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This paper proposes an equilibrium theory of nominal exchange rates, which offers a new perspective on various issues in open economy macroeconomics. The nominal exchange rate and portfolio choices are jointly determined in equilibrium, thus providing a new approach to overcoming the indeterminacy results in Kareken and Wallace (1981). The distinctive features of this theory are that the nominal exchange rate is determined in international financial markets, that the risk premium and UIP deviations are fully endogenous equilibrium objects and that the real exchange rate inherits its properties from the nominal exchange rate. In terms of policy, this novel theory implies that a country with an exchange rate peg and free asset mobility faces a tetralemma and not a trilemma, because it loses not only monetary policy independence but also fiscal policy independence.

JEL Classification: D52, E31, E43, E52, E62, E63

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This paper proposes an equilibrium theory of nominal exchange rates, which offers a new perspective on various issues in open economy macroeconomics. The nominal exchange rate and portfolio choices are jointly determined in equilibrium, thus providing a new approach to overcoming the indeterminacy results in Kareken and Wallace (1981). The distinctive features of this theory are that the nominal exchange rate is determined in international financial markets, that the risk premium and UIP deviations are fully endogenous equilibrium objects and that the real exchange rate inherits its properties from the nominal exchange rate. In terms of policy, this novel theory implies that a country with an exchange rate peg and free asset mobility faces a tetralemma and not a trilemma, because it loses not only *monetary* policy independence but also *fiscal* policy independence.

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1 Introduction

This paper proposes a new equilibrium theory of nominal exchange rates and country portfolios in a world where monetary policy operates in the standard way, that is, through setting nominal interest rates instead of the money supply. A distinctive feature of this theory is that the nominal exchange rate, international bond portfolios and risk premia are determined in international financial markets in response to fundamental and policy shocks (Figure 1). Equilibrium requires households to make optimal portfolio decisions, taking the stochastic nominal exchange rate process as given, and at the same time exchange rates and risk premia clear the bond markets. The real exchange rate then inherits its properties, such as its volatility, from the properties of the nominal exchange rate. While the magnitude of response of the real exchange rate and of the goods market depends on parameters such as price rigidity and the country's trade openness, the causality always runs from the nominal to the real exchange rate. In conventional open macroeconomic models, this causality is reversed. The real exchange rate is then determined in international goods markets, and the nominal exchange rate inherits its properties from real exchange rates.[1] Of course, both directions - from nominal to real and from real to nominal - can be present in an equilibrium model, but here the first direction is the dominant one, whereas the latter direction dominates in the traditional approach. Similarly, international portfolios are also determined in financial markets in response to intertemporal and precautionary savings motives, and to diversify exchange rate and fundamental risks. The resulting net asset flows in the financial market then determine the current account in the goods market.

One implication of the financial-market-based determination of exchange rates is that the nominal exchange rate is priced like an asset and is thus an order of magnitude more volatile than consumption and output, as observed in the data in freely floating exchange rate regimes. In the traditional approach, however, the real exchange rate clears the goods market and is therefore closely connected to only mildly volatile consumption and output, leading to several puzzles in international macroeconomics (Obstfeld and Rogoff, 2000).

Another main theoretical result is that, in the absence of specific policy interventions as discussed below, a positive level of fundamental risk rules out a constant exchange rate and instead implies that equilibrium exchange rates are risky. This is the case simply because different fundamental shocks require different exchange rates to clear international asset markets.

[1] Kollmann (2005) and Itskhoki and Mukhin (2019) show that these "goods-market-based" models are inconsistent with the data, arguing that instead, UIP deviations are needed to account for the data. The theory proposed here delivers these UIP deviations endogenously.

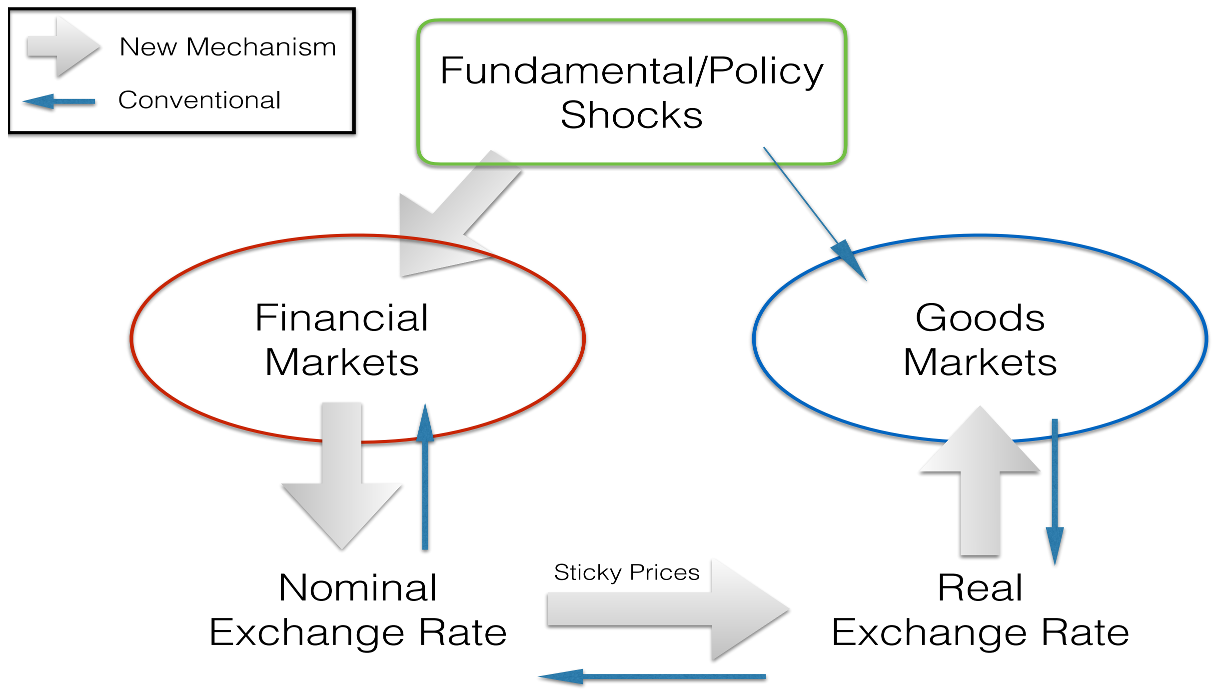


Figure 1: Nominal and Real Exchange Rate Determination: New And Conventional Mechanism

A volatile exchange rate and non-vanishing risk premia are then an equilibrium outcome and a necessary feature of any equilibrium in the presence of fundamental risk.[2] The endogenous risk premia also rationalize deviations from uncovered interest rate parity (UIP). In equilibrium, exchange rates and risk premia adjust endogenously, so that currency excess returns are explained through differences in risk exposure. The marginal investor is then indifferent between investing in a risky high-interest-rate country and a less risky low-interest-rate country. Lustig and Verdelhan (2007), Lustig et al. (2014) and Verdelhan (2018) provide empirical support for this mechanism. These authors show that risk indeed accounts for most of the variation in currency excess returns in the data. My paper is not only consistent with their empirical findings, but in addition, provides a theory in which these risk premia/UIP deviations are an endogenous equilibrium object.[3] In addition, the theory proposed here also delivers an equilibrium theory of international asset flows, simply because different fundamental shocks lead to different international portfolio choices.[4] And again, empirical evidence supports the theoretical mechanism.

[2]The amount of equilibrium risk and risk premia are endogenous to monetary and fiscal policies, as in several closed-economy models (see, for instance, Caballero and Farhi, 2017; Drechsler et al., 2018; Silva, 2019; Kekre and Lenel, 2019), where heterogeneity in risk preferences generates cyclical (equity) risk premia.

[3]AsMaggiori (2021) emphasizes, risk premia are unlikely to be the sole variable to explain all puzzles, and market segmentation and risk-based explanations are likely to be complementary.

[4]My theory thus offers an answer to the open question raised by Maggiori (2021) as to what drives international asset flows.

As documented in the data by Broner et al. (2020), an increase in foreign purchases of home government bonds is associated with an appreciation of the home currency.

A comparison with the literature further elucidates the properties of the proposed theory. From a reduced-form perspective, the endogenous risk premium movements look like financial shocks to the uncovered interest-rate parity (UIP) condition in Itskhoki and Mukhin (2019). The key differences are that here the risk premium, namely the financial shock, and the exchange rate, are both endogenous, that both move in response to fundamental shocks, and that one variable does not drive the other. From an empirical perspective, this difference between the two approaches allows me to match not only unconditional moments in the data, as in Itskhoki and Mukhin (2019), but also conditional ones, for example responses of the economy that are conditional on technology or monetary shocks.[5] Asset flows and portfolio rebalancing are optimal endogenous responses to aggregate risks in this paper and are not driven by noise traders as in Gabaix and Maggiori (2015) and Itskhoki and Mukhin (2019). Risk premia are endogenous, as in the segmented markets model in Alvarez et al. (2009). However here, the properties of the nominal exchange rate carry over to the real exchange rate, whereas movements in real exchange rates are essential in Alvarez et al. (2009) to account for movements of nominal exchange rates in the data. Similarly, the models in Gabaix and Maggiori (2015) and Itskhoki and Mukhin (2019) are also essentially real, so that movements in real exchange rates cause movements in nominal exchange rates.[6]

The benchmark model is intentionally simple, so as to focus on the new mechanism for jointly determining the nominal exchange rate and portfolio choices. I use a two-country, two-generation overlapping-generations (OLG) model without capital and two internationally traded nominal bonds, which delivers tractability and at the same time features all three ingredients necessary for the theory to operate: Markets are incomplete within each country, aggregate country risk is non-diversifiable and each country issues nominal government bonds denominated in its own currency. There is also only one good, such that the real exchange rate is equal to one, which makes it clear that movements in nominal exchange rates are not driven by movements in real exchange rates.

I then add non-tradable goods and sticky prices to the benchmark model to derive three results. First, the same mechanism as in the benchmark model determines nominal exchange rates and portfolios, establishing that the assumptions of the benchmark are *not* essential

[5] I thank Gernot Müller for pointing this out.

[6] A related literature considers optimal exchange rate policies with frictional international financial markets, but imposes exogeneity on international real interest rates (Liu and Spiegel, 2015; Chang and Velasco, 2016; Fanelli and Straub, 2019) or risk premia (Farhi and Werning, 2014) or on portfolio choices through noise traders (Cavallino, 2019). In contrast to this literature, returns on home and foreign bonds, risk premia and portfolio choices are all endogenous here.

elements of the theory proposed here. Second, the real exchange rate is not constant and is highly correlated with the nominal exchange rate as in flexible exchange rate regimes. Third, prices are substantially less volatile than exchange rates, as in the data.

I then consider policies that implement an exchange rate peg, which, in addition to its empirical relevance, also illustrates clearly how the approach proposed in this paper differs from existing ones. A major finding is that the classic policy trilemma in international economics - that at most two of the following three policies are simultaneously feasible: (i) unrestricted capital mobility; (ii) setting the nominal interest rate independently (monetary policy independence); and (iii) a fixed exchange rate - turns into a tetralemma, because fixed exchange rates and free capital mobility imply the loss not only of monetary policy independence, but also of fiscal policy independence. The argument is simple. Interest rate parity implies that domestic monetary policy has to track foreign monetary policy to be consistent with a constant exchange rate. But this monetary policy does not peg the exchange rate yet, which still moves in response to unanticipated shocks to ensure asset market clearing. Fiscal policy then has to ensure that the exchange rate remains unchanged in response to those unanticipated shocks. This restriction on fiscal policy is absent in the standard trilemma, since the exchange rate is not determined as clearing the asset market, and can thus be pegged irrespective of shocks to asset markets. Here, in contrast, I show that monetary policy cannot stabilize the exchange rate on its own through setting nominal interest rates, and that fiscal policy has to step in when unanticipated shocks move asset demands and thus also the equilibrium level of the exchange rate.

Practical experience with many exchange rate regimes suggests that the tetralemma accurately describes the boundaries of policies.[7] The frequent interventions by central banks to peg the exchange rate, of which many ended in debacles, support the claim that the nominal interest rate is insufficient to peg the exchange rate on its own (Obstfeld and Rogoff, 1995). This need for policies in addition to setting interest rates as suggested by the tetralemma, becomes particularly relevant at the zero lower bound (Caballero et al., 2016; Amador et al., 2019a,b). It is also generally accepted that an exchange rate regime cannot be sustained if it is inconsistent with overall macroeconomic policy (Fischer, 2001). This view is in line with the tetralemma which requires fiscal policy adjustments to be consistent with an exchange rate peg. By contrast, in the trilemma world, fiscal policy is irrelevant in maintaining a pegged exchange rate.

The results for an exchange rate peg are qualitatively consistent with the Mussa (1986)

[7]For example, Fratzscher et al. (2019) find that foreign exchange rate interventions are widely used and are an effective policy tool for smoothing the path of exchange rates in countries where the narrow band regime is consistent with macroeconomic policy. See Cavallino (2019) for a short survey and further references.

puzzle, which entails the volatility of the real exchange rate falling (significantly) when moving from a freely floating regime to a pegged nominal exchange rate. Indeed, since the real exchange rate inherits its properties from the nominal exchange rate, moving from a flexible to a pegged regime renders both exchange rates in the model constant. I also show that the new mechanism for determining exchange rates could be a promising avenue to address several puzzles in open economy macroeconomics, such as the (Fama, 1984; Engel, 2016) puzzle and the Kollmann-Backus-Smith Consumption-Real Exchange rate puzzle (Backus and Smith, 1993; Kollmann, 1995). The empirical success is not particularly surprising, since Kollmann (2005) and Itskhoki and Mukhin (2019) show that UIP shocks are needed to account for the data. And as explained above, endogenous risk premium movements in my model and UIP shocks are equivalent, from a reduced form perspective.

The deeper theoretical reason underlying the findings in this paper is that the model is built on a new way to jointly determine both the nominal exchange rate and portfolios. To better understand the underlying mechanism and the implications for the issues motivating this research, it is instructive to recall the indeterminacy result of Kareken and Wallace (1981) (KW).[8] Consider two countries where monetary policy sets nominal interest rates. The uncovered interest rate parity condition then determines the expected change in the exchange rate only, but leaves the exchange rate *level* indeterminate. An equivalent type of price level indeterminacy also arises in closed economies (Sargent and Wallace (1975)), but as pointed out in KW, the open economy framework adds another subtle type of indeterminacy. The KW indeterminacy arises if assets are fully mobile across borders, and household portfolio choices and net foreign asset positions are indeterminate. At the aggregate level, this portfolio indeterminacy turns into an indeterminacy of asset demands and thus of the market-clearing exchange rates.

The solution to the KW indeterminacy in the textbook Mundell Fleming model is a normalization of the future expected exchange rate. Modern dynamic models build on the seminal work of Gali and Monacelli (2005) and use an interest rate rule satisfying the Taylor principle (Benigno and Benigno, 2008). Others (e.g. Alvarez et al., 2009; Gabaix and Maggiori, 2015) deviate from the consensus in monetary economics and central banks that monetary policy operates through setting nominal interest rates, and instead assume that monetary policy determines money supply. The nominal exchange rate is then determined if in addition, money is not freely mobile across countries, so that agents cannot use any currency in every country without transaction costs.

This paper offers a different solution. Monetary policy sets nominal interest rates. House-

[8] Cavallo and Ghironi (2002) and Ghironi (2008) adopt an overlapping generations model instead of a representative agent one (within a country), mainly to ensure stationarity. This assigns a role to the stock of real net foreign assets, but does not deliver nominal exchange rate determinacy.

holds are not indifferent between home and foreign bonds, but instead use them to diversify aggregate risk. The determinacy of portfolio choices then carries over to exchange rates which have to adjust in order to clear bonds markets in all countries. The nominal exchange rate and portfolio choices are then jointly determined in equilibrium. Section 3 explains the theoretical foundations of my approach within a large class of models, including incomplete markets (within countries) and OLG models, with aggregate risk.[9,10]

The remainder of the paper is organized as follows. Section 2 develops the simple OLG model with a constant real exchange rate and flexible prices, and explains the workings of this new theory and how it jointly determines exchange rates and asset choices. I extend the benchmark model in Section 2.7, where the real exchange rates is volatile and prices are sticky. I discuss nominal exchange rate pegs and the tetralemma in Section 2.8. Section 3 explains the theoretical foundations within a large model class, including heterogeneous agents incomplete markets and OLG models. Section 4 provides some concluding remarks. Most derivations and proofs are relegated to the appendix.

2 Model

In this section, I explain the approach to exchange rate determination and how it is related to portfolio choices, using a simple (partially linearized) OLG model. Households can invest in home and foreign nominal bonds, two assets which have endogenously different returns due to different stochastic prices across countries. To ensure a well-defined portfolio, I also assume that investing in the foreign bonds is subject to a (small) transaction cost.[11] This results in a trade-off - foreign bonds feature more attractive insurance properties than domestic (home) bonds, but are subject to a transaction cost, unlike domestic bonds - and households in both countries are willing to hold positive amounts of bonds of both countries.

[9]Clarida (1990), Willen (2004) and Mendoza et al. (2009) were the first among many other contributions to integrating the Bewley-Imrohoroglu-Huggett-Aiyagari incomplete markets model into an open economy model, and to show that this model class helps us to understand global capital flows and trade imbalances. Here, I use this same type of model and show that this model class, in addition to its well-documented appealing quantitative predictions, provides an additional benefit over complete markets models, namely a new approach to nominal exchange rate determinacy.

[10]Kollmann (2012) and Coeurdacier et al. (2011) use a different class of incomplete markets models - limited participation in asset markets - to address the Kollmann-Backus-Smith Consumption-Real Exchange rate anomaly. Corsetti et al. (2008) address the same anomaly in a model with internationally incomplete, but nationally complete markets.

[11]An alternative and possibly appealing alternative would be to allow for a risk of default when investing abroad. This would however add some history-dependence to the model which is interesting but would render the model analytically intractable.

2.1 Open Economy OLG Model

The world economy consists of two countries, (H)ome and (F)oreign, in which at each point of time t two generations, (y)oung and (o)ld, are alive. The state of the home country is $s_t^H = s_t \in \mathcal{N}(0, \sigma^2)$ and independent over time. The state of the foreign country $s_t^F = -s_t$ is perfectly negatively correlated with the home state, which in the symmetric benchmark below is equivalent to the assumption of no world risk. There is a single good such that the law of one price implies a real exchange rate equal to one. This assumption, which I relax in Section 2.7, ensures that movements in the nominal exchange rate are not driven by movements in the real exchange rate. The nominal exchange rate is the domestic price of foreign currency such that an increase is a depreciation,

$$\epsilon_s = \frac{q_s^F}{q_s^H}, \tag{1}$$

where $q_{H,s}(q_{F,s})$ is the inverse of the price level in country $H(F)$ in state s. I consider a cashless economy (Woodford, 2003) where monetary policy in each country H and F sets nominal interest rates i^H and i^F respectively. Fiscal policy sets nominal bonds B^H, B^F (denominated in their own currency), operates a social security system and sets taxes such that the government budget constraints hold in all states of the world.

The real endowment of the young generation in countries H and F is

$$y_{s_t}^H = y^H + \kappa_y^H s_t^H = y^H + \kappa_y s_t \quad \text{and} \quad y_{s_t}^F = y^F + \kappa_y^F s_t^F = y^F - \kappa_y s_t, \tag{2}$$

with the same cyclical component, $\kappa_y^H = \kappa_y^F = \kappa_y$, such that the Period t budget constraint for young home households equals

$$c_{y,s_t}^H + A_{H,s_t}^H q_{s_t}^H + A_{F,s_t}^H q_{s_t}^F \leq y^H + \kappa_y s_t - T_{s_t}^H, \tag{3}$$

where household consumption is $c_{y,s}^H$, $A_{H,s}^H \geq 0$ are home and $A_{F,s}^H \geq 0$ are foreign nominal government bond holdings of the home young generation. The real value of acquired home bonds is $A_{H,s}^H q_s^H$ and since there is only one good, the real value of acquired foreign bonds is $(A_{F,s}^H \epsilon_s) q_s^H = A_{F,s}^H q_s^F$. Households have to pay real taxes T_s^H which are used to cover government interest rate expenditures, so that at equilibrium in each state s, $T_s^H = i^H B^H q_s^H$.[12] The Period

[12]Tractability requires eliminating any history-dependence so that young households' decision problem depends on the current state s_t only and not on previous states s_{t-1}. I therefore assume (implicitly) that the social security contributions of the young generation are constant and that government expenditure is adjusted to balance the social security budget. Household endowment is thus net of the constant social security contributions

t budget constraint for young foreign households equals

$$c^F_{y,s_t} + A^F_{F,s_t} q^F_{s_t} + A^F_{H,s_t} q^H_{s_t} \leq y^F - \kappa_y s_t - T^F_{s_t}, \tag{4}$$

where $A^F_{F,s} \geq 0$ are foreign and $A^F_{H,s} \geq 0$ are home bond holdings of the foreign young generation, $c^F_{y,s}$ is consumption of the young, and T^F_s are taxes imposed on foreign households to cover the interest rate payments on foreign bonds, $T^F_s = B^F i^F q^F_s$.[13] Domestic consumption when old in Period $t+1$, c^H_{o,s_{t+1},s_t}, equals

$$c^H_{o,s_{t+1},s_t} = y^H_{o,s_{t+1},s_t} + (1+i^H) A^H_{H,s_t} q^H_{s_{t+1}} + (1+i^F)(1-\chi) A^H_{F,s_t} q^F_{s_{t+1}}, \tag{5}$$

where old age income, $y^H_{o,s_{t+1},s_t} = y^H_o + \kappa_o s_{t+1} + \kappa_s s_t$, is the sum of social security benefits $\kappa_s s_t$ (linked to the previous period's income state s_t) and labor income $y^H_o + \kappa_o s_{t+1}$.[14] The transaction cost for investing in the foreign country is χ. The constraint $A^H_{F,s_t} \geq 0$ implies that this is indeed a cost and not a subsidy for short-selling foreign bonds. I will only consider parametrizations where the short-selling constraint $A^H_{F,s_t} \geq 0$ is not binding around $s_t = 0$ so that households hold diversified portfolios and linearizing around $s_t = 0$ is appropriate. Foreign consumption when old, c^F_{o,s_{t+1},s_t}, thus equals

$$c^F_{o,s_{t+1},s_t} = y^F_{o,s_{t+1},s_t} + (1+i^F) A^F_{F,s_t} q^F_{s_{t+1}} + (1+i^H)(1-\chi) A^F_{H,s_t} q^H_{s_{t+1}}, \tag{6}$$

for old age income $y^F_{o,s_{t+1},s_t} = y^F_o - \kappa_o s_{t+1} - \kappa_s s_t$ and where the transaction cost χ is now on home bonds (the foreign bonds for foreign investors) and foreign bonds feature zero transaction costs. The young generation in period t derives utility

$$u(c_{y,t}) + E_t u(c_{o,t+1}), \tag{7}$$

where $u(c) = -exp(-\gamma c)$.

I consider linear approximations of prices and portfolio choices, implying that period t prices

and net of a constant tax payment.

[13]Fiscal policy is passive here while the Fiscal theory of the price level (FTPL), developed by Sargent and Wallace (1981) , Leeper (1991), Sims (1994, 1997), Woodford (1995, 1997, 1998a,b), Dupor (2000) and Cochrane (1999, 2001, 2005), requires an active policy (Leeper, 1991). Passive fiscal policy basically means that fiscal policy is assumed to balance the government budget for all prices. For a detailed discussion of the differences between my theory and the FTPL see Hagedorn (2019).

[14]Linking old age income to the previous period's state adds some persistence which, as will become clear below, helps when quantifying the model but at the same time maintains independence of shocks across time. However, this modeling choice of income is irrelevant for the theoretical results.

are linear in $s = s_t$,

$$\begin{aligned} q_s^H &= \bar{q}^H + \lambda^H s, & (8) \\ q_s^F &= \bar{q}^F + \lambda^F s^F = \bar{q}^F - \lambda^F s, & (9) \end{aligned}$$

and that both q_s^H and q_s^F are normally distributed. The exchange rate then equals

$$\epsilon_s = \frac{q_s^F}{q_s^H} = \frac{\bar{q}^F - \lambda^F s}{\bar{q}^H + \lambda^H s}. \quad (10)$$

The utility function therefore simplifies to

$$u(c_{y,s_t}^H) - e^{-\gamma \mu_{s_t}^H + \frac{(\gamma \sigma \Sigma_{s_t}^H)^2}{2}}, \quad (11)$$

where

$$\mu_{s_t}^H = y_o^H + \kappa_s s_t + (1 + i^H) A_{H,s_t}^H \bar{q}^H + (1 + i^F)(1 - \chi) A_{F,s_t}^H \bar{q}^F \quad (12)$$

is the mean and

$$\sigma \Sigma_{s_t}^H = \sigma \big[\kappa_o + (1 + i^H) A_{H,s_t}^H \lambda^H - (1 + i^F)(1 - \chi) A_{F,s_t}^H \lambda^F \big] \quad (13)$$

is the standard deviation of old age home consumption so that old age consumption can be written as[15]

$$c_{o,s_{t+1},s_t}^H = \mu_{s_t}^H + \Sigma_{s_t}^H s_{t+1}.$$

Similarly, foreign young households expected utility equals

$$\begin{aligned} & & u(c_{y,s_t}^F) - e^{-\gamma \mu_{s_t}^F + \frac{(\gamma \sigma \Sigma_{s_t}^F)^2}{2}}, & \quad (14) \\ \text{where} \quad \mu_{s_t}^F &=& y_o^H - \kappa_s s_t + (1 + i^F) A_{F,s_t}^F \bar{q}^F + (1 + i^H)(1 - \chi) A_{H,s_t}^F \bar{q}^H, & \quad (15) \\ \sigma \Sigma_{s_t}^F &=& \sigma \big[\kappa_o + (1 + i^F) A_{F,s_t}^F \lambda^F - (1 + i^H)(1 - \chi) A_{H,s_t}^F \lambda^H \big] & \quad (16) \end{aligned}$$

are the mean and the standard deviation of old-age foreign consumption.

[15]To be precise, the standard deviation is the absolute value of (13).

