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JEL Classification: N/A

Keywords: Entry, captive consumers, price discrimination, Product Innovation

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Challenging the incumbent: Entry in markets with captive consumers and taste heterogeneity

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14 May 2021

Abstract

We analyze entry of a firm with a new and differentiated product into a market with two properties: An existing incumbent has a captive consumer base, and all consumers have heterogeneous tastes. The interaction of the share of captive consumers with the degree of taste heterogeneity leads to non-monotone effects of both parameters on entry. In particular, a higher captive share can support entry when heterogeneity is low but not when it is high, and higher taste heterogeneity (i.e., less product substitutability) can impede entry in the presence of captive consumers. Considering these effects together leads to new insights on entry, horizontal product innovation, and price discrimination.

Keywords: entry, captive consumers, asymmetric competition, product innovation

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1 Introduction

The provision of new products is a core function of markets. A larger variety of products enables consumers to find goods that satisfy their wants better. Knowing which factors influence entry of new products into markets is thus important from a welfare perspective. In this paper, we analyze two determinants of entry: incumbency advantage and intensity of competition. We will show that their interaction generates complex non-monotone effects on entry.

The incumbency advantage we consider manifests itself in exclusive consumer awareness. An incumbent firm has been present and perceived by consumers for a longer time, so that some of them are unable or unwilling to consider emerging alternatives. They are captive to the incumbent. This generates an asymmetry that can impede entry. If the share of captive consumers is large, then potential demand for the entrant is low and so are her expected profits. The intensity of competition is determined by consumer taste heterogeneity. If this heterogeneity is large, consumers have stronger preferences regarding their optimal product and are thus less willing to substitute one good for the other. Less consumer substitution means less intense competition and therefore more potential for a firm to generate profits, making entry more feasible in the absence of captive consumers.[1]

On their own, incumbency advantage and competition intensity thus each have clear implications for entry prospects. Standard considerations suggest that, when analyzed in isolation, entry is difficult in markets with many consumers who are captive to the incumbent and in markets where products are close substitutes. In this paper, we show that the interaction between the share of captive consumers and the degree of substitution leads to more complex, non-monotone effects: an increase in the share of captive consumers may *favor* entry for some parameters, while making it more difficult for others. Likewise, a decline in product substitutability may *impede* entry for some parameter constellations, and make it easier for others. This results from counteracting influences of one factor on the other. A change in one of the factors that would reduce the entrant's profit on its own can increase it through the interaction with the other factor, and vice versa.

Crucially, while an increase in the share of captive consumers reduces the market size for the entrant, and thereby her demand potential, it may also lead the incumbent to set his price less aggressively. In order to better exploit his captive base, he sets a higher price, in turn permitting the entrant to set her price higher. In this situation, an increase in the share of captive consumers may foster entry, in spite of its negative effect on potential demand. This results in a non-monotonicity, where entry is unprofitable for low shares of captive consumers, becomes profitable for intermediate values, and unprofitable again for high shares as the demand potential for the entrant becomes too small. Whether this happens depends on the level of consumer heterogeneity. When it is low, competition for non-captive consumers is intense and, at the same time, captive consumers are more valuable. An increase in the captive share then incentivizes the incumbent to set a higher price, and this will alleviate the competitive pressure that the entrant faces. The demand-reducing

[1] Variants of this argument have been used in different contexts, see, for instance, Sutton (1991), Boone (2000), Raith (2003), Vives (2008).

effect of captive consumers is thus overcome by the competition-softening effect if consumer heterogeneity is sufficiently low.

In turn, while without captive consumers an increase in heterogeneity softens competition, it may intensify it in the presence of a captive base. As heterogeneity increases, the incumbent may want to price more aggressively, because many of his captive consumers now have a weaker taste for his product. This effect emerges already for arbitrarily low levels of the captive consumer share. It generates a non-monotonicity where entry is feasible for low values, infeasible for higher values of heterogeneity, and feasible again for even larger heterogeneity because the softer competition in the non-captive segment now dominates. This non-monotonicity is only present because of the interaction of heterogeneity and a captive consumer base.

Our model features a fundamental asymmetry between incumbent and entrant. The former considers the impact of his pricing on the captive monopoly segment as well.[2] Any effect of a parameter change that is different from the standard duopoly case without captive consumers originates in the incumbent's changed pricing incentives. We first investigate these modified pricing incentives in a general model and outline the channel through which captive consumers impact entry prospects. To gain insights on the relevant market conditions, we then proceed to analyze a model of price competition on the circle (Salop, 1979). We describe the post-entry equilibria for the possible combinations of captive consumer share and consumer taste heterogeneity. Depending on parameters, different types of equilibria emerge, characterized by the combined influence of the two variables. They differ in the extent to which the incumbent focuses on his captive market or engages in competition with the entrant, and in whether the presence of the captive market segment makes the incumbent set his price more or less aggressively relative to a standard duopoly. For wide parameter ranges, the equilibria are in pure strategies. When heterogeneity is sufficiently small, the incumbent's pricing incentives give rise to a mixed-strategy equilibrium. He alternates between high and low prices to resolve the tension between undercutting the entrant and focusing on his captive base.

The circle model of product differentiation uncovers the specific role of consumer heterogeneity for pricing and thus for profits. For each of the incumbent's two market segments, a change in heterogeneity affects his pricing incentives in a different way. In the duopolistic segment, low heterogeneity implies weak product differentiation. This incites the incumbent to price more aggressively. Conversely, when heterogeneity is high, the incumbent has incentives to set his price higher. In the monopolistic segment, low heterogeneity means consumers have similar valuation for the product and can be reached with one high price. As heterogeneity becomes higher, in order to still reach the consumers with a weaker taste for the product, the incumbent lowers his price until, ultimately, he gives up on consumers with weak preferences for his product and again sets a higher price to profit from those with stronger preferences. Together, the price effects in the two segments lie behind the non-monotone relation between heterogeneity and profits discussed above. Similarly, the profit effects of increasing the share of captive consumers can be understood by comparing

[2]We restrict the incumbent to set a uniform price for all consumers. If he could set separate prices, the presence of captive consumers would have no effect on the entrant beyond the reduction in market size.

the incumbent's pricing incentives from the two segments in isolation. If he were able to set different prices, his price in the monopolistic segment would be higher than the duopolistic price when heterogeneity is low, and lower when heterogeneity is high.[3] Thus, the pricing incentives work against the adverse demand effect for the entrant in the former case (leading to the inverse U-relation discussed above), but they exacerbate it in the latter (leading to a strictly negative relation).

It is useful to discuss this non-monotonicity result in the light of the large literature on the relation between competition and process innovation (cost reductions) or vertical product innovations (product improvements). The literature provides various arguments why this relation should not be expected to be monotone, and why it depends on firm or market characteristics.[4] Empirical observations are broadly consistent with this view.[5] In the theoretical literature, the intensity of competition is often captured by the degree of substitution between the products. The effects of competition on process innovations and vertical product innovation can be positive, negative or non-monotone. By contrast, we focus on *horizontal* product innovations, that is, the introduction of new products. As argued above, when captive consumers do not exist, the increasing intensity of competition associated with greater substitutability makes innovative entry less attractive. Our analysis shows that this insight should be taken with a grain of salt in the presence of captive consumers as the relation between substitutability and horizontal innovation is non-monotone. This also expands on the literature on horizontal product innovation, like, for example, Chen and Schwartz (2013).

Our analysis has several welfare implications. First, governments may want to reduce the share of captive consumers, for instance, by engaging in measures to foster transparency. While such measures often increase consumer surplus, there are notable exceptions. We find large parameter regions where increases in the captive market share (i) foster entry by softening competition or (ii) lead to lower prices without influencing entry. Thus, reducing the share of captive consumers does not necessarily increase consumer surplus. On a closely related note, rather than reducing the share of captive consumers itself, the government might want to intervene when an incumbent engages in activities to create captive consumers.[6] The above arguments might suggest that such actvities are not necessarily detrimental for consumers. However, the incumbent will only engage in them if they are in his interest. This is not the case for activities that foster entry. Thus, we can rule out that a monopolist will increase the share of captive consumers if this induces entry. Surprisingly, however, activities that lead to lower prices given that entry has taken place may be in the incumbent's interest. When consumer heterogeneity is sufficiently large, increasing the share of captive consumers increases incumbent profits in spite of the decreasing prices because of the favorable demand effect. At the same time, the price reduction contributes to higher consumer surplus. Even

[3]The possibility that duopoly prices may be above monopoly prices with differentiated goods is well known. See Chen and Riordan (2008).

[4]See Aghion et al. (2005), Gilbert (2006), Vives (2008), Schmutzler (2010, 2013).

[5]For instance, Aghion et al. (2005) argue that the relation between the intensity of competition and innovation is inverse-U shaped, whereas Aghion et al. (2009) highlight the difference between the effects on stronger and weaker firms.

[6]Examples for such activities are advertising or search engine optimization.

though captive consumers are always worse off than non-captive ones, this price effect leads to the existence of a small region where a greater share of captive consumers increases total consumer surplus.[7]

As a second policy contribution, we detail the effects of price discrimination between captive and non-captive consumer segments on entry. The existing debate emphasizes the competition-softening effect of captive consumers which implies that banning price discrimination will be good for entry. By adding taste heterogeneity to the analysis, we find that this holds only for certain parameter values. If heterogeneity is sufficiently high, so that the separate price for the captive segment would lie below that for the non-captive consumers, allowing price discrimination fosters entry.

The analysis of competition with captive consumers has produced a vast literature that goes back to Varian (1980) who was interested in price dispersion in a symmetric model.[8] Reflecting its interest in price dispersion, this literature focuses on mixed-strategy equilibria with homogeneous goods, akin to those emerging in our setting for low heterogeneity. More closely related to our analysis are Sinitsyn (2008, 2009). These papers introduce heterogeneity among non-captive consumers and characterize the corresponding pricing equilibria. Their focus lies on the description of the complex mixed-strategy equilibria that emerge for low levels of heterogeneity. In Sinitsyn (2009), firms are symmetric in their captive base. Sinitsyn (2008), like our paper, analyzes an asymmetric setup where only one of two firms has captive consumers. While his focus is on price dispersion, we are concerned with entry. Further, the captive consumers in his model differ from switchers not only in their product awareness, but they also have uniform valuations for the good. We consider a market setup where consumers, captive or not, always have heterogeneous tastes. As we show, this has important consequences for the pricing equilibria and the feasibility of entry.

Entry when an incumbent has the advantage of a separate monopolistic market has been analyzed first by Armstrong and Vickers (1993). They focus on the effects of price discrimination on entry prospects and highlight the competition-softening effect of a larger captive segment.[9] In our model, captive consumers and switchers are part of the same market, and we add heterogeneity. This permits us to describe the complex interaction effects of the characteristics of a market that a firm considers to enter.

The idea that consumers fail to consider alternative products is supported by a broad empirical literature. Barroso and Llobet (2012), analyzing the introduction of new car models, document that consumer awareness takes time to build and is dynamically affected by advertising. A product which has been in the market for longer thus has had more chances to build

[7]Similarly, one can analyze measures of the entrant to reduce the share of captive consumers. While these measures are usually good for consumers, the alignment of interests is not perfect: Most importantly, entrants may refrain from such measures in certain parameter regions in which, contrary to consumers, they benefit from softer competition.

[8]More recently, Armstrong and Vickers (2019) have analyzed asymmetric mixed-strategy equilibria where consumers differ with respect to the firms they take into consideration when deciding on their purchases. Their analysis allows for rich types of competitive interactions, but focuses on homogeneous-goods oligopolies.

[9]Anton, Vander Weide, and Vettas (2002) analyze the competition-softening effect in the context of auctions for the right to serve a new market.

awareness. This highlights a channel through which an incumbency advantage emerges.[10] Hortaçsu, Madanizadeh, and Puller (2017) identify inattentiveness as a significant factor in preventing consumers from switching to a newly available electricity provider. Ho, Hogan, and Scott Morton (2017) identify similar consumer inattention with insurance plans.

