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JEL Classification: L40, L10, D43

Keywords: vertical integration, platform markets, network effects, foreclosure

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The Competitive Effects of Vertical Integration in Platform Markets*

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Abstract

We analyze vertical integration between platforms providing operating systems to manufacturers of devices in presence of indirect network effects between buyers of devices and developers of applications. Vertical integration creates market power over non-integrated manufacturers and application developers. That market power provides the merged entity with the ability to coordinate pricing decisions across both sides of the market, which allows to better internalize network effects. Vertical integration does not systematically lead to foreclosure and can benefit all parties, even in the absence of efficiency gains. Its competitive impact depends on the strength and the structure of indirect network effects.

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1. Introduction

Motivation. Software platform industries have recently witnessed many changes in the nature of the relationship between software and hardware producers. Traditional suppliers of operating systems have ventured into the hardware market and prominent hardware manufacturers have developed their own operating systems. In the smartphone market, while Apple further intensified its hallmarked integration between hardware and

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software, Google launched in 2016 the Pixel, its first device conceptualized and engineered in-house, and recently acquired a major handset manufacturer, HTC.[1] Samsung and Huawei, subjugated to Google for the use of its Android platform while delivering Google substantial money through services installed on their phones, have started to equip some of their devices with their own operating systems. In the online retail sector, Amazon sells devices powered by FireOS, an operating system built on Android's technology but stripped from Google's applications. Microsoft, once praised for its software-only model, has recently ventured in the electronic devices market with the Surface brand. Other industries are witnessing a similar momentum of integration along the value chain.[2] These changes are scrutinized closely by regulators and competition authorities, and whether the usual competitive assessment of integration could readily be applied to platform markets remains an on-going debate in the antitrust arena.[3]

In this article, we address the following question: what are the competitive effects of vertical integration between platforms and device manufacturers? We show that indirect network effects, which are prevalent in digital markets, substantially impact the competitive assessment of vertical integration. Although vertical integration still creates some market power, as the common wisdom has it, the sources of such market power are different than in one-sided markets. Perhaps more importantly, the exercise of such market power does not necessarily harm either consumers or non-integrated competitors. For instance, when indirect network effects are sufficiently strong and asymmetric (in a sense to be defined properly later on), vertical integration can benefit all parties, even in the absence of efficiency gains. A recurrent intuition of our analysis is that vertical integration enables to coordinate several pricing decisions and such coordination sometimes allows a better internalization of indirect network effects.

The Model. Several platforms compete to license their operating systems to two manufacturers of devices. Manufacturers equip their devices with an operating system and then compete to sell devices to buyers. Developers pay fees to platforms to publish their applications on the operating systems. Therefore, a device gives its buyers access to applications developed for the operating system it is equipped with. This interaction between buyers of devices and developers of applications is the source of indirect network effects in our analysis.

Our benchmark is the situation in which none of the platforms are integrated with a manufacturer. There, competition between equally-efficient platforms leads to marginal cost pricing, both for the royalty paid by manufacturers for the operating systems and for the fee paid by developers to publish their applications. Competition prevents platforms from exerting any market power, either on manufacturers or on developers.

Vertical Integration. We then consider vertical integration between a platform and a manufacturer. Because it faces competition from equally-efficient platforms, the

[1]Google initially maintained arm's-length relationships with several smartphone hardware producers to build the Nexus range, even after the acquisition of Motorola in 2011. Whereas some experts argued that Google's primary objective was to strengthen its patent portfolio, many now retrospectively think that this was also a test of the feasibility of a more integrated business model.

[2]For instance, whereas Google is actively promoting its self-driving technology through its division Waymo, traditional car manufacturers such as BMW have developed their own in-house technology.

[3]See, for instance, OECD (2018) and the FTC Hearing #3 regarding 'Competition and Consumer Protection in the 21st Century' (www.ftc.gov/news-events/events-calendar/2018/10/ftc-hearing-3-competition-consumer-protection-21st-century).

vertically-integrated platform is forced to license its operating system at marginal cost. Such a result is standard from the literature on strategic vertical integration (see, e.g., Salop and Scheffman, 1983, Ordover et al., 1990 or Chen, 2001): absent any competitive advantage (in the form of efficiency gains for instance), vertical integration does not create market power over non-integrated manufacturers.[4]

Vertical integration creates, however, market power over developers because the integrated platform has monopoly power over the access to the buyers of its device. This is a new source of market power, which comes from the two-sided nature of our model. The next step of our analysis consists in assessing how the integrated firm exercises such market power. To do so, observe that the integrated platform has two pricing instruments: the fee paid by developers to publish their applications on its operating system and the price paid by buyers for its device. The integrated firm's prices are guided by two forces: a one-sided logic, according to which increasing prices (above their pre-merger levels) allows to extract more profit from developers and from buyers; a two-sided logic, according to which setting an asymmetric price structure allows to better internalize network effects between buyers and developers. Which logic prevails overall depends on the strength and the structure of indirect network effects. For instance, and in the spirit of the literature on two-sided markets (Armstrong, 2006, Rochet and Tirole, 2006 and Caillaud and Jullien, 2003), when buyers value strongly applications, the integrated platform finds it optimal to decrease its developer fee and increase the price for its device.

Next, we analyze the consequences of that market power.

FORECLOSURE. Because the integrated firm sets the same royalty as in the pre-merger benchmark, foreclosure cannot be the result of a 'raise the rival's cost' effect. Foreclosure may arise, or not, because the integrated firm has some market power over developers, which ultimately impacts the non-integrated manufacturers' profit. For instance, when buyers value strongly applications, the integrated platform subsidizes developers and increases the price of its device, which boosts the non-integrated manufacturers' demand. As a result, the vertical merger benefits non-integrated manufacturers. A reverse conclusion obtains when developers value more the participation of buyers. Summarizing, foreclosure of non-integrated manufacturers is neither systematic nor the result of a 'raise the rival's cost' effect. It is, rather, the mere collateral damage of the integrated firm's market power over developers that, sometimes, depending on the strength and the structure of network effects, leads to an asymmetric pricing structure.

WELFARE. Assuming a linear specification of our model, we fully characterize the impact of vertical integration on buyer and developer surpluses. In a nutshell, when indirect network effects are strong and sufficiently asymmetric, large social gains can be generated by implementing an asymmetric pricing structure that internalizes these effects. This is precisely what the integrated platform does, and vertical integration benefits buyers and developers. Otherwise, when network effects are balanced or weak, the pricing structure

[4]We focus on the literature that determines circumstances under which vertical integration creates some market power and thus may lead both to softer downstream competition at the expense of final customers and to harmful foreclosure of non-integrated competitors. Another strand, following Hart and Tirole (1990), shows that vertical integration may be used as a mean not to create but rather to restore the upstream market power that was eroded by a lack of commitment; see Rey and Tirole (2007) and Riordan (2008) for surveys.

chosen by the integrated platform aims more to extract surplus from buyers and developers rather than to internalize indirect network effects. In these cases, developers and buyers tend to be harmed by the vertical merger.

Importantly, there is no obvious correlation between buyer/developer harm and foreclosure of non-integrated manufacturers. For instance, when network effects are much stronger on the developers side than on the buyers side of the market, buyers and developers may benefit from the merger; internalization of indirect network effects by the integrated platform may require to increase the developer fee, which hurts the non-integrated manufacturer.

Efficiency Gains. Last, we consider that vertical integration creates synergies. In a traditional one-sided framework, efficiency gains have two facets. They are pro–competitive because they are passed through partly to buyers in the form of a lower price for the integrated platform's device. They are anti-competitive because they create some market power that allows the integrated firm to command a higher royalty from non-integrated manufacturers, thereby softening competition on the buyers' market through a 'raise the rival's cost' effect.

The analysis becomes more complex in our two-sided framework. Although a more efficient integrated platform is able to command a higher royalty from non-integrated manufacturers, it is not always willing to do so. This holds because, again, the integrated platform uses its pricing instruments to extract surplus (from buyers, developers and non-integrated manufacturers) but also to internalize network effects across both sides of the market. Such internalization requires, sometimes, to set a royalty lower than its pre-merger level. To illustrate, when network effects are stronger on the developers' side than on the buyers' side, subsidizing buyers can be done by setting a low price for the integrated platform's device and charging a low royalty on non-integrated manufacturers. Whether the developer fee and the royalty increase or decrease following the vertical merger depends, again, on the structure and the strength of indirect network effects.

We then study whether vertical integration leads to foreclosure and harms buyers or developers. Overall, when indirect network effects are strong and asymmetric, a situation that may characterize more infant platform markets, vertical integration tends to be beneficial to buyers and developers because it allows a better internalization of those network effects. When network effects are balanced, a situation that may characterize more mature platform markets, vertical integration tends to be detrimental to welfare.

Related literature. To the best of our knowledge, our paper is the first to link, on the one hand, the literature on two-sided markets and, on the other hand, the literature on strategic vertical integration in the specific context of platform-manufacturer relationships.

From the literature on two-sided markets, we borrow the general insight that indirect network effects are key to understanding platform pricing and competition (Caillaud and Jullien, 2003; Armstrong, 2006; Rochet and Tirole, 2006; Weyl, 2010). That literature has considered the effect of exclusive dealing between a platform and content providers (that is, developers in our model): Evans (2013) discusses the antitrust of such vertical relations in platform industries; Doganoglu and Wright (2010) and Hagiu and Lee (2011) provide a rationale for why platforms sign exclusive contracts with content providers; Church and Gandal (2000) describe the incentives of a manufacturer that is integrated with a developer to make its applications compatible with the hardware of a rival manufacturer; Hagiu and Spulber (2013) show that investment in first-party content (that is, vertical integration with one side of the market) depends on whether a platform faces a 'chicken-and-egg' coordination problem; in the video game industry, Lee (2013) finds that exclusivity tends to be pro-competitive, in that it benefits an entrant platform more than an incumbent platform. While we share with these papers the issue of the competitive impact of vertical restraints in two-sided markets, our work also differs substantially, for we are interested in the interactions between platforms/operating systems and manufacturers when devices are an essential link to connect buyers and developers.

Our analysis also belongs to the strategic approach of vertical integration initiated by Ordover et al. (1990). A message conveyed by that literature is that vertical integration can lead to input foreclosure and be detrimental to consumer surplus. Analyses that feature trade-offs between the pro- and the anti-competitive effects of vertical integration include the following: Ordover et al. (1990) and Reiffen (1992), in which integration generates an extra commitment power; Riordan (2008) and Loertscher and Reisinger (2014), in which the integrated firm is dominant; Chen (2001), in which manufacturers have switching costs; Choi and Yi (2000), in which upstream suppliers can choose the specification of their inputs; Chen and Riordan (2007), in which exclusive dealing can be used in combination with integration; Nocke and White (2007) and Normann (2009), in which upstream suppliers tacitly collude; Hombert et al. (2019), in which there are more manufacturers than upstream suppliers; and Hunold and Stahl (2016), in which integration can be either controlling or passive.[5] None of these papers address multi-sided markets, and our analysis provides several new insights. For instance, vertical integration does not systematically lead to input foreclosure and may benefit both consumers and non-integrated manufacturers even in the absence of efficiency gains.

Organization of the Paper. Section 2 describes the model. Section 3 provides two benchmarks, the Ramsey optimum and the separation case. Section 4 analyzes the impact of vertical integration when there are no efficiency gains. Section 5 studies the role of efficiency gains. Section 6 concludes. All proofs are relegated to an Appendix.

2. Model

We consider a two-sided market where buyers of devices and developers of applications may interact. These interactions require: buyers to purchase devices from manufacturers; developers to decide which operating systems to develop for; manufacturers to choose an operating system and a price for their devices; platforms to set royalties for the use of their operating systems and fees to publish applications on their operating systems.

Platforms. Platforms compete to license their operating systems to manufacturers of devices and attract developers of applications. Platforms are symmetric: the marginal cost to provide an operating system is given by $c > 0$ and there are no costs to publish applications on an operating system. There are $N + 1$ (with $N \geq 2$) such platforms, denoted by I, E_1, ..., E_N. Platform I will be the one contemplating the merger with a

[5] For empirical analyses, see, e.g., Lafontaine and Slade (2007) and Crawford et al. (2018) and the references therein.

manufacturer. In the following, we will sometimes refer to platforms E_1, ..., E_n as the fringe of platforms.

Platforms levy royalties from manufacturers. Denote by w_i the royalty charged by platform i to a manufacturer for each device using that platform's operating system and sold to buyers. Alternatively, the royalty can be viewed as the means through which the monetization of user-generated data is shared between the manufacturer and the platform.[6]

Platforms also charge fees to application developers.[7] Denote by a_i the fee charged by platform i to allow a developer to make its application available on that platform's operating system.

APPLICATION DEVELOPERS. Developers bear a development cost but no additional platform-specific cost. Once an application is developed, it can therefore be made available on any operating system at no further cost. Let a denote the total fee paid by developers to platforms to reach a mass n_B of buyers. Let $Q_S(a, n_B)$ denote the number of developers willing to develop an application. Assume that application developers value positively the number of buyers ($\partial Q_S/\partial n_B > 0$) and that the number of developers decreases with the fees charged by platforms ($\partial Q_S/\partial a < 0$). To compute developer surplus, we consider that there exists a representative developer with utility function $U_S(q_S, n_B)$ such that $Q_S(a, n_B)$ is the solution of $\max_{q_S \geq 0} U_S(q_S, n_B) - aq_S$. Let $V_S(a, n_B)$ denote the corresponding indirect utility.

Developers want to reach the buyers of devices. Developers are thus willing to pay a positive fee to a platform if that platform's operating system has been chosen by some manufacturers. Developers are also willing to publish their applications on a platform that subsidizes them through a negative developer fee, even if that platform has attracted no manufacturers. The total developer fee a has thus two components: the fee paid to platforms that have attracted some manufacturers; the subsidy, if any, offered by platforms that have attracted no manufacturers.

MANUFACTURERS. There are two manufacturers, denoted by M_1 and M_2, which are symmetric and produce at the same constant marginal cost, normalized to nil. In our two-sided framework, the buyers' demand for manufacturer M_k's device depends on the number of applications running on that device, denoted by n_S^k, and on the prices charged by manufacturers to buyers, denoted by p_k and p_ℓ, $k \neq \ell \in \{1, 2\}$. Hence, the demand for device k may be written as $Q_B^k(p_k, p_\ell, n_S^k)$. Assume that: devices are demand substitutes for buyers, or $\partial Q_B^k/\partial p_k < 0 < \partial Q_B^k/\partial p_\ell$; the direct price effect is stronger than the indirect one, or $\partial Q_B^k/\partial p_k + \partial Q_B^k/\partial p_\ell < 0$; buyers value positively the number of applications, or $\partial Q_B^k/\partial n_S^k > 0$. We also assume demands are symmetric: if there is the same number of applications n_S available on each device, as will be the case in equilibrium, then $Q_B^k(p_k, p_\ell, n_S) = Q_B(p_k, p_\ell, n_S)$ and $Q_B^\ell(p_\ell, p_k, n_S) = Q_B(p_\ell, p_k, n_S)$.

[6]Contracts between a manufacturer and a platform typically specify which party owns the data and, accordingly, who can monetize these data through advertising for instance.

[7]Software platforms often charge developers on participation (Google charges developers $25 for each application published on the Play Store) or on transaction each time an application is sold on the platform (both Apple and Google charge a 30% royalty on each transaction on their respective applications stores).

To compute the buyer surplus in equilibrium, we consider that there exists a representative buyer with utility function $U_B(q_1, q_2, n_S)$, where n_S is the number of applications running on each device, such that $Q_B(p_1, p_2, n_S)$ and $Q_B(p_2, p_1, n_S)$ are solutions of $\max_{(q_1 \geq 0, q_2 \geq 0)} U_B(q_1, q_2, n_S) - p_1 q_1 - p_2 q_2$. Let $V_B(p_1, p_2, n_S)$ denote the corresponding indirect utility.

Figure 1 summarizes the structure of the model.

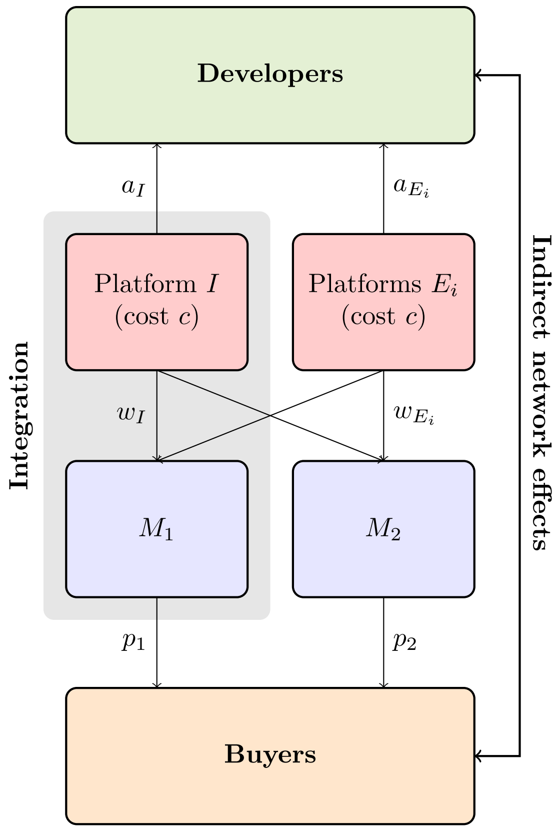


Figure 1 – The model.

Last, we assume throughout our analysis that royalties must be larger than the platforms' marginal cost c (but developer fees may be either positive or negative). This assumption allows us to bypass the following issue. From the perspective of the platform-manufacturer relationship, the royalty and developer fee act jointly as an 'imperfect' two-part tariff offered to the manufacturer (since the developer fee indirectly affects the manufacturer's profit through its impact on developers). In the framework of upstream suppliers competing in two-part tariffs to supply several downstream manufacturers, Schutz (2012) shows that a pure strategy equilibrium may fail to exist and discusses issues related to its characterization. Restricting the fixed component of the two-part tariff to be positive eliminates these issues. Our assumption on the royalties plays a similar role.

RUNNING EXAMPLE. We sometimes use the following specification of the model.

