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Abstract

In bilateral negotiations between a principal and two agents, we show that the agents' bargaining strengths are crucial for the determination of the bargaining sequence and the efficiency of decisions. In a general framework with externalities between agents, we find that the surplus is highest if the principal negotiates with the stronger agent first, regardless of externalities being positive or negative. The principal chooses the efficient sequence with negative externalities, but often prefers the inefficient sequence with positive externalities. We show that our results extend to a general number of agents and provide conditions for simultaneous negotiations to be optimal.

JEL Classification: C72, C78, D62, L14

Keywords: bargaining power, Sequential negotiations, Externalities, Bilateral contracting, Endogenous timing

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1 Introduction

There are many economic situations in which a principal negotiates with several agents, and the outcome of the negotiation between the principal and one agent imposes externalities on other agents. Examples include the following situations:
(i) Vertical relations between a supplier and retailers, which sell in the final consumer market. Externalities between the retailers are negative if they sell substitutes, but positive if they sell complements.
(ii) A vaccine manufacturer contracts with research labs to develop or improve a vaccine. Externalities between labs can be negative (e.g., because they provide similar quality improvements) or positive (e.g., because they work on complementary methods).[1]
(iii) An entrepreneur negotiates with different venture capitalist firms. The latter either benefit from each other, as additional money allows the entrepreneur to secure higher profits (positive externalities), or compete against each other (negative externalities).

A salient feature in these settings is that the principal often bargains with each agent bilaterally—e.g., because multi-lateral agreements are too costly or precluded by antitrust law.[2] As pointed out by many negotiation consultants (e.g., Wheeler, 2005; Lax and Sebenius, 2012), the proper sequence of these negotiations is one of most crucial choices of the principal. For instance, Sebenius (1996) provides examples of several different situations—ranging from land acquisitions over political endorsements to military coalitions—in which negotiation leaders were confronted with the question whether to approach "harder" or "easier" players first.

The negotiation sequence does not only determine the principal's success, but also the efficiency of agreements and the total welfare generated by the bargaining process, as the decision in each negotiation affects all agents.[3]

A key variable driving the sequencing choice is the difference in agents' bargaining power. Several recent studies indeed show that there are substantial asymmetries in the bargaining strengths of (otherwise similar) players. For example, Grennan (2013, 2014), using data from the market for medical devices, finds that the bargaining ability of hospitals in the negotiation with suppliers is responsible for almost 80% of the price variation across hospitals. De los Santos et al. (2021) obtain a similar result in the e-book market—i.e., publishers, such as Harper Collins or Penguin Random House, are highly asymmetric in their bargaining power

[1] For more details an vaccine production, see e.g., Smith et al. (2011).

[2] McAfee and Schwarz (1994) and Dequiedt and Martimort (2015), among others, provide detailed justifications for the impossibility of multi-lateral contracts.

[3] Understanding how such sequencing decisions are made is also important for structural empirical analysis of negotiations. For empirical papers explicitly modeling negotiations between parties, see e.g. Crawford and Yurokoglu (2012), Gowrisankaran et al. (2015), or Ho and Lee (2017).

in the negotiation with e-book sellers Amazon and Barnes & Noble.

Yet, the literature on contracting with externalities has so far not considered the impact of such bargaining asymmetries. Instead, it has mainly focused on situations in which bargaining power is concentrated on one side: either the principal has all bargaining power (e.g., Segal, 1999; Genicot and Rey, 2006; Möller, 2007) or the agents make offers to the principal (e.g., Bernheim and Whinston, 1986).

In this paper, we study intermediate values of bargaining power between the principal and the agents, allowing for asymmetric bargaining power of agents. In particular, we analyze how the bargaining strength of the agents affects the negotiation sequence. This begs several interesting questions, such as: How does the sequence chosen by the principal compare to the surplus-maximizing sequence? How do externalities between agents shape the results? Under which conditions does the principal prefer simultaneous negotiations?

To answer these questions, we consider a general framework in which a principal negotiates with two agents who differ in their bargaining power.[4] Bargaining power of a player is captured by the probability of making a (take-it-or-leave-it) offer to the counter party.[5] The principal chooses the bargaining sequence, that is, whether to negotiate with the weak or the strong agent first. Each negotiation is over a contract that fixes a decision—e.g., a quantity to be traded—and a transfer. While, in general, there may be an incentive to renegotiate a contract signed in the first negotiation after the second negotiation, in practice, requirements of time or legal costs often preclude renegotiation, and we therefore focus on this case.

We allow for positive and negative externalities between agents, and also interaction of the traded quantities in the principal's utility function.[6] For the sake of exposition, we assume that, if the principal fails to reach an agreement with an agent, this agent's payoff is independent of the outcome in the other negotiation. This assumption seems natural e.g. in the situations described above.[7] To trace out the effect of asymmetric bargaining strengths, we derive our main results under the assumption that agents are symmetric except for bargaining power.

We first show that the sequence which maximizes the joint surplus of the players is the one in which the principal bargains first with the agent who has *high bargaining power*. This result holds *regardless* of whether externalities between agents are positive or negative. The

[4] Our main insights carry over to a general number of agents (as we show in Section 7.1).

[5] Such a random proposer structure is commonly used to represent bargaining strength in trading situations (e.g., Zingales, 1995; Ali, 2015; Auster et al. 2021) or political processes (e.g., Baron and Ferejohn, 1989; Eraslan and Evdokimov, 2019), and is sometimes referred to as the recognition probability of a player. In our game, each bilateral negotiation is also equivalent to asymmetric Nash bargaining with the respective bargaining weights, and can be interpreted as such.

[6] For example, the latter naturally occurs if the principal is a supplier with a non-linear cost function.

[7] Our results, however, are robust to such externalities (see Section 7.2).

intuition is based on the effect that the principal obtains only a fraction of the surplus in the second-stage negotiation. The bargainers in the first stage, therefore, consider the second-stage surplus only partially. However, they do so to a larger extent if the principal has more bargaining power in the second stage. Due to this *partial-surplus effect*, the distortion in the first negotiation relative to the efficient outcome is smaller when negotiating first with the strong agent.

We then study the sequence chosen by the principal. We find that the principal chooses the surplus-maximizing sequence if externalities are negative, but may choose an inefficient timing when externalities are positive. In addition to the partial-surplus effect, the principal's preferred sequence is driven by two additional effects: the *anticipated-externality effect* and the *outside-option effect*.

The anticipated-externality effect occurs because the payoff of the agent with whom the principal bargains first depends on the outcome in the second negotiation. The agent anticipates that an agreement will be reached in the second stage. Therefore, if the principal proposes in the first negotiation, she can demand a transfer that depends on this anticipated externality.[8] Instead, when the agent proposes, the principal obtains her outside option, which is the payoff from rejecting the offer of the first agent and negotiating only with the remaining agent. This payoff does not depend on any externality. When the principal bargains with the weak agent first, she proposes with a high probability and, hence, is likely to bear the externality herself. Therefore, bargaining with the weak agent first is attractive for the principal when externalities are positive, and the reverse holds for negative externalities.

The outside-option effect occurs because the first-stage bargainers fix a decision, anticipating that an agreement will be reached in the second stage. Therefore, in case the principal makes an offer in the first stage, the decision does not maximize her outside option, which is the surplus she gets in case no agreement will be reached in the second stage. By contrast, the bargainers in the second stage know whether an agreement was reached in the first stage. Hence, in case of no first-stage agreement, the principal obtains her maximal outside option if she makes an offer in the second stage. As a consequence, the principal's outside option is higher in the second stage than in the first stage. The effect therefore works in favor of the sequence in which the principal bargains with the strong agent first, as she then gets to make an offer in the second stage with a higher probability.

The principal's preferred sequence depends on the interplay of the three effects. With negative externalities, all three effects favor the sequence of negotiating first with the strong agent. The chosen sequence is therefore the efficient one. The same result occurs if externalities between agents are absent, but the decisions interact in the principal's utility

[8] The feminine pronoun denotes the principal and the masculine ones the agents.

function. Then, the anticipated-externality effect is not present, but the partial-surplus and the outside-option effect are at work. This holds regardless of the type of interaction between the decisions (i.e., whether the principal's utility function is super-modular or sub-modular).

With positive externalities, all three effects are at work and point in opposite directions, as the anticipated-externality effect favors bargaining with the weak agent first. Therefore, the result is no longer clear-cut. If externalties are small and the principal's utility function is additively separable, however, the anticipated-externality effect dominates. The intuition is that the partial-surplus and the outside-option effect lead to distortions in the first-stage decision, which are of second order if externalities are small. By contrast, the anticipated-externality effect has a first-order effect on the agents' utility.[9] The principal therefore chooses an inefficient sequence, as this allows her to obtain a larger share of a smaller pie.

If externalities are positive and large, all three effects are sizable. The principal still prefers to bargain with the weaker agent first if e.g. equilibrium decisions are bounded, as the importance of the partial-surplus and the outside-option effect is then limited. However, the opposite result may also occur, leading to a non-monotonicity in the sequence, as the principal prefers to bargain with the strong agent first if externalities are negative and highly positive, but with the weak agent first if externalities are moderately positive.

In summary, the externalities between agents and the interaction of the decisions in the principal's utility function shape the sequence of negotiations. Whereas the sign of the externalities (i.e., negative or positive) is crucial for the principal's preferred sequence, the type of interaction in the principal's utility function is not. Although, for instance, positive externalities between agents and super-modularity of the principal's utility function both increase the joint surplus, their effect on the equilibrium sequence is highly different: the former favors negotiating with the weak agent first through the anticipated-externality effect, whereas the latter unambiguously favors negotiating with the strong agent first.

Our insights are consistent with practical negotiation guidance, but also go beyond it. In particular, Sebenius (1996) and Lax and Sebenius (2012) point out that the principal should "(s)eek to get the easy parties on board first" in case players benefit from agreements with other players. This is in line with our finding for positive externalities between agents. We provide additional guidance by showing that the reverse order is optimal if a player is harmed when the principal reaches an agreement with other players. In addition, we demonstrate that the result for positive externalities no longer holds if the anticipated-externality effect is dominated by the partial-surplus effect and the outside-option effect.

[9] A similar result holds if decisions are binary (i.e., the agent decides whether or not to participate in a joint project) and participation is always optimal. In this setting, there is no distortion in decisions and only the anticipated-externality effect is at work. The principal then prefers to bargain with the strong agent first when externalities are negative, and with the weak agent first when externalities are positive.

We also consider simultaneous negotiations. We show that for negative externalities, sequential negotiations in which the principal bargains with the stronger agent first dominate simultaneous negotiations. Instead, when externalities are positive, simultaneous negotiations may be optimal for the principal. The intuition is rooted in the fact that, with simultaneous bargaining, agents cannot observe the outcome in the other negotiation. Each agent supposes that an agreement will be reached there. Instead, with sequential negotiations, the agent bargaining at the second stage observes if the bargainers in the first stage failed to reach an agreement. With positive externalities, this effect leads to a higher outside option of the principal with simultaneous negotiations.

Our paper contributes to the literature on contracting with externalities, pioneered by Segal (1999, 2000). He considers the situation in which a principal simultaneously offers contracts to agents in the presence of multilateral externalities. In this context, Möller (2007) allows for sequential contracting, and shows that the principal prefers sequential over simultaneous contracting only if externalities on non-traders are sufficiently large.[10] Genicot and Rey (2006) also analyze contracting over time and demonstrate how the principal extracts most surplus from agents by combining simultaneous and sequential offers.[11] These papers consider an *offer game* where the principal has all the bargaining power. Instead, Bernheim and Whinston (1986) and Martimort and Stole (2002, 2003) consider a *bidding game* where the agents simultaneously propose contracts to the principal. Contrary to these papers, we study a situation with intermediate bargaining power and demonstrate how the bargaining power affects the optimal negotiation sequence.[12]

Several papers consider sequential negotiations in different environments, but take the bargaining sequence as given and assume symmetry between agents. For example, Cai (2000) considers the case in which the principal needs to reach an agreement with all agents to obtain a positive surplus (e.g., a railroad must get permission from all landowners) and Bagwell and Staiger (2010) analyze trade agreement negotiations between countries when most-favored nation clauses are in place. Iaryczower and Oliveros (2017) study collective action problems in which competing principals bargain to seek support of a majority of agents.

A few papers analyze the sequencing of negotiations, but focus on different issues than

[10]Jehiel and Moldovanu (1995a, 1995b) show that externalities on non-traders can also cause substantial delay in reaching in agreement, both with a finite and an infinite horizon. This result also occurs if renegotiation among agents is possible (Jehiel and Moldovanu, 1999).

[11]Horn and Wolinsky (1988) and Marshall and Merlo (2004) study the difference between simultaneous and sequential negotiations in the context of a union negotiating over wages with two competing firms, and find that sequential bargaining is always preferred by the union.

[12]Galasso (2008) combines the offer and the bidding game in a sequential bargaining model along the lines of Rubinstein (1982). He focuses on negative externalities between symmetric agents and shows that the principal's payoff can be decreasing in her bargaining power, but does not analyze sequencing of negotiations.

our paper. Noe and Wang (2004) consider a situation in which the principal can keep the order of negotiations confidential, and determine conditions for secrecy being more profitable than public negotiations.[13] Krasteva and Yildirim (2012b) analyze a model in which two sellers offer complementary products and the buyer's payoff with only a single agreement is uncertain. In addition, the buyer can decide whether to execute the contracts after both negotiations are finished. They show that the sequence only matters in case of uncertainty, and then depends on the degree of complementarity and the extent of asymmetry between buyers.[14] Xiao (2015) endogenizes the bargaining sequence in a model in which a buyer negotiates with several sellers who own complementary goods (similar to Cai, 2000). He shows that the buyer wants to negotiate with small sellers first. None of these papers analyzes how the principal's optimal sequence depends on the direction of the externalities or the interplay between the agents' bargaining strength and the externalities. In addition, they all consider binary decisions in a negotiation, whereas we allow for continuous decisions.[15]

The remainder of the paper is organized as follows: Section 2 sets out the model and presents preliminary results. Section 3 considers the bargaining sequence that maximizes the surplus of all players. Section 4 analyzes the sequence chosen by the principal. Section 5 considers simultaneous negotiations. Section 6 presents extensions to contract disclosure and exclusive dealing contracts. Section 7 considers a general number of agents and externalities on non-traders. Section 8 concludes. All proofs are relegated to the Appendix.

2 The Model

Assumptions. There are three players: a principal A ("she") and two agents B and C ("he").[16] A and B negotiate over a decision $b \in \mathcal{B} \subseteq \mathbb{R}_+$, with $0 \in \mathcal{B}$, and a monetary transfer $t_B \in \mathbb{R}$ from B to A. Similarly, A and C negotiate over a decision $c \in \mathcal{C} \subseteq \mathbb{R}_+$, with $0 \in \mathcal{C}$, and a transfer $t_C \in \mathbb{R}$.[17] The payoff of the principal is $u_A(b,c) + t_B + t_C$, and the payoffs of the agents are $u_B(b,c) - t_B$ and $u_C(b,c) - t_C$, respectively.

Negotiations are bilateral, and the sequence is chosen by A. Within each stage, there is random proposer take-it-or-leave-it bargaining. Bargaining power is modelled as the proba-

[13]Krasteva and Yildirim (2012a) provide a similar analysis, and also distinguish between exploding and non-exploding offers.

[14]Marx and Shaffer (2007, 2010) and Raskovich (2007) also analyze a situation with one buyer who bargains with two sellers, but consider the case in which there are no direct externalities between sellers.

[15]Sequencing has also been studied in the literature on agenda formation (e.g. Winter 1997). However, sequencing here refers to the order of different issues.

[16]In Section 7.1, we analyze the case with N agents and show that our main insights carry over.

[17]To simplify the exposition, we restrict b and c to one-dimensional variables. However, many of our results extend to the case in which b and c are multi-dimensional.

bility of making the offer: B proposes with probability $\beta \in [0,1]$, C proposes with $\gamma \in [0,1]$.[18] Without loss of generality, $\beta \geq \gamma$, that is, B is the stronger bargainer among the agents.[19] As it is the objective of the paper to analyze which agent the principal will approach first, we follow the literature on sequencing decisions and rule out renegotiation. For instance, Iaryczower and Oliveros (2017) point out that sequential contracting without renegotiation is prevalent in inter-firm bargaining or political endorsements. Möller (2007) and Montez (2014) explain that renegotiation is often infeasible, as it involves high legal costs.

The timing of the game is as follows. In stage 0, A chooses whether to bargain with B first (sequence BC) or with C first (sequence CB). In sequence BC, in stage 1, A bargains with B. With probability β, B proposes a contract $(b, t_B) \in \mathcal{B} \times \mathbb{R}$, and A either accepts or rejects. With probability $1-\beta$, A proposes, and B then accepts or rejects. If A and B reach an agreement on a contract (b, t_B), the decision b is implemented and the transfer t_B is made. In case of rejection, $b = t_B = 0$. In $t = 2$, C observers the outcome of stage 1.[20] Then, A and C bargain. With probability γ, C proposes a contract $(c, t_C) \in \mathcal{C} \times \mathbb{R}$; with probability $1-\gamma$, A proposes. If they reach an agreement on a contract (c, t_C), the decision c is implemented and the transfer t_C is paid. Otherwise, $c = t_C = 0$. Sequence CB is similar, except that A bargains with C in stage 1 and with B in stage 2.

In our bargaining game, the principal negotiates with one agent at a time. This is a highly relevant situation because negotiations often require physical presence of the principal, and it is too costly to communicate with all agents at the same time.[21] However, there can be circumstances in which the principal can delegate the negotiations, which allows for the possibility of simultaneous bargaining. We will consider this case in Section 5.

Our assumption on the contract space implies that the contract negotiated in stage 1 cannot condition on actions chosen in stage 2. This is motivated by the fact that such contingent contracts are rare and difficult to enforce (see e.g., Möller, 2007). In addition, if A is an upstream firm serving two retailers B and C, a contract between A and B that conditions on c is usually prohibited by competition law (Dequiedt and Martimort, 2015). An exception are exclusive dealing contracts, which allow the principal to commit to deal with only one agent. In our main analysis, we assume that such commitments are infeasible;

[18]We could also determine the outcome of each negotiation via the Nash bargaining solution with bargaining weights β for B and $1-\beta$ for A in the negotiation between A and B, and γ for C and $1-\gamma$ for A in the negotiation between A and C (see, for example, Muthoo 1999). All our results then continue to hold.

[19]As outlined in the Introduction, capturing different negotiation strengths through differences in the bargaining power of agents is natural in many situations. We note that it is not equivalent to different outside options by the agents, and explain this at the end of this section.

[20]In Section 6.1, we consider the situation in which C does not observe this outcome, and A can decide whether to disclose it or not.

[21]Due to this reason, many studies on bargaining such as Noe and Wang (2004), Krasteva and Yildirim (2012a,b), and Iaryczower and Oliveros (2017) analyze sequential negotiations.

Section 6.2 enriches the contract space to allow for exclusive contracts and shows that our results still hold.

We assume that there are no externalities on non-traders: $u_B(0,c)$ is constant in c, $u_C(b,0)$ is constant in b, and we normalize the utility functions such that $u_A(0,0) = u_B(0,c) = u_C(b,0) = 0$. This is a natural assumption in most of the examples given in the Introduction (e.g., a firm's profit in a market in which the firm is not active, is zero and does not depend on the quantity traded in that market).[22]

Externalities between agents are negative if $u_B(b,c) \leq u_B(b,0)$ and $u_C(b,c) \leq u_C(0,c)$ for all b and c. Externalities are positive if the inequalities are reversed, and there are no externalities if the inequality signs are replaced with equality signs.[23] Moreover, externalities are strictly negative (strictly positive) if $u_B(b,c) < (>)\, u_B(b,0)$ and $u_C(b,c) < (>)\, u_C(0,c)$, whenever $b > 0$ and $c > 0$. At this point, we do not make assumptions on whether externalities are stronger or weaker if b or c are larger, but only whether they are positive or negative as defined above. Towards the end of Section 4, we will put more structure on the externalities to derive further results.

To isolate the impact of differences in bargaining power, our main results assume some degree of symmetry between B and C. We say that *agents are symmetric except for bargaining power* if (i) $\mathcal{B} = \mathcal{C}$, (ii) u_A is a symmetric function, that is, $u_A(b,c) = u_A(c,b)$, and (iii) $u_C(c,b) = u_B(b,c)$.

We define the joint surplus of all three players as $S(b,c) := \sum_{i\in\{A,B,C\}} u_i(b,c)$. Finally, we impose the tie-breaking rule that, if A is indifferent, but surplus is strictly higher in one of the sequences, A selects the surplus-maximizing sequence.

We point out that our basic set-up is quite general. For example, it does not assume differentiability of the utility functions or monotonicity of the externalities in b or c. Similarly, the sets of possible decisions $\mathcal{B}$ and $\mathcal{C}$ could be discrete or continuous. Some of our result assume more structure, as will be explicitly indicated below.

Example with a Supplier and Retailers. To provide a concrete example for our framework, suppose that b and c are quantities of an input good sold by supplier A to retailers B and C who pay fixed amounts t_B and t_C for receiving their respective quantity. Retailers transform the input to output, and their production technology is one-to-one. Retailers compete in quantities in the downstream market. Their payoff functions are then $u_B = bp_B(b,c)$

[22]In Section 7.2, we nevertheless show that our results extend to the case where externalities on non-traders are present, but not too large.

[23]For some results, we only need to specify whether the decision with one agent $i \in \{B,C\}$ has negative, positive, or no externalities on the utility of the other agent $j \neq i$, without an assumption on the externality of the decision with agent j on agent i.

and $u_C = cp_C(c, b)$, respectively, where p_B and p_C are the prices of the products, which depend on the quantities of both retailers. (For simplicity, we set retail costs to zero.) If retailers sell substitutes, p_B is falling in c (and, similarly, p_C is falling in b), hence externalities are negative. By contrast, if products are complements, externalities are positive since prices are increasing in the quantity of the other retailer. The function u_A describes the supplier's production costs and could be $u_A = -k(b, c)$.[24]

Preliminaries. Within each stage, there is take-it-or-leave-it-bargaining with transferable utilities. Thus, the decision maximizes the joint expected surplus of the two bargainers. Moreover, the proposer chooses a transfer such that the respondent is just willing to accept.

Consider sequence BC (sequence CB can be analyzed similarly). In stage 2, the decision b and transfer t_B are already fixed. The decision reached in stage 2 maximizes the joint surplus of A and C, given b. We assume that, for any b, there exists a unique

$$c^* (b) := \arg\max_{c \in \mathcal{C}} \{u_A (b, c) + u_C (b, c)\}.$$

The existence of an optimal second-stage decision is ensured when (i) the sets $\mathcal{B}$ and $\mathcal{C}$ are finite, or (ii) the utility functions u_i ($i = A, B, C$) are continuous on $\mathcal{B} \times \mathcal{C}$ and the sets $\mathcal{B}$ and $\mathcal{C}$ are compact. A sufficient condition for uniqueness in case (ii) is that $u_A (b, c) + u_B (b, c)$ is strictly quasiconcave in b, and $u_A (b, c) + u_C (b, c)$ is strictly quasiconcave in c.

The expected payoff of A in stage 2 of sequence BC is

$$(1 - \gamma) (u_A (b, c^* (b)) + u_C (b, c^* (b))) + \gamma u_A (b, 0) + t_B$$

because, with probability $1-\gamma$, A proposes and extracts C's utility, whereas, with probability γ, C proposes and holds A down to her reservation utility of $u_A (b, 0)$. If $b = t_B = 0$, the expected payoff of A in stage 2 is

$$O_A^{BC} = (1 - \gamma) \max_{c \in \mathcal{C}} \{u_A (0, c) + u_C (0, c)\}.$$

This is the expected utility of A in case the first-stage negotiation with B fails; it is therefore A's outside option in the first stage.

In the first stage of sequence BC, the joint surplus of A and B consists of B's payoff and

[24] Our setting can also accommodate strategic interaction after stage 2. For example, b and c could represent the unit prices constituting the variable parts of a two-part tariff, whereas t_B and t_C constitute the fixed parts. To be consistent with our notation that $b = 0$ denotes no trade, we can use an inverse scale of the unit prices. As long as all subgames after stage 2 have unique subgame equilibrium payoffs that (net of transfers) depend only on the decisions (b, c) taken in stages 1 and 2, these payoffs can be expressed by functions u_A, u_B, and u_C, and our model applies.

A's expected payoff in stage 2 (net of t_B):

$$S_{AB}^{BC}(b) := u_B(b, c^*(b)) + (1-\gamma)(u_A(b, c^*(b)) + u_C(b, c^*(b))) + \gamma u_A(b, 0). \tag{1}$$

In any equilibrium of sequence BC, A and B reach a decision $b^{BC} \in \arg\max_{b\in\mathcal{B}} S_{AB}^{BC}(b)$,[25] and A's expected payoff of is

$$U_A^{BC} = (1-\beta) S_{AB}^{BC}\left(b^{BC}\right) + \beta O_A^{BC}.$$

In case several $b \in \arg\max_{b\in\mathcal{B}} S_{AB}^{BC}(b)$ exist, note that they all lead to the same payoffs for A and B. In case they lead to a different joint surplus, we assume that a decision that maximizes $S(b, c^*(b))$ is selected. Therefore, the joint surplus in any equilibrium of sequence BC is unique, even if first-stage decisions are not unique. We impose the corresponding assumptions on sequence CB, and denote the equilibrium first-stage decision in sequence CB by c^{CB}.

Finally, we note that if agents were symmetric and had the same bargaining power but differed in their outside options, the principal would be indifferent between both sequences. To see this, denote the outside option of agent $i \in \{B, C\}$ by u_i^0, with $u_B^0 > u_C^0$. Then, in the expressions for U_A^{BC} and U_A^{CB}, besides setting $\beta = \gamma$, $u_i(\cdot,\cdot)$ needs to be replaced by $u_i(\cdot,\cdot) - u_i^0$ because the principal, when being selected as the proposer, can only extract the agent's payoff minus his respective outside option. It is then straightforward to check that the outside options enter U_A^{BC} and U_A^{CB} in the same way. If agents are symmetric otherwise, this implies $U_A^{BC} = U_A^{CB}$. As we will show in the next sections, this is not true if agents differ in their bargaining power; hence, differences in bargaining power are not equivalent to differences in outside options.

3 The Surplus-Maximizing Sequence

Before analyzing the principal's optimal sequence, we determine whether sequence BC or CB generates a higher joint surplus. The first-best surplus is

$$S^{FB} = \max_{b\in\mathcal{B},\, c\in\mathcal{C}} \{u_A(b,c) + u_B(b,c) + u_C(b,c)\}.$$

[25] Existence of a maximum of $S_{AB}^{BC}(b)$ is ensured under the conditions discussed above (in case (ii), $c^*(b)$ is continuous by the Maximum Theorem; thus, $S_{AB}^{BC}(b)$ is continuous, and a solution to $\max_{b\in\mathcal{B}} S_{AB}^{BC}(b)$ exists by the Weierstrass Theorem).

In general, there are two reasons why the equilibrium decisions in a given sequence are not surplus-maximizing. The first is that the negotiation in the second stage maximizes the surplus of the two players involved, but does not take into account the effect of the decision on the agent with whom A has negotiated in the first stage. This effect works through the externality of c on B in sequence BC (and through the externality of b on C in sequence CB). For instance, in the supplier-retailers example with substitute products, negotiating a larger quantity in the second stage has a negative effect on the agent with whom A bargained first.

The second reason why equilibrium decisions are not surplus-maximizing is due to the fact that A only receives a fraction of the surplus in the second-stage negotiation. This implies that, in the first stage, the two bargainers only partially consider the second-stage surplus. Therefore, first-stage decisions may be distorted away from the surplus-maximizing outcome. This *partial-surplus effect* works through two channels. First, through the externality of b on C in sequence BC (and through the externality of c on B in sequence CB). In the supplier-retailer example, if A signs a contract with a large quantity in the first stage, the surplus that A and her negotiation partner can achieve in the second stage is lower if goods are substitutes. Second, through interaction of b and c in A's utility function. This occurs because the agent with whom A bargains in the second stage extracts A's utility with some probability. In the example, suppose that A's cost is given by a convex function of the total quantity $b + c$ and the negotiation sequence is BC. Then, the first-stage decision b will be chosen too high from a joint-surplus perspective because, with some probability, A will not be the proposer in the second stage, implying that C has to bear this higher cost.

Remark 1 illustrates that these two effects are indeed the *only* reasons for inefficiencies. Denote the joint surplus in sequence BC by S^{BC} and in sequence CB by S^{CB}.

