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On the Transmission of Small and Large Shocks

Abstract

We analyze how small and large demand and supply shocks are transmitted in the economy. We use a behavioural macroeconomic model that is characterized by the fact that individuals lack the cognitive ability to understand the underlying model and to know the distribution of the shocks that hit the economy. We find, first, that when shocks are small the trajectory taken after the shock by output gap and inflation is unpredictable. In this case the signal provided by the shock is overwhelmed by the noise produced by the initial disequilbria at the moment of the shock. Second, when the shock is large (more than 5 standard deviations like in the case of the covid-shock) the subsequent trajectories taken by output gap and inflation typically coalesce around a good and a bad trajectory. The way this result comes about is that different initial conditions force the monetary authorities into making different choices about the interest rate. Sometimes these choices are bad so that the economy is forced into a bad trajectory and sometimes they are good pushing the economy into a benign trajectory. We also find that when the shocks are large the initial conditions in particular expectations have strong power in predicting which trajectory will be chosen.

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ON THE TRANSMISSION OF SMALL AND LARGE SHOCKS[1]

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Abstract
We analyze how small and large demand and supply shocks are transmitted in the economy. We use a behavioural macroeconomic model that is characterized by the fact that individuals lack the cognitive ability to understand the underlying model and to know the distribution of the shocks that hit the economy. We find, first, that when shocks are small the trajectory taken after the shock by output gap and inflation is unpredictable. In this case the signal provided by the shock is overwhelmed by the noise produced by the initial disequilbria at the moment of the shock. Second, when the shock is large (more than 5 standard deviations like in the case of the covid-shock) the subsequent trajectories taken by output gap and inflation typically coalesce around a good and a bad trajectory. The way this result comes about is that different initial conditions force the monetary authorities into making different choices about the interest rate. Sometimes these choices are bad so that the economy is forced into a bad trajectory and sometimes they are good pushing the economy into a benign trajectory. We also find that when the shocks are large the initial conditions in particular expectations have strong power in predicting which trajectory will be chosen.

[1] We are grateful to Jean-Philippe Bouchaud for his very useful comments and suggestions.

1. Introduction

During the last 50 years most advanced countries have experienced three large shocks. The first one occurred in the early 1970s when oil prices surged. This supply side shock led for many years to slow economic growth and high inflation termed as 'stagflation'. The second one occurred during 2008-09 when the world was hit by a financial crisis that led to large drops in aggregate demand, output and employment. An even largeger shock occurred in 2020 when the Covid-pandemic hit the world on both demand and supply sides. This shock led to pronounced declines in output and employment. The size of these shocks was sizable as they exceeded by far three standard deviations from the mean typically observed historically. For example, the Covid-shock led to declines in GDP in the first quarter of 2020 of 10% or more in many countries. This is equivalent to 10 standard deviations from the mean quarterly variation of GDP observed in normal times. If these shocks were normally distributed, we should not observe them more than once in a million years. The shock of the Great Recession in 2008-09 was not as intense but surely exceeded three standard deviations implying that they should not be observed anymore for a couple of centuries.

Traditionally, standard macroeconomic DSGE-models have analyzed the transmission of shocks using impulse responses and assuming a one standard deviation of the shock. This has typically been done after linearizing around the equilibrium an underlying non-linear model. The question has usually not been asked how large shocks affect the economy and whether these large shocks display fundamentally different transmission mechanisms than small ones. This is the question we want to analyze in this paper.

In order to do so we will use a behavioural macroeconomic model (see De Grauwe(2012, and De Grauwe and Ji(2019)). This is a model which assumes that agents have cognitive limitations. They do not know the underlying structure of the model nor do they know the distribution of the shocks that affect the economy. It seems to us that this is the appropriate assumption to make. In a world where occasionally but unpredictably, large shocks by far exceeding three standard deviations ("black swans") occur, agents find it difficult if not impossible to understand the distribution of these shocks. This uncertainty has also been called "Knightian uncertainty".

Behavioural macroeconomic models generate an endogenous dynamics of booms and busts in economic activity. This dynamics is driven by self-fulfilling movements of optimism and pessimism (animal spirits). The fundamental reason of the emergence of such a dynamics is the fact that individuals have cognitive limitations preventing them from having rational expectations, i.e. preventing them from understanding the complexity of the underlying model. This lack of understanding provides the basis of a mechanism in which individuals find it rational to use simple rules of behaviour, check ex post how well these rules have worked and are willing to experiment with other rules when they observe that these work better. It also turns out that the shifting in the rules of behaviour at the individual level generates a collective process of herding based on the fact that successful rules will be copied by others. It is this collective process that is at the core of the waves of optimism and pessimism driving the business cycle movements.

In this paper we will analyze how demand and supply shocks are propagated. It will be shown that there is a fundamental uncertainty about this propagation. This is not the uncertainty arising from the fact that we (economists and policymakers) do not know the true value of the parameters of the model, but because the endogenous dynamics of booms and busts creates a veil of uncertainty. We will show that as a result, and even if we knew the parameters of the model with certainty, the transmission process of exogenous shocks is very unpredictable.

We will focus on the size of these shocks. It will be shown, first, that when the shocks are small (one standard deviation) the noise surrounding the initial conditions overwhelms the signal provided by the shock. As a result, the transmission of such small shocks is essentially unpredictable. Second, it will be shown that as the size of the shocks increases the transmission path after the shock will tend to coalesce around two possible trajectories, a good one and a bad one, as if there are two attractors around which the transmission dynamics is organized. This feature will allow us to focus on the importance of initial conditions in guiding the economy towards the good or the bad trajectories. This will also make it possible to show that initial conditions can be good predictors of subsequent trajectories.

The rest of the paper is structured as follows. Section 2 presents the behavioural macroeconomic model. We will develop the essence of that model and put some more technical material in appendix. Section 3 presents the impulse responses of small and large demand and supply shocks. Section 4 analyzes the power of initial conditions in predicting the subsequent trajectories of output gap, inflation and interest rate. In section 5 we perform an econometric analysis on the predictive power of initial conditions. Section 6 performs a sensitivity analysis allowing us to trace the transition from small to large shocks. Section 7 concludes and provides policy implications.

2. The model

2.1 The basic behavioral macroeconomic model consists of an aggregate demand equation, an aggregate supply equation and a Taylor rule as described by De Grauwe (2011) and De Grauwe and Ji(2019).

The aggregate demand equation can be expressed in the following way:

$$y_t = a_1 \widetilde{\mathrm{E}}_t y_{t+1} + (1 - a_1) y_{t-1} + a_2 \left(r_t - \widetilde{\mathrm{E}}_t \pi_{t+1} \right) + v_t \qquad (1)$$

where y_t is the output gap in period t, r_t is the nominal interest rate, π_t is the rate of inflation and two forward looking components , $\widetilde{\mathrm{E}}_t \pi_{t+1}$ and $\widetilde{\mathrm{E}}_t y_{t+1}$. The tilde above *E* refers to the fact that expectations are not formed rationally. How exactly these expectations are formed will be specified subsequently.

The aggregate supply equation is represented in (2). This New Keynesian Philips curve includes a forward looking component, $\widetilde{\mathrm{E}}_t \pi_{t+1}$, and a lagged inflation variable. Inflation π_t is sensitive to the output gap y_t. The parameter b_2 measures the extent to which inflation adjusts to changes in the output gap.

$$\pi_t = b_1 \widetilde{\mathrm{E}}_t \pi_{t+1} + (1 - b_1) \pi_{t-1} + b_2 y_t + \eta_t \qquad (2)$$

The aggregate demand and supply equations in (1) and (2) can be derived from expected utility maximization of consumers and expected profit maximization of firms (Hommes and Lustenhouwer(2019) and De Grauwe and Ji(2020)). See Appendix 1 where we provide for a microfoundation.

The Taylor rule describes the central bank's behaviour in setting the interest rate. This behavior can be described as follows:

$$r_t = (1 - c_3)[c_1(\pi_t - \pi^*) + c_2 y_t] + c_3 r_{t-1} + u_t \tag{3}$$

where r_t is the interest rate in period t, π_t is the inflation rate, π^* is the target rate of inflation and y_t is the output gap.

This Taylor rule tells us that the central bank increases (reduces) the interest rate when currently observed inflation exceeds (falls short of) the target and when the currently observed output gap is positive (negative). We assume that the central bank wants to smoothen interest rate changes (see Levin et al. (1999) and Woodford (1999, 2003)). This is shown by including a lagged interest rate. When no smoothing occurs c_3 = 0 we obtain the original Taylor rule. Note also that we set the natural rate of interest equal to zero.

We have also added error terms in each of the equations (1) to (3a&b). These describe the nature of the different shocks that can hit the economy. There are demand shocks, v_t, supply shocks, η_t and interest rate shocks, u_t. It is assumed that these shocks are normally distributed with mean zero and a constant standard deviation.

2.2 Expectations formation

In this section we analyze how the forecast of output gap $\widetilde{\mathrm{E}}_t y_{t+1}$ and inflation $\widetilde{\mathrm{E}}_t \pi_{t+1}$ are formed in the model. The rational expectations hypothesis requires agents to understand the complexities of the underlying model and to know the frequency distributions of the shocks that will hit the economy. We take it that agents have cognitive limitations that prevent them from understanding and processing this kind of information. These cognitive limitations have been confirmed by laboratory experiments and survey data (see Carroll, 2003; Branch, 2004; Pfajfar, D. and B. Zakelj, (2011 &2014); Hommes, 2011).

Forecasting output gap

We assume two types of rules agents follow to forecast the output gap. A first rule is called a "fundamentalist" one. Agents estimate the steady state value of the output gap (which is normalized at 0) and use this to forecast the future output gap. A second forecasting rule is a "naïve" one. This is a rule that does not presuppose that agents know the steady state output gap. They are agnostic about it. Instead, they extrapolate the previous observed output gap into the future. There is ample evidence from laboratory experiments that support these assumptions that agents use simple heuristics to forecast output gap and inflation. See Pfajfar and Zakelj, (2011 &2014), Kryvtsov and Petersen (2013) and also Assenza et al.(2014a) for a literature survey. The fundamentalist and extrapolator rules for output gap are specified as follows:

$$\widetilde{\mathrm{E}}_{\mathrm{t}}^{\mathrm{f}} \mathrm{y}_{\mathrm{t}+1} = 0 \tag{4}$$

$$\widetilde{\mathrm{E}}_{\mathrm{t}}^{\mathrm{e}} y_{\mathrm{t}+1} = y_{t-1} \tag{5}$$

This kind of simple heuristic has often been used in the behavioral macroeconomics and finance literature where agents are assumed to use fundamentalist and chartist rules (see Brock and Hommes(1997), Branch and Evans(2006), De Grauwe and Grimaldi(2006), Brazier et al. (2008)). It is probably the simplest possible assumption one can make about how agents who experience cognitive limitations, use rules that embody limited knowledge to guide their behavior. They only require agents to use information they understand, and do not require them to understand the whole picture. In De Grauwe (2012) more complex rules are used, e.g. it is assumed that agents do not know the steady state output gap with certainty and only have biased estimates of it. This is also the approach taken by Hommes and Lustenhouwer (2019).

The market forecast can be obtained as a weighted average of these two forecasts, i.e.

$$\widetilde{\mathrm{E}}_{\mathrm{t}} y_{t+1} = \alpha_{f,t} \widetilde{\mathrm{E}}_{\mathrm{t}}^{\mathrm{f}} \mathrm{y}_{\mathrm{t}+1} + \alpha_{e,t} \widetilde{\mathrm{E}}_{\mathrm{t}}^{\mathrm{e}} \mathrm{y}_{\mathrm{t}+1} \tag{6}$$

$$\alpha_{f,t} + \alpha_{e,t} = 1 \tag{7}$$

where $\alpha_{f,t}$ and $\alpha_{e,t}$ are the probabilities that agents use the fundamentalist and the naïve rule respectively.