- *Buyers.*

Demand for device k is given by[8]

$$Q_B(p_k, p_\ell, n_S) = v + u_B n_S - p_k - \gamma \left(p_k - \frac{p_k + p_\ell}{2} \right),$$

where $\gamma \geq 0$ measures the degree of substitutability between products (or of product market competition) and u_B the strength of indirect network effects from the buyers' side. Assume $v > c$.

The utility function of the representative buyer is given by

$$U_B(q_1, q_2, n_S) = q_0 + (v + u_B n_S) \sum_{k=1,2} q_k - \frac{1}{2} \frac{1}{2(1+\gamma)} \left(2 \sum_{k=1,2} q_k^2 + \gamma (\sum_{k=1,2} q_k)^2 \right),$$

where q_0 is the numéraire and q_k is the quantity of device k bought.

- *Developers.*

  The number of applications is given by $Q_S(a, n_B) = u_S n_B - a$, where u_S measures the strength of network effects from the developers' side. The representative developer's utility function is given by $U_S(q_S, n_B) = q_0 + u_S n_B q_S - \frac{1}{2} q_S^2$ where q_0 is the numéraire and q_S is the number of applications developed.

Timing. In stage 1, platforms set royalties to license their operating systems to manufacturers and fees charged to developers. In stage 2, manufacturers choose the operating system for their devices. Once operating systems are chosen, manufacturers set the prices of their devices in stage 3. Last, in stage 4, buyers decide whether to buy a device, and, simultaneously, developers decide whether to develop an application. All decisions are public and we look for the subgame-perfect equilibrium of the game.

2.1. Participation Decisions

Consider the last stage of the game. Given devices prices p_1 and p_2 and a total developer fee a, the number of buyers of each device and the number of developers must be consistent with each other and solve the following system

$$\begin{cases} n_{B1} = Q_B(p_1, p_2, n_S), \\ n_{B2} = Q_B(p_2, p_1, n_S), \\ n_S = Q_S(a, n_{B1} + n_{B2}). \end{cases} \tag{2.1}$$

Assume that the solution of (2.1) is interior for the relevant range of prices.[9] That solution defines, as functions of the prices of the devices and the developer fee only, the buyers' demands for devices, denoted by $D_k(p_k, p_\ell, a)$ with $k \neq \ell \in \{1, 2\}$, and the number of applications developed (also called the developers' demand), denoted by $D_S(p_1, p_2, a)$. As shown in Appendix A.1, the following usual properties hold: the developers' demand is decreasing in the prices of devices and in the developer fee ($\partial D_S / \partial p_k < 0$ and $\partial D_S / \partial a < 0$); the demand for a device is decreasing in its own price and in the developer fee

[8] This is Shubik and Levitan (1980)'s linear demands system, to which we append indirect network effects additively.

[9] As shown in Appendix A.1, this requires that indirect network effects are not too strong.

($\partial D_k/\partial p_k < 0$ and $\partial D_k/\partial a < 0$). We further impose that the demand for a device is more responsive to its own price than to the price of the other device ($\partial D_k/\partial p_k + \partial D_k/\partial p_\ell < 0$). In the running example, all these properties hold provided that $2u_B u_S < 1$.

Perhaps more surprising is the fact that indirect network effects impact the nature of the interaction between manufacturers on the product market. The demand for device faced by a manufacturer may, indeed, either increase or decrease with the price of the rival manufacturer, depending on the strength of indirect network effects relative to the degree of product market competition. Formally, using the system (2.1), it follows immediately that (omitting arguments)

$$\frac{\partial D_k}{\partial p_\ell} = \frac{\partial Q_B}{\partial p_\ell} + \frac{\partial Q_B}{\partial n_S}\frac{\partial D_S}{\partial p_\ell},$$

which can be positive or negative. The intuition is as follows. If p_ℓ, say, increases, then some buyers are diverted from M_ℓ, and M_k's demand increases by $\partial Q_B/\partial p_\ell$. This is a standard rivalry effect created by product market competition between manufacturers. The increase in p_ℓ has, moreover, a negative impact on the total number of buyers, since the direct price effect on buyers of device ℓ is stronger than the indirect price effect on buyers of device k ($\frac{\partial Q_B}{\partial p_\ell}(p_\ell, p_k, n_S) + \frac{\partial Q_B}{\partial p_\ell}(p_k, p_\ell, n_S) < 0$). Since there are less buyers overall, there are fewer applications too, for developers find it less attractive to develop. Because buyers value applications, this negatively affects M_k's demand by $\partial Q_B/\partial n_S$. We therefore expect that when indirect network effects are small (that is, when $(\partial Q_B/\partial n_S)(\partial D_S/\partial p_\ell) \approx 0$), the rivalry effect created by product market competition dominates and $\partial D_k/\partial p_\ell \leq 0$, that is, devices are demand substitutes. By contrast, when product market competition is weak (that is, when $\partial Q_B/\partial p_\ell \approx 0$), then the interaction created by indirect network effects dominates and $\partial D_k/\partial p_\ell \geq 0$, that is, devices are demand complements. In the running example, devices are demand substitutes when $\gamma - 2u_B u_S(1+\gamma) > 0$, and demand complements otherwise.

In the sequel, we shall focus on the case studied by the bulk of the literature on strategic vertical integration, namely the case where manufacturers' products are demand substitutes:

ASSUMPTION 1. *Indirect network effects are not too strong relative to product market competition so that manufacturers' products are demand substitutes: for $k \neq \ell$, for all (p_k, p_ℓ, a)*

$$\frac{\partial D_k}{\partial p_\ell}(p_k, p_\ell, a) \leq 0.$$

In the running example, this amounts to $\sigma \equiv \gamma - 2u_B u_S(1+\gamma) \geq 0$.

2.2. *Competition between Manufacturers*

In stage 3, manufacturers compete on the product market. Given a royalty w_k that M_k pays for an operating system and a fee a paid by developers, let $\pi_k(w_k, p_k, p_\ell, a) = (p_k - w_k)D_k(p_k, p_\ell, a)$ denote M_k's profit. We make some assumptions on manufacturers' best responses in prices that ensure the price competition subgame is 'well-behaved.'

M_k's best response, denoted by $R_k(w_k, p_\ell, a)$, is uniquely characterized by the first-order condition $\frac{\partial \pi_k}{\partial p_k}(w_k, R_k, p_\ell, a) = 0$. Moreover, $0 \leq \frac{\partial R_k}{\partial p_\ell} < 1$ for all (w_k, p_k, p_ℓ, a), so that prices of devices are strategic complements and best responses satisfy the usual stability assumption.[10] Last, M_k's best response decreases with a, that is, $\frac{\partial R_k}{\partial a} \leq 0$ for all (w_k, p_k, p_ℓ, a).[11] This assumption seems reasonable since an increase in the developer fee negatively impacts the demand for device k. Together, these assumptions ensure that there exists a unique pair of prices $(\hat{p}_1(w_1, w_2, a), \hat{p}_2(w_2, w_1, a))$ that form the Nash equilibrium of stage 3 of the game, and that the equilibrium price of a manufacturer is increasing in its royalty and in the developer fee, or $\frac{\partial \hat{p}_k}{\partial w_k} > 0$ and $\frac{\partial \hat{p}_k}{\partial a} \leq 0$. We further assume that cost pass-throughs are smaller than 1, or $\frac{\partial \hat{p}_k}{\partial w_k} < 1$.[12] All these assumptions are satisfied in our running example as we show in Appendix A.2.

3. Benchmarks

We study two benchmarks: the welfare-maximizing outcome; the situation of 'separation,' in which none of the platforms are integrated with manufacturers.

3.1. Ramsey Pricing

Let $\Pi(p_1, p_2, a) = (p_1 - c)D_1(p_1, p_2, a) + (p_2 - c)D_2(p_2, p_1, a) + aD_S(p_1, p_2, a)$ denote the industry profit, that is, the sum of the platforms' and the manufacturers' profits. Welfare is then given by $W(p_1, p_2, a) = V_B(p_1, p_2, D_S(p_1, p_2, a)) + V_S(a, D_1(p_1, p_2, a) + D_2(p_2, p_1, a)) + \Pi(p_1, p_2, a)$.

Welfare-maximizing Prices. Maximizing welfare requires to set prices for the devices and a developer fee that are below the corresponding marginal costs in order to internalize network effects across buyers and developers. Simple computations show indeed that, in an interior optimum, the socially optimal prices are given by (omitting some notations)[13]

$$\begin{aligned} p_1 - c = p_2 - c &= -\frac{\partial U_S}{\partial n_B}(D_S, D_1 + D_2), \\ a &= -\frac{\partial U_B}{\partial n_S}(D_1, D_2, D_S). \end{aligned}$$

Although these prices maximize welfare, they provide the industry with a loss. Absent the possibility to make transfers between, on the one hand, buyers and developers, and, on the other hand, platforms and manufacturers, prices of devices and the developer fee must ensure that the industry breaks even.

Ramsey Prices. Imposing that the industry breaks even prevents from setting prices below marginal costs on both sides of the market. Suppose that the break-even constraint binds

$$(p_1 - c)D_1 + (p_2 - c)D_2 + aD_S = 0. \tag{3.1}$$

This implies that either devices are sold at a price below the marginal cost ($p_1 - c < 0$

---

[10] This holds when $0 < \frac{\partial^2 \pi_k}{\partial p_k \partial p_\ell} < -\frac{\partial^2 \pi_k}{\partial p_k^2}$ for all (w_k, p_k, p_ℓ, a). See Seade (1980) and Dixit (1986).

[11] Observe that $\frac{\partial R_k}{\partial a} < 0$ amounts to $\frac{\partial^2 \pi_k}{\partial p_k \partial a} = \frac{\partial D_k}{\partial a} + D_k \frac{\partial^2 D_k}{\partial p_k \partial a}(-\frac{\partial D_k}{\partial p_k})^{-1} < 0$. Hence, R_k decreases with a if either $\frac{\partial^2 D_k}{\partial p_k \partial a} < 0$ or if $\frac{\partial^2 D_k}{\partial p_k \partial a} > 0$ but small enough.

[12] This is a usual assumption that can be linked to the log-curvature of demand functions as discussed in Weyl and Fabinger (2013) and Ritz (2015) for instance.

[13] See Appendix A.3.

and $p_2 - c < 0$) and application developers are charged a strictly positive fee ($a > 0$); or, devices are sold at a strictly positive margin ($p_1 - c > 0$ and $p_2 - c > 0$) and developers are given a subsidy ($a < 0$). Intuitively, developers should be subsidized when they create a strong externality on buyers (that is, when buyers have a stronger valuation for applications than developers' valuation for the number buyers), while keeping an eye on how such subsidies impact the break-even constraint (3.1). This intuition is made rigorous in Appendix A.3, in which we show that developers are subsidized if and only if (omitting some notations)

$$\frac{1}{\eta_S}\left(\frac{\partial U_S}{\partial n_B} + \frac{\partial U_B}{\partial n_S}\frac{\partial Q_S}{\partial n_B}\right) < \frac{1}{\eta_B}\left(\frac{\partial U_B}{\partial n_S} + \frac{\partial U_S}{\partial n_B}\frac{\partial Q_B}{\partial n_S}\right), \tag{3.2}$$

where $\eta_B = -\frac{1}{n_B}(\frac{\partial Q_B}{\partial p_1}(p,p,n_S) + \frac{\partial Q_B}{\partial p_2}(p,p,n_S))$ and $\eta_S = -\frac{1}{n_S}\frac{\partial Q_S}{\partial a}(a, n_B)$. There are three terms in the left-hand side of Equation (3.2). The first term ($1/\eta_S$) is the inverse of the semi-elasticity of the developers' demand. The second term ($\frac{\partial U_S}{\partial n_B}$) measures the extent to which developers benefit directly from an increase in the participation of buyers. The third term ($\frac{\partial U_B}{\partial n_S}\frac{\partial Q_S}{\partial n_B}$) measures the extent to which buyers benefit from an increase in their own participation through a feedback effect: an increase in the number of buyers boosts the number of developers, which ultimately benefits buyers. Therefore, Equation (3.2) shows that developers are subsidized (and buyers are thus taxed) when they are the 'high-elasticity group of users', that is, when, relative to buyers, their demand is more price elastic, and when they benefit less from the participation of buyers and more from their own participation through the feedback effect.

The next Lemma confirms this intuition in our running example.

LEMMA 1. *Consider the running example and assume that $2(u_B + u_S)^2 < 1$.*[14] *The price of devices p^R and the developer fee a^R that maximize welfare W subject to the industry break-even constraint (3.1) are such that $a^R > 0$ and $p^R < c$ if and only if $u_B < 2u_S^3$.*

Proof. See Appendix A.3. □

At the optimum, the industry's break-even constraint binds even though there are no fixed costs of production. This emphasizes that indirect network effects makes the social planner willing to operate cross-subsidies between the various sides of the market.

3.2. Separation

The situation of 'separation,' in which none of the manufacturers are integrated with either platforms, serves as our benchmark to assess the impact of vertical integration.

Let $\hat{\pi}_k(w_k, w_\ell, a) = \pi_k(w_k, \hat{p}_k(w_k, w_\ell, a), \hat{p}_\ell(w_\ell, w_k, a), a)$ denote M_k's profit at the equilibrium of the subgame starting at stage 3. From the assumptions made on the best responses, an increase in the developer fee reduces the number of applications and acts thus as a negative shock on the demands faced by manufacturers. It thus lowers manufacturers' profits, or $\frac{\partial \hat{\pi}_k}{\partial a}(w_k, w_\ell, a) \leq 0$ for all (w_k, w_ℓ, a). From the assumption made on the cost pass-throughs, a manufacturer's profit decreases with the royalty it pays, or $\frac{\partial \hat{\pi}_k}{\partial w_k}(w_k, w_\ell, a) < 0$ for all (w_k, w_ℓ, a). Roughly speaking, the assumptions made in Section 2.2 ensure that the direct shift in the profit function (associated to a change in the royalty

[14] The assumption $2(u_B + u_S)^2 < 1$ ensures that the Ramsey problem is concave.

or in the developer fee) is stronger than the indirect shift in the marginal profit, which in turn changes the equilibrium between manufacturers.[15]

Consider now stages 1 and 2 of the game. We show that, in the separation benchmark, platforms set royalties and developer fees equal to their respective marginal cost in equilibrium.

To provide some intuition, suppose all platforms set a royalty equal to marginal cost c and a nil developer fee. M_k's profit is then given by $\hat{\pi}_k(c, c, 0)$. Platforms make no profit, developers are willing to publish their applications on all platforms and, consequently, manufacturers are indifferent between any operating systems.

Consider now a deviation by, say, platform I, which sets $a_I \gtrless 0$ and $w_I > c$. Each manufacturer has then to choose whether to adopt I's or one of the E_i's operating system. If both manufacturers choose one of the E_i's operating system, each of them earns $\hat{\pi}_k(c, c, \min\{0, a_I\})$ because developers might still be willing to publish their applications on I if $a_I < 0$. If both manufacturers choose I's operating system, each of them earns $\hat{\pi}_k(w_I, w_I, a_I)$. If M_k chooses I and M_ℓ chooses one of the E_i, then profits are given by $\hat{\pi}_k(w_I, c, a_I)$ and $\hat{\pi}_\ell(c, w_I, a_I)$. Since a manufacturer's profit is decreasing both in the royalty it pays and in the developer fee, $\hat{\pi}_k(c, c, \min\{0, a_I\}) \geq \hat{\pi}_k(w_I, c, a_I)$ and $\hat{\pi}_k(c, w_I, a_I) \geq \hat{\pi}_k(w_I, w_I, a_I)$: it is a dominant strategy for each manufacturer to choose one of the E_i's operating system. Hence, platform I's deviation is not profitable.

In the Appendix, we show that there are no other profitable deviations and that licensing the operating system at marginal cost and charging no developer fee is the unique equilibrium outcome in the separation benchmark. The following lemma summarizes this discussion.

LEMMA 2. *In equilibrium under separation, developers pay no fee ($a^S = 0$), platforms license their operating systems at marginal cost ($w^S = c$), and platforms make no profits.*

Proof. See Appendix A.4. □

Figure 2 summarizes the Ramsey and the separation benchmarks for the running example. Under separation, competition between symmetric platforms prevents the exercise of any market power at the platform level. Competition, however, also limits the ability of platforms to implement some implicit redistribution between the different groups of users. Put differently, competition prevents non-integrated platforms from harnessing indirect network effects between the various groups of users through an asymmetric price structure, a finding that echoes Rochet and Tirole (2006) for instance. This will prove important to understand the consequences of vertical integration, which we study now.

[15] Although intuitive, these properties may not always hold. In a Cournot oligopoly, Seade (1985), Kimmel (1992) and Linnemer (2003), among others, find conditions under which an increase in the marginal cost of several firms increases or decreases equilibrium profits; see Février and Linnemer (2004) for a unifying framework. Cowan (2004) extends the analysis to demand shocks. See also Dixit (1986) and Leahy and Neary (1997) for the case of Bertrand oligopolies.

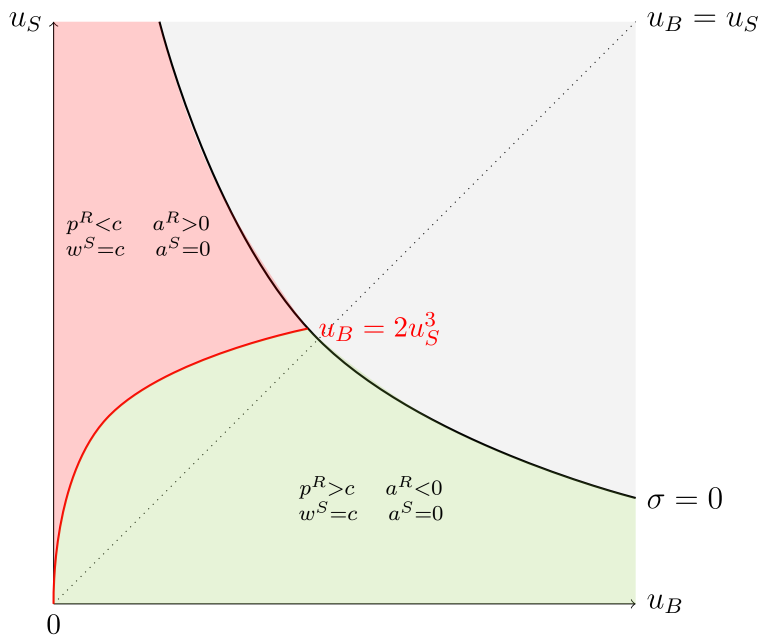


Figure 2 – Ramsey and separation benchmarks in the running example. Note: the grey area corresponds to non admissible values of the parameters ($\sigma < 0$).

