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ATTRIBUTE DEPENDENCE AND THE PROVISION OF QUALITY

Hans Degryse and Andreas Irmen

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Centre for Economic Policy Research
25–28 Old Burlington Street
London W1X 1LB
Tel: (44 171) 878 2900
Fax: (44 171) 878 2999
Email: cepr@cepr.org

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ABSTRACT

Attribute Dependence and the Provision of Quality*

A quality improvement often necessitates modifications of varietal product features. This paper studies firms' incentives to provide quality when this decision affects the goods' degree of horizontal differentiation. Intuitively, one is inclined to argue that private incentives to provide quality are insufficient relative to the social optimum if a quality improvement reduces horizontal differentiation. We find that this argument depends on whether the game is simultaneous or sequential. In the former case private incentives prove excessive relative to the social optimum, and in the latter case, insufficient. As a result, a regulator might want to impose either minimum or maximum quality standards.

JEL Classification: D21, L15
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Hans Degryse
CentER
Tilburg University
PO Box 90153
5000 LE Tilburg
THE NETHERLANDS
Tel: (31 13) 466 8210
Fax: (31 13) 466 3066
Email: h.degryse@kub.nl

Andreas Irmen
DEEP-HEC
BFSH–1
Université de Lausanne
CH–1015 Lausanne
SWITZERLAND
Tel: (41 21) 692 3474
Fax: (41 21) 692 3365
Email: andreas.irmen@hec.unil.ch

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NON-TECHNICAL SUMMARY

Before marketing a product a firm has to determine its attributes. It was Hotelling in his influential paper *Stability in Competition* who argued that two considerations drive this decision. On the one hand, firms equip their products with attributes that are close to consumers' preferences since this guarantees a high demand (demand effect). On the other hand, they take into account the attributes of competing products and offer a variant not too similar such that price competition is relaxed (strategic effect).

In spite of the fact that products consist of several attributes the above mechanisms are usually analysed in models of one-dimensional product differentiation where all product features are summarized in a single attribute. It is just recently that the literature on product differentiation has taken up the topic of firms' product placement decisions in multi-dimensional product spaces. So far its major focus has been the study of the pure effect of competitive positioning along each dimension. This requires the assumption of attribute independence: firms can pick each attribute independently of the choice of other product characteristics.

This paper argues that the latter assumption may indeed be restrictive. The car industry is a case in point. Under attribute independence a car producer can choose the attribute 'low gas consumption' without taking into account the weight of the material, the power of the engine, or the shape of the car. This seems unrealistic, however. Better gas mileage requires at least a mix of light materials, less powerful engines, and aerodynamic shapes. In other words, the attribute 'low gas consumption' is dependent on whether other attributes are chosen in an appropriate way.

This example (others are given in the main text) suggests that in real world situations attributes are often dependent. The paper focuses explicitly on the interaction of dependent attributes. In particular, we are interested in the question of how firms' incentives to provide quality change when the level of quality affects the degree of horizontal differentiation. How can this interaction come about? Clearly, better gas mileage is a quality feature of a car since less consumption reduces operational costs for consumers. One way of achieving lower gas consumption is by making the car more aerodynamic. Windtunnel tests show that only certain shapes guarantee better aerodynamics, however. Hence, one would expect shapes of cars with lower gas consumption to become more and more similar. The differences in design will be less pronounced. Therefore, an increase in the quality of cars via better aerodynamics may reduce the variety offered in the market. We say that in this case quality and variety interact negatively.

The results of the analysis are determined by the effect attribute dependence has on the two mechanisms identified by Hotelling. First, a negative dependence of a product's quality and variety attribute strengthens the demand effect and therewith the firms' incentives to provide quality. Consumers are more easily attracted by quality improvements than by price reductions. We show that this demand force drives the results when firms simultaneously choose quality and price. It leads to excessive private incentives to provide quality. Second, the strategic effect is also stronger under negative dependence. We find that price competition along the horizontal characteristic becomes fiercer, thus stimulating underinvestment in quality. It is then shown that the effect of negative dependence on the strategic effect more than outweighs that on the demand effect if firms can commit to a quality level before competing in price. As a consequence, an unregulated market underprovides quality such that a minimum quality standard is welfare improving. We also analyse the case of positive dependence for which the results tend to be the opposite.

The results are derived in a simple duopoly model of product innovation where products are described by a quality and a variety attribute. The quality offered by a firm influences consumers' willingness to pay for its product in two ways: it shifts this function outward and determines the transportation rate. This allows an interpretation of the latter as an indicator of the degree of horizontal product differentiation in the market.

I. INTRODUCTION

Improving the quality of one's product often necessitates modifications of other product features. The incentive to provide quality may therefore depend on how the goods' entire position in the characteristics space is affected. The purpose of this paper is to study this interaction in a simple model where quality improvements strengthen or weaken the products varietal differentiation with respect to a competing good. How does this link affect firms' incentives to provide quality? In the course of our analysis we identify circumstances where unregulated markets may not offer a satisfactory quality level such that minimum quality standards are called for. Before we get precise on that, let us motivate our analysis a little further and illustrate the interaction this paper is about.

First, consider cars. A better gas mileage is undoubtedly a quality feature of a car since less consumption reduces operational costs for the consumers. One way of achieving a lower gas consumption is by making the car more aerodynamic. However, windtunnel tests show that only certain shapes guarantee better aerodynamics. Hence, one would expect that the shapes of cars with lower gas consumption become more and more similar. The differences in design will be less pronounced. Therefore, an increase in the quality of cars via better aerodynamics reduces the variety offered in the market. We say that in this case quality and variety interact negatively.

Banking products are a second example. The characteristics of banking products are often negatively correlated. Matutes and Padilla (1994) discuss shared ATM networks and banking competition. Compatibility of networks is valued by depositors since it facilitates access to deposits. The reason is that not only network size matters but also the location of the bank's proprietary network is a source of differentiation. In other words, signing a compatibility agreement implies a reduction of geographical differentiation. Another example concerning the financial sector is phonebanking. This technology increases quality and at the same time reduces the importance of transportation costs (Bouckaert and Degryse (1995), or Degryse (1996)). Therefore, banks cannot become vertically differentiated without affecting horizontal differentiation in a negative way. A third example concerns the after sale service provided by firms. The latter may be performed on the telephone. This implies that clumsy users located further away from the store value phone support more than their clumsy colleagues at a shorter distance. As a result compatibility of networks, phonebanking, and after sale service negatively affect the degree of horizontal differentiation.

Fourth, advertising expenditure is often modeled as a quality feature (see, e.g., Economides (1989) or Schulz and Stahl (1994)). However, the extent to which advertising affects the consumers' willingness to pay may depend on their preferences. D'souza and Rao's (1995) findings are in support of this. They state that the effect of repeated advertising on the individual choice probabilities interacts positively with prior preferences. In a static context of our analysis, this comes close to saying that advertising may have a stronger impact on those consumers feeling 'closer' to the brand than on those feeling 'further away'. Thus, quality-like effects via advertising may be positively correlated with 'proximity.'

