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Pietro Reichlin

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Centre for Economic Policy Research
33 Great Sutton Street, London EC1V 0DX, UK
Tel: +44 (0)20 7183 8801
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Abstract

I study the optimal redistributive tax structure on capital and labor in a version of the Judd (1985)'s model supplemented by skill biased technology and perfect correlation between skills and wealth. Assuming that the planner is forced to implement a log-linear (progressive) tax and transfer function of pre-tax labor income (often used in public finance), and that low skilled households are hand to mouth consumers, I show that the optimal long-run capital tax rate is positive and the labor marginal tax rate can be positive or negative, depending on demand elasticities as well as on the impact of capital on the skill premium. A positive capital tax serves the purpose of reducing tax distortions arising from redistribution, and it survives for any parametrization of the log-linear tax scheme except for a fully progressive system.

JEL Classification: E21, E62, H2, H21

Keywords: Dynamic optimal taxation

Pietro Reichlin - pietro.reichlin@gmail.com
LUISS Guido Carli and CEPR

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1. Introduction

A large part of wage inequality observed in the past thirty years is accounted for by an unprecedented rise in the skill premium, mostly explained by the adoption of skilled biased technologies[1]. This phenomenon has stimulated an intense debate about the benefits of redistribution through labor and capital taxation. Assuming that lump-sum taxation is unavailable, what is the optimal tax design in this world? How do the tax structure resolve the efficiency-inequality trade-off? Is capital taxation a good idea as a way to contain the inequality generated by skill biased technologies?

To tackle these issues, I consider an extension of the Judd (1985)'s capitalists-workers model to an economy where workers are endowed with different skills, and characterize the optimal distribution of net income obtained by implementing a parametric tax transfer scheme. The latter is characterized by a log linear after tax labor income with an arbitrary degree of progressivity and a linear (flat) tax rate on capital income. In order to make the distributional problem more transparent, I follow the Judd's dichotomy of the set of households between *wealth-poor*, *i.e.*, "hand to mouth" consumers, and *wealth-rich*. Differently from Judd's dichothomy, both type of households work, and there exists

Pietro Reichlin, Department of Economics, LUISS 'Guido Carli', 32 Viale Romania, I-00198 Rome (Italy); CEPR. *E-mail:* `preichlin@luiss.it`. I thank seminar participants at NYU Abu Dhabi and Marco D'Amico for helping me on the simulations of the model.

[1]See Katz and Murphy (1992), Krusell et al. (2000).

perfect correlation between wealth and skill level. One way to rationalize the inability of the unskilled individuals to accumulate wealth and acquire skills is to assume imperfect financial markets (credit constraints) coupled with a low labor income or high impatience. This assumption is admittedly quite extreme, but it serves the purpose of providing a sharp characterization of the optimal tax structure, and it is not at odd with the available evidence. In fact, a rising concentration of capital income is also contributing to the overall increase in inequality, and there seems to be a high correlation between income, wealth and skill acquisition (see Roine and Waldestrom (2014), Smith et al. (2019)). According to Milanovic (2019), in the US *"people who are capital-rich now tend also to be labor-rich (or to put it in more contemporary terms, they tend to be individuals with high human capital)"* (p. 17), whereas Hoffmann et al. (2020) claim that *"most of the growth in labor and capital income inequality can be linked to education"* (p. 65). In line with these assertions, Dynan et al. (2004) find a strong positive association between lifetime income and saving rates in U.S. data. This evidence implies that a two classes model of the type considered in Judd (1985), where workers are "hand to mouth" individuals and capitalists pure *rentiers* is far from being a realistic picture of contemporary economies.

Within this setting, I consider a standard Ramsey problem of finding the optimal tax structure for the given parametric tax scheme. The basic findings are the following.

(a) The optimal long-run capital tax is positive and the labor tax may be positive or negative, depending on a set of parameter values arising from preferences and technology. In particular, if the degree of skill bias and the intertemporal elasticity of substitution in consumption for the wealth-rich/skilled individuals is sufficiently strong, the marginal labor tax rate may be negative.

(b) The higher the degree of progressivity of the tax system, the lower the optimal capital tax rate.

(c) A larger after tax skilled labor income as a share of consumption makes the labor tax more distortionary (more elastic labor supply) and, then, it calls for a higher capital tax. Hence, somewhat counterintuitively, the optimal capital tax rate falls when the wealth-rich individuals derive a higher share of their disposable income from capital.

The intuition for the positivity of long-run capital taxation is that, under a uniform labor tax schedule across skills, any effort by the government to increase the unskilled after tax wage results in a higher after tax wage for the skilled and, then, it generates lower net revenues and higher distortions. When the production function is skilled neutral, the optimal capital tax is zero at steady state because consumption demand elasticities across periods are equal and there is no benefit from an intertemporal reallocation of the capitalists' consumption when their marginal rate of substitution between current and next period consumption equals the marginal product of capital. If, instead, the production function is skilled biased, a lower capital stock reduces the skill premium and, in this way, it makes redistribution across workers with different skill levels less costly. Essentially, a wedge between the wealth-rich individuals' marginal rate of substitution between consecutive consumptions and the marginal rate of transformation along the production possibility frontier is required to compensate for the "excess taxation" of the poor (due to uniform labor taxes) and the reduced government's revenue caused by the positive impact on the skill premium of a higher capital stock. Quite clearly, the higher is the degree of tax progressivity, the smaller is the redistributional role of the capital tax. Turning to the quantitative exercise in the model with separable and CRRA utility, and constant unskilled labor marginal productivity, I show that the optimal capital tax rate ranges from 13%, when the the average tax progressivity is at 25%, to 23%, when labor tax rates are flat, and, for any given degree of progressivity, higher capital tax rates have the effect of increasing the skilled workers' labor supply (as a way to compensate a falling capital income). Within the selected specification of the model, this implies that a higher labor tax progressivity reduces inequality but is welfare reducing (although this is not true more generally). Regarding the impact on the optimal capital tax of a higher relative productivity of skill (vs. unskilled) labor, it is important to stress that this has to be evaluated in terms of labor supply distortions. In particular, I show that the distortions arising from taxing labor increase with the after tax labor income of the skilled individuals as a share of their consumption (and, then, they decrease with capital income as a share of consumption). Then, as the wealth-rich households become more productive, it is optimal for the planner to switch the burden of taxation from labor to capital. It turns out that the impact of a rising skill premium on the rich households' labor income is ambiguous. In the simulated version of the model the skilled individuals' labor supply and after tax labor income increase with the skill premium, and this calls for a shift of the tax burden away from labor.

Note that, if the planner is allowed to impose flat labor tax rates contingent on the workers' skill levels and a non negative lump-sum transfer to the unskilled, the optimal plan is characterized by a zero capital tax and a zero labor tax on the unskilled. I call this a *second best characterization.* If there is enough fiscal space, the unskilled workers' consumption can be sustained through the lump sum transfer. Since the latter has a negative effect on labor supply, the typical optimal allocation, in this case, is characterized by a sort of "exploitation" of the rich (or more productive), in the sense that these agents are exerting a high labor effort and their labor income (not their wealth) may be heavily taxed at or near steady states.

The possibility of a positive capital tax when labor taxes are uniform across type of workers with different productivities has been noticed in Chari and Kehoe (1998). More recently, the issue has been addressed with reference to the inequality generated by automation (robots) in Slavík and Yazici (2014), Ales et al. (2015), Thuemmel (2019) and Guerreiro et al. (2020). All of these contributions provide an argument for the taxation of labor saving capital equipments (as opposed to structures) based on the idea that this type of capital has a positive effect on the skill premium. Since the latter affect the distribution of income in the face of uninsurable shocks, capital taxation can be welfare improving because it provides some insurance or more equality. The results of these papers are based on the government's limited information about the individuals' effective labor effort and allow for more general tax schedules following the seminal contribution by Mirrlees. Essentially, the lower wage inequality generated by a tax on robots allows for lighter income taxation and, then, lower distortions arising from a changing labor effort.

Relative to the existing literature, my contribution is more focused on the issue of post-transfers distortionary redistribution of income across skilled and unskilled and wealth polarization. This specific goal allows for a sharper characterization of the tax formulas. However, our analysis has two obvious limitations. First, due to the extreme polarization of wealth and the perfect correlation of the latter with skill levels, I cannot justify the government's inability to make taxes contingent on skills based on imperfect knowledge. The adoption of a uniform tax-transfer scheme based on pre-tax income only (and not identity or profession) is, however, a realistic representation of existing tax codes. Arguments based on horizontal equity and non-discrimination usually prevent the use of more sophisticated tax schemes[2]. The second limitation is that the skill composition of the labor force is exogenous. A plausible conjecture is that allowing individuals to chose their skills based on education costs and ability would create a trade-off between equality and efficiency, *i.e.*, a fall in the skill premium would discourage skill acquisition and decrease the skill intensity of the labor force. This would limit the benefits of the capital tax.

The paper is organized as follows. Section 2 describes the model and defines the competitive equilibria. Section 3 sets up the social welfare problem and provides a characterization of second best (linear tax rates skill contingent) and third best (uniform linear tax rates) allocations. Section

[2]According to Musgrave (1967), the principle of horizontal equity implies that "people in equal position should pay equal amounts of tax" (p. 45), and, then, he adds that "No distinction is to be made between either sources or uses of income" (p. 47).

4 presents and simulates a specific example of the model to convey more intuition, and section 5 concludes.

