

# REAL INTEREST RATES, NOMINAL SHOCKS, AND REAL SHOCKS

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## ABSTRACT

### Real Interest Rates, Nominal Shocks, and Real Shocks\*

This paper uses a structural time-series analysis to analyse the properties of *ex-ante* real interest rates of the five major OECD economies in relation to temporary and permanent shocks to real output. Following Blanchard and Quah (1989) we refer to these innovations as 'nominal' and 'real' shocks respectively. The relationships of rates to these shocks appear to be qualitatively consistent with predictions of stochastic general equilibrium models of business cycles driven by both real and nominal disturbances. Real and nominal shocks originating in the United States are found to be the most important causes of persistence in *ex-ante* real interest rates, but of the two, only nominal shocks cause dynamic movements in rates that are coherent across all countries. Further results indicate that the rise in real interest rates experienced by these countries in the early 1980s was mainly due to nominal shocks in all five countries; real shocks played little or no role.

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## NON-TECHNICAL SUMMARY

This paper uses a structural time-series analysis to analyse the properties of *ex-ante* real interest rates of the five major OECD economies (France, Germany, Japan, the United Kingdom and the United States) in relation to temporary and permanent shocks to real output. The relationships of rates to these shocks appear to be qualitatively consistent with predictions of stochastic general equilibrium models of business cycles driven by both real and nominal disturbances. Real and nominal shocks originating in the United States are found to be the most important causes of persistent movements in *ex-ante* real interest rates, but of the two only nominal shocks cause movements in rates that are coherent across all countries. Further results indicate that the rise in real interest rates experienced by these countries in the early 1980s was mainly due to nominal shocks in all five countries; real shocks played little or no role. The data consist of current and lagged real output growth, current and lagged inflation, and nominal short-term interest rates (treasury bill rates where markets for these existed), quarterly for the period 1957(1) to 1994(4). A measure of *ex-ante* real interest rates is obtained for each country using an inflation forecast for that country derived from vector auto-regression (VAR) in inflation and output growth.

In the early 1980s, rates appeared to be very high, following a period of low and maybe negative real rates in the inflationary 1970s. A number of factors have been considered as possible causes of high real rates, including the effects of US fiscal expansion, tight monetary policies around the world, higher profitability of capital investment, and shifts in portfolios. Robert Barro and Xavier Sala-i-Martin explored the effects of many of these, using a model based on the balance between the supply of and demand for real saving. Their results suggest that, with a one-year lag, the rate of return on the stock market, the real price of oil, the lagged investment-to-income ratio, and the world interest rate all have significant positive effects on the *ex-ante* world real interest rate. Money growth appeared to have a significantly negative effect. The world debt-to-GDP ratio and the ratio of deficits to GDP had insignificant effects. Empirically, this model has proved to be highly robust. Others have tested similar models.

While many analyses (including Barro and Sala-i-Martin's) have used a measure of a 'world' real rate of interest, it is not clear that there exists such a thing. Real interest rates in different countries tend to be different, whether measured *ex ante* or *ex post*, on any conventionally used measure. They would be equal across countries if goods and capital markets were perfectly

integrated, such that the law of one price or purchasing power parity was valid (at least in expectation), and rates of return on assets (expressed in a common currency) were uniform, i.e. the uncovered interest parity condition was valid. This would imply that the expected rate of appreciation of the exchange rate was equal both to the difference in the expected rates of inflation between two countries, and to the difference in their nominal interest rates. It is a notorious fact that neither of these conditions holds true, however. There are long-lasting and substantial deviations from purchasing power parity (PPP). Indeed it is far from clear that PPP has any validity even as a long-run relationship. There are also persistent deviations from uncovered interest parity (UIP). There is furthermore no tendency for deviations from PPP and UIP to be mutually offsetting, such that the real rate of interest would be uniform across countries despite the failure of both PPP and UIP. Part of the reason for the differences in conventionally measured real interest rates across countries may be the use of an inappropriate price index which includes both traded and non-traded goods. Purchasing power parity would only be expected to apply to tradable goods.

This paper avoids the issue of estimating a 'world real interest rate'. Instead we use a structural time-series model for real *ex-ante* interest rates, real output growth and inflation to analyse the interactions between the real interest rates of a number of countries and investigate the extent to which real interest rates appear to respond to economy-specific real and nominal shocks. This exercise might be interpreted as an attempt to adopt an empirical structure that nests the kind of model for real interest rates that is implied by a real business cycle plus cash-in-advance theory of the economy. Hence, it allows real rates to have properties arising from a conceptually coherent theory. The empirical structure yields estimated responses of *ex-ante* rates to real and nominal shocks that appear to be consistent with this body of theory. It is then used to analyse the empirical issues about interest rates raised in the literature and discussed above. In particular, we: (a) identify the major sources of persistence in rates; (b) find that one of these persistent components is 'common' to all rates in the sense that it has coherent dynamic effects on rates across countries; and (c) identify the sources of the general rise in rates in the early 1980s. The persistence in real interest rates is found to emanate predominantly from real and nominal shocks originating in the United States. The common component in rates, which is found to be driven by a nominal shock originating in the United States, is analogous to a 'world real interest rate'. Finally, the general rise in rates in the early 1980s appears to be a consequence of nominal shocks; real shocks played little or no part.



# Real Interest Rates, Nominal shocks and Real Shocks

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## 1. Introduction

This paper uses a structural time-series analysis to expose the properties of ex ante real interest rates of the five major OECD economies in relation to temporary and permanent shocks to real output. Following Blanchard and Quah(1989) we refer to these innovations as "nominal" and "real" shocks respectively. The relationships of rates to these shocks appear to be qualitatively consistent with predictions of stochastic general equilibrium models of business cycles driven by both real and nominal disturbances. Real and nominal shocks originating in the US are found to be the most important causes of persistence in ex ante real interest rates but of the two only nominal shocks cause dynamic movements in rates that are coherent across all countries. Further results indicate that the rise in real interest rates experienced by these countries in the early nineteen eighties was mainly due to nominal shocks in all five countries; real shocks played little or no role.

There has been varying interest in real interest rates over the years. The experience of what appeared to be very high real rates in the early 1980's, following low and maybe negative real rates in the inflationary 1970's caused a burst of interest, for example. Blanchard and Summers (1984) reviewed a number of possible causes of high real rates, including the effects of US fiscal expansion, tight monetary policies around the world, higher profitability of capital investment, and shifts in portfolios. However, real interest rates frequently do not appear among the macroeconomic variables whose time series properties are compared in calibration exercises carried out within the stochastic general equilibrium framework.

More recently, Barro and Sala-i-Martin (1990) modeled real interest rates, both for the world as a whole and for individual countries, in a rather eclectic way. They proposed a model of world real interest rates based on the balance between

the supply of and demand for real saving. The demand for saving (investment) depends on the world rate of return on the stock market, and changes in the ex ante real rate of interest. The desired saving rate is assumed to depend on current income relative to its long run value, a variable which is captured by the relative price of crude oil, and the ex ante real interest rate. They also allow monetary and fiscal policy variables to influence saving. Their regression results suggest that the lagged rate of return on the stock market, the lagged real price of oil, the lagged investment-to-income ratio, and the lagged world interest rate all have significant positive effects on the ex ante world real interest rate. The lagged rate of growth of the money supply appears to have a significantly negative effect. The world debt to GDP ratio and the ratio of deficits to GDP prove to have insignificant effects. The model of the economy lying behind their empirical estimates might be described as basically real, with effects of monetary and fiscal policy bolted on to pick up the consequences of possibly short-term wage and price stickiness.

Empirically, the model of Barro and Sala-i-Martin proves to be highly robust. Nevertheless, not all scholars are willing to welcome its hybrid nature. Lucas, in commenting on their paper, remarked "In evaluating these conclusions, I did not find the theoretical framework offered in the paper especially helpful." He argued for a more thorough-going Fisherian approach to real interest rates. Barro and Sala-i-Martin's model might also be criticised for one of its choices of explanatory variables, in that it includes the real rate of return on stock markets, lagged one period. To the extent that financial markets are integrated, it would be expected that a rise in returns on stock would be reflected throughout the markets, explaining a rise in interest rates. If innovations in stock returns have persistent effects, it is unsurprising that the lagged return on stock markets would be associated with the current ex ante real rate of interest. To this extent, it might be argued that the paper assumes what it aims to explain.

Phelps (1992) models the world real rate of interest as a function of the world stock of public debt, the stock of capital, government expenditures, temporary changes in oil prices, and inflation surprises.