2.2 Portfolio Choice and Exchange Rates

The first-order condition for home bonds acquired by home households, $A^H_{H,s}$, is then[16]

$$\begin{aligned} & (\bar{q}^H + \lambda^H s_t) \qquad (17) \\ = \; & E[(e^{-\gamma(c^H_{o,s_{t+1},s_t} - c^H_{y,s_t})})(1+i^H)(\bar{q}^H + \lambda^H s_{t+1})] \\ = \; & E[(e^{-\gamma(c^H_{o,s_{t+1},s_t} - c^H_{y,s_t})})](1+i^H)\bar{q}^H + Cov[e^{-\gamma(c^H_{o,s_{t+1},s_t} - c^H_{y,s_t})}, (1+i^H)\lambda^H s_{t+1}] \\ = \; & \underbrace{[e^{-\gamma(\mu^H_{s_t} - c^H_{y,s_t}) + \frac{(\gamma\sigma\Sigma^H_{s_t})^2}{2}}]}_{\text{E(SDF)}} \underbrace{[(1+i^H)\bar{q}^H]}_{\text{E(Payoff)}} - \underbrace{[e^{-\gamma(\mu^H_{s_t} - c^H_{y,s_t}) + \frac{(\gamma\sigma\Sigma^H_{s_t})^2}{2}}](1+i^H)\lambda^H\gamma\sigma^2\Sigma^H_{s_t}}_{\text{Cov(SDF, Payoff)}}, \end{aligned}$$

which delivers the standard decomposition into the expected stochastic discount factor, $E(SDF)$, the expected payoff, $E(Payoff)$, and a covariance term. Since prices and portfolios are linear, the first-order condition needs to be approximated. As will become clear below, a linear approximation with respect to s_t is sufficient for determining portfolio choices:[17]

Home Investors investing in Home Bonds A^H_{H,s_t}:

$$\underbrace{\bar{q}^H + \lambda^H s_t}_{\text{Price}} = \underbrace{[\bar{m}^H + \hat{m}^H s_t]}_{\text{E(SDF)}} \underbrace{(1+i^H)\bar{q}^H}_{\text{E(Payoff)}} \underbrace{-\lambda^H\gamma\sigma^2(1+i^H)[(\bar{m}^H + \hat{m}^H s_t)\bar{\Sigma}^H + \bar{m}^H\hat{\Sigma}^H s_t]}_{+\text{ Cov(SDF, Payoff)}}, \quad (18)$$

where the expected stochastic discount factor (SDF) is approximated as

$$e^{-\gamma(\mu^H_{s_t} - c^H_{y,s_t}) + \frac{(\gamma\sigma\Sigma^H_{s_t})^2}{2}} \approx \bar{m}^H + \hat{m}^H s_t =: m^H_{s_t}, \quad (19)$$

$\Sigma^H_{s_t}$ and $\Sigma^F_{s_t}$ are approximated as

$$\Sigma^H_{s_t} \approx \bar{\Sigma}^H + \hat{\Sigma}^H s_t \qquad \text{and} \qquad \Sigma^F_{s_t} \approx \bar{\Sigma}^F - \hat{\Sigma}^F s_t, \quad (20)$$

[16]Note that for a lognormal distribution $X = exp(s)$ with mean 0 and variance σ^2 and coefficients c_0, c_1, c_2,

$$E[e^{c_0 + c_1 log(X)} c_2 log(X)] = e^{c_0 + \frac{(c_1\sigma)^2}{2}} c_1 c_2 \sigma^2,$$

so that the covariance $Cov(e^{-\gamma(c^H_{o,s_{t+1},s_t})}, s_{t+1})$ equals

$$Cov(e^{-\gamma(\mu^H_{s_t} + \Sigma^H_{s_t} s_{t+1})}, s_{t+1}) = -e^{-\gamma\mu^H_{s_t} + \frac{(\gamma\sigma\Sigma^H_{s_t})^2}{2}} \gamma\sigma^2\Sigma^H_{s_t}.$$

[17]For a variable $x(s)$, $\bar{x} = x(s=0)$ denotes the value of x at $s = 0$ and $\hat{x}$ the deviation, so that $x(s)$ is approximated as $\bar{x} + \hat{x}\log(1+s) \approx \bar{x} + \hat{x}s$.

and the SDF at the point of approximation $s = 0$ equals

$$\bar{m}^H = e^{\gamma(c^H_{y,s=0} - \mu^H_{s=0}) + \frac{(\gamma\sigma\Sigma^H_{s=0})^2}{2}}. \tag{21}$$

The covariance equals

$$-[e^{-\gamma(\mu^H_{s_t} - c^H_{y,s_t}) + \frac{(\gamma\sigma\Sigma^H_{s_t})^2}{2}}](1 + i^H)\lambda^H\gamma\sigma^2\Sigma^H_{s_t} \tag{22}$$

and is approximated as

$$-\lambda^H\gamma\sigma^2(1 + i^H)[(\bar{m}^H + \hat{m}^H s_t)\bar{\Sigma}^H + \bar{m}^H\hat{\Sigma}^H s_t]. \tag{23}$$

Note that future old age uncertainty is fully incorporated and the linearization is w.r.t. the state s_t which is known when young households make their portfolio decisions. Old age consumption in period $t + 1$ depends on s_t, however, since first, social security payments are mechanically linked to s_t and second, period t portfolio choices depend on s_t. Also, $\bar{m}^H + \hat{m}^H s_t$ is the expected SDF, as it incorporates old age uncertainty, but depends on the state of the world s_t when young. The approximation is valid only when s_t is small enough, which I assume to be the case. In particular s_t is small enough so that prices q^H_s and q^F_s are positive.

If households hold a diversified portfolio (which I show below to be true for a sufficiently small χ) the remaining first order conditions are

Home Investors: Foreign Bonds A^H_{F,s_t}

$$(\bar{q}^F - \lambda^F s_t) = (1 + i^F)(1 - \chi)\{[\bar{m}^H + \hat{m}^H s_t]\bar{q}^F + \lambda^F\gamma\sigma^2[(\bar{m}^H + \hat{m}^H s_t)\bar{\Sigma}^H + \bar{m}^H\hat{\Sigma}^H s_t]\} \tag{24}$$

Foreign Investors: Foreign Bonds A^F_{F,s_t}

$$(\bar{q}^F - \lambda^F s_t) = (1 + i^F)\{[\bar{m}^F - \hat{m}^F s_t]\bar{q}^F - \lambda^F\gamma\sigma^2[(\bar{m}^F - \hat{m}^F s_t)\bar{\Sigma}^F - \bar{m}^F\hat{\Sigma}^F s_t]\} \tag{25}$$

Foreign Investors: Home Bonds A^F_{H,s_t}

$$(\bar{q}^H + \lambda^H s_t) = (1 + i^H)(1 - \chi)\{[\bar{m}^F - \hat{m}^F s_t]\bar{q}^H + \lambda^H\gamma\sigma^2[(\bar{m}^F - \hat{m}^F s_t)\bar{\Sigma}^F - \bar{m}^F\hat{\Sigma}^F s_t]\}, \tag{26}$$

where the expected foreign stochastic discount factor is approximated as

$$e^{-\gamma(\mu^F_{s_t} - c^F_{y,s_t}) + \frac{(\gamma\sigma\Sigma^F_{s_t})^2}{2}} \approx \bar{m}^F - \hat{m}^F s_t =: m^F_s \tag{27}$$

and at the point of approximation $s = 0$ equals

$$\bar{m}^F = e^{\gamma(c^F_{y,s=0} - y^F_o) - \gamma\mu^F_{s=0} + \frac{(\gamma\sigma\Sigma^F_{s=0})^2}{2}}. \tag{28}$$

Note that the SDFs m_s^H and m_s^F depend on the endogenous prices $\bar{q}^H, \bar{q}^F, \lambda^H, \lambda^F$, which renders the computation of the full equilibrium - prices and portfolio decisions jointly - a non-linear problem although all first-order conditions are linear in s. To obtain a trend-free exchange rate, monetary policy in both countries is assumed to be identical, $i = i^H = i^F$, but I allow below for temporary monetary shocks. Before computing the equilibrium, I will establish several properties of this new model, in order to highlight its main mechanisms.

2.3 Theoretical Analysis

The portfolio choices of the home investor, the young generation in period t, are approximated around $s_t = 0$ as

$$A^H_{H,s} \approx \bar{A}^H_H + B^H \hat{A}^H_H s, \tag{29}$$

$$A^H_{F,s} \approx \bar{A}^H_F + B^F \hat{A}^H_F s, \tag{30}$$

and similarly for the foreign investor,

$$A^F_{F,s} \approx \bar{A}^F_F - B^F \hat{A}^F_F s, \tag{31}$$

$$A^F_{H,s} \approx \bar{A}^F_H - B^H \hat{A}^F_H s. \tag{32}$$

Using this notation allows me to write the zero- and first-order component of Σ_{H,s_t} as

$$\bar{\Sigma}_H = \kappa_o + (1+i^H)\bar{A}^H_H \lambda^H - (1+i^F)(1-\chi)\bar{A}^H_F \lambda^F, \tag{33}$$

$$\hat{\Sigma}_H = (1+i^H)B^H \hat{A}^H_H \lambda^H - (1+i^F)(1-\chi)B^F \hat{A}^H_F \lambda^F, \tag{34}$$

and the zero- and first-order component of $\Sigma^F_{s_t}$ as

$$\bar{\Sigma}_F = \kappa_o + (1+i^F)\bar{A}^F_F \lambda^F - (1+i^H)(1-\chi)\bar{A}^F_H \lambda^H, \tag{35}$$

$$\hat{\Sigma}_F = (1+i^F)B^F \hat{A}^F_F \lambda^F - (1+i^H)(1-\chi)B^H \hat{A}^F_H \lambda^H. \tag{36}$$

To derive the portfolio choices, I proceed in two steps. First, I consider the zero-order component of the portfolio, the choices $\bar{A}^H_H, \bar{A}^F_H, \bar{A}^F_F, \bar{A}^H_F$, at the point of approximation $s_t = 0$. The standard first-order approximation approach where the non-stochastic steady state is used as the point of approximation cannot be used to compute the zero-order component (for example Coeurdacier, 2009; Devereux and Sutherland, 2010, 2011; Tille and van Wincoop, 2010). A first-order approximation with respect to s_{t+1} would eliminate the covariance term and thus the risk, implying that any portfolio choice would be consistent with equilibrium. This problem is overcome here, since no approximation (for s_{t+1}) is considered, but instead, the full nonlinear solution, so that the four first-order conditions evaluated at $s_t = 0$ are sufficient to solve for the four zero-order components, two for the home investor ($\bar{A}^H_H, \bar{A}^H_F$) and two for the foreign investor ($\bar{A}^F_F, \bar{A}^F_H$).[18]

Result 1. *(Portfolio Choice – zero-order component)*

The zero-order components of the portfolio, $\bar{A}^H_H, \bar{A}^H_F, \bar{A}^F_F, \bar{A}^F_H$ solve

$$\bar{q}^H = \bar{m}^H(1+i^H)\bar{q}^H - \lambda^H\gamma\sigma^2(1+i^H)\bar{m}^H\bar{\Sigma}^H, \tag{37}$$

$$\bar{q}^F = \bar{m}^H(1+i^F)(1-\chi)\bar{q}^F + \lambda^F\gamma\sigma^2(1+i^F)(1-\chi)\bar{m}^H\bar{\Sigma}^H, \tag{38}$$

$$\bar{q}^F = (1+i^F)\bar{m}^F\bar{q}^F - \lambda^F\gamma\sigma^2(1+i^F)\bar{m}^F\bar{\Sigma}^F, \tag{39}$$

$$\bar{q}^H = (1+i^H)(1-\chi)\bar{m}^F\bar{q}^H + \lambda^H\gamma\sigma^2(1+i^H)(1-\chi)\bar{m}^F\bar{\Sigma}^F, \tag{40}$$

where $\bar{m}^H, \bar{m}^F, \bar{\Sigma}^H, \bar{\Sigma}^F$ are defined in (21), (28), (33) and (35).

In the second step, I derive the four first-order components, two for the home investor ($\hat{A}^H_H, \hat{A}^H_F$) and two for the foreign investor ($\hat{A}^F_F, \hat{A}^F_H$). To this end, I use the linear components of the four first-order conditions, which are sufficient to solve for the first-order components.

Result 2. *(Portfolio Choice – first-order component)*

The first-order components of the portfolio, $\hat{A}^H_H, \hat{A}^H_F, \hat{A}^F_F, \hat{A}^F_H$ solve

$$\lambda^H = \hat{m}^H(1+i^H)\bar{q}^H - \lambda^H\gamma\sigma^2(1+i^H)[\hat{m}^H\bar{\Sigma}^H + \bar{m}^H\hat{\Sigma}^H], \tag{41}$$

$$\lambda^F = (1+i^F)(1-\chi)\hat{m}^H\bar{q}^F + \lambda^F\gamma\sigma^2(1+i^F)(1-\chi)[\hat{m}^H\bar{\Sigma}^H + \bar{m}^H\hat{\Sigma}^H], \tag{42}$$

$$\lambda^F = (1+i^F)\hat{m}^F\bar{q}^F - \lambda^F\gamma\sigma^2(1+i^F)[\hat{m}^F\bar{\Sigma}^F + \bar{m}^F\hat{\Sigma}^F], \tag{43}$$

$$\lambda^H = (1+i^H)(1-\chi)\hat{m}^F\bar{q}^H + \lambda^H\gamma\sigma^2(1+i^H)(1-\chi)[\hat{m}^F\bar{\Sigma}^F + \bar{m}^F\hat{\Sigma}^F]. \tag{44}$$

where $\hat{m}^H, \hat{m}^F, \hat{\Sigma}^H, \hat{\Sigma}^F$ are defined in (19), (27) and (34) and (36).

The two previous results establish a partial equilibrium result, which maps prices into portfolio choices. Before moving to the equilibrium results, I first show that autarky - each country holds its own bonds only, $A^H_{H,s} = B^H, A^F_{F,s} = B^F$ - cannot constitute an equilibrium if transactions costs χ are not too high and $\kappa_y \neq \kappa_s$. The role of transaction costs in preventing an autarky

[18] Devereux and Sutherland (2011) use second-order approximations of the same first-order conditions to determine the zero-order component. The assumptions with respect to the utility function and the distribution allow me to fully incorporate risk about s_{t+1} in the second period without any need for approximation beyond linearly approximating prices. Appendix A.II discusses the equivalence of the two approaches.

equilibrium is clear, as $\chi = 1$ would clearly induce such an equilibrium. The reason for the second condition, $\kappa_y \neq \kappa_s$, is related to the result in Constantinides and Duffie (1996), namely that an income process following a random walk ensures an autarkic equilibrium.[19] Since the income of the period t generation is $y^H + \kappa_y s_t$ when being young and $y_o^H + \kappa_o s_{t+1} + \kappa_s s_t$ when old in period $t+1$, $\kappa_y = \kappa_s$ means that the persistence of the s_t shock is one. Home investors would then be willing to hold home bonds only, implying an autarkic equilibrium with risk-free portfolios and a constant exchange rate.[20]

Result 3 (Ruling out Autarky)**.** *If $\kappa_y \neq \kappa_s$, fundamental risk $\sigma > 0$, and for sufficiently small transaction costs χ, autarky*

$$A^H_{H,s} = B^H, \qquad A^F_{F,s} = B^F$$

is **not** *an equilibrium.*

The logic behind this result is simple. The "no-random-walk" assumption $\kappa_y \neq \kappa_s$ implies that $\hat{m}^H \neq 0$, such that the output uncertainty carries over to prices, and bonds are risky even in autarky. The riskiness in bond returns induces an incentive to deviate from autarky and to diversify through holding both home and foreign bonds. This diversification strategy enhances utility if the transaction cost χ is not too high and the foreign bond is sufficiently cheap relative to the home bond.

However, the diversified portfolio does not eliminate all the risk but balances the higher mean return (from home bonds) and the lower variance (from foreign bonds), implying a non-constant volatility $\Sigma_s \neq 0$, a non-constant SDF and a non-constant exchange rate. In a symmetric world a particularly simple characterization of this risk is available (the appendix provides the asymmetric case). A symmetric world is defined as two identical countries but where the states are perfectly negatively correlated, $s^F = -s^H$, so that $B = B^F = B^H$, $\bar{q} = \bar{q}^H = \bar{q}^F$, $\lambda = \lambda^H = \lambda^F$, $\bar{m} = \bar{m}^H = \bar{m}^F$, $\hat{m} = \hat{m}^H = \hat{m}^F$, $\bar{\Sigma} = \bar{\Sigma}^H = \bar{\Sigma}^F$ and $\hat{\Sigma} = \hat{\Sigma}^H = \hat{\Sigma}^F$.

Result 4. *(Presence of Risk - symmetric world)*

$$\textbf{Portfolio Volatility:} \quad \bar{\Sigma} + s\hat{\Sigma} = \frac{\bar{q}\chi}{(2-\chi)\lambda\gamma\sigma^2} - s\frac{4(1-\chi)}{\gamma\sigma^2(2-\chi)^2} \neq 0$$

$$\textbf{SDF:} \quad \bar{m} + s\hat{m} = \frac{1}{1+i}\frac{2-\chi}{2(1-\chi)} - s\frac{\lambda\chi}{2\bar{q}(1-\chi)(1+i)} \neq \frac{1}{1+i}$$

$$\textbf{Exchange Rate Volatility:} \quad Var[\bar{\epsilon} + s\hat{\epsilon}] = \sigma^2(2\frac{\lambda}{\bar{q}})^2 > 0$$

[19]See also Storesletten et al. (2007), Heathcote et al. (2014) and Krebs (2003) for a similar setup.

[20]In the OLG model the cross-sectional income dispersion within a generation is zero such that no further adjustments of the income process as in the incomplete markets model in Constantinides and Duffie (1996) are necessary to ensure an autarkic equilibrium.

The nominal risk renders all assets risky so that the SDF at $s = 0$

$$\bar{m} = \frac{1}{1+i}\frac{2-\chi}{2(1-\chi)} > \frac{1}{1+i} \tag{45}$$

is larger than $1/(1+i)$, echoing the well-known result that the interest rate is lower in incomplete market models with precautionary savings than if markets were complete and allowed to fully insure against aggregate country risk.

The source of the state-contingency in the portfolio is exchange rate risk. To reduce this risk, households also invest in foreign bonds, such that at the margin, the marginal gain from lower risk is balanced with the transaction costs. Is an equilibrium without exchange rate risk possible? No, since then home bonds would return-dominate the foreign bonds and portfolios would be fully home-biased. But Result 3 shows that in this case, prices are not constant in equilibrium, contradicting the assumption that there is no exchange rate risk, ruling out a fully home-biased portfolio as an equilibrium outcome.

The results so far are partial equilibrium, describing the mapping from prices to portfolio choices, but not yet imposing asset market clearing. A stationary equilibrium entails state-contingent portfolio choices and prices for the two countries such that optimal portfolio choices are as characterized in Results 1 and 2 and asset markets clear for all states s:

Asset market clearing

Home bond market: $$\underbrace{(\bar{A}^H_H + B^H \hat{A}^H_H s)}_{\text{Home demand}} + \underbrace{(\bar{A}^F_H - B^H \hat{A}^F_H s)}_{\text{Foreign demand}} = \underbrace{B^H}_{\text{Supply}} \tag{46}$$

Foreign bond market: $$\underbrace{(\bar{A}^H_F + B^F \hat{A}^H_F s)}_{\text{Home demand}} + \underbrace{(\bar{A}^F_F - B^F \hat{A}^F_F s)}_{\text{Foreign demand}} = \underbrace{B^F}_{\text{Supply}} \tag{47}$$

While the non-linearity of $\bar{m}$ and $\bar{\Sigma}$ prevents solving for the equilibrium $\bar{q}$ and λ explicitly, the portfolio choices, taking this asset market clearing into account, can be solved for. The choice of the zero-order components in state $s = 0$ is characterized through the trade-off between a higher expected return of home bonds because of transaction costs, and a lower riskiness of foreign bonds relative to home bonds.

Evaluating the first-order conditions (18) and (24) at $s = 0$ and solving for $\bar{A}^H_H$ as a function of $\bar{A}^H_F$ reveals this trade-off,

$$\bar{A}^H_H \lambda^H = (1-\chi)\bar{A}^H_F \lambda^F + \frac{\kappa_o}{(1+i)} + \frac{\chi}{(2-\chi)(1+i)\gamma\sigma^2}\frac{\bar{q}^H}{\lambda^H}.$$

To understand this, consider the alternative portfolio choice $\bar{A}^H_H \lambda^H = (1-\chi)\bar{A}^H_F \lambda^F + \frac{\kappa_o}{(1+i)}$, which would eliminate all old-age consumption risk, such that both home and foreign bonds are (equally) riskless. This is not an equilibrium though, since home bonds return-dominate foreign bonds, due to transaction costs. Due to these transaction costs an equilibrium requires home bonds to be risky and riskier than foreign bonds. If the price of the home bond increases in a boom, $\lambda^H > 0$ (as in the numerical analysis below), increasing the portfolio share of home bonds increases their riskiness. So see this, note that increasing the share of home bonds increases the correlation of old-age consumption with the return on home bonds, rendering home bonds riskier. The return-risk trade-off thus implies a higher home bond share than $\bar{A}^H_H \lambda^H = (1-\chi)\bar{A}^H_F \lambda^F + \frac{\kappa_o}{(1+i)}$. Therefore the last term in (48) is added, which corrects the portfolio shares such that the higher return of home bonds is balanced with higher risk.

For the cyclical component, I use again that the price of the home bond increases in a boom, $\lambda^H > 0$. The higher price leads to a reduction in home investors' exposure to home bonds in booms, $\hat{A}^H_H < 0$, implying a reduction in home bonds' riskiness and thus a lower risk premium, using the same arguments as above. Asset market clearing then implies that foreign investors acquire more home bonds, that is $\hat{A}^F_H < 0$.[21] By symmetry, the price of foreign bonds falls in a home boom, requiring their risk premium to increase. For home investors, foreign bonds are a means of diversification. Buying fewer foreign bonds, $\hat{A}^H_F < 0$, thus increases their riskiness as an equilibrium requires. Asset market clearing then again implies that foreign investors increase their exposure, $\hat{A}^F_F < 0$. Finally symmetry implies that $\hat{A}^H_H = \hat{A}^F_F$.

The model also features a neutrality result with respect to the supply of nominal bonds. Let $\bar{q}^H_{sym} = \bar{q}^F_{sym}, \lambda^H_{sym} = \lambda^F_{sym}$ be the equilibrium prices in the symmetric world with the same amount of bonds in both countries, $B = \frac{B^F + B^H}{2}$. Then the nominal prices in the asymmetric case with bonds supplies $B^H \neq B^F$, are $\bar{q}^H = \bar{q}^H_{sym} \frac{(B^F+B^H)/2}{B^H}, \lambda^H = \lambda^H_{sym} \frac{(B^F+B^H)/2}{B^H}, \bar{q}^F = \bar{q}^F_{sym} \frac{(B^F+B^H)/2}{B^F}$ and $\lambda^F = \lambda^F_{sym} \frac{(B^F+B^H)/2}{B^F}$. All real variables, including real portfolio holdings, are the same in the symmetric case, $B^H = B^F$, and in the asymmetric case, $B^H \neq B^F$. For example, $\bar{A}^H_H$ home nominal bond holdings in the symmetric world turn into $\bar{A}^H_H \frac{B^H}{(B^F+B^H)/2}$ in the asymmetric case.

Taking into account this neutrality, that is $\bar{q}^H B^H = \bar{q}^F B^F$ and $\lambda^H B^H = \lambda^F B^F$, and the asset market clearing conditions $\bar{A}^F_H = B^H - \bar{A}^H_H$ and $\bar{A}^H_F = B^F - \bar{A}^F_F$ yields

[21] The sign is negative since the states are perfectly negatively correlated, $s^F = -s^H$.

Result 5. *(Portfolio Choices) In a symmetric world but allowing $B^H \neq B^F$,*

$$\begin{aligned}
\bar{A}_H^H &= \frac{1-\chi}{2-\chi}B^H + \frac{\bar{q}^H\chi}{(\chi-2)^2(1+i)\gamma\sigma^2(\lambda^H)^2} + \frac{\kappa_o}{(\chi-2)\lambda^H(1+i)} = B^H - \bar{A}_H^F \\
\bar{A}_F^F &= \frac{1-\chi}{2-\chi}B^F + \frac{\bar{q}^F\chi}{(\chi-2)^2(1+i)\gamma\sigma^2(\lambda^F)^2} + \frac{\kappa_o}{(\chi-2)\lambda^F(1+i)} = \bar{A}_H^H\frac{B^F}{B^H} = B^F - \bar{A}_H^F \\
B^H\hat{A}_H^H &= \frac{4(\chi-1)}{\gamma\chi(1+i)\lambda^H\sigma^2(\chi-2)^2} = B^H\hat{A}_H^F \\
B^F\hat{A}_F^F &= \frac{4(\chi-1)}{\gamma\chi(1+i)\lambda^F\sigma^2(\chi-2)^2} = B^F\hat{A}_F^H
\end{aligned}$$

The neutrality result is a combination of familiar monetary textbook results and of unrestricted savings and portfolio decisions by households, echoing the results in Backus and Kehoe (1989). Other types of policy intervention break this neutrality, but would require a different fiscal policy.

2.4 Limit portfolio and exchange rate

The equilibrium determination of exchange rates and portfolios combines two partial equilibrium parts. The first is the mapping from exchange rates to asset and portfolio choices. This is standard finance theory. The second new part is the mapping from assets to the exchange rate, which together with the portfolio choices, determines the exchange rate.

