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Keywords: quality signaling, consumer reviews, reputation, loss leadership

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Underpromise and Overdeliver? - Online Product Reviews and Firm Pricing

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May 7, 2021

Abstract

We consider a signaling model capturing the introductory and the mature phase of a product. Information concerning product quality is transmitted between consumers through reviews, which partially depend on the expectations consumers had prior to their purchase. When future sales are sufficiently important, a novel tension arises: High-quality types may want to underpromise and overdeliver by imitating low types in order to get a better review. We show the existence of a Pareto-improving separating equilibrium. Both more informative reviews and price transparency can lead to higher prices. Our analysis reveals a new rationale for loss-leadership.

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1 Introduction

For many consumers it has become natural to gather information about products online prior to purchase. The reviews of previous consumers are an especially important part of the available information. There is growing empirical evidence on the relationship between consumer reviews, demand, and revenue (Chevalier and Mayzlin, 2006; Anderson and Magruder, 2012; Luca, 2016; Jin et al., 2018; Zhu and Zhang, 2010; Liu, 2006; Dellarocas et al., 2007; Reinstein and Snyder, 2005; see De Maeyer, 2012, and Magnani, 2020, for detailed surveys).

Therefore, it is increasingly the case that firms' pricing and marketing choices need to take into account how consumers write reviews. There are many ways in which firms can influence reviews for their products or services. These practices are poorly understood theoretically. This is one of the first papers to model how consumers write reviews and how firms respond to the review creation process with their pricing strategies.

We start with the premise that consumer reviews depend on two main ingredients. The first is the *true quality* of a product, as is usually the case in the theoretical literature on the topic (see e.g., Jullien and Park, 2014). Additionally, reviews also convey how the product fares relative to the consumers' *prior expectation*, which may depend on the price paid and other marketing-related activities. For instance, Luca and Reshef (2020) show that a restaurant's price increase of 1% decreases average ratings by 3-5%. Hui et al. (2021) find that consumers are more likely to create reviews when their expectations were not fulfilled. The incorporation of expectations in the review creation process is also motivated by anecdotal evidence that expectations matter for reviews. Many "5-star" reviews on websites such as Amazon.com do not try to say that this is the best product one could have purchased for the need, but that it is very good for the price (and thus relative to the prior expectation). Similarly, otherwise well-functioning products frequently get 1-star reviews not because they are bad, but simply because they fall short of very high expectations commanded by a high price.

Starting with this novel view of review creation, we model a monopolist who introduces an experience good. Quality to consumers consists of two parts q and e. The first part q is known to the firm, whereas the extra part e is not. Initially, neither is known to consumers. The monopolist's pricing choice influences the quality expectations of the consumers and hence the introductory-period demand. Introductory-period consumers create a review after consumption. Naturally, a good review is more likely if the true quality is high, but less likely if prior expectations are high since consumers are less likely to be satisfied with the product. Post-introductory period consumers observe the review, but do not observe the introductory period prices, and update their beliefs about product quality accordingly.[1]

Our main objects of interest are introductory period prices conditional on the firm-known quality component q. We derive a separating equilibrium which perfectly reveals the product quality q to introductory-period consumers. This equilibrium is supported by the prospect of favorable reviews, which are beneficial for firms and can be shaped by the expectations the firm induces in the first period. As long as there is some degree of uncertainty concerning the extra quality component e in the mature phase, our reviews have this property. Hence, the presence of expectations-depending reviews and a post-introductory period creates endogenous incentives to obtain a good review in the introductory period.

The monopolist faces a novel trade-off in the choice of the introductory-period price. Charging a higher price increases introductory-period profit but also leads to higher expectations. When the product inevitably falls short of these elevated expectations, early consumers are likely to respond with a bad review, which reduces post-introductory period profits. This tension is common to many signaling models, but in our model there is an additional incentive to underpromise and overdeliver since reviews also alter the profitability of downward deviations. When a high-quality firm charges below its equilibrium price, it suffers introductory-period loss. But, by inducing lower expectations upon which it is bound to overdeliver, it also increases the probability of a good review and thus higher profits in the future. Our separating equilibrium guarantees that deviations in either direction are not profitable. Equilibrium prices are increasing in quality and depend on the price premium for a good review, which in turn depends on review informativeness. Importantly, we find that reviews that depend on expectations lead to a Pareto improvement.

[1]Some reviews explicitly mention the price aspect, and some consumers may read reviews in detail and can thus separate the expectation channel when digesting reviews. As we make clear in extensions in Section 5.2 and in Appendix B, our main findings are robust to modified information structures. We only need some degree of residual uncertainty concerning product quality, which arguably is present in any review creation and processing format.

Furthermore, we offer a novel explanation for loss leadership (Farrell and Klemperer, 2007; Lal and Matutes, 1994): Introductory prices can be below marginal cost for low-quality types, particularly when the mature phase is relatively important. High types are tempted to cut their price in the first period in order to induce a good review and capitalize in the mature phase, which can only be mitigated by below-cost pricing by low types. The low types are willing to suffer first-period losses because deviation to higher (positive) prices jeopardizes good reviews and thus mature-phase profits.

The separating equilibrium of the introductory phase has several interesting features. Equilibrium prices are non-monotone in the mature-phase market size n. For an initially low n, increasing n results in higher first-period prices, whereas the opposite is true when n is already high. The reason is the following. When n is low, imitation of low types is deterred by high types charging below-monopoly prices. The lowest quality type does not distort prices at all. When the market size n increases, low types' deviations are less profitable since there is stronger punishment through bad reviews. This allows high types to charge higher prices. Exactly the opposite applies when n is initially high. In that case price setting is primarily constrained by the incentives of high types to underpromise and overdeliver. There is no distortion for the highest quality type and low-quality types charge below-monopoly prices in order to deter deviations by high types. Thus, prices are maximized for an intermediate level of n, where all types charge the full information monopoly price.

Our framework lends itself naturally to exploring the relationship between platform design and prices. In particular, we first study the effect of review informativeness on introductory-period prices. This can be influenced by review aggregation platforms by asking consumers specifically about certain features when creating a review. Again, we find a non-monotone relationship. When informativeness increases from initially low levels, introductory prices increase. The disciplinary force of bad reviews is relatively weak, so high types need to charge relatively low prices in order to deter low types from imitation. As informativeness increases, reviews increasingly substitute as an imitation deterrent, allowing high types to charge higher prices. Conversely, increasing informativeness from already high levels leads to lower introductory prices because, in that case, a good review is very profitable for the firm. Hence, imitation of high types needs to be deterred through the competitive pricing of low types, and more so the more informative the reviews are. Thus, for platforms that make profits proportional to firms' profits, there is an inherent barrier to the maximal amount of information these platforms want to allow to be revealed.

The second platform design aspect we study concerns the effect of price transparency, i.e., allowing second-period consumers to observe first-period prices on top of the review. We find that this may be detrimental for consumers and may lead to higher prices, in particular when the mass of second-period consumers n is high. In that case, price setting is constrained by high types who may want to underpromise and overdeliver. On a more transparent platform where also first-period prices are transmitted, there is less temptation to deviate downwards, allowing low types to charge higher prices.[2]

In the next subsection, we show how our paper relates to the existing literature. In Section 2 we describe our main model, which we analyze in Section 3. We study implications for platform design in Section 4. We consider a number of interesting extensions in Section 5, where we relax our baseline assumption that all quality types have the same marginal costs and show that qualitative results of the model are unaltered. We also establish the robustness of our main results to richer review information structures and show that the incentive to underpromise and overdeliver persists in a multiple-periods settings. Finally, we conclude in Section 6.

Related literature

There is a large literature on quality signaling through prices, enabled e.g., through repeat purchases (Shapiro, 1983; Milgrom and Roberts, 1986) or differences in production costs (Milgrom and Roberts, 1986; Bagwell and Riordan, 1991). Although we allow for marginal cost differences in an extension, in our baseline specification costs of imitation arise endogenously due to the punishment of disappointed reviewers. Rhodes and Wilson (2018) and Janssen and Roy (2020) consider an exogenous fine of misreporting. In our paper, the penalty arises endogenously through bad reviews upon disappointment. Additionally, in our model there is 'negative penalty,' namely a pressure from high-quality types on low-quality types.

The reputation framework of Jullien and Park (2014) is particularly close to our study,

[2]The finding that full platform transparency is not optimal for consumers also arises in Vellodi (2018), albeit through a different channel than ours, namely shaping the incentives for firm entry.

in the sense that firms strategically influence consumer expectations. However, as opposed to our model, expectations in their model do not result from prices but from a cheap-talk message of the firm about the quality, and consumer expectations do not influence review creation. As in our paper, in Stenzel et al. (2020) higher prices also lead to worse reviews. In their setting, endogeneous selection of consumers is key and consumers make non-Bayesian inference about quality. In our paper, consumers are perfectly rational but residual uncertainty about quality makes a good review attractive for firms.

Our study describes situations in which a firm has already obtained an experience good as a result of research and development efforts, and is now faced with the decision of introducing it optimally. This is in contrast to the literature on certification and disclosure (see, e.g., Albano and Lizzeri, 2001, or Dranove and Jin, 2010, for an extensive review of this literature). The implied trade-off is very different to that of sellers on platforms like eBay, who may engage in high effort initially in order to build up a good reputation and reap that reputation later by shirking (Jullien and Park, 2014; Klein et al., 2016). We strictly separate the life cycle of the product into two periods: an introductory period in which reviews do not yet carry sufficient information to override the quality signaled through early-period prices; and a second period in which the firm can profit from having gained a high reputation, but importantly cannot 'shirk' or engage in any other action that may be harmful for consumers.

