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Do Low Interest Rates Harm Innovation, Competition, and Productivity Growth?

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Abstract

The answer to this question crucially depends on the nature of creative destruction. In Schumpeterian models, if innovation by market laggards only incrementally refines their existing technology, then, as the interest rate falls to very low levels, growth declines with low-R&D market leaders becoming entrenched. However, if market laggards have some chance to innovate radically and immediately catch up to the leading technology, low interest rates boost productivity growth. Using micro data, we structurally estimate a Schumpeterian model that nests these alternative possibilities. In the estimated model, laggards have a meaningful chance to innovate radically, implying that low interest rates increase growth and market competition. Incorporating firm entry, optimal patent policy, and financial frictions strengthens our results.

JEL Classification: E2, O31, O34

Keywords: Real interest rate, Innovation, growth, Markups

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Do Low Interest Rates Harm Innovation, Competition, and Productivity Growth?

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(Federal Reserve Board)

April 5, 2021

Abstract

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1. Introduction

The long and persistent decline of real interest rates in developed economies is one of the central macroeconomic trends of recent decades. This decline in interest rates has coincided with low productivity growth, rising market power, and declining business dynamism. Productivity growth has fallen, except for a temporary burst in the late 1990s and early 2000s (Fernald et al., 2017). Markups, market concentration, and productivity gaps between leaders and laggards have been increasing (Hall, 2018; Autor et al., 2020; Andrews et al., 2016).[1] An ongoing scholarly effort aims to understand the linkages among these trends.[2] In a recent contribution, Liu, Mian and Sufi (2020) develop a Schumpeterian model in which low interest rates reduce productivity growth by entrenching low-R&D market leaders. However, the effects of interest rates on productivity growth, market power, and innovation remain far from well understood.

Our paper evaluates whether low interest rates can explain the trends of low productivity growth and declining business dynamism. We study a parsimonious Schumpeterian model. Firms in each industry compete in the product market and invest in R&D to improve productivity. To begin, we show how the effects of lower interest rates on growth and market power depend on the nature of creative destruction. Much of the literature assumes that innovation by market laggards is sometimes radical or creative, allowing the laggard to immediately catch up to the leading technology (Aghion et al., 2001; Acemoglu and Cao, 2015). Other research assumes that laggards innovate only by incrementally refining their existing technologies (Harris and Vickers, 1987; Liu et al., 2020). These alternative views about creative destruction have very different macroeconomic implications.

In Liu et al. (2020), laggards can innovate only incrementally, making leadership

[1] Akcigit and Ates (2021) review the literature on declining dynamism.

[2] Potential explanations include the rising importance of intangible or organizational capital (Aghion et al., 2019b; De Ridder, 2019), anti-competitive practices that reduce knowledge diffusion (Akcigit and Ates, 2019), declining innovativeness of market laggards (Olmstead-Rumsey, 2020), and low population growth (Peters and Walsh, 2020).

persistent. A lower interest rate increases the value of being a persistent market leader and therefore creates a costly R&D race in competitive industries. Because achieving leadership requires winning this costly R&D race, a lower interest rate discourages laggards, allowing leaders to remain entrenched despite innovating little. As a result, starting from a low interest rate, further declines in the interest rate lead to *lower* growth. At the other extreme, in Acemoglu and Akcigit's (2012) quick catch-up model, laggard innovation is always radical (rather than incremental), making leadership less persistent. In their model, a lower interest rate increases productivity growth by reducing the required return on R&D.[3]

Because the effects of low interest rates on growth and competition depend on the nature of creative destruction, we structurally estimate a Schumpeterian model that encompasses these alternatives. Our model nests laggard innovation that is always incremental or always radical, as well as intermediate cases. We also nest a range of assumptions about patent policy, which also shapes creative destruction. We identify our model's parameters using salient moments characterizing market power, innovation, and firm dynamics. Our estimated model provides a good fit to the US data on reallocation and the cross section of markups, profit volatility, R&D, and innovation output.[4] These moments are informative about the nature of creative destruction.

To evaluate whether low interest rates can explain recent trends in innovation and market power, our central exercise studies the aggregate implications of a lower household discount rate in the estimated model.[5] Our parameter estimates imply a meaningful chance that innovating laggards quickly catch-up to leaders. Correspondingly, as the discount rate falls, productivity growth rises and market power declines; the interest rate falls less than one-for-one with the discount rate. These

[3]Radical innovation by laggards can be considered a type of "advantage of backwardness" (Gerschenkron, 1962). Acemoglu et al. (2020) studies the origins of incremental and radical innovation.

[4]When we extend the model to include entry, the model fits well the employment share of firms of different ages and the Foster et al. (2001) growth decomposition.

[5]Changes in the discount rate represent factors such as demographics and risk appetite that are not modeled and affect the demand for claims on firm profits.

results suggest that recent key macroeconomic trends of low productivity growth and rising markups are explained by factors other than low interest rates.[6]

Unpacking the mechanism with the innovation multiplier. A lower discount rate affects productivity growth through three channels. First, it contributes to higher innovation and growth by reducing the required return on R&D ("valuation channel"). Second, these valuation-driven increases in innovation induce strategic interactions among firms that can affect aggregate productivity growth ("strategic channel").[7] Third, valuation-driven increases in innovation affect the distribution of leader-laggard productivity gaps, which impacts aggregate productivity growth because R&D is highest in competitive industries ("composition channel"). Depending on the nature of creative destruction, strategic interactions among firms and composition effects can amplify, dampen, or even overturn the boost to R&D through the valuation channel.

To characterize and quantitatively evaluate these channels, we introduce an *innovation multiplier* that maps the cross section of valuation-driven increases in firm innovation into the general equilibrium effect on aggregate growth. Based solely on the direct effect of valuation-driven increases in innovation, a 1 percentage point decline in the discount rate would increase annual growth by 15 basis points. We find that the strategic channel subtracts 6 basis points of growth. The composition channel adds 1 basis point. Thus, the strategic and composition channels on balance dampen, but do not overturn, the valuation effect of lower interest rates on growth.

Robustness and entry. To assess the robustness of our benchmark model's results, we introduce firm entry, inelastic labor supply, and imperfect substitution across each industry's varieties. We also introduce alternative assumptions about the elasticity of innovation to R&D, and alternative targeted values for key moments including the distribution of markups. We re-estimate the model after each of these

[6] Aside from low interest rates, potential explanations for low aggregate productivity growth and rising markups that have been proposed in the literature are detailed in footnote 2.

[7] Section 5 explains these strategic interactions in detail. For example, we show how valuation-driven increases in laggard innovation can generate higher leader innovation (through an "escape competition" effect) or lower leader innovation (through a "trickle down" effect).

modifications. Our results are robust. All of the re-estimated models feature some radical innovation for the laggard or entrant, with growth increasing as the discount rate declines. Our results are also robust to incorporating optimal patent policy and financial frictions, as discussed next.

Patent policy. Patent policy is another key aspect of creative destruction that shapes the relation of growth, competition, and the interest rate. We therefore ask: How should patent policy be optimally adjusted when there is a persistent change in the interest rate? And, if patent policy is adjusted optimally, what are the implications for the relation of interest rates, productivity growth, and market power? In our estimated model, the social planner faces a trade-off: Weaker patent protection reduces markups and production distortions, but also reduces innovation and productivity growth. As the discount rate falls, this trade-off changes in a way that leads the social planner to strengthen patent protection: Growth, but not production distortions, becomes much more sensitive to the patent expiry rate.[8] As a result, if the social planner adjusts patent policy optimally, the growth-interest rate relation remains negative and slightly steepens.

To understand how these results depend on the nature of creative destruction, we also study an economy with purely gradual laggard advancement. In this sclerotic economy, at low interest rates, laggards are discouraged and leaders innovate only enough to counteract patent expiry. As a result, at low interest rates, this economy offers a free lunch to the social planner: Weakening patent protection reduces markup distortions *and* increases growth, by forcing leaders to innovate more to maintain their advantage. Moreover, as the interest rate declines, the planner prefers to *weaken* patent protection (the opposite of what obtains in our estimated benchmark model), to counter the growth-reducing, anti-competitive effects of a low interest rate. In this economy, if patent policy is held constant, then, as the interest rate approaches very low levels, growth declines. Remarkably, if the social planner adjusts patent policy

[8]This result holds whether the social planner focuses only on steady-state welfare or also takes into account transition dynamics.

optimally, then growth rises even as the interest rate approaches very low levels.

Credit access. Financial frictions are another factor that can affect the relation of interest rates, productivity growth, and market power. We therefore introduce a limited commitment constraint that can restrict R&D. As expected, the financial constraint does not bind for leaders with large technology advantages, which have low desired R&D and high pledgeable assets. However, the constraint can bind for leaders in competitive industries (which, absent the constraint, choose high R&D but have lower pledgeable profits than leaders with large advantages) and for laggards. Again, the consequences for aggregate productivity growth depend on the nature of creative destruction. In our estimated benchmark model, tighter credit access reduces productivity growth, by constraining the R&D of leaders with small technology advantages; these leaders contribute importantly to aggregate growth. However, in the economy with only incremental laggard advancement, when the discount rate is low, tighter credit access can *increase* growth, because constraints on leaders with small technology advantages increase the share of high-R&D, low-market-power industries. Moreover, when access to credit is very restricted, it is no longer the case that growth declines as the interest rate falls to very low levels, because lower discount rates relax the financial constraint by increasing the value of pledgeable profits.

Related literature. Low interest rates and low productivity growth pose challenges to monetary and fiscal policy. The literature on "secular stagnation" emphasizes how a decline in aggregate consumption demand is exacerbated by leverage and the zero lower bound (Hansen, 1939; Eggertsson and Krugman, 2012; Summers, 2014). Such negative effects of lower aggregate demand would be further amplified if lower demand triggers a decline in productivity growth through its effects on firms' strategic R&D races, as in Liu et al. (2020). However, we find in our estimated model that lower aggregate consumption demand boosts growth, *dampening* the transmission of lower aggregate demand to the interest rate. Benigno and Fornaro (2017) shows that lower demand can reduce innovation and productivity growth if the zero lower bound creates a stagnation trap. Our paper also contributes to the literature on misallocation (Hsieh and Klenow, 2009; Peters, 2020), by studying optimal patent policy in a low interest rate environment.

The paper is organized as follows. Section 2 presents the model. Section 3 studies how the nature of creative destruction affects growth and the interest rate. Section 4 describes the model estimation and presents the main results. Section 5 unpacks the economic mechanism by introducing the innovation multiplier. Sections 6 and 7 study how the relation of growth, market power, and the interest rate depends on patent policy and credit access. Section 8 concludes. The Internet Appendix provides additional material, including extensions of the model, proofs, and robustness exercises.

2. Model

2.1 Preferences and final goods

The economy admits a representative household with utility function

$$\int_{t=0}^{\infty} e^{-\rho t}\big(\ln(C(t)) - L(t)\big)dt, \tag{1}$$

where $C(t)$ is consumption of the final good, $L(t)$ is labor, and $\rho > 0$ is the discount rate.

A continuum of intermediate-goods industries is indexed by $j \in [0,1]$. Each industry includes two firms. The final good is produced using intermediate goods according to the production function, $\ln Y(t) = \int_0^1 \ln[y_1(j,t)^{\frac{\kappa-1}{\kappa}} + y_2(j,t)^{\frac{\kappa-1}{\kappa}}]^{\frac{\kappa}{\kappa-1}} dj$, where $y_i(j,t)$ is the quantity produced by firm $i \in \{1,2\}$ in industry j. The production function is a Cobb-Douglas aggregator across industries with a constant elasticity of substitution $\kappa > 1$ between the two varieties within each industry. The final good is the numeraire and sold in a perfectly competitive market.

2.2 Intermediate goods market

Each firm uses a linear production technology. At time t, firm i in industry j produces $y_i(j,t) = q_i(j,t)l_i(j,t)$, where $q_i(j,t)$ is the firm's labor productivity and $l_i(j,t)$ is labor hired at wage $w(t)$. We next drop the time t and industry j indexes to characterize equilibrium in the intermediate goods market, taking each firm's productivity as given. Firms compete in prices, à la Bertrand. Firm i sets price p_i. Cobb-Douglas aggregation implies that sales are equal across industries. Firm i solves $\Pi_i = \max_{p_i}(p_i - \frac{w}{q_i})y_i$, subject to: $\frac{p_1}{p_2} = (\frac{y_1}{y_2})^{-\frac{1}{\kappa}}$ and $Y = p_1 y_1 + p_2 y_2$.

When it causes no confusion, we denote the leader by i and the laggard by $-i$. Equilibrium gross markups are $\psi_i = \frac{\kappa+\nu^{1-\kappa}}{\kappa-1}$ and $\psi_{-i} = \frac{\kappa\nu^{1-\kappa}+1}{(\kappa-1)\nu^{1-\kappa}}$, where the relative price $\nu \equiv \frac{p_i}{p_{-i}}$ is defined implicitly by $\nu^{\kappa} = \frac{q_{-i}}{q_i}\frac{\kappa\nu^{\kappa-1}+1}{\kappa+\nu^{\kappa-1}}$. The equilibrium profits and labor demand are

$$\pi_z = 1 - \frac{\kappa}{\kappa-1}\psi_z^{-1} \qquad\qquad l_z = \frac{\pi_z}{\psi_z - 1}\frac{1}{\omega}, \tag{2}$$

for $z \in \{i, -i\}$ and where $\pi_z = \frac{\Pi_z}{Y}$ and $\omega = \frac{w}{Y}$ are scaled profits and wages, respectively. The leader's markup ψ_i and profit π_i are weakly increasing in the leader's relative productivity $\frac{q_i}{q_{-i}}$. The laggard's markup ψ_{-i} and profit π_{-i} are weakly increasing in the laggard's relative productivity $\frac{q_{-i}}{q_i}$. In industries with neck-and-neck competition ($q_i = q_{-i}$), each firm earns a profit $\pi_i = \pi_{-i} = \frac{1}{\kappa+1}$ and the markup is $\psi_i = \psi_{-i} = \frac{\kappa+1}{\kappa-1}$. When within-industry varieties are perfect substitutes, limit pricing obtains and only the leader produces, with leader markup $\psi_i = \frac{q_i}{q_{-i}}$ and profit $\pi_i = 1 - \psi_i^{-1}$.

Productivity ladder. Firms in each industry are ordered on a quality ladder. Each rung represents a proportional productivity improvement of scale $\lambda > 1$. The number of rungs separating leader and laggard is $s \in \{0, 1, ..., \bar{s}\} \equiv S^+$. We assume that the maximum possible technology gap between leader and laggard within an industry is $\bar{s}$. The *technology position* of a firm at time t is denoted by $\sigma \in \{-\bar{s}, \ldots, \bar{s}\} \equiv S$. In an industry with gap $s > 0$, the leader has technology position $\sigma = s$ and the laggard's position is $\sigma = -s$. A firm with position $\sigma = 0$ is tied.

An innovating leader advances one rung and its productivity increases by a factor λ. An innovating laggard can advance incrementally or radically. With probability $(1-\phi)$, an innovating laggard advances one rung—a *slow catch-up* innovation (Acemoglu and Akcigit, 2012). With probability ϕ, an innovating laggard obtains a *quick catch-*

up innovation, closing the gap completely.[9] The expected number of rungs that an innovating laggard advances, $1 + \phi(s-1)$, is strictly increasing in the industry gap if the catch-up speed ϕ is strictly positive. Thus, an innovation catch-up speed greater than zero embeds an "advantage of backwardness" (Gerschenkron, 1962). A firm in position σ innovates at rate x_σ by hiring $G(x_\sigma; B) = [x_\sigma/B]^{\frac{1}{\gamma}}$ workers as R&D scientists. $B > 0$ is an R&D cost scaling parameter and $\gamma > 0$ captures the convexity of R&D costs in the arrival rate.

Patent expiry also allows a laggard to partly or completely catch up with its competitor. Patent expiry occurs at rate $\eta \geq 0$. With probability $(1-\zeta)$, patent expiry shrinks the gap by one rung. With probability ζ, patent expiry collapses the gap to zero.

2.3 Equilibrium

We focus on Markov perfect equilibria. We characterize the balanced growth path (BGP) in which output grows at a constant rate g with a stationary distribution of technology gaps. All growing variables are scaled by $Y(t)$. The scaled value function of a firm in position σ is $v_\sigma(t) \equiv \frac{V_\sigma(t)}{Y(t)}$, where $V_\sigma(t)$ is the firm's discounted expected net profits. Scaled operating profits for a firm in position σ are π_σ, from (2).

Household and firm maximization. Under preferences (1), household maximization implies the Euler equation

$$\frac{\dot{C}}{C} = \frac{\dot{Y}}{Y} = g = r - \rho \tag{3}$$

and scaled wage $\omega = 1$.

Denote the innovation strategy of a firm's competitor by $\{x_\sigma^c\}_{\sigma \in S}$, where x_σ^c is the innovation rate of a competitor in technology position σ. Each firm chooses its innovation strategy to maximize discounted expected future profits, taking as given its competitor's innovation strategy and the wage rate. The firm trades off R&D costs against the future discounted profits from an innovation. Operating profits are taxed

[9] Internet Appendix Section A.2 extends the model to include the possibilities of the laggard advancing an intermediate number of rungs or even leapfrogging the leader.

at rate τ.

For a leader (i.e., a firm with $\sigma > 0$), using the Euler equation, the steady state value function satisfies

$$\rho v_\sigma = \max_{x_\sigma} \underbrace{(1-\tau)\pi_\sigma}_{\text{Operating profits}} - \underbrace{G(x_\sigma)\omega}_{\text{R\&D costs}} + \underbrace{x_\sigma \Delta v_\sigma}_{\text{Own innovation}} + \\ \underbrace{x^c_{-\sigma}(\phi v_0 + (1-\phi)v_{\sigma-1} - v_\sigma)}_{\text{Competitor innovation}} + \underbrace{\eta(\zeta v_0 + (1-\zeta)v_{\sigma-1} - v_\sigma)}_{\text{Patent expiry}}. \quad (4)$$

The leader value function has three main parts. First, firms earn after-tax operating profits $(1-\tau)\pi_\sigma$. Second, firms incur R&D costs and, if successful in R&D, earn the expected capital gain from own innovation, Δv_σ, with $\Delta v_\sigma = v_{\sigma+1} - v_\sigma$ for $\sigma \geq 0$. Third, leader firms can experience capital losses from a competitor innovation or patent expiry. Competitor innovation, with arrival rate $x^c_{-\sigma}$, reduces the leader's technology position to zero (with probability ϕ) or by one rung (with probability $1-\phi$). Patent expiry, with arrival rate η, also reduces the leader firm's technology position by at least one step.

Similarly, the value function for a laggard (i.e., a firm with $\sigma < 0$) is

$$\rho v_\sigma = \max_{x_\sigma} \underbrace{(1-\tau)\pi_\sigma}_{\text{Operating profits}} - \underbrace{G(x_\sigma)\omega}_{\text{R\&D costs}} + \underbrace{x_\sigma \Delta v_\sigma}_{\text{Own innovation}} + \\ \underbrace{x^c_{-\sigma}(v_{\sigma-1} - v_\sigma)}_{\text{Competitor innovation}} + \underbrace{\eta(\zeta v_0 + (1-\zeta)v_{\sigma+1} - v_\sigma)}_{\text{Patent expiry}}. \quad (5)$$

For a firm in technology position $\sigma < 0$, the capital gain from a successful innovation is $\Delta v_\sigma = \phi v_0 + (1-\phi)v_{\sigma+1} - v_\sigma$. Equation (5) also holds for tied firms if one substitutes $\eta = 0$.

For a firm in any position σ, profit maximization implies

$$x_\sigma = G'^{-1}\left(\frac{\Delta v_\sigma}{\omega}\right). \quad (6)$$

Because G is twice differentiable and strictly convex, the firm innovation rate x_σ is increasing in the capital gain from innovation and decreasing in the cost of R&D measured by the scaled wage ω.

Market clearing and aggregate growth. Let μ_s denote the share of industries with technology gap s. The steady-state industry distribution $\{\mu_s\}_{s\in S^+}$ is obtained by equating outflows and inflows for each gap s. For $s > 0$,

$$\underbrace{(x_s + x_{-s} + \eta)\mu_s}_{\text{Outflow from gap } s} = \underbrace{\mu_{s+1}((1-\phi)x_{-(s+1)} + (1-\zeta)\eta) + \mu_{s-1}x_{s-1}(1 + \mathbb{1}_{s=1})}_{\text{Inflow to gap } s}. \tag{7}$$

For an industry with a positive gap s, outflows occur because of leader or laggard innovation or patent expiry. Inflows to gap s occur when there is an incremental laggard innovation or incremental patent expiry in an industry with gap $s+1$. An additional source of inflows to gap s is a leader innovation in an industry with gap $s-1$. The indicator function for $s = 1$, or $\mathbb{1}_{s=1}$, is used to take into account that a tied industry shifts to a one-rung gap when either of the two tied firms innovates.

The stationary distribution of gaps is determined by (7), which applies for $s > 0$, together with the normalization $\sum_{s\in S^+} \mu_s = 1$. For completeness, the outflow-inflow identity for tied industries is

$$\underbrace{2x_0\mu_0}_{\text{Outflows}} = \underbrace{\mu_1(x_{-1} + \eta) + \sum_{\sigma\in(2,\ldots,\bar{s})} \mu_\sigma(\phi x_{-\sigma} + \zeta\eta)}_{\text{Inflows}}. \tag{8}$$

Outflows from neck-and-neck competition occur when either of the two firms innovates. Inflows to neck-and-neck competition occur due to any innovation or patent expiry in industries with a one-rung gap and due to quick catch-up innovation or patent expiry in industries with a gap of more than one rung.

Aggregate growth, g, is

$$g = \ln\lambda \sum_{s\in S^+} \mu_s(1 + \mathbb{1}_{s=0})x_s = \ln\lambda \sum_{s=1}^{\bar{s}} \mu_s\big[(\phi s + 1 - \phi)x_{-s} + (\zeta s + 1 - \zeta)\eta\big]. \tag{9}$$

Growth equals tied and leader firms' productivity gains, which accrue through innovation. Equivalently, growth equals laggards' productivity gains, which accrue though innovation and patent expiry. Laggard innovations do not directly advance the productivity of technologies in use. However, in a BGP, laggard advances must equal leader advances, highlighting the connection between creative destruction and growth.

The aggregate demand for labor is

$$L = \sum_{s \in S^+} \mu_s \left[G(x_s) + G(x_{-s}) + l_{i,s} + l_{-i,s} \right], \tag{10}$$

where $l_{i,s}(l_{-i,s})$, given by (2), is the demand for production labor of a leader (laggard) in an industry with gap s. In an industry with gap s, the demand for R&D workers is $G(x_s) + G(x_{-s})$.

Definition. A balanced growth path equilibrium is, for every t, the tuple $\Upsilon \equiv (g, \{\mu_s\}_{s\in S^+}, \{x_\sigma\}_{\sigma\in S})$ and $(p_{j,z}, y_{j,z})_{j\in[0,1],z\in\{i,-i\}}$ such that (i) prices and quantities of intermediate goods $(p_{j,z}, y_{j,z})$ satisfy the intermediate goods production function and the profit-maximization conditions (2); (ii) $\forall\sigma$, x_σ is a best response to $\{x^c_\iota\}_{\iota\in S}$; (iii) $\forall\sigma$, $x^c_\sigma = x_\sigma$ (symmetry); (iv) growth is determined by (9); (v) the distribution of technology gaps $\{\mu_s\}_{s\in S^+}$ is stationary; and (vi) labor and goods markets clear, with aggregate labor given by (10).

3. Growth-interest rate relation: A first pass

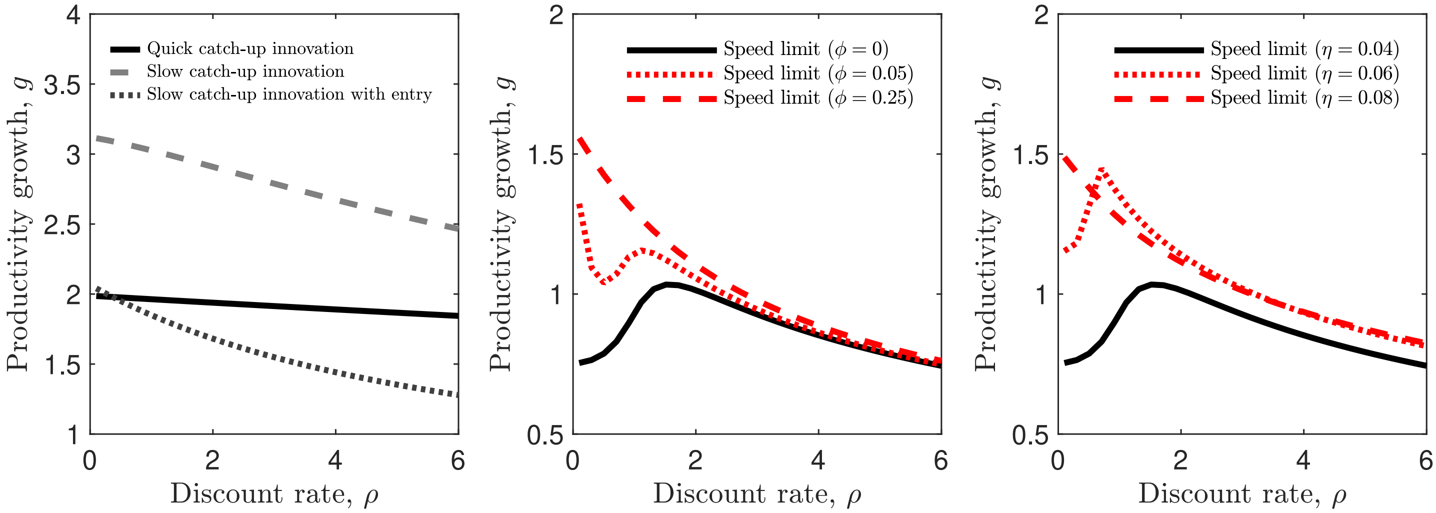
This section studies the relation of growth and the interest rate in four models with starkly different assumptions about creative destruction. In Acemoglu and Akcigit (2012) and Akcigit and Ates (2019), there is at least some chance of quick catch-up through innovation or patent expiry.[10] In Liu et al. (2020), innovation and patent expiry are both "pure" slow catch-up $(\phi = 0, \zeta = 0)$.

