuations.

Figures 6–16 show the impulse responses of the observed variables (YGR_t, π_t, R_t, YGR_t^*, π_t^*, R_t^*, d_t), nominal interest rate differential ($R_t - R_t^*$), and the excess return on the foreign currency ($d_{t+1} + R_t^* - R_t$) to the other shocks that are not reported in Section 5.2, given the posterior mean estimates of parameters in the baseline estimation.

Table 5: Posterior distributions of parameters under alternate model specifications

	Linear		2nd order	
Parameter	Mean	90% interval	Mean	90% interval
ε	5.127	[4.308, 5.999]	5.007	[4.389, 5.705]
σ	2.184	[1.875, 2.501]	2.180	[1.962, 2.419]
θ	0.594	[0.495, 0.707]	0.710	[0.665, 0.761]
ι	0.193	[0.048, 0.313]	0.143	[0.048, 0.236]
θ^*	0.672	[0.603, 0.748]	0.633	[0.581, 0.680]
ι^*	0.119	[0.030, 0.199]	0.140	[0.047, 0.234]
ϕ_r	0.790	[0.754, 0.831]	0.817	[0.785, 0.850]
ϕ_π	1.946	[1.715, 2.190]	1.947	[1.703, 2.160]
ϕ_y	0.274	[0.164, 0.383]	0.207	[0.139, 0.275]
ϕ_r^*	0.768	[0.717, 0.815]	0.771	[0.732, 0.816]
ϕ_π^*	2.017	[1.812, 2.244]	2.113	[1.911, 2.307]
ϕ_y^*	0.249	[0.147, 0.347]	0.207	[0.130, 0.288]
ρ_A	0.667	[0.494, 0.813]	0.652	[0.560, 0.732]
ρ_g	0.943	[0.910, 0.977]	0.839	[0.786, 0.884]
ρ_A^*	0.618	[0.530, 0.722]	0.551	[0.453, 0.643]
ρ_g^*	0.954	[0.927, 0.979]	0.968	[0.947, 0.989]
ρ_Ω	0.997	[0.995, 0.999]	0.997	[0.996, 0.999]
$100\sigma_A$	2.138	[1.337, 2.969]	3.003	[2.126, 3.868]
$100\sigma_g$	8.339	[6.913, 9.566]	8.864	[7.495, 10.060]
$100\sigma_{\epsilon_R}$	0.159	[0.135, 0.185]	0.154	[0.133, 0.176]
$100\sigma_A^*$	2.980	[1.916, 4.115]	2.781	[2.055, 3.417]
$100\sigma_g^*$	7.781	[6.613, 8.969]	4.706	[4.108, 5.333]
$100\sigma_{\epsilon_R}^*$	0.160	[0.137, 0.185]	0.161	[0.140, 0.183]
$100\sigma_\Omega$	6.885	[6.059, 7.711]	8.591	[7.538, 9.648]

Note: This table shows the posterior mean and 90 percent highest posterior density intervals based on 2,000 particles from the final importance sampling in the SMC algorithm.

Table 6: Posterior distributions of parameters under alternate model specifications (cont.)

	3rd order		No risk-sharing shock	
Parameter	Mean	90% interval	Mean	90% interval
ε	4.388	[4.129, 4.625]	4.139	[3.775, 4.439]
σ	2.615	[2.502, 2.774]	2.427	[2.200, 2.628]
θ	0.708	[0.675, 0.742]	0.521	[0.469, 0.565]
ι	0.140	[0.053, 0.256]	0.587	[0.482, 0.673]
θ^*	0.495	[0.439, 0.539]	0.840	[0.824, 0.858]
ι^*	0.330	[0.260, 0.416]	0.616	[0.471, 0.792]
ϕ_r	0.749	[0.715, 0.793]	0.685	[0.632, 0.725]
ϕ_π	2.208	[2.041, 2.360]	1.803	[1.655, 1.946]
ϕ_y	0.123	[0.096, 0.152]	0.103	[0.069, 0.139]
ϕ_r^*	0.745	[0.714, 0.772]	0.699	[0.655, 0.739]
ϕ_π^*	1.428	[1.329, 1.489]	1.380	[1.245, 1.499]
ϕ_y^*	0.085	[0.054, 0.116]	0.089	[0.056, 0.122]
ρ_A	0.542	[0.456, 0.620]	0.332	[0.126, 0.473]
ρ_g	0.983	[0.965, 1.000]	0.553	[0.356, 0.701]
ρ_A^*	0.562	[0.486, 0.644]	0.930	[0.903, 0.953]
ρ_g^*	0.947	[0.920, 0.988]	0.581	[0.502, 0.649]
ρ_Ω	0.997	[0.995, 0.999]	-	-
ρ_{σ_A}	-	-	0.251	[0.090, 0.373]
ρ_{σ_g}	-	-	0.386	[0.268, 0.512]
$\rho_{\sigma_{\epsilon_R}}$	-	-	0.378	[0.304, 0.462]
$\rho_{\sigma_A}^*$	-	-	0.105	[0.061, 0.146]
$\rho_{\sigma_g}^*$	-	-	0.241	[0.156, 0.335]
$\rho_{\sigma_{\epsilon_R}}^*$	-	-	0.356	[0.196, 0.501]
ρ_{σ_Ω}	-	-	-	-
$100\sigma_A$	2.948	[2.218, 3.630]	1.396	[1.014, 1.728]
$100\sigma_g$	8.108	[6.955, 9.136]	4.616	[3.417, 5.520]
$100\sigma_{\epsilon_R}$	0.217	[0.172, 0.268]	0.200	[0.143, 0.253]
$100\sigma_A^*$	1.749	[1.370, 2.117]	11.140	[9.235, 13.468]
$100\sigma_g^*$	4.038	[3.405, 4.580]	8.034	[6.393, 9.945]
$100\sigma_{\epsilon_R}^*$	0.285	[0.148, 0.430]	0.179	[0.133, 0.227]
$100\sigma_\Omega$	6.589	[5.940, 7.360]	-	-
τ_A	-	-	1.087	[0.782, 1.427]
τ_g	-	-	1.227	[0.851, 1.573]
τ_{ϵ_R}	-	-	0.736	[0.570, 0.888]
τ_A^*	-	-	0.987	[0.894, 1.121]
τ_g^*	-	-	1.430	[1.142, 1.725]
$\tau_{\epsilon_R}^*$	-	-	1.245	[0.930, 1.591]