These examples show that the assumption of independent attributes may indeed be restrictive. This paper focuses explicitly on the interaction of dependent attributes. In particular, we are interested in the question of how firms' incentives to invest in quality change when this decision affects the degree of horizontal differentiation in the market. First, let us focus on negative interaction. The results of our analysis are determined by two forces. The first is a demand effect. A negative dependence of a product's quality and variety characteristic strengthens firms' incentives to provide quality since consumers are more easily attracted by quality improvements than by price reductions. This demand force drives the results when firms simultaneously choose quality and price. It leads to excessive private incentives to provide quality in the presence of negative characteristic dependence. The second force is strategic. Thus it plays only a role if firms can commit to a quality level before competing in price. We show that in this setting a negative correlation renders price competition along the horizontal characteristic fiercer, thus stimulates underinvestment in quality. Private incentives to provide quality are dampened when a quality increase weakens the product's varietal differentiation. In the sequential game 'first quality then price' we show that the effect of negative dependence on the strategic effect more than outweighs the one on the demand effect. Consequently, an unregulated market underprovides quality such that a minimum quality standard is welfare improving.

We also analyze the case of positive dependence for which the results tend to be the opposite. In particular the model recommends a maximum quality standard if an increase in quality sufficiently decreases competition along the varietal characteristic. The reason is that firms heavily overinvest in quality which implies substantial R&D costs and high transportation costs for the average consumers.

We present these results in a simple duopoly model of product innovation in the context of dependent characteristics.[1] Clearly, we need a model of at least two-dimensional product differentiation in order to represent the product's quality and varietal characteristics. Within this framework, there are at least two ways of how one can introduce characteristic dependence into the picture. First, a product's position in the characteristics space may be directly affected by the choice of quality. This formulation fits the car example sketched out above. Given that there is an optimal shape that minimizes gas consumption, firms are forced to approach this shape when improving the quality of their product. Second, the quality offered by a firm can determine the preference intensity consumers attach to its product. Phonebanking is a case in point, so is after sale service, and advertising. By offering a higher product quality via phonebanking, banks reduce the importance consumers attach to geographical differentiation of competing branches. In other words, the quality decision affects the transportation costs associated with the dimension of horizontal product differentiation. In the following we restrict attention to the second variant of interaction between the vertical and the horizontal dimension of product differentiation.[2]

Recently, the product differentiation literature has taken up the topic of firms' product placement decisions in multi-dimensional product spaces (See Anderson et. al. (1992) and the literature cited therein). However, the empirically relevant case of attribute dependence has not yet been analyzed. Our paper presents a simple approach to tackle this problem. The literature closest to our study includes von Ungern-Sternberg (1988), Dos Santos Ferreira and Thisse (1996), Economides (1989), Canoy and Peitz (1995), and Degryse (1996). The first

[1] Notice that our model of quality provision can be interpreted as a model of deterministic R&D decisions. In both cases the key feature is that by incurring a higher fixed cost a firm enhances all consumers' marginal willingness to pay for its product (see also Sutton (1991)). However, some specific aspects of R&D such as process innovation, spillovers to rivals, imperfect patents, uncertainty about success of projects, or the timing of innovation lie out of the scope of this paper. For an overview on that see Tirole (1988).

[2] We solved a model close to the car example. It confirms the insights and the qualitative results contained in section III of this paper. A version of the 'car example model' is available upon request.

two papers endogenize the determination of the level of transportation costs. Von Ungern-Sternberg interprets this as competition in the degree of the products' « general purposeness ». He solves for the simultaneous equilibrium in price and transportation cost when firms are equidistantly located on a circle and incur costs that are decreasing in the transportation rate. In equilibrium firms overprovide quality, i.e., fix transportation cost too low compared to the social optimum. Our findings for the simultaneous game with negative interaction confirm this result. However, our study suggests that if firms can commit to a transportation cost level before competing in prices the private incentive to provide « general purposeness » would fall short of the social optimum. Dos Santos Ferreira and Thisse analyze this case where firms commit to a transportation cost level before competing in prices. Firms can increase the quality of their product by offering lower transportation rates, i.e., by providing a better transportation technology. The main difference to our setup is that we allow for a quality improvement to have two effects. It increases the willingness to pay for a product and changes the transportation rate. This allows us to interpret the transportation rate as an indicator of the toughness of price competition along the horizontal product characteristic.

Economides (1989) studies quality variations and maximal variety differentiation. He shows that there is minimal quality differentiation in equilibrium. We obtain a similar result but unlike Economides, our analysis emphasizes that the level of quality offered depends heavily on the interaction between quality and variety. It is worth mentioning Canoy and Peitz (1995) who support our conjecture that characteristic dependence is an important phenomenon for the analysis of market structure. They start from the observation that submarkets for high-quality and low-quality variants are markedly different from each other. This is reflected in their model where, exogenously given, variants in the low-quality range are identical for all customers whereas they strongly disagree about the value of variants in the high-quality segment. In other words, they deal with positive interaction between quality and variety for given product characteristics. In contrast, the present study endogenizes the quality decision of firms taking into account the effect of quality on horizontal differentiation. Therefore, the disagreement among customers between variants partly results from the strategic behavior of firms. Finally, Degryse (1996) discusses interaction between horizontal and vertical differentiation with an application to banking. In his paper, banks face a discrete choice between a high and a low quality. In addition, he only considers negative interaction. In contrast, this paper allows for a continuous quality choice and incorporates positive interaction.

This paper is organized in the following way. Section II introduces the model. Section III studies the unregulated quality decision. We discuss a noncooperative simultaneous and sequential game as well as the cooperative quality choice. Section IV compares these results with the socially optimal quality standard. Section V contains some final remarks.

II. THE MODEL

In this section, we present a model of two-dimensional product differentiation. We consider two firms whose products are vertically and horizontally differentiated. Firms compete in quality and price. We distinguish among three different ways of how firms' decisions come about. First, we study the case where quality and price are simultaneously determined. Second, we analyze the sequential choice of both decision variables. Firms play a two-stage game. At the first stage, they determine the quality of their product, at the second stage they compete in prices. The two stage nature of the game reflects the fact that quality is often less flexible than prices. Third, we deal with an R&D cartel which at the first stage determines quality collusively before competing in prices. The particularity of all these scenarios is that firms' quality decision affects the degree of horizontal product differentiation.

Firms' investments in product quality affect the indirect utility of customers in two ways. First, a higher quality is preferred by all consumers giving rise to vertical product differentiation. Second, depending on the type of interaction an increase in the quality level of one product may be higher (lower) valued by consumers with a high preference for its variety and less by others. More precisely, consider two firms A and B located at the endpoints of a line with length one over which consumers are uniformly distributed with density one. Each consumer buys one item of either product. Her indirect utility of a purchase of brand A is given by (a similar expression holds for a purchase of brand B)[3]

[3] Our model encompasses other models of product differentiation. If $\delta = 0$ our model becomes a special case of Economides (1989). If in addition we removed the quality variable from the indirect utility function, our model were a standard Hotelling (1929) model with a per unit transportation rate equal to one. If we took distance away we would obtain a special case of Mussa-Rosen (1978).

$$V = v + s_A - p_A - (1 - \delta s_A)z \text{ where } \delta \leq \frac{1}{\bar{s}},$$

and v = gross surplus

s_A = quality provided by firm A

p_A = firm A's price

δ = interaction parameter

$\bar{s}$ = maximum quality

z = consumer location.