In the Acemoglu and Akcigit (2012) and Akcigit and Ates (2019) models, growth rises monotonically as the discount rate falls, with the slope varying notably across models (Figure 1, left panel). In the Liu et al. (2020) model, growth has an inverted-U relation with the discount rate (black lines in the middle and right panels).[11] Correspondingly, there is a growth "speed limit"—a maximum growth rate such that further declines in the interest rate reduce growth. However, introducing a 5% chance

[10]See Figure 1 for details. Internet Appendix A extends our model to nest Akcigit and Ates (2019), which includes entry. Regarding entry, see also Section 4.5.

[11]As detailed in Internet Appendix B, the middle and right panels of Figure 1 incorporate a modification to the microfoundation for profits, as in the quantitative model of Liu et al. (2020).

Figure 1: **Growth and the discount rate under alternative assumptions about creative destruction.**



Left panel: The quick catch-up innovation (solid line, $\phi = 1$, $\eta = 0$) and slow catch-up innovation (dashed line, $\phi = 0$, $\zeta = 1$, $\eta = 0.02$) models are from Acemoglu and Akcigit (2012). The slow catch-up innovation with entry (dotted line, $\phi = 0.0423$, $\zeta = 1$, $\eta = 0.0109$) model is from Akcigit and Ates (2019). Middle and right panels: The growth "speed limit" (solid black line, $\phi = \zeta = 0$, $\eta = 0.039$) model is from Liu et al. (2020). The other speed limit models are identical except with regard to the innovation catch-up speed, ϕ, or the patent expiry rate, η. Internet Appendix B lists the full set of parameter values for each model.

of quick catch-up through laggard innovation, the inverted-U relation effectively disappears (middle panel, dotted red line). With a 25% chance of quick catch-up, as the discount rate goes to zero, growth rises sharply. When there is no chance of quick catch-up innovation, laggard firms become discouraged as the discount rate falls, because leadership can only be achieved through a prolonged period of high R&D and many successive innovations. However, with a small chance of quick catch-up innovation, laggard firms increase R&D as the discount rate falls, rather than becoming discouraged. Invigorated laggards force the leaders to increase their R&D to maintain their advantage, boosting growth along the intensive margin. Moreover, when a laggard achieves a quick catch-up, its industry returns to the vigorous R&D race characterizing competitive industries, boosting growth along the extensive margin.

Patent policy is another key aspect of creative destruction that shapes the relation of growth, competition, and the interest rate. Like an increase in innovation catch-up speed, a higher patent expiry rate in the Liu et al. (2020) model also implies that laggards do not become discouraged as the discount rate falls; with a higher patent expiry rate, fewer laggard innovations are required (in expectation) to close the gap with the leader. Thus, if the patent expiry rate is raised from 4% to 8%, the relation of growth and the interest rate becomes monotonically negative. To explore this further, section 6 studies how the growth-interest rate relation changes if the social planner optimally adjusts patent policy as the discount rate changes.

These results point to the importance of the nature of creative destruction for the relation between growth and the interest rate. Our model nests innovation that is purely quick ($\phi = 1$), slow ($\phi = 0$), or in between ($0 < \phi < 1$). We similarly nest alternative assumptions about the rate and speed of patent expiry. In the next section, we estimate the model parameters by matching cross-sectional moments informative about the nature of creative destruction.

4. Quantitative analysis

Tighter restrictions on creative destruction are expected to generate higher markups, reduced reallocation, and lower profit volatility. Extremely tight restrictions on creative destruction are also expected to imply market leaders need very little R&D to maintain their advantage over discouraged laggards. We therefore identify our model's parameters using using salient moments characterizing reallocation and the cross section of markups, innovation output, profit volatility, and R&D.

4.1 Parameters and moment conditions

We set the discount rate to 2% and the corporate tax rate to 20%. In our benchmark model, the two varieties within each industry are perfect substitutes. We externally calibrate (γ, η, ζ). The R&D curvature parameter γ is 0.5, as in Peters (2020) and many others and consistent with empirical estimates discussed in Acemoglu et al. (2018). We assume full patent protection $\eta = 0$, as in Acemoglu and Akcigit's

Table 1: **Parameters and Moments.**

Parameter estimates		Moments used in estimation		
Parameter	Value	Description	Model	Data
ϕ	0.283	Productivity growth	1.03%	1.03%
λ	1.018	Markups		
B	2.463	Mean	19.42%	19.40%
		50th percentile	13.12%	13.64%
		90th percentile	41.81%	42.62%
		Innovation output		
		Mean	5.90%	6.75%
		50th percentile	0.00%	0.00%
		90th percentile	18.17%	19.49%
		Profit volatility		
		All firms	39.30%	45.09%
		Top profit quintile	16.25%	21.89%
		R&D to sales		
		All firms	5.50%	5.35%
		Top profit quintile	3.59%	2.52%
		FHK within	89.85%	90.77%

The parameters (ϕ, λ, B) minimize the criterion (12). FHK within is the adjusted within moment, described in Section 4.1.

(2012) benchmark model.[12] Full patent protection implies that the patent expiry speed ζ does not affect the equilibrium. We estimate (ϕ, λ, B). Section 4.4 provides robustness. The targeted moments, listed in Table 1, are:

Productivity growth, markups, and innovation output. We target productivity growth equal to average total factor productivity growth for 1960–2019, from Fernald et al. (2017). We target the mean, median, and 90th percentile of the industry-level distribution of net markups, from Hall (2018), and the firm-level distribution of annual innovation output, from Kogan et al. (2017), defined as the sum of the economic value of all patents earned (based on stock market reaction to patent grants), normalized by firm value.

Growth decomposition. Foster et al. (2001) decompose productivity growth into

[12]Section 4.4 extends the set of estimated parameters to include the patent parameters η and ζ.

five components using the identity

$$\Delta\Theta_t = \underbrace{\sum_{i\in\mathbb{C}_t}\xi_{it-1}\Delta\theta_{it}}_{\text{within}} + \underbrace{\sum_{i\in\mathbb{C}_t}(\theta_{it-1}-\Theta_{t-1})\Delta\xi_{it}}_{\text{between}} + \underbrace{\sum_{i\in\mathbb{C}_t}\Delta\theta_{it}\Delta\xi_{it}}_{\text{cross}} + \underbrace{\sum_{i\in\mathbb{N}_t}\xi_{it}(\theta_{it}-\Theta_{t-1})}_{\text{entry}} + \underbrace{\sum_{i\in\mathbb{X}_t}\xi_{it-1}(\Theta_{t-1}-\theta_{it-1})}_{\text{exit}}, \quad (11)$$

where $\mathbb{C}_t$ is the set of continuing firms, $\mathbb{N}_t$ of entering firms, and $\mathbb{X}_t$ of exiting firms in industry j between $t-1$ and t. In addition, $\theta_{it} = \ln(\frac{y_{it}}{l_{it}})$ is the log productivity of firm i at time t, $\xi_{it} = \frac{p_{it}y_{it}}{\sum_i p_{it}y_{it}}$ is the revenue share of firm i at time t, and $\Theta_{it} = \sum_i \xi_{it}\theta_{it}$. As we have not yet introduced entry, we target the *adjusted* within share, defined as within divided by the sum of within, between, and cross, obtained from Foster et al. (2008). The adjusted within statistic is the share of continuing firms' productivity growth accounted for by those firms' productivity improvements holding market shares constant (i.e., absent reallocation).[13]

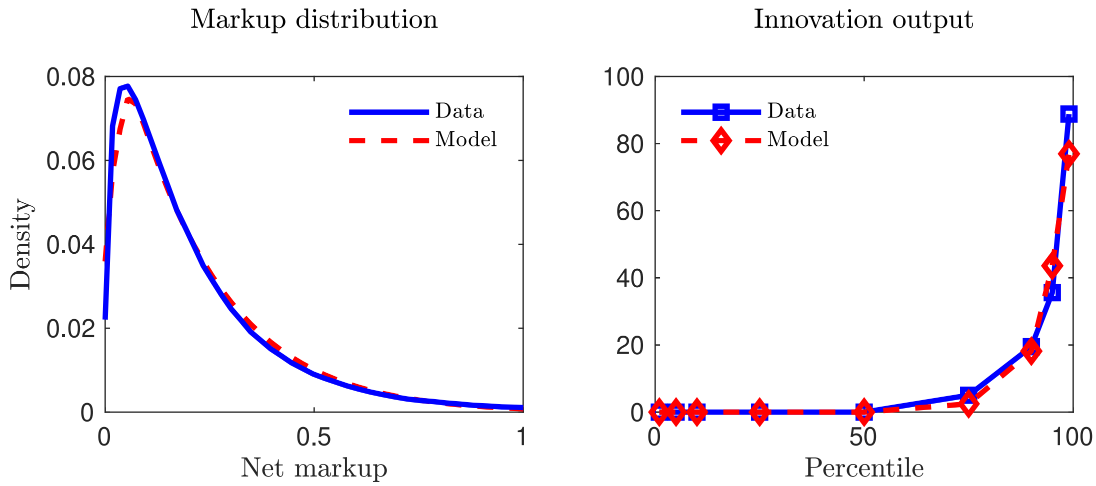
Profit volatility and R&D to sales. We calculate profit volatility and R&D intensity using COMPUSTAT data. (Internet Appendix C discusses the data in detail.) Operating profits are sales (SALE) minus cost of goods sold (COGS). We target unconditional profit volatility (the standard deviation of all firms' profit growth rates between year y and $y+1$) as well as profit volatility for the top quintile of firms ranked by profits in the base year y. We similarly target R&D to sales unconditionally and for firms in the highest profit quintile.

We identify the parameters (ϕ, λ, B) using the simulated method of moments. We obtain some model moments directly from the solution for the balanced growth path equilibrium Υ. For the remaining moments, we compute model values using a simulation of $N = 50,000$ industries for $T = 12$ years.[14] We compare moments in the

[13] In Section 4.5 we introduce entry and re-estimate the model to match the unadjusted within component as well as the entry component, with the remaining components as untargeted moments.

[14] Our simulation method builds on Lentz and Mortensen (2008), Akcigit and Kerr (2018) and Acemoglu et al. (2018).

Figure 2: **Innovation output and markup distribution.**



Data on markups and innovation output are from Hall (2018) and Kogan et al. (2017).

model and the data, choosing the parameters that minimize the criterion

$$\min \sum_{m=1}^{14} weight_m \left(\frac{|\text{model}(m) - \text{data}(m)|}{\frac{1}{2}|\text{model}(m)| + \frac{1}{2}|\text{data}(m)|} \right). \tag{12}$$

We give aggregate growth and the average markup a weight five times the weight of the other moments.[15] Section 4.4 discusses robustness to alternative values for the targeted growth rate and markup distribution.

4.2 Identification

The estimated innovation catch-up speed parameter ϕ indicates a 30% chance that an innovating laggard closes the technology gap completely (Table 1). Each rung on the ladder represents a proportional productivity improvement that is fairly small, 1.8%, or $\ln(\lambda)$.

There is a very good fit between the model-implied moments and the data (Table 1). The model replicates well the entire distribution of markups and innovation output (Figure 2).

[15]Because median innovation output is zero, the normalized absolute error for this moment in the expression (12) is undefined whenever the model value is equal to zero. In this case, we set the normalized absolute error for this moment equal to zero.

Table 2: **Sensitivity Matrix.**

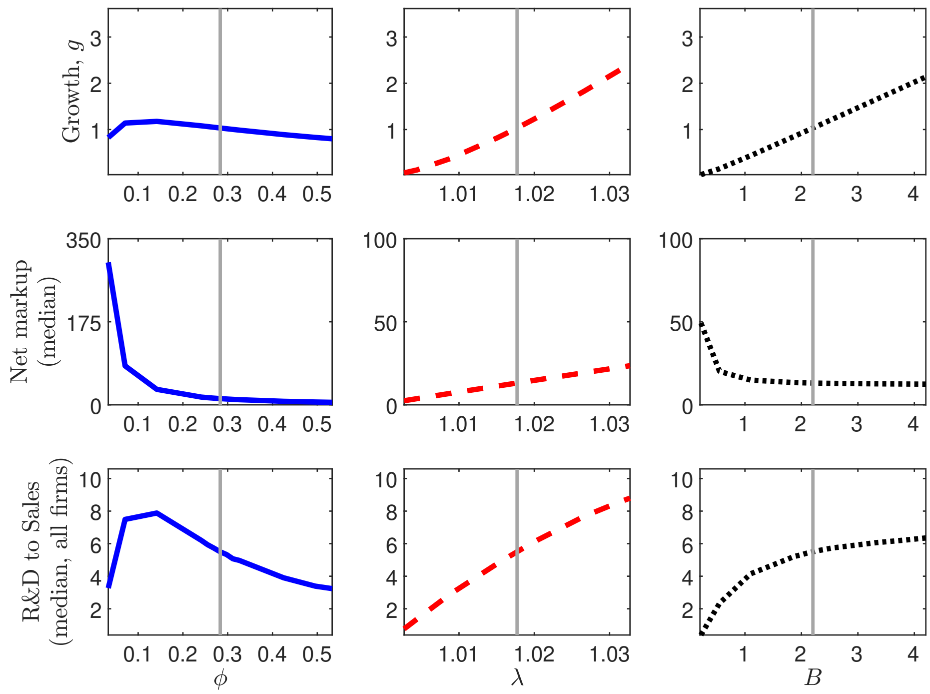
	Change in ...			
Moments	ϕ	λ	B	Initial level
Productivity growth	-1.43%	7.19%	5.83%	1.03
Markups				
Mean	-7.99%	4.71%	-0.93%	19.42
50th percentile	-9.14%	4.69%	-0.93%	13.12
90th percentile	-7.86%	4.63%	-1.00%	41.81
Innovation output				
Mean	1.20%	1.62%	3.67%	5.90
90th percentile	4.80%	0.96%	7.97%	18.17
FHK within	-0.35%	-0.15%	-0.29%	89.85
Profit volatility				
All firms	1.78%	1.53%	3.63%	39.30
Top profit quintile	2.56%	0.67%	2.87%	16.25
R&D to sales				
All firms	-3.29%	4.36%	1.59%	5.50
Top profit quintile	-2.13%	2.70%	1.72%	3.59

This table reports the effect on the targeted moments of a 5% increase in the catch-up speed ϕ, productivity increase $(\lambda - 1)$, and R&D cost scaling parameter B. Higher B corresponds to lower R&D costs. The 50th percentile of the innovation output distribution is omitted because it is equal to zero for parameter values close to the estimated parameter values.

Table 2 contains a "sensitivity matrix" that reports the change in the targeted moments for a 5% increase in each structural parameter (Peters, 2020). An increase in catch-up speed ϕ reduces markups and increases profit volatility. Lower markups arise because a higher catch-up speed increases creative destruction mechanically and encourages laggard R&D. Profit volatility rises because quick catch-up innovation by laggards sharply reduces leader profits. An increased catch-up speed also raises average innovation output, with more high-value innovations by laggards. The adjusted *within* term falls, as more turnover in market shares implies a larger role for reallocation in the productivity growth decomposition.

As shown in Figure 3, growth and R&D intensity vary non-monotonically with ϕ. As ϕ increases locally from its estimated value, leaders face a higher risk that their future profits from an innovation will be wiped out by quick catch-up laggard innovation, reducing the expected value of leader innovations and thereby inducing

Figure 3: **Mapping from parameters to moments.**



The figure shows how selected moments vary with catch-up speed ϕ (left panels), innovation step size λ (middle panels), and R&D cost scaling parameter B (right panels), holding other parameters constant. All moments are shown in percent.

lower leader R&D. However, if ϕ decreases to a very low level, growth falls to a low level, as low-R&D leaders become entrenched.

Increasing the productivity step size λ leads to higher growth and markups, both mechanically (higher λ implies a greater increase in productivity and markup from rising one rung on the ladder) and through an endogenous increase in R&D. Increasing λ also leads to higher innovation output, slightly higher reallocation (or, equivalently, a slightly lower *within* term), and higher profit volatility. Increasing λ has an effect on markups of the *opposite* sign as increasing ϕ. Increasing λ also has an effect on growth and top-quintile R&D of the opposite sign as increasing ϕ locally to its estimated value. However, increasing λ has effects on innovation output and reallocation of the *same* sign as increasing ϕ, and varying λ has very little effect on top-quintile profit volatility.

Table 3: **Innovation catch-up speed ϕ and the markup distribution.**

	Hall (2018) estimates (Data)	Calibrated model	Model with severe restrictions on creative destruction (ϕ = 0.05)
Mass of industries below 15%	53.80	55.20	6.14
Mass of industries above 40%	11.50	11.30	82.60

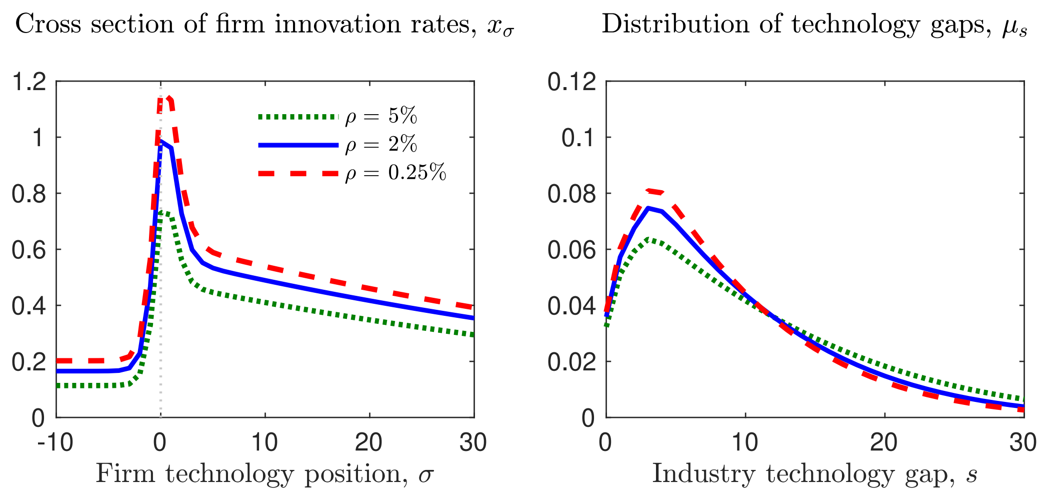
A higher value for the R&D cost parameter B corresponds to lower costs of achieving a given innovation rate. Increasing B increases growth, innovation output, R&D to sales, and profit volatility, as expected. Increasing B reduces the R&D cost for all firms, whether leader, laggard, or tied. The resulting increases in R&D for laggards would lead to smaller markups, but are roughly offset by the increases in R&D for leaders, which would lead to higher markups. Increasing B has effects on growth and R&D to sales of the opposite sign as increasing ϕ locally. However, increasing B has a relatively small effect on markups and has effects on innovation output, the *within* term, and profit volatility of the same sign as increasing ϕ.

Table 3 illustrates how the catch-up speed parameter ϕ affects the distribution of markups. From Hall (2018), 54% of industries have an average markup below 15%, while 12% have an average markup above 40%. The estimated model, with $\phi = 0.283$, matches these distributional moments closely. In contrast, if we set $\phi = 0.05$, the markup distribution acquires a right tail that is counterfactually large, with 83% of industries having markups above 40%.

4.3 Equilibrium properties

Figure 4, left panel, shows the innovation strategy as a function of a firm's technology position, for three different values of the discount rate. R&D is highest for tied firms, due to *escape competition* effects. A tied firm is spurred to innovate because, if successful, the firm begins to earn operating profits and reduces its R&D costs. As a leader's productivity advantage grows, the leader reduces R&D further. If a

Figure 4: **Firm innovation strategy and the distribution of technology gaps.**

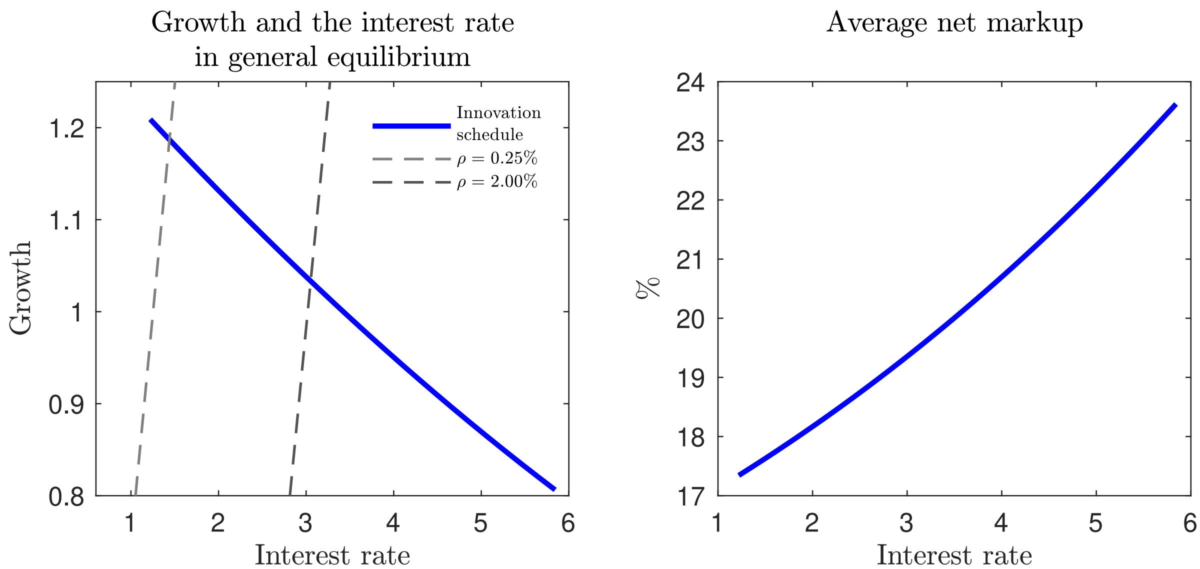


leader has attained an advantage of at least a few steps, the leader faces a partly discouraged laggard and, absent a quick catch-up laggard innovation, the leader can expand its advantage even while reducing its R&D expenses. As a laggard falls behind, its innovation rate declines because the number of incremental innovations needed to close the technology gap rises. However, laggards, no matter how far behind, maintain an annual innovation rate of at least 20% per year. Because of the possibility of quick catch-up, even far-behind laggards do not give up completely on R&D.

With a decrease in the discount rate from 2% to 0.25%, innovation rates increase for all firms, including laggards. Because innovating laggards obtain a mix of slow and quick catch-up gains in productivity, the higher laggard innovation rate induces a pro-competitive shift in the distribution of technology gaps (Figure 4, right panel). Growth therefore increases from 1.03% to 1.19%, reflecting an intensive margin effect (higher innovation rates conditional on a firm's position) and an extensive margin effect (a greater share of industries in high R&D, competitive industries). With a lower interest rate, there is a decline in the average productivity gap between leader and laggard.

Figure 5 displays the two key relations that determine growth and the interest

Figure 5: **Growth, markups, and the interest rate.**



rate. The blue line, or *innovation schedule*, captures profit maximization by firms. In the estimated model, productivity growth rises monotonically and approximately linearly as the interest rate falls. The dashed lines in Figure 5 represent the *Euler equation*, or the household savings side of the economy, for different values of the household discount rate ρ. Conditional on a discount rate, the equilibrium growth and interest rate are determined so that, given that interest rate, firms' innovation decisions lead to output growth of g, while households increase their consumption at the same rate. As the discount rate declines, equilibrium growth increases and the interest rate falls. The interest rate declines less than one-for-one with the household discount rate, because growth increases.

A decline in the discount rate leads to a more competitive distribution of technology gaps, as shown in Figure 4. As a result, the average markup declines as the interest rate falls (Figure 5, right panel). Market concentration also declines modestly and the profit share falls.[16]

[16]As in Akcigit and Ates (2019), sales concentration measures such as the Herfindahl-Hirschman index depend on the share of tied industries, where sales are equalized due to Bertrand competition, and the share of non-tied industries, where perfect substitution across the two varieties implies that

Thus, in our estimated Schumpeterian model, which fits well the firm- and industry-level micro data, the aggregate implications of a lower interest rate are at odds with important recent macroeconomic trends. Productivity growth has experienced a protracted bust, amid rising markups, increasing concentration, larger productivity gaps between leaders and laggards, and a rising profit share. Thus, our results cast doubt on low interest rates as an explanation of these trends.

4.4 Robustness

Internet Appendix D reports in detail a number of exercises undertaken to gauge the robustness of these results. A first set of exercises alters an assumption about an exogenously set parameter and re-estimates the other parameters. Our results are not substantially affected by the choices of innovation curvature parameter γ, the elasticity of intertemporal substitution (EIS, heretofore assumed to be one), and the elasticity of substitution, κ, among each industry's varieties. In each exercise, the growth-interest rate relation remains negative.

Our benchmark model assumes full patent protection, or $\eta = 0$, as in Acemoglu and Akcigit (2012). We extend the set of three estimated parameters to include the patent expiry rate η and the patent catch-up speed ζ. The estimated patent expiry rate is zero, to three decimal places. In a separate exercise, we assume a patent expiry rate of 3% and re-estimate the model. We also re-estimate the model assuming inelastic labor supply. In addition, we study robustness with respect to values of the underlying moments. We set the annual growth rate target to 0.75% and re-estimate the model. We also estimate the model targeting markups much higher than in our benchmark estimation. We target net markups with a mean of 60%, a median of 30%, and a 90th percentile of 150%.[17] Finally, following most of the literature, innovations in our benchmark model advance the laggard only one rung or close the gap completely. We extend the model to include the possibilities of the laggard

only the leader produces. Internet Appendix Section G shows the relation of the profit share and the interest rate.