Remark 1 *Suppose that* $1 \geq \beta > \gamma = 0$, *and* c *has no externalities on* B. *Then,* $S^{BC} = S^{FB} \geq S^{CB}$.

The remark shows that the equilibrium decisions maximize joint surplus in sequence BC if C has no bargaining power and the decision that A reaches with C does not affect B's utility. The first assumption (i.e., $\gamma = 0$) shuts down the partial-surplus effect because A receives the full surplus in the negotiation with C, and the second assumption shuts down the effect that the decision in the second stage is suboptimal for the agent with whom A negotiates first. We point out that Remark 1 does not assume any symmetry between the agents. In addition, the result holds no matter which externality b imposes on C.

The next proposition shows that sequence BC dominates sequence CB from a surplus perspective also if C has some bargaining power (i.e., $\gamma > 0$), in case agents are symmetric except for bargaining power.

Proposition 1 *Suppose that agents are symmetric except for bargaining power, and* $1 \geq \beta > \gamma \geq 0$*. Then,* $S^{BC} \geq S^{CB}$*.*

Therefore, if agents only differ in their bargaining strength, surplus is higher when the principal bargains with the *stronger agent first*. It is important to note that this result holds *irrespective* of whether externalities are negative or positive.[26] Moreover, it also does not matter how the decisions interact in the principal's utility function u_A. The intuition is rooted in the partial-surplus effect. If the principal negotiates with the weaker agent in the second stage, she receives a larger share of the surplus in this stage. Therefore, the utility of the agent with whom the principal bargains in the second stage is taken into consideration to a larger extent in the first-stage negotiation. The first-stage decision therefore leads to a higher surplus than in the case in which the principal bargains with the weaker agent first. Instead, the other effect described above (i.e., the second-stage decision ignores the utility of the agent with whom the principal bargained first) plays out similarly in the two sequences when agents are symmetric except for bargaining power. As a consequence, the joint surplus is higher in sequence BC than in CB.[27]

This intuition explains why the surplus-maximizing sequence is BC, *regardless* of the externalities. In contrast, the sequence preferred by the principal crucially depends on the externalities, as we will show in the next section.

4 The Sequence Preferred by the Principal

We start this section by considering the special case in which $\beta = 1$, that is, B has all bargaining power. This case shows in a particularly transparent way how the externalities affect the principal's preference over the bargaining sequences.

Remark 2 *Suppose that* $\beta = 1$ *and* $\gamma \in [0, 1)$*. If* b *has negative externalities on* C*, then* $U_A^{BC} \geq U_A^{CB}$*; instead, if* b *has positive externalities on* C*, then* $U_A^{BC} \leq U_A^{CB}$*.*[28] *If* b *has no externalities on* C*, then* $U_A^{BC} = U_A^{CB}$*.*

[26]Interestingly, it also does not matter whether the principal has more or less bargaining power than the agents, or one of them.

[27]The inequality in the proposition is strict (i.e., $S^{BC} > S^{CB}$), if first-stage decisions differ in the two sequences. Sufficient (but not necessary) conditions for this are that either externalities are strict or b and c interact in A's utility function, and that the bargaining problems are smooth in the sense that all utility functions are differentiable and equilibrium decisions are unique, interior, and differentiable.

[28]When externalities are strictly negative or strictly positive and equilibrium decisions are not zero, then the inequalities are strict.

The remark shows that for $\beta = 1$, the principal's preference is solely driven by the sign of the externalities. In fact, only the externality of b on C matters for the principal because B has all bargaining power and fully bears the externality of c by himself. The remark also allows for asymmetries between agents over and above their different bargaining powers.

What is the intuition behind this result? As the principal only receives a payoff in the negotiation with C, we can focus on this negotiation. Consider first sequence CB. When A negotiates with C in the first stage, the two bargainers anticipate the decision $b^*(c)$ taken between A and B in the second stage. Therefore, in case A proposes in the first stage, C is willing to make a payment up to $u_C(b^*(c), c)$—an amount that depends on the externality of b on C. In contrast, in sequence BC, B will drive the principal down to her outside option in the first stage. This outside option is equal to the expected payoff that A achieves in the negotiation with C, given that she rejected B's offer in the first stage, and consequently $b = t_B = 0$. Therefore, the outside option does not depend on the externality from b on C. The principal then prefers sequence CB if externalities are positive because in this sequence she can (with positive probability) gain the positive external effect of b on C for herself. By contrast, if externalities are negative, she prefers sequence BC, which insulates her from the externality. Finally, if there are no externalities, the principal is indifferent.

This *anticipated-externality effect* is the crucial driver for the principal's preferred sequence in case she does not receive a payoff in the negotiation with B. In fact, the partial-surplus effect described above does not matter then, as A does not receive a surplus in the negotiation with B; hence, internalization plays no role.

Remark 2 already shows that the efficiency of the sequence chosen by the principal depends on the externalities. Combining Remark 2 and Proposition 1 directly imply that, if agents are symmetric except for bargaining power and $\beta = 1$, the equilibrium sequence is *efficient* when externalities are *negative* but *inefficient* when they are *positive*.

We now turn to the analysis of the case in which the bargaining power of both agents is strictly below 1. In particular, we are interested in whether and, if yes, how the conclusions of Remark 2 need to be modified if $\beta < 1$. To isolate the effect of differing bargaining power, we focus our analysis on the symmetric case, that is, agents are symmetric except for bargaining power.

If the bargaining power of all players is strictly between 0 and 1, the partial-surplus effect and the anticipated-externality effect are both at work. In addition, there is a third effect, which we call the *outside-option effect*. This effects occurs because the first-stage decision does, in general, not maximize A's outside option in the second-stage negotiation, because it maximizes the joint utility of the bargainers in the first stage under the assumption that an agreement will be reached in the second stage. Therefore, if A gets to make an offer exactly once, it is better for her to do so in the second stage. Her decision then maximizes the joint surplus in this negotiation. By contrast, if she makes an offer in the first stage, her decision will take into account that she may receive a fraction of the surplus in the negotiation at the second stage. If A turns out not to be the proposer in the second stage, the first-stage decision was ex post suboptimal, since it was not set to maximize her second-stage outside option. Other things being equal, this outside option effect favors sequence BC, where it is more likely that A proposes only in stage 2.

We now determine how these three effects play out for different signs of the externalities. We start with negative externalities, than move to no externalities, and finally consider positive externalities.

With negative externalities, we obtain a clear-cut result:

Proposition 2 *Suppose that the agents are symmetric except for bargaining power,* $1 > \beta > \gamma$*, and externalities are negative. Then,* $U_A^{BC} \geq U_A^{CB}$*, with strict inequality if externalities are strictly negative and equilibrium decisions are not zero.*

With negative externalities, all three effects point in the same direction. First, as explained in Section 3, the partial-surplus effect favors sequence BC, as, in this sequence, the second-stage surplus is considered to a larger extent in the first negotiation than in sequence BC. Second, as explained above, the outside-option effect also favors sequence BC. Third, the anticipated-externality effect points in favor of sequence BC, as well. In contrast to the case of Remark 2, in which A received no payoff when bargaining with B, the anticipated-externality effect is now present in both negotiations. However, due to the fact that $\beta > \gamma$, A's expected payoff is larger in the negotiation with C. The anticipated-externality effect is therefore more important in the negotiation with C, which speaks in favor of sequence BC. As a consequence, if agents are symmetric except for bargaining power, with negative externalities the principal prefers the sequence BC as in Remark 2, even if B does not have full bargaining power.

We now turn to the case in which there are no externalities between agents. As demonstrated in Remark 2, if $\beta = 1$, the principal is indifferent between the two sequences. The next proposition shows that this is no longer the case if $\beta < 1$.

Proposition 3 *Suppose agents are symmetric except for bargaining power,* $1 > \beta > \gamma$*, and there are no externalities. Then,* $U_A^{BC} \geq U_A^{CB}$*, with strict inequality if* $b^{BC} \neq c^{CB}$*. A sufficient condition for* $b^{BC} \neq c^{CB}$ *is that (i)* u_A *is strictly super-modular or strictly sub-modular, (ii) equilibrium decisions are interior, and (iii)* u_i $(i = A, B, C)$ *and* $c^*(b)$ *are differentiable.*

Even without externalities, the two negotiation problems are not independent of each other because the decisions b and c interact in the principal's utility function. Although the anticipated-externality effect is not at work, the other two effects are for $\beta < 1$. Since the principal obtains a strictly positive expected payoff in both negotiations, the partial-surplus effect and the outside-option effect indeed matter for her. As both effects favor sequence BC compared to sequence CB, she prefers the former.

By contrast, when $\beta = 1$ (as in Remark 2), both the partial-surplus effect and the outside-option effect are not relevant. The equilibrium decision in the negotiation between A and C maximizes their joint surplus even in sequence CB because A will be negotiated down to her outside option in the second stage. This shows that, in case of no externalities, $\beta = 1$ is only a special case. In general, the interaction of the decision variables in A's utility function induce a non-equivalence of the sequences, even if externalities are absent.

Because of the partial-surplus and the outside-option effect, the principal's preference for sequence BC is strict if first-stage decisions differ across sequences. Conditions (i)-(iii) ensure that this is indeed the case. The role of Condition (i) is to ensure interaction between the bargaining problems. The condition holds in many economic applications—e.g., it is satisfied in our supplier-retailers example if A has strictly increasing marginal costs. If the retailers sell substitute products, A then proposes a lower quantity in sequence BC than in sequence CB in order to be able to better use the opportunities that arise when proposing in the second stage as well. Condition (i) alone is not sufficient to rule out the possibility that first-stage decisions might be identical in the two sequences, be it because they occur at a boundary of the feasible set, or because the utility functions are not differentiable. Conditions (ii) and (iii) serve to rule these possibilities out.[29]

It is important to note that the way how b and c interact in the principal's utility function does not drive the result; instead, the *mere* fact that the utility function is not additively separable is enough for the principal to prefer sequence BC. This implies—e.g., in the supplier-retailers example—that not only strictly increasing marginal costs, but also strictly decreasing marginal costs lead to the same result.[30] Therefore, whereas the *direction* of the externalities is crucial for the sequencing decision, this does not hold for the *type of interaction* in u_A. For example, suppose there are complementarities in the production of joint surplus. How this affects the sequence preferred by the principal depends on whether the complementarities stem from the principal's utility function u_A, or from externalities between agents. Super-modularity of u_A (e.g., the supplier's cost is a concave function of

[29]Similarly, Edlin and Shannon (1998) rely on interiority and differentiability assumptions for strictly monotone comparative statics.

[30]The principal's preference for BC is strict as long as the cost function is not highly concave, as otherwise the solution would be at the boundary.

the sum $b + c$) favors sequence BC through the outside-option effect. In contrast, positive externalities between agents may favor sequence CB through the anticipated-externality effect, as was noted above (Remark 2).

Before considering positive externalities, we illustrate the importance of the anticipated-externality effect in an example in which decisions are binary. Binary decisions occur, for instance, in political processes where decisions are often either "Yes" or "No", or when objects are indivisible and buyers either choose to buy or not.

Example with binary decisions. Assume that agents are symmetric except for bargaining power, and $0 \leq \gamma < \beta \leq 1$. Suppose that $\mathcal{B} = \mathcal{C} = \{0, 1\}$, with the possible interpretation that $b = 1$ ($c = 1$) indicates that B (C) participates in a joint project, or the sale of an indivisible object between A and B (C). Moreover, suppose that participation is optimal in every subgame (i.e., regardless of the decision made in the other negotiation).

By assumption, the equilibrium first-stage decision is to participate (i.e., $b^{BC} = c^{CB} = 1$). This implies that there are no distortions in the first-stage decision. As a consequence, the partial-surplus effect and the outside-option effect are not present, and the principal's preferences are pinned down solely by the anticipated-externality effect: if externalities are strictly positive, A strictly prefers CB, whereas if they are strictly negative, A strictly prefers BC. If there are no externalities, A is indifferent between both sequences.

We now turn to the analysis of positive externalities. In the case $\beta = 1$, and in the example with binary decisions, only the expected externality effect is present, whereas the other two effects are not at work. As a consequence, with positive externalities, the principal unambiguously prefers sequence CB in these settings. In general, however, all three effects are present. As the partial-surplus effect and the outside-option effect work in favor of sequence BC (regardless of the direction of the externalities), the optimal sequence with positive externalities depends on how the opposing effects play out. Both sequences BC and CB can emerge in equilibrium. Indeed, as we will show below, this ambiguity exists regardless of the strength of the externalities.

We know from above that, if u_A is not additively separable, the principal prefers sequence BC when externalities are absent; by continuity, she may also prefer sequence BC with small positive externalities. As we will show next, however, if u_A is additively separable and the bargaining problems are sufficiently smooth, the principal prefers sequence CB when externalities are positive but small.

To make this precise, we assume the agents' utility functions include a parameter $k \in \mathbb{R}$, which captures the importance of the externalities, but does not affect the principal's utility function. We assume that the bargaining problems are smooth in the sense that all utility functions are differentiable and equilibium decisions are unique, interior, and differentiable.

We also make the following assumptions how k affects the agents' utility functions: (i) if $k = 0$, there are no externalities, (ii) if $c = 0$ ($b = 0$), then k has no effect on u_B (u_C), (iii) if $b > 0$ and $c > 0$, u_B and u_C are strictly increasing in k. We call this the case of parametric externalities. It allows us to derive the following proposition:

Proposition 4 *Consider the case of parametric externalities. Suppose agents are symmetric except for bargaining power, $1 > \beta > \gamma$, and u_A is additively separable. Then, there exists a $\hat{k} > 0$ such that $U_A^{CB} > U_A^{BC}$ for all $k \in (0, \hat{k})$.*

The proposition shows that for small positive externalities, sequence CB dominates BC if the decisions b and c do not interact in A's utility function. The intuition behind this result is as follows: The partial-surplus and the outside-option effect occur because they lead to a sub-optimal choice of the first-stage decision, which may differ in the two sequences. In particular, the former effect arises because the second-stage surplus is considered only partially in the first stage, which distorts the first-stage decision compared to the surplus-maximizing one, whereas the latter effect arises because the first-stage decision does not maximize A's outside option. Suppose now that $k = 0$. Because u_A is additively separable, the bargaining problems do not interact, which implies that all decisions are chosen in an optimal way. When increasing k slightly (i.e., moving from no to small positive externalities), due to the Envelope Theorem, the effect of the changing first-stage decisions on A's payoff is only of second order. Therefore, the partial-surplus and the outside-option effect are also of second order. By contrast, the anticipated-externality effect, which does not depend on the sub-optimality of the first-state decisions, but on the direction of the externalities, is still of first order. The effect is strictly positive because decisions are strictly positive. It follows that, for small positive externalities, the anticipated-externality dominates the other two effects. Therefore, the principal strictly prefers to bargain with the weaker agent first.

If externalities are not small, all three effects are of first order. A main question is then whether sequence CB remains optimal when externalities are large. The next proposition considers the opposite case to Proposition 4 by focusing on the case in which the positive externalities grow beyond bounds. As in Proposition 4, we consider the case of parametric externalities.[31] Crucially, we assume that while the externality grows without bounds, the equilibrium decisions converge to finite limits. The latter is a natural assumption in many settings; for example, it holds when equilibrium decisions are monotone in k and the set of feasible decisions is bounded.

[31] We can relax the smoothness assumptions and require only that all utility functions and equilibrium decisions are continuous.

Proposition 5 *Consider the case of parametric externalities. Assume that agents are symmetric except for bargaining power, and* $1 > \beta > \gamma$. *Suppose that, as* $k \to \infty$, $u_B(b, c, k) \to \infty$ *whenever* $b > 0$ *and* $c > 0$, *but equilibrium decisions converge to finite limits. Then, there exists a* $\check{k} \in \mathbb{R}_+$ *such that* $U_A^{CB} > U_A^{BC}$ *for all* $k > \check{k}$.

The intuition for the result is that the anticipated-externality effect is stronger than the other two effects under the conditions of the proposition. Although the partial-surplus effect and the outside-option effect are present, their size is bounded because the decisions are bounded. Instead, the size of the anticipated-externality grows without bounds, which implies that the principal prefers sequence CB over BC if k is large enough.

In general, however, the principal's preference is not unambiguous for large positive externalities. We illustrate this with the help of the following simple example.

Assume that $1 > \beta > \gamma$. Agents are symmetric except for bargaining power, and $\mathcal{B} = \mathcal{C} = \mathbb{R}_+$. Moreover, all utility functions are additively separable with $u_A = -v(b) - v(c)$, $u_B = g(b) + kc$, and, by symmetry, $u_C = g(c) + kb$. The functions v and g are strictly increasing and differentiable, with $v(0) = g(0) = 0$, $g'(0) > v'(0)$, and $g''(b) \leq 0 < v''(b)$ for all b. Furthermore, $v'(b)$ is finite for all $b \in (0, \infty)$, with $\lim_{b\to\infty} v'(b) = \infty$.

In this example, the first-stage decisions go to infinity as k goes to infinity; hence, Proposition 5 does not apply. The next remark shows that the principal's preferred sequence for large positive externalities depends on the limit behavior of her cost function v.

Remark 3 *(i) If*

$$\lim_{x\to\infty} \frac{v'(x)}{v(x)} = 0, \tag{2}$$

there exists a $\bar{k} \in \mathbb{R}_+$ *such that* $U_A^{BC} > U_A^{CB}$ *for all* $k > \bar{k}$.
(ii) If

$$\lim_{x\to\infty} \frac{v'(x)}{v(x)} = \infty, \tag{3}$$

there exists a $\tilde{k} \in \mathbb{R}_+$ *such that* $U_A^{CB} > U_A^{BC}$ *for all* $k > \tilde{k}$.

Remark 3 shows that the principal's cost function is the important driver for the optimal sequence. In particular, this function determines the relative importance of the different effects. In case (i), the value of the cost grows faster than its derivative. This implies that the cost function has a stronger impact on A's payoff than the externalities in the agents' utility functions if k grows large. As a consequence, the outside-option effect grows faster and eventually dominates the anticipated-externality effect. Therefore, the principal strictly prefers sequence BC for large positive externalities. Note that case (i) applies, for example, whenever v is a polynomial function. In contrast, in case (ii), the cost function is highly convex. This slows down the growth of the first-stage decisions, and hence the growth of the outside-option effect, as k gets large. The outside-option effect then grows slower than the anticipated-externality effect, and the principal prefers CB for large positive externalities. Note that case (ii) applies, for example, when $v(b) = \exp(h(b)) - 1$, where h is a strictly increasing and strictly convex function. Finally, if neither (2) nor (3) holds, then the principal's cost function is not enough to pin down the principal's preferred sequence.

The result shows that, with large positive externalities, the optimal sequence depends on the details of the players' utility functions. This also holds if externalities have medium size. In fact, it is possible that the preferred sequence switches between CB and BC multiple times, as externalities increase. Such multiple switching points, however, do not occur in a simple specification of the supplier-retailers example, as we show next.

Example with a supplier and two retailers in Cournot competition. Consider the example outlined in Section 2, in which A is a supplier contracting with two retailers, B and C. Suppose that retailers compete in quantities and that their utility functions are $u_B(b,c) = (1-b+kc)b$ and $u_C(b,c) = (1-c+kb)c$, respectively, with $k \in [-1,1]$. Therefore, if $k=-1$, the retailers sell perfect substitutes, whereas if $k=1$, the two goods are perfect complements. If $k=0$, the profit functions are independent of each other (i.e., there are no externalities). The supplier's utility function is $u_A(b,c) = -y(b+c) - x(b+c)^2/2$, with $0 \le y \le 1$ to ensure that, in equilibrium, $b, c > 0$, and $x \ge 0$. This implies that the supplier's cost function has a linear and a convex term. For $x = 0$, u_A is additively separable.

After solving for the optimal quantities and the respective utilities in both sequences, we obtain that the utility of the principal in sequence BC is

$$U_A^{BC} = \frac{(1-\beta)(1-y)^2\left[\gamma x^2 + 2(2(1+k)-\gamma k)(2+x) + k^2\right]}{2(2+x)\left[\gamma x^2 + 2(2(1+k)-\gamma k)x + 4 - (3-\gamma)k^2\right]} + \frac{\beta(1-\gamma)(1-y)^2}{2(2+x)}, \tag{4}$$

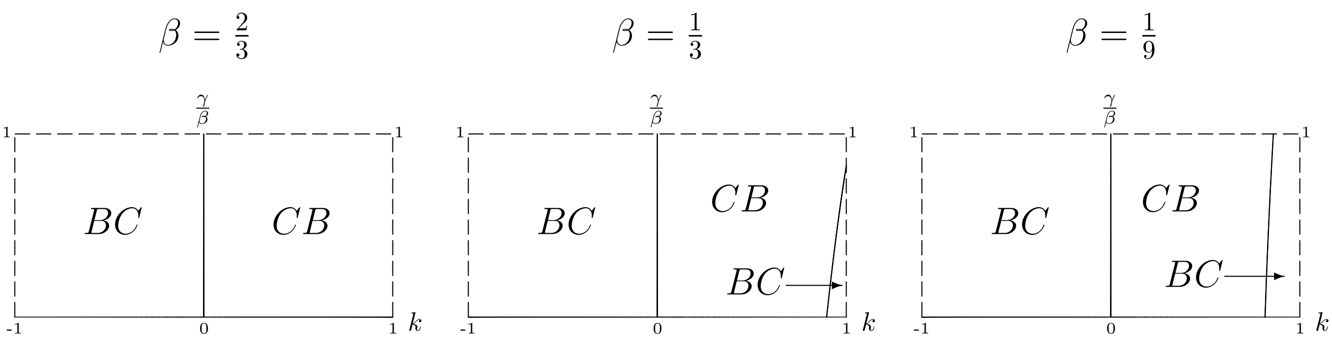
whereas her utility in sequence CB is

$$U_A^{CB} = \frac{(1-\gamma)(1-y)^2\left[\beta x^2 + 2(2(1+k)-\beta k)(2+x) + k^2\right]}{2(2+x)\left[\beta x^2 + 2(2(1+k)-\beta k)x + 4 - (3-\beta)k^2\right]} + \frac{\gamma(1-\beta)(1-y)^2}{2(2+x)}. \tag{5}$$

Comparing U_A^{BC} with U_A^{CB}, it is easy to check that for all $k \le 0$ and $x \ge 0$, the principal prefers sequence BC over CB, strictly so if k is strictly negative and/or x is strictly positive, following Propositions 2 and 3. For $k > 0$ and $x = 0$, the principal prefers sequence CB for k close to 0 (as stated by Proposition 4). Instead, for $k = 1$, we obtain that $\operatorname{sign}\left\{U_A^{BC} - U_A^{CB}\right\} = \operatorname{sign}\left\{9(1+\beta\gamma) - 16(\beta+\gamma)\right\}$, which implies that the principal prefers sequence BC if $\beta \le (9-16\gamma)/(16-9\gamma)$. Because $\beta \ge \gamma$, this inequality can be fulfilled only if $\gamma \le \left(16 - 5\sqrt{7}\right)/9 \approx 0.308$. If the inequality holds, there is a unique threshold value for k

between 0 and 1, such that the principal prefers sequence CB for k below this threshold and sequence BC for k above this threshold. Instead, if the inequality does not hold, sequence CB is optimal for all $k \in (0, 1]$. These results confirm, first, that either sequence can be optimal if externalities are positive, and, second, that in this example, the optimality of the sequences switches at most once with positive externalities.

Figure 1 displays the different equilibrium regions for $x = 0$ and three values of β (i.e., $\beta = 2/3$, $\beta = 1/3$, and $\beta = 1/9$). It is evident from the figure that the range in which sequence BC is optimal for positive externalities is larger if β is lower. Indeed, the partial-surplus and the outside-option effect, which both favor sequence BC, are the more substantial, the lower the agents' bargaining powers.



Each diagram shows the equilibrium timing for the respective value of β.
In all diagrams, the horizontal axis displays $k \in [-1, 1]$ and the vertical axis $\gamma/\beta \in [0, 1]$.

Figure 1: Equilibrium timing in the supplier-retailers example

We conclude this section by summarizing three main insights that result from our analysis in this and the previous section: (i) The surplus-maximizing sequence is the one in which the principal bargains with the stronger agent first, *regardless* of the externalities, whereas the privately-optimal sequence depends on the externalities. (ii) If externalities are negative, the sequence chosen by the principal is *efficient* whereas the equilibrium sequence may be *inefficient* if externalities are positive. (iii) The effect of the externalities on the equilibrium sequence can be *non-monotonic:* as externalities change from negative to positive, the optimal sequence may switch from BC to CB, but can switch back to BC for stronger positive externalities.

5 Simultaneous Negotiations

We so far focused on the optimal timing of sequential negotiations. Under some circumstances, simultaneous bilateral negotiations with the two agents are also possible. This occurs, if, for example, the principal can outsource the negotiations to delegates (or representatives) who act on her behalf. In this section, we analyze whether the principal may prefer such simultaneous to sequential negotiations. We extend our game by allowing the principal, in stage 0, to choose between simultaneous and sequential negotiations.

In simultaneous negotiations, the two delegates bargain independently of each other, which implies that they do not observe the outcome in the other negotiation. The same holds for the agents.[32] The optimal decisions in each negotiation therefore depend on the beliefs about the agreed outcome in the other negotiation. Because beliefs are arbitrary in a (weak) Perfect Bayesian Equilibrium in case of out-of-equilibrium offers, multiple equilibria exist. We follow the literature (e.g., Horn and Wolinsky, 1988; McAfee and Schwartz, 1994; Segal, 1999; Marshall and Merlo, 2004; Nocke and Rey, 2018) and focus on "passive beliefs": even after an out-of-equilibrium offer, a player believes that the pair of bargainers in the other negotiation play as on the equilibrium path. These beliefs are often referred to as Nash-in-Nash conjectures (Collard-Wexler et al., 2019); they are the most reasonable ones in our case because the two delegates act independently.

With passive beliefs, in the negotiation between A and B, the solution b^* is given by

$$b^*(c) := \arg\max_{b\in\mathcal{B}} \{u_A(b,c) + u_B(b,c)\},$$

where c is the belief about the outcome in the other negotiation.[33] As in case of sequential negotiations, we assume that b^* is unique for any c. Similarly, in the negotiation between A and C, c^* is given by

$$c^*(b) := \arg\max_{c\in\mathcal{B}} \{u_A(b,c) + u_C(b,c)\},$$

where b is the belief about the outcome in the other negotiation. In equilibrium, expectations are correct; the equilibrium values $b^\star$ and $c^\star$ are jointly determined by the two equations above.

Turning to the transfers, if A's delegate is drawn as the proposer in the negotiation with B, she sets $t_B = u_B(b^\star, c^\star)$. Similarly, in the negotiation with C, she sets $t_C = u_C(b^\star, c^\star)$. By contrast, if B is selected as the proposer in the negotiation with A, he offers $t_B = -u_A(b^\star, c^\star) + u_A(0, c^\star)$. This occurs because the principal (or her delegate) obtains $u_A(0, c^\star)$ when rejecting B's contract. By the same argument, if C is selected as the proposer in the negotiation with A, he sets $t_C = -u_A(b^\star, c^\star) + u_A(b^\star, 0)$.

The expected payoff of the principal with simultaneous negotiations can then be written

[32]In the next section, we analyze the case of sequential negotiations in which *only* the agent in the second negotiation does not observe the outcome in the first negotiation, but the principal does. The principal can then choose whether or not to disclose the first-stage decision.

[33]Note that passive beliefs imply that c is independent of b.

as

$$U_A^{sim} = (1-\beta)(1-\gamma)\left\{u_A(b^\star, c^\star) + u_B(b^\star, c^\star) + u_C(b^\star, c^\star)\right\} + (1-\beta)\gamma\left\{u_A(b^\star, 0) + u_B(b^\star, c^\star)\right\}$$

$$+\beta(1-\gamma)\left\{u_A(0, c^\star) + u_C(b^\star, c^\star)\right\} + \beta\gamma\ \left\{u_A(b^\star, 0) + u_A(0, c^\star) - u_A(b^\star, c^\star)\right\}. \qquad (6)$$

We can now compare the principal's payoff in the simultaneous negotiations with the one in the sequential timing. As above, we start with the case of negative externalities. We focus on sequence BC because we know from Proposition 2 that it dominates CB in case of negative externalities.