4. VERTICAL INTEGRATION

Consider now that platform I is integrated with manufacturer M_1. To streamline the exposition, we directly consider that competition between non-integrated platforms $E_1, ..., E_N$ leads them to set a nil developer fee and a royalty equal to marginal cost c, a result that follows from the logic of Lemma 2.

Our analysis proceeds as follows. First, we show that vertical integration creates market power over developers (Section 4.1). Second, we study how such market power is exercised (Section 4.2) and its consequences on welfare (Section 4.3).

4.1. Price Competition and Choice of Operating System

At stage 3 of the game, the integrated platform and the non-integrated manufacturer compete in prices to sell their devices to buyers. The outcome of that price competition subgame depends on whether the non-integrated manufacturer chooses one the non-integrated platform's operating system (in the following, to ease exposition, we will refer to this case by saying that M_2 chooses 'platform E') or that of the integrated platform:

(E) When M_2 chooses E's operating system, its profit is $(p_2 - c)D_2$ and the integrated platform's profit is

$$(p_1 - c)D_1 + a_I D_S \tag{4.1}$$

because developers are willing to reach buyers of device 1.

(I) When M_2 chooses the integrated platform's operating system, its profit writes now as $(p_2 - w_I)D_2$ and that of the integrated platform is given by

$$(p_1 - c)D_1 + a_I D_S + (w_I - c)D_2 \tag{4.2}$$

because it perceives some royalties from the non-integrated manufacturer on top of the revenues earned from developers.

For both cases, we adopt implicitly the same assumptions on best responses as those made in Section 2.2 to ensure that the price competition subgame is 'well-behaved.' In case (E) (resp. case (I)), price competition on the product market then leads to equilibrium prices denoted by $p_1^E(c, c, a_I)$ and $p_2^E(c, c, a_I)$ (resp. $p_1^I(c, w_I, a_I)$ and $p_2^I(w_I, c, a_I)$). Let $\pi_1^E(c, c, a_I) = (p_1^E - c)D_1(p_1^E, p_2^E, a_I)$ $\pi_2^E(c, c, a_I) = (p_2^E - c)D_2(p_2^E, p_1^E, a_I)$ (resp. $\pi_1^I(c, w_I, a_I) = (p_1^I - c)D_1(p_1^I, p_2^I, a_I) + (w_I - c)D_2(p_2^I, p_1^I, a_I) + a_I D_S(p_1^I, p_2^I, a_I)$ and $\pi_2^I(w_I, c, a_I) = (p_2^I - w_I)D_2(p_2^I, p_1^I, a_I)$) denote the corresponding profits.

We now show two related facts. First, because it changes the objective pursued by the integrated platform, the choice of an operating system by the non-integrated manufacturer impacts competition on the buyers' market. Second, a consequence of this first fact is that the integrated platform is empowered with some market power over the non-integrated manufacturer.

Let us then focus on the integrated firm's pricing incentives at stage 3 of the game. These incentives depend on the revenues raised from developers, namely $a_I D_S$. Intuitively, a low price for device 1 boosts the overall number of devices, which then increases the number of developers. This effect is present both when the integrated firm supplies the non-integrated manufacturer (see Equation (4.2)) and when it does not (see Equation (4.1)).

Pricing incentives also depend on whether the integrated firm licenses its operating system to the non-integrated manufacturer. Indeed, when this is the case, lowering the price of device 1 increases the demand for device 2 since manufacturers' products are demand substitutes, and, therefore, increases the revenues $(w_I - c)D_2$ generated by the licensing of the operating system. Therefore, when it licenses its operating system, the integrated firm tends to become more accommodating on the product market, that is, it tends to set a higher price p_1 for its devices so as to increase the revenues associated to the licensing of its operating system. Formally, the integrated firm's best response in price changes from

$$D_1 + (p_1 - c)\frac{\partial D_1}{\partial p_1} + a_I \frac{\partial D_S}{\partial p_1} = 0$$

when M_2 buys from the fringe, to

$$D_1 + (p_1 - c)\frac{\partial D_1}{\partial p_1} + a_I \frac{\partial D_S}{\partial p_1} + (w_I - c)\frac{\partial D_2}{\partial p_1} = 0$$

when M_2 chooses the integrated firm's operating system. Since devices are demand substitutes, $\partial D_2/\partial p_1 > 0$ and the integrated firm's best response shifts upward when it licenses its operating system. Put differently, the integrated firm becomes a softer competitor if the non-integrated manufacturer chooses its operating system. This is the so-called 'accommodation effect' found in the literature on strategic vertical integration. This effect suggests that the non-integrated manufacturer may be willing to pay a royalty above the fringe's price to make the integrated manufacturer a softer competitor, or, put differently, that the integrated platform has some market power over the non-integrated manufacturer.

That market power is, however, constrained by the competitive pressure exerted by platforms from the fringe. Indeed, the non-integrated manufacturer always has the option of buying the fringe's operating system at a royalty c. Therefore, the non-integrated manufacturer chooses the integrated platform's operating system if

$$\pi_2^I(w_I, c, a_I) \geq \pi_2^E(c, c, a_I). \tag{4.3}$$

4.2. The Integrated Platform's Pricing Policy

At the first stage of the game, the integrated platform's profit writes as $\pi_1^I(c, w_I, a_I) = (p_1^I - c)D_1(p_1^I, p_2^I, a_I) + (w_I - c)D_2(p_2^I, p_1^I, a_I) + a_I D_S(p_1^I, p_2^I, a_I)$. Formally, w_I and a_I are solution of

$$\begin{aligned} \max_{(w_I, a_I)} \quad & \pi_1^I(c, w_I, a_I) \\ \text{s.t.} \quad & \pi_2^I(w_I, c, a_I) \geq \pi_2^E(c, c, a_I), \\ & w_I \geq c. \end{aligned} \tag{4.4}$$

To study this problem, we proceed in two steps. First, we consider a relaxed problem in which none of the constraints are taken into account. This allows to understand the logic underlying the exercise of market power by the integrated platform. Second, we solve for the constrained problem.[16]

THE RELAXED PROBLEM. Maximizing the integrated firm's profit requires to extract revenues from developers and from the non-integrated manufacturer: this calls for increasing a_I and w_I above the respective marginal costs, while keeping an eye on how this impacts competition on the buyer's market. But it also requires to take advantage of network effects across both sides of the market: when, for instance, buyers value strongly the applications offered by developers, the integrated platform wants to boost the number of applications available on its device with a low developer fee and extract some of the buyer surplus through a high royalty paid by the non-integrated manufacturer.

Let us assume that the solution of this relaxed problem is interior and denote by (w_I^*, a_I^*) the solution of the system formed by the two first-order conditions $\partial\pi_1^I/\partial w_I = 0$ and $\partial\pi_1^I/\partial a_I = 0$. In Appendix A.5, we characterize this solution for the running example. Figure 3 below represents the two curves $w_I^* = c$ and $a_I^* = 0$ (which does not depend on marginal cost c) in the (u_B, u_S)-space to allow a comparison with the separation benchmark.

It is instructive to consider first the case analyzed in the extant literature, namely the case with no network effects or $u_B = u_S = 0$. It comes immediately that the developer fee is then nil because developers have no intrinsic value to participate and bring no value to buyers. The royalty is strictly above the marginal cost because this raises the revenues from licensing the operating system and softens competition on the downstream market.

[16]Some conditions are required to ensure that the maximization problems we study are quasi-concave. These conditions are fully characterized in Appendix A.5 in the case of the running example.

Considering network effects between both sides of the market leads to distinguish three broad cases:

- 'Strong and buyer-skewed network effects.' This corresponds to the green region in Figure 3, in which buyers' valuation for number of applications is larger than developers' valuation for the number of buyers (that is, $u_B > u_S$). There, the solution of the relaxed problem calls for setting a negative developer fee (that is, $a_I^* < 0$) to boost the participation of developers and extracting the buyer surplus thereby created with a high royalty (that is, $w_I^* > c$).

- 'Strong and developer-skewed network effects.' This corresponds to the red region in Figure 3, in which $u_S \gg u_B$. There, the solution of the relaxed problem leads to a royalty below the marginal cost (that is, $w_I^* < c$) to boost the participation of buyers and a high developer fee to extract the developer surplus (that is, $a_I^* > 0$).

- In the blue region, network effects are rather balanced and weak across both sides of the market. There, the integrated platform sets a positive developer fee (that is, $a_I^* > 0$) and a royalty above the marginal cost (that is, $w_I^* > c$).

The comparison between the outcome under separation (in which $a_I = a^S = 0$ and $w_I = w^S = c$) and the optimal pricing policy obtained in the relaxed problem illustrates a central feature of our model. The way that the integrated firm exercises its market power on developers and on the non-integrated manufacturer depends on the strength and the structure of network effects across both sides of the market. In a nutshell, when network effects are strong and asymmetric, a two-sided market logic is at work: the integrated firm implements an asymmetric pricing structure, subsidizing one side and taxing the other, to harness those network effects. When network effects are weak and balanced, a one-sided market logic is at work: the integrated platform exercises its market power by raising both the developer fee and the royalty above the corresponding marginal costs.

OPTIMAL PRICING POLICY. Let us now come back to the platform's problem as defined in (4.4) and denote by (w_I^{**}, a_I^{**}) its solution. With respect to the relaxed problem, the first constraint is that the royalty must be larger than the marginal cost c. This implies that subsidizing buyers with a royalty below cost is no longer possible.

The second constraint is the participation constraint of the non-integrated manufacturer, namely (4.3). Observe that the non-integrated manufacturer's profit if it buys the fringe's operating system, namely $\pi_2^E(c, c, a_I)$, coincides with its profit if it buys the integrated platform's operating system at royalty $w_I = c$: this holds because there is no accommodation effect when the integrated platform licenses at its marginal cost. Therefore, $\pi_2^E(c, c, a_I) = \pi_2^I(c, c, a_I)$ and participation constraint (4.3) can thus be rewritten as $\pi_2^I(w_I, c, a_I) \geq \pi_2^I(c, c, a_I)$ or, equivalently, $w_I \leq c$.[17] Therefore, the competitive pressure exerted by the fringe prevents the integrated platform from raising the royalty above marginal cost. This illustrates another standard result from the literature on strategic integration: absent efficiency gains, vertical integration does not create market power over non-integrated manufacturers.

Combining the two observations above implies that the integrated firm must set a royalty equal to the marginal cost, i.e., $w_I^{**} = c$. Since royalties of the integrated platform

[17] Under our assumption, the profit of a manufacturer is decreasing in its marginal cost.

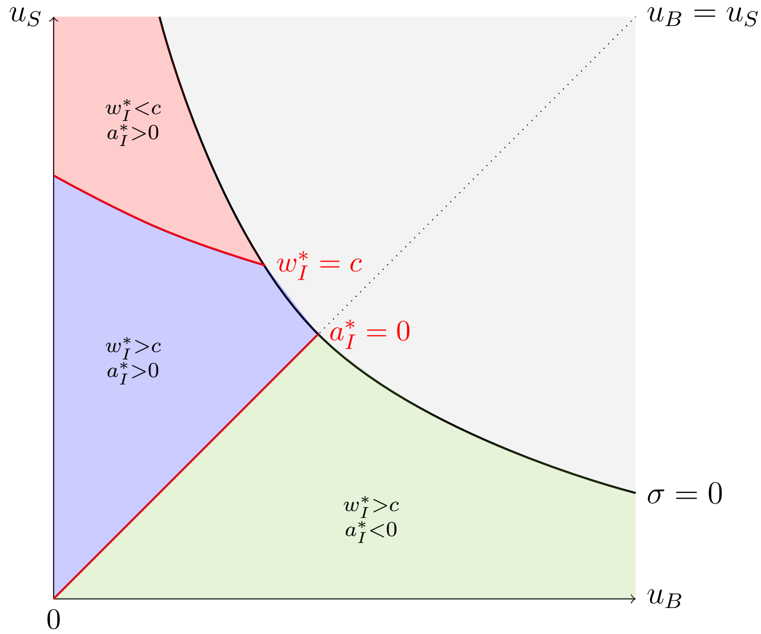


Figure 3 – The solution of the relaxed problem in the running example.

and of the fringe are equal to the marginal cost, the only variable of interest is the developer fee a_I. To make the notation simpler, prices and profits are written as function of this variable only from now on.

The integrated platform still has some market power over developers who want to access the buyers of its device. The optimal developer fee maximizes its profit, which writes as $\pi_1^I(a_I) = (p_1^I(a_I) - c)D_1(p_1^I(a_I), p_2^I(a_I), a_I) + a_I D_S(p_1^I(a_I), p_2^I(a_I), a_I)$, and is given by the first-order condition $d\pi_1^I(a_I^{**})/da_I = 0$, or

$$(4.5)\qquad \left(\left[D_S + a_I\left(\frac{\partial D_S}{\partial a_I} + \frac{\partial D_S}{\partial p_2}\frac{dp_2^I}{da_I}\right)\right] + \left[(p_1^I - c)\left(\frac{\partial D_1}{\partial a_I} + \frac{\partial D_1}{\partial p_2}\frac{dp_2^I}{da_I}\right)\right]\right)\Big|_{a_I = a_I^{**}} = 0.$$

Increasing the developer fee has both a direct impact on the number of applications and the demand for the integrated manufacturer's device, and an indirect impact through the strategic effect on the non-integrated manufacturer's price. Remember that, under our assumptions, $dp_k^I/da_I \leq 0$. Hence, Equation (4.5) shows that increasing the developer fee allows to capture revenues from developers (first bracketed term) but depreciates the profit earned from buyers (second bracketed term). The first effect call for increasing the developer fee above the marginal cost, but the second one calls for decreasing it. Even though the integrated platform is led to license its operating system at marginal cost, it may still be willing to subsidize developers if this boosts sufficiently the demand for its own device.

The next proposition characterizes the integrated firm's optimal pricing policy in our running example.

PROPOSITION 1. *Consider the running example. The integrated platform's optimal pricing policy is as follows:*

- *The royalty is equal to the marginal cost:* $w_I^{**} = c$.

- *The developer fee is the solution a_I^{**} of (4.5) and is such that $a_I^{**} > 0$ if and only if $u_B < \underline{h}(u_S)$ (with $\underline{h}(u_S) > u_S$ for all $u_S > 0$).*

Proof. See Appendix A.6. □

Figure 4 represents graphically Proposition 1. The integrated platform boosts the number of applications with a negative developer fee only when network effects are sufficiently strong and buyer-skewed. Otherwise, it sets a positive developer fee. Indeed, with respect to the relaxed problem, the competitive pressure exerted by platforms from the fringe forces the integrated platform to license its operating system at marginal cost. It is therefore less profitable to subsidize developers because the integrated firm cannot recoup such a loss with a royalty above its marginal cost.

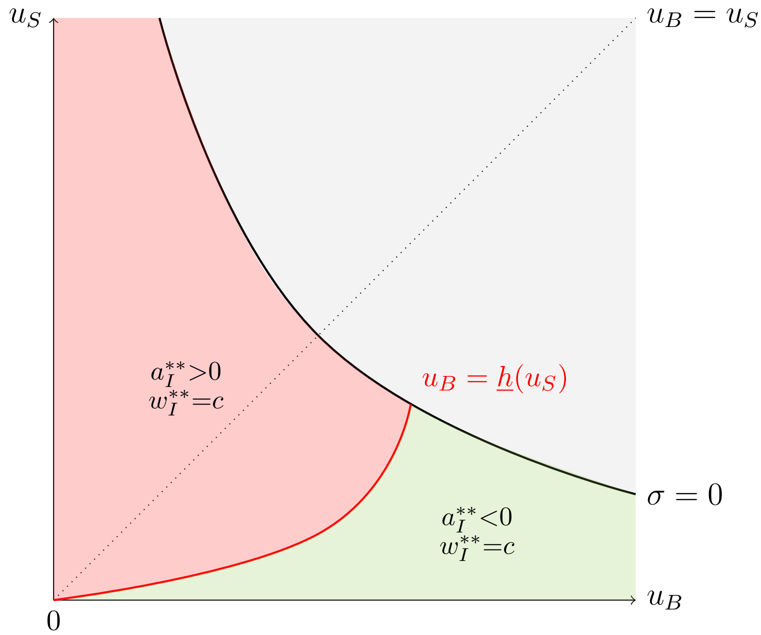


Figure 4 – The integrated platform's optimal pricing policy in the running example.

4.3. Competitive Impact of Vertical Integration

We now assess the welfare impact of vertical integration. Since the integration outcome coincides with that under separation when $a_I = 0$, we only need to study how the non-integrated manufacturer's profit $\pi_2^I(a_I) = (p_2^I(a_I) - c)D_2(p_2^I(a_I), p_1^I(a_I), a_I)$, the buyer surplus $V_B(p_1^I(a_I), p_2^I(a_I), D_S^I(a_I))$ (with $D_S^I(a_I) = D_S(p_1^I(a_I), p_2^I(a_I), a_I)$), and the developer surplus $V_S(a_I, D_1(p_1^I(a_I), p_2^I(a_I), a_I) + D_2(p_2^I(a_I), p_1^I(a_I), a_I))$ vary with the developer fee a_I.

To study whether vertical integration leads to foreclosure of the non-integrated manufacturer, we can differentiate the non-integrated manufacturer's profit with respect to the developer fee to get

$$\frac{d\pi_2^I}{da_I}(a_I) = (p_2^I - c)\left(\frac{\partial D_2}{\partial a_I} + \frac{\partial D_2}{\partial p_1}\frac{dp_1^I}{da_I}\right) < 0.$$

Since prices of devices decrease with the developer fee under our assumptions, we obtain immediately the next proposition.

PROPOSITION 2. *Vertical integration creates foreclosure if and only if the developer fee increases above the pre-merger level. Therefore, in the running example, the non-integrated manufacturer is hurt by the vertical merger if and only if* $u_S > \underline{h}(u_B)$.