[17]These markup targets are from De Loecker et al. (2020), for 2016.

advancing an intermediate number of rungs or even leapfrogging the leader (Akcigit et al., 2018; Acemoglu and Cao, 2015).

Across all of these exercises, the estimated catch-up speed ϕ is between 0.16 and 0.82. Thus, the result that the catch-up speed is meaningfully above zero, with at least some advantage of backwardness, is robust to a wide set of choices regarding exogenously set parameters, moment values, and the elasticity of labor supply. Across all of these exercises, growth rises as the discount rate falls, while the average markup declines or is little changed.

4.5 Entry

Entrants are an important source of creative destruction. We extend the model to include entry and re-estimate it to match a set of moments enlarged to identify the entry parameters.

At every time t, a mass one of potential entrants choose an entry innovation rate x_E and hire $G(x_E; B_E)$ R&D workers. B_E is an R&D cost scaling parameter, which may differ from the scaling parameter for incumbent firms. Upon a successful innovation, the entrant's industry is drawn stochastically from the set of all industries $j \in [0, 1]$. The entrant displaces the industry's follower (or one of the two incumbents, with equal probability, if the entrant joins a neck-and-neck industry). With probability $1 - \phi_E$, an entrant begins life one step ahead of the firm that it displaces. With probability ϕ_E, an entrant begins life in position $l_E \in \{0, ..., \bar{s}\}$, a leapfrogging parameter.[18] If a potential entrant does not innovate, we assume, as in Akcigit and Ates (2019), that the entrant disappears and is replaced by a new potential entrant. R&D efforts by potential entrants are *undirected* in the sense of Klette and Kortum (2004) and Akcigit and Kerr (2018). Because entrants do not know ex ante the industry they will join, incumbent leaders cannot strategically "escape" the risk of entry—no matter an incumbent's lead, the probability of entry is x_E.[19]

[18] Acemoglu and Cao (2015) discusses empirical research emphasizing the radical nature of entrant innovation.

[19] The details of the model with entry are presented in the Internet Appendix.

We jointly estimate the innovation parameters $(\phi, \lambda, B, B_E, \phi_E, l_E)$. To help identify the entry parameters, we target additional moments in the estimation: the share of productivity growth accounted for by entrants in the growth decomposition (11) and the employment shares of firms less than 6 years old and less than 11 years old. These moments are emphasized by Acemoglu et al. (2018) and Garcia-Macia et al. (2019), among others. Employment shares by firm age are from the US Census Business Dynamics Statics, taking an average of values for the earliest and most recent years with publicly available age data (1987 and 2014).

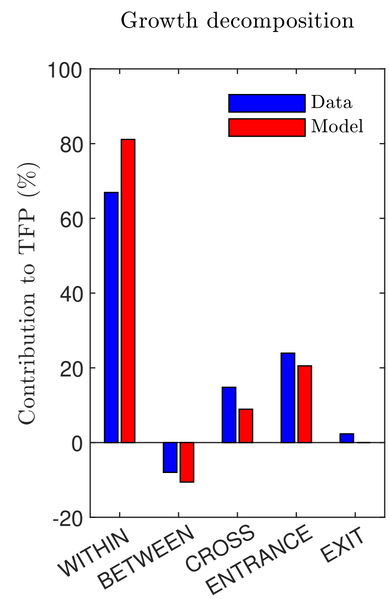
Table 4 shows that the model matches well the aggregate growth rate, average markup, and the new entry moments. The Internet Appendix shows that the quality of fit is high as well for the remaining moments. Entrant innovation is more radical than laggard innovation, in that entrants can leapfrog the leader by one rung ($l_E = 1$) and have a higher chance of a quick catch-up innovation (almost 50%, relative to 20% for laggards). Figure 6 shows that the model matches well the Foster et al. (2001) growth decomposition. Internet Appendix Section D.4 shows that growth rises and the average markup declines as the interest rate falls.

Recent work by Akcigit and Ates (2019) and Olmstead-Rumsey (2020) emphasizes that, since the 2000s, productivity growth and business dynamism have been especially low. Examples of reduced dynamism include a lower employment share of young firms (Decker et al., 2014) and lower patent quality for small firms (Olmstead-Rumsey, 2020). Therefore, as a final robustness exercise, we re-estimate the model, targeting moment values for 2004 onward, as in Olmstead-Rumsey (2020). As detailed in Appendix D, targeted productivity growth is only 0.56% and the targeted average markup is 44%. When we re-estimate the model with entry, laggard innovation generates almost pure slow catch-up ($\phi = 0.07$), the R&D productivity of laggards and entrants falls (i.e., lower B and B_E), and the estimated entrant catch-up speed is little changed, compared with the estimation using the longer sample period (Table 4). Thus, this economy, estimated on post-2004 data, has elements of sclerosis. However, innovating entrants still have a high chance of quickly catching-up. As a result, as the discount rate falls, productivity growth rises and the average markup falls.

Table 4: **Estimation with Entry.**

Moments	Model With Entry	Data
Productivity growth	1.03%	1.03%
Mean markup	19.42%	19.40%
FHK decomposition (Entry)	20.53%	23.90%
Employment share, ≤ 10 years	29.33%	26.20%
Employment share, ≤ 5 years	17.77%	15.70%
Parameters		
ϕ	0.188	
λ	1.019	
B	2.317	
ϕ_E	0.479	
B_E	0.678	
l_E	1.000	

Figure 6: **FHK decomposition.**



5. The innovation multiplier

A lower discount rate is expected to increase growth by raising the valuation of the profits from innovation. However, Schumpeterian models feature an important role for strategic interactions among firms. In addition, an economy's competitiveness affects growth, because more competitive industries generally feature higher innovation rates, as firms seek to escape competition. This section develops—and applies to our estimated model—an analytical framework to evaluate the valuation, strategic, and composition channels that determine how a lower discount rate affects growth, competition, and innovation.

We consider a change in the discount rate $d\rho$ and begin with the valuation channel. Let $\boldsymbol{\partial x} = [\partial x_{-\bar{s}} \dots \partial x_{\bar{s}}]'$ define the effect of a lower discount rate on the cross section of firm innovation when each firm holds its competitor's strategy constant.[20] The

[20]Specifically, consider the system of equations given by (4)–(5), taking as given the competitor's strategy $\{x_\sigma^c\}_{\sigma \in S}$. We obtain $\frac{\partial x_\sigma}{\partial \rho}$ using the implicit function theorem and define $\partial x_\sigma \equiv \frac{\partial x_\sigma}{\partial \rho} d\rho$, where $d\rho$ is a change in the discount factor.

theorem below introduces an innovation multiplier that maps this valuation-driven change in firm innovation into the general equilibrium (GE) effect on growth, the distribution of technology gaps, and firm innovation strategies.

Theorem 1 ***(The Innovation Multiplier)***. *Consider a change in the discount rate ρ. There exists a general equilibrium multiplier matrix $\mathbb{M}$ such that the effect of this change on the BGP, $\boldsymbol{\Upsilon} \equiv (g, \{\mu_s\}_{s \in S^+}, \{x_\sigma\}_{\sigma \in S})'$, to a first order, is*

$$\boldsymbol{d\Upsilon} = \mathbb{M}\, \boldsymbol{\partial x}, \tag{13}$$

where $\boldsymbol{\partial x}$ is the valuation-driven change in firm innovation.

See Internet Appendix E for a proof.

Each row of the matrix $\mathbb{M}$ maps the cross section $\boldsymbol{\partial x}$ into the GE effect on a particular element of the equilibrium vector $\boldsymbol{\Upsilon}$. For instance, the row corresponding to growth, denoted $\mathbb{M}_g$, is a (row) vector of length $2\bar{s}+1$ that maps $\boldsymbol{\partial x}$, a column vector of the same length, into the GE effect on growth. Similarly, $\mathbb{M}_{x_\sigma}$ and $\mathbb{M}_{\mu_\sigma}$ define two rows of $\mathbb{M}$ mapping $\boldsymbol{\partial x}$ into the GE changes in the innovation rates, x_σ, and the distribution of industries, μ_σ, respectively. From Theorem 1, we can decompose the GE effect on growth as follows:

$$dg = \mathbb{M}_g \boldsymbol{\partial x} = \left[\ln\lambda \sum_{\sigma \in S^+} (1 + \mathbb{1}_{\sigma=0}) \Bigg(\underbrace{\mu_\sigma e_\sigma}_{\substack{\text{Direct}\\\text{effect}}} + \underbrace{\mu_\sigma(\mathbb{M}_{x_\sigma} - e_\sigma)}_{\substack{\text{Strategic}\\\text{channel}}} + \underbrace{\mathbb{M}_{\mu_\sigma} x_\sigma}_{\substack{\text{Composition}\\\text{channel}}} \Bigg) \right] \underbrace{\boldsymbol{\partial x}}_{\substack{\text{Valuation-driven}\\\text{change in}\\\text{innovation}}}, \tag{14}$$

where $\mathbb{1}_{\sigma=0}$ is an indicator function for $\sigma = 0$ and e_σ is a row vector of length $(2\bar{s}+1)$, with a 1 in position $(\bar{s}+1+\sigma)$ and zeros elsewhere. In (14), the growth multiplier $\mathbb{M}_g$ is the term in square brackets and it is formed by three components. The first is the *direct effect*: Leader and tied firm innovations (but not laggards') contribute directly to the productivity of technologies in use. Absent strategic and composition effects, a valuation-driven change $\partial x_\sigma, (\sigma \in S^+)$ in the innovation rate of a tied or leader firm increases aggregate productivity growth by $(1 + \mathbb{1}_{\sigma=0})\mu_\sigma \ln\lambda$, or the mass of firms in position σ times the log innovation step size.

The second component is the *strategic channel*: The GE innovation rate of any tied or leader firm is affected (through strategic interactions) by the entire cross section of valuation-driven changes in innovation, according to $dx_\sigma = \mathbb{M}_{x_\sigma} \boldsymbol{\partial x}$. Finally, the last term is the composition channel: The GE effect on the mass of industries with technology gap $\sigma \in S^+$ is affected by the entire cross section of valuation-driven increases in innovation, according to $d\mu_\sigma = \mathbb{M}_{\mu_\sigma} \boldsymbol{\partial x}$. These changes in the technology gap distribution affect growth because innovation is higher in more competitive industries.

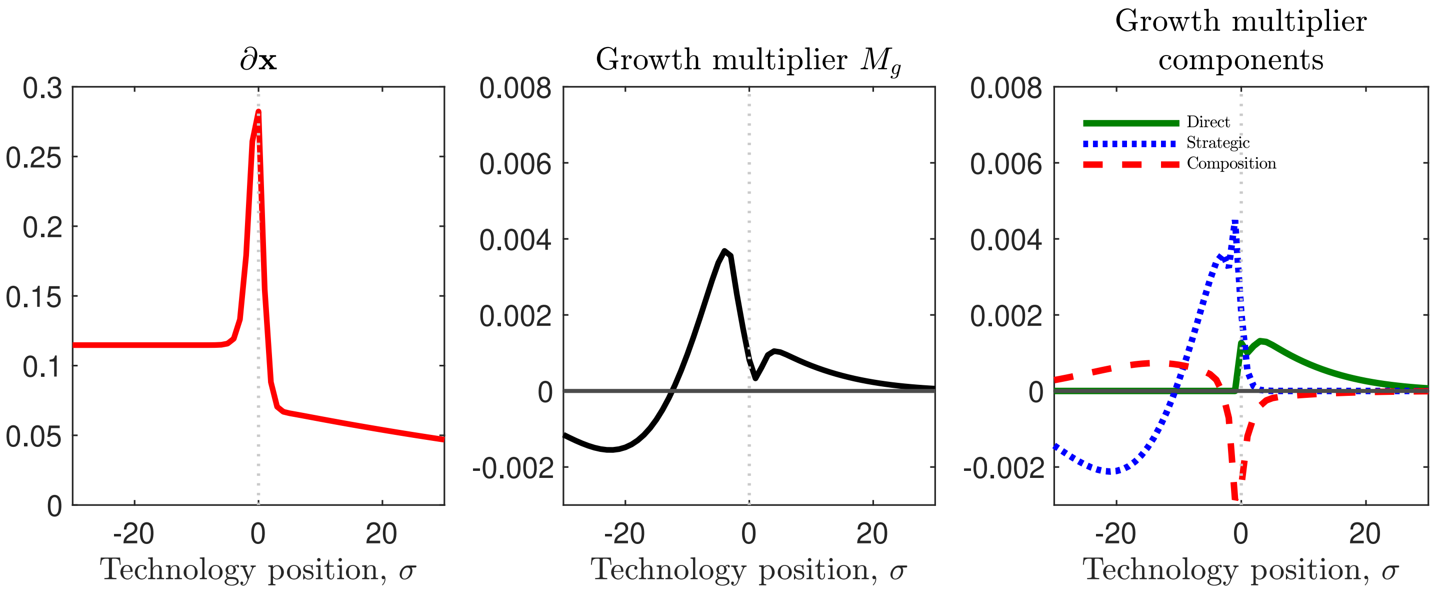
The left panel of Figure 7 shows $\boldsymbol{\partial x}$, the cross section of valuation-driven changes in firm innovation. Holding constant the competitor's strategy, a decline in the discount rate implies an increase in innovation for all firms, in every technology position.[21] The growth multiplier $\mathbb{M}_g$ maps this cross section of valuation-driven changes in firm innovation into the GE change in growth. For our benchmark estimated model, the growth multiplier $\mathbb{M}_g$ is shown by the thick black line in the middle panel. Valuation-driven increases in laggards' innovation in *uncompetitive* industries contribute negatively to growth, while increases in laggards' innovation in *competitive* industries contribute positively.[22] These results are remarkable because, absent strategic and composition effects, laggard innovations have *no* effect on growth. The GE effect of laggard innovation on growth, whether positive or negative, arises mostly through strategic interactions, as shown by the bottom panel. Valuation-driven increases in tied and leader firm innovation contribute positively to growth, mostly because of their direct effect on growth (right panel of Figure 7).

The right panel shows the direct, strategic, and composition components of the growth multiplier. Valuation-driven increases in laggard innovation in uncompeti-

[21]As shown in Internet Appendix F, the sign of this valuation-driven change in firm innovation, in principle, could be negative if an innovation from a certain technology position leads to back-loaded losses through increased R&D expenditures. The uniformly positive valuation-driven effect of a lower discount rate on innovation is therefore dependent on the model estimation.

[22]To interpret the magnitude of the growth multiplier's elements, note that if laggards in competitive industries (with $s \leq 10$) increase their annual innovation rate by 5 percentage points in response to a lower discount rate when taking as given their competitors' strategy, then the effect on annual growth would be $0.05 \times \mathbb{M}_g \sum_{\sigma \in \{-10,..,-1\}} e'_\sigma = 0.0005$, or 5 basis points.

Figure 7: **Growth multiplier.**



The left panel shows ∂x, the cross section of valuation-driven changes in innovation from a 100 bps decline in the discount rate. The middle panel shows the growth multiplier $\mathbb{M}_g$ that maps the cross section of valuation-driven increases in innovation ∂x into the GE effect on growth. The right panel shows the components of the growth multiplier, as defined in Equation (14).

tive industries have a negative effect on growth through a *trickle-down* effect. For example, a leader that is currently 4 steps ahead has less to gain from innovation, the greater the innovation rates of laggards 5 or more steps behind, with whom the leader would compete if the leader were to innovate. Valuation-driven increases in laggard innovation in competitive industries have a negative effect on growth through an *escape-competition* effect. If a laggard 4 or fewer steps behind increases its innovation, then leaders that are 4 steps ahead are spurred to innovate to escape competition.[23]

Valuation-driven increases in leader and tied firm innovation have a negative effect on aggregate growth through the composition channel (blue dotted line, right panel of Figure 7). Conversely, valuation-driven increases in laggard innovation generally have a positive effect on aggregate growth through the composition channel.

[23]The trickle-down and escape-competition effects are unpacked further in Section E of the Internet Appendix.

These composition effects have the expected signs: higher leader innovation contributes to a less competitive economy, by enlarging technology gaps, while higher laggard innovation contributes to a less competitive economy.

Next, we quantify the valuation, strategic, and composition channels through which a lower discount rate affects growth. From (14), with only the valuation channel (i.e., absent strategic and composition effects), a 100 basis point decline in the discount rate increases aggregate productivity by $[\ln\lambda\sum_{\sigma\in S^+}(1+\mathbb{1}_{\sigma=0})\mu_\sigma e_\sigma]\boldsymbol{\partial x}$, or 15 basis points. However, in general equilibrium, the boost to aggregate growth is only 10 basis points. Overall, the strategic and composition channels *dampen*, but do not overturn, the boost to growth from the valuation channel. On balance, the strategic channel decreases growth by $-[\ln\lambda\sum_{\sigma\in S^+}(1+\mathbb{1}_{\sigma=0})(\mu_\sigma(\mathbb{M}_{x_\sigma}-e_\sigma))]\boldsymbol{\partial x}$, or 6 basis points. The composition channel increases growth by $[\ln\lambda\sum_{\sigma\in S^+}(1+\mathbb{1}_{\sigma=0})(\mathbb{M}_{\mu_\sigma}x_\sigma)]\boldsymbol{\partial x}$, or 1 basis point, because of the pro-competitive shift induced by a lower discount rate.

Generality of the multiplier. Our multiplier methodology can be used to study the GE effects of a broad range of policies. For example, consider the effect $\boldsymbol{\partial x}$ on the cross section of firm innovation from a change in the patent expiry rate, holding constant the competitor's strategy.[24] The proof of Theorem 1 in the Internet Appendix shows that the exact same multiplier that maps the valuation effect on firm innovation of a change in ρ into the GE change in growth also maps the valuation effect on firm innovation of a change in η into the GE change in growth.

6. Optimal patent policy, interest rates, and growth

Patent policy is another key aspect of creative destruction that shapes the relation of growth, competition, and the interest rate. We therefore ask: How should patent policy be optimally adjusted when there is a persistent change in the interest rate?

[24]This notion of a "valuation effect" arising from changes in patent expiry is linked to firms discounting expected profits from innovation taking into account the patent expiry rate. Specifically, the valuation effect on the cross section of firm innovation of a change in the patent expiry rate is $\partial x_\sigma \equiv \frac{\partial x_\sigma}{\partial\eta}d\eta$, where $d\eta$ is the change in the patent expiry rate and $\frac{\partial x_\sigma}{\partial\eta}$ is obtained by the implicit function theorem from (4)–(5).

And, if patent policy is adjusted optimally, what are the implications for the relation of interest rates, productivity growth, and market power? This section addresses these questions; in Section 6.1, we focus on the steady state. Section 6.2 incorporates transition dynamics.

6.1 Steady-state welfare

Welfare at time $t = 0$ is: $\mathbb{W}(0) = \int_{t=0}^{\infty} e^{-\rho t}(\ln C(t) - L(t))dt$. Along a BGP, the representative household's consumption is: $C(t) = Y(t) = Y(0)e^{gt}$. Correspondingly,

$$\mathbb{W}(0) = \frac{\ln(Y(0)) - L}{\rho} + \frac{g}{\rho^2}. \tag{15}$$

We define the quality index $Q(t) \equiv \exp(\int_0^1 \ln q_i(j;t)dj)$, where $q_i(j,t)$ is the productivity of the leader firm in industry j (or the tied firms in tied industries). When the two varieties in each industry are perfect substitutes, the ratio of output $Y(t)$ to the quality index $Q(t)$ is declining in the average net markup:

$$\ln\Bigg(\underbrace{\frac{Y(0)}{Q(0)}}_{\substack{\text{Ratio of}\\ \text{output to}\\ \text{potential}}}\Bigg) = -\underbrace{\sum_{s=0}^{\bar{s}} \mu_s \ln \psi_s}_{\text{Average net markup}}. \tag{16}$$

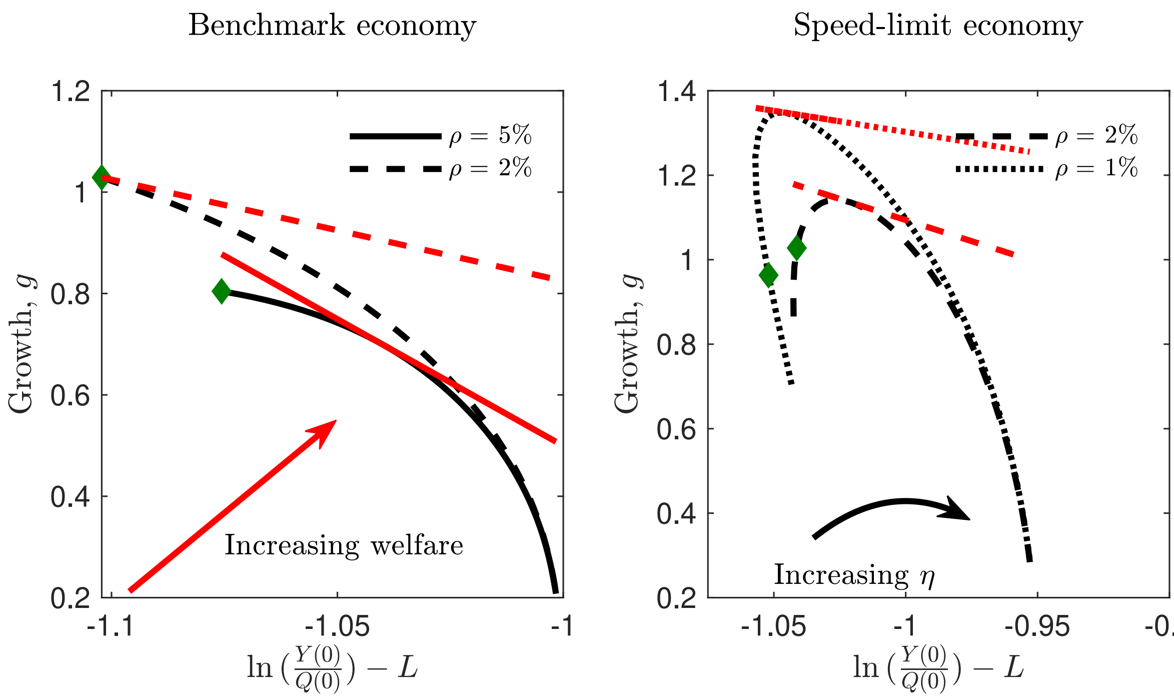
Markups generate a wedge between actual output $Y(t)$ and the quality index $Q(t)$. We refer to the quality index as potential output because it is the output that obtains if all firms have a zero net markup.[25]

Expressions (15) and (16) capture the core trade-off in designing patent policy: Generally, reducing patent protection reduces markups and production distortions but also reduces R&D and growth (Nordhaus, 1969).

Figure 8, left panel, illustrates the problem of a social planner choosing the patent expiry rate to maximize welfare. Each black line traces out how growth and the ratio of output-to-potential (less the disutility of labor) vary with the patent expiry rate η, conditional on a discount rate. The green diamonds show outcomes at the estimated value of η. The red lines are iso-welfare lines, showing combinations of growth and

[25] The Internet Appendix provides a proof of (16) and also characterizes output to potential output when varieties are imperfect substitutes.

Figure 8: **Optimal patent expiry rate.**



Black lines trace out the frontier, as the patent expiry rate changes, of growth and output-to-potential $\ln \frac{Y(0)}{Q(0)}$ less the disutility of labor. Iso-welfare lines (in red) show combinations of growth and output-to-potential that provide the same level of welfare. The green diamonds show growth and output-to-potential conditional on the patent expiry rate equal to its estimated value.

output-to-potential that deliver the same total welfare. The slope of the iso-welfare curve is $-\rho$, capturing that growth becomes relatively more important to the social planner as the discount rate falls.

In the benchmark economy, the social planner faces the Nordhaus trade-off: as patent protection is strengthened, growth rises but output-to-potential falls (i.e., market power rises). With the discount rate at its calibrated value of 2%, the estimated patent expiry rate $\eta = 0$ is in fact the optimal expiry rate; welfare is maximized at this corner solution. If the patent expiry rate η were set (suboptimally) to 5%, the consumption-equivalent welfare loss would be 4.5%.

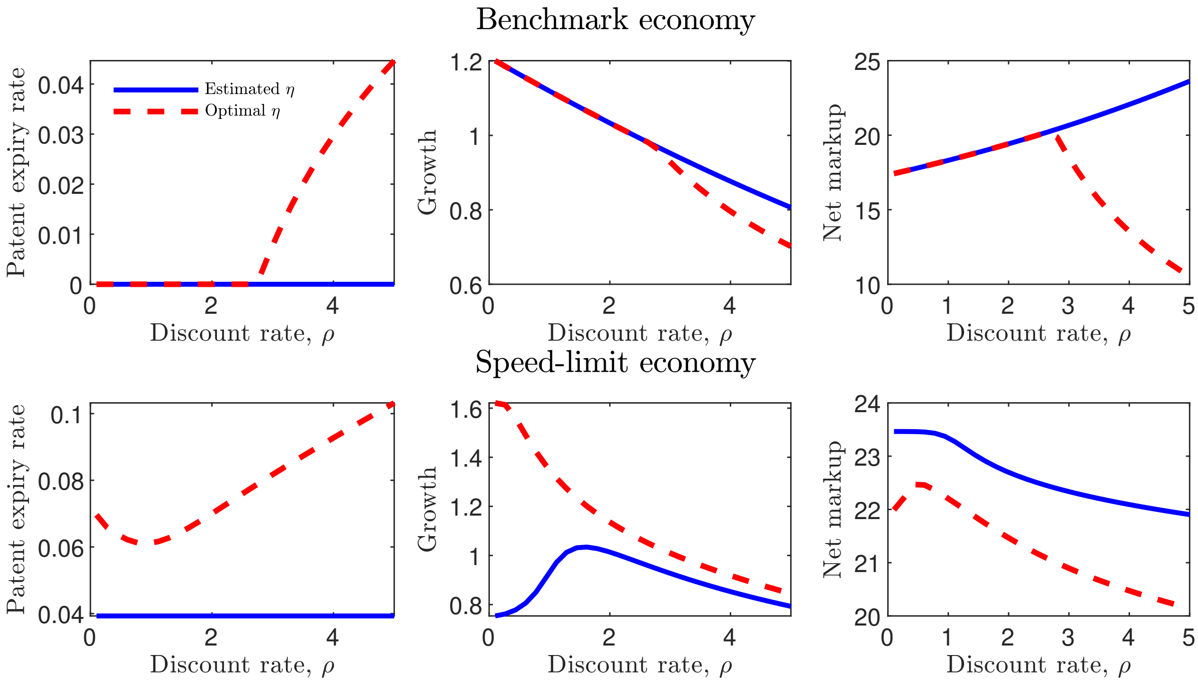
Next, we consider optimal patent policy at a higher discount rate. With ρ equal to 5%, the optimal patent expiry rate is positive and the estimated patent expiry rate is too low. For a discount rate of 5%, outcomes under the optimal patent expiry rate are shown by the tangency of the growth and output-to-potential frontier with the iso-welfare line.