To "zoom in" on this mapping from portfolios to exchange rates, I consider the limit economy when both the uncertainty and the transaction cost vanish, $\sigma^2 \to 0, \chi \to 0$. The previous analysis shows that I obtain a well defined equilibrium portfolio choice and an exchange rate for each combination of strictly positive σ and χ, implying well-defined limits

$$\begin{gathered}
\lim_{\sigma,\chi\to 0} \epsilon_{s=0}(\sigma,\chi) = \epsilon \\
\lim_{\sigma,\chi\to 0} \bar{q}^H(\sigma,\chi)\bar{A}_H^H(\sigma,\chi) = S_H^H \qquad \lim_{\sigma,\chi\to 0} \bar{q}^F\bar{A}_F^H(\sigma,\chi) = S_F^H \\
\lim_{\sigma,\chi\to 0} \bar{q}^F\bar{A}_F^F(\sigma,\chi) = S_F^F \qquad \lim_{\sigma,\chi\to 0} \bar{q}^H\bar{A}_H^F(\sigma,\chi) = S_H^F
\end{gathered} \tag{48}$$

The demand of home households for home real bonds converges to S_H^H and for foreign real bonds to S_F^H . The demand of foreign households for foreign real bonds converges to S_F^F and for home real bonds to S_H^F. The limit real asset demand in the home and the foreign country are $S^H = S_H^H + S_F^H$ and $S^F = S_F^F + S_H^F$ respectively. Similar to Judd and Guu (2001), considering the limit of vanishing uncertainty and transaction costs delivers the zero-order component of the portfolio as well as of prices.

Considering two special cases is instructive. First, without any fundamental uncertainty but with positive transaction costs, each country is in autarchy, so that it only holds its own bonds and the exchange rate is determined to be one:

Result 6. *(No Fundamental Risk) Without fundamental risk,* $\sigma \equiv 0$*, each country is in autarchy and the exchange rate is constant,*

$$A^H_{H,s} = B^H, \quad A^F_{F,s} = B^F, \quad \epsilon_s = 1. \tag{49}$$

I can also derive the limit of vanishing transaction costs, $\chi \to 0$, while keeping fundamental risk unchanged. Somewhat surprisingly, the limit results are the same independent of whether in addition fundamental risk vanishes or not. The explanation is simple though. In equilibrium the transaction cost advantage of home bonds must be exactly balanced by the risk advantage of foreign bonds. The transaction cost advantage vanishes if $\chi \to 0$ so that in equilibrium prices and portfolios adjust such that the risk advantage also vanishes, independently of the amount of fundamental risk present.[22] As a result the limit does not depend on the fundamental risk:

Result 7. *(Vanishing Transaction Costs) The limit for vanishing transactions costs* $(\chi \to 0)$ *coincides with the limit limit of vanishing uncertainty and transactions costs* $(\chi \to 0, \sigma \to 0)$*, which is characterized in Result 8.*

The limit of vanishing uncertainty and transaction costs can be characterized precisely:

Result 8. *(Limit of Vanishing Uncertainty & Transactions Costs)*
In a symmetric world, but allowing $B^F \neq B^H$ *the limits are:*

Limit Nominal Portfolios

$$\lim_{\sigma,\chi \to 0} \bar{A}^H_H(\sigma, \chi) = \lim_{\sigma,\chi \to 0} \bar{A}^F_H(\sigma, \chi) = \frac{B^H}{2}, \tag{50}$$

$$\lim_{\sigma,\chi \to 0} \bar{A}^F_F(\sigma, \chi) = \lim_{\sigma,\chi \to 0} \bar{A}^H_F(\sigma, \chi) = \frac{B^F}{2}. \tag{51}$$

[22] Note, that the volatility of the exchange rate does not vanish but *increases* when transaction costs vanish (Result 4).

Limit Prices and Exchange Rate

$$\lim_{\sigma,\chi\to 0} \bar{q}^H(\sigma,\chi) = \frac{\gamma(y-y_o) - ln(1/(1+i))}{2\gamma B^H(1+i)}, \tag{52}$$

$$\lim_{\sigma,\chi\to 0} \bar{q}^F(\sigma,\chi) = \frac{\gamma(y-y_o) - ln(1/(1+i))}{2\gamma B^F(1+i)}, \tag{53}$$

$$\lim_{\sigma,\chi\to 0} \epsilon(\sigma,\chi) = \frac{B^H}{B^F}. \tag{54}$$

Limit Real Portfolios

$$S_H^H = S_F^H = \frac{\gamma(y-y_o) - ln(1/(1+i))}{4\gamma(1+i)}, \tag{55}$$

$$S_F^F = S_H^F = \frac{\gamma(y-y_o) - ln(1/(1+i))}{4\gamma(1+i)}. \tag{56}$$

At the limit, when all differences between countries have vanished, home and foreign investors hold equal amounts of bonds of both countries. The demand for home bonds is increasing in the income difference of the young and the old generation, $y - y_o$, such that asset market clearing requires the price of the bond to increase. Note that since $\bar{q}^H$ are $\bar{q}^F$ are the inverses of price levels, a higher output level thus leads to a lower price level, as in New Keynesian models. The bond price $\bar{q}^H$ is also inversely related to the amount of bonds B^H, echoing the neutrality result discussed above. Again, while nominal variables change, the real value of assets, $\bar{q}^H \bar{A}_H^H$ and $\bar{q}^H \bar{A}_H^F$, is invariant to changes in B^H. The identical neutrality result holds for foreign assets. Result 8 also shows that the home country can engineer a depreciation through expanding the amount of home bonds, and an appreciation through a contraction of home bond supply. Such policy measures leave the real exchange rate unaffected, as both the nominal exchange rate and the price level increase by x percent if B^H increases by x percent.

2.5 Monetary Policy and Exchange Rate Pegs

In this Section I establish two results. First, I extend Result 3, which shows that autarky is not an equilibrium when the nominal interest rate is constant. I now allow the nominal interest to respond to fundamental shocks,

$$i_s^H = \bar{i} + \phi_i s; \qquad i_s^F = \bar{i} - \phi_i s \tag{57}$$

and establish that autarky is also not an equilibrium in this richer environment. Second, I establish a main result of this paper on the inability of monetary policy to implement an exchange rate peg.

To understand the no-autarky result recall the reasoning behind Result 3 for constant nominal interest rates. Since fundamental output uncertainty would also be present in autarky, equilibrium prices would not be constant in autarky, rendering both home and foreign bonds risky. Home households then deviate from autarky to diversify their portfolio and they hold both home and foreign bonds, showing that autarky is not an equilibrium.

For an interest rate rule like (57) to induce autarky it is thus necessary to neutralize the effect of fundamental uncertainty on prices such that prices are constant. If prices are constant, positive transaction costs imply that home households do not hold any foreign bonds, which are return dominated by home bonds and provide no insurance. If the home country was a closed economy, implementing a constant price through monetary policy is possible. For example, a negative home output shock leads to lower asset demand, requiring a decrease in the price of home bonds or equivalently an increase in the home price level. To keep the price constant, the nominal interest rate has to be raised. This interest rate increase by itself raises the demand for home bonds and induces an increase in the price of home bonds or equivalently a decrease in the home price level. Based on this mechanism it is clear that the nominal interest rate can be adjusted so as to keep prices unchanged. But the home and the foreign country are not closed economies. Showing that autarky is an equilibrium would in addition require that foreign investors do not want to invest in home bonds. In the above example this is not the case since the rise in home nominal interest rates makes investing in home bonds attractive for foreign investors. The proof in the appendix extends this intuition and establishes the no-autarky result in my model:[23]

Result 9 (Ruling out Autarky with Monetary Policy)**.** *If $\kappa_y > \kappa_s$, fundamental risk $\sigma > 0$, and for sufficiently small χ and κ_o, autarky*

$$A^H_{H,s} = B^H, \qquad A^F_{F,s} = B^F$$

is **not** *an equilibrium for any interest rate rule*

$$i^H_s = \bar{i} + \phi_i s; \qquad i^F_s = \bar{i} - \phi_i s. \tag{58}$$

[23]The assumption $\kappa_y > \kappa_s$ rules out a random walk by requiring the income process to be less persistent than a unit root, which allows me to theoretically determine the sign of the price responses. The assumption that κ_o is small allows me to put bounds on the covariance of old age consumption and the price q.

The next result builds on the no-autarky intuition to show that an interest rate rule (57) cannot be used to implement an exchange rate peg. The key step in establishing this result is to show that any equilibrium features risky prices and exchange rates or equivalently that an equilibrium with a constant exchange rate does not constitute an equilibrium for any specification of monetary policy.

This is an important conceptual difference between my paper and a large literature that uses interest rate rules to ensure a determinate exchange rate. In this literature there are potentially many equilibria if the interest rate is constant, that is the economy is indeterminate. The objective is then to find an interest rate rule that renders the economy determinate. The seminal paper in this literature, Benigno et al. (2007), shows that a clever choice of the interest rate rule, $i_t^H = i_t^F + \phi(\epsilon_t/\epsilon^* - 1)$ with $\phi > 0$, not only ensures determinacy but also allows the home country to maintain a fixed and determinate exchange rate ϵ^*. The home central bank sets the interest rate in response to the foreign nominal interest rate i_t^F and the exchange rate such that a constant exchange rate is the only equilibrium and all other equilibria are eliminated.

In my paper the issue is different. For any choice of monetary policy including a constant nominal interest rate, the economy is determinate and there is a unique equilibrium with a unique exchange rate. The policy question is then not whether a constant exchange rate is the unique equilibrium but whether a constant exchange rate is an equilibrium at all. I show in this Section that it is not. All equilibria feature a volatile exchange rate no matter what the interest rate rule is.

This difference also explains why the interest rate rules (57) do not include endogenous variables such as the exchange rate. The reason is that the exchange rate ϵ_s and all other endogenous variables are a function of the state s in my environment. Using an interest rate rule as in Benigno et al. (2007) which includes the exchange rate in my model is thus equivalent to an interest rate rule which only depends on the state s. One just has to write the endogenous variable as a function of the state s and then for example replace ϵ_t with $\bar{\epsilon} + s_t\hat{\epsilon}$. This is not possible in Benigno et al. (2007) since this model features indeterminacy for exogenous interest rate rules so that the exchange rate cannot be written as a function of the state s. Instead, for each realization of the state s, there are potentially infinitely many exchange rates. Appendix A.III explains the logic underlying the determinacy result in Benigno et al. (2007) and provides further intuition why their results do not carry over to my model with an endogenous risk premium.

The proof and the intuition proceed in three steps. To prove by contradiction, assume an equilibrium with a constant nominal exchange rate exists. The first step is to recognize that a constant exchange rate would render both home and foreign bonds risk-free. Positive transaction costs then imply that home investors prefer home bonds and foreign investors prefer foreign bonds if both bonds pay the same nominal return. Second, this preference ranking is unchanged if the return differential between home and foreign bonds is small such that autarky is the only candidate for an equilibrium in this case. If the return differential is large then world demand fully switches to the high interest rate bonds and the low interest rate bond market does not clear, ruling out this possibility. The third step then shows that the autarkic allocation of step 2 is not an equilibrium, establishing that an equilibrium necessarily features a volatile exchange rate.

Result 10 (Ruling out Exchange Rate Peg with Monetary Policy)**.** *If $\kappa_y \neq \kappa_s$ and fundamental risk $\sigma > 0$, then the exchange rate is not constant for all interest rate rules*

$$i_s^H = \bar{i} + \phi_i s; \qquad i_s^F = \bar{i} - \phi_i s. \tag{59}$$

2.6 Numerical Analysis

While $\bar{m}$ and $\bar{\Sigma}$ do not depend on the state s, they are non-linear functions of price variables $\bar{q}$ and λ, rendering the computation of an equilibrium a non-linear problem. I therefore resort to a numerical analysis to illustrate the workings of the model.

I restrict the analysis to ± 2 standard deviations interval for the state s to ensure that prices are positive. Output $y_H = 100$ and $y_F = 100$, nominal bond supplies $B^H = 10$ and $B^F = 10$ and the standard deviation $\sigma = 1$ is normalized to one. A one standard deviation in s changes output by $\kappa_y = \kappa_y^H = \kappa_y^F = 0.4$ (percent). The simple model has some features which are irrelevant for exchange rate and portfolio determinacy but allow me to ensure that the model is well behaved in spite of linear prices. For example, to increase the correlation of young and old income, I assumed that social security is linked to previous income, implying that the strength of intertemporal substitution is not excessively state-dependent. I set old age non-asset income for home households $12.2 + 0.5\kappa_y s + \kappa_y s_{-1}$ and symmetrically for foreign households $12.2 - 0.5\kappa_y s - \kappa_y s_{-1}$.[24] The nominal interest $i = 0.7$, corresponding to about 2 percent per year for the 30-year model period. The transaction cost $\chi = \chi^H = \chi^F = 0.89$ to match the home bias bond measure in Coeurdacier and Rey (2013), $1 - \frac{\bar{A}_F^H}{B^F/(B^H+B^F)} = 1 - 2\bar{A}_F^H = 0.75$. As shown before, the theoretical results only require $\chi > 0$ and do not rely on χ being large and χ can be arbitrarily small as Result 8 shows. Generating the observed home-bias requires

[24] CARA utility implies that setting the intercept 12.2 is equivalent to setting a discount factor to calibrate the steady state. The remaining parameters are chosen to ensure that the $\pm 2\sigma$ range is consistent with my assumptions, e.g. positive prices.

χ to be large though. For example, a low value of $\chi = 0.01$ would yield a home bias measure of -0.007. The perfect negative correlation of home and foreign states implies that foreign bonds are an excellent hedge for home investors. A large cost χ renders foreign bonds sufficiently unattractive such that home investors prefer home bonds to foreign bonds. The risk aversion parameter $\gamma = 5$ for the old generation (implying a relative risk aversion of about 3) and for numerical reasons, I choose a different risk aversion of 1 for the young generation. Using this parametrization, the unique solution for prices is

$$\bar{q}^H + \lambda^H s = 1 + 0.12s, \tag{60}$$

$$\bar{q}^F - \lambda^F s = 1 - 0.12s. \tag{61}$$

Consistent with Result 8, a home output expansion, $s > 0$, leads to an increase in the home bond price. Higher income leads to higher savings, and because of the home bias, to a higher demand for home bonds. Asset market clearing then requires that the home bond price increases. Panel a) of Figure 2 shows the resulting exchange rate,

$$\epsilon_s = \frac{\bar{q}^F - \lambda^F s}{\bar{q}^H + \lambda^H s} = \frac{1 - 0.12s}{1 + 0.12s}, \tag{62}$$

which appreciates in good (home) states, $s > 0$.

Due to varying exchange rates and risk-premia, the portfolio choices,

$$\bar{A}^H_H + B^H \hat{A}^H_H s = (0.875 - 0.061s)B^H, \tag{63}$$

$$\bar{A}^H_F + B^F \hat{A}^H_F s = (0.125 - 0.061s)B^F, \tag{64}$$

$$\bar{A}^F_F + B^F \hat{A}^F_F s = (0.875 + 0.061s)B^F, \tag{65}$$

$$\bar{A}^F_H + B^H \hat{A}^F_H s = (0.125 + 0.061s)B^H, \tag{66}$$

vary considerably with state s, as illustrated in Panel b) of Figure 2. As characterized in Result 5, an appreciation due to an output increase in the state s decreases home's holding of both home and foreign bonds.

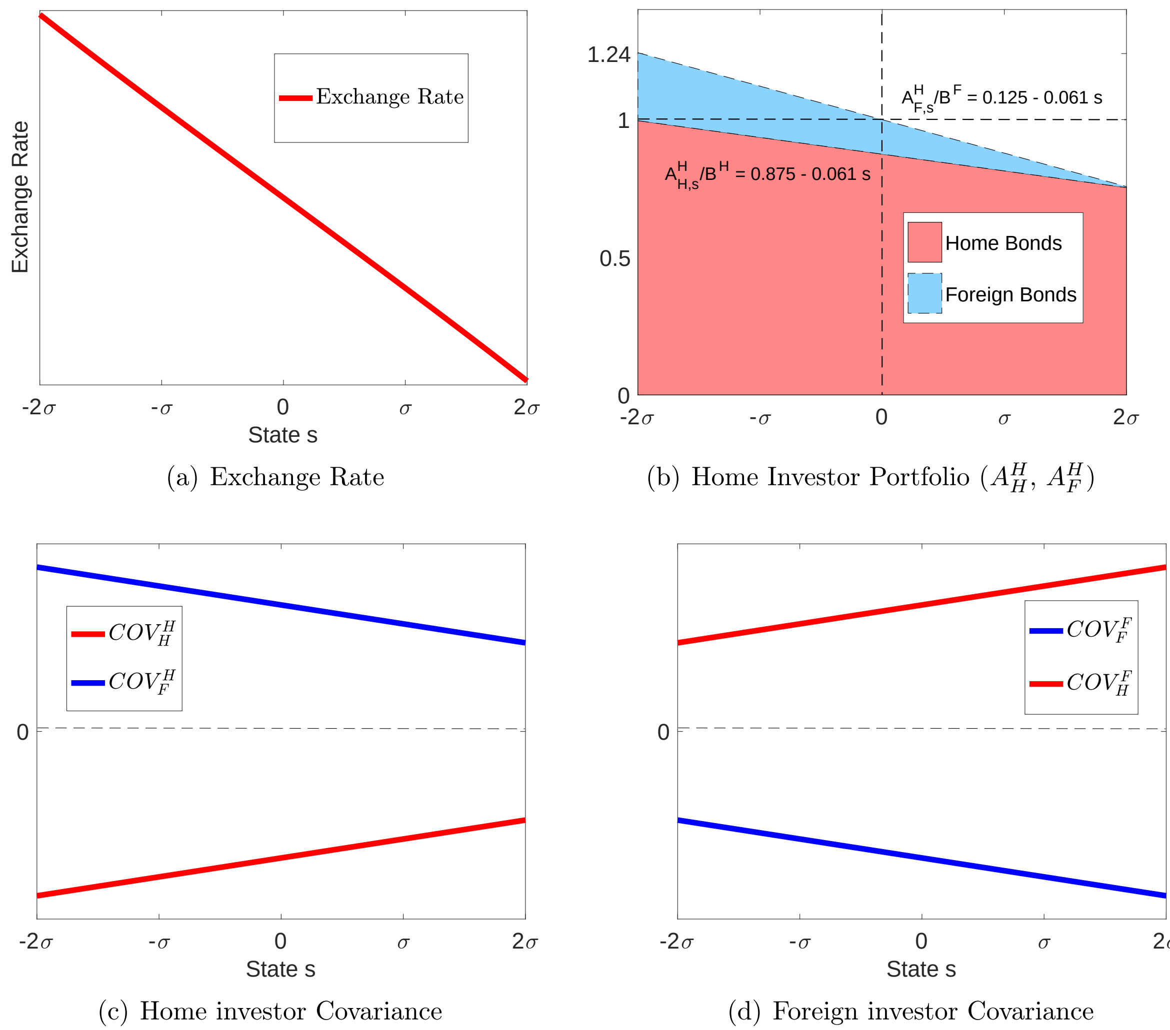


Figure 2: (a) Nominal Exchange Rates; (b) Home Portfolio Choices; (c) Home Investor Covariance: Home Bonds (COV^H_H), Foreign Bonds (COV^H_F); (d) Foreign Investor Covariance: Foreign Bonds (COV^F_F), Home Bonds (COV^F_H)

The appreciation lowers the expected return of home bonds for both home and foreign investors, such that both countries want to sell home bonds and buy foreign bonds. Market clearing, however, rules out all agents being sellers, and requires that the risk properties of home bonds improve and make them a more attractive investment.

For home investors, home bonds are risky - the payoff is positively correlated with consumption, and the covariance term is negative - and selling home bonds reduces their risk. For foreign investors on the other hand home bonds are risk-reducing - the payoff is negatively correlated with consumption and the covariance term is positive - and buying home bonds reduces their risk. As a result, foreign investors buy home bonds and home investors sell them. Panels c) and d) of Figure 2 confirm that this trading strategy reduces the risk-premium of home bonds

- increases the covariance terms COV_H^H and COV_H^F (the red lines) - for both countries.This is the case for the home country since they reduce their exposure to the risky investment and for the foreign country since they increase risk-reducing investments, such that the risk premium of home bonds falls for both countries and makes buying them more attractive for both countries. The same panels also confirm that home bonds require a positive risk premium for home investors as $Cov_H^H < 0$ and a negative one for foreign investors, $Cov_H^F > 0$.

For foreign bonds, the situation is symmetric taking into account that the foreign currency depreciates, which raises the expected return of foreign bonds for home and foreign investors and renders both countries buyers. To ensure market clearing, the risk premium for foreign bonds has to increase in order to reduce the demand. For home investors, foreign bonds are risk reducing, so that selling them increases the risk. For foreign investors, foreign bonds are risky, so that buying them increases the risk. As Panel c) and d) of Figure 2 show, this trade increases the risk-premium - decreasing the covariance terms Cov_F^H and Cov_F^F (the blue lines) - for both countries. The two panels also confirm that foreign bonds require a positive risk premium for foreign investors, $Cov_F^F < 0$ and a negative one for home investors, $Cov_F^H > 0$.

These model properties are consistent with the findings in Molodtsova and Papell (2009) and Engel et al. (2019) that higher home inflation predicts an appreciation during the following period. Indeed, home inflation between period t and $t-1$ is high if q_s^H is low, that is the currency has depreciated, and is thus expected to appreciate between periods t and $t+1$.

Permanent output changes

The effect of permanent changes in output on exchange rates is similar to that for temporary shocks. Indeed, Figure 3 shows that a permanent increase in home output with foreign output unchanged, leads to a permanent appreciation of the home currency. Again, higher output implies a higher demand for home bonds and thus a higher price of home bonds at equilibrium, that is, an appreciation. While these experiments compare the steady-state levels of the exchange rate $\epsilon_{s=0}$ for different values of y^H, productivity still fluctuates, but now around different steady-state values. As explained above, this uncertainty is necessary to ensure well-defined portfolio choices for each steady-state value of y^H and thus a determinate exchange rate.

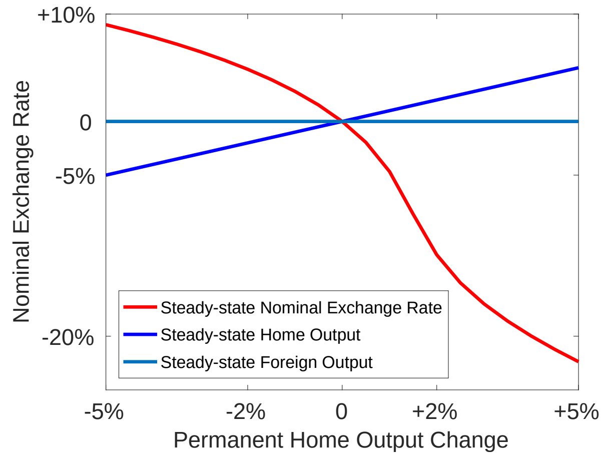
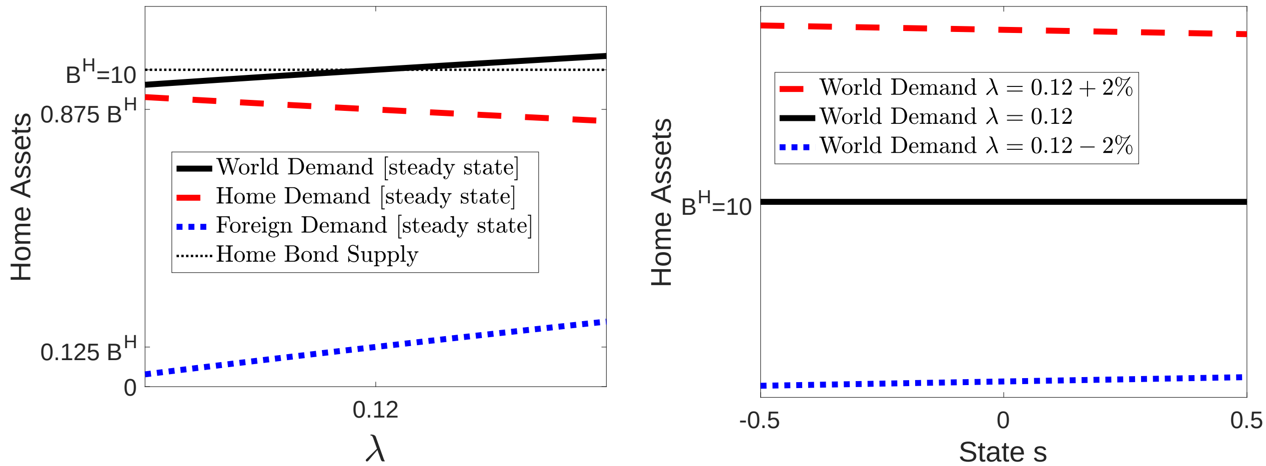


Figure 3: Steady State: Nominal Exchange Rate and Permanent Home Output Change

Price formation

While Figure 2 describes portfolio choices and the behavior of risk premia at equilibrium prices, Figures 4 and 5 aim at understanding equilibrium price formation and visualize demand at non-equilibrium prices. Panel a) of Figure 4 shows world demand $\bar{A}_H^H + \bar{A}_H^F$ for home bonds at $s = 0$ to be an increasing function of $\lambda = \lambda^F = \lambda^H$ while $\bar{q} = \bar{q}^H = \bar{q}^F = 1$ is fixed at its equilibrium value. Both home and foreign households make optimal savings and portfolio decisions taking prices $\bar{q}$ and λ as given but now I do not impose a market clearing condition but instead report world demand, the sum of home, $\bar{A}_H^H$, and foreign demand, $\bar{A}_H^F$.

World demand for home bonds exceeds home bond supply B^H if $\lambda > 0.12$ and falls short of B^H if $\lambda < 0.12$. The explanation hast two parts. First, a higher λ renders payoffs more risky and households save more. This is the standard precautionary savings response in an incomplete markets model with one (non-state contingent) bond. Precautionary savings plus more risk leads to more savings, $\bar{q}\bar{A}_H^H(\lambda) + \bar{q}\bar{A}_F^H(\lambda)$ increases in λ. The same logic operates here but in addition households adjust their portfolios since the risk properties of home and foreign bonds change. A higher λ renders home bonds more risky and foreign bonds less risky for home investors, implying a reduction in their home bond holdings, $\bar{A}_H^H(\underset{(-)}{\lambda})$, and an increase in their foreign bond holdings, $\bar{A}_F^H(\underset{(+)}{\lambda})$. Symmetrically for foreign investors, a higher λ renders foreign bonds more risky and home bonds less risky, implying a reduction in their foreign bond holdings, $\bar{A}_F^F(\underset{(-)}{\lambda})$, and an increase in their home bond holdings, $\bar{A}_H^F(\underset{(+)}{\lambda})$. Combining the portfolio and the precautionary savings arguments shows that the home demand for home bonds falls but by less than their demand for foreign bonds increases. Symmetrically the foreign demand for foreign bonds falls but by less than their demand for home bonds increases. Symmetry - $\bar{A}_H^H(\lambda) = \bar{A}_F^F(\lambda)$ and $\bar{A}_F^H(\lambda) = \bar{A}_H^F(\lambda)$ - and the precautionary savings arguments then imply



(a) Steady state: $\bar{A}_H^H(\lambda) + \bar{A}_H^F(\lambda)$, $\bar{A}_H^H(\lambda)$ and $\bar{A}_H^F(\lambda)$ (b) Cyclical: $\bar{A}_H^H + \bar{A}_H^F + B^H(\hat{A}_H^H + \hat{A}_H^F)\mathbf{s}$

Figure 4: Demand for Home Assets: Varying λ

that $\bar{A}_H^H(\lambda) + \bar{A}_H^F(\lambda) = \bar{A}_H^H(\lambda) + \bar{A}_F^H(\lambda)$ is increasing in λ. Again, by symmetry the world demand for foreign bonds, $\bar{A}_F^F(\lambda) + \bar{A}_F^H(\lambda)$, is increasing in λ.

