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JEL Classification: D15, D52, E22, G21

Keywords: Liquidity Provision, Instability, incomplete markets

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Abstract

We establish that the provision of intertemporal liquidity is fundamentally prone to instability. Not only are banks subject to coordination failures but also asset markets are inherently unstable. These findings challenge the notion of optimal private provision of liquidity.

Keywords liquidity provision · instability · market incompleteness

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Declaration of interests None

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1 Liquidity Provision in Dynamic Economies

The private provision of liquidity in *dynamic economies* with *incomplete markets* has become a focus of contentious debate. According to the bank-based view, banks are institutions that bypass market incompleteness by centralizing allocation decisions. However, liquidity provision by banks is prone to instability. The contrasting market-based view stresses that simple trading opportunities are sufficient to provide liquidity as efficiently as unfettered, competitive banking sectors. However, as we show, market equilibria may be also inherently unstable, multiple, and even indeterminate. Our findings thus highlight that instability seems integral to the private provision of liquidity in dynamic economies with incomplete markets.

2 Background

The canonical view on private liquidity provision (Bryant, 1980; Diamond and Dybvig, 1983) considers a two-period economy with one generation of consumers. Consumers face a liquidity risk for they may become impatient and need to access their investment before its full returns are realized. The management of such liquidity risk is naturally bound to some form of intra-generational risk sharing in this quasi-static model.

In a dynamic framework with overlapping replicas of this quasi-static model, the possibility of risk-sharing across generations of consumers arises. Specifically, the returns on long-term investments can be shared among patient consumers, who originally funded these investments, and the impatient consumers of the following generation. Such inter-generational risk-sharing Pareto-dominates the intra-generational solution (Fulghieri and Rovelli, 1998). However, only centralized, infinitely-lived, mutual financial intermediaries implement the first-best provided they can discriminate consumers by their age and no further trading opportunities for consumers or banks exist (Qi, 1994). Neither competitive banks nor asset markets provide efficient outcomes. If consumers cannot be discriminated, outcomes with competitive asset markets are less efficient than with banks (Fulghieri and Rovelli, 1998), but their outcomes are equivalent if trading opportunities are unfettered (Bhattacharya and Padilla, 1996) and some well-defined government redistribution may indeed allow asset markets to outperform banks (Bhattacharya et al., 1998).

Such allocative efficiency considerations aside, a major drawback of inter-generational risk sharing through banks stems from its vulnerability to coordination failures. Not only could patient consumers withdraw early if they believe other patient consumers to do so — a risk already well-known from the quasi-static banking framework, and which does not apply to its asset-market counterpart (Jacklin, 1987). There is the additional risk that a new generation of consumers coordinates on not depositing their endowments in (already operating) banks in the first place (Qi, 1994). This inherent instability of banks has been argued to tilt the balance in favor of asset markets. We revisit the so far overlooked stability properties of asset market equilibria in the overlapping generations framework and find that the notion of stable asset markets is also generally unjustified.

3 Model

Setup Time is discrete with $-\infty \leq t \leq +\infty$, and there is a single good at every date. There is a constant-returns-to-scale technology to produce the good. Production can be initiated at any date t and yields a gross return of R per unit after two periods at date $t+2$ and nothing ever after. At every date t, a generation of consumers of unit mass is born, each consumer with an endowment of 1 unit of the good. A consumer born at date t can initiate production when young. She becomes either impatient and values consumption at $t+1$ when middle-aged, or patient and values consumption at $t+2$ when old. At date $t+1$ consumers learn their type, which is private information. Patience among consumers is uncorrelated and the share of impatient consumers $\lambda \in]0,1[$ is deterministic and common knowledge. Let $c_t^m \geq 0$ denote what a consumer born at date t consumes as a middle-aged consumer at $t+1$ (she will consume when middle-aged only if she is impatient), and $c_t^o \geq 0$ what she consumes as an old consumer at $t+2$ (she will consume when old only if she is patient).

Her *Bernoulli* utility function u is strictly increasing and concave with expected life-time utility

$$U(c_t) = \lambda u(c_t^m) + (1-\lambda) u(c_t^o). \tag{1}$$

Good market The supply of the good at every date t is given by the endowment of the generation t and by the production initiated two dates prior to date t. These goods are used to initiate new production at date t and for consumption by impatient consumers of generation $t-1$ and by patient consumers of generation $t-2$.