An important concern about online reviews is strategic review manipulation. In their empirical study, Mayzlin et al. (2014) find some evidence of potentially fraudulent online reviews. However, they do not model the review creation of consumers explicitly. While we acknowledge that review manipulation might affect online reviews, we abstract from manipulation and focus on the reviews created by consumers instead.

An alternative way to interpret our model is that our first-period consumers and their reviews serve as a particular form of advertising (Moraga-González, 2000; Zhao, 2000; Grunewald and Kräkel, 2017; Niu et al., 2019; Chen and Xie, 2008). The analog of 'advertising cost' in our model is charging a lower price in the introductory phase in order to induce more favorable reviews for the mature phase of the product. In this literature, firms typically have both prices and explicit advertising at their disposal. High-quality types need to distort their actions in order to deter imitation by low types, i.e., they need to price and advertise more aggressively. In our setting, there is no role for additional advertising expenditures, since all the information the firm has is fully extracted from prices in the separating equilibrium. We also show that when the audience of advertising, i.e., consumers in the mature phase, is sufficiently important, then it is no longer true that high types need to incur the biggest 'advertising cost' (distort their prices). Instead, low types have to charge lower prices, possibly below marginal costs, in order to deter imitation by high types.

In the marketing literature, a trade-off similar to ours appears as 'de-marketing', i.e., lowering marketing effort (Miklós-Thal and Zhang, 2013; Joshi and Musalem, 2012; Kopalle and Lehmann, 2006; Kuksov et al., 2013). Miklós-Thal and Zhang (2013) is close to our paper since, also there, de-marketing only affects second-period consumers' quality expectations. However, in this paper, precisely observing the firm's first-period de-marketing makes consumers more optimistic about the quality. We show that incentives to underpromise and overdeliver are present independently of whether first-period actions are observable or not. In Berger et al. (2010), a negative review can be beneficial for the firm as it increases the awareness of the product.

2 Model

Consider a monopolist firm selling a single good. Consumers are ex-ante uninformed about quality, which consists of two parts. The firm learns the main part q, which is distributed according to $F(q)$ on $[\underline{q}, \overline{q}]$. The extra part e is a mean zero random variable distributed according to $G(e)$ on $[\underline{e}, \overline{e}]$. The firm does not know e and neither do consumers. Total willingness to pay for the good is identical across all consumers and given by $v = q + e$, so $E(v|q) = q$. If a consumer purchases the good at a price p, she receives utility $v - p$. Consumers derive zero utility from not purchasing anything. We allow that in case consumers are indifferent between buying or not, only a fraction of them purchase.[3]

Marginal cost of production is assumed to be zero for any realization of q. Naturally, since the firm does not observe e, costs cannot depend on it either. This assumption can be interpreted as a firm which has already invested in research and development, and

[3]We would like to thank an anonymous referee for suggesting this possibility.

for reasons outside of the scope of the model is marketing a product of quality q.[4] We further assume that $\underline{q}+\underline{e} > 0$, so even the lowest possible quality level is valued above its marginal cost.

Consumption takes place during the *introductory* and the *mature* phase of the product, which we refer to as periods 1 and 2, respectively. The second period can be interpreted as a reduced form for the entire mature phase of the product. Consumers do not overlap over periods and consumers do not choose the timing of their consumption. As will become clear, there is no incentive to strategically delay consumption in our setting. The mass of consumers in the first period is normalized to 1, and there is a mass n of consumers in the second period. The firm does not discount the future.

After the first period, a good (G) or bad (B) review is created, so that $R \in \{G, B\}$. Other than that, there is no flow of information from first to second-period consumers.[5] The probability of a good review is given by a twice-continuously differentiable function $h(\delta, e)$, where $\delta = q - q^e$ and q^e denotes the point belief about q that the first-period consumers hold prior to purchase.[6] If the firm does not sell in the first period, then a bad review is created with certainty.[7] We assume that h is increasing in δ and e, which assures that second-period consumers interpret a good review as positive news regarding quality.

Since neither the firm nor the consumers are informed about the second quality component e, we denote the expected good review probability conditional on δ by

$$r(\delta) = \int_{\underline{e}}^{\bar{e}} h(\delta, e) dG(e). \tag{1}$$

Instead of working with the function h, from now on we use r instead and make the following assumptions about r directly:

Assumption 1. *r is a strictly increasing and strictly concave function, i.e., $r'(\delta) > 0$ and $r''(\delta) < 0$.*

[4]This assumption is also made, for instance, in Stock and Balachander (2005), which includes several additional arguments, and in Miklós-Thal and Zhang (2013) and Rhodes and Wilson (2018). Although we believe this is a reasonable assumption in a setting in which quality is not chosen, we relax this assumption in an extension in Section 5.1.

[5]We consider a richer review signal in Section 5.2 and observability of first-period prices in Section 4.2.

[6]The support of δ is $[\underline{\delta}, \overline{\delta}]$ where $\underline{\delta} = \underline{q} - \overline{q}$ and $\overline{\delta} = \overline{q} - \underline{q}$, and the support of e is $[\underline{e}, \overline{e}]$. So h maps from $[\underline{\delta}, \overline{\delta}] \times [\underline{e}, \overline{e}]$ to $[0, 1]$.

[7]Qualitatively similar results would emerge if we relax this assumption, for instance, by introducing an additional signal as in Section 5.2, as long as this signal is interpreted as bad news by consumers.

The first part of the assumption is intuitive. It postulates that holding consumer belief fixed, higher quality q results in higher probability of a good review. Conversely, holding the true quality fixed, increasing consumers' anticipated quality q^e reduces the probability of a good review. In essence, δ measures how surprises regarding quality feed into review creation. Consumers react favorably toward the firm if they are positively surprised and unfavorably if they are disappointed.

The second part of the assumption regarding the concavity of r is a necessary condition for price separation in the first period. Note that it is reminiscent of the usual positive cross-derivative in signaling models since

$$\frac{\partial^2 r(\delta)}{\partial q \partial q^e} = -r''(\delta) > 0.$$

Note that by the virtue of r depending on δ, the review probability does not depend on the true quality of a product that meets consumers' expectations, i.e., when $q = q^e$.[8]

In summary, the timing of the game is as follows. The game starts with nature drawing q and e from CDFs $F(q)$ and $G(e)$, respectively. The firm becomes informed about q but not about e. The firm then charges its first-period price p_1 that is observed by first-period consumers, who form a point belief $q^e(p_1)$.[9] Consumers then compare $q^e(p_1)$ and p_1 and purchase if $q^e(p_1) > p_1$ (or may randomize between purchasing or not when $q^e(p_1) = p_1$). The review R is generated according to the probability defined by the function $h(\delta, e)$. Second-period consumers and the firm observe R, after which the firm sets p_2. Consumers decide to buy or not, and may again randomize between purchasing or not if indifferent. After this the game ends.

We stress that all consumers, as well as the firm, are perfectly rational and understand the firm's strategic pricing and the review creation process, and make correct Bayesian inference. Residual noise in the review creation and interpretation are the main ingredients in our subsequent analysis. Our solution concept is (weak) Perfect Bayesian equilibrium (PBE, see Mas-Colell et al., 1995).

[8] This assumption is made for analytical convenience and does not qualitatively change our results.

[9] Again, this assumption is made for analytical convenience. In principle, dispersed beliefs can also be considered, but then the good review probability has to depend on an entire distribution.

3 Analysis

3.1 Introductory phase (Period 1)

Our main focus is on identifying separating equilibria in the first period. Instead of proceeding with backward induction, for the moment we simply assume that the second period gives rise to reduced-form profits as follows. Assume for now that: (i) having a good review is valuable for the firm so that it earns additional profit Δ in the second period if the review is G rather than B; and (ii) the incremental value Δ is independent of q. (i) stems from the assumptions we have made so far and is intuitive. (ii) is less obvious and holds in our model, as we show in the next section when we formally find equilibria of the second period.

Formally, let $\pi_2(q, R)$ denote the expected profit of a firm with quality q with a review R. Note that the first-period belief q^e is irrelevant for these profits because, conditional on a realized review, it plays no role for the second-period equilibrium. We can then define the aforementioned Δ as

$$\Delta = \pi_2(q, G) - \pi_2(q, B),$$

which, as noted earlier, is assumed to be independent of q. Note that Δ depends on second-period consumers' beliefs concerning first-period equilibrium, but once these are set, in the first period the firm can take Δ as given.

We now proceed to characterizing separating equilibria of the first period. The firm charges its first-period price according to an increasing and differentiable price schedule $p_1^*(q)$. While other types of equilibria (e.g., pooling equilibria) may exist, in line with the prior literature on quality signaling, we focus on this simple and most interesting class. Given that such an equilibrium is separating, for any price p_1 in the equilibrium support of prices, there is a unique quality q such that $p_1^*(q) = p_1$. Thus, instead of writing the expected profits in terms of prices, it is more convenient to write the firm's maximization problem in terms of induced belief $q^e \in [\underline{q}, \bar{q}]$, so that if a firm induces belief q^e it has to charge a price $p_1^*(q^e)$.[10]

[10] As we show in Lemma 1 in the appendix, no firm will deviate to a price outside of the equilibrium support of prices $[p_1^*(\underline{q}), p_1^*(\bar{q})]$ provided that consumers' out-of-equilibrium beliefs are sufficiently pessimistic.