2. The Model

Preferences and Technologies. I consider an economy with two types of infinitely lived individuals (or households), indexed by $i \in \{s, u\}$, and defined as *skilled* and *unskilled*, respectively. All types are endowed with some amount of type-specific labor quality, *skilled* or *unskilled*, that they offer elastically, and have unit mass. Each type of individual consumes c_t^i units of the unique consumption good and supplies $n_t^i \in [0, \bar{n}]$ units of labor. The latter is used, together with capital, k, to generate the unique consumption good, with a technology represented by the production function

$$f(k, n^s, n^u).$$

Assumption 1. *The production function, $f(k, n^s, n^u)$, exhibits constant returns to scale, it is increasing, strictly concave and twice differentiable. In particular, letting f_k, f_s, f_u be the partial derivatives of f with respect to k, n^s, n^u, respectively, I impose $f_j > 0$, $f_{jj} \leq 0$ for $j = k, s, u$ and $f_{su} \geq 0$.*

The ratio between the before tax skilled and unskilled marginal productivities,

$$\pi = \frac{f_s(k, n^s, n^u)}{f_u(k, n^s, n^u)} \tag{1}$$

is called the *skill premium.* Note that, because of constant returns to scale, marginal productivities are homogeneous of degree zero in (k, n^s, n^u). Then, letting the capital (to skilled) labor ratio be $x = k/n^s$, and the unskilled to skilled ratio $u = n^u/n^s$, we have

$$f_j(k, n^s, n^u) = f_j(x, 1, u), \qquad \text{for } j = s, u.$$

It follows that the skilled premium is a function of x and u only, which I write as $\pi(x, u)$. I say that the technology is *skilled neutral* if $\pi_x = 0$. This property holds for a Cobb-Douglas specification. If, instead, we have $\pi_x > 0$, I say that the technology is *skilled biased.* For later use, it is convenient to define the elasticities

$$\eta_x^\pi = \pi_x x/\pi, \qquad \eta_u^\pi = \pi_u u/\pi,$$

as the units-free measures by which the risk premium responds to changes in x and u. By direct computation we have

$$\eta_x^\pi = x\left(\frac{f_{sk}}{f_s} - \frac{f_{uk}}{f_u}\right), \qquad \eta_u^\pi = u\left(\frac{f_{su}}{f_s} - \frac{f_{uu}}{f_u}\right).$$

The above expressions together with assumption 1 show that $\pi_u \geq 0$, whereas the effect of x on π is ambiguous. In particular, $\pi_u \geq 0$ and

$$\pi_x \geq 0 \qquad \Leftrightarrow \qquad f_{sk}/f_s \geq f_{uk}/f_u.$$

Krusell et al. (2000) consider the following CES specification

$$f(k, n^s, n^u) = \left(A\left(\alpha k^\theta + (1-\alpha)(n^s)^\theta\right)^{\xi/\theta} + B(n^u)^\xi\right)^{1/\xi}, \tag{2}$$

for some positive A and B, α in $(0,1)$ and ξ, $\theta < 1$. In this case, $1/(1-\theta)$ is the elasticity of substitution between skilled labor and capital, and $1/(1-\xi)$ the elasticity of substitution between unskilled labor and capital. One can easily verify that

$$\pi(x,u) = \left(\frac{A}{B}\right)(1-\alpha)u^{1-\xi}\left(\alpha x^\theta + (1-\alpha)\right)^{(\xi-\theta)/\theta}. \tag{3}$$

Then, $\eta_u^\pi = 1-\xi$ and we have skilled bias technological progress if and only if $\xi > \theta$. In a simulation at the end of the paper I will use this specification for $\xi = 1$ and $\theta = 0$.

Preferences are represented by the following lifetime utilities:

$$V^i = \sum_{t=0}^{\infty} \beta_i^t U(c_t^i, n_t^i), \tag{4}$$

where $\beta_i \in (0,1)$ and the per period utility function, $U(.)$, verifies the following assumptions.

Assumption 2. *$U(.)$ is increasing in c, decreasing in n, twice differentiable, strictly concave and verifying normality of consumption, c, and leisure, $\bar{n} - n$. More formally, $U_c > 0$, $U_n < 0$, $U_{cc} < 0$, $U_{nn} \leq 0$, and*

$$U_{cc}U_{nn} - U_{cn}^2 > 0, \quad U_{nn}U_c - U_{cn}U_n \leq 0, \quad U_{cc}U_n - U_{cn}U_c > 0. \tag{5}$$

Furthermore, I assume that $U_c \to \infty$ as $c \to 0$.

For later use, I define the elasticities of marginal utilities

$$\sigma_c^i = -U_{cc}^i c^i/U_c^i, \qquad \sigma_{cn}^i = U_{nc}^i c^i/U_n^i, \qquad \sigma_n^i = U_{nn}^i n^i/U_n^i.$$

Note that, if $U(c,n)$ is separable, $(\sigma_c^i)^{-1}$ is the intertemporal elasticity of consumption and $(\sigma_n^i)^{-1}$ is the Frisch elasticity of labor supply.

Assuming that capital fully depreciate in one period, the resource constraint at any period $t \geq 0$ is:

$$\sum_i c_t^i + k_{t+1} + g_t \leq f(k_t, n_t^s, n_t^u), \tag{6}$$

where $g_t > 0$ is an exogenous flow of public spending. A positive sequence $\mathcal{A} = \{c_t^i, n_t^i, k_{t+1}; i = s, u\}_{t=0}^{\infty}$ verifying (6) and such that $n_t^i \leq \bar{n}$ for all $t \geq 0$ is called a *feasible allocation.*

Competitive Equilibria. Firms are perfectly competitive and maximize profits by setting

$$f_k(k_t, n_t^s, n_t^u) = \hat{R}_t, \qquad f_s(k_t, n_t^s, n_t^u) = w_t^s, \qquad f_u(k_t, n_t^s, n_t^u) = w_t^u, \tag{7}$$

where $\hat{R}_t$ denotes the before tax gross interest rate and, for $i = s, u$, w_t^i are the type-specific before tax wage rates.

Now let $y_t^i = w_t^i n_t^i$ be the t-period before tax income of individual i, and $\hat{y}^i$ her after tax labor income. At all time t, the government has access to a redistribution (or tax-transfer) scheme such that

$$\hat{y}_t^i = N(y_t^i), \tag{8}$$

where

$$N(y) = (1 - \bar{\tau}^n) y^{1-\rho}, \qquad \rho, \bar{\tau}^n \in [0, 1]. \tag{9}$$

This log linear parametrization has been used in Feldstein (1969), Benabou (2002) and Heathcote et al. (2017), among others. The parameter $\bar{\tau}^n$ captures the level of taxation, while the parameter ρ is a measure of tax progressivity. In particular, for $\rho = 0$ we have a flat tax system, while $\rho = 1$ implies complete equalization of after tax income. A recent literature has suggested that the above scheme is a good approximation of existing tax codes. Similarly to most tax codes of advanced economies I assume that financial wealth is subject to a linear tax rate $\tau^k \in [0, 1]$, so that

$$R_t = (1 - \tau_t^k) f_{k,t}.$$

Lump-sum transfers are unavailable and the government issues one-period bonds at the market rate $\hat{R}_{t+1}$. Then, letting a_t^i be the individual i's after tax net asset position at time t, I define her per-period budget constraints as

$$a_{t+1}^i / R_{t+1} + c_t^i = N(y_t^i) + a_t^i, \tag{10}$$

and the government's per period budget constraint as

$$b_{t+1}/R_{t+1} = g_t + b_t - \tau_t^k f_{k,t} k_t + \sum_i \left(N(y_t^i) - y_t^i \right), \tag{11}$$

where b_t denotes the after tax government debt at time t. Asset market clearing requires

$$\sum_i a_t^i = R_t k_t + b_t. \tag{12}$$

Finally, I assume that net claims must be non-negative at all periods,

$$a_{t+1}^i \geq 0. \tag{13}$$

For a given set of prices, $\{R_t, w_t^i\}_{t=0}^{\infty}$, any individual $i \in \{s, u\}$ selects a sequence, $\{c_t^i, n_t^i, a_{t+1}^i\}_{t=0}^{\infty}$, that maximizes V_0^i subject to the non-negativity constraints, the budget constraints, (10), and the debt limit (13), for all $t \geq 0$, and for given initial assets, a_0^i. From now on, to simplify the notation, I set $U_{c,t}^i = U_c(c_t^i, n_t^i)$, $U_{n,t}^i = U_n(c_t^i, n_t^i)$. Then, by a standard variational argument, a first order characterization of an interior V^i-optimal plan is provided by the following conditions

$$U_{c,t}^i - R_{t+1}\beta_i U_{c,t+1}^i \geq 0, \tag{14}$$

$$U_{c,t}^i (1-\rho) N(y^i) + n^i U_{n,t}^i = 0, \tag{15}$$

$$\left(U_{c,t}^i - R_{t+1}\beta_i U_{c,t+1}^i \right) a_{t+1}^i = 0, \tag{16}$$

$$\lim_{t \to \infty} \beta_t^i U_{c,t}^i a_t^i = 0. \tag{17}$$

Note that the total tax paid by the type-i individuals is $T(y^i) = y^i - N(y^i)$, and, then, the marginal labor tax rates are

$$\tau^n(y_t^i) = T'(y^i) = 1 - N'(y^i) = 1 - (1-\rho)N(y^i)/y^i.$$

By the first order conditions (14), (15), the implicit marginal tax rates are

$$\tau_t^k = 1 - \left(\frac{1}{f_{k,t}} \right) \frac{U_{c,t-1}^s}{\beta U_{c,t}^s}, \qquad \tau^n(y_t^i) = 1 + \frac{U_{n,t}^i}{U_{c,t}^i w_t^i}, \qquad i = s, u. \tag{18}$$

The inability of the government to make the tax scheme skill-contingent and, instead, apply the assumed log linear tax code, implies the following *uniform labor tax restriction*

$$\frac{U_{n,t}^s (n_t^s)^\rho}{U_{c,t}^s} = \frac{U_{n,t}^u (n_t^u)^\rho}{U_{c,t}^u} \times \pi_t^{1-\rho}. \tag{19}$$

Definition 1. *Given some initial distribution of net claims,* $(a_0^i; i = s, u)$, *some initial levels of government debt and capital,* (b_0, k_0), *and a sequence of government spending,* $\{g_t\}_{t=0}^{\infty}$, *a competitive equilibrium is a feasible allocation,* $\mathcal{A} = \{c_t^i, n_t^i, k_{t+1}; i = s, u\}_{t=0}^{\infty}$, *a sequence of net factor prices,* $\{w_t^i, R_{t+1}\}_{t=0}^{\infty}$, *and a policy* $\{\bar{\tau}_t^n, \tau_t^k; i = s, u\}_{t=0}^{\infty}$, *verifying equations* (7)–(18), *for some sequence of public debt,* $\{b_{t+1}\}_{t=0}^{\infty}$.