Thus, as the discount rate falls, the social planner prefers to strengthen patent protection (i.e., the optimal patent expiry rate declines). As the discount rate falls, the Nordhaus trade-off changes in a way that leads the planner to strengthen protection: Equilibrium growth becomes more sensitive to the patent expiry rate, but there is little change in the mapping from the patent expiry rate to equilibrium production distortions. That is, as the discount rate falls, the black frontier in the left panel of Figure 8 steepens, such that, even holding constant the slope of the iso-welfare curve, the social planner would choose higher growth at the expense of higher markups. A second reason that the social planner strengthens protections is the rotation of the iso-welfare line: Growth is more important for welfare as the discount rate falls.

To understand how these results depend on the nature of creative destruction, we also study an economy with purely gradual laggard advancement. Figure 8, right panel, shows the frontier of growth and output-to-potential in this speed-limit economy, in which creative destruction is severely restricted by assumption (Section 3). In this economy, when patent protection is very weak, the Norhaus trade-off is present. However, for patent expiry rates near the calibrated rate in Liu et al. (2020), the trade-off disappears: Weakening patent protection increases output-to-potential (as expected) *and* growth. In this sclerotic economy, laggards are discouraged and leaders innovate only enough to counteract patent expiry; weakening patent protection forces leaders to innovate more to maintain their advantage. This "free lunch" for the social planner generates economically important welfare gains: by raising the expiry rate optimally, the social planner increases growth by 11 basis points, with output-to-potential rising 2 percentage points and a consumption-equivalent welfare gain of 8.5%. The free-lunch gains in growth and output-to-potential are even larger at lower discount rates.

In the benchmark economy (top panels of Figure 9), the optimal expiry rate is

Figure 9: **Optimal patent expiry rate and its implications for growth and markups.**



Solid blue lines pertain to the estimated patent expiry rate. Dashed red lines pertain to the optimal expiry rate conditional on the discount rate ρ.

weakly decreasing as the discount rate falls. In the speed-limit economy (bottom panels), the opposite pattern obtains at very low discount rates: the optimal expiry rate rises as the discount rate falls, to counter the growth-reducing anti-competitive effects of a lower discount rate. In the speed-limit economy, with a constant patent expiry rate, such anti-competitive effects give rise to an inverted-U relation between growth and the discount rate. Remarkably, with optimal patent policy, the growth speed limit disappears (bottom middle panel). Varying the expiry rate optimally also implies that, at low discount rates, further declines in the discount rate *decrease* markups. These results underscore that the nature of creative destruction, including assumptions about patent policy, are crucial for the relation of growth, markups, and the interest rate.

6.2 Transition dynamics

Next, we study an economy initially on the BGP for a discount rate of 2%. At time $t = 0$, the discount rate falls to 0.25%, permanently. First, we assume that patent policy is unchanged, with a zero expiry rate. Second, we allow the social planner to adjust the patent expiry rate in response to the decline in the discount rate, taking into account the transition dynamics.

The solid line in Figure 10 shows the transition dynamics in the benchmark economy. When the discount rate falls at $t = 0$, the growth of the quality index $Q(t)$ immediately jumps from 1.03% to 1.16% and then rises gradually to the new BGP rate, 1.19%. On impact, the increase in quality-index growth is entirely along the intensive margin. The innovation strategy (mapping a firm's technology position to its innovation rate) shifts immediately to the innovation strategy of the new BGP.[26] In contrast, the distribution of technology gaps is unchanged on impact and evolves over time to a new, more competitive distribution of gaps. As the economy becomes more competitive, markups decline and become less dispersed, reducing production distortions and causing consumption growth to initially overshoot growth of the quality index.

Next, we assume that at time $t = 0$, concurrently with the decline in the discount rate, the social planner can make a one-time permanent change to the patent expiry rate. The planner solves

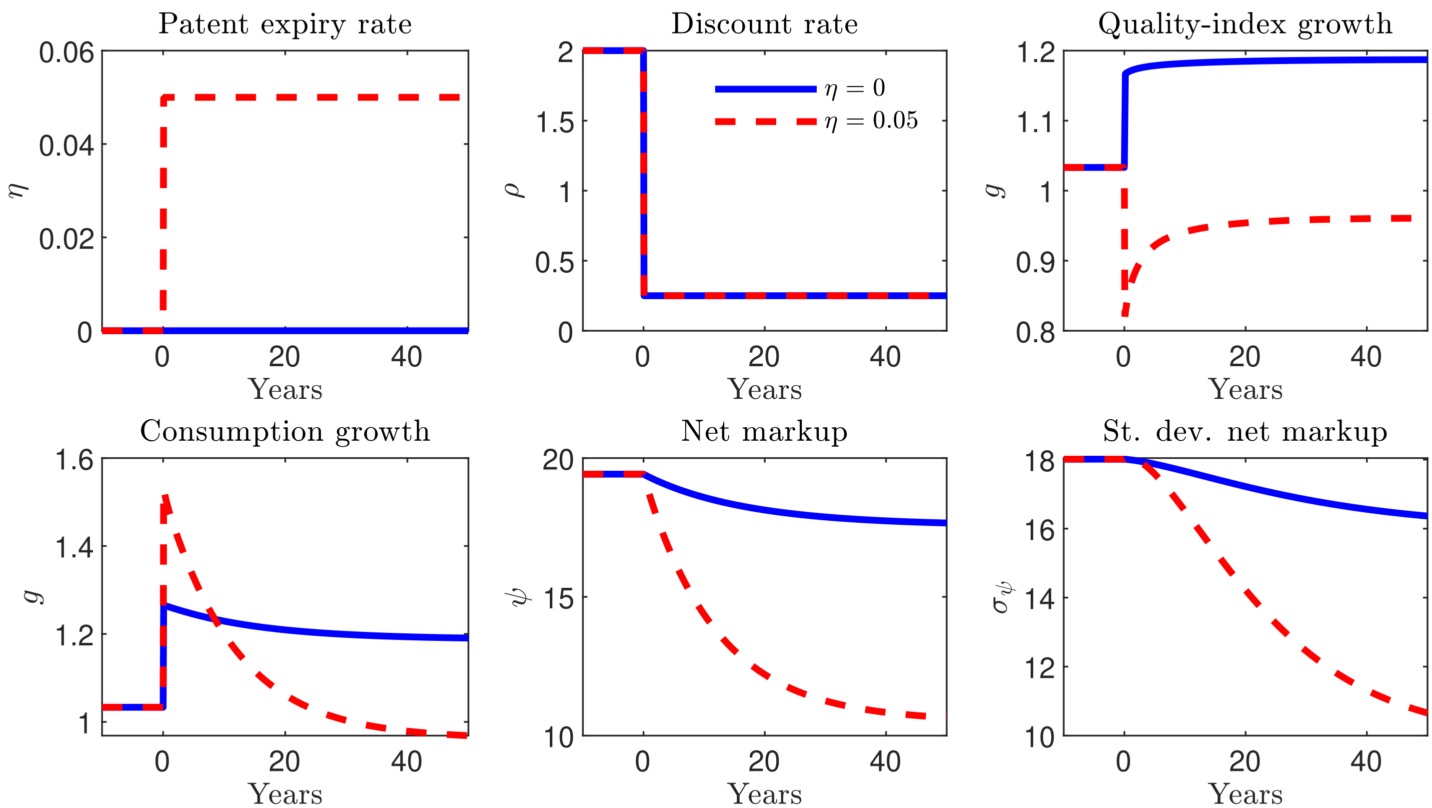
$$\max_{\eta} \int_{t=0}^{\infty} \exp(-\rho t)\Big(\ln(C(t;\eta)) - L(t;\eta)\Big)dt, \tag{17}$$

where the discount rate ρ in (17) is the new discount rate of the household 25 basis points beginning in period 0, and $\{C(t;\eta), L(t;\eta)\}_{t\in[0,\infty)}$ is the *equilibrium* path of aggregate consumption and labor conditional on the discount rate ρ and the patent expiry rate η.[27] To illustrate the planner's problem, if the planner were to increase the

[26] See Internet Appendix Section A for a proof, and for the definition of the dynamic general equilibrium.

[27] The initial conditions of the economy are the distribution of technology gaps and the initial quality index $Q(0)$. We set the initial distribution of technology gaps equal to the distribution in the initial BGP (before the decline in the discount rate). The initial quality index $Q(0)$ is a normalization,

Figure 10: **Transition dynamics under alternative patent policies.**



patent expiry rate to a suboptimal 5%, the transition dynamics shown by the dashed line in Figure 10 would obtain. For this exercise (ρ falling from 2% to 0.25%), taking into account the transition dynamics, the optimal patent expiry rate is zero, as was also the case when the planner maximized steady-state welfare. However, taking transition dynamics into account *does* matter, for example, for a similar exercise where the discount rate rises from 2% to 5%. In such an exercise, the optimal patent expiry rate is 4.5% when the planner focuses on steady-state welfare only, and lower, 2.2%, when the planner takes into account transition dynamics.

7. Financial Frictions

Restrictions on credit access might be expected to limit creative destruction and hence foster conditions for an inverted-U relation between growth and the interest

which shifts multiplicatively the entire consumption path but does not affect the patent policy chosen by the planner.

rate. To investigate this possibility, we introduce a limited commitment constraint. We assume that firms finance their R&D wage bill (on an intra-period basis) and that this financing is limited to a share α of firm value (Aghion et al., 2019a; Jermann and Quadrini, 2012).[28] Specifically,

$$\omega G(x_\sigma) \leq \alpha v_\sigma \tag{18}$$

implying that the innovation rate for a firm in position σ is

$$x_\sigma = \min\left\{G'^{-1}\left(\frac{\Delta v_\sigma}{\omega}\right), G^{-1}\left(\frac{\alpha v_\sigma}{\omega}\right)\right\}. \tag{19}$$

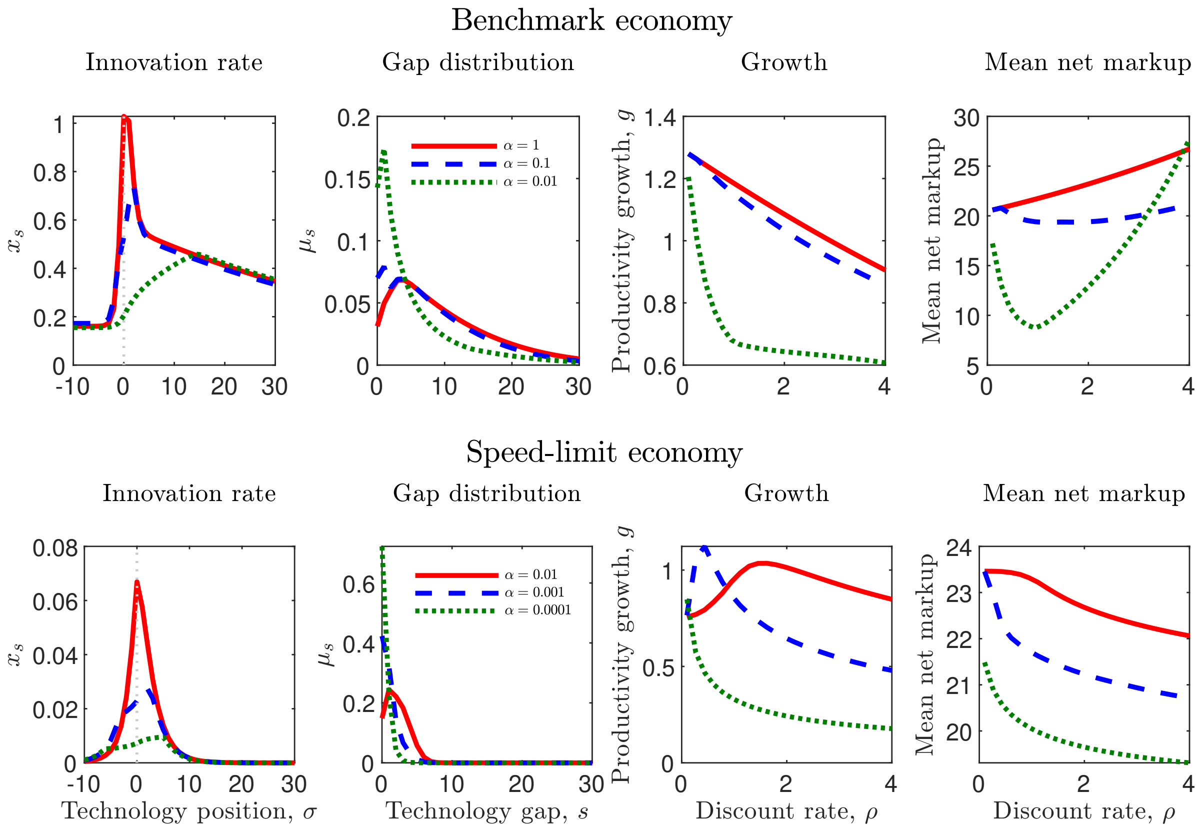
With full pledgeability ($\alpha = 1$), the BGP is unaffected (solid red lines in Figure 11). However, firm value derives from intangible technological knowledge that might not be pledgeable at all. To assess robustness, we re-estimate the model conditional on $\alpha = 0.1$ and obtain a good fit (Internet Appendix D). Note that the constraint $\alpha = 0.1$ is not very restrictive, in that it only limits annual R&D expenses to less than 10% of the expected discounted value of all future firm profits. The top row of Figure 11 shows key equilibrium features under the estimated parameter values if $\alpha = 0.1$ (dashed blue lines). As the discount rate declines, growth rises and the average markup falls.

At a 2% discount rate, the constraint binds for tied firms and leaders in competitive industries, but not for far-ahead leaders which have high firm values and low desired innovation rates (top left panel). Laggards are also not constrained because their desired innovation expenditures are low, even relative to firm value. With leaders in competitive industries innovating less, growth is lower than under full pledgeability. The average markup is also lower than under full pledgeability, because the constraint inhibits leaders in competitive industries from innovating to accumulate market power. If credit access is reduced further (dotted green lines), growth and markups fall further.

In the speed-limit economy, reduced credit access also limits the accumulation

[28]The limited commitment constraint arises if, in the event of default, firms can renege and make a take-it-or-leave-it offer to creditors, which can only seize a fraction of the firm's value.

Figure 11: **Credit access.**



Lower values of the limited-commitment parameter α correspond to reduced credit access.

of market power. With only gradual laggard advancement, reduced credit access constrains the innovation of leaders and laggards in competitive industries, making patent expiry a relatively more potent force and contributing to a more competitive economy. Reduced credit access therefore supports growth through a composition effect. At low interest rates, this pro-growth composition effect is sufficiently large that reduced credit availability *increases* growth. Moreover, when credit access is severely restricted, the speed limit vanishes completely: Starting from a low interest rate, a further decline in interest rate no longer fosters the accumulation of market power that is necessary for the growth speed-limit to obtain.

8. Conclusion

Recent decades have seen low productivity growth amid rising market power and falling business dynamism. Our paper evaluates the persistent decline in the real interest rate as a potential explanation for these trends. If innovation and patent expiry advance market laggards only incrementally, and if there is no entry, then starting from a low interest rate, a further decline in the interest rate can trigger strategic interactions among firms that reduce growth and competition. However, if market laggards or entrants have some chance of innovating radically, a lower interest rate boosts all firms' innovation rates and growth increases. Thus, the effect of lower interest rates on productivity growth and competition depend crucially on the *nature* of creative destruction.

With this motivation, we estimate the model using micro data on markups, R&D, innovation output, profit volatility, reallocation, and, in the model with entry, employment share by firm age. The data favor a parametrization with laggards or entrants having at least some chance of more-than-incremental innovation. Correspondingly, as the interest rate declines, productivity growth rises and markups decline. Our results are robust to incorporating entry, optimal patent policy, and financial frictions. Overall, these findings suggest that recent key macroeconomic trends of low productivity growth and rising markups are explained by factors other than low interest rates.

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Internet Appendix for

"Interest Rates, Innovation, and Creative Destruction"

Craig A. Chikis, Jonathan Goldberg, and David López-Salido

A. Generalized exposition of the model

This section develops a model nesting the assumptions used throughout the main text. This general model is then used for the proofs.

A.1 Intermediate goods market

This part of the Appendix proves the claims in Section 2 of the main text. We begin with $\kappa < \infty$, or imperfect substitution across each industry's two varieties. The effective demand elasticity facing firm $z \in \{i, -i\}$ is $\epsilon_z \equiv -\frac{d \ln y_z}{d \ln p_z} = \kappa(1 - \delta_z) + \delta_z$, where δ_z is the market share of firm z. The gross markup for firm z is therefore $\psi_z = \epsilon_z/(\epsilon_z - 1)$. With linear production technology, the marginal cost for firm z is $\frac{w}{q_z}$. The demand equation $\frac{p_i}{p_{-i}} = (\frac{y_i}{y_{-i}})^{-\frac{1}{\kappa}}$, implies that the ratio of the leader's market share to the laggard's market share is $\frac{\delta_i}{\delta_{-i}} = \nu^{1-\kappa}$, where $\nu \equiv \frac{p_i}{p_{-i}}$ is the ratio of the leader's price to the laggard's price. Combining the equations for the effective demand elasticity, the gross markup, the marginal cost, and relative market shares, one obtains that the ratio of the leader's price to the laggard's price, $\nu \equiv \frac{p_i}{p_{-i}}$, satisfies $\nu = \frac{q_i}{q_{-i}}(\frac{\kappa+\nu^{1-\kappa}}{\kappa+\nu^{\kappa-1}})$. From the equation for the relative market shares and the identity that the market shares sum to 1 (i.e., $\delta_i + \delta_{-i} = 1$), one obtains $\delta_i = \frac{\nu^{1-\kappa}}{1+\nu^{1-\kappa}}$. Substituting for δ_i in the equation for gross markup, one obtains the gross markup as a function of the relative price, $\psi_i = \frac{\kappa+\nu^{1-\kappa}}{\kappa-1}$. The expression for ψ_{-i} is derived analogously, by substituting for δ_{-i} in the equation for gross markup.

The leaders profit's can be written $\Pi_i = (p_i - \frac{w}{q_i})y_i$, or $\Pi_i = (1 - \frac{w}{p_i q_i})p_i y_i$. From the definition of market share and using the fact that industry sales are equal to Y in each industry, we have $p_i y_i = \delta_i Y$. Substituting this expression into the equation for profits and using the definition of gross markup ($\psi_i = \frac{p_i}{\frac{w}{q_i}}$), one obtains $\pi_i = \frac{\nu^{1-\kappa}}{\kappa+\nu^{1-\kappa}}$. The labor demand $l_i = \frac{(\kappa-1)\nu^{1-\kappa}}{(\kappa+\nu^{1-\kappa})(1+\nu^{1-\kappa})}$. Using the definition of the markup, ψ_i, it follows, after some simple algebra, that the labor demand and the profit functions for firm i can be expressed as expression (2). The expression for π_{-i} is derived analogously, using $\Pi_{-i} = (1 - \frac{w}{p_{-i}q_{-i}})p_{-i}y_{-i}$.. For the laggards, the expressions for labor demand and gross markup are $l_{-i} = \frac{(\kappa-1)\nu^{1-\kappa}}{(\kappa+\nu^{1-\kappa})(1+\kappa\nu^{1-\kappa})}$ and $\psi_{-i} = \frac{1+\kappa\nu^{1-\kappa}}{(\kappa-1)\nu^{1-\kappa}}$, respectively. Similarly, and after some algebra, it is easy to manipulate these expressions to obtain the labor demand and profits of the laggards included in expression (2).

Scaled operating profits for a firm in position σ are π_σ, obtained from (2) and $\frac{q_i}{q_{-i}} = \lambda^{|\sigma|}$, implying

$$\pi_\sigma = \begin{cases} \frac{\nu_{|\sigma|}^{1-\kappa}}{\kappa+\nu_{|\sigma|}^{1-\kappa}} & \text{if } \sigma > 0 \\ \frac{1}{\kappa\nu_{|\sigma|}^{1-\kappa}+1} & \text{if } \sigma < 0 \\ \frac{1}{\kappa+1} & \text{if } \sigma = 0. \end{cases} \tag{IA.1}$$

Finally, with perfect substitution across each industry's two varieties, limit pricing obtains, with $\pi_\sigma = 1 - \lambda^{-\sigma}$ for $\sigma \in S^+$ and $\pi_\sigma = 0$ otherwise. Correspondingly, $l_\sigma = \frac{1}{\omega\lambda^s}$ for $s \in S^+$ and $l_\sigma = 0$ otherwise. Gross markups are $\psi_\sigma = \lambda^\sigma$ for $s \in S^+$ and $\psi_\sigma = 0$ otherwise.

A.2 Advancement functions

We present an *advancement* on the quality (productivity) ladder in a way that can embed a rich set of assumptions about the nature of technological progress—and hence it is a flexible way of capturing different elements of creative destruction.

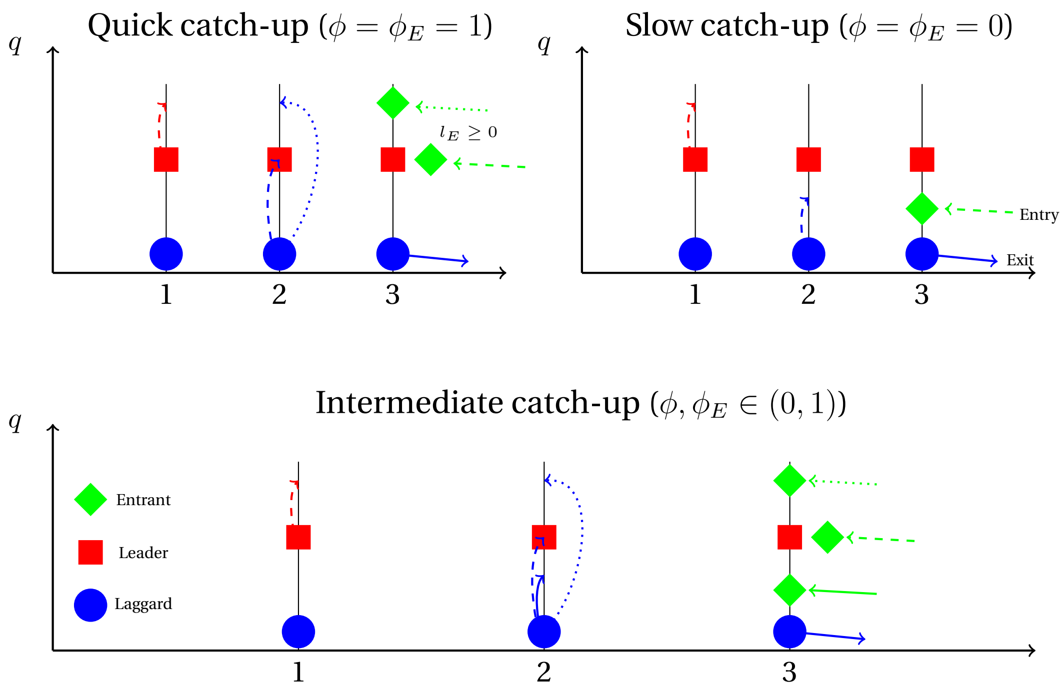
For an incumbent in position σ that obtains an innovation, let $F_{\sigma,\hat{\sigma}}$ represent the probability distribution over its new position, $\hat{\sigma}$. Thus, an innovating firm with productivity $q(t)$ obtains a new productivity $q(t+\Delta t) = \lambda^{\hat{\sigma}-\sigma}q(t)$. Modeling advancement using the $F_{\sigma,\hat{\sigma}}$ function allows us to nest models with innovations that advance at least one rung and as many as $\bar{s} - \sigma$ rungs, with a strictly positive probability of advancing an intermediate number of rungs, as in Akcigit and Kerr (2018), Akcigit et al. (2018), and Olmstead-Rumsey (2020). We refer to such models as "multiple innovation step size" models.