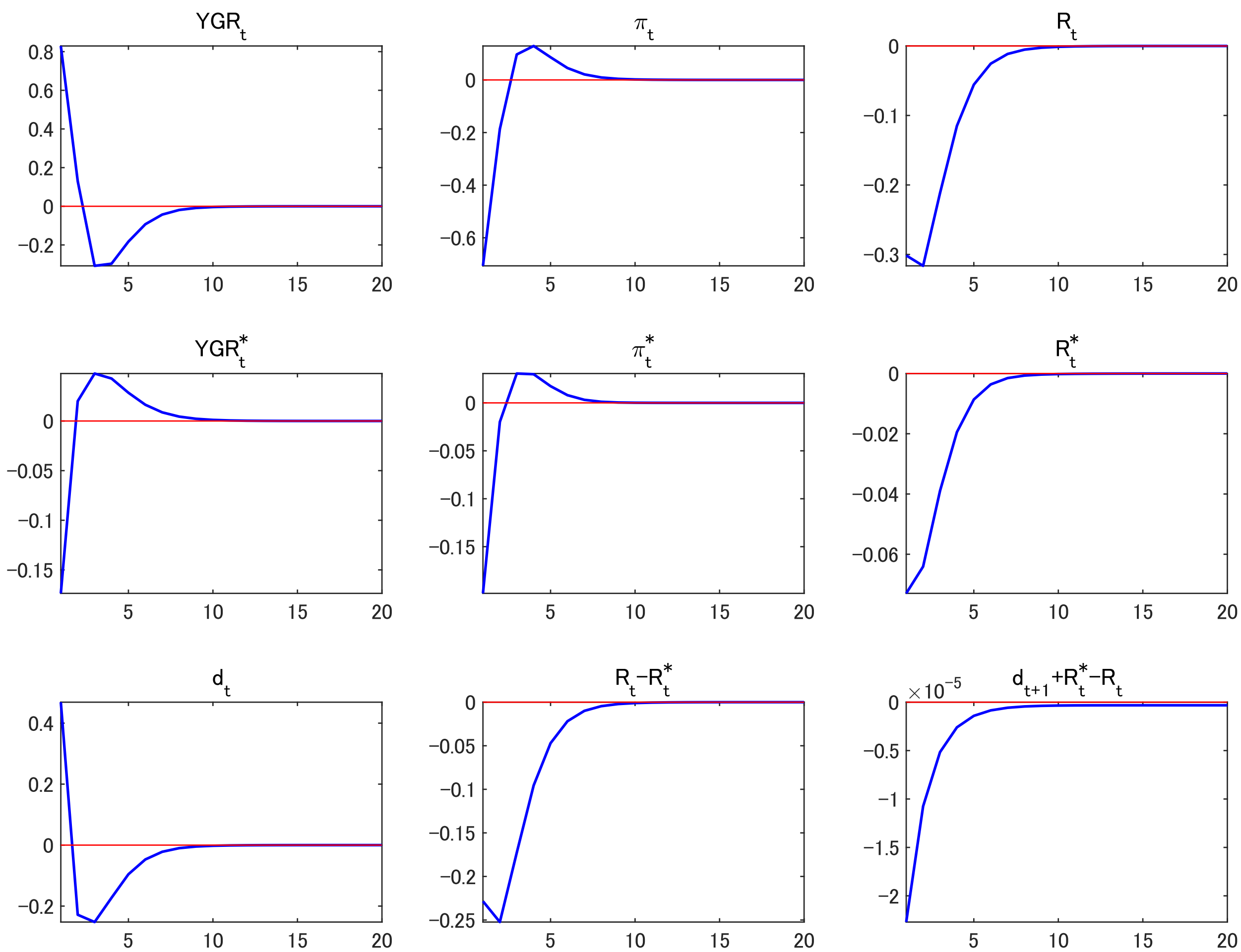
Note: This table shows the posterior mean and 90 percent highest posterior density intervals based on 2,000 particles from the final importance sampling in the SMC algorithm.

Table 7: Relative variances excluding each shock under linear model estimates

		$\Delta \log Y_t$	$\log \pi_t$	$\log R_t$	$\Delta \log Y_t^*$	$\log \pi_t^*$	$\log R_t^*$	d_t
Linear Approx. w/o Stochastic Volatilities								
w/o:	u_A	0.690	0.280	0.382	0.994	0.954	0.940	0.977
	u_g	0.423	0.962	0.793	1.000	0.992	0.997	0.970
	u_{ϵ_R}	0.959	0.920	0.984	1.000	0.996	1.000	0.983
	u_A^*	0.985	0.933	0.919	0.667	0.242	0.307	0.963
	u_g^*	0.999	0.995	0.997	0.456	0.966	0.840	0.968
	$u_{\epsilon_R}^*$	1.000	0.996	0.999	0.955	0.952	0.985	0.989
	u_Ω	0.925	0.920	0.940	0.929	0.896	0.921	0.141

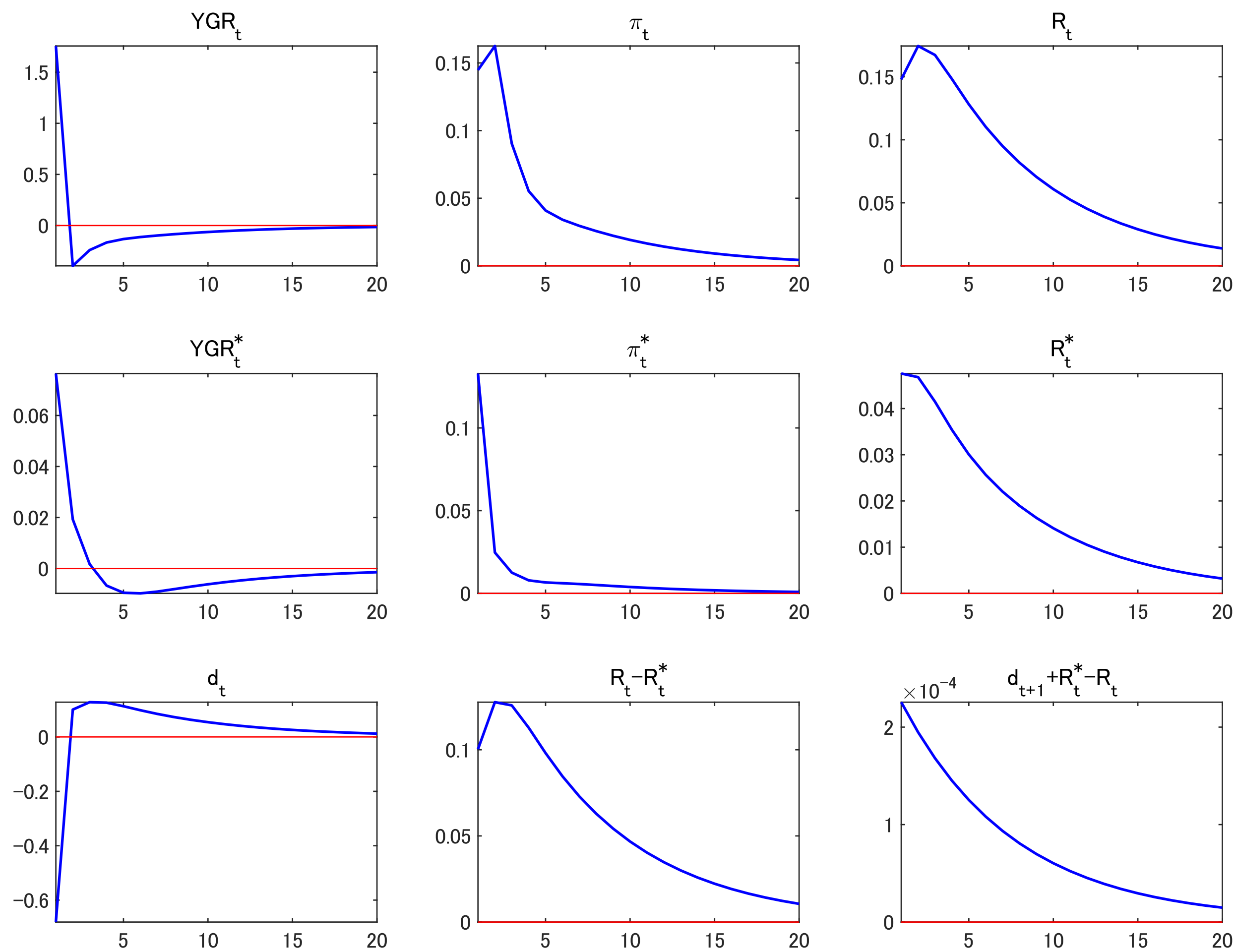
Note: This table shows the variances of the output growth rate, the inflation rate, the nominal interest rate in the home and foreign countries, and the nominal exchange rate depreciation excluding each shock, relative to those with all the shocks, given the posterior mean estimates of parameters in the model under linear approximation.