As indicated earlier, agents in our model are willing to learn, i.e. they continuously evaluate their forecast performance. We specify a switching mechanism of how agents adopt specific rule. As shown in Appendix 2, we follow the discrete choice theory (see Anderson, de Palma, and Thisse, (1992) and Brock & Hommes (1997)) to work out the probability of choosing a particular rule. We obtain:

$$\alpha_{f,t} = \frac{exp\left(\gamma U_{f,t}\right)}{exp\left(\gamma U_{f,t}\right) + exp(\gamma U_{e,t})} \tag{8}$$

$$\alpha_{e,t} = \frac{exp(\gamma U_{e,t})}{exp\left(\gamma U_{f,t}\right) + exp(\gamma U_{e,t})} \tag{9}$$

where $U_{f,t}$ and $U_{e,t}$ the past forecast performance (utility) of using the fundamentalist and the naïve rules. The parameter γ measures the "intensity of choice". It can also be interpreted as expressing a willingness to learn from past performance. When $\gamma = 0$ this willingness is zero; it increases with the size of γ.

The forecast performance affects the probability of using a particular rule. For example, as shown in Equation (8), as the past forecast performance (utility) of the fundamentalist rule improves relative to that of the naïve rule, agents are more likely to select the fundamentalist rule for their forecasts of the output gap.

Agents also have to forecast inflation. Similar heuristics rules as in the case of output forecasting are described in Appendix 3.

Forecasting inflation

Agents also forecast inflation. A similar simple heuristics is used as in the case of output gap forecasting, with one rule that could be called a fundamentalist rule and the other a naïve rule. (See Brazier et al. (2008) for a similar setup). We assume an institutional set-up in which the central bank announces an explicit inflation target. The fundamentalist rule then is based on this announced inflation target, i.e. agents using this rule have confidence in the credibility of this rule and use it to forecast inflation. Agents who do not trust the announced inflation target use the naïve rule, which consists in extrapolating inflation from the past into the future.

The fundamentalist rule will be called an "inflation targeting" rule. It consists in using the central bank's inflation target to forecast future inflation, i.e.

$$\widetilde{\mathrm{E}}_t^f \pi_{t+1} = \pi^* \tag{10}$$

where the inflation target is π^*. The "naive" rule is defined by

$$\widetilde{\mathrm{E}}_t^e \pi_{t+1} = \pi_{t-1} \tag{11}$$

The market forecast is a weighted average of these two forecasts, i.e.

$$\widetilde{\mathrm{E}}_t \pi_{t+1} = \beta_{f,t} \widetilde{\mathrm{E}}_t^f \pi_{t+1} + \beta_{e,t} \widetilde{\mathrm{E}}_t^e \pi_{t+1} \tag{12}$$

$$\beta_{f,t} + \beta_{e,t} = 1 \tag{13}$$

Where $\beta_{f,t}$ and $\beta_{e,t}$ are the probabilities that agents use the fundamentalist and the naïve rule respectively. The same selection mechanism is used as in the case of output forecasting to determine the probabilities of agents trusting the inflation target and those who do not trust it and revert to extrapolation of past inflation. This inflation forecasting heuristics can be interpreted as a procedure of agents to find out how credible the central bank's inflation targeting is. If this is very credible, using the announced inflation target will produce good forecasts and as a result, the probability that agents will rely on the inflation target, $\beta_{f,t}$, will be high. If on the other hand the inflation target does not produce good forecasts (compared to a simple extrapolation rule) the probability that agents will use it will be small. Use the switching mechanism similar to the one specified equations (8) and (9), we can compute the probability of choosing a particular rule.

$$\beta_{f,t} = \frac{exp\left(\gamma U'_{f,t}\right)}{exp\left(\gamma U'_{f,t}\right)+exp\left(\gamma U'_{e,t}\right)} \qquad (14)^2$$

$$\beta_{e,t} = \frac{exp\left(\gamma U'_{e,t}\right)}{exp\left(\gamma U'_{f,t}\right)+exp\left(\gamma U'_{e,t}\right)} \qquad (15)$$

2.3. Calibration

The procedure to solve the model is shown in Appendix 3. As our model has strong non-linear features we use numerical methods to analyze the dynamics created by the model. In order to do so, we have to calibrate the model, i.e. to select numerical values for the parameters of the model. In Table 2 we show these numerical values with the references from the literature. The model was calibrated in such a way that the time units can be considered to be quarters. The three shocks (demand shocks, supply shocks and interest rate shocks) are independently and identically distributed (i.i.d.) with standard deviations of 0.5%. These shocks produce standard deviations of the output gap and inflation that mimic the standard deviations found in the empirical data using quarterly observations for the US and the Eurozone. The way we did this is be described in more detail in De Grauwe and Ji(2020). It should also be mentioned that the parameter values in Table 1 ensure local stability of the steady state. Finally, to simplify our analysis, we remove the structural inertia components in the demand and the

[2] Note $U'_{f,t} = -\sum_{k=0}^{\infty} \omega_k \left[\pi_{t-k-1} - \widetilde{E}_{f,t-k-2}\pi_{t-k-1}\right]^2$ and $U'_{e,t} = -\sum_{k=0}^{\infty} \omega_k \left[\pi_{t-k-1} - \widetilde{E}_{e,t-k-2}\pi_{t-k-1}\right]^2$

supply equations. Hence, we set a1=1 and b1=1. It turns out that our results are not fundamentally affected by this assumption.

Table 2: Parameter values of the calibrated model

a1 = 0.5	coefficient of expected output in output equation (Smets and Wouters(2003))
a2 = -0.2	interest elasticity of output demand (McCallum and Nelson (1999)).
b1 = 0.5	coefficient of expected inflation in inflation equation (Smets and Wouters (2003))
b2 = 0.05	coefficient of output in inflation equation,
π^*=0	inflation target level
c1 = 1.5	coefficient of inflation in Taylor equation (Blattner and Margaritov(2010))
c2 = 0.5	coefficient of output in Taylor equation assuming a dual Mandate Central Bank (Blattner and Margaritov(2010))
c3 = 0.5	interest smoothing parameter in Taylor equation (Blattner and Margaritov(2010))
γ = 2	intensity of choice parameter, see Kukacka, Jang and Sacht (2018)
σ_v = 0.5	standard deviation shocks output
σ_η = 0.5	standard deviation shocks inflation
σ_u = 0.5	standard deviation shocks Taylor
ρ = 0.5	memory parameter (see footnote 3)

3. The results of the model

In this section we present impulse responses of demand and supply shocks. One important feature of impulse responses in a (non-linear) behavioural model is that these responses are sensitive to initial conditions. Thus, the transmission of, say, a demand shock will be influenced by the values of output, inflation, interest rate and the expectations of these variables at the moment the shock occurs. This also means that the timing of the shock matters. The same shock at one point in time may be transmitted very differently from one that occurs at a different time, as will be made clear in this section.

The way we computed the impulse responses to a particular shock was the following. We first run a base simulation using a particular realization of all the stochastic variables (the error terms in the demand, supply and Taylor rule equations). We then rerun the model with exactly the same realizations of these stochastic variables except for the fact that at period t = 100 a shock is introduced in the demand or in the supply equation. We then computed the differences between the output gap in the series with the shock and the series obtained in the base simulation. We also expressed these differences as ‘multipliers’, i.e. we divided them by the size of the shock. This yielded one particular impulse response for a given set of realizations of the stochastic variables. We repeated this 1000 times, each time with another realization of the stochastic variables in the model. This then yielded 1000 different impulse responses to the same shock, but with different initial conditions of the endogenous variables of the model.

This procedure also implies that at the moment the shock occurs the system is out of equilibrium. Thus, each of the 1000 impulse responses will have as a starting point a different disequilibrium. Put differently, the initial conditions each reflect different initial disequilibria. We will show that this has important implications for the subsequent trajectories the impulse responses take.

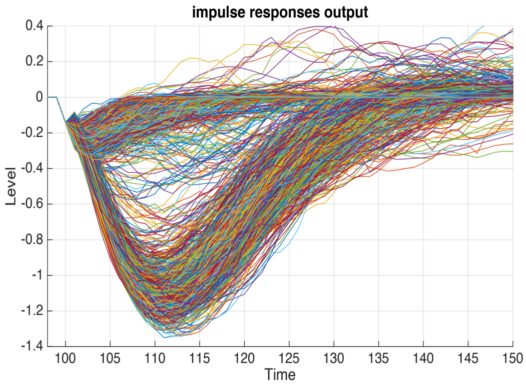
3.1 Impulse response to supply shocks

We first discuss the impulse responses to a supply shock. We will distinguish two shocks, a large one and a small one. The large shock is a 10 standard deviation shock; a small one is a 1 standard deviation shock. The large one is truly large but it corresponds to the size of the shock observed in early 2020 when GDP dropped by 10% to 20% in many countries as a result of the worldwide shutdown of production. We show the results in Figures 1 and 2.

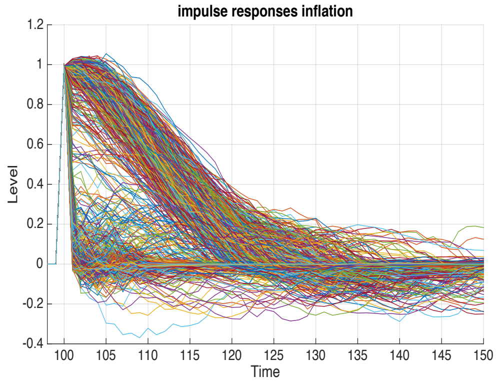
A first thing to note is the large differences in the trajectories of the endogenous variables after the supply shock. Over time these impulse responses tend to converge, but it takes a long time for convergence to be reached. During the transition the trajectories can be very different. For example, take the trajectories of the output gap after a 'arge supply shock in Figure 1. We observe the existence of two trajectories. The first one, a "good" trajectory, implies a mild decline of the output gap and a relatively quick return to the steady state value; the second trajectory, a "bad" one, follows a very deep decline in output and a long and slow recovery. These two trajectories seem to be related the interest rate trajectory where we observe a bifurcation immediately after the shock into a trajectory of declining and a trajectory of increasing interest rates.

Figure 1: Impulse responses to a large supply shock (10 std)

(a)



(b)



(c)

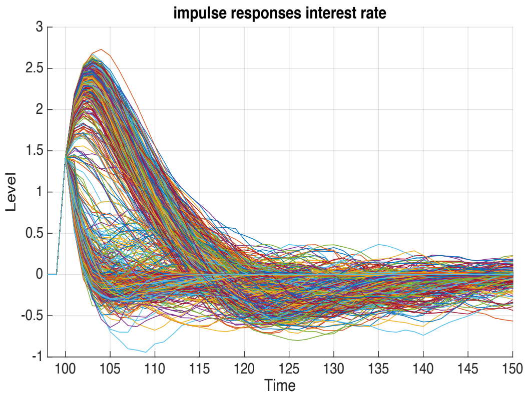
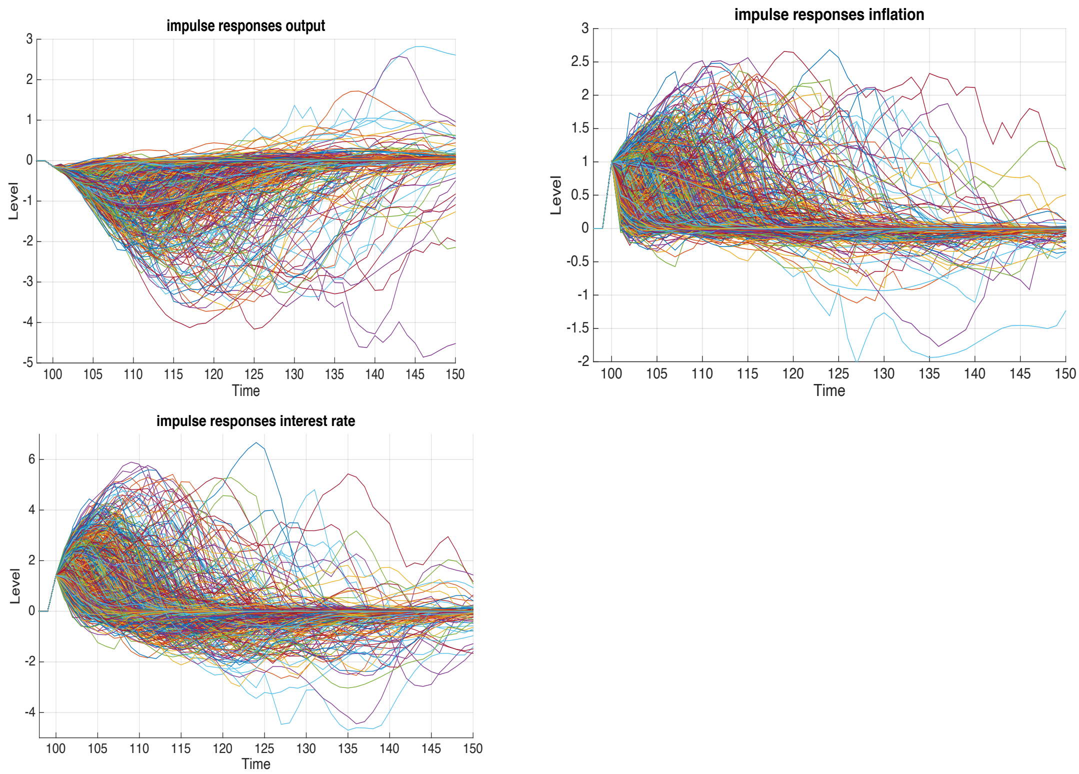