While both these authors have used a measure of a "world" real rate of interest, it is not clear that there exists such a thing. Real interest rates in different countries tend to be different, whether measured ex ante or ex post, on any conventionally used measure. They would be equal across countries if goods and capital markets were perfectly integrated, such that the law of one price or purchasing power parity was valid (at least in expectation), and rates of return on assets (expressed in a common currency) were uniform, i.e., the uncovered interest parity condition was valid. This would imply that the expected rate of appreciation of the exchange rate was equal both to the difference in the expected rates of inflation between two countries, and to the difference in their nominal inter-



est rates. However, it is a notorious fact that neither of these conditions holds true. There are long lasting and substantial deviations from purchasing power parity. Indeed it is far from clear that PPP has any validity even as a long-run relationship. There are also persistent deviations from UIP. There is furthermore no tendency for deviations from PPP and UIP to be mutually offsetting, such that the real rate of interest would be uniform across countries despite the failure of both PPP and UIP. Several authors have asked whether any meaning can be given to the concept of a world real rate of interest, including Gagnon and Unferth (1993) and Driffill, Gilbert and Yeoward (1993).

Part of the reason for the differences in conventionally measured real interest rates across countries may be the use of an inappropriate price index which includes both traded and non-traded goods. Purchasing power parity would only be expected to apply to tradeable goods. Dutton (1993) argues that providing the price indices are chosen appropriately so as to include only traded goods, there is less evidence against equality of ex ante real interest rates across countries. Her tests are based on the non-predictability of ex ante real rate differentials, using monthly data for the US, UK, Japan, Canada, and France.

Different approaches to the measurement of the "world real rate of interest" have been taken. As against Barro and Sala-i-Martin (1990) who use a weighted average of individual countries, using real GDP as weights, Gagnon and Unferth (1993) extract a world rate from time series data on nine countries by using a panel regression. Here the data are allowed to determine the influence that each country has on the world rate. Their model is  $RR_{it} = \alpha + \beta_i + \gamma_t + \nu_{it}$  in which the  $\beta$ 's are country effects, and the  $\gamma$ 's are time effects. The world real rate is  $\alpha + \gamma_t$ , and the  $\beta$ 's are constrained to sum to zero. It is effectively assumed that the real interest rate of each country differs from the world rate by a constant amount plus a random term.

Gregory and Watt (1995) pursue a similar approach, although they allow for a richer dynamic structure, and allow for influences from inflation rates onto *ex post* real interest rates. They examine monthly data for nine countries for the period 1975 to 1990. They propose that real interest rates in each country are driven by both world and local factors, and for each of these is a factor associated with inflation and a real factor. The inflation rate in each country is therefore modeled as  $\pi_{it} = \beta_i \pi_t^w + \eta_{it}^{\pi}$ , in which  $\pi_t^w$  is the "world inflation rate" factor, and  $\eta_{it}^{\pi}$  is the local inflation factor, while the real interest rate is modeled as  $r_{it} = \phi_i r_t^w + \delta_i \pi_t^w + \alpha_i \eta_{it}^{\pi} + \eta_{it}^r$ , in which  $r_t^w$  denotes the world real interest rate factor, and  $\eta_{it}^r$  denotes the local real interest rate factor. The dynamic structure of the model is restricted by assuming that each of the four factors follow AR(1) processes with mutually independent innovations, and by making identifying assumptions on the variance of the innovations in the world real interest rate and inflation

rate factors. Having imposed this structure on the data, the model yields some clear patterns. The world factors are highly persistent, much more so than are the local factors. An increase in world inflation causes a reduction in countries' real interest rates (which they interpret as a manifestation of a Mundell-Tobin effect), whereas a rise in the local inflation factor is associated with a rise in that country's real interest rate.

There exist yet other approaches to measuring a world real rate which start from the statistical properties of real interest rates. It has been observed by a number of authors that the hypothesis that real rates contain a unit root cannot be rejected empirically. (Indeed, while Gregory and Watt proceed on the reasonable assumption of stationarity, having considered a lot of evidence, there is much statistical evidence which is not inconsistent with there being a unit root in real interest rates.) If, however, the real rates for individual countries do not depart from each other by too much, it may be that there is a common trend in real interest rates. The common trend might be extracted from the data and identified with the world real interest rate.

In this paper we avoid the issue of estimating a "world real interest rate". Instead we use a structural time series model for real ex ante interest rates, real output growth and inflation to analyse the interactions between the real interest rates of a number of countries and investigate the extent to which real interest rates appear to respond to economy specific real and nominal shocks which have similar properties to those derived in Blanchard and Quah(1989). This exercise might be interpreted as an attempt to adopt an empirical structure that nests the kind of model for real interest rates that is implied by a real business cycle plus cash-in-advance theory of the economy. Hence, it allows real rates to have properties arising from a conceptually coherent theory. Our empirical structure yields estimated responses of ex ante rates to real and nominal shocks that appear to be consistent with this body of theory. It is then used to analyse the empirical issues about interest rates raised in the literature and discussed above. In particular we a) identify the major sources of persistence in rates, b) find that one of these persistent components is "common" to all rates in the sense that it has coherent dynamic effects on rates across countries and c) identify the sources of the general rise in rates in the early 1980's. The persistence in rates documented by Rose and others, is found to emanate predominantly from real and nominal shocks originating in the US. The common component in rates, which is found to be driven by a nominal shock originating in the US, is analogous to the "world real interest rate" of Barro and Martin, etc. except that there, the "world rate" was constructed as an ad hoc weighted average of country rates whilst here it emerges naturally from our structural time series model. Finally the general rise in rates in the early 1980's discussed by Blanchard and Summers and others appears to

be a consequence of nominal shocks; real shocks played little or no part.

We consider ex ante real rates of interest for five countries: the United States, Japan, Germany, France, and the UK. The data consist of current and lagged real output growth, current and lagged inflation, and nominal short-term interest rates (treasury bill rates where markets for these existed) quarterly for the period 1957(1) to 1994(4)<sup>1</sup>. A measure of ex ante real interest rates is obtained for each country using an inflation forecast for that country derived from a VAR in inflation and output growth.

## 2. Real and nominal interest rates in real business cycle and cash-in-advance models

### 2.1. Real interest rates in the simplest rbc model

The nature of the relationship between the world real interest rate and world productivity shocks (as measured for example by Solow residuals) is suggested by real business cycle models. A positive productivity shock should on impact raise the marginal productivity of capital and hence real interest rates. It is likely also to induce greater capital accumulation, and thus its effects should gradually die away, whether the original productivity shock has permanent effects on productivity or not.

This is borne out by considering the simplest textbook model (Blanchard and Fischer, 1989). It is assumed that the economy contains a representative consumer who maximizes expected utility over an infinite horizon. Utility is logarithmic in consumption:  $u(c_t) = \ln(c_t)$ . There is a discount factor  $\beta$ . Production is a Cobb-Douglas function:  $Y_t = A_t K_t^\theta$  in which  $0 < \theta < 1$ , and  $A_t$  is a productivity shock which follows a stochastic process. For tractability it is assumed that there is a 100% per period depreciation rate. As a result,  $K_{t+1} = I_t$  and  $Y_t = C_t + I_t$ . The solution to the problem involves consumption being a fixed share of output each period:  $C_t = \alpha Y_t$  and  $K_{t+1} = (1 - \alpha)Y_t$ , where  $\alpha = 1 - \beta\theta$ . As a result  $K_{t+1} = \beta\theta A_t K_t^\theta$ , or in logs,  $k_{t+1} = \log(\beta\theta) + a_t + \theta k_t$ . The marginal productivity of capital  $MPK_t$  is  $\theta A_t K_t^{\theta-1}$  or, in logs,  $\log(MPK_t) = \log(\theta) + a_t + (\theta - 1)k_t$ . With some substitution this leads to:  $\log(MPK_t) = -\log(\beta) + (1 - \theta L)^{-1}(a_t - a_{t-1})$ .

If the log of the productivity parameter,  $a_t$ , is a random walk with drift, then the log of the MPK is an AR(1) process with mean  $-\log(\beta) + (1 - \theta)^{-1}\mu$ , where  $\mu$  is the drift of the random walk.

Thus this very simple model derives the MPK as a stationary process, while consumption and income are integrated. Investment and the MPK go up after a

<sup>1</sup>Throughout the paper, the estimation period was 1959(2) to 1994(1), to make allowances for lags and missing observations.

big productivity shock. The increase in investment raises the capital stock onto a higher steady state growth path (or level, where there is no long run growth), and the MPK is gradually pushed back down towards its long run value. The effect of a productivity shock on the real interest rate is transitory, not permanent. The speed with which the effect of the shock on the real interest rate dies away may be an artefact of one of the simplifications of the model, viz., the assumption of 100% depreciation. If the capital stock were to depreciate slowly, and with irreversible investment, the amount of investment would on average be a much smaller fraction of the capital stock. Changes to the capital stock would be a smaller fraction of it (the capital stock); and capital would be more persistent. Hence the MPK would be more persistent. The theoretical prediction of a persistent real interest rate may then not be inconsistent with the empirical finding that the real rate is non-stationary. The data may contain a stable but persistent real rate.