Proposition 6 *Suppose externalities are negative and u_A is super-modular. Then, $U_A^{BC} \geq U_A^{sim}$, with strict inequality if externalities are strictly negative or u_A is strictly super-modular, and equilibrium decisions are not zero.*

The intuition behind this result is driven by three effects. The first one is related to the intuition given in the section on the surplus-maximizing sequence. In the sequential timing BC, the two bargainers take the utility of agent C partially into account because the principal receives a share of it. By contrast, in simultaneous negotiations, the delegate of the principal and agent B do not consider the utility of agent C when negotiating with respect to b because a change in b will not affect the outcome in the negotiation between the principal and agent C. As a consequence, the decision made by A and B leads to a smaller overall cake in the simultaneous negotiations.

The second effect, which is inherent in the simultaneous timing, is rooted in the fact that the bargainers in each negotiation cannot observe the outcome of the other negotiation (because negotiations take place simultaneously). In particular, agent C cannot observe if an agreement was reached between A and B. She supposes—correctly so on the equilibrium path—that the decision in the other negotiation was $b^\star > 0$. In the sequence BC, agent C instead observes whether there was an agreement in the negotiation between A and B. This difference affects the expected transfer that A obtains: in case A and B failed to reach an agreement, the principal can demand a transfer from C that equals C's utility given that $b = b^\star$ in the simultaneous negotiations, whereas in the sequence BC she can demand a transfer from C that equals C's utility given that $b = 0$. With negative externalities, the latter is larger than the former, thereby favoring the sequential negotiation.

The third effect, which is also inherent in the simultaneous bargaining, is that it now matters how b and c interact in u_A (i.e., whether u_A is super-modular or sub-modular). This can be seen from the last term of (6): with probability $\beta\gamma$—i.e., the probability with which both agents are selected as the proposers—A's payoff is $u_A(b^\star, 0) + u_A(0, c^\star) - u_A(b^\star, c^\star)$.

This occurs because in the negotiation with, say, agent B, the principal's outside option is to reject B's offer and obtain a payoff of $u_A(0, c^\star)$. Therefore, the agent will claim a payment from the principal equal to $u_A(b^\star, c^\star) - u_A(0, c^\star)$. The same reasoning holds for agent C. It is evident that if u_A is super-modular, A's payoff is negative, which works again in favor of the sequential negotiations. For the same reason, BC is not necessarily optimal for A if u_A is sub-modular.[34]

We now turn to the case without externalities. Focussing on sequence BC (since it dominates sequence CB by Proposition 3), we obtain the following proposition:

Proposition 7 *Suppose there are no externalities. (i) If u_A is super-modular, then $U_A^{BC} \geq U_A^{sim}$. (ii) If u_A is sub-modular, then there exists a threshold $\hat{\gamma} \geq 0$ such that $U_A^{BC} > U_A^{sim}$ if $\gamma \leq \hat{\gamma}$. Moreover, there exists a threshold $\tilde{\gamma} \leq 1$ such that $U_A^{sim} \geq U_A^{BC}$ if $\gamma \geq \tilde{\gamma}$.*

Without externalities but interaction of b and c in the principal's utility function, the first effect described after Proposition 6 is still present. This works in favor of sequence BC. In addition, the third effect is present as well, which favors sequence BC if u_A is super-modular, but simultaneous negotiations if u_A is sub-modular. Therefore, the principal prefers sequence BC if u_A is super-modular. Instead, of u_A is sub-modular, the result depends on the bargaining power of the agents. If the principal has a lot of bargaining power, (e.g., γ is relatively small), the event that both agents make the offer is unlikely. The first effect is then dominant, which implies that the principal prefers sequential negotiations. By contrast, if both agents have high bargaining power (i.e., γ is relatively large, which also implies that β is relatively large due to the fact that $\beta \geq \gamma$), the sub-modularity of u_A is the dominating effect, and the principal favors simultaneous negotiations.[35]

The result in case of sub-modularity can be illustrated with the help of the supplier-retailers example considered in the previous section. Recall that for all $x > 0$, u_A is sub-modular. If $k = 0$ (i.e., there are no externalities), the utility of the principal in sequence BC is

$$\frac{(1-y)^2\left[(1-\beta\gamma)(4+\gamma x^2)+4(2-\beta\gamma)\right]}{2(2+x)(4(1+x)+\gamma x^2)}. \tag{7}$$

[34] We note that Proposition 6 results from a comparison of sequential versus simultaneous negotiations, which neither relies on agents being symmetric nor on $\beta > \gamma$. Indeed, the same argument adopted to sequence CB also shows that $U^{CB} \geq U_A^{sim}$.

[35] As Proposition 6, Proposition 7 does not depend on agents being symmetric, and a similar result can be established for sequence CB. Moreover, all inequalities in Proposition 7 are strict if the super- or sub-modularity of u_A is strict, provided that equilibrium decisions are not zero and differ in the sequential versus the simultaneous timing.

In the simultaneous negotiations, the principal's utility is

$$\frac{(1-y)^2\left[4(1+x)-(\beta+\gamma)(2+x)\right]}{8(1+x)^2}. \tag{8}$$

Subtracting (8) from (7) and letting $\gamma \to 0$ yields $((1-y)^2\beta x^2)\,/\,(8(2+x)(1+x)^2)$, which is strictly positive. Hence, sequential negotiations are preferred over simultaneous ones. By contrast, if $\gamma \to 1$, which implies that also $\beta \to 1$, the difference between (7) and (8), becomes $-\,((1-y)^2 x)\,/\,(4(1+x)^2)$, which is strictly negative. It is easy to show that there is a unique threshold value of γ, such that simultaneous negotiations are preferred for γ above this threshold.

Finally, we turn to the case of positive externalities and derive a result for small externalities. We consider the case of parametric externalities, as in Proposition 4 above, and focus on sequence CB (since it dominates BC by Proposition 4).

Proposition 8 *Consider the case of parametric externalities. Suppose agents are symmetric except for bargaining power,* $1 > \beta > \gamma > 0$, *and* u_A *is additively separable. Then, there exists a* $\underline{k} > 0$ *such that* $U_A^{sim} > U_A^{CB}$ *for all* $k \in (0, \underline{k})$.

The intuition why simultaneous negotiations dominate sequential negotiations if externalities are positive, but small, follows from the logic of the second effect described after Proposition 6. In any sequential negotiation, the bargainers in the second stage know the outcome of the first stage. If there was no agreement reached in the first stage, the principal, when being selected as the proposer, can extract from the agent in the second stage an amount that equals her payoff, given that the decision in the first stage is 0. By contrast, in the simultaneous negotiations, an agent does not observe the outcome of the other bargaining game and supposes that an agreement was reached there. If externalities are positive, this implies that the principal can demand more from the agent. Although rejections do not happen on the equilibrium path, this effect increases the outside option of the principal.[36]

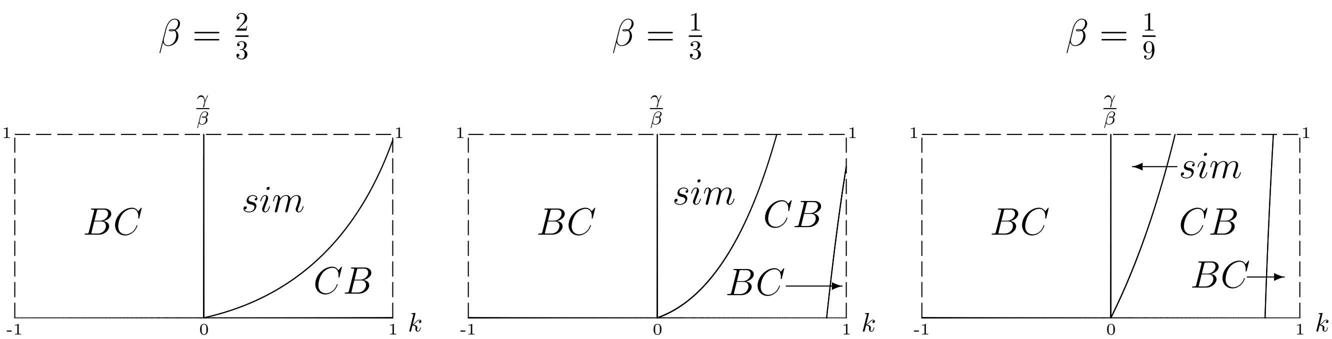
Finally, we can illustrate the result with positive externalities in our example with a supplier and two retailers. The utility of the principal with simultaneous negotiations is

$$\frac{(1-y)^2\left[4(1+x)-(\beta+\gamma)(2+x)\right]}{2(2(1+x)-k)^2}.$$

If $x = 0$, so that u_A is additively separable, the supplier-retailers example fulfills the requirements of Proposition 8. As illustrated in Figure 2, there exists a threshold $\underline{k}$, such that

[36]The first effect explained after Proposition 6 is only of second order if externalities are small and u_A is additively separable. The third effect is not present due to the additive separability of u_A.

$U_A^{sim} > U_A^{CB}$ if and only if $k < \underline{k}$. Moreover, $\underline{k}$ is always below the threshold value of k at which the sequence switches from CB to BC (in case such a switching point exists). Therefore, when externalities are positive, three different timings may be optimal for the principal, dependent on the level of k: if k is small, the principal chooses simultaneous negotiations, for intermediate values of k, she chooses the sequence CB, and for large values of k, she chooses the sequence BC.



Each diagram shows the equilibrium timing for the respective β, including simultaneous negotiations. In all diagrams, the horizontal axis displays $k \in [-1, 1]$ and the vertical axis $\gamma/\beta \in [0, 1]$.

Figure 2: Equilibrium timing with simultaneous negotiations

6 Extensions

In this section, we consider two extensions of our model. In Subsection 6.1, we analyze whether the principal has the incentive to disclose the contract of the first negotiation to the second bargainer. In Subsection 6.2, we consider the situation in which exclusivity contracts are possible, that is, the bargainers can negotiate an exclusive-dealing contract in which the principal commits not to contract with the the other agent. To keep the section concise, we focus on our main question whether the principal prefers the sequence BC or CB, restrict attention to the case in which agents are symmetric except for bargaining power, and assume $1 > \beta > \gamma > 0$.

6.1 Disclosure

In our main model, we analyze the situation in which the contract signed in the first negotiation (if any) is known to the agent with whom the principal bargains second. An important question is therefore if the principal indeed has the incentive to disclose the first-stage contract in case she also has the option to keep the contract secret.

In order to explore this question, we augment our main model by an intermediate stage (e.g., stage 1.5), in which A decides whether or not to disclose the contract signed in the first stage before the second-stage negotiation. In case of disclosure, the game proceeds as in the main model. In case of non-disclosure, the agent with whom A bargains in the second stage forms a belief about the first-stage decision; bargaining in the second stage then takes place as in the main model. The solution concept is (weak) Perfect Bayesian Equilibrium (PBE).

Without loss of generality, we consider the sequence BC when analyzing the principal's disclosure decision. All our results regarding disclosure hold for sequence CB as well. We focus on the case in which the set of decisions $\mathcal{B}$ is compact and the bargaining problems are smooth in the sense defined above (i.e., utility functions are differentiable, and in the game where the first-stage contract is observed by the second agent, equilibrium decisions are unique, interior, and differentiable). We also restrict attention to pure strategies.

We start with the case in which u_A is additively separable.

Proposition 9 *If u_A is additively separable, there exists a PBE in which A discloses the first-stage contract. If, in addition, u_B is strictly increasing in c whenever $b > 0$, and u_B is super-modular, there is no non-disclosure PBE.*

The first part of the proposition shows that, if the principal's utility is additively separable, a disclosure equilibrium always exists, regardless of the externalities. The intuition behind this result is relatively simple: As non-disclosure does not occur on the equilibrium path, C's belief is not restricted in a PBE. If C interprets non-disclosure as a 'bad' signal and believes that the first-stage decision was the one that minimizes the joint surplus of A and C in the second stage, A's optimal strategy is to disclose. As a consequence, all our results on the optimality of the different sequences also hold in the extended situation in which the principal endogenously decides whether or not to disclose the contract.

The second part of the proposition provides sufficient conditions for the disclosure equilibrium to be the unique equilibrium. These conditions are strictly monotone positive externalities, and super-modularity of the agents' utility functions. Because of the positive externalities, A prefers to disclose a first-stage decision that is sufficiently high. This implies that there exists an upper bound for decisions that A prefers not to disclose. As a consequence, if a non-disclose equilibrium existed, it can only occur for sufficiently low first-stage decisions. However, super-modularity of u_B then ensures that the optimal first-stage decision in case of disclosure is always above this threshold, which implies that disclosure is optimal.

It is important to note that the conditions of the second part of the proposition are only sufficient but not necessary. To illustrate this, we use the supplier-retailers example given above, with $x = 0$, as the principal's utility function is then additively separable. Strictly monotone positive externalities and super-modularity of u_B are satisfied if $k > 0$. Instead, if $k < 0$, the second part of Proposition 9 does not apply. The disclosure equilibrium is still the unique equilibrium, however, if $k > -2(1-\gamma)$, that is, if the sub-modularity of u_B is not particularly strong.[37] As k is bounded below by -1, for $\gamma < 1/2$, the disclosure equilibrium is the unique equilibrium in the entire parameter range with $k \neq 0$. By contrast, if $k < -2(1-\gamma)$, there exist multiple non-disclosure equilibria in addition to the disclosure equilibrium.

In Proposition 9, the negotiations are interdependent because of externalities between agents, but u_A is additively separable. We now turn to the opposite case in which interaction between b and c occurs only in A's utility function. We then obtain the following result:

Proposition 10 *Suppose there are no externalities. If u_A is super-modular or sub-modular, there exists a PBE in which A discloses the first-stage contract. Moreover, if u_A is strictly super-modular or strictly sub-modular, there is no non-disclosure PBE.*

In case of no externalities, the disclosure decision is only important for the case in which C is the proposer in the second stage (recall that sequence BC is optimal without externalities). If A makes an offer, C's decision whether to accept or reject does not depend on b, which implies that disclosure plays no role. Instead, if C proposes, he will base the offer on his expectation about b in case of non-disclosure, but on the true value of b in case of disclosure. If the optimal c does not depend on the first-stage decision, A is indifferent between disclosing or not. However, if the optimal c depends on b, which holds true if u_A is strictly super-modular or strictly sub-modular, A prefers to disclose, which implies that disclosure is again the unique equilibrium.

In summary, our discussion shows that disclosure is the optimal strategy of the principal under several natural circumstances. The results of our basic model then carry over to this extended game.

6.2 Exclusive Contracts

We so far assumed that contracts between the principal and an agent cannot condition on the outcome that the principal reaches in the negotiation with the other agent. This assumption is reasonable in many real-world situations—e.g., such conditional contracts between firms are often illegal by antitrust law or difficult and costly to write, as they require the specification of several different contingencies (Dequiedt and Martimort, 2015).

[37] The derivation of this result is shown in the Appendix. In case $k = 0$, the bargaining problems do not interact, and disclosure and non-disclosure equilibria are equivalent.

An exception are, however, exclusive dealing contracts, which, in some settings, are both legal and feasible. In this section, we therefore consider the robustness of our results to exclusive contracts.

There are two ways to study exclusive contracts. The first is the single-contract scenario, which is considered by e.g. Rasmusen et al. (1991), Fumagalli and Motta (2006), and Marx and Shaffer (2007) in different environments. In this case, the two bargainers can either sign a contract that leaves the principal free to trade with the other agent (as in our main model), or an exclusive-dealing contract that commits the principal not to trade with the other agent. The second scenario is a menu of contracts, in which the proposer in the negotiation offers two contracts, one for the case in which the principal also trades with the other agent, and one for the case of exclusive dealing. This scenario is considered by e.g. Segal and Whinston (2000) or Rey and Whinston (2013).[38] We consider the two scenarios in turn.

Single contract. In the single-contract scenario, we extend the contract space of the main model in the following way: in the negotiation in the first stage, the two bargainers can either sign an exclusive contract or a non-exclusive contract. The latter contract is the same as in our main model above and specifies a decision and a transfer, without restricting the contract that the principal may sign with the other agent. The exclusive contract also specifies a decision and a transfer, but restricts the principal to not negotiate with the second agent.[39]

Analyzing this scenario, we obtain the following result:

Proposition 11 *If the contract space is extended to the scenario in which the bargainers can either sign an exclusive or a non-exclusive contract, the sequences given in Propositions 1-5 are still optimal.*

The proposition shows that the possibility to negotiate an exclusive dealing contract does not affect our main insights. In what follows, we explain the intuition behind this result. Without an exclusive contract, the principal is free to negotiate with the second agent. Her expected payoff must therefore be (weakly) higher than the payoff without reaching an agreement in the second stage because she will reject any offer that gives her a lower payoff. Consider now the case without externalities. In this case, the payoff of the first agent is not affected by the second-stage decision. Therefore, the joint surplus of the principal and the first agent is (weakly) higher without an exclusive contract. With positive

[38]The second scenario can also be interpreted as one in which the contract specifies a penalty for contract breach from exclusivity; see Fumagalli et al. (2018).

[39]Such a contract is often called a 'naked-exclusion' contract.

externalities between agents, excluding the second agent has the additional disadvantage that this externality on the first agent is not realized. As a consequence, with positive and no externalities, exclusion does not occur in equilibrium.

By contrast, with negative externalities, exclusive contracts may increase the joint surplus of the principal and the first agent, as the principal does not consider the negative externality on the first agent when bargaining with the second agent. Without exclusion, however, the principal benefits from bargaining with the second agent. These benefits are the larger the lower the bargaining power of the second agent. Therefore, if exclusion is used in the sequence BC (where the second agent is weak), it will also be used in CB (where the second agent is strong), but not vice versa.

The principal's expected payoff from an exclusive contract is, however, independent of the sequence of negotiations. The reason is that, if an exclusive contract maximizes the joint surplus of those who bargain in stage 1, each player proposes the exclusive contract in each negotiation, which implies that the principal obtains the respective payoff with probability $1-\beta\gamma$.[40] Therefore, if an exclusive contract is optimal in both sequences, A is indifferent between the two sequences. By the argument above, exclusion is, however, more profitable in sequence CB than in sequence BC. It follows that if exclusion maximizes the joint surplus of the first-stage negotiation in sequence CB but not in sequence BC, the joint surplus of A and the first agent must be higher in the latter sequence. Therefore, A prefers sequence BC and no exclusive contract.

These considerations show that our key results on A's preferences over the negotiation sequences also hold when exclusive contracts are feasible. A minor caveat is that, with negative externalities, the principal's preference for sequence BC is not strict if there is exclusion in both sequences, while she typically has a strict preference for BC if exclusive contracts are not feasible (see Proposition 2).

Menu of contracts We now turn to the scenario in which the proposer in each negotiation can offer a menu of contracts. This menu consists of two contracts, (d_i^e, t_i^e) and (d_i, t_i), where the exclusive contract (d_i^e, t_i^e) is executed if the decision reached with the other agent $j \neq i$ is zero ($d_j = 0$), and (d_i, t_i) becomes relevant otherwise. We focus on the case in which, without exclusive contracts, the second-stage decision is not zero, after any first stage-decision. In this scenario, there is an optimal sequence independent of the externalities.

[40] Formally, in timing BC, A's payoff is $\max_b \{u_A(b,0) + u_B(b,0)\}$ if she offers an exlusive deal in stage 1. If B proposes an exclusive deal, A's payoff is her outside option $(1-\gamma)\max_c \{u_A(0,c) + u_C(0,c)\}$. In expectation, A therefore obtains $(1-\beta\gamma)\max_b \{u_A(b,0) + u_B(b,0)\}$, which depends only on the product $\beta\gamma$, and is equal to A's expected payoff in timing CB.

Proposition 12 *If the contract space is extended to a menu of contracts, the principal prefers the sequence CB.*

With a menu of contracts, the exclusive-dealing contract negotiated with the first agent matters in the second-stage bargaining, despite the fact that the respective transfer and decision may not be implemented on the equilibrium path. The reason is that this contract determines the principal's outside option against the second bargainer. By choosing this contract in an appropriate way, the surplus of the second agent can be fully extracted, independent of his bargaining power.[41] This implies that the principal and the first agent fully internalize the effect of their decision on u_A and on the utility of the second agent. As a consequence, both sequences lead to the same joint surplus of all three players. Because the second agent obtains no rent regardless of the sequence and the principal receives a higher share of the joint surplus if she bargains with the weaker agent first, her optimal sequence is CB.

7 Generalizations

In this section, we consider two generalizations of our baseline model. In Subsection 7.1, we analyze the case with more than 2 agents, and in Subsection 7.2, we allow for externalities on non-traders (i.e., agents with whom the principal did not reach an agreement). As in the last section, we focus on the principal's preferred sequence and suppose that agents are symmetric except for bargaining power.

7.1 N Agents

In this section, we show that our insights are robust to the case with a larger number of agents. Suppose that there are $N \geq 2$ agents. We assume that all agents differ in their bargaining power. As the main model, the principal decides about the sequence of the negotiations (i.e., with which agent to bargain first, second, etc.). To simplify the exposition, we consider the case in which the principal commits to a sequence before the negotiations start.[42] Moreover, we assume that each subgame has a unique equilibrium, after breaking ties, such that a player accepts when indifferent between accepting and rejecting an offer.

[41] This type of rent extraction via a menu of contracts is similar to the one shown by Aghion and Bolton (1987) and Segal and Whinston (2000).

[42] All our results extend to the case without commitment (i.e., the principal chooses with which agent to bargain after each round of negotiations): then there exists an equilibrium where the principal approaches the agents in the order given in the Propositions 13 and 14. The proof is available from the authors.

The analysis with N agents is considerably more involved than with two agents, because changing the sequence in any two time periods may affect the decisions taken in other stages of the game. We first show that if there are no externalities between agents, but the bargaining decisions interact in A's utility function, the additional complication is still tractable in a general framework.

Proposition 13 *Suppose that there are no externalities, but decisions interact in $u_A(\cdot)$. The principal then optimally negotiates with agents in decreasing order of their bargaining power.*

The result shows that the insight derived in the case with two agents (Proposition 3) carries over to the case with N agents. As in the case with two agents, the intuition is based on the partial-surplus and the outside option effect, which both make it favorable for A to bargain with weaker agents later. Although the intuition is based on the same effects as in Section 3, the proof is more complicated. We show the result by an "adjacent pairwise interchange" argument (see, for example Baker and Trietsch, 2009): this involves—starting from an arbitrary bargaining sequence—to check whether A prefers to change the position of agents in two consecutive time periods. To prove the proposition, we present an algorithm that sequentially assigns decisions and transfers to all agents, such that A has the same payoff from the algorithm as in the equilibrium of the bargaining game. Our algorithm uses only the equilibrium decisions, but not the equilibrium transfers, as an input, which makes it comparatively simple and tractable and may make the method useful for the analysis of other dynamic bargaining games as well.

We next turn to the case with externalties between agents. To avoid the complication that changes in the bargaining sequence affect decisions in other stages of the game, we consider the case of binary decisions, following the example presented in Section 3. With binary decisions, $d_i \in \mathcal{D} = \{0, 1\}$ for all $i = 1, ..., N$, and participation of all agents is always optimal. The latter implies that a change in the sequence does not change equilibrium decisions. As explained above, this case allows us to bring out the role of externalities on the bargaining sequence in the clearest way, as the optimal sequence is then only driven by the direction of the externalities. As the next proposition shows, we obtain a clear-cut result:

Proposition 14 *Consider the case with binary decisions, in which participation by all agents is optimal. With negative externalities, A optimally negotiates with agents in decreasing order of their bargaining power. By contrast, with positive externalities, A optimally negotiates with agents in increasing order of their bargaining power.*

The result shows again that one of the main insights of the case with two agents carries over to the case with a larger number of agents: negative externalities imply that the principal prefers to bargain with weaker agents later, whereas with positive externalities, the opposite holds true. As in the two-agent case, the result is driven by the anticipated-externality effect.

7.2 Externalities on Non-Traders

We so far assumed that there are no externalities on non-traders, that is, $u_B(0, c)$ is constant in c, and likewise $u_C(b, 0)$ is constant in b. This is a realistic assumption in most of the applications discussed above. For instance, in the supplier-retailers example, if a retailer does not reach an agreement with the supplier, he obtains a profit of zero in this market. In some cases, however, there are externalities on non-traders, so that the payoff of an agent who does not reach an agreement with the principal depends on the outcome of the negotiation between the principal and the other agent. Indeed, externalities on non-traders have thoroughly been explored in the literature (see, for example, Jehiel and Moldovanu 1995a, 1995b, Segal 1999, and Möller 2007).

In this section, we include externalities on non-traders in our model, focussing on the case of two agents who differ only in bargaining power. We show that, while externalities on non-traders may qualify results, the main insights are robust to such externalities as long as they are—in a well-defined sense—weaker than those on the traders.

To make precise how the externalities on non-traders must be weaker than those on traders, denote by $f(\cdot)$ the optimal second-stage decision, that is, $f(\cdot)$ is given by—exemplified with sequence BC—$f(b) := \arg\max_c \{u_A(b, c) + u_C(b, c)\}$. Using this definition, we obtain the following result, which generalizes Proposition 2:

Proposition 15 *If*

$$u_B(b, 0) - u_B(b, f(b)) \geq u_B(0, 0) - u_B(0, f(0)) \tag{9}$$

for all b, then $U_A^{BC} \geq U_A^{CB}$.

Without externalities on non-traders, $u_B(0, 0) = u_B(0, f(0))$ and (9) reduces to $u_B(b, 0) \geq u_B(b, f(b))$, which is true when there are negative externalities between agents; hence, Proposition 15 provides a generalization of Proposition 2. In addition, it shows that A prefers BC if externalities on traders and non-traders are both negative, but traders are affected more in the following sense: the second-stage decision $f(b)$ has a more negative impact on B's utility when reaching an agreement than the second-stage decision $f(0)$ has on B's utility as a non-trader.[43]

[43] Proposition 15 also implies that, if externalities on traders are negative but those on non-traders are absent or positive, A prefers BC.

Proposition 15 also generalizes Proposition 3 by showing that A prefers BC if there are no externalities on traders but positive externalities on non-traders. Therefore, our analysis demonstrates that the main insights derived in Section 3 for negative and no externalities are robust with respect to externalities on non-traders. We focus here on these two cases as the optimal sequence in case of positive externalities can go either way even without externalities on non-traders.

To conclude this section, we consider the case of binary decisions where participation is always optimal. Then $f(0) = f(1) = b^{BC} = c^{CB} = 1$. It is straightforward to show that in this case

$$U_A^{BC} - U_A^{CB} = (\beta - \gamma)(u_B(0,1) - u_B(0,0) - (u_B(1,1) - u_B(1,0))). \tag{10}$$

In (10), $u_B(1,1) - u_B(1,0)$ measures the externality on a trader, whereas $u_B(0,1) - u_B(0,0)$ measures the externality on a nontrader. If externalities are negative and stronger on traders than on non-traders—i.e., $u_B(1,1) - u_B(1,0) < u_B(0,1) - u_B(0,0) < 0$—then A prefers sequence BC. Conversely, if externalities are positive (both on traders and on non-traders), but they are stronger on traders, A prefers CB.

It is evident from (10) that A's preference over sequences depends on whether u_B is sub-modular or super-modular.[44] We therefore obtain the following proposition:

Proposition 16 *Suppose decisions are binary and participation of all agents is optimal. If u_B is strictly sub-modular, A strictly prefers sequence BC over CB. Instead, if u_B is strictly super-modular, A strictly prefers sequence CB over BC.*

8 Conclusion

The situation in which a principal bargains with multiple agents bilaterally, and the decision in each negotiation has an effect on the payoff of other agents, is prevalent in many economic environments. The principal then has the choice how to sequence the negotiations with the agents. This paper has shown that the difference in the agents' bargaining power is crucial for the optimal sequence, and therefore the efficiency of decisions.

To understand the driving forces behind the sequencing decision, we considered a general framework with one principal and two agents. We show that the optimal sequence is driven by the interplay of three effects: the *partial-surplus effect*, the *anticipated-externality effect*, and

[44] The inequality in (9) also has a flavor of sub-modularity, but differs because the second-stage decision in general depends on whether the first agent participates. If, however, the second-stage decision is independent of the first-stage decision (i.e., $f(b) = f(0)$), then sub-modularity of u_B implies (9); hence A prefers BC.

the *outside-option effect*. Because of the partial-surplus effect, the sequence that generates the highest joint surplus is the one on which the principal bargains with the strong agent first, independent of whether externalities are positive or negative. By contrast, the sequence chosen by the principal depends on the externalities.