Horizontal innovation, i.e. the introduction of new products, has been analyzed mostly in light of market and cost structure. Chen and Schwartz (2013) compare the innovation incentives of an existing monopolist with those of a new entrant. The monopolist does not have an incumbency advantage and they do not focus on the effects of horizontal product substitutability, but rather on vertical aspects like quality differences between new and existing good. Boone (2000) considers the interaction of horizontal product innovation incentives with firms' cost efficiency levels.

In Section 2, we introduce a general model which provides conditions for incumbent and entrant profits to increase in the share of captive consumers. In order to analyze the effects of market conditions on entry in more detail, we proceed with a specific model of competition with horizontally differentiated products in Section 3, which is a variant of Salop's competition on the circle. We characterize the equilibrium regions and show how parameters affect post-entry equilibria and, in particular, the profitability of entry. In Section 4, we discuss policy implications. Section 5 concludes.

2 The general framework

We consider a market in which an established incumbent faces potential competition from an entrant. Consumers differ in two ways. First, they have heterogeneous tastes. Second, a share of consumers only considers the incumbent's product, while the remainder chooses between both products. We call members of the first group *captive consumers* and the second *switchers*. We investigate how taste heterogeneity and captive share affect whether entry into the market takes place. Each of these factors affects the entrant's profit directly as well as indirectly through its influence on the firms' equilibrium prices.

The effects of the share of captive consumers can be described in a general setting, which we do in this section. The captive market share determines the weights of the incumbent's pricing incentives from the two market segments. The incumbent's pricing, in turn, is decisive for the resulting equilibrium prices and hence the entrant's profits. Consumer heterogeneity determines the demand conditions in the two market segments. To assess its role in detail, we will then proceed with a specific demand model in Section 3.

2.1 Assumptions

We use a two-stage game with two firms: the incumbent $i = 0$, and a potential entrant $i = 1$. Each firm produces a single product. In the first stage, firm 1 decides whether to incur a fixed cost $F \geq 0$ to enter the market. In the second stage, if entry has taken place, firms

[10] In a theoretical model with two periods, Chioveanu (2008) has firms engaging in advertising in the first period to create captive consumers.

compete in a duopoly; otherwise the incumbent serves consumers as a monopolist. After observing the entry decision, the incumbent sets a price p_0 and, if firm 1 has entered, the latter simultaneously sets a price p_1. Profits from sales to consumers are then realized.[11] Variable production costs are assumed to be constant and equal to 0 for both firms. If the incumbent is a monopolist, his market demand function is given as $D_M(p_0;\tau)$, where τ is a demand or policy parameter, taken from a non-degenerate subinterval $\mathcal{T}$ of the real numbers. We assume that the demand function is twice continuously differentiable in both variables almost everywhere and decreasing in the price wherever demand is positive. The entrant's product is differentiated from the one produced by the incumbent. When firms compete for consumers, the demand function is $D_i(p_i, p_j;\tau)$, $i = 1, 2$. We assume that this demand function is also twice continuously differentiable in the interior (wherever prices and demands are positive). Further, we make the following assumptions:

Assumption 1.

(a) *D_i is decreasing in p_i and increasing in p_j, $j \neq i$, whenever $D_i > 0$.*

(b) $\frac{\partial D_i}{\partial p_j} + p_i\left(\frac{\partial^2 D_i}{\partial p_i \partial p_j}\right) \geq 0\ \forall i, j \in \{0, 1\},\ j \neq i.$

(c) $D_0(p_0, p_1;\tau) \leq D_M(p_0, \tau)$ *for all* $(p_0, p_1;\tau) \in \mathbb{R}^+ \times \mathbb{R}^+ \times \mathcal{T}$.

These assumptions mean that the goods are imperfect substitutes, prices are strategic complements and the incumbent's duopoly demand, for fixed p_0, is weakly lower than his monopoly demand. Like the remainder of the set-up, Assumption 1 is compatible with standard models of horizontally and/or vertically differentiated price competition. Note that we allow for the possibility that τ affects both the monopoly and duopoly demand functions (like a shift in income and consumer characteristics) and the possibility that it only affects one of the two demand functions (like an improvement in the quality of the entrant's good, which has no effect on the monopoly demand).

We suppose that a fraction $\mu \in [0, 1]$ of consumers will not buy from the entrant under any circumstances. Notice that being captive is not correlated with any other consumer characteristics, in particular the consumer's preferences over products. A consequence is the possibility that some consumers are captive even when they do not consume the incumbent's product. This is in line with research on consumers' "consideration sets", which finds that they are formed separately from preferences over products.[12]

Crucially, we assume that the incumbent cannot price discriminate between captive con-

[11] As a tie-breaking rule, we will assume that entry takes place if the entrant is indifferent between entering and not entering.

[12] Kardes et al. (1993) analyze the decomposition of brand choice into distinct steps, from awareness, to inspection, to eventual choice. They also show that a brand that was present earlier in the market has a higher likelihood to be included in the early awareness step, producing an advantage of incumbency. Campbell (2013) presents a model of a social network in which consumers have to be informed before they can decide whether or not to purchase a product. Their probability of being informed is independent of their valuation for the good.

sumers and switchers. The incumbent's and entrant's total demand functions are thus not symmetric. They are given by

$$\begin{aligned}\widetilde{D}_0\left(p_0, p_1; \tau, \mu\right) &= \mu D_M\left(p_0; \tau\right) + \left(1 - \mu\right) D_0\left(p_0, p_1; \tau\right) \\ \widetilde{D}_1\left(p_1, p_0; \tau, \mu\right) &= \left(1 - \mu\right) D_1\left(p_1, p_0; \tau\right)\end{aligned}$$

We denote the resulting profit functions for $i = 0, 1$; $j \neq i$, as

$$\Pi_i\left(p_i, p_j; \tau, \mu\right) = p_i \widetilde{D}_i\left(p_i, p_j; \tau, \mu\right).$$

The polar cases $\mu = 1$ and $\mu = 0$ correspond to the monopoly with demand function D_M and the duopoly with demand functions D_0 and D_1, respectively. We refer to the latter case as the *pure duopoly.*

We focus our analysis in this section on the case that the pricing game has a unique pure-strategy equilibrium.[13] We denote the resulting equilibrium prices as $p_0^*\left(\tau, \mu\right)$ and $p_1^*\left(\tau, \mu\right)$ and the equilibrium profits as $\Pi_0^*\left(\tau, \mu\right)$ and $\Pi_1^*\left(\tau, \mu\right)$, where we drop $\left(\tau, \mu\right)$ wherever appropriate. In addition to Assumption 1, we assume that the demand functions are such that the functions Π_i are concave and that the stability condition $\frac{\partial^2 \Pi_0}{\partial p_0^2} \frac{\partial^2 \Pi_1}{\partial p_1^2} > \frac{\partial^2 \Pi_0}{\partial p_0 \partial p_1} \frac{\partial^2 \Pi_1}{\partial p_1 \partial p_0}$ holds.

2.2 Results

Our results concern the profit effects of a change in a parameter $\theta \in \{\tau, \mu\}$. We use the simple fact that the total effect of a marginal change of θ on the profit of firm i is

$$\frac{d\Pi_i^*}{d\theta} = \frac{\partial \Pi_i}{\partial \theta} + \frac{\partial \Pi_i}{\partial p_j} \frac{dp_j^*}{d\theta} \text{ for } j \neq i. \tag{1}$$

Thus, the total profit effect $\frac{d\Pi_i^*}{d\theta}$ of a parameter change consists of a direct effect (which ignores price adjustments), captured by the partial derivative $\frac{\partial \Pi_i}{\partial \theta}$, and an indirect effect induced by the adjustments in the rival's equilibrium price.[14] To obtain a better understanding of the total profit effect, it is essential to understand how the parameter under consideration affects prices. For the demand parameter τ, this will be most usefully analyzed in a specific model (see Section 3). For the share μ of captive consumers, however, there is a simple general characterization of the sign of the price effect:

Lemma 1. *p_0^* and p_1^* are both increasing in μ if and only if*

$$D_M\left(p_0^*\right) - D_0\left(p_0^*, p_1^*\right) > p_0^* \left(\frac{\partial D_0}{\partial p_0}(p_0^*, p_1^*) - \frac{\partial D^M}{\partial p_0}(p_0^*) \right), \tag{2}$$

or, equivalently,

$$D_M\left(p_0^*\right) - D_0\left(p_0^*, p_1^*\right) > \varepsilon_M\left(p_0^*\right) D_M\left(p_0^*\right) - \varepsilon_0\left(p_0^*, p_1^*\right) D_0\left(p_0^*, p_1^*\right), \tag{3}$$

[13] In the example in Section 3, in some parameter regions only mixed-strategy equilibria exist.

[14] Using the logic of the envelope theorem, there is no indirect effect of θ coming from a change in p_i^*.

where ε_M and ε_0 are the (absolute values of the) elasticities of D_M and D_0, respectively, with respect to the incumbent's price. If (2) or (3) is violated with strict inequality, both prices are decreasing in μ.

Intuitively, μ does not affect the entrant's reaction function directly, as it only scales her profit by a constant. However, μ affects the incumbent's reaction function. Thus, by strategic complements, whenever an increase in μ shifts the incumbent's reaction curve out, both prices increase, otherwise they both decrease. As the parameter increases, the incumbent pays relatively more attention to the monopoly market. He sets higher prices after an increase in μ if and only if monopoly prices are higher than pure duopoly prices (those set in the absence of captive consumers). Condition (2) makes this requirement precise: As μ increases, the incumbent has more demand from captive consumers and less demand from switchers; the marginal effect is $D_M - D_0$. As he has to share the switchers with the competitor, $D_M - D_0$ will typically be positive (as captured in Assumption 1(c)). However, the term $\left(\frac{\partial D_0}{\partial p_0} - \frac{\partial D^M}{\partial p_0}\right)$ on the right-hand side of (2) is usually also positive in parameterized examples. With competition, the demand losses from higher prices will also be lower than in monopoly.[15] If, as required by (2), this *elasticity effect* is smaller than the former *demand effect*, then prices are increasing in the share of captive consumers. Condition (3) illustrates the elasticity effect. The incumbent price rises when the direct gains from existing demands outweigh the effects of demand changes induced by the monopolist's price change. This can, but need not be the case. Intuitively, the relative size of the effects determines whether monopoly prices are higher or lower than pure duopoly prices. A stronger focus on captive consumers increases prices only in the former case. As argued by Chen and Riordan (2008), however, with differentiated goods the latter case can arise as well; this possibility will also arise in our specific model in Section 3.

The following result gives conditions for firms' profits to increase with the share of captive consumers.

Lemma 2.

(i) The entrant's profit Π_1^* is increasing in μ if and only if

$$\frac{\mathrm{d}p_0^*}{\mathrm{d}\mu} > \frac{D_1\left(p_1^*, p_0^*\right)}{(1-\mu)\frac{\partial D_1}{\partial p_0}(p_1^*, p_0^*)}. \tag{4}$$

.

(ii) The incumbent's profit Π_0^* is increasing in μ if and only if

$$\frac{\mathrm{d}p_1^*}{\mathrm{d}\mu} > \frac{D_0\left(p_0^*, p_1^*, \right) - D_M\left(p_0^*\right)}{(1-\mu)\,\frac{\partial D_0}{\partial p_1}(p_0^*, p_1^*)}. \tag{5}$$

To understand (i), note that $D_1\left(p_1^*, p_0^*\right)$ captures the entrant's demand loss from a marginal increase in μ, the negative direct effect. As the denominator $(1-\mu)\,\frac{\partial D_1}{\partial p_0}$ is positive by

[15] This is for instance true in the example in Section 3.

Assumption 1(a), the entrant can only benefit from an increase in the share of captive consumers if the incumbent responds with a sufficiently high price increase.

Result (ii) is similar to (i) in that the critical condition for a positive profit effect is a lower bound on the effect of μ on the opponent's price. However, contrary to (4), the right-hand side of (5) is negative because the direct effect on the incumbent's profit, the demand increase $D_M(p_0^*) - D_0(p_0^*, p_1^*)$, is positive. Thus, as long as it does not lead to a substantial drop in the entrant's price (a very negative indirect effect), an increase in the incumbent's share of captive consumers increases the incumbent's profits.

To assess the interaction of the captive market share with underlying competitive demand conditions in more detail, we introduce a parameterized model in the next section.