The stochastic productivity process which drives the above RBC model is  $I(1)$ , and it imparts a stochastic trend to GDP, Consumption, etc. As we saw above, innovations in productivity feed in to real interest rates through their effect on the marginal productivity of capital. The equation above implies that real interest rates will be given as a distributed lag function of themselves and the innovation in the productivity process.

## 2.2. Differences among countries' real interest rates

While a very simple model implies a connection between productivity shocks and real interest rates, it says nothing about the relationship between real interest rates in different countries. In practice, real interest rates, as conventionally measured, tend to be different in different countries. This need not be inconsistent with perfectly flexible wages and prices, full information, and a complete set of contingent claims markets (the set of assumptions routine in much of the real business cycle literature). Lucas (1982), and Stockman and Dellas (1989) consider cash-in-advance models of open economies. Lucas considers economies each of which produces a different good. Stockman and Dellas consider economies which produce a common traded good and also a non-traded good. Real rates of interest are not uniquely defined in worlds containing many goods. The own-rates of interest on different goods need not be the same. A conventionally measured real rate which is the rate of return on a nominal asset deflated by the increase in the price of a basket of goods need not be uniform across countries. The cash-in-advance framework implies that monetary disturbances may have effects on real interest rates. However, as Christiano and Eichenbaum (1995) point out, these may be of a pattern over time and of a size which do not conform to facts.

### 2.3. Liquidity effects in real business cycle models

Christiano (1991) and Christiano and Eichenbaum (1995) develop a number of models in which monetary disturbances have real effects which conform to broad facts, in that increases in money cause interest rates to fall in the short run and stimulate production. They obtain this result by introducing several modifications to earlier cash-in-advance models. These include "sluggish saving", developed from work of Lucas and Fuerst, the idea that households have to make savings decisions before knowing the realisation of the current period productivity or money supply shock; "sluggish investment" which applies the same to capital, in that firms make investment decisions without knowing the current shocks; and thirdly the idea that firms need cash to finance their working capital — payment of wages in advance of production being completed and sold — and have short-run inflexibilities in their production plans, which are fixed in part before the realisation of shocks are known. With these features a fairly simple cash-in-advance model can reproduce some features of data for the US economy. In particular, the liquidity effect of an increase in the money supply can dominate the expected inflation effect and generate at least a short term fall in interest rates.

Consequently, one might conclude from this analysis that observed movements in real interest rates among countries might plausibly be viewed as the consequence of real and nominal disturbances — productivity and money supply shocks — working themselves out through the decisions of firms and households in a stochastic general equilibrium model of the world economy. With this kind of model in mind we proceed to consider the data at hand.

## 3. Data

### 3.1. Sources

The data we use are taken from the International Financial Statistics. Data for output are nominal GDP, IFS line 99b.c. These are all seasonally adjusted figures for quarterly GDP at annual rates expressed in national currency units (billions of US \$, Japanese yen, French francs, Deutschmarks, and pounds). Quarterly data for France were not available for the period 1960 to 1964 inclusive, and so the figures used are interpolated from annual data. The price level data is in each case the consumer price index, IFS line 64, an index number with 1990 = 100. <sup>2</sup> The real output measure for each country has been computed from the ratio of the

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<sup>2</sup>The results of this paper are not substantially altered when a GDP deflator is used as a price measure instead of the CPI.

nominal GDP figure to the CPI measure. The interest rates are the three month treasury bill rate, IFS line 60c, for the US and the UK. For France, Germany, and Japan, the interest rate data is the money market rate, IFS line 60b, which is a rate on much shorter term instruments than three months. However the data are in all cases averages over the periods (quarters) in question. The data are for the period 1957(1) to 1994(4).

### 3.2. Time series properties of real interest rates

Before moving to a structural model of real interest rates, we examine the time-series properties of the real rates themselves.

The ex ante real interest rates we compute turn out to be fairly persistent. For most of them, it is not possible to reject the hypothesis that they contain a unit root. ADF tests on individual ex ante real rates are reported in Table 1, along with the estimated largest autoregressive root.

The results depend on the number of lags, but for most of the countries, the data are not inconsistent with real interest rates being  $I(1)$ . This is unappealing, from a number of points of view. As Rose (1988) points out, it implies a contradiction between the apparent behaviour of real rates and the broad facts about data on aggregate consumption. It may however be the case that real rates are in fact stationary but rather persistent. The statistical evidence on the stationarity or otherwise of real interest rates is far from clear-cut. Gregory and Watt (1995) discuss this issue at some length, and decide finally to proceed on the assumption that real interest rates and inflation rates are stationary. By contrast, Limosani (1995) proceeds on the assumption of non-stationarity and looks for cointegration among real interest rates for various groups of countries. In view of the ambiguity of the statistical evidence, we explore the persistence and co-integration properties of rates in the next section. However, the underlying hypothesis of the paper is that rates are stationary but highly persistent, and subsequent sections of the paper proceed under this hypothesis.

### 3.3. Co-integration of real interest rates and a common trend

If real rates were indeed non-stationary, the idea of there being a "world real interest rate" in any sense would only have any content if the real rates of individual countries were cointegrated in a particular way. If none of the rates moves too far from any of the others, then they must contain a single common trend. If for example each pairwise difference of real rates were stationary, then there would be  $n-1$  independent cointegration vectors and an independent vector of real rates which contains the common stochastic trend. We therefore investigate the presence of a possible common trend in real interest rates. Whilst testing for co-integration

creates a tension with our underlying hypothesis that rates are stationary, this is more apparent than real. In finite samples, we would expect approximately the same distributions for the test statistics for both non-stationary and for nearly non-stationary series.

First consider results of a co-integration test. This is Johansen's maximum  $\lambda$  test for the number of co-integration vectors, and is based on estimates of the eigenvalues in a VAR in real interest rates.

Once again the results depend on the lag length, but there is some evidence for 4 cointegrating vectors, i.e., for a single common trend in real interest rates, at a lag order less than or equal to 2. Diagnostics for 3 of these 5 interest rates indicate that a lag order of 2 is adequate, but in the remaining cases, the appropriate lag order was found to be 3.

If there is a common trend, the largest principal component (LPC) will provide a (non-unique) estimate of it. Regressing real interest rates on the LPC gave residuals that were far more mean reverting than the original series. Table 3 shows that the largest autoregressive root for these residual deviations varies between 0.57 and 0.77, whereas for the interest rates themselves the figures are 0.73 and 0.91. Although no longer asymptotically valid, the ADF tests reflect this decline in persistence with values between -2.86 and 5.0, as compared with -1.96 and -3.4 for the interest rates themselves.

These results are not sensitive to the use of ex post rather than ex ante real interest rates. Similar results to those in table 3 emerge when the data are instead residual deviations from an average ("world") real rate or residual deviations from the US rate rather than from the largest principal component.

Two lessons can be drawn from these results. The first is that because of the persistence of the series, the small sample properties of many test statistics will differ from their asymptotic counterparts, and inferences based on asymptotic properties will have to be treated with care. Accordingly, results that may depend on decisions from such tests are subjected to sensitivity analyses. The second is that the structural time series model should be able to identify one or two common persistent components or "trends" in the series. In fact, this turns out to be the case.

We now turn to a structural model of interest rates in which an attempt is made to explain their movements in terms of innovations in real GDP growth and inflation.

## 4. A Structural Time-Series Model

### 4.1. Real and nominal shocks

The most general empirical model of the relationship between real interest rates, inflation, and real GDP growth for our five countries that we consider can be written as an unrestricted VAR in all fifteen variables. Viz.,

$$\Phi(L)z_t = e_t, \quad (4.1)$$

in which  $z_t = (\Delta p_{1t}, \Delta y_{1t}, \Delta p_{2t}, \Delta y_{2t}, \dots, \Delta p_{5t}, \Delta y_{5t}, r_{1t}, \dots, r_{5t})'$ ,  $\Phi(L) = I + \Phi_1 L + \Phi_2 L^2 + \dots + \Phi_p L^p$ , for some lag length  $p$ , and  $e_t$  is a  $15 \times 1$  vector of errors. The various models we explore below can be viewed within this framework as restricted versions of this model. The inflation forecasts used to construct our real ex ante interest rates are generated from five country-specific bivariate VARs in inflation and real GDP growth. These forecasts are consistent with the restricted VAR used to identify nominal and real shocks.