Asset market All consumers alive at date t can trade already initiated, but yet-to-materialize production of goods (henceforth assets) in exchange for the good. The price payable at date t for assets originated at date $t-1$ is q_t (in units of the consumption good). Impatient consumers of generation $t-1$ sell the assets they have originated when young. Existing assets are held by patient consumers of the same generation, using the returns from assets they bought when young, and by young consumers of the following generation t.

4 Analysis

The consumer problem Let $x_t \geq 0$ and $z_t \geq 0$ be the investment of a consumer born at date t in new production and in existing assets, respectively. Then, her problem reads

$$\max_{(x_t, z_t, c_t^m, c_t^o) \in \mathbb{R}_+^4} \quad U(c_t)$$

$$\text{s.t.} \quad \begin{cases} x_t + q_t z_t & \leq & 1 \\ c_t^m & \leq & q_{t+1} x_t + R z_t \\ c_t^o & \leq & R\left(x_t + \frac{R z_t}{q_{t+1}}\right) \end{cases} \tag{2}$$

The first constraint is the consumer's budget constraint when young. Her spending on initiating new production, $x_t \geq 0$, and on buying assets, $z_t \geq 0$, at price q_t cannot exceed her endowment. The second constraint is the budget constraint for impatient consumers. They cannot consume more than their earnings from the assets they bought when young and the proceeds from selling the assets they originated when young. The last constraint is the budget constraint for patient consumers. They initiated production investing x_t when young. They can also buy more assets when middle-aged using the proceeds Rz_t earned from the assets bought when young, which at current prices q_{t+1} will buy them additional Rz_t/q_{t+1} assets.

Non-satiation implies all budget constraints hold with equality. The budget constraints for c_t^m and c_t^o imply that both, c_t^m and c_t^o are strictly increasing in x_t iff $\frac{R}{q_t} < q_{t+1}$, strictly decreasing in x_t iff $\frac{R}{q_t} > q_{t+1}$, and independent from x_t iff $\frac{R}{q_t} = q_{t+1}$. Together with $x_t \in [0,1]$, a consumer's optimum consumption plan $(\hat{c}_t^m, \hat{c}_t^o, \hat{x}_t, \hat{z}_t) \in \mathbb{R}_+^2 \times [0,1] \times [0, q_t^{-1}]$ thus satisfies

$$
\begin{aligned}
(\hat{x}_t, \hat{z}_t) &= \begin{cases} \left(0, q_t^{-1}\right) & \text{if } \frac{R}{q_t} > q_{t+1} \\ (1,0) & \text{if } \frac{R}{q_t} < q_{t+1} \\ \left(\check{x}_t, (1-\check{x}_t) q_t^{-1}\right) & \text{if } \frac{R}{q_t} = q_{t+1} \end{cases} \\
(\hat{c}_t^m, \hat{c}_t^o) &= \left(q_{t+1}\hat{x}_t + R\hat{z}_t, \frac{R}{q_{t+1}}(q_{t+1}\hat{x}_t + R\hat{z}_t)\right)
\end{aligned}
$$

where $\check{x}_t$ can be any value in $[0,1]$.

First best The first-best investments satisfy

$$
x_t^{\text{fb}} \in \begin{cases} \{1\} & \text{for all } t \text{ if } \quad R > 1, \\ \{0\} & \text{for all } t \text{ if } \quad R < 1, \\ [0,1] & \text{for all } t \text{ if } \quad R = 1. \end{cases} \tag{3}
$$

This follows because, for all $t \in \mathbb{Z}$, a mass λ of impatient consumers born at date $t-1$ and a mass $1-\lambda$ of patient consumers born at $t-2$ will consume. Therefore the total mass of consumers consuming at date t is one for all $t \in \mathbb{Z}$. For any given level of resources, risk aversion implies that their utility is highest if there is full insurance against their idiosyncratic liquidity risk, i.e. $c_t^m = c_t^o$. Total resources available for consumption at every date t are $1 + Rx_{t-2} - x_t$. Hence, utility is maximized provided total resources are highest throughout time, which holds as stated in (3).