Polarized Equilibria and Implementability. In the rest of this paper I concentrate on a specific set of competitive equilibria to be called the *polarized equilibria.* The latter are such that the unskilled individuals have zero and the skilled have positive net wealth at all periods. This assumption may be rationalized by assuming, for instance, that the unskilled are more impatient, *i.e.*, $\beta_u < \beta_s$, but I am not making this assumption explicitly. My aim is to identify the unskilled with a class of poor households following the idea that there may be a positive correlation between labor income and wealth.

Following a common approach in optimal tax theory, in the sequel I will merge the first order conditions (14)–(17) and the individuals' budget constraint. In particular, assuming that $n_t^i > 0$ for all i and $t \geq 0$, multiplying (10) by $U_{c,t}^i$, and exploiting (14) and (16), we get

$$\beta_i U_{c,t+1}^i a_{t+1}^i + U_{c,t}^i c_t^i + \frac{1}{1-\rho} U_{n,t}^i n_t^i = U_{c,t}^i a_t^i. \tag{20}$$

By defining

$$E_\rho(c^i, n^i) = U_c^i \left(c^i + \frac{1}{1-\rho} \frac{U_n^i}{U_c^i} n^i \right),$$

solving (20) forward, and exploiting the transversality condition (17), we obtain the present value representation

$$\sum_{t=0}^{\infty} \beta_i^t E_\rho(c_t^i, n_t^i) = U_{c,0}^i a_0^i, \qquad i = s, u. \tag{21}$$

The function $E_\rho(.)$ will be called the individual i's *net expenditure function.* Since the unskilled individuals have no wealth at any time, their period-by-period budget constraint reduces to

$$E_\rho^u(c_t^u, n_t^u) = 0 \tag{22}$$

for all $t \geq 0$. Note that the after tax income, $N(.)$, is strictly increasing in the before tax income, it is invertible in this range at any interior solution, and, then, from (22) we can derive n^u as a function of c^u, for all $c^u \geq 0$. In particular, I replace (22) with

$$n_t^u = n_\rho(c_t^u). \tag{23}$$

Condition (23) and

$$\sum_{t=0}^{\infty} \beta_s^t E_\rho(c_t^s, n_t^s) = U_{c,0}^s a_0^s. \tag{24}$$

will be called the unskilled and skilled individuals' *implementability constraints*. Note that, if we use the market clearing conditions in the good and asset markets (equations (6) and (12)) and profit maximization (equation (7)), the skilled individuals' lifetime present value budget constraint (24) corresponds to the government's *long-run present value budget constraint*, *i.e.*, the equality between the government's initial liabilities (right hand side of (24)) and the present value of all future stream of primary surpluses (left hand side of (24)). Evidently, any feasible allocation, $\mathcal{A}' = \{c_t^j, n_t^s, k_{t+1}; j = s, u\}_{t=0}^{\infty}$, satisfying the implementability conditions (23), (24) and the uniform labor tax restriction (19) is a polarized equilibrium for some initial levels of government debt and capital, (b_0, k_0), and some sequence of public spending, $\{g_t\}_{t=0}^{\infty}$. Note that $\mathcal{A}'$ differs from $\mathcal{A}$ because it does not contain the unskilled individuals' labor supplies, n_t^u, as these can be deduced from the implementability conditions (23).

3. The Ramsey Problem

To set up a *Ramsey problem* for this economy, I assume that the planner's social welfare function is

$$\mathcal{U} = \sum_{t=0}^{\infty} \beta^t \left(\gamma U(c^s, n^s) + U(c_t^u, n_t^u)\right) \tag{25}$$

Note that per period utilities are discounted at a rate β. In principle, this may differ from the subjective discount rates, but, for simplicity, I assume that $\beta = \beta_s \geq \beta_u$. If $\beta_u \neq \beta$, this implies that the planner's welfare function is characterized by some form of paternalism. The benefit of using the skilled individuals' discount rate in the planner's welfare function is to allow for steady state solutions.

Now observe that the optimal decisions in (23) can be used to define the unskilled households' per period indirect utility

$$\tilde{U}_\rho(c^u) = U(c^u, n_\rho(c^u)), \tag{26}$$

which can be shown to be an increasing function of c^u. Then, the Ramsey Problem is defined as the choice of an allocation $\mathcal{A}'$ maximizing $\mathcal{W}$ subject to resource feasibility and the implementability constraints. In particular, I state the planning problem as

(PP) $$\max_{\mathcal{A}'\geq 0} \sum_{t=0}^{\infty} \beta^t \left(\gamma U(c^s, n^s) + \tilde{U}_\rho(c_t^u) \right) \qquad \text{subject to:}$$

(27) $$f(k_t, n_t^s, n(c_t^u)) - c_t^s - c_t^u - k_{t+1} \geq 0,$$

the implementability constraint (24) and the uniform labor tax restriction (19). To make the analysis interesting, I will only consider the cases where the solution to the Ramsey problem is not a first-best solution and assume (as usually done in the literature) that the initial stocks of capital and public debt held by households cannot be taxed above a given level. This makes it impossible for the government to eliminate initial liabilities through non-distortionary taxation.

Second Best Plans. To understand the importance of the restrictions implied by the assumed tax-transfer scheme (8), here I give a sketchy characterization of the optimal plan under the assumption that the government can set skill contingent linear tax rates independently, together with a lump-sum transfer to the unskilled individuals. In particular, suppose that the planner has access to a policy

$$\mathcal{P} = \{\tau_t^k, \tau_t^{n,s}, \tau_t^{n,u}, z_t\}_{t=0}^{\infty},$$

where τ_t^k is the linear capital tax that I have already defined above, $\tau_t^{n,i}$ are skill-contingent linear tax rates on labor and $z_t \geq 0$ is non negative lump sum transfer to the unskilled. By modifying the first order conditions (14), (15) accordingly (just set $\rho = 0$ and replace $\bar{\tau}^n$ with $\tau^{n,s}$ for the skilled and with $\tau^{n,u}$ for the unskilled) I obtain the implementability conditions

(28) $$U_{c,t}^u(c_t^u - z_t) + n_t^u U_{n,t}^u = 0,$$

(29) $$\sum_{t=0}^{\infty} \beta^t (U_{c,t}^s c_t^s + n_t^u U_{n,t}^u) = 0.$$

A *second best plan* is a sequence $\{c_t^i, n_t^i, z_t, k_{t+1}\}_{t=0}^{\infty}$ maximizing the welfare function defined in (25) subject to the resource feasibility condition (6), the implementability constraints (28), (29) and the non negativity constraint $z_t \geq 0$. Note that (29) is obtained from (24) by setting $\rho = 0$. Using a Lagrange method, it can shown that the optimal tax structure at steady state is characterized by a zero capital tax, a positive tax rate on the skilled workers' wage and, if the non negativity constraint on lump sum transfers is non binding, a zero tax rate on the unskilled workers' wage. A full characterization of the plan is provided in appendix A. Assuming, for simplicity, that $U(c, n)$ is separable with constant elasticities, σ_c, σ_n, and that the solution is interior, one can easily derive that, at the second best outcome,

$$(30) \qquad \pi = \gamma \left(\frac{n^s}{n^u}\right)^{\sigma_n} \left(\frac{(\sigma_c + \sigma_n)}{(\sigma_c + \sigma_n) - (1 + \sigma_n)\tau^{n,s}}\right).$$

I interpret the above as saying that a higher skill premium tilts the balance of the workforce towards the skilled individuals, as the latter are more able to transform their labor effort into labor income. A higher lump sum transfer to the unskilled, by reducing their incentive to work, may accomplish this objective and, also, reduce *ex post* inequality.