3 Competition on the circle

While maintaining the two-stage structure introduced above, we now specify the demand functions D_M, D_0 and D_1 by assuming that they are generated by a model of spatial competition, allowing us to characterize the price equilibrium explicitly. Using this characterization, we then derive our main results about the effects of the market environment on entry.

3.1 Setup and post-entry equilibrium

We first provide the assumptions. Thereafter, we identify the post-entry equilibria, which will determine entry profits.

3.1.1 Assumptions

We assume that the incumbent $i = 0$ is located at point 0 on a circle with circumference 1. Transportation costs are linear, with parameter t. The total mass of consumers is equal to one. They are uniformly distributed along the circle. A consumer located at position x has net valuation for product i given by $v - p_i - td_{xi}$, where v is gross valuation, p_i is product i's price, and d_{xi} is the distance from the consumer to the product. Each consumer buys at most one unit, either from firm 0 or from firm 1. We normalize $v = 1 + \frac{t}{4}$ so that the average consumers (located at 1/4 or 3/4) have net valuation (after subtracting transport cost) of 1. Thus, the net valuations of the consumers for the incumbent good are uniformly distributed on $[1 - t/4, 1 + t/4]$, so that an increase in t increases the taste dispersion without affecting the average consumer valuation. We thus focus on heterogeneity in consumer tastes and shut down effects of t on the overall desirability of a product.[16] As long as there is no competition, the incumbent is a monopolist with demand function $D_M(p_0, t) = \min\left\{2\frac{1+t/4-p_0}{t}, 1\right\}$. The parameter t rotates the demand functions around the point with price 1 and quantity 1/2. We assume that firm 1 can enter at position 1/2, thus maximizing the distance from the

[16] The standard Salop model fixes the *maximal* valuation, so that an increase in t results in a reduction of almost all consumers' valuations.

incumbent.[17] The fixed cost of entry is $F > 0$. We maintain the following assumption:

Assumption 2. $t \leq 2$.

This assumption will rule out equilibria in which the market for switchers is not covered after entry and both firms would act independently.

3.1.2 Benchmarks

As benchmarks, we consider the pure duopoly without captive consumers ($\mu = 0$) and the monopoly ($\mu = 1$). Here and in the entire following analysis, we restrict firms' price choices to $p_i \in [0, 1 + t/4]$. This is without loss of generality, as $1 + t/4$ is the highest possible consumer valuation. The following points are useful for future reference:

Observation:

(i) *In a pure duopoly ($\mu = 0$), equilibrium prices and profits are increasing in t.*

(ii) *In a monopoly ($\mu = 1$), prices are increasing in t only when the market is not fully covered ($t > 4/3$); they are decreasing for $t < 4/3$. Profits are decreasing in t.*[18]

(iii) *Prices are higher (lower) in the monopoly than in the pure duopoly if $t < (>) 4/3$.*

Observation (i) captures the idea that, in the absence of captive consumers, an increase in consumer heterogeneity unambiguously fosters entry: For $\mu = 0$, the standard Salop equilibrium with symmetric prices $p_i = t/2$ emerges. Entry takes place if and only if $t \geq 4F$. Thus an increase in consumer heterogeneity favors entry in the pure duopoly as it softens competition and thereby increases entrant profits.

Our analysis of the effects of consumer heterogeneity will rely critically on the contrast between observations (i) and (ii): In the monopoly (μ=1), the incumbent charges $p = 1 - \frac{t}{4}$ when $t \leq 4/3$, in which case he serves the entire market. For $t > 4/3$, he chooses price $p = \frac{1}{2} + \frac{t}{8}$, and not all consumers are served. Crucially, consumer heterogeneity therefore only has a positive effect on the monopoly price when the market is not fully covered. As long as the incumbent serves all consumers, prices are decreasing in t because it becomes harder to keep all consumers on board as heterogeneity increases.

Observation (iii) will play an important role for the complex effects that the share of switchers has on the equilibrium outcome. Reflecting the logic of the general model, prices can be higher in duopoly than in monopoly, because in the former case the demand elasticity may be lower than in the latter.

[17] This assumption simplifies the analysis because it guarantees that demand expressions are symmetric on either side of the firms' locations. It is justified by the principle of maximum differentiation, as an entrant would optimally locate as far away as possible from her competitor after entry.

[18] The statement on profits relies on Assumption 2; beyond $t > 4$, monopoly profits are increasing in t. This non-monotonicity corresponds to Johnson and Myatt (2006) who make the general point that profits are high for very high and very low heterogeneity.

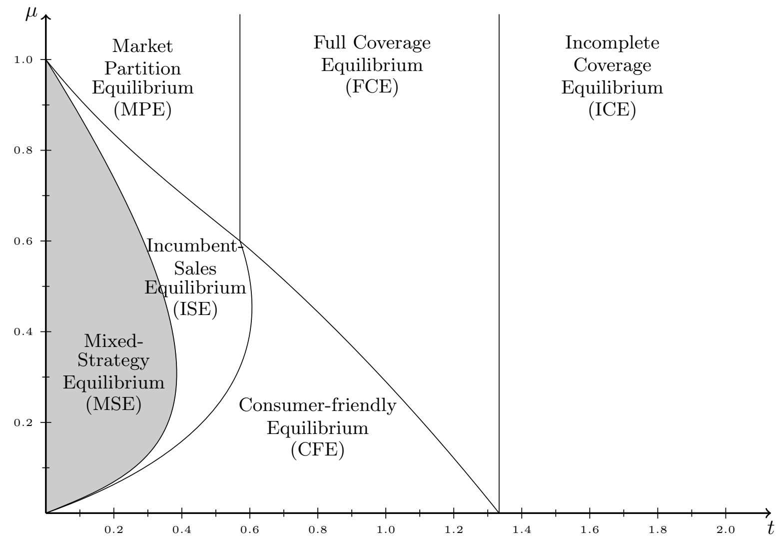


Figure 1: Equilibrium Regions

3.1.3 Pricing Equilibria

We now analyze the equilibria in the post-entry pricing stage. The pure-strategy equilibria (PSE) fall into four classes.

Definition 1.

(i) *In an* ***incomplete-coverage equilibrium (ICE)****, the incumbent does not serve all captive consumers. All switchers are served; both firms have positive switcher demand.*

(ii) *In a* ***full-coverage equilibrium (FCE)****, the incumbent serves all captive consumers at the maximal price ensuring participation. All switchers are served; both firms have positive switcher demand.*

(iii) *In a* ***consumer-friendly equilibrium (CFE)****, the incumbent serves all captive consumers at a price that gives strictly positive surplus to all of them. All switchers are served; both firms have positive switcher demand.*

(iv) *In a* ***market partitioning equilibrium (MPE)****, all switchers are served by the entrant, whereas the monopolist serves all captive consumers.*

The following result gives existence conditions for each type of PSE and provides equilibrium prices. Moreover, it gives conditions under which only mixed-strategy equilibria (MSE) exist. Among those we characterize an **incumbent-sales equilibrium (ISE)** where he randomizes between a high and a low price whereas the entrant chooses a single price.

Proposition 1. *Suppose $\mu < 1$. With competition on the circle, the following equilibria arise:*

(i) For $t > 4/3$, an ICE arises. Prices are

$$p_0^* = \frac{(3-\mu)\,t + 8\mu}{2\,(5\mu+3)},\ p_1^* = \frac{(3+2\mu)\,t + 4\mu}{2\,(5\mu+3)}.$$

(ii) For $\frac{4}{7} < t < \frac{4}{3}$ and $\mu > \frac{12-9t}{12-t}$, an FCE emerges. Prices are

$$p_0^* = 1 - \frac{t}{4},\ p_1^* = \frac{1}{2} + \frac{t}{8}.$$

(iii) For $\frac{36\mu-36\mu^2}{15\mu-8\mu^2+9} < t < \frac{12-12\mu}{9-\mu}$ and $\mu \leq 0.6$, a CFE arises. Prices are

$$p_0^* = \frac{(3+\mu)\,t}{(3-3\mu)\,2},\ p_1^* = \frac{(3-\mu)\,t}{(3-3\mu)\,2}.$$

(iv) For $\frac{4-4\mu}{1+3\mu} < t < \frac{4}{7}$ and $\mu > 0.6$, an MPE arises. Prices are

$$p_0^* = 1 - \frac{t}{4},\ p_1^* = 1 - \frac{3t}{4}.$$

(v) There are no PSE except those in (i)-(iv). Thus, there exists no PSE with $\mu \in (0,1)$ and $t < \min\left(\frac{36\mu-36\mu^2}{15\mu-8\mu^2+9}, \frac{4-4\mu}{3\mu+1}\right)$. In this region, an MSE exists.

(vi) There exists a function $b(\mu)$ such that for all $\mu \in (0,1)$ and $b(\mu) \leq t < \min\left(\frac{36\mu-36\mu^2}{15\mu-8\mu^2+9}, \frac{4-4\mu}{3\mu+1}\right)$, an ISE arises. The incumbent randomizes between the monopoly price $1-\frac{t}{4}$ and a lower price and the entrant sets a single price.

Figure 1 depicts the equilibrium regions. For sufficiently high heterogeneity, the incumbent does not serve all captive consumers, so that an ICE emerges. Conversely, for sufficiently low heterogeneity, the incumbent reaches all of his captive consumers. At the same time, competition for switchers is more intense. If there are not too many captive consumers, prices are low enough that there is a CFE. Beyond a certain threshold number of captive consumers, the incumbent focuses on them and sets the monopoly price. If heterogeneity is low, the entrant finds it optimal to set a price just low enough to attract all the remaining switchers; an MPE emerges. If heterogeneity is not too low, the entrant sets a higher price and some switchers buy from the incumbent; an FCE emerges.

The PSE regions described in (i)-(iv) of Proposition 1 do not cover the entire parameter region $t \leq 2$. For low values of t as in (v) and intermediate values of μ, there is no PSE. The non-existence of PSE in models with captive consumers has been well known for the case of perfect competition since Varian (1980). As Proposition 1 shows, it extends to the case of imperfect competition as long as the degree of substitution is high enough. Intuitively, competition for switchers is intense when consumer heterogeneity is low. The incumbent wants to focus on exploiting the captive consumers by setting a high price, to which the entrant would respond by also setting a high price. As t is low, switchers are sensitive to price differences, and the incumbent would be tempted to react with a low price to the entrant's high price. Intermediate values of μ mean that there is a large group of both consumer types. As a result, the described reasoning has most bite: The conflict between the incumbent's incentives to attract switchers and to exploit captive consumers is particularly pronounced and the region without PSE is large.

(vi) shows that, in a large part of the MSE region adjacent to the PSE region, a particularly simple equilibrium, the ISE, exists. The incumbent randomizes between the monopoly price and a lower competitive (sales) price. The entrant, competing only for the captive consumers, sets one price which is a best reply to the incumbent's expected price. This mirrors the approach of Sinitsyn (2008) who shows that in an asymmetric duopoly where one firm has a captive consumer base, this stronger firm will typically randomize between two prices while the weak firm uses one price. We did not explicitly characterize the equilibria in the remainig MSE region when $t > 0$.[19]

We mention in passing that without Assumption 2 ($t \leq 2$), preference heterogeneity would be so large that firms would no longer compete for the switchers with weaker preferences for their goods, leading to a *Doubly Incomplete Coverage Equilibrium* where neither the switcher market nor the captive market are covered. In Appendix A.3, we show what, for more general discrete-choice games, the equilibrium structure is similar. We find that, except for the Doubly Incomplete Coverage Equilibrium, no additional types of equilibria can arise other than those in Proposition 1.

3.1.4 Entry Behavior

We are now ready to characterize the conditions under which entry takes place. We focus on the parameters for which a PSE or the ISE exists in the pricing game.

Proposition 2. *Entry takes place if and only if the following conditions apply.*

(i) In the ICE Region, $\Pi_1 = (1-\mu)\frac{t}{4}\left(\frac{3t+2t\mu+4\mu}{3t+5t\mu}\right)^2 \geq F$.

(ii) In the CFE Region, $\Pi_1 = (1-\mu)\frac{t}{4}\left(\frac{3-\mu}{3-3\mu}\right)^2 \geq F$.

(iii) In the FCE Region, $\Pi_1 = (1-\mu)\frac{t}{4}\left(\frac{t+4}{4t}\right)^2 \geq F$.