We now want to obtain measures of real and nominal disturbances which might have influenced real interest rates. There are a number of candidates for these. Much of the real business cycle literature has used Solow residuals as a measure of the exogenous productivity disturbances which are assumed to be the (only) driving variables of the system. However, in quarterly data these residuals contain a seasonal pattern which appears unrelated to any pattern in real interest rate data. They are also autocorrelated which implies that they may not be interpreted directly as underlying productivity shocks. Further, the Solow residuals give only a measure of a real shock, and not a nominal shock, whereas it is clear that nominal disturbances affect real interest rates, at least in the short run. (Barro and Sala-i-Martin (1990), for example, found evidence of this.) As an alternative we use a "structural VAR" method similar to that proposed by Blanchard and Quah (1989).

We estimate 5 VARs, one for each country, in its inflation and real GDP growth rates. Prior specification testing suggested that a lag order of 4 was adequate. Seasonal dummies were also included. We impose the restriction that the nominal shock has no permanent effect on the real GDP. This restriction is enough to identify uniquely and separately a "real" and a "nominal" shock. Inverting the VARs to give the MA representation we have

$$\begin{pmatrix} \Delta p_{it} \\ \Delta y_{it} \end{pmatrix} = \Omega_i(L) \begin{pmatrix} \xi_{it}^n \\ \xi_{it}^r \end{pmatrix} \quad (4.2)$$

in which  $\Omega_i(1) = \begin{pmatrix} \cdot \\ 0 \end{pmatrix}$



and the nominal and real innovations  $\xi_{it}^n$  and  $\xi_{it}^r$  are by construction mutually orthogonal and of unit variance. Henceforth we refer to these temporary and permanent shocks to output ( $\xi_{it}^n$  and  $\xi_{it}^r$ , respectively) as BQ nominal and real shocks.

Comparing equations 4.1 and 4.2 we see that this structure imposes Granger non-causality from real interest rates and other countries' GDP growth and inflation rates on to each country's GDP and inflation. These are strong restrictions, which are imposed in order to enable us to obtain the BQ residuals, and because the identification restrictions that would be needed in a completely general VAR would be large in number and would probably be arbitrary in nature. Also a completely general VAR is likely to be highly over parameterized.

The restrictions severely limit the way in which the transmission of business cycles may occur in the model. In particular the structure allows for transmissions only through contemporaneous correlations of real (nominal) shocks across countries. Strictly speaking, the restrictions are only exactly valid when the dynamic response path of output to domestic real (nominal) shocks is proportional to those of real (nominal) shocks originating elsewhere. Whilst this scenario is unlikely to hold exactly, it may be approximately true if the real (nominal) shocks that originate in the domestic economy are qualitatively similar in nature to the foreign real (nominal) shocks. For example the real shocks may be exogenous movements in the productivity of capital and labour generated by technical progress. In this case it is not unreasonable to assume that the time path response of a country's output to such a shock will be approximately the same (up to a multiplicative constant) regardless of the geographical origin of the shock. Similarly, if nominal shocks are primarily monetary disturbances perturbing aggregate demand then again it is not unreasonable to suppose that the time path of responses of home output to a shock of domestic origin will be just a scaled up version of that to a monetary shock originating elsewhere.<sup>3</sup>

In order to test whether these restrictions are justifiable empirically, omitted variable tests are carried out. Results are reported in table 4. These results are for LM tests in which the residual from each equation above is regressed on two lags of other countries' output and inflation, and two lags of real ex ante interest rates are used. Thus the test statistics are all distributed  $\chi^2(26)$ .

The exclusion restrictions (26 excluded lags from each of 10 equations) given are tested on an equation by equation basis. It would also be possible to test using a single statistic such as the likelihood ratio. It might have appeared natural to start with the unrestricted VAR and use a LR test to test the validity of the restrictions implied in equation 4.2 above. However, simulations by the authors

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<sup>3</sup>Of course the scaling could be zero as in the case of freely floating exchange rates and full monetary insulation.

suggest that the latter is an unwise policy. Taking the estimated coefficients and error variance covariance matrix as "true" values and adding the assumption that the errors are normally distributed, we simulated 500 data sets for inflation and output growth for the five countries and performed tests of the exclusion restrictions on each data set. The actual test size for a nominal size of 5% turned out in the simulations to be in excess of 70%. The problem appears to be that the ML estimates of the covariance matrix of the errors from the unrestricted model regressions severely underestimates the true matrix. Estimated 95th percentiles from these simulations for the LM tests on an equation by equation basis are given in table 4. By contrast with the LR test, we see that these tests had sizes closer to the nominal size of 5%. Although still larger than the nominal test size, the rejection rate was now around 10%. This motivates the use of equation by equation tests. Also, in the tables below, nominal critical values for all the various exclusion restrictions are supplemented by their simulated counterparts.

As can be seen from Table 4, there are 3 significant statistics using the standard critical value and only two using the simulated ones. We also tested for excluded lags of up to order three and the results from these tests (available on request) were qualitatively similar.

The above test statistics and simulated finite sample critical values are strictly valid only if the residuals in these equations are homoskedastic and normally distributed respectively. We test for GARCH errors using a Breusch-Pagan test (Hamilton, 1994, p664) and for non-normality using a goodness-of-fit chi-squared test. Results are reported in tables 5 and 6. There is little evidence of either heteroskedasticity or non-normality.

As well as testing exclusion restrictions, we also examine the explanatory power of the included variables. The joint significance of the four included lagged inflation rates and GDP growth rates is high in all but three cases, as table 7 shows:

The VARs therefore capture much of the variance in GDP growth and inflation very parsimoniously. A dynamic specification test, in which lagged residuals from all countries were added to the VARs, was passed in all but one case as Table 8 shows

The structural stability of the VAR estimates was examined via an asymptotic Chow test taking 1972(2) as the break point. This corresponds roughly with the end of the post-war float. The results are given in Table 9. Although there is only one rejection at the nominal significance level, many of the statistics are quite high. This suggests that we should be careful to examine the sensitivity of our main results to dropping the fixed rate (pre 1972(2)) observations.

In the light of these tests, we argue that, while the country VAR's may not be the best representation of the data, the exclusion restrictions that we impose are unlikely to have a fundamental bearing on the qualitative nature of our results.

The BQ VAR set-up is in sympathy with RBC models with country-specific productivity shocks and monetary shocks. These feed through to interest rates which are driven by these shocks. We also of course allow for unexplained components to interest rates in anticipation of failing to explain them purely in terms of BQ nominal and real shocks.

#### 4.2. The relationship between inflation, growth, and real interest rates.

We examine an unrestricted VAR model for the ex ante real interest rates, inflation rates, and real GDP growth rates for the five countries. Initially we chose a lag order of two on all variables. Diagnostic tests for the significance of a third lag are given in table 10. Before discussing these and other tests, we should note that the statistics given here (plus those in Tables 11 to 14) are subject to the problems caused by generated regressors (Pagan and Nicholls, 1984) and near unit root behaviour in interest rates (discussed above). These factors together with the relatively few degrees of freedom that we have mean that the asymptotic critical values are unlikely to be very accurate. The results must therefore be treated with caution and the tests may only be used as heuristic devices. Given that these tests establish the number of lagged terms appearing in the VAR for rates we shall be careful later to examine the sensitivity of any empirical conclusions to changing the lags in the VAR.

Table 10 shows that the third lag is important for all but France and Germany. However, to include three lags would imply regressions containing 45 explanatory variables, which is unlikely to give reliable estimates. We therefore looked for restrictions which would allow us to omit some of these variables. Tables 11 and 12 show separately the significance of the third lag of interest rates, and inflation and output growth respectively. It is clear from these tables that the latter are far more important in explaining rates than are the former.

A further test of the second lag of interest rates given in table 13 shows that these also are relatively unimportant. As a result, they were dropped from the VAR, leaving a final specification that has once lagged rates and three lags in growth and inflation as explanatory variables for interest rates.

As a further specification test of this final form for rates we explore the structural stability of the VAR using the same asymptotic Chow test as we used above in Section 4.1. The results of this test are in table 14. Like their counterparts for output and growth, these tests show only one rejection at a nominal 5% level of significance (the case of Japan) but again the number of high statistics reinforces the need to check the sensitivity of our results with respect to dropping observations from the fixed exchange rate period.