Figure 2: Impulse responses to a small supply shock (1 std)



A second thing to note is that the predictability of the trajectories appears to be different between large and small shocks. When the supply shock is large there appears to be some predictability. We observe that trajectories tend to coalesce around the two main trajectories, the good one and the bad one mentioned earlier. In the next section we will analyze the question of whether and how these different adjustment paths can be predicted by the initial conditions.

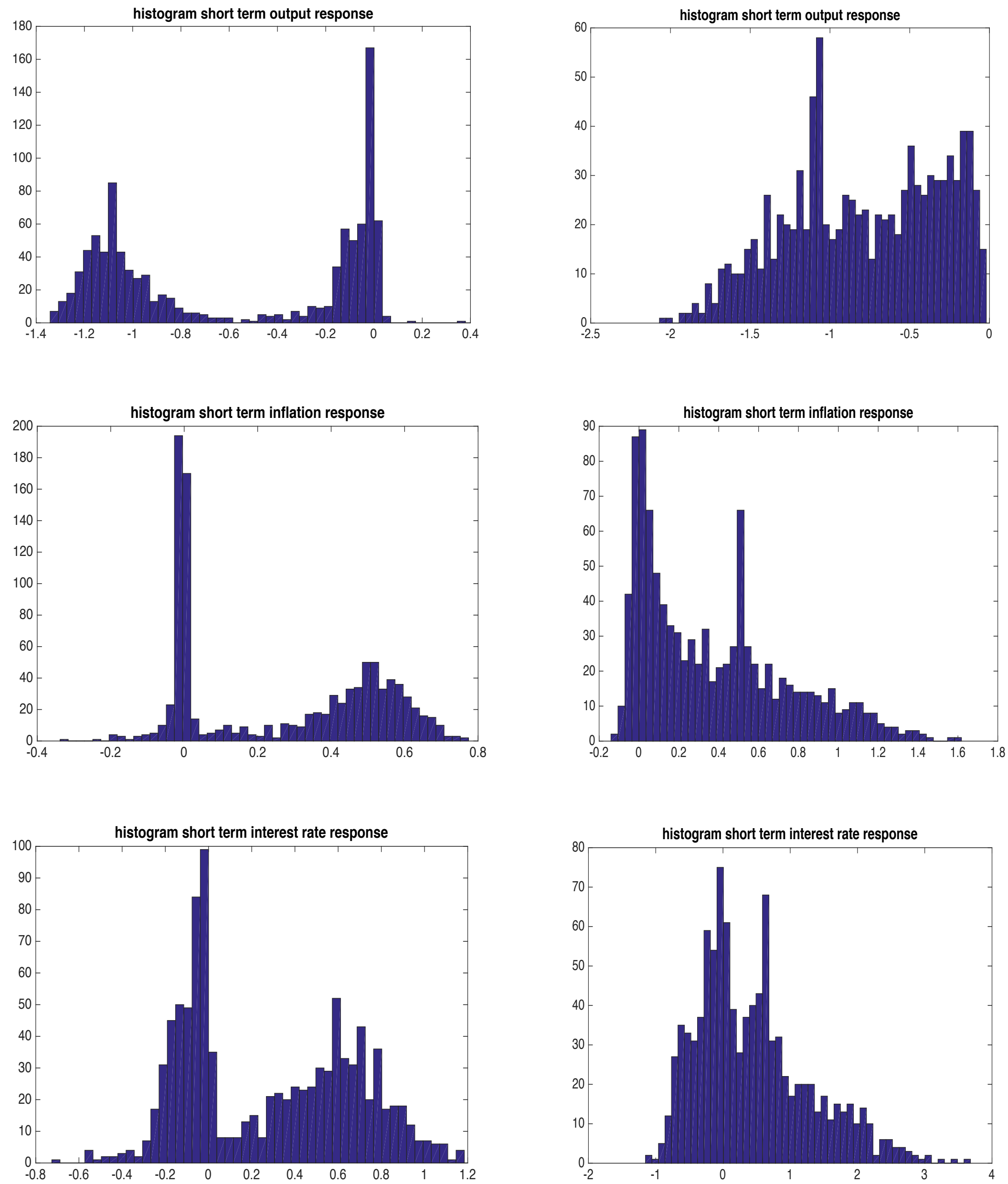
We observe from Figure 2 that the trajectories after a 'small' shock do not tend to coalesce around well-defined adjustment paths. Thus, it appears that when the supply shock is small the predictability of the transmission paths following the shock is much lower than with a large shock. We show the contrast between the two shocks by presenting the distribution of the responses of the output gap in period 12 after the shock. This is obtained by taking a cross-section of the impulse responses of the output gap at period 112 in Figures 1 and 2 and plotting the frequency distributions. These are shown in Figure 3.

We observe very little structure in the distribution of the impulse responses of the output gap after the small shock. This stands in sharp contrast with the distribution after the large supply shock. The latter is clearly bi-modal with peaks around -1.1 and -0.1.

Another important difference to note is that when the shock is small the variance of the distribution of the impulse multipliers is larger than when the shock is large. This also implies that there is a range of stronger negative multipliers. Note again that these are the multipliers, i.e. we divide the output and inflation responses by the size of the shock.

Figure 3: Frequency distribution of impulse responses (12 periods after shock)

Large supply shock Small supply shock



3.2 Impulse responses to demand shocks

We now turn to the impulse responses to demand shocks. We will make the same distinction between small and large shocks. We show the results in Figures 4 and 5. Comparing these with Figures 1 and 2 we find similar results. The large demand shock leads to a similar bifurcation of the output trajectories into a good and a bad one. In the good trajectory output returns very quickly to the steady state; in the bad trajectory the recovery after the shock is much slower. This seems to be related to a similar bifurcation of the interest rate trajectories. We return to this result later we analyze the importance of initial conditions.

Figure 4: Impulse responses to a large demand shock (10 std)

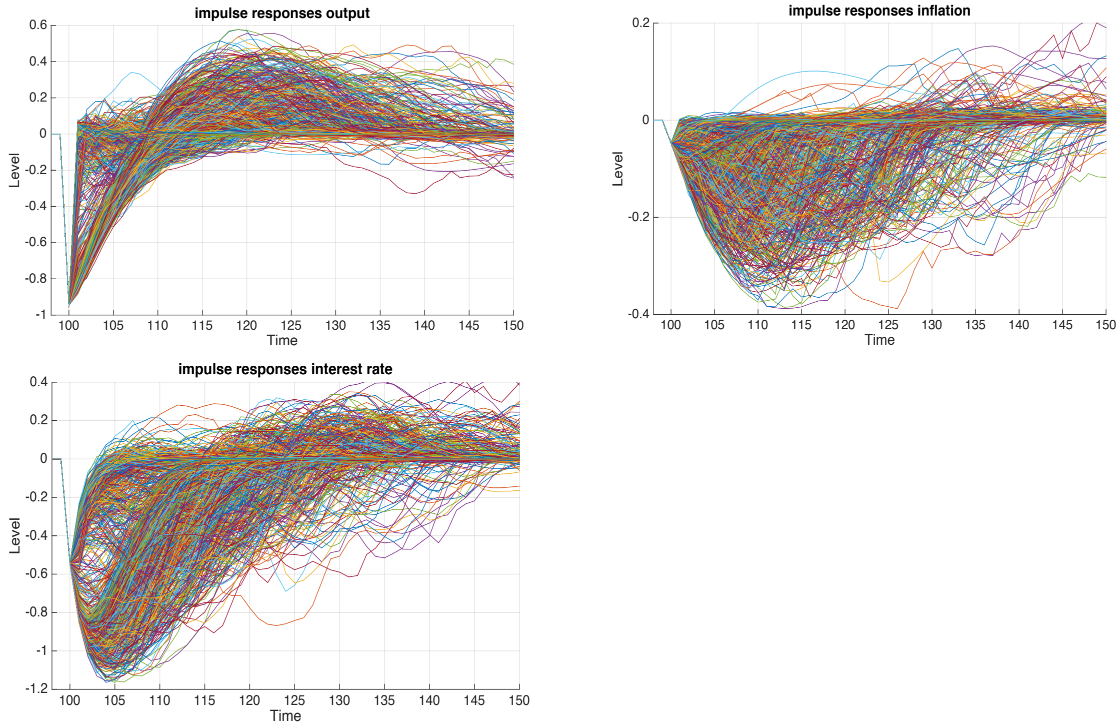
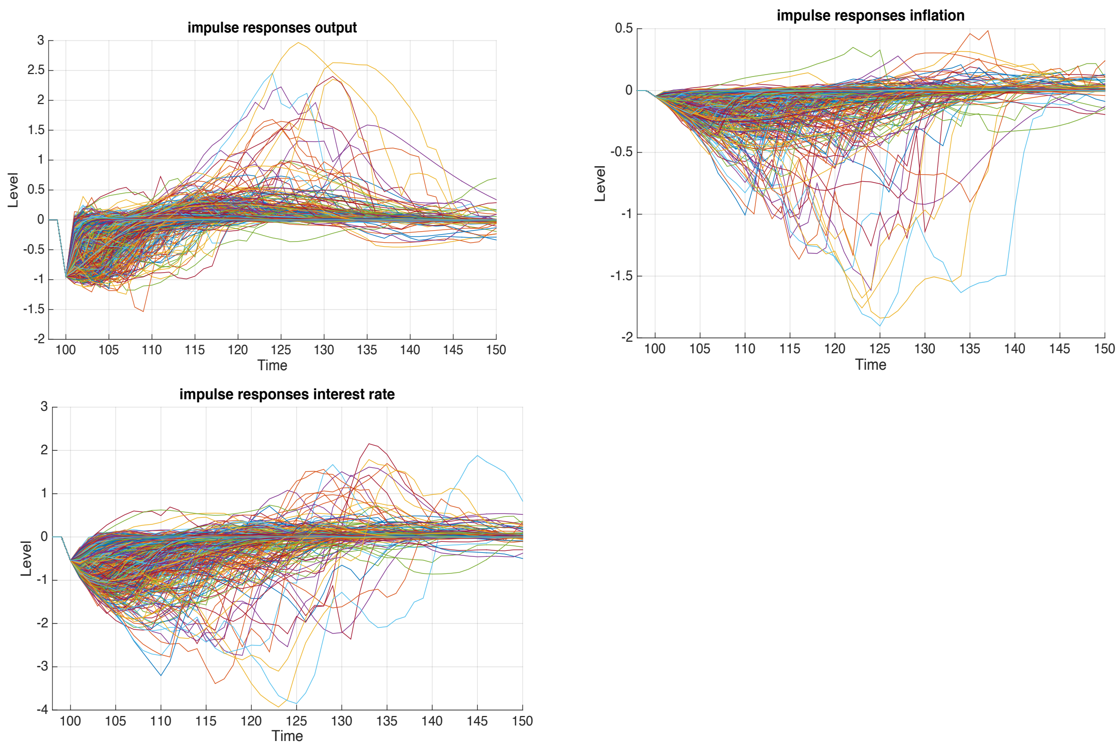


Figure 5: Impulse responses to a small demand shock (1 std)



We also find similar results after a small demand shock: the impulse responses do not tend to coalesce around well-defined adjustment paths. As a result, the predictability of the transmission paths following the small shock is lower than after a large shock.