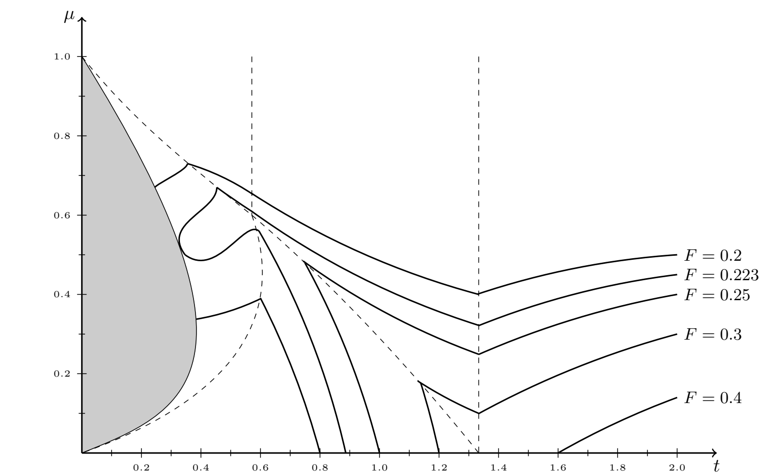
(iv) In the MPE Region, $\Pi_1 = (1-\mu)\frac{t}{4}\left(\frac{4}{t}-3\right) \geq F$.

[19] For $t = 0$, we provide a straightforward characterization in the Appendix. In his setting related to ours, Sinitsyn (2008) shows for $t > 0$ that both firms keep adding prices to their strategy support step by step as heterogeneity decreases. He solves for them numerically, which becomes ever more complex with lower heterogeneity. We conjecture that in our setup similar equilibria with larger price supports will exist for lower levels of heterogeneity.

(v) In the ISE Region,

$$\mathbb{E}\Pi_1 = (1-\mu)\frac{t}{4}\frac{2\left((1+10\mu-3\mu^2)t-24\mu(1-\mu)+\frac{1-3\mu}{1-\mu}\sqrt{\mu(1-\mu)^3t(4-t)}\right)^2}{\left(36\mu(1-\mu)-(1+15\mu)t\right)\left((1+2\mu-3\mu^2)t+3\sqrt{\mu(1-\mu)^3t(4-t)}\right)} \geq F.$$

The proof is straightforward computation of the profits given the equilibrium prices determined in Proposition 1. Figure 2 illustrates the result for different levels F. It provides the entry regions for different values of t and μ. We will argue below that entry profit is maximal ($\Pi_1 = 0.5$) when $(t, \mu) = (2, 0)$. For $F = 0.4$, the entry region consists of the region in the lower right corner inside the ICE. Next, for $F = 0.3$ and $F = 0.25$, entry also takes place in parts of the CFE and FCE regions. Finally, for $F = 0.223$ and $F = 0.2$, the entry region extends into the MPE and ISE regions.



Note: The dashed lines correspond to the boundaries of the equilibrium regions in Proposition 1 (see Figure 1)

Figure 2: Entry Regions

To sum up, entry is most likely when t is large and μ is low (near $(t, \mu) = (2, 0)$). However, the shape of the entry regions shows that neither the effect of μ nor the effect of t on entry is monotone. In the following, we explore the logic behind these non-monotonicities.

3.2 The effects of captive consumers

We now investigate how the share of captive consumers affects the equilibrium, focusing throughout on pure-strategy equilibria. We first analyze the price effects. Then we provide conditions under which increasing μ induces and deters entry, respectively. Recall that *pure duopoly* refers to a situation with two firms in the market and no captive consumers ($\mu = 0$).

3.2.1 Price Effects

Proposition 1 immediately implies the following result.

Lemma 3. *Both prices are increasing in μ in the CFE region and decreasing in the ICE region. Elsewhere, μ has no effect on prices.*

As the general analysis of Section 2 suggested, the price effects of captive consumers can be positive or negative. Intuitively, in the consumer-friendly equilibrium, heterogeneity is low enough that relatively high pure monopoly prices would be sustainable, whereas competitive pressure is noticeable in a duopoly. When μ is low, the incumbent focuses on the switcher market. As μ increases, the incumbent puts more focus on the captive market and his price rises. By contrast, in the incomplete-coverage equilibrium, t is high. The relationship between monopoly and pure duopoly prices is now inverse. High consumer heterogeneity makes it difficult for a monopolist to serve all consumers. At the same time, a high t softens competition and thus causes high pure duopoly prices. The incumbent thus decreases the price as captive consumers become more important.

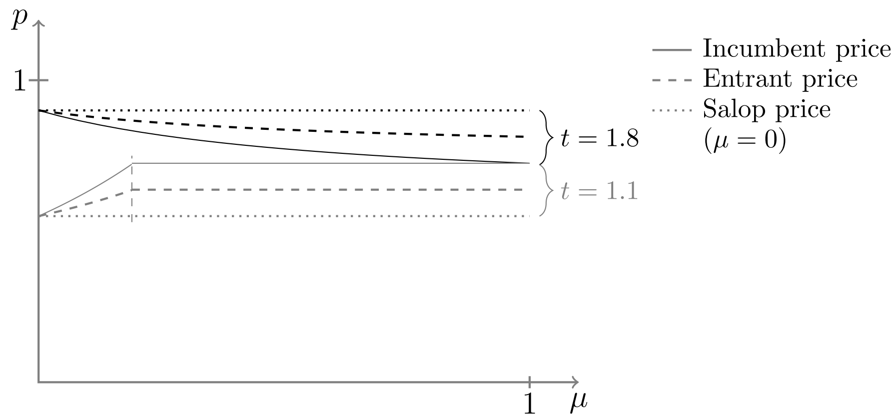


Figure 3: Price effects of μ

Figure 3 illustrates these effects in more detail. For $t = 1.1$ and sufficiently low shares of captive consumers, the CFE arises, and an increase in μ softens competition (increases prices). For $t = 1.8$, in the ICE region, pure duopoly prices are higher than monopoly prices and, as the monopolist focuses more on captive consumers with higher μ, prices fall.[20] In line with the discussion of Lemma 1, the region with an increasing (decreasing) price corresponds to a situation where the pure monopoly price is above (below) the pure duopoly price. When heterogeneity is high, captive consumers force the incumbent to set a lower price and counteract the competition-softening effect of higher heterogeneity.

3.2.2 Entry Effects

Figure 2 shows that increases in μ do not unambiguously work against entry. In the following, we therefore delineate the conditions under which a greater share of captive consumers fosters entry. The general analysis of Section 2 shows that an increase in μ affects the entrant's profit in the PSE regions negatively unless it has a strong positive effect on the incumbent's price.

[20]By strategic complements, the entrant follows the behavior of the incumbent, but reduces its price by a smaller amount.

Lemma 3 thus implies that μ must have a negative effect on entry in the PSE regions except possibly in the CFE region. The following central result confirms that, reflecting positive effects of captive consumers on entry profits in the CFE region, a higher captive share can actually foster entry in this region. It clearly delineates the conditions under which this happens. To state the result, we define a function $\mu^B(t)$ on $(0, 4/3)$ by the requirement that $(t, \mu^B(t))$ lies on the lower boundary of the MPE or the FCE region, respectively.[21]

Proposition 3.

(i) Suppose $0 < t < 4/3$. *If* $F \leq t/4$, *there exists a threshold level* $\overline{\mu}(t) \in [0, 1)$ *such that entry in PSE only occurs for* (t, μ) *such that* $\mu \in [0, \overline{\mu}(t)]$.

(a) Suppose $4/7 \leq t < 4/3$. *If* $t/4 < F < \frac{1}{8}\frac{(t+4)^2}{12-t}$, *then entry takes place in an open neighborhood of* $(t, \mu^B(t))$. *If* $F > \frac{1}{8}\frac{(t+4)^2}{12-t}$, *then there is no entry for any value of* μ.

(b) Suppose $0 < t < 4/7$. *If* $t/4 < F \leq t\frac{4-3t}{3t+4}$, *then entry takes place in an open neighborhood of* $(t, \mu^B(t))$. *If* $F > t\frac{4-3t}{3t+4}$, *then there is no entry for any value of* μ.

(ii) Suppose $4/3 \leq t$. *Then, if* $F > t/4$, *there is no entry for any* μ *for this* t; *otherwise there exists a threshold level* $\overline{\mu} \in [0, 1)$ *such that entry only occurs for* $\mu \in [0, \overline{\mu}]$.

The proposition highlights the interaction between the effects of captive consumers and taste heterogeneity. For high levels of heterogeneity, $t > 4/3$ (case (ii)), prices are decreasing in μ, thus reinforcing the adverse potential demand effect of captive consumers on the entrant and reducing her profits. Thus, in line with Figure 2, entry takes place only for sufficiently low values of μ.

For low heterogeneity ($0 < t < 4/3$; case (i)), introducing captive consumers increases entrant profits in the CFE, as the competition-softening effect dominates the negative potential demand effect. Thus, when fixed costs are so high that there is no entry in pure duopoly, it may nevertheless take place above a threshold level of μ near the boundary line defined by $\mu^B(t)$. By contrast, as μ approaches 1, profits must fall, so that, in the MPE and FCE regions there is a threshold beyond which entry is not feasible. The proposition gives conditions on F and t guaranteeing that entry arises near the boundary line.

Figure 2 illustrates the situations described by Proposition 3. For $F = 0.3$ and $F = 0.25$, there exist values of t for which $t < 4/3$ and, according to case (i), entry takes place below a threshold level of μ. For slightly lower values of t, entry occurs only at intermediate values, which corresponds to cases (a) and (b) of (i). Finally, for low values of t, there is no entry at all. Part (ii) summarizes the much simpler behavior in the ICE region.

Note that, by continuity, the condition applies not only to the PSE regions, but also to the

[21] The graph of the function is the downward-sloping line, moving from (0,1) to (4/3,0) in Figure 1. It is (implicitly) defined by $\frac{4-4\mu}{1+3\mu} = t$ for $t \in [0, 4/7]$ and $\mu = \frac{12-9t}{12-t}$ for $t \in [4/7, 4/3]$.

ISE region just below $\mu^B(t)$.[22] Figure 2 shows this clearly for $F = 0.2$. As we can see for $F = 0.223$, it is even possible that, when t is low, entry occurs in the ISE region, but not in a pure-strategy equilibrium. All told, we have therefore established that entry takes place only for intermediate values of μ as long as competition is sufficiently intense (taste heterogeneity is low enough).[23]

3.3 The Effects of Taste Heterogeneity

We now shift attention towards the effect of increasing taste heterogeneity, as captured in t, on entry. Again, we focus on the regions in which pure-strategy equilibria exist. Without captive consumers, the relation between heterogeneity and entry profits would be trivial: According to Observation 1(i), the effect of an increase in the degree of substitution (a reduction in t) on entry profits is negative when there are no captive consumers. Thus, increasing competition in this sense reduces entry profits.[24] With captive consumers, a reduction in t, and hence more intense competition in a duopoly, may have a positive effect on the post-entry profits of the entrant. We will now show how this can happen for intermediate levels of t.

3.3.1 Price effects

Again relying on Proposition 1, we first state how taste heterogeneity affects prices. Crucially, we find that, contrary to the case without captive consumers, for an arbitrary positive share of captive consumers, heterogeneity reduces prices over some interval.

Lemma 4. *Increasing heterogeneity has positive price effects in the CFE and ICE regions and negative effects in the MPE region. In the FCE region, heterogeneity reduces the incumbent price, but increases the entrant price.*

Two countervailing forces are at work: Consumer heterogeneity softens competition for switchers, but it may induce the incumbent to set lower prices to serve all captive consumers (see the discussion in Section 3.1.2). Figure 4 shows that, as in the standard Salop model ($\mu = 0$), for $\mu = 0.1$ both prices are higher when there is much heterogeneity than when consumers are very homogeneous. However, the relation is not monotone, as the incumbent price is decreasing in t in the intermediate FCE region. Intuitively, this comes from the incumbent's attempts to keep all captive consumers on board. For a high share of captive consumers ($\mu = 0.8$), the incumbent focuses on this group for low values of t, setting prices so high that the switchers all buy from the entrant. As t increases, the incumbent lowers his price to continue selling to all captive consumers. This mechanism generates the downward

[22] The entrant's expected profit in the ISE converges to her profit in the MPE at the boundary between the two regions. Hence, the continuity argument applies.