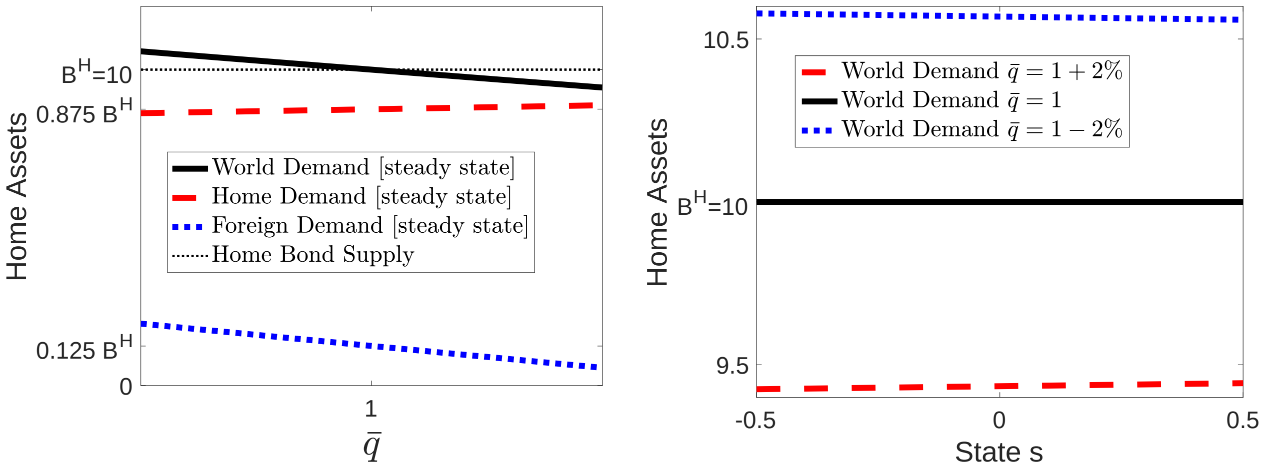
The same arguments explain that world demand for home and foreign bonds increases in λ, uniformly in all states s as panel b) of Figure 4 shows. At the equilibrium price $\lambda = 0.12$ world demand equals world supply of home bonds. Increasing λ by 2% uniformly shifts demand up whereas decreasing λ by 2% uniformly shifts it down.

Figure 5 shows the outcome of the second experiment, which keeps $\lambda = \lambda^F = \lambda^H = 0.12$ fixed at its equilibrium value and instead varies the second price component $\bar{q} = \bar{q}^H = \bar{q}^F$. An increase in the home bond price $\bar{q}$ lowers the risk since $\lambda/\bar{q}$ falls, explaining why home households demand more home bonds, $\bar{A}_H^H(\underset{(+)}{\bar{q}})$. For foreign households, a fall in $\lambda/\bar{q}$ means that home bonds are less risk-reducing, explaining why foreign households demand fewer home bonds in response to a higher $\bar{q}$, $\bar{A}_H^F(\underset{(-)}{\bar{q}})$. The home bias implies that an increase in $\bar{q}$ renders households' portfolios less risky and thus their precautionary savings demand falls, explaining why households save less if $\bar{q}$ increases: both $\bar{A}_H^H(\bar{q}) + \bar{A}_F^H(\bar{q})$ and $\bar{A}_F^F(\bar{q}) + \bar{A}_H^F(\bar{q})$ are decreasing in $\bar{q}$. Symmetry - $\bar{A}_H^F(\bar{q}) = \bar{A}_F^H(\bar{q})$ - and the precautionary savings argument then imply that world demand for home bonds, $\bar{A}_H^H(\bar{q}) + \bar{A}_H^F(\bar{q})$, is also decreasing in $\bar{q}$.

An equilibrium requires combining these partial equilibrium considerations such that the portfolio choices are optimal given prices and exchange rates and at the same time these prices clear all asset markets. As illustrated above, a key equilibrium aspect is that prices, exchange rates and risk premia have to adjust jointly, to ensure market clearing.

Monetary Policy

The model has so far emphasized productivity movements as the driving force. I now



(a) Steady state: $\bar{A}^H_H(\bar{q}) + \bar{A}^F_H(\bar{q})$, $\bar{A}^H_H(\bar{q})$ and $\bar{A}^F_H(\bar{q})$

(b) Cyclical: $\bar{A}^H_H + \bar{A}^F_H + B^H(\hat{A}^H_H + \hat{A}^F_H)\mathbf{s}$

Figure 5: Demand for Home Assets: Varying $\bar{q}$

switch the focus and consider variation in monetary policy, modeled as exogenous fluctuations in nominal interest rates,

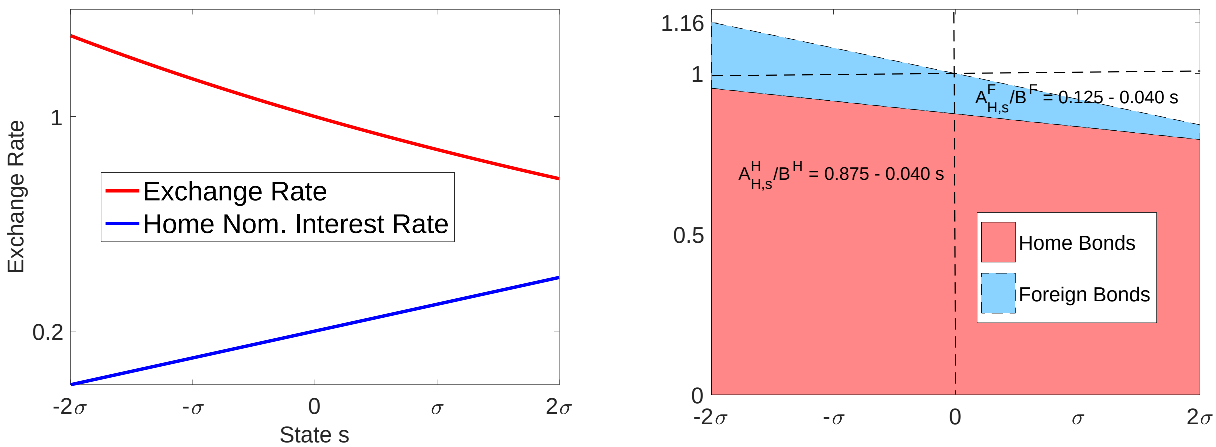
$$i^H_s = \bar{i} + \phi_i s \tag{67}$$

$$i^F_s = \bar{i} - \phi_i s \tag{68}$$

where I maintain, for simplicity, the assumption that $s^F = -s^H$. I assume that all other sources of variation are negligible to focus on the effects of monetary policy, which requires adjusting the parametrization.[25] Note that exchange rates and prices are determined here for constant nominal interest rates or for exogenous interest rate rules.[26] Tractability forbids linking monetary policy to past events as this would add state-variables to the model and would thus complicate it substantially. While this is not relevant for most of the analysis in this paper, it prevents considering realistic dynamics of variables of interest beyond the impact period of the policy shock.

[25] The changed parameter values are $\chi = \chi_H = \chi_F = 0.3388$, $\kappa_y, \kappa_o, \kappa_s \approx 0$, $y^o_H = y^o_F = 0.07 y_H$, $\phi_i = 0.1$ and $\bar{i} = 0.2$.

[26] Since prices are linear functions of the state s, the rules (67) and (68) could be equivalently rewritten as price targeting rules. See Section 2.5 for why determinacy implies that a policy rule written in terms of endogenous variables can be rewritten as a function of state s.



(a) Exchange Rate / Mon. Policy

(b) Home Portfolio (A_H^H, A_H^F)

Figure 6: Portfolio Choices and Nominal Exchange Rates: Monetary Policy

A tightening of home monetary policy - an increase in the nominal interest rate in states $s > 0$ - leads to an appreciation (see panel a) of Figure 6), and the home portfolio is adjusted in the direction of less home and foreign bonds in response to an increase in the nominal interest rate i^H, as panel b) of Figure 6) illustrates. Asset market clearing then requires foreign investors to buy more home and foreign bonds. The associated price of home bonds is $q_s^H = 1 + 0.066s$ and of foreign bonds is $q_s^F = 1 - 0.066s$.

To better understand the underlying mechanism, consider an increase $i^H = 0.3$ and the associated decrease $i^F = 0.1$, that is $s = 1$. As a result, the price for home bonds, q_s^H, increases and the price for foreign bonds, q_s^F, falls, implying an appreciation of the exchange rate. For home investors the nominal return from home bonds is $(1 + i^H)$ and increases by 0.1 and the expected return from foreign bonds is $(1 + i^F)(1 - \chi)\frac{E_t \epsilon_{t+1}}{\epsilon_t}$ and increases by 0.036, suggesting a violation of the interest rate parity condition which can be exploited through investing in home bonds. Similarly for the foreign country the expected return from home bonds is $(1 + i^H)(1 - \chi)E_t \frac{\epsilon_t}{\epsilon_{t+1}}$ and falls by 0.04 and the expected return from foreign bonds is $(1 + i^F)$ and decreases by 0.1 so that home bonds return-dominate foreign bonds for investors in both countries.

As in the case with technological shocks, risk premia have to adjust to ensure market clearing and that the UIP condition holds. Here, the UIP condition requires the home bond risk premium to increase, whereas the risk premium of foreign bonds has to fall. While the risk adjustments in response to technological shocks only occur through portfolio re-balancing, changes in nominal interest rates have an additional direct risk impact. The increase in i^H has a second moment effect, as it increases the variance of the home bond payoff, which is proportional to the interest rate $1 + i^H$. Similarly, the lower interest rate on foreign bonds i^F decreases the variance of the payoff of foreign bonds.

These direct risk adjustments overshoot here, requiring portfolio changes to reduce the risk premium on home bonds. Home investors therefore acquire fewer home bonds, since this decreases their risk premium. For home investors the overall increase in the home bond risk premium is thus the sum of a direct increase induced by a higher i^H and a decrease due to a portfolio adjustment in the direction of fewer home bonds. Symmetrically for foreign investors, the overall increase in the home bond risk premium is the sum of a direct increase induced by a higher i^H and a decrease due to a portfolio adjustments in the direction of more home bonds. Similarly, the risk of foreign bonds decreases since the volatility of their payoff falls due to a lower i^F, which lowers their risk, and home investors hold fewer foreign bonds and foreign investors more of them, which increases their riskiness. The increase in the excess return of home as opposed to foreign bonds is thus compensated for by an increase in the riskiness of home relative to foreign bonds.

Monetary Policy: Exchange Rate Management

While the theoretical Result 10 in Section 2.5 shows that an interest rate rule cannot fully peg the exchange rate, monetary policy can affect the volatility of the exchange rate. Indeed, the volatility of the exchange rate decreases if the monetary policy parameter ϕ_i is lowered as Figure 7 shows. This is expected since a lower ϕ_i means a smaller increase of the nominal interest rate in states $s > 0$. Dampening the nominal interest rate response, a lower ϕ_i, then leads to a smaller appreciation while the appreciation is larger for a stronger nominal interest rate response, a higher ϕ_i. This illustrates that, although fiscal policy is needed to fully peg the exchange rate as I show below, monetary policy is still effective in managing the exchange rate.

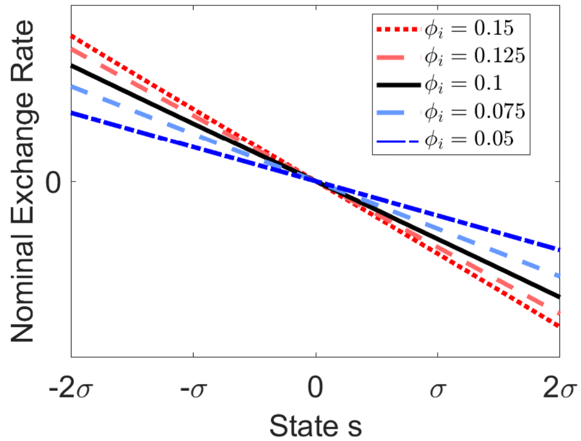


Figure 7: Log Nominal Exchange Rate and Monetary Policy for different interest rate responses $\phi_i = 0.05, 0.075, 0.1, 0.125, 0.15$.

The co-movement of the exchange rate and the interest-rate differential is also consistent with the data. Engel (2016) refines the Fama (1984) finding that a positive interest rate differential is associated with an exchange rate appreciation. Engel (2016) shows that the exchange rate appreciates on impact and eventually depreciates again. This is consistent with the model as Figure 6 shows. The nominal exchange rate appreciates in period t in response to a period t increase in the home nominal interest rate and then depreciates in period $t+1$. Interestingly, this response is conditional on a monetary shock, showing that the model is able to match not only unconditional but also conditional moments in the data. However, the two-period model setup and its simplicity do not enable tracing the full dynamic adjustment path documented in Engel (2016). Achieving this would require a richer model in which adjustment costs in prices or portfolios add slow-moving state variables to the model, such that the adjustment process to shocks is spread out over several periods. However, Hassan and Mano (2018) correct for the uncertainty about future mean interest rates and show that investors expect a high interest rate currency to depreciate, consistent with my model. They also find that the high interest rate currency does not depreciate proportionately to the interest rate differential. The model replicates this finding as the risks of home and foreign bonds adjust endogenously.

Monetary Policy: Permanent Differences

The theory also allows for permanent differences in nominal interest rates as Figure 8 illustrates: A higher steady-state nominal interest rate in the home country, $i^H - i^F > 0$, leads to a depreciation of the home currency. A permanent increase in i^H results in a permanent increase in interest rate payments from the home to the foreign country and thus a permanent wealth transfer from home to foreign. In response home households save less and foreign households save more. The home biases imply that this wealth transfer affects aggregate bond demands:

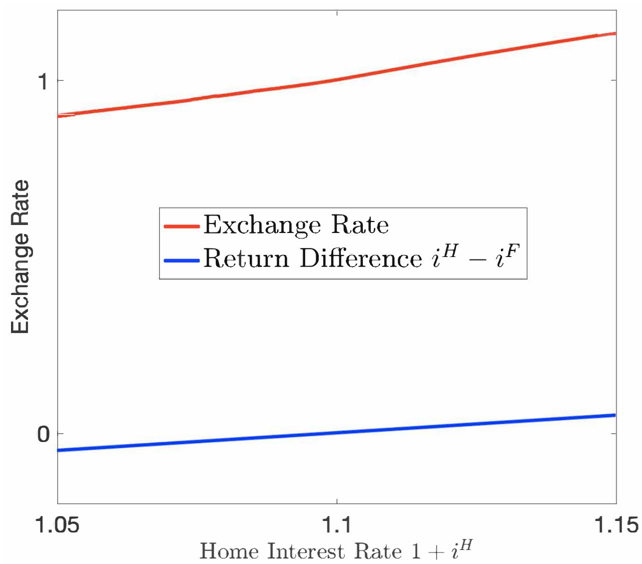


Figure 8: Permanent differences in i^H and i^F

Aggregate demand for home bonds falls and increases for foreign bonds. Asset market clearing then requires a depreciation of the home currency to stimulate home bond demand such that it equals its supply. For the benchmark parametrization, these wealth effects dominate the effects of the higher return on home bonds. Equilibrium then entails not only a depreciation of the home currency but also a reduction of home bonds' riskiness. Both adjustments - exchange rate and risk - stimulate the demand for home bonds and ensure asset market clearing. To understand the role of the wealth transfer, suppose now that the return on home bonds only increases for foreign investors due to a reduction of their transaction costs but all interest rates and other transaction costs remain unchanged. Under these assumptions the home currency appreciates. The reason is that now the aggregate demand for home bonds increases as the wealth transfer of the previous experiment is absent and the foreign investors demand more home bonds due to their higher after transaction costs return. Market clearing then requires an appreciation of the home currency and an increase in home bonds' riskiness.

2.7 Real Exchange Rates

In order to focus on determining the nominal exchange rate, portfolio choices and their interaction, the analysis has so far assumed a single good, implying that the real exchange rate is equal to one. This assumption also ensures that the movements in the nominal exchange rate are not caused by movements in the real exchange rate.

However, one reason to consider nominal exchanges rate is their strong co-movement with real exchange rates in the data, and it is the latter rate that matters for trade decisions. One purpose of this section of the work is to show that the theory does not rely on a constant real exchange rate and that instead, the real exchange rate inherits the volatility of the nominal exchange rate and that both rates are highly correlated. I therefore extend the model by allowing for non-tradable goods in both countries, and assume prices to be sticky in the non-tradable sector. Since the objective is not to develop a full quantitative model, but to show that the assumption of constant real exchange rate is not essential, I assume one tradable good, implying that its price is the same in both countries. This provides substantial tractability for the cost of constant terms of trade which is not essential here.[27]

I show first that the results for nominal exchange rates from the previous sections carry over to the richer model in this section and secondly, that the real exchange rate is volatile and strongly correlated with its nominal counterpart. Movements in the nominal exchange rate now carry over to the real exchange rate, since prices are sticky.

Another implication of a constant real exchange rate is that the volatility of prices and nominal exchange rates is related one-to-one. One might thus be inclined to think that the theory relies on equally volatile price levels and nominal exchange rates, whereas in the data, the latter one is more volatile. A second objective of this section is to show that adding price stickiness overcomes this tight relationship. The volatility of the price index is dampened while at the same time the volatility of the nominal exchange rate is unaffected, showing that the theory does not rely on those volatilities being equal.

2.7.1 A Model with Non-Tradables

Young home households purchase tradable consumption goods $c_{y,s}^{H,T}$ at price $p_s^{H,T}$ and non-tradable goods $c_{y,s}^{H,N}$ at price $p_s^{H,N}$ where both prices are in home currency. Old households consume only tradables $c_{o,s,s_{-1}}^{H,T}$. The young home generation has an after-tax real tradable good endowment $y_s^H = y^H + \kappa_y s$, provides labor l_s^H to the non-tradable sector at a competitive wage w_s^H and receives dividends d_s^H, both in the non-tradable good. Define q_s^H and q_s^F as the inverse of the prices of tradables $p_s^{H,T}$ and $p_s^{F,T}$ and not the inverse of the price level as in Section 2.1 above. The prices for both tradables and non-tradables are in their country's currency, so that the real price of the non-tradable good in terms of the tradable good is $p_s^{H,N} q_s^H$ for the home and $p_s^{F,N} q_s^F$ for the foreign country.

The budget constraint for young home households expressed in terms of the tradable good

[27] Adding home and foreign tradables would render the analysis unnecesarily complex but would allow the model to account for the Engel (1999) criticism that almost all of the variance of the real exchange rate arises from movements in the relative price of tradable goods in flexible exchange rate regimes.

then equals, where ϵ_s is the nominal exchange rate,

$$c_{y,s}^{H,T} + A_{H,s}^{H} q_s^H + A_{F,s}^{H} \epsilon_s q_s^H + p_s^{H,N} q_s^H c_{y,s}^{H,N} \leq y_H + \kappa_y s + p_s^{H,N} q_s^H (w_{H,s} l_s^H + d_s^H). \tag{69}$$

Prices q_s^H and q_s^F are again assumed to be linear in s,

$$q_s^H = \bar{q}^H + \lambda^H s, \tag{70}$$
$$q_s^F = \bar{q}^F + \lambda^F s^F = \bar{q}^F - \lambda^F s. \tag{71}$$

The non-tradable sector is subject to price adjustment costs and I assume them to be infinite. This eliminates the price of non-tradbales from the list of unknowns and allows for an easy mapping between the model with a varying and a constant real exchange rate. While this assumption is unnecessary for the substantive results in this section, it substantially enhances tractability. Prices in the non–tradable sector are therefore constant

$$\overline{p}^{H,N} = p_s^{H,N}, \qquad \overline{p}^{F,N} = p_s^{F,N}. \tag{72}$$

Firms in the non-tradable sector produce using a linear technology $y^N = l$ with labor l as the only input and have to satisfy all demand at these constant prices, such that the market clears and firms make profits (in non-tradable goods) $d_s^H = c_{y,s}^{H,N} - w_s^H l_s^H$ and $d_s^F = c_{y,s}^{F,N} - w_s^F l_s^F$ respectively, for wages w_s^H and w_s^F and labor choices l_s^H and l_s^F in the two countries. Households' expenditure on non-tradables thus equals their labor plus dividend income from this sector, implying

$$\overline{p}^{H,N} q_s^H c_{y,s}^{H,N} = \overline{p}^{H,N} q_s^H (w_s^H l_s^H + d_s^H), \tag{73}$$
$$\overline{p}^{F,N} q_s^F c_{y,s}^{F,N} = \overline{p}^{F,N} q_s^F (w_s^F l_s^F + d_s^F), \tag{74}$$

where $\overline{p}^{H,N} q_s^H$ and $\overline{p}^{F,N} q_s^F$ transform the non-tradable good into units of the tradable good. Tradable goods are identical across countries so that the nominal exchange rate is the ratio of the nominal prices of tradables,

$$\epsilon_s = \frac{q_s^F}{q_s^H} = \frac{\bar{q}^F - \lambda^F s}{\bar{q}^H + \lambda^H s}. \tag{75}$$

For home households, the real value in terms of tradable goods of home bonds is $A_{H,s}^H q_s^H$ and of foreign bonds is $A_{F,s}^H \epsilon q_s^H = A_{F,s}^H q_s^F$, so that the young home household budget constraint simplifies to

$$c_{y,s}^{H,T} + A_{H,s}^{H} q_s^H + A_{F,s}^{H} q_s^F \leq y^H + \kappa_y s. \tag{76}$$

Similarly, foreign households acquire $A_{F,s}^F q_s^F$ foreign and $A_{H,s}^F q_s^F / \epsilon = A_{H,s}^F q_s^H$ home bonds, so that the young foreign household budget constraint simplifies to

$$c_{y,s}^{F,T} + A_{F,s}^{F} q_s^F + A_{H,s}^{F} q_s^H \leq y^F - \kappa_y s. \tag{77}$$

Consumption of the home old generation equals as before

$$c_{o,s.s_{-1}}^H = y_o^H + \kappa_o s + \kappa_s s_{-1} + (1+i^H) A_{H,s_{-1}}^H q_s^H + (1+i^F)(1-\chi) A_{F,s_{-1}}^H q_s^F, \tag{78}$$

where their non-asset income, $y_o^H + \kappa_o s + \kappa_s s_{-1}$, is in tradables and nominal interest rates are constant. Similarly, foreign old generation consumption is

$$c_{o,s,s_{-1}}^F = y_o^F - \kappa_o s - \kappa_s s_{-1} + (1+i^F) A_{F,s_{-1}}^F q_s^F + (1+i^H)(1-\chi) A_{H,s_{-1}}^F q_s^H \tag{79}$$

and non-asset income, $y_o^F - \kappa_o s - \kappa_s s_{-1}$, is in tradables.

The utility function is as before,

$$u(c_{y,s_t}) + E_t u(c_{o,s_{t+1},s_t}), \tag{80}$$

where $u(c) = -exp(-\gamma c)$ with the only difference that now the young generation consumption $c_{y,s}$ is an aggregation of tradable and non-tradable consumption for $0 < \alpha < 1$,

$$c_{y,s} = (c_{y,s}^T)^{\alpha} (c_{y,s}^N)^{1-\alpha}. \tag{81}$$

Young household optimization yields

$$c_{y,s}^{H,N} = c_{y,s}^{H,T} \frac{1-\alpha}{\alpha} \frac{1}{\overline{p}^N q_s^H} \tag{82}$$

and thus

$$c_{y,s}^{H} = (c_{y,s}^{H,T})^{\alpha} (c_{y,s}^{H,N})^{1-\alpha} = c_{y,s}^{H,T} \Big(\frac{1-\alpha}{\alpha} \frac{1}{\overline{p}^N q_s^H} \Big)^{1-\alpha}. \tag{83}$$

The expected stochastic discount factor (SDF) equals

$$e^{\gamma(c^H_{y,s}-y^H_o)-\gamma\mu^H_s+\frac{(\gamma\sigma\Sigma^H_s)^2}{2}} \approx \bar{m}^H + \hat{m}^H s, \tag{84}$$

but where now, $c^H_{y,s}$ is the consumption aggregator of tradable and non-tradable consumption defined in (83). The corresponding consumption aggregator of foreign households is denoted $c^F_{y,s}$.

The linearized equations characterizing this model, including the budget constraints (76) and (77), are identical to those in the previous section with a constant real exchange rate. This is where the simplifying assumption of infinite price adjustment costs plays a role, as it implies that the amount of non-tradable consumption can be solved for, after all other model variables are known.

The price indexes in the home and foreign country are (omitting multiplicative constants)

$$P^H_s = (q^H_s)^{-\alpha}(\bar{p}^H)^{1-\alpha}, \tag{85}$$
$$P^F_s = (q^F_s)^{-\alpha}(\bar{p}^F)^{1-\alpha}. \tag{86}$$

The real exchange rate equals, given the nominal exchange rate ϵ_s,

$$rer_s = \frac{\epsilon_s P^F_s}{P^H_s}, \tag{87}$$

which simplifies using $\bar{p}^H = \bar{p}^F$ and $\epsilon_s = q^F_s/q^H_s$,

$$rer_s = \epsilon_s(\frac{q^H_s}{q^F_s})^{\alpha}(\frac{\bar{p}^F}{\bar{p}^H})^{1-\alpha} = (\frac{q^F_s\bar{p}^F}{q^H_s\bar{p}^H})^{1-\alpha} = (\frac{q^F_s}{q^H_s})^{1-\alpha} = (\epsilon_s)^{1-\alpha}. \tag{88}$$

We thus obtain a simple relationship between the nominal and the real exchange rate, where the log real exchange rate is the nominal exchange rate scaled by $1-\alpha$.