One difference with the supply shock is that the output gap tends to return to the steady state much quicker after the demand shock than after the supply shock. This is true both for the large and the small shocks. This is related to the fact that in contrast to a supply shock, a demand shock does not put the central bank in a dilemma situation. Both output and inflation decline and therefore give an unequivocal signal to the central bank that the interest rate should decline. This contrasts with a supply shock that produces and increase in inflation and a decline in output (stagflation). This creates a mixed signal for the central bank: the increase in inflation signals a required interest increase and the decline in output a required interest rate decline. We will return to this interpretation when we discuss the importance of initial conditions.

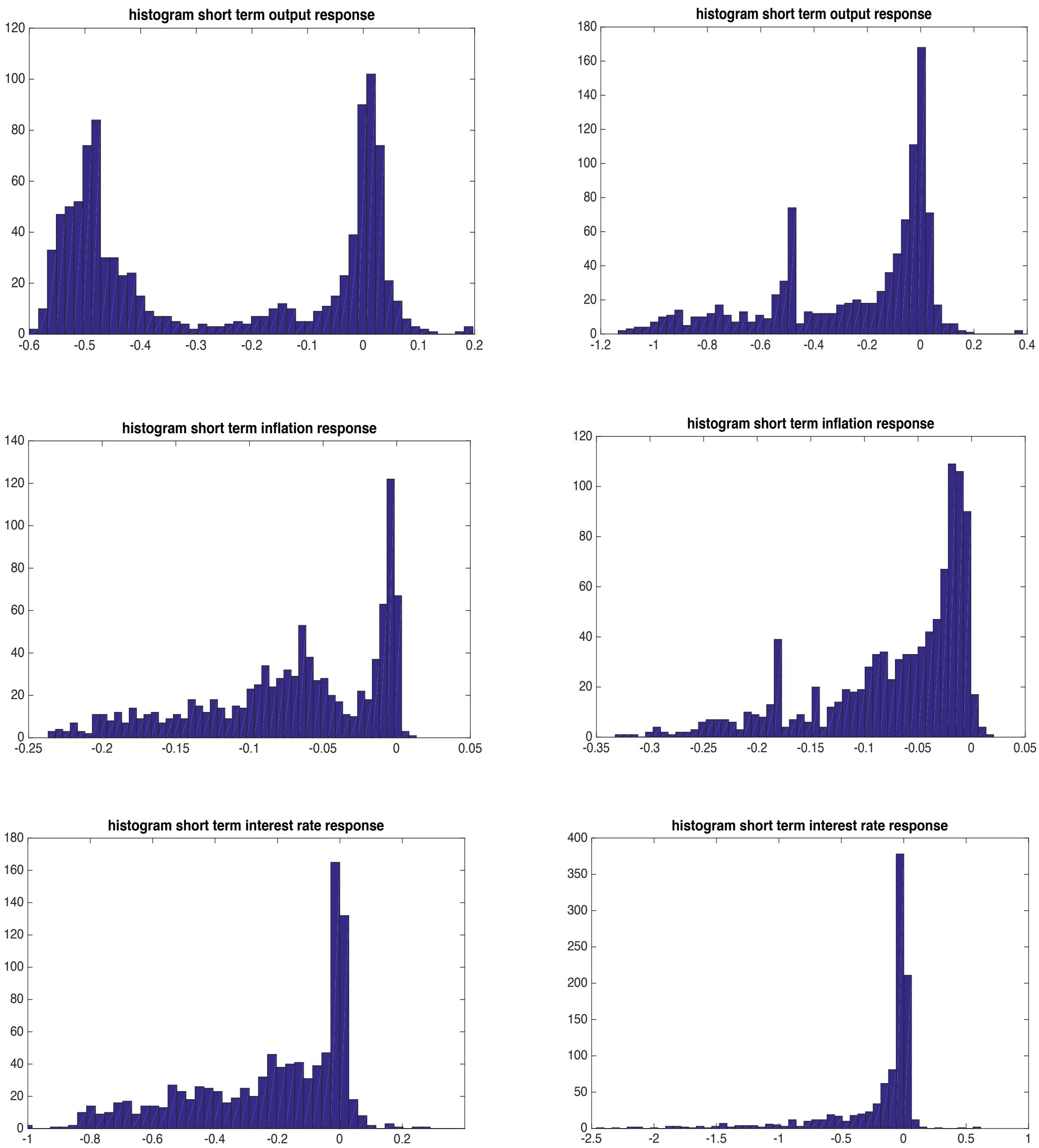
We show the contrast between the large and the small shocks by also presenting the distribution of the responses of the output gap in period 4 after the shock (Figure 6). We chose 4 quarters because after the demand shock the output gap is much quicker to return to its long-term equilibrium than after the supply shock. One way to put this difference is that the sort-term is much shorter after a demand shock than after a supply shock.

Like in the case of the supply shock, we observe very little structure in the distribution of the impulse responses of the output gap after the small shock. This contrasts with the distribution after the large demand shock which appears to be bi-modal. In addition, when the shock is small the variance of the distribution of the impulse multipliers is larger than when the shock is large.

Figure 6: Frequency distribution of impulse responses (4 periods after shock)

Large demand shock

Small demand shock



4. The power of initial conditions

How do the initial conditions affect the output trajectories following the demand and the supply shocks? This is the question we analyze in this section. We will first present simple charts, and in the next section we turn to the use of econometric methods.

4.1 Supply shocks

We start with the supply shocks. In Figure 7 we present two of the initial conditions, i.e. inflation expectations and output gap forecasts prevailing just before the shock on the horizontal axes, and the output gap 12 periods after the supply shock on the vertical axes. We show this for both the large and the small shock. It is striking to find that after a large supply shock the initial expectations of inflation appear to be a very good predictor of the subsequent trajectory of the output gap. More specifically, when initially inflationary expectations exceeded the central bank's inflation target (normalized at 0) the output gap multiplier after 12 periods settles around -1.1. In other words, the subsequent output trajectory is always the bad one, i.e. the output gap is pushed down further as shown in Figure 1. In contrast when initially the inflation expectations are below the central bank's inflation target, the output gap multiplier after 12 periods settles close to 0. Thus, in this case subsequent trajectory is always a good one. In Figure 1 this corresponds to the trajectory that quickly leads the output gap to return to equilibrium.

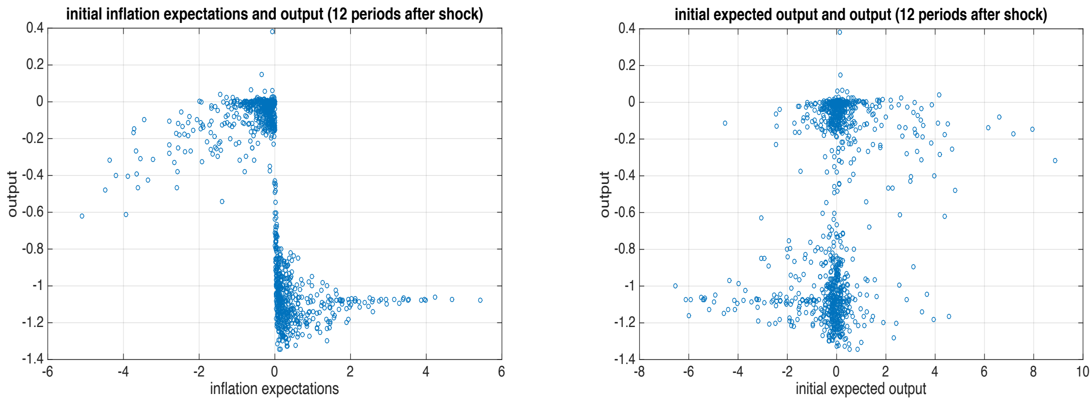
As can be seen from Figure 7, the predictive power of the initial predictions of the output gap is much weaker. Optimistic forecasts of the output gap appear to lead to both a good and bad subsequent trajectory.

The contrast between large and small shocks is striking. In the case of a small shock, the initial inflationary expectations have a much weaker predictive power than in the case of a large shock. Visually we see very little relation between, the initial inflationary expectations and the output gap multiplier after 12 periods. The same holds for the predictive power of the forecasts of the output gap.

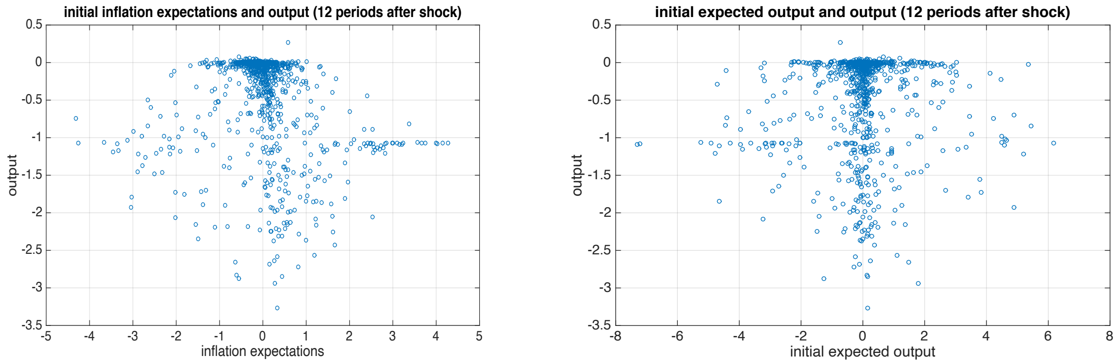
In the next section we return to this issue using econometric techniques to estimate the predictive power of initial conditions (including others than inflation and output expectations) for both the large and small supply shocks.

Figure 7: Initial inflation and output expectations, and output gap 12 periods after supply shock

Large shock



Small shock



What is the underlying mechanism that explains the strong power of the initial inflationary expectations to predict the subsequent trajectory of the output gap when the supply shock is large? To answer this question, we have to analyse the reactions of the central bank to the supply shock. In order to do so it is useful to turn to Figure 1 again.

We note the following. There is a quick bifurcation in the interest rate path after the shock (Figure 1(c)). One path goes up quickly; the other goes down. The upward interest rate path corresponds to the high inflation initial condition. This unfavourable initial condition has the effect of keeping the inflation rate at a high level after the supply shock (See Figure 1 (b)). As a result, the central bank that attaches a relatively high weight on inflation in the Taylor rule feels obliged to raise the interest rate. This in turn pushes the output gap further down (Figure 1(a)). The economy is pushed into a bad trajectory because the unfavourable inflation expectations that existed prior to the shock force the central bank to tighten up after the shock, thereby enhancing the downturn in output.

In contrast, when the inflationary expectations are initially favourable (below the inflation target), the upward movement of the inflation immediately after the shock is quickly corrected. As a result, the central bank observing a relatively favourable inflation outcome reacts by reducing the interest rate to deal with the negative effect of the supply shock on output. This mitigates the negative output effect of the supply shock and pushes the economy onto the good trajectory with a quick return of the output gap to its equilibrium level. Here again the initial favourable inflation expectations tend to reduce the inflation effect of the supply shock “freeing the hands” of the central bank to fight the decline in output by a reduction of the interest rate.