[23] The statement also holds for the MSE in the case $t = 0$, where the maximal expected profit arises for $\mu = 0.5$.

[24] This is essentially a variant of the standard insight that more intense competition increases industry concentration (e.g., Sutton 1991).

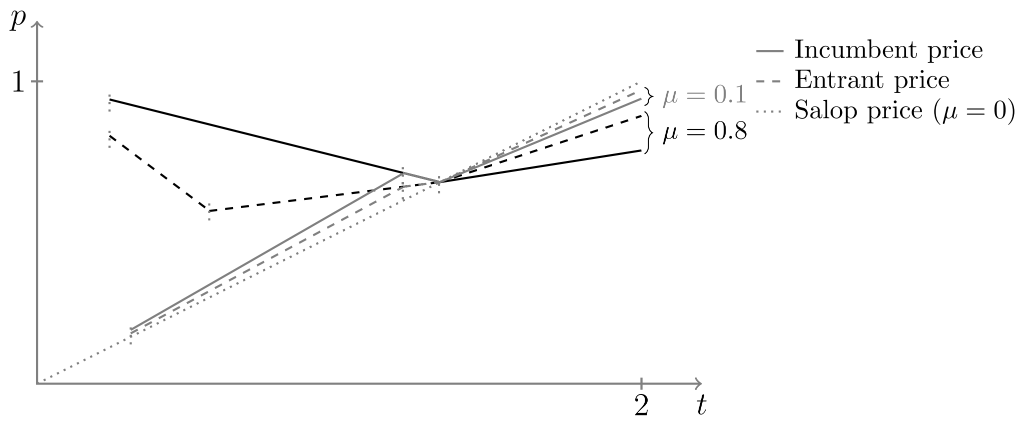


Figure 4: Price effects of t

sloping part of the price curves.[25]

3.3.2 Effects on entrant profits

Figure 2 identifies non-monotone effects of heterogeneity on entry, with the details depending on the share of captive consumers. For instance, for $F = 0.25$ and $\mu = 0.3$, both the no-entry and the entry region are non-convex, consisting of two intervals. Such patterns reflect non-monotone relations between t and entry profits, summarized as follows.

Proposition 4. *Consider the PSE regions.*

(i) Suppose $\mu > 0.6$. There is a U-shaped relation between t and entry profits. The minimum is obtained at $t = \frac{4}{3}$.

(ii) Suppose $0 < \mu < 0.6$. For given μ, entry profits have four extreme values $t_1(\mu) < t_2(\mu) < t_3(\mu) < t_4(\mu)$, where t_1 and t_3 are local minima, whereas t_2 and t_4 are local maxima: $t_1 = \frac{36\mu - 36\mu^2}{15\mu - 8\mu^2 + 9}$ defines the leftern-most point in the CFE regime, $t_2 = \frac{12-12\mu}{9-\mu}$ is the boundary between the CFE regime and the FCE regime, $t_3 = 4/3$ is the boundary between the FCE regime and the ICE regime, and $t_4 = 2$.

The result shows that the relation between t and entry profits is either U-shaped or N-shaped. It is an immediate implication of Corollary 4 in Appendix A.2.2, which shows how entrant profits depend on t within each equilibrium region.[26] Contrary to the case without captive consumers, for every $\mu \in (0, 1)$, there exists a non-degenerate subinterval of $[0, 4/3]$ on which entry profits are decreasing in t. The negative effect of heterogeneity on entry profits in intermediate parameter regions reflects the price effects discussed above.

[25] For the entrant, the decline comes from the reaction to the incumbent's price reduction.

[26] The effect of increasing consumer heterogeneity on the incumbent's profit is similarly complex as for the entrant (see Proposition 4). In particular, there is an intermediate interval of t-values for which the relation between t and incumbent profits is negative.

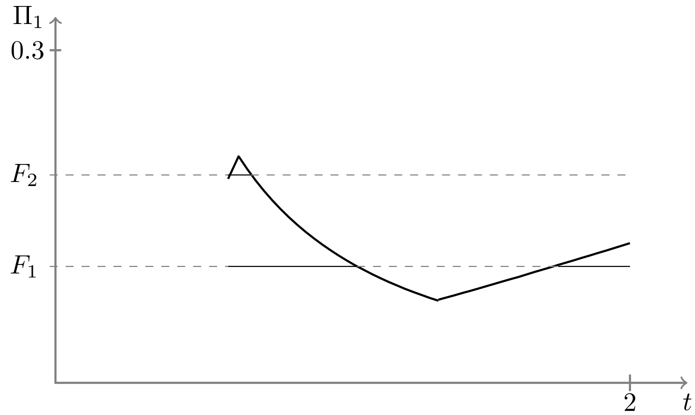


Figure 5: Nonconvexity of entry regions, $\mu = 0.55$, PSE regions

This non-monotone relation between t and entry profits explains the complex relation between t and entry observed in Figure 2. Proposition 4 provides information on how this relation depends on μ. For $\mu > 0.6$ and suitable levels of fixed costs, entry will take place for very low and very high values of t, but not for intermediate levels. For $\mu < 0.6$, Figure 5 shows (fixing $\mu = 0.55$ for illustration) how the observed entry pattern also depends on fixed cost. For $F = F_1$ there are two isolated convex intervals with entry, whereas for $F = F_2$, entry takes place in one intermediate interval.

Differentiated entry is an instance of horizontal product innovation, and taste heterogeneity in our model is inversely related to product substitutability – a standard measure of the intensity of competition. Our paper thus contributes to the literature on the relation between competition and innovation. We find that increased competition intensity from higher product substitutability can lead to more innovation. This would not be the case without captive consumers. These results relate to studies of the effects of suitable competition parameters on cost-reducing or demand-enhancing innovations, which show that even the qualitative nature of the relation depends on market-specific details and that different types of monotonicities are conceivable (Gilbert, 2006; Vives, 2008; Schmutzler, 2010, 2013).[27]

4 Policy Implications

The analysis of captive consumers gives rise to several policy issues. First, policy measures directed at increasing the transparency of markets can serve to reduce the share of captive consumers. Second, the literature has discussed the effects of price discrimination between captive consumers and switchers on entry. Our analysis sheds new light on this discussion in the context of differentiated goods.

[27] By contrast, Aghion et al. (2005) provide some empirical support for an inverse-U shape.

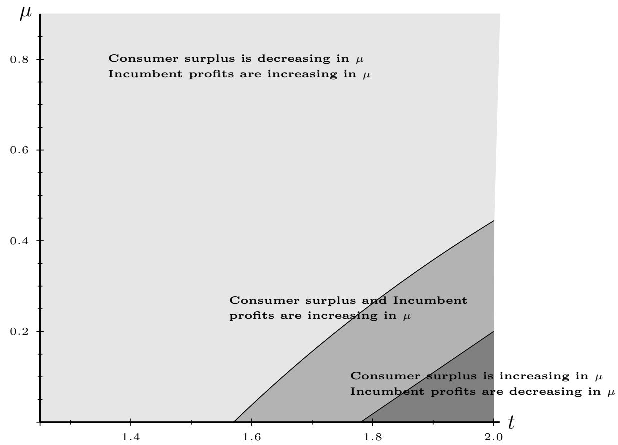


Figure 6: Consumer Surplus vs. Profits in the ICE Region

4.1 Policy towards Captive Consumers

Governments can take measures towards reducing the share of captive consumers themselves, for instance by supporting the dissemination of information about new products.[28] Less directly, governments can decide whether they should restrict activities of incumbents to create captive consumers or whether they should foster activities of entrants to reduce the number of captive consumers.

We first suppose the government can directly influence the share of captive consumers itself. Increases in μ affect consumer surplus through several channels. First, keeping prices fixed, there is a direct weakly negative effect for those consumers who change from being switchers to being captive, as their set of available choices shrinks. Second, there are indirect effects for the consumers who keep their status as captive or flexible, because of induced changes on duopoly prices or market structure. These indirect effects can potentially outweigh the direct effect. The following result summarizes the interplay of direct and indirect effects.

Corollary 1.

(i) Conditional on entry having taken place, an increase in μ reduces total consumer surplus except in a subset of the ICE region that is contained in $[1.56, 2] \times [0, 0.46]$.

(ii) Suppose a marginal increase in μ changes the entry decision. Total consumer surplus falls in regions FCE and MPE; it increases in region CFE. In ICE, the effect on total consumer surplus is negative, except in a subset of $[1.903, 2] \times [0, 0.110]$.

More details on this result are presented in Appendix A.2.4. In particular, the analysis relies on Lemma 6, which gives explicit expressions for consumer surplus. Part (i) relies on the implication of Proposition 1 that in regions CFE, FCE and MPE, the price effect on consumer surplus induced by increasing μ is either negative or zero, so that the adverse

[28] As an illustration, the European Competition Authorities recommended the adoption of transparent and objective price comparison sites for retail banking (OECD (2006)).

direct effect of increasing the captive population determines the overall result. In the ICE region, duopoly prices fall as μ increases. This leads to the possibility that an increase in the share of captive consumers increases consumer surplus in spite of the adverse direct effect that some consumers have less choice. Figure 6 depicts the subset of the ICE region where the effect of μ on consumer surplus is positive.

Result (ii) relies on the previous result that an increase in the share of captive consumers works against entry except in the CFE region. Together with the fact that total consumer surplus in CFE, FCE and MPE is higher in duopoly than in monopoly, the statements for these regions follow. In a small part of region ICE, total consumer surplus increases if a marginal increase in the share of captive consumers deters entry. This reflects the fact that prices in a pure duopoly would be higher than in monopoly in this region.

So far, we have treated the share of captive consumers as directly under the control of the government. Alternatively, we can think of the incumbent as influencing μ. For instance, he may use persuasive advertising to make consumers believe that his product is the only solution to their needs. We now ask two questions. First, under which circumstances would an incumbent want to engage in activities to create captive consumers? Second, how do the incumbent's interests compare with those of consumers? To deal with these issues, one could assume that, before the entrant decides about entry, there is an initial stage in which the incumbent can increase the share of captive consumer at some cost.[29] The incumbent would do so only if this leads to a profit increase that outweighs the costs. In the following, we detail the benefits.

Corollary 2.

(i) A marginal increase in the share of captive consumers that leaves market structure unaffected increases incumbent profits, except in a subset of the ICE region contained in $[1.777, 2] \times [0, 0.2]$.[30]

(ii) A marginal increase in the share of captive consumers that affects market structure leads to higher incumbent profits, except in the CFE region.

The captive share μ thus affects the incumbent's profit positively unless it induces entry or has a strong negative effect on the entrant's price in post-entry equilibrium (see Lemma 2). The latter possibility requires that consumers are sufficiently heterogeneous that the ICE region emerges. In addition, it turns out that the share of captive consumers has to be very small (as in the darkly shaded area in the lower right corner of Figure 6).[31] The former possibility can only arise in the CFE ($t < 4/3$).

We can now deal with the second question: If an incumbent engages in activities to create

[29] This is similar to Chioveanu (2008) who has ex-ante symmetric firms who invest in advertising to create a stock of captive consumers. She finds that firms will chose asymmetric advertising intensities in equilibrium.

[30] For fixed market structure, a negative effect in the ICE region requires $t > \frac{72\mu+40\mu^2+48}{24\mu+5\mu^2+27}$. See Corollary 4.

[31] Corollary 4 in Appendix A.2.2 derives the exact condition.

captive consumers, should one be concerned about adverse effects on consumer surplus? According to our previous arguments, this is not obvious, given that consumers benefit from an increase in captive consumers in two cases: (i) In CFE such an increase can induce entry and (ii) in a small part of the ICE region (for substantial heterogeneity and low initial share of switchers), total consumer surplus increases in μ even if there is no market structure effect. However, Corollary 2 shows that Case (i) will not result from an incumbent's strategy, as he would have lower profit after entry. Case (ii) can arise in a very small part of the ICE region: By Corollary 1, for (t, μ) near $(2, 0)$, consumer surplus is increasing in μ. This region has a non-empty intersection with the area where incumbent profits are increasing in μ (see Figure 6). With this small exception, therefore, any increase in μ that the incumbent would want to bring about is therefore detrimental for consumers. Even though the relation between μ and consumer surplus is not unambiguously negative, it therefore seems warranted for policy makers to keep a careful eye on strategies that create captive consumers.