However, except for exercises with multiple innovation step sizes in Internet Appendix Section IA.II (discussed in Section 4.4 of the main text), all models in our paper can be nested using

$$F_{\sigma,\hat{\sigma}}(\phi, l) = \begin{cases} 1-\phi & \text{if } \sigma < l-1 \text{ and } \hat{\sigma} = \sigma + 1 \\ \phi & \text{if } \sigma < l-1 \text{ and } \hat{\sigma} = l \\ 1 & \text{if } \sigma \geq l-1 \text{ and } \hat{\sigma} = \sigma + 1, \end{cases} \tag{IA.2}$$

and $F_{\sigma,\hat{\sigma}} = 0$ otherwise. The advancement function $F_{\sigma,\hat{\sigma}}$ has two parameters: the leapfrog parameter l and the catch-up speed ϕ. The leapfrog parameter $l \in \{0, ..., \bar{s}\}$ is the maxi-

Figure IA.1: **Quality ladder.**



Top left panel (QUICK CATCH UP): An innovating leader advances one rung (Line 1). An innovating laggard advances to a tied or leadership position depending on the leapfrogging parameter l (Line 2). An entrant arrives l_E steps ahead of the leader, displacing the laggard (Line 3). Top right panel (SLOW CATCH UP): Leader innovation is the same as with quick catch up (Line 1). An innovating laggard advances one rung (Line 2). An entrant arrives one step ahead of the exiting laggard (Line 3). Bottom panel: In the intermediate case between these extremes, an innovating laggard advances a stochastic number of rungs (Line 2). An entrant arrives a stochastic number of steps ahead of the displaced laggard (Line 3).

mum technology position that a laggard can potentially obtain upon innovation. If $l = 0$, at most, an innovating laggard catches up to, but does not surpass, the productivity of the leader. If $l > 0$, an innovating laggard might surpass the leader on the quality ladder. A higher value of ϕ implies a greater probability of an innovating laggard jumping all the way to position l. Figure IA.1 illustrates advancement on the quality ladder under these assumptions.

Similarly, for a firm in position σ, conditional on a patent expiry, let $\hat{\sigma}$ is $F^p_{\sigma,\hat{\sigma}}$ define the probability distribution over its new position, $\hat{\sigma}$. The patent expiry assumption in our paper is captured by $F^p_{\sigma,\hat{\sigma}}(\zeta) = F_{\sigma,\hat{\sigma}}(\zeta, 0)$. The next section introduces entry. We keep the symmetry and define that an entrant replacing an incumbent firm with position σ begins its life in a random technology position $\hat{\sigma}$ with probability $F^E_{\sigma,\hat{\sigma}} = F_{\sigma,\hat{\sigma}}(\phi_E, l_E)$.

A.3 Entry

This section of the Internet Appendix extends the main text's benchmark model to include two types of entry: directed and undirected. With directed entry, the potential entrant making its R&D decision already knows which industry it will enter upon a successful innovation. With undirected entry, the potential entrant does not know ex-ante which industry it will enter. We include directed entry to nest the Akcigit and Ates (2019) model of Section 3 and Internet Appendix Section B. We include undirected entry to nest the extended version of the benchmark model in Section 4.5 of the main text.

Let $\{\mu_s(t)\}_{s\in S^+}$ denote the share of industries with a technology gap s at time t. With directed entry, at every time t and for each $s \in S$, a mass $\mu_s(t)$ of potential entrants choose an entry rate $x_{E,s}$ and hire $G(x_{E,s}; B_{E,s})$ R&D workers. Under undirected entry, at every time t, a mass 1 of potential entrants choose an entry rate x_E and hire $G(x_E; B_E)$ R&D workers. A successful undirected entrant begins life in an industry with gap s with probability μ_s. Conditional on entering an industry with gap s, an entrant occupies position $\hat{s}$ with probability $F^E_{-s,\hat{s}}$.

A directed potential entrant innovating for an industry with gap s has an expected capital gain from entry $\Delta v_{E,s}(t) = \sum_{\hat{s}=-s+1}^{\bar{s}} F^E_{-s,\hat{s}} v_{\hat{s}}(t)$. For an undirected potential entrant, the expected capital gain from entry is

$$\Delta v_E(t) = \sum_{s=0}^{\bar{s}} \mu_s(t)\Bigg(\sum_{\hat{s}=-s+1}^{\bar{s}} F^E_{-s,\hat{s}} v_{\hat{s}}(t)\Bigg). \tag{IA.3}$$

The entrant trades off R&D costs against the increased likelihood of successfully entering an industry. We introduce an R&D subsidy $\tau_{R\&D}$ to enable us to nest Akcigit and Ates (2019). The directed entrant solves:

$$\max_{x_{E,s}(t)} -(1-\tau_{R\&D})G(x_{E,s}(t))\omega + x_{E,s}(t)\Delta v_{E,s}(t), \tag{IA.4}$$

while, similarly, the undirected entrant solves:

$$\max_{x_E(t)} -(1-\tau_{R\&D})G(x_E(t))\omega + x_E(t)\Delta v_E(t). \tag{IA.5}$$

A.4 Equilibrium

For a firm in position σ at time t, the (unscaled) discounted expected value of profits satisfies the Hamilton–Jacobi–Bellman equation

$$\rho V_\sigma(t) - \dot{V}_\sigma(t) = \max_{x_\sigma(t)} (1-\tau)\pi_\sigma Y(t) - (1-\tau_{R\&D}) G(x_\sigma(t)) w(t) + x_\sigma(t) \sum_{\hat{\sigma}=s+1}^{\bar{\sigma}} (F_{\sigma,\hat{\sigma}} V_{\hat{\sigma}}(t) - V_\sigma(t)) +$$
$$x^c_{-\sigma}(t) \sum_{\hat{\sigma}=-\bar{s}}^{\sigma-1} (F_{-\sigma,-\hat{\sigma}} V_{\hat{\sigma}}(t) - V_\sigma(t)) + x_{E,|\sigma|}(t) \sum_{\hat{\sigma}=-\bar{s}}^{\sigma-1} ((\mathbb{1}_{\sigma>0} + \frac{1}{2}\mathbb{1}_{\sigma=0}) F^E_{-\sigma,-\hat{\sigma}} V_{\hat{\sigma}}(t) - V_\sigma(t)) +$$
$$\eta_\sigma \sum_{\hat{\sigma}=-\bar{s}}^{\bar{s}} (F^p_{\sigma,\hat{\sigma}} V_{\hat{\sigma}}(t) - V_\sigma(t)),$$

where $\dot{V}_\sigma(t)$ is the derivative of $V_\sigma(t)$ with respect to time. Here, $\eta_\sigma = 0$ for $\sigma = 0$ (reflecting no patent expiry in tied industries) and $\eta_\sigma = \eta$ otherwise. $x_{E,|\sigma|}$ is the entry rate of the directed entrant in industry $|\sigma|$, with $x_{E,|\sigma|} = x_E$ in the case of directed entry and $x_{E,|\sigma|} = 0$ in the case of no entry. Dividing by $Y(t)$ and using the Euler equation (3), one obtains

$$r(t)v_\sigma(t) - \dot{v}_\sigma(t) = \max_{x_\sigma(t)} (1-\tau)\pi_\sigma(t) - (1-\tau_{R\&D}) G(x_\sigma(t)) \omega(t) + x_\sigma(t) \sum_{\hat{\sigma}=s+1}^{\bar{\sigma}} F_{\sigma,\hat{\sigma}} v_{\hat{\sigma}}(t) - v_\sigma(t) +$$
$$x^c_{-\sigma}(t) \sum_{\hat{\sigma}=-\bar{s}}^{\sigma-1} (F_{-\sigma,-\hat{\sigma}} v_{\hat{\sigma}}(t) - v_\sigma(t)) + x_E(t) \sum_{\hat{\sigma}=-\bar{s}}^{\sigma-1} ((\mathbb{1}_{\sigma>0} + \frac{1}{2}\mathbb{1}_{\sigma=0}) F^E_{-\sigma,-\hat{\sigma}} v_{\hat{\sigma}}(t) - v_\sigma(t)) +$$
$$\eta_\sigma \sum_{\hat{\sigma}=-\bar{s}}^{\bar{s}} (F^p_{\sigma,\hat{\sigma}} v_{\hat{\sigma}}(t) - v_\sigma(t)). \quad \text{(IA.6)}$$

Imposing that $\dot{v}_\sigma(t) = 0$ along the BGP equilibrium and using the Euler equation (3), one obtains (4)–(5).

Firms' first order conditions. For a firm in technology position $\sigma \in S$, define the capital gain from a successful innovation Δv_σ:

$$\Delta v_\sigma(t) = \sum_{\hat{\sigma}=\sigma+1}^{\bar{\sigma}} F_{\sigma,\hat{\sigma}} v_{\hat{\sigma}}(t) - v_\sigma(t) \quad \text{(IA.7)}$$

For an incumbent firm in position $\sigma \in S$

$$x_\sigma(t) = G'^{-1}\left(\frac{1}{1-\tau_{R\&D}} \frac{\Delta v_s(t)}{\omega(t)}\right). \quad \text{(IA.8)}$$

The *directed* entrant's first-order condition is

$$x_{E,s}(t) = G_E'^{-1}\left(\frac{1}{1-\tau_{R\&D}} \frac{\Delta v_{E,s}(t)}{\omega(t)}\right). \quad \text{(IA.9)}$$

Similarly, the *undirected* entrant's first-order condition is

$$x_E(t) = G_E'^{-1}\left(\frac{1}{1-\tau_{R\&D}}\frac{\Delta v_E(t)}{\omega(t)}\right). \tag{IA.10}$$

Outflow-inflow equations. The share of industries with each gap $s \in S^+ \setminus \{0\}$ satisfies

$$\begin{aligned}(x_s + x_{-s}(1-F_{-s,s}) + x_E(1-F^E_{-s,s}) + \eta_s)\mu_s = \dot{\mu}_s(t) + \\ \sum_{\sigma \in S^+ \setminus s} \mu_\sigma \left(x_\sigma F_{\sigma,s} + x_{-\sigma}(F_{-\sigma,s} + F_{-\sigma,-s}) + x_E(F^E_{-\sigma,s} + F^E_{-\sigma,-s}) + \eta_\sigma F^p_{\sigma,s} \right),\end{aligned} \tag{IA.11}$$

and, for $s = 0$,

$$(2x_0 + x_E)\mu_0 = \dot{\mu}_0 + \sum_{\sigma \in S^+ \setminus 0} \mu_\sigma \left(x_\sigma F_{\sigma,0} + x_{-\sigma} F_{-\sigma,0} + \eta_\sigma F^p_{\sigma,0} + x_E F^E_{-\sigma,0} \right). \tag{IA.12}$$

Growth of the quality index. The quality index is $Q(t) = \exp(\int_0^1 \ln q_i(j;t))$, where $q_i(j;t)$ is the productivity of the leader in industry j at time t, or the the productivity of each firm in industry j at time t if firms are tied. In the model without leapfrogging $l = 0$ and, if there is entry, $l_E = 0$, growth of the quality index is

$$g(t) = \ln \lambda \sum_{s \in S^+} \mu_s(t)(1 + \mathbb{1}_{s=0}) x_s(t). \tag{IA.13}$$

With leapfrogging by laggards $l > 0$ or entrants $l > 0$, growth of the quality index is

$$g(t) = \sum_{s=0}^{\bar{s}} \mu_s(t) g_s(t), \tag{IA.14}$$

where $g_s(t)$ is the expected growth rate of the frontier technology in an industry with gap s, or

$$g_s = (\ln \lambda)\left(\sum_{\hat{s}=1}^{\bar{s}} (x_{E,s} F^E_{-s,\hat{s}} + x_{-s} F_{-s,\hat{s}})\hat{s} + x_s \sum_{\hat{s}=s+1}^{\bar{s}} F_{s,\hat{s}}(\hat{s} - s) \right) \tag{IA.15}$$

To explain (IA.14)–(IA.15), in an industry with gap $s > 0$, there are two potential sources of frontier-advancing innovation. The first is leapfrogging by the entrant or the incumbent laggard. The arrival rate of entrants in each industry is $x_{E,s}$. Conditional on entry, the frontier advances $\hat{s} > 0$ steps if the entrant leapfrogs the incumbent leader to achieve position $\hat{s}$. Conditional on entry into an industry with technological gap s, the probability of the entrant achieving position $\hat{s}$ is $F^E_{-s,\hat{s}}$. The arrival rate of laggard innovation in an industry with gap s is x_{-s}. Similarly to the case with entry, the innovating laggard can advance the technological frontier by $\hat{s} > 0$ steps if it leapfrogs the incumbent leader to achieve position $\hat{s}$. Conditional on a laggard innovation in an industry with gap s, the probability of advancing to position $\hat{s}$ is $F_{-s,\hat{s}}$. The second source of frontier-advancing innovation is coming from the leader in each industry. A leader innovation arrives at rate x_s in an industry with gap s and advances the leader to position $\hat{s}$ with probability $F_{s,\hat{s}}$, thereby pushing forward the frontier by $\hat{s} - s$ steps.

To obtain (IA.13)–(IA.15) in the case of imperfect substitution across within-industry varieties ($\kappa < \infty$), substitute $y_z = q_z l_z$, for $z \in \{i, -i\}$, into the final-good production function $\ln Y(t) = \int_0^1 \ln[y_i(j;t)^{\frac{\kappa-1}{\kappa}} + y_{-i}(j;t)^{\frac{\kappa-1}{\kappa}}]^{\frac{\kappa}{\kappa-1}} dj$. Further, substitute for l_i and l_{-i} from Section A.1. Substituting $q_{-i} = \lambda^{-s(j;t)}$, with some algebra, one obtains:

$$\ln \frac{Y(t)}{Q(t)} = \int_0^1 \ln \left(\frac{\kappa - 1}{1 + \nu_{s(j;t)}^{\kappa-1}} \left[(\kappa + \nu_{s(j;t)}^{1-\kappa})^{\frac{1-\kappa}{\kappa}} + (\lambda^s(\kappa \nu_{s(j;t)}^{1-\kappa} + 1))^{\frac{1-\kappa}{\kappa}} \right]^{\frac{\kappa}{\kappa-1}} \right) dj. \tag{IA.16}$$

Along a BGP, the distribution of industry gaps is stationary, and hence the right hand side of (IA.16) is constant. Therefore along a BGP, $Y(t)$ grows at the same rate as $Q(t)$, with

$$g \equiv \lim_{\Delta t \to 0} \frac{\ln Q(t + \Delta t) - \ln Q(t)}{\Delta t}. \tag{IA.17}$$

During an interval of length Δt, for $s \geq 1$, in the fraction μ_s of industries with gap $s \geq 1$, (i) leaders innovate at rate $x_s(t)\Delta t + o(\Delta t)$; (ii) laggards innovate at rate $x_{-s}(t)\Delta t + o(\Delta t)$; (iii) potential entrants innovate at rate $x_{E,s} + o(\Delta t)$ (with directed entry) or $x_E + o(\Delta t)$ (with undirected entry); and (iv) these innovations increase the productivity of the leader technology as described above, with each rung advanced increasing productivity by a factor λ. Thus, $\lim_{\Delta t \to 0} \frac{\ln Q(t+\Delta t) - \ln Q(t)}{\Delta t}$ is given by (IA.14)–(IA.15).

For the case of perfect substitution across within-industry varieties, (IA.14)–(IA.15) is obtained similarly using (16).

Representative household's first order conditions and labor supply. The representative household maximizes the utility function (1) subject to the budget constraint,

$$C(t) + \dot{A}(t) = w(t)L(t) + r(t)A(t) + T(t),$$

where $w(t)$ is the wage rate, $r(t)$ is the real interest rate, and $T(t)$ denotes lump-sum taxes paid by consumers to the government. The households are the shareholders of the firms and households' total assets $A(t)$ are the sum of the firms' asset values: $\int_{\mathbb{F}} V_{\mathbb{F}} df$, with $\mathbb{F}$ denoting the set of firms in the economy. The first order condition for consumption implies the standard Euler equation (3) and, combined with the first order condition for labor supply, implies

$$\frac{w(t)}{C(t)} = 1.$$

Accordingly, in equilibrium, the scaled real wage ω is equal to one.

Equilibrium definition. A dynamic general equilibrium is a sequence

$$\{g(t), \omega(t), L(t), \{\mu_s(t)\}_{s\in S^+}, \{x_\sigma(t)\}_{\sigma\in S}, \{x_{E,s}(t)\}_{s\in S^+}, \{p_j(t), y_j(t)\}_{j\in[0,1]}\}_{t\in[0,\infty)} \quad \text{(IA.18)}$$

such that (i) the sequence of prices and quantities $\{p_j(t), y_j(t)\}_{j\in[0,1],t\in[0,\infty)}$ satisfy the intermediate goods production function and the profit-maximization conditions (2); (ii) $\forall\sigma$, $x_\sigma(t)$ is a best response to $\{x^c_\sigma(t)\}_{\sigma\in S}$; (iii) $\forall\sigma$, $x^c_\sigma = x_\sigma$ (symmetry); (iv) labor and goods markets clear; (v) the distribution of technology gaps $\{\mu_s(t)\}_{s\in S^+}$ satisfies (IA.11)–(IA.12); (vi) growth is determined by (IA.14)-(IA.15); and (vii) $x_{E,s}(t) = 0, \forall s \in S^+$, if there is no entry, $\{x_{E,s}\}_{s\in S^+}$ satisfies (IA.9) if there is directed entry, and $x_{E,s} = x_E, \forall s \in S^+$ satisfies (IA.10) if there is undirected entry. With perfectly elastic labor supply $\omega(t) = 1$ and $L(t)$ is given by (10). With inelastic labor supply, $L(t) = 1$ and $\omega(t)$ satisfies (10).

Balanced growth path (BGP) definition. A BGP equilibrium is a dynamic general equilibrium with: constant growth g, scaled wage ω, aggregate labor L, incumbent innovation rates $\{x_\sigma\}_{\sigma\in S}$, directed-entrant innovation rates $\{x_{E,s}\}_{s\in S^+}$ if there is directed entry, and an undirected-entrant innovation rate x_E if there is undirected entry; and a stationary distribution of technology gaps $\{\mu_s\}_{s\in S^+}$.

Denote a BGP by the vector $(g, \omega, L, \{\mu_s\}_{s\in S^+}, \{x_s\}_{s\in S})'$, augmented by $\{x_{E,s}\}_{s\in S}$ in the case of directed entry and by x_E in the case of undirected entry, satisfying the growth equation (9) (one equation), labor market clearing (10) (one equation) with $\omega = 1$ (one equation) in the case of perfectly elastic labor supply or $L = 1$ (one equation) in the case of inelastic labor supply, incumbent first-order conditions ($2\bar{s} + 1$ equations of the form (IA.8)), directed-entrant first-order conditions ($2\bar{s}+1$ equations of the form (IA.9)) if there is directed entry, and the undirected-entrant first-order condition (IA.10) (one equation) if there is undirected entry, the outflow-inflow condition (IA.11) for $s \in \{1, ..., \bar{s}\}$ with $\dot{\mu}_s = 0$ ($\bar{s}$ equations), and the normalization of industry weights ($\sum_{s\in S} \mu_s = 1$, one equation). Thus, a BGP has $3\bar{s} + 5$ elements if there is no entry, with an equal number of equations characterizing the BGP; $4\bar{s}+6$ elements if there is directed entry, with an equal number of equations characterizing the BGP; and, similarly, $3\bar{s}+6$ elements if there is undirected entry.

Equilibrium dynamics. Here we present a key result for obtaining the transition dynamics in Section 6.2 of the main text.

Lemma. Suppose that there is no entry or that entry is directed. Then, the time-t innovation rate of a firm in position $\sigma \in S$ is equal to the BGP innovation rate of a firm in position σ. That is, $x_\sigma(t) = x_\sigma, \forall \sigma \in S, \forall t \in [0,\infty)$. The time-$t$ value function $\{v_\sigma(t)\}_{\sigma \in S}$ is equal to the BGP value function, with $v_\sigma(t) = v_\sigma$. With directed entry, the time-t entry rate is equal to the BGP entry rate, with $x_{E,s}(t) = x_{E,s}, \forall s \in S^+$.

Proof. Conjecture that $v_\sigma(t)$ is equal to the BGP value function. Then $x_\sigma(t)$ satisfying (IA.8) is the BGP x_σ. Moreover, $v_\sigma(t) = v_\sigma$ and $\dot{v} = 0$ satisfy (IA.6). Finally, $x_{E,s} = x_E$ satisfies (IA.9).

A.5 Computing consumption-equivalent welfare gains

Consider two patent expiry rates, η_A and η_B. Denote steady state welfare under each patent expiry rate by $\mathbb{W}_A(0)$ and $\mathbb{W}_B(0)$, respectively. Steady-state welfare is obtained from (15) and (16).

Welfare comparisons across steady states. Following Acemoglu and Akcigit (2012), Acemoglu et al. (2018), and Akcigit et al. (2020), Section 6 of the main text compares welfare across steady states. As in Acemoglu and Akcigit (2012), we make comparisons across steady states conditional on an exogenous initial quality index $Q(0)$. Comparing across steady states, the consumption-equivalent welfare gain of switching from policies B to policies A is denoted by χ, where $\mathbb{W}_A(0) = \frac{1}{\rho}\ln(1+\chi) + \mathbb{W}_B(0)$.

Welfare comparisons taking into account transition dynamics. Section 6.2, in contrast, studies welfare and optimal policy taking into account transition dynamics. For a given initial distribution of technology gaps $\{\mu_s(0)\}_{s\in S^+}$ and initial quality index $Q(0)$, we calculate ex-ante welfare $W(0)$ as

$$W(0) = \int_{t=0}^{T} \exp(-\rho t)\Big(\ln(C(t)) - L(t)\Big) + \exp(-\rho T)\mathbb{W}(T), \tag{IA.19}$$

where $\{C(t), L(t)\}_{t\in[0,T)}$ is the path of aggregate consumption and labor induced by the patent policy η and other model parameters, $\mathbb{W}(\mathrm{T})$ is steady state welfare, and T is sufficiently large such that the economy has converged to the steady state at time T. An assumption of an alternative initial quality index $\tilde{Q}(0) \neq Q(0)$ implies an alternative time-0 welfare $\tilde{W}(0) = \frac{1}{\rho}(\ln(\tilde{Q}(0)) - \ln(Q(0))) + W(0)$. Thus, the choice of initial quality index $Q(0)$ does not affect comparisons of welfare for different policies; this result applies whether one is comparing steady state welfare or taking into account transition dynamics.

Denote time-0 welfare under each patent expiry rate by $W_A(0)$ and $W_B(0)$, respectively. Taking into account transition dynamics, the consumption-equivalent welfare gain of switching from policies B to policies A is denoted by χ, where $W_A(0) = \frac{1}{\rho}\ln(1+\chi) + W_B(0)$.

A.6 Extension: Elasticity of intertemporal substitution.

The benchmark model's assumption of log preferences can be modified to allow the EIS to differ from 1. Assume the household maximizes

$$\int_{t=0}^{\infty} e^{-\rho t} \frac{C(t)^{1-\frac{1}{\varphi}} - 1}{1 - \frac{1}{\varphi}} dt. \tag{IA.20}$$

where $\varphi > 0$ is the EIS and $\frac{1}{\varphi}$ is the coefficient of relative risk aversion. The Euler equation becomes $g = \varphi(r - \rho)$. Substituting into the firm value function, one obtains, for $\sigma \in S$,

$$\begin{aligned}(\varphi\rho + (1-\varphi)r)v_\sigma(t) - \dot{v}_\sigma(t) = \max_{x_\sigma(t)} (1-\tau)\pi_\sigma - (1-\tau_{R\&D})G(x_\sigma(t))\omega(t) \\ + x_\sigma(t) \sum_{\hat{\sigma}=s+1}^{\bar{\sigma}} (F_{\sigma,\hat{\sigma}} v_{\hat{\sigma}}(t) - v_\sigma(t)) + \\ x^c_{-\sigma}(t) \sum_{\hat{\sigma}=-\bar{s}}^{\sigma-1} (F_{-\sigma,-\hat{\sigma}} v_{\hat{\sigma}}(t) - v_\sigma(t)) + x_E(t) \sum_{\hat{\sigma}=-\bar{s}}^{\sigma-1} ((\mathbb{1}_{\sigma>0} + \frac{1}{2}\mathbb{1}_{\sigma=0})F^E_{-\sigma,-\hat{\sigma}} v_{\hat{\sigma}}(t) - v_\sigma(t)) + \\ \eta_\sigma \sum_{\hat{\sigma}=-\bar{s}}^{\bar{s}} (F^p_{\sigma,\hat{\sigma}} v_{\hat{\sigma}}(t) - v_\sigma(t)).\end{aligned} \tag{IA.21}$$

With elastic labor supply, we obtain the BGP for given ρ by searching for an interest rate r such that innovation rates satisfy (IA.21) and the Euler equation is satisfied. With inelastic labor supply, we obtain the BGP by searching for (r, ω) pair such that (IA.21), the Euler equation, and labor market clearing is satisfied.

A.7 Model solution and estimation

Model solution (for given parameters). Following Acemoglu and Akcigit (2012), we solve for the value functions $\{v_\sigma\}_{\sigma \in S}$ for given parameters from (4)-(5) using value function iteration and uniformization. For the parameterizations in the main text and Internet Appendix, we verified that increasing $\bar{s}$ does not alter the equilibrium properties.

Model estimation (finding parameters that minimize the SMM criterion). For our benchmark model, to find parameters that minimize the SMM criterion, we use the DIviding RECTangles (DIRECT) algorithm (Jones, Perttunen and Stuckmann, 1993). DIRECT is a global, deterministic search algorithm based on systematically dividing the parameter space into smaller and smaller hyperrectangles. For models with entry, due to the higher number of estimated parameters, we use the Multi-Level Single-Linkage (MLSL) algorithm (Kan and Timmer, 1987; Kucherenko and Sytsko, 2005). MLSL is a global algorithm that performs a sequence of local optimizations from random starting points. For the local optimizations within MLSL, we used the Constrained Optimization BY Linear Approximations (COBYLA) algorithm (Powell, 1994). We used code for these algorithms from NLOpt Version 2.6.2 (Johnson, 2021).

B. Models from Figure 1

Our model nests the models studied in Section 3, using the parameter values listed in Table IA.I. Our analysis of Liu et al.'s (2020) quantitative model also incorporates an additional assumption in their paper regarding the microfoundation for profits.[29]

The R&D scale parameters B and B_E, the discount rate ρ, and the patent expiry rate η are annualized. That is, to solve these models at a monthly frequency, one would divide the reported parameter (e.g., B and η) by 12. All other parameters are invariant to the frequency at which one solves the model.