Proof. Immediate from the text. □

This foreclosure effect is different from the one found in the literature for two reasons.

First, it does not stem from a 'raise the rival's cost' effect. In our setting, foreclosure is a collateral damage of the integrated platform's market power on developers, but not the result of its desire to soften competition on the buyer's market.

Second, foreclosure is not systematic. When network effects are strong and buyer-skewed, Proposition 1 has shown that the integrated firm lowers the developer fee (with respect to the pre-merger level), which increases the non-integrated manufacturer's profit. A reverse result obtains when the integrated firm raises the developer fee above the pre-merger level.

Consider now the impact of vertical integration on buyer and developer surpluses. The analysis is slightly more involved since these surpluses are intertwined through indirect network effects. We consider a small variation of the developer fee around its value under separation (that is, 0). Simple manipulations lead to (omitting some arguments)

$$\left.\frac{dV_B}{da_I}\right|_{a_I=0} = -Q_B\left(\frac{dp_1^I}{da_I}+\frac{dp_2^I}{da_I}\right)+\frac{\partial U_B}{\partial n_S}\frac{dD_S}{da_I}, \tag{4.6}$$

$$\left.\frac{dD_S}{da_I}\right|_{a_I=0} = \frac{1}{1-2\frac{\partial Q_B}{\partial n_S}\frac{\partial Q_S}{\partial n_B}}\left(\frac{\partial Q_S}{\partial n_B}\left(\frac{\partial Q_B}{\partial p_k}+\frac{\partial Q_B}{\partial p_\ell}\right)\left(\frac{dp_1^I}{da_I}+\frac{dp_2^I}{da_I}\right)+\frac{\partial Q_S}{\partial a}\right). \tag{4.7}$$

Equation (4.7) describes how the number of applications varies when the developer fee charged by the integrated platform increases. First, since publishing applications becomes more costly, developers are less willing to participate; this corresponds to the term $\partial Q_S/\partial a < 0$. Second, the prices paid by buyers decrease, so that there are more buyers overall, which benefits developers through indirect network effects; this corresponds to the term $(\partial Q_B/\partial p_k + \partial Q_B/\partial p_\ell)(dp_1^I/da_I + dp_2^I/da_I) > 0$.

Equation (4.6) describes how the surplus of buyers varies when the developer fee increases. There are two effects again. First, the prices paid by buyers decrease, which boosts the demand from those buyers (this corresponds to the first term in the right-hand side). Second, fewer or more applications are developed, which impacts negatively or positively buyers through indirect network effects.

This suggests that the impact of vertical integration on buyers and on developers may be ambiguous. For instance, taxing developers with a positive fee may actually improve the surpluses of buyers and developers, if doing so sufficiently reduces the prices of devices and boosts the number of buyers. Next proposition provides a complete characterization of the impact of vertical integration on buyer and developer surpluses in our running example.

PROPOSITION 3. *Consider the running example. Vertical integration*[18]

- *increases buyer surplus if either* $u_B \geq \underline{h}(u_S)$ *or* $u_S \geq \overline{h}_B(u_B)$*;*
- *increases developer surplus if and only if either* $u_B \geq \underline{h}(u_S)$ *or* $u_S \geq \overline{h}_S(u_B)$*.*

Proof. See Appendix A.7. □

Figure 5 represents graphically Proposition 2 and Proposition 3.

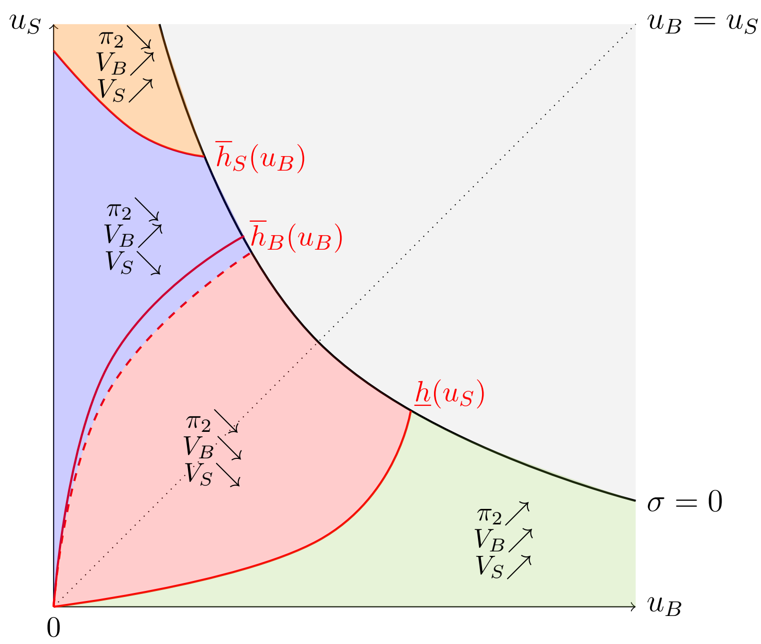


Figure 5 – Impact of vertical integration on the non-integrated manufacturer's profit (π_2), buyer surplus (V_B), and developer surplus (V_S) in the running example.

Several comments are worth making. First, and quite remarkably, vertical integration can improve buyer and developer surpluses simultaneously even in the absence of efficiency gains. The intuition is that, when indirect network effects are sufficiently strong and asymmetric, the integrated firm's market power over developers leads to an asymmetric pricing structure that better internalizes network effects. The price structure is thus closer to the one that would be socially optimal, that is, prices under vertical integration are closer to their Ramsey counterparts than under separation.

More precisely, with buyer-skewed network effects ($u_B > \underline{h}(u_S)$), the integrated platform subsidizes developers (see Proposition 1). Although the prices of devices increases, buyers benefit from an increase in the number of applications, which explains that their surplus increases following the merger. By contrast, with developer-skewed network effects ($u_S > \overline{h}_B(u_S)$), the integrated platform taxes developers, which tends to reduce the number of applications. However, the prices of devices decrease and buyer surplus increases following the merger. In these two cases of strongly-skewed indirect network

[18]In Appendix A.7, we show that $\overline{h}_B(u_B) = ((\gamma+2)u_B)/(2(\gamma+1)u_B^2+1)$ and $\overline{h}_S(u_B) = (1/2)(\sqrt{2(\gamma+4)+(\gamma+3)^2u_B^2} - (\gamma+3)u_B)$.

effects, which correspond to the green and orange regions in Figure 5, the surpluses of buyers and developers increase because the integrated firm implements an asymmetric pricing structure that is more in line with the Ramsey optimum (see Figure 2).

Second, the impact of vertical integration on foreclosure is now disconnected from its impact on buyers or on developers. We already know from Proposition 2 that foreclosure is related to the developer fee chosen by the integrated platform. Proposition 3 shows that what matters for buyer and developer surpluses is the asymmetry between network effects. Figure 5 illustrates that buyers and developers gain from the vertical merger when network effects are either strongly buyer-skewed ($u_B > \underline{h}(u_S)$), a region where the developer fee decreases, or strongly developer-skewed ($u_S > \overline{h}_B(u_S)$, a region where the developer fee increases.

Third, when network effects become smaller or more symmetric, there is less value to create through an asymmetric pricing structure and the integrated firm's market power is more likely to be detrimental. To illustrate, suppose that $u_B = u_S$. The Ramsey optimum requires then to subsidize developers and tax buyers (see Figure 2). By contrast, the integrated firm exercises its market power over developers by increasing the developer fee, which leads to a lower price for its device (see Figure 4). In Appendix A.7, we show that, when $u_B < \underline{h}(u_S)$ and $u_S < \overline{h}_B(u_S)$, the buyer surplus may either increase or decrease.[19]

4.4. Discussion

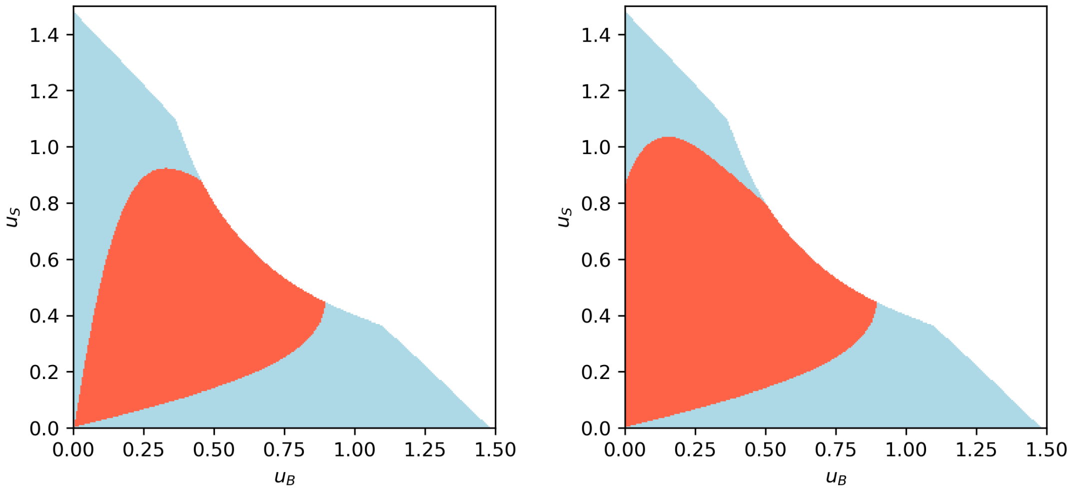
Our analysis provides therefore a new efficiency defense for vertical integration in platform markets. Vertical integration creates market power on the developer side of the market. That market power is used by the integrated firm to extract more profit from developers and from the non-integrated manufacturer; a potentially harmful effect for buyers. In a two-sided market, however, that market power is also used to internalize indirect network effects between the two sides of the market; a potentially beneficial effect for buyers.

Although consumer surplus seems to be the standard pursued by antitrust authorities, it is also interesting to briefly look at total welfare. Figure 6 computes total welfare for a particular specification of our running example.[20] It shows that welfare also improves following vertical integration when network effects are sufficiently strong and asymmetric.

These simulations can be used to study the role of parameter γ that describes the degree of substitutability between manufacturers' products on the downstream market (Figure 7). Intuitively, as γ increases, products become more substitutes and price competition between manufacturers intensifies. As a result, prices become more rigid and cost-based. For the integrated platform, this implies that internalizing network effects

[19]In Proposition 3, the conditions for the buyer surplus to increase are sufficient only. In Appendix A.7, we show that the intermediate region $\{(u_B, u_S) : u_B \leq \underline{h}(u_S) \text{ and } u_S \leq \overline{h}_B(u_B)\}$ can be divided into two subsets, one in which the buyer surplus increases and the other in which it decreases. The frontier between these two subsets is represented by the red dashed curve in Figure 5.

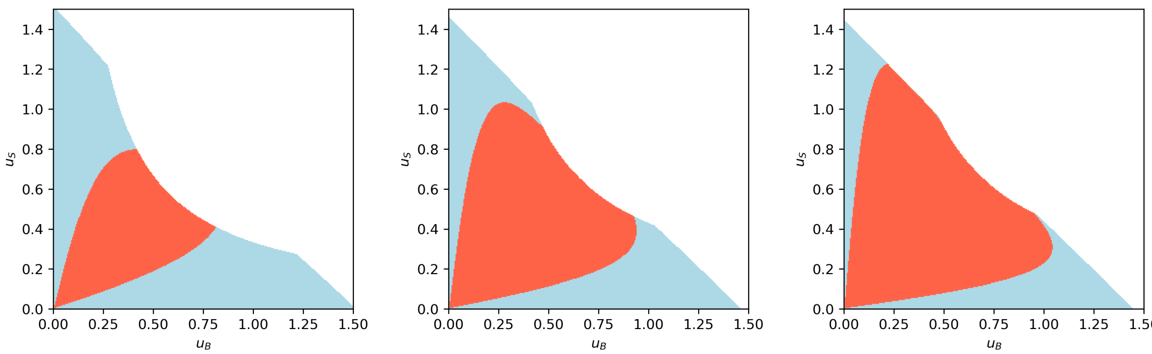
[20]The Python code of the simulations is available on the authors' webpages, as well as other simulations. If not specified in the figure legend, then we use the following set of parameters values: $\gamma = 4$, $v = 1$, $c = 0.4$.



(a) ΔV_B (b) ΔW

Figure 6 – Impact of vertical integration on the buyers surplus (V_B, left panel) and on total welfare (W, right panel): V_B and W decrease (resp. increase) following integration in the red area (resp. the blue area).

through an asymmetric price structure becomes less interesting: there is no point in subsidizing developers if the price charged to buyers cannot be raised. Therefore, as the simulations reported in Figure 7 suggest, vertical integration becomes more harmful to buyers when manufacturers' products are more demand substitutes. Both network effects and the intensity of competition between manufacturers matter to determine whether vertical integration benefits or hurts buyers.



(a) $\gamma = 2$ (b) $\gamma = 6$ (c) $\gamma = 10$

Figure 7 – Impact of vertical integration on buyers surplus (V_B) for different degrees of substitutability between manufacturers (γ): V_B decreases (resp. increases) following integration in the red area (resp. the blue area).

5. VERTICAL INTEGRATION WITH EFFICIENCY GAINS

We now study vertical mergers that bring efficiency gains. Efficiency gains are modeled as follows: following the merger, the integrated firm's marginal cost to supply the operating system becomes c_0 instead of c, with $0 < c_0 < c$.[21] The separation benchmark is unchanged (see Section 3.2).

From now on, we consider that platform I is integrated with manufacturer M_1. Under vertical integration, we can still apply the logic of Lemma 2 to show that, in equilibrium, platforms from the fringe license their operating systems at a royalty equal to their marginal cost and set a nil developer fee: $w_E = c$ and $a_E = 0$.

As in Section 4, pricing incentives at stage 3 of the game depend on the choice of operating system by the non-integrated firm. Let $(p_1^E(c_0, c, a_I), p_2^E(c, c_0, a_I))$ (resp. $(p_1^I(c_0, w_I, a_I), p_2^I(w_I, c_0, a_I)))$ be the prices of devices when the non-integrated manufacturer adopts the fringe's (resp. the integrated platform's) operating system and $(\pi_1^E(c_0, c, a_I), \pi_2^E(c, c_0, a_I))$ (resp. $(\pi_1^I(c_0, w_I, a_I), \pi_2^I(w_I, c_0, a_I)))$ be the corresponding profits. For future reference, we have in particular $\pi_1^I(c_0, w_I, a_I) = (p_1^I - c_0)D_1(p_1^I, p_2^I, a_I) + (w_I - c_0)D_2(p_2^I, p_1^I, a_I) + a_I D_S(p_1^I, p_2^I, a_I)$, $\pi_2^I(w_I, c_0, a_I) = (p_2^I - w_I)D_2(p_2^I, p_1^I, a_I)$ and $\pi_2^E(c, c_0, a_I) = (p_2^E - c)D_2(p_2^E, p_1^E, a_I)$.

The roadmap of our analysis is as follows. First, efficiency gains provide the integrated firm with some market power over the non-integrated manufacturer (Section 5.1). Second, how such market market power is exercised (Section 5.2) is not necessarily detrimental either to buyers and developers or to the non-integrated manufacturer (Sections 5.3 and 5.4).

5.1. Efficiency Gains Create Market Power

Efficiency gains create market power vis-à-vis the non-integrated manufacturer. To understand why, observe that, at stage 3 of the game, the integrated platform's profit when it supplies the non-integrated manufacturer writes now as

$$(p_1 - c_0)D_1 + (w_I - c_0)D_2 + a_I D_S. \tag{5.1}$$

Comparing the integrated platform's profit with efficiency gain (Equation (5.1)) and without (Equation (4.2)) shows that, with efficiency gains, even when the integrated platform sets the same royalty as the fringe ($w_I = c$), it earns some strictly positive profit from licensing its operating system ($(c - c_0)D_2 > 0$). Hence, even when both operating systems are licensed at the fringe's marginal cost c, the non-integrated manufacturer strictly prefers adopting the integrated firm's operating system because this makes that firm more accommodating on the buyers market. Put differently, we now have $\pi_2^I(w_I = c, c_0, a_I) > \pi_2^E(c, c_0, a_I)$.

This implies that the integrated platform is now able to license its operating system at a royalty w_I strictly above its marginal cost c_0. Whether it is willing to do so will be studied next. Before, notice that the integrated firm's market power remains constrained by the fringe's behavior. The following participation constraint ensures that the non-integrated manufacturer is willing to adopt the integrated platform's operating system

$$\pi_2^I(w_I, c_0, a_I) \geq \pi_2^E(c, c_0, a_I). \tag{5.2}$$

[21] Alternatively, we could have assumed that synergies between platform I and manufacturer M_1 increases the quality of I's operating system. The formulation of efficiency gains in terms of cost reduction allows to relate clearly our results with the extant literature.

5.2. The Integrated Firm's Pricing Policy

The integrated platform's problem can be written as follows

$$\begin{aligned} \max_{(w_I,a_I)} \quad & \pi_1^I(c_0, w_I, a_I) \\ \text{s.t.} \quad & \pi_2^I(w_I, c_0, a_I) \geq \pi_2^E(c, c_0, a_I), \\ & w_I \geq c_0. \end{aligned} \tag{5.3}$$

The main novelty brought by efficiency gains is that the integrated firm has market power not only on developers but also on the non-integrated manufacturer. It is again useful to introduce the solution of the relaxed problem (that is, when none of the constraints in the above problem are taken into account), which we denote by (w_I^*, a_I^*). Up to the fact that the integrated firm's cost is now c_0, the outcome of the relaxed problem can be represented in a similar way as in Figure 3.

Consider now the constrained problem (5.3). Figure 8 describes the solution of that problem in our running example. There are three regions of interest depending on which constraints are binding.