Finally, the entrant can try to reduce the share of captive consumers by engaging in informative advertising. Straightforward arguments show that there is substantial overlap in the interests of consumers and the entrant: Usually, if the entrant benefits from reductions in the share of captive consumers, so do the consumers.[32] However, there is obviously one important case where the interests are not aligned: In the CFE region, the entrant may abstain from reducing the share of captive consumers to keep competition soft, even though consumers would benefit from such a reduction.

4.2 Price discrimination

Up to now, we have considered the situation in which the incumbent is restricted to charging a uniform price to all his buyers. An important and active debate discusses the impact of price discrimination on entry. One of the main contributions is Armstrong and Vickers (1993). They show that banning an incumbent's possibility to discriminate between captive and switching consumers will help entry through the effect of relaxed competition. The incumbent's uniform price is higher than the competitive one because he exploits the captive consumers. Similar results have, for instance, been obtained by Anton, Vander Weide, and Vettas (2002) and Bouckaert, Degryse, and van Dijk (2013).

We now demonstrate that consumer heterogeneity plays a significant role also for the effects of price discrimination. Depending on the extent of heterogeneity, price discrimination will have opposite effects on entry. We have already shown that, in the absence of price discrimination, a larger captive market share can help entry for low values of consumer heterogeneity. When price discrimination is possible, so that the incumbent can set different prices for the captive and switching consumers, this effect vanishes. Each of these two prices is now independent of μ, and since the captive market previously took competitive pressure off the switcher market, the incumbent now prices more aggressively in the switcher market, thereby reducing the entrant's attainable profit. When consumer heterogeneity is high, however, this result is reversed, because the incumbent needs to set a low price in order to attract his captive

[32] An exception is the part of the ICE region that consists of the two shaded areas in the lower right corner of Figure 6. Here the entrant would want a reduction in μ, but the consumers would not.

consumers. At the same time, the price in the duopolistic segment would be relatively high because consumers substitute the products less willingly. It follows that in the ICE, where heterogeneity is large, price discrimination makes entry more viable.

To see this more formally, notice that with price discrimination, the two firms compete on the circle as in the standard oligopoly model. Denote the incumbent's price for switchers by $p_{0,s}^{pd}$ and the entrant's price as p_1^{pd}. Under Assumption 2, the equilibrium prices in the switcher market will be

$$p_{0,s}^{pd} = p_1^{pd} = \frac{t}{2}.$$

Each firm sells to half of the switchers, so that the entrant makes a profit of

$$\Pi_1^{pd} = (1-\mu)\frac{t}{4}.$$

Lemma 5 in the Appendix shows that the entrant's profit without discrimination in CFE, FCE, and MPE is higher than $(1-\mu)\frac{t}{4}$, but in ICE, it is lower than $(1-\mu)\frac{t}{4}$. We thus have the following result:

Corollary 3. *When price discrimination by the incumbent is allowed, it reduces the entrant's post-entry profits when* $t < \frac{4}{3}$*, but increases them when* $t \geq \frac{4}{3}$*.*

5 Conclusion

Incumbent firms often possess advantages over entrants stemming from consumer inattention. This paper asks under which circumstances entry arises despite this. We propose a framework to analyze incentives for innovative entry with differentiated goods into a market previously dominated by a monopolist. We show how the share of captive consumers and consumer taste heterogeneity interact to determine entry. For low initial levels of taste heterogeneity and captive consumers, an increase in the share of captive consumers is conducive to entry, reflecting its competition-softening effect. As the share of captive consumers increases beyond a certain level, further increases are detrimental to entry as a result of the reduction in the entrant's potential demand. By contrast, for high initial levels of heterogeneity, entry profits are monotone decreasing in the share of captive consumers.

Our analysis also contributes to the literature on competition and innovation. For arbitrary positive shares of captive consumers, there exists an intermediate interval on which increasing taste heterogeneity (equivalent to decreasing product substitutability) reduces entry profits. This is in marked contrast to the case without captive consumers, where greater consumer heterogeneity always fosters entry. Thus, once one introduces captive consumers, the relation between consumer heterogeneity and horizontal product innovation becomes similarly complex as the relation between consumer heterogeneity and "vertical" (cost-reducing or quality-enhancing) innovation.

In spite of the ambiguous comparative statics conclusions, our analysis has clear competition policy implications. First, while, for low consumer heterogeneity, forbidding price discrimination fosters entry, the inverse relation holds for large heterogeneity. Second, if an incumbent engages in activities to create captive consumers, this lowers consumer welfare except in a very small parameter region. Consumer heterogeneity matters for both of these results because it determines whether entry increases or decreases prices.

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A Appendix

A.1 General model

A.1.1 Proof of Lemma 1

Total differentiation of the first-order conditions implies that, for $i = 0, 1$ and $j \neq i$,

$$\frac{dp_i}{d\mu} = \frac{\frac{\partial^2 \Pi_i}{\partial p_i \partial p_j} \frac{\partial^2 \Pi_j}{\partial p_j \partial \mu} - \frac{\partial^2 \Pi_i}{\partial p_i \partial \mu} \frac{\partial^2 \Pi_j}{\partial p_j^2}}{\frac{\partial^2 \Pi_i}{\partial p_i^2} \frac{\partial^2 \Pi_j}{\partial p_j^2} - \frac{\partial^2 \Pi_i}{\partial p_i \partial p_j} \frac{\partial^2 \Pi_j}{\partial p_j \partial p_i}},$$

where all expressions are evaluated at the equilibrium. By the stability condition, the denominator is positive. Using the entrant's first-order condition, $\frac{\partial^2 \Pi_1}{\partial p_1 \partial \mu} = 0$ at the equilibrium. Hence, $dp_0/d\mu > 0$ if and only if $-\frac{\partial^2 \Pi_0}{\partial p_0 \partial \mu} \frac{\partial^2 \Pi_1}{\partial p_1^2} > 0$, that is, using concavity of Π_1, $\frac{\partial^2 \Pi_0}{\partial p_0 \partial \mu} > 0$. Similarly, $dp_1/d\mu > 0$ if and only if $\frac{\partial^2 \Pi_1}{\partial p_0 \partial p_1} \frac{\partial^2 \Pi_0}{\partial p_0 \partial \mu} > 0$, that is, using strategic complements (A1(b)), $\frac{\partial^2 \Pi_0}{\partial p_0 \partial \mu} > 0$. Finally, $\frac{\partial^2 \Pi_0}{\partial p_0 \partial \mu} > 0$ if and only if

$$D_M - D_0 + p_0 \left(\frac{\partial D_M}{\partial p_0} - \frac{\partial D_0}{\partial p_0} \right) > 0.$$

Simple rearrangements show that this condition is equivalent to conditions (2) and (3) in the proposition.

A.1.2 Proof of Lemma 2

(i) Using (1), it suffices to show that (4) is equivalent to $\frac{\partial \Pi_1}{\partial \mu} + \frac{\partial \Pi_1}{\partial p_0} \frac{\partial p_0^*}{\partial \mu} > 0$. This equivalence follows from $\frac{\partial \Pi_1}{\partial \mu} = -p_1 D_1$ and $\frac{\partial \Pi_1}{\partial p_0} = p_1 (1 - \mu) \frac{\partial D_1}{\partial p_0}$ after simple rearrangements, using $\frac{\partial D_1}{\partial p_0} > 0$ (by A1(i)).

(ii) Using (1), it suffices to show that (5) is equivalent to $\frac{\partial \Pi_0}{\partial \mu} + \frac{\partial \Pi_0}{\partial p_1} \frac{\partial p_1^*}{\partial \mu} > 0$. This equivalence follows from $\frac{\partial \Pi_0}{\partial \mu} = p_0 (D_M - D_0)$ and $\frac{\partial \Pi_0}{\partial p_1} = p_0 (1 - \mu) \frac{\partial D_0}{\partial p_1}$ after simple rearrangements, using $\frac{\partial D_0}{\partial p_1} > 0$ (implied by A1(i)).

A.2 Salop model

A.2.1 Proof of Proposition 1

(i) For the proposed equilibrium price $p_0 = \frac{(3-\mu)t + 8\mu}{2(5\mu+3)}$, $D_0^M(p_0) = 2\mu \frac{1 + \frac{t}{4} - p_0}{t} < \mu$ when $t > \frac{4}{3}$, so that the incumbent does not serve all captive consumers. Further, given the suggested prices p_0 and p_1, the indifferent switcher has non-negative valuation and is located between positions 0 and $\frac{1}{2}$ for $t \leq 2$. The demand from switchers is derived from the indifferent consumer, who is located at $x = \frac{\frac{t}{2} - p_0 + p_1}{2t}$. The profit of the incumbent is thus $\Pi_0 = p_0 \left[\mu \left(2 \frac{1 + \frac{t}{4} - p_0}{t} \right) + (1 - \mu) \left(\frac{\frac{t}{2} - p_0 + p_1}{t} \right) \right]$. The entrant's demand comes only from switchers. Her profit is thus $\Pi_1 = (1 - \mu) p_1 \frac{\frac{t}{2} + p_0 - p_1}{t}$. The entrant's interior best-response condition is thus: $p_1 = \frac{t+2p_0}{4}$. First-order conditions give equilibrium price candidates $p_0 = \frac{(3-\mu)t+8\mu}{2(5\mu+3)}$ and $p_1 = \frac{(3+2\mu)t+4\mu}{2(5\mu+3)}$. Second-order conditions are easily checked.

(ii) At the proposed price $p_0 = 1 - \frac{t}{4}$, the captive market is exactly covered as the net utility of the captive consumer located at $\frac{1}{2}$ equals zero. Moreover, the indifferent switcher has non-negative utility because all consumers would be prepared to buy from the incumbent. Further, at the proposed prices the indifferent consumer is located between 0 and $\frac{1}{2}$ because $t > \frac{4}{7}$. As in case (i), the entrant reacts to p_0 according to the reaction function $p_1 = \frac{t+2p_0}{4}$ and thus sets $p_1 = \frac{1}{2} + \frac{t}{8}$. We now need to check for possible profitable deviations of the incumbent. For an upward deviation, the elastic demand from case (i) applies, for a downward deviation, the demand of captive consumers will remain fixed at one. Considering this, the incumbent has no profitable upward deviation when $t \leq \frac{4}{3}$. It has no profitable downward deviation when $t > 12\frac{1-\mu}{9-\mu}$, which is equivalent to $\mu > \frac{9t-12}{t-12}$.