The final specification of the model therefore is equation 4.2 for growth and

inflation, and for rates it is

$$\Gamma(L)r_t = \psi(L)x_t + \gamma'\widehat{\xi}_t + \xi_t^e \quad (4.3)$$

in which  $x_t = (x'_{1t}, x'_{2t}, \dots, x'_{5t})'$ ,  $x_{it} = \begin{pmatrix} \Delta p_{it} \\ \Delta y_{it} \end{pmatrix}$ ,  $r_t = (r_{1t}, r_{2t}, \dots, r_{5t})$ ,  $\widehat{\xi}_t = (\widehat{\xi}_{1t}^n, \dots, \widehat{\xi}_{5t}^n, \widehat{\xi}_{1t}^r, \dots, \widehat{\xi}_{5t}^r)'$  are the estimated contemporaneous Blanchard-and-Quah innovations for inflation and real growth for each country, as obtained above, and  $\xi_t^e$  is a residual. The lag polynomials  $\Gamma(L)$  and  $\psi(L)$  are:  $\Gamma(L) = I + \Gamma_1 L$ ;  $\psi(L) = \psi_1 L + \psi_2 L^2 + \psi_3 L^3$ . The inclusion of the BQ innovations in the regression is an innocuous device. It is done to make the error term  $\xi_t^e$  in the VAR orthogonal to them, and to estimate their impact effects on interest rates.

#### 4.3. An unobserved components model for real growth and inflation

Our aim is to decompose interest rates into orthogonal components that we may identify with real and nominal shocks. Unfortunately the BQ shocks as they stand do not qualify for this role because they themselves are likely to be correlated across countries. For example, some productivity shocks are likely to originate in the global economy, whilst others may originate from shocks to output in the various countries. We estimate the following unobserved components model, in order to extract a measure of common and country-specific real and nominal disturbances. It assumes that innovations in real GDP growth for the five countries share a common component and also have an idiosyncratic component. Innovations in inflation are assumed to have the same structure. The vector of five real GDP growth innovations is denoted  $\xi_t^r = (\xi_{1t}^r, \xi_{2t}^r, \dots, \xi_{5t}^r)$ , and the inflation innovations analogously as  $\xi_t^n$ . Thus we have

$$\begin{aligned} \xi_t^r &= \alpha_r \bar{\xi}_t^r + u_{rt} \\ \xi_t^n &= \alpha_n \bar{\xi}_t^n + u_{nt} \end{aligned} \quad (4.4)$$

in which  $\alpha_r, \alpha_n$  are  $5 \times 1$  vectors of coefficients,  $\bar{\xi}_t^r, \bar{\xi}_t^n$  are common innovations, and  $u_{rt}, u_{nt}$  are  $5 \times 1$  vectors of idiosyncratic components. The restrictions  $E(u_{nt}u_{nt}') = \text{diag}\{\sigma_{ni}^2\}$ ,  $E(u_{rt}u_{rt}') = \text{diag}\{\sigma_{ri}^2\}$  for all  $t$ , are imposed, and the common and idiosyncratic components are assumed to be independent. These equations each contain 11 parameters, but yield 15 moments and cross moments. Thus Generalised Method of Moments yields 4 over-identifying restrictions. They are estimated by GMM and results with heteroskedasticity consistent estimates of the covariance matrix are reported below in table 15.

The coefficients in the unobserved components model for the nominal shocks are not estimated with much precision. Most are not significantly different from

zero. However, the coefficients (the  $a_{rt}$ 's) in the model for real shocks are all significant, positive, and of plausible sizes. Tests of the overidentification restrictions in the models give  $\chi_2(4)$  test statistics of 2.81 for the real shocks and 7.36 for the nominal shocks, both of which are well inside the critical region at conventional test sizes.

By construction, the BQ residuals possess the property that for each country within sample the nominal and the real shocks are uncorrelated. There is however nothing to guarantee that the real shock of one country is uncorrelated with the nominal shock of another, although this would be required if the unobserved components in the above decomposition are to be genuinely uncorrelated for all possible pairs of shocks. Table 16 displays pairwise correlation coefficients between real and nominal shocks for the five countries.

The only significant covariances are those between the US real shock and the German nominal shock, and between the French real and the Japanese nominal. All the other entries in the table are relatively small. The correlations indicate that the components identified in the model above will if estimated be only approximately uncorrelated. Variance decompositions based on this model will therefore only be approximate.

#### 4.4. Impulse Response Functions

How do the real and nominal shocks associated with each country, and also the estimated world nominal and real shocks from the unobserved components models, affect each country's real interest rate? To answer this question, we examine impulse response functions. By substituting for growth and inflation using equation 4.2 in equation 4.3, and inverting the left hand side lag polynomial in equation 4.3, the estimated VAR in real interest rates can be written in MA form as

$$r_t = \theta_{nr}(L)\xi_t^{nr} + \theta_e(L)\xi_t^e$$

where  $\theta_{nr}(L)$  and  $\theta_e(L)$  are infinite lag polynomials, where  $\xi_t^{nr} = (\xi_t^{nr} | \xi_t^{nr'})'$  are the re-ordered vector of BQ residuals and where the columns of  $\theta_{nr}(L)$  have been re-ordered conformably.

Finally, using equation 4.4, the common and idiosyncratic shocks are substituted into the expression to give

$$r_t = \theta_n(L)\alpha_n\bar{\xi}_t^n + \theta_n(L)u_{nt} + \theta_r(L)\alpha_r\bar{\xi}_t^r + \theta_r(L)u_{rt} + \theta_e(L)\xi_t^e \quad (4.5)$$

We have consistent estimates of all of the lag polynomial terms in equation 4.5. From it the impulse responses of real rates to the common and idiosyncratic real and nominal shocks can be derived.

Figure 1 shows the responses of each country's real ex ante interest rate to a one unit innovation in the common nominal shock  $\xi_t^c$  and real shock  $\xi_t^r$ . (The figures include confidence intervals around the point estimates. The confidence intervals were obtained from simulations, using the same method as for critical values in table 4 above. They are asymptotically 95% confidence intervals.) The responses for the five countries share common features. The effect of the nominal shock is a reduction in real interest rates during the early periods, which eventually dies away to zero. Exceptionally, the United States real interest rate rises significantly in the first three quarters, before falling below zero. The effect of the real shock is to raise real interest rates in the initial quarters (for up to around 20 quarters) before they fall back towards zero, in the US, France and the UK. In Japan, the effect becomes negative after eight quarters. In Germany the effect in the second quarter is significantly negative; otherwise the German response follows a similar pattern to those of the US, France and the UK. These appear to be consistent with the kinds of responses that would be produced in RBC models with liquidity effects, such as those of Christiano, Eichenbaum, and others.

Figure 2 shows each country's response to its own idiosyncratic nominal and real shock. In response to their nominal shocks, Japan, France, and the UK show an initial fall in the rate, whereas the US and Germany show initially small and statistically insignificant rises. The responses to the real shocks are predominantly positive, although between nine quarters and seventeen the US response is significantly negative. Thus typically, the common and the idiosyncratic nominal shock produce an effect on rates in the same direction. (The exception being the US.) This contrasts with results of Gregory and Watt who found that the own-country nominal shock produced an increase in rates, whereas the common nominal shock produced a decrease.

We now turn to a consideration of sources of persistence and common components in rates, and the factors causing the general rise in rates in the early 1980s. Consider the issue of common persistent components in rates. There are seventeen (theoretically) independent innovations in the moving average representation in equation 4.5. The equation shows each rate to be the sum of seventeen infinite moving averages in each of these innovations which may be interpreted as (theoretically) orthogonal components. Hence each innovation forms a sole source of five interest rate components (one for each country). For example, the 1,5 element of  $\theta_n(L)u_{nt}$  is the component of US rates arising from the UK's idiosyncratic nominal shock.

If a component is to form something like a common trend for all five interest rates, its time path should have two properties: it should be highly correlated across countries and should make a large contribution to each rate's variance. The correlations of the components across countries depend only on the moving

average parameters in equation 4.5 and the variances of the shocks both of which we have estimated already. For each component therefore, we may compute the ten ( $=C_3^2$ ) correlation coefficients between all possible country pairs. To assess contributions to persistence we may also estimate the variance of each component under the theoretical assumption that all components are mutually uncorrelated. In theory, the variance of rates should equal the sum of the variance of the twelve components on the right of equation 4.5 plus the variance of the residuals. In practice however this will not be the case because as we noted in Section 4.3 above the uncorrelatedness of the components was not fully imposed during estimation so that they will in general be correlated within the sample.<sup>4</sup> To assess the relative contribution of each component, we divide each component's estimated variance by the sum of the variances of all components excluding residuals. The latter sum approximates the amount of variance in rates accounted for by real and nominal shocks so that the proportions we compute represent estimates of each component's contribution to explained variance. For completeness, we also tabulate the exact empirical explained variance as a proportion of the total variance of rates. This was computed as  $1-(A/B)$  where A is the variance of the residual components and B is the variance of rates. The correlation coefficients are given in Table 17 and the results from the decomposition of variance are given in Table 18.