If externalities are negative, all three effects point in the same direction and favor the efficient sequence. Instead, with positive externalities, we identify conditions under which the equilibrium timing is to bargain with the weak agent first. This is the case if, for instance, externalities are small and the principal's utility function is additively separable. The anticipated-externality effect then dominates the other two effects, which induces the principal to choose the sequence that does not maximize the joint surplus. The equilibrium sequence can thus be inefficient, but only if externalities are positive. In addition, we also compare sequential with simultaneous bargaining. We show that simultaneous negotiations can be optimal if externalities are positive.

In our study, we have focused on the role of bargaining power, as we think it is an important, yet understudied, topic in the context of contracting with externalities. To bring out the effects of the bargaining power in a clear way, we have derived our main results for the case in which agents are symmetric except for bargaining power.[45] As our framework is relatively general, it lends itself naturally to explore the effects of other dimensions on optimal negotiations. For example, agents may differ in their contribution to the total surplus instead of the bargaining power. Also, agents may be asymmetric in the externalities they exert on each other. These differences can affect the surplus-maximizing sequence and the one chosen by the principal. In particular, asymmetries in those other dimensions may bring in new effects that could strengthen or qualify the effects shown in this paper. We think this is an interesting avenue for future research.

[45]Without the assumption of symmetry, we have derived some results for limiting cases of bargaining power. In particular, if one agent has all the bargaining power, the principal will negotiate with this agent first if externalities are negative, but with the weaker agent first if externalities are positive.

A Appendix

A.1 Surplus-maximizing Sequence

A.1.1 Proof of Remark 1

Proof. Consider sequence BC. In the second stage, the decision reached is

$$\begin{aligned} c^*(b) &= \arg\max_{c\in\mathcal{C}} \{u_A(b,c) + u_C(b,c)\} \\ &= \arg\max_{c\in\mathcal{C}} \{u_A(b,c) + u_B(b) + u_C(b,c)\} \\ &= \arg\max_{c\in\mathcal{C}} S(b,c), \end{aligned}$$

where the second equality is due to the fact that u_B is independent of c, and b is predetermined from the first stage; thus, the term $u_B(b)$ can be treated as a constant in the maximization problem, and adding it does therefore not change the location of the maximum. In the first stage, the decision maximizes the joint surplus $S_{AB}^{BC}(b)$ of A and B. Since $\gamma = 0$, $S_{AB}^{BC}(b) = S(b, c^*(b))$. Therefore, $S^{BC} = \max_{b\in\mathcal{B}} S(b, c^*(b)) = S^{FB} \geq S^{CB}$. ■

A.1.2 Proof of Proposition 1

The proof of the proposition uses the following lemma:

Lemma 1 *Suppose that $w : \mathcal{B} \to \mathbb{R}$ and $v : \mathcal{B} \to \mathbb{R}$ are functions, $0 \leq \gamma_0 < \gamma_1 \leq 1$ and*

$$b_i \in \arg\max_{b\in\mathcal{B}} \{(1-\gamma_i) w(b) + \gamma_i v(b)\} \text{ for } i = 0, 1.$$

Then $w(b_1) \leq w(b_0)$.

Proof. If $\gamma_0 = 0$, $b_0 \in \arg\max_{b\in\mathcal{B}} w(b)$, hence $w(b_0) \geq w(b_1)$. The rest of the proof considers the case $\gamma_0 > 0$. Towards a contradiction, suppose that $w(b_1) > w(b_0)$. By assumption, $(1-\gamma_0) w(b_0) + \gamma_0 v(b_0) \geq (1-\gamma_0) w(b_1) + \gamma_0 v(b_1)$, or equivalently,

$$(1-\gamma_0)(w(b_0) - w(b_1)) \geq \gamma_0 (v(b_1) - v(b_0)). \tag{11}$$

Since $w(b_1) > w(b_0)$ and $1 \geq \gamma_1 > \gamma_0 > 0$, the left side of inequality (11) is strictly negative; hence, $v(b_1) < v(b_0)$.

Similarly, $(1-\gamma_1) w(b_1) + \gamma_1 v(b_1) \geq (1-\gamma_1) w(b_0) + \gamma_1 v(b_0)$, or equivalently

$$-(1-\gamma_1)(w(b_0) - w(b_1)) \geq -\gamma_1 (v(b_1) - v(b_0)). \tag{12}$$

Adding (12) to (11) shows that

$$(\gamma_1 - \gamma_0)(w(b_0) - w(b_1)) \geq (\gamma_0 - \gamma_1)(v(b_1) - v(b_0)).$$

This is a contradiction since the left-hand side is strictly negative and the right-hand side strictly positive. ■

Proof of Proposition 1. We first show that S^{BC} is decreasing in γ and constant in β. It is evident from equation (1) that the equilibrium decisions $\left(b^{BC}, c^*\left(b^{BC}\right)\right)$ do not depend on β. Therefore, $S^{BC} = S\left(b^{BC}, c^*\left(b^{BC}\right)\right)$ is constant in β. Moreover, $b^{BC} \in \arg\max_{b\in\mathcal{B}} S_{AB}^{BC}(b)$, where

$$\begin{aligned} S_{AB}^{BC}(b) &= u_B(b, c^*(b)) + (1-\gamma)(u_A(b, c^*(b)) + u_C(b, c^*(b))) + \gamma u_A(b, 0) \\ &= (1-\gamma) S(b, c^*(b)) + \gamma [u_A(b, 0) + u_B(b, c^*(b))]. \end{aligned}$$

Applying Lemma 1 with $w(b) = S(b, c^*(b))$ and $v(b) = u_A(b, 0) + u_B(b, c^*(b))$ shows that $S\left(b^{BC}, c^*\left(b^{BC}\right)\right)$ is decreasing in γ. In a similar way, we can establish that S^{CB} is decreasing in β and constant in γ.

Suppose now agents are symmetric. If $\beta = \gamma$, sequences BC and CB differ only in the names of the agents. Since equilibrium surplus is unique, $S^{BC} = S^{CB}$. Because S^{BC} is constant in β and decreasing in γ, whereas S^{CB} is decreasing in β and constant in γ, it follows that, for $\beta > \gamma$, $S^{BC} \geq S^{CB}$. ■

A.2 Sequence Preferred by the Principal

A.2.1 Proof of Remark 2

Proof. Since $\beta = 1$, $U_A^{BC} = O_A^{BC} = (1-\gamma)\max_{c\in\mathcal{C}}\{u_A(0, c) + u_C(0, c)\}$. In contrast, in sequence CB, $U_A^{CB} = (1-\gamma)\max_{c\in\mathcal{C}}\{u_A(0, c) + u_C(b^*(c), c)\}$, where

$$b^*(c) = \arg\max_{b\in\mathcal{B}}\{u_A(b, c) + u_B(b, c)\}.$$

Therefore,

$$U_A^{BC} - U_A^{CB} = (1-\gamma)\left(\max_{c\in\mathcal{C}}\{(u_A(0, c) + u_C(0, c))\} - \max_{c\in\mathcal{C}}\{u_A(0, c) + u_C(b^*(c), c)\}\right).$$

When there are negative externalities of b on C, then $u_C(0,c) \geq u_C(b,c)$ for all b, c. Therefore, $U_A^{BC} \geq U_A^{CB}$.[46] The results on positive and no externalities can be established similarly. ■

A.2.2 Proof of Proposition 2

Proof. The symmetry of the agents has two implications that will be used in the proof. First,

$$\arg\max_{c\in\mathcal{C}} \{u_A(x,c) + u_C(x,c)\} = \arg\max_{b\in\mathcal{B}} \{u_A(b,x) + u_C(b,x)\} =: f(x) \tag{13}$$

for all $x \in \mathcal{B} = \mathcal{C}$. The function $f(x)$ defined in (13) gives the second-stage decision that ensues after a first-stage decision x; under symmetry, it is the same function in both sequences. Second, symmetry implies that $\max_{c\in\mathcal{C}} \{u_A(0,c) + u_C(0,c)\} = \max_{b\in\mathcal{B}} \{u_A(b,0) + u_B(b,0)\}$. Since the outside options of A in stage 1 are, respectively, $O_A^{BC} = (1-\gamma)\max_{c\in\mathcal{C}}\big\{u_A(0,c) + u_C(0,c)\big\}$ and $O_A^{CB} = (1-\beta)\max_{b\in\mathcal{B}} \{u_A(b,0) + u_B(b,0)\}$, it follows that symmetry implies that

$$\beta O_A^{BC} - \gamma O_A^{CB} = (\beta - \gamma) \max_{c\in\mathcal{C}} \{u_A(0,c) + u_C(0,c)\}. \tag{14}$$

The surplus of A and B in sequence BC as a function of b is

$$S_{AB}^{BC}(b) = (1-\gamma)(u_A(b, f(b)) + u_C(b, f(b))) + \gamma u_A(b,0) + u_B(b, f(b)).$$

In equilibrium of sequence BC, $b = b^{BC} \in \arg\max_{b\in\mathcal{B}} S_{AB}^{BC}(b)$. Similarly, the surplus of A and C in sequence CB as a function of c is

$$S_{AC}^{CB}(c) = (1-\beta)(u_A(f(c), c) + u_B(f(c), c)) + \beta u_A(0,c) + u_C(f(c), c).$$

In equilibrium of sequence CB, $c = c^{CB} \in \arg\max_{c\in\mathcal{C}} S_{AC}^{CB}(c)$. The expected payoffs of A in sequences BC and CB are, respectively, $U_A^{BC} = (1-\beta) S_{AB}^{BC}\left(b^{BC}\right) + \beta O_A^{BC}$ and $U_A^{CB} = (1-\gamma) S_{AC}^{CB}\left(c^{CB}\right) + \gamma O_A^{CB}$.

Since $b^{BC} \in \arg\max_{b\in\mathcal{B}} S_{AB}^{BC}(b)$,

$$S_{AB}^{BC}\left(b^{BC}\right) \geq S_{AB}^{BC}\left(c^{CB}\right). \tag{15}$$

Moreover, by symmetry,

$$S_{AB}^{BC}\left(c^{CB}\right) = (1-\gamma)\left(u_A\left(f\left(c^{CB}\right), c^{CB}\right) + u_B\left(f\left(c^{CB}\right), c^{CB}\right)\right) + \gamma u_A\left(0, c^{CB}\right) + u_C\left(f\left(c^{CB}\right), c^{CB}\right)$$

[46] Moreover, when externalities are strictly negative and $c^{CB} \neq 0 \neq b^*(c)$, then $U_A^{BC} > U_A^{CB}$.

and therefore

$$(1-\beta)\,S_{AB}^{BC}\left(c^{CB}\right)-(1-\gamma)\,S_{AC}^{CB}\left(c^{CB}\right)=(\gamma-\beta)\left(u_A\left(0,c^{CB}\right)+u_C\left(f\left(c^{CB}\right),c^{CB}\right)\right). \quad (16)$$

From (14), (15), and (16),

$$\begin{aligned}
& U_A^{BC}-U_A^{CB} && (17)\\
= \;& (1-\beta)\,S_{AB}^{BC}\left(b^{BC}\right)-(1-\gamma)\,S_{AC}^{CB}\left(c^{CB}\right)+\beta O_A^{BC}-\gamma O_A^{CB}\\
\geq \;& (1-\beta)\,S_{AB}^{BC}\left(c^{CB}\right)-(1-\gamma)\,S_{AC}^{CB}\left(c^{CB}\right)+\beta O_A^{BC}-\gamma O_A^{CB}\\
= \;& (\beta-\gamma)\left(\max_{c\in\mathcal{C}}\left\{u_A(0,c)+u_C(0,c)\right\}-\left(u_A\left(0,c^{CB}\right)+u_C\left(f\left(c^{CB}\right),c^{CB}\right)\right)\right)\\
\geq \;& (\beta-\gamma)\left(\max_{c\in\mathcal{C}}\left\{u_A(0,c)+u_C(0,c)\right\}-\max_{c\in\mathcal{C}}\left\{\left(u_A(0,c)+u_C(f(c),c)\right)\right\}\right)\geq 0.
\end{aligned}$$

The final inequality follows from negative externalities, which imply $u_C(0,c)\geq u_C(b,c)$ for all $b\geq 0$. Moreover, whenever externalities are strictly negative and $b>0$, $u_C(0,c)>u_C(b,c)$ for all $c>0$; hence, $U_A^{BC}>U_A^{CB}$. ■

We briefly pause to point out how the three inequalities in (17) are related to the three effects discussed in the main text. The partial-surplus effect is driven by different properties of the first stage decisions b^{BC} and c^{CB}. These are used in (15) to obtain the first inequality in (17). The outside-option effect results from the first stage decision not maximizing A's outside option in the second stage. This drives the second inequality in (17). Finally, the anticipated-externality effect drives the third inequality in (17).

A.2.3 Proof of Proposition 3

Proof. The chain of inequalities at the end of the proof of Proposition 2 establishes that $U_A^{BC}\geq U_A^{CB}$ also in case of no externalities. Moreover, when $b^{BC}\neq c^{CB}$, then inequality (15) is strict.[47] Since $\beta<1$, it follows that $U_A^{BC}>U_A^{CB}$ whenever $b^{BC}\neq c^{CB}$. We show that (i)-(iii) imply that $b^{BC}\neq c^{CB}$.

By (iii), $S_{AB}^{BC}(b)$ and $S_{AC}^{CB}(c)$ are differentiable. Since any $b^{BC}\in\arg\max_{b\in\mathcal{B}}S_{AB}^{BC}(b)$ is

[47] In case equilibrium first-stage decisions are not unique, the inequality $b^{BC}\neq c^{CB}$ is to be understood to hold for any equilibrium selection; i.e. $\arg\max_{c\in\mathcal{C}}S_{AC}^{CB}(c)\cap\arg\max_{b\in\mathcal{B}}S_{AB}^{BC}(b)=\emptyset$.

interior by (ii), it satisfies the first-order condition

$$\begin{aligned}\frac{\partial S_{AB}^{BC}\left(b^{BC}\right)}{\partial b} &= \frac{\partial u_B\left(b^{BC}\right)}{\partial b}+(1-\gamma)\frac{\partial}{\partial b}u_A\left(b^{BC},f\left(b^{BC}\right)\right)+\gamma\frac{\partial}{\partial b}u_A\left(b^{BC},0\right)\\ &\quad+(1-\gamma)\left(\frac{\partial}{\partial c}\left(u_A\left(b^{BC},f\left(b^{BC}\right)\right)+u_C\left(f\left(b^{BC}\right)\right)\right)\frac{df\left(b^{BC}\right)}{db}\right)=0.\end{aligned}$$

Since $f\left(b^{BC}\right)$ is interior by (ii), and $u_A(b,c)+u_C(c)$ is differentiable by (iii), the first-order condition

$$\frac{\partial}{\partial c}\left(u_A\left(b^{BC},f\left(b^{BC}\right)\right)+u_C\left(f\left(b^{BC}\right)\right)\right)=0$$

holds. Therefore,

$$\frac{\partial u_B\left(b^{BC}\right)}{\partial b}+(1-\gamma)\frac{\partial}{\partial b}u_A\left(b^{BC},f\left(b^{BC}\right)\right)+\gamma\frac{\partial}{\partial b}u_A\left(b^{BC},0\right)=0.$$

In addition, $f\left(b^{BC}\right)>0$ since it is interior by (ii). Condition (i) then implies

$$\frac{\partial}{\partial b}u_A\left(b^{BC},f\left(b^{BC}\right)\right)\neq\frac{\partial}{\partial b}u_A\left(b^{BC},0\right).$$

Since $\beta>\gamma$,

$$\begin{aligned}&\frac{\partial u_B\left(b^{BC}\right)}{\partial b}+(1-\gamma)\frac{\partial}{\partial b}u_A\left(b^{BC},f\left(b^{BC}\right)\right)+\gamma\frac{\partial}{\partial b}u_A\left(b^{BC},0\right)\\ \neq\;&\frac{\partial u_B\left(b^{BC}\right)}{\partial b}+(1-\beta)\frac{\partial}{\partial b}u_A\left(b^{BC},f\left(b^{BC}\right)\right)+\beta\frac{\partial}{\partial b}u_A\left(b^{BC},0\right)\\ =\;&\frac{\partial u_C\left(b^{BC}\right)}{\partial c}+(1-\beta)\frac{\partial}{\partial c}u_A\left(f\left(b^{BC}\right),b^{BC}\right)+\beta\frac{\partial}{\partial c}u_A\left(0,b^{BC}\right)\\ =\;&\frac{\partial S_{AC}^{CB}\left(b^{BC}\right)}{\partial c},\end{aligned}$$

where the first equality is due to symmetry. We have thus shown that $\partial S_{AC}^{CB}\left(b^{BC}\right)/\partial c\neq 0$.

Since any $c^{CB}\in\arg\max_{c\in\mathcal{C}}S_{AC}^{CB}(c)$ is interior by (ii), it satisfies the first-order condition $\partial S_{AC}^{CB}\left(c^{CB}\right)/\partial c=0$. Therefore, $b^{BC}\neq c^{CB}$. ∎

A.2.4 Proof of Proposition 4

In what follows, we denote the first-stage decision in sequence BC, which depends on k, by $b^{BC}(k)$. We also denote the second-stage decision in sequence BC by $c^*(b,k):=\arg\max_{c\in\mathcal{C}}\{u_A(b,c)+u_C(b,c,k)\}$, and define $b^*(c,k)$ similarly as the optimal second-stage decision in sequence CB. By symmetry, for any given first-stage decision x,

$$c^*(x,k) = b^*(x,k) =: f(x,k).$$

As a first step, we determine how the joint surplus of the principal and the first agent changes in k. The proof rests on the Envelope Theorem, which we can use although the choices in the second stage do, in general, not maximize the joint surplus of those who bargain in the first stage. Under the assumptions of Proposition 4, however, at $k=0$, the second-stage decision also maximizes the surplus of the negotiation in the first stage, therefore the corresponding terms disappear.

Lemma 2 *Under the assumptions of Proposition 4,* $S_{AB}^{BC}(k) = \max_{b\in\mathcal{B}} S_{AB}^{BC}(b, c^*(b,k), k)$ *and* $S_{AC}^{CB}(k) = \max_{c\in\mathcal{C}} S_{AC}^{CB}(b^*(c,k), c, k)$ *are differentiable in* k, *and*

$$\frac{d}{dk}\left((1-\gamma) S_{AC}^{CB}(k) - (1-\beta) S_{AB}^{BC}(k)\right)|_{k=0} = (\beta-\gamma)\frac{\partial}{\partial k} u_B(b,c,k)\Big|_{\substack{k=0\\ b=c=f(0,0)}} > 0.$$

Proof. At $k=0$, the bargaining problems are completely independent, and (by symmetry) all equilibrium decisions are identical. That is, for any b and c,

$$b^{BC}(0) = c^{CB}(0) = b^*(c,0) = c^*(b,0) = f(0,0).$$

By our smoothness assumptions, $S_{AB}^{BC}(k)$ is differentiable in k. By the envelope theorem,

$$\begin{aligned}\frac{d}{dk}\max_{b\in\mathcal{B}} S_{AB}^{BC}(b, c^*(b,k), k)|_{k=0} &= \frac{\partial}{\partial k}\left(u_B(b,c,k) + (1-\gamma) u_C(b,c,k)\right)\Big|_{\substack{k=0\\ b=c=f(0,0)}} \\ &\quad + \left(\frac{\partial S_{AB}^{BC}(b,c,k)}{\partial c}\frac{\partial c^*(b,k)}{\partial k}\right)\Big|_{\substack{k=0\\ b=c=f(0,0)}}.\end{aligned}$$

The first term of the right-hand side is the direct effect of k, keeping b and c constant, whereas the second term captures that the second-stage equilibrium decision depends on k.

We next show that

$$\frac{\partial S_{AB}^{BC}(b,c,k)}{\partial c}\Big|_{\substack{k=0\\ b=c=f(0,0)}} = 0.$$

We have

$$\frac{\partial S_{AB}^{BC}(b,k)}{\partial c} = \frac{\partial}{\partial c} u_B(b, c^*(b), k) + (1-\gamma)\frac{\partial}{\partial c}\left(u_A(b, c^*(b,k)) + u_C(b, c^*(b,k), k)\right).$$

Evaluated at $k=0$, u_B does not depend on c; hence, the first term on the right-hand side is zero. Moreover, $c^*(b,k)$ maximizes $u_A(b,c) + u_C(b,c,k)$, and by the first-order condition of this maximization problem, the second term on the right hand side is also zero.

It follows that

$$\frac{d}{dk}S_{AB}^{BC}(k)\,|_{k=0} = \frac{\partial}{\partial k}\left(u_B(b,c,k) + (1-\gamma)\,u_C(b,c,k)\right)\Big|_{\substack{k=0\\ b=c=f(0,0)}}.$$

Similarly,

$$\begin{aligned}\frac{d}{dk}S_{AC}^{CB}(k)\,|_{k=0} &= \frac{\partial}{\partial k}\left(u_C(b,c,k) + (1-\beta)\,u_B(b,c,k)\right)\Big|_{\substack{k=0\\ b=c=f(0,0)}}\\ &= \frac{\partial}{\partial k}\left(u_B(b,c,k) + (1-\beta)\,u_C(b,c,k)\right)\Big|_{\substack{k=0\\ b=c=f(0,0)}},\end{aligned}$$

where the second line uses symmetry. Therefore,

$$\frac{d}{dk}\left((1-\gamma)\,S_{AC}^{CB}(k) - (1-\beta)\,S_{AB}^{BC}(k)\right)|_{k=0} = (\beta-\gamma)\,\frac{\partial}{\partial k}u_B(b,c,k)\Big|_{\substack{k=0\\ b=c=f(0,0)}},$$

which is strictly positive since $b = c = f(0,0) > 0$.

■

We are now in a position to prove Proposition 4.

Proof. Since u_A and $u_C(0,c,k)$ do not depend on k,

$$O_A^{BC} = (1-\gamma)\max_{c\in\mathcal{C}}\{u_A(0,c) + u_C(0,c,k)\}$$

does not depend on k. Similarly, O_A^{CB} is independent of k. The payoff of A in sequences BC and CB is

$$U_A^{BC}(k) := (1-\beta)\,S_{AB}^{BC}(k) + \beta O_A^{BC} \quad \text{and} \quad U_A^{CB}(k) := (1-\gamma)\,S_{AC}^{CB}(k) + \gamma O_A^{CB},$$

respectively. Therefore, Lemma 2 implies that

$$\frac{\partial}{\partial k}\left(U_A^{CB}(k) - U_A^{BC}(k)\right)|_{k=0} > 0.$$

If $k = 0$, the bargaining problems do not interact, and $U_A^{BC}(0) = U_A^{CB}(0)$. It follows that for sufficiently small $k > 0$, $U_A^{CB}(k) > U_A^{BC}(k)$. ■

A.2.5 Proof of Proposition 5

We start with a similar argument as in the proof of Proposition 2. Since $c^{CB} \in \arg\max_{c\in\mathcal{C}} S_{AC}^{CB}(c)$, $S_{AB}^{CB}\left(c^{CB}\right) \geq S_{AB}^{CB}\left(b^{BC}\right)$, and, hence,

$$\begin{aligned}
& U_A^{CB} - U_A^{BC} \\
= \; & (1-\gamma) S_{AC}^{CB}\left(c^{CB}\right) - (1-\beta) S_{AB}^{BC}\left(b^{BC}\right) + \gamma O_A^{CB} - \beta O_A^{BC} \\
\geq \; & (1-\gamma) S_{AC}^{CB}\left(b^{BC}\right) - (1-\beta) S_{AB}^{BC}\left(b^{BC}\right) + \gamma O_A^{CB} - \beta O_A^{BC} \\
= \; & (\beta-\gamma)\left(\left(u_A\left(b^{BC},0\right) + u_B\left(b^{BC}, f\left(b^{BC}\right)\right)\right) - \max_{b\in\mathcal{B}}\left\{u_A(b,0) + u_B(b,0)\right\}\right).
\end{aligned}$$

Therefore it is sufficient to establish that, for sufficiently large k,

$$u_A\left(b^{BC},0\right) + u_B\left(b^{BC}, f\left(b^{BC}\right)\right) > \max_{b\in\mathcal{B}}\left\{u_A(b,0) + u_B(b,0)\right\}. \tag{18}$$

By assumption, u_B is constant in k if $c = 0$, and u_A does not depend on k. Therefore, the right-hand side of (18) is constant in k. In contrast, the left-hand side grows beyond all bounds as $k \to \infty$, as we show next.

By assumption, equilibrium decisions b^{BC} and $f\left(b^{BC}\right)$ converge to finite limits $\bar{b} = \lim_{k\to\infty} b^{BC}$ and $\bar{c} = \lim_{k\to\infty} f\left(b^{BC}, k\right)$. Making the dependence of u_B and f on k explicit, and using continuity, we have $\lim_{k\to\infty} u_B\left(b^{BC}, f\left(b^{BC}, k\right), k\right) = \lim_{k\to\infty} u_B\left(\bar{b}, \bar{c}, k\right) = \infty$. By contrast, $\lim_{k\to\infty} u_A\left(b^{BC}, 0\right) = u_A\left(\bar{b}, 0\right)$ is finite. Therefore, (18) holds for sufficiently large k.

A.2.6 Proof of Remark 3

In sequence BC, the second-stage decision solves

$$\max_c -v(c) + g(c).$$

Due to the separability of the utility functions, this decision neither depends on b nor on k, and we denote it by c^*. It is implicitly given by the first-order condition $g'(c^*) = v'(c^*)$. Similarly, the second-stage decision in sequence CB is $b^* = c^*$.

In the first stage of sequence BC, the decision maximizes

$$\begin{aligned}
& u_B(b,c^*) + (1-\gamma)\left(u_A(b,c^*) + u_C(b,c^*)\right) + \gamma u_A(b,0) \\
= \; & g(b) + kc^* + (1-\gamma)\left(-v(b) - v(c^*) + g(c^*) + kb\right) - \gamma v(b).
\end{aligned}$$

The first-stage decision b^{BC} is then given by $g'\left(b^{BC}\right) + (1-\gamma)k - v'\left(b^{BC}\right) = 0$. Similarly,

c^{CB} is given by $g'\left(c^{CB}\right)+(1-\beta)k-v'\left(c^{CB}\right)=0$. Note that c^{CB} is increasing in k, with $\lim_{k\to\infty}c^{CB}=\infty$. (To see this, note that if c^{CB} converges to a finite limit $\bar{c}$, then $g'\left(c^{CB}\right)-v'\left(c^{CB}\right)\to g'(\bar{c})-v'(\bar{c})$ which is finite, but $(1-\beta)k\to\infty$, which contradicts the first-order condition.)

We are now in a position to prove Part (i). As shown in the Proof of Proposition 2, A strictly prefers BC if (see inequality (17))

$$\max_{c\in\mathcal{C}}\left\{u_A(0,c)+u_C(0,c)\right\}>\left(u_A\left(0,c^{CB}\right)+u_C\left(f\left(c^{CB}\right),c^{CB}\right)\right). \tag{19}$$

In the current setting,

$$\begin{aligned}
& u_A\left(0,c^{CB}\right)+u_C\left(f\left(c^{CB}\right),c^{CB}\right)\\
=\; & -v\left(c^{CB}\right)+g\left(c^{CB}\right)+kb^*\\
=\; & -v\left(c^{CB}\right)\left[1-\left(\frac{g\left(c^{CB}\right)}{v\left(c^{CB}\right)}+b^*\frac{k}{v\left(c^{CB}\right)}\right)\right]
\end{aligned} \tag{20}$$

We show this diverges to minus infinity as $k\to\infty$ (and, consequently, $c^{CB}\to\infty$). Note first that then $v\left(c^{CB}\right)\to\infty$. We will show next that the term in squared brackets converges to one.

First, the term $g\left(c^{CB}\right)/v\left(c^{CB}\right)$ converges to zero. Recall that $g'(c)>0\geq g''(c)$ for all $c>0$. If $g\left(c^{CB}\right)$ converges to some finite limit, $\lim_{k\to\infty}\left(g\left(c^{CB}\right)/v\left(c^{CB}\right)\right)=0$. Moreover, if $\lim_{k\to\infty}g\left(c^{CB}\right)=\infty$, then, by L'Hopital's rule,

$$\begin{aligned}
\lim_{k\to\infty}\frac{g\left(c^{CB}\right)}{v\left(c^{CB}\right)} &= \lim_{k\to\infty}\frac{g'\left(c^{CB}\right)}{v'\left(c^{CB}\right)}\\
&= \lim_{k\to\infty}\frac{g'\left(c^{CB}\right)}{g'\left(c^{CB}\right)+(1-\beta)k}\\
&= \frac{1}{1+(1-\beta)\lim_{k\to\infty}\frac{k}{g'\left(c^{CB}\right)}}=0.
\end{aligned}$$

The second equality uses the first-order condition defining c^{CB}. The last equality follows since g is strictly increasing and concave; hence, $0\leq\lim_{k\to\infty}g'\left(c^{CB}\right)\leq g'(0)<\infty$.