(iii) In the proposed equilibrium, the captive market is covered and the captive consumer at $\frac{1}{2}$ obtains a positive surplus. The incumbent thus has an inelastic captive demand of μ. $t < 12\frac{1-\mu}{9-\mu}$ guarantees that all captive consumers get a positive surplus. The indifferent switcher is located between 0 and $\frac{1}{2}$ when $\mu < 0.6$, and the incumbent's demand from switchers thus is $(1-\mu)\left(\frac{1}{2}\frac{t-2p_0+2p_1}{t}\right)$. Its profit is $\Pi_0 = p_0\left(\mu + (1-\mu)\left(\frac{1}{2}\frac{t-2p_0+2p_1}{t}\right)\right)$. The entrant's profit function remains the same as in the previous cases ($\Pi_1 = p_1(1-\mu)\left(\frac{1}{2}\frac{t-2p_1+2p_0}{t}\right)$). The proposed equilibrium candidate prices $p_0 = \frac{(3+\mu)t}{(3-3\mu)2}$ and $p_1 = \frac{(3-\mu)t}{(3-3\mu)2}$ thus fulfill the firm's respective first-order conditions. We further need to check deviations by the incumbent to $\widehat{p}_0 = 1 - \frac{t}{4}$ where he only serves the captive market. When $t > \frac{36\mu-36\mu^2}{15\mu-8\mu^2+9}$, no profitable deviation of this type exists. First note that for $t > \frac{12\mu-12}{11\mu-15}$ (which implies $t > \frac{36\mu-36\mu^2}{15\mu-8\mu^2+9}$), the price $1-\frac{t}{4}$ is so low that the incumbent would still serve switchers: To see this, compare the net utility of the consumer located at 0, which is $1+\frac{t}{4}-1+\frac{t}{4} = \frac{t}{2}$ when buying from the deviating incumbent, whereas at the entrant it is $1 - \frac{t}{4} - \frac{1}{6}t\frac{3-\mu}{1-\mu} = \frac{1}{12}\frac{-9t-12\mu+5t\mu+12}{1-\mu}$. After some rearrangements, it follows that the consumer located at 0 still prefers to buy from the incumbent. Thus suppose $t < \frac{12\mu-12}{11\mu-15}$, so that the deviation is feasible. As the profit before deviation is $t\frac{\mu+3}{6-6\mu}\left(\mu + (1-\mu)\left(\frac{\frac{t}{2} - t\frac{\mu+3}{6-6\mu} + t\frac{3-\mu}{6-6\mu}}{t}\right)\right) = \frac{1}{36}t\frac{(\mu+3)^2}{1-\mu}$, the net benefit from deviation is $\mu\left(1-\frac{t}{4}\right) - \frac{1}{36}t\frac{(\mu+3)^2}{1-\mu} = \frac{1}{36}\frac{-9t+36\mu-15t\mu-36\mu^2+8t\mu^2}{1-\mu}$. This expression remains negative as long as $-9t+36\mu-15t\mu-36\mu^2+8t\mu^2 < 0$, or equivalently, $t > \frac{36\mu-36\mu^2}{15\mu-8\mu^2+9}$.

(iv) At the proposed prices, the incumbent serves all captive consumers; with the marginal consumer at $t = 1/2$ obtaining a net surplus of zero. The entrant serves all switchers at the highest possible price such that all switchers buy from it: the switcher at position zero is indifferent between buying from the entrant and buying from the incumbent. The incumbent's profit is $\mu\left(1-\frac{t}{4}\right)$; the entrant's profit is $(1-\mu)\left(1-\frac{3t}{4}\right)$. We now test for deviations. As $t < 4/7 < 4/3$, the incumbent cannot benefit from an increase of the prices to captive consumers. Arguing as in (iii), if the incumbent lowers his prices to some lower value p_0, his profit becomes $\Pi_0 = p_0\left(\mu + (1-\mu)\left(\frac{1}{2}\frac{t-2p_0+2p_1}{t}\right)\right)$. Such a deviation is not profitable when $t \geq 4\frac{1-\mu}{3\mu+1}$. The entrant's profit function remains the same as in the previous cases ($\Pi_1 = p_1 (1-\mu) \left(\frac{1}{2}\frac{t-2p_1+2p_0}{t}\right)$). The entrant has no incentive to lower its price. An upward deviation is not profitable when $t \leq \frac{4}{7}$.

(v) First, consider equilibria where the switcher market is not completely covered (and thus the captive market is not covered either). A firm that marginally increases prices would thus not lose any demand to the competitor. Hence, its optimal price must correspond to the price of a monopolist with demand $2\frac{1+t/4-p}{t}$. Under Assumption 2, maximization of this function would lead to a price that is so low that demand would we greater than 1, a contradiction. Thus, in the following, we can rule out equilibria with incomplete coverage in the switcher market. Second, consider an equilibrium with incomplete coverage in the captive market, but not in the switcher market. Contrary to the ICE in (i), first assume that there is market partition. Then the price of the incumbent must correspond to the optimum on the monopolistic captive market. As $t < 2$ by Assumption 2, this price is inconsistent with incomplete coverage. Thus any equilibrium with incomplete coverage in the captive market must have both firms competing actively in the switcher market. Then the first-order conditions imply that it must have the form of the equilibrium in (i). Thus, we can rule out additional incomplete coverage equilibria. Third, we rule out additional equilibria with full coverage in both markets. There can be no such equilibrium where market partition takes place and, unlike in (iii), the marginal consumer earns positive surplus. In this case, the monopolist could profitably raise prices. Thus, any equilibrium with full coverage and market partition must give zero surplus to the marginal captive consumers, as in (iii). This fixes both prices as as in (iii). As to equilibria with full coverage, but without market partition, they either give zero surplus to marginal captive consumers as in (ii) or a positive surplus as in (iv). In the former case, the zero surplus requirement immediately determines both prices as in (ii). In the latter case, the equilibrium conditions must necessarily hold for both firms. Finally, it is straightforward to see that the region where none of the conditions (i)-(iv) holds is given by $\mu \in (0,1)$ and $t < \min\left(\frac{36\mu-36\mu^2}{15\mu-8\mu^2+9}, \frac{4-4\mu}{3\mu+1}\right)$. Thus, by the previous analysis, first note that the demand functions under pure duopoly are given as[33]

$$D_i(p_i, p_j) = \begin{cases} 0 & \text{if} & p_i \geq p_j + \frac{t}{2} \\ \frac{t/2-p_i+p_j}{t} & \text{if} & \frac{t}{2} - p_j < p_i < \frac{t}{2} + p_j \leq 1 + \frac{t}{4} \\ 1 & \text{if} & p_i \leq p_j - \frac{t}{2} \\ \frac{2(1+t/4-p_i)}{t} & \text{otherwise.} \end{cases} \tag{6}$$

As these functions (and thus profit functions) are continuous and strategy spaces are non-empty and compact, the existence of an MSE immediately follows from Theorem 3 in Dasgupta and Maskin (1986).[34] For homogeneous goods, $t = 0$, this result no longer applies because the payoff functions are not continuous. However, standard calculations show that

[33] The last case below corresponds to the monopolistic situations because the market is not fully covered.
[34] Without loss of generality, we can restrict price choices to the interval $[0, 1 + t/4]$

the pricing game has a unique MSE with price distributions of the entrant given as

$$G(p) = \begin{cases} 0 & \text{if} \quad p < \mu \\ \frac{p-\mu}{(1-\mu)p} & \text{if} \quad p \geq \mu \end{cases}$$

and for the incumbent as

$$F(p) = \begin{cases} 0 & \text{if} \quad p < \mu \\ 1 - \frac{\mu}{p} & \text{if} \quad \mu \leq p < 1 \\ 1 & \text{if} \quad p = 1. \end{cases}$$

Thus, there must be an atom of size $(1-\mu)$ *at* $p = 1$.

(vi) We now show that the following prices and mixing probabilities constitute an equilibrium. The incumbent mixes between two prices, p_0^h and p_0^l, with probability α and $1-\alpha$, respectively. The entrant sets the single price p_1. Prices and probabilities are:

$$p_0^h = 1 - \frac{t}{4},$$

$$p_0^l = \frac{t}{2} \cdot \frac{3\mu(1-\mu)^2(4-t) + (1+3\mu)\sqrt{\mu(1-\mu)^3 t(4-t)}}{(1-\mu)\left[t(2\mu - 3\mu^2 + 1) + 3\sqrt{\mu(1-\mu)^3 t(4-t)}\right]},$$

$$\alpha = \frac{t\,(19\mu - 6\mu^2 + 3) - 36\mu(1-\mu) + 6\sqrt{\mu(1-\mu)^3 t(4-t)}}{t(15\mu + 1) - 36\mu(1-\mu)},$$

and

$$p_1 = \frac{t}{2} \cdot \frac{24\mu(1-\mu)^2 - t(9\mu - 13\mu^2 + 3\mu^3 + 1) - (1-3\mu)\sqrt{\mu(1-\mu)^3 t(4-t)}}{(1-\mu)\left[t(2\mu - 3\mu^2 + 1) + 3\sqrt{\mu(1-\mu)^3 t(4-t)}\right]}.$$

Given price p_1, the incumbent serves only the captive consumers when he sets the high price p_0^h. It is thus indeed optimal for the incumbent to choose the monopoly price, i.e. $p_0^h = 1 - \frac{t}{4}$. Further, we now know that the incumbent's profit when he sets p_0^h is equal to $\mu\left(1 - \frac{t}{4}\right)$. When the incumbent set his low price p_0^l, he competes with the entrant for switchers. p_0^l now needs to satisfy the following two conditions. (a) p_0^l gives the incumbent the same profit as p_0^h, given p_1. (b) p_0^l is a best reply to p_1. This is indeed the case.

The entrant's price p_1, in turn, needs to be a best reply to the incumbent's expected price, $\overline{p}_0 = \alpha p_0^h + (1-\alpha)p_0^l$. This is verified as well.

It is readily verified that $\overline{p}_0$ and p_1 converge to the incumbent's and entrant's respective unique prices in the bordering PSE regions (MPE and CFE) exactly at those borders. The right-hand side delimiters of the MSE region are thus confirmed.

The incumbent's optimal deviation is to set the monopoly price $1 - \frac{t}{4}$ with probability 1. In the online supplementary material to this paper, we show that this happens to the left of the locus of (t, μ)-points where the incumbent's switcher demand at p_0^l will be equal to one.

The entrant's optimal deviation is to set the higher price $1 - \frac{3}{4}t$, which is her optimal reply to incumbent price $1 - \frac{t}{4}$ (see the case of the MPE). This happens to the left of the (t, μ) locus at which her expected profit from doing so is equal to her profit from the proposed equilibrium. This locus can be represented by a function $t = b(\mu)$. In the supplementary material, we show that this locus lies everywhere to the right of the incumbent's critical deviation line. This establishes the left-hand side delimiter of the MSE region.

Thus, the proposed prices and choice probability constitute an equilibrium for

$$b(\mu) \leq t \leq \begin{cases} \frac{36\mu(1-\mu)}{15\mu - 8\mu^2 + 9}, & \text{if } \mu \leq 0.6, \\ \frac{4(1-\mu)}{3\mu+1}, & \text{if } \mu > 0.6. \end{cases}$$

A.2.2 Profits

As an immediate corollary of Proposition 1, we obtain the following characterization of equilibrium profits.

Lemma 5. *The profits in the PSE regions are given as follows:*

(i) ICE: $\Pi_0 = \frac{1+\mu}{4t} \frac{(3t+8\mu-t\mu)^2}{(5\mu+3)^2}$ *and* $\Pi_1 = (1-\mu)\frac{t}{4}\left(\frac{3t+2t\mu+4\mu}{3t+5t\mu}\right)^2$

(ii) CFE: $\Pi_0 = (1-\mu)\left(\frac{3+\mu}{3-3\mu}\right)^2 \frac{t}{4}$ *and* $\Pi_1 = (1-\mu)\frac{t}{4}\left(\frac{3-\mu}{3-3\mu}\right)^2$

(iii) FCE: $\Pi_0 = \frac{7t+4\mu+t\mu-4}{8t}\left(1-\frac{t}{4}\right)$ *and* $\Pi_1 = (1-\mu)\frac{t}{4}\left(\frac{t+4}{4t}\right)^2$

(iv) MPE: $\Pi_0 = \mu\left(1-\frac{t}{4}\right)$ *and* $\Pi_1 = (1-\mu)\frac{t}{4}\left(\frac{4}{t}-3\right)$

Proof. Proposition 1 gives the equilibrium prices. Inserting these prices into the profit expressions using the demand functions 6 gives the results. ■

Lemma 5 is useful to understand how profits depend on parameters.

Corollary 4.

(i) Π_0 *is increasing in* μ *everywhere except in the ICE region for* $t > \frac{72\mu+40\mu^2+48}{24\mu+5\mu^2+27}$. Π_0 *is increasing in t in the CFE region and decreasing in the MPE region. It is increasing in t in the ICE region if and only if* $\mu < \frac{3t}{t+8}$. *It is increasing in the FCE region if and only if* $\mu < \frac{16-7t^2}{t^2+16}$

(ii) Π_1 *is increasing in* μ *in CFE, decreasing in the remaining regions.* Π_1 *is increasing in t in ICE and CFE, decreasing in the remaining regions.*

Proof. (i) From Proposition 1, it follows that p_1 is non-decreasing in μ except in the ICE region. Hence Π_0 is increasing in μ everywhere except possibly in the ICE region. There, $\Pi_0 = \frac{1+\mu}{4t}\frac{(3t+8\mu-t\mu)^2}{(5\mu+3)^2}$. Hence

$$\frac{\partial \Pi_0}{\partial \mu} = \frac{1}{4}\left(-3t - 8\mu + t\mu\right)\frac{27t - 72\mu + 24t\mu - 40\mu^2 + 5t\mu^2 - 48}{t\left(5\mu+3\right)^3}.$$

This expression is positive if and only if the numerator is negative or $t < \frac{72\mu+40\mu^2+48}{24\mu+5\mu^2+27}$.