The consumer's indirect utility depends on her most preferred variety z which determines the distance to be covered when purchasing brand A or B. More importantly, it depends on the product quality s_i in two ways. First, *independent* of the consumer's location an increase in quality augments utility by the same amount. Second, the quality offered by firm i determines the per unit transportation rate towards firm i's product. The impact of the latter effect is *dependent* on the interaction parameter δ and on the consumer's location. If δ is positive, an increase in quality decreases the per unit transportation rate. In this case, we say that quality and variety are *negatively* correlated. Both dimensions are said to be *positively* correlated when δ is negative, and *independent* when $\delta = 0$. The consumer location z determines the marginal indirect utility of quality, i.e. $\partial V / \partial s_i = 1 + \delta z$. Hence, under negative (positive) correlation, consumers located far from firm i gain more (less) from a marginal quality improvement. We assume that at fixed prices no consumer is worse off when the quality of either firm increases, i.e. $\partial V / \partial s_i \geq 0$ for all $z \in [0,1]$. This introduces a lower bound on the interaction parameter, i.e. $\delta \geq (-1)$. When δ is zero, the indirect utility function becomes separable in quality and variety. Finally, $\delta \leq 1/\bar{s}$ guarantees that per unit transportation costs are nonnegative.

Clearly, one could criticize the chosen form of consumers' preferences on grounds of its specificity. However, as we show below it yields demand functions with very intuitive properties. Let consumers compare their preference for the horizontal characteristic, the quality level, and the price of both products. This trade-off gives rise to a marginal consumer location that determines firms' demand functions in a standard way. Explicitly, one obtains

$$D_A(p_A, p_B, s_A, s_B)=\begin{cases}1 & \text{for } p_B - p_A \geq 1+s_B - s_A(1+\delta) \\ \dfrac{1+s_A - s_B(1+\delta)+p_B - p_A}{2-\delta(s_A+s_B)} & \text{for } 1+s_B - s_A(1+\delta) \geq p_B - p_A \geq s_B(1+\delta)-s_A-1 \\ 0 & \text{for } s_B(1+\delta)-s_A-1 \geq p_B - p_A\end{cases} \quad (1)$$

Firm B's demand is implicitly given as $D_B = 1 - D_A$. Demands are linear in both prices, increasing in own quality, and convex (concave) if δ is positive (negative). The higher δ the lower become transportation costs, thus more consumers switch brands in response to price and quality changes, i.e. $i, j = A, B; i \neq j$

$$\frac{\partial^2 D_i}{\partial p_j \partial \delta} \geq 0, \quad \frac{\partial^2 D_i}{\partial p_i \partial \delta} \leq 0$$

$$\frac{\partial^2 D_i}{\partial s_i \partial \delta} \geq 0, \quad \frac{\partial^2 D_i}{\partial s_j \partial \delta} \leq 0.$$

Since quality changes also affect the level of transportation costs, we have

$$\frac{\partial^2 D_i}{\partial p_j \partial s_i} > 0, \quad if\, \delta > 0$$

$$\frac{\partial^2 D_i}{\partial p_i \partial s_j} < 0, \quad if\, \delta < 0.$$

In the former case the full cost function shifts downwards and becomes flatter when $\Delta s_i > 0$. This implies lower transportation costs, thus more price sensitive demands. The opposite holds for $\delta < 0$.

We assume production is such that the level of quality does not affect variable costs which are normalized to zero.[4] There is only a fixed cost of producing quality given by $C(s_i) = cs_i^2 / 2$. The quadratic form of the cost function reflects diminishing returns of investment in quality. Accordingly, profit functions are

$$\Pi_i(p_i, p_j, s_i, s_j) = p_i D_i(p_i, p_j, s_i, s_j) - C(s_i), \; i = A, B, \; i \neq j. \qquad (2)$$

[4] The case of quality dependent variable costs is discussed in Section V.

III. THE UNREGULATED QUALITY DECISION

In this section we discuss the noncooperative and the cooperative choice of price and quality. First, in Section III.1. we study the noncooperative simultaneous and sequential games. Second, in Section III.2. we turn to the choice of an R&D cartel.

III.1. NONCOOPERATIVE CHOICE

III.1.1. SIMULTANEOUS EQUILIBRIUM: PRICES AND QUALITIES

In this section we study the symmetric simultaneous equilibrium in prices and qualities. Given the profit functions of equation (2) both firms $i = A, B$ solve simultaneously

$$\frac{\partial \Pi_i}{\partial p_i} = 0 \Leftrightarrow p_i = \frac{D_i}{\left(-\frac{\partial D_i}{\partial p_i}\right)} \tag{3}$$

$$\frac{\partial \Pi_i}{\partial s_i} = 0 \Leftrightarrow p_i \frac{\partial D_i}{\partial s_i} = \frac{\partial C}{\partial s_i}. \tag{4}$$

Eliminating p_i, a Nash equilibrium requires for both firms that

$$D_i \left.\frac{dp_i}{ds_i}\right|_{D_i = const.} = \frac{\partial C}{\partial s_i}, \tag{5}$$

where $dp_i / ds_i|_{D_i = const.} > 0$ measures by how much firm i can push up its price in response to a marginal increase in quality while keeping its demand constant. Accordingly, the LHS of (5) is the additional revenue that firm i obtains from a marginal increase in quality without affecting its demand. In equilibrium the latter equals the marginal costs of providing higher quality. Intuitively, changes in s_i and p_i can only be « demand neutral » if they leave the position of the marginal consumer unaffected. Hence, $dp_i / ds_i|_{D_i = const.}$ must be equal to the marginal consumer's marginal utility. Proposition 1 gives the symmetric equilibrium resulting from evaluation of (5) by using $c_1 \equiv \max\{0, (1 + 2\delta(2 + \delta)) / 4\}$ and $c_2 = 1/4 + \delta(20 + 13\delta)/16$.

Proposition 1:

The simultaneous game in quality and price has a unique symmetric equilibrium in pure strategies if $c \geq \max\{c_1, c_2\}$. The equilibrium quality is $s^{M} = (2+\delta)/(4c)$ while equilibrium prices are $p^{M} = 1 - \delta s^{M}$.

Proof: See Appendix.

Proposition 1 shows that the simultaneous equilibrium price and quality depend on the interaction parameter δ. Compared to the case of independence we find that s^{M} is lower (higher) under positive (negative) correlation. In other words, $ds^{M} / d\delta > 0$. The intuition is simple. Higher values of δ increase the marginal consumer's marginal valuation of a quality improvement which is entirely captured by the firms.