Equilibrium An equilibrium is a sequence of prices and consumption plans

$$((\bar{q}_t), (\bar{c}_t^m, \bar{c}_t^o, \bar{x}_t, \bar{z}_t))_{t \in \mathbb{Z}}$$

such that for every t

- consumers maximize utility: $(\bar{c}_t^m, \bar{c}_t^o, \bar{x}_t, \bar{z}_t)$ solves the problem of a consumer in generation t;
- the good market clears:

$$1 + R\bar{x}_{t-2} = \bar{x}_t + \lambda \bar{c}_{t-1}^m + (1-\lambda)\bar{c}_{t-2}^o; \tag{4}$$

- the asset market clears:

$$\lambda \bar{x}_{t-1} = (1-\lambda)\frac{R\bar{z}_{t-1}}{\bar{q}_t} + \bar{z}_t. \tag{5}$$

Taking into account the budget constraints from program (2), and since there is no date t at which prices satisfy $R/\bar{q}_t > \bar{q}_{t+1}$ or $R/\bar{q}_t < \bar{q}_{t+1}$ (see Appendix), any equilibrium thus solves:

$$\begin{aligned}
\frac{R}{\bar{q}_{t-1}} &= \bar{q}_t, \\
\bar{x}_t &= 1 - \bar{q}_t \bar{x}_{t-1} + (1-\lambda)\bar{q}_t.
\end{aligned}$$

Proposition 1 *A* steady state *exists with* $\bar{x}_t = \bar{x}_{t+1} = \tilde{x} = \frac{1+\sqrt{R}(1-\lambda)}{1+\sqrt{R}}$ *and* $\bar{q}_t = \bar{q}_{t+1} = \tilde{q} = \sqrt{R}$ *for all* t*. Provided* $R < 1$*, the steady state is* locally asymptotically stable *and there is* overinvestment *in production. Provided* $R > 1$*, the steady state is* unstable *and there is* underinvestment *in production.*

Proof: Steady state requires $\bar{c}_t^m$ and $\bar{c}_t^o$ constant across time. Since $R/\bar{q}_{t-2} \leq \bar{q}_{t-1}$, $\bar{c}_t^m$ is constant iff $\bar{q}_t, = \bar{q}_{t+1} = \tilde{q}$ for all t. Therefore $R/\tilde{q} = \tilde{q}$, i.e. $\tilde{q} = \sqrt{R}$. Let $f : [0,1] \to \mathbb{R}$ with $f(x) = 1 - \sqrt{R}x + (1-\lambda)\sqrt{R}$. Then, the asset market clears if $\bar{x}_{t+1} = f(\bar{x}_t)$. Solving $\tilde{x} = f(\tilde{x})$ yields $\tilde{x} = \frac{1+\sqrt{R}(1-\lambda)}{1+\sqrt{R}}$. Stability properties follow since $|f'(\tilde{x})| = \sqrt{R}$. Overinvestment and underinvestment, respectively, follow since $\tilde{x} \in]0,1[$ while $x^{\text{fb}} = 1$ if $R > 1$ and $x^{\text{fb}} = 0$ if $R < 1$. □

Instability implies that the asset market economy is lacking a basic self-correction property. Whenever unexpected errors occur in the behavior of some generation t, the economy will never regain its steady-state despite future generations behaving consistently with equilibrium conditions.[1] But asset markets have another unpleasant feature for other, periodic solutions to the dynamic system also exist.

Proposition 2 *Suppose* $R > 1$*. There exist* two-period-cycles *with* $\bar{x}_{t+2k} = \frac{1+\sqrt{R}(1-\lambda)}{1+\sqrt{R}} + \frac{\lambda}{R-1}\left(\bar{q}_t - \sqrt{R}\right)$ *and* $\bar{x}_{t+2k+1} = \frac{1+\sqrt{R}(1-\lambda)}{1+\sqrt{R}} + \frac{\lambda}{R-1}\left(\bar{q}_{t+1} - \sqrt{R}\right)$ *for some* t *and for all* $k \in \mathbb{Z}$*. Asset prices* $(\bar{q}_{t+2k}, \bar{q}_{t+2k+1}) \in [1,R] \times [1,R]$ *satisfy* $\bar{q}_{t+2k+1} = R/\bar{q}_{t+2k}$*. Cycles are* unstable*. No further equilibria exist.*