Note that, if in equilibrium $p_1^*(q) = q$, then consumers are indifferent between purchasing or not and hence may randomize their purchase decision. In Appendix A we characterize a separating equilibrium where $p_1^*(q) = q$ and separation is achieved via different purchase probabilities for different qualities. We call such equilibria *separating equilibria with quantity reductions*. In *separating equilibria with price reductions*, consumers always buy when they receive a non-negative utility from doing so, and separation is achieved by distorting prices downward relative to the monopoly level. As we show in Proposition 8 in the appendix, separating equilibria with price reductions and quantity reductions yield the same profit if they co-exist. However, consumer surplus and hence also total welfare is higher in equilibria with price reductions, which always exist, whereas equilibria with quantity reductions do not always exist. Reviews that depend on expectations enable equilibria with price reductions and hence a Pareto improvement. Thus, in the main part of our analysis, we focus on separating equilibria with price reductions, and we discuss quantity reductions during the introductory phase in Appendix A. Throughout, we focus on the most profitable equilibrium of a certain class in case of co-existence.

We now solve for the separating equilibria in our chosen class. For a given induced quality belief q^e, we can write the expected profits at the beginning of the first period as

$$\pi(q, q^e) = p_1^*(q^e) + n(\pi_2(q, B) + r(q - q^e)\Delta). \tag{2}$$

We define a threshold level n_1, which will play an important role in the subsequent analysis, as

$$n_1 = \frac{1}{\Delta r'(0)}. \tag{3}$$

We now formally describe the equilibrium price schedule for separating equilibria with price reductions and show that such an equilibrium always exists.

Proposition 1. *A separating equilibrium with price reductions always exists. In the most profitable equilibrium in this class, first-period prices are given by*

$$p_1^*(q) = \begin{cases} \overline{q} - n(\overline{q} - q)r'(0)\Delta & \text{if } n \geq n_1 \\ \underline{q} + n(q - \underline{q})r'(0)\Delta & \text{if } n < n_1. \end{cases} \tag{4}$$

All consumers buy for any $p \leq p_1^*(\overline{q})$, and do not buy for $p > p_1^*(\overline{q})$ (supported by beliefs $q^e(p) = \underline{q}$). In equilibrium, expected profits equal $E\pi(q) = p_1^*(q) + n\pi_2(q, B)$.

Proof. We construct an equilibrium where $p_1(q) \leq q$ for all q. Taking the derivative with respect to q^e in (2) and imposing $q = q^e$ we obtain

$$0 = p_1'(q) + nr'(0)\Delta \frac{\partial \delta}{\partial q^e}$$
$$p_1^*(q) = n\Delta r'(0)(q - \underline{q}) + C$$

where C is a constant yet to be determined.

Each firm type maximizes profits in case the second derivative is negative. Evaluating the second-order condition using the equilibrium price function, we obtain

$$p_1''(q^e) + nr''(q - q^e)\Delta = n\Delta r''(q - q^e) < 0$$

where the last inequality follows from the concavity of r. We also used $p_1''(q^e) = 0$ which follows from the first-order condition that imposes linearity on $p_1(q^e)$.

In order to have an equilibrium, we need consumers to buy at the prices charged, i.e., $p_1^*(q) \leq q$ for all q. Furthermore, we are looking for the most profitable equilibrium for the firm so that the equilibrium price has to be the highest possible given the constraint. Since the price function is linear, this expression is binding either for $\underline{q}$ (when the slope of the price function is less than one) or for $\overline{q}$ (otherwise). So, it is binding for $\underline{q}$ if $n\Delta r'(0) < 1$ which can be rewritten as $n < n_1$. In that case we set the intercept of the price function such that $p_1(\underline{q}) = \underline{q}$, and otherwise such that $p_1(\overline{q}) = \overline{q}$, resulting in the equilibrium price function stated in the proposition.

Finally, we need all types, in both cases, to make a positive expected profit and hence prefer selling to leaving the market. Consider first the case where $n \geq n_1$. In that case, low-quality types may sell in the first period below marginal cost. However, by construction of $p_1^*(q)$, they prefer selling at these prices to selling at $p_1^*(\overline{q}) = \overline{q}$, in which case they would make a positive profit in both periods. In detail,

$$\pi_1(q, q) \geq \pi_1(q, \overline{q}) = \overline{q} + nE\pi_2(q, \overline{q}) > 0$$

for all q since $\overline{q} \geq 0$ and also $E\pi_2(q, \overline{q}) \geq 0$ for all q. Now consider the case where $n < n_1$. Again, all types make a positive profit in the second period and also in the first period since

$$\underline{q} + n(q - \underline{q})r'(0)\Delta \geq \underline{q} > 0$$

holds for all q. □

Some explanations are in order regarding the structure of the equilibrium. In many quality signaling models, the inherent tension is that at full information prices, low types wants to pretend to be a high type. In a one-period game in our context with prices as the only choice variable, any quality type wants to imitate a higher type as long as consumers still buy. The firm wants to *overpromise* and *underdeliver*.

This intuition still holds when we add a second period which is relatively unimportant, i.e., when n is relatively small. The firm is now incentivized to get a good review that induces higher beliefs about e and hence higher profits. The price schedule is mostly constrained by the incentives of relatively low types to imitate high types. To avoid deviations, imitating high types is made less attractive by lowering the high types' profitability, which is achieved by lowering the high types' prices. Since reviews react negatively to disappointment (recall that r increases in δ), the presence of future consumers serves as a deterrent to imitation. The more important the future, the less inclined low types are to imitate, because the adverse effects of imitation become relatively stronger. Thus, as n increases, the less the profitability of high types needs to be distorted in equilibrium. Prices increase with n, and at $n = n_1$, all types charge full information prices.

The situation changes once n exceeds n_1, i.e., once the future is sufficiently important for the firm. In this case a good review becomes so important that the price schedule is constrained not by the low types who want to imitate the high types, but rather by the high types who want to imitate the low types. When doing so, the high types forgo first-period profits, but positively surprise consumers who reward them with a good review, which in turn increases second-period profits. High types are tempted to *underpromise* and *overdeliver*. Now the price schedule is close to full-information prices for high types and far below such prices for low types, which ensures that high types do not want to deviate to low prices.

When n is sufficiently high, so that the price schedule is shifted downward for low types, whose monopoly prices are inherently low, the lowest prices may well fall below marginal cost (normalized to zero in our model). This happens when n exceeds a threshold so that the lowest and adjacent types' prices are below zero. We highlight this in the following corollary, where we make use of a threshold

$$n_2 = \frac{1}{\Delta r'(0)} \frac{\overline{q}}{\overline{q} - \underline{q}} \tag{5}$$

where $n_1 < n_2$ always holds by $\frac{\overline{q}}{\overline{q}-\underline{q}} > 1$.

Corollary 1. *For any $n > n_2$, there is loss leadership in the first period, i.e., there are some types q such that $p_1^*(q) < 0$.*

Proof. This follows directly since $p_1^*(\underline{q}) = 0$ if $n = n_2$ and prices decrease in n for $n \geq n_2 > n_1$. For $n \geq n_2$, if $q < \left(1 - \frac{1}{n\Delta r'(0)}\right)\bar{q}$ then $p_1^*(q) < 0$. □

Our analysis reveals a novel channel through which loss leadership can occur: low types initially charge prices below marginal costs in order to deter the imitation of high types. The prospect of high sales in the future compensates low types for the low introductory period prices. Interestingly, in our model cross-subsidization that usually occurs in loss-leadership models (Lal and Matutes, 1994) is not across products but rather across periods. In this regard, it is more akin to low introductory prices in switching cost models (Klemperer, 1987; Cabral, 2016).

3.2 Mature phase (Period 2)

We now discuss pricing in the mature phase of the product. There are pooling and separating equilibria in the second period, and we focus on the most profitable equilibrium respectively. We will show that irrespective of the equilibrium chosen in the second period, our assumptions regarding Δ are satisfied.[11]

Regardless of the type of equilibrium, second-period consumers observe the realized review $R \in \{G, B\}$. They do not observe p_1 directly, so the equilibrium of the second period will only depend on R.[12] Second-period consumers take into account that in the first period there is a separating equilibrium so that $q = q^e$ and hence $\delta = 0$, and update their beliefs about e conditional on R with Bayes' Rule, which yields

$$e_2(G) = E(e|G) = \frac{\int_{\underline{e}}^{\overline{e}} e h(0,e) dG(e)}{\int_{\underline{e}}^{\overline{e}} h(0,e) dG(e)}$$

and

$$e_2(B) = E(e|B) = \frac{\int_{\underline{e}}^{\overline{e}} e(1 - h(0,e)) dG(e)}{\int_{\underline{e}}^{\overline{e}} (1 - h(0,e)) dG(e)}.$$

[11]Standard equilibrium refinement concepts concerning off-equilibrium beliefs do not have any bite in our second period. Therefore, we do not select among these equilibria, and instead only use certain properties they have in common.

[12]We relax this assumption in Section 4.2.