Second, consider the term $b^*k/v\left(c^{CB}\right)$. As b^* does not depend on k, we can use the first-order condition defining c^{CB} to get

$$k=\frac{v'\left(c^{CB}\right)-g'\left(c^{CB}\right)}{1-\beta},$$

which implies

$$\frac{k}{v\left(c^{CB}\right)}=\frac{1}{1-\beta}\left(\frac{v'\left(c^{CB}\right)}{v\left(c^{CB}\right)}-\frac{g'\left(c^{CB}\right)}{v\left(c^{CB}\right)}\right).$$

By assumption, $v'\left(c^{CB}\right)/v\left(c^{CB}\right)\to 0$. Moreover, $\lim_{k\to\infty} g'\left(c^{CB}\right)<\infty=\lim_{k\to\infty} v\left(c^{CB}\right)$; hence, $g'\left(c^{CB}\right)/v\left(c^{CB}\right)\to 0$.

We have shown that the squared bracket in (20) converges to 1. Thus

$$\lim_{k\to\infty}\left(u_A\left(0,c^{CB}\right)+u_C\left(f\left(c^{CB}\right),c^{CB}\right)\right)=-\infty.$$

Therefore, for large enough k, (19) is satisfied, and the principal strictly prefers BC.

It remains to prove Part (ii). As in the proof of the previous proposition, it is sufficient to show that (18) holds. In the current setting,

$$\begin{aligned} u_A\left(b^{BC},0\right)+u_B\left(b^{BC},f\left(b^{BC}\right)\right) &= -v\left(b^{BC}\right)+g\left(b^{BC}\right)+kc^* \\ &= -v\left(b^{BC}\right)\left[1-\left(\frac{g\left(b^{BC}\right)}{v\left(b^{BC}\right)}+\frac{kc^*}{v\left(b^{BC}\right)}\right)\right]. \end{aligned}$$

By the same argument as above, $g\left(b^{BC}\right)/v\left(b^{BC}\right)\to 0$, as $k\to\infty$. Moreover,

$$\frac{k}{v\left(b^{BC}\right)}=\frac{1}{1-\gamma}\frac{v'\left(b^{BC}\right)-g'\left(b^{BC}\right)}{v\left(b^{BC}\right)}\to\infty,$$

because $g'\left(b^{BC}\right)/v\left(b^{BC}\right)\to 0$ (as shown above), but $v'\left(b^{BC}\right)/v\left(b^{BC}\right)\to\infty$ by (3). Therefore,

$$\lim_{k\to\infty}\left(u_A\left(b^{BC},0\right)+u_B\left(b^{BC},f\left(b^{BC}\right)\right)\right)=+\infty,$$

which implies that for k large enough, A strictly prefers CB.

A.3 Simultaneous Negotiations

A.3.1 Proof of Proposition 6

Proof. A's payoff in sequence BC is

$$U_A^{BC}=(1-\beta)\,S_{AB}^{BC}\left(b^{BC}\right)+\beta O_A^{BC}, \tag{21}$$

with

$$S_{AB}^{BC}\left(b^{BC}\right)=u_B\left(b^{BC},c^*\left(b^{BC}\right)\right)+(1-\gamma)\left(u_A\left(b^{BC},c^*\left(b^{BC}\right)\right)+u_C\left(b^{BC},c^*\left(b^{BC}\right)\right)\right)+\gamma u_A\left(b^{BC},0\right)$$

and

$$O_A^{BC} = (1-\gamma)\max_{c\in\mathcal{C}}\left\{u_A(0,c)+u_C(0,c)\right\}.$$

Inserting the last two expressions into (21) yields

$$(1-\beta)\left\{u_B\left(b^{BC},c^*\left(b^{BC}\right)\right)+(1-\gamma)\left(u_A\left(b^{BC},c^*\left(b^{BC}\right)\right)+u_C\left(b^{BC},c^*\left(b^{BC}\right)\right)\right)+\gamma u_A\left(b^{BC},0\right)\right\}$$

$$+\beta(1-\gamma)\left\{u_A(0,c^*(0))+u_C(0,c^*(0))\right\}.$$

This can be written as

$$U_A^{BC} = (1-\beta)(1-\gamma)\left\{u_A\left(b^{BC},c^*\left(b^{BC}\right)\right)+u_B\left(b^{BC},c^*\left(b^{BC}\right)\right)+u_C\left(b^{BC},c^*\left(b^{BC}\right)\right)\right\} \quad (22)$$

$$+(1-\beta)\gamma\left\{u_A(b^{BC},0)+u_B\left(b^{BC},c^*\left(b^{BC}\right)\right)\right\}+\beta(1-\gamma)\left\{u_A(0,c^*(0))+u_C(0,c^*(0))\right\}.$$

We now compare (22) with (6). We start with the last term of (6). If u_A is weakly super-modular, then $u_A(b,c) \geq u_A(b,0)+u_A(0,c)$ for all b and c. Therefore, the last term of (6) is weakly negative.

Looking at the first and the second term of (22), it is easy to see that the structure is the same as the one of the first two terms of (6). However, the arguments are different. In (6), they are $b^\star$ and $c^\star$ or $b^\star$ and 0, whereas in (22) they are b^{BC} and $c^*\left(b^{BC}\right)$ or b^{BC} and 0. If b^{BC} were equal to $b^\star$, then $c^*\left(b^{BC}\right)$ would also be equal to $c^\star$ because the maximization problem with respect to c is then the same in the simultaneous and the sequential timing. However, b^{BC} is chosen to maximize the first two terms of (22) (i.e., taken into account the reaction of c in the second stage). Therefore, by a revealed preference argument, if b^{BC} differs from $b^\star$, the first two terms of (22) must be larger than the corresponding ones of (6).

Finally, we need to compare the last term of (22) (i.e., $\beta(1-\gamma)\left\{u_A(0,c^*(0))+u_C(0,c^*(0))\right\}$), with the third term of (6) (i.e., $\beta(1-\gamma)\{u_A(0,c^\star)+u_C(b^\star,c^\star)\}$). Since $b^\star \geq 0$ and $c^*(0)$ maximizes $u_A(0,c)+u_C(0,c)$, it is evident that the latter term is lower then the former if externalities are negative. It follows that all terms in (6) are weakly lower than those in (22) if externalities are negative and u_A is super-modular. In addition, unless equilibrium decisions are zero, (6) is strictly lower than (22) if externalities are strictly negative and/or u_A is strictly super-modular. ■

A.3.2 Proof of Proposition 7

Proof. We again compare (22) with (6). Starting with the first two terms of each expressions, the argument made in the previous proof does not depend on externalities: as b^{BC} is chosen to maximize these terms whereas $b^\star$ is not, these terms must be weakly larger in (22) than in

(6). Comparing the third term of (6) with the last term of (22), as there are no externalities, the difference in the first argument of u_C in both terms is irrelevant. Therefore, the driving force in the difference between these two terms is that $c^*(0)$ maximizes $u_A(0,c) + u_C(0,c)$ but $c^\star$ does not necessarily do so. As a consequence, this term is also weakly larger in (22) than in (6). Finally, as in the proof of the last proposition, if u_A is super-modular, the last term of (6) is negative. It follows that sequence BC is preferred by the principal if u_A is super-modular. Moreover, the principal strictly prefers sequence CB if u_A is strictly super-modular and equilibrium decisions are not zero.

We now turn to the case in which u_A is sub-modular. Then the difference in the first three terms between (22) and (6) is non-negative (see the proof of Proposition 6). If, in addition, $\gamma = 0$, the last term in (6) drops out; hence, $U_A^{BC} \geq U_A^{sim}$. Moreover, at $\gamma = 0$ we have $U_A^{BC} > U_A^{sim}$ if equilibrium decisions differ in sequential versus simultaneous negotiations—i.e., $b^{BC} \neq b^\star$. By continuity, the result then also holds for γ close to 0.

Finally, suppose $\gamma = 1$, which implies that $\beta = 1$ (since $\gamma \leq \beta$). By (22), $U_A^{BC} = 0$. By (6), $U_A^{sim} = u_A(b^\star, 0) + u_A(0, c^\star) - u_A(b^\star, c^\star) \geq 0$ because u_A is sub-modular. Hence $U_A^{sim} \geq U_A^{BC}$. Moreover, at $\gamma = 1$ we have $U_A^{sim} > U_A^{BC}$ if u_A is strictly sub-modular and equilibrium decisions are not zero. By continuity, the result then also holds for γ close to 1.
■

A.3.3 Proof of Proposition 8

Proof. From the proof of Proposition 4 we know that, when evaluated at $k = 0$,

$$\frac{d}{dk} U_A^{CB}(k)\,|_{k=0} = \frac{\partial}{\partial k}\left((1-\gamma) u_C(b,c,k) + (1-\gamma)(1-\beta) u_B(b,c,k)\right)\Big|_{\substack{k=0 \\ b=c=f(0,0)}}.$$

Applying the same logic to (6) , we obtain

$$\begin{aligned}\frac{d}{dk} U_A^{sim}(k)\,|_{k=0} &= \frac{\partial}{\partial k}\Big((1-\beta)(1-\gamma)\left(u_B(b,c,k) + u_C(b,c,k)\right) \\ &\quad + (1-\beta)\gamma u_B(b,c,k) + \beta(1-\gamma) u_C(b,c,k)\Big)\Big|_{\substack{k=0 \\ b=b^\star \\ c=c^\star}} \\ &= \frac{\partial}{\partial k}\left((1-\gamma) u_C(b,c,k) + (1-\beta) u_B(b,c,k)\right)\Big|_{\substack{k=0 \\ b=b^\star \\ c=c^\star}}\end{aligned}$$

Symmetry of agents, no externalities ($k = 0$), and u_A being additively separable imply that $b^\star = c^\star = f(0,0)$. As a consequence,

$$\frac{d}{dk}\left\{U_A^{sim}(k) - U_A^{CB}(k)\right\}|_{k=0} = \frac{\partial}{\partial k}\left(\gamma(1-\beta)u_B(b,c,k)\right)\Big|_{\substack{k=0 \\ b=c=f(0,0)}} > 0 \; .$$

If $k = 0$, the bargaining problems do not interact, and $U_A^{sim}(0) = U_A^{CB}(0)$. It follows that for sufficiently small $k > 0$, $U_A^{sim}(k) > U_A^{CB}(k)$. ■

A.4 Disclosure

This appendix contains the proof of Proposition 9, details on the supplier-retailer example mentioned in the text after Proposition 9, and the proof of Proposition 10.

A.4.1 Proof of Proposition 9

By separability and symmetry, $u_A(b,c) = v_A(b) + v_A(c)$ where $v_A(x) := u_A(x,0)$. Note that $v_A(0) = u_A(0,0) = 0$. Without loss of generality, consider sequence BC.[48]

Existence of a disclosure equilibrium We will construct an equilibrium where A discloses the first stage contract, after any first stage decision. Strategies for the first stage are as in our main model. Moreover, non-disclosure is an off-equilibrium event, and C's beliefs about the first stage decision—if it is not disclosed by A—can be chosen arbitrarily.

The equilibrium is suppoted by the following beliefs: if A does not disclose the first stage contract, C believes with probability 1 that the first stage decision was

$$b' = \arg\min_b \left\{v_A(c^*(b)) + u_C(b, c^*(b))\right\}$$

where

$$c^*(b) = \arg\max_c \left\{v_A(c) + u_C(b,c)\right\}.$$

Note that this belief does not depend on A's offer in case that the principal proposes in stage 2.

We now show that such a b' exists. The functions v_A and u_C are differentiable by assumption; hence, they are continuous. Because c is chosen from a compact set, by the maximum theorem, the value function

$$\max_c \left\{v_A(c) + u_C(b,c)\right\} = v_A(c^*(b)) + u_C(b, c^*(b))$$

[48]The same argument holds when changing the sequence to CB.

is continuous in b (see e.g. Jehle and Reny 2011, Theorem A2.21). Since b is chosen from the set $\mathcal{B}$, which is compact, a minimum exists by the Weierstrass theorem.

Next, we show that, given this belief, disclosure is optimal for A, after any given first-stage decision $b = b_0$ and transfer t_B. Suppose the first-stage decision was b_0. If A discloses, her payoff is

$$v_A(b_0) + (1-\gamma)(v_A(c^*(b_0)) + u_C(b_0, c^*(b_0))) + t_B.$$

If A does not disclose, C believes that the first stage decision was b'. Thus C will accept any offer such that $u_C(b', c) \geq t_C$. Therefore, if A proposes in stage 2, she will demand $t_C = u_C(b', c)$. Moreover, A will propose the decision

$$\begin{aligned} & \arg\max_c \{u_A(b_0, c) + u_C(b', c)\} \\ = & \arg\max_c \{v_A(c) + u_C(b', c)\} \\ = & c^*(b'). \end{aligned}$$

The payoff of A is

$$v_A(b_0) + t_B + v_A(c^*(b')) + u_C(b', c^*(b')).$$

Moreover, A will accept any offer such that $v_A(c) + t_C \geq 0$; the true first-stage decision b_0 is not relevant for A 's decision to accept or reject C's offer due to the separability of u_A. If C proposes in stage 2, he will demand $t_C = -v_A(c)$; the payoff of A is

$$v_A(b_0) + t_B.$$

In expectation, A's payoff from nondisclosure is

$$\begin{aligned} & v_A(b_0) + t_B + (1-\gamma)(v_A(c^*(b')) + u_C(b', c^*(b'))) \\ = & v_A(b_0) + t_B + (1-\gamma)\min_b \{v_A(c^*(b)) + u_C(b, c^*(b))\} \\ \leq & v_A(b_0) + t_B + (1-\gamma)\, v_A(c^*(b_0)) + u_C(b_0, c^*(b_0)). \end{aligned}$$

Therefore, disclosure is sequentially optimal for A, after any first stage decision b_0 and transfer t_B.

There is no non-disclosure equilibrium Because the proof is somewhat lengthy, we break it down in several steps, and informally preview the key arguments here.

The proof is by contradiction. Suppose there is a PBE without disclosure and denote the first-stage decision in the supposed equilibrium by b^0. In step 1, we derive A's payoff, and the joint surplus of A and B, in the supposed equilibrium. Step 2 derives A's payoff, and the joint surplus of A and B, after a deviation to a first stage decision $b^{dev} \neq b^0$, assuming this deviation is kept secret.

Step 3 analyzes A's disclosure decision. On the equilibrium path of the supposed nondisclosure equilibrium, A is indifferent between disclosing b^0 and nondisclosure. Roughly speaking, the intuition is that upon nondisclosure C believes that $b = b^0$ anyhow, so nothing is gained or lost by disclosure.[49] Moreover, with positive externalities between agents, A will disclose first stage decisions $b > b^0$, and A will not disclose after $b < b^0$. The reason is that A can demand a higher transfer from C when C knows that the first stage decision is higher, because of the positive externalities. (When C proposes in stage 2, C's belief about (or knowledge of) b does not matter for A anyhow because u_A is additively separable.)

Step 4 shows that the joint surplus of A and B must be nondecreasing in b at b^0, holding the second-stage behavior constant. Otherwise, A and B achieve a higher joints surplus by agreeing on some $b^{dev} < b^0$—a deviation that A will keep secret by step 3.

Step 5 completes the proof by showing that a small upward deviation to some $b^{dev} > b^0$ close to b^0 increases the joint surplus of A and B. By step 3, such a deviation is disclosed. By step 4, the direct effect on the joint surplus of A and B is nonnegative. Moreover, C is willing to pay more to participate because of the positive externalities, so the transfer A can achieve in case she proposes in stage 2 is higher. In addition, because u_C is super-modular and u_A is additively separable, the resulting second-stage decision is higher, which benefits B. Taken together, these considerations imply that there is a profitable upward deviation.

Step 1: supposed equilibrium payoffs Suppose there is a PBE in pure strategies in which A does not disclose on the equilibrium path. Denote the first stage decision in the supposed equilibrium by b^0. We will show that, in any such equilibrium, the payoff of A is[50]

$$v_A\left(b^0\right) + t_B + (1-\gamma)\left(v_A\left(c^*\left(b^0\right)\right) + u_C\left(b^0, c^*\left(b^0\right)\right)\right). \tag{23}$$

Similarly, the joint surplus of A and B is

$$v_A\left(b^0\right) + u_B\left(b^0, c^*\left(b^0\right)\right) + (1-\gamma)\left(v_A\left(c^*\left(b^0\right)\right) + u_C\left(b^0, c^*\left(b^0\right)\right)\right). \tag{24}$$

In the supposed equilibrium, nondisclosure is on the equilibrium path. Therefore, C's

[49] This intuition is only 'rough' since we must rule out that out of equilibrium beliefs of C would force A to make generous offers in her negotiation with C if she does not disclose; we do this in step 1.

[50] To be precise, this is A's expected payoff after the negotiation in the first stage has been completed, but before stage 2 begins. The expectation is over who proposes in stage 2. Similarly, the joint surplus of A and B laid out below is their expected joint surplus after stage 1, but before stage 2.

belief about b must be consistent with the equilibrium strategies and derived via Bayes' rule. Therefore, C believes that $b = b^0$ with probability 1 whenever A does not disclose and C makes the offer in stage 2. When A makes the offer in stage 2, C's beliefs may depend on the offer. Denote the offer on the equilibrium path by (t_C^0, c^0).

Consider stage 2. Suppose the decision in stage 1 was b^0, and A has not disclosed the decision. When C proposes, he will set $t_C = -v_A(c)$ and propose the decision

$$\arg\max_c \left\{v_A(c) + u_C\left(b^0, c\right)\right\} = c^*\left(b^0\right).$$

The payoff of A is

$$v_A\left(b^0\right) + t_B, \tag{25}$$

and the joint surplus of A and B is $u_A(b^0) + u_B(b^0, c^*(b^0))$. Note that these conclusions hold independently of C's belief about b due to the additive separability of u_A.

When A proposes, C's belief determines which contracts he will accept. We start by considering candidate equilibria where C believes that $b = b^0$ with probability 1, no matter what A proposes.[51] When A proposes, she knows that C will accept any (c, t_C) such that $t_C \leq u_C(b^0, c)$. Hence, she will propose $t_C = u_C(b^0, c)$ and the decision c is equal to

$$\arg\max_c \left\{v_A(c) + u_C\left(b^0, c\right)\right\} = c^*\left(b^0\right).$$

The payoff of A is

$$v_A\left(b^0\right) + v_A\left(c^*\left(b^0\right)\right) + u_C\left(b^0, c^*\left(b^0\right)\right) + t_B. \tag{26}$$

The joint surplus of A and B is

$$v_A\left(b^0\right) + v_A\left(c^*\left(b^0\right)\right) + u_B\left(b, c^*\left(b^0\right)\right) + u_C\left(b^0, c^*\left(b^0\right)\right).$$

Taking the expectation over who proposes in stage 2, it follows from (25) and (26) that in any equilibrium in which, after nondisclosure, C believes that $b = b^0$ indepedently of A's offer, the payoff of A is given by (23). Similarly, the joint surplus of A and B is given by (24).

To complete step 1, it remains to show that (23) and (24) are also valid if C's beliefs depends on the offer (c, t_C).

Informally, A's payoff cannot be lower than (23), since A can secure herself the payoff (23) if she discloses. Moreover, A's payoff cannot be higher than (23) as well: if C proposes,

[51] We consider the possibility that C's beliefs depend on A's offer below.

A is brought down to her outside option (25) no matter what C believes; and if A proposes, (26) gives her the highest surplus that A and C can generate. The remainder of Step 1 spells out these arguments formally.

Suppose that, after nondisclosure and an offer (c, t_C) from A, the beliefs of C are given by the cumulative distribution function

$$F(x; t_C, c) = \Pr(b \leq x \,| t_C, c).$$

Then C will accept any offer such that

$$t_C \leq \int_{\mathcal{B}} u_C(b, c)\, dF(b; t_C, c)$$

and A will choose (c, t_C) to

$$\max_{c, t_C} v_A(c) + t_C$$

subject to the constraint that C accepts.[52]

As above, let (c^0, t_C^0) denote A's offer on the equilibrium path. Since beliefs need to be consistent with strategies on the equilibrium path, $F(b; t_C^0, c^0)$ puts all probability mass on b^0 : on the equilibrium path, C believes that $b = b^0$ with probability one. Thus $t_C^0 \leq u_C(b^0, c^0)$. This implies

$$\begin{aligned} v_A\left(c^0\right) + t_C^0 &\leq v_A\left(c^0\right) + u_C\left(b^0, c^0\right) \\ &\leq \max_c v_A(c) + u_C\left(b^0, c\right) \\ &= v_A\left(c^*\left(b^0\right)\right) + u_C\left(b^0, c^*\left(b^0\right)\right) \end{aligned}$$

Suppose that

$$v_A\left(c^0\right) + t_C^0 < v_A\left(c^*\left(b^0\right)\right) + u_C\left(b^0, c^*\left(b^0\right)\right) \tag{27}$$

This cannot hold in equilibrium, since then A would then prefer disclosing b^0. We thus have

$$\begin{aligned} v_A\left(c^0\right) + t_C^0 &= v_A\left(c^*\left(b^0\right)\right) + u_C\left(b^0, c^*\left(b^0\right)\right) \\ &= \max_c \left\{v_A(c) + u_C\left(b^0, c\right)\right\} \end{aligned}$$

[52] As in the main part of the paper, assuming that C accepts is without loss of generality, since A could propose $c = t_C = 0$ if she does want to deal with C. Acceptance and rejection then have the same consequences.

We also have $t_C^0 \leq u_C\left(b^0, c^0\right)$ hence

$$\begin{aligned} & v_A\left(c^0\right)+u_C\left(b^0, c^0\right) \\ \geq\ & v_A\left(c^0\right)+t_C^0 \\ =\ & \max_c\left\{v_A(c)+u_C\left(b^0, c\right)\right\} \\ \geq\ & v_A\left(c^0\right)+u_C\left(b^0, c^0\right) \end{aligned}$$

which implies that the above inequalities hold with equality. It follows that $t_C^0 = u_C\left(b^0, c^0\right)$ and

$$c^0 = \arg\max_c\left\{u_A(c)+u_C\left(b^0, c\right)\right\} = c^*\left(b^0\right)$$

because $c^*\left(b^0\right)$ is unique by assumption.

As a consequence, in any non-disclosure equilibrium with first-stage equilibrium decision b^0, when A proposes, she proposes $c^*\left(b^0\right)$ and $t_C = u_C\left(b^0, c^*\left(b^0\right)\right)$, and C accepts. The payoff of A is then given by (26). This completes the proof that that A's payoff in any non-disclosure equilibrium is given by (23). Similarly, the joint surplus of A and B is given by (24).

For future reference, we point out that C's beliefs F must be such that A cannot gain by offering any $(t_C, c) \neq (t_C^0, c^0)$. Otherwise A would offer the contract (t_C, c), which contradicts that (t_C^0, c^0) is an equilibrium.

Step 2: payoffs after non-disclosed deviations in the first stage We now consider deviations where A and B agree on a decision $b^{dev} \neq b^0$ in the first stage and A does not disclose the first stage decision.

Consider stage 2. If C proposes, he is not aware of the deviation. As in step 1, he proposes the decision $c^*\left(b^0\right)$ and transfer $t_C = -v_A\left(c^*\left(b^0\right)\right)$. In case A proposes, we have already established at the end of step 1 that A cannot do better than offering (t_C^0, c^0). Our objective is to show that a profitable deviation exists, and for this purpose it is without loss of generality to assume that after an undisclosed deviation to $b^{dev} \neq b^0$, A proposes $\left(t_C^0, c^0\right)$.[53]

[53] A might be indifferent between proposing $\left(t_C^0,\ c^0\right)$ and some other proposal $\left(t_C^1, c^1\right) \neq \left(t_C^0,\ c^0\right)$. The reason is that $\left(t_C^1, c^1\right)$ is an off-equilibrium offer, so C's belief $F\left(b; t_C^1, c^1\right)$ is arbitrary. As argued at the end of step 1, equilibrium implies that A is not better off with offering $\left(t_C^1, c^1\right)$. For A's payoff, it is therefore irrelevant whether she proposes $\left(t_C^1, c^1\right)$ or $\left(t_C^0, c^0\right)$. If such a $\left(t_C^1, c^1\right)$ exists, we impose the mild equilibrium refinement that A chooses the proposal that gives the first agent the highest utility. In fact, A can credibly promise to behave this way in the negotiation with the first agent. A will then choose $\left(t_C^1, c^1\right)$ only if this gives B at least as high a payoff as $\left(t_C^0, c^0\right)$. Thus their joint payoff from the deviation in stage 1 is at least as high as when A proposes $\left(t_C^0, c^0\right)$ after the deviation.

The payoff of A from such a deviation is

$$v_A\left(b^{dev}\right) + t_B + (1-\gamma)\left(v_A\left(c^*\left(b^0\right)\right) + u_C\left(b^0, c^*\left(b^0\right)\right)\right),$$

and the joint surplus of A and B is

$$v_A\left(b^{dev}\right) + u_B\left(b^{dev}, c^*\left(b^0\right)\right) + (1-\gamma)\left(v_A\left(c^*\left(b^0\right)\right) + u_C\left(b^0, c^*\left(b^0\right)\right)\right).$$

Step 3: the disclosure decision This step analyzes A's disclosure decision. First, suppose that decision b^0 (i.e., the decision on the supposed equilibrium path with non-disclosure) has been agreed upon. If A discloses, the second stage is as in our main model. If A does not disclose, by our analysis above, her payoff is given by (23). Thus, A is indifferent between disclosure and nondisclosure.

Next, suppose the first-stage decision was $b^{dev} \neq b^0$. Consider stage 2. If A does not disclose, her payoff is $v_A\left(b^{dev}\right) + t_B + (1-\gamma)(v_A(c^*(b^0)) + u_C(b^0, c^*(b^0)))$ by step 2. If A discloses, her payoff is $v_A\left(b^{dev}\right) + t_B + (1-\gamma)\left(v_A\left(c^*\left(b^{dev}\right)\right) + u_C\left(b^{dev}, c^*\left(b^{dev}\right)\right)\right)$ as in our main model. The difference (disclose - hide) is $(1-\gamma)$ times

$$v_A\left(c^*\left(b^{dev}\right)\right) + u_C\left(b^{dev}, c^*\left(b^{dev}\right)\right) - \left(v_A\left(c^*\left(b^0\right)\right) + u_C\left(b^0, c^*\left(b^0\right)\right)\right)$$

By assumption, there are positive externalities between agents, and u_C is strictly increasing in b for all $c > 0$. Thus A has a strict incentive to disclose whenever $b^{dev} > b^0$. To see this, note that the inequality

$$\begin{aligned} & v_A\left(c^*\left(b^{dev}\right)\right) + u_C\left(b^{dev}, c^*\left(b^{dev}\right)\right) = \max_c \left\{v_A(c) + u_C\left(b^{dev}, c\right)\right\} \\ > \ & \max_c \left\{v_A(c) + u_C\left(b^0, c\right)\right\} = v_A\left(c^*\left(b^0\right)\right) + u_C\left(b^0, c^*\left(b^0\right)\right) \end{aligned}$$

holds iff $b^{dev} > b^0$. Therefore, A will disclose a deviation $b^{dev} > b^0$, and A will not disclose a deviation $b^{dev} < b^0$.

An immediate implication is that b^0 must be strictly greater than zero. The joint payoff of A and B in a non-disclosure equilibrium with $b^0 = 0$ would be the same as in the game with an observable first-stage decision after $b = 0$, namely $S_{AB}^{BC}(0)$ (see (1) for the definition of $S_{AB}^{BC}(b)$). By assumption, however, the equilibrium first-stage decision in the game with observable decisions is unique and interior. Therefore, $b^{BC} > 0$ and $S_{AB}^{BC}\left(b^{BC}\right) > S_{AB}^{BC}(0)$. Consider now a candidate equilibrium without disclosure where $b^0 = 0$. A deviation to $b^{BC} > 0$ will be disclosed, and gives A and B a strictly higher joint surplus $S_{AB}^{BC}\left(b^{BC}\right) > S_{AB}^{BC}(0)$, contradicting equilibrium. Therefore, if a non-disclosure equilibrium exists, it must involve $b^0 > 0$.