Figure 6: Responses to home technology shock



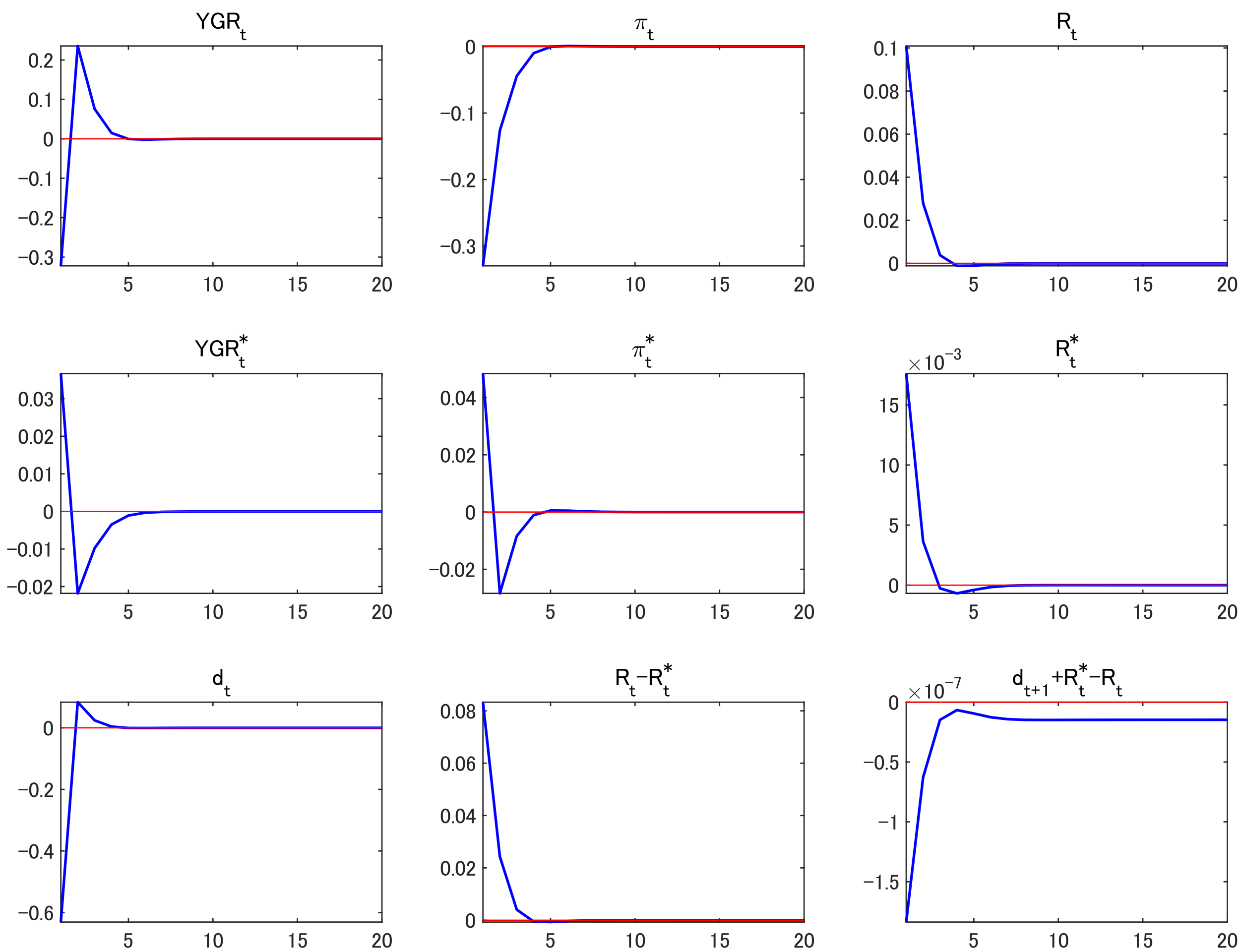
Note: This figure shows the impulse responses of output growth, inflation, the nominal interest rate in both countries, depreciation of the nominal exchange rate, nominal interest rate differential, and deviation from the UIP, in terms of deviations from steady-state values, to a one-standard-deviation shock to home technology, given the posterior mean estimates of parameters in the baseline model.