Consider pooling equilibria in the second period. In any such equilibrium consumers will buy as long as the price does not exceed $E(v|R) = E(q)+e_2(R)$. In the most profitable pooling equilibrium, all firms charge $p_2^* = E(q) + e_2(R)$ for $R \in \{G, B\}$. Profits are the same for all types q and given by

$$\pi_{2,Pool}(R) = E(q) + e_2(R). \tag{6}$$

In order to support such an equilibrium, for any $p_2 \neq E(q) + e_2(R)$, we assume that consumers believe $q^e = \underline{q}$, so that they do not buy because $E(q) > \underline{q}$.

In addition to this pooling equilibrium, there is also a separating equilibrium with quantity reductions, since we allow that only a fraction of consumers purchase in case they are indifferent. Let $\rho_2^*(q^e)$ be the probability that a consumer purchases the product when she believes that the quality is q^e and she is charged $p_2 = q^e$ (see Appendix A for details on this approach).

We can write second-period profits for type q with belief q^e and review R as

$$\pi_{2,Sep}(q, q^e, R) = \rho_2(q^e)(q^e + e_2(R)).$$

Assuming $\rho_2(q^e)$ is differentiable, the necessary first-order condition for the equilibrium is

$$\rho_2'(q^e)(q^e + e_2(R)) + \rho_2(q^e) = 0.$$

We can now impose $\rho_2(\underline{q}) = 1$ for the most profitable such equilibrium to solve the above differential equation and get the equilibrium purchase probability

$$\rho_2^*(q) = \frac{\underline{q} + e_2(R)}{q + e_2(R)}.$$

Hence, in the most profitable separating equilibrium in the second period, for type q profits are given by

$$\pi_{2,Sep}(q, q, R) = \rho_2^*(q)(q + e_2(R)) = \frac{\underline{q} + e_2(R)}{q + e_2(R)}(q + e_2(R)) = \underline{q} + e_2(R),$$

so again, in equilibrium all types make the same profit, but strictly less than in the most profitable pooling equilibrium. The latter follows because $\underline{q} < E(q)$.

Irrespective of whether we focus on the separating or the pooling equilibrium of the second period, the value of having a good review is equal to

$$\Delta = \pi_2(q, G) - \pi_2(q, B) = e_2(G) - e_2(B).$$

As assumed in our analysis of the introductory phase, the value is independent of q.

Note that in the absence of the second quality component e, there would be no separating equilibrium with price reductions in the first period. This is because in this case $\Delta = 0$ and so $n_1 = \infty$. The separating equilibrium with quantity reductions would still exist, and would take the same form as the separating equilibrium of the second period we have just described. Recall that $E(e) = 0$, so in expectation $e_2(R)$ is zero. Thus, $E\pi_{2,Pool}(q) = E(q)$ and $E\pi_{2,Sep}(q) = \underline{q}$. Again, both expressions are independent of q.

In summary, irrespective of whether we focus on pooling or separating equilibria in the second period, a good review is beneficial for the firm, but the relative profitability is not type-dependent. Thus, the assumptions we made in the previous section are satisfied.

4 Platform design

We now explore several implications for platform design. We have in mind a review aggregation platform that makes profits through firms' profit or revenue, e.g., due to a transaction fee that is proportional to prices. In that case, the platform's interests are aligned with those of the firm, but possibly not with those of the consumers.

A review aggregation platform has several tools at its proposal that influence the review creation and review interpretation process. For instance, it could query consumers about certain features of the product and it can specify a minimum and a maximum length of the review text. It can choose to highlight certain aspects when presenting reviews (or aggregates thereof) to future consumers. As we have already established, reviews have an effect on firm pricing, so how should we expect a platform to design the review environment? In particular, we consider two aspects in the next two subsections, namely the informativeness of reviews and price transparency.

4.1 Review informativeness

In order to give meaning to *review informativeness*, we introduce a particular class of review functions that satisfy the required properties, namely *additively separable review functions*, specified as follows:

$$h(\delta, e; \alpha) = \frac{1}{2} + (1 - \alpha)\hat{h}(\delta) + \alpha\gamma e \tag{7}$$

where $\hat{h}(\delta)$ is an increasing and concave function with $\hat{h}(0) = 0$, and γ is a scaling parameter.[13]

Our main parameter of interest is α. Since in equilibrium $\delta = 0$, reviews are more informative the higher α. α may reflect, for instance, the fraction of consumers creating a review based on the extra quality component e, as opposed to realized deviations from expected quality (measured by δ). More broadly, α is the relative weight of true quality instead of expectation-based measures. A high α corresponds to *highly informative* reviews, since these reviews put less weight on expectations and more weight on the extra quality component e instead. α could be influenced by review aggregation platforms by querying consumers for certain types of information when they create a review.

For this review function, we have

$$r'(0) = (1 - \alpha)\hat{h}'(0),$$

and

$$\Delta = 4\alpha\gamma Var(e).$$

The premium of a good review is increasing in e's variance and in the weight α on the component e in the review function.

The effect of α on equilibrium prices is non-trivial because it affects both Δ and $r'(0)$. Δ increases in α and whereas $r'(0)$ decreases in α. The key issue here is the following. As informativeness α increases, consumers put less weight on the firm-controlled component δ in their reviews. This makes reviews less sensitive to expectations, but it also increases the attractiveness of a good review in the second period due to an increase in Δ. For our first-period equilibrium the two play a role via $\Delta r'(0)$, which depends on α in the form $\alpha(1 - \alpha)$. The latter is increasing (decreasing) in α for low (high) initial levels, that is, when $\alpha < \frac{1}{2}$ ($\alpha > \frac{1}{2}$).

This relationship with α drives the comparative statics we derive next. In order to do so, let $\bar{\alpha}$ and $\underline{\alpha}$ denote the larger and the smaller roots of

$$4\gamma Var(e)(1 - \alpha)\alpha\hat{h}'(0)n = 1$$

(which is the expansion of $\Delta r'(0)n = 1$) for $n > \frac{1}{\gamma Var(e)\hat{h}'(0)}$, otherwise let $\bar{\alpha} = \underline{\alpha} = \frac{1}{2}$.

[13] As before, h maps from $[\underline{\delta}, \overline{\delta}] \times [\underline{e}, \overline{e}]$ to $[0, 1]$. All parameters need to be admissible in the sense of inducing a probability on the entire support.

Proposition 2. *For additively separable review functions, first-period prices are non-monotonic in informativeness α. In particular, first-period prices $p_1^*(q)$ are increasing in α for $\alpha < \underline{\alpha}$ and decreasing in α for $\alpha > \bar{\alpha}$.*

Proof. By definition of $\underline{\alpha}$ and $\bar{\alpha}$ and Proposition 1, for both $\alpha < \underline{\alpha}$ and $\alpha > \bar{\alpha}$ we have $n < n_1$, so that $p_1^*(q) = \underline{q} + 4n(q - \underline{q})\alpha(1-\alpha)\gamma Var(e)\hat{h}'(0)$. Then, $p_1^*(q)$ is increasing in α for $\alpha < \underline{\alpha} \leq \frac{1}{2}$ and decreasing in α for $\alpha > \bar{\alpha} \geq \frac{1}{2}$. For $\alpha \in [\underline{\alpha}, \bar{\alpha}]$ we have $n \geq n_1$ so that $p_1^*(q) = \bar{q} - 4n(\bar{q} - q)\alpha(1-\alpha)\gamma Var(e)\hat{h}'(0)$. It then follows that $p_1^*(q)$ is increasing in α for $\alpha \in (\frac{1}{2}, \bar{\alpha})$ and decreasing in α for $\alpha \in (\underline{\alpha}, \frac{1}{2})$. □

The intuition behind the non-monotonicity result in Proposition 2 is as follows. When α is initially very low or very high, then $n < n_1$, so the equilibrium first-period price schedule is constrained by the deviation incentives of the lowest types. If α is low initially, then getting a good review is not attractive since Δ is almost zero, and then the dominant effect is through an increase in Δ, which increases prices. Conversely, if α is high initially, then Δ remains at a high level, and the dominant effect is a decrease in the review sensitivity to disappointment. The threat of bad reviews is less threatening for low types, so high types need to make stronger downward price reductions in order to deter deviations of low types, leading to lower prices.

The general conclusion here is that more informative reviews lead to higher prices if this component initially plays a small role, but does the opposite if it plays initially a large role. Second-period consumers do not benefit from informative reviews, since they always pay a price which equals the expected quality conditional on the review. Thus, consumer welfare unambiguously decreases when review informativeness increases from initially low levels.

These comparative statics have important implications for platform design. Our analysis reveals that it is not in the platform's interest to induce consumers to create perfectly informative reviews. Instead, we should expect review aggregation platforms to allow only the revelation of partial information. The profit-maximizing interests of the platform provide a natural barrier to the maximal amount of information we can expect review aggregation platforms to accumulate.

4.2 Price transparency

In this section we consider an extension where second-period consumers are able to observe the first-period price. This extension is related to a notion of transparency on review platforms. For example, alongside a consumer's review, a platform may decide to show the price at which the consumer purchased the good. Consumers reading such a review may use the price information to infer the expectations against which the review was written.

More specifically, we now assume that p_1 is observed in the second period. Conditional on a separating equilibrium in the first period, now second-period consumers also infer quality $q^e(p_1)$. In the off-equilibrium event that a price $p_2 > q^e$ is charged, we assume that consumers are sufficiently pessimistic about quality such that they do not buy.