2.7.2 Results: Real Exchange Rates

The same linearization of the FOC for $A^H_H, A^H_F, A^F_F, A^F_H$ in equations (18), (24), (25) and (26) for the model with a constant real exchange rate applies here in the model with a varying real exchange rate. The only difference is that the linearized SDFs $\bar{m}^H + \hat{m}^H s$ and $\bar{m}^F + \hat{m}^F s$ could have a different mean and variance, since the underlying consumption process of young households does. The converse applies, if the SDF is the same in the two models; the same set of equations then describe equilibrium prices q_s and the nominal exchange rate ϵ_s, implying the identical solution for ϵ_s in both models.

A simple reparametrization indeed ensures the same SDF in the model with non-tradables as in the model with a fixed real exchange rate. I set the two new parameters, $\alpha = 0.5$, and the constant price of non-tradabales equal to the steady-state price of tradables, $\bar{p}^H = \bar{p}^F = 1/\bar{q}^H = 1/\bar{q}^F$. The standard deviation of the young-age endowment of tradables is increased from 0.4 percent to 5.75 percent, so that $y_s^H = 100 + 5.75s$ and $y_s^F = 100 - 5.75s$. Although the volatility of the tradable endowment is much higher now, the standard deviation of young age consumption is unchanged at 0.67 percent. A higher endowment in tradables leads to a fall in its price (an increase in q_s). This decline in the relative price $\bar{p}/q_s$ induces a demand shift from non-tradables to tradables, implying a drop in the supply of demand-determined non-tradables, which leaves aggregate consumption largely unaffected. Figure 9 shows the result. The log real exchange rate is perfectly correlated with the nominal exchange rate but less volatile.

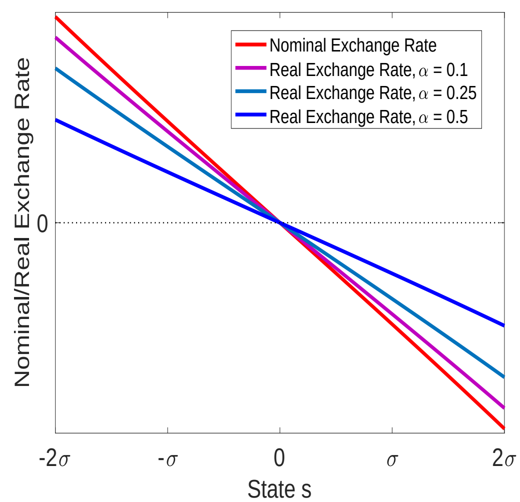


Figure 9: Log Nominal and Real Exchange Rates for different degrees of aggregate price rigidity, $\alpha = 0.5$ (benchmark), $\alpha = 0.25$ (medium rigidity), $\alpha = 0.1$ (high rigidity).

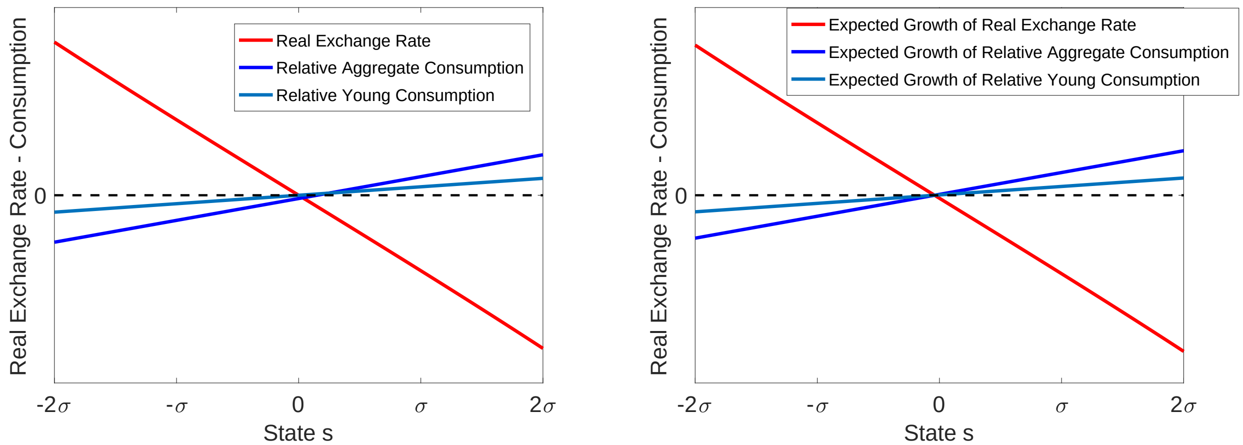
The volatility of the real exchange rate is inversely related to the rigidity of prices, which is parameterized through α. A lower value for α implies a more rigid aggregate price level since the share of the constant price non-tradables increases. Figure 9 also shows the result when the price rigidity is increased to $\alpha = 0.25$ and $\alpha = 0.1$. Not surprisingly, the volatility of the real exchange rate increases when moving from $\alpha = 0.5$ to $\alpha = 0.25$ and $\alpha = 0.1$. For each value of α, I reparameterize the model to obtain the same SDF as in the benchmark, implying that the volatility of the nominal exchange rate is the same across all degrees of price rigidity. The volatility of the price index changes though, establishing that it is unrelated to the nominal exchange rate volatility.

Several conclusions can be drawn from this analysis. First, the theory of nominal exchange rate determination does not rely on a constant real exchange rate. Instead, this assumption merely serves to show that the nominal exchange rate movements are not caused by real exchange rate movements. Second, the nominal and the real exchange rate are highly correlated. Third, the volatility of the price index is unrelated to the volatility of the nominal exchange rate, and inversely related to the volatility of the real exchange rate. In the extreme case when prices are almost fully sticky, namely α close to 0, the aggregate price is basically constant while the nominal and the real exchange rate become almost equally volatile.

Furthermore, the Kollmann-Backus-Smith Consumption-Real-Exchange-Rate anomaly disappears here. If markets are complete, international risk sharing implies that relative consumption is high when relative prices are low, such that relative consumption across countries is strongly positively correlated with the real exchange rate. However, this risk-sharing condition does not hold in the data where instead, the correlation is slightly negative (Backus and Smith, 1993; Kollmann, 1995). This puzzle is overcome here as the left panel of Figure 10 shows. The correlation between relative consumption and the real exchange rate is negative in the model and thus qualitatively consistent with the data. This negative relationship holds both for aggregate consumption and consumption of the young, implying that the result is not driven by some carefully engineered redistribution across generations. The result also does not rely on the magnitude of the trade elasticity (Corsetti et al., 2008) since the model features only one tradable good or on a cost function that depends on the amount of foreign assets held (Benigno and Thoenissen, 2008) since these costs are constant here. In addition these two models are real models which cannot speak to the determination of the nominal price of tradables or the nominal exchange rate, establishing that the mechanism in my paper is different.

Indeed, the result follows directly from the different way of determining the nominal exchange rate here. As explained above, financial market determination of exchange rates implies that a positive productivity shock requires a nominal appreciation, so as to ensure market clearing, and unsurprisingly, consumption increases when output increases. This comovement of the nominal exchange rate and output is present both in the benchmark model with $rer \equiv 1$ in Section 2 and in this Section's model with a varying real exchange rate. In the latter case, the real exchange rate inherits its properties from the nominal exchange rate, implying that the nominal appreciation in a boom translates into a real appreciation.[28]

[28]Note that the production of non-tradables adjusts to satisfy demand and clear goods markets. Note also that here there is a feedback from non-tradable consumption to the nominal exchange rate since preferences are non-separable in tradable and non-tradable consumption. With separable preferences this feedback would disappear and causality runs from nominal exchange rates to real exchange rates to consumption without any feedback.



(a) Levels

(b) Growth Rates

Figure 10: Kollmann-Backus-Smith Consumption-Real Exchange rate anomaly disappears.

For an appreciation to constitute an equilibrium, the risk-premium has to adjust, which, from a reduced form perspective, renders the mechanism similar to that in Itskhoki and Mukhin (2019). In their work, an exogenous shock to the frictional financial sector, which is isomorphic to a UIP shock, is the main driving force of exchange rate movements. From this perspective, the risk premium movements in my model are the financial shocks which co-move with the exchange rate. The key difference is that here, both the risk premium (=the financial shock) and the exchange rate are both endogenous, both move in response to fundamental shocks and that one variable does not drive the other one. Furthermore, the movements of consumption and real exchange rates are conditional on fundamental shocks, showing that the new mechanism can rationalize both conditional and unconditional moments.

The right panel of Figure 10 also shows that the arguments extend from levels to growth rates. Consistent with Itskhoki and Mukhin (2019) and the data, and in contrast for example to Corsetti et al. (2008), the correlation between relative consumption growth and the change in the real exchange rate is negative.[29] Figure 10 also shows that the exchange rate is significantly more volatile than consumption, again consistent with the data.

2.8 Exchange Rate Peg

In this Section, I consider an exchange rate peg and which policies are needed to implement a constant nominal exchange rate. I argue that the innovative way of determining exchange rates proposed here transforms the open macroeconomics policy trilemma into a tetralemma:

[29] See Appendix A.8 in Itskhoki and Mukhin (2019) for a detailed discussion of alternative resolutions of the Kollmann-Backus-Smith Consumption-Real-Exchange-Rate anomaly.

A country with a fixed exchange rate and free capital mobility loses both monetary and fiscal policy independence.[30]

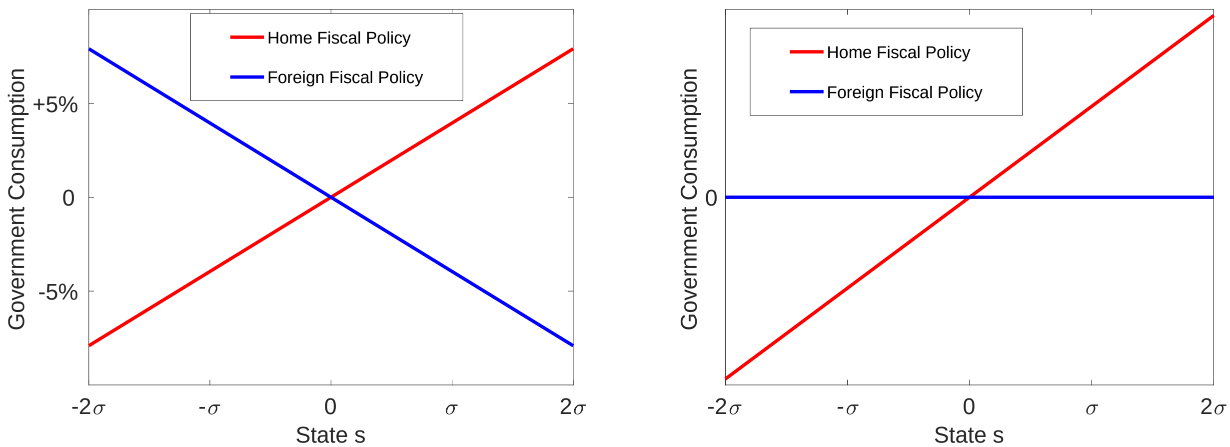
2.8.1 Fiscal Policy

I use the model with a non-constant real exchange rate introduced in the previous section, and add a richer fiscal policy with real government consumption $g_s^H = \bar{g} + \hat{g}s$ in state s financed through lump-sum taxes on the young generation, such that their real tax obligations are now $T_s^H = i^H B^H q_s^H + \bar{g} + \hat{g}s$. Fiscal policy can adjust government consumption in different states s through choosing $\hat{g}$. A positive $\hat{g} > 0$ means that fiscal policy is pro-cyclical, government spending and thus taxes are increased in high income states $s > 0$ and lowered in low income states $s < 0$. Fiscal policy in the foreign country is symmetric with real government consumption $g_s^F = \bar{g} - \hat{g}s$ in state s. Note that foreign policy is also pro-cyclical if $\hat{g} > 0$, since $s^F = -s$, so that $s < 0$ is a high-income state in the foreign country. Household utility is assumed to be separable in private and government consumption, implying that the previous analysis applies.

I now compute the cyclical component of fiscal policy, the parameter $\hat{g}$, which implements a constant nominal exchange rate. The left panel of Figure 11 shows that a pro-cyclical fiscal policy renders the exchange rate constant. This is not surprising, since the economy is driven by supply shocks, so that $s > 0$ implies high productivity. As a result, the supply of goods increases, home prices fall (note that q^H is the inverse price level and increases) and the home currency appreciates. Increasing government expenditure on the other hand stimulates demand and implies an increase in the home price level and a depreciation. An appropriate pro-cyclical fiscal policy then stimulates demand sufficiently, such that it matches the higher supply and neutralizes the price and exchange rate movements. Other fiscal policies, say deficit financing, could also be used to manage aggregate nominal demand and thus the exchange rate, but the one considered here is the most simple one for making this point.

The simple fiscal policy requires cooperation between the two countries, as each country uses fiscal policy to neutralize the impact of shocks on their country. Fixing the exchange rate does not require cooperation however, and can be achieved through fiscal policy in the home country only, while at the same time, foreign policy does not respond to shocks. Since foreign policy is constant and thus the countries are not symmetric, prices in the two countries will

[30] The tetralemma argues that both fiscal and monetary policy lose independence if exchange rates are fixed but does not restrict policies if exchange rates are freely floating. Rey (2015) considers this latter floating exchange rate scenario and argues that monetary policy is not independent even in a world with floating exchange rates and free capital mobility invalidating the trilemma and leading to a dilemma. The tetralemma and the dilemma are thus consistent, since the first considers policy for fixed exchange rates and the latter for flexible exchange rates.



(a) Coordinated Fiscal Policy

(b) Only Home Fiscal Policy

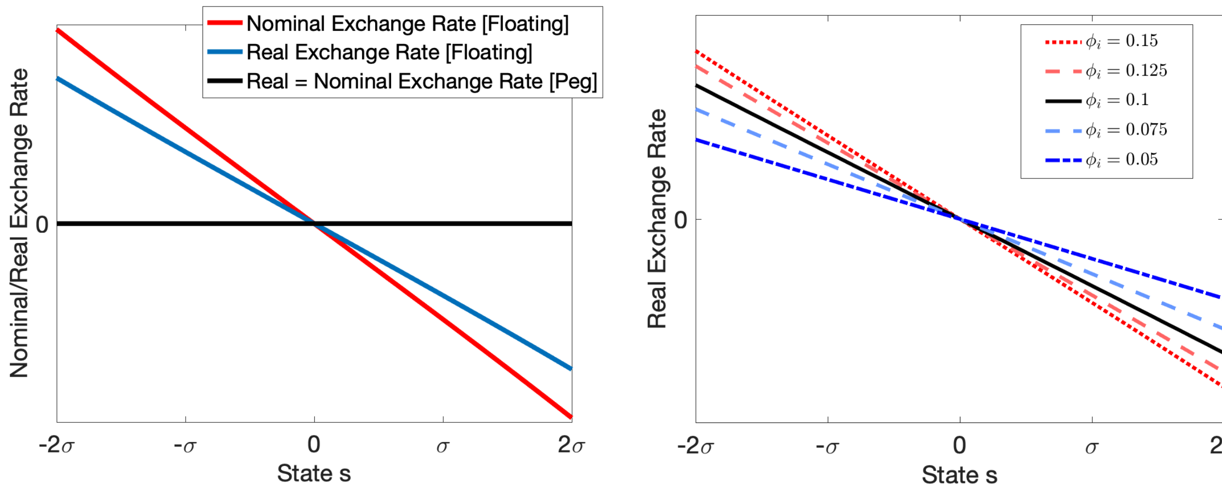
Figure 11: Exchange Rate Peg : Fiscal Policy

not be constant and both countries still face risk, even when the exchange rate is pegged. A constant nominal exchange rate therefore requires fiscal policy to implement the same non-constant prices in both countries. The right panel of Figure 11 shows the result. The home country again engages in procyclical policy but now operates at a larger scale due to the foreign country not contributing.

The results are qualitatively consistent with the Mussa (1986) puzzle, that the volatility of the real exchange rate falls when moving from a freely floating regime to a pegged nominal exchange rate. Note that the reduction in nominal exchange rate volatility occurs endogenously as a consequence of a policy change and not by assumption.

Figure 12 illustrates this finding. The real exchange rate is almost as volatile as the nominal exchange rate in the floating regime whereas it is constant when the nominal exchange rate is pegged.[31] The Mussa result not only holds for a comparison with a pegged exchange rate but operates continuously. If the volatility of the nominal exchange rate is reduced due to a change in monetary policy as in Figure 7, the same reduction of the volatility is obtained for the real exchange rate. Indeed, reducing the interest rates response from $\phi_i = 0.15$ to $\phi_i = 0.05$ decreases the volatility of the nominal and the real exchange rate. The reason is that the stochastic properties of the real exchange rate are inherited from the nominal exchange rate, implying that a change in the regime for a nominal variable carries over to a real variable - the Mussa puzzle.

[31] Note that in the model the peg of the nominal exchange rate is perfect such that the volatility is zero whereas it is positive in the data in a "peg regime". The model also allows to model a tightening of capital controls through a higher χ. I keep χ fixed across regimes to isolate the effects of an exchange rate peg.



(a) Real Exchange Rate: Flexible and Fixed Regime (b) Real Exchange Rate and Monetary Policy

Figure 12: Mussa Puzzle

2.8.2 Tetralemma

The classic policy trilemma in international economics is that at most two of the following three policies are simultaneously feasible: (i) unrestricted capital mobility; (ii) setting nominal interest rates independently (monetary policy independence); and (iii) a fixed exchange rate. The underlying logic is quite simple. Free asset flows imply that the uncovered interest rate parity holds, such that a fixed exchange rate regime requires setting the domestic nominal interest rate equal to the foreign nominal interest rate.

The interest rate parity condition with zero transaction costs, $\chi = 0$, and when the future exchange rate ϵ_{t+1} is known at time t,

$$\epsilon_t = \frac{1 + i^F_{t+1}}{1 + i^H_{t+1}} \epsilon_{t+1}, \tag{89}$$

demonstrates this logic. If the exchange rate is constant, $\frac{\epsilon_{t+1}}{\epsilon_t} = 1$, the interest rate parity condition simplifies further and implies that $(1 + i^H) = (1 + i^F)$.

In this paper, but with a different mechanism for exchange rate determination, giving up an independent monetary policy is necessary, but not sufficient to stabilize the level of the exchange rate. Neither a constant interest rate, nor an interest rate rule responding to the state s nor an interest rate rule as in Benigno et al. (2007) does accomplish the task of pegging the exchange rate.

For a constant nominal interest rate, the reason is that the above logic neglects the fact that shocks, for example to output, move the current exchange rate ϵ_t without necessarily moving

ϵ_{t+1} by the same magnitude. In the model, the nominal interest rates in both the home and the foreign country are constant and equal but as Figure 2 shows the exchange rate is quite volatile and responds to unanticipated output shocks.

The above interest rate parity condition logic does not hold in the presence of risk, as a covariance term needs to be added to this equation. For example, a period t positive output shock leads to a fall of ϵ_t (a period t appreciation) and to an expected increase in period $t+1$, $E_t\epsilon_{t+1} > \epsilon_t$. This does not violate households' first-order conditions since the covariance - the risk premium - adjusts together with prices to clear the market. As a result, a varying nominal exchange rate is an equilibrium outcome, despite constant nominal interest rates.

While a constant nominal interest is not effective in pegging the exchange rate, the finding in Section 2.6 that monetary policy, through changing the nominal interest rate, can move the nominal exchange rate, raises the possibility that a careful designed monetary policy might ensure a constant exchange rate. Such an argumentation would however be a logical fallacy. Although home monetary policy can move the exchange rate and manage its volatility ("a managed exchange rate regime"), it cannot be used to implement a fixed exchange rate. The simple reason is that a fixed exchange rate requires an interest rate peg in accordance with the interest rate parity condition, that is $i^H = i^F$. To see this, suppose that there is an interest rate response function that manages to peg the exchange rate. The interest rate parity condition for a constant exchange rate implies that this response function has to be the constant nominal interest rate. But if the interest rate is constant, then Section 2.6 shows that the exchange rate is not constant, which is a contradiction. Result 10 in Section 2.5 confirms this intuition and proves that monetary policy cannot fully peg the nominal exchange rate. Section 2.5 also explains why an interest rate rule as in Benigno et al. (2007) does not peg the exchange rate here, simply because this rule is about ensuring determinacy and selecting the desired equilibrium whereas here it is about affecting the properties of the unique equilibrium.

This suggests that a country faces a tetralemma. Unrestricted capital mobility and a fixed exchange rate imply that a country loses both monetary and fiscal policy independence, or more generally loses its ability to manage aggregate domestic demand. An exchange rate peg requires fiscal policy to absorb shocks hitting the economy, so as to stabilize the exchange rate, while home monetary policy perfectly tracks foreign monetary policy. In a world with cooperation between the two countries, each uses fiscal policy to eliminate the impact of shocks on households. For example, as shown in the previous Section 2.8.1, in response to a positive output shock government consumption and taxes are increased so as to fully reverse the price movements, and vice versa in response to a negative output shock, where government consumption and taxes are decreased. Similarly, the home country can use fiscal policy to peg the exchange rate without any cooperation of the foreign country. But again, fiscal policy - coordinated or uncoordinated - has to be used. These arguments together establish that monetary policy can affect the volatility of the exchange rate but never fully stabilize it. A full stabilization requires fiscal policy in addition.

3 Exchange Rates and Portfolios: General Case

The deeper theoretical reason for the findings on the properties of nominal and real exchange rates including new answers to several puzzles and the tetralemma is a novel way to jointly determine the equilibrium exchange rate and portfolios. In this Section, I explain the underlying theory in more detail and argue that it applies quite generally, and extends beyond OLG economies. Specifically, I show that the assumption of incomplete asset markets within each country can replace the OLG assumption. I first explain why incomplete markets and OLG models deliver exchange rate determinacy before turning to the role of my assumptions. To focus on the theoretical fundamentals, I only consider steady states, that is, the equilibria in a world without uncertainty. It is important to keep in mind though that this steady state is the limit of vanishing aggregate uncertainty and transaction costs, since otherwise, we would run into indeterminacy issues, as I will explain below.

Building on the simple OLG model of the previous Section, I use a graphical representation, which applies both to OLG and heterogenous agents incomplete (within countries) markets models, in order to highlight the determination of exchange rates. Using the notation introduced in Section 2.4 on limit portfolios and exchange rates, the asset market clearing condition for home real bonds is rewritten as[32]

$$\frac{B^H}{\mathbf{P^H}} = S_H^H + S_H^F, \tag{90}$$

where $S_H^H + S_H^F$ is the sum of the home and the foreign country demand for home real bonds. For foreign bonds the market clearing condition is

$$\frac{B^F}{\mathbf{P^F}} = S_F^F + S_F^H. \tag{91}$$

where $S_F^F + S_F^H$ is the sum of the home and the foreign country demand for foreign real bonds.

[32]For the ease of exposition, I assume that both B^H and B^F are constant. It is straightforward to relax this, since in a steady-state, the real value of bonds is constant and $\frac{B_t^H}{P_t^H} = \frac{B^H(1+\pi^H)^t}{P^H(1+\pi^H)^t} = \frac{B^H}{P^H}$ and $\frac{B_t^F}{P_t^F} = \frac{B^F(1+\pi^F)^t}{P^F(1+\pi^F)^t} = \frac{B^F}{P^F}$.

As above, the equilibrium steady-state price levels $P^H (= 1/q^H)$ and $P^F (= 1/q^F)$ and thus the exchange rate $\epsilon = P^H/P^F$ are characterized as the solution to these two asset market clearing conditions (90) and (91). All variables - $S^H, S^F, S^H_H, S^H_F, S^F_F, S^F_H, P^F, P^H$ - are well-defined as the limit of vanishing aggregate uncertainty and transactions costs, where the limit is approached along an equilibrium path with nonzero uncertainty and transaction costs.