Figure 7: Responses to home aggregate demand shock



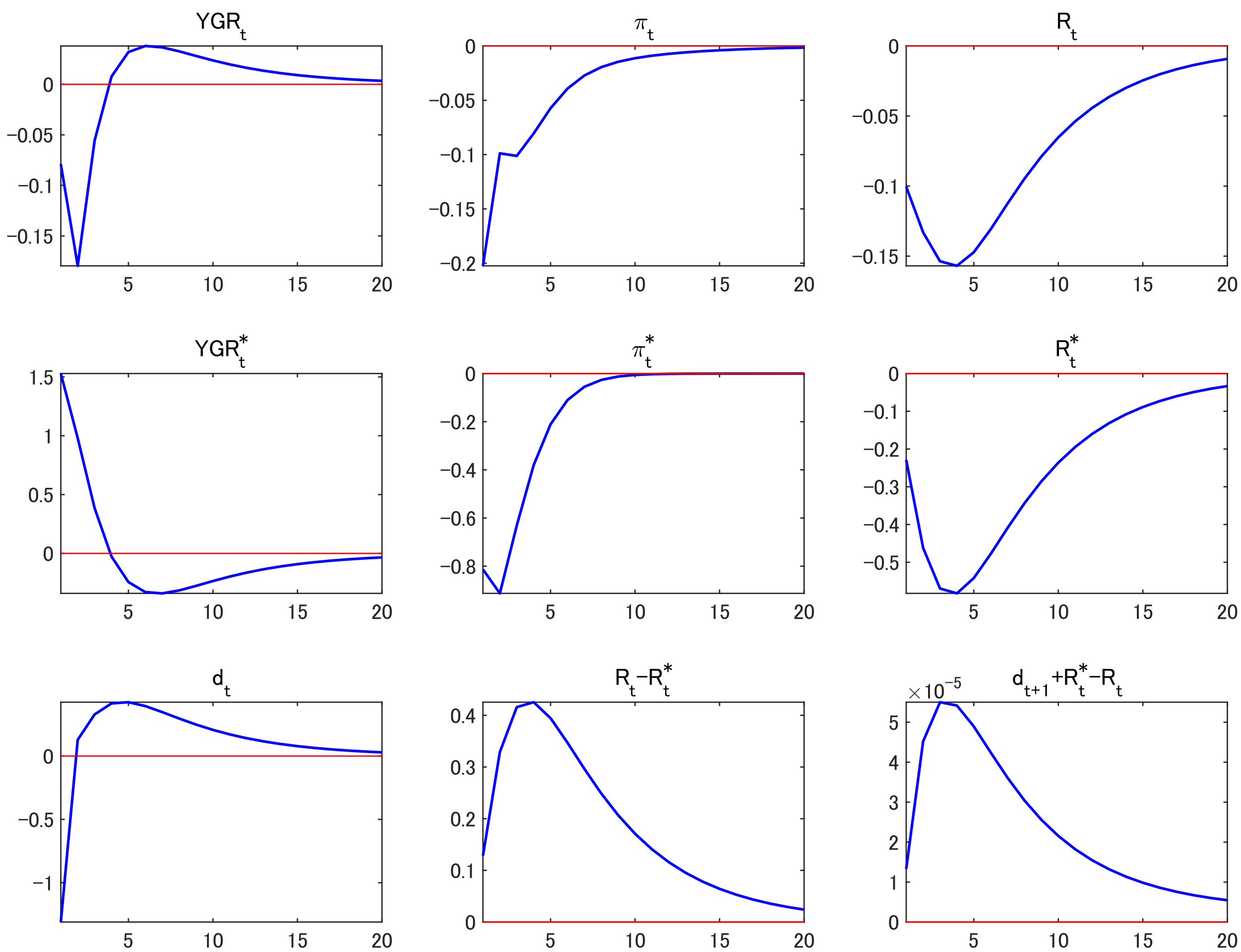
Note: This figure shows the impulse responses of output growth, inflation, the nominal interest rate in both countries, depreciation of the nominal exchange rate, nominal interest rate differential, and deviation from the UIP, in terms of deviations from steady-state values, to a one-standard-deviation shock to home aggregate demand, given the posterior mean estimates of parameters in the baseline model.

Figure 8: Responses to home monetary policy shock



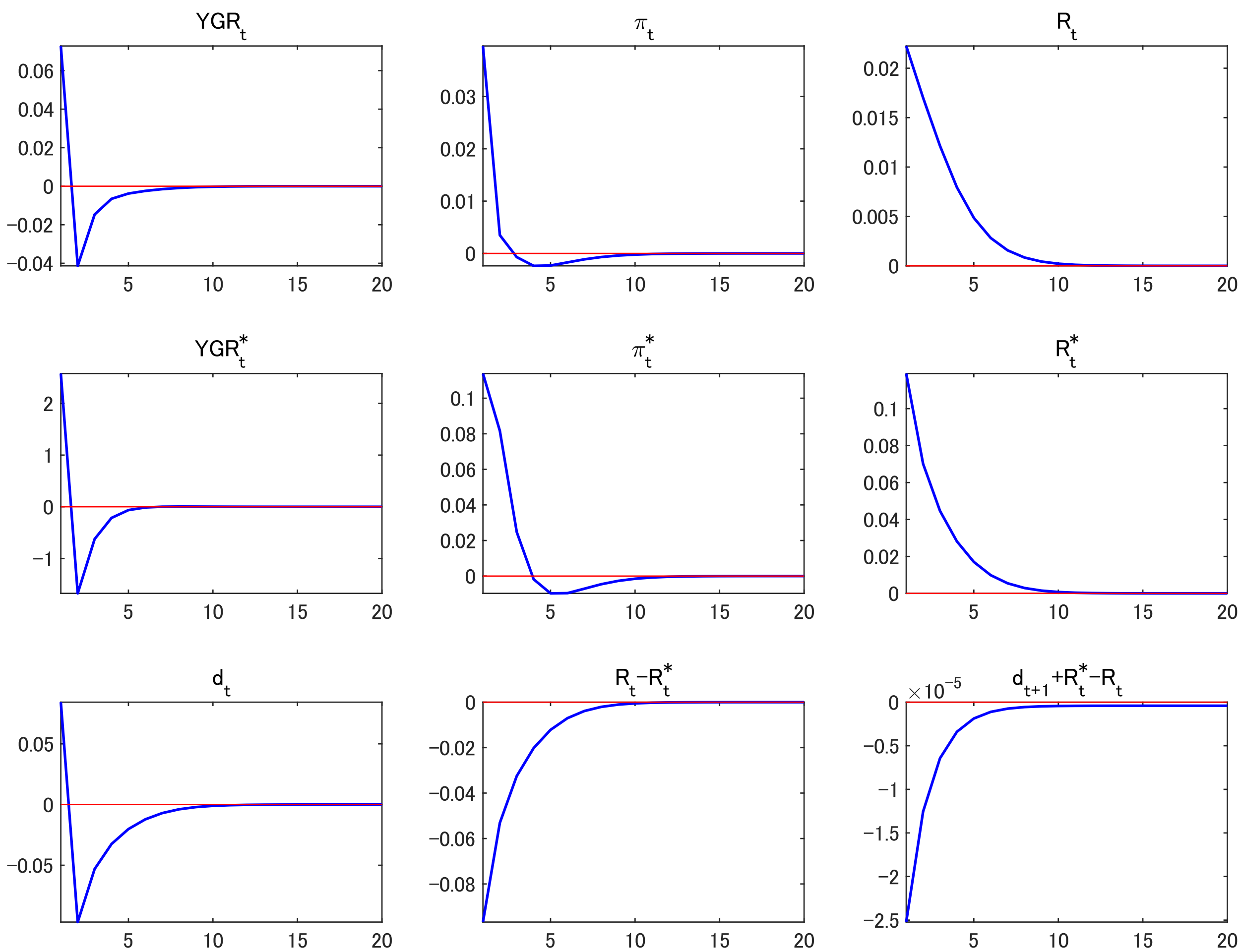
Note: This figure shows the impulse responses of output growth, inflation, the nominal interest rate in both countries, depreciation of the nominal exchange rate, nominal interest rate differential, and deviation from the UIP, in terms of deviations from steady-state values, to a one-standard-deviation shock to home monetary policy, given the posterior mean estimates of parameters in the baseline model.