In terms of high correlations across countries, two components stand out from Table 17, namely those components due to the US and UK idiosyncratic nominal shocks respectively. In the case of the first, the correlations all lie above .76 and those for the UK above .87. In terms of volatility, Table 18 shows that the components due to the US real and nominal idiosyncratic shocks are dominant. Multiplying each of columns 2 to 13 by column 1 gives an estimate of the proportion of variance in rates accounted for by each of the 12 components. Doing so, we estimate that the component in the US idiosyncratic nominal shock accounts for approximately 11%, 4%, 21%, 14% and 9% of the variance in US, Japanese, German, French and UK rates respectively. Given that there are 17 components contributing to the variance in rates, then excepting the figure for Japan, these proportions are quite high. This underlines the importance of real shocks in driving interest rates but it does not indicate that this real shock is the source of a "common trend" because most of the correlations across countries of this component of rates (Table 17) are quite low. Repeating the computation for the US nominal idiosyncratic shock we see that this component accounts for around 6%, 15%, 20%, 17% and 9% of the variance of rates in the US, Japan, Germany, France and the UK respectively. The component of rates due to the US

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<sup>4</sup>The problem is that the parameters associated with the decomposition in equation 4.4 had to be estimated from an overidentified system so that sample variances do not correspond one to one with their theoretical counterparts.

idiosyncratic nominal shock therefore, satisfies the criterion for a "common trend" being both relatively volatile and highly correlated across countries. Perhaps this is not a surprising result because during the early part of the sample period the US was responsible for exporting inflation worldwide although this phenomena was obviously less predominant after the break up of Bretton Woods. To the extent that inflation shocks affect real *ex ante* interest rates we would therefore expect inflation shocks originating in the US to have a pervasive influence on the general level of world real interest rates.

In a final experiment, we set all real shocks (plus unexplained residuals) to zero and simulated the resulting real rate series for the five countries. The time series of a straight forward cross country average of these simulated rates are compared with the actual average for real rates in Figure 3. The simulated series seems to track well the rise in rates that occurred in the later part of our sample. The experiment was repeated for all the real shocks (setting nominal and unexplained shocks to zero) and the corresponding graphs are presented on the right of figure 3. By contrast, this graph shows that real shocks cannot explain the general rise in rates during the 1980's. Further experiments were undertaken to estimate the individual influence of each component on the rise in rates. Using signal extraction techniques we estimated the means of the components conditional on the BQ estimates but found that no single component was able on its own to account for the general rise in rates. Detailed results of these extra experiments are available on request.

Finally we note the results of the sensitivity analyses proposed in Sections 4.1 and 4.2 above. With respect to lags in the VAR equations for interest rates, we dropped the third lag in output growth and inflation and added a second lag in rates. As we would expect, the contributions to persistence of the real and nominal shocks declined slightly with the "unexplained" residuals becoming correspondingly more important. Despite this, nominal and real shocks originating in the US remained a dominant source of persistence and US nominal shocks still had a coherent effect on rates across countries. All the other results were qualitatively unchanged. Dropping the first 47 observations of the sample increased coefficient standard errors and widened confidence intervals but did not qualitatively alter the main results.

## 5. Conclusions

Stochastic general equilibrium models, such as models of real business cycles which allow for liquidity effects, suggest that real interest rate movements should be the result of real and nominal disturbances to the economy. In this paper, we have examined the dynamics of real *ex ante* interest rates for five major OECD



economies (the US, Japan, Germany, France and the UK) within the context of a VAR in real interest rates, inflation, and real GDP growth. Real interest rates have then been resolved into components explained by real and nominal shocks arising in these countries, as well as unexplained components. Some of these real and nominal shocks appear to be common sources of variation and persistence in real interest rates, particularly the US real and nominal shock, each of which accounts for a substantial fraction of real interest rate variation in all five countries.

The model suggests that the the source of the widespread rise in real interest rates around the world in the early nineteen eighties was nominal shocks rather than real shocks whose contribution in this respect is negligible.

Impulse response functions show that the typical response to a nominal shock is an initial reduction in interest rates, followed by a gradual return to normal levels. The typical response to a real shock is an increase interest rates, followed by a gradual return to normal levels. This is a general pattern, to which there are exceptions. It is broadly consistent with a real business cycle world in which there is short run price stickiness, in which an inflationary shock does not lead to higher future expected inflation, and thus the so-called Mundell-Tobin effect outweighs any Fisher effect.

An attractive feature of analysing real interest rates in this was is that it avoids the difficult issue of whether or not there exists a "world real interest rate" and how to measure it. It also puts the analysis in a coherent framework, provided by stochastic general equilibrium models. There are however a number of limitations. If an effort is to be made to explore the interrelations between real interest rates in a number of countries, and the effects of output and inflation innovations, the limitations on the length of data series mean that it is necessary to restrict the number of variables in the VAR. This can be done by restricting the number of variables analysed for each country, or restricting the number of countries analysed, or imposing other restrictions on the VAR. We have taken a mixture of all three procedures. A drawback of this VAR analysis is that it does not identify whether or not one or two popular suspects were the culprits in interest rate movements. In particular, it has said nothing about whether government expenditure or borrowing, or oil price shocks had major effects on real interest rates, except in so far as they were associated with innovations in inflation or real GDP.

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## TABLES

Table 1. ADF Tests and largest AR root in ex ante real interest rates

Country	US	Japan	Germany	France	UK
ADF(2)	-2.3	-3.14	-3.03	-2.05	-2.47
$\rho(2)$	.88	.84	.78	.90	.84
ADF(4)	-2.00	-1.93	-3.48	-1.90	-2.51
$\rho(4)$	.90	.84	.73	.91	.81

Note: the 5% critical value for the ADF test is -2.99.  $\rho(n)$  is the largest AR root in a model with n+1 lags.

Table 2. Co-integration tests for ex ante real interest rates

Null versus alternative hypothesis				95%
	4 lags	3 lags	2 lags	critical value
0 versus 1	49.1	47.3	55.1	34.4
1 versus 2	23.3	27.4	39.3	28.1
2 versus 3	15.3	13.6	25.1	22.0
3 versus 4	6.7	10.8	15.4	15.7
4 versus 5	3.9	4.3	4.2	9.2

Note: The null hypothesis is that there are  $x$  cointegration vectors against the alternative that there are  $x + 1$ . "4 lags" etc. refers to the number of lags in the VAR.

Table 3. ADF tests and largest root for residual deviations from Largest Principal Component

Country	US	Japan	Germany	France	UK
ADF(2)	-3.2	-3.8	-3.6	-3.3	-5.0
$\rho(2)$	.75	.77	.65	.61	.57
ADF(4)	-2.8	-3.7	-3.4	-2.8	4.6
$\rho(4)$	.77	.75	.64	.75	.60

Table 4. Omitted variable tests for Blanchard-Quah residuals

	US	Japan	Germany	France	UK
$\Delta p$	45.2* (41.5)	37.8 (42.4)	26.7 (42.6)	32.1 (42.3)	44.6* (43.4)
$\Delta y$	32.7 (43.0)	40.5* (41.9)	32.9 (40.6)	31.8 (41.2)	38.2 (42.5)

Note: the 95% critical value for the  $\chi^2(26)$  test statistics is 38.9 (\* = significant at the 5% level). The numbers in parentheses are the simulated 95% critical values for the test statistics.

Table 5. Tests for GARCH(1,1) error process

	US	Japan	Germany	France	UK
$\Delta p$	0.68	0.005	2.38	1.63	4.92
$\Delta y$	0.33	1.00	7.34**	0.29	0.37

The test statistics are distributed as  $\chi^2(2)$  with a 95% critical value of 5.99. Only the statistic for the German output growth equation is significant at 5%.

Table 6. Tests for normality of residuals

	US	Japan	Germany	France	UK
$\Delta p$	4.77	20.00	13.99	16.87	22.54
$\Delta y$	15.30	15.73	18.76	19.02	12.70

The test statistics are distributed as  $\chi^2(14)$ , the 95% critical value of which is 23.7. Consequently none of these is significant at 5%.

Table 7.  $\chi^2(11)$  test statistics for the included variables in the BQ VARs

	US	Japan	Germany	France	UK
$\Delta p$	101.6**	65.9**	89.9**	100.7**	92.0**
$\Delta y$	39.1**	33.9**	17.8	20.8*	26.3*

Note: the 5% critical value for  $\chi^2(11)$  is 19.7. (\*\* denotes significance at 1%; \* denotes significance at 5%) The tests are the sample size multiplied by the  $R^2$  from each regression. Each regression includes four lags of inflation and output growth and three seasonal dummies.

Table 8

	US	Japan	Germany	France	UK
$\Delta p$	16.9	19.4*	6.2	9.8	16.0
$\Delta y$	12.1	14.6	13.9	14.1	7.3

Note: 95% critical value is 18.3 ( $\chi^2(10)$ ). 10 once-lagged residuals from the 5 countries' BQ VAR's were added to the VARs and their significance tested to give the  $\chi^2(10)$ 's. Apart from the equation for inflation for Japan, which gave a marginally significant test, all the statistics are satisfactory.