Proof: Let $\bar{q}_t = \bar{q}_{t+2k}$ and $\bar{q}_{t+1} = \bar{q}_{t+2k+1}$ for some t and all $k \in \mathbb{Z}$. Provided $R/\bar{q}_{t+2k} = \bar{q}_{t+2k+1}$, consumers are indifferent between all $\hat{x}_t \in [0,1]$, such that $\{\bar{x}_t\}_{t \in \mathbb{Z}}$ is determined solely by market clearing:

$$\bar{x}_{t+2k} = 1 - \bar{q}_{t+2k}\bar{x}_{t+2k+1} + (1-\lambda)\bar{q}_{t+2k}$$

$$\bar{x}_{t+2k+1} = 1 - \bar{q}_{t+2k+1}\bar{x}_{t+2k} + (1-\lambda)\bar{q}_{t+2k+1}$$

[1] This does not include the possibility of mistakes as part of the equilibrium concept itself, as in *approximate equilibria* (Anderson et al., 1982).

Substituting $\bar{x}_{t+2k+1}$ from the second into the first line, and $\bar{q}_{t+2k+1}$ by $R/\bar{q}_{t+2k}$ yields

$$\begin{aligned}
\bar{x}_{t+2k} &= \tfrac{1+\sqrt{R}(1-\lambda)}{1+\sqrt{R}} + \tfrac{\lambda}{R-1}\left(\bar{q}_{t+2k} - \sqrt{R}\right), \\
\bar{x}_{t+2k+1} &= \tfrac{1+\sqrt{R}(1-\lambda)}{1+\sqrt{R}} + \tfrac{\lambda}{R-1}\left(\bar{q}_{t+2k+1} - \sqrt{R}\right).
\end{aligned}$$

In a cycle, $\bar{x}_{t+2k}$ and $\bar{x}_{t+2k+1}$ are both bounded above by 1. Therefore, prices $\bar{q}_{t+2k}$ and $\bar{q}_{t+2k+1}$ are both bounded above by R and, as $R/\bar{q}_{t+2k} = \bar{q}_{t+2k+1}$, bounded below by 1. Instability of cycles follows because $|f'(\bar{x}_{t+2k}) f'(\bar{x}_{t+2k+1})| = R > 1$. Since $R/\bar{q}_{t+2k} = \bar{q}_{t+2k+1}$ holds in any equilibrium and supports only cycles, no further equilibria exist. □

Accordingly, coordination problems do also arise in decentralized market economies where multiple, even indeterminate periodic equilibria exist (which are also unstable). The perceived comparative stability advantage of asset market equilibria over banking equilibria, which commonly holds in the single-generation framework, thus no longer exists here. However, unlike bank-run equilibria, the various dynamic asset market equilibria cannot be welfare-ranked, as we show next.

5 Implications for Investment and Welfare

In periods when asset prices are high, investment is higher than in periods when asset prices are low. Moreover, in cyclical equilibria, consumption plans satisfy $\bar{c}^m_{t+2k} = R/\bar{q}_{t+2k}$ and $\bar{c}^m_{t+2k+1} = \bar{q}_{t+2k}$ as well as $\bar{c}^o_{t+2k} = \bar{c}^o_{t+2k+1} = R$ for some t and for all $k \in \mathbb{Z}$. Hence, the burden arising from asset prices fluctuating around $\sqrt{R}$ is fully borne by impatient consumers.