From a comparative static analysis we learn that the impact of δ on the equilibrium quality can be decomposed into two opposing effects. Denote by $f(p(s,\delta),\delta,s,c) = 0$ the implicit function obtained through evaluation of (5) at the symmetric equilibrium. Total differentiation of $f(\cdot)$ yields

$$\frac{ds^{M}}{d\delta} = -\frac{\overbrace{\overbrace{\frac{\partial p^{M}}{\partial \delta}}^{(-)}\Big|_{s=s^{M}} \overbrace{\frac{\partial D_I}{\partial s_I}}^{(+)}}^{(-)} + \overbrace{p^{M} \overbrace{\frac{\partial^2 D_A}{\partial s_A \partial \delta}}^{(+)}}^{(+)}}{\underbrace{\frac{\partial f}{\partial s}}_{(-)}} > 0. \qquad (6)$$

Clearly, it is the numerator which determines the sign of $ds^{M} / d\delta$. It tells us how the marginal revenue of an additional quality improvement reacts to changes in δ. Two effects are at work. First, there is a negative price competition effect. Since higher values of δ make price competition fiercer a quality improvement pays-off less. Second, there is a positive effect via demand. New consumers are more easily attracted via quality improvements when δ is high. Since the second effect dominates the first, quality augments. This is to be compared with the scenario of sequential choice.

Equilibrium prices only reflect marginal transportation costs. The intuition is that products are homogeneous with respect to quality but maximally differentiated with respect to their horizontal characteristic. Quality has then no direct influence on the determination of equilibrium prices. However, it enters indirectly via the interdependence of both dimensions of product differentiation. Equilibrium prices fall with δ, i.e.

$$\frac{dp^{s_l}}{d\delta} = \overbrace{\frac{\partial p^{s_l}}{\partial \delta}}^{(-)}\Bigg|_{s=s^{s_l}} + \overbrace{\frac{\partial p^{s_l}}{\partial s}}^{sign[-\delta]}\overbrace{\frac{\partial s^{s_l}}{\partial \delta}}^{(+)} \leq 0 . \tag{7}$$

There is a *direct* and an *indirect* effect of changes in δ on the equilibrium price level. The direct effect is part of the price competition effect in (6). Transportation costs fall such that more consumers switch brands when prices change, thus competition along the horizontal characteristic intensifies. The indirect effect is positive since the equilibrium quality increases as δ augments. This may relax price competition depending on $sign[-\delta]$. The indirect and direct effect have the same sign in the case of negative correlation. The direct effect outweighs the indirect effect if positive correlation prevails. As a result, the total effect is always negative.

III.1.2. SEQUENTIAL CHOICE: QUALITY → PRICE

This section deals with the solution of the following two stage game. At the first stage, firms simultaneously determine the quality of their products. The first stage decisions become observable before firms compete in prices at the second stage. We are interested in the symmetric subgame perfect Nash equilibrium. Accordingly, we solve for the second stage price game first. The noncooperative solution of the first stage quality game is presented afterwards.

The Price Game

Given demands and profit functions of (1) and (2), standard computations yield the unique stage two price equilibrium as

$$p_i^* = 1 + \frac{s_i(1-\delta) - s_j(1+2\delta)}{3}, i = A, B; i \neq j. \tag{8}$$

Three remarks are in order. First, observe that price competition becomes tougher as δ increases ($\partial p_i^* / \partial\delta \leq 0,\ i = A, B$). Lower transportation costs make consumers more likely to switch from one brand to the other. Accordingly, demand functions become more price elastic. Second, unlike standard models of quality choice here a quality improvement of one's product does not necessarily allow for a higher price. In fact, we have

$$\frac{\partial p_i^*}{\partial s_i} = \frac{(1-\delta)}{3} < 0 \text{ if } \delta > 1,\ i = A, B.$$

The reason is the following. In general an increase in s_i shifts firm i's best reply outward. However, if $\delta > 1$ it also considerably reduces the per unit transportation rate. Hence, price competition along the varietal dimension becomes so intense that p_i^* must fall in equilibrium.[5]

Third,

$$\frac{\partial p_j^*}{\partial s_i} = \frac{-(1+2\delta)}{3} \tag{9}$$

describes the change of the rival's price when own quality increases. Notice that $sign[\partial p_j^* / \partial s_i] = sign[-(1+2\delta)]$. Hence the rival responds friendly (aggressively) when $\delta < (>) -0.5$. It is then interesting to study how the interaction of characteristics affects firms' choice of quality at the first stage of the game.

[5] In technical terms, δ>1 implies an inward shift of j's best reply at least twice as big as the outward shift of i's best reply.

Noncooperative Quality Game

The demand function for firm A at the price equilibrium is given by

$$D_A(s_A, s_B) = \begin{cases} 1 & \text{for } s_A - s_B \geq 3 - \delta(2s_A + s_B) \\ \dfrac{3 + s_A(1-\delta) - s_B(1+2\delta)}{3(2 - \delta(s_A + s_B))} & \text{for } 3 - \delta(2s_A + s_B) \geq s_A - s_B \geq \delta(s_A + 2s_B) - 3 \\ 0 & \text{for } \delta(s_A + 2s_B) - 3 \geq s_A - s_B \end{cases} \quad (10)$$

The demand for firm B equals $1 - D_A$. Firms maximize profits

$$\Pi_i = \frac{[3 + s_i(1-\delta) - s_j(1+2\delta)]^2}{9(2 - \delta(s_i + s_j))} - \frac{c}{2}s_i^2 \ , i = A, B; i \neq j$$

simultaneously with respect to their quality on $s_i \in [0, \bar{s}]$, where $\bar{s} \leq \frac{1}{\delta}$ if $\delta > 0$. This leads to an overall equilibrium characterized in Proposition 2. Define $\bar{c}_2 \equiv 2/9 + \delta(20-\delta)/36$, $c_3 \equiv 4/27 + \delta(352 + 133\delta)/432$, and $c_5 \equiv \max\{0, 1/9 + \delta(20-\delta)/72\}$.

Proposition 2:

The sequential game 'first quality then price' has a unique symmetric subgame perfect equilibrium in pure strategies if for $\delta > 0$, $c \geq \max\{\bar{c}_2, \bar{c}_3\}$ and if for $-1 \leq \delta \leq 0$, $c \geq \bar{c}_5$. The equilibrium quality is $s^{nc} = \max\{(4-\delta)/(12c), 0\}$ while equilibrium prices are $p^{nc} = 1 - \delta s^{nc}$.

Proof: See Appendix.

Proposition 2 emphasizes that the noncooperative equilibrium quality and price level depend crucially on the interaction parameter δ. Compared to the independence case s^{nc} is higher under positive and lower under negative correlation, thus $ds^{nc}/d\delta < 0$.

The workings of this result can be better understood if one decomposes the effect of a marginal increase in quality on own profits into the direct, the strategic, and the cost effect. Formally,

$$\frac{d\Pi_i}{ds_i} = p_i \frac{\partial D_i}{\partial s_i} + p_i \frac{\partial D_i}{\partial p_j}\frac{dp_j}{ds_i} - \frac{\partial C_i}{\partial s_i}. \tag{11}$$

The first term on the RHS is known as the demand effect. It is easily verified that this term coincides with the LHS of (5). It is positive since it reflects the marginal consumer's marginal utility of a quality increase. The second term represents the strategic effect. It describes the rival's price response to an increase in own quality. It has the sign determined in (9). Hence the (under-) overinvestment prescription depends on whether $\delta\,(>) < -0.5$. In the (former) latter case firms capture (less) more than the marginal consumer's marginal benefit of a quality increase and $s^{N}(>) < s^{W}$ (see Figure 1 for an illustration).