Step 4: first stage deviations that will not be disclosed Suppose that A and B agree on a decision $b^{dev} < b^0$. By step 3, A will not disclose. The joint surplus of A and B after this deviation (derived in step 2) must be weakly smaller than in the candidate equilibrium (derived in step 1):

$$\begin{aligned} & v_A\left(b^{dev}\right) + u_B\left(b^{dev}, c^*\left(b^0\right)\right) + (1-\gamma)\left(v_A\left(c^*\left(b^0\right)\right) + u_C\left(b^0, c^*\left(b^0\right)\right)\right) \\ \leq \; & v_A\left(b^0\right) + u_B\left(b^0, c^*\left(b^0\right)\right) + (1-\gamma)\left(v_A\left(c^*\left(b^0\right)\right) + u_C\left(b^0, c^*\left(b^0\right)\right)\right) \end{aligned}$$

It follows that, for all $b \leq b^0$

$$v_A(b) + u_B\left(b, c^*\left(b^0\right)\right) \leq v_A\left(b^0\right) + u_B\left(b^0, c^*\left(b^0\right)\right).$$

For future reference, note that this implies

$$\frac{\partial}{\partial b}\left(v_A(b) + u_B(b,c)\right)\Big|_{b^0, c^*(b^0)} \geq 0. \tag{28}$$

Step 5: there is a profitable upward deviation We have shown in step 3 that any upward deviation to a $b > b^0$ will be disclosed. Moreover, the joint surplus of A and B when their decison b is disclosed is

$$S_{AB}(b) = v_A(b) + u_B(b, c^*(b)) + (1-\gamma)\left(v_A(c^*(b)) + u_C(b, c^*(b))\right)$$

as in the main part of the paper. In step 1, we have shown that in the supposed equilibrium, the joint surplus of A and B is $S_{AB}(b^0)$ (see (24)).

We will show that $S_{AB}(b)$ is strictly increasing in b at $b = b_0$. By the assumption that the bargaining problems are smooth, $S_{AB}(b)$ is differentiable, and

$$\begin{aligned} S'_{AB}(b) &= \frac{\partial}{\partial b}\left(v_A(b) + u_B(b, c^*(b))\right) + (1-\gamma)\frac{\partial}{\partial b}u_C(b, c^*(b)) \\ &\quad + \frac{dc^*(b)}{db}\left(\frac{\partial u_B(b, c^*(b))}{\partial c} + (1-\gamma)\frac{\partial}{\partial c}\left(v_A(c^*(b)) + u_C(b, c^*(b))\right)\right) \end{aligned}$$

By definition, $c^*(b)$ maximizes $v_A(c) + u_C(b,c)$. Since we assumed that $c^*(b)$ is interior, the first-order condition

$$\frac{\partial}{\partial c}\left(v_A\left(c^*\left(b\right)\right)+u_C\left(b,c^*\left(b\right)\right)\right)=0$$

holds. Thus,

$$S'_{AB}\left(b\right)=\frac{\partial}{\partial b}\left(v_A\left(b\right)+u_B\left(b,c^*\left(b\right)\right)\right)+\left(1-\gamma\right)\frac{\partial}{\partial b}u_C\left(b,c^*\left(b\right)\right)+\frac{dc^*\left(b\right)}{db}\frac{\partial u_B\left(b,c^*\left(b\right)\right)}{\partial c}$$

By super-modularity of u_C and additive separability of u_A, $dc^*\left(b\right)/db\geq 0$. Moreover, by positive externalities and strict monotonicity of u_B in c (and of u_C in b, by symmetry),

$$\frac{\partial}{\partial b}u_C\left(b,c^*\left(b\right)\right)>0 \quad \text{and} \quad \frac{\partial u_B\left(b,c^*\left(b\right)\right)}{\partial c}>0.$$

Hence,

$$S'_{AB}\left(b\right)>\frac{\partial}{\partial b}\left(v_A\left(b\right)+u_B\left(b,c^*\left(b\right)\right)\right).$$

Evaluated at b^0

$$S'_{AB}\left(b^0\right)>\frac{\partial}{\partial b}\left(v_A\left(b\right)+u_B\left(b,c\right)\right)\big|_{b^0,c^*(b^0)}\geq 0$$

where the last inequality has been established in Step 4 (see equation (28)).

To summarize, in any supposed nondisclosure equilibrium, a small upward deviation to some $b^{dev}>b^0$ close to b^0 leads to a strictly higher joint surplus for A and B, which is a contradiction to the supposed equilibrium. Hence, no nondisclosure equilibrium exists.

A.4.2 Example with a supplier and retailers in Cournot competition

In this section, we show that there is a unique disclosure equilibrium even if agents' utility functions are sub-modular, as long as the sub-modularity is sufficiently small, in the supplier-retailers example presented towards the end of Section 4.

In this example, retailers compete in Cournot fashion and their utility functions are $u_B(b,c)=(1-b+kc)b$ and $u_C(b,c)=(1-c+kb)c$. For $k\in[-1,0)$, the retailers produce substitutes, which implies that their utility functions are sub-modular. The supplier's utility function is $u_A(b,c)=-y(b+c)$, with $0\leq y\leq 1$, which exhibits additive-separability (since we set $x=0$). In case of disclosure, we know from Section 4 that sequence BC is optimal for the supplier. Determining the optimal first-stage decision b^{BC} yields

$$b^{BC}=\frac{(1-y)(2(1+k)-\gamma k)}{4-(3-\gamma)k^2}. \tag{29}$$

To determine the conditions for the non-existence of a non-disclosure equilibrium, we follow the same steps as in the proof of Proposition 9. Denoting the first-stage quantity in a candidate equilibrium with non-disclosure by b^0, the equilibrium payoff of the supplier is given by (23). With the demand and cost function of the example, this payoff net of t_B can be written as

$$-yb^0 + (1-\gamma)\left(-\frac{y(1+kb^0-y)}{2} + \frac{(1+kb^0+y)(1+kb^0-y)}{4}\right).$$

If A and B deviate in the first-stage negotiation to a quantity $b^{dev} \neq b^0$ and A does not disclose this deviation to C, A's payoff (again net of t_B) is

$$-yb^{dev} + (1-\gamma)\left(-\frac{y(1+kb^0-y)}{2} + \frac{(1+kb^0+y)(1+kb^0-y)}{4}\right). \tag{30}$$

Instead, if A discloses the deviation, her payoff is

$$-yb^{dev} + (1-\gamma)\left(-\frac{y(1+kb^{dev}-y)}{2} + \frac{(1+kb^{dev}+y)(1+kb^{dev}-y)}{4}\right). \tag{31}$$

Comparing (30) with (31), A prefers non-disclosure to disclosure if

$$-\frac{1}{4}k\left(b^{dev}-b^0\right)(1-\gamma)\left(2(1-y)+k(b^{dev}+b^0)\right) > 0 \quad \Leftrightarrow \quad b^{dev} > b^0.$$

This result is the equivalent result to the one obtained in step 3 of the proof of Proposition 9, but in this case with negative externalities. The principal then prefers non-disclosure iff $b^{dev} > b^0$.

Since the supplier will not disclose first-stage quantities that are above the quantity in the candidate equilibrium (i.e., b^0), the latter quantity must be sufficiently large to render a deviation unprofitable. This implies that the joint surplus of A and B in the candidate equilibrium must be larger than the one after a deviation. Using the formula in step 4 above, this is equivalent to

$$-yb^0 + (1-b^0+kc(b^0))b^0 \geq -yb^{dev} + (1-b^{dev}+kc(b^0))b^{dev},$$

with $c(b^0) = \left(1-y+kb^0\right)/2$.[54] From this inequality, we obtain that a deviation to $b^{dev} > b^0$ is profitable if $b^{dev} \in (b^0, (1-y)(2-k))$. Therefore, for a non-disclosure equilibrium with a

[54]The second-stage decision $c(b^0)$ is strictly positive in equilibrium. The reason is that the optimal first-stage decision leaves a positive profit for A and B, which implies that per-unit margin is positive. As the quantities sold by B and C are (imperfect) substitutes, C will also obtain a positive margin.

first-stage decision of b^0 to exist, we must have

$$b^0 \geq \frac{1-y}{2-k}. \tag{32}$$

We can now compare the threshold value of b^0 given by the right-hand side of (32) with b^{BC} given by (29). Doing so yields

$$\frac{1-y}{2-k} > \frac{(1-y)(2(1+k)-\gamma k)}{4-(3-\gamma)k^2} \quad \Leftrightarrow \quad k < -2\,(1-\gamma)\,.$$

Therefore, if the last inequality holds, b^{BC} is lower than the threshold of b^0. This implies that it is profitable for A and B to deviate from negotiating $b = b^0$ and A not disclosing it to negotiating $b = b^{BC}$ and A disclosing it. As a consequence, a non-disclosure equilibrium does not exist and the unique equilibrium is the disclosure one.[55]

A.4.3 Proof of Proposition 10

Existence of a disclosure equilibrium Suppose that A proposes in stage 2. Then, C will accept any offer such that $t_C \leq u_C\,(c)$. A will propose $c^*\,(b)$ and A's payoff is

$$\max_c \{u_A\,(b,c) + u_C\,(c)\} + t_B.$$

Note this is true no matter whether A has disclosed or not.

Suppose that C proposes in stage 2. Suppose in the off-equilibrium event that A has not disclosed, C believes that $b = 0$ if u_A is sub-modular (and $b = \max \mathcal{B}$ if u_A is super-modular).[56] For the rest of this proof, we will focus on the case where u_A is sub-modular. A similar argument applies in case u_A is super-modular.

Given his belief, C expects A to accept iff $t_C \geq -u_A\,(0,c)$, thus C will set $t_C = -u_A\,(0,c)$ and propose

$$\begin{aligned} c^*\,(0) &= \arg\max_c \{u_C\,(c) + u_A\,(0,c)\}\,, \\ t_C &= -u_A\,(0, c^*\,(0))\,. \end{aligned}$$

Then, the payoff of A from accepting is $u_A\,(b, c^*\,(0)) - u_A\,(0, c^*\,(0)) + t_B$, and A's payoff from

[55] Analogously to step 5 above, one can also check that the derivative of the joint disclosure surplus of A and B with respect to b is strictly negative at $b = (1-y)/(2-k)$ if $k < -2\,(1-\gamma)$.

[56] Because $\mathcal{B}$ is compact, a $b^{\max} \in \mathcal{B}$ such that $b^{\max} \geq b$ for all $b \in \mathcal{B}$ exists.

rejecting is $u_A(b,0)+t_B$. Note that

$$u_A(b,c^*(0))-u_A(0,c^*(0))\leq u_A(b,0)-u_A(0,0)=u_A(b,0)$$

for all b by sub-modularity. Thus A's payoff is $u_A(b,0)+t_B$. This is also her payoff if she discloses b and C proposes. This implies that there is an equilibrium in which A discloses. The equilibrium is not strict, because A has the same payoff if she does not disclose.

No non-disclosure equilibrium if u_A is strictly super-modular or strictly sub-modular The proof is by contradiction. We first determine the payoffs in a candidate equilibrium in which A does not disclose. Denote the equilibrium path first stage decision by b^0.

We first show that A's payoff in the supposed equilibrium is

$$\gamma u_A\left(b^0,0\right)+(1-\gamma)\left(u_A\left(b^0,c^*\left(b^0\right)\right)+u_C\left(c^*\left(b^0\right)\right)\right)+t_B. \tag{33}$$

Consider stage 2. Suppose that A proposes. C will accept any offer such that $t_C\leq u_C(c)$. Note that C's beliefs about the first-stage decisions are irrelevant here, since u_C does not depend on b. Thus, A will set $t_C=u_C(c)$ and solve $\max_c\{u_A(b^0,c)+u_C(c)\}$. The solution is $c^*(b^0)$. The payoff of A is

$$u_A\left(b^0,c^*\left(b^0\right)\right)+u_C\left(c^*\left(b^0\right)\right)+t_B.$$

Now suppose that C proposes. Since nondisclosure is on the equilibrium path, C believes that $b=b^0$ with probability one. Thus, C believes that A will accept any offer with $u_A(b^0,c)+t_C\geq u_A(b^0,0)$. Therefore, he will set $t_C=u_A(b^0,0)-u_A(b^0,c)$ and propose the decision

$$\arg\max_c\left\{u_C(c)-\left(u_A\left(b^0,0\right)-u_A\left(b^0,c\right)\right)\right\}=c^*\left(b^0\right)$$

and the transfer

$$t_C=u_A\left(b^0,0\right)-u_A\left(b^0,c^*\left(b^0\right)\right).$$

A then obtains the payoff $u_A(b^0,0)+t_B$.

Taking the expectation over who proposes in stage 2, it follows that A's payoff in the supposed equilibrium is given by (33). Moreover, the joint surplus of A and B is

$$u_B\left(b^0\right)+\gamma u_A\left(b^0,0\right)+(1-\gamma)\left\{u_A\left(b^0,c^*\left(b^0\right)\right)+u_C\left(c^*\left(b^0\right)\right)\right\}. \tag{34}$$

We will next show that there always exists a profitable deviation from $b = b^0$ in the first stage. We denote this deviation by b^{dev} and assume that A does not disclose this deviation.[57]

Consider the negotiation in stage 2. Suppose A proposes. Since u_C does not depend on b, C will accept any offer such that $u_C(c) \geq t_C$, independently of his belief about b. A will propose $c^*\left(b^{dev}\right)$ and $t_C = u_C\left(c^*\left(b^{dev}\right)\right)$, C will accept, and A's payoff is $u_A\left(b^{dev}, c^*\left(b^{dev}\right)\right) + u_C\left(c^*\left(b^{dev}\right)\right) + t_B$. Note that A bases her offer on the true decision b^{dev}.

Suppose C proposes. Since non-disclosure is on the equilibrium path, C believes that $b = b^0$ with probability 1. Thus C proposes $c^*(b^0)$ and $t_C = u_A(b^0, 0) - u_A(b^0, c^*(b^0))$. If A accepts this offer, her payoff is

$$u_A\left(b^{dev}, c^*\left(b^0\right)\right) + u_A\left(b^0, 0\right) - u_A\left(b^0, c^*\left(b^0\right)\right) + t_B.$$

If A rejects, her payoff is $u_A\left(b^{dev}, 0\right) + t_B$. So, the payoff of A is

$$\max\left\{u_A\left(b, c^*\left(b^0\right)\right) + u_A\left(b^0, 0\right) - u_A\left(b^0, c^*\left(b^0\right)\right), u_A(b, 0)\right\}|_{b=b^{dev}} + t_B.$$

Taking the expectation over who proposes in stage 2, the payoff of A from an undisclosed first-stage deviation is

$$\begin{aligned}&\gamma \max\left\{u_A\left(b^{dev}, c^*\left(b^0\right)\right) + u_A\left(b^0, 0\right) - u_A\left(b^0, c^*\left(b^0\right)\right), u_A\left(b^{dev}, 0\right)\right\} \\ &+ (1-\gamma)\left(u_A\left(b^{dev}, c^*\left(b^{dev}\right)\right) + u_C\left(c^*\left(b^{dev}\right)\right)\right) + t_B.\end{aligned}$$

Similarly, the joint payoff of A and B, when they agree on a first-stage decision b and A does not disclose it, is given by

$$\begin{aligned}S_{AB,N}(b) \quad : \quad &= u_B(b) + \gamma \max\left\{u_A\left(b, c^*\left(b^0\right)\right) + u_A\left(b^0, 0\right) - u_A\left(b^0, c^*\left(b^0\right)\right), u_A(b, 0)\right\} \\ &+ (1-\gamma)\left\{u_A(b, c^*(b)) + u_C(c^*(b))\right\}.\end{aligned}$$

The additional subscript N stands for non-disclosure.

For similar reasons as in step 3 of the proof of Proposition 9, if a non-disclosure equilibrium exists, it must involve $b^0 > 0$. To see this, suppose that $b^0 = 0$. The joint surplus of A and B in the supposed equilibrium is $S_{AB}^{BC}(0)$ by (1) and (34). In the game with observable first-stage contracts, equilibrium decisions are unique and interior by assumption, thus $b^{BC} > 0$ and $S_{AB}^{BC}\left(b^{BC}\right) > S_{AB}^{BC}(0)$. A deviation to b^{BC} gives A and B a joint surplus of $S_{AB,N}\left(b^{BC}\right) \geq S_{AB}^{BC}\left(b^{BC}\right) > S_{AB}^{BC}(0)$, contradicting equilibrium.

[57]Since there are no externalities between agents, A never has a strict incentive to disclose. Moreover, the first agent is not affected by what happens in the second stage. Thus it is without loss of generality to assume that the deviation is not disclosed.

Strict super-modularity or strict sub-modularity of u_A implies that

$$\frac{\partial u_A\left(b, c^*\left(b^0\right)\right)}{\partial b}\Big|_{b=b^0} \neq \frac{\partial u_A\left(b, 0\right)}{\partial b}\Big|_{b=b^0}.$$

It follows that the payoff of A, considered as a function of b^{dev}, has a kink at b^0, with

$$\lim_{b\uparrow b_0} g'\left(b\right) < \lim_{b\downarrow b^0} g'\left(b\right) \tag{35}$$

where

$$g\left(b\right) := \max\left\{u_A\left(b, c^*\left(b^0\right)\right) + u_A\left(b^0, 0\right) - u_A\left(b^0, c^*\left(b^0\right)\right), u_A\left(b, 0\right)\right\}.$$

Since $\gamma > 0$, inequality (35) implies that

$$\lim_{b\uparrow b_0} S'_{AB,N}\left(b\right) < \lim_{b\downarrow b^0} S'_{AB,N}\left(b\right). \tag{36}$$

In the supposed equilibrium, the joint payoff of A and B is $S_{AB,N}\left(b^0\right)$, see equation (34). Moreover, we must have $\lim_{b\uparrow b_0} S'_{AB,N}\left(b\right) \geq 0$, otherwise deviating to a $b < b^0$ in some vicinity of b^0 is a profitable deviation. Similarly, we must have $\lim_{b\downarrow b^0} S'_{AB,N}\left(b\right) \leq 0$, otherwise a small upward deviation to some $b > b^0$ is profitable. Hence, $\lim_{b\uparrow b_0} S'_{AB,N}\left(b\right) \geq 0 \geq \lim_{b\downarrow b^0} S'_{AB,N}\left(b\right)$. This contradicts inequality (36).

A.5 Exclusive Contracts

A.5.1 Single contract: Proof of Proposition 11

Call the agent who negotiates in stage $i = 1, 2$ agent i, with bargaining power β_i. Let d_i denote the decision with agent i, and t_i the transfer of agent i. A contract in stage 1 is either non-exclusive and fixes (d_1, t_1) independent of what happens in stage 2, or exclusive and fixes (d_1, t_1) together with the exclusion clause that $d_2 = 0$, which fully binds A.

Consider stage 2. If in stage 1 an exclusive contract was signed, then $d_2 = t_2 = 0$. If in stage 1 a non-exclusive contract was signed, stage 2 is as in our main model.

Consider stage 1. The principal and agent 1 will agree on an exclusive contract if

$$\begin{aligned} &\max_{d_1}\left\{u_1\left(d_1, 0\right) + u_A\left(d_1, 0\right)\right\} \\ > \; &\max_{d_1}\left\{u_1\left(d_1, f\left(d_1\right)\right) + \beta_2 u_A\left(d_1, 0\right) + \left(1-\beta_2\right)\left(u_A\left(d_1, f\left(d_1\right)\right) + u_2\left(d_1, f\left(d_1\right)\right)\right)\right\} \end{aligned} \tag{37}$$

The left hand side is their joint surplus from an exclusive contract, whereas the right hand side is their joint surplus without exclusion (as in our main model). Without loss of generality, we break the tie in favor of non-exclusion.

Note that

$$\begin{aligned} u_A(d_1, f(d_1)) + u_2(d_1, f(d_1)) &= \max_{d_2}\{u_A(d_1, d_2) + u_2(d_1, d_2)\} \\ &\geq u_A(d_1, 0) + u_2(d_1, 0) = u_A(d_1, 0). \end{aligned}$$

Therefore, the joint surplus of A and agent 1 without exclusion (given by the right-hand-side of (37)) is decreasing in β_2.

An exclusive contract commits A not to deal with agent 2. Thus the joint surplus of A and agent 1 from an exclusive contract does not depend on β_2. By contrast, their joint payoff from a non-exclusive contract is decreasing in β_2, as shown above. It follows that, if exclusive contracts are used in sequence BC (where $\beta_2 = \gamma$) they are also used in sequence CB (where $\beta_2 = \beta > \gamma$). This leaves three possible cases: (i) there is exclusion both in BC and in CB, (ii) exclusion in CB but not in BC, and (iii) in both sequences there is no exclusion.

We begin with case (i): exclusion in BC and in CB.

Lemma 3 *Suppose that*

$$\begin{aligned} &\max_b\{u_B(b,0) + u_A(b,0)\} \\ > \ &\max_b\{u_B(b, f(b)) + \gamma u_A(b,0) + (1-\gamma)(u_A(b, f(b)) + u_C(b, f(b)))\}. \end{aligned} \tag{38}$$

Then there is exclusion in sequence BC and in sequence CB; hence, $U_A^{BC} = U_A^{CB}$ and $S^{BC} = S^{CB}$.

Proof. By (37), there is exclusion in both sequences. Consider sequence BC. If the principal proposes, agent B is willing to pay up to $t_B = u_B(b,0)$, and A's payoff is $\max_b\{u_A(b,0) + u_B(b,0)\}$. When agent B proposes, the principal's payoff is given by her outside option

$$(1-\gamma)\max_c\{u_A(0,c) + u_C(0,c)\}.$$

In expectation, therefore, the principal's payoff in sequence BC is

$$\begin{aligned} U_A^{BC} &= (1-\beta)\max_b\{u_A(b,0) + u_B(b,0)\} + \beta(1-\gamma)\max_c\{u_A(0,c) + u_C(0,c)\} \\ &= (1-\beta\gamma)\max_b\{u_A(b,0) + u_B(b,0)\}. \end{aligned}$$

Note this depends only on the product $\beta\gamma$. Therefore, A's expected payoff from exclusive contracts is the same in sequences BC and CB : $U_A^{BC} = U_A^{CB}$. Moreover, the joint surplus of A, B and C is

$$S^{BC} = S^{CB} = \hat{S} := \max_{d_1} \{u_A(d_1, 0) + u_1(d_1, 0)\}$$

in both sequences. ■

We now turn to case (ii): exclusion in CB but not in BC.

Lemma 4 *Suppose that*

$$\begin{aligned} & \max_b \{u_B(b, f(b)) + \gamma u_A(b, 0) + (1-\gamma)(u_A(b, f(b)) + u_C(b, f(b)))\} \\ \geq & \max_b \{u_B(b, 0) + u_A(b, 0)\} \\ > & \max_b \{u_B(b, f(b)) + \beta u_A(b, 0) + (1-\beta)(u_A(b, f(b)) + u_C(b, f(b)))\} \end{aligned} \tag{39}$$

Then, there is exclusion in sequence CB, but not in BC. Moreover, $U_A^{BC} \geq U_A^{CB}$ and $S^{BC} > S^{CB}$.

Proof. By (37), there is exclusion in sequence CB, but not in BC. Since there is exclusion in sequence CB,

$$U_A^{CB} = (1-\beta\gamma) \max_b \{u_A(b, 0) + u_B(b, 0)\}$$

and $S^{CB} = \hat{S}$ as in case (i).

In sequence BC, our main analysis applies:

$$\begin{aligned} U_A^{BC} = & (1-\beta) \max_b \{u_B(b, f(b)) + \gamma u_A(b, 0) + (1-\gamma)(u_A(b, f(b)) + u_C(b, f(b)))\} \\ & +\beta(1-\gamma) \max_b \{u_A(b, 0) + u_B(b, 0)\} \end{aligned}$$

Thus,

$$\begin{aligned} & U_A^{BC} - U_A^{CB} \\ = & (1-\beta) [\max_b \{u_B(b, f(b)) + \gamma u_A(b, 0) + (1-\gamma)(u_A(b, f(b)) + u_C(b, f(b)))\} \\ & - \max_b \{u_A(b, 0) + u_B(b, 0)\}]. \end{aligned}$$

By the first inequality in (39), it follows that $U_A^{BC} \geq U_A^{CB}$. Moreover, the joint surplus of A and B is

$$\begin{aligned} & \max_b \{u_B(b, f(b)) + \gamma u_A(b, 0) + (1-\gamma)(u_A(b, f(b)) + u_C(b, f(b)))\} \\ \geq & \max_b \{u_B(b, 0) + u_A(b, 0)\} = \hat{S}. \end{aligned}$$

The joint surplus of all three players, S^{BC}, is the joint surplus of A and B, plus the payoff of C, which is non-negative. Thus, $S^{BC} \geq \hat{S} = S^{CB}$. ■

Finally, if

$$\begin{aligned} & \max_b \{u_B(b, f(b)) + \beta u_A(b, 0) + (1-\beta)(u_A(b, f(b)) + u_C(b, f(b)))\} \\ \geq \quad & \max_b \{u_B(b, 0) + u_A(b, 0)\}, \end{aligned} \tag{40}$$

exclusive contracts are not used in equilibrium, irrespective of the sequence (case (iii)), and our main analysis applies.

In all three possible cases, the joint surplus is weakly higher in sequence BC than CB, as in Proposition 1. Which of these three cases applies depends on the externalities between agents. Suppose that externalities between agents are positive, or absent. Then

$$\begin{aligned} & \max_{d_1} \{u_1(d_1, f(d_1)) + \beta_2 u_A(d_1, 0) + (1-\beta_2)(u_A(d_1, f(d_1)) + u_2(d_1, f(d_1)))\} \\ \geq \quad & \max_{d_1} \{u_1(d_1, f(d_1)) + u_A(d_1, 0)\} \\ \geq \quad & \max_{d_1} \{u_1(d_1, 0) + u_A(d_1, 0)\} \end{aligned}$$

where the first inequality uses that the joint payoff is decreasing in β_2, and the second inequality uses positive or no externalities between agents. This chain of inequalities holds regardless of the sequence. It implies that (40) is fulfilled: the joint surplus of A and agent 1 is higher without exclusion, and they will agree on a non-exclusive contract in equilibrium. To summarize, with positive or no externalities between agents, there is no exclusion in equilibrium; hence, our analysis in the main text above applies. In particular, Propositions 3, 4 and 5 hold.

Now consider negative externalities between agents. Then all three cases are possible. In cases (i) and (ii), A prefers sequence BC by the Lemmas above. In case (iii), there is no exclusion; hence, our main analysis applies. Therefore, with negative negative externalities between agents, A prefers sequence BC, as in Proposition 2 above.

A.5.2 Menu of contracts: Proof of Proposition 12

A contract in stage 1 is a menu specifying (d_1^e, t_1^e, d_1, t_1), where the exclusive dealing contract (d_1^e, t_1^e) is executed when the decision reached with agent 2 is zero $(d_2 = 0)$, and (d_1, t_1) is relevant otherwise.

Consider stage 2. The joint surplus of A and agent 2 is $u_A(d_1^e, 0) + t_1^e$ if $d_2 = 0$, and

$u_A(d_1, d_2) + u_2(d_1, d_2) + t_1$ if $d_2 > 0$. The second stage decision will be[58]

$$f(d_1) := \arg\max_{d_2} \{u_A(d_1, d_2) + u_2(d_1, d_2)\} > 0$$

if

$$\max_{d_2} \{u_A(d_1, d_2) + u_2(d_1, d_2)\} + t_1 \geq u_A(d_1^e, 0) + t_1^e, \tag{41}$$

and $d_2 = 0$ otherwise. (As above, we break ties in favor of non-exclusivity.)

Suppose A proposes in stage 2. If (41) holds, A will propose $d_2 = f(d_1)$ and $t_2 = u_2(d_1, d_2)$; A's payoff is $u_A(d_1, f(d_2)) + u_2(d_1, f(d_2)) + t_1$. If (41) does not hold, then $d_2 = 0$ and A's payoff is $u_A(d_1^e, 0) + t_1^e$.

Suppose agent 2 proposes in stage 2. If (41) holds, agent 2 will propose $d_2 = f(d_1)$ and t_2 such that A is indifferent between accepting and rejecting the offer. The payoff of A is $u_A(d_1^e, 0) + t_1^e$. This is also A's payoff if (41) is violated.