The previous analysis can be complemented by introducing the inflation credibility of the central bank. This notion has a ready interpretation in our model. We define the central bank’s credibility as the fraction of agents that use the announced inflation target as their forecasting rule. With 100% credibility this fraction is 1, i.e. all agents rely on the inflation target to forecast next period’s inflation. With 0% credibility no agents use the inflation target as their forecasting rule; instead all of them use extrapolation of the past inflation rate as their forecasting rule.

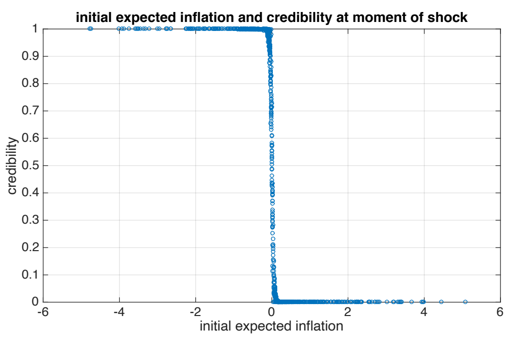
We now find the following results, represented in Figure 8. Figure 8(a) shows the relation between the inflation expectations before the shock (horizontal axis) and the central bank’s credibility at the time of the shock. We observe that when in the past inflation expectations were very high, i.e. above the target, the effect of the supply shock is to reduce the central bank’s credibility dramatically to zero. This dramatic loss of credibility then also becomes a very good predictor of the subsequent decline in the output gap as is shown in Figure 8(b). We observe that when the central bank’s credibility is very low the output gap is very likely to follow the bad trajectory with a steep decline in period 12 after the supply shock.

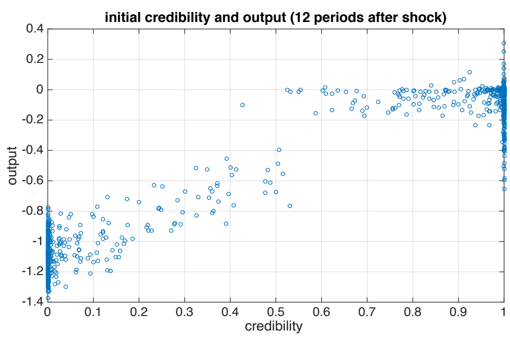
The reverse happens when the initial inflation expectations are below the inflation target. The supply shock then actually improves the central bank’s credibility and this guides the output gap into the good trajectory with a low decline in output.

Our results have some relevance to understand the experience of the 1970s with the supply shocks and the recent covid supply shock. Preceding the supply shocks of the 1970s there had been a buildup of inflation and inflationary expectations. Our model predicts that with these initial conditions, the recovery would take a long time. This is also what happened for many countries with a prior history of significant inflation, especially after the second oil shock of 1979. According to the World Bank(2021) the world GDP growth rate took five years to return to its pre-1979 level of 4.2%. This growth rate was only reached in 1984 again. The trajectory of this protracted recovery also followed the prediction of our model: given the inflationary environment the supply shock of 1979 “forced” many central banks, in particular the US Federal Reserve under Paul Volcker, to raise the interest rates thereby intensifying the economic downturn.

The Covid supply shock of 2020 was preceded by a period of low inflation and low inflationary expectations. Our model predicts that this should make a quick recovery possible, mainly because the central banks did not worry about the inflationary consequences and therefore could actually follow expansionary monetary policies. It appears today that a relatively quick recovery is likely to happen. The European Commission(2021), for example, predicts that in 2022 most EU-countries will have returned to their pre-pandemic GDP growth path.

Figure 8: (a) (b)





How can one interpret the low predictive power of initial conditions when the supply shock is small? Here is the answer. A supply shock equal to 1 standard deviation of the stochastic shocks hitting the economy is of the same order of magnitude as these stochastic shocks. As a result, the initial conditions create departures from the equilibrium that have the same order of magnitude as the supply shock. This does two things. First, it leads to a very low signal to noise ratio, the signal being the supply shock and the noise being the initial condition. As a result, the signal will tend to be overwhelmed by the noise. Second, as the initial conditions are departures from equilibrium, the supply shock has a similar magnitude as the departures from equilibrium. These initial departures from equilibrium can steer the economy in a different direction than the supply shock does. It can also be that the initial conditions and the supply shock reinforce each other. Since the forces of the initial conditions and of the supply shock are of a similar magnitude, it becomes near impossible to separate them out by computing impulse responses. All this produces a low predictability of the initial conditions on the output trajectory after the shock. This problem of unpredictability does not occur when the supply shock is very large relative to the size of the stochastic shocks, or put differently, when the signal to noise ratio is very large.

This result puts into question the standard procedure in modern macroeconomics that consists in presenting impulse responses of one standard deviation shocks. These create an "illusion of knowledge", i.e. a perception that we can predict how a small shock will be transmitted into the economy. When the shocks are small relative to the noise the subsequent output and inflation responses are overwhelmed by the noise. We really need large shocks to see through the noise.

4.2 Demand shocks

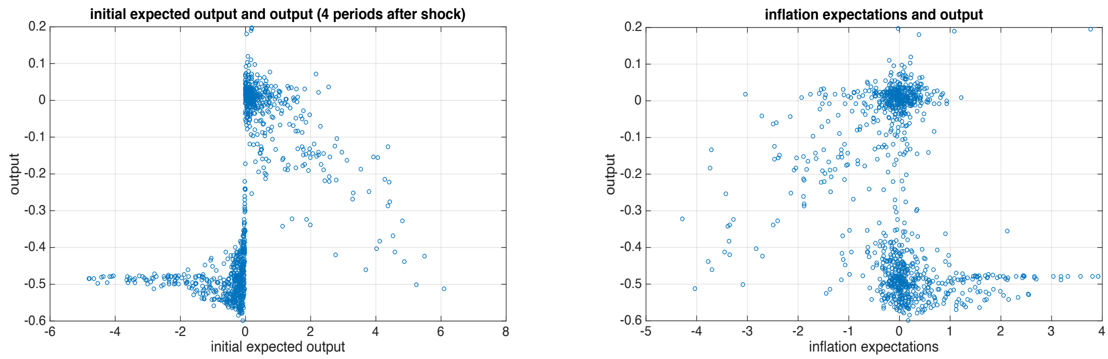
We proceed as in the previous section and show two initial conditions, i.e. inflation expectations and output gap forecasts prevailing just before the shock on the horizontal axes, and the output gap 4 periods after the demand shock on the vertical axes. We show this for both the large and the small shock in Figure 9.

In contrast with the supply shock we now find that it is the initial output expectation that has the strongest predictive power. More precisely, when initially agents are optimistic about future output, the output gap 4 periods later clusters around 0%. In other words, initial optimism about the future business cycle forces the economy along the good trajectory after the demand shock. In contrast, when output forecasts are negative, the output gap 4 periods later settles around -0.5. Thus, pessimism about the future business cycle pushes the economy along the bad trajectory after the demand shock. Note that inflation forecasts have only limited power to predict which trajectory will be chosen.

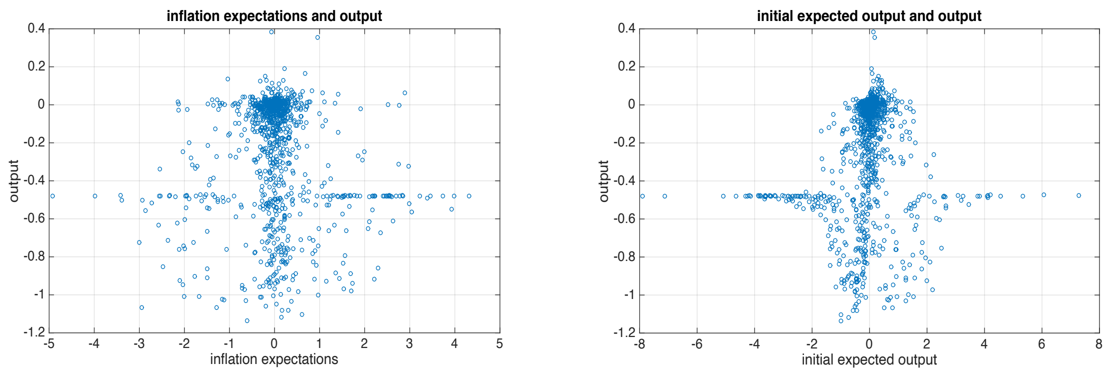
Finally, when the demand shock is small there is very limited predictability of the initial conditions (output and inflation forecasts) .

Figure 9: Initial inflation and output expectations, and output gap 4 periods after demand shock

Large shock



Small shock



Comparing these results with the impulse responses after a large demand shock as shown in Figure 4 leads to the following paradox. The good trajectory is one where output recovers very quckly after the demand shock while at the same time the central bank quickly increases the interest rate. The bad trajectory is the one where output continues to decline after the shock while the central bank reduced the interest rate further. How can one explain this?

The answer has to do with the initial conditions, in particular with output expectations and initial output (not shown here, but to be included in the econometric analysis). When the economy is in a boom to start with (optimistic output expectations and positive output gap), the negative demand shock has a subdued effect on output. Also inflation does not decline much. As a result, the central bank quickly restores its pre-shock policy stance. When the economy is in a recession to start with (negative output gap and pessimistic outlook)) the negative shock has a much stronger negative effect on output and inflation forcing the central bank to drastically reduce the interest rate. This sharp decline then makes it possible to return to equilibrium but with a delay. Thus, it is the intial business cycle situation that uniqually determines the trajectory subsequent to the shock. The central bank is purely reactive.

5. The predictive power of initial conditions: econometric analysis

In this section, using the impulse responses data from our theoretical model, we analyze econometrically what the power is of initial conditions to predict in which cluster the output gap will be pushed 12 periods after the supply shock and 4 periods after the demand shock. As before we distinguish between a large and a small shock.

From Figures 3 and 6 we observe that the output gap after a large shock appears to be following two separate distributions. Standard models will generally not be helpful when one has data from more than one distribution with no information to identify which observation goes with which distribution,. However, Finite Mixture Models (FMM) might come to the rescue (McLachlan and Peel, 2000)). They use a mixture of parametric distributions to model data, estimating both the parameters for the separate distributions and the probabilities of component membership for each observation.

To formally test these ideas, we collect the simulation data under different supply and demand shocks assuming either small (=1 std) or large (=10 std) shocks. In each specific shock, we fit these data with finite mixture models. As discussed earlier, the data generated from our theoretical model suggest that the output gap may be from more than one distribution. We fit the data using the finite model with two classes (assuming two distributions) and then a similar model with only one class (assuming one distribution).

Both the Akaike information criterion (AIC) and the Bayesian information criterion (BIC) from the two models favour the two-class model. This confirms that our theoretical model in fact generate output gaps with more than one distribution. We proceed with the two-class model in analyzing how expectations and other initial conditions are associated with the future output gap. The estimates results are shown in Tables 1 and 2.

In Table 1, we find that in the case of a large demand shock the output gap expectation is a perfect predictor of the future output. The coefficient is -94.4068 indicating that as long as the output gap expectation is positive, the probability of falling in a bad cluster (meaning at -0.4685) is almost at zero. Similarly, in the case of a large supply shock inflation expectation perfectly predicts the future output gap. The coefficient of 164.1634 informs us that when the inflation expectation is positive, the probability of being in a bad cluster (meaning at -1.0358) is close to 1. By contract, when shocks are small, expectations are much less powerful in predicting the future clustering of the output gap. We observe from table 1 that output expectations in the case of a small demand shock is a significant factor associated with the probabilities of the clusters. In a small supply shock, we do not find any significant factors that can be associated with the probabilities of the clusters.