Observe that quality is an aggressive variable under independence. If correlation is negative, i.e. $\delta > 0$, increasing one's quality triggers an even more aggressive price response of the competitor. This is in line with intuition since the rival gains more consumers for a given price change if δ is high. The strategic (underinvestment) effect is then more pronounced. Using the terminology of Fudenberg and Tirole (1984), firms behave as *puppy dogs* thus generating a lower equilibrium quality. On the other hand, if $\delta < 0$ the incentive to underinvest is dampened since a higher quality renders products more heterogeneous. If the positive correlation is substantial the strategic character of the game switches. Unlike often modeled quality can become a soft variable. Firms overinvest in quality thus becoming *fat cats*. In all these cases, the profitability of quality is limited by the cost function which exhibits increasing marginal costs of quality.

An important result of our analysis is that equilibrium quality unequivocally falls as δ increases. Let $g(s,\delta,c)=0$ be the implicit function resulting from evaluation of (11) at the symmetric equilibrium. Total differentiation results in

$$\frac{ds^{sc}}{d\delta} = -\frac{\overbrace{\overbrace{\frac{\partial p^{sc}}{\partial \delta}}^{(-)}\Bigg|_{s=s^{sc}} \overbrace{\left[\frac{\partial D_i}{\partial s_i} + \frac{\partial D_i}{\partial p_j}\frac{dp_j}{ds_i}\right]}^{+}}^{(-)} + p^{sc}\overbrace{\left[\overbrace{\frac{\partial^2 D_i}{\partial s_i \partial \delta}}^{(+)} + \overbrace{\frac{\partial^2 D_i}{\partial p_i \partial \delta}\frac{dp_j}{ds_i} + \frac{\partial D_i}{\partial p_j}\frac{d^2 p_j}{ds_i d\delta}}^{(-)}\right]}^{(-)}}{\underbrace{\frac{\partial g}{\partial s}}_{(-)}} < 0 . \qquad (12)$$

It is easily verified that the denominator is negative. Consequently, the numerator of (12) determines the sign of $ds^{sc} / d\delta$. Observe that the first term is negative since the equilibrium price decreases as δ rises (direct price competition effect). The sign of the second term depends on how changes in δ affect the demand and the strategic effects. As explained before the demand effect increases in δ and the strategic effect is more pronounced. Since the strategic effect reacts stronger on changes in δ the second term in the numerator of (12) is also negative. Hence, $ds^{sc} / d\delta < 0$. This result is in sharp contrast to what we found for the simultaneous game. A comparison of (6) and (12) reveals that the impact δ has on the strategic effect is responsible for this result.

Again only marginal transportation costs are reflected in the equilibrium prices. Comparative statics have the same structural form as in (7). However $ds^{sc} / d\delta$ is negative such that

$$\frac{dp^{sc}}{d\delta} = \frac{1}{6c}[\delta - 2] \geq 0 \Leftrightarrow \delta \geq 2 .$$

If $\delta \geq 2$ the indirect effect dominates the direct effect implying that the full cost function becomes steeper.

III.2. COOPERATIVE SOLUTION

In this subsection, we study the case where the duopolists form an R&D cartel, i.e. coordinate their decisions concerning quality without sharing costs. In other words, firms agree on the quality but have separate labs. They anticipate the competitive solution of the second stage price competition and choose quality cooperatively in order to maximize total profits.[6] Hence, total profits at the price equilibrium are given by

$$\begin{aligned}\Pi_A + \Pi_B = p_A(s_A, s_B)D_A(p_A(s_A, s_B), p_B(s_A, s_B), s_A, s_B) - C(s_A) \\ + p_B(s_A, s_B)D_B(p_A(s_A, s_B), p_B(s_A, s_B), s_A, s_B) - C(s_B)\end{aligned} \tag{13}$$

where $p_A(s_A, s_B)$, $p_B(s_A, s_B)$ stem from equation (8). The cooperative equilibrium is characterized in Proposition 3.

__Proposition 3:__

The cooperative quality level is $s^c = \max\left\{\frac{-\delta}{2c}, 0\right\}$ and equilibrium prices are

$p^c = 1 - \delta s^c$. [7]

Proof: Maximizing (13) with respect to both qualities leads to two first order conditions. By making use of the fact that a higher quality cannot increase aggregate demand these can be written as

$$\frac{1}{2}\left[\frac{dp_i}{ds_i} + \frac{dp_j}{ds_i}\right] = cs_i,\ i = \text{A,B}; i \neq j\ . \tag{14}$$

Solving for s^c yields the solution. The second derivative is always negative ensuring a maximum. Q.E.D.

Hence, firms in an R&D cartel may invest in quality even if they face an inelastic demand. Proposition 3 emphasizes that this is only the case if a higher quality strengthens horizontal differentiation. Indeed, first order conditions in (14) show that, apart from costs, firms care only about the influence of quality on own and rival's prices. More horizontal differentiation,

[6] This is one type of semi-collusion (see also Fershtman and Gandal (1994)). Another type is cooperation at the price setting stage and competition at the quality stage. The latter variant could be appropriate for analyzing banking issues as banks often collude(d) in interest rates. The optimal quality would positively depend on the reservation value v. A higher δ would stimulate the optimal quality as it enhances the demand effect.

[7] Equilibrium quality is doubled if firms share R&D costs (Research Joint Venture cartelization).

i.e. when $\delta < 0$, allows for a higher rise in own price and induces a more friendly response from the rival following an increase in one's quality .

It is straightforward to show that p^c decreases in δ. More interestingly, $p^c = p^{nc} \Leftrightarrow \delta = -4/5$. By construction, this implies that $s^c = s^{nc} \Leftrightarrow \delta = -4/5$ and we obtain

$$-1 \leq \delta \leq -4/5: \; p^c \geq p^{nc} \text{ and } s^c \geq s^{nc}$$
$$\delta > -4/5: \; p^{nc} > p^c \text{ and } s^{nc} > s^c.$$

Hence, if positive correlation sufficiently relaxes price competition the cartel provides a higher quality and sets a higher price than the non cooperating firms. This result is usually found in the presence of R&D spillovers (see, e.g. d'Aspremont and Jacquemin (1988) and Motta (1992)). Still, the welfare implications remain to be analyzed.

IV. REGULATED QUALITY DECISION AND WELFARE ANALYSIS

In this section, we first undertake a simple welfare analysis by looking at an optimal quality standard. Then, the noncooperative and cooperative outcomes are compared to the optimal quality standard. The latter is shown to be either a minimal or a maximal quality standard depending on how quality and variety interact.

IV.1. OPTIMAL QUALITY STANDARD

In this subsection, we discuss the behavior of a regulator maximizing social welfare by imposing a quality standard s^{sw}. It is assumed that the regulator does not fix prices but faces a break-even constraint for both firms. The welfare measure is a simple addition of consumer and producer surplus.

The consumer surplus CS boils down to

$$CS = v + s - (1-\delta s) - \left(\frac{1-\delta s}{4}\right).$$

The first term is the gross surplus each consumer enjoys from purchasing one unit of the good. The second component measures the consumers' utility derived from the quality s . The last two terms are the price customers pay and the average transportation cost a customer incurs when purchasing either brand.

The producer surplus PS consists of two components and equals

$$PS = [1-\delta s] - c(s)^2 \geq 0 \ .$$

The term in brackets represents the total revenues whereas the second is the costs of developing the quality level s.