We now turn to stage 1. Consider first a contract menu such that (41) holds. With such contracts, it is possible to bring down agent 2 to his outside utility of zero, even when agent 2 proposes. To do so, choose (d_2^e, t_1^e) such that (41) holds with equality:

$$u_A(d_1, f(d_1)) + u_2(d_1, f(d_1)) + t_1 = u_A(d_1^e, 0) + t_1^e.$$

The principal's payoff is then $u_A(d_1, f(d_1)) + u_2(d_1, f(d_1)) + t_1$, no matter who proposes in stage 2. Moreover, the joint surplus of A and agent 1 is

$$S(d_1, f(d_1)) := u_A(d_1, f(d_1)) + u_1(d_1, f(d_1)) + u_2(d_1, f(d_1)).$$

If A proposes in stage 1, she will propose $d_1^* = \arg\max S(d_1, f(d_1))$ and $t_1 = u_1(d_1^*, f(d_1^*))$, giving her a payoff

$$S^* := \max_{d_1} S(d_1, f(d_1)).$$

If agent 1 proposes, he will propose d_1^*, and t_1 such that A is brought down to her outside option,

$$(1 - \beta_2) \max_{d_2} \{u_A(0, d_2) + u_2(0, d_2)\}.$$

In either case, the joint payoff of A and agent 1 from a contract that satisfies (41) is S^*.

A contract menu that violates (41) leads to $d_2 = 0$. The joint surplus of A and agent 1

[58] As mentioned, we focus on the case where the second stage decision without an exclusive contract is not zero, hence $f(d_1) > 0$.

is then

$$\max_{d_1^e} \left\{ u_A \left(d_1^e, 0 \right) + u_1 \left(d_1^e, 0 \right) \right\} =: \hat{S}.$$

We now consider whether A and agent 1 will agree on a contract that satisfies or violates (41). There are three cases. First, if $\hat{S} < S^*$, the joint surplus of A and agent 1 from a contract that satisfies (41) is higher. Then the joint surplus of all three players is the same in both sequences, $S^{BC} = S^{CB} = S^*$. Moreover, $U_A^{BC} = (1-\beta) S^* + \beta (1-\gamma) \hat{S}$, $U_A^{CB} = (1-\gamma) S^* + \gamma (1-\beta) \hat{S}$, and therefore,

$$U_A^{CB} - U_A^{BC} = (\beta - \gamma) \left(S^* - \hat{S} \right) > 0;$$

hence, A prefers sequence CB.

Second, if $\hat{S} > S^*$, the joint surplus of A and agent 1 from a contract that violates (41) is higher. Then the joint surplus of all three players is $S^{BC} = S^{CB} = \hat{S}$. Moreover,

$$U_A^{BC} = (1-\beta) \hat{S} + \beta (1-\gamma) \hat{S} = (1-\beta\gamma) \hat{S} = U_A^{CB};$$

hence, A is indifferent between the sequences.

Third, if $\hat{S} = S^*$, satisfying or violating (41) gives A and agent 1 the same joint surplus, which implies that they are indifferent. No matter how they break the tie, the joint surplus of all three players is $S^{BC} = S^{CB} = \hat{S}$, and $U_A^{BC} = U_A^{CB} = (1-\beta\gamma) \hat{S}$; hence, A is indifferent between the sequences.

To summarize, in all three cases, $S^{BC} = S^{CB}$, and $U_A^{CB} \leq U_A^{BC}$ so A (weakly) prefers sequence CB.

A.6 N agents

We call the agent with whom A negotiates in stage t agent t. The bargaining power of agent t in sequence T is denoted by $\beta_{T,t}$. The decision in stage t is denoted by $d_{T,t}$. We omit the reference to the sequence T when it is clear from the context. We assume that any subgame has a unique subgame perfect Nash equilibrium (SPNE), after breaking ties that a player accepts when indifferent between accepting or rejecting an offer. Moreover, A commits to a sequence of negotiations ex ante. Prior to the negotiations, the sequence chosen by A becomes common knowledge.

The section proceeds in several steps. We first derive a tractable expression for the principal's expected payoff in a given sequence (Section A.6.1). To do so, we show that this payoff has a recursive structure, which can be used to find an algorithm that determines

A's payoff. We next compare two different sequences with each other, in which we exchange the positions of two adjacent agents (Section A.6.2). We finally use the insights from this comparison to prove Propositions 13 and 14 (Section A.6.3).

A.6.1 The principal's expected payoff

This subsection investigates the structure of A's equilibrium payoff in a given sequence. We allow for externalities between agents, interactions of the decisions in u_A, and do not assume symmetry.[59] We will only use symmetry later in the proofs of Propositions 13 and 14.

Recursion relation for A's payoff To obtain a recursive structure for A's payoff, we determine A's expected payoff in the subgame starting at the beginning of stage t — i.e., before nature draws the proposer in stage t. We denote by h_{t-1} the history of decisions until this stage, that is, $h_{t-1} = (d_1, ..., d_{t-1})$. Let then $U_A^t \big|_{h_{t-1}} (T)$ be A's expected SPNE payoff in the subgame starting at stage t, given sequence T. This payoff is net of the transfers from the agents who have already bargained with A, but includes expected future transfers.[60] Let h_0 denote the beginning of the game. For example, $U_A^1 |_{h_0} (T)$ is A's expected payoff at the beginning of the game. Let $f_T (h_t)$ denote the path of SPNE decisions in a subgame starting after history h_t in sequence T.

Consider stage t, and a history of decisions h_{t-1}. With probability β_t, the agent proposes. In this case, the principal's payoff is given by her outside option, which is to continue the game in $t+1$ with a decision $d_t = 0$ and a transfer $t_t = 0$. With probability $(1 - \beta_t)$, the principal proposes. The principal will then set the transfer equal to the largest transfer the agent is willing to accept, $t_t = u_t (h_{t-1}, d_t, f_T (h_{t-1}, d_t))$. She will choose the decision d_t to maximize her payoff, which is (again net of transfers prior to stage t, which are sunk in stage t) :

$$u_t (h_{t-1}, d_t, f_T (h_{t-1}, d_t)) + U_A^{t+1} \big|_{(h_{t-1}, d_t)} (T) .$$

These considerations show that the following recursion relation for A's payoffs holds for

[59] The precise definition of symmetry in case of N agents is that (i) u_A is symmetric in that any permutation of $(d_1, ..., d_n)$ leaves u_A constant, (ii) for any two agents i and $j \neq i$, $u_i (d') = u_j (d)$ where d' denotes the vector of decisions obtained from $d = (d_1, ..., d_n)$ by exchanging d_i with d_j, and (iii) any permutation of the decisions of agents $k \neq i$ leaves u_i constant.

[60] Note that the history of decisions h_{t-1} is comprised only of the parts of the history up to stage t, which are relevant for the equilibrium in the subgame. In contrast, the past transfers, who proposed in which stage, and the acceptance or rejection decisions do not influence the equilibrium of the subgame after a history of decisions h_{t-1}.

$t = 1, ..., \ N-1$:

$$U_A^t \left|_{h_{t-1}} (T) \right. \tag{42}$$
$$= \beta_{T,t} U_A^{t+1} \left|_{(h_{t-1},0)} (T) \right. + (1-\beta_{T,t}) \max_{d_t} \left\{ u_t (h_{t-1}, d_t, f_T (h_{t-1}, d_t)) + U_A^{t+1} \left|_{(h_{t-1},d_t)} (T) \right. \right\}$$

A similar reasoning applied to stage N shows that the following initial condition holds:

$$U_A^N \left|_{h_{N-1}} (T) \right. \tag{43}$$
$$= \beta_{T,N} u_A (h_{N-1}, 0) + (1-\beta_{T,N}) \max_{d_N} \{ u_N (h_{N-1}, d_N) + u_A (h_{N-1}, d_N) \}.$$

Let $f_{T,t} (h_{t-1})$ denote the SPNE decision taken in stage t of sequence T, after a history of decisions h_{t-1}. Note that $f_{T,t} (h_{t-1})$ is the first component of the vector $f_T (h_{t-1})$.[61] In a SPNE, the decision taken in stage t maximizes the joint surplus of the principal and agent t, regardless who proposes. Therefore,

$$f_{T,N} (h_{N-1}) = \arg\max_{d_N} \{ u_N (h_{N-1}, d_N) + u_A (h_{N-1}, d_N) \} \tag{44}$$

and

$$f_{T,t} (h_{t-1}) = \arg\max_{d_t} \left\{ u_t (h_{t-1}, d_t, f_T (h_{t-1}, d_t)) + U_A^{t+1} \left|_{(h_{t-1},d_t)} (T) \right. \right\}. \tag{45}$$

Expected payoff in different subgames We now define the *fictitious history*, a concept we will use to find a succinct expression for the principal's equilibrium payoff. Let $r_t = 1$ if A responds in stage t, and $r_t = 0$ if A proposes in stage t. An event r is a vector $r = (r_1, ..., r_N)$ describing who proposes in which stage. Using that $f_{T,1} (h_0)$ is the SPNE decision in $t = 1$ in sequence T, we now recursively define a *fictitious history* as follows. Let

$$\eta_{T,1} = \begin{cases} f_{T,1} (h_0), & \text{if } r_1 = 0, \\ 0, & \text{if } r_1 = 1; \end{cases}$$

moreover, for $t = 2, ..., N$:

$$\eta_{T,t} = \begin{cases} (\eta_{T,t-1}, f_{T,t} (\eta_{T,t-1})), & \text{if } r_t = 0, \\ (\eta_{T,t-1}, 0), & \text{if } r_t = 1, \end{cases}$$

where $\eta_{T,t}$ is a vector of t decisions. In general, it differs across sequences and events. The relevant sequence is indicated in the first subscript. We sometimes write $\eta_{T,t} (r)$ to indicate

[61] Recall that $f_T (h_{t-1})$ is a path of SPNE decisions. Since h_{t-1} is a history of $t-1$ decisions, $N-(t-1)$ agents remain, thus $f_T (h_{t-1})$ is vector of $N-(t-1)$ decisions.

the relevant event. Note that $\eta_{T,t}(r)$ depends on $(r_1, ..., r_t)$, but not on $(r_{t+1}, ..., r_N)$.

In words, the fictitious history is built up as follows. If A proposes in a stage t ($r_t = 0$), the decision in t is the SPNE decision $f_{T,t}(\eta_{t-1})$. If A responds in a stage t ($r_t = 1$), the decision in t is 0.

We can similarly define a fictitious history after a given history of decisions h_{t-1}: let

$$\eta_{T,t}(h_{t-1}, r) = \begin{cases} (h_{t-1}, f_{T,t}(h_{t-1})), & \text{if } r_t = 0, \\ (h_{t-1}, 0), & \text{if } r_t = 1, \end{cases}$$

and for $\tau = t+1, ..., N$

$$\eta_{T,\tau}(h_{t-1}, r) = \begin{cases} (\eta_{T,\tau-1}(h_{t-1}, r), f_{T,\tau}(\eta_{T,\tau-1}(h_{t-1}, r))), & \text{if } r_\tau = 0, \\ (\eta_{T,\tau-1}(h_{t-1}, r), 0), & \text{if } r_\tau = 1. \end{cases}$$

For example, $\eta_{T,\tau}(h_{t-1}, r)$ is a vector of length τ consisting of the decisions h_{t-1} in the first $t-1$ stages, and the fictitious history for decisions in stage t up to stage τ. Note that for $\tau = t+1, ..., N$, $\eta_{T,\tau}(h_{t-1}, r)$ depends on $(r_t, ..., r_\tau)$, but is independent of $(r_1, ..., r_{t-1})$ and $(r_{\tau+1}, ..., r_N)$, as the former is included in h_{t-1} and the latter is not decided yet. This implies that $\eta_{T,N}(h_{t-1}, r)$ denotes a history of N decisions where the first $t-1$ decisions are given by h_{t-1}, and the decisions in the remaining stages are given by the fictitious history, started at t.

Iteratively applying the recursion relation (42) to the initial condition (43) shows that the expected payoff of A at the start of a subgame in stage t (net of transfers already received from agents $1, ..., t-1$) is

$$\begin{aligned} U_A^t\big|_{h_{t-1}}(T) &= \sum_{r \in \{0,1\}^{N-t+1}} \prod_{i=t}^{N} \beta_{T,i}^{r_i} (1-\beta_{T,i})^{1-r_i} [u_A(\eta_{T,N}(h_{t-1}, r)) \\ &\quad + \sum_{j=t}^{N} u_j(\eta_{T,j}(h_{t-1}, r), f_T(\eta_{T,j}(h_{t-1}, r)))]. \end{aligned} \tag{46}$$

In particular, using the definitions of the fictitious history, whenever $r_t = 1$ (i.e., whenever the agent proposes), the decision that enters the fictitious history $\eta_{T,t}$ as the second argument equals zero, which implies $\eta_{T,t}(h_{t-1}, 1) = (h_{t-1}, 0)$. This occurs with probability $\beta_{T,t}$. Instead, with probability $1 - \beta_{T,t}$, the principal proposes. This implies $r_t = 0$ and $\eta_{T,t}(h_{t-1}, 0) = (h_{t-1}, f_{T,t}(h_{t-1}))$.

Since equations (42) and (43) uniquely pin down the expected payoff in any subgame, net of previous transfers, A's expected payoff is as given in equation (46).[62] Evaluating (46) at $t = 1$ then yields A's ex ante expected payoff:

$$\begin{aligned} U_A(T) &= U_A^1|_{h_0}(T) = \qquad (47) \\ &= \sum_{r\in\{0,1\}^N} \prod_{i=1}^{N} \beta_{T,i}^{r_i} (1-\beta_{T,i})^{1-r_i} \left(u_A(\eta_{T,N}(r)) + \sum_{j=1}^{N} u_j(\eta_{T,j}(r), f_T(\eta_{T,j}(r))) \right). \end{aligned}$$

Following this procedure, which automatically sets the decision d_i to zero, if the agent i proposes in stage i, and sets $d_i = f_{T,i}(\eta_{T,i-1}(r))$ if A proposes in stage i, is not a game, but rather an algorithm that helps to tractably analyze A's payoff for different sequences.

A.6.2 Comparison between different sequences

In this section, we use (47) to compare two sequences, denoted by S and T. Recall from above that the SPNE decisions in the negotiation with agent i in case of sequence T are denoted by $f_T(\cdot)$ whereas the ones in case of sequence S are $f_S(\cdot)$, and the two may differ.

To compare the two sequences, suppose that the SPNE decisions of sequence T are used in sequence S. We denote A's expected payoff in this case by $V_A(S,T)$, with

$$V_A(S,T) := \sum_{r\in\{0,1\}^N} \prod_{i=1}^{N} \beta_{S,i}^{r_i} (1-\beta_{S,i})^{1-r_i} \left(u_A(\eta_{T,N}(r)) + \sum_{i=1}^{N} u_i(\eta_{T,i}(r), f_T(\eta_{T,i}(r))) \right). \qquad (48)$$

By construction, if S and T are the same sequence, then the expected payoff equals A's expected payoff from this sequence of the bargaining game. Thus $V_A(T,T) = U_A(T)$. Moreover, as we will show in Subsection A.6.3 below, under the assumptions of Propositions 13 and 14, $V_A(S,T)$ is a lower bound on A's equilibrium payoff in the sequence S: $U_A(S) \geq V_A(S,T)$.[63] Therefore, to show that A's expected payoff is higher in sequence S, it is enough to show that $V_A(S,T) \geq V_A(T,T)$.

We next provide two preliminary steps (shown in Lemmas 5 and 6) that lead to a tractable formula, which will help us to determine the conditions under which the principal bargains with the agents in descending order of their bargaining power. Suppose that in sequence T, the principal does not bargain in this order. This implies that there exist stages $\hat{\imath}$ and $\hat{\imath}+1$ such that $\beta_{T,\hat{\imath}} < \beta_{T,\hat{\imath}+1}$. We will show that exchanging agents $\hat{\imath}$ and $\hat{\imath}+1$, keeping the sequence of agents constant otherwise, increases A's payoff. The new sequence after this

[62] Indeed, evaluating (46) at $t = N$ yields (43).

[63] This is trivial under the assumptions of Proposition 14, where equilibrium decisions do not depend on the negotiation sequence and thus $U_A(S) = V_A(S,S) = V_A(S,T)$. Under the assumptions of Proposition 13, we prove that $V_A(S,S) \geq V_A(S,T)$ below.

adjacent pairwise interchange will be denoted by S. Our aim is to show that $U_A(S) \geq U_A(T)$.

To do so, we first use the relation between the sequences S and T to derive a result on the difference $V_A(S,T) - V_A(T,T)$. Let

$$\begin{aligned} V_A^t(S,T)\big|_{(r_1,\ldots,r_{t-1})} \quad : \quad = & \sum_{(r_t,\ldots,r_N)\in\{0,1\}^{N-t+1}} \prod_{i=t}^{N} \beta_{S,i}^{r_i}(1-\beta_{S,i})^{1-r_i} \\ & * \left(u_A(\eta_{T,N}(r)) + \sum_{i=1}^{N} u_i(\eta_{T,i}(r), f_T(\eta_{T,i}(r)))\right), \end{aligned} \tag{49}$$

that is, $V_A^t(S,T)$ is A's interim expected payoff starting from stage t when using sequence S, but the decisions are the SPNE decisions from sequence T. Note that

$$V_A(S,T) = \sum_{(r_1,\ldots,r_{t-1})\in\{0,1\}^{t-1}} \prod_{i=1}^{t-1} \beta_{S,i}^{r_i}(1-\beta_{S,i})^{1-r_i} V_A^t(S,T)\big|_{(r_1,\ldots,r_{t-1})}. \tag{50}$$

Lemma 5 *Consider a sequence T such that $\beta_{T,\hat{\imath}} < \beta_{T,\hat{\imath}+1}$, and a sequence S which is identical to sequence T except that agents $\hat{\imath}$ and $\hat{\imath}+1$ change places. Then*

$$\begin{aligned} & V_A(S,T) - V_A(T,T) \\ = & \prod_{i=1}^{\hat{\imath}-1} \beta_{T,i}^{r_i}(1-\beta_{T,i})^{1-r_i}(\beta_{T,\hat{\imath}+1} - \beta_{T,\hat{\imath}})\left(V_A^{\hat{\imath}+2}(T,T)\big|_{(r_1,\ldots,r_{\hat{\imath}-1},1,0)} - V_A^{\hat{\imath}+2}(T,T)\big|_{(r_1,\ldots,r_{\hat{\imath}-1},0,1)}\right). \end{aligned}$$

Proof. From (50),

$$V_A(T,T) = \sum_{(r_1,\ldots,r_{\hat{\imath}-1})\in\{0,1\}^{\hat{\imath}-1}} \prod_{i=1}^{\hat{\imath}-1} \beta_{T,i}^{r_i}(1-\beta_{T,i})^{1-r_i} V_A^{\hat{\imath}}(T,T)\big|_{(r_1,\ldots,r_{\hat{\imath}-1})}.$$

Similarly,

$$\begin{aligned} V_A(S,T) &= \sum_{(r_1,\ldots,r_{\hat{\imath}-1})\in\{0,1\}^{\hat{\imath}-1}} \prod_{i=1}^{\hat{\imath}-1} \beta_{S,i}^{r_i}(1-\beta_{S,i})^{1-r_i} V_A^{\hat{\imath}}(S,T)\big|_{(r_1,\ldots,r_{\hat{\imath}-1})} \\ &= \sum_{(r_1,\ldots,r_{\hat{\imath}-1})\in\{0,1\}^{\hat{\imath}-1}} \prod_{i=1}^{\hat{\imath}-1} \beta_{T,i}^{r_i}(1-\beta_{T,i})^{1-r_i} V_A^{\hat{\imath}}(S,T)\big|_{(r_1,\ldots,r_{\hat{\imath}-1})}, \end{aligned}$$

where the second equality follows from $\beta_{S,i} = \beta_{T,i}$ for $i < \hat{\imath}$. Thus,

$$\begin{aligned} & V_A(S,T) - V_A(T,T) \qquad (51) \\ = & \sum_{(r_1,...,r_{\hat{\imath}-1})\in\{0,1\}^{\hat{\imath}-1}} \prod_{i=1}^{\hat{\imath}-1} \beta_{T,i}^{r_i} (1-\beta_{T,i})^{1-r_i} \left(V_A^{\hat{\imath}}(S,T)\big|_{(r_1,...,r_{\hat{\imath}-1})} - V_A^{\hat{\imath}}(T,T)\big|_{(r_1,...,r_{\hat{\imath}-1})} \right). \end{aligned}$$

By the construction of (49), we can write

$$V_A^{\hat{\imath}}(T,T)\big|_{(r_1,...,r_{\hat{\imath}-1})} = \beta_{T,\hat{\imath}} V_A^{\hat{\imath}+1}(T,T)\big|_{(r_1,...,r_{\hat{\imath}-1},1)} + (1-\beta_{T,\hat{\imath}}) V_A^{\hat{\imath}+1}(T,T)\big|_{(r_1,...,r_{\hat{\imath}-1},0)}.$$

Iterating one more time gives

$$\begin{aligned} V_A^{\hat{\imath}}(T,T)\big|_{(r_1,...,r_{\hat{\imath}-1})} = & \ \beta_{T,\hat{\imath}}\beta_{T,\hat{\imath}+1} V_A^{\hat{\imath}+2}(T,T)\big|_{(r_1,...,r_{\hat{\imath}-1},1,1)} \\ & + \beta_{T,\hat{\imath}}(1-\beta_{T,\hat{\imath}+1}) V_A^{\hat{\imath}+2}(T,T)\big|_{(r_1,...,r_{\hat{\imath}-1},1,0)} \\ & + (1-\beta_{T,\hat{\imath}})\beta_{T,\hat{\imath}+1} V_A^{\hat{\imath}+2}(T,T)\big|_{(r_1,...,r_{\hat{\imath}-1},0,1)} \\ & + (1-\beta_{T,\hat{\imath}})(1-\beta_{T,\hat{\imath}+1}) V_A^{\hat{\imath}+2}(T,T)\big|_{(r_1,...,r_{\hat{\imath}-1},0,0)}. \end{aligned}$$

Similarly,

$$\begin{aligned} V_A^{\hat{\imath}}(S,T)\big|_{(r_1,...,r_{t-1})} = & \ \beta_{S,\hat{\imath}}\beta_{S,\hat{\imath}+1} V_A^{\hat{\imath}+2}(S,T)\big|_{(r_1,...,r_{\hat{\imath}-1},1,1)} \\ & + \beta_{S,\hat{\imath}}(1-\beta_{S,\hat{\imath}+1}) V_A^{\hat{\imath}+2}(S,T)\big|_{(r_1,...,r_{\hat{\imath}-1},1,0)} \\ & + (1-\beta_{S,\hat{\imath}})\beta_{S,\hat{\imath}+1} V_A^{\hat{\imath}+2}(S,T)\big|_{(r_1,...,r_{\hat{\imath}-1},0,1)} \\ & + (1-\beta_{S,\hat{\imath}})(1-\beta_{S,\hat{\imath}+1}) V_A^{\hat{\imath}+2}(S,T)\big|_{(r_1,...,r_{\hat{\imath}-1},0,0)}. \end{aligned}$$

For agents $i = \hat{\imath}+1, ..., N$, we have $\beta_{S,i} = \beta_{T,i}$. Thus for for any given $(r_1, ..., r_{\hat{\imath}+1})$,

$$V_A^{\hat{\imath}+2}(S,T)\big|_{(r_1,...,r_{\hat{\imath}+1})} = V_A^{\hat{\imath}+2}(T,T)\big|_{(r_1,...,r_{\hat{\imath}+1})}.$$

By assumption, $\beta_{S,\hat{\imath}} = \beta_{T,\hat{\imath}+1}$ and $\beta_{S,\hat{\imath}+1} = \beta_{T,\hat{\imath}}$. Thus

$$\begin{aligned} V_A^{\hat{\imath}}(S,T)\big|_{(r_1,...,r_{t-1})} = & \ \beta_{T,\hat{\imath}}\beta_{T,\hat{\imath}+1} V_A^{\hat{\imath}+2}(T,T)\big|_{(r_1,...,r_{\hat{\imath}-1},1,1)} \\ & + \beta_{T,\hat{\imath}+1}(1-\beta_{T,\hat{\imath}}) V_A^{\hat{\imath}+2}(T,T)\big|_{(r_1,...,r_{\hat{\imath}-1},1,0)} \\ & + (1-\beta_{T,\hat{\imath}+1})\beta_{T,\hat{\imath}} V_A^{\hat{\imath}+2}(T,T)\big|_{(r_1,...,r_{\hat{\imath}-1},0,1)} \\ & + (1-\beta_{T,\hat{\imath}})(1-\beta_{T,\hat{\imath}+1}) V_A^{\hat{\imath}+2}(T,T)\big|_{(r_1,...,r_{\hat{\imath}-1},0,0)}. \end{aligned}$$

Therefore,

$$\begin{aligned} & V_A^{\hat{\imath}}(S,T)\big|_{(r_1,...,r_{\hat{\imath}-1})} - V_A^{\hat{\imath}}(T,T)\big|_{(r_1,...,r_{\hat{\imath}-1})} \\ = \ & (\beta_{T,\hat{\imath}+1} - \beta_{T,\hat{\imath}}) \left(V_A^{\hat{\imath}+2}(T,T)\big|_{(r_1,...,r_{\hat{\imath}-1},1,0)} - V_A^{\hat{\imath}+2}(T,T)\big|_{(r_1,...,r_{\hat{\imath}-1},0,1)} \right). \end{aligned}$$

Together with (51), this concludes the proof of the lemma. ■

Next, we will investigate the difference $V_A^{\hat{\imath}+2}(T,T)\big|_{(r_1,...,r_{\hat{\imath}-1},1,0)} - V_A^{\hat{\imath}+2}(T,T)\big|_{(r_1,...,r_{\hat{\imath}-1},0,1)}$. Consider the events $(r_1,...,r_{\hat{\imath}-1},1,0)$ and $(r_1,...,r_{\hat{\imath}-1},0,1)$. In stages $1,...,\hat{\imath}-1$, A accumulates the same transfers from the agents in the two events under consideration, since these two events differ only in later stages. Moreover, in the fictitious history, A proposes and collects a transfer in $\hat{\imath}+1$ in event $(r_1,...,r_{\hat{\imath}-1},1,0)$, but does not receive a transfer in $\hat{\imath}$, where she responds. In contrast, in event $(r_1,...,r_{\hat{\imath}-1},0,1)$, A receives a transfer in $\hat{\imath}$ but not in $\hat{\imath}+1$. The next lemma uses this difference to determine an expression for $V_A^{\hat{\imath}+2}(T,T)\big|_{(r_1,...,r_{\hat{\imath}-1},1,0)} - V_A^{\hat{\imath}+2}(T,T)\big|_{(r_1,...,r_{\hat{\imath}-1},0,1)}$.