Intuitively, this alternative specification makes upward deviations even more attractive than before. Now, imitating a high type is beneficial both in the first period (through a higher price) and in the second period (through higher beliefs about q, induced by the higher price p_1, and hence also a higher price in the second period). The only countervailing force that remains is the threat of bad reviews. Consumers who can perfectly infer q using p_1 still rely on reviews because they are also interested in learning the level of the extra quality component e.

Therefore, second-period profits now only depend on q^e (which equals claimed quality q^e from period 1) and not on the true type q. In the second period, it is always optimal to charge the (possibly misleading) monopoly price $p_2(q^e) = q^e$. Conditional on q^e and a review R, second-period profits are given by

$$\pi_2(q, q^e, R) = q^e + e_2(R)$$

and expected profits are given by

$$E\pi_2(q, q^e) = q^e + r(q, q^e)\Delta$$

and we use this alternative profit expression, which has an additional dependence on q^e, in the total profit function (2) in the following. We define

$$n_3 = \frac{1}{\Delta r'(0) - 1} \tag{8}$$

where $n_1 < n_3$ is implied when $\Delta r'(0) > 1$.

We now characterize the most profitable first-period separating equilibrium under price observability.

Proposition 3. *Suppose first-period prices are observed by second-period consumers and* $\Delta r'(0) > 1$. *Then a separating equilibrium with price reductions exists. In the most profitable equilibrium in this class, first-period prices are given by*

$$p^*_{1,obs}(q) = \begin{cases} \overline{q} - n(\overline{q} - q)(\Delta r'(0) - 1) & \text{if } n \geq n_3 \\ \underline{q} + n(q - \underline{q})(\Delta r'(0) - 1) & \text{if } n < n_3. \end{cases} \tag{9}$$

All consumers buy for all in-equilibrium prices. In equilibrium, expected profits equal $E\pi_{obs}(q) = p^*_{1,obs}(q) + nq$.

Proof. Taking derivatives in the profit function and imposing $q = q^e$, we now obtain

$$0 = p_1'(q) + n(r'(0)\Delta\frac{\partial \delta}{\partial q^e} + 1)$$
$$p_1(q) = C + n(\Delta r'(0) - 1)q$$

where C is a constant of integration yet to be determined.

Note that for a separating equilibrium to exist, we need a price function that strictly increases in q, i.e., $\Delta r'(0) > 1$. A price schedule decreasing in q cannot be an equilibrium, since it would entail that $p(\overline{q}) < p(\underline{q})$. But then there would always be a type q that would want to deviate to a price arbitrarily close above $p(\overline{q})$, and there are no off-equilibrium beliefs that could deter such a deviation.

The remainder of the proof is analogous to the proof of Proposition 1 and is thus omitted. □

Proposition 3 shows that contrary to the baseline case in which prices are not observable by second-period consumers, now a separating equilibrium with price reductions only exists when $\Delta r'(0) > 1$. The left-hand side expression of this inequality, $\Delta r'(0)$, measures the expected loss a type q incurs per second-period consumer by pretending to have a slightly higher quality $q^e > q$ (the probability of a good review, which is worth Δ, is diminished by $r'(0)$ by a small deviation). The right-hand side, 1, is the expected gain per second-period consumer. Absent price observability, this expression is 0 on the right-hand side, so it always holds. With price observability, however, deviating upwards entails the additional benefit of being perceived to be of higher quality also in the second period.

The loss $\Delta r'(0)$ needs to be sufficiently strong in order to have any hope of deterrence; if it is not, there is no possibility for separation, no matter how strong the distortion is.

We can now readily compare prices and profits with and without price observability.

Proposition 4. *Suppose $\Delta r'(0) > 1$. If $n < n_1$, then price transparency leads to lower prices and lower total profits, i.e., $p_1^*(q) > p_{1,obs}^*(q)$ and $E\pi(q) > E\pi_{obs}(q)$ for all quality levels q. If $n > n_3$, then price transparency leads to higher prices and higher total profits, i.e., $p_1^*(q) < p_{1,obs}^*(q)$ and $E\pi(q) < E\pi_{obs}(q)$ for all quality levels q.*

Proof. This follows right away from comparing the equilibrium prices and profits in Proposition 1 and Proposition 3. □

The intuition for the result in Proposition 4 is as follows. Allowing price observability makes upward deviations more tempting, because then there is also a second-period gain on top of the first-period gain. When n is sufficiently small, the future is relatively unimportant, so mostly low types have to be deterred from imitation, and even more so under price observability. Thus, the price reduction of high types has to be even stronger, resulting in lower prices.

Conversely, when n is sufficiently high, the future is very important, so high types want to underpromise and overdeliver. For them, introducing price observability makes downward deviations less attractive, because they are also accompanied by a second-period loss. Hence, even a moderate price reduction of low types deters imitation by high types, resulting in higher prices. Thus, when the incentives to underpromise and overdeliver are sufficiently strong, more transparent environments in which consumers can get easier access to the reasons of review creation result in higher prices.

Altogether, when the mature-phase market size n is large, firms benefit both in the introductory period and also in total from transparency (price observability). For online platforms deriving revenue proportional to firm's profit, we should thus expect increasingly more transparent review platform designs when the mature-phase market is sufficiently large, which harms consumers. Conversely, when n is relatively small, we should expect less transparent review platform designs.

5 Extensions

5.1 Marginal cost differences

Up to now we assumed that all types have the same marginal costs of production $c = 0$. We now demonstrate that our main analysis of introductory prices still holds when we allow for cost differences. We consider the following linear cost function:

$$c(q) = \beta_0 + \beta_1 q$$

where we assume $0 < \beta_1 < 1$, implying that high types are socially more efficient, and $\underline{q} \geq \frac{\beta_0 - e}{1 - \beta_1}$ such that all types prefer selling to not selling even in the worst possible scenario.

We now start our analysis with the second period. In the most profitable pooling equilibrium,[14] all types charge a second-period price $E(q) + e_2(R)$ and hence

$$\pi_{2,Pool}(q, R) = E(q) + e_2(R) - c(q)$$

and therefore the incremental profitability of a good review is given by

$$\Delta_{Pool} = e_2(G) - e_2(B)$$

which crucially is, as before, independent of q. Expected profits in that case are

$$E\pi_{2,Pool}(q, q^e) = E(q) + e_2(B) - c(q) + r(q - q^e)\Delta_{Pool}.$$

The total profit function (2) now becomes

$$\pi(q, q^e) = (p_1(q^e) - c(q)) + nE\pi_{2,Pool}(q, q^e) \tag{10}$$

and we can readily compare first-period profits for the model with and without marginal cost differences.

Proposition 5. *If a pooling equilibrium is played in the second period, then first-period prices are identical with and without marginal cost differences.*

Proof. The total profit function in the model without cost differences is given by (2), i.e.:

$$\pi(q, q^e) = p_1^*(q^e) + n(\pi_2(q, B) + r(q - q^e)\Delta).$$

[14] One could, in principle, also consider separating equilibria with quantity reductions in the second period. In that case, Δ depends on q, which breaks the linearity in the pricing function and a closed form solution is no longer available.

Taking derivatives with respect to q^e and imposing $q = q^e$ yields

$$0 = p_1'(q) + nr'(0)\Delta \frac{\partial \delta}{\partial q^e}.$$

In the model with cost differences, the total profit function is given by (10), i.e.:

$$\pi(q, q^e) = (p_1(q^e) - c(q)) + nE\pi_{2,Pool}(q, q^e)$$

and taking derivatives and imposing $q = q^e$ yields

$$0 = p_1'(q) + nr'(0)\Delta \frac{\partial \delta}{\partial q^e}.$$

Since the terms involving $c(q)$ drop out in the first-order condition, the first-order conditions, which pin down optimal prices, are identical. Thus, the first-period prices are also identical. □

Proposition 5 shows the robustness of our main results concerning introductory pricing under an alternative model with marginal cost differences. In our model, there is the endogenous threat of punishment through bad reviews. Even if there are marginal cost differences on top of our main channel through reviews, the threat of bad reviews persists – as well as the prospect of good reviews in case consumers are positively surprised, and hence an incentive to underpromise and overdeliver.

5.2 More detailed reviews

In our baseline model, we assumed that reviews can only take two levels, namely good or bad. Our analysis can be extended to more detailed reviews. Suppose that the possible reviews are denoted by R_k for $k = \{0, 1, \ldots, K\}$, with the interpretation that a higher k corresponds to a better review. Then there are a total of $K + 1$ review levels.

Conditional on δ and e, we need to specify the entire distribution of reviews. Denote with $h_k(\delta, e)$ the probability that a review $R = R_k$ is created, and let h_k be given by

$$h_k(\delta, e) = \frac{1 + \left(k - \frac{K}{2}\right)(m_\delta(\delta) + m_e(e))}{K + 1}$$

where m_δ and m_e are known functions that guarantee that $0 \leq h_k(\delta, e) \leq 1$ for all δ and all e. We assume $m_\delta' > 0$, $m_e' > 0$, $m_\delta'' < 0$ and $m_\delta(0) = 0$. Note that $\sum_{k=0}^{K} h_k(\delta, e) = 1$, so this is indeed a distribution over reviews.