Adding consumer surplus CS and producer surplus PS gives the total surplus TS

$$TS = CS + PS = v + s - \left(\frac{1-\delta s}{4}\right) - c(s)^2$$

Notice that TS does not reflect revenues since these are a pure redistribution among producers and customers. A higher quality standard has then three effects. First, it positively affects the indirect utility of the customer. Second, it changes the total amount of transportation costs customers face. In particular, it increases (decreases) total transportation costs depending on $\delta<(>)0$. Third, it increases the development costs $c(s)^2$.

The optimal quality standard is given in Proposition 4.

Proposition 4:

The optimal quality standard s^{sw} chosen by a regulator under a break-even constraint is equal to $s^{sw} = \min\left\{\frac{\sqrt{4c+\delta^2}-\delta}{2c}, \frac{4+\delta}{8c}\right\}$ yielding $p^{sw} = 1-\delta s^{sw}$ as equilibrium prices.

Proof: The optimal quality standard s^{sw} solves

$$\max_s \ TS = v + s - \left(\frac{1-\delta s}{4}\right) - c(s)^2 \tag{15}$$

subject to $PS \geq 0$.

The constraint implies that $s^{sw} \in [0, (\sqrt{4c+\delta^2} - \delta)/(2c)]$. It is then easily verified that the break-even constraint is not binding if $-1 \leq \delta \leq -4/9$. For $\delta \geq -4/9$, the break-even constraint is binding if $c \leq (4+\delta)(4+9\delta)/64$. QED

Clearly, the optimal quality standard depends on the interaction of quality and variety.[8] For an interior solution we obtain $ds^{sw}/d\delta > 0$. The intuition for this is not hard to find. The regulator is interested in maximizing the utility of the average consumer located at $z = 1/4$. Indeed the first order condition associated with (15) sets the marginal benefit for the average consumer equal to the social marginal cost. Since the former increases in δ the socially optimal quality also does.

IV.2. COMPARISON AND QUALITY STANDARDS

In this section, we compare the quality levels associated with the noncooperative and cooperative scenarios investigated above with the optimal quality standard. We restrict attention to the unconstrained s^{sw}.[9] Figure 1 depicts the four different quality levels as a function of δ.

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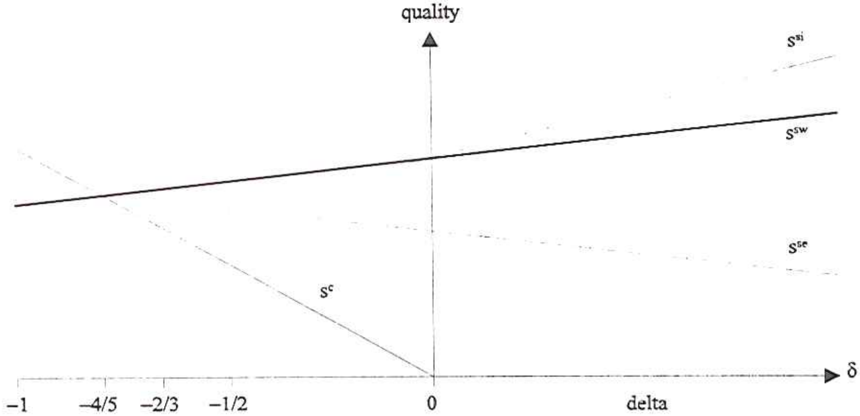
[8] Notice that imposing a minimum quality standard 'at stage zero' under the cost conditions given in Propositions 1 and 2 does not necessarily imply that s^{sw} is the firm's equilibrium choice. If this is the case s^{sw} should be interpreted as the level of quality firms are forced to provide. Indeed, the question of how a minimum quality standard may perturb a symmetric equilibrium in qualities remains an interesting one left for future research. See for asymmetric equilibria the studies by Ronnen (1991), Crampes and Hollander (1995).

[9] The main qualitative results are confirmed when working with the constrained s^{sw}.

Figure 1: Optimal quality standard versus cooperative and noncooperative quality levels.



The quality level is presented on the vertical axis and δ on the horizontal axis. Recall that the noncooperative sequential and simultaneous equilibrium quality intersect where the strategic effect equals zero, i.e. at $\delta = -1/2$, and that $s^{se} = s^c$ when $\delta = -4/5$. Proposition 5 summarizes the comparison of quality levels.

Proposition 5:

The ranking of the optimal quality standard, the cooperative, and noncooperative quality levels depends on the nature of interaction in the following way.

If $\delta \geq 0$, then $s^{si} \geq s^{sw} > s^{se} > s^c$.

$$\text{If } \delta < 0\text{, then } \begin{cases} s^{sw} > s^{si} > s^{se} > s^c & \text{if } -1/2 < \delta < 0, \\ s^{sw} > s^{se} \geq s^{si} > s^c & \text{if } -2/3 < \delta \leq -1/2, \\ s^{sw} > s^{se} > s^c \geq s^{si} & \text{if } -4/5 < \delta \leq -2/3, \\ s^c \geq s^{se} \geq s^{sw} > s^{si} & \text{if } -1 \leq \delta \leq -4/5. \end{cases}$$

Let us first deal with s^{si} and s^{sw}. Under negative (positive) correlation the simultaneous noncooperative equilibrium quality level turns out to be higher (lower) than the socially optimal one. The reason is that the marginal benefit of a higher quality accruing to the marginal consumer exceeds (falls short of) the one for the average consumer when $\delta > (<) 0$. Hence, firms have an excessive (insufficient) incentive to provide quality. Therefore, a minimum (maximum) quality standard is called for in case of positive (negative) correlation.

In what follows, we compare the different sequential outcomes to s^{sw}. The most clear-cut results of this comparison are obtained for the case of negative correlation. Private incentives lead to socially insufficient quality levels and the gap between s^{sw} and s^{se} grows the higher δ. Both findings are linked to the presence of strategic effects. Clearly, one expects $s^{si} > s^{se}$ if the strategic effect is negative. In addition, from equation (12) we know that changes in the strategic effect dominate changes in the demand effect as δ increases. Hence, whereas the social incentives to provide quality increase in δ the private incentives fall.

The qualitative results are unchanged when correlation is not too positive. More precisely, $s^{sw} \geq s^{se}$ as long as $0 > \delta > -4/5$. The intuition comes in two steps. First, if $0 > \delta \geq -1/2$, we have $s^{sw} \geq s^{si} \geq s^{se}$. That is because the average consumer's marginal benefit of quality exceeds the one of the marginal consumer if $\delta < 0$, and the strategic effect is still negative. Second, if $-1/2 > \delta > -4/5$ the strategic effect becomes positive but private incentives fall short of social incentives in spite of the overinvestment effect. Finally, if $-4/5 \geq \delta \geq -1$ the overinvestment effect becomes so strong that $s^{se} \geq s^{sw} \geq s^{si}$.

The R&D cartel does not provide quality unless this relaxes price competition which requires $\delta < 0$. Observe however, that s^{c} is determined by price effects only and that the LHS of (14) is more sensitive to changes in δ than the revenue part in (11). Accordingly, s^{c} grows faster than s^{se} when δ falls. The sum of both price effects in (14) is equal to the marginal benefit accruing to the average consumer iff $\delta = -4/5$, hence $s^{c} \geq s^{se} \geq s^{sw}$ if $-4/5 \geq \delta \geq -1$.