Table 2 presents the predictive power of the initial conditions conditional on being in the good, respectively the bad cluster. We find significance of the initial output and initial forecasts of output in the case of a large and a small demand shock. Similarly, we find significance of initial inflation and inflation forecasts in the case of both a large and small supply shock. Thus, initial conditions continue to matter once the economy is traveling along one of these two trajectories.

Table 1. Determinants of being in bad cluster in the FMM models

	Demand shock 10	Demand shock 1	Supply shock 10	Supply shock 1
Output gap expectation	**-94.4068*****	-3.3598***	0.7623	0.0689
	(14.2671)	(0.5197)	(1.0044)	(0.1527)
Inflation expectation	1.1099	-0.4987	**164.1634*****	0.0859
	(0.9903)	(0.4920)	**(32.6687)**	(0.2203)
Initial output gap	-0.2375	-1.8531***	-1.9343	-0.0890
	(0.5799)	(0.3755)	(1.3259)	(0.1367)
Initial inflation	0.9540	1.4470***	-0.0785	-0.2044
	(0.8563)	(0.4436)	(1.3743)	(0.2091)
Initial interest rate	-0.2348	-0.0438	0.3840	0.1305
	(0.4023)	(0.2186)	(0.8432)	(0.1063)
Constant	-1.2669***	-3.3598***	-0.9060**	0.1180
	(0.3179)	(0.3201)	(0.4351)	(0.0719)
Good cluster (mean)	0.0175	-0.0860	-0.0276	-0.0073
	(0.0029)	(0.0075)	(0.0044)	(0.0008)
Bad cluster (mean)	-0.4685	-0.7537	-1.0358	-0.7150
	(0.0032)	(0.0166)	(0.0080)	(0.0324)
Observations	1000	1000	1000	1000

Standard errors in parentheses
* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Table 2. Impacts of initial conditions and output gap in each class: Finite mixture models

	Demand shock 10	Demand shock 1	Supply shock 10	Supply shock 1
Good cluster				
Output gap expectation	-0.0336***	-0.0493***	0.0107	-0.0017
	(0.0055)	(0.0164)	(0.0078)	(0.0017)
Inflation expectation	0.0220***	0.0264	0.0398***	-0.0078***
	(0.0076)	(0.0224)	(0.0117)	(0.0028)
Initial output gap	-0.0193***	-0.0608***	0.0032	0.0014
	(0.0050)	(0.0143)	(0.0073)	(0.0014)
Initial inflation	0.0194***	0.0362*	0.0032***	0.0085***
	(0.0075)	(0.0213)	(0.0104)	(0.0022)
Initial interest rate	0.0030	-0.0012	0.0002	-0.0021**
	(0.0038)	(0.0109)	(0.0056)	(0.0011)
Constant	0.0159***	-0.0927***	-0.0295***	-0.0073***
	(0.0029)	(0.0074)	(0.0043)	(0.0008)
Bad cluster				
Output gap expectation	-0.0119**	-0.0525**	0.02349*	-0.0011
	(0.0060)	(0.0244)	(0.0142)	(0.0701)
Inflation expectation	0.0064	0.0064	0.0636***	0.0463
	(0.0091)	(0.0338)	(0.0222)	(0.0988)
Initial output gap	0.0258***	-0.0110	-0.0104	0.0081
	(0.0059)	(0.0253)	(0.0141)	(0.0648)
Initial inflation	0.0045	0.0243	-0.0456**	-0.0915
	(0.0084)	(0.0306)	(0.0201)	(0.0984)
Initial interest rate	-0.0060	-0.0153	-0.0228**	0.0028
	(0.0043)	(0.0158)	(0.0104)	(0.0499)
Constant	-0.4693***	-0.7583***	-1.0333***	-0.7162***
	(0.0032)	(0.0171)	(0.0083)	(0.0323)
Observations	1000	1000	1000	1000

6. Sensitivity analysis

We have presented results highlighting the strong differences in the transmission of small and large shocks. It is important to know, however, where the demarcation lies between small and large shocks. In order to answer this question, we perform a sensitivity analysis. We start with a one standard deviation shock and we gradually increase the size of the shock. We present the results for the supply shock in Figure 9.

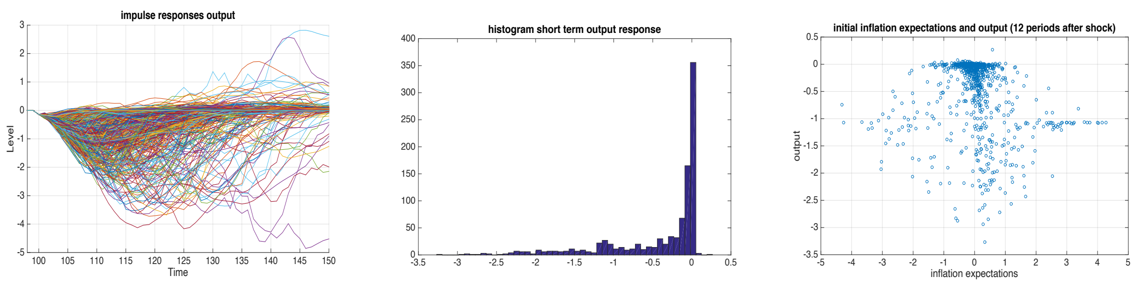
Concentrating first on the impulse responses (a) we find that as the size of the shock is increased, the bifurcation in the trajectories start to become visible with a shock exceeding 5 standard deviations. This is also made clear from the (b) column in Figure 9. This shows the frequency distribution of the output response after 12 periods. We observe that from a supply shock equal to 5 on the distribution starts showing a bi-modal distribution indicating that the impulse responses tend to bifurcate.

Finally, the evidence in column (c) confirms this. With a supply shock of 5 we achieve some measure of predictability of the subsequent output responses by the initial inflationary expectations. This also suggests that with a shock of size 5 the signal of the shock is sufficiently large with respect to the noise so as to create some predictability of the future output responses.

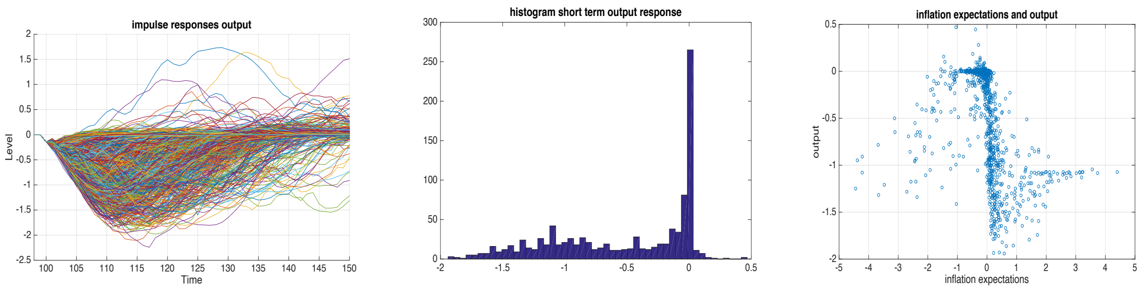
Figure 9: Increasing the size of the supply shock

(a) (b) (c)

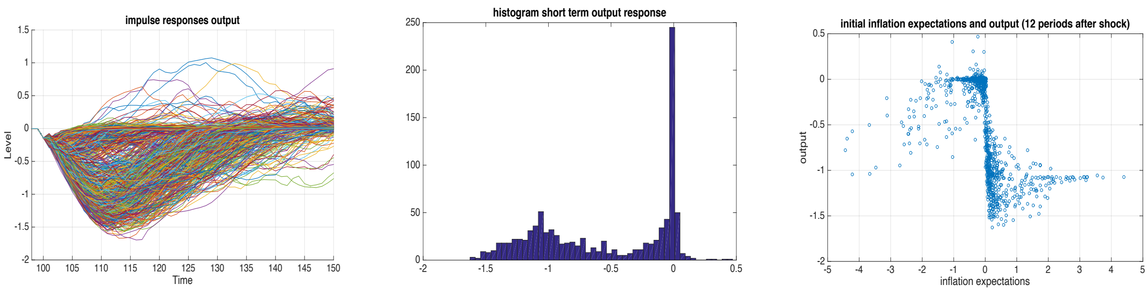
Supply shock = 1



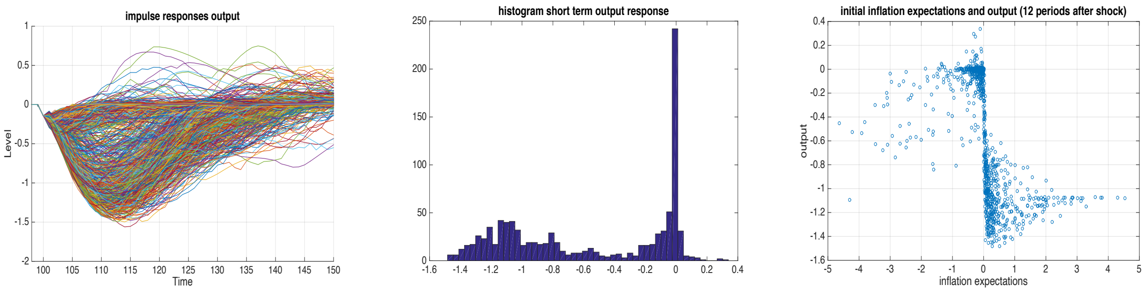
Supply shock = 3



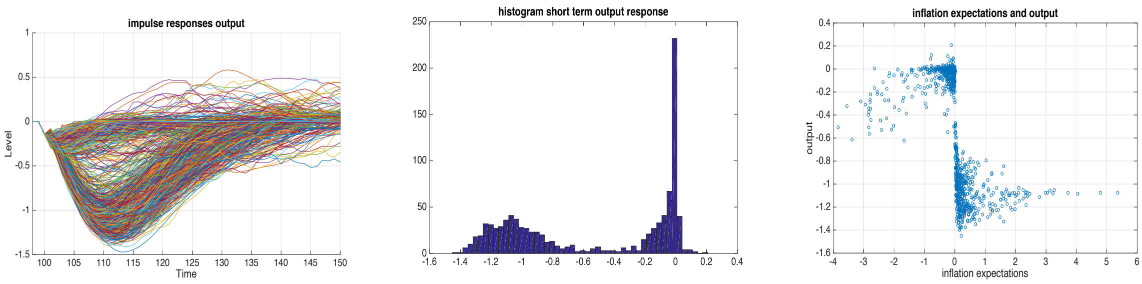
Supply shock = 5



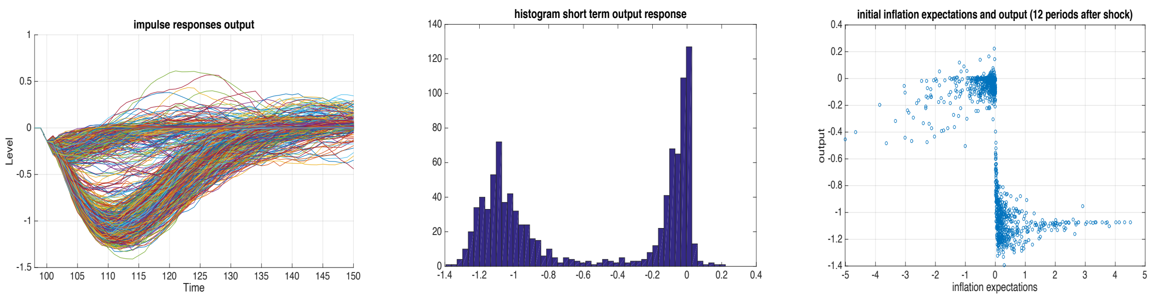
Supply shock = 6



Supply shock = 8



Supply shock = 10



7. Conclusion

The size of shocks matters and the initial conditions prevailing when the shocks occur also matter. These are the two insights provided by this paper.