Some simple algebra yields an equivalent, but empirically more applicable characterization of prices and the exchange rate in terms of each countries observed asset positions. Observe first that by definition nominal net foreign asset holdings by the home country, NFA^H (denominated in the home currency), satisfy

$$\frac{NFA^H}{P^H} = S^H_F - S^F_H, \tag{92}$$

and by the foreign country, NFA^F (denominated in the foreign currency), satisfy

$$\frac{NFA^F}{P^F} = S^F_H - S^H_F = -\frac{NFA^H}{P^H}. \tag{93}$$

Using this in (90) and (91) and rearranging yields:

$$\frac{B^H + NFA^H}{\mathbf{P^H}} = S^H_H + S^H_F = S^H, \tag{94}$$

$$\frac{B^F + NFA^F}{\mathbf{P^F}} = S^F_F + S^F_H = S^F, \tag{95}$$

which defines a mapping from assets to prices and exchange rates. The advantage of this characterization is that it is stated in terms of empirically observable assets B^H, NFA^H, B^F, NFA^F and depends only on a country's total savings S^H, S^F but not on the portfolio decisions $S^H_H, S^H_F, S^F_F, S^F_H$ separately. In the OLG model in Section 2, total asset demand in the limit equals, using Result 8,

$$S^H = \frac{\gamma(y - y_o) - ln(1/(1+i))}{2\gamma(1+i)}, \tag{96}$$

$$S^F = \frac{\gamma(y - y_o) - ln(1/(1+i))}{2\gamma(1+i)}. \tag{97}$$

Another advantage of the second characterization is that it also allows using the Metzler diagram for a graphical depiction. Figure 13 shows how prices and the exchange rate are derived. The left and right panels report the home and foreign savings curves S^H and S^F as a function of the world real interest rate $1 + r$. In incomplete markets models, these curves are upward sloping under standard assumptions while they can be downward sloping in OLG models depending on the magnitudes of income, substitution, and wealth effects. Although the slope is irrelevant, it is important that the curve not be horizontal, as I explain below. On the horizontal axis, the figures also show the real value of home assets, $B^H/P^H + NFA^H/P^H$, and the real value of foreign assets, $B^F/P^F + NFA^F/P^F = B^F/P^F - NFA^H/P^H$, where I used that $NFA^F/P^F = -NFA^H/P^H$. The right panel tells us that the price level P^H can be determined as clearing the home market,

$$B^H/\mathbf{P^H} + NFA^H/\mathbf{P^H} = S^H(1+r,\ldots), \tag{98}$$

which then determines the real value of net foreign assets, $NFA^F/P^F = -NFA^H/P^H$. Using this in the left panel determines the price level P^F from asset market clearing in the foreign country,

$$B^F/\mathbf{P^F} - NFA^H/P^H = S^F(1+r,\ldots). \tag{99}$$

Therefore, the exchange rate $\epsilon = P^H/P^F$ is determinate and solves[33]

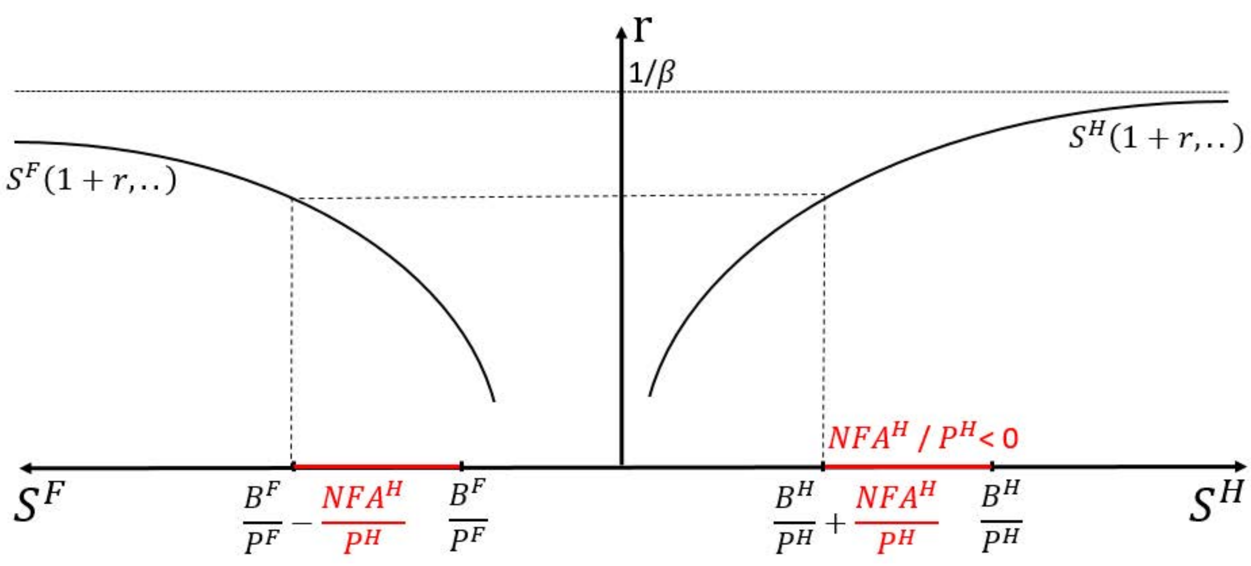


Figure 13: Exchange Rate Determination in Metzler Diagram

$$\epsilon = \frac{B^H + NFA^H}{S^H(1+r,\ldots)} \frac{S^F(1+r,\ldots)}{B^F - NFA^H/\epsilon}. \tag{100}$$

[33]If home bonds grow at rate $1+\pi^H$ in steady state, different from the growth rate of foreign bonds, $1+\pi^F$, then the exchange rate is $\epsilon\frac{(1+\pi^H)^t}{(1+\pi^F)^t}$.

I will now argue that the determinacy result does not depend on specific model properties, and applies to a wide class of models with three properties:

1. Market incompleteness/Ricardian equivalence fails

   $\hookrightarrow$ Well-defined aggregate savings within each country.

2. Non-diversifiable aggregate risk

   $\hookrightarrow$ Well-defined international portfolios for each country.

3. Nominal assets

   $\hookrightarrow$ Assigns a role for nominal prices.

The need for the later property - assets are (partially) nominal - is clear. If assets were fully price-indexed, there would be no role for nominal prices, since the whole economy would be specified in real terms only. It is however sufficient that assets are partially nominal, i.e. a fraction less than 100% could be indexed.

The role of the other two assumptions - market incompleteness and aggregate risk - is more subtle. To understand this, it is useful to first consider a frictionless world without aggregate risk and where markets are complete. In such a world indeterminacies of the Sargent and Wallace (1975) (SW) and the Kareken and Wallace (1981) (KW) type arise. The steady-state nominal interest rates i^H and i^F merely determine the expected change in the nominal exchange rate, $E_t \frac{\epsilon_{t+1}}{\epsilon_t}$, but not the levels ϵ_t and ϵ_{t+1}. The uncovered interest rate parity condition,

$$1 + i^H = (1 + i^F) E_t \frac{\epsilon_{t+1}}{\epsilon_t}, \tag{101}$$

if satisfied for a pair $(\epsilon_t, \epsilon_{t+1})$, is also satisfied for any multiple $(\lambda\epsilon_t, \lambda\epsilon_{t+1})$ for all $\lambda > 0$. This is the analog for exchange rates of the price level indeterminacy revealed by SW. Accordingly, the derivation illustrated in Figure 13 no longer applies. With complete markets the steady-state savings curve is degenerate and becomes a horizontal line at the steady-state real interest rate $1/\beta$ (for a discount factor β). As Figure 14 illustrates, asset market clearing in both countries is consistent with a continuum of prices, e.g. P_1^H, P_2^H, P_3^H for the home country and P_1^F, P_2^F, P_3^F for the foreign country, and hence with a continuum of exchange rates $\epsilon = P^H/P^F$.

What incomplete markets contribute are well defined steady-state aggregate savings functions S^H and S^F.[34] Although adding incomplete markets overcomes the SW indeterminacy, it still does not deliver determinacy, as now, the KW type indeterminacy comes into effect. Since

[34]For a textbook treatment of incomplete markets models and their steady states, see Ljungqvist and Sargent (2012).

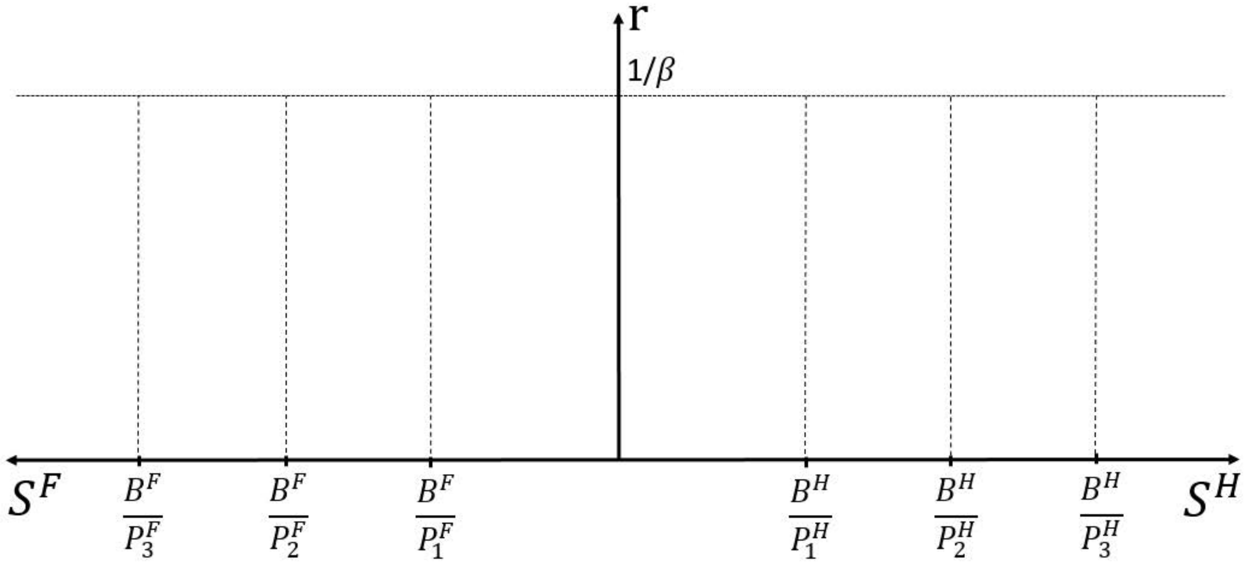


Figure 14: Complete Markets: Exchange Rate Indeterminacy of Sargent and Wallace (1975) type

bonds are freely mobile across borders and without transactions costs, the world asset market clears when

$$S^H + S^F = \frac{B^H}{P^H} + \frac{B^F}{P^F} = \frac{B^H}{P^H} + \epsilon \frac{B^F}{P^H}, \tag{102}$$

which, for every exchange rate $\epsilon > 0$, has a different solution P^H.[35] However, all of these different exchange rates and price levels are associated with different net foreign asset positions,

$$\frac{NFA^H}{P^H} = S^H - \frac{B^H}{P^H}. \tag{104}$$

[35]If mobility were restricted, as an extreme example if each country can only hold its own bonds, then the exchange rate would be determined. This mobility restriction implies separate asset market clearing conditions for each country H and F,

$$S^H = \frac{B^H}{P^H} \quad \text{and} \quad S^F = \frac{B^F}{P^F}, \tag{103}$$

which determine price levels P^H and P^F and thus the nominal exchange rate $\epsilon = P^H/P^F$. However, in this case $NFA \equiv 0$ prevents a meaningful discussion of cross-border asset flows.
That part of the literature which assumes that monetary policy sets money supply instead of interest rates, makes similar assumptions and typically restricts the usage of a country's currency to this particular country (The assumption is that households derive utility only from holding their own currency). Imposing such a full home bias for bonds would counterfactually imply $NFA \equiv 0$.

For example: one can choose price levels P_-^H and P_-^F such that the world asset markets clear

$$S^H + S^F = \frac{B^H}{P_-^H} + \frac{B^F}{P_-^F}, \tag{105}$$

and that the associated net foreign asset positions are

$$NFA_-^H = P_-^H S^H - B^H < 0 \tag{106}$$
$$NFA_-^F = P_-^F S^F - B^F > 0 \tag{107}$$

and the exchange rate equals $\epsilon_- = P_-^H / P_-^F$. Similarly: one can pick world asset market clearing prices P_0^H and P_0^F such that

$$NFA_0^H = P_0^H S^H - B^H = 0 \tag{108}$$
$$NFA_0^F = P_0^F S^F - B^F = 0 \tag{109}$$

or prices P_+^H and P_+^H such that

$$NFA_+^H = P_+^H S^H - B^H > 0 \tag{110}$$
$$NFA_+^F = P_+^F S^F - B^F < 0 \tag{111}$$

and again world asset markets clear. All these choices are equilibrium outcomes but are associated with different exchange rates $\epsilon^- = P_-^H/P_-^F < \epsilon^0 = P_0^H/P_0^F < \epsilon^+ = P_+^H/P_+^F$, different prices $P_-^H < P_0^H < P_+^H$ and $P_-^F > P_0^F > P_+^F$ and different NFAs.[36]

This is where assumption 2 (aggregate risk) becomes relevant. Aggregate country risk delivers well-defined portfolio choices regarding how to split a country's savings between home and foreign bonds. This adds NFAs to the list of equilibrium objects and eliminates it as a free parameter. In particular, total assets $A^H = B^H + NFA^H$ is an outcome of agent diversification of aggregate risk. Figure 13 then illustrates the mapping from $A^H = B^H + NFA^H$ to P^H and of $A^F = B^F + NFA^F$ into prices P^H and P^F and the exchange rate $\epsilon = P^H/P^F$.

In the OLG model Result 8 shows that at the limit of symmetric equilibria, households divide their savings equally between home and foreign bonds, $S_H^H = S_F^H = S^H/2$ and $S_F^F = S_H^F = S^F/2$. Knowing these portfolio decisions and using equations (96) and (97) then allows solving for the

[36]Different NFA positions mean different wealth transfers across regions. For example $NFA_+^H < 0$ means that the home country transfers interest rate payments to the foreign country. These wealth transfers change countries' asset demands, but are omitted here, as they are irrelevant for the indeterminacy argument. The world asset market clearing condition is still one equation in two unknowns, which causes the indeterminacy.

equilibrium limit exchange rate as described above using the Metzler diagram.

4 Concluding Remarks

This paper proposes a new equilibrium theory in which nominal and real exchange rates and international portfolio choices are jointly determined in international financial markets. This entails different properties of exchange rates than in standard approaches and a new role for fiscal and monetary policy. The nominal exchange rate is priced like an asset and is thus more volatile than in traditional approaches. The real exchange rate inherits its properties from the nominal exchange rate, rendering it volatile and implying a strong co-movement with the nominal exchange rate. Without specific policy interventions, the equilibrium exchange rate is not constant and instead is state-dependent in order to clear international asset markets. Monetary policy operating through setting nominal interest rates, however, cannot implement an exchange rate peg, and fiscal policy intervention is necessary for a successful peg, transforming the open macroeconomics policy trilemma into a tetralemma. A country with a fixed exchange rate and free capital mobility is confronted with a tetralemma, because it not only loses monetary policy independence as in the trilemma, but in addition also fiscal policy independence.

The novel approach to determining the exchange rate also offers a new promising approach to accounting for several puzzles in the data. These include the co-movement of exchange rates and interest rate differentials, the Backus-Smith-Kollmann Consumption-Real Exchange Rate puzzle, and the Mussa puzzle on the volatility of real and nominal exchange rates under freely floating and pegged exchange rate regimes.

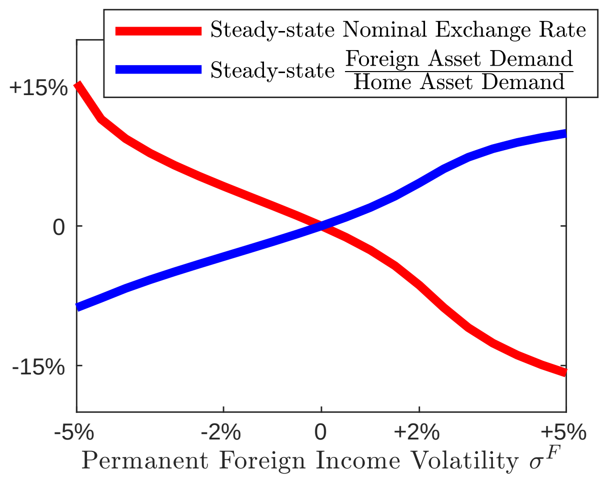
Determining exchange rates in financial as opposed to in goods markets also offers a new perspective on the "transfer problem", which describes the relationship between wealth transfers across countries and exchange rates.[37] Here, foreign investors pay an equilibrium risk premium on home bonds, which entails a state-dependent transfer from the home country to the foreign country in bad foreign states. At the same time, although wealth flows out of the home country in these bad foreign states, the home currency appreciates. On the other hand, permanent non-state-dependent transfers from the home country to the foreign country - the Keynes/Ohlin scenario - lead to a higher demand for foreign bonds, requiring an appreciation of the foreign currency to clear asset markets. By contrast, in traditional models all transfers - state dependent and state independent - from the home to the foreign country induce a shift in demand towards foreign goods, so that the foreign currency always appreciates to clear the goods market.

[37] See Keynes (1929a,b,c) and Ohlin (1929a,b) for the Keynes/Ohlin debate and Maggiori (2017) for the related "reserve currency paradox".

The new theory also relates to international asset flows, as both portfolio choices and prices jointly respond to changes in aggregate risks. The proposed theory can therefore be used to discuss several questions, thereby motivating a large literature in open economy macroeconomics. How does a sudden asset outflow affect the exchange rate? How does an increase in asset demand in the rest of the world affect asset flows and the exchange rate? Can a country divorce itself from such global financial flows? And more generally, how can a country manage its exchange rate?

For example, consider an increase in income volatility in the rest of the world, while keeping income volatility at home unchanged, exposing households in the rest of the world to higher risk. What does the model imply for such an experiment? In response to this higher risk, the rest of the world increases its precautionary savings through accumulating more home bonds. The model predicts that this surge in demand for domestic assets leads to an appreciation of the domestic currency, as the model simulation results in Figure 15 confirm.

Figure 15: Steady-State Nominal Exchange Rates, Asset Demands and Foreign Income Volatilities



Note - The Figure shows the steady-state nominal exchange rates $\bar{\epsilon}$ and foreign-to-home total real asset demand $\frac{q^H \bar{A}_H^H + q^F \bar{A}_F^H}{q^F \bar{A}_F^F + q^H \bar{A}_H^F}$ (both in logs) for a foreign income inequality σ^F deviating $\pm 5\%$ from the benchmark, while keeping the home income inequality σ^H unchanged. For example, an increase in foreign income uncertainty by 5 percent leads to an 15.8% appreciation of the home country and an 10% percent increase in foreign to home total real asset demand.

The shift in world asset demand towards home bonds is associated with an inflow of assets into the home country, and as the theory implies, an inflow of assets leads to a currency appreciation. In terms of policy, the model suggests that undoing this appreciation would require increasing the supply of domestic bonds.

Applying this scenario to the US, higher precautionary savings in developing countries, due to greater world risk, is expected to be absorbed by developed countries. Due to the depth of US financial markets and the US dollar being the leading reserve currency, this most likely means being disproportionately absorbed by the US. The model then predicts that these asset flows lead to an appreciation of the US dollar. The US can reverse this appreciation through issuing more government bonds. This suggests that a larger asset demand by the rest of the world (ROW) for US bonds can be accommodated without any effects on US exchange rates, provided that the ROW's demand does not persistently increase at a faster rate than US GDP. If it does, stabilizing the exchange rate will then require an exploding US debt/GDP ratio, which is infeasible due to the limited US fiscal capacity. The US would then have to accept falling prices and an appreciation of its currency, a flexible exchange rate post-Bretton-Woods version of Triffin's dilemma. Alternatively, the ROW diverts its savings to other currencies, the Euro or the Yuan.

The present paper enables studying this and many more questions in open economy macroeconomics on asset flows and exchange rates. A key aspect of studying these issues is that asset flows, exchange rates and country portfolios are jointly determined in equilibrium, a novel and distinct feature of this paper.

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APPENDIX

A.I Proofs and Derivations

Derivation of Result 1 [Zero Order Portfolio].

The first two equations of the four first-order conditions, (18), (24), (25), (26), in the main text,

$$\begin{aligned}
\bar{q}^H + \lambda^H s_t &= (1+i^H)\{[\bar{m}^H + \hat{m}^H s_t]\bar{q}^H - \lambda^H \gamma\sigma^2[(\bar{m}^H + \hat{m}^H s_t)\bar{\Sigma}^H + \bar{m}^H \hat{\Sigma}^H s_t]\} \\
\bar{q}^F - \lambda^F s_t &= (1+i^F)(1-\chi)\{[\bar{m}^H + \hat{m}^H s_t]\bar{q}^F + \lambda^F \gamma\sigma^2[(\bar{m}^H + \hat{m}^H s_t)\bar{\Sigma}^H + \bar{m}^H \hat{\Sigma}^H s_t]\} \\
\bar{q}^F - \lambda^F s_t &= (1+i^F)\{[\bar{m}^F - \hat{m}^F s_t]\bar{q}^F - \lambda^F \gamma\sigma^2[(\bar{m}^F - \hat{m}^F s_t)\bar{\Sigma}^F - \bar{m}^F \hat{\Sigma}^F s_t]\} \\
\bar{q}^H + \lambda^H s_t &= (1+i^H)(1-\chi)\{[\bar{m}^F - \hat{m}^F s_t]\bar{q}^H + \lambda^H \gamma\sigma^2[(\bar{m}^F - \hat{m}^F s_t)\bar{\Sigma}^F - \bar{m}^F \hat{\Sigma}^F s_t]\}.
\end{aligned}$$

allow solving for $\bar{m}^H, \hat{m}^H, \bar{\Sigma}^H$ and $\hat{\Sigma}^H$ and the last two equations allow solving for $\bar{m}^F, \hat{m}^F, \bar{\Sigma}^F$ and $\hat{\Sigma}^F$. The result for the home country is

$$\bar{m}^H = \frac{\bar{q}^H(\chi-1)\lambda^F - \bar{q}^F\lambda^H}{(1+i)(\chi-1)(\bar{q}^F\lambda^H + \bar{q}^H\lambda^F)}, \quad \text{(A1)}$$

$$\hat{m}^H = \frac{\chi\lambda^F\lambda^H}{(1+i)(\chi-1)(\bar{q}^F\lambda^H + \bar{q}^H\lambda^F)}, \quad \text{(A2)}$$

$$\bar{\Sigma}^H = \frac{\chi\bar{q}^H\bar{q}^F}{\sigma^2\gamma(\bar{q}^F\lambda^H - (\chi-1)\bar{q}^H\lambda^F)}, \quad \text{(A3)}$$

$$\hat{\Sigma}^H = -\frac{(1-\chi)(\bar{q}^F\lambda^H + \bar{q}^H\lambda^F)^2}{\sigma^2\gamma(\bar{q}^F\lambda^H - (\chi-1)\bar{q}^H\lambda^F)^2}, \quad \text{(A4)}$$

and symmetrically for the foreign country.

$$\bar{m}^F = \frac{\bar{q}^F(\chi-1)\lambda^H - \bar{q}^H\lambda^F}{(1+i)(\chi-1)(\bar{q}^F\lambda^H + \bar{q}^H\lambda^F)}, \quad \text{(A5)}$$

$$\hat{m}^F = \frac{\chi\lambda^F\lambda^H}{(1+i)(\chi-1)(\bar{q}^F\lambda^H + \bar{q}^H\lambda^F)}, \quad \text{(A6)}$$

$$\bar{\Sigma}^F = \frac{\chi\bar{q}^H\bar{q}^F}{\sigma^2\gamma(\bar{q}^H\lambda^F - (\chi-1)\bar{q}^F\lambda^H))}, \quad \text{(A7)}$$

$$\hat{\Sigma}^F = -\frac{(1-\chi)(\bar{q}^F\lambda^H + \bar{q}^H\lambda^F)^2}{\sigma^2\gamma(\bar{q}^H\lambda^F - (\chi-1)\bar{q}^F\lambda^H)^2}. \quad \text{(A8)}$$

The zero-order components of the home portfolio, $\bar{A}^H_H$ and $\bar{A}^H_F$ solve

$$\bar{m}^H = e^{\gamma(c^H_{y,s=0}-\mu^H_{s=0})+\frac{(\gamma\sigma\bar{\Sigma}^H)^2}{2}} \tag{A9}$$

$$\bar{\Sigma}^H = \kappa_o + (1+i)\bar{A}^H_H\lambda^H - (1+i)(1-\chi)\bar{A}^H_F\lambda^F, \tag{A10}$$

where, since $T^H_{s=0} = B^H\bar{q}^H i$,

$$\mu^H_{s=0} = y^H_o + (1+i)\bar{A}^H_H\bar{q}^H + (1+i)(1-\chi)\bar{A}^H_F\bar{q}^F \tag{A11}$$

$$c^H_{y,s=0} = y^H - B^H\bar{q}^H i - \bar{A}^H_H\bar{q}^H - \bar{A}^H_F\bar{q}^F \tag{A12}$$

Solving (A10) for $\bar{A}^H_H$ yields

$$\bar{A}^H_H = \frac{\bar{\Sigma}^H + (1+i)(1-\chi)\bar{A}^H_F\lambda^F - \kappa_o}{(1+i)\lambda^H} \tag{A13}$$

Substituting (A11) and (A12) into (A9), using (A13) and taking logs yields

$$\begin{aligned}\log\big(\bar{m}^H\big) &= \gamma(y^H - y^H_o - B^H\bar{q}^H i) - \gamma\Big((2+i)\bar{A}^H_H\bar{q}^H + (1+(1+i)(1-\chi))\bar{A}^H_F\bar{q}^F\Big) + \frac{(\gamma\sigma\bar{\Sigma}^H)^2}{2} \\ &= \gamma(y^H - y^H_o - B^H\bar{q}^H i) + \frac{(\gamma\sigma\bar{\Sigma}^H)^2}{2} - \gamma(1+(1+i)(1-\chi))\bar{A}^H_F\bar{q}^F \\ &\quad -\gamma(2+i)\bar{q}^H[\frac{\bar{\Sigma}^H + (1+i)(1-\chi)\bar{A}^H_F\lambda^F - \kappa_o}{(1+i)\lambda^H}],\end{aligned}$$

a linear equation in $\bar{A}^H_F$ that can be solved for $\bar{A}^H_F$ since the coefficient multiplying $\bar{A}^H_F$ is strictly negative in the symmetric case or more generally if λ^F and λ^H have the same sign. Note that at this point $\bar{\Sigma}^H$ is a number already solved for in (A3). Using this solution for $\bar{A}^H_F$ in (A13) yields a solution for $\bar{A}^H_H$. Equivalent arguments apply for the foreign country.