Figure 9: Responses to foreign technology shock



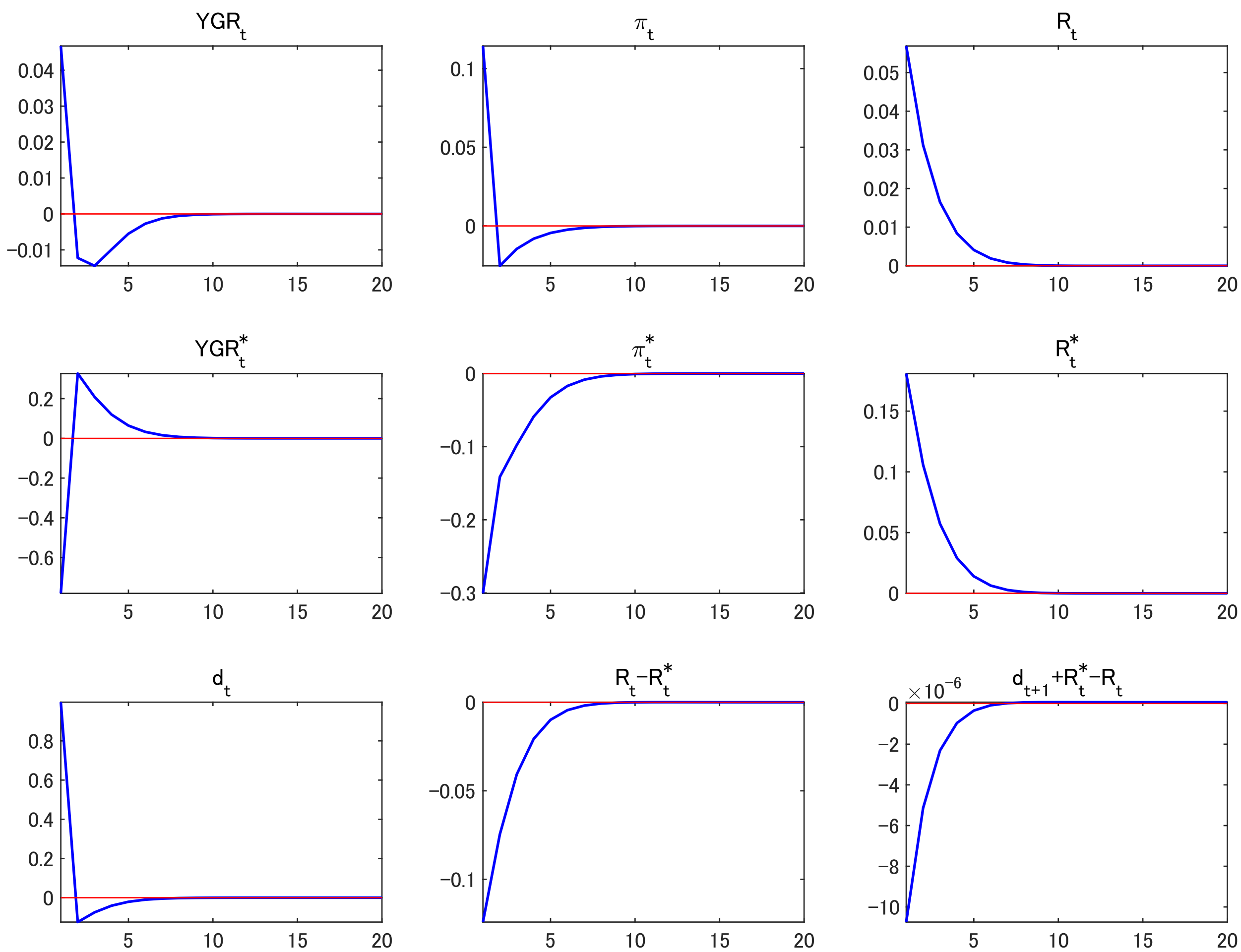
Note: This figure shows the impulse responses of output growth, inflation, the nominal interest rate in both countries, depreciation of the nominal exchange rate, nominal interest rate differential, and deviation from the UIP, in terms of deviations from steady-state values, to a one-standard-deviation shock to foreign technology, given the posterior mean estimates of parameters in the baseline model.