Observe that not all consumers are necessarily better off under a minimum quality standard. In fact, using (1) it is easily seen that for all unregulated equilibria $j = si, se, c$ we have

$$V^{sw} - V^{j} \geq 0 \Leftrightarrow (s^{sw} - s^{j})[1 + \delta + \delta z] \geq 0 \text{ for } z \leq 1/2,$$

where the term in brackets measures the marginal benefit of a quality improvement accruing to a consumer located at z. Accordingly, some consumers are worse off under a minimum quality standard if $-4/5 < \delta < -2/3$. All consumers are worse off under a maximum quality standard when $\delta \geq 0$.

V. DISCUSSION AND CONCLUSIONS

Real world observations show that product attributes are often dependent. In spite of this, the existing literature on product differentiation has not yet integrated this phenomenon into the study of firms' product placement decisions. This paper is a first attempt to fill this gap by considering a situation where the quality offered by a firm influences the transportation costs associated with horizontal product differentiation.

We approach the problem by incorporating attribute dependence into the consumers' indirect utility function. Some consumers benefit more than others from a quality improvement depending on their tastes for a horizontal product characteristic, and the form of interaction between quality and variety. When interaction is negative (positive) the marginal consumer benefits more (less) from a higher quality than the inframarginal consumers. Although this modelization is specific, we have argued that it yields demand functions with very intuitive properties. Alternatively, one could have introduced an explicit technological constraint on the supply side. We have studied this case and found the main qualitative results of the model of this paper confirmed.

Our analysis assumes that all quality dependent costs are fixed costs incurred at the first stage of the game. This enables us to determine how quality improvements influence price competition depending on the form of interaction. Clearly, variable costs can be quality dependent implying profits of the form

$$\Pi_i\left(p_i, p_j, s_i, s_j\right) = [p_i - c_i(s_i)]D_i(p_i, p_j, s_i, s_j),\ i = A, B,\ i \neq j .$$

We have studied this scenario by specifying $c_i(s_i) = c s_i^2 / 2$. The results are even more clear cut: all comparative statics maintain their sign and everything switches at $\delta = 0$, that is $s^{st} \geq s^{sw} \geq s^{sc}$, if $\delta \geq 0$, and $s^{sc} > s^{sw} > s^{st}$, if $\delta < 0$.

This brings us back to our main findings and their policy implications. Welfare losses are due to two forces and their dependence on the nature of interaction between quality and variety. First, there is the difference between the benefit of a marginal quality improvement accruing to the marginal and to the average consumer (see, e.g., Spence (1975)). This is the driving force behind our results when the game is simultaneous. Negative (positive) correlation strengthens the private incentives to overprovide quality since the marginal consumer gains more (less) from a quality improvement than under independence. Accordingly, a minimum quality standard is called for when correlation is positive and a maximum quality standard under negative correlation.

The second force is strategic, thus plays a role only when the game is sequential. Negative (positive) correlation renders price competition along the horizontal characteristic fiercer (more relaxed), thus strengthening (lessening) the importance of strategic considerations for the firms' quality choice. As a result the private incentives to provide quality become insufficient under negative correlation. A minimum quality standard is then desirable. Notice however that the presence of a negative strategic effect is sufficient but not necessary to generate inadequate private incentives. However, if correlation is strongly positive private incentives to provide quality become again excessive relative to the social optimum. A maximum quality standard is then welfare improving.

APPENDIX

Proof of Proposition 1:

We proceed as follows. First, we solve for the simultaneous symmetric equilibrium in prices and qualities and give the condition under which profit functions reach a local maximum at this equilibrium. Second, we find the most profitable deviation from this equilibrium and determine a condition on c such that this deviation becomes unprofitable. We deal explicitly with firm A. By symmetry, the same results hold for firm B.

i) Explicitly, the first order conditions of equations (3) and (4) for firm A are

$$\frac{1-2p_A+p_B+s_A-s_B(1+\delta)}{(2-\delta(s_A+s_B))}=0$$

$$\frac{p_A\left[(2+\delta)(1-\delta s_B)+\delta(p_B-p_A)\right]}{(2-\delta(s_A+s_B))^2}=cs_A.$$

Imposing symmetry yields $s^N=\dfrac{2+\delta}{4c}$. Second order sufficient conditions for a local maximum, i.e.,

$$\left.\frac{\partial^2\Pi_A}{\partial p_A^2}\right|_{s=s^N}=\frac{(-1)}{(1-\delta s)}<0$$

$$\left.\frac{\partial^2\Pi_A}{\partial s_A^2}\right|_{\substack{s=s^N\\ p_i=p^N}}=\frac{\delta(2+\delta)}{4(1-\delta s)}-c<0$$

$$\left[\frac{\partial^2\Pi_A}{\partial p_A^2}\frac{\partial^2\Pi_A}{\partial s_A^2}-\left(\frac{\partial^2\Pi_A}{\partial p_A\partial s_A}\right)^2\right]_{\substack{s=s^N\\ p_i=p^N}}=\frac{c}{(1-\delta s)}-\frac{1+\delta^2}{4(1-\delta s)^2}>0$$

are satisfied if

$$c>c_1=\max\left\{0,\frac{1+2\delta(2+\delta)}{4}\right\}.$$

ii) We show that firm A cannot increase its profit by choosing $s_A \neq s^M, p_A \neq p^M$ given that $s_B = s^M, p_B = p^M$. Observe that $\partial^3\Pi_A / \partial s_A^3 \geq 0$, hence $\partial\Pi_A / \partial s_A$ is convex. If the second derivative is negative when evaluated at $s_A = s^M$, a profitable deviation may only arise for $s^M < s_A$. Two cases must be distinguished.

1. Let $\delta > 0$. Two subcases have to be considered depending on whether the optimal deviation lies on the bound given by $D_A = 1$, or on the upper bound of the strategy space, i.e., $\bar{s} = 1/\delta$.

a) Assume that s_A lies on the boundary defined by $D_A\left(p_A, s_A, s_B = s^M, p_B = p^M\right) = 1$. From the latter we obtain the optimal deviation price as $\hat{p}_A(s_A) = (1+\delta)(s_A - s^M)$. Hence deviation profits if $D_A \leq 1$ are

$$\hat{\Pi}_A(\hat{p}_A(s_A), s_B = s^M, p_B = p^M) = (1+\delta)(s_A - s^M) - \frac{c}{2}s_A^2.$$

Maximizing with respect to s_A yields a unique maximum at $\hat{s}_A = (1+\delta)/c$.[10]

Evaluate $\hat{\Pi}_A$ at $\hat{s}_A$. One can then show that

$$\Pi_A(p^M, s^M) - \hat{\Pi}_A \geq 0 \text{ if } c \geq c_2 = \frac{1}{4} + \frac{\delta(20+13\delta)}{16}.$$

Observe that the above condition is only valid if $\hat{s}_A = (1+\delta)/c \leq \bar{s} = 1/\delta$. The case where this does not hold is considered next.

b) Assume that $(1+\delta)/c > \bar{s} = 1/\delta$. Then we necessarily have $\hat{s}_A = 1/\delta$. The highest deviation profit as a function of p_A is given by

$$\hat{\Pi}_A(p_A, \hat{s}_A = 1/\delta, s_B = s^M, p_B = p^M) = p_A \frac{(2(1-\delta s^M) - p_A - s^M + 1/\delta)}{(1-\delta s^M)} - \frac{c}{2\delta^2}.$$

Maximizing $\hat{\Pi}_A$ w.r.t. p_A subject to $D_A \leq 1$ yields

$$\hat{p}_A = (1+\delta)(1/\delta - s^M) .$$

[10] Intuitively marginal costs of a quality improvement must be equal to the marginal benefit of it accruing to the marginal consumer located at $z = 1$.