Revenue Effects. I start the analysis of the third best plans by stating some important properties of the skilled and unskilled individuals' implementability constraints. First, note that, by standard arguments, the unskilled labor supply function, $n_\rho^u(c^u)$, is a continuous, differentiable function with elasticity

$$(31) \qquad \frac{n'_\rho(c^u)c^u}{n^u} = \frac{1 + \sigma_{cn}^u - \sigma_c^u}{1 + \sigma_n^u + (1-\rho)\sigma_{cn}^u} \equiv \varepsilon_n^u.$$

Hence, the unskilled workers' labor supply and consumption are positively correlated if and only if the consumption intertemporal elasticity of substitution is sufficiently large or, more formally, if and only if

$$\frac{1}{\sigma_c^u} \geq \frac{1}{1 + \sigma_{cn}^u}.$$

Furthermore, $n_\rho(c^u)$ is decreasing in ρ and such that $n_\rho \to 0$ as $\rho \to 1$. Now observe that, by the definition of skill premium in (1), by the uniform labor tax restriction (19), and by the unskilled implementability condition (23), we have

$$(32) \qquad E_\rho(c^s, n^s) = U_c^s c^s + \frac{1}{1-\rho} U_n^s n^s = U_c^s \left(c^s - \left(\frac{n^s \pi}{n_\rho(c^u)}\right)^{1-\rho} c^u\right).$$

where, using (23), we have

$$(33) \qquad \pi = \pi \left(\frac{k}{n^s}, \frac{n(c^u)}{n^s}\right).$$

Hence, the expenditure function is affected by all the components of the government's choice set, *i.e.*, (c^s, n^s, c^u, k). For this reason, at interior allocations, I let

$$U_c^s \left(c^s - \left(\frac{n^s \pi(x,u)}{n_\rho(c^u)}\right)^{1-\rho} c^u\right) \equiv \mathcal{E}(c^s, n^s, c^u, k).$$

As stated by Erosa and Gervais (2001), the above function "*gives a bonus to date-t allocations that bring in extra government revenues, thereby relieving other periods from distortionary taxation, and* *the same term imposes a penalty in the opposite situation*". Hence, the response of this flow of revenues to changes in the endogenous variables determines the optimal tax structure when the government budget constraint is binding. We know from standard optimal Ramsey taxation that the impact of a higher c^s on E is ambiguous, as it depends on the elasticity of the skilled individuals' demand of the consumption good. We are on the increasing part of the Laffer curve when this elasticity is relatively small, so that a higher capital tax increases the government's revenue. On the other hand, the effect of a higher labor supply, n^s, or a higher net wage, w^s, generates a lower E^s, *i.e.*, a loss in government revenue, as it should be expected. However, since the government cannot differentiate the tax rates based on skilled levels, the net expenditure, E^s, is also affected by the unskilled individuals' consumption, c^u, as well as the skill premium, π, which, in turn, is a function of n^s, n^u and k.

To evaluate the impact of a change in the endogenous variables on E^s, I start by studying their effects on the skill premium. In particular, remember that $\eta_x^\pi = \pi_x x/\pi$, $\eta_u^\pi = \pi_u u/\pi$ are the elasticities of the skill premium with respect to the capital-to-skilled labor ratio and the unskilled-to-skilled ratio. Then, using (33), we obtain

$$\frac{\partial \pi/\pi}{\partial n^s/n^s} = -\left(\eta_x^\pi + \eta_u^\pi\right), \tag{34}$$

$$\frac{\partial \pi/\pi}{\partial k/k} = \eta_x^\pi, \tag{35}$$

$$\frac{\partial \pi/\pi}{\partial c^u/c^u} = \eta_u^\pi \varepsilon_n^u. \tag{36}$$

By assumption 1, $\eta_u^\pi \geq 0$ and $\eta_x^\pi + \eta_u^\pi \geq 0$. Hence, the above show that, with skilled bias technologies, the skill premium, π, is decreasing in skilled employment, increasing in capital and it is increasing in the unskilled consumption if and only if their labor supply is increasing in c^u.

By defining

$$\omega^s = \frac{c^s - N(f_s n^s)}{c^s},$$

and using (34)–(36), I obtain the following expressions for the partial derivatives of the expenditure function with respect to c^s, n^s, c^u and k

$$\mathcal{E}_{c^s} = U_c^s\left(1 - \sigma_c^s \omega^s\right), \tag{37}$$

$$\mathcal{E}_{n^s} = U_n^s\left(1 + \sigma_{cn}^s \omega^s\right) - U_n^s(\eta_x^\pi + \eta_u^\pi), \tag{38}$$

$$\mathcal{E}_{c^u} = -U_c^s\left(\frac{n^s \pi}{n^u}\right)^{1-\rho}\left(1 + (1-\rho)\varepsilon_n^u \eta_u^\pi\right), \tag{39}$$

$$\mathcal{E}_k = -U_c^s(1-\rho)\left(\frac{n^s \pi}{n^u}\right)^{1-\rho}\frac{c^u}{k}\eta_x^\pi. \tag{40}$$

Except for the terms η_x^π and η_u^π, equations (37) and (38) are relatively standard in the literature on optimal taxation and they measure the social benefits (through a relaxation of the implementability constraint) of increasing each of the variables affecting $\mathcal{E}$. In particular, $\mathcal{E}_{n^s}$ has two parts: the first defines the "private" effect on the implementability constraint of a higher n^s for given skilled wage (a negative effect), and the second part arises from the "external" effect of a larger labor effort by the skilled individuals through the skill premium (a positive effect). The literature on optimal taxation (cf. Chari and Kehoe (1999), Atkeson et al. (1999)) defines the terms

$$g_c^s \equiv \mathcal{E}_{c^s}/U_c^s = 1 - \sigma_c^s \omega^s, \qquad g_n^s \equiv \mathcal{E}_{n^s}/U_n^s = 1 + \sigma_{cn}^s \omega^s - (\eta_x^\pi + \eta_u^\pi) \tag{41}$$

as the *general equilibrium elasticities* related, respectively, to the tax rates on consumption and labor. They represent the distortions in the skilled individuals' consumption and labor supply caused by changes in, respectively, U_c^s (price of consumption) and U_n^s (price of labor). I will show later on in this section that a higher value of g_n^s and a lower value of g_c^s imply more scope for labor taxation. The basic insight is that former variable is positively correlated with the wage elasticity of labor supply and the latter with the wage elasticity of consumption demand (see appendix B). More precisely, the sign and size of the labor tax wedge can be derived from

$$g_n^s - g_c^s = (\sigma_c^s + \sigma_{cn}^s)\omega^s - (\eta_x^\pi + \eta_u^\pi), \tag{42}$$

so that higher values of σ_c^s (lower elasticity of intertemporal substitution (EIS)), higher values of ω^s (lower after tax skilled labor income) and lower values of the skill premium elasticities, η_x^π, η_u^π are all contributing to an increase in the optimal tax on labor. Note that, by evaluating the budget constraint (24) at a steady state, we derive

$$\omega^s = 1 - \frac{N(f_s n^s)}{c^s} = \frac{(1-\beta)a^s}{c^s} \in [0,1], \tag{43}$$

which represents the skilled individuals' capital income to consumption ratio. Then, at steady state, the case for a labor tax is enhanced by larger values of the skilled individuals' capital income (relative to their consumption). To understand the remaining effects in (39) and (40), it is sufficient to note that, for given c^s and n^s, a larger skilled wage (due to a rise in the unskilled wage and/or a larger skill premium) affect negatively the net expenditure $\mathcal{E}$. In particular, $\mathcal{E}_{c^u} < 0$ and, if the technology is skilled biased, $\mathcal{E}_k < 0$.

The argument for Capital Taxes. Drawing from the above finding, I offer an argument for why long run capital tax rates may be positive. For simplicity, I assume (temporarily) that individuals' labor supplies are inelastic and normalized to one, so that the skilled individuals' net expenditure function depends on c_t^s, c_t^u and k_t only. Hence, with some abuse of notation, I write $E_\rho(c^s, n^s) = \mathcal{E}(c_t^s, c_t^u, k_t)$. Furthermore, in order to avoid non interior solutions[3], I assume that $\sigma_c^s \leq 1$. Starting from some stationary feasible allocation, $\mathcal{A}'$, I seek a resource feasible and implementable reallocation of the skilled and unskilled individuals' consumption, at some dates $t > 1$ and $t+1$ that makes the unskilled individuals better off. This reallocation is such that all components of $\mathcal{A}'$ are left unchanged except for $(c_t^s, c_{t+1}^s, c_t^u, k_{t+1})$. In particular, I consider small change

$$dc_t^s = -\alpha^s, \qquad dc_t^u = \alpha^u > 0,$$

followed by a change in c_{t+1}^s and k_{t+1} such that the sequence of resource feasibility constraints (27) at t and $t+1$, as well as the implementability constraint,

$$\Gamma = \sum_{t=0}^{\infty} \beta^t \mathcal{E}(c_t^s, c_t^u, k_t) - U_{c,0}^s a_0^s \geq 0, \tag{44}$$

are left unchanged. In this case I say that the assumed perturbation of $\mathcal{A}'$ is feasible and implementable and, because $\alpha^u > 0$, it makes the unskilled better off. The assumed perturbation of $\mathcal{A}'$ verifies resource feasibility at t and $t+1$ under the conditions

$$dk_{t+1} = \alpha^s - \alpha^u, \qquad dc_{t+1}^s = f_k dk_{t+1} = f_k(\alpha^s - \alpha^u). \tag{45}$$

Now I need to verify that this perturbation of the given allocation is implementable, *i.e.*, it does not reduce the present value of the government's net revenues. This condition can be written as follows

$$d\Gamma = -\beta^t \left(\mathcal{E}_{c^s}\alpha^s - \mathcal{E}_{c^u}\alpha^u\right) + \beta^{t+1}\left(\mathcal{E}_{c^s} dc_{t+1}^s + \mathcal{E}_k dk_{t+1}\right) \geq 0.$$

Using (45) and recalling that, at steady state, $\beta = 1/R$, the above is equivalent to

$$\alpha^s \left((f_k - R)\mathcal{E}_{c^s} + \mathcal{E}_k\right) \geq \alpha^u \left(f_k \mathcal{E}_{c^s} - R\mathcal{E}_{c^u} + \mathcal{E}_k\right), \tag{46}$$

where, by assumption, $\mathcal{E}_{c^s} > 0$, $\mathcal{E}_{c^u} < 0$ and $\mathcal{E}_k < 0$. Now suppose that the capital tax is zero at steady state, so that $f_k = R$. Note that, if the technology is skill-neutral, we have $\mathcal{E}_k = 0$ and, then, (46) implies

$$0 \geq \alpha^u R(\mathcal{E}_{c^s} - \mathcal{E}_{c^u}).$$

[3] Straub and Werning (2020) have remarked that long run capital taxation may not be zero when the intertemporal elasticity of substitution is below one in the Judd (1985)'s model with homogeneous labor.