We have analyzed how small and large demand and supply shocks are transmitted in the economy. In order to do so, we used a behavioural macroeconomic model that is characterized by the fact that individuals lack the cognitive ability to understand the underlying model and to know the distribution of the shocks that hit the economy. In such a world it is rational for these individuals to use simple forecasting rules (heuristics) and to subject these rules to a regular fitness test. As a result, these agents will frequently switch to the best performing rule. This creates a dynamics of booms and busts driven by waves of optimism and pessimism (animal spirits).

It is in this context that we analyze the transmission of large and small shocks to demand and supply. Our main results can be summarized as follows. First, when shocks are small (one standard deviation from the mean) the trajectory taken by output gap and inflation after the shock is unpredictable. In this case the signal provided by the shock is overwhelmed by the noise produced by the inititial disequilbria at the moment of the shock.

Second, when the shock is large (more than 5 standard deviations) we find that the subsequent trajectories taken by output gap and inflation typically coalesce around a good and a bad trajectory. The way this result comes about is that different initial conditions force the monetary authorities into making different choices about the interest rate. Sometimes these choices are bad so that the economy is forced into a bad trajectory and sometimes they are good pushing the economy into a benign trajectory. We also find that when the shocks are large the initial conditions have some power in predicting which trajectory will be chosen.

Our results have some relevance to understand the experience of the 1970s with the supply shocks and the recent covid supply shock. Preceding the supply shocks of the 1970s there had been a buildup of inflation and inflationary expectations. Our model predicts that with these initial conditions, the recovery would take a long time. This is also what happened for many countries with a prior history of significant inflation, especially after the second oil shock of 1979. The Covid supply shock of 2020 was preceded by a period of low inflation and low inflationary expectations. Our model predicts that this should make a quick recovery possible. It appears today that this is likely to happen.

Appendix 1. Microfoundations of the basic behavioral model

We show that the aggregate demand and supply equations used in the main text can be micro-founded on individual utility maximization of households and profit maximization of firms.

1. Aggregate demand

Let us start by modeling the demand side of the model. This will be based on the maximization of individual's utility of consumption over an infinite horizon. The individual consumer, *i*, maximizes the following function over an infinite horizon:

$$E_0 \sum_{t=0}^{\infty} \beta^t \, U(C_t^i, N_t^i) \qquad \text{(A1)}$$

where β is discount factor, U is the utility function which is assumed to have the same form every period and every agent, C_t^i is consumption of agent i in period t and N_t^i is hours worked by agent i.

The budget constraint faced by consumer *i* is:

$$P_t C_t^i + Q_t B_t^i \le B_{t-1}^i + W_t N_t^i - T_t \qquad \text{(A2)}$$

where B_t^i is the quantity of one period discount bond purchased by agent i in period t and maturing in *t+1*; Q_t is price of bond, P_t is the price of consumption goods, W_t is the wage rate and T_t is a lump-sum tax

First order condition for consumer's optimum (Euler equation):

$$-\frac{\mathrm{U_{n,t}}}{\mathrm{U_{c,t}}} = \frac{\mathrm{W_t}}{\mathrm{P_t}} \qquad \text{(A3)}$$

$$Q_t = \beta \tilde{E}_t^i \left[\frac{U_{c,t+1}}{U_{c,t}} \frac{P_t}{P_{t+1}} \right] \qquad \text{(A4)}$$

where $\tilde{E}_t^i$ is the forecast made by agent i in period t using the behavioral heuristic explained in the main text.

Assume a CES utility function

$$U\left(C_t^i, N_t^i\right) = \frac{C_t^{i,1-\sigma}}{1-\sigma} - \frac{N_t^{i,1+\varphi}}{1+\varphi} \qquad \text{(A5)}$$

First order optimality conditions can then be written as:

$$\frac{W_t}{P_t} = C_t^{i,\sigma} N_t^{i,\varphi} \tag{A6}$$

$$Q_t = \beta \tilde{E}_t^i \left[\left(\frac{C_{t+1}^i}{C_t^i} \right)^{-\sigma} \frac{P_t}{P_{t+1}} \right] \tag{A7}$$

Log-linearization (A6) yields the labour supply equation

$$w_t - p_t = \sigma c_t^i + \varphi n_t^i \tag{A8}$$

where lower case letters are natural logarithms. We will not use this labour supply equation as I currently assume full wage flexibility.

Similarly log-linearization (A7) yields the consumption equation (Euler equation) for individual i:

$$c_t^i = \tilde{E}_t^i c_{t+1}^i - \frac{1}{\sigma}\left(r_t - \tilde{E}_t^i \pi_{t+1} - \rho\right) \tag{A9}$$

where the interest rate $r_t = -\log(Q_t)$, the inflation rate $\pi_t = \log P_{t+1} - \log P_t$ and $\rho = -\log \beta$.

We follow Hommes and Lustenhouwer (2016) in assuming that the probability to follow a particular forecasting rule (heuristic) in period t is the same across agents, and independent of the heuristic they followed in the past. This follows from the fact that agents are not inherently different, but that each of them is confronted with the same choice between being following a naïve or fundamentalist forecasting rule. In addition, as in Hommes and Lustenhouwer(2016), we assume "agents know that all agents have the same probability to follow a particular heuristic in the future, and that they know that consumption decisions only differ between households in so far as their expectations are different". In this case households' forecasts about their individual consumption must be the same as their forecast of the consumption of any other individual. It follows that the individual's forecast of his own consumption will coincide with the forecast of aggregate consumption:

$$\tilde{E}_t^i c_{t+1}^i = \tilde{E}_t^i c_{t+1} \tag{A10}$$

where c_{t+1} is aggregate consumption.

This allows us to rewrite the Euler equation as

$$c_t^i = \tilde{E}_t^i C_{t+1} - \frac{1}{\sigma}\left(r_t - \tilde{E}_t^i \pi_{t+1} - \rho\right) \tag{A11}$$

In equilibrium aggregate demand = aggregate supply of output, i.e. $c_t = y_t$

Assume that agents understand market clearing. As a result, their forecast of consumption coincides with their forecast of output. It can be written as

$$\tilde{E}_t^i c_{t+1} = \tilde{E}_t^i y_{t+1} \qquad \text{(A12)}$$

As a result, (A11) can be written as

$$c_t^i = \tilde{E}_t^i y_{t+1} - \frac{1}{\sigma}\left(r_t - \tilde{E}_t^i \pi_{t+1} - \rho\right) \qquad \text{(A13)}$$

Aggregating this expression over all agents i, and using the market clearing condition, yield

$$y_t = \tilde{E}_t y_{t+1} - \frac{1}{\sigma}\left(r_t - \tilde{E}_t \pi_{t+1} - \rho\right) \qquad \text{(A14)}$$

This is the aggregate demand equation used in the main text (without inertia). Thus the aggregate demand equation in our behavioral model can be micro-founded.

The aggregate demand equation used in the main text includes a lagged output gap. Such a lagged output gap can be introduced by assuming habit formation (see Fuhrer(200), Dennis(2008)). We then have a utility function of the form:

$$E_0 \sum_{t=0}^{\infty} \beta^t \, U(C_t^i, H_t^i, N_t^i) \qquad \text{(A15)}$$

where H_t^i is the habit stock of agent i.

Assuming the habit stock obeys the expression

$$H_t^i = \eta C_{t-1}^i$$

where it is assumed that external habit formation (see Dennis(2008). This allows us to derive an aggregate demand equation of the form (see Dennis(2008))

$$y_t = \frac{1}{1+\eta} \tilde{E}_t y_{t+1} + \frac{\eta}{1+\eta} y_{t-1} - \frac{1}{\sigma}\left(r_t - \tilde{E}_t \pi_{t+1} - \rho\right) \qquad \text{(A16)}$$

which is the aggregate demand equation (1) used in the main text where

$$a_1 = \frac{1}{1+\eta}$$

2. Aggregate supply

There is a continuum of firms each producing a differentiated good j in monopolistically competitive markets.

The production function of firm j is specified as follows

$$Y_t^j = A_t N_t^{j,1-\alpha} \tag{A17}$$

We introduce a "New-Keynesian" feature in the model which is that prices are sticky. It is customary to assume so-called Calvo pricing. It is possible to micro-found the behavioral model under the same assumptions about price rigidity as in the standard DSGE-models.

Calvo pricing assumes that each firm will reset prices in period t with probability $1-\theta$, where θ is the fraction of firms that keep their prices fixed. Thus, θ can be considered as a measure of prices stickiness.

Each period firms that have drawn the "Calvo lottery ticket", i.e. are allowed to change their price, will set that price, P_t^* , such that it maximizes the current value of profits generated while that price remains effective

Firms maximize expected profits with respect to P_t^*

$$\sum_{k=0}^{\infty}(\beta\theta)^k \tilde{E}_t^j \left[P_t^{j,*} Y_{t+k/t}^j - \Psi_{t+k}\left(Y_{t+k/t}^j\right)\right] \tag{A18}$$

subject to the demand constraints

$$Y_{t+k/t}^j = \left(\frac{P_t^*}{P_{t+k}}\right)^{-\varepsilon} C_{t+k}^j \tag{A19}$$

Where $\Psi_{t+k}\left(Y_{t+k/t}^j\right)$ is the cost function, $Y_{t+k/t}^j$ is the output of the firm that last reset its price in period t.

The first order condition of an optimum is:

$$\sum_{k=0}^{\infty}(\beta\theta)^k \tilde{E}_t^j \left[Y_{t+k/t}^j\left(P_t^{j,*} - M\psi_{t+k/t}^j\right)\right] = 0 \tag{A20}$$

where $\psi_{t+k/t}$ is the marginal cost in *t+k* for firm that last reset its price in *t* and *M* is mark-up, i.e.

$$M = \frac{\varepsilon}{\varepsilon-1} \tag{A21}$$

Log-linearizing and solving for the price, yields

$$p_t^{j,*} = \mu + (1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k \tilde{E}_t^j\left[mc_{t+k/t}^j + p_{t+k}^j\right] \qquad \text{(A22)}$$

where μ is the desired mark-up, $mc_{t+k/t}$ is the (real) marginal cost. Note that $mc_{t+k/t} + p_{t+k}$ is the nominal marginal cost.

Equation (A21) says that firms resetting their price will choose a price equal to desired (equilibrium) mark-up plus expected marginal costs that will prevail as long as the price is effective (is not changed).

Using

$$\pi_t = (1-\theta)(p_t^* - p_{t-1}) \qquad \text{(A23)}$$

We obtain

$$\pi_t^j = \lambda \sum_{k=0}^{\infty} \beta^k \tilde{E}_t^j\left[\widehat{mc}_{t+k}^j\right] \qquad \text{(A24)}$$

where $b_2 = \frac{(1-\theta)(1-\beta\theta)}{\theta}\frac{1-\alpha}{1-\alpha\varepsilon}$ and $\widehat{mc}_{t+k}^j$ is the marginal cost expressed as a deviation from the steady state.

Thus when the deviation of marginal cost from steady state is positive a fraction of prices is adjusted upwards, leading to more inflation.

Just as in the case of the demand equation the discrete choice model of the selection of forecasting rules implies that

$$\tilde{E}_t^j\left[\widehat{mc}_{t+k}^j\right] = \tilde{E}_t^j\left[\widehat{mc}_{t+k}\right] \qquad \text{(A25)}$$

Finally we can rewrite

$$\pi_t^j = \beta \hat{E}_t^j[\pi_{t+1}] + \lambda\widehat{mc}_t \qquad \text{(A26)}$$

Aggregating over all firms j (see Hommes and Lustenhouwer(2016), we obtain

$$\pi_t = \beta\widetilde{E_t}[\pi_{t+1} + \lambda mc_t] \qquad \text{(A27)}$$

The last step consists in relating marginal cost to the output gap

$$\pi_t = \beta\tilde{E}_t[\pi_{t+1}] - \lambda\hat{\mu}_t \qquad \text{(A28)}$$

where $\hat{\mu}_t = \mu_t - \mu = -\widehat{mc}_t$ and $\lambda = \frac{(1-\theta)(1-\beta\theta)}{\theta}\,\frac{1-\alpha}{1-\alpha+\alpha\mathcal{E}}$

In a way analogous to the assumptions on the price-setting constraints facing firms, assume that for each period only a fraction $1-\theta_w$ of households drawn randomly from the population reoptimize their posted nominal wage. We now consider how households choose the wage for their labour when allowed to reoptimize that wage. The household will choose w_t at period t in order to maximize.

$$\tilde{E}_t^i\{\sum_{t=0}^{\infty}(\beta\theta_w)^k\, U(C_{t+k|t}^i, N_{t+k|t}^i)\} \quad \text{(A29)}$$

Where $C_{t+k|t}^i$ and $N_{t+k|t}^i$ repectively denote the consumption and labour supply in period t+k of a household I that last rest its wage in period t. Note that the utility generated under any other wage set in the future is irrelevant from the point of view of the optimal setting of the current wage, and thus can be ignored in (A28).