The assumptions we make here are a straightforward extension from our baseline model to an environment with a richer review structure. The first two assumptions imply that the distribution rotates counter-clockwise as δ and e increase, which shifts probability mass to the right (in a first-order stochastic dominance sense). The concavity assumption on m_δ is equivalent to the concavity assumption we had on the r function in the baseline model.

Analysis proceeds much in the same fashion as with two review levels. Given a particular review realization R_k, and assuming consumers expect a separating equilibrium to be played in the first period, consumers in the second period compute the conditional expectation of e given R_k, which is given by

$$E(e|R_k) = \frac{\int_{\underline{e}}^{\overline{e}} e h_k(0,e)dG(e)}{\int_{\underline{e}}^{\overline{e}} h_k(0,e)dG(e)} = 1 + \left(k - \frac{K}{2}\right) E_e(m_e(e)^2)$$

since $E(e) = 0$. We then consider a pooling equilibrium in the second period where the firm earns $E(q)+E(e|R_k)$. Since, as before, $E(e|R_k)$ increases in k, having a better review is desirable for the firm.

Let $r_k(\delta) = \int_{\underline{e}}^{\overline{e}} h_k(\delta,e)dG(e)$. In the first period the firm then maximizes

$$\pi(q,q^e) = p_1(q^e) + n(E(q) + \sum_{k=0}^{K} r_k(q-q^e)E(e|R_k)). \tag{11}$$

Analogous to the definition of the cutoff point n_1 for the baseline case, we now define

$$\tilde{n}_1 = \frac{1}{\sum_{k=0}^{K} r_k'(0)E(e|R_k)}$$

for subsequent analysis. Now we can state our main result for this section.

Proposition 6. *Suppose there are $K+1$ review levels. A separating equilibrium with price reductions always exists. In the most profitable equilibrium in this class, first-period prices are given by*

$$\tilde{p}_1^*(q) = \begin{cases} \overline{q} - n(\overline{q} - q)\sum_{k=0}^{K} r_k'(0)E(e|R_k) & \text{if } n \geq \tilde{n}_1 \\ \underline{q} + n(q - \underline{q})\sum_{k=0}^{K} r_k'(0)E(e|R_k) & \text{if } n < \tilde{n}_1. \end{cases} \tag{12}$$

All consumers buy for any $p \leq \tilde{p}_1^*(\overline{q})$, and do not buy for $p > \tilde{p}_1^*(\overline{q})$ (supported by beliefs $q^e(p) = \underline{q}$).

Proof. We proceed analogous to the proof in Proposition 1. We first take derivatives with respect to q^e in the total profit expression (11) and impose $q = q^e$. We obtain

$$0 = p_1'(q) - n\sum_{k=0}^{K} r_k'(0)E(e|R_k)$$
$$\tilde{p}_1^*(q) = (q - \underline{q})n\sum_{k=0}^{K} r_k'(0)E(e|R_k) + \tilde{C}.$$

In Lemma 2 in the appendix we show that $\sum_{k=0}^{K} r_k'(0)E(e|R_k) > 0$ and $\sum_{k=0}^{K} r_k''(0)E(e|R_k) < 0$. Thus, the price schedule candidate is increasing in q, and the second-order condition is satisfied.

Finally, we need to impose $p_1(q) \leq q$ and pin down $\tilde{C}$ in order to have the most profitable equilibrium. The slope of the price function is given by $n\sum_{k=0}^{K} r_k'(0)E(e|R_k)$, so depending on whether n is below or above $\tilde{n}_1$, we obtain the equilibrium price schedule specified in the proposition. □

Observe the close similarity between Proposition 1, resulting from our baseline model, and Proposition 6 from our extension with more detailed reviews. The price schedule is determined by the endogenous information revealed through reviews. Depending on whether the mature-phase market size n is small or large, price setting is constrained by deviation incentives of either low or high types.

We conclude that qualitatively similar results hold when we extend our binary review setting to more than two review levels, as long as at least some degree of residual uncertainty remains.

5.3 Multiple periods

It is natural to think that toward the end of the life cycle of a product, the firm no longer has any interest in increasing its reputation (see also Miklós-Thal and Zhang, 2013). In our two-period model, this implies that during the mature phase of the product, reputational concerns do not play a role in the pricing decision, which is the main reason we focus on the introductory phase of the product. In this section we illustrate that the main channel we describe remains relevant also in a setting with multiple periods.

Suppose that there are now three periods instead of two. We are interested in separating equilibria through price reductions in the first and in the second period. Denote the mass of consumers per period as n_i for $i \in \{1, 2, 3\}$. We assume that, as before, the firm learns its first quality component q before setting the first-period price p_1, but does not observe the realization of e. First-period consumers generate a review $R_1 \in \{G, B\}$ as before. Second-period consumers observe R_1, but, as in our baseline specification, do not observe p_1. Thus, they update beliefs based on the review and form expectations $E(e|R_1)$. The firm also observes the review and sets a price $p_2(q; R_1)$. After consumption, second-period consumers also generate a review R_2, which depends on the expectations they had. Finally, third-period consumers observe only the second review R_2, and the firm sets a price p_3 accordingly.

In this specification, the now final period 3 is similar to the previously final period 2: There is a pooling equilibrium (conditional on all the information available to period 3 consumers) and also a separating equilibrium (again, conditional on all the information available to period 3 consumers) with quantity reductions. The per-consumer profit for the firm is hence given by $E(q) + E(e|R_2)$.

In period 2, the firm starts with a certain reputation determined by the first-period review R_1. As in our previous period 1, consumers have a certain prior about quality. This prior is induced by $E(e|R_1)$, which is based on the assumption that in a separating equilibrium of the first period, $q = q_1^e$. The firm additionally knows whether it deviated in the first period, so it updates its expectations about e differently, namely through $E(e|R_1, q, q_1^e)$.

The continuation payoff of the firm upon inducing belief q_2^e in period 2 is given by

$$\pi_2(q_2^e; R_1) = n_2 p_2(q_2^e) + n_3(r(q + E(e|R_1, q, q_1^e) - q_2^e - E(e|R_1))\Delta + E(e|B)).$$

Notice that q_1^e enters this profit expression directly (we discuss the relevant trade-offs in the next paragraph). As before, taking first-order conditions determines the equilibrium price schedule $p_2^*(q, R_1, q_1^e)$.

Expected profits before the beginning of period 1 are given by

$$\begin{aligned}\pi_1(q, q_1^e) = {} & n_1 p_1(q_1^e) + n_2 p_2^*(q, R_1(q_1^e), q_1^e) \\ & + n_3(r(q + E(e|R_1, q, q_1^e) - p_2^{-1*}(q, R_1(q_1^e), q_1^e) - E(e|R_1))\Delta + E(e|B)).\end{aligned}$$

As this profit expression makes clear, an additional trade-off arises in period 1. As before, getting a bad review (induced by unrealistically high expectations) in period 1 lowers second-period profits. Exactly these worse reviews make it easier to positively surprise consumers in period 2! This would generate a better review R_2 and result in higher profits in the possibly more relevant period 3. In a separating equilibrium these considerations cancel out, but the incentives to deviate in either direction are still relevant for the equilibrium price schedule.

We close this section with our main takeaway. Allowing for more than two periods may introduce interesting price non-monotonicities over time, depending on the relative market size in each period. Either way, the incentive to underpromise and overdeliver persists, although we do not pursue this point further and refer the interested reader to Stenzel et al. (2020) for an environment with non-Bayesian consumers.

6 Conclusion

Our model describes a novel mechanism explaining why consumer reviews are valuable for firms, and how firms can strategically introduce an experience good in order to unfold a stream of favorable reviews. When choosing its introductory price, a firm needs to take into account that because a higher price causes higher expectations, consumers are willing to pay more for the product, but importantly are also less likely to write a good review. Anticipating future demand for the product, the firm's choice needs to balance between profits in the introductory and in the mature phase.

In this environment, firms of high quality may pretend to have a low quality in order to get a better review. Thus, when the number of consumers whose purchase decision is influenced by reviews is sufficiently large, then low types need to distort their prices downwards in order to deter deviations by high types. We show that reviews which depend both on true quality but also on expectations enable separating equilibria in prices that are Pareto improving. Although we start with a relatively simple setting, we demonstrate the robustness of this key tension in several extensions.

We describe a novel rationale for loss leadership. Moreover, we identify important implications for platform design. In particular, we find that introductory prices are non-monotonic in review informativeness. The sign of the effect of price transparency in reviews on prices depends on the market size. Thus, review aggregation platforms that derive revenue proportional to a firm's profit have no interest in perfectly informative, transparent reviews. Instead, we should expect that these platforms ensure that some uncertainty in the review creation process remains.

In the separating equilibrium of our model, higher quality firms charge higher prices. This is also predicted by standard models of vertical differentiation with perfect information. Our model differs from these models since it also provides a rationale for charging different prices during the introductory and the mature phase of a product. Empirical analysis of these results is left for future research.

References

Albano, G. L. and Lizzeri, A. (2001). Strategic certification and provision of quality. *International Economic Review*, 42(1):267–283.

Anderson, M. and Magruder, J. (2012). Learning from the crowd: Regression discontinuity estimates of the effects of an online review database. *The Economic Journal*, 122(563):957–989.

Bagwell, K. and Riordan, M. H. (1991). High and declining prices signal product quality. *The American Economic Review*, pages 224–239.

Berger, J., Sorensen, A. T., and Rasmussen, S. J. (2010). Positive effects of negative publicity: When negative reviews increase sales. *Marketing Science*, 29(5):815–827.