Since $(\mathcal{E}_{c^s} - \mathcal{E}_{c^u}) > 0$, the above can only be verified for $\alpha^u = 0$. In other words, if the technology is skilled neutral, there is no way to make a resource feasible reallocation of the skilled individuals' consumption across two consecutive periods so as to increase the unskilled workers' after tax wage at a steady state with zero capital tax. If, on the other hand, the technology is skill biased, we have $\mathcal{E}_k > 0$, and, then, (46) provides

$$\alpha^s \mathcal{E}_k \geq \alpha^u R \left(\mathcal{E}_{c^s} - \mathcal{E}_{c^u} + \mathcal{E}_k / R\right),$$

which allows for $\alpha^u > 0$ if either α^s and $\mathcal{E}_{c^s} - \mathcal{E}_{c^u} + \mathcal{E}_k/R$ are both positive or they are both negative. In other words, with skilled biased technology and a zero capital tax at steady state, there exists a reallocation of the skilled individuals' consumption across two consecutive periods that generates a higher consumption (and utility) for the unskilled workers.

Optimal Tax Rates. Now we are ready to characterize the optimal tax plan by using a Lagrange approach. Define the *pseudo welfare function*

$$\mathcal{W}_\rho(c^s, n^s, c^u, k, \mu) = \gamma U(c^s, n^s) + \tilde{U}_\rho(c^u) + \mu \mathcal{E}(c^s, n^s, c^u, k),$$

where $\mu \geq 0$ represents the lagrange multiplier associated to the implementability condition (23). Then, the Lagrange function is

$$\mathcal{L}_\rho(\mathcal{A}', \mu, \Lambda) = \sum_{t=0}^{\infty} \beta^t \{\mathcal{W}_\rho(c_t^s, n_t^s, c_t^u, k_t, \mu)$$

$$+\lambda_t(f(k_t, n_t^s, n_\rho(c_t^u)) - c_t^s - c_t^u - k_{t+1} - g_t)\} - \mu U_c(c_0^s, n_0^s) a_0^s,$$

where $\Lambda = \{\lambda_t\}_{t=0}^{\infty}$ is a non negative sequence of (discounted) Lagrange multipliers. The first order conditions at an interior allocation for all $t \geq 1$ are

$$U_{c,t}^s \left(\gamma + \mu \mathcal{E}_{c^s,t} / U_{c,t}^s\right) - \lambda_t = 0, \tag{47}$$

$$-U_{n,t}^s \left(\gamma + \mu \mathcal{E}_{n^s,t} / U_{n,t}^s\right) - \lambda_t f_{s,t} = 0, \tag{48}$$

$$\tilde{U}_\rho'(c_t^u) + \mu \mathcal{E}_{c^u,t} - \lambda_t (1 - f_{u,t} n_\rho'(c^u)) = 0, \tag{49}$$

$$\beta \left(\lambda_{t+1} f_{k,t+1} + \mu \mathcal{E}_{k,t+1}\right) - \lambda_t = 0, \tag{50}$$

where the terms $\mathcal{E}_{c^s,t}$, $\mathcal{E}_{n^s,t}$, $\mathcal{E}_{c^u,t}$, $\mathcal{E}_{k,t}$ have been defined in (37)–(40). As I have said earlier, the left hand sides of the above conditions describe the net social benefits of increasing each of the relevant variables (c_t^s, n_t^s, c_t^u, k_{t+1}). A peculiar feature of the present model is that the elasticities of the skill premium with respect to capital and unskilled labor intensity (in terms of skilled labor) affect negatively the general equilibrium elasticity of labor for the skilled individuals. It is well known (as recently emphasized in Straub and Werning (2020)) that a large enough EIS (a small σ_c) is necessary to obtain interior solutions in the Judd (1985)'s model. In our case, the existence of a steady state allocation (with time invariant Lagrange multipliers) for all $\gamma \geq 0$ implies

$$\sigma_c^s \omega^s < 1, \qquad \eta_x^\pi + \eta_u^\pi < 1 + \sigma_{cn}^s \omega^s, \tag{51}$$

i.e., small enough EIS and skill premium elasticities. The above inequalities are assumed to be verified at steady state in the following analysis. Intuitively, this assumption means that there it is not possible to squeeze indefinitely the capital owners's consumption by taxing away their income without reaching a point at which the government revenue starts falling due to a Laffer effect.

In a first best allocation, the marginal rates of substitution between labor and consumption for each type of worker is equal to the marginal product of the corresponding type of labor and the marginal rate of substitution between current and next period consumption is equal to the marginal product of capital. This is not so in with distortionary taxation. A measure of the distortions that should be imposed to finance a given stream of spending when the planner can only use the assumed tax code is given by the implicit marginal tax rates that we can derive from (47)–(50), *i.e.*,

$$\tau_t^{n,i} = 1 + \frac{U_{n,t}^i}{f_{i,t} U_{c,t}^i}, \qquad \tau_t^k = 1 - \frac{U_{c,t}^s}{f_{k,t} \beta U_{c,t+1}^s}. \tag{52}$$

Note that, because of the uniform labor tax restriction (19),

$$\tau^{n,u} = 1 - (1 - \tau^{n,s}) \left(\frac{\pi n^s}{n^u} \right)^\rho,$$

so that the unskilled labor distortion is increasing in the skilled labor distortion, but it is decreasing with the skill premium. I now derive the optimal tax wedges at steady state based on the first order conditions (47)–(50). By solving equations (47), (48) for λ_t, we obtain

$$\tau^{n,s} = \mu \left(\frac{(\sigma_c^s + \sigma_{cn}^s)\omega^s - (\eta_x^\pi + \eta_u^\pi)}{\gamma + \mu(1 + \sigma_{cn}^s \omega^s - (\eta_x^\pi + \eta_u^\pi))} \right), \tag{53}$$

implying that the skilled labor tax wedge is positive only if $(\sigma_c^s + \sigma_{cn}^s)\omega^s > \eta_x^\pi + \eta_u^\pi$. Recall that, at steady state, $\omega^s = (1-\beta)a^s/c^s > 0$, so that the above shows that there may be a case for a negative labor tax wedge for the skilled near steady states only if the external effects η_x^π and η_u^π are sufficiently large. More generally, and a part from these external effects, the case for labor taxation follows from standard arguments based on demand elasticities. I have have shown above in equation

(42) that the gap between the general equilibrium elasticity of labor supply and consumption for the skilled individuals is affected positively by σ_c^s and ω^s. In other words, larger values of these two variables imply that labor taxation generates less distortions than capital taxation. However, due to the effects of changing the capital and the unskilled labor intensity, a concentration of the entire burden of taxation on labor is not optimal. Using (47) in (50), we derive the following expression for the capital wedge

$$\tau^k = \frac{\eta_x^\pi c^s}{k}(1-\rho)\frac{\mu(1-\omega^s)}{\gamma + \mu(1-\sigma_c^s\omega^s)}. \tag{54}$$

Note that the optimal capital tax is zero when $\rho = 1$, *i.e.*, when there is maximum tax progressivity, and when $\omega^s = 1$, *i.e.*, the skilled individuals have no labor income. Now let us turn to the first order condition (49). To get more intuition, we can use (39) and rearrange terms to rewrite it as

$$\underbrace{\tilde{U}'_\rho(c^u)}_{\text{extra utility}} + \lambda \underbrace{f_u n'_\rho(c^u)}_{\text{extra output}} = \lambda + \mu \underbrace{U_c^s \left(\frac{n^s\pi}{n^u}\right)^{1-\rho} (1 + (1-\rho)\epsilon_n^u \eta_u^\pi)}_{\text{extra revenue cost}}. \tag{55}$$

In other words, optimality requires that the combined positive effect of some extra unskilled consumption arising from their utility and marginal productivity of labor equals the combined negative effect arising from resource dissipation and the extra tax revenues that must be raised to increase the unskilled consumption.

We may get a better understanding of the optimal tax rule by assuming a separable-constant elasticity utility function

$$U(c,n) = \begin{cases} c^{1-\sigma_c}/(1-\sigma_c) - n^{1+\sigma_n}/(1+\sigma_n) & \text{if } \sigma_c \neq 1, \\ \ln c - n^{1+\sigma_n}/(1+\sigma_n) & \text{otherwise.} \end{cases} \tag{56}$$

More specifically, according to a popular simplification in the optimal tax literature (cf. Saez and Stantcheva (2018)) one may set $\sigma_c = 0$ and derive

$$n_\rho(c^u) = ((1-\rho)c^u)^{\frac{1}{1+\sigma_n}},$$

so that, by rearranging the optimal tax conditions obtained above, we derive

$$\tau^{n,s} = -\frac{\mu(\eta_x^\pi + \eta_u^\pi)}{\gamma + \mu(1 - \eta_x^\pi - \eta_u^\pi)} < 0,$$

i.e., the optimal marginal tax on skilled labor is negative. The intuition is based on the definition of the general equilibrium elasticities in (41). Because there is no income effect, the general equilibrium elasticity related to consumption, g_c^s, is equal to one, which implies a very large elasticity of labor supply with respect to the net wage income.