Derivation of Result 2 [First Order Portfolio]

The first-order components of the home portfolio, $\hat{A}^H_H$ and $\hat{A}^H_F$ solve

$$\hat{m}^H = \bar{m}^H[\gamma(\hat{c}^H_y - \hat{\mu}^H) + \gamma^2\sigma^2\bar{\Sigma}^H\hat{\Sigma}^H], \tag{A14}$$

$$\hat{\Sigma}^H = (1+i)B^H\hat{A}^H_H\lambda^H - (1+i)(1-\chi)B^F\hat{A}^H_F\lambda^F, \tag{A15}$$

where, since $\hat{T}^H = iB^H\lambda^H$,

$$\hat{c}^H_y = \kappa_y - iB^H\lambda^H - B^H\hat{A}^H_H\bar{q}^H - \bar{A}^H_H\lambda^H - B^F\hat{A}^H_F\bar{q}^F + \bar{A}^H_F\lambda^F, \tag{A16}$$

$$\hat{\mu}^H = \kappa_s + (1+i)B^H\hat{A}^H_H\bar{q}^H + (1+i)(1-\chi)B^F\hat{A}^H_F\bar{q}^F. \tag{A17}$$

Solving (A15) for $\hat{A}_H^H$ yields

$$\hat{A}_H^H = \frac{\hat{\Sigma}^H + (1+i)(1-\chi)B^F \hat{A}_F^H \lambda^F}{(1+i)B^H \lambda^H}. \tag{A18}$$

Substituting (A16) and (A17) into (A14) and using (A18)

$$\begin{aligned}
&\frac{\hat{m}^H}{\bar{m}^H} \qquad\qquad (A19)\\
&= \gamma\Big(\kappa_y - \kappa_s - iB^H\lambda^H - (2+i)B^H\hat{A}_H^H\bar{q}^H - \bar{A}_H^H\lambda^H - (1+(1+i)(1-\chi))B^F\hat{A}_F^H\bar{q}^F + \bar{A}_F^H\lambda^F\Big)\\
&\quad + \gamma^2\sigma^2\bar{\Sigma}^H\hat{\Sigma}^H\\
&= \gamma\Big(\kappa_y - \kappa_s - iB^H\lambda^H - (2+i)[\frac{\hat{\Sigma}^H + (1+i)(1-\chi)B^F\hat{A}_F^H\lambda^F}{(1+i)B^H\lambda^H}]\bar{q}^H\Big)\\
&\quad - \gamma\Big(\bar{A}_H^H\lambda^H + (1+(1+i)(1-\chi))B^F\hat{A}_F^H\bar{q}^F - \bar{A}_F^H\lambda^F\Big) + \gamma^2\sigma^2\bar{\Sigma}^H\hat{\Sigma}^H
\end{aligned}$$

a linear equation in $\hat{A}_F^H$ that can be solved for $\hat{A}_F^H$ since the coefficient multiplying $\hat{A}_F^H$ is strictly negative in the symmetric case or more generally if λ^F and λ^H have the same sign. Note that at this point $\bar{\Sigma}^H$ and $\hat{\Sigma}^H$ are numbers already solved for in (A3) and (A4) and the zero-order portfolios $\bar{A}_H^H$ and $\bar{A}_F^H$ were solved for in the proof of Result 1. Using this solution for $\hat{A}_F^H$ in (A18) yields a solution for $\hat{A}_H^H$. Equivalent arguments apply for the foreign country.

Derivation of Result 3 [Ruling out Autarky]

Result 3 claims that autarky,

$$A_{H,s}^H = B^H, \qquad A_{F,s}^F = B^F$$

is not an equilibrium.

Suppose it is, and that investors follow an autarky strategy and hold their own bonds only, $A_H^H = B^H, A_F^H = 0$ and $A_F^F = B^F, A_H^F = 0$. Suppose furthermore first that prices are not risky, $\lambda^H = \lambda^F = 0$. The first-order condition for home investors in home bonds would then be

$$\bar{q}^H = (1+i^H)(\bar{m}^H + s\hat{m}^H\bar{q}^H), \tag{A20}$$

which is a contradiction, since in autarky with $\lambda^H = \lambda^F = 0$ we have $\hat{c}_y^H = \kappa_y$ and $\hat{\mu}^H = \kappa_s$, so that $\hat{m}^H = \kappa_y - \kappa_s$ and thus $\kappa_y \neq \kappa_s$ implies $\hat{m}^H \neq 0$ and thus, condition (A20) cannot hold for all s.

Thus $\lambda^H = \lambda^F \neq 0$ for autarky to be an equilibrium. Autarky implies that $\hat{\Sigma}^H = \hat{\Sigma}^F = 0$ and thus the FOC (18) for home investors buying home bonds implies using symmetry:

$$\bar{q} = (1+i)\bar{m}[\bar{q} - \lambda\gamma\sigma^2\bar{\Sigma}] \quad \text{and}$$
$$\lambda = (1+i)\hat{m}[\bar{q} - \lambda\gamma\sigma^2\bar{\Sigma}],$$

so that $[\bar{q} - \lambda\gamma\sigma^2\bar{\Sigma}] = \frac{\bar{q}}{(1+i)\bar{m}}$ and

$$\lambda = (1+i)\hat{m}[\bar{q} - \lambda\gamma\sigma^2\bar{\Sigma}] = (1+i)\hat{m}\frac{\bar{q}}{(1+i)\bar{m}} = \frac{\bar{q}\hat{m}}{\bar{m}} \tag{A21}$$

Similarly, the FOC (24) for home investors buying foreign bonds implies using symmetry:

$$\bar{q} = (1+i)(1-\chi)\bar{m}[\bar{q} + \lambda\gamma\sigma^2\bar{\Sigma}] \quad \text{and}$$
$$-\lambda = (1+i)(1-\chi)\hat{m}[\bar{q} + \lambda\gamma\sigma^2\bar{\Sigma}],$$

so that $[\bar{q} + \lambda\gamma\sigma^2\bar{\Sigma}] = \frac{\bar{q}}{(1+i)(1-\chi)\bar{m}}$ and

$$-\lambda = (1+i)(1-\chi)\hat{m}[\bar{q} + \lambda\gamma\sigma^2\bar{\Sigma}] = (1+i)(1-\chi)\hat{m}\frac{\bar{q}}{(1+i)(1-\chi)\bar{m}} = \frac{\bar{q}\hat{m}}{\bar{m}}. \tag{A22}$$

Equations (A21) and (A21) together impliy $\lambda = -\lambda$ and thus $\lambda = 0$, which was already ruled out above, establishing that autarky is not an equilibrium.

Derivation of Result 4 [Presence of Risk]

The result for the symmetric case follows from the proof of Result 1 for the asymmetric case using $\bar{q} = \bar{q}^H = \bar{q}^F$ and $\lambda = \lambda^H = \lambda^F$ in equation (A1) - (A4). Note that symmetry implies that equations (A1) - (A4) and (A5) - (A8) are now identical.

The linearized exchange rate is

$$\epsilon_s = \frac{\bar{q} - \lambda s}{\bar{q} + \lambda s} \approx \bar{\epsilon} + s\hat{\epsilon} = 1 - \frac{2\lambda}{\bar{q}}s,$$

such that the variance equals

$$Var[\bar{\epsilon} + s\hat{\epsilon}] = \sigma^2(\frac{2\lambda}{\bar{q}})^2 > 0.$$

The same arguments but without invoking the symmetry assumption establish the asymmetric case.

Derivation of Result 5 [Portfolio Choices (Symmetric World)]:

Inspecting the first-order conditions (18), (24), (25) and (26) and the market clearing conditions (46) and (47) shows that in an equilibrium, that allows for $B^H \neq B^F$ but is otherwise symmetric, we have $\bar{q}^F = \bar{q}^H \frac{B^H}{B^F}$, $\lambda^F = \lambda^H \frac{B^H}{B^F}$, $\bar{m}^H = \bar{m}^F$, $\hat{m}^H = \hat{m}^F$, $\hat{\Sigma}^H = \hat{\Sigma}^F$ and $\bar{\Sigma}^H = \bar{\Sigma}^F$ and for the portfolio choices $\bar{A}^H_H = B^H - \bar{A}^F_H = \bar{A}^F_F \frac{B^H}{B^F}$, $\bar{A}^H_F = B^F - \bar{A}^F_F = (B^H - \bar{A}^H_H)\frac{B^F}{B^H} = \bar{A}^F_H \frac{B^F}{B^H}$ and $\hat{A}^H_H = \hat{A}^H_F = \hat{A}^F_H = \hat{A}^F_F$.

Using symmetry, the first-order condition of home investors in home bonds (18) is

$$\bar{q}^H + \lambda^H s = [\bar{m}^H + \hat{m}^H s](1+i^H)\bar{q}^H - \lambda^H \gamma\sigma^2(1+i^H)[(\bar{m}^H + \hat{m}^H s)\bar{\Sigma}^H + \bar{m}^H \hat{\Sigma}^H s],$$

and the FOC for home investors investing in foreign bonds, (24), after multiplying with B^F/B^H, simplifies to

$$\begin{aligned} & \bar{q}^H - \lambda^H s \\ = {} & \frac{B^F}{B^H}(\bar{q}^F - \lambda^F s) \\ = {} & \frac{B^F}{B^H}\Big\{(1+i^F)(1-\chi)\{[\bar{m}^H + \hat{m}^H s]\bar{q}^F + \lambda^F\gamma\sigma^2[(\bar{m}^H + \hat{m}^H s)\bar{\Sigma}^H + \bar{m}^H\hat{\Sigma}^H s]\}\Big\} \\ = {} & (1+i^H)(1-\chi)\{[\bar{m}^H + \hat{m}^H s]\bar{q}^H + \lambda^H\gamma\sigma^2[(\bar{m}^H + \hat{m}^H s)\bar{\Sigma}^H + \bar{m}^H\hat{\Sigma}^H s]\} \end{aligned}$$

To compute the zero-order portfolio $\bar{A}^H_H$ evaluate these two first-order conditions at $s = 0$, such that their difference is, using $i^H = i^F$,

$$0 = (1+i)\bar{m}^H[\chi\bar{q}^H - (2-\chi)\lambda^H\gamma\sigma^2\bar{\Sigma}^H]. \tag{A23}$$

Using equation (33) for $\bar{\Sigma}^H$ and that symmetry implies $\bar{A}^H_F\lambda^F = (B^H - \bar{A}^H_H)\lambda^H$,

$$\begin{aligned} \bar{\Sigma}^H &= \kappa_o + (1+i^H)\bar{A}^H_H\lambda^H - (1+i^H)(1-\chi)\bar{A}^H_F\lambda^F \\ &= \kappa_o + (1+i^H)\bar{A}^H_H\lambda^H - (1+i^H)(1-\chi)(B^H - \bar{A}^H_H)\lambda^H \\ &= \kappa_o + (1+i^H)\bar{A}^H_H\lambda^H(2-\chi) - (1+i^H)(1-\chi)\lambda^H B^H, \end{aligned}$$

allows solving (A23) for

$$\bar{A}^H_H = B^H\frac{1-\chi}{2-\chi} + \frac{\kappa_o}{(\chi-2)\lambda^H(1+i)} + \frac{\bar{q}^H\chi}{(\chi-2)^2(1+i)\gamma\sigma^2(\lambda^H)^2}.$$

To compute the first-order component, I again use the symmetry of the same first-order conditions for home investors, but now consider the s-terms only:

$$\begin{aligned} 2\lambda^H &= (1+i^H)\hat{m}^H[\chi\bar{q}^H - (2-\chi)\lambda^H\gamma\sigma^2\bar{\Sigma}^H] - (1+i^H)\bar{m}^H(2-\chi)\lambda^H\gamma\sigma^2\hat{\Sigma}^H \\ &= -(1+i^H)\bar{m}^H(2-\chi)\lambda^H\gamma\sigma^2\hat{\Sigma}^H, \end{aligned} \tag{A24}$$

where the last equality used (A23). Using equation (34), $B^F \hat{A}_F^H \lambda^F = B^H \hat{A}_H^H \lambda^H$ and setting $i^H = i$,

$$\hat{\Sigma}^H = (1+i)B^H \hat{A}_H^H \lambda^H - (1+i)(1-\chi)B^H \hat{A}_H^H \lambda^H = (1+i)\chi B^H \hat{A}_H^H \lambda^H,$$

such that, using (A24),

$$B^H \hat{A}_H^H = \frac{\hat{\Sigma}^H}{(1+i)\chi\lambda^H} = \frac{-2}{(1+i)^2(2-\chi)\chi\lambda^H \bar{m}^H \gamma\sigma^2}.$$

Using $\bar{m}^H = \frac{1}{1+i}\frac{2-\chi}{2(1-\chi)}$ (see Result 4) yields

$$B^H \hat{A}_H^H = \frac{4(\chi-1)}{\chi\gamma(1+i)\lambda^H\sigma^2(2-\chi)^2}.$$

Analogous derivations yield the claims for $\bar{A}_F^F$ and $B^F \hat{A}_F^F$.

Derivation of Result 8 [Limit Portfolios and Exchange Rates]

I first compute the limit nominal portfolios, that is, the portfolio choices when $\chi, \sigma \to 0$ and in a second step turn to exchange rates. Note that the shock s is normalized by its standard deviation σ that is, $s = 1$ means a one-standard deviation shock of size σ. Taking the limit of $\sigma \to 0$ requires making this implicit normalization explicit, so that the linearized first order conditions are

$$\begin{aligned}
\bar{q}^H + \lambda^H \sigma s_t &= [\bar{m}^H + \hat{m}^H s_t\sigma](1+i^H)\bar{q}^H - \lambda^H\gamma\sigma^2(1+i^H)[(\bar{m}^H + \hat{m}^H s_t\sigma)\bar{\Sigma}^H + \bar{m}^H\hat{\Sigma}^H s_t\sigma], \\
\bar{q}^F - \lambda^F \sigma s_t &= (1+i^F)(1-\chi)\{[\bar{m}^H + \hat{m}^H s_t\sigma]\bar{q}^F + \lambda^F\gamma\sigma^2[(\bar{m}^H + \hat{m}^H s_t\sigma)\bar{\Sigma}^H + \bar{m}^H\hat{\Sigma}^H s_t\sigma]\} \\
\bar{q}^F - \lambda^F \sigma s_t &= (1+i^F)\{[\bar{m}^F - \hat{m}^F s_t\sigma]\bar{q}^F - \lambda^F\gamma\sigma^2[(\bar{m}^F - \hat{m}^F s_t\sigma)\bar{\Sigma}^F - \bar{m}^F\hat{\Sigma}^F s_t\sigma]\} \\
\bar{q}^H + \lambda^H \sigma s_t &= (1+i^H)(1-\chi)\{[\bar{m}^F - \hat{m}^F s_t\sigma]\bar{q}^H + \lambda^H\gamma\sigma^2[(\bar{m}^F - \hat{m}^F s_t\sigma)\bar{\Sigma}^F - \bar{m}^F\hat{\Sigma}^F s_t\sigma]\}.
\end{aligned}$$

Note first that equations (A1) - (A4) and (A5) - (A8) imply that $\bar{m}^H$, $\bar{m}^F$, $\sigma\hat{m}^H$, $\sigma\hat{m}^F$, $\sigma\bar{\Sigma}^H$, $\sigma\bar{\Sigma}^F$, $\sigma^2\hat{\Sigma}^H$ and $\sigma^2\hat{\Sigma}^F$ depend on $\tilde{\lambda}^H := \lambda^H\sigma$ and $\tilde{\lambda}^F := \lambda^F\sigma$ but not on λ^H, λ^F or σ separately. The same is then also true for the first-order conditions, which also depend on $\tilde{\lambda}^H$ and $\tilde{\lambda}^F$ but not on λ^H, λ^F or σ separately. This means that if the solution for prices $\bar{q}^H$, λ^H, $\bar{q}^F$ and λ^F for $\sigma = 1$ is known, solutions for other values of σ are simply $\bar{q}^H$, λ^H/σ, $\bar{q}^F$ and λ^F/σ. That is, the product of the amount of risk σ and its price λ remains constant, although both individually change with σ.

For the symmetric case, this allows for a solution for the limit portfolios using Result 5, where I use the fact that both $\tilde{\lambda} = \tilde{\lambda}^H = \tilde{\lambda}^F$ and $\tilde{\lambda}/\bar{q}$ are unbounded from above if $\chi \to 0$: assuming for now that $\bar{q}$ is bounded and suppose to the contrary that $\tilde{\lambda}/\bar{q}$ was bounded and since $\bar{q}$ is bounded, $\tilde{\lambda}$ would also be bounded. Symmetry implies that $\hat{A}_H^H = \hat{A}_F^F = \hat{A}_F^H$ and the previous analysis shows that

$$B^H \bar{q} \hat{A}_H^H \sigma = \frac{4(\chi-1)\bar{q}}{\chi\gamma(1+i)\tilde{\lambda}(2-\chi)^2}$$

and $B^H \bar{q} \hat{A}_F^H \sigma$ would both become unbounded when $\chi \to 0$. Since the two unbounded terms have the same sign and all remaining terms in equation (A25) multiplied with σ,

$$\begin{aligned} & \sigma\frac{\hat{m}^H}{\bar{m}^H} \qquad \text{(A25)} \\ = \; & \gamma\Big((\kappa_y-\kappa_s)\sigma - iB^H\tilde{\lambda}^H - (2+i)B^H\hat{A}_H^H\sigma\bar{q}^H - \bar{A}_H^H\tilde{\lambda}^H - (1+(1+i)(1-\chi))B^F\hat{A}_F^H\sigma\bar{q}^F + \bar{A}_F^H\tilde{\lambda}^F\Big) \\ & +\gamma^2\sigma^2\bar{\Sigma}^H\hat{\Sigma}^H\sigma \end{aligned}$$

are bounded since $(\sigma\hat{m}, \sigma\bar{\Sigma}, \sigma^2\hat{\Sigma}, \bar{A}_H^H, \bar{A}_F^H)$ are bounded, this cannot be a solution to the first-order condition (A25). Thus $\tilde{\lambda}/\bar{q}$ is unbounded and since $\bar{q}$ is bounded, $\tilde{\lambda}$ is also unbounded. Using this result and taking the limit:

$$\lim_{\sigma,\chi\to 0} \bar{A}_H^H = \lim_{\sigma,\chi\to 0}\{B^H\frac{1-\chi}{2-\chi} + \frac{\kappa_o}{(\chi-2)\tilde{\lambda}(1+i)} + \frac{\bar{q}^H\chi}{(\chi-2)^2(1+i)\gamma\tilde{\lambda}^2}\} = \frac{B^H}{2}.$$

The claim for $\bar{A}_H^F$ follows since $\bar{A}_H^F = B^H - \bar{A}_H^H$. Equivalent arguments show that

$$\lim_{\sigma,\chi\to 0} \bar{A}_F^F = \frac{B^F}{2} = \lim_{\sigma,\chi\to 0} \bar{A}_F^H.$$

Furthermore,

$$\begin{aligned} \lim_{\sigma,\chi\to 0} \bar{m}^H &= \lim_{\sigma,\chi\to 0} \frac{\bar{q}^H(\chi-1)\tilde{\lambda}^F - \bar{q}^F\tilde{\lambda}^H}{(1+i)(\chi-1)(\bar{q}^F\tilde{\lambda}^H + \bar{q}^H\tilde{\lambda}^F)} = \frac{1}{1+i}, \\ \lim_{\sigma,\chi\to 0} \sigma\bar{\Sigma}^H &= \lim_{\sigma,\chi\to 0} \frac{\chi\bar{q}^H\bar{q}^F}{\gamma(\bar{q}^F\tilde{\lambda}^H - (\chi-1)\bar{q}^H\tilde{\lambda}^F)} = 0, \end{aligned}$$

using the fact that $\tilde{\lambda}/\bar{q}$ and $\tilde{\lambda}$ are unbounded. Using that $\bar{q}^H B^H = \bar{q}^F B^F$,

$$\begin{aligned} \lim_{\sigma,\chi\to 0} \mu_{H,s=0} &= \lim_{\sigma,\chi\to 0}\{y_o + (1+i)\bar{A}_H^H\bar{q}^H + (1+i)(1-\chi)\bar{A}_F^H\bar{q}^F\} = y_o + (1+i)B^H \lim_{\sigma,\chi\to 0} \bar{q}^H \\ \lim_{\sigma,\chi\to 0} c_{y,s=0}^H &= \lim_{\sigma,\chi\to 0}\{y - B^H\bar{q}^H i - \bar{A}_H^H\bar{q}^H - \bar{A}_F^H\bar{q}^F\} = y - B^H(1+i) \lim_{\sigma,\chi\to 0} \bar{q}^H, \end{aligned}$$

so that

$$\lim_{\sigma,\chi\to 0} e^{\gamma(c_{s=0}^H - \mu_{s=0}^H) + \frac{(\gamma\sigma\bar{\Sigma}^H)^2}{2}} = \lim_{\sigma,\chi\to 0} e^{\gamma(y - B^H\bar{q}^H(1+i) - y_o - (1+i)B^H\bar{q}^H) + 0}.$$

The price $\bar{q}^H$ then solves

$$\lim_{\sigma,\chi\to 0} \bar{m}^H = \frac{1}{1+i} \quad = \quad \lim_{\sigma,\chi\to 0} e^{\gamma(y-y_o-2B^H\bar{q}^H(1+i))},$$

implying

$$\lim_{\sigma,\chi\to 0} \bar{q}^H = \frac{y-y_o-\log\left(\frac{1}{1+i}\right)/\gamma}{2B^H(1+i)}.$$

This also confirms that $\bar{q}^H$ is bounded. If it was not, $\tilde{\lambda}$ would still be unbounded, $\bar{q}^H$ solves

$$\frac{1}{1+i} \quad = \quad \lim_{\sigma,\chi\to 0} e^{\gamma(c^H_{y,s=0}-\mu^H_{s=0})+\frac{(\gamma\sigma\bar{\Sigma}^H)^2}{2}}$$

and the same derivations show that all terms on the RHS are finite, so that a finite $\bar{q}$ solves this equation, contradicting the notion that $\bar{q}$ is unbounded.

Similarly, for the foreign country

$$\lim_{\sigma,\chi\to 0} \bar{q}^F = \frac{y-y_o-\log\left(\frac{1}{1+i}\right)/\gamma}{2B^F(1+i)}.$$

The results for S^H_H, S^H_F, S^F_F and S^F_H follow from the definitions in (48).

Derivation of Result 9 [Ruling out Autarky with Monetary Policy]

Before showing that autarky cannot be an equilibrium we have to extend the model to allow for an interest rate rule. In the model with an interest rate rule and endowment shocks the first-order condition for home bonds acquired by home households, $A^H_{H,s}$, is

$$\begin{aligned} & (\bar{q}^H+\lambda^H s_t) \qquad\qquad (A26)\\ = \; & E\Big[(e^{-\gamma(c^H_{o,s_{t+1},s_t}-c^H_{y,s_t})})[(1+i^H+\phi^H_i s_t)(\bar{q}^H+\lambda^H s_{t+1})]\Big]\\ = \; & E[(e^{-\gamma(c^H_{o,s_{t+1},s_t}-c^H_{y,s_t})})][(1+i^H+\phi^H_i s_t)\bar{q}^H]+Cov[e^{-\gamma(c^H_{o,s_{t+1},s_t}-c^H_{y,s_t})},(1+i^H+\phi^H_i s_t)\lambda^H s_{t+1}]\\ = \; & \underbrace{[e^{-\gamma(\mu^H_{s_t}-c^H_{y,s_t})+\frac{(\gamma\sigma\Sigma^H_{s_t})^2}{2}}]}_{\text{E(SDF)}}\underbrace{[(1+i^H+\phi^H_i s_t)\bar{q}^H]}_{\text{E(Payoff)}}-\underbrace{[e^{-\gamma(\mu^H_{s_t}-c^H_{y,s_t})+\frac{(\gamma\sigma\Sigma^H_{s_t})^2}{2}}](1+i^H+\phi^H_i s_t)\lambda^H\gamma\sigma^2\Sigma^H_{s_t}}_{\text{Cov(SDF, Payoff)}},\end{aligned}$$

where now

$$c^H_{y,s_t} = y^H+\kappa_y s_t - A^H_{H,s_t}(\bar{q}^H+\lambda^H s_t) - A^H_{F,s_t}(\bar{q}^F-\lambda^F s_t) - T^H_{s_t}, \qquad (A27)$$

$$\mu^H_{s_t} = y^H_o+\kappa_s s_t+(1+i^H+\phi^H_i s_t)A^H_{H,s_t}\bar{q}^H+(1+i^F-\phi^F_i s_t)(1-\chi)A^H_{F,s_t}\bar{q}^F \qquad (A28)$$

is the mean and

$$\sigma\Sigma^H_{s_t} = \sigma[\kappa_o + (1+i^H+\phi^H_i s_t)A^H_{H,s_t}\lambda^H - (1+i^F-\phi^F_i s_t)(1-\chi)A^H_{F,s_t}\lambda^F] \tag{A29}$$

is the standard deviation of old age home consumption, which is approximated as

$$\Sigma^H_{s_t} \approx \bar{\Sigma}^H + \hat{\Sigma}^H s_t, \tag{A30}$$

with

$$\bar{\Sigma}^H = \kappa_o + (1+i^H)\bar{A}^H_H\lambda^H - (1+i^F)(1-\chi)\bar{A}^H_F\lambda^F, \tag{A31}$$

$$\hat{\Sigma}^H = (1+i^H)B^H\hat{A}^H_H\lambda^H + \phi^H_i\bar{A}^H_H\lambda^H - (1+i^F)(1-\chi)B^F\hat{A}^H_F\lambda^F + \phi^F_i(1-\chi)\bar{A}^H_F\lambda^F \tag{A32}$$

Assume again $T^H_s = i^H B^H q^H_s$ so that government expenditure adjusts to absorb some of the changes in interest rate payments equal to $B^H\bar{q}^H\phi^H_i s_{t-1}$, eliminating history dependence in these payments which is needed for tractability. However, this does not mean that all interest rate payments are absorbed since total (linearized) period t interest rate payments are

$$B^H\bar{q}^H(i^H+\phi^H_i s_{t-1}) + B^H i^H\lambda^H s_t = \underbrace{B^H i^H q^H_s}_{=T^H_s} + B^H\bar{q}^H\phi^H_i s_{t-1}$$

and $T^H_s = i^H B^H q^H_s$ taxes are raised to cover the remaining interest rate payments.

Using these results, a linear approximation of the FOC with respect to s_t yields

$$\begin{aligned}&\bar{q}^H + \lambda^H s_t\\ = \;& (1+i^H)\{[\bar{m}^H(1+\frac{\phi^H_i}{1+i^H}s_t) + \hat{m}^H s_t]\bar{q}^H - \lambda^H\gamma\sigma^2[(\bar{m}^H(1+\frac{\phi^H_i}{1+i^H}s_t) + \hat{m}^H s_t)\bar{\Sigma}^H + \bar{m}^H\hat{\Sigma}^H s_t]\}.\end{aligned}$$

The other first-order conditions can be derived analogously and yield the linearizations:

Home Investors: Foreign Bonds A^H_{F,s_t}

$$\begin{aligned}&\bar{q}^F - \lambda^F s_t\\ = \;& (1+i^F)(1-\chi)\{[\bar{m}^H(1-\frac{\phi^F_i}{1+i^F}s_t) + \hat{m}^H s_t]\bar{q}^F + \lambda^F\gamma\sigma^2[(\bar{m}^H(1-\frac{\phi^F_i}{1+i^F}s_t) + \hat{m}^H s_t)\bar{\Sigma}^H + \bar{m}^H\hat{\Sigma}^H s_t]\}\end{aligned}$$

Foreign Investors: Foreign Bonds A^F_{F,s_t}

$$\begin{aligned}&\bar{q}^F - \lambda^F s_t\\ = \;& (1+i^F)\{[\bar{m}^F(1-\frac{\phi^F_i}{1+i^F}s_t) - \hat{m}^F s_t]\bar{q}^F - \lambda^F\gamma\sigma^2[(\bar{m}^F(1-\frac{\phi^F_i}{1+i^F}s_t) - \hat{m}^F s_t)\bar{\Sigma}^F - \bar{m}^F\hat{\Sigma}^F s_t]\}\end{aligned}$$

Foreign Investors: Home Bonds A^F_{H,s_t}

$$
\begin{aligned}
& \bar{q}^H + \lambda^H s_t \\
= \quad & (1+i^H)(1-\chi)\{[\bar{m}^F(1+\frac{\phi_i}{1+i^H}s_t) - \hat{m}^F s_t]\bar{q}^H + \lambda^H \gamma\sigma^2[(\bar{m}^F(1+\frac{\phi_i}{1+i^H}s_t) - \hat{m}^F s_t)\bar{\Sigma}^F - \bar{m}^F\hat{\Sigma}^F s_t]\},
\end{aligned}
$$

I can now show that autarky,

$$
A^H_{H,s} = B^H, \qquad A^F_{F,s} = B^F
$$

is not an equilibrium.