Cabral, L. (2016). Dynamic pricing in customer markets with switching costs. *Review of Economic Dynamics*, 20:43–62.

Chen, Y. and Xie, J. (2008). Online consumer review: Word-of-mouth as a new element of marketing communication mix. *Management Science*, 54(3):477–491.

Chevalier, J. A. and Mayzlin, D. (2006). The effect of word of mouth on sales: Online book reviews. *Journal of marketing research*, 43(3):345–354.

De Maeyer, P. (2012). Impact of online consumer reviews on sales and price strategies: A review and directions for future research. *Journal of Product & Brand Management*.

Dellarocas, C., Zhang, X. M., and Awad, N. F. (2007). Exploring the value of online product reviews in forecasting sales: The case of motion pictures. *Journal of Interactive marketing*, 21(4):23–45.

Dranove, D. and Jin, G. Z. (2010). Quality disclosure and certification: Theory and practice. *Journal of Economic Literature*, 48(4):935–63.

Farrell, J. and Klemperer, P. (2007). Coordination and lock-in: Competition with switching costs and network effects. *Handbook of industrial organization*, 3:1967–2072.

Grunewald, A. and Kräkel, M. (2017). Advertising as signal jamming. *International Journal of Industrial Organization*, 55:91–113.

Hui, X., Klein, T. J., and Stahl, K. (2021). When and why do buyers rate in online markets? Technical report, University of Bonn and University of Mannheim, Germany.

Janssen, M. C. and Roy, S. (2020). Regulating product communication. *American Economic Journal: Microeconomics*, forthcoming.

Jin, G., Lee, J., Luca, M., et al. (2018). Aggregation of consumer ratings: an application to yelp. com. *Quantitative Marketing and Economics*, 16(3):289–339.

Joshi, Y. V. and Musalem, A. (2012). Underpromising and overdelivering: Strategic implications of word of mouth. *Robert H. Smith School Research Paper No. RHS-06-142*.

Jullien, B. and Park, I.-U. (2014). New, like new, or very good? reputation and credibility. *The Review of Economic Studies*, 81(4):1543–1574.

Klein, T. J., Lambertz, C., and Stahl, K. O. (2016). Market transparency, adverse selection, and moral hazard. *Journal of Political Economy*, 124(6):1677–1713.

Klemperer, P. (1987). Markets with consumer switching costs. *The quarterly journal of economics*, 102(2):375–394.

Kopalle, P. K. and Lehmann, D. R. (2006). Setting quality expectations when entering a market: What should the promise be? *Marketing Science*, 25(1):8–24.

Kuksov, D., Shachar, R., and Wang, K. (2013). Advertising and consumers' communications. *Marketing Science*, 32(2):294–309.

Lal, R. and Matutes, C. (1994). Retail pricing and advertising strategies. *Journal of business*, pages 345–370.

Liu, Y. (2006). Word of mouth for movies: Its dynamics and impact on box office revenue. *Journal of marketing*, 70(3):74–89.

Luca, M. (2016). Reviews, reputation, and revenue: The case of yelp. com. *Com (March 15, 2016). Harvard Business School NOM Unit Working Paper*, (12-016).

Luca, M. and Reshef, O. (2020). The impact of prices on firm reputation. *Harvard Business School NOM Unit Working Paper*, (20-128).

Magnani, M. (2020). The economic and behavioral consequences of online user reviews. *Journal of Economic Surveys*.

Mas-Colell, A., Whinston, M. D., Green, J. R., et al. (1995). *Microeconomic theory*, volume 1. Oxford university press New York.

Mayzlin, D., Dover, Y., and Chevalier, J. (2014). Promotional reviews: An empirical investigation of online review manipulation. *American Economic Review*, 104(8):2421–55.

Miklós-Thal, J. and Zhang, J. (2013). (de) marketing to manage consumer quality inferences. *Journal of Marketing Research*, 50(1):55–69.

Milgrom, P. and Roberts, J. (1986). Price and advertising signals of product quality. *The Journal of Political Economy*, pages 796–821.

Moraga-González, J. L. (2000). Quality uncertainty and informative advertising. *International Journal of Industrial Organization*, 18(4):615–640.

Niu, B., Yue, H., Luo, H., and Shang, W. (2019). Pricing for newly-launched experience products: Free trial or not? *Transportation Research Part E: Logistics and Transportation Review*, 126:149–176.

Reinstein, D. A. and Snyder, C. M. (2005). The influence of expert reviews on consumer demand for experience goods: A case study of movie critics. *The journal of industrial economics*, 53(1):27–51.

Rhodes, A. and Wilson, C. M. (2018). False advertising. *The RAND Journal of Economics*, 49(2):348–369.

Shapiro, C. (1983). Optimal pricing of experience goods. *The Bell Journal of Economics*, pages 497–507.

Stenzel, A., Wolf, C., and Schmidt, P. (2020). Pricing for the stars-dynamic pricing in the presence of rating systems. Technical report, University of Bonn and University of Mannheim, Germany.

Stock, A. and Balachander, S. (2005). The making of a “hot product”: A signaling explanation of marketers’ scarcity strategy. *Management science*, 51(8):1181–1192.

Vellodi, N. (2018). Ratings design and barriers to entry. *Available at SSRN 3267061*.

Zhao, H. (2000). Raising awareness and signaling quality to uninformed consumers: A price-advertising model. *Marketing Science*, 19(4):390–396.

Zhu, F. and Zhang, X. (2010). Impact of online consumer reviews on sales: The moderating role of product and consumer characteristics. *Journal of marketing*, 74(2):133–148.

Appendix

A Separation with quantity reductions

In our baseline model, we focused on separating equilibria in the first period in which all first-period consumers purchased and prices were distorted in order to deter deviations. In this section, we consider a different type of separating equilibria: all firm types charge full information prices, making consumers indifferent between purchasing or not. Imitation can then be deterred when only a certain fraction of consumers purchase.

For this we define $\rho_1^*(q)$ as the probability a consumer purchases if $p^*(q) = q$ so that a consumer receives zero surplus. In such separating equilibria with *quantity reductions* the separation is achieved by distorting purchase probabilities so that no type wants to deviate.

Separating equilibria with quantity reductions exist even in the simplest one-shot game with unit demand and no marginal cost differences. Each type simply charges the full information price $p^*(q) = q$. All consumers buy when observing $p(\underline{q}) = \underline{q}$, but only a fraction $\rho^*(q) = \underline{q}/q$ buy for all other prices on the equilibrium support. No type has an incentive to deviate since profits are the same for all prices.

In our game with reviews the equilibrium is more involved. First, we show in Proposition 7 that such an equilibrium exists as long as n is sufficiently small. In Proposition 8 we provide the main rationale for considering separating equilibria with distorted prices in most of the paper: Profits are the same across equilibria, but consumer surplus is higher, making these equilibria Pareto-superior. Since in equilibria with quantity distortions consumers are indifferent between purchasing or not, consumer welfare is zero.

More specifically, in this equilibrium, $p_1^*(q) = q$ and hence consumers are indifferent between buying or not, so $\rho_1(q) \in (0, 1]$. Then the total profit expression (2) becomes

$$\pi(q, q^e) = q^e \rho_1(q^e) + n(\pi_2(q, B) + r(q - q^e)\Delta). \tag{13}$$

Proposition 7. *If $n < n_2$, then a separating equilibrium with quantity reductions exists. In the most profitable equilibrium in this class, $p_1^*(q) = q$ and a fraction $\rho_1^*(q)$ of consumers purchase where*

$$\rho_1^*(q) = \begin{cases} n\Delta r'(0) - (n\Delta r'(0) - 1)\overline{q}/q & \text{if } n \geq n_1 \\ n\Delta r'(0) + (1 - n\Delta r'(0))\underline{q}/q & \text{if } n < n_1. \end{cases} \tag{14}$$

If $n \geq n_1$, then $\rho(\overline{q}) = 1$ and ρ is increasing in q, and if $n < n_1$ then $\rho(\underline{q}) = 1$ and ρ is decreasing in q.

Proof. Taking derivatives with respect to q^e in the total profit expression (13) and imposing $q = q^e$ and $\rho_1^*(\underline{q}) = \underline{\rho}$ we obtain

$$0 = \rho_1'(q)q + \rho_1(q) + nr'(0)\Delta\frac{\partial\delta}{\partial q^e}$$
$$\rho(q) = n\Delta r'(0) + C/q.$$

where C is a constant yet to be determined.

We first consider equilibria in this class in which ρ_1^* is a decreasing function. In that case we impose $\rho_1^*(\underline{q}) = \underline{\rho}$, for $\underline{\rho}$ yet to be determined, and obtain

$$\rho_1(q;\underline{\rho}) = n\Delta r'(0) - (n\Delta r'(0) - \underline{\rho})\underline{q}/q.$$

We need that the partial derivative of ρ_1^* is negative:

$$\frac{\partial\rho_1^*(q)}{\partial q} = (n\Delta r'(0) - \underline{\rho})\underline{q}/q^2 < 0$$

which holds if and only if

$$n\Delta r'(0) - \underline{\rho} < 0$$

which is least stringent when $\underline{\rho} = 1$ and then holds as long as $n < n_1$. Note that in that case $\rho_1^*(\overline{q}) > 0$ always holds.