An other important specification of the utility function in (56), is obtained by setting $\sigma_c = 1$. This example will be considered in more detail in the next section. By recalling (31), labor supply is independent of consumption and equal to the constant

$$n_\rho(c^u) = (1-\rho)^{\frac{1}{1+\sigma_n}}.$$

By (53) and (54), we obtain

$$\tau^{n,s} = \frac{\mu(\omega^s - (\eta_x^\pi + \eta_u^\pi))}{\gamma + \mu(1 - \eta_x^\pi - \eta_u^\pi)}. \tag{57}$$

In this case, the marginal labor tax on skilled workers is positive as long as their capital income to consumption ratio, ω^s, is relatively large. To get a better understanding of the capital tax formula, assume that $\gamma = 0$. Then, by the formula for τ^k in (54), we derive $\tau^k = \eta_x^\pi(1-\rho)c^s/k$. Furthermore, remember that $c^s = (1-\beta)a^s/\omega^s$, so that, at steady state,

$$c^s = (1-\beta)(k/\beta + b)/\omega^s.$$

Hence, if $b = 0$, we derive

$$\tau^k = \eta_x^\pi \left(\frac{1-\beta}{\beta}\right)\frac{(1-\rho)}{\omega^s}, \qquad \tau^{n,s} = \frac{\omega^s - (\eta_x^\pi + \eta_u^\pi)}{1 - (\eta_x^\pi + \eta_u^\pi)}. \tag{58}$$

The above suggests that the capital tax grows with the elasticity of the skill premium with respect to capital and it falls with the capital income to consumption ratio, ω^s. By the previous discussion, we know that a higher value of ω^s goes along with lower distortions from labor taxation. Then, a higher value of ω^s means that there are social gains from shifting taxes from capital to labor. Since, at steady state, $\omega^s = 1 - N(y^s)/c^s$, in this particular case we can claim that the optimal capital tax increases with the skilled individuals' net labor income as a share of their consumption. Loosely speaking, the higher is the human capital component of the rich individuals' wealth, the higher is the optimal capital tax (and the lower the labor tax on the skilled).

Changing Tax Progressivity. I consider now the effect on the unskilled workers' utility of changing the parameter ρ, the degree of tax progressivity, at an optimal steady state plan. Let $\partial c^u/\partial\rho$ be the total effect of increasing ρ on the optimal unskilled workers' consumption and $\partial n_\rho(c^u)/\partial\rho|_{c^u}$ the effect of increasing ρ on these workers' labor supply for given consumption. Then, the welfare effect of a higher ρ for the uskilled is given by

$$\frac{\partial U^u}{\partial \rho} = \left(U_c^u + U_n^u n_\rho'(c^u)\right) \frac{\partial c^u}{\partial \rho} + U_n^u \left.\frac{\partial n_\rho(c^u)}{\partial \rho}\right|_{c^u},$$

where

$$\left.\frac{\partial n_\rho(c^u)}{\partial \rho}\right|_{c^u} = -\frac{n^u}{(1-\rho)(1+\sigma_n^u+(1-\rho)\sigma_{cn}^u)}.$$

Using (31), we obtain

$$\frac{\partial U^u}{\partial \rho} = \frac{U^u c^u}{1+\sigma_n^u+(1-\rho)\sigma_{cn}^u}\left((\rho+\sigma_n^u+(1-\rho)\sigma_c^u)\frac{\partial \ln c^u}{\partial \rho}+1\right), \tag{59}$$

from which we derive

$$\left.\frac{\partial U^u}{\partial \rho}\right|_{\rho=0} > 0 \quad \Leftrightarrow \quad \frac{1}{\sigma_c^u+\sigma_n^u} > -\frac{\partial \ln c^u}{\partial \rho}.$$

Note that the direct effect of increasing ρ on labor supply for given consumption is negative. Hence, we can expect a higher tax progressivity to have a utility benefit in terms of reduced labor effort and a utility cost in terms of the reduced consumption that may follow from a lower employment of both skilled and unskilled individuals. Both of these two effects are magnified by a higher elasticity of labor supply, *i.e.*, by a lower value of σ_n. In the next section I will provide some simulations of a special version of the model with CRRA utility showing that a rising tax progressivity is not welfare improving for realistic values of σ_c and σ_n.

4. Numerical Analysis at Steady State

In this section I analyze the steady state allocations when utility is sparable in consumption and labor and characterized by constant elasticities. I analyze the steady state (polarized) equilibria with uniform labor taxation, study the effects of changing both τ^k and ρ, and compare these to the second best outcome. As a preliminary observation, note that, if we suppress labor supply of the wealthy individuals, our model becomes a special case of the traditional Judd (1985)'s capitalists-workers economy with non linear labor taxation. In this case, we obtain that workers' utility is decreasing in the capital tax rate and in the degree of tax progressivity, for all well defined preferences and technology (cf. appendix C). The example analyzed in this section shows that this is result does not hold in the present model.

I consider a special case of the more general specification of technology and preferences defined by the production function (2) and the utility function (56). In particular, I let $\xi = 1$, $\theta = 0$, $\sigma_c = 1$.

This parametrization implies

$$f(k, n^s, n^u) = Ak^\alpha (n^s)^{1-\alpha} + Bn^u, \qquad U(c, n) = \ln c - n^{1+\sigma_n}/(1 + \sigma_n). \tag{60}$$

A key feature of this example is that the intertemporal elasticity of substitution is exactly one, the marginal productivity of the unskilled labor is constant and, by the linearity of f in n^u, the skill premium, $\pi = f_s/f_u$, is independent of u. In particular,

$$f_u = B, \qquad f_s = A(1 - \alpha)x^\alpha. \tag{61}$$

In other words, unskilled labor is highly replaceable with capital and non essential, in the sense that its marginal productivity is bounded even when no unskilled is employed. This assumption is in line with some of the most pessimists predictions about the effect of automation and technical change in advanced economies. By the steady state Euler equation,

$$(1 - \tau^k)f_k = (1 - \tau^k)A\alpha x^{\alpha-1} = 1/\beta, \tag{62}$$

which delivers the equilibrium value of x as a function of τ^k. Using (61), equation (62) implies that the equilibrium skill premium is a decreasing function of the capital tax rate, *i.e.*,

$$\pi = (1 - \alpha)(A\alpha\beta(1 - \tau^k))^{\frac{\alpha}{1-\alpha}}. \tag{63}$$

From now on, the values x and π will be assumed to be verifying equations (62), (63), *i.e.*, to be strictly decreasing functions of the capital tax rate, τ^k.

Consider, first, the second best allocation for the example studied above. In this case, the capital tax should be set to zero, so that both x and π are determined by the time discount rate and the technological parameters. Moreover, the unskilled labor is also untaxed, so that, by the implementability constraint for this type of workers, we derive

$$c^u = Bn^s + z, \qquad c^u(n^u)^{\sigma_n} = B.$$

The above imply that the unskilled labor supply is a decreasing function of z. Recalling the second best characterization in (30), with the above specifications, we derive

$$\pi^* = \gamma \left(\frac{n^s}{n^u}\right)^{\sigma_n} \left(\frac{1}{1 - \tau^{n,s}}\right),$$

where π^* denotes the skill premium defined in (63) for $\tau^k = 0$ and $\tau^{n,s}$ is the second best optimal labor tax on the skilled. Evidently, the indirect utility of the unskilled is increasing in z and $\tau^{n,s}$ must be set so as to guarantee the feasibility of the second best allocation. Hence, the planner should raise $\tau^{n,s}$ so as to increase z and the unskilled workers' utility as much as it is postulated by the skilled welfare weight and resource feasibility. The above expression says that the cost of raising $\tau^{n,s}$ is a higher unskilled intensity of the work force.

Now I consider the same specification of the model for the case of uniform log linear tax schedule defined in (9) across the two type of individuals. Note that, by the implementability conditions (23), (24) and the first order condition (15), at a polarized steady state equilibrium, we have

$$n^u = (1-\rho)^{\frac{1}{1+\sigma_n}}, \tag{64}$$

$$n^s = ((1-\rho)(1-\omega^s))^{\frac{1}{1+\sigma_n}}. \tag{65}$$

where, I recall, ω^s is the skilled individuals' capital income to consumption ratio. In appendix D I provide some more details about the equilibrium structure. In particular, it is shown that the model typically features the co-existence of two equilibria for a given parameter specification: a low tax rate equilibrium and a high tax rate equilibrium, the former being Pareto dominant compared with the latter. This multiplicity appears to be a consequence of the non monotonicity of the government revenue with respect to tax rates. The simulation is based on the following parameter values:

A	B	α	g	b	σ_n	β
7.5	5	0.45	9.8	0	0.66	0.97

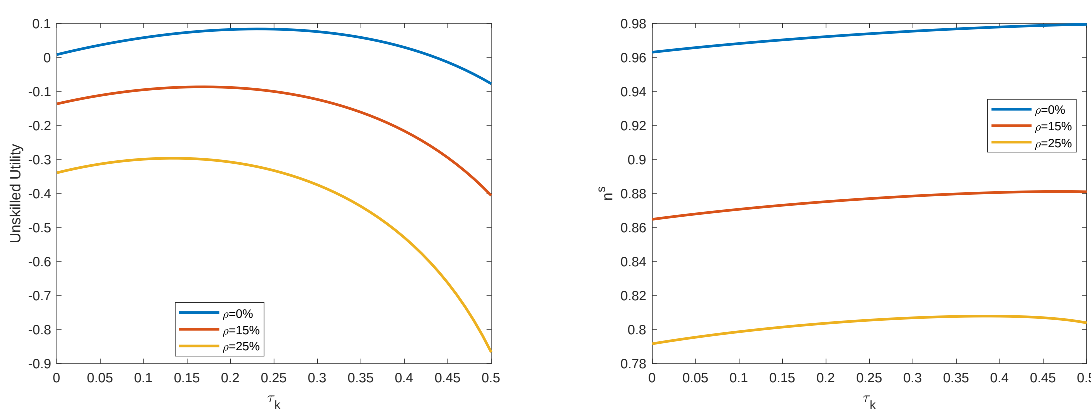
The values of A, B and g insure that a first best outcome cannot be achieved at steady state, the value of α matches a common estimate of the capital share of income and the value of σ_n is about half way between the most common estimates based on macro and micro data[4]. I provide some quantitative evaluation of the effects of changing the capital tax rate on welfare, labor supply and the marginal labor tax rate, some estimates of the optimal values of the capital tax for different values of the degree of progressivity, ρ, and different values of A. Finally, I conduct some robust checks by changing the the value of σ_n from 0.66 to 0.29 and 1.5. Note that, according to Heathcote et al. (2020), the estimated value of ρ in advanced economies range from a minimum of 9% (USA) to a maximum of 23% (Denmark).