[29] In their quantitative analysis, Liu et al. (2020) use Bertrand competition but modify the microfoundation for profits by assuming that in an industry with gap s, the production cost of the follower is $\lambda^{\min\{s,1\}}$ times the cost of the leader. Under this assumption, a larger gap s implies a laggard must advance more rungs to catch-up with the leader, but the laggard's cost disadvantage is the same for all gaps $s \geq 1$. We incorporate this assumption in the right panel of Figure 1 of the main text.

Model	ϕ	λ	B	γ	η	ϕ_E	B_E	l_E
Acemoglu and Akcigit (2012) Quick catch-up	1.00	1.05	1.20	0.35	0.00			
Acemoglu and Akcigit (2012) Slow catch-up	0.00	1.05	1.20	0.35	0.02			
Liu et al. (2020)	0.00	1.21	0.36	0.50	0.04			
Akcigit and Ates (2019) post-1980s	0.04	1.04	0.72	0.35	0.01	0.04	3.03	0.00

Table IA.I: **Parameters for the models in Section 3.**

C. Data appendix

Markup distribution. We target moments characterizing the distribution of markups. We obtain data values from Hall (2018), who estimates the distribution of Lerner indexes. The Lerner index is defined as the ratio of price minus marginal cost to price. To obtain markup moments, we make many draws from the distribution of Lerner indexes in Hall (2018) and convert each Lerner index draw to a markup. The mean, median, and 90th percentile of the Lerner index in Hall (2018) are 15%, 12%, and 30%. These untargeted moments in our model are 15%, 12%, and 29%, indicating a very good fit.

Innovation output, profit volatility, and R&D to sales. Three types of moments are calculated at least in part using COMPUSTAT data: innovation output, profit volatility, and R&D to sales. The data frequency is annual (except for the use of quarterly data to calculate the number of quarters with positive profits). To focus on innovative firms, data moments are calculated for firm-years with positive R&D. (All firms have positive R&D in the model, due to the Inada-type condition that $G'(0) = 0$.) In line with Kogan et al. (2017): we omit financial firms (SIC codes 6000 to 6799) and utilities (SIC codes 4900 to 4949); we restrict attention to firm-year observations with nonmissing values for book assets and SIC codes; and we winsorize at the 1% level using yearly breakpoints. Specifically, we winsorize innovation output, profit growth (used to calculate profit volatility), and R&D to sales, using yearly data, in the model and in the data. All COMPUSTAT-related statistics are calculated for the same sample period as the productivity growth target, 1960–2019.

Following Bloom, Schankerman and van Reenen (2013) and De Ridder (2019), R&D

is the Compustat variable XRD. Innovation output is the sum of the economic value of all patents earned (based on stock market reaction to patent grants), normalized by firm value. The economic value of patents is obtained from the website of Dimitris Papanikolaou based on Kogan, Papanikolaou, Seru and Stoffman (2017).[30] As in their paper, firm value in the data is the book value of firm assets AT. Firm value in the model is the discounted expected value of firm profits, v_σ. The innovation output distribution is similar when using firms' enterprise value in COMPUSTAT as a measure of firm value. Enterprise value is the sum of a firm's equity market capitalization, preferred stock outstanding, and the book value of debt. For R&D to sales, we calculate, in the model and the data, the median ratio of R&D to sales, for all firms and for firms in the top quintile of firms ranked by profits. To reduce outliers, we restrict attention, in calculating profit-volatility moments (i.e., for all firms or a subset of firms, the standard deviation of profit growth between year y and $y + 1$) in the model and in the data, to firms with two quarters of positive operating profits in the base year y.

Internet Appendix D conducts four robustness exercises related to the data construction: (i) changing the targeted value of the growth rate; (ii) targeting much higher values of the mean, median, and 90th percentile markup, using estimates from De Loecker et al. (2020) for 2016; (iii) including firm-years with zero R&D when calculating COMPUSTAT-related moments; and (iv) eliminating the two-quarter-positive-profits restriction when calculating profit volatility.

D. Robustness

This Internet Appendix presents a number of robustness exercises, which are summarized Section 4.4 of the main text. Section D.1 reports the results of exercises in which an exogenously set parameter is altered and the model is re-estimated. Section D.2 describes exercises related to patent policy. Section D.3 reports the results of exercises in which target values for key moments are changed. Section D.4 discusses the results from estimating an extended version of the model with entry. Section D.5 studies an extended version of the model in which the laggard or entrant can advance an intermediate number

[30] See https://github.com/KPSS2017/Technological-Innovation-Resource-Allocation-and-Growth-Extended-Data

of rungs or even leapfrog the leader. Section D.6 gauges robustness with respect to how moment target values are calculated in the model and data.

D.1 Varying an exogenously set parameter and re-estimating.

A first set of exercises alters an assumption about an exogenously set parameter and re-estimates the other parameters. In the benchmark model, the innovation elasticity parameter $\gamma = 0.5$, as in Akcigit and Kerr (2018), Liu et al. (2020), Peters (2020), and De Ridder (2019). The first column of Table IA.II shows the results from re-estimating the model conditional on $\gamma = 0.33$, as in Acemoglu and Akcigit (2012) and Akcigit and Ates (2019). The second column shows the results from re-estimating the model conditional on $\gamma = 0.66$. The model fit is quite good for both alternative values of γ. Fitting the data with lower values of γ requires lower values of the catch-up speed ϕ and higher R&D costs (i.e., lower values of the R&D cost scaling parameter B). Overall, the growth-interest rate relation and the average markup-interest rate relation are only modestly affected when altering γ and re-estimating the model (Figure IA.2, top panels). With lower γ, the growth-interest rate relation is a bit flatter, as is the average markup-interest rate relation. Next, we re-estimate the model when varying κ, the elasticity of substitution across the two varieties in each industry. Much of the literature, including Acemoglu and Akcigit (2012), Akcigit and Ates (2019), Aghion and Howitt (1992), and Acemoglu et al. (2018), assumes perfect substitution across varieties ("$\kappa = \infty$"). With imperfect substitution across varieties, both firms in each industry produce, as shown in Section 2.2 of the main text. For these exercises, we calculate the industry markup as the revenue-weighted average markup.[31] The third to fifth columns of Table IA.II show the results for $\kappa \in \{12, 24, 36\}$. (The quantitative model of Liu et al. (2020) assumes $\kappa = 12$.) Re-estimating the model with imperfect substitution across varieties, the model fit is good except that profit volatility is too low, unconditionally and for the top profit quintile. Imperfect substitution across varieties implies that, conditional on the innovation step size λ, profits π_σ are generally less sensitive to a firm's technology position σ, as shown in

[31] The revenue-weighted average firm markup is generally higher than the equal-weighted average firm markup. For example, for the model estimated with $\kappa = 24$, the mean revenue-weighted markup is 19.4%; the mean equal-weighted markup is 14.0%. The revenue-weighted and equal-weighted averages converge as κ approaches 1.

Figure IA.3. Altering the value of κ implies little change in the estimated value of ϕ. Figure IA.2, second row of panels, shows that the growth-interest rate relation is little changed when varying κ. For each value of κ, the average markup declines or is little changed as the interest rate falls.

Section A.6 of the Internet Appendix extends the model to include EIS not equal to one. In our next exercises, we assume alternative values for the EIS $\varphi \in \{0.25, 0.5, 1.5\}$ and re-estimate the remaining parameters. The results are reported in the sixth to eighth columns of Table IA.II. The quality-of-fit is good for each EIS value and the estimated values of (B, ϕ, λ) are little affected by varying the EIS. The growth-interest rate relation is negative in each case, and steeper when the EIS is higher. The net markup-interest rate relation varies little across these EIS exercises.

We also re-estimate the model assuming a discount factor of $\rho = 1.5\%$, rather than $\rho = 2\%$ as in the benchmark model in the main text. The results are, overall, little changed. The final exercise reported in Table IA.II and Figure IA.2 assumes inelastic labor supply, as in much of the literature, including Aghion and Howitt (1992), Acemoglu and Akcigit (2012), Akcigit and Ates (2019), Acemoglu et al. (2018), and Akcigit and Kerr (2018). The model with inelastic labor supply is presented in Internet Appendix Section A. With a discount rate of 2%, the market-clearing scaled wage is $\omega = 0.936$, implying a pure-profits share of 6.4%. The scaled wage is close to its value with perfectly elastic labor supply (i.e., $\omega = 1$) and therefore the estimated parameter values are quite similar when assuming perfectly elastic or perfectly inelastic labor supply. Moreover, with inelastic labor supply, the scaled wage varies little as the discount rate changes, implying that the relation of growth, the average markup, and the interest rate is little affected by assuming inelastic labor supply.

D.2 Robustness and patent policy

The benchmark model in the main text assumes that the patent expiry rate is zero, as in the benchmark model of Acemoglu and Akcigit (2012). Table IA.III reports results from two exercises to gauge the robustness of our results with respect to this assumption. The first column reports an exercise in which we assume the annual patent expiry rate is 3% and re-estimate the remaining parameters, including the patent expiry catch-up

Moments	$\gamma = 0.33$	$\gamma = 0.66$	$\kappa = 12$	$\kappa = 24$	$\kappa = 36$	$\varphi = 0.25$	$\varphi = 0.50$	$\varphi = 1.50$	$\rho = 1.5\%$	Inelastic labor	Target
Productivity Growth	1.04%	1.03%	1.03%	1.03%	1.03%	1.03%	1.03%	1.03%	1.03%	1.03%	1.03%
Markup											
Mean	19.19%	19.25%	22.05%	19.40%	19.40%	19.38%	19.44%	18.62%	19.40%	19.45%	19.40%
50th percentile	12.02%	13.69%	18.46%	12.06%	11.11%	12.87%	13.02%	12.55%	13.02%	13.09%	13.64%
90th percentile	43.12%	39.78%	26.68%	36.19%	40.11%	42.19%	42.05%	39.96%	41.60%	41.85%	42.62%
Innovation output											
Mean	5.26%	6.49%	1.24%	2.81%	3.46%	7.76%	6.74%	5.50%	5.07%	5.82%	6.75%
50th percentile	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
90th percentile	20.20%	16.46%	6.32%	12.41%	14.05%	21.71%	19.97%	17.72%	17.29%	18.34%	19.49%
FHK within	91.22%	87.80%	90.25%	89.29%	88.74%	89.50%	89.41%	89.34%	89.24%	89.45%	90.77%
Profit volatility											
All firms	40.31%	35.43%	12.12%	19.90%	29.62%	39.60%	39.45%	40.02%	38.90%	39.32%	45.09%
Top profit quintile	15.04%	16.80%	12.38%	15.58%	16.31%	16.75%	16.56%	16.53%	16.24%	16.43%	21.89%
R&D to sales											
All firms	4.74%	6.31%	4.09%	5.69%	6.10%	3.88%	4.82%	5.62%	5.80%	5.51%	5.35%
Top profit quintile	3.09%	3.71%	2.65%	4.84%	4.77%	2.52%	3.17%	3.74%	3.88%	3.64%	2.52%
Parameters											
ϕ	0.163	0.536	0.499	0.347	0.376	0.333	0.312	0.302	0.309	0.300	
λ	1.019	1.019	1.072	1.041	1.037	1.019	1.019	1.019	1.020	1.019	
B	1.537	3.331	0.872	1.195	1.264	2.804	2.475	2.290	2.073	2.220	

Table IA.II: **Robustness: Varying a parameter value and re-estimating.** Each column, except the last column, reports the results of an exercise in which the indicated exogenously set parameter value is altered and then the model is re-estimated to minimize the criterion (12). The final column gives the targeted values of the moments. In this table, FHK within is the adjusted within moment, described in Section 4.1 of the main text.

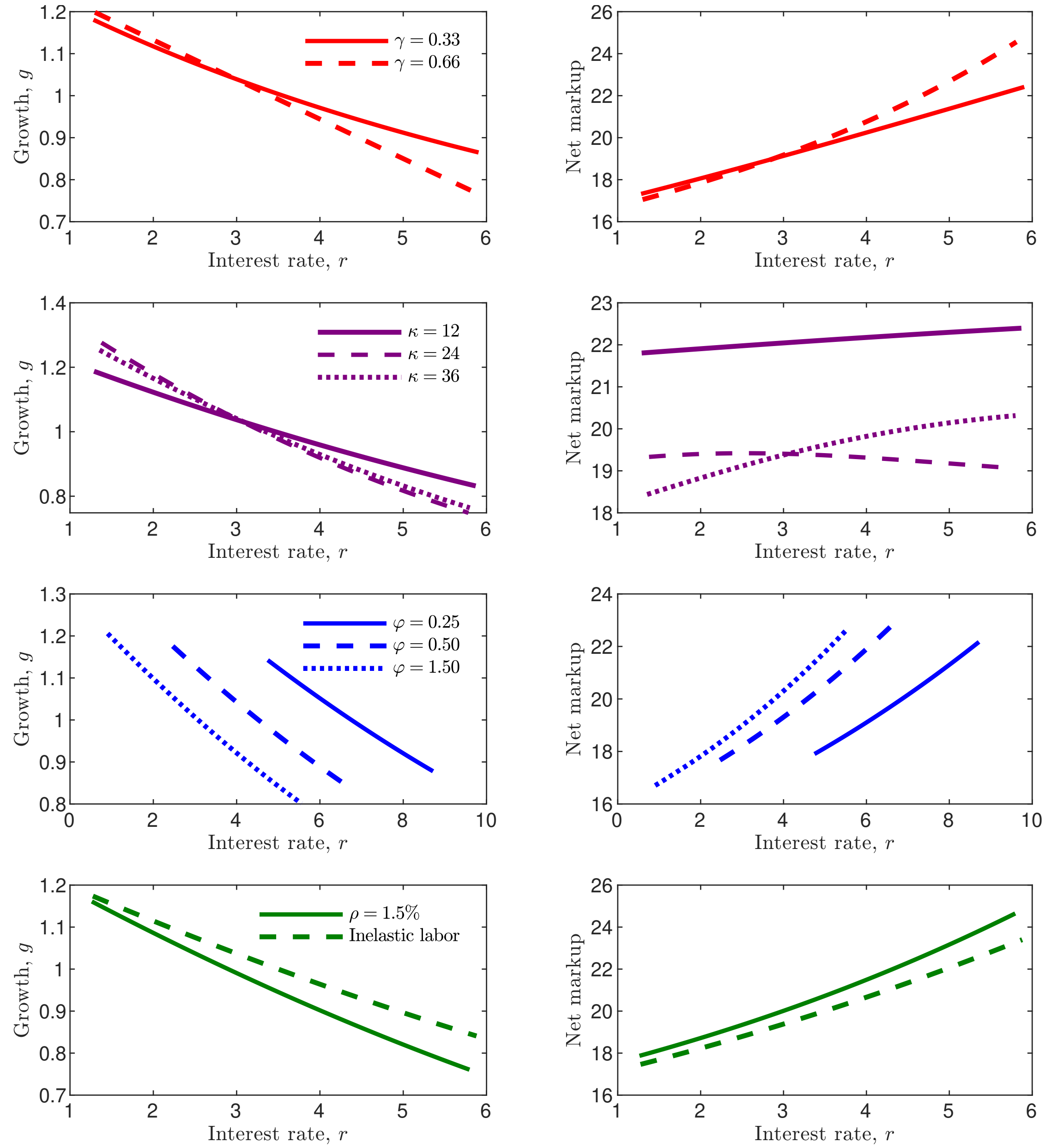


Figure IA.2: **Growth, average markup, and the interest rate, in robustness exercises that vary an exogenously set parameter and then re-estimate the model.** The parameters altered in these exercises are the R&D cost curvature parameters γ , the elasticity of substitution across varieties within each industry κ, the elasticity of intertemporal substitution φ, and the discount rate ρ. We also estimate a model with perfectly inelastic labor supply.

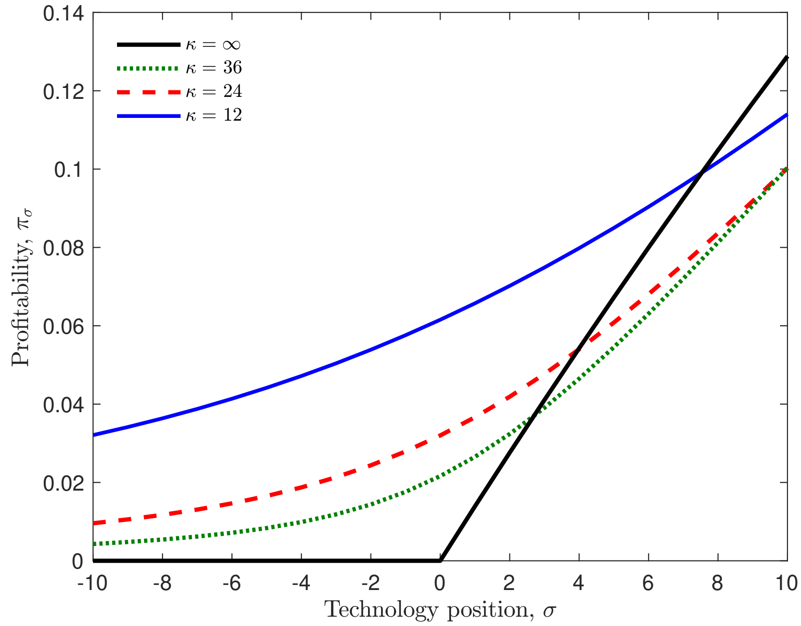


Figure IA.3: **Profit π_σ as a function of firm technology position σ, for different elasticities of substitution across within-industry varieties.**

speed ζ. The estimated innovation catch-up speed ϕ declines somewhat relative to the benchmark model. The growth-interest rate relation is negative and slightly steeper than in the benchmark model (Figure IA.4, left panel). The average markup-interest rate relation is positive and slightly shallower than in the benchmark model (right panel). Next, we extend the set of three estimated parameters to include the patent expiry rate η and the expiry catch-up speed ζ. The estimated patent expiry rate is zero, to thee decimal places.

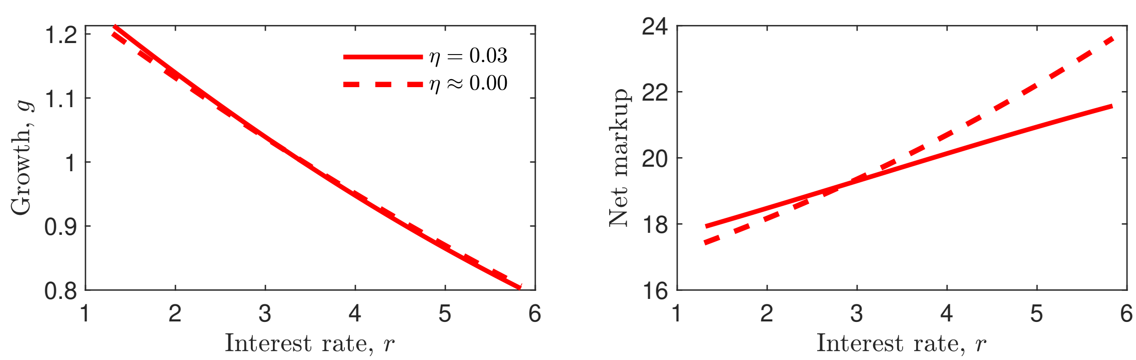


Figure IA.4: **Growth, average markup, and the interest rate, in robustness exercises related to patent policy.**

Moments	$\eta = 0.03$	η estimated	Target
Productivity Growth	1.03%	1.03%	1.03%
Markup			
Mean	19.35%	19.42%	19.40%
50th percentile	13.18%	13.12%	13.64%
90th percentile	41.78%	41.81%	42.62%
Innovation output			
Mean	5.38%	5.90%	6.75%
50th percentile	0.00%	0.00%	0.00%
90th percentile	17.05%	18.17%	19.49%
FHK within	90.20%	89.85%	90.77%
Profit volatility			
All firms	39.68%	39.30%	45.09%
Top profit quintile	15.91%	16.25%	21.89%
R&D to sales			
All firms	5.59%	5.50%	5.35%
Top profit quintile	3.67%	3.59%	2.52%
Parameters			
ϕ	0.228	0.283	
λ	1.016	1.018	
B	2.761	2.463	
η	0.030	0.000	
ζ	0.378	1.000	

Table IA.III: **Robustness: Patent policy.** The first column reports the results of an exercise in which the annual patent expiry rate is set to 3% and then the model parameters, including the patent expiry catch-up speed ζ, are re-estimated. The second columns reports the results of an exercise in which all five parameters shown $(\phi, \lambda, B, \eta, \zeta)$ are estimated jointly. The final column gives the targeted values of the moments. In this table, FHK within is the adjusted within moment, described in Section 4.1 of the main text.

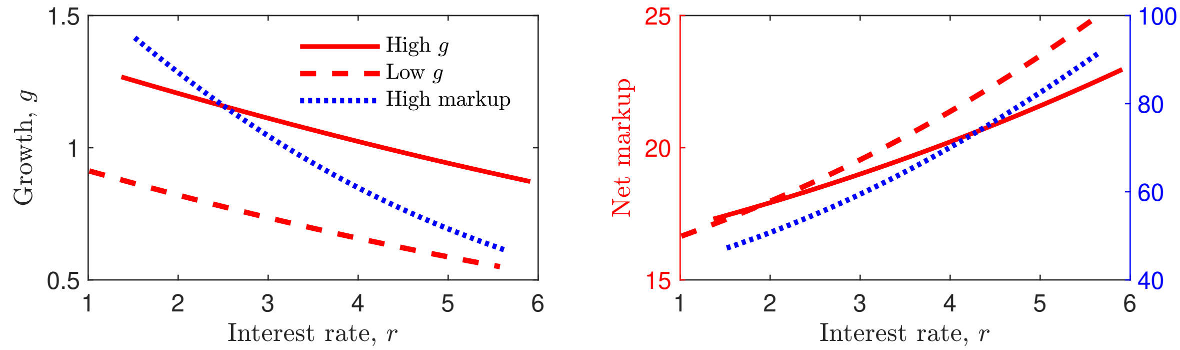


Figure IA.5: **Growth, average markup, and the interest rate, in robustness exercises varying the values of targeted moments.**

D.3 Varying targeted moment values.

Next, we study robustness with respect to values of the underlying moments. We set the annual growth rate g to 0.75% and re-estimate the model. We repeat this exercise with g equal to 1.10%. We also estimate the model targeting markups much higher than in our benchmark estimation. We target net markups with a mean of 60%, a median of 25%, and a 90th percentile of 150%. These markup targets are from De Loecker et al. (2020), for 2016. The results are reported in Table IA.IV and Figure IA.5. A higher growth target is met primarily by increasing the innovation step size λ and reducing R&D costs (i.e, increasing B). For the high-markups estimation, the growth rate and average markup target are achieved, but moments related to innovation output, profit volatility, and R&D to sales are matched notably worse than when the model is estimated using markup targets from Hall (2018). The estimated innovation catch-up speed ϕ declines, as expected. Targeting a higher growth rate shifts the growth-interest rate relation upward (by construction) with little change in the slope. When high markups are targeted, the growth-interest rate relation remains negative and the average markup-interest rate relation remains positive, with both are steeper relative to these relations in the benchmark model.

D.4 Entry

Section 4.5 of the main text discusses results from estimating a version of the model with undirected entry. Here we present additional detail regarding those results. Table IA.V lists values of all targeted moments in the model and in the data. Figure IA.6 shows that,

Moments	High g		Low g		High markup	
	Model	Target	Model	Target	Model	Target
Productivity Growth	1.10%	1.10%	0.75%	0.75%	1.03%	1.03%
Markup						
Mean	19.14%	19.40%	19.16%	19.40%	59.97%	60.00%
50th percentile	12.91%	13.64%	12.99%	13.64%	41.75%	25.00%
90th percentile	41.16%	42.62%	41.45%	42.62%	130.65%	150.00%
Innovation output						
Mean	6.09%	6.75%	4.88%	6.75%	3.03%	6.75%
50th percentile	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
90th percentile	18.52%	19.49%	13.39%	19.49%	5.78%	19.49%
FHK within	89.52%	90.77%	90.76%	90.77%	93.47%	90.77%
Profit volatility						
All firms	40.65%	45.09%	32.78%	45.09%	22.04%	45.09%
Top profit quintile	16.94%	21.89%	14.06%	21.89%	9.45%	21.89%
R&D to sales						
All firms	5.57%	5.35%	5.02%	5.35%	7.51%	5.35%
Top profit quintile	3.66%	2.52%	3.27%	2.52%	3.14%	2.52%
Parameters						
ϕ	0.296		0.272		0.172	
λ	1.019		1.016		1.027	
B	2.486		2.102		1.449	

Table IA.IV: **Robustness: Values of Targeted Moments.** The first two columns show model and target moments when the target growth rate g is 1.10%. The third and fourth columns present the same exercise, but with a target growth rate of 0.75%. The final two columns present an exercise in which the markup distribution is shifted significantly to the right. In this table, FHK within is the adjusted within moment, described in Section 4.1 of the main text.

Moments	Model	Target
Productivity Growth	1.03%	1.03%
Markup		
Mean	19.42%	19.40%
50th percentile	13.15%	13.64%
90th percentile	41.37%	42.62%
Innovation output		
Mean	7.15%	6.75%
50th percentile	0.00%	0.00%
90th percentile	15.48%	19.49%
FHK within	81.12%	66.93%
Profit volatility		
All firms	41.06%	45.09%
Top profit quintile	16.66%	21.89%
R&D to sales		
All firms	5.24%	5.35%
Top profit quintile	3.47%	2.52%
FHK entry	20.53%	23.93%
Employ share, ≤ 10 years	29.33%	26.20%
Employ share, ≤ 5 years	17.77%	15.70%
Parameters		
ϕ	0.188	
λ	1.019	
B	2.317	
l	0.000	
ϕ_E	0.479	
B_E	0.678	
l_E	1.000	

Table IA.V: **Robustness: Entry.** This table reports results from estimating the model with undirected entry from Section A.3 of the Internet Appendix.

with entry, the growth-interest rate relation remains negatively sloped and the average markup-interest rate relation remains positively sloped.