Figure 10: Responses to foreign aggregate demand shock



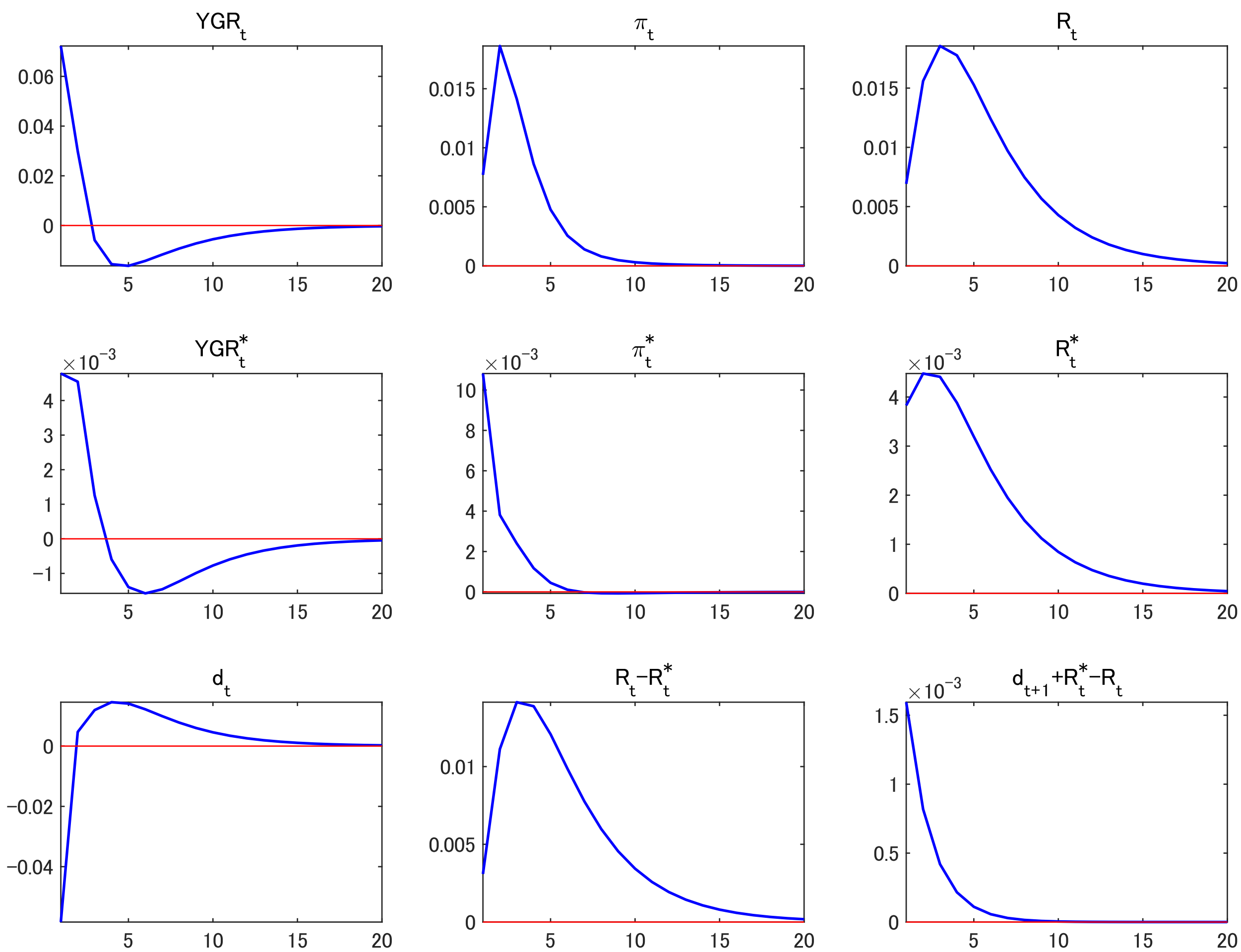
Note: This figure shows the impulse responses of output growth, inflation, the nominal interest rate in both countries, depreciation of the nominal exchange rate, nominal interest rate differential, and deviation from the UIP, in terms of deviations from steady-state values, to a one-standard-deviation shock to foreign aggregate demand, given the posterior mean estimates of parameters in the baseline model.

Figure 11: Responses to foreign monetary policy shock



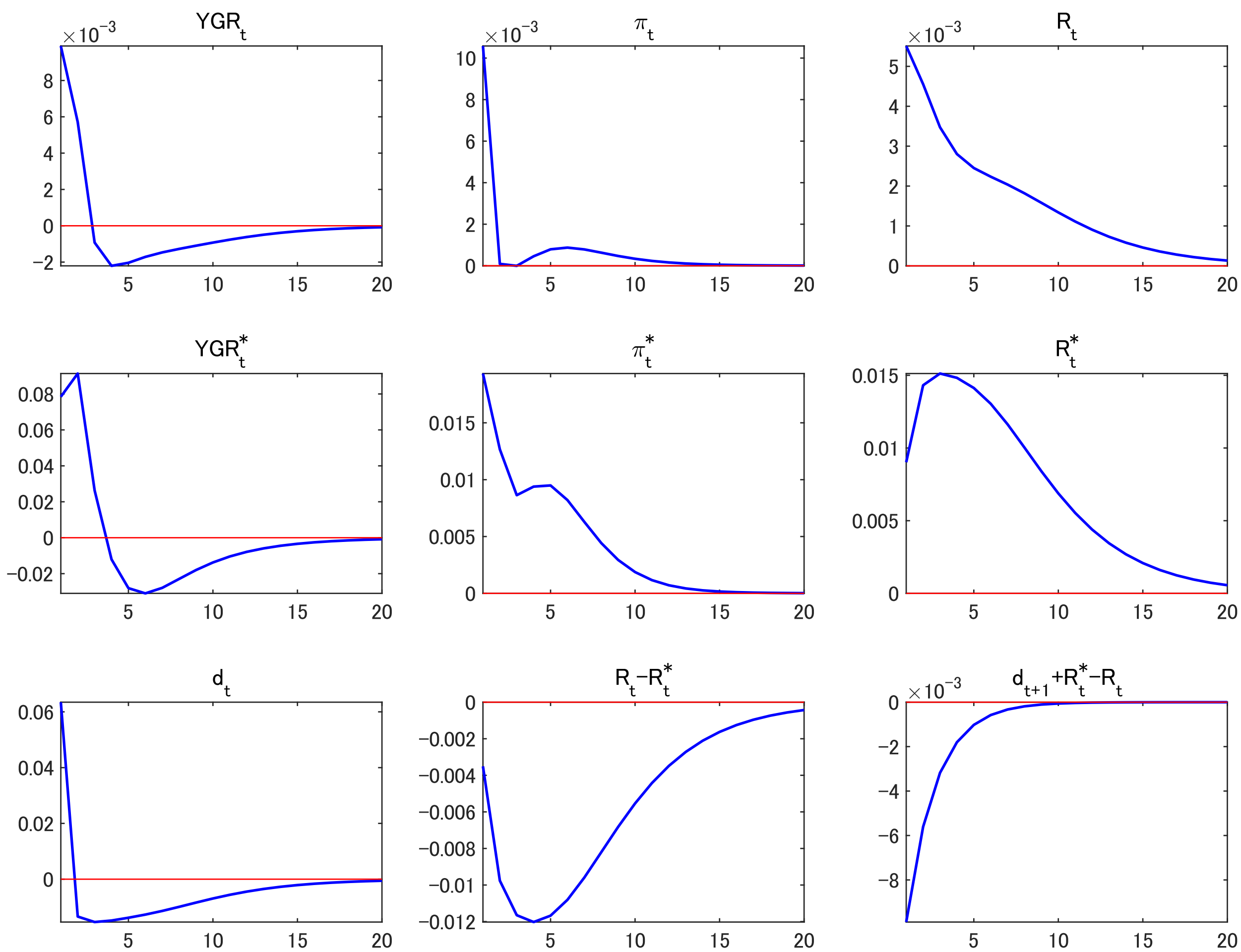
Note: This figure shows the impulse responses of output growth, inflation, the nominal interest rate in both countries, depreciation of the nominal exchange rate, nominal interest rate differential, and deviation from the UIP, in terms of deviations from steady-state values, to a one-standard-deviation shock to foreign monetary policy, given the posterior mean estimates of parameters in the baseline model.