Next, we explore equilibria where ρ_1 is an increasing function. In that case we impose $\rho_1(\overline{q}) = \overline{\rho}$, for $\overline{\rho}$ yet to be determined, and obtain

$$\rho_1(q;\overline{\rho}) = n\Delta r'(0) - (n\Delta r'(0) - \overline{\rho})\overline{q}/q.$$

We need that the derivative of ρ_1 is positive:

$$\rho_1' = (n\Delta r'(0) - \overline{\rho})\overline{q}/q^2 > 0$$

which holds if and only if

$$n\Delta r'(0) - \overline{\rho} > 0.$$

We additionally need that each type makes positive sales:

$$\rho_1(\underline{q};\overline{\rho}) > 0$$
$$n\Delta r'(0) - (n\Delta r'(0) - \overline{\rho})\overline{q}/\underline{q} > 0$$

which is least stringent when $\overline{\rho} = 1$ since $\overline{q}/\underline{q} > 1$ and then holds as long as

$$n < \frac{1}{\Delta r'(0)} \frac{\overline{q}}{\overline{q} - \underline{q}} = n_2.$$

So for $n \in [n_1, n_2)$, this equilibrium exists since in that case also $n\Delta r'(0) - \overline{\rho} > 0$ holds.

In both cases, as in the proof of Proposition 1, evaluating the second-order condition using the equilibrium ρ_1 function, we find that concavity of r implies that each type q maximizes profit by setting a price $p(q) = q$.

□

The basic structure of the quantity distortions is very similar to price distortions. Consider first the case where $n < n_1$. In that case a positive review is not very important for the firm, so low types are very tempted to imitate high types. Thus, the profitability of high types has to be reduced. Since all types charge full information prices, low types are deterred from imitation by high types selling lower quantities.

When $n \geq n_1$, the incentives are reversed: Now a good review is very valuable and hence high types are tempted to underpromise and overdeliver. In that case, the sales of low types have to be distorted in order to deter deviation.

Note that an equilibrium with quantity distortions only exists as long as $n < n_2$ (where $n_2 > n_1$). As described in the main text, n_2 is precisely the threshold above which high types are so tempted to imitate low types that low types have to charge prices below marginal cost (loss leadership). In equilibria with quantity distortions, prices are fixed at full information levels and quantities are distorted. However, there is a natural bound to quantity distortions: Sales cannot be negative. At $n = n_2$, the lowest type does not sell at all ($\rho(\underline{q}) = 0$), and when n becomes even larger, the only possibility to deter high types from imitation would be through negative sales, which is not feasible.

Proposition 8. *Suppose $n < n_2$ such that both the most profitable separating equilibrium with price and quantity reductions co-exist. Then, (i) profits are the same in both equilibria for all types, (ii) consumer surplus is higher under distorted prices, and (iii) total welfare is higher under distorted prices.*

Proof. (i) Second-period profits are the same either way, so we only need to focus on the first period. First-period profits in the equilibrium with distorted prices are given by $p_1^*(q)$ in (4) in Proposition 1 since in that case, all consumers buy. In the equilibrium with distorted quantities, prices are given by $p_1(q) = q$, but only a fraction $\rho_1(q)$ as in (14) in Proposition 7 purchase.

Consider first the case where $n < n_1$. In that case:

$$\begin{aligned}
\rho_1(q)q &= (n\Delta r'(0) + (1 - n\Delta r'(0))\underline{q}/q)q \\
&= n\Delta r'(0)q + (1 - n\Delta r'(0))\underline{q} \\
&= \underline{q} + n\Delta r'(0)(q - \underline{q}) \\
&= p_1^*(q)
\end{aligned}$$

and hence profits are the same across equilibria. Now, consider the case where $n \geq n_1$. In that case:

$$\begin{aligned}
\rho_1(q)q &= (n\Delta r'(0) - (n\Delta r'(0) - 1)\overline{q}/q)q \\
&= n\Delta r'(0)q + (1 - n\Delta r'(0))\overline{q} \\
&= \overline{q} - n\Delta r'(0)(\overline{q} - q) \\
&= p_1^*(q)
\end{aligned}$$

and hence profits are the same across equilibria.

(ii) In equilibria with quantity distortions, consumers are indifferent between buying or not, and hence consumer surplus is zero. When prices are distorted, valuations strictly exceed prices and hence expected consumer surplus is positive.

(iii) This follows right away from combining (i) and (ii). □

Proposition 8 first shows that profits are the same under price and quantity distortions (as long as these two co-exist). Consumers benefit from price distortions (relative to full information prices), but do not benefit at all from quantity reductions. All consumers are indifferent between buying or not, hence consumer welfare is zero. Combining these two considerations, we have that total welfare is higher under price distortions. Moreover, price distortions are a Pareto improvement relative to quantity distortions.

Finally, we show that firms cannot be better off in equilibria in which some types charge full information prices (with quantity distortions) and the other types charge distorted prices.

B Reviews depending on first-period prices

Reviews in our baseline model depend on first-period prices p_1, namely through the expectations $q^e(p_1)$ they induce for first-period consumers. In principle, first-period prices

p_1 might also enter the review creation process directly. For instance, the probability of getting a good review could be given by $h(\delta_2, e)$ instead of $h(\delta, e)$, where δ_2 is the net consumer surplus, i.e., $\delta_2 = q - p_1$ (instead of $\delta = q - q^e$).

The similarities to our baseline specification are best highlighted by considering the model with quantity reductions, which yield the same profit as in equilibria with price reductions (see Proposition 8 in Appendix A).

When reviews depend on expectations and there are quantity reductions such that a fraction $\rho_1(q)$ purchase when a price $p_1 = q$ is charged, then profits are given by (13), i.e.:

$$\pi(q, q^e) = q^e \rho_1(q^e) + n(\pi_2(q, B) + r(q - q^e)\Delta).$$

When reviews depend on prices instead, profits are given by

$$\pi(q, q^e) = q^e \rho_1(q^e) + n(\pi_2(q, B) + r(q - p_1(q^e))\Delta).$$

This makes it clear right away that a model in which reviews are based on prices is identical, since in equilibrium all types charge full information prices $p_1(q) = q$, and also in case of potential deviations the effect is exactly the same. Thus, one of the implications of Proposition 7 applies right away: When the mature-phase market size n is relatively small, high types' profit needs to be distorted downwards in order to deter imitation by low types. When the mature-phase market is large, high types may want to imitate low types, and hence low types' profit needs to be distorted downwards. Therefore, even when reviews depend directly on first-period prices, our main tension between underpromise and overdeliver remains.

C Additional Proofs

Lemma 1. *Regardless of beliefs, no type wants to deviate to a price p with $p < p_1^*(\underline{q})$. No type wants to deviate to a price p with $p > p_1^*(\overline{q})$ when beliefs are sufficiently pessimistic such that consumers do not buy.*

Proof. In terms of post-introductory period profits, the most favorable belief is that $q = \underline{q}$. Even if consumers hold such a belief when observing $p < p_1^*(\underline{q})$, no firm will wish to deviate to p because each one would instead deviate to $p^*(\underline{q})$ which induces the same belief (and thus post-introductory period profits) and earns higher profit in the first period.

Now consider upward deviations to a price p with $p > p_1^*(\overline{q})$. When beliefs are sufficiently pessimistic, i.e., $q^e(p) < p$, then consumers do not buy in the first period, which is strictly worse than deviating to $p_1^*(\overline{q})$. Not selling at all in the first period definitely triggers a bad review, which again is strictly worse than deviating to $p_1^*(\overline{q})$. Thus, no firm will wish to deviate to p because each one would instead deviate to $p^*(\overline{q})$.

□

Lemma 2. *Suppose there are $K+1$ review levels. Then $\sum_{k=0}^{K} r_k'(0)E(e|R_k) > 0$ and $\sum_{k=0}^{K} r_k''(0)E(e|R_k) < 0$.*

Proof. By definition of h_k and r_k, we have that

$$r_k'(\delta) = \frac{k - K/2}{K+1} m_\delta'(\delta)$$

and

$$r_k''(\delta) = \frac{k - K/2}{K+1} m_\delta''(\delta)$$

and recall that

$$E(e|R_k) = 1 + \left(k - \frac{K}{2}\right) E_e(m_e(e)^2).$$

Then

$$\begin{aligned}
\sum_{k=0}^{K} r_k'(0)E(e|R_k) &= \sum_{k=0}^{K} \frac{k - K/2}{K+1} m_\delta'(0) \left(1 + \left(k - \frac{K}{2}\right) E_e(m_e(e)^2)\right) \\
&= \frac{m_\delta'(0)}{K+1} \sum_{k=0}^{K} \left(\left(k - \frac{K}{2}\right) + \left(k - \frac{K}{2}\right)^2 E_e(m_e(e)^2)\right) \\
&= \frac{m_\delta'(0)}{K+1} \sum_{k=0}^{K} \left(\left(k - \frac{K}{2}\right)^2 E_e(m_e(e)^2)\right) > 0
\end{aligned}$$

since $\sum_{k=0}^{K}(k - K/2) = 0$, $m_\delta'(0)$ is positive by assumption and all the terms inside the sum are also positive.

Analogously, the sign of $\sum_{k=0}^{K} r_k''(0)E(e|R_k)$ is given by $m_\delta''(0)$, which is negative by assumption, and hence $\sum_{k=0}^{K} r_k''(0)E(e|R_k) < 0$.

□