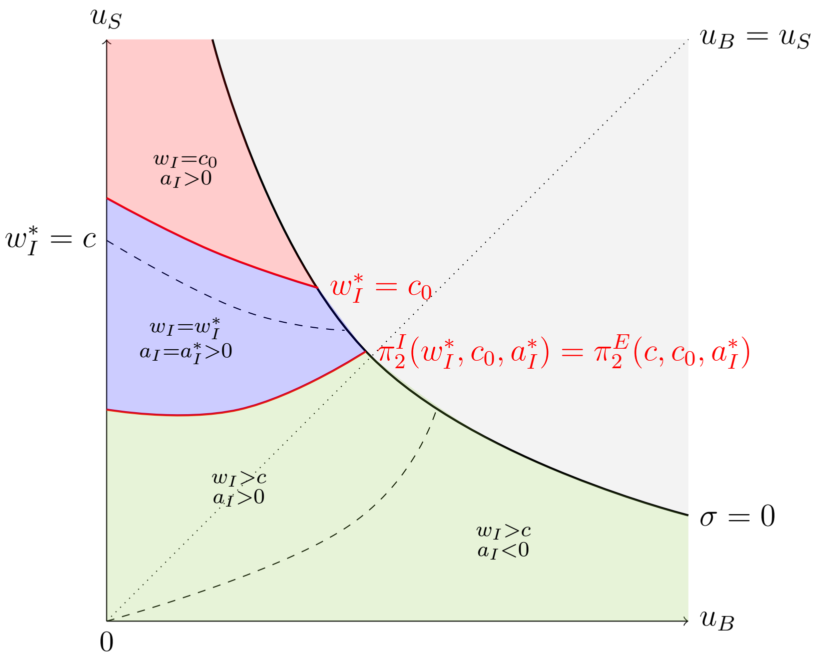


Figure 8 – The integrated platform's optimal pricing policy in the running example with efficiency gains.

Suppose that network effects are sufficiently buyer-skewed (right part of the green region in Figure 8). In that case, absent any constraints the integrated platform would like to set a royalty above the pre-merger level and a negative developer fee ($w_I^* > c$ and $a_I^* < 0$). With respect to Section 4, setting a royalty strictly above the pre-merger level is now feasible thanks to the efficiency gains that create upstream market power. Hence, with buyer-skewed network effects, we expect that the non-integrated manufacturer's participation constraint becomes binding (that is, $\pi_2^I(w_I, c_0, a_I) = \pi_2^E(c, c_0, a_I)$) and that the integrated platform charges a royalty above the pre-merger level but subsidizes developers.

Suppose now that indirect network effects are sufficiently developer-skewed (red region in Figure 8). The integrated firm then wants to subsidize buyers with a royalty below cost and tax developers with a positive fee ($w_I^* < c_0$ and $a_I^* > 0$). Hence, we expect that the integrated firm charges a royalty equal to its marginal cost (that is, $w_I = c_0$) and taxes developers. Observe furthermore that when $w_I = c_0$, the non-integrated manufacturer's participation constraint is strictly satisfied thanks to the accommodation effect (that is, $\pi_2^I(c_0, c_0, a_I) > \pi_2^E(c, c_0, a_I)$).

Last, it is also possible that none of the constraints are binding (blue region in Figure 8), in which case the solution of the integrated platform's problem is actually the solution of the relaxed problem. This arises with rather balanced network effects. In that case, the integrated firm moderately increases the royalty and the non-integrated firm benefits from the accommodation effect.

5.3. Competitive Impact of Vertical Integration with Efficiency Gains: Polar Cases

With efficiency gains, the impact of vertical integration on buyers, developers and the non-integrated manufacturer is less straightforward to assess because both the developer fee and the royalty change with respect to the separation benchmark. For instance, when the developer fee increases beyond its pre-merger level, this does not necessarily imply foreclosure, for the royalty could be lowered leading to a net gain for the non-integrated manufacturer (red region in Figure 8). We explore this intuition by analyzing two polar cases.

Consider first that developers do not value the participation of buyers (that is, $u_S = 0$ in our running example). Without efficiency gains, buyers and developers, as well as the non-integrated manufacturer, benefit from integration. With efficiency gains, the integrated platform sets a royalty above the pre-merger level ($w_I > c$) and a negative fee for developers ($a_I < 0$). Since developers do not value the participation of buyers, they are better off following integration because their participation is subsidized. Things are more complicated for buyers and the non-integrated manufacturer. On the one hand, both benefit from the fact that the participation of developers is subsidized. On the other hand, the integrated platform exploits its competitive advantage and charges a royalty above the pre-merger level. Intuitively, this latter effect prevails when network effects are weak overall, that is, when buyers also value weakly the participation of developers. Next proposition formalizes this intuition.

PROPOSITION 4. *Consider the running example and assume $u_S = 0$. Following integration,*

- *there is foreclosure if and only if u_B is small enough;*
- *buyers are better off if and only if u_B is large enough;*
- *developers are always better off.*

Proof. See the Online Appendix. □

Let us now consider another polar case in which buyers do not value the participation of developers (that is, $u_B = 0$ in our running example). The integrated platform sets a developer fee above the pre-merger level ($a_I > 0$), as in the case without efficiency gains. The difference is that efficiency gains provide incentives to lower the royalty. When u_S is small (resp. large), the integrated platform sets a royalty beyond (resp. below) the pre-merger level. Therefore, intuitively, not only buyers and developers but also the non-integrated manufacturer may benefit from integration when developers value strongly the participation of buyers. Next proposition formalizes this intuition.

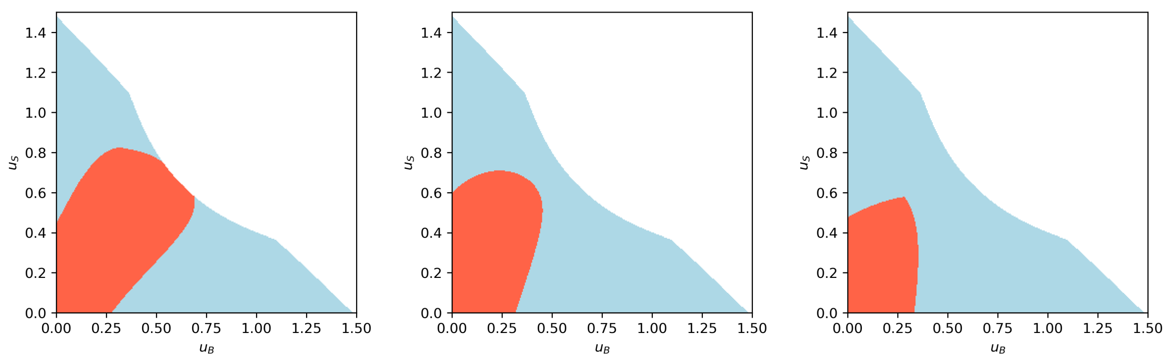
Proposition 5. *Consider the running example and assume $u_B = 0$. Following integration,*

- *there is foreclosure if and only if u_S is either large enough or small enough;*
- *buyers are better off if and only if u_S is large enough;*
- *developers are better off if and only if u_S is large enough.*

Proof. See the Online Appendix. □

5.4. *Competitive Impact of Vertical Integration with Efficiency Gains: Numerical Simulations*

In this Section, we extend Propositions 4 and 5 using numerical simulations.[22] The results are depicted in Figures 9, 10 and 11, which represent the range of parameters u_B and u_S for which integration hurts or benefits, the buyers, the developers and the non-integrated manufacturer respectively.

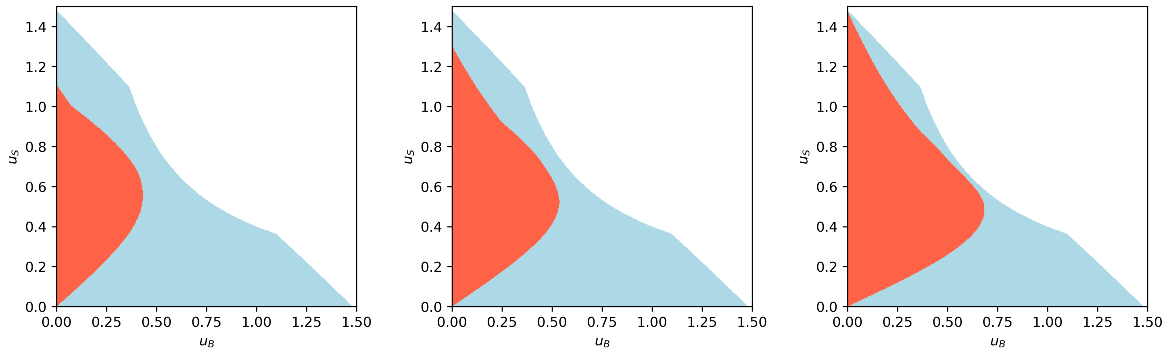


(a) $c - c_0 = 0.1$ (b) $c - c_0 = 0.2$ (c) $c - c_0 = 0.3$

Figure 9 – Impact of vertical integration on buyer surplus (V_B) for different levels of efficiency gains ($c - c_0$): V_B decreases (resp. increases) following integration in the red area (resp. in the blue area).

Overall, the results of the simulations are in line with those of Section 4. First, there is no foreclosure when network effects are skewed toward buyers (see Figure 11). Second, buyers and developers benefit from integration when network effects are sufficiently asymmetric (see Figure 9 and 10 respectively). Third, when network effects are rather balanced, the buyers, the developers and the non-integrated manufacturer all lose from the vertical merger (see Figure 9, 10 and 11 respectively).

[22]The Python code of the simulations is available on the authors' webpages, as well as other simulations. As in Section 4.4, we use the following set of parameters values: $\gamma = 4$, $v = 1$, $c = 0.4$.



(a) $c - c_0 = 0.1$ (b) $c - c_0 = 0.2$ (c) $c - c_0 = 0.3$

Figure 10 – Impact of vertical integration on developer surplus (V_S) for different levels of efficiency gains ($c - c_0$): V_S decreases (resp. increases) following integration in the red area (resp. the blue area).

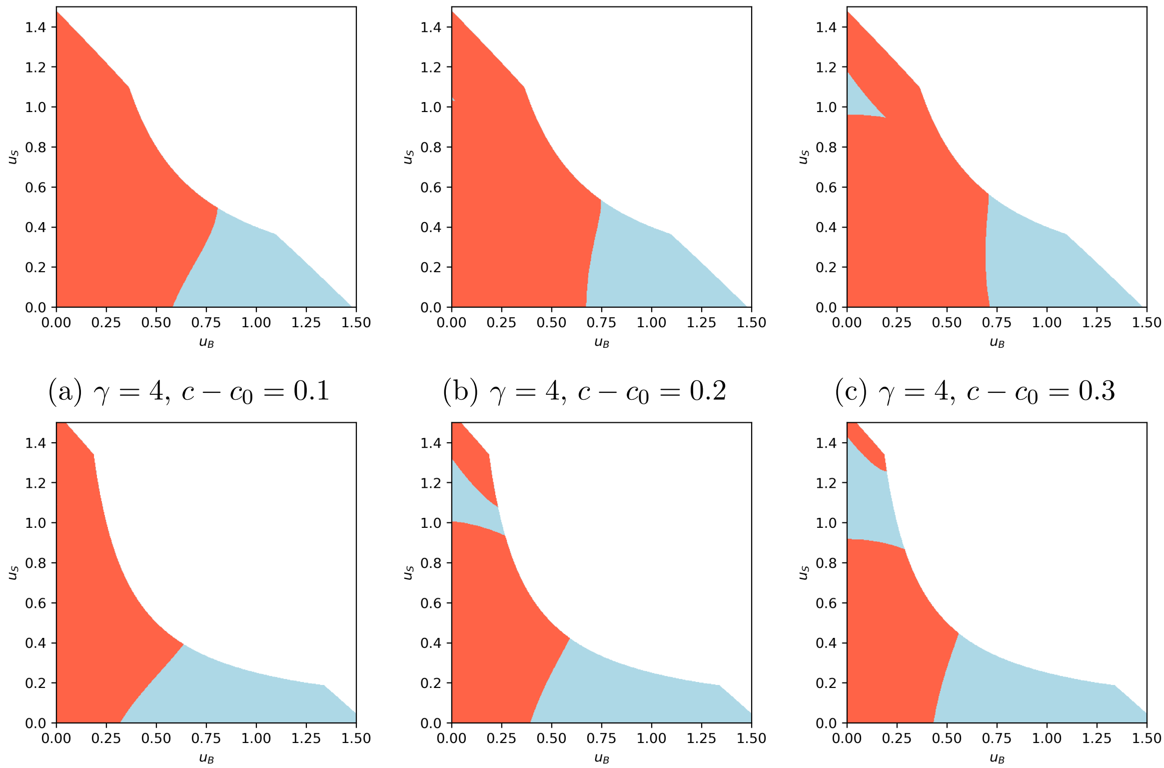
Perhaps more interestingly, the simulations allow to better assess the role of efficiency gains beyond the special cases studied in Propositions 4 and 5.

Consider first that network effects are developer-skewed. As efficiency gains increase, Figure 11 suggests that foreclosure becomes less of an issue. This arises because the integrated platform sets a royalty close or equal to its marginal cost. Put differently, efficiency gains are passed through almost entirely to the non-integrated manufacturer. This effect can be strong enough to compensate for the increase in the developer fee. This happens when efficiency gains are sufficiently large and when the manufacturers' products are weak substitutes, that is when γ is small (Figures 11d, 11e and 11f). In this case, competition on the downstream market is weak and the non-integrated manufacturer benefits fully from the lower royalty.

Second, the non-integrated manufacturer is foreclosed when network effects are small, that is in the neighborhood of $(u_B, u_S) = (0, 0)$ (see Figure 11). This is reminiscent of the literature on the strategic effects of vertical integration: when vertical integration creates efficiency gains, the non-integrated rival is (partially) foreclosed (see, e.g., Chen, 2001).

Third, buyers and developers are better off following integration when network effects are strong and asymmetric, that is when either u_S is large and u_B is small or the opposite (see Figures 9 and 10). These are situations where a platform would like to internalize network effects through an asymmetric price structure. In Section 4, we have shown that both buyers and developers benefit from such an asymmetric price structure. When it creates efficiency gains, vertical integration allows the platform to implement an even more asymmetric price structure (see Section 5.2). Buyers and developers benefit as well from such a more asymmetric price structure.

The results from the simulations reinforce the main message of our analysis. A vertically integrated platform is empowered with some market power on users on both sides of the market and on the non-integrated manufacturer. However, because vertical integration allows for a better internalization of network effects, the exercise of this market power is not necessarily at the expense of the non-integrated firm. Buyers and developers may also benefit from integration.



(a) $\gamma = 4$, $c - c_0 = 0.1$ (b) $\gamma = 4$, $c - c_0 = 0.2$ (c) $\gamma = 4$, $c - c_0 = 0.3$

(d) $\gamma = 1$, $c - c_0 = 0.1$ (e) $\gamma = 1$, $c - c_0 = 0.2$ (f) $\gamma = 1$, $c - c_0 = 0.3$

Figure 11 – Impact of vertical integration on the non-integrated manufacturer's profit (π_2) for different levels of efficiency gains ($c - c_0$): π_2 decreases (resp. increases) following integration in the red area (resp. the blue area).

6. Conclusion

We develop a model of a platform market, in which platforms interact with manufacturers of devices and there are indirect network effects between buyers of devices and developers of applications. We study the consequences of vertical integration between one of the platforms and one of the manufacturers.

The sources of upstream market power, and their consequences in terms of foreclosure or consumer surplus, are different from those unveiled in the extant literature. Even absent any efficiency gains, vertical integration creates market power over developers who want to access the buyers of the integrated manufacturer's device. With efficiency gains, vertical integration creates additionally some market power over the non-integrated manufacturer. However, what is key is how the integrated firm exploits these sources of market power. We show that this heavily depends on the strength and the structure of indirect network effects. When network effects are strong in level but also sufficiently asymmetric, the integrated firm implements an asymmetric pricing structure to harness network effects, which may well improve the buyer and developer surpluses as well as the non-integrated manufacturer's profit. Our analysis therefore warns policy-makers against blind application of the traditional view about foreclosure when addressing platform markets.

As in standard markets, antitrust authorities may want to limit the anti-competitive effects of vertical integration by constraining the pricing of the royalty. In the context of platform markets, this remedy raises, however, several issues. First, with strong indirect network effects, capping the royalty reduces the price decrease of the integrated manufacturer. Second, a cap on the royalty is likely to impact the pricing on the developer side of the market. A more complete assessment of such a behavioral remedy is left for future research.

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A. Appendix

A.1. Demand Functions with Indirect Network Effects

In the last stage of the game, given prices (p_1, p_2, a), the number of buyers and the number of developers solve

$$\left\{ \begin{array}{rcl} n_{B1} & = & Q_B(p_1, p_2, n_S), \\ n_{B2} & = & Q_B(p_2, p_1, n_S), \\ n_S & = & Q_S(a, n_{B1} + n_{B2}). \end{array} \right. \tag{A.1}$$

To avoid 'cornered-market' solutions, in which all buyers or all developers participate in equilibrium, we assume that indirect network effects are not too strong so that, in the relevant range, each manufacturer faces a demand that is locally elastic with respect to prices.

Assumption A.1 (Indirect Network Effects Are Not Too Strong). *In the relevant range of prices (p_1, p_2, a), the total number of buyers $n_{B1} + n_{B2}$ and the number of developers n_S satisfy the following condition*

$$\frac{\partial Q_S}{\partial n_B}(a, n_{B1} + n_{B2}) \left(\frac{\partial Q_B}{\partial n_S}(p_1, p_2, n_S) + \frac{\partial Q_B}{\partial n_S}(p_2, p_1, n_S) \right) < 1.$$

We can then show the following result.

Lemma A.1. *In the relevant range of prices (p_1, p_2, a), system (A.1) has a unique interior solution.*

Proof. Let $D(p_1, p_2, a) = D_1(p_1, p_2, a) + D_2(p_2, p_1, a)$. From system (A.1), we have

$$D(p_1, p_2, a) = Q_B(p_1, p_2, Q_S(a, D(p_1, p_2, a))) + Q_B(p_2, p_1, Q_S(a, D(p_1, p_2, a))). \tag{A.2}$$

For a given (p_1, p_2, a), $D(p_1, p_2, a)$ is thus a fixed point of $\psi(x) = Q_B(p_1, p_2, Q_S(a, x)) + Q_B(p_2, p_1, Q_S(a, x))$. Notice then that

$$\psi'(x) = \frac{\partial Q_S}{\partial n_B}(a, x) \left(\frac{\partial Q_B}{\partial n_S}(p_1, p_2, Q_S(x)) + \frac{\partial Q_B}{\partial n_S}(p_2, p_1, Q_S(x)) \right).$$

Assumption A.1 implies that $|\psi'(\cdot)| < 1$, so that $\psi(\cdot)$ is a contraction mapping and Equation (A.2) has a unique solution. It follows that $D_S(p_1, p_2, a) = Q_S(a, D(p_1, p_2, a))$ is uniquely defined, as well as $D_k(p_k, p_\ell, a) = Q_B(p_k, p_\ell, D_S(p_1, p_2, a))$. □

We can then show the following result.