Let

$$\begin{aligned} r_{(0,1)} &:= (r_1,...,r_{\hat{\imath}-1},0,1,r_{\hat{\imath}+2},...,r_n) \\ r_{(1,0)} &:= (r_1,...,r_{\hat{\imath}-1},1,0,r_{\hat{\imath}+2},...,r_n) \end{aligned}$$

Note that $\eta_{T,\hat{\imath}-1}(r_{(0,1)}) = \eta_{T,\hat{\imath}-1}(r_{(1,0)})$. To shorten notation, let

$$\hat{\eta}_{\hat{\imath}-1} := \eta_{T,\hat{\imath}-1}(r_{(0,1)}) = \eta_{T,\hat{\imath}-1}(r_{(1,0)}).$$

Lemma 6 *For all* $(r_1,...,r_{\hat{\imath}-1})$,

$$\begin{aligned} & V_A^{\hat{\imath}+2}(T,T)\big|_{(r_1,...,r_{\hat{\imath}-1},1,0)} - V_A^{\hat{\imath}+2}(T,T)\big|_{(r_1,...,r_{\hat{\imath}-1},0,1)} \\ = \ & u_{\hat{\imath}+1}(\hat{\eta}_{\hat{\imath}-1},0,f_T(\hat{\eta}_{\hat{\imath}-1},0)) + U_A^{\hat{\imath}+2}\Big|_{(\hat{\eta}_{\hat{\imath}-1},0,f_{T,\hat{\imath}+1}(\hat{\eta}_{\hat{\imath}-1},0))}(T) \\ & - \left(u_{\hat{\imath}}(\hat{\eta}_{\hat{\imath}-1},f_T(\hat{\eta}_{\hat{\imath}-1})) + U_A^{\hat{\imath}+2}\Big|_{(\hat{\eta}_{\hat{\imath}-1},f_{T,\hat{\imath}}(\hat{\eta}_{\hat{\imath}-1}),0)}(T) \right). \end{aligned}$$

Proof. By definition,

$$\begin{aligned} V_A^t(T,T)\big|_{(r_1,...,r_{\hat{\imath}-1},1,0)} &= \sum_{(r_{\hat{\imath}+2},...,r_N)} \prod_{i=\hat{\imath}+2}^{N} \beta_{T,i}^{r_i}(1-\beta_{T,i})^{1-r_i} \\ & \quad * \left(u_A(\eta_{T,N}(r_{(1,0)})) + \sum_{i=1}^{N} u_i(\eta_{T,i}(r_{(1,0)}), f_T(\eta_{T,i}(r_{(1,0)}))) \right) \end{aligned}$$

and

$$V_A^t(T,T)\,\big|_{(r_1,\ldots,r_{\hat{\imath}-1},0,1)} = \sum_{(r_{\hat{\imath}+2},\ldots,r_N)} \prod_{i=\hat{\imath}+2}^{N} \beta_{T,i}^{r_i}(1-\beta_{T,i})^{1-r_i} * \left(u_A\left(\eta_{T,N}\left(r_{(0,1)}\right)\right) + \sum_{i=1}^{N} u_i\left(\eta_{T,i}\left(r_{(0,1)}\right), f_T\left(\eta_{T,i}\left(r_{(0,1)}\right)\right)\right)\right)$$

The vectors $r_{(0,1)}$ and $r_{(1,0)}$ do not differ in their components 1 to $\hat{\imath}-1$. Therefore,

$$\sum_{i=1}^{\hat{\imath}-1} u_i\left(\eta_{T,i}\left(r_{(0,1)}\right), f_T\left(\eta_{T,i}\left(r_{(0,1)}\right)\right)\right) = \sum_{i=1}^{\hat{\imath}-1} u_i\left(\eta_{T,i}\left(r_{(1,0)}\right), f_T\left(\eta_{T,i}\left(r_{(1,0)}\right)\right)\right).$$

Moreover, in the event $r_{(1,0)}$, A responds in $\hat{\imath}$; hence, in the fictitious history, she does not obtain a transfer in this stage. Similarly, in event $r_{(0,1)}$, A does not obtain a transfer in the fictitious history in stage $\hat{\imath}+1$. Therefore,

$$\begin{aligned}
& V_A^t(T,T)\,\big|_{(r_1,\ldots,r_{\hat{\imath}-1},1,0)} - V_A^t(T,T)\,\big|_{(r_1,\ldots,r_{\hat{\imath}-1},0,1)} \\
= & \sum_{(r_{\hat{\imath}+2},\ldots,r_N)\in\{0,1\}^{N-t}} \prod_{i=\hat{\imath}+2}^{N} \beta_{T,i}^{r_i}(1-\beta_{T,i})^{1-r_i} \\
& *[u_A\left(\eta_{T,N}\left(r_{(1,0)}\right)\right) \\
& +u_{\hat{\imath}+1}\left(\eta_{T,\hat{\imath}+1}\left(r_{(1,0)}\right), f_T\left(\eta_{T,\hat{\imath}+1}\left(r_{(1,0)}\right)\right)\right) + \sum_{i=\hat{\imath}+2}^{N} u_i\left(\eta_{T,i}\left(r_{(1,0)}\right), f_T\left(\eta_{T,i}\left(r_{(1,0)}\right)\right)\right) \\
& -u_A\left(\eta_{T,N}\left(r_{(0,1)}\right)\right) \\
& -u_{\hat{\imath}}\left(\eta_{T,\hat{\imath}}\left(r_{(0,1)}\right), f_T\left(\eta_{T,\hat{\imath}}\left(r_{(0,1)}\right)\right)\right) - \sum_{i=\hat{\imath}+2}^{N} u_i\left(\eta_{T,i}\left(r_{(0,1)}\right), f_T\left(\eta_{T,i}\left(r_{(0,1)}\right)\right)\right)].
\end{aligned}$$

The fictitious history $\left(\eta_{T,\hat{\imath}+1}\left(r_{(1,0)}\right), f_T\left(\eta_{T,\hat{\imath}+1}\left(r_{(1,0)}\right)\right)\right)$ does not depend on $(r_{\hat{\imath}+2},...,r_N)$; hence, the transfers paid in stage $\hat{\imath}+1$ in event $r_{(1,0)}$, which is $u_{\hat{\imath}+1}\left(\eta_{T,\hat{\imath}+1}\left(r_{(1,0)}\right), f_T\left(\eta_{T,\hat{\imath}+1}\left(r_{(1,0)}\right)\right)\right)$, can be taken out of the expectation. Similarly, the transfer paid in stage $\hat{\imath}$ in event $r_{(0,1)}$ can be taken out of the expectation. Thus

$$\begin{aligned}
& V_A^t\left(T,T\right)\big|_{(r_1,\ldots,r_{\hat{\imath}-1},1,0)} - V_A^t\left(T,T\right)\big|_{(r_1,\ldots,r_{\hat{\imath}-1},0,1)} \\
= \; & u_{\hat{\imath}+1}\left(\eta_{T,\hat{\imath}+1}\left(r_{(1,0)}\right), f_T\left(\eta_{T,\hat{\imath}+1}\left(r_{(1,0)}\right)\right)\right) - u_{\hat{\imath}}\left(\eta_{T,\hat{\imath}}\left(r_{(0,1)}\right), f_T\left(\eta_{T,\hat{\imath}}\left(r_{(0,1)}\right)\right)\right) \\
& + \sum_{(r_{\hat{\imath}+2},\ldots,r_N)} \prod_{i=\hat{\imath}+2}^{N} \beta_{T,i}^{r_i}\left(1-\beta_{T,i}\right)^{1-r_i} \left(u_A\left(\eta_{T,N}\left(r_{(1,0)}\right)\right) + \sum_{i=\hat{\imath}+2}^{N} u_i\left(\eta_{T,i}\left(r_{(1,0)}\right), f_T\left(\eta_{T,i}\left(r_{(1,0)}\right)\right)\right)\right) \\
& - \sum_{(r_{\hat{\imath}+2},\ldots,r_N)} \prod_{i=\hat{\imath}+2}^{N} \beta_{T,i}^{r_i}\left(1-\beta_{T,i}\right)^{1-r_i} \left(u_A\left(\eta_{T,N}\left(r_{(0,1)}\right)\right) + \sum_{i=\hat{\imath}+2}^{N} u_i\left(\eta_{T,i}\left(r_{(0,1)}\right), f_T\left(\eta_{T,i}\left(r_{(0,1)}\right)\right)\right)\right).
\end{aligned}$$

The result then follows from (46). To see this, evaluate (46) at $t = \hat{\imath}+2$ and history $\left(\hat{\eta}_{\hat{\imath}-1}, 0, f_{T,\hat{\imath}+1}\left(\hat{\eta}_{\hat{\imath}-1}, 0\right)\right)$. Note that by definition

$$\eta_{T,i}\left(\left(\hat{\eta}_{\hat{\imath}-1}, 0, f_{T,\hat{\imath}+1}\left(\hat{\eta}_{\hat{\imath}-1}, 0\right)\right), r\right) = \eta_{T,i}\left(r_{(1,0)}\right)$$

for $i = \hat{\imath}+2, \ldots, N$. This shows

$$\begin{aligned}
U_A^{\hat{\imath}+2}\Big|_{\left(\hat{\eta}_{\hat{\imath}-1},0,f_{T,\hat{\imath}+1}\left(\hat{\eta}_{\hat{\imath}-1},0\right)\right)}(T) \; = \; & \sum_{(r_{\hat{\imath}+2},\ldots,r_N)} \prod_{i=\hat{\imath}+2}^{N} \beta_{T,i}^{r_i}\left(1-\beta_{T,i}\right)^{1-r_i} \\
& * \left(u_A\left(\eta_{T,N}\left(r_{(1,0)}\right)\right) + \sum_{i=\hat{\imath}+2}^{N} u_i\left(\eta_{T,i}\left(r_{(1,0)}\right), f_T\left(\eta_{T,i}\left(r_{(1,0)}\right)\right)\right)\right).
\end{aligned}$$

Similarly, evaluating (46) at $t = \hat{\imath}+2$ and history $\left(\hat{\eta}_{\hat{\imath}-1}, f_{T,\hat{\imath}}\left(\hat{\eta}_{\hat{\imath}-1}\right), 0\right)$ shows

$$\begin{aligned}
U_A^{\hat{\imath}+2}\Big|_{\left(\hat{\eta}_{\hat{\imath}-1},f_{T,\hat{\imath}}\left(\hat{\eta}_{\hat{\imath}-1}\right),0\right)}(T) \; = \; & \sum_{(r_{\hat{\imath}+2},\ldots,r_N)} \prod_{i=\hat{\imath}+2}^{N} \beta_{T,i}^{r_i}\left(1-\beta_{T,i}\right)^{1-r_i} \\
& * \left(u_A\left(\eta_{T,N}\left(r_{(0,1)}\right)\right) + \sum_{i=\hat{\imath}+2}^{N} u_i\left(\eta_{T,i}\left(r_{(0,1)}\right), f_T\left(\eta_{T,i}\left(r_{(0,1)}\right)\right)\right)\right).
\end{aligned}$$

■

Recall that $\beta_{T,\hat{\imath}+1} > \beta_{T,\hat{\imath}}$ by assumption. By the last two lemmas, $V_A\left(S,T\right) \geq V_A\left(T,T\right)$ if

$$\begin{aligned}
& u_{\hat{\imath}+1}\left(\hat{\eta}_{\hat{\imath}-1}, 0, f_T\left(\hat{\eta}_{\hat{\imath}-1}, 0\right)\right) + U_A^{\hat{\imath}+2}\Big|_{\left(\hat{\eta}_{\hat{\imath}-1},0,f_{T,\hat{\imath}+1}\left(\hat{\eta}_{\hat{\imath}-1},0\right)\right)}(T) \\
\geq \; & u_{\hat{\imath}}\left(\hat{\eta}_{\hat{\imath}-1}, f_T\left(\hat{\eta}_{\hat{\imath}-1}\right)\right) + U_A^{\hat{\imath}+2}\Big|_{\left(\hat{\eta}_{\hat{\imath}-1},f_{T,\hat{\imath}}\left(\hat{\eta}_{\hat{\imath}-1}\right),0\right)}(T).
\end{aligned} \tag{52}$$

We will next show that this inequality is fulfilled under the assumptions of Propositions 13, and 14 with negative externalities. As mentioned above, we will also show that $U_A(S) \geq V_A(S,T)$ under these assumptions.

A.6.3 Proof of Propositions 13 and 14

Proof of Proposition 13 Suppose there are no externalities between agents. Then (52) simplifies to

$$\begin{aligned} & u_{\hat{\imath}+1}\left(f_{T,\hat{\imath}+1}\left(\hat{\eta}_{\hat{\imath}-1},0\right)\right)+U_A^{\hat{\imath}+2}\Big|_{\left(\hat{\eta}_{\hat{\imath}-1},0,f_{T,\hat{\imath}+1}\left(\hat{\eta}_{\hat{\imath}-1},0\right)\right)}(T) \\ \geq\ & u_{\hat{\imath}}\left(f_{T,\hat{\imath}}\left(\hat{\eta}_{\hat{\imath}-1}\right)\right)+U_A^{\hat{\imath}+2}\Big|_{\left(\hat{\eta}_{\hat{\imath}-1},f_{T,\hat{\imath}}\left(\hat{\eta}_{\hat{\imath}-1}\right),0\right)}(T). \end{aligned} \tag{53}$$

We now prove inequality (53).

By (45), evaluated at $t=\hat{\imath}+1$ and $h_{t-1}=(\hat{\eta}_{\hat{\imath}-1},0)$, and the fact that there are no externalities between agents,

$$\begin{aligned} & u_{\hat{\imath}+1}\left(f_{T,\hat{\imath}+1}\left(\hat{\eta}_{\hat{\imath}-1},0\right)\right)+U_A^{\hat{\imath}+2}\Big|_{\left(\hat{\eta}_{\hat{\imath}-1},0,f_{T,\hat{\imath}+1}\left(\hat{\eta}_{\hat{\imath}-1},0\right)\right)}(T) \\ =\ & \max_{d_{\hat{\imath}+1}}\left\{u_{\hat{\imath}+1}\left(d_{\hat{\imath}+1}\right)+U_A^{\hat{\imath}+2}\big|_{\left(\hat{\eta}_{\hat{\imath}-1},0,d_{\hat{\imath}+1}\right)}(T)\right\} \\ \geq\ & u_{\hat{\imath}+1}\left(f_{T,\hat{\imath}}\left(\hat{\eta}_{\hat{\imath}-1}\right)\right)+U_A^{\hat{\imath}+2}\Big|_{\left(\hat{\eta}_{\hat{\imath}-1},0,f_{T,\hat{\imath}}\left(\hat{\eta}_{\hat{\imath}-1}\right)\right)}(T) \\ =\ & u_{\hat{\imath}}\left(f_{T,\hat{\imath}}\left(\hat{\eta}_{\hat{\imath}-1}\right)\right)+U_A^{\hat{\imath}+2}\Big|_{\left(\hat{\eta}_{\hat{\imath}-1},f_{T,\hat{\imath}}\left(\hat{\eta}_{\hat{\imath}-1}\right),0\right)}, \end{aligned}$$

where the last line uses symmetry.

It remains to show that A's utility in the sequence S (i.e., $U_A(S)=V_A(S,S)$) is indeed (weakly) larger than $V_A(S,T)$. To do so, recall first that $V_A(S,S)$ was obtained by automatically setting the decision to $d_t=f_{S,t}(h_{t-1})$ and the transfer to $t_t=u_t(d_t)$,[64] in case the principal proposes in stage t after a history of decisions h_{t-1}, whereas both the decision and the transfer are zero in case the agent proposes.

Suppose that instead of automatically assigning these decisions and transfers, when $r_t=0$, the principal *chooses* a decision d_t which automatically creates a transfer $t_t=u_t(d_t)$. However, whenever $r_t=1$, the decision d_t and the transfer t_t are automatically set to zero.

This defines a dynamic decision problem for the principal.[65] It is not a game since

[64] The transfer depends only on d_t because there are no externalities between agents.

[65] It might be useful to describe the dynamic decision problem of A in a way that makes it clear it is a standard finite horizon stochastic dynamic programming problem. For this, we have to define a state variable, a set of admissible controls (decisions) for each stage, and the payoff of A in a given stage.

In a standard dynamic programming problem problem, the payoff of the decision maker in any stage t depends only on the value of the state variable in t, on controls taken in t, and possibly random terms. In our setting, A's payoff in the final stage depends on the whole history of all decisions, because u_A depends on all the decisions. We use state augmentation to deal with this problem of time delay (Bertsekas 2017, the agents are replaced by automata. It has two features that are crucial here. First, by construction, the optimal decision of the principal in stage t after a history of decisions h_{t-1} is just $f_{S,t}(h_{t-1})$. Thus, the principal's expected payoff is equal to $V_A(S,S)$ in the solution to the dynamic decision problem, as in the bargaining game in sequence S. Second, by Bellmann's principal of optimality, the optimal decisions of the principal are time consistent. If the principal could commit to any other decision rule ex ante, she would not gain from this.[66] Therefore, replacing the decision rule f_S by f_T can only lead to a lower payoff. This shows that $V_A(S,S) \geq V_A(S,T)$. ■

We point out that the proof of inequality (53) follows the logic of the outside option effect discussed in the two-agent case in the main part of the paper.

Note that the time consistency argument for $V_A(S,S) \geq V_A(S,T)$ depends on there being no externalities between agents. In contrast, if there are externalities between agents, the transfer from an agent depends also on the future equilibrium decisions, and a reduction to a decision problem as in the last paragraphs of the proof is no longer possible.

Proof of Proposition 14 We now prove inequality (52) under the assumptions of Proposition 14. Consider the case of binary decisions, $d_i \in \{0,1\}$, and assume that participation is always optimal. That is, $f_{T,t}(h_{t-1}) = f_{S,t}(h_{t-1}) = 1$ for all t and h_{t-1}. By symmetry,

$$U_A^{\hat{\imath}+2}\Big|_{\left(\hat{\eta}_{\hat{\imath}-1},0,f_{T,\hat{\imath}+1}(\hat{\eta}_{\hat{\imath}-1},0)\right)}(T) = U_A^{\hat{\imath}+2}\Big|_{\left(\hat{\eta}_{\hat{\imath}-1},f_{T,\hat{\imath}}(\hat{\eta}_{\hat{\imath}-1}),0\right)}(T),$$

since the same number of agents has participated before stage $\hat{\imath}+2$ in history $(\hat{\eta}_{\hat{\imath}-1}, 0, f_{T,\hat{\imath}+1}(\hat{\eta}_{\hat{\imath}-1}, 0))$ as in history $(\hat{\eta}_{\hat{\imath}-1}, f_{T,\hat{\imath}}(\hat{\eta}_{\hat{\imath}-1}), 0)$. Therefore, inequality (52) simplifies to

$$u_{\hat{\imath}+1}(\hat{\eta}_{\hat{\imath}-1}, 0, f_T(\hat{\eta}_{\hat{\imath}-1}, 0)) \geq u_{\hat{\imath}}(\hat{\eta}_{\hat{\imath}-1}, f_T(\hat{\eta}_{\hat{\imath}-1})). \tag{54}$$

Chapter 1.4): in stage t, define the state as $s_t = (h_{t-1}, r_t)$. As above, h_0 denotes the beginning of the game. For $t > 1$, h_{t-1} is a vector of $t-1$ decisions. At the beginning of stage t, the binary random variable r_t realizes. The distribution of r_t is given by $\Pr(r_t = 1) = \beta_{S,t}$ (which does not depend on any decisions and can thus be treated as an uncontrollable state component).

The set of decisions available in stage t is

$$D_t = \begin{cases} \{0\}, & \text{if } r_t = 1, \\ \mathcal{D}, & \text{if } r_t = 0, \end{cases}$$

where $\mathcal{D}$ is the set of decisions in the bargaining game.

In stage t, the random variable r_t realizes, A observes the realization, and chooses a decision $d_t \in D_t$. Note that when $r_t = 1$, then A has no other choice than $d_t = 0$. In stage t, r_{t+1} realizes, and the state updates according to $s_{t+1} = (h_{t-1}, d_t, r_{t+1})$.

In $t = 1, ..., N-1$, the instantaneous payoff of A in t is $u_t(d_t)$. In $t = N$, the instantaneous payoff of A is $u_N(d_N) + u_A(h_N)$. The payoff of A is the sum of the instantaneous payoffs $u_A(h_N) + \sum_{t=1}^{N} u_t(d_t)$.

[66]See, for example, Proposition 1.3.1 in Bertsekas (2017).

We focus on the case of negative externalities. (The proof for positive externalities is similar.) In the history $(\hat{\eta}_{\hat{\imath}-1}, 0, f_T(\hat{\eta}_{\hat{\imath}-1}, 0))$, agent $\hat{\imath}$ does not participate, whereas in $(\hat{\eta}_{\hat{\imath}-1}, f_T(\hat{\eta}_{\hat{\imath}-1}))$, agent $\hat{\imath}$ does participate; in all other respects these histories are identical. By negative externalities and symmetry, it follows that

$$u_{\hat{\imath}+1}(\hat{\eta}_{\hat{\imath}-1}, 0, f_T(\hat{\eta}_{\hat{\imath}-1}, 0)) \geq u_{\hat{\imath}+1}(\hat{\eta}_{\hat{\imath}-1}, f_T(\hat{\eta}_{\hat{\imath}-1})) = u_{\hat{\imath}}(\hat{\eta}_{\hat{\imath}-1}, f_T(\hat{\eta}_{\hat{\imath}-1})).$$

Finally, with binary decisions and participation of all agents, $f_{T,t}(h_{t-1}) = f_{S,t}(h_{t-1}) = 1$, which implies $V_A(S,S) = V_A(S,T)$. This concludes the proof of Proposition 14. ■

We point out that the proof of inequality (54) follows the logic of the expected-externality effect discussed in the main part of the paper.

A.7 Externalities on non-traders

A.7.1 Proof of Proposition 15

Consider stage 2 of sequence BC after first-stage decision b. If A proposes, she proposes the contract $c = f(b)$ and $t_C = u_C(b, f(b)) - u_C(b, 0)$, C accepts, and A's payoff is $u_A(b, f(b)) + u_C(b, f(b)) - u_C(b, 0) + t_B$. If C proposes, he proposes the contract $c = f(b)$ and $t_C = u_A(b, 0) - u_A(b, f(b))$, A accepts, and A 's payoff is $u_A(b, 0) + t_B$. In expectation over who proposes, A's payoff is

$$(1-\gamma)(u_A(b, f(b)) + u_C(b, f(b)) - u_C(b, 0)) + \gamma u_A(b, 0) + t_B.$$

Now consider stage 1 of sequence BC. If A proposes, she sets $t_B = u_B(b, f(b)) - u_B(0, f(0))$, so that B accepts, and chooses b to maximize the joint surplus of A and B,

$$S_{AB}^{BC}(b) = u_B(b, f(b)) + (1-\gamma)(u_A(b, f(b)) + u_C(b, f(b)) - u_C(b, 0)) + \gamma u_A(b, 0).$$

Let $b^{BC} := \arg\max_b S_{AB}^{BC}(b)$. Note that, here, b also influences C 's outside option, and therefore his willingness to pay to participate if A proposes in stage 2, and this will be taken into account in the first stage negotiation. A's payoff is

$$S_{AB}^{BC}(b^{BC}) - u_B(0, f(0)).$$

If B proposes, he proposes b^{BC} and t_B such that A is brought down to her outside option,

$$(1-\gamma)\left(u_A\left(0, f\left(0\right)\right) + u_C\left(0, f\left(0\right)\right) - u_C\left(0,0\right)\right).$$

Therefore,

$$\begin{aligned} U_A^{BC} &= (1-\beta)\left(S_{AB}^{BC}\left(b^{BC}\right) - u_B\left(0, f\left(0\right)\right)\right) \\ &\quad +\beta\left(1-\gamma\right)\left(u_A\left(0, f\left(0\right)\right) + u_C\left(0, f\left(0\right)\right) - u_C\left(0,0\right)\right). \end{aligned}$$

Similarly, using symmetry, the joint surplus of A and C in the first stage of sequence CB is

$$S_{AC}^{CB}\left(c\right) = u_B\left(c, f\left(c\right)\right) + (1-\beta)\left(u_A\left(c, f\left(c\right)\right) + u_C\left(c, f\left(c\right)\right) - u_C\left(c,0\right)\right) + \beta u_A\left(c,0\right).$$

Let $c^{CB} = \arg\max_c S_{AC}^{CB}\left(c\right)$. Then

$$\begin{aligned} U_A^{CB} &= (1-\gamma)\left(S_{AC}^{CB}\left(c^{CB}\right) - u_B\left(0, f\left(0\right)\right)\right) \\ &\quad +\gamma\left(1-\beta\right)\left(u_A\left(0, f\left(0\right)\right) + u_C\left(0, f\left(0\right)\right) - u_C\left(0,0\right)\right). \end{aligned}$$

We have $S_{AB}^{BC}\left(b^{BC}\right) \geq S_{AB}^{BC}\left(c^{CB}\right)$. Therefore,

$$\begin{aligned} & U_A^{BC} - U_A^{CB} \\ \geq\ & (1-\beta)\, S_{AB}^{BC}\left(c^{CB}\right) - (1-\gamma)\, S_{AC}^{CB}\left(c^{CB}\right) \\ & + (\beta-\gamma)\left(u_A\left(0, f\left(0\right)\right) + u_C\left(0, f\left(0\right)\right) - u_C\left(0,0\right) + u_B\left(0, f\left(0\right)\right)\right) \\ =\ & (\beta-\gamma)\left[-u_B\left(c^{CB}, f\left(c^{CB}\right)\right) - u_A\left(c^{CB}, 0\right)\right. \\ & \left. + u_A\left(0, f\left(0\right)\right) + u_C\left(0, f\left(0\right)\right) - u_C\left(0,0\right) + u_B\left(0, f\left(0\right)\right)\right] \\ =\ & (\beta-\gamma)\left[u_B\left(0, f\left(0\right)\right) - u_B\left(c^{CB}, f\left(c^{CB}\right)\right) - u_A\left(c^{CB}, 0\right)\right. \\ & \left. + u_A\left(0, f\left(0\right)\right) + u_C\left(0, f\left(0\right)\right) - u_C\left(0,0\right)\right]. \end{aligned}$$

From (9), we have $u_B\left(0,0\right) - u_B\left(b,0\right) \leq u_B\left(0, f\left(0\right)\right) - u_B\left(b, f\left(b\right)\right)$. Evaluating this at $b = c^{CB}$ and using it in the last expression of the displayed formula, we obtain that this last expression is (weakly) larger than

$$\begin{aligned} & (\beta-\gamma)\left(u_B\left(0,0\right) - u_B\left(c^{CB}, 0\right) - u_A\left(c^{CB}, 0\right) + u_A\left(0, f\left(0\right)\right) + u_C\left(0, f\left(0\right)\right) - u_C\left(0,0\right)\right) \\ =\ & (\beta-\gamma)\left(-u_B\left(c^{CB}, 0\right) - u_A\left(c^{CB}, 0\right) + u_A\left(0, f\left(0\right)\right) + u_C\left(0, f\left(0\right)\right)\right) \geq 0 \end{aligned}$$

where the inequality is due to the fact that

$$\begin{aligned} & u_A(0, f(0)) + u_C(0, f(0)) = \max_c \{u_A(0, c) + u_C(0, c)\} \\ \geq \;& u_A\left(0, c^{CB}\right) + u_C\left(0, c^{CB}\right) = u_B\left(c^{CB}, 0\right) + u_A\left(c^{CB}, 0\right). \end{aligned}$$

It follows that $U_A^{BC} \geq U_A^{CB}$. ■

A.7.2 Proof of Proposition 16

Consider stage 2 of sequence BC after first stage decision b. If A proposes, she proposes the contract $c = 1$ and $t_C = u_C(b, 1) - u_C(b, 0)$, C accepts, and A's payoff is $u_A(b, 1) + u_C(b, 1) - u_C(b, 0) + t_B$. If C proposes, he proposes the contract $c = 1$ and $t_C = u_A(b, 0) - u_A(b, 1)$, A accepts, and A's payoff is $u_A(b, 0) + t_B$. In expectation over who proposes, A's payoff is

$$(1 - \gamma)(u_A(b, 1) + u_C(b, 1) - u_C(b, 0)) + \gamma u_A(b, 0) + t_B.$$

Now consider stage 1 of sequence BC. If A proposes, she proposes the contract $b = 1$ and $t_B = u_B(1, 1) - u_B(0, 1)$, and B accepts. A's payoff is

$$(1 - \gamma)(u_A(1, 1) + u_C(1, 1) - u_C(1, 0)) + \gamma u_A(1, 0) + u_B(1, 1) - u_B(0, 1)$$

If B proposes, he proposes $b = 1$ and t_B such that A is brought down to her outside option,

$$(1 - \gamma)(u_A(0, 1) + u_C(0, 1) - u_C(0, 0)).$$

Therefore,

$$\begin{aligned} U_A^{BC} &= (1 - \beta)((1 - \gamma)(u_A(1, 1) + u_C(1, 1) - u_C(1, 0)) + \gamma u_A(1, 0) + u_B(1, 1) - u_B(0, 1)) \\ &\quad + \beta(1 - \gamma)(u_A(0, 1) + u_C(0, 1) - u_C(0, 0)). \end{aligned}$$

Similarly, using symmetry

$$\begin{aligned} U_A^{CB} &= (1 - \gamma)((1 - \beta)(u_A(1, 1) + u_C(1, 1) - u_C(1, 0)) + \beta u_A(1, 0) + u_B(1, 1) - u_B(0, 1)) \\ &\quad + \gamma(1 - \beta)(u_A(0, 1) + u_C(0, 1) - u_C(0, 0)). \end{aligned}$$

Determining the difference $U_A^{BC} - U_A^{CB}$ and using symmetry leads to equation (10).

Super-modularity of a function $f(\cdot, \cdot)$ implies that, for $x > y$, $f(x, x) - f(y, x) \geq f(x, y) - f(y, y)$. Using $x = 1$ and $y = 0$, this implies $f(0, 1) - f(0, 0) \leq f(1, 1) - f(1, 0)$. By contrast, sub-modularity implies $f(0,1) - f(0,0) \geq f(1,1) - f(1,0)$. Using this definition in (10) leads to the statement of Proposition 16. ■

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