Table 9. Test of structural stability of VAR's in  $\Delta p$  and  $\Delta y$  using forecast errors

	US	Japan	Germany	France	UK
$\Delta p$	15.2	41.0**	18.7	21.0	18.9
$\Delta y$	18.3	18.7	9.2	21.0	19.2

Note: The test statistics are asymptotically distributed as  $\chi^2(12)$ , the 95% critical value of which is 21.1. \*\* denotes significant at 5% using the distribution of  $\chi^2(12)$ .

Table 10. Test of significance of a third lag in a VAR of real interest rates, inflation, and real GDP growth.

US	Japan	Germany	France	UK
45.5**	49.4**	21.2	33.9*	84.2**

Note: The test statistics are distributed as  $\chi^2(15)$ , the 95% point of which is 25.0. \*\* denotes significance at 1% and \* denotes significance at 5%.

Table 11. Test of including an extra lag of real interest rates

US	Japan	Germany	France	UK
10.9	11.5*	5.6	15.3**	14.5**

The test statistic is distributed as  $\chi^2(5)$ , with a 95% critical value of 11.1. \*\* denotes significance at 1%, \* at 5%.

Table 12. Test of including an extra lag of inflation and real GDP growth

US	Japan	Germany	France	UK
28.9**	42.3**	16.5	26.1**	77.9**

The test statistic is distributed as  $\chi^2(10)$ , which has a 95% value of 18.3.

Table 13. Test of significance of 2nd lag of real interest rates in the VAR

US	Japan	Germany	France	UK
6.2	9.2	17.4**	12.1*	8.0

Note: The test statistics are  $\chi^2(5)$  once again, with a 95% critical value of 11.1. \*\* denotes significance at 1%, \* at 5%.

Table 14. Predictive failure tests for the interest rate VAR

US	Japan	Germany	France	UK
41.1	58.2	50.4	39.9	50.8

The test statistics are  $\chi^2(36)$ , the upper 5% point of which is 51.5.

Table 15.

GMM estimates of parameters from the unobserved components model for nominal and real shocks<sup>5</sup>

<sup>5</sup>The variance of the common real and common nominal shocks are unidentified and are normalised to unity.

	coefficient	std error	t-ratio		coefficient	std error	t-ratio
$a_{n1}$	.15	.16	.89	$a_{r1}$	.33	.13	2.46
$a_{n2}$	.43	.17	2.61	$a_{r2}$	.31	.12	2.56
$a_{n3}$	.33	.20	1.66	$a_{r3}$	.62	.14	4.53
$a_{n4}$	.42	.19	2.17	$a_{r4}$	.43	.11	4.08
$a_{n5}$	.18	.20	.89	$a_{r5}$	.61	.17	3.59
$\sigma_{n1}^2$	.92	.15	6.01	$\sigma_{r1}^2$	.74	.11	6.52
$\sigma_{n2}^2$	.79	.22	3.62	$\sigma_{r2}^2$	.69	.12	5.78
$\sigma_{n3}^2$	.85	.14	6.02	$\sigma_{r3}^2$	.59	.15	3.82
$\sigma_{n4}^2$	.61	.23	2.64	$\sigma_{r4}^2$	.74	.14	5.10
$\sigma_{n5}^2$	.89	.12	7.54	$\sigma_{r5}^2$	.55	.14	3.95
$\sigma_n^2$	1.00			$\sigma_r^2$	1.00		

Note:  $a_{ni}$  and  $a_{ri}$  are the  $i$ th elements of  $\alpha_n$  and  $\alpha_r$  respectively.

Table 16. Correlation coefficients between nominal and real disturbances across countries

		Real shocks				
		US	Japan	Germany	France	UK
Nominal Shocks	US	0	0.10	-0.01	-0.005	0.14
	Japan	-0.07	0	-0.07	-0.17	-0.12
	Germany	-0.28	-0.002	0	0.006	-0.09
	France	-0.13	-0.13	0.03	0	-0.11
	UK	-0.02	0.14	0.01	0.11	0

Under the null hypothesis of zero correlation, each element is  $N(0, n^{-1/2})$  the 95% critical value of which is 0.17.

Table 17. Pairwise correlations of simulated real interest rate movements produced by individual components

		Common nominal shock				
		us	ja	ge	fr	uk
us	1.00					
ja	-.42	1.00				
ge	.92	-.40	1.00			
fr	.87	-.53	.97	1.00		
uk	.93	-.31	.89	.83	1.00	

Common real shock					
	us	ja	ge	fr	uk
us	1.00				
ja	-.26	1.00			
ge	.91	-.39	1.00		
fr	.73	.30	.71	1.00	
uk	.88	-.50	.74	.41	1.00

us idiosyncratic nominal shock					
	us	ja	ge	fr	uk
us	1.00				
ja	-.89	1.00			
ge	.82	-.76	1.00		
fr	.85	-.80	.98	1.00	
uk	.85	-.76	.99	.98	1.00

Japanese idiosync. nom. shock					
	us	ja	ge	fr	uk
us	1.00				
ja	.80	1.00			
ge	.46	-.06	1.00		
fr	.95	.78	.52	1.00	
uk	.97	.81	.35	.91	1.00

German idiosync. nom. shock					
	us	ja	ge	fr	uk
us	1.00				
ja	.47	1.00			
ge	.54	.27	1.00		
fr	.12	.13	.63	1.00	
uk	.94	.57	.61	.25	1.00

French idiosync. nom. shock					
	us	ja	ge	fr	uk
us	1.00				
ja	.75	1.00			
ge	-.44	-.03	1.00		
fr	-.59	-.21	.95	1.00	
uk	-.15	-.06	.85	.74	1.00

UK idiosyncratic nominal shock					
	us	ja	ge	fr	uk
us	1.00				
ja	-.89	1.00			
ge	.93	-.96	1.00		
fr	.92	-.91	.94	1.00	
uk	.93	-.97	.95	.87	1.00

US idiosyncratic real shock					
	us	ja	ge	fr	uk
us	1.00				
ja	-.37	1.00			
ge	.36	-.92	1.00		
fr	.49	-.95	.94	1.00	
uk	.56	-.90	.97	.93	1.00

Japanese idiosyncratic real shock					
	us	ja	ge	fr	uk
us	1.00				
ja	.45	1.00			
ge	.70	.12	1.00		
fr	.20	-.16	.02	1.00	
uk	.11	-.66	.10	.55	1.00

German idiosyncratic real shock					
	us	ja	ge	fr	uk
us	1.00				
ja	.60	1.00			
ge	.54	-.06	1.00		
fr	.85	.12	.62	1.00	
uk	.95	.60	.40	.82	1.00



French idiosyncratic real shock						UK idiosyncratic real shock					
	us	ja	ge	fr	uk		us	ja	ge	fr	uk
us	1.00					us	1.00				
ja	.96	1.00				ja	-.89	1.00			
ge	-.38	-.51	1.00			ge	.95	-.90	1.00		
fr	-.69	-.79	.73	1.00		fr	.80	-.75	.90	1.00	
uk	.78	.58	-.02	-.32	1.00	uk	.89	-.80	.80	.67	1.00