Evaluate $\hat{\Pi}_A$ at $\hat{p}_A$. One can then show that

$$\Pi_A(p^M, s^M) - \hat{\Pi}_A \geq 0 \text{ if } c > c_3 = \frac{3\delta(2+\delta)}{4}.$$

2. Let $\delta \leq 0$. The strategy space constraint is no longer binding such that the reasoning and the results of 1a) apply.

The comparison of the cost conditions shows that $c_2 > c_3$. Hence the symmetric simultaneous equilibrium exists if $c \geq \max\{c_1, c_2\}$. QED

Proof of Proposition 2:

We proceed as follows. First, we solve for the symmetric equilibrium in quality and develop a condition on costs such that the profit functions attain a maximum at this equilibrium. Second, we show that under the condition on c there is no profitable deviation in the strategy space. We deal explicitly with firm A. By symmetry, the same results hold for firm B.

i) Firm A's profit at the price equilibrium is of the form

$$\Pi_A(s_A, s_B) = p_A(s_A, s_B) D_A \ (p_A(s_A, s_B), p_B(s_A, s_B), s_A, s_B) - C(s_A).$$

Deriving with respect to s_A yields the following first order condition

$$\frac{(3 + s_A(1-\delta) - s_B(1+2\delta))(4 - \delta(s_A + 3s_B + 1) + \delta^2 s_A)}{9(2 - \delta(s_A + s_B))^2} = cs_A. \tag{A1}$$

Evaluation at the symmetric equilibrium yields $s^{sc} = \max\left\{\frac{4-\delta}{12c}, 0\right\}$. The second order sufficient condition requires that

$$\left.\frac{\partial^2 \Pi_A}{\partial s_A^2}\right|_{s_A = s_B = s^{sc}} = -\frac{2c(18c + \delta^2 - 8\delta - 2)}{3(12c + \delta(\delta - 4))} \leq 0. \tag{A2}$$

It is straightforward to show that condition (A2) is satisfied if

$$c \geq \bar{c}_1 = \frac{1}{9} + \frac{\delta(8-\delta)}{18}.$$

ii) We show that firm A cannot increase its profit by choosing $s_A \neq s^{se}$ given that $s_B = s^{se}$. The argument hinges on the sign of the third derivative of profits which is given by

$$\frac{\partial^3 \Pi_A}{\partial s_A^3} = \frac{2\delta(1-\delta s_B)^2(2+\delta)^2}{3(2-\delta(s_A+s_B))^4}.$$

(A3)

Hence, $sign[\partial^3 \Pi_A / \partial s_A^3] = sign[\delta]$ and two cases have to be considered:

1. Let $\delta > 0$. The third derivative is positive implying that the first derivative is convex in s_A. Since the second derivative is negative when evaluated at $s_A = s^{se}$ we know that the most profitable deviation must arise for $\hat{s}_A > s^{se}$. Two subcases can arise. The distinction stems from the fact that the deviations at $\hat{s}_A = 1/\delta$ can either lie within the s_A – interval for which the middle piece of D_A given in (10) applies or outside of it. This distinction depends on δ. Evaluation of the middle piece of D_A at $s_A = 1/\delta$ and $s_B = s^{se}$ gives

$$D_A(s_A = 1/\delta, s_B = s^{se}) = \frac{2\delta+1}{3\delta}\begin{cases} <1 \text{ if } \delta > 1 \\ \geq 1 \text{ if } 0 < \delta \leq 1. \end{cases}$$

Accordingly, two subcases arise.

a) Let $\delta > 1$. We have to show that

$$\Pi_A\left(s^{se}, s^{se}\right) - \hat{\Pi}_A\left(\hat{s}_A = 1/\delta, s^{se}\right) \geq 0$$

by using the middle piece of (10). The above is satisfied if

$$c \geq \bar{c}_2 = \frac{2}{9} + \frac{\delta(20-\delta)}{36} > \bar{c}_1.$$

Hence, for $\delta > 1$ the symmetric equilibrium exists if $c \geq \bar{c}_2$.

b) Let $0 > \delta \geq 1$. The price equilibrium associated with the first demand piece in (10) is given by $p_B^* = 0$, $p_A^* = s_A(1+d) - s^{se} - 1$, where p_A^* is the highest price allowing firm A to attract the entire demand. Accordingly, the deviation profit is

$$\hat{\Pi}_A\left(s_A, s^{se}\right) = \left[s_A(1+\delta) - s^{se} - 1\right]1 - cs_A^2 / 2$$

which has an interior solution $\hat{s}_A = (1+\delta)/c < 1/\delta$. One can then show that

$$\Pi_A\left(s^{se}, s^{se}\right) - \hat{\Pi}_A\left(\hat{s}_A = (1+\delta)/c, s^{se}\right) \geq 0$$

if

$$c \geq \bar{c}_3 = \frac{4}{27} + \frac{\delta(352+133\delta)}{432}.$$

If $\hat{s}_A = 1/\delta < (1+\delta)/c$ then one obtains a corner solution and we have to show that

$$\Pi_A\left(s^{se}, s^{se}\right) - \hat{\Pi}_A\left(\hat{s}_A = 1/\delta, s^{se}\right) \geq 0.$$

The latter is satisfied if

$$c \geq \bar{c}_4 = \frac{\delta(20-11\delta)}{12}.$$

Hence, for $\delta > 0$ the symmetric subgame perfect equilibrium exists if $c \geq \max\{\bar{c}_2, \bar{c}_3\}$.

2. Let $-1 \leq \delta \leq 0$. The third derivative given in (A3) becomes negative implying that the first derivative is concave for non-negative s_A. Suppose the second derivative of the profit function w.r.t. s_A is negative at $s_A = s^{se}$, i.e., $c \geq \bar{c}_1$. Then we know that the most profitable deviation arises at $\hat{s}_A = 0 < s^{se}$. Consequently, we have to show that

$$\Pi_A\left(s^{se}, s^{se}\right) - \hat{\Pi}_A\left(\hat{s}_A = 0, s^{se}\right) \geq 0.$$

The latter holds if

$$c \geq \bar{c}_5 = \max\left\{0, \frac{1}{9} + \frac{\delta(20-\delta)}{72}\right\} \geq \bar{c}_1.$$

Hence, for $-1 \leq \delta \leq 0$ the symmetric subgame perfect equilibrium exists if $c \geq \bar{c}_5$. Adding i) and ii) completes the proof.

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