LEMMA A.2. *The following properties hold:* $\frac{\partial D_k}{\partial p_k}(p_k, p_\ell, a) < 0$, $\frac{\partial D_S}{\partial p_k}(p_k, p_\ell, a) < 0$, $|\frac{\partial D_k}{\partial p_k}(p, p, a)| > |\frac{\partial D_k}{\partial p_\ell}(p, p, a)|$, $\frac{\partial D_k}{\partial a}(p_k, p_\ell, a) < 0$ *and* $\frac{\partial D_S}{\partial a}(p_k, p_\ell, a) < 0$.

Proof. To avoid any confusion, let $Q_{B1}(p_1, p_2, n_S) = Q_B(p_1, p_2, n_S)$ and $Q_{B2}(p_2, p_1, n_S) = Q_B(p_2, p_1, n_S)$. By the implicit function theorem, $D(p_1, p_2, a) = D_1(p_1, p_2, a) + D_2(p_2, p_1, a)$ is continuously differentiable. Differentiating Equation (A.2) with respect to p_1 and rearranging terms, we find (omitting some arguments)

$$\frac{\partial D}{\partial p_1} \left[1 - \frac{\partial Q_S}{\partial n_B}(a, D) \left(\frac{\partial Q_{B1}}{\partial n_S}(p_1, p_2, D) + \frac{\partial Q_{B2}}{\partial n_S}(p_2, p_1, D) \right) \right] = \frac{\partial Q_{B1}}{\partial p_1}(p_1, p_2, D) + \frac{\partial Q_{B2}}{\partial p_1}(p_2, p_1, D).$$

By Assumption A.1, the term in squared brackets is positive. Therefore, $\partial D/\partial p_1$ is negative. Similarly, $\partial D/\partial p_2 < 0$. Since $\partial Q_S/\partial n_B(a, n_B) > 0$, $D_S(p_1, p_2, a) = Q_S(a, D(p_1, p_2, a))$ is decreasing in both p_1 and p_2. Then $\partial D_1/\partial p_1 = \partial Q_{B1}/\partial p_1 + (\partial D/\partial p_1)(\partial Q_{B1}/\partial n_S)\partial Q_S/\partial n_B$, which shows that $\partial D_1/\partial p_1 < 0$. Similarly, $\partial D_1/\partial p_2 = \partial Q_{B1}/\partial p_2 + (\partial D/\partial p_2)(\partial Q_{B1}/\partial n_S)\partial Q_S/\partial n_B$. For symmetric prices, $\partial D/\partial p_1 = \partial D/\partial p_2$, and therefore, $|\partial D_1/\partial p_1| - |\partial D_1/\partial p_2| = |\partial Q_{B1}/\partial p_1| - |\partial Q_{B1}/\partial p_2| < 0$, which is negative under our assumptions.

From Equation (A.1), the developer demand solves

$$D_S(a, p_1, p_2) = Q_S(a, Q_{B1}(p_1, p_2, D_S(a, p_1, p_2)) + Q_{B2}(p_2, p_1, D_S(a, p_1, p_2))). \tag{A.3}$$

By the implicit function theorem, $D_S(a, p_1, p_2)$ is continuously differentiable. Differentiating Equation (A.3) with respect to a and rearranging terms, we find (omitting some arguments)

$$\frac{\partial D_S}{\partial a} = \frac{\frac{\partial Q_S}{\partial a}}{1 - \frac{\partial Q_S}{\partial n_B} \left(\frac{\partial Q_{B1}}{\partial n_S} + \frac{\partial Q_{B2}}{\partial n_S} \right)}.$$

By Assumption A.1, the denominator is positive, and therefore, $\partial D_S/\partial a$ has the sign of $\partial Q_S/\partial a$, which is negative. Since $D_k(p_k, p_\ell, a) = Q_B(p_k, p_\ell, D_S(p_k, p_\ell, a))$ and $\partial Q_B/\partial n_S > 0$, $\partial D_k/\partial a$ is also negative. □

A.2. Assumptions in the Running Example

The running example satisfies the assumptions made in Section 2. Assumption A.1, which ensures that there exists a unique and interior solution to system (2.1), writes as $2u_B u_S < 1$.

Solving for n_{Bk}, $k = 1, 2$, and n_S in (2.1), the buyers' and developers' demands are given by

$$D_k(p_k, p_\ell, a) = \frac{2v - (2+\sigma)p_k + \sigma p_\ell - 2u_B a}{2(1 - 2u_B u_S)}, \tag{A.4}$$

$$D_S(p_1, p_2, a) = \frac{u_S(2v - p_1 - p_2) - a}{1 - 2u_B u_S}, \tag{A.5}$$

where $\sigma = \gamma - 2u_B u_S(1+\gamma)$, which is positive by assumption. From (A.4), and since $\sigma \geq 0$ and $0 \leq 2u_B u_S < 1$, D_k is strictly decreasing in p_k and a. Manufacturers' products are demand substitutes since $\partial D_k/\partial p_\ell = \sigma/(2(1-2u_B u_S)) \geq 0$. From (A.5), we have that D_S is decreasing in p_1, p_2 and a. Finally, the direct price effect is stronger than the indirect one since $-\partial D_k/\partial p_k - \partial D_k/\partial p_\ell = 1/(1-2u_B u_S) > 0$.

We now check that the price competition subgame is 'well-behaved' (see Section 2.2). Consider the separation benchmark in which manufacturers pay royalties (w_k, w_ℓ), developers pay a fee a, and M_k earns a profit equal to $\pi_k = (p_k - w_k)D_k$. $\partial^2\pi_k/\partial p_k^2 = -(2+\sigma)/(1-2u_B u_S) < 0$, so that π_k is strictly concave in p_k and M_k's best response is uniquely characterized by the first-order condition $\partial\pi_k/\partial p_k(p_k, p_\ell) = 0$. The best response is given by $R_k(p_\ell, w_k, a) = (2(v+w_k) + \sigma(p_\ell + w_k) - 2u_B a)/(2(2+\sigma))$. We then have $0 < \partial R_k/\partial p_\ell < 1$ and $\partial R_k/\partial a \leq 0$. Equilibrium prices are given by $\hat{p}_k = (-2au_B(4+3\sigma) + v(8+6\sigma) + (2+\sigma)(w_\ell\sigma + 2w_k(2+\sigma)))/((4+\sigma)(4+3\sigma))$. One can then check that $\partial\hat{p}_k/\partial w_k = (2(2+\sigma)^2)/((4+\sigma)(4+3\sigma))$, which belongs to $[1/2, 2/3)$ for $\sigma \geq 0$, $\partial\hat{p}_k/\partial w_\ell = (\sigma(2+\sigma))/((4+\sigma)(4+3\sigma))$, which belongs to $[0, 1/3)$ for $\sigma \geq 0$, and $\partial\hat{p}_k/\partial a = -\frac{2u_B}{(4+\sigma)} \leq 0$.

Last, we check that these assumptions are satisfied when I and M_1 are integrated and the integrated platform sets a developer fee a_I and licenses its operating system to M_2 at a royalty w_I. The integrated firm's best response is $R_1(p_2, w_I, a_I) = (2(v + c - (u_B + u_S)a_I) + \sigma(p_2 + w_I))/(2(2+\sigma))$, which is increasing with a slope smaller than 1 in p_2 and decreasing in a_I. M_2's best response is given by $R_2(p_1, w_I, a_I) = (2(v + w_I) + \sigma(p_1 + w_I) - 2u_B a_I)/(2(2+\sigma))$. Those best responses satisfy our assumptions. Additionally, equilibrium prices are given by $p_1^I = w_I + (c - a_I(2u_B + u_S) + 2v - 3w_I)/(4+\sigma) + (c - a_I u_S - w_I)/(4+3\sigma)$ and $p_2^I = w_I + (c - a_I(2u_B + u_S) + 2v - 3w_I)/(4+\sigma) + (-c + a_I u_S + w_I)/(4+3\sigma)$. These prices are increasing in w_I at a rate smaller than 1 and decreasing in a_I.

A.3. Ramsey Prices

We first analyze Ramsey prices in the general case and then in our running example.

RAMSEY PRICES IN THE GENERAL CASE. Note that we can assume that $p_1 = p_2 \equiv p$ without loss of generality. To alleviate notations and streamline the exposition, consider the following notations: $D_B(p,a) = D_1(p,p,a) + D_2(p,p,a)$, $D_S(p,a) = D_S(p,p,a)$, $\Pi(p,a) = (p-c)D_B(p,a) + aD_S(p,a)$, $W(p,a) = V_B(p,p,D_S(p,a)) + V_S(a, D_B(p,a)) + \Pi(p,a)$. Ramsey prices solve the following problem

$$\begin{aligned} \max_{(p,a)} \quad & W(p,a) \\ \text{s.t.} \quad & \Pi(p,a) \geq 0. \end{aligned} \tag{A.6}$$

Let $\lambda \geq 0$ be the Lagrange multiplier associated to the constraint. Assume that this problem is well-behaved so that its solution can be characterized through first-order conditions.

Consider the following change of variables: $\varphi : (p,a) \mapsto (n_B, n_S) = (D_B(p,a), D_S(p,a))$. It is a $\mathcal{C}^1$-diffeomorphism since, under assumption A.1, the system of equations $n_B = 2Q_B(p,p,n_S)$ and $n_S = Q_S(a, n_B)$ has a unique solution, namely $(n_B, n_S) = (D_B(p,a), D_S(p,a))$, in the relevant range of parameters. Let $(P(n_B,n_S), A(n_B,n_S)) = \varphi^{-1}(n_B, n_S)$. Problem (A.6) then rewrites

$$\begin{aligned}\max_{(n_B,n_S)} \quad W = \quad & U_B(1/2n_B, 1/2n_B, n_S) - P(n_B,n_S)n_B + U_S(n_S,n_B) - A(n_B,n_S)n_S \\ & +(P(n_B,n_S)-c)n_B + A(n_B,n_S)n_S \\ \text{s.t.} \quad & (P(n_B,n_S)-c)n_B + A(n_B,n_S)n_S = 0.\end{aligned}$$

The first order conditions on n_B and n_S can be written as follows (omitting notations)

$$\text{(A.7)} \qquad \begin{aligned} \tfrac{1}{2}\left(\tfrac{\partial U_B}{\partial q_1} + \tfrac{\partial U_B}{\partial q_2}\right) - c + \tfrac{\partial U_S}{\partial n_B} + \lambda\left(\tfrac{\partial P}{\partial n_B} n_B + P - c + \tfrac{\partial A}{\partial n_B} n_S\right) &= 0, \\ \tfrac{\partial U_B}{\partial n_S} + \tfrac{\partial U_S}{\partial q_S} + \lambda\left(\tfrac{\partial P}{\partial n_S} n_B + \tfrac{\partial A}{\partial n_S} n_S + A\right) &= 0.\end{aligned}$$

Then, noticing that the maximization problems of the representative buyer and developer give $\frac{\partial U_B}{\partial q_1} = P$, $\frac{\partial U_B}{\partial q_2} = P$, and $\frac{\partial U_S}{\partial q_S} = A$, Equations (A.7) rewrite

$$\text{(A.8)} \qquad \begin{aligned} P - c + \tfrac{\partial U_S}{\partial n_B} + \lambda\left(\tfrac{\partial P}{\partial n_B} n_B + P - c + \tfrac{\partial A}{\partial n_B} n_S\right) &= 0, \\ \tfrac{\partial U_B}{\partial n_S} + A + \lambda\left(\tfrac{\partial P}{\partial n_S} n_B + \tfrac{\partial A}{\partial n_S} n_S + A\right) &= 0.\end{aligned}$$

With no break-even constraint, the welfare-maximizing prices are obtained by setting $\lambda = 0$ in (A.8): $P - c = -\frac{\partial U_S}{\partial n_B}$ and $A = -\frac{\partial U_B}{\partial n_S}$ as stated in the text. This condition obviously violates the break-even constraint.

Suppose now that $\lambda > 0$. Since the constraint is binding, we now have

$$\text{(A.9)} \qquad (P-c)n_B + An_S = 0,$$

which shows in particular that either $P - c \leq 0$ and $A \geq 0$, or $P - c > 0$ and $A < 0$ at the optimum. Then, combining Equations (A.8) and (A.9), we obtain the following equation on λ

$$\text{(A.10)} \qquad \lambda = -\frac{n_S \frac{\partial U_B}{\partial n_S} + n_B \frac{\partial U_S}{\partial n_B}}{n_B^2 \frac{\partial P}{\partial n_B} + n_S^2 \frac{\partial A}{\partial n_S} + n_B n_S \left(\frac{\partial P}{\partial n_S} + \frac{\partial A}{\partial n_B}\right)}.$$

Let D denote the denominator in (A.10). Since $\lambda > 0$ and the numerator in (A.10) is positive, we have $D < 0$. Then, by combining Equations (A.8) and (A.9), we obtain the following expression for the margin $P - c$

$$\text{(A.11)} \qquad P - c = \frac{n_S\left(\frac{\partial U_B}{\partial n_S}(n_B \frac{\partial P}{\partial n_B} + n_S \frac{\partial A}{\partial n_B}) - \frac{\partial U_S}{\partial n_B}(n_B \frac{\partial P}{\partial n_S} + n_S \frac{\partial A}{\partial n_S})\right)}{D - n_S \frac{\partial U_B}{\partial n_S} - n_B \frac{\partial U_S}{\partial n_B}}.$$

Since the denominator is negative, Equation (A.11) shows that $P-c$ has the sign of $f(n_B,n_S) = \frac{\partial U_S}{\partial n_B}(n_B \frac{\partial P}{\partial n_S} + n_S \frac{\partial A}{\partial n_S}) - \frac{\partial U_B}{\partial n_S}(n_B \frac{\partial P}{\partial n_B} + n_S \frac{\partial A}{\partial n_B})$.

Then, noticing that $\frac{\partial P}{\partial n_B} = \frac{1}{\partial Q_B/\partial p}$ and $\frac{\partial P}{\partial n_S} = -\frac{\partial Q_B/\partial n_S}{\partial Q_B/\partial p}$ (where, with a slight abuse of notations, $\partial Q_B/\partial p = \partial Q_B/\partial p_1 + \partial Q_B/\partial p_2$), $\frac{\partial A}{\partial n_B} = -\frac{\partial Q_S/\partial n_B}{\partial Q_S/\partial a}$ and $\frac{\partial A}{\partial n_S} = \frac{1}{\partial Q_S/\partial a}$, we have after rearranging terms

$$f(n_B,n_S) = \frac{1}{\eta_B}\left(\frac{\partial U_B}{\partial n_S} + \frac{\partial U_S}{\partial n_B}\frac{\partial Q_B}{\partial n_S}\right) - \frac{1}{\eta_S}\left(\frac{\partial U_S}{\partial n_B} + \frac{\partial U_B}{\partial n_S}\frac{\partial Q_S}{\partial n_B}\right),$$

where $\eta_B = -\frac{1}{n_B}\frac{\partial Q_B}{\partial p}(p, n_S)$ and $\eta_S = -\frac{1}{n_S}\frac{\partial Q_S}{\partial a}(a, n_B)$.

Next, we study the same problem but in the context of our running example. This allows, first, to determine the conditions under which the constrained-maximization problem is concave, and, second, to obtain a neat characterization of which side is taxed/which side is subsidized as function of network effects.

PROOF OF LEMMA 1 (RAMSEY PRICES IN THE RUNNING EXAMPLE). Consider the unconstrained problem $\max_{(p,a)} W$. Assume $2(u_B + u_S)^2 < 1$, which ensures that the Hessian is negative definite so that W is strictly concave. We have indeeed $\partial^2 W/\partial p^2 = -(2 - 4u_S(2u_B + u_S))/(1 - 2u_B u_S)^2 < 0$, $\partial^2 W/\partial a^2 = -(1 - 2u_B(u_B + 2u_S))/(1 - 2u_B u_S)^2 < 0$ and $(\partial^2 W/\partial p^2)(\partial^2 W/\partial a^2) - (\partial^2 W/\partial a \partial p)^2 = (2 - 4(u_B + u_S)^2)/(1 - 2u_B u_S)^2 > 0$.

Solving for the first-order conditions, we obtain $p = -(2u_S(u_B + u_S)v - c(1 - 2u_B(u_B + u_S)))/(1 - 2(u_B + u_S)^2)$ and $a = -(2u_B(v - c))/(1 - 2(u_B + u_S)^2)$, which yields $p - c \leq 0$ and $a \leq 0$.

Consider now problem (A.6). Π is strictly concave under the assumption $2(u_B + u_S)^2 < 1$. We have indeed $\partial^2 \Pi/\partial p^2 = -4/(1 - 2u_B u_S) < 0$, $\partial^2 \Pi/\partial a^2 = -2/(1 - 2u_B u_S) < 0$ and $(\partial^2 \Pi/\partial p^2)(\partial^2 \Pi/\partial a^2) - (\partial^2 \Pi/\partial a \partial p)^2 = (8 - 4(u_B + u_S)^2)/(1 - 2u_B u_S)^2 > 0$.

The Lagragean $\mathcal{L} = W + \lambda \Pi$, with $\lambda \geq 0$, is thus strictly concave as the sum of two strictly concave functions. The optimum is then characterized by the first-order conditions $\partial \mathcal{L}/\partial p = 0$ and $\partial \mathcal{L}/\partial a = 0$, and the complementary slackness condition $\lambda \partial \mathcal{L}/\partial \lambda = 0$. The constraint must bind at the optimum since the unconstrained outcome violates the break even constraint (except in the degenerate case $u_B = u_S = 0$). Therefore, $\lambda > 0$ at the optimum.