Figures 1 shows the behavior of the unskilled utility and the skilled labor supply for different values of τ^k and ρ. Note that, under the assumed parameter configuration, the unskilled individuals' welfare is increasing in τ^k, as the latter rises from zero up to 23% if $\rho = 0$, up to 17% for $\rho = 0.15$ and up

[4]According to Whalen and Reichling (2017), estimates based on macro data for σ_n are between 0.25 and 0.38, whereas the values derived from micro data are range from 1.9 to 3.7 (see also Peterman (2016)).

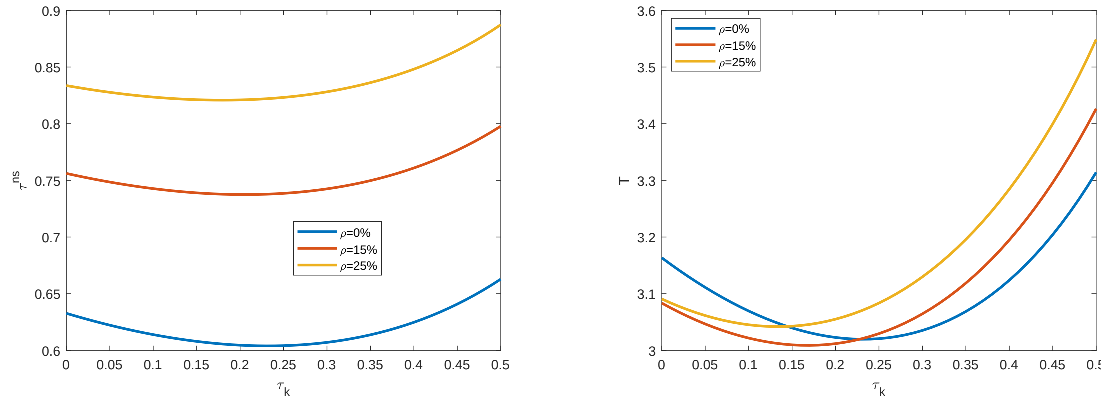
to 13% when $\rho = 0.25$. Hence, as I have anticipated in the previous section, the optimal capital tax rate falls with the degree of tax progressivity. To have a sense of the welfare impact of increasing the capital tax rate from zero to the optimal value, it may be useful to consider the percentage increase in the unskilled individuals' consumption that matches the welfare gain, *i.e.*, the *consumption equivalent welfare gain*. The latter is evaluated at 10% when $\rho = 0$, at 4% when $\rho = 0.15$ and at 3.5% when $\rho = 0.25$. For the present specification of the model, a rise in the degree of tax progressivity is welfare reducing for both skilled and unskilled, at any given capital tax rate, although it generates more equality. The second graph in figure 1 shows that the skilled workers labor supply is increasing with the capital tax. This effect goes along with a lower capital income and is one of the main reasons why the capital tax generates more welfare for the unskilled. Figure 2 shows the effect of a rising capital tax rate on the marginal labor tax on the skilled workers and the shape of the total tax-transfer schedule on the unskilled. The former appears to be rather large (ranging between 0.65 and 0.85) and increasing in the degree of tax progressivity. The curves describing the marginal tax rates are U-shaped and they have a minimum around the optimal capital tax. In other words, an optimal capital tax allows for a minimization of the marginal tax on the skilled workers labor. The second graph in figure 2 shows that, for this model specification, the unskilled workers pay positive taxes for all the selected levels of tax progressivity. Finally, I consider the case of a rising relative labor productivity of skilled labor, A/B, from 1.3 to 1.6, due to a rise in A only. Following this productivity improvement, the optimal capital tax rises from 0.12 to 0.19. The intuition is that a rising value of A, by increasing the skill premium, generates a higher labor supply by the skilled workers and this, in turn, makes labor taxation less distortionary.

FIGURE 1. Steady state effects of changing τ^k: Utility and Labor Supply



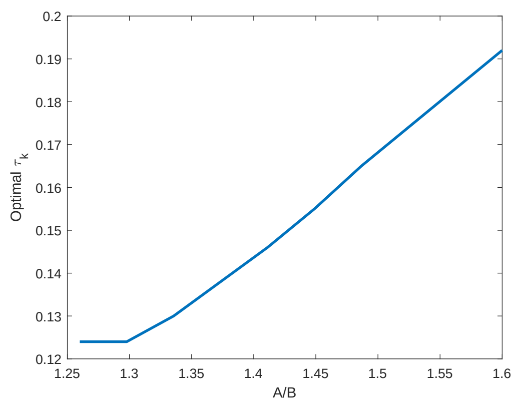
Notes: This figure plots the steady state values for the unskilled workers' utility and the the skilled workers' labor supply for different values of the capital tax rate τ^k and three different values of tax progressivity.

Figure 2. Steady state effects of changing τ^k: Labor Taxes



Notes: This figure plots the steady state values for the marginal labor tax rate on the skilled and the total tax on the unskilled, for different values of the capital tax rate τ^k and three different values of tax progressivity.

Figure 3. Optimal Capital Tax for Changing Skill Labor Productivity



Notes: This figure plots the level of the optimal capital tax rate for different values of the relative skill labor productivity.

5. Conclusions

I have analyzed the optimal taxation problem in a model with skilled biased technology and wealth polarization when labor taxes allow for arbitrary degrees of progressivity and, possibly, positive transfers to poor individuals, but cannot be contingent on skills. In this case, capital should be taxed and labor should be taxed only if skilled premium is not too sensitive with respect to capital and unskilled labor. A capital tax serves the role of mitigating the distortions that arise in the attempt to reduce income inequality. One of the main shortcomings of this model is the assumption that the distribution of skills is exogenous. I conjecture that, if workers can able to improve their skill level by investing in human capital, the case for a capital tax would diminish, as the latter may have adverse effects on incentives.

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Appendix A. The Second Best Allocation

From the unskilled implementability constraint, we derive a differentiable labor supply function $n^u = \phi(c^u, z)$. Letting $\phi_{c,t}$ and $\phi_{z,t}$ be the partial derivatives with respect to consumption and transfer, one can readily verify that $\phi_{z,t} < 0$. Now, let the unskilled indirect utility be

$$\hat{U}(c^u, z) \equiv U(c^u, \phi(c^u, z)),$$

and note that

$$\partial \hat{U}/\partial c_t^u = U_{c,t}^u + \phi_{c,t} U_{n,t}^u > 0.$$

I write the Lagrangean function for the second best problem as

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^t \Big(\gamma U(c_t^s, n_t^s) + \hat{U}(c_t^u, z_t) + \mu (U_{c,t}^s c_t^s + U_{n,t}^s n_t^s)$$
$$+ \lambda_t (f(k_t, n_t^s, \phi(c_t^u, z_t)) - c_t^s - c_t^u - g_t - k_{t+1}) + \xi_t z_t \Big),$$

where ξ_t is the Lagrange multiplier associated to the constraint $z_t \geq 0$. To provide a first order characterization of these plans, it is convenient to define the *general equilibrium elasticities* of consumption and labor for the skilled individuals

$$g_c^s = 1 + \frac{U_{cc}^s c^s + U_{cn}^s n^s}{U_c^s} = 1 - \sigma_c^s + \sigma_{cn}^s U_n^s n^s / U_c^s c^s, \qquad g_n^s = 1 + \frac{U_{cn}^s c^s + U_{nn}^s n^s}{U_n^s} = 1 + \sigma_n^s + \sigma_{cn}^s.$$

These represent the distortions in the skilled individuals' consumption and labor supply caused by changes in, respectively, U_c^s (price of consumption) and U_n^s (price of labor). Assuming that the solution for $\{c_t^s, n_t^s, c_t^u\}_{t=0}^{\infty}$, the first order conditions are

$$U_{c,t}^s \left(\gamma + \mu g_{c,t}^s\right) - \lambda_t = 0, \tag{66}$$
$$-U_{n,t}^s \left(\gamma + \mu g_{n,t}^s\right) - \lambda_t f_{s,t} = 0, \tag{67}$$
$$U_{c,t}^u + \phi_{c,t} U_{n,t}^u + \lambda_t (f_{u,t} \phi_{c,t} - 1) = 0, \tag{68}$$
$$U_{n,t}^u \phi_{z,t} + \lambda_t f_{u,t} \phi_{z,t} + \xi_t = 0, \tag{69}$$
$$\beta \lambda_{t+1} f_{k,t+1} - \lambda_t = 0. \tag{70}$$