Figure 12: Responses to volatility shock to home aggregate demand



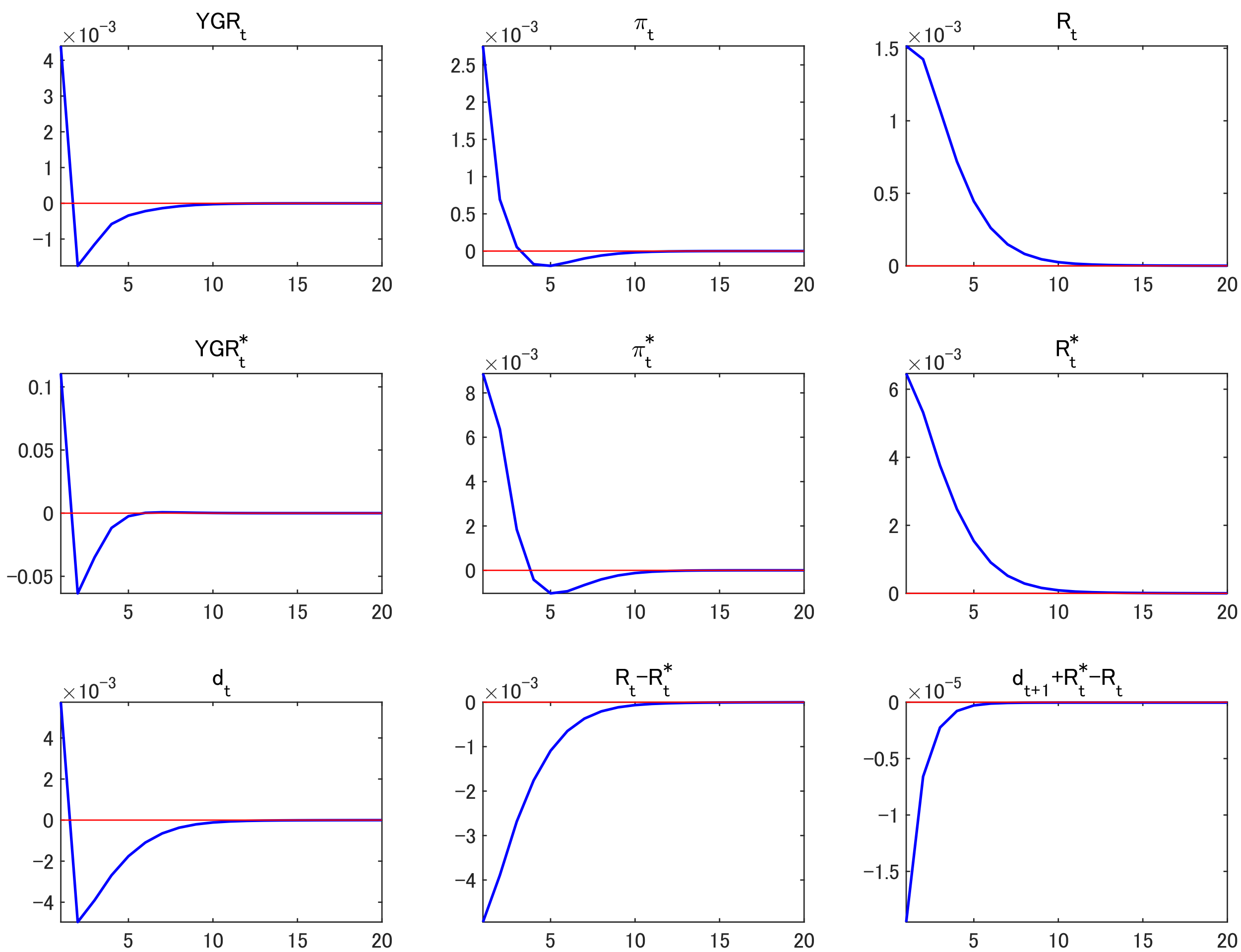
Note: This figure shows the impulse responses of output growth, inflation, the nominal interest rate in both countries, depreciation of the nominal exchange rate, nominal interest rate differential, and deviation from the UIP, in terms of deviations from steady-state values, to a one-standard-deviation volatility shock to home aggregate demand, given the posterior mean estimates of parameters in the baseline model.

Figure 13: Responses to volatility shock to foreign technology



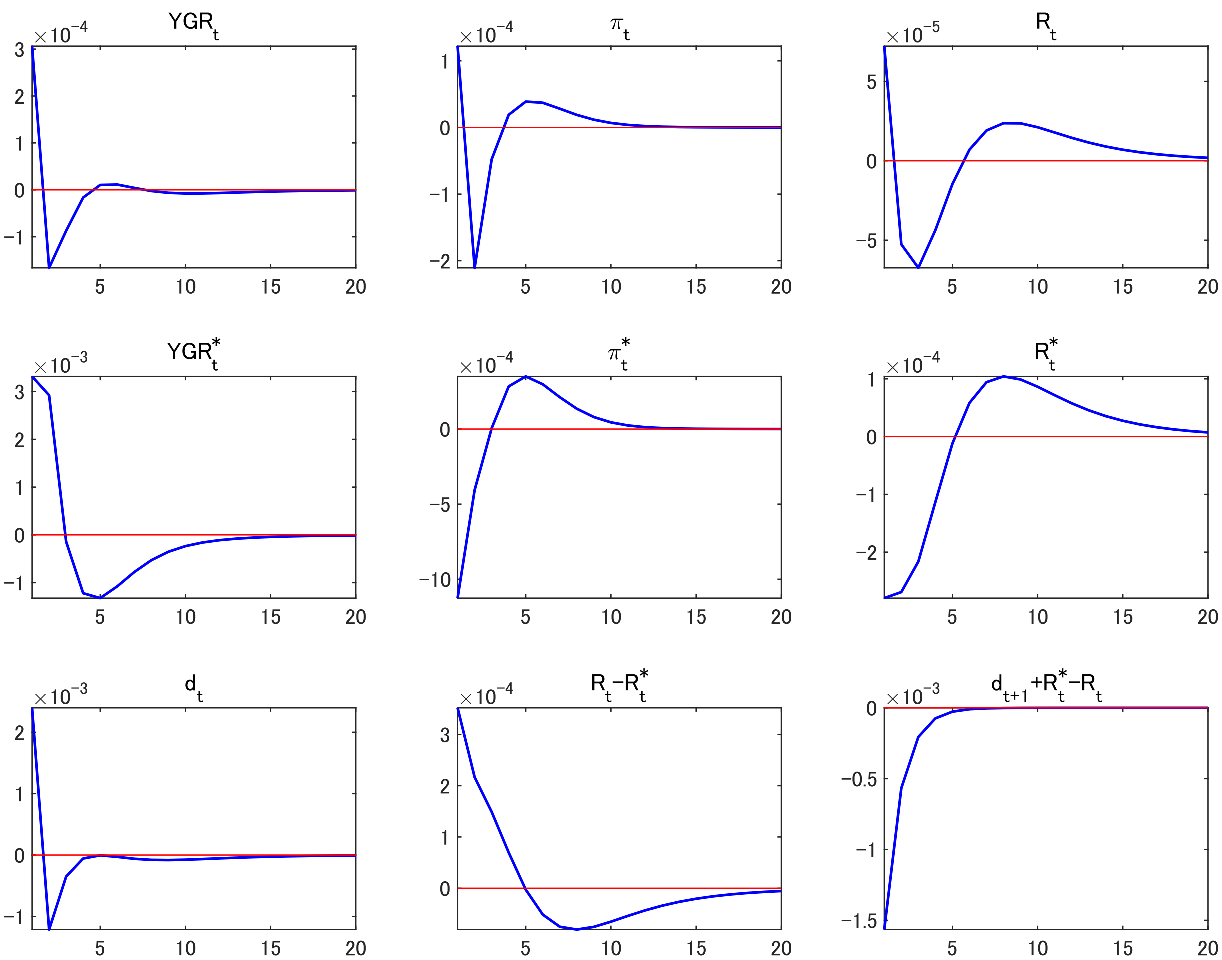
Note: This figure shows the impulse responses of output growth, inflation, the nominal interest rate in both countries, depreciation of the nominal exchange rate, nominal interest rate differential, and deviation from the UIP, in terms of deviations from steady-state values, to a one-standard-deviation volatility shock to foreign technology, given the posterior mean estimates of parameters in the baseline model.