Using the first-order conditions $\partial \mathcal{L}/\partial p = 0$ and $\partial \mathcal{L}/\partial a = 0$, we can express the optimal price p^R and developer fee a^R as functions of the multiplier λ

$$a^R = \frac{2(\lambda + 1)(v - c)(\lambda u_B + u_B - \lambda u_S)}{-4\lambda + 2\left(\lambda^2\left((u_B + u_S)^2 - 2\right) + 2\lambda(u_B + u_S)^2 + (u_B + u_S)^2\right) - 1}, \tag{A.12}$$

$$p^R - c = \frac{(v - c)\left(-\lambda(2\lambda + 1) + 2(\lambda + 1)^2 u_B u_S + 2(\lambda + 1)^2 u_S^2\right)}{-4\lambda + 2\left(\lambda^2\left((u_B + u_S)^2 - 2\right) + 2\lambda(u_B + u_S)^2 + (u_B + u_S)^2\right) - 1}. \tag{A.13}$$

Replacing in the constraint $\Pi = 0$, the multiplier λ satisfies

$$(u_B + u_S)^2 = \frac{\lambda(2\lambda + 1)^2}{2(\lambda + 1)^3}. \tag{A.14}$$

The right-hand side in (A.14) is strictly increasing and takes values in $[0, 2)$ for $\lambda \in [0, +\infty)$. Therefore, (A.14) has a unique strictly positive solution in λ when $(u_B + u_S)^2 < 2$, which is ensured by our assumption $2(u_B + u_S)^2 < 1$. Denote that solution by λ^R. We can use (A.14) to replace $(u_B + u_S)^2$ as a function of λ^R in the denominator of (A.12) (which is the same as the denominator of (A.13)) to show that this denominator is equal to $-4\lambda^R - 1/(\lambda^R + 1) < 0$. As a consequence, and using again (A.14), it is straightforward to show that

$$\text{Sign}(a^R) = -\text{Sign}(p^R - c) = -\text{Sign}(\lambda^R u_B + u_B - \lambda^R u_S).$$

Observe now that $a^R = 0$ (or equivalently $p^R = c$) amounts to $\lambda^R = u_B/(u_S - u_B)$ with λ^R the unique positive solution of (A.14). Plugging this expression in (A.14), we obtain that the following condition between u_B and u_S must hold to have $a^R = 0$ (or equivalently $p^R = c$): $u_B = 2u_S^3$.

A.4. Proof of Lemma 2 (Separation Outcome)

First, it cannot be an equilibrium that a platform attracts manufacturers and charges a positive developer fee because another platform can profitably undercut on both sides of the market.

Second, suppose that, say, platform I subsidizes developers ($a_I < 0$) and attracts both manufacturers with a positive royalty ($w_I > 0$). A platform from the fringe can then profitably attract manufacturers by slightly undercutting I's royalty ($w_E = w_I - \varepsilon$ with $\varepsilon > 0$ small) and setting a nil developer fee ($a_E = 0$) because, then, developers can still benefit from the subsidy offered by I and pay no fee to access buyers.

A.5. Unconstrained Outcome with No Efficiency Gains in the Running Example

We study the unconstrained outcome and derive formally the curves drawn in Figure 3. Let $\pi_1^I(w_I, a_I) = (p_1^I - c)D_1(p_1^I, p_2^I, a_I) + (w_I - c)D_2(p_2^I, p_1^I, a_I) + a_I D_S(p_1^I, p_2^I, a_I)$, where prices p_1^I and p_2^I are given in Appendix A.2.

CONDITIONS FOR CONCAVITY. We find conditions that ensure the concavity of the maximization problem $\max_{(w_I,a_I)} \pi_1^I(w_I, a_I)$. One can show that a sufficient condition is $2(u_B + u_S)^2 < 1$. This is, however, an overly restrictive condition that prevents from studying situations with quite asymmetric network effects. In the sequel, we establish a set of necessary and sufficient conditions that ensure the concavity of the previous problem.

Computations show that:

(i) $\partial^2\pi_1^I/\partial w_I^2 = -[4(\sigma+1)(\sigma+2)(\sigma(9\sigma+32)+32)]/[(\sigma+4)^2(3\sigma+4)^2(1-2u_Bu_S)]$, which is strictly negative.

(ii) $\partial^2\pi_1^I/\partial a_I^2 = -[2(\sigma+4)^2(3\sigma+4)^2 - 4(\sigma+2)(3\sigma+4)^2u_B^2 - 4(3\sigma+4)(\sigma(7\sigma+32)+32)u_Bu_S - 4(\sigma(\sigma(7\sigma+40)+64)+32)u_S^2]/[(\sigma+4)^2(3\sigma+4)^2(1-2u_Bu_S)]$.

(iii) $(\partial^2\pi_1^I/\partial w_I^2)(\partial^2\pi_1^I/\partial a_I^2) - (\partial^2\pi_1^I/\partial a_I\partial w_I)^2 = -4(\sigma+1)[5\sigma^2(9u_B^2 + 22u_Bu_S + 9u_S^2 - 20) + 16\sigma(5u_B^2 + 14u_Bu_S + 5u_S^2 - 12) + 9\sigma^3((u_B+u_S)^2 - 2) + 16((3u_B+u_S)(u_B+3u_S) - 8)]/((\sigma+4)^2(3\sigma+4)^2(1-2u_Bu_S)^2)$.

Simplifying these expressions further, the Hessian is negative definite if and only if $-2(\sigma+4)^2(3\sigma+4)^2 + 4(\sigma+2)(3\sigma+4)^2u_B^2 + 4(3\sigma+4)(\sigma(7\sigma+32)+32)u_Bu_S + 4(\sigma(\sigma(7\sigma+40)+64)+32)u_S^2 < 0$ and $5\sigma^2(9u_B^2 + 22u_Bu_S + 9u_S^2 - 20) + 16\sigma(5u_B^2 + 14u_Bu_S + 5u_S^2 - 12) + 9\sigma^3((u_B+u_S)^2 - 2) + 16((3u_B+u_S)(u_B+3u_S) - 8) < 0$.

Instead of working with (u_B, u_S), it turns out to be easier to work with (σ, u_S) with $\sigma = \gamma - 2(1+\gamma)u_Bu_S$ (which is possible since such transformation is a $\mathcal{C}^1$-diffeomorphism). Remind that $\sigma \geq 0$ by assumption and $\sigma \leq \gamma$ by definition. Equipped with this change of variables, we have that $\partial^2\pi_1^I/\partial a_I^2 < 0$ and $(\partial^2\pi_1^I/\partial w_I^2)(\partial^2\pi_1^I/\partial a_I^2) - (\partial^2\pi_1^I/\partial a_I\partial w_I)^2 > 0$ are equivalent to

$$\text{(A.15)} \quad f(x) = \frac{1}{x(1+\gamma)(1+\sigma)(4+\sigma)^2(4+3\sigma)^2}\big(4x^2(1+\gamma)^2(32+\sigma(64+\sigma(40+7\sigma))) \\ -2x(1+\gamma)(4+3\sigma)(2(2+\sigma)(16+5\sigma(4+\sigma)) + \gamma(32+3\sigma(16+\sigma(7+\sigma)))) \\ +(\gamma-\sigma)^2(2+\sigma)(4+3\sigma)^2\big) < 0$$

and

$$\text{(A.16)} \quad g(x) = \frac{1}{x(1+\sigma)(4+\sigma)^2(4+3\sigma)^2}\big(4x^2(1+\gamma)^2(4+3\sigma)(12+\sigma(11+3\sigma)) \\ -4x(1+\gamma)(128+\gamma(4+3\sigma)(12+\sigma(11+3\sigma)) + \sigma(272+\sigma(212+\sigma(73+9\sigma)))) \\ +(\gamma-\sigma)^2(4+3\sigma)(12+\sigma(11+3\sigma))\big) < 0$$

where $x = u_S^2$. Notice that both the denominators in (A.15) and (A.16) are positive. Let N_1 and N_2 denote the numerators in (A.15) and (A.16) respectively. We have

$$\text{(A.17)} \quad N_2 - N_1 = (\gamma-\sigma)^2(4+\sigma)(4+3\sigma) + 4x^2(1+\gamma)^2(16+\sigma(16+\sigma(5+2\sigma))) \\ +2x(1+\gamma)(\gamma(4+3\sigma)(8+\sigma(26+3\sigma(5+\sigma))) + 2\sigma(48+\sigma(76+\sigma(37+6\sigma)))) > 0$$

Put differently, Condition (A.16) is more demanding than Condition (A.15), that is $N_2 < 0$ is a necessary and sufficient condition for the concavity of the maximization problem. Then, simple computations show that condition $N_2 < 0$ amounts to

$$\frac{(u_B + u_S)^2 - 2}{(u_S - u_B)^2} < \frac{16 + \sigma(16 + 5\sigma)}{(2 + \sigma)(32 + \sigma(32 + 9\sigma))}. \tag{A.18}$$

Since the right-hand side in (A.18) is positive, a sufficient condition for the concavity of the maximization problem is $(u_B + u_S)^2 < 2$. When $(u_B + u_S)^2 > 2$, condition (A.18) is more likely to be satisfied if $(u_S - u_B)^2$ large, that is, roughly speaking when network effects are sufficiently 'skewed'.

Let us now describe the set of (u_B, u_S) such that (A.18) is satisfied, which as we have seen amounts to $g(x) < 0$. The numerator in $g(x)$ is a polynomial of degree 2 in x, whose discriminant is equal to $64(1+\gamma)^2(1+\sigma)(2+\sigma)(32+\sigma(32+9\sigma))(\gamma(4+3\sigma)(12+\sigma(11+3\sigma))+2(32+\sigma(56+\sigma(33+7\sigma)))) > 0$. Therefore, it has two distinct real roots $\underline{x}(\sigma)$ and $\overline{x}(\sigma)$. Since the numerator in $g(x)$ is positive when $x = 0$, the smallest root is positive: $\underline{x}(\sigma) > 0$. It follows that, for a given $\gamma \geq 0$, condition $g(x) < 0$ amounts to $\underline{x}(\sigma) < u_S^2 < \underline{x}(\sigma)$. Define $\underline{u}_S(\sigma) = \sqrt{\underline{x}(\sigma)}$, $\underline{u}_B(\sigma) = \frac{\gamma-\sigma}{2(1+\gamma)\underline{u}_S(\sigma)}$, $\overline{u}_S(\sigma) = \sqrt{\overline{x}(\sigma)}$ and $\overline{u}_B(\sigma) = \frac{\gamma-\sigma}{2(1+\gamma)\overline{u}_S(\sigma)}$. By construction, the set of (u_B, u_S) such that condition (A.18) is satisfied is the set of (u_B, u_S) whose frontiers are given by the two parametric curves $\underline{\mathcal{C}} = (\underline{u}_B(\sigma), \underline{u}_S(\sigma))$ and $\overline{\mathcal{C}} = (\overline{u}_B(\sigma), \overline{u}_S(\sigma))$ for all $\sigma \in [0, \gamma]$. Curves $\underline{\mathcal{C}}$ and $\overline{\mathcal{C}}$ are represented in Figure 12.

Simple computations show that the slope of the parametric curves is given by

$$-\frac{2(1+\gamma)x(\sigma)}{(\gamma - \sigma) + 2\frac{x(\sigma)}{x'(\sigma)}}, \tag{A.19}$$

where $x(\sigma) = \underline{x}(\sigma)$ for the curve $\underline{\mathcal{C}}$ and $x(\sigma) = \overline{x}(\sigma)$ for the curve $\overline{\mathcal{C}}$. Computations show that $\underline{x}'(\sigma) < 0 < \overline{x}'(\sigma)$. Plugging this in Equation (A.19) shows that the curve $\overline{\mathcal{C}}$ is downward slopping in the plane (u_B, u_S). Computations then show that $(\gamma - \sigma) + 2\frac{\underline{x}(\sigma)}{\underline{x}'(\sigma)} > 0$, which proves that the curve $\underline{\mathcal{C}}$ is downward slopping in the plane (u_B, u_S).

Figure 12 represents the sufficient condition (area below the red curve) and the necessary and sufficient condition (area below the blue curves) for the concavity of the maximization problem.

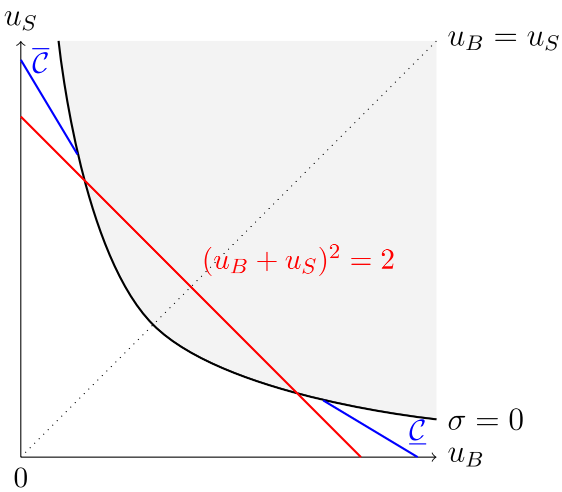

Figure 12 – Necessary and sufficient conditions for the concavity of the maximization problem in the running example.

UNCONSTRAINED OPTIMUM. Denote now by (w_I^*, a_I^*) the unique solution of the system formed by the two first-order conditions $\partial\pi_1^I/\partial w_I = 0$ and $\partial\pi_1^I/\partial a_I = 0$. Simple computations lead to

$$w_I^* - c = \frac{(v-c)(3\sigma+4)\left(-3\sigma(\sigma+4) + (\sigma(3\sigma+13)+20)u_B u_S + (\sigma(3\sigma+11)+12)u_S^2 - 16\right)}{D},$$
$$a_I^* = \frac{(v-c)(3\sigma+4)(\sigma(3\sigma+11)+12)(u_B-u_S)}{D},$$

with $D<0$ since one can show that it is proportional to $-(\partial^2\pi_1^I/\partial w_I^2)(\partial^2\pi_1^I/\partial a_I^2)+(\partial^2\pi_1^I/\partial a_I\partial w_I)^2 < 0$.

First, $a_I^* = 0$ is equivalent to $u_B = u_S$.

Second, $w_I^* = c$ amounts to $-3\sigma(\sigma+4) + (\sigma(3\sigma+13)+20)u_B u_S + (\sigma(3\sigma+11)+12)u_S^2 - 16 = 0$ with $\sigma = \gamma - 2(1+\gamma)u_B u_S$. Expressing u_B as a function of σ, $w_I^* = c$ amounts to $(\sigma(3\sigma+13)+20)(\gamma-\sigma))/(2(\gamma+1)) - 3\sigma(\sigma+4) + (\sigma(3\sigma+11)+12)u_S^2 - 16 = 0$, a second degree polynomial in u_S with a positive root equal to

$$\hat{u}_S^*(\sigma) = \sqrt{\frac{\gamma + \frac{8\sigma}{9\sigma^2+33\sigma+36} + \sigma + \frac{8}{3}}{2(\gamma+1)}}.$$

Define $\hat{u}_B^*(\sigma) = \frac{\gamma-\sigma}{2(\gamma+1)\hat{u}_S(\sigma)}$. Then, the set of (u_B, u_S) such that $w_I^* = c$ is described by the curve associated to the parametric equations $(u_B = \hat{u}_B^*(\sigma), u_S = \hat{u}_S^*(\sigma))$ for all $\sigma \in [0,\gamma]$.

Straightforward computations show that $(\hat{u}_S^*(\sigma))^2 > (\hat{u}_B^*(\sigma))^2$ for all σ, so that the set of (u_B, u_S) such that $w_I^* = c$ lies strictly above the 45°-degree line. We have $(\hat{u}_B^*(\gamma), \hat{u}_S^*(\gamma)) = (0, \sqrt{(3\gamma(\gamma+4)+16)/(\gamma(3\gamma+11)+12)})$. Simple computations then show that $\hat{u}_S^{*2}(\gamma) \in [\underline{x}(\gamma), \overline{x}(\gamma)]$ and $\hat{u}_S^{*2}(0) \in [\underline{x}(0), \overline{x}(0)]$.

Last, since $\hat{u}_S(\sigma)^2$ is increasing in σ, Equation (A.19) shows that the derivative of the curve associated to the parametric equations is negative for all σ in the relevant range.

Simple computations also show that: (i) $\hat{u}_S(0)^2 = (8+3\gamma)/(6(1+\gamma))$, which belongs to $[\underline{x}(0) = (8+3\gamma-4\sqrt{4+3\gamma})/(6(1+\gamma)), \overline{x}(0) = (8+3\gamma+4\sqrt{4+3\gamma})/(6(1+\gamma))]$; (ii) $\hat{u}_S(\gamma)^2 = (\gamma+4)/(\gamma(3\gamma+11)+12)+1 > \overline{x}(\gamma) = \frac{2}{3}(\gamma/(\gamma(3\gamma+11)+12)+4/(3\gamma+4)+3)$.

To summarize, $w_I^* = c$ describes a curve in the (u_B, u_S)-space that is always above the 45°-degree line and is strictly decreasing, starts and ends within the sets of admissible values, as depicted in Figure 3.

A.6. Proof of Proposition 1 (Pricing Policy under Vertical Integration with No Efficiency Gains)

We consider here the constrained outcome with no efficiency gains in the running example. Let $\pi_1^I(a_I) = (p_1^I - c)D_1(p_1^I, p_2^I, a_I) + a_I D_S(p_1^I, p_2^I, a_I)$, where the prices p_1^I and p_2^I are given in Appendix A.2. From the analysis of Appendix A.5, this is a strictly concave function of a_I provided that (A.18) holds. Therefore, as shown in Appendix A.5, the objective is concave in a_I iff $u_S^2 \in [\underline{x}(\sigma), \overline{x}(\sigma)]$ for $\sigma \in [0,\gamma]$.

The first-order condition leads to

$$a_I^{**} = -\frac{(3\sigma+4)(v-c)(2(\sigma+2)(3\sigma+4)u_B - (3\sigma(\sigma(\sigma+7)+16)+32)u_S)}{H}$$

with $H = (\sigma+4)^2(3\sigma+4)^2 - 2(\sigma+2)(3\sigma+4)^2u_B^2 - 2(3\sigma+4)(\sigma(7\sigma+32)+32)u_B u_S - 2(\sigma(\sigma(7\sigma+40)+64)+32)u_S^2 > 0$ when $u_S^2 \in [\underline{x}(\sigma), \overline{x}(\sigma)]$.