By (66) and (67), we readily obtain the implicit labor tax on the skilled from

$$\frac{-U_{n,t}^s}{U_{c,t}^s} = f_{s,t} \left(\frac{\gamma + \mu g_{c,t}^s}{\gamma + \mu g_{n,t}^s} \right), \tag{71}$$

which is positive because $g_c^s < g_n^s$. Since $\phi_{z,t} < 0$, equation (69) provides

$$\lambda_t \geq -U_{n,t}^u / f_{u,t}.$$

Recalling (**??**), the above inequality and equation (68) imply

$$U_{c,t}^u + \phi_{c,t} U_{n,t}^u \geq -\frac{U_{n,t}^u}{f_{u,t}} (1 - f_{u,t} \phi_{c,t}),$$

i.e.,

$$f_{u,t} \geq -U_{n,t}^u / U_{c,t}^u, \tag{72}$$

where the strict inequality holds only if $\xi_t > 0$ and $z_t = 0$. Finally, note that the above, together with (66) provides

$$U^u_{c,t}/U^s_{c,t} = \gamma + \mu g^s_{c,t}. \tag{73}$$

Appendix B. Interpretation of the General Equilibrium Elasticities

To get more intuition about the role of the general equilibrium elasticities, consider the following static optimal tax problem. A representative consumer maximizes utility $U(c,n)$ over the budget constraint

$$c \leq \hat{w}n,$$

where $\hat{w} = w(1-\tau^n)$ and w is some exogenous labor productivity. By the first order conditions for utility maximization, we derive

$$U_c\hat{w} + U_n = 0.$$

The economy lasts for one period only and allocations of consumption and labor are subject to the resource feasibility constraint

$$c + g \leq wn,$$

where g defines a given amount of government spending. The government budget constraint is equivalent to the individual's budget constraint, so that, by using the first order conditions, the optimal tax problem reduces to

$$\max_{c,n} U(c,n) \quad \text{s.t.: } c + g \leq wn,\, U_c c + U_n n \geq 0.$$

The second constraint is what I have called implementability. Then, a solution of the planning problem verifies:

$$-\frac{U_n}{U_c} = w\left(\frac{1+\mu g_c}{1+\mu g_n}\right),$$

where

$$g_c = 1 + \frac{U_{cc}c + U_{cn}n}{U_c}, \qquad g_n = 1 + \frac{U_{cn}c + U_{nn}n}{U_n}$$

are the general equilibrium elasticities and $\mu \geq 0$ the Lagrange multiplier associated to the implementability constraint. It follows that the implicit tax on labor is $\tau^n = \mu(g_n - g_c)/(1+\mu g_n)$. To see how the general equilibrium elasticities are related to the demand elasticities, let $c(\hat{w})$ and $n(\hat{w})$ be the consumption demand and labor supply decisions and differentiate the first order conditions and the budget constraints to get

$$c'(\hat{w}) = -\frac{U_n g_n}{\Delta}, \qquad n'(\hat{w}) = \frac{U_c g_c}{\Delta},$$

where $\Delta = -\hat{w}U_{cc} - 2U_{cn} - U_{nn}/\hat{w} > 0$. Then, by exploiting again the first order conditions and the budget constraint,

$$\frac{c'(\hat{w})\hat{w}}{c} - \frac{n'(\hat{w})\hat{w}}{n} = \frac{-U_n}{n\Delta}(g_n - g_c).$$

Hence, $g_n - g_c > 0$ if and only if the wage elasticity of consumption demand is larger than the wage elasticity of labor supply. As we have seen in the above section, a large value of $g_n - g_c$ calls for a large labor tax. It follows that a large labor tax is associates to a much larger elasticity of consumption demand relative to labor supply.

Appendix C. Comparative Statics at Steady State of the Judd's Model

To understand the role of capital taxes and tax progressivity, it is useful to start with a version of the model that replicates a steady state allocation of the Judd (1985)'s capitalists-workers economy. Namely, suppose that the skilled individuals are replaced by a set of non working capitalists whose only source of income capital and financial ownership, while the unskilled are hand-to-mouth workers. The steady version of (23) and (24) under these assumptions provide

$$n^u = n_\rho(c^u), \tag{74}$$

$$c^s = (1-\beta)(k/\beta + b), \tag{75}$$

where k and b are the steady state levels of capital and public debt. Letting $n = n^u$, $w^u = w$ and $x = k/n$, the optimal allocation of the capitalists' consumption at steady state and profit maximization imply

$$(1-\tau^k)f_x(x,1) = 1/\beta, \qquad w = f(x,1) - xf_x(x,1). \tag{76}$$

The above determine the equilibrium value of the capital-labor ration, x, as a decreasing function of τ^k. Now I use (75) to write market clearing as

$$c^u + (g + (1-\beta)b) = n_\rho(c^u)\Phi(x), \tag{77}$$

where

$$\Phi(x) = f(x,1) - x/\beta$$

is the per capita output net of investment. I assume that the above equation has a solution $c^u(x)$ and, in case of multiple solutions, I select the largest value as a steady state equilibrium of the Judd (1985)'s capitalists-workers economy, since the workers' indirect utility is increasing in their consumption. Note that this value is such that the left hand side of (77) crosses the right hand side from below, so that

$$n'_\rho\Phi(x) < 1. \tag{78}$$

Now I differentiate the steady state equilibrium, $c^u(x)$ with respect to x and ρ to obtain

$$\frac{\partial c^u}{\partial x} = \frac{n\tau^k f_x}{1 - n'_\rho(c^u)\Phi(x)} \geq 0. \qquad \frac{\partial c^u}{\partial \rho} = \left.\frac{\partial n_\rho}{\partial \rho}\right|_{c^u} \frac{(f - x/\beta)}{1 - n'_\rho(c^u)(f - x/\beta)} < 0.$$

By (78) and since x is decreasing in τ^k, it follows that the workers' utility is decreasing in the capital tax rate. This result is a restatement of Judd's celebrated proposition that the efficiency loss due to a higher tax on capital more than compensate the welfare gain the workers may obtain from redistributing the burden of taxation away from labor.

Appendix D. Solving the Model

For any tax policy, $(\bar{\tau}^n, \rho)$, equalization of the marginal rates of substitution between consumption and labor across individuals implies

$$c^u\pi^{1-\rho} = c^s(1-\omega^s)^{\frac{\rho+\sigma_n}{1+\sigma_n}}. \tag{79}$$

The above says that consumption inequality grows with π and ω^s and it establishes a relation between the unskilled and the skilled individuals' consumption. Now note that the market clearing condition can be stated as

$$c^s + c^u + g = Bn^u + n^s(Ax^\alpha - x). \tag{80}$$

Using (64), (65) and (79) into (80) and letting $\Phi(x) = Ax^\alpha - x$, we derive the following one-to-one relation between ω^s and c^s,

$$c^s = \frac{((1-\rho)^{\frac{1}{1+\sigma_n}}B - g) + \Phi(x)((1-\rho)(1-\omega^s))^{\frac{1}{1+\sigma_n}}}{1 + \pi^{\rho-1}(1-\omega^s)^{\frac{\rho+\sigma_n}{1+\sigma_n}}} \equiv G(\omega^s), \tag{81}$$

for $\omega^s \in [0,1]$. The above is called the *good market clearing condition*, and it shows that the effect of larger capital income to consumption ratio, ω^s, on the skilled consumption is ambiguous. In fact, for any given c^s, a higher ω^s decreases both the unskilled consumption and the skilled labor supply. Now recall that

$$a^s = (n^s x/\beta + b). \tag{82}$$

Then, since $\omega^s = (1-\beta)a^s/c^s$, using (65) into (79), and letting $r = (1-\beta)/\beta$ be the steady state (before tax) real interest rate, we obtain

$$c^s = \frac{((1-\rho)(1-\omega^s))^{\frac{1}{1+\sigma_n}} rx + rb/(1+r)}{\omega^s} \equiv H(\omega^s). \tag{83}$$

The latter is called the *financial market clearing condition*, and it establishes an inverse relation between ω^s and c^s. In fact, a higher ω^s decreases the skilled individuals' labor supply and, then, it lowers the equilibrium capital stock, along with ω^s. This is coherent with a higher ω^s only if c^s falls.

I assume that there exists a positive solution $(\hat{c}^s, \hat{\omega}^s)$ with $\hat{\omega}^s \in (0,1)$ such that

$$\hat{c}^s = G(\hat{\omega}^s) = H(\hat{\omega}^s).$$

For each of these solutions, equation (79) generate a unique value $\hat{c}^u$. Together with the values x, π, n^s and n^u defined in (62), (63), (64) and (65), these solutions form an equilibrium for this economy. Note that, since $G(0) < \lim_{\omega^s \to 0} H(\omega^s) = +\infty$ and

$$G(1) = (1-\rho)^{\frac{1}{1+\sigma_n}}B - g, \qquad H(1) = rb/(1+r),$$

the existence of at least one equilibrium is guaranteed for $(1-\rho)^{\frac{1}{1+\sigma_n}}B > g + rb/(1+r)$. On the other hand, if the latter inequality is not verified, equations (81), (83) generate at least two equilibria (if any). Multiplicity follows from a sort of Laffer effect, whereby the government is able to raise the same revenue from a high marginal tax rate and a low tax base or from a small tax rate and large tax base.

To give a sense of the outcomes of the model, I provide a simulation based on the parameter values specified in section 4. In this case, setting $\rho = 0.15$ and $\tau^k = 0.23$, the model generates two equilibria, E_1 and E_2. The former is characterized by a higher consumption of both skilled and unskilled and by a lower capital income to consumption ratio, ω^s. More generally, since $H(.)$ is decreasing, the set of equilibria can be ranked in ascending (descending) order in terms of the value of c^s (ω^s). By (79), the equilibrium characterized by the largest value of c^s (lowest value of ω^s) generates the largest value of c^s. Then, since the individuals' indirect utility functions are increasing in consumption, the solution characterized by the largest value of consumptions, c^s and c^u, and the lowest value of capital income to consumption ratio, ω^s, Pareto dominates any other equilibrium.