D.5 Intermediate step sizes.

Innovations in our benchmark model advance the laggard only one rung or close the gap completely. We next extend the model to include the possibilities of the laggard advancing an intermediate number of rungs or even leapfrogging the leader.

Together with the leapfrogging parameter l, a second parameter ϕ_M captures the *speed* at which an innovating laggard moves ahead. A higher value of ϕ_M implies a greater probability of an innovating laggard jumping all the way to position l. Note that $\phi_M \in (-\infty, \infty)$ is distinct from the catch-up speed parameter ϕ, as we will explain next. For $\phi_M \in (-\infty, \infty)$, the advancement function $F_{\sigma,\hat{\sigma}}$ is defined as follows. A firm

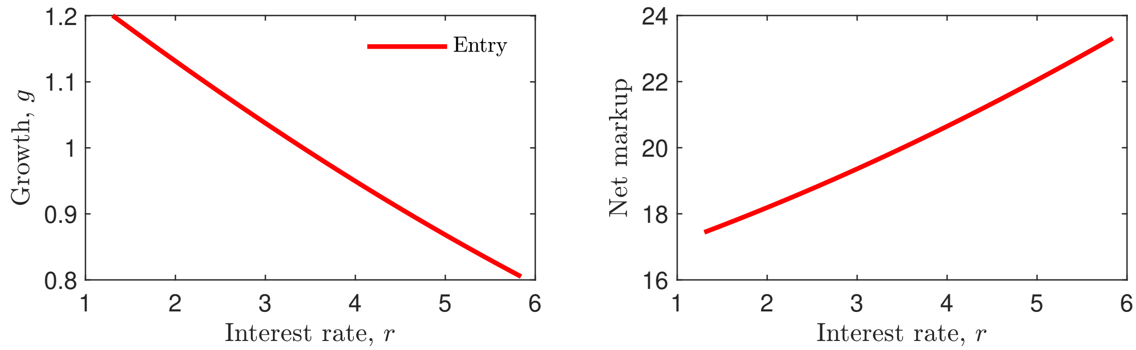


Figure IA.6: **Growth, average markup, and the interest rate, when the model is extended to include entry.**

in position $\sigma < l$ (that is, a laggard firm or a leader firm less than l steps ahead) has a probability of advancing to position $\hat{\sigma}$

$$F_{\sigma,\hat{\sigma}}(\phi_M, l) = \begin{cases} f_0 \sum_{\tilde{\sigma}=-\bar{s}}^{\sigma+1} \exp(\phi_M \tilde{\sigma}) & \text{if } \hat{\sigma} = \sigma + 1 \\ f_0 \exp(\phi_M \hat{\sigma}) & \text{if } \hat{\sigma} > \sigma + 1 \text{ and } \hat{\sigma} \leq l, \end{cases} \quad \text{(IA.22)}$$

and $F_{\sigma,\hat{\sigma}} = 0$ otherwise. The constant f_0 ensures that $\sum_{\hat{\sigma}\in\{-\bar{s},\ldots,\bar{s}\}} F_{\sigma,\hat{\sigma}} = 1, \forall\sigma$. We assume that leaders at or beyond position l advance (only) one step at a time. That is, for $\sigma \geq l$, $F_{\sigma,\sigma+1} = 1$. This setup provides enough flexibility to capture a range of assumptions about leader innovation, including the common assumption that leader innovation is incremental ($l = 0$).

Our specification of the advancement function F nests several approaches in the literature. The "quick catch-up" setup of Aghion et al. (2001), with laggards catching up with a single innovation but unable to leapfrog, corresponds to the case of $\phi_M = \infty$ and $l = 0$.[32] We nest two additional quick catch-up settings: leapfrogging as in Acemoglu and Akcigit (2012) is obtained under $\phi_M = \infty$ and $l = 1$; and "radical" innovation as in Acemoglu and Cao (2015) corresponds to $\phi_M = \infty$ and $l > 1$. With $\phi_M = -\infty$, we nest the "slow catch-up" assumption of Acemoglu and Akcigit (2012) and Liu et al. (2020), in which an innovating firm advances only one step at a time. With $\phi_M \in (-\infty, \infty)$, the step size is stochastic as in Akcigit and Kerr (2018). We accommodate a range of intermediate

[32]Formally, we define $F_{\sigma,\hat{\sigma}}(-\infty, l)$, with $F_{\sigma,\hat{\sigma}}(-\infty, l) = 1$ if and only if $\hat{\sigma} = \sigma + 1$. We similarly define $F_{\sigma,\hat{\sigma}}(\infty, l)$, with $F_{\sigma,\hat{\sigma}}(\infty, l) = 1$ if and only if $\hat{\sigma} = \sigma + l$. Note that $F_{\sigma,\hat{\sigma}}(\phi_M, l)$ converges uniformly to $F_{\sigma,\hat{\sigma}}(-\infty, l)$ as $\phi_M \to -\infty$ and similarly $F_{\sigma,\hat{\sigma}}(\phi_M, l)$ converges uniformly to $F_{\sigma,\hat{\sigma}}(\infty, l)$ as $\phi_M \to \infty$.

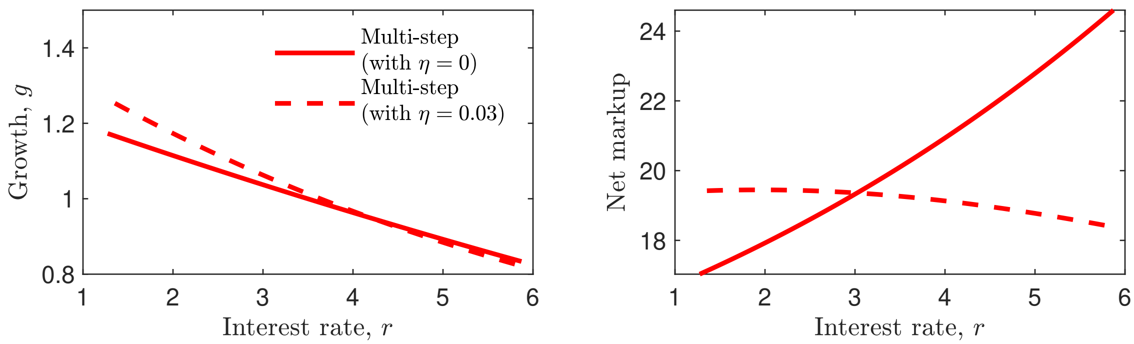


Figure IA.7: **Growth, average markup, and the interest rate, in the multi-step models.**

catch-up speeds, with $\phi_M = 0$ corresponding to a uniform probability of landing on each step between $\sigma + 2$ and l. We approximate the setup of Akcigit et al. (2018) with $\phi_M \in (0, \infty)$ and $l = \bar{s}$.

Table IA.VI reports the results of two exercises in this multi-step setting. We estimate the model conditional on full patent protection (first column) and conditional on a patent expiry rate of 3% (second column). Because the estimated values of ϕ_M are positive, the estimated model features a form of "advantage of backwardness": an innovating laggard in position σ is more likely to advance to $\hat{\sigma} + 1$ than to $\hat{\sigma}$, for $\hat{\sigma} > \sigma$. In the estimation with a 3% patent expiry rate, another aspect of the estimated model that fosters creative destruction is leapfrogging ($l = 2$). Overall, the model fit is fairly good, but not as good as with the simpler functional form assumption for laggard advancement in the main text (i.e., (IA.2) with $l = 0$, which is used in the main text estimations). Figure IA.7 shows that, in both of the multi-step estimated models, as the interest rate falls, growth rises monotonically and the average markup falls or is little changed.

D.6 Robustness and moment construction

This section reports robustness exercises related to how certain moments are calculated in the model and in the data. As described in the data appendix (Internet Appendix C), we calculate moments related to innovation output, profit volatility, and R&D to sales for firm-years with positive R&D. This restriction only affects the calculation of the targeted values in the data. (In the model, all firms have positive R&D because of the Inada-type condition that $G'(0) = 0$.) The first two columns of Table IA.VII report the

Moments	Multi-step (with $\eta = 0$)	Multi-step (with $\eta = 0.03$)	Target
Productivity Growth	1.03%	1.06%	1.03%
Markup			
Mean	19.40%	19.35%	19.40%
50th percentile	12.71%	13.77%	13.64%
90th percentile	36.40%	37.69%	42.62%
Innovation output			
Mean	3.63%	3.10%	6.75%
50th percentile	0.00%	0.00%	0.00%
90th percentile	12.33%	10.88%	19.49%
FHK within	85.76%	85.28%	90.77%
Profit volatility			
All firms	32.34%	33.36%	45.09%
Top profit quintile	17.07%	16.17%	21.89%
R&D to sales			
All firms	5.08%	5.33%	5.35%
Top profit quintile	2.28%	2.49%	2.52%
Parameters			
ϕ_M	0.604	0.050	
λ	1.054	1.038	
B	0.877	1.183	
η	0.000	0.030	
ζ	–	1.477	
l	0.000	2.000	

Table IA.VI: **Robustness: Alternative advancement functions.** This table reports results from estimating the model under an extension in which an innovating laggard in technology position σ has a positive probability of advancing to any position $\hat{\sigma} \in \{\sigma + 1, ..., l\}$. In this table, FHK within is the adjusted within moment, described in Section 4.1 of the main text.

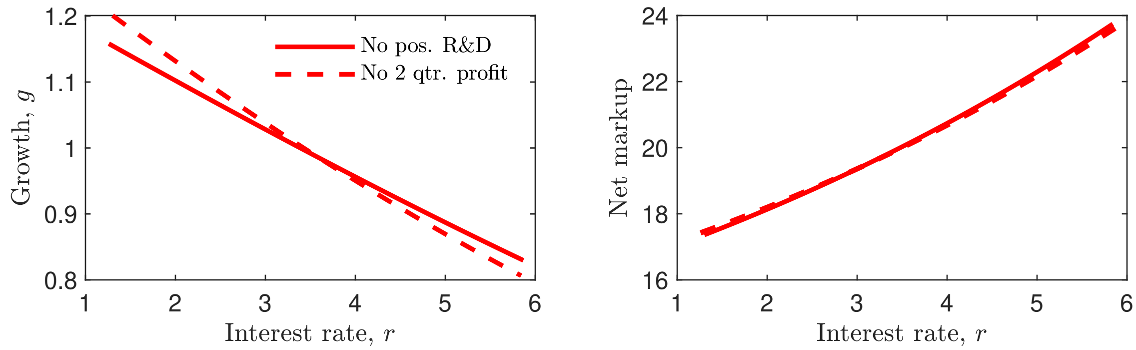


Figure IA.8: **Growth, average markup, and the interest rate, in robustness exercises related to moment construction.**

results of estimating the model when dropping the restriction to firm-years with positive R&D. Without the positive R&D restriction, the targeted innovation-output distribution shifts markedly inward, the targeted profit volatility profit volatility increases somewhat for high-profit firms, and, as expected, targeted R&D to sales declines. Re-estimating the model to fit these targets, the speed of catch-up rises to $\phi = 0.82$, the step size increases to $\ln \lambda = 6.3\%$, and R&D costs are shifted upward (i.e., the R&D cost parameter B falls, corresponding to higher R&D costs). The third and fourth columns of Table IA.VII present an exercise in which we calculate data target values when dropping the restriction, in the construction of model and data moments, that only firms with positive profits for two quarters in the "base year" are included when calculating profit volatility. This exercise increases only slightly the target values for the profit volatility moments, with target values for all other moments unchanged. The estimated parameters change little; unconditional profit volatility is somewhat too high relative to the data, because of outliers that have very high profit growth in the simulation due to having only one or two months of positive profits in the base year.

D.7 Robustness and financial frictions

Section 7 of the main text extends the model to include a limited commitment problem à la Aghion et al. (2019a). As described there, we set the limited commitment parameter $\alpha = 0.1$ and re-estimate the model. The results are shown in Table IA.VIII. (See Section 7 of the main text for a detailed analysis of how the limited commitment problem affects firms' innovation decisions and the distribution of technology gaps.) As shown in Figure

	No pos. R&D		No 2 qtr. profit	
Moments	Model	Target	Model	Target
Productivity Growth	1.02%	1.03%	1.03%	1.03%
Markup				
Mean	19.43%	19.40%	19.42%	19.40%
50th percentile	10.49%	13.64%	13.11%	13.64%
90th percentile	40.33%	42.62%	41.80%	42.62%
Innovation output				
Mean	3.40%	3.38%	5.89%	6.75%
50th percentile	0.00%	0.00%	0.00%	0.00%
90th percentile	7.94%	7.90%	18.18%	19.50%
FHK within	81.22%	90.77%	89.81%	90.77%
Profit volatility				
All firms	33.89%	50.21%	58.76%	48.57%
Top profit quintile	17.24%	25.48%	16.25%	20.28%
R&D to sales				
All firms	4.52%	3.58%	5.50%	5.35%
Top profit quintile	2.79%	1.89%	3.59%	2.52%
Parameters				
ϕ	0.817		0.284	
λ	1.063		1.018	
B	0.720		2.459	

Table IA.VII: **Robustness: Alternative approaches to calculating data moments.** This table reports the results of two exercises in which we alter choices made in calculating data moments and then re-estimate the model. In the first exercise, we calculate data target values when dropping the restriction to firm-year observations with positive R&D. This exercise, reported in the first two columns, thereby alters the target values for innovation output, profit volatility, and R&D to sales moments. In the second exercise, we calculate data target values when dropping the restriction, in the construction of model and data moments, that only firms with positive profits for two quarters in the "base year" are included when calculating profit volatility. This exercise, reported in the last two columns, alters only the profit volatility moments. In this table, FHK within is the adjusted within moment, described in Section 4.1 of the main text.

Moments	$\alpha = 0.1$	Targets
Productivity Growth	1.03%	1.03%
Markup		
Mean	19.40%	19.40%
50th percentile	12.85%	13.64%
90th percentile	42.97%	42.62%
Innovation output		
Mean	5.57%	6.75%
50th percentile	0.00%	0.00%
90th percentile	19.37%	19.49%
FHK within	90.04%	90.77%
Profit volatility		
All firms	41.17%	45.09%
Top profit quintile	16.18%	21.89%
R&D to sales		
All firms	5.97%	5.35%
Top profit quintile	3.84%	2.52%
Parameters		
ϕ	0.261	
λ	1.019	
B	2.315	

Table IA.VIII: **Robustness: Financial frictions.** This table reports the results from re-estimating the model with limited commitment parameter $\alpha = 0.1$. Section 7 extends the model to include limited commitment. In this table, FHK within is the adjusted within moment, described in Section 4.1 of the main text.

IA.9, in this re-estimated model, the growth-interest rate relation is negatively sloped. The the average markup is U-shaped with respect to the interest rate, with the average markup remaining within a range of 19 to 23% for a wide range of interest rates.

D.8 Robustness and time period used for moment targets

Recent work by Akcigit and Ates (2019) and Olmstead-Rumsey (2020) emphasizes that, since the 2000s, productivity growth and business dynamism have been especially low. We therefore undertake exercises in which we target moment values calculated for 2004 onward. Our first exercise has no firm entry, as in the benchmark model of the main text. Because the evidence of reduced business dynamism includes lower employment shares of young firms (Decker et al. (2014)), we conduct a second exercise where we estimate the model with firm entry.

Our target values are calculated identically to the targets in the main text, except that: we use a sample period of 2004–2019 for productivity growth (Fernald et al. (2017)) and R&D to sales; the markup distribution is for 2015, from Hall (2018); and the employment share by firm age is for 2014. The resulting target values are shown in Table IA.IX. The

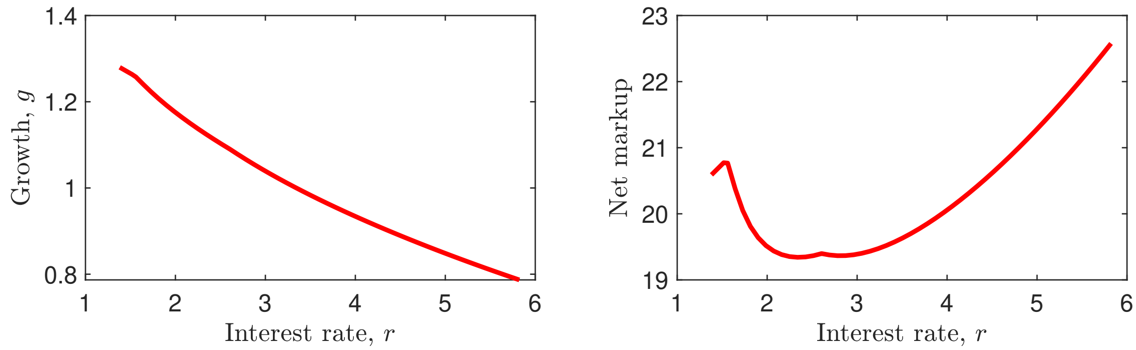


Figure IA.9: **Growth, average markup, and the interest rate, in a robustness exercise including a limited commitment constraint.**

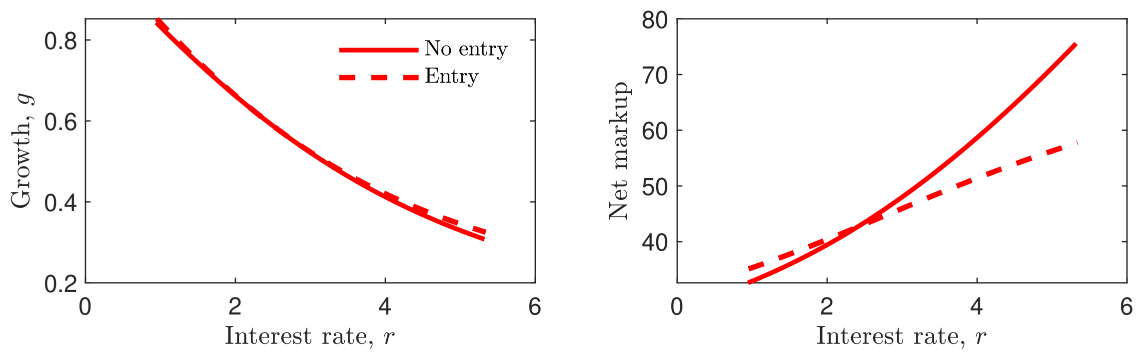


Figure IA.10: **Growth, average markup, and the interest rate, in robustness exercises using a different sample period.**

economy as depicted by these data moments have some elements of sclerosis: annual growth is only 0.56%, despite little change in R&D to sales, and the 90th percentile markup is 97%. The employment share of young firms is also lower than in the benchmark analysis.

The re-estimations point to lower R&D productivity for incumbents (i.e, lower B) and entrants (i.e, lower B_E in the model with entry), relative to the models estimated using a longer sample period (see Tables 1 and 4 of the main text). Laggard and entrant innovation are less likely to be radical (ϕ and ϕ_E decline). However, in each model, innovating laggards or entrants still have a meaningful chance of quickly catching-up. As a result, as the discount rate falls, productivity growth declines and the average markup rises, as shown in Figure IA.10.

Moments	No entry	Target	Entry	Target
Productivity Growth	0.58%	0.58%	0.58%	0.58%
Markup				
Mean	44.20%	44.20%	43.63%	44.20%
90th percentile	97.04%	83.77%	89.35%	83.77%
FHK entry	0.00%	–	22.66%	23.93%
R&D to sales				
Top profit quintile	2.63%	3.04%	2.62%	3.04%
Employ share, ≤ 10 years	–	–	14.50%	19.12%
Employ share, ≤ 5 years	–	–	8.19%	10.82%
Parameters				
ϕ	0.176		0.073	
λ	1.019		1.018	
B	1.352		1.385	
η	0.000		0.000	
ζ	–		–	
ϕ_E	–		0.167	
B_E	–		0.518	
l	0.000		0.000	
l_E	–		5.000	

Table IA.IX: **Robustness: Changing the sample period.** This table reports the results of exercises in which the sample period in the data is 2004–2019, a period with low productivity growth and high markups, relative to prior decades. In this table, for the model without entry (first two columns), FHK within is the "adjusted" within moment described in Section 4.1 of the main text; for the model with entry (last two columns), FHK within is the unadjusted within moment.

E. Innovation multiplier

E.1 Proof of Theorem 1

We first prove Theorem 1 under the assumptions of elastic labor supply and no entry. Theorem 1 in the main text is stated under these assumptions, and these assumptions hold in all exercises discussed in the main text except for some robustness exercises. However, as shown subsequently, Theorem 1 is readily extended to include inelastic labor supply and entry. Suppose that labor supply is perfectly elastic. As in the main text, denote $\Upsilon = (g, \{\mu_s\}_{s \in S^+}, \{x_\sigma\}_{\sigma \in S})$. Υ contains all the elements of the BGP, as defined in Internet Appendix A, except for ω and L. We exclude ω because $\omega = 1$ with perfectly elastic labor supply. We exclude L, for convenience, because L enters only one of the equations characterizing the BGP, specifically, the aggregate labor demand equation (10). Let H denote the stack of the equations characterizing Υ. (These equations include all equations characterizing the BGP described in Internet Appendix A.4, except for labor market clearing and $\omega = 1$). This stack is a set of $K = 3\bar{s}+3$ possibly non-linear equations, with the k-th equation of the form $h(\Upsilon; k) = 0$.

In a neighborhood of ρ, if H_Υ is invertible, the implicit function theorem implies

$$d\Upsilon = -H_\Upsilon^{-1} H_\rho d\rho. \tag{IA.23}$$

H_Υ is a $K \times K$ matrix with element (k_1, k_2) corresponding to $\frac{\partial h(\Upsilon;k_1)}{\partial \Upsilon_{k_2}}$, where Υ_{k_2} is the k_2-th element of Υ. In our quantitative analyses, we always find that H_Υ is invertible. H_ρ is a $K \times 1$ matrix. Element k of H_ρ is $\frac{\partial h(\Upsilon;k)}{\partial \rho}$.

Consider (IA.8), written as $G'^{-1}(\frac{1}{1-\tau_{R\&D}} \frac{\Delta v_\sigma}{\omega}) - x_\sigma = 0$ for $\sigma \in S$. The corresponding element of H_ρ is $\frac{\partial x_s}{\partial \rho}$. All other elements of H are 0, because ρ does not enter into other equations in H. Therefore, (13) holds, with $\mathbb{M} = -H_\Upsilon^{-1} M_1$ where M_1 is a matrix that maps $\{x_\sigma\}_{\sigma \in S}$ into Υ.