The curve $a_I^{**} = 0$ is given by $-(2(\sigma+2)(3\sigma+4)u_B - (3\sigma(\sigma(\sigma+7)+16)+32)u_S) = 0$ and can be expressed as a function of (u_S, σ) as $(3\sigma(\sigma(\sigma+7)+16)+32)u_S - (\sigma+2)(3\sigma+4)(\gamma -$

$\sigma)/((\gamma+1)u_S) = 0$, which has a unique positive root

$$\hat{u}_S^{**}(\sigma) = \frac{\sqrt{(\sigma+2)(3\sigma+4)(\gamma-\sigma)}}{\sqrt{(\gamma+1)(3\sigma(\sigma(\sigma+7)+16)+32)}}.$$

Define $\hat{u}_B^{**}(\sigma) = \frac{\gamma-\sigma}{2(\gamma+1)\hat{u}_S^{**}(\sigma)}$. Then, the set of (u_B, u_S) such that $a_I^{**} = 0$ is characterized by the parametric equations $(u_B = \hat{u}_B^{**}(\sigma), u_S = \hat{u}_S^{**}(\sigma))$ for all σ in the relevant range. We can check that $\hat{u}_S^{**}(\sigma) \in [\underline{x}(\sigma), \overline{x}(\sigma)]$ (so that the curve associated to the parametric equations always lies within the set of admissible values) using brute force computations that are similar to those performed in Appendix A.5 and are not reported here.

We have $\lim_{\sigma\to\gamma}(\hat{u}_B^{**}(\gamma), \hat{u}_S^{**}(\gamma)) = (0,0)$, and $(\hat{u}_B^{**}(0), \hat{u}_S^{**}(0)) = (\sqrt{\gamma/(\gamma+1)}, (1/2)\sqrt{\gamma/(\gamma+1)})$. Simple computations also show that $(\hat{u}_S^{**}(\sigma))^2 \leq (\hat{u}_B^{**}(\sigma))^2$ for all $\sigma \in [0,\gamma]$, which shows that the curve $a_I^{**} = 0$ lies below the 45°-degree line in the (u_B, u_S)-space. Its derivative is given by

$$\frac{\frac{d\hat{u}_S^{**}}{d\sigma}(\sigma)}{\frac{d\hat{u}_B^{**}}{d\sigma}(\sigma)} = \frac{I}{J},$$

with $I = 2(\sigma+2)(3\sigma+4)(\gamma(3\sigma(\sigma(\sigma(3\sigma+20)+46)+48)+64)+\sigma(3\sigma(\sigma(11\sigma+80)+200)+640)+256) > 0$ and $J = (3\sigma(\sigma(\sigma+7)+16)+32)((3\sigma(\sigma+4)+8)(3\sigma(\sigma(2\sigma+9)+16)+32)-\gamma(3\sigma(\sigma(\sigma(3\sigma+20)+46)+48)+64))$. The numerator I is strictly positive. The denominator J is equal to zero when $\gamma = [(3\sigma(\sigma+4)+8)(3\sigma(\sigma(2\sigma+9)+16)+32)]/[3\sigma(\sigma(\sigma(3\sigma+20)+46)+48)+64]$. The right-hand side is strictly increasing in σ and is equal to 4 for $\sigma = 0$. Hence, if $\gamma < 4$, the slope is always positive. If $\gamma \geq 4$, there is a vertical asymptote and the slope is first positive and then negative. Notice also that

$$\left.\frac{\frac{d\hat{u}_S^{**}}{d\sigma}(\sigma)}{\frac{d\hat{u}_B^{**}}{d\sigma}(\sigma)}\right|_{\sigma=0} = \frac{\gamma+4}{2(4-\gamma)} \text{ and } \left.\frac{\frac{d\hat{u}_S^{**}}{d\sigma}(\sigma)}{\frac{d\hat{u}_B^{**}}{d\sigma}(\sigma)}\right|_{\sigma=\gamma} = \frac{2(\gamma+2)(3\gamma+4)}{3\gamma(\gamma(\gamma+7)+16)+32}.$$

To summarize, $a_I^{**} = 0$ describes a curve in the (u_B, u_S)-space such that: it goes through $(0,0)$, it is always below the 45°-degree line, and it is strictly increasing if $\gamma < 4$ or increasing then decreasing if $\gamma \geq 4$. Therefore, there exists a uniquely defined function $\underline{h} : u_S \mapsto \underline{h}(u_S)$ such that, for all (u_B, u_S), $a_I^{**}(u_B, u_S) > 0$ if and only if $u_B < \underline{h}(u_S)$.

A.7. *Proof of Proposition 3 (Impact of Vertical Integration on Buyers and Developers)*

IMPACT ON BUYERS. Let $V_B^I(a_I)$ be the buyer surplus under vertical integration when the royalty is equal to c and the developer fee is set at some value a_I. Let V_B^S be the buyer surplus under separation. Let $\Delta V_B(a_I) = V_B^I(a_I) - V_B^S$. Computations show that

$$\Delta V_B(a_I) = a_I\left(a_I - \frac{2(\sigma+2)(3\sigma+4)^2(v-c)((\sigma+2)u_B - u_S)}{K}\right)\frac{K}{(\sigma+4)^2(3\sigma+4)^2(1-2u_Bu_S)^2}$$

where $K = (\sigma+2)^2(3\sigma+4)^2u_B^2 + \left[(\sigma+2)(\sigma(\sigma+16)+16) - 2u_Bu_S(\sigma+1)(\sigma+4)^2\right]u_S^2 - 2u_Bu_S(\sigma+2)(3\sigma+4)^2$. We show first that $K > 0$. Since $2u_Bu_S < 1$, the term in brackets in K is strictly greater than $(\sigma+2)(\sigma(\sigma+16)+16) - (\sigma+1)(\sigma+4)^2 = (3\sigma+4)^2$. Plugging this in K then gives $K > (3\sigma+4)^2(u_S - u_B(2+\sigma))^2 \geq 0$. This implies that the sign of $\Delta V_B(a_I)$ is given by the sign of $a_I(a_I - 2(\sigma+2)(3\sigma+4)^2(v-c)((\sigma+2)u_B - u_S)/K)$.

FIRST SUFFICIENT CONDITION. Consider that $a_I^{**} < 0$, which amounts to $u_B > \underline{h}(u_S)$, with $\underline{h}$ defined in Appendix A.6 and such that $\underline{h}(u_S) > u_S$. Consequently, $a_I^{**} < 0$ implies $(\sigma+2)u_B -$

$u_S > 0$, so that $\Delta V_B(a_I^{**}) > 0$. Therefore, we have established a first sufficient condition

$$a_I^{**} < 0 \Rightarrow \Delta V_B(a_I^{**}) > 0.$$

SECOND SUFFICIENT CONDITION. Consider that $a_I^{**} \geq 0$. If $(\sigma+2)u_B - u_S \leq 0$, then $\Delta V_B(a_I^{**}) \geq 0$. Notice that $(\sigma+2)u_B - u_S \leq 0$ is equivalent to

$$u_S \geq \overline{h}_B(u_B) \equiv \frac{(\gamma+2)u_B}{2(\gamma+1)u_B^2+1}.$$

$\overline{h}_B$ is first increasing then decreasing, reaching a maximum at $u_B = 1/\sqrt{2(\gamma+1)}$ and leading to $u_S = (\gamma+2)/(2\sqrt{2(\gamma+1)})$. These values of (u_B, u_S) satisfy $\sigma \geq 0$ iff $\gamma \geq 2$.

$\overline{h}_B$ can also be represented with the parametric equations $(u_B = \sqrt{(\gamma-\sigma)/(2(\gamma+1)(\sigma+2))}$, $u_S = \hat{\tilde{u}}_S(\sigma) \equiv \sqrt{(\sigma+2)(\gamma-\sigma)/(2(\gamma+1))})$ for $\sigma \in [0,\gamma]$. This rewriting allows to get immediately that $\overline{h}_B$ is strictly above the 45°-degree line, and goes through $(0,0)$ (for $\sigma=\gamma$) and $(\sqrt{\gamma/(4(\gamma+1))}, \sqrt{\gamma/(\gamma+1)})$ (for $\sigma = 0$). This also shows that the conditions $a_I^{**} \geq 0$ (i.e., $u_B \leq \underline{h}(u_S)$) and $(\sigma+2)u_B - u_S \leq 0$ (i.e., $u_S \geq \overline{h}_B(u_B)$) define a non-empty set. Last, straightforward manipulations show that $\hat{\tilde{u}}_S^2(\sigma)$ belongs to $[\underline{x}_1(\sigma), \overline{x}_1(\sigma)]$ for all σ, so that it always belong to the admissible set.

INTERMEDIATE REGION. It remains to study the sign of $\Delta V_B(a_I^{**})$ when $a_I^{**} \geq 0$ and $(\sigma+2)u_B - u_S \geq 0$. Observe that $(\sigma+2)u_B - u_S \geq 0$ amounts to $u_S^2 \leq \overline{x}_a(\sigma) \equiv (\sigma+2)(\gamma-\sigma)/(2(\gamma+1))$. Similarly, $a_I^{**} \geq 0$ amounts to $u_S^2 \geq \underline{x}_a(\sigma) \equiv (\sigma+2)(3\sigma+4)(\gamma-\sigma)/((\gamma+1)(3\sigma(\sigma(\sigma+7)+16)+32))$ (see Appendix A.6). Simple computations show that $\overline{x}_a(\sigma) > \underline{x}_a(\sigma)$ for all σ in $[0,\gamma)$, so the interval in non empty.

On this interval, the sign of $\Delta V_B(a_I^{**})$ is given by the sign of

$$a_I^{**} - \frac{2(\sigma+2)(3\sigma+4)^2(v-c)((\sigma+2)u_B - u_S)}{K}.$$

Replacing u_B by $(\gamma-\sigma)/(2(\gamma+1)u_S)$ and up to some positive multiplicative terms (namely, $v-c$, H and K), the previous expression has the same sign as $h_1(x) = (\sigma+2)^3(3\sigma+4)^3(\gamma-\sigma)^3 + 4(\gamma+1)(\sigma+2)(3\sigma+4)x^2[\gamma^2(\sigma(\sigma(\sigma(23\sigma+183)+507)+584)+240) + \gamma(\sigma(\sigma(\sigma(\sigma(49-5\sigma)+562)+1618)+1840)+736) + \sigma(\sigma(\sigma(\sigma(37-4\sigma)+422)+1184)+1312)+512] - 4(\gamma+1)^2x^3(\gamma(3\sigma+4)(\sigma(\sigma+4)(\sigma(19\sigma+65)+88)+128) - \sigma(\sigma(\sigma+3)(\sigma+8)+16)(3\sigma(\sigma(\sigma+6)+10)+16)) - (\gamma+1)(\sigma+2)^2(3\sigma+4)^2x(\gamma-\sigma)(3\gamma(3\sigma(\sigma(\sigma+7)+16)+32) + \sigma(\sigma(\sigma(3\sigma+61)+288)+480)+256)$ with $x = u_S^2$. One can then show that $h_1(\underline{x}_a) < 0 < h_1(\overline{x}_a)$ and also $h_1'(\underline{x}_a) < 0 < h_1'(\overline{x}_a)$. Since $h_1'(x)$ is a polynomial of degree 2, it has at most two real roots and only one of these roots belongs to $[\underline{x}_a, \overline{x}_a]$. Since $h_1'(\underline{x}_a) < 0 < h_1'(\overline{x}_a)$, this implies that there exists a unique $\tilde{x} \in (\underline{x}_a, \overline{x}_a)$ such that h_1 decreases for $x \in [\underline{x}_a, \tilde{x}]$ and increases for $x \in [\tilde{x}, \overline{x}_a]$. This finally implies that there exists a unique $x_0^B(\sigma) \in (\underline{x}_a, \overline{x}_a)$ such that $h_1(x_0^B(\sigma)) = 0$.

Let $\tilde{u}_S(\sigma) = \sqrt{x_0^B(\sigma)}$ and $\tilde{u}_B(\sigma) = (\gamma-\sigma)/((1+\gamma)\tilde{u}_S(\sigma))$. By construction, the parametric curve $\tilde{\mathcal{C}} = (\tilde{u}_B(\sigma), \tilde{u}_S(\sigma))$, $\sigma \in [0,\gamma]$, is the frontier of the set of (u_B, u_S) such that $\Delta V_B(a_I^{**}) \leq 0$ when $a_I^{**} \geq 0$ and $(\sigma+2)u_B - u_S \geq 0$. Since $x_0^B(\sigma)$ is continuous and differentiable with respect to σ and the function $\sigma \mapsto (\tilde{u}_B(\sigma), \tilde{u}_S(\sigma))$ is injective, the curve $\tilde{\mathcal{C}}$ divides the (u_B, u_S)-space into two connected subsets, one in which the buyer surplus increases, the other in which it decreases.

IMPACT ON DEVELOPERS. Let $V_S^I(a_I)$ be the developer surplus under vertical integration when the royalty is equal to c and the developer fee is set at some value a_I. Let V_S^S be the developer surplus under separation. Let $\Delta V_S(a_I) = V_S^I(a_I) - V_S^S$. Computations show that

$$\Delta V_S(a_I) = \frac{(\sigma - 2u_S(2u_B+u_S)+4)^2}{2(\sigma+4)^2(1-2u_Bu_S)^2} a_I \left(a_I - \frac{4(\sigma+2)u_S(v-c)}{\sigma - 2u_S(2u_B+u_S)+4} \right).$$

Using $\sigma = \gamma - 2(1+\gamma)u_B u_S$, we rewrite $\sigma - 2u_S(2u_B + u_S) + 4 = 0$ as $4 + \gamma - 2u_S^2 - 2u_S u_B(\gamma + 3) = 0$ and denote the unique positive solution of this second degree polynomial equation in u_S by

$$\overline{h}_S(u_B) = \frac{1}{2}\left(\sqrt{2(\gamma+4) + (\gamma+3)^2 u_B^2} - (\gamma+3)u_B\right).$$

We thus have $\sigma - 2u_S(2u_B + u_S) + 4 > 0 \Leftrightarrow u_S < \overline{h}_S(u_B)$. The following facts are easily established: (i) $\overline{h}_S$ is strictly decreasing and strictly convex; (ii) $\overline{h}_S(0) = \sqrt{2+\gamma/2} > 0$; (iii) $\overline{h}_S(u_B) = u_B \Leftrightarrow u_B = u_S = 1/\sqrt{2} \Leftrightarrow \sigma = -1$. This implies that, in the (u_B, u_S)-space, and for $\sigma \geq 0$, $\overline{h}_S$ is strictly decreasing and above the 45°-degree line.

First Sufficient Condition. Therefore, if $a_I^{**} < 0$ (which amounts to $u_B > \underline{h}(u_S)$, with $\underline{h}$ below the 45°-degree line), then we also have $\sigma - 2u_S(2u_B + u_S) + 4 > 0$ (which amounts to $u_S < \overline{h}_S(u_B)$, with $\overline{h}_S$ above the 45°-degree line). As a consequence, we obtain a first sufficient condition

$$a_I^{**} < 0 \Rightarrow \Delta V_S(a_I^{**}) > 0.$$

Second Sufficient Condition. Next, we want to show the existence of another sufficient condition

$$\sigma - 2u_S(2u_B + u_S) + 4 < 0 \Rightarrow \Delta V_S(a_I^{**}) > 0.$$

Notice that $\sigma - 2u_S(2u_B + u_S) + 4 < 0$ (which amounts to $u_S > \overline{h}_S(u_B)$ and thus implies $u_S > u_B$) implies $a_I^{**} > 0$ (because $a_I^{**} < 0$ amounts to $\underline{h}(u_S) < u_B$ and thus implies $u_S < u_B$). Therefore, $\sigma - 2u_S(2u_B + u_S) + 4 < 0$ implies $\Delta V_S(a_I^{**}) > 0$.

It remains to show that the condition $\sigma - 2u_S(2u_B + u_S) + 4 < 0$ is compatible with the conditions for concavity. The condition $u_S > \overline{h}_S(u_B)$ can be equivalently expressed in terms of (u_S, σ) as $u_S > \tilde{u}_S(\sigma) \equiv \sqrt{(\gamma(\sigma+2) + 3\sigma + 4)/(2(\gamma+1))}$ with $\sigma \in [0, \gamma]$. Simple computations show then that $\tilde{u}_S^2(\sigma) \geq \underline{x}(\sigma)$ for all σ and $\tilde{u}_S^2(\sigma) \leq \overline{x}(\sigma)$.

Intermediate Region. Last, we establish that when $a_I^{**} > 0$ and $\sigma - 2u_S(2u_B + u_S) + 4 > 0$, $\Delta V_S(a_I^{**})$ is strictly negative. From the computations made to analyze the buyer surplus, $a_I^{**} > 0$ is equivalent to $u_S^2 > \underline{x}_a(\sigma)$. Simple computations show that $\sigma - 2u_S(2u_B + u_S) + 4 > 0$ amounts to $u_S^2 < \tilde{x}_a(\sigma) \equiv (\gamma(\sigma+2) + 3\sigma + 4)/(2(\gamma+1))$. Computations show that $\Delta V_S(a_I^{**}) = 0$ amounts to $h_2(x) \equiv (\sigma+2)(3\sigma+4)^2(\gamma-\sigma)(\gamma(\sigma+2) - \sigma(2\sigma+7) - 4) + 2(\gamma+1)^2(\sigma(\sigma+4)(\sigma(19\sigma + 65) + 88) + 128)x^2 - (\gamma+1)(3\sigma+4)x(\gamma(\sigma+2)(3\sigma(3\sigma(\sigma+7) + 46) + 88) + \sigma(\sigma(\sigma(31\sigma + 251) + 720) + 880) + 384) = 0$, with $x = u_S^2$ and $x \in [\underline{x}_a, \tilde{x}_a]$. The previous expression is a strictly convex second degree polynomial in x with a strictly positive discriminant. Hence it admits two reals roots. Computations show that $h_2(\underline{x}_a) < 0$ and $h_2(\tilde{x}_a) < 0$, which proves that $h_2(x) < 0$ for all $x \in [\underline{x}_a, \tilde{x}_a]$. Hence, if $a_I^{**} > 0$ and $\sigma - 2u_S(2u_B + u_S) + 4 > 0$, then $\Delta V_S(a_I^{**}) < 0$.