Finally, for average investment in production we obtain

$$\frac{1}{2}\left(\bar{x}_{t+2k} + \bar{x}_{t+2k+1}\right) = \frac{R-1-\lambda R}{R-1} + \frac{\lambda}{R-1}\frac{\frac{R}{q_{t+2k+1}} + q_{t+2k+1}}{2}$$

which is a convex function of $\bar{q}_{t+2k+1}$ with a minimum at $\bar{q}_{t+2k+1} = \sqrt{R}$ and strictly increasing for all $\bar{q}_{t+2k+1} > \sqrt{R}$ and strictly decreasing for all $\bar{q}_{t+2k+1} < \sqrt{R}$. Not only do infinitely many possible cycles exist. Also, average investment in any cycle is higher than in the steady state (and thus closer to the first-best), and the most volatile cycle actually allows for the largest average investment. However, cycles provide no Pareto-improvements over steady states. While patient consumers receive the same as in the steady state, impatient consumers of every other generation will receive lower consumption. This affects the generation, for which the asset price at the time when they are middle-aged is low. The other generation, for which the asset price at the time when they are middle-aged is high, gain from a greater magnitude of the cycle.

6 Concluding Remarks

We utilized the framework of Qi (1994), Bhattacharya and Padilla (1996), Bhattacharya et al. (1998), and Fulghieri and Rovelli (1998), which builds upon three implicit assumptions: First, short-selling production is not possible, i. e. production cannot be traded if it has not been initiated before. Second, only young consumers can initiate production. Third, frictions in credit markets preclude young consumers from borrowing, so middle-aged patient consumers cannot contribute to new production. It is arguably important to understand whether either assumption is key to (in)stability, and if so which additional trading opportunities may allow for stable and predictable market outcomes.

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Online Appendix: The Dynamic System

Consider the market clearing conditions for the good and the asset market, respectively, see main text, Eqs. (4) and (5). Taking into account the budget constraints from program (2), these the market clearing conditions read

$$1+R\bar{x}_{t-2}=\bar{x}_t+\lambda\bar{q}_t\left(\bar{x}_{t-1}+\frac{R(1-\bar{x}_{t-1})}{\bar{q}_t\bar{q}_{t-1}}\right)+(1-\lambda)R\left(\bar{x}_{t-2}+\frac{R(1-\bar{x}_{t-2})}{\bar{q}_{t-1}\bar{q}_{t-2}}\right), \tag{6a}$$

$$\lambda\bar{x}_{t-1}=(1-\lambda)\frac{R(1-\bar{x}_{t-1})}{\bar{q}_t\bar{q}_{t-1}}+\frac{1-\bar{x}_t}{\bar{q}_t}. \tag{6b}$$

There is no date t at which prices satisfy $R/\bar{q}_t>\bar{q}_{t+1}$. This is because for $R/\bar{q}_t>\bar{q}_{t+1}$ consumers set $\hat{x}_t=0$. The asset market thus cannot clear at date $t+1$ for any $x_{t+1}\in[0,1]$ and any finite q_{t+1}. Furthermore, there is no date t at which prices satisfy $R/\bar{q}_t<\bar{q}_{t+1}$. To understand why, suppose $R/\bar{q}_t<\bar{q}_{t+1}$ for some t. Then, consumers born at date t set $\hat{x}_t=1$. Equilibrium on the asset market at date t requires $\hat{x}_{t-1}=1-\lambda$. However, as $\hat{x}_{t-1}\in]0,1[$, we have $R/\bar{q}_{t-1}=\bar{q}_t$. The good market clears at date t if and only if $\hat{x}_{t-2}=\lambda\bar{q}_t/R+(1-\lambda)$, which satisfies $\hat{x}_{t-2}\leq 1$ only if $R/\bar{q}_t\geq 1$. As prices are supposed to satisfy $R/\bar{q}_t<\bar{q}_{t+1}$, this implies $\bar{q}_{t+1}>1$ and $R/\bar{q}_{t-1}=\bar{q}_t$ implies $\bar{q}_{t-1}\geq 1$. The asset market clears at date $t+1$ if and only $\hat{x}_{t+1}=1-\lambda q_{t+1}$. Given that $\hat{x}_{t-1}=1-\lambda$, one can check that the good market clears for any $(\hat{x}_{t+1},q_{t+1})$ satisfying $\hat{x}_{t+1}=1-\lambda q_{t+1}$. The asset market clears at date $t+2$ if and only $\lambda\,(1-\bar{q}_{t+1})=(1-\hat{x}_{t+2})/\bar{q}_{t+2}$ which cannot have a solution since $\bar{q}_{t+1}>1$.