Figure 14: Responses to volatility shock to foreign aggregate demand



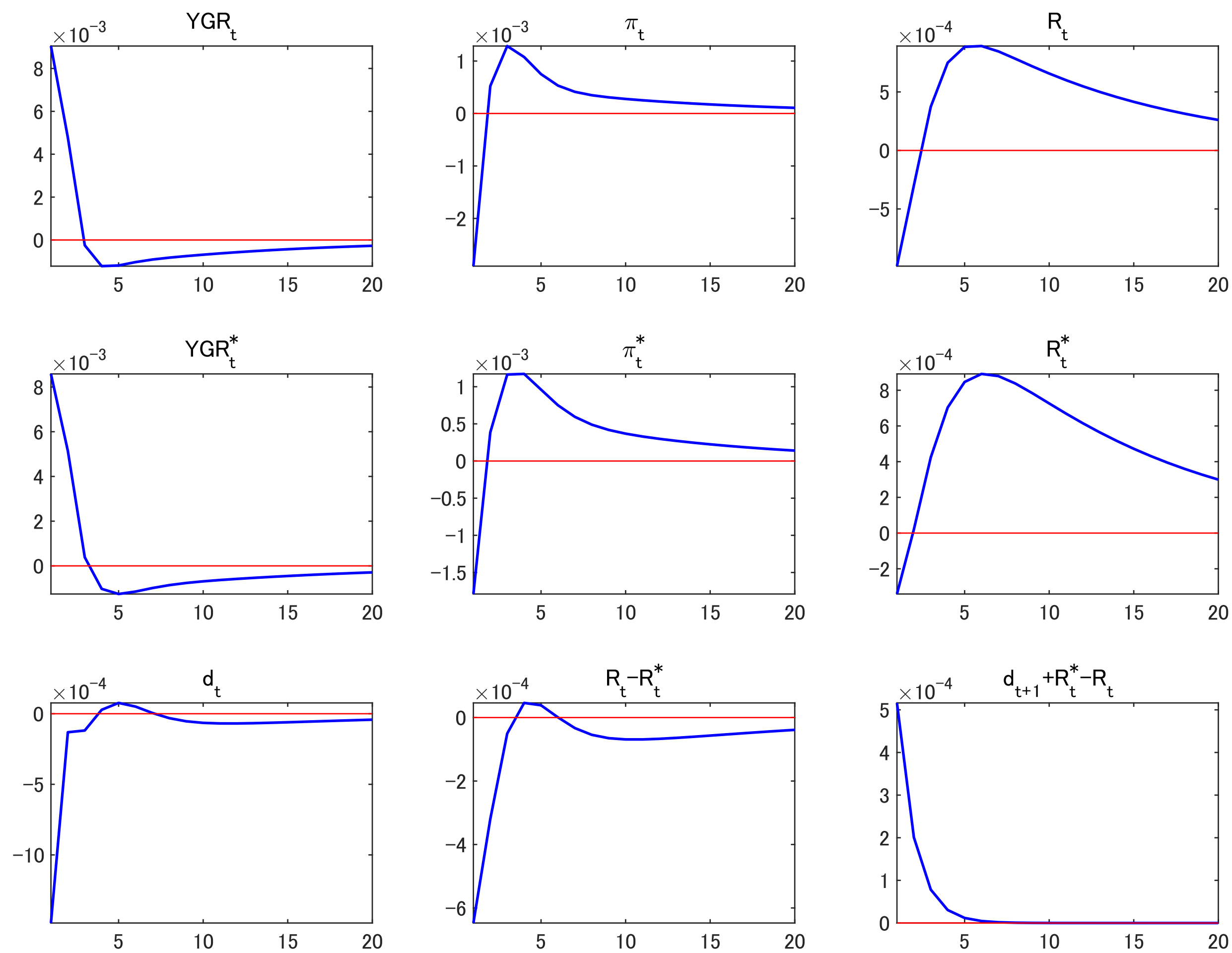
Note: This figure shows the impulse responses of output growth, inflation, the nominal interest rate in both countries, depreciation of the nominal exchange rate, nominal interest rate differential, and deviation from the UIP, in terms of deviations from steady-state values, to a one-standard-deviation volatility shock to foreign aggregate demand, given the posterior mean estimates of parameters in the baseline model.

Figure 15: Responses to volatility shock to foreign monetary policy



Note: This figure shows the impulse responses of output growth, inflation, the nominal interest rate in both countries, depreciation of the nominal exchange rate, nominal interest rate differential, and deviation from the UIP, in terms of deviations from steady-state values, to a one-standard-deviation volatility shock to foreign monetary policy, given the posterior mean estimates of parameters in the baseline model.

Figure 16: Responses to volatility shock to risk sharing condition



Note: This figure shows the impulse responses of output growth, inflation, the nominal interest rate in both countries, depreciation of the nominal exchange rate, nominal interest rate differential, and deviation from the UIP, in terms of deviations from steady-state values, to a one-standard-deviation volatility shock to the risk-sharing condition, given the posterior mean estimates of parameters in the baseline model.