Table 18. Variances of simulated interest rate series

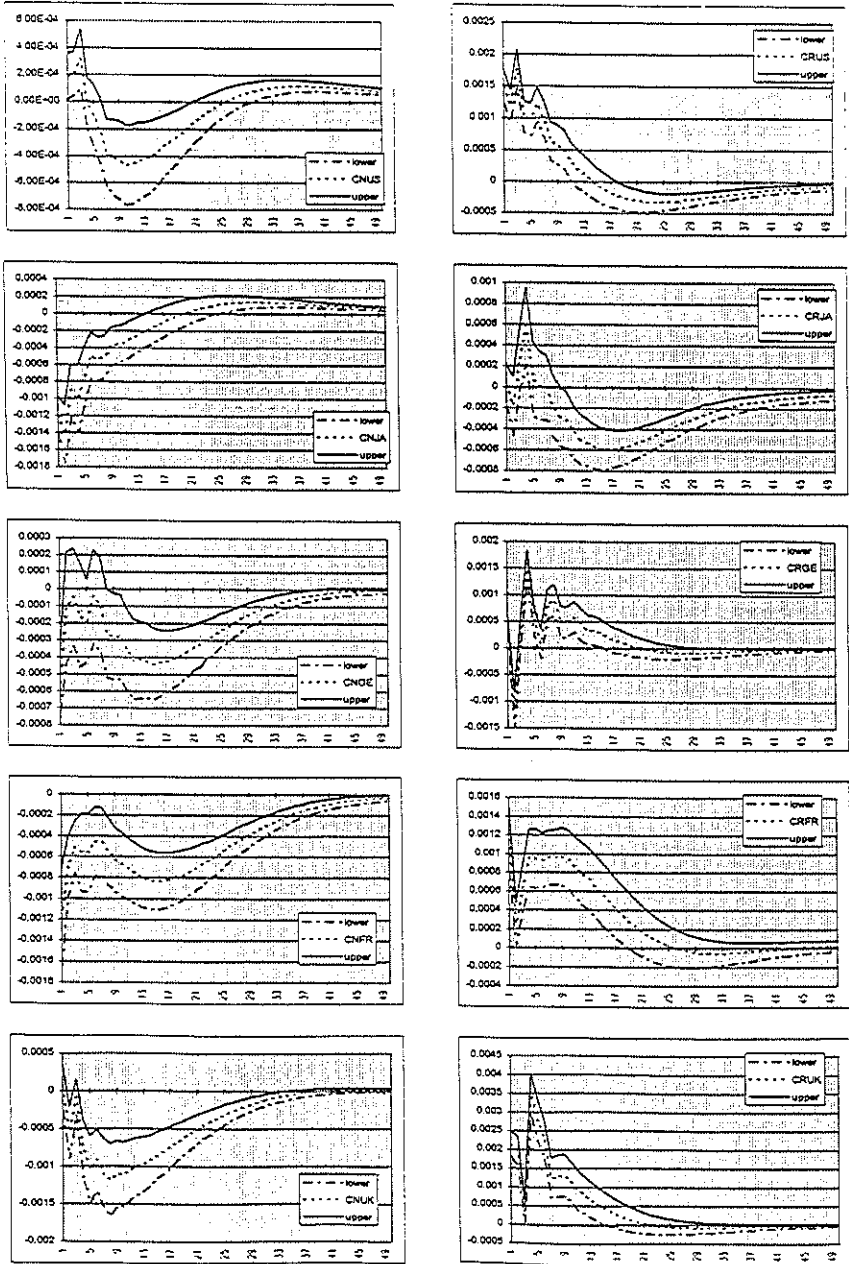
	pexp	pcn	pcr	pin1	pin2	pin3	pin4	pin5
US	69.26	18.7	25.4	8.0	3.5	1.9	2.0	10.6
Japan	77.3	2.9	3.6	18.5	19.0	.6	4.5	9.7
Germany	75.4	11.7	10.8	25.4	1.4	2.6	3.0	9.6
France	69.4	15.9	10.4	24.2	3.4	2.6	6.5	8.6
UK	84.0	27.7	19.7	10.3	1.7	1.2	.3	19.4

	pir1	pir2	pir3	pir4	pir5
US	15.8	0.6	1.9	7.6	4.0
Japan	4.8	21.0	.5	12.8	2.2
Germany	27.2	.8	3.9	1.1	2.6
France	19.4	.2	1.8	4.7	2.4
UK	10.9	.4	1.8	.1	6.6

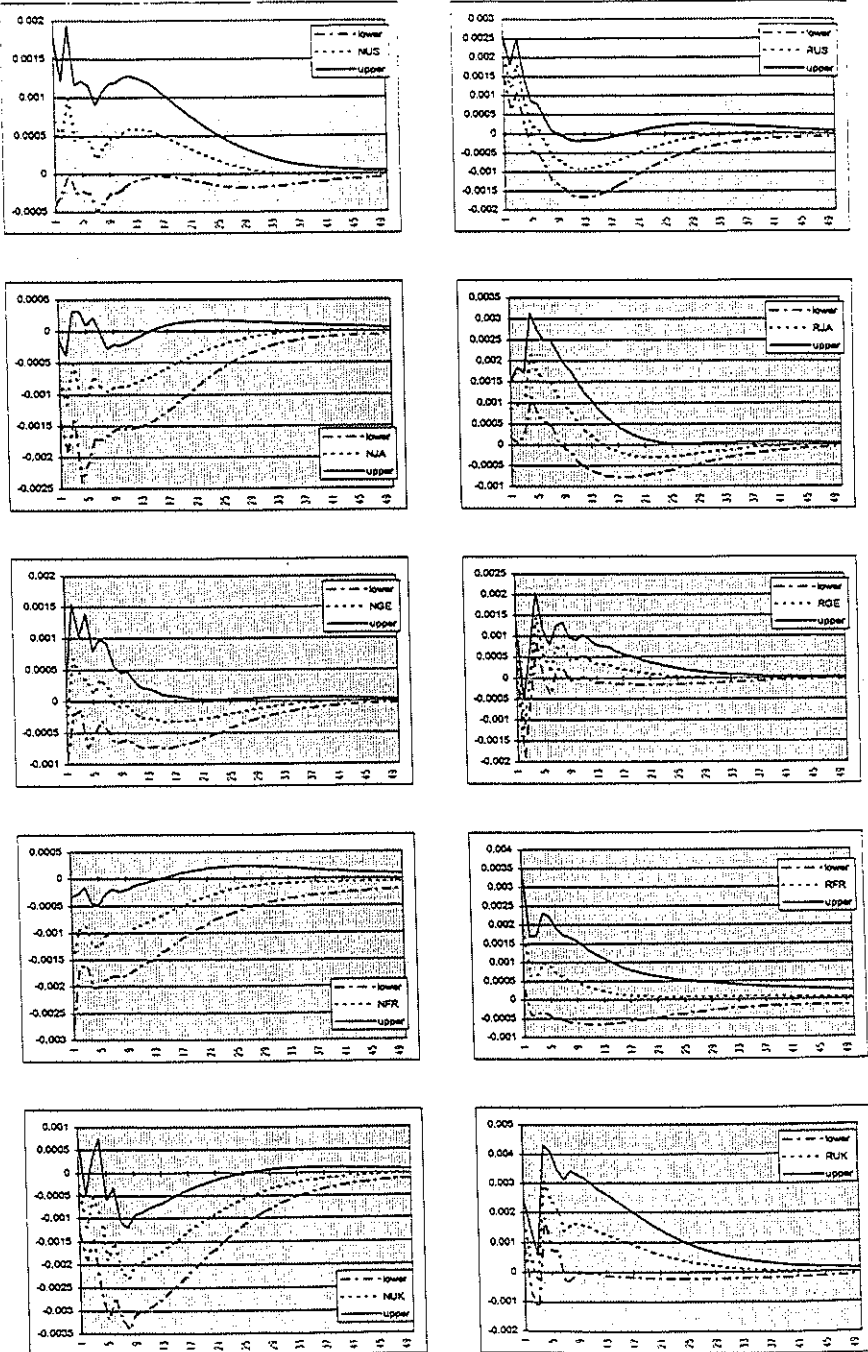
Note: pexp denotes the proportion of variance in the interest rate of the country in the left hand column explained by all real and nominal shocks; pcn and pcr are the estimated proportions of pexp accounted for by common nominal and common real shocks respectively; pinj and pirj are the estimated proportions of pexp accounted for by idiosyncratic nominal and real shocks originating in country j where j=1 denotes the US, 2 is Japan, 3 Germany, 4 France, and 5 the UK. All numbers are percentages.

Figure 1: Responses to common nominal and real shocks



$NR^X$  ( $RR^X$ ) = response of country X's rate to common nominal (real) shocks. Upper and lower lines give approximate 95% confidence interval.

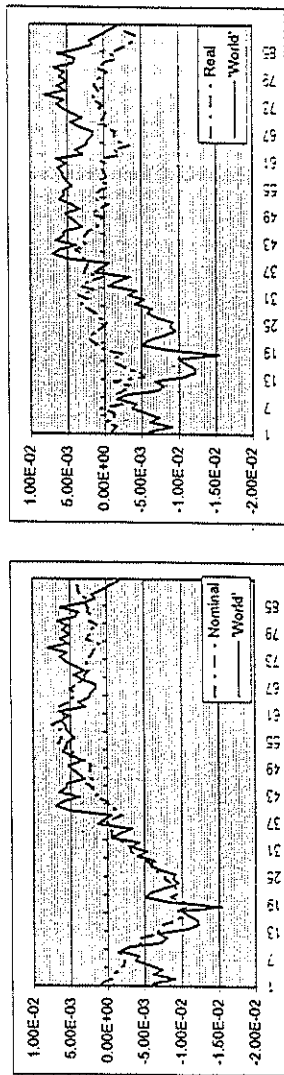
Figure 2: Responses to idiosyncratic nominal and real shocks



N<sup>X</sup>\* (R<sup>X</sup>\*) = response of country X's rate to idiosyncratic nominal (real) shocks. Upper and lower lines give approximate 95% confidence interval.

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Figure 3: The effect of nominal (left graph) and real (right graph) shocks on average rates



Nominal (Real) is the average of simulated rates absent all except nominal (real) shocks. 'World' is the average of real ex ante rates.







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