Given the utility function specified in (A5), the first-order condition associated with the problem above is given by

$$\sum_{k=0}^{\infty}(\beta\theta_w)^k\, \tilde{E}_t^i\left\{N_{t+k|t}\, U_c(C_{t+k|t}^i, N_{t+k|t}^i)(\frac{w_t}{p_{t+k}} - M_w \mathrm{MRS}_{t+k|t})\right\} = 0 \quad \text{(A30)}$$

$MRS_{t+k|t}^i = -\frac{U_n(C_{t+k|t}^i, N_{t+k|t}^i)}{U_c(C_{t+k|t}^i, N_{t+k|t}^i)}$ denote the marginal rate of substitution between consumption and labour in period t+k for the household resetting the wage in period t and $M_w = \frac{\mathcal{E}_w}{1-\mathcal{E}_w}$. Note that $\mathcal{E}_w$measures the elasticity of substitution among labour varieties.

Log-linearizing (A30) around the steady state (zero inflation) yields the following approximate wage setting rule

$$w_t^* = \mu^w + (1-\beta\theta_w)\sum_{k=0}^{\infty}(\beta\theta_w)^k \tilde{E}_t^i\{mrs_{t+k|t} + p_{t+k}\} \quad \text{(A31)}$$

Where μ^w is household markup. $mrs_{t+k|t}$ is the (log) marginal rate of substitution in period t+k for a household that reset its wage in period t.

Using $\pi_t^w = (1-\theta_w)(w_t - w_{t-1})$

We obtain:

$$\pi_t^w = \beta\tilde{E}_t^i\{\pi_{t+1}^w\} - \lambda_w\hat{\mu}_t^w \quad \text{(A32)}$$

Where $\lambda_w = \frac{(1-\theta_w)(1-\beta\theta_w)}{\theta_w(1+\mathcal{E}_w\varphi)}$ and $\hat{\mu}_t^w = \mu_t^w - \mu^w$

Just as in the demand equation and price setting equations, the discrete choice model of the selection of forecasting rules (concerning wages inflation) implies that it is feasible to aggregate over all households *i*, hence:

$$\pi_t^w = \beta\tilde{E}_t\{\pi_{t+1}^w\} - \lambda_w \hat{\mu}_t^w \quad \text{(A33)}$$

To obtain the Philips curve used in our model, we follow Gali (2008):

Define real wage $\omega_t = w_t - p_t$, real natural wage $\omega_t^n = w_t^n - p_t^n$, and real wage gap $\widetilde{\omega}_t = \omega_t - \omega_t^n$,

$$\hat{\mu}_t^w = \widetilde{\omega}_t - (\sigma + \frac{\varphi}{1-\alpha})y_t$$

$$\hat{\mu}_t = -\widetilde{\omega}_t - \frac{\alpha}{1-\alpha}y_t$$

Referring to Equation (A26), the New Keynesian Philips curve is

$$\begin{aligned}\pi_t &= \beta\tilde{E}_t[\pi_{t+1}] - \lambda\hat{\mu}_t \\ &= \beta\tilde{E}_t[\pi_{t+1}] + \lambda(\widetilde{\omega}_t + \tfrac{\alpha}{1-\alpha}y_t) \\ &= \beta\tilde{E}_t[\pi_{t+1}] + \lambda(\hat{\mu}_t^w + (\sigma + \tfrac{\varphi}{1-\alpha})y_t + \tfrac{\alpha}{1-\alpha}y_t) \\ &= \beta\tilde{E}_t[\pi_{t+1}] + \lambda\left(\tfrac{\sigma(1-\alpha)+\varphi+\alpha}{1-\alpha}\right)y_t + \lambda\hat{\mu}_t^w \\ &= \beta\tilde{E}_t[\pi_{t+1}] + b_2 y_t + \lambda\hat{\mu}_t^w\end{aligned}$$

where $b_2 = \frac{(1-\theta)(1-\beta\theta)}{\theta}\,\frac{\sigma(1-\alpha)+\varphi+\alpha}{1-\alpha+\alpha\mathcal{E}}$

To obtain an aggregate supply equation with a lagged inflation, as we have in the main text an indexation scheme has to be introduced. In such an indexation scheme the prices that in the context of the Calvo rule cannot be optimized in period t are indexed to inflation in period t-1. This is done in Smets and Wouters(2003). It is shown there that with indexation the aggregate supply curve is of the form:

$$\pi_t = \frac{\beta}{1+\beta\wedge}\tilde{E}_t[\pi_{t+1}] + \frac{\xi}{1+\beta\wedge}\pi_{t-1} + b_2 y_t + \lambda\hat{\mu}_t^w$$

where $\wedge$ expresses the degree of indexation. When $\wedge = 0$ there is no indexation and we obtain an aggregate supply curve without lagged inflation. When $\wedge = 1$ there is full indexation and we obtain the aggregate supply curve used in the main text. In that case the coefficients on the forward looking and lagged inflation add up to 1. This leads to equation (2) in the main text.

APPENDIX 2: Selecting the forecasting rules in output forecasting

We define the forecast performance (utility) of a using particular rule as follows[3].

$$U_{f,t} = -\sum_{k=0}^{\infty} \omega_k \left[y_{t-k-1} - \widetilde{E}_{f,t-k-2} y_{t-k-1}\right]^2 \qquad \text{(B1)}$$

$$U_{e,t} = -\sum_{k=0}^{\infty} \omega_k \left[y_{t-k-1} - \widetilde{E}_{e,t-k-2} y_{t-k-1}\right]^2 \qquad \text{(B2)}$$

where $U_{f,t}$ and $U_{e,t}$ are the utilities of the fundamentalist and naïve rules, respectively. These are defined as the negative of the mean squared forecasting errors (MSFEs) of the forecasting rules; ω_k are geometrically declining weights. We make these weights declining because we assume that agents tend to forget. Put differently, they give a lower weight to errors made far in the past as compared to errors made recently. The degree of forgetting turns out to play a major role in our model. This was analyzed in De Grauwe(2012).

Agents evaluate these utilities in each period. We apply discrete choice theory (see Anderson, de Palma, and Thisse, (1992) and Brock & Hommes(1997)) in specifying the procedure agents follow in this evaluation process. If agents were purely rational they would just compare $U_{f,t}$ and $U_{e,t}$ in (10) and (11) and choose the rule that produces the highest value. Thus under pure rationality, agents would choose the fundamentalist rule if $U_{f,t} > U_{e,t}$, and vice versa. However, psychologists have stressed that when we have to choose among alternatives we are also influenced by our state of mind (see Kahneman(2002)). The latter can be influenced by many unpredictable things. One way to formalize this is that the utilities of the two alternatives have a deterministic component (these are $U_{f,t}$ and $U_{e,t}$) and a random component $\xi_{f,t}$ and $\xi_{e,t}$ The probability of choosing the fundamentalist rule is then given by

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[3] (B1) and (B2) can be derived from the following equation:

$$U_t = \rho U_{t-1} + (1-\rho)[y_{t-1} - \tilde{E}_{t-2} y_{t-1}]^2 \quad \text{(B1')}$$

where ρ can be interpreted as a memory parameter. When $\rho = 0$ only the last period's forecast error is remembered; when $\rho = 1$ all past periods get the same weight and agents have infinite memory. We will generally assume that $0 < \rho < 1$. Using (9') we can write

$$U_{t-1} = \rho U_{t-2} + (1-\rho)[y_{t-2} - \tilde{E}_{t-3} y_{t-2}]^2 \text{(B1'')}$$

Substituting (B1") into (B1') and repeating such substitutions ad infinitum yields the expression (B1) where

$$\omega_k = (1-\rho)\rho^k$$

$$\alpha_{f,t} = P\left[(U_{f,t} + \xi_{f,t}) > (U_{e,t} + \xi_{e,t})\right] \quad \text{(B3)}$$

In words, this means that the probability of selecting the fundamentalist rule is equal to the probability that the stochastic utility associated with using the fundamentalist rule exceeds the stochastic utility of using the naïve rule. In order to derive a more precise expression one has to specify the distribution of the random variables $\xi_{f,t}$ and $\xi_{e,t}$. It is customary in the discrete choice literature to assume that these random variables are logistically distributed. One then can obtain the probabilities specified in (8) and (9).

The parameter γ measures the "intensity of choice". It is related to the variance of the random components. Defining $\xi_t = \xi_{f,t} - \xi_{e,t}$ we can write (see Anderson, Palma and Thisse(1992)):

$$\gamma = \frac{1}{\sqrt{var(\xi_t)}}$$

When var(ξ_t) goes to infinity, γ approaches *0.* In that case agents' utility is completely overwhelmed by random events making it impossible for them to choose rationally between the two rules. As a result, they decide to be fundamentalist or extrapolator by tossing a coin and the probability to be fundamentalist (or extrapolator) is exactly 0.5. When $\gamma = \infty$ the variance of the random components is zero (utility is then fully deterministic) and the probability of using a fundamentalist rule is either 1 or 0.

Appendix 3 Solving the model

The solution of the model is found by first substituting equation (3) into (1) and rewriting in matrix notation. This yields:

$$\begin{bmatrix} 1 & -b_2 \\ -a_2c_1 & 1-a_2c_2 \end{bmatrix}\begin{bmatrix} \pi_t \\ y_t \end{bmatrix} = \begin{bmatrix} b_1 & 0 \\ -a_2 & a_1 \end{bmatrix}\begin{bmatrix} \widetilde{\mathrm{E}}_{\mathrm{t}}\pi_{t+1} \\ \widetilde{\mathrm{E}}_{\mathrm{t}}y_{t+1} \end{bmatrix} + \begin{bmatrix} 1-b_1 & 0 \\ 0 & 1-a_1 \end{bmatrix}\begin{bmatrix} \pi_{t-1} \\ y_{t-1} \end{bmatrix} + \begin{bmatrix} 0 \\ a_2c_3 \end{bmatrix} r_{t-1} + \begin{bmatrix} \eta_t \\ a_2u_t+\varepsilon_t \end{bmatrix}$$

i.e.

$$\boldsymbol{AZ_t} = \boldsymbol{B\widetilde{E_t}}\,\boldsymbol{Z_{t+1}} + \boldsymbol{CZ_{t-1}} + \boldsymbol{b}r_{t-1} + \boldsymbol{v_t} \tag{C1}$$

where bold characters refer to matrices and vectors. The solution for $\boldsymbol{Z_t}$ is given by

$$\boldsymbol{Z_t} = \boldsymbol{A^{-1}}\left[\boldsymbol{B\widetilde{E_t}}\,\boldsymbol{Z_{t+1}} + \boldsymbol{CZ_{t-1}} + \boldsymbol{b}r_{t-1} + \boldsymbol{v_t}\right] \tag{C2}$$

The solution exists if the matrix $\boldsymbol{A}$ is non-singular, i.e. $(1-a_2c_2)-a_2b_2c_1 \neq 0$. The system (C2) describes the solutions for y_t and π_t given the forecasts of y_t and π_t discussed in equations (6) and (15). The solution for r_t is found by substituting y_t and π_t obtained from (C2) into (3).

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