Suppose it is, and that investors follow an autarky strategy and hold their own bonds only, $A^H_H = B^H, A^H_F = 0$ and $A^F_F = B^F, A^F_H = 0$. Suppose furthermore first that prices are not risky, $\lambda^H = \lambda^F = 0$. The first-order condition for home investors in home bonds would then be, using $i = i^H = i^F$ and $\phi_i = \phi^H_i = \phi^F_i$,

$$
\bar{q}^H = (1+i)[\bar{m}^H(1+\frac{\phi_i}{1+i}s_t) + \hat{m}^H s_t]\bar{q}^H,
$$

which requires $\bar{m}^H\frac{\phi_i}{1+i} + \hat{m}^H = 0$ for the FOC to hold for all s_t.

The first-order condition for home investors in foreign bonds would be

$$
\bar{q}^F = (1+i)(1-\chi)[\bar{m}^H(1-\frac{\phi_i}{1+i}s_t) + \hat{m}^H s_t]\bar{q}^F,
$$

which is equivalent, using $\hat{m}^H = -\bar{m}^H\frac{\phi_i}{1+i}$, to

$$
\bar{q}^F = (1+i)(1-\chi)\{[\bar{m}^H(1-\frac{\phi_i}{1+i}s_t) - \bar{m}^H\frac{\phi_i}{1+i}s_t]\bar{q}^F,
$$

implying $\phi_i = 0$ and thus $\hat{m}^H = 0$, which is, as in the proof of Result 3, a contradiction, since $\kappa_y \neq \kappa_s$ implies $\hat{m}^H \neq 0$.

Thus $\lambda^H \neq 0$ and similarly for foreign investors, $\lambda^F \neq 0$, where symmetry implies $\lambda = \lambda^F = \lambda^H \neq 0$.

A necessary condition for an autarkic equilibrium is that for home investing in home bonds, using $\bar{q} = \bar{q}^H = \bar{q}^F$, $B = B^H = B^F$ and $i = i^H = i^F$, is that the FOC holds at $s = 0$,

$$
\bar{q} = (1+i)[\bar{m}^H\bar{q} - \lambda\gamma\sigma^2\bar{m}^H\bar{\Sigma}^H]. \tag{A33}
$$

If $\lambda\bar{\Sigma}^H > 0$, then $\lambda\gamma\sigma^2\bar{m}^H\bar{\Sigma}^H > 0$ and thus (A33) implies $(1+i)\bar{m}^H > 1$. Autarky also means that home investors do not find it profitable to invest in foreign bonds, requiring

$$
\bar{q} > (1+i)(1-\chi)\{\bar{m}^H\bar{q} + \lambda\gamma\sigma^2\bar{m}^H\bar{\Sigma}^H\} \tag{A34}
$$

Since $(1+i)\{\bar{m}^H\bar{q}+\lambda\gamma\sigma^2\bar{m}^H\bar{\Sigma}^H\} > \bar{q}$ from (A33) , condition (A34) is not satisfied for sufficiently small transaction costs χ, $\lambda\bar{\Sigma}^H > 0$ and λ bounded away from zero, establishing that autarky is not an equilibrium under these conditions.

Note that since $\lambda\bar{\Sigma}^H > 0$ for all $\lambda > 0$, autarky is not an equilibrium if λ is larger than an arbitrarily small positive bound.

What remains to be shown is that autarky is also not an equilibrium if $\lambda\bar{\Sigma} \leq 0$ or if λ is positive but small. To this aim, I first use that the first-order condition for home households investing in home bonds implies

$$\lambda = (1+i)\{[\bar{m}^H\frac{\phi_i}{1+i} + \hat{m}^H]\bar{q} - \lambda\gamma\sigma^2[(\bar{m}^H\frac{\phi_i}{1+i} + \hat{m}^H)\bar{\Sigma}^H + \bar{m}^H\hat{\Sigma}^H]\}. \tag{A35}$$

Using (A33) this is equivalent to

$$\lambda = \bar{q}\frac{\phi_i}{1+i} + (1+i)\{\hat{m}^H\bar{q} - \lambda\gamma\sigma^2[\hat{m}^H\bar{\Sigma}^H + \bar{m}^H\hat{\Sigma}^H]\}. \tag{A36}$$

Furthermore,

$$\hat{m}^H = \bar{m}^H[\gamma(\hat{c}_y^H - \hat{\mu}^H) + \gamma^2\sigma^2\bar{\Sigma}^H\hat{\Sigma}^H], \tag{A37}$$

where, since $\hat{T}^H = iB\lambda$,

$$\hat{c}_y^H = \kappa_y - (1+i)B\lambda, \tag{A38}$$

$$\hat{\mu}^H = \kappa_s. \tag{A39}$$

This implies that for $\lambda \leq 0$ or small positive λ, since $\kappa_y > \kappa_s$,

$$\tilde{m}^H := \gamma(\hat{c}_y^H - \hat{\mu}^H) = \gamma(\kappa_y - \kappa_s - (1+i)B\lambda) > 0. \tag{A40}$$

In autarky, $\hat{\Sigma}^H = \phi_i B\lambda$, so that rewriting (A36) yields

$$\lambda = \bar{q}\frac{\phi_i}{1+i} + (1+i)\{[\bar{m}^H\tilde{m}^H + \underbrace{\phi_i\gamma^2\sigma^2\bar{m}^H\bar{\Sigma}^H B\lambda}_{=\gamma^2\sigma^2\bar{m}^H\bar{\Sigma}^H\hat{\Sigma}^H}][\bar{q} - \lambda\gamma\sigma^2\bar{\Sigma}^H] - \underbrace{\phi_i\lambda\gamma\sigma^2\bar{m}^H B\lambda}_{=\lambda\gamma\sigma^2\bar{m}^H\hat{\Sigma}^H}\},$$

which is equivalent, using (A33), to

$$\lambda = \bar{q}\tilde{m}^H + \phi_i\Big\{\frac{\bar{q}}{1+i} + \gamma^2\sigma^2\bar{\Sigma}^H B\lambda\bar{q} - \lambda\gamma\sigma^2(1+i)\bar{m}^H B\lambda\Big\}. \tag{A41}$$

Autarky also requires that home investors do not hold foreign bonds. Since this has to hold for all states s it is necessary that

$$-\lambda = (1+i)(1-\chi)\{[-\bar{m}^H \frac{\phi_i}{1+i} + \hat{m}^H]\bar{q} + \lambda\gamma\sigma^2[(-\bar{m}^H \frac{\phi_i}{1+i} + \hat{m}^H)\bar{\Sigma}^H + \bar{m}^H\hat{\Sigma}^H]\}, \quad \text{(A42)}$$

which is equivalent to, using $\hat{m}^H = \bar{m}^H[\tilde{m}^H + \gamma^2\sigma^2\bar{\Sigma}^H\hat{\Sigma}^H]$ and $\hat{\Sigma}^H = \phi_i B\lambda$,

$$\begin{aligned}
-\lambda &= (1+i)(1-\chi)\bar{m}^H\tilde{m}^H[\bar{q} + \lambda\gamma\sigma^2\bar{\Sigma}^H] \\
&+ (1-\chi)\phi_i\Big\{ -\bar{m}^H\bar{q} - \lambda\gamma\sigma^2\bar{m}^H\bar{\Sigma}^H + (1+i)\gamma^2\sigma^2\bar{m}^H\bar{\Sigma}^H B\lambda[\bar{q} + \lambda\gamma\sigma^2\bar{\Sigma}^H] + (1+i)\lambda\gamma\sigma^2\bar{m}^H B\lambda\Big\} \\
&= (1-\chi)\bar{q}\tilde{m}^H[2\bar{m}^H(1+i) - 1] \qquad \text{(A43)} \\
&+ (1-\chi)\phi_i\Big\{\bar{q}[\frac{1}{1+i} - 2\bar{m}^H] + \gamma^2\sigma^2\bar{\Sigma}^H B\lambda\bar{q}[2\bar{m}^H(1+i) - 1] + (1+i)\lambda\gamma\sigma^2\bar{m}^H B\lambda\Big\},
\end{aligned}$$

where I used that (A33) implies $(1+i)\lambda\gamma\sigma^2\bar{m}^H\bar{\Sigma}^H = \bar{q}[\bar{m}^H(1+i) - 1]$.

I can solve (A41) and (A43) for ϕ as a function of all other parameters, resulting in the definitions

$$\begin{aligned}
\Phi^H &:= \frac{\lambda - \bar{q}\tilde{m}^H}{\frac{\bar{q}}{1+i} + \gamma^2\sigma^2\bar{\Sigma}^H B\lambda\bar{q} - \lambda\gamma\sigma^2(1+i)\bar{m}^H B\lambda}, \\
\Phi^F &:= \frac{1}{1-\chi}\frac{\lambda - (1-\chi)\bar{q}\tilde{m}^H[1 - 2(1+i)\bar{m}^H]}{\frac{-\bar{q}}{1+i}[1 - 2(1+i)\bar{m}^H] + \gamma^2\sigma^2\bar{\Sigma}^H B\lambda\bar{q}[1 - 2(1+i)\bar{m}^H] - (1+i)\lambda\gamma\sigma^2\bar{m}^H B\lambda}.
\end{aligned}$$

The remainder of the proof shows that $\Phi^F - \Phi^H \neq 0$ for small κ_o and $\lambda\bar{\Sigma} < 0$ (or small positive λ), implying that there is no combination of (λ, ϕ) that solves both necessary equilibrium conditions (A35) and (A42).

I first establish $\Phi^F - \Phi^H \neq 0$ for $\kappa_o = 0$ and $\lambda = 0$ and then use a continuity argument to show the result for small κ_o and λ. For $(\kappa_o, \lambda) = (0, 0)$ we obtain using $\tilde{m}^H > 0$ from (A40),

$$\begin{aligned}
\Phi^F &= \frac{1}{1-\chi}\frac{(1-\chi)\bar{q}\tilde{m}^H[2(1+i)\bar{m}^H - 1]}{\frac{\bar{q}}{1+i}[2(1+i)\bar{m}^H - 1]} = (1+i)\tilde{m}^H > 0, \\
\Phi^H &= \frac{-\bar{q}\tilde{m}^H}{\frac{\bar{q}}{1+i}} = -(1+i)\tilde{m}^H < 0,
\end{aligned}$$

implying $\Phi^F(\kappa_o = 0, \lambda = 0) - \Phi^H(\kappa_o = 0, \lambda = 0) > 0$.

Since the inequality is strict and the function $\Phi^F - \Phi^H$ is continuous in (κ_o, λ) around $(0, 0)$ the result holds if both κ_o and λ are small. First, κ_o is small by assumption. Second, λ is also small or satisfies $\lambda\bar{\Sigma} = \lambda(\kappa_o + (1+i)B\lambda) \leq 0$, which implies that λ is small if κ_o is small. It thus follows that $\Phi^F - \Phi^H > 0$ for small κ_o on the set $\lambda\bar{\Sigma} \leq 0$ (or small positive λ).

Combining all arguments, I have shown that for small κ_o and small χ no solution (λ, ϕ) exists, which solves the autarky equilibrium conditions, implying that autarky is not an equilibrium.

Derivation of Result 10 [Ruling out Exchange Rate Pegs with Monetary Policy]

Pegging the exchange rate in this symmetric world means that $\lambda = \lambda^H = \lambda^F = 0$. For home-investors, the first-order condition (A26) for investing in home bonds then equals

$$\bar{q} \;=\; e^{-\gamma(\mu^H_{s_t} - c^H_{y,s_t}) + \frac{(\gamma\sigma\Sigma^H_{s_t})^2}{2}} [(1 + i + \phi_i s_t)\bar{q}^H] \tag{A44}$$

and for investing in foreign bonds

$$\bar{q} \geq (1-\chi) e^{-\gamma(\mu^H_{s_t} - c^H_{y,s_t}) + \frac{(\gamma\sigma\Sigma^H_{s_t})^2}{2}} [(1 + i - \phi_i s_t)\bar{q}^H], \tag{A45}$$

The inequality sign in the FOC for foreign bonds reflects the possibility that home investors investors do not necessarily hold foreign bonds. The two equations imply that

$$e^{-\gamma(\mu^H_{s_t} - c^H_{y,s_t}) + \frac{(\gamma\sigma\Sigma^H_{s_t})^2}{2}} [(1 + i + \phi_i s_t)\bar{q}^H] \geq e^{-\gamma(\mu^H_{s_t} - c^H_{y,s_t}) + \frac{(\gamma\sigma\Sigma^H_{s_t})^2}{2}} [(1 + i - \phi_i s_t)\bar{q}^H](1-\chi),$$

which is equivalent to

$$[(1 + i + \phi_i s_t)] \geq [(1 + i - \phi_i s_t)](1-\chi).$$

Since this equation has to hold for all s_t, $\phi_i = 0$, that means the nominal interest rate has to be constant. The same arguments hold for general interest rates functions (for the home country) $1+i+\phi(s_t)$ for a function ϕ with normalization $\phi(0) = 0$. The foreign country nominal interest rate then equals $1 + i - \phi(s_t)$. This more general specification of monetary policy leads to the condition

$$[(1 + i + \phi(s_t)] \geq [(1 + i - \phi(s_t)](1-\chi),$$

Assuming for now and verifying below that home investors hold home and foreign bonds (no autarky), the inequality becomes an equality:

$$[(1 + i + \phi(s_t)] = [(1 + i - \phi(s_t)](1-\chi),$$

implying that $\frac{\partial\phi(s)}{\partial s} = 0$ and thus $\phi \equiv 0$.

Assuming that home households invest in home and foreign bonds and using $\lambda = \phi_i = 0$ would imply that the two first-order conditions (A44) and (A45) hold with equality and thus would result in a contradiction for every $\chi > 0$:

$$e^{-\gamma(\mu^H_{s_t} - c^H_{y,s_t}) + \frac{(\gamma\sigma\Sigma^H_{s_t})^2}{2}} (1 + i)\bar{q}^H \;=\; \bar{q} = (1-\chi) e^{-\gamma(\mu^H_{s_t} - c^H_{y,s_t}) + \frac{(\gamma\sigma\Sigma^H_{s_t})^2}{2}} (1 + i)\bar{q}^H. \tag{A46}$$

This contradiction shows that a constant nominal interest rate and $\lambda = 0$ leave autarky as the only possibility, that is home households hold all home bonds and foreign households hold all foreign bonds. To rule out autarky for all states $s \neq 0$, note that in autarky with $\lambda = \phi_i = 0$, $\hat{\Sigma}^H = 0$,$\Sigma^H_{s_t} = \bar{\Sigma}^H = \kappa_o$ and

$$\hat{c}^H_y - \hat{\mu}^H \quad = \quad \kappa_y - \kappa_s \neq 0, \tag{A47}$$

implying that the FOC for home investors in home bonds,

$$\bar{q} \quad = \quad e^{-\gamma(\mu^H_{s_t} - c^H_{y,s_t}) + \frac{(\gamma\sigma\Sigma^H_{s_t})^2}{2}}[(1+i)\bar{q}^H]$$

holds only for one state, $s_t = 0$, and does not hold for all other states $s_t \neq 0$, establishing that autarky is not an equilibrium and thus that an equilibrium with a fixed exchange rate ($\lambda = 0$) does not exist.

For completeness note, that it is not possible that the home investor holds foreign bonds but not necessarily home bonds and symmetrically for the foreign investor holding home bonds but not necessarily foreign bonds. This would imply for home investors that

$$[(1 + i + \phi(s_t)] \leq [(1 + i - \phi(s_t)](1 - \chi),$$

and for foreign investors

$$[(1 + i - \phi(s_t)] \leq [(1 + i + \phi(s_t)](1 - \chi),$$

which for all $\chi > 0$ leads to the contradiction:

$$[(1 + i + \phi(s_t)] \leq [(1 + i + \phi(s_t)](1 - \chi)^2.$$

A.II Zero-order portofolio in Devereux and Sutherland (2011)

Devereux and Sutherland (2011) use a second-order approximation of households' first-order conditions to derive the zero-order portfolio. While this approach is different from the one considered in this paper, I argue now that both approaches yield the same result.

Devereux and Sutherland (2011) consider an environment with zero transaction costs and only real assets, which all have a price of one. In my OLG model their first-order conditions are

$$E_t u(c^H_{o,t+1}(1 + r^H_{t+1})) = E_t u(c^H_{o,t+1}(1 + r^F_{t+1})) \tag{A48}$$
$$E_t u(c^F_{o,t+1}(1 + r^H_{t+1})) = E_t u(c^F_{o,t+1}(1 + r^F_{t+1})) \tag{A49}$$

where $1+r^H_{t+1}$ and $1+r^F_{t+1}$ are the non-constant gross real returns of the real home and foreign bonds respectively.

The first-order approximation of the real returns are

$$(1 + r^H_{t+1}) \approx \bar{r}^H + \rho^H s \tag{A50}$$
$$(1 + r^F_{t+1}) \approx \bar{r}^F - \rho^F s. \tag{A51}$$

and of old age consumption are

$$c^H_{o,t+1} \approx \mu^H_{s=0} + \bar{\Sigma}^H s_{t+1} \tag{A52}$$
$$c^F_{o,t+1} \approx \mu^F_{s=0} - \bar{\Sigma}^F s_{t+1}. \tag{A53}$$

Taking second-order approximations of the first-order conditions (A48) and (A49) and combining them yields

$$E_t[(\bar{\Sigma}^H + \bar{\Sigma}^F)(\rho^H + \rho^F)] = 0, \tag{A54}$$

which is condition (14) in Devereux and Sutherland (2011) using the notation introduced here. Condition (A54) is equivalent to

$$\bar{\Sigma}^H = 0. \tag{A55}$$

since symmetry implies $\bar{\Sigma}^H = \bar{\Sigma}^F$ and $\rho^H = \rho^F \neq 0$.

The approach in my paper yields the first-order conditions

$$1 = (1+i)\{[\bar{m}^H + \hat{m}^H s_t]\bar{r}^H - \rho^H \gamma\sigma^2[(\bar{m}^H + \hat{m}^H s_t)\bar{\Sigma}^H + \bar{m}^H \hat{\Sigma}^H s_t]\} \tag{A56}$$
$$1 = (1+i)\{[\bar{m}^H + \hat{m}^H s_t]\bar{r}^F + \rho^F \gamma\sigma^2[(\bar{m}^H + \hat{m}^H s_t)\bar{\Sigma}^H + \bar{m}^H \hat{\Sigma}^H s_t]\} \tag{A57}$$
$$1 = (1+i)\{[\bar{m}^F - \hat{m}^F s_t]\bar{r}^F - \rho^F \gamma\sigma^2[(\bar{m}^F - \hat{m}^F s_t)\bar{\Sigma}^F - \bar{m}^F \hat{\Sigma}^F s_t]\} \tag{A58}$$
$$1 = (1+i)\{[\bar{m}^F - \hat{m}^F s_t]\bar{r}^H + \rho^H \gamma\sigma^2[(\bar{m}^F - \hat{m}^F s_t)\bar{\Sigma}^F - \bar{m}^F \hat{\Sigma}^F s_t]\}. \tag{A59}$$

Evaluating at $s_t = 0$,

$$\begin{aligned} 1 &= (1+i)\{\bar{m}^H \bar{r}^H - \rho^H \gamma \sigma^2 \bar{m}^H \bar{\Sigma}^H\} && \text{(A60)} \\ 1 &= (1+i)\{\bar{m}^H \bar{r}^F + \rho^F \gamma \sigma^2 \bar{m}^H \bar{\Sigma}^H\} && \text{(A61)} \\ 1 &= (1+i)\{\bar{m}^F \bar{r}^F - \rho^F \gamma \sigma^2 \bar{m}^F \bar{\Sigma}^F\} && \text{(A62)} \\ 1 &= (1+i)\{\bar{m}^F \bar{r}^H + \rho^H \gamma \sigma^2 \bar{m}^F \bar{\Sigma}^F\}. && \text{(A63)} \end{aligned}$$

Taking differences of the first two equations, (A60) and (A61), and using symmetry yields

$$0 = -(\rho^H + \rho^F)\gamma\sigma^2 \bar{m}^H \bar{\Sigma}^H = -2\rho^H \gamma \sigma^2 \bar{m}^H \bar{\Sigma}^H, \quad \text{(A64)}$$

which is equivalent to

$$\bar{\Sigma}^H = 0. \quad \text{(A65)}$$

Similarly, taking differences of the last two equations, (A62) and (A63), and using symmetry yields

$$\bar{\Sigma}^F = \bar{\Sigma}^H = 0, \quad \text{(A66)}$$

the same condition as in (A55) above to determine the zero-order portfolios.

The equivalence is not surprising in view of Property 1 in Devereux and Sutherland (2011). This property states that the first-order behavior of consumption, given by $\bar{\Sigma}^H$ in (A52) and $\bar{\Sigma}^F$ in (A53) , and of excess returns, given by ρ^H in (A50) and ρ^F in in (A51), is sufficient to determine the zero-order portfolio, so that higher-order approximations of consumption and returns are not necessary. I choose a different approach, which involves making stronger assumptions on the stochastic environment and on utility functions, since this enables a tractable way to deal with transactions costs and nominal assets, which are absent in the model of Devereux and Sutherland (2011).

A.III Interest rate rules and exchange rate pegs (Benigno et al., 2007)

In this Section, I first recapitulate how a properly designed interest rate rule maintains a fixed exchange rate in Benigno et al. (2007). I then illustrate why this property does not carry over to the model in this paper.

To understand the arguments in Benigno et al. (2007), I start from the log-linearized UIP condition (without transaction costs)

$$i_t^H - i_t^F = E_t \epsilon_{t+1} - \epsilon_t \tag{A67}$$

and assume that the home country is the follower which aims to peg the exchange rate at ϵ^*, taking the foreign exchange rate path as given. Benigno et al. (2007) then show that the interest rate rule

$$i_t^H = i_t^F + \phi(\epsilon_t - \epsilon^*), \tag{A68}$$

with $\phi > 0$, which reacts both to the foreign nominal interest rate i_t^F and the exchange rate deviations from the target ϵ^*, is consistent with a fixed exchange rate. The logic is simple. Using this interest rate rule in the UIP condition yields,

$$\phi \hat{\epsilon}_t = E_t \hat{\epsilon}_{t+1} - \hat{\epsilon}_t, \tag{A69}$$

where $\hat{\epsilon}$ denotes the log deviation of the exchange rate from its target level ϵ^*, i.e. $\hat{\epsilon}_t = \epsilon_t - \epsilon^*$ (Note that ϵ is the log exchange rate). The exchange rate is then determined by

$$(1 + \phi)\hat{\epsilon}_t = E_t \hat{\epsilon}_{t+1}, \tag{A70}$$

which has a unique solution $\hat{\epsilon}_t$ for all t since $\phi > 0$.

This logic does not carry over to this paper, which adds an endogenous risk premium ψ_t to the UIP condition,

$$i_t^H - i_t^F = E_t \epsilon_{t+1} - \epsilon_t + \psi_t. \tag{A71}$$

The previous derivation now yields

$$(1 + \phi)\hat{\epsilon}_t = E_t \hat{\epsilon}_{t+1} + \psi_t. \tag{A72}$$

which prevents us from concluding that there is a unique solution with a fixed exchange rate, $\hat{\epsilon}_t = 0$, since the risk premium $\psi_t \neq 0$. Indeed, the risk premium is not constant in the model simulations in which the nominal interest rate is constant (Figure 2) and in which it is responding to the state s (Figure 6).

A clever extension of the Benigno et al. (2007) rule[38] would be to add a risk-premium term

[38] Pointed out by Luca Dedola

to the rule:

$$i_t^H = i_t^F + \phi(\epsilon_t - \epsilon^*) + \psi_t. \tag{A73}$$

Using this interest rate rule in (A71) now yields

$$i_t^F + \phi(\epsilon_t - \epsilon^*) + \psi_t - i_t^F = E_t\epsilon_{t+1} - \epsilon_t + \psi_t, \tag{A74}$$

implying again

$$(1 + \phi)\hat{\epsilon}_t = E_t\hat{\epsilon}_{t+1}. \tag{A75}$$

This yields a unique solution $\hat{\epsilon}_t = 0$ for all t only if $\psi_t = 0$ since the rule (A73) implies

$$i_t^H = i_t^F + \phi\hat{\epsilon}_t + \psi_t = i_t^F + \psi_t \tag{A76}$$

and the UIP condition (A68) at the same time implies

$$i_t^H = i_t^F + \phi(\epsilon_t - \epsilon^*) = i_t^F. \tag{A77}$$

Obviously the last two conditions are consistent only if $\psi_t = 0$, so that the model would not feature a risk premium as in the first model considered above where the Benigno et al. (2007) rule works.

However, Result 10 establishes that any equilibrium features risk that is $\psi_t \neq 0$ in any equilibrium and thus here the Benigno et al. (2007) rule does not work. This echos the arguments of Section 2.5 that the Benigno et al. (2007) interest rate rule selects the constant exchange rate equilibrium out of many potential equilibria whereas here there is only one equilibrium and an interest rate rule changes the properties of this unique equilibrium. But cannot implement an equilibrium with a constant exchange rate, i.e. there is no interest rate rule which delivers this property.