Extensions of Theorem 1. To extend Theorem 1 to apply to the case of inelastic labor supply, augment Υ to include ω and augment H to include (10). The remainder of the proof is unchanged, because the new element of H_ρ (corresponding to (10)) is equal to zero. To extend Theorem 1 to apply to models with entry, augment Υ to include $\{x_{E,s}\}_{s \in S}$ in the case of directed entry and x_E in the case of undirected entry. Similarly, augment H to include entrant first order conditions (of the form (IA.9) in the case of directed entry and (IA.10) in the case of undirected entry). The row of H_ρ corresponding to (IA.10), $G_E'^{-1}(\frac{\Delta v_E}{\omega}) - x_E = 0$, is $\frac{\partial x_E}{\partial \rho}$. The row of H_ρ corresponding to (IA.10), $G_E'^{-1}(\frac{\Delta v_{E,s}}{\omega}) - x_{E,s} = 0$, is $\frac{\partial x_E}{\partial \rho}$. Thus, the valuation-driven change in firm innovation becomes $\boldsymbol{\partial x} = [\partial x_{-\bar{s}} \ldots \partial x_{\bar{s}} x_{E,-\bar{s}} \ldots \partial x_{E,\bar{s}}]'$ in the case of directed entry, and $\boldsymbol{\partial x} = [\partial x_{-\bar{s}} \ldots \partial x_{\bar{s}} x_E]'$ in the case of undirected entry. Here, the valuation-driven change in the directed entrant's innovation is $\partial x_{E,\sigma} = \frac{\partial x_{E,\sigma}}{\partial \rho} d\rho$, where $\frac{\partial x_{E,\sigma}}{\partial \rho}$ is obtained from (IA.4). Similarly, the valuation-driven change in the undirected entrant's innovation is $\partial x_E = \frac{\partial x_E}{\partial \rho} d\rho$, where $\frac{\partial x_E}{\partial \rho}$ is obtained from (IA.5).

$$H_\Upsilon = \begin{bmatrix} 1 & -2\ln(\lambda)x_0 & -\ln(\lambda)x_1 & 0 & -2\ln(\lambda)\mu_0 & -\ln(\lambda)\mu_1 \\ 0 & 2x_0 & -(x_{-1}+\eta) & -\mu_1 & 2\mu_0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \left[\sum_{\hat{s}=-1}^{1} F_{-1,\hat{s}}\left(\frac{\partial v_{\hat{s}}}{\partial x^c_{-1}} - \frac{\partial v_{-1}}{\partial x^c_{-1}}\right)\right]-1 & \sum_{\hat{s}=-1}^{1} F_{-1,\hat{s}}\left(\frac{\partial v_{\hat{s}}}{\partial x^c_{0}} - \frac{\partial v_{-1}}{\partial x^c_{0}}\right) & \sum_{\hat{s}=-1}^{1} F_{-1,\hat{s}}\left(\frac{\partial v_{\hat{s}}}{\partial x^c_{1}} - \frac{\partial v_{-1}}{\partial x^c_{1}}\right) \\ 0 & 0 & 0 & \sum_{\hat{s}=-1}^{1} F_{0,\hat{s}}\left(\frac{\partial v_{\hat{s}}}{\partial x^c_{-1}} - \frac{\partial v_{0}}{\partial x^c_{-1}}\right) & \left[\sum_{\hat{s}=-1}^{1} F_{0,\hat{s}}\left(\frac{\partial v_{\hat{s}}}{\partial x^c_{0}} - \frac{\partial v_{0}}{\partial x^c_{0}}\right)\right]-1 & \sum_{\hat{s}=-1}^{1} F_{0,\hat{s}}\left(\frac{\partial v_{\hat{s}}}{\partial x^c_{1}} - \frac{\partial v_{0}}{\partial x^c_{1}}\right) \\ 0 & 0 & 0 & 0 & 0 & -1 \end{bmatrix} \quad \text{(IA.24)}$$

Obtaining the H_Υ matrix. To compute H_Υ, one needs values for $\{\frac{\partial v_\sigma}{\partial \rho}\}_{\sigma \in S}$ and $\{\frac{\partial v_\iota}{\partial x^c_\sigma}\}_{(\iota,\sigma)\in S\times S}$. In the case of no entry, the valuation terms $\{\frac{\partial v_s}{\partial \rho}\}_{s\in S}$ are obtained as the solution to the following system of $2\bar{s}+1$ equations, which are linear in model parameters and $\{v_\sigma\}_{\sigma\in S}$. For any $\sigma \in S$, taking the partial derivative of the value function (IA.8) with respect to ρ,

$$v_\sigma + \rho\frac{\partial v_\sigma}{\partial \rho} = x_\sigma \frac{\partial \Delta v_\sigma}{\partial \rho} + x^c_{-\sigma}\sum_{\hat{s}=-\bar{s}}^{\sigma-1}(F_{-\sigma,-\hat{\sigma}}\frac{\partial[v_{\hat{\sigma}} - v_\sigma]}{\partial \rho}) + \eta\sum_{\hat{\sigma}=0}^{\sigma-1}(F^p_{\sigma,\hat{\sigma}}\frac{\partial[v_{\hat{\sigma}} - v_\sigma]}{\partial \rho}). \quad \text{(IA.25)}$$

The strategic terms $\{\frac{\partial v_\iota}{\partial x^c_\sigma}\}_{(\iota,\sigma)\in S\times S}$ are the solution to

$$\rho\frac{\partial v_\iota}{\partial x^c_\sigma} = x_s\frac{\partial \Delta v_\iota}{\partial x^c_\sigma} + x^c_{-\iota}\sum_{\hat{\iota}=-\bar{s}}^{\iota-1}(F_{-\iota,-\hat{\iota}}\frac{\partial[v_{\hat{\iota}} - v_\iota]}{\partial x^c_\sigma}) + \eta\sum_{\hat{\iota}=0}^{\iota-1}(F^p_{\iota,\hat{\iota}}\frac{\partial[v_{\hat{\iota}} - v_\iota]}{\partial x^c_\sigma}) + \mathbb{1}_{-\iota=\sigma}(\sum_{\hat{\iota}=-\bar{s}}^{\iota-1}(F_{-\iota,-\hat{\iota}}v_{\hat{\iota}} - v_\iota)). \quad \text{(IA.26)}$$

E.2 Multiplier magnitudes

To interpret the magnitude of the elements of the growth multiplier (Figure 7), note that if laggards in competitive industries (with $s \leq 10$) increase their annual innovation rate by 5 percentage points in response to a lower discount rate when taking as given their competitors' strategy, then the effect on annual growth would be $0.05 \times \mathbb{M}_g \sum_{\sigma\in\{-10,..,-1\}} e'_\sigma = 0.0005$, or 5 basis points. (Recall, e_σ is a row vector of length $(2\bar{s}+1)$, with a 1 in position $\bar{s}+1+\sigma$.). In contrast, if laggard firms in uncompetitive industries (with $s > 10$) have a valuation-driven increase in innovation of 5 percentage points, then the effect on annual growth would be $0.05 \times \mathbb{M}_g \sum_{\sigma\in\{-10,..,-1\}} e'_\sigma$, equal to -5 basis points. Because laggard innovation has no direct effect on aggregate growth, these results show the powerful escape-competition and trickle-down effects in the model.

E.3 Unpacking the strategic interactions.

To shed light on the strategic channel, Figure IA.11, top panel, shows the multiplier $\mathbb{M}_{x_\iota}$ for a firm in position $\iota = 4$, or how the cross section of valuation-driven increases in innovation rates $\{x_\sigma\}_{\sigma \in S}$ translate into the GE effect on the innovation of a firm 4 steps ahead of its competitor. The spike at $\sigma = 4$ reflects that valuation-driven increases in the innovation rate of a firm 4 steps ahead directly contribute to the GE increase in this firm's innovation rate (as shown by the solid green line in the bottom panel). The large positive spike at $\sigma = -4$ reflects that valuation-driven increases in the innovation rate of a firm 4 steps spur its competitor to increase innovation to *escape competition* (as shown by the dotted blue line in the bottom panel).[33] Negative values for farther-behind laggards reflect, in contrast, a strategic *trickle-down* effect: A leader currently 4 steps ahead has less to gain from innovation, the greater the innovation rates of laggards 5 or more steps behind, with whom the leader would compete if the leader were to innovate.

F. Unpacking the valuation-driven change in firm innovation, ∂x

This section unpacks—qualitatively and quantitatively—the determinants of the valuation-driven change in firm innovation, $\boldsymbol{\partial x} = [\partial x_{-\bar{s}} \ldots \partial x_{\bar{s}}]'$, the effect of a lower discount rate on the cross section of firm innovation when each firm holds its competitor's strategy constant.[34] The valuation-driven change in firm innovation captures the "standard" channel by which a lower discount rate affects innovation by reducing the required return on R&D.

Using the first order condition (6) from the main text, the valuation-driven change in the innovation rate of a firm in position σ due to a discount rate change $d\rho$ is, to first

[33] Specifically, the strategic component of $\mathbb{M}_{x_\iota}$ is defined as $\frac{\partial x_\iota}{\partial x^c_\sigma}$, which isolates the strategic channel through which a firm in position ι responds to its competitor's innovation rate when in position σ. The strategic term $\frac{\partial x_\iota}{\partial x^c_\sigma}$ is obtained using implicit differentiation of the system of equations (4)–(5), taking the discount rate ρ as given.

[34] Specifically, $\partial x_\sigma \equiv \frac{\partial x_\sigma}{\partial \rho} d\rho$, where $d\rho$ is a change in the discount factor and $\frac{\partial x_\sigma}{\partial \rho}$ is obtained using the implicit function theorem for the system of equations (4)–(5), as discussed in detail in Internet Appendix Section E

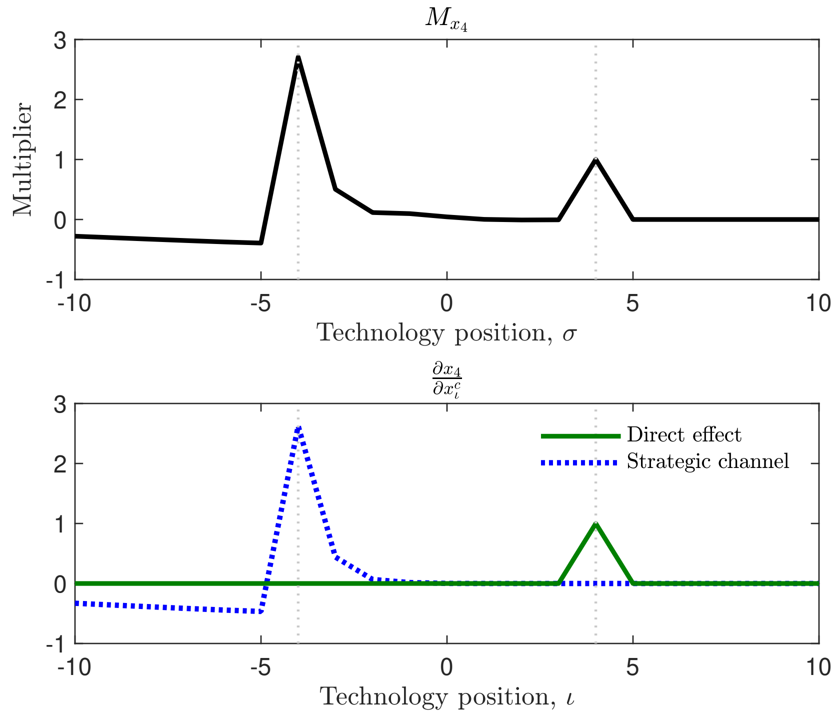


Figure IA.11: **Strategic interactions.** The top panel shows the multiplier $\mathbb{M}_{x_4}$ mapping the cross section of valuation-driven increases in innovation $\boldsymbol{\partial x}$ into the GE effect on the innovation rate of leader firms 4 steps ahead of their competitors. The solid green line in the bottom panel shows the direct effect: valuation-driven increases in innovation for firms 4 steps ahead contribute directly to the GE change in innovation for these firms. The dotted blue line shows the strategic component of $\mathbb{M}_{x_4}$, as described in footnote 33.

order,

$$\partial x_\sigma = \frac{\partial x_\sigma}{\partial \rho} d\rho = (-c_\sigma \times \mathscr{D}_\sigma \times \Delta v_\sigma) d\rho, \tag{IA.27}$$

where $c_\sigma = ((1 - \tau_{R\&D}) G''(x_\sigma)\omega)^{-1} > 0$ is a curvature term capturing how the marginal cost of innovation changes with the innovation rate, $\mathscr{D}_\sigma \equiv -\frac{\partial \Delta v_\sigma}{\partial \rho} \frac{1}{\Delta v_\sigma}$ is the duration of profits from innovation, and Δv_σ is the capital gain from innovation, described in (IA.7).

The *curvature term* is positive because R&D costs are convex semi-elasticity of the value of an asset with respect to the discount rate. This term is increasing in the innovation rate, x_σ if and only if $\gamma \leq 0.5$. In the benchmark model, $\gamma = 0.5$, implying that the curvature term is invariant across σ.

The *duration of expected profits from innovation*, $\mathscr{D}_\sigma$, is connected to, but distinct from, the widely cited metric of the *duration of firm profits*, or, $\mathbb{D}_\sigma \equiv -\frac{\partial v_\sigma}{\partial \rho} \frac{1}{v_\sigma}$. We turn now to the mapping between these two definitions of duration.

Non-recursive value function and the duration of profits from innovation. Using the recursive Hamilton–Jacobi–Bellman equations (4)–(**??**), the non-recursive value function along a BGP equilibrium is:

$$v_{s(t)}(t) = \int_t^{\infty} e^{-\rho(z-t)} \mathbb{E}_{s(t),t} \Pi^N_{\varsigma(z)} dz, \tag{IA.28}$$

where $\mathbb{E}_{s(t),t}$ is the expectation operator over a firm's position $\varsigma(z)$ at time $z > t$ conditional on having position $s(t)$ at time t. Net operating profits conditional on having position $\varsigma(z)$ are given by $\Pi^N_{\varsigma(z)}$.[35] The capital gain from innovation can therefore be written non-recursively as

$$\Delta v_{s(t)} = \int_t^{\infty} e^{-\rho(z-t)} \underbrace{\left(\sum_{\hat{s}=s+1}^{\bar{s}} F_{s(t),\hat{s}} \mathbb{E}_{\hat{s},t} - \mathbb{E}_{s(t),t} \right) \Pi^N_{\varsigma(z)}}_{\text{Expected profit at time } z > t \text{ from innovation}} dz. \tag{IA.29}$$

Using expressions (IA.28) and (IA.29), we obtain the following relation:

$$\mathscr{D}_\sigma = \frac{1}{\Delta v_\sigma} \left(\sum_{\hat{s}=s+1}^{\bar{s}} F_{\sigma,\hat{s}} v_{\hat{s}} \mathbb{D}_{\hat{s}} - v_\sigma \mathbb{D}_\sigma \right). \tag{IA.30}$$

Duration a concept from asset pricing is the (negative) semi-elasticity of the value of an asset with respect to the discount rate: the percent change in the asset's value from a marginal decline in the discount rate. (a formal definition of the expected profits from innovation is provided at the end of this section of the Internet Appendix.) Duration is measured in time units and captures the amount of time that elapses before an asset holder receives the asset's cash flows.

Discussion. Holding constant the competitor's innovation rate, the sign and magnitude of the effect of a lower discount rate on an individual firm's R&D depends on the *time pattern* of the expected profits from innovation, as shown by expressions (IA.28) and (IA.29). With respect to the sign, if the expected profits from innovation are weakly positive at all horizons $z > t$, then their duration must be positive. That is, holding factors external to the firm constant, a lower discount rate is associated with higher R&D. However, if the expected profits from innovation are negative at some future dates $z > t$, then the discount-rate valuation effect can in principle be negative even as the capital

[35] Define $\Pi^N_{\varsigma(z)} : (S \cup \varnothing) \to R$ as the net operating profit of a firm in position $\varsigma(z)$, with $\Pi^N_{\varsigma(z)} = (1 - \tau)\mathscr{L}_{\varsigma(z)} \mathbb{1}_{\varsigma(z)>0} - (1 - \tau_{R\&D}) G(x_{\varsigma(z)})$ for $\varsigma(z) \in S$ and $\Pi^N_{\varsigma(z)} = 0$ if $\varsigma(z) = \varnothing$, with $\varnothing$ connoting that a firm has been displaced by entry prior to time z.

gain from innovation is positive. This outcome can obtain when an innovation leads to positive profits shortly after the innovation, followed by losses later. With a negative duration of profits from innovation, an individual firm facing a lower discount rate would decrease R&D.

Valuation effect on firm innovation, ∂x, in the benchmark model. In the estimated model, holding constant the competitor's strategy, a decline in the discount rate implies a rise in innovation for all firms, in every technology position (grey dashed line in the top-left panel of Figure IA.12). The sign of this valuation-driven effect on firm innovation, in principle, could be negative if an innovation from a certain technology position leads to back-loaded losses through increased R&D expenditures. In the estimated model, for some positions σ, an innovation does lead to losses, due to an endogenous increase in R&D and therefore R&D costs, but these losses are front-loaded, occurring immediately after an innovation. Thus, the uniformly positive valuation-driven effect of a lower discount rate on innovation is therefore a result of the model calibration. A 100 basis point decline in the discount rate leads to valuation-driven increase in the innovation rate of far-behind laggards of 11 percentage points per year; for tied firms, the valuation-driven increase is almost 30 percentage points per year, while for far-ahead leaders ($\sigma \in \{20, ..., 30\}$), the increase is 5 percentage points.

The large magnitude of valuation-driven increases in innovation in tied industries reflects two forces. First, *tied firms' capital gain from innovation* is high: An innovating tied firm starts to earn operating profits and, as its lead increases further, reduces its R&D expenditures (bottom left panel of Figure IA.12). Second, *tied firms' duration of profits from innovation* is higher than leaders' because tied firms' profits from innovation are relatively backloaded. After innovating, a tied firm initially incurs high R&D expenses, to try to build a lead sufficient to discourage its competitor. Thus, for tied firms, the expected profits from innovation accrue steadily over a long period (bottom right panel of Figure IA.12, dashed line). In contrast, an innovating leader even a couple of steps ahead immediately sees an increase in net profits, but this increase is small, reflecting the concavity of leaders' operating profits in their technology advantage and a small decline in R&D expenses. These leaders' advantage is eroded over time by laggard innovations

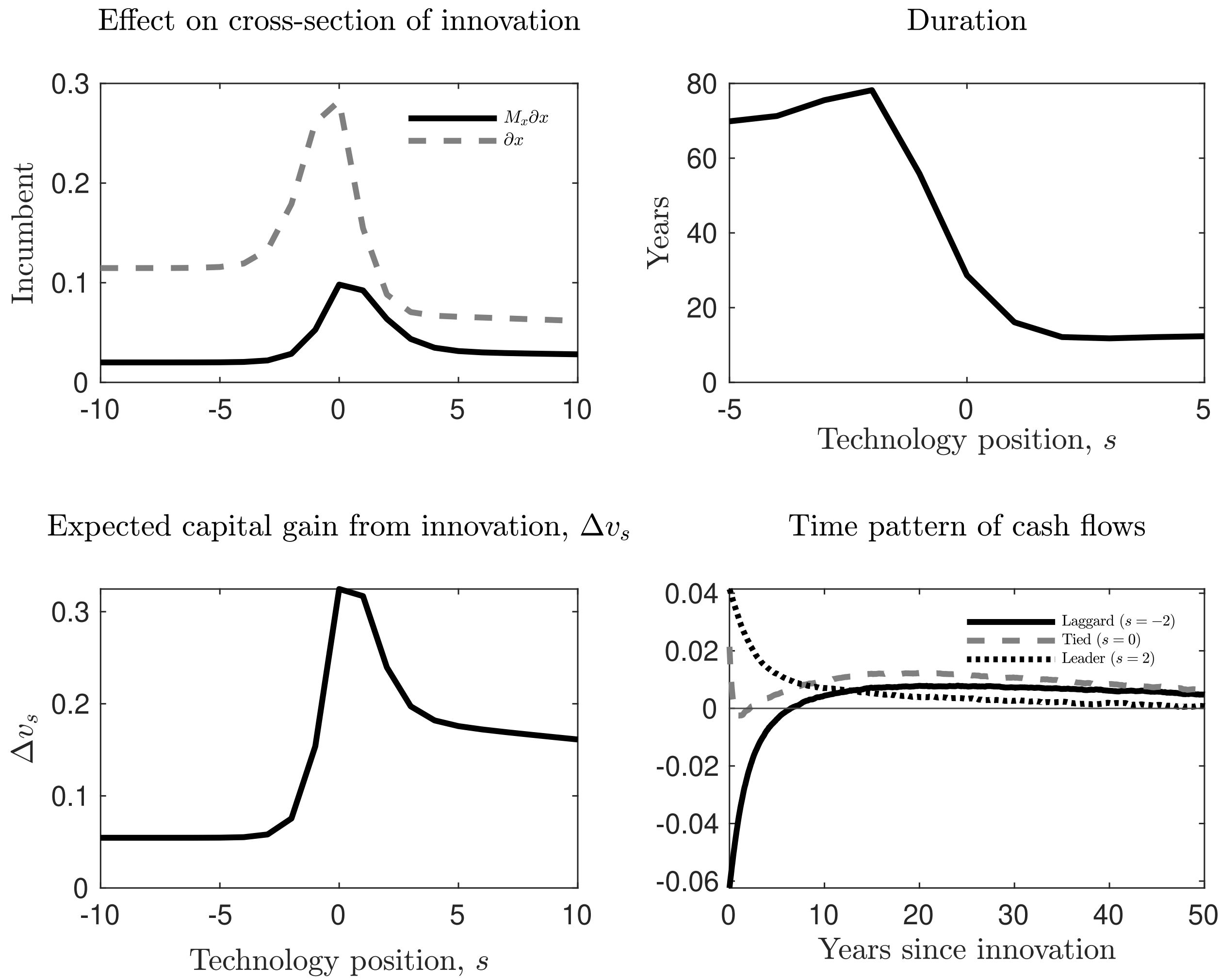


Figure IA.12: **Effects on firm innovation of a lower discount rate.** The grey dashed line in the top left panel shows the effect of a 100 bps decline in the discount rate, holding all other factors constant (∂x). The solid line shows the general equilibrium effect ($dx = M_x \partial x$). The capital gain from innovation (bottom left) and the duration of profits from innovation (top right) are key determinants of ∂x (equation (IA.27)). The bottom right panel shows the time pattern of the expected profits from innovation.

and patent expiries, making these leaders' profits from innovation relatively front-loaded (dotted line). Compared with nearly tied leaders (in competitive sectors), further-ahead leaders (in less competitive sectors) therefore have a lower capital gain from innovation and a shorter duration of expected profits from innovation. Thus, the valuation-driven increase in leaders' innovation declines progressively with leaders' technology advantage.

Laggards have much lower capital gains from innovation than tied firms but the duration of their profits from innovation is very high. This high duration reflects that, for laggards, an innovation is associated with immediately higher R&D expenditures and no change in operating profits. Thus, for laggards, an innovation on impact generates expected losses followed by an increase in expected profits further in the future (bottom right panel of Figure IA.12, solid line). As laggards fall behind from the tied state, their capital gain from innovation initially declines quickly (faster than the duration of profits from innovation rises). Thus, laggard innovation declines as an industry becomes less competitive. However, because of the possibility of quick catch-up, the expected capital gain remains strictly positive for all laggards, and consequently laggards do not become completely discouraged even if far behind.

G. Additional results

Mapping from parameters to moments. Figure 3 of the main text shows how three targeted moments vary with the estimated parameters (ϕ, λ, B). For completeness, Figure IA.13 presents how the remaining targeted moments vary with these parameters.

Profit share. Section 4.3 states that the profit share falls as the interest rate declines, as shown in Figure IA.14.

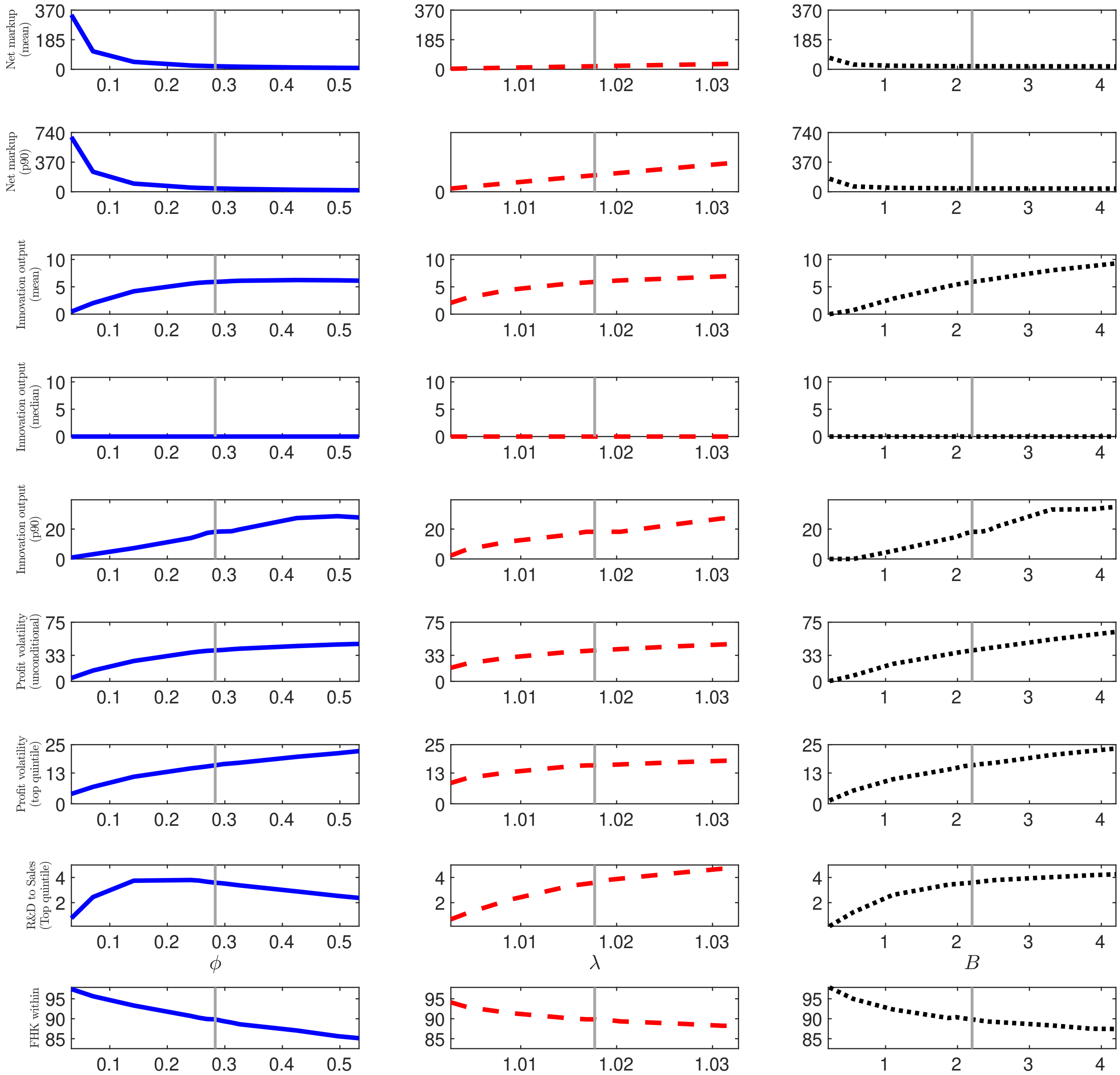


Figure IA.13: **Mapping from parameters to moments.** The figure shows how selected moments vary with catch-up speed ϕ (left panels), innovation step size λ (middle panels), and R&D cost scaling parameter B (right panels), holding other parameters constant. All moments are shown in percent.

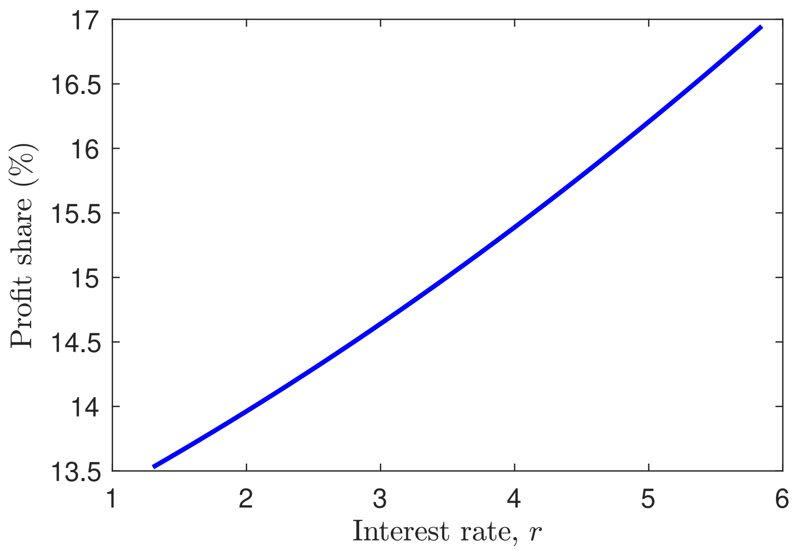


Figure IA.14: **Profit share and the interest rate, in the benchmark economy.**

Internet Appendix References

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