Next, we consider the relation between t and Π_0. In the CFE region, $\Pi_0 = (1-\mu)\left(\frac{3+\mu}{3-3\mu}\right)^2 \frac{t}{4}$, which is increasing in t. In the MPE region $\Pi_0 = \mu\left(1 - \frac{t}{4}\right)$, which is is decreasing in t. In the ICE region, $\frac{\partial \Pi_0}{\partial t} = \frac{1}{4}\left(\mu+1\right)\left(-3t - 8\mu + t\mu\right)\frac{-3t+8\mu+t\mu}{t^2(5\mu+3)^2}$. This is positive only if $\mu < 3\frac{t}{t+8}$ In the FCE region, $\Pi_0 = \frac{7t+4\mu+t\mu-4}{8t}\left(1 - \frac{t}{4}\right)$. Thus $\frac{\partial \Pi_0}{\partial t} = -\frac{1}{32t^2}\left(16\mu + t^2\mu + 7t^2 - 16\right)$. This is positive only if $\mu < \frac{16-7t^2}{t^2+16}$.

(ii) From Proposition 1, it follows that p_0 is non-increasing in μ except in the CFE region. Hence, Π_1 is decreasing in μ everywhere except possibly in the CFE region. There $\Pi_1 = (1-\mu)\left(\frac{3-\mu}{3-3\mu}\right)^2 \frac{t}{4}$. Hence, $\frac{\partial \Pi_1}{\partial \mu} = \frac{1}{36}\frac{t}{(\mu-1)^2}\left(-\mu^2 + 2\mu + 3\right) > 0$. Now consider the relation between t and Π_1. In the ICE region, $\Pi_1 = \frac{1-\mu}{4t}\frac{(3t+4\mu+2t\mu)^2}{(5\mu+3)^2}$. Hence $\frac{\partial \Pi_1}{\partial t} = \frac{1-\mu}{4t}\frac{(3t+4\mu+2t\mu)^2}{(5\mu+3)^2} > 0$. In the FCE region $\frac{\partial \Pi_1}{\partial t} = \frac{1}{64t^2}\left(1-\mu\right)\left(t^2 - 16\right) < 0$. In the CFE region, $\Pi_1 = (1-\mu)\left(\frac{3-\mu}{3-3\mu}\right)^2 \frac{t}{4}$, which is increasing in t. In the MPE region, $\Pi_1 = (1-\mu)\left(1 - \frac{3}{4}t\right)$, which is decreasing in t. ■

A.2.3 Entry

Proof of Proposition 3 (i) By Proposition 1, for $0 < t < 4/3$, the CFE arises for $\mu = 0$. In the CFE, $\Pi_1 = (1-\mu)\left(\frac{3-\mu}{3-3\mu}\right)^2 \frac{t}{4}$. If $F \leq t/4$, $\Pi_1 \geq F$ for $\mu = 0$. Thus, entry takes place for $\mu = 0$. As Π_1 is increasing in μ, it takes its maximum in the CFE region at the upper boundary of this region and entry takes place in the entire CFE region. In the MPE and FCE regions, Π_1 is monotonically decreasing to zero. Thus entry takes place up to a cut-off value of μ for which (t, μ) is in the MPE or FCE region.
Next, suppose $t/4 < F$. Then there is no entry for $\mu = 0$. As Π_1 is increasing in the CFE region, but decreasing in the MPE and FCE regions, if entry takes place, it must arise for the highest μ in the CFE region or the lowest μ in the MPE or FCE region.

(a) First consider the subcase that $4/7 \leq t < 4/3$. Then the highest μ in the CFE region and the lowest μ in the FCE region coincide, so that the maximum profit arises for $t = \frac{12-12\mu}{9-\mu}$ or equivalently $\mu = \mu^B(t) = \frac{9t-12}{t-12}$ by Proposition 1. Inserting this into the entrant's profit expression from Lemma 5 gives $\frac{1}{8}\frac{(t+4)^2}{12-t}$. The condition that $F \leq \frac{1}{8}\frac{(t+4)^2}{12-t}$ gives the result: For given t, entry takes place if and only if (t, μ) is in a suitable neighborhood of the regime boundary.

(b) If instead $t < 4/7$, the lower boundary of the MPE region and the upper boundary of the CFE region do not coincide, so that they both are candidates for the maximal entry profit. Comparing the entrant's profits at both boundaries using the profit expressions from Lemma 5 and the regime boundaries from Proposition 1 shows that they are always higher at the lower boundary of the MPE region than at the upper boundary of the CFE region. The entry condition becomes $(1-\mu)\left(1-\frac{3}{4}t\right) \geq F$ for the value of μ at the lower boundary of the MPE region, which satisfies $\frac{4-4\mu}{3\mu+1} = t$ or equivalently $\mu = \frac{4-t}{3t+4}$; thus $\left(1-\frac{4-t}{3t+4}\right)\left(1-\frac{3}{4}t\right) = t\frac{4-3t}{3t+4} \geq F$ must hold for entry to take place at the lower boundary of the MPE region.

(ii) follows because the entrant's profits are decreasing in μ in the ICE region ($t > 4/3$), with the minimum of 0 attained for $\mu = 1$.

A.2.4 Consumer surplus

This section collects the computations of consumer surplus in the (pure-strategy) equilibria of the Salop model. They support the claims made in Section 4.1 and, in particular Corollary 1. Recalling the fact that a consumer's surplus from buying product i is given by $1+\frac{t}{4}-p_i-td_i$, straightforward calculations yield the following results.

Lemma 6. *Consumer surplus in each equilibrium region is given by the following terms.*

(i) In the ICE region,

$$CS = \frac{16\mu(9+8\mu-\mu^2)+8t(18+15\mu-10\mu^2-7\mu^3)-t^2(54+9\mu-78\mu^2-\mu^3)}{16t(3+5\mu)^2}$$

(ii) In the FCE region,

$$CS = \mu\frac{t}{4}+(1-\mu)\frac{16+8t+33t^2}{128t}$$

(iii) In the CFE region,

$$CS = 1-\frac{t(27+18\mu-\mu^2)}{72(1-\mu)}$$

(iv) In the MPE region,

$$CS = \mu\frac{t}{4}+(1-\mu)3\frac{t}{4}$$

Inspecting these expressions, it is clear that total consumer surplus is decreasing in μ in the FCE, CFE, and MPE regions. Part (i) of Corollary 1 then follows by noting the comparative statics in the ICE region: When t lies above a critical threshold (roughly $t \approx 1.5683$), consumer surplus is first increasing in μ and then falling once μ increases above a threshold that is itself increasing in t (when $t = 2$, the threshold is roughly $\mu \approx 0.4599$); the parameter region is depicted in Figure 6.[35]

As to part (ii) of Corollary 1, entry of the new product clearly increases consumer surplus in the FCE, CFE, and MPE regions, reflecting lower prices in duopoly than in monopoly. In the ICE region, however, there exists a region for high t and low μ in which entry decreases consumer surplus. To see this, note that, in the ICE region, the consumer surplus under monopoly is $\frac{(4+t)^2}{64t}$. Comparing this expression with the expression in (i) above, it is straightforward to show that the set of all (t, μ) for which the consumer surplus under monopoly is lower than under duopoly is bounded above by an upward-sloping line that intersects the t-axis at approximately 1.9038 and the μ-axis at approximately 0.10906. Thus, the set is a subset of $[1.903, 2] \times [0, 0.110]$.

A.3 Equilibrium structure with general demand

In this section, we analyze the equilibrium structure in a general discrete choice model that contains the Salop example as a special case, but is itself a special case of the general model of Section 2. We replace the assumptions on consumers in Section 3 as follows.

Assumption 3. *(i) There is continuum of consumers with mass 1.*

(ii) Each consumer has unit demand.

(iii) There is an atomless joint valuation distribution $F(v_0, v_1)$ with support $\mathcal{V} \subseteq \left[\underline{V}, \overline{V}\right] \times \left[\underline{V}, \overline{V}\right]$ for some $\overline{V} > \underline{V} \geq 0$ and joint density $f(v_0, v_1)$. F has positive mass in every neighborhood of $\left(\underline{V}, \overline{V}\right)$; similarly for $\left(\overline{V}, \underline{V}\right)$.

(iv) The distribution of valuation differences $v_1 - v_0$ has a density that is everywhere positive.

Note that we do not assume that all valuation profiles in $\left[\underline{V}, \overline{V}\right] \times \left[\underline{V}, \overline{V}\right]$ have positive density. The positivity requirements are sufficiently weak that the model of Section 3 fulfills Assumption 4 with $\left(\underline{V}, \overline{V}\right) = \left(1 - \frac{t}{4}, 1 + \frac{t}{4}\right)$ We write $\Delta = \overline{V} - \underline{V}$ to denote the maximal valuation advantages.

We show that there are no other conceivable types of PSE except those discussed in Proposition 1, except for a *Doubly Incomplete Coverage Equilibrium (DICE)*, in which neither the captive market nor the switcher market is covered.

Lemma 7. *Suppose $t > 0$. (i) There can be no PSE such that $p_i^* = 0$ or $p_i^* \geq \overline{V}$ for $i = 0$ or $i = 1$.*
(ii) In any equilibrium with $0 < p_0^* < \underline{V}$, both firms must be active in the switcher market; the entrant's price must satisfy $p_1^* \in (p_0^* - \Delta, p_0^* + \Delta)$. Thus, the equilibrium must be a CFE.
(iii) Any equilibrium with $p_0^* = \underline{V}$ must satisfy one of the two following constellations:

[35] The boundary of the parameter region is given by the equation $5t^2\mu^3 + 9t^2\mu^2 + 513t^2\mu - 280t\mu^3 - 504t\mu^2 - 1080t\mu - 1080t - 80\mu^3 - 144\mu^2 + 48\mu + 432 = 0$.

(a) $p_1^* = p_0^* - \Delta = \underline{V} - \Delta$*, which corresponds to an MPE;*[36]
(b) $p_1^* \in (p_0^* - \Delta, p_0^* + \Delta) = \left(\underline{V} - \Delta, \overline{V}\right)$*, which results in an FCE.*
(iv) In any equilibrium with $\overline{V} > p_0^* > \underline{V}$*, the captive market is incompletely covered. One of the following cases can arise:*
(a) $p_1^* \in (p_0^* - \Delta, \underline{V}]$*: In this case, the switcher market is jointly covered by the two firms. Thus, an ICE exists.*
(b) If $p_1^* \in \left(\underline{V}, \min\left\{p_0^* + \Delta, \overline{V}\right\}\right)$*, the switcher market is also incompletely covered; thus a DICE arises.*

Proof. (i) As there is a positive measure of consumers with $0 \leq v_0 < v_1$ and $0 \leq v_1 < v_0$ respectively, there can be no equilibrium with $p_0^* = 0$. If $p_i^* \geq \overline{V}$, then the firm earns zero profits. It can avoid this, for instance by setting positive prices sufficiently close to zero.
(ii) By contradiction. If $p_1^* \leq p_0^* - \Delta$, then firm 1 would get the entire switcher demand.[37] Firm 0 could therefore deviate by increasing the price, thereby increasing profits in the loyal market without any losses in the switcher market (where profits are zero anyway). If $p_1^* \geq p_0^* + \Delta$, then the entrant would have zero demand. A deviation to any positive price below $p_0^* + \Delta$ would be profitable. This is a CFE because even the consumers with the lowest valuations obtain a positive net surplus.
(iii) By contradiction. $p_1^* \geq p_0^* + \Delta = \overline{V}$ is inconsistent with Result (i). If $p_1^* < p_0^* - \Delta = \underline{V} - \Delta$, then market partition arises, but firm 1 could increase the price without losing demand. Thus the only remaining possibilities are $p_1^* = p_0^* - \Delta$, which corresponds to an MPE, or $p_1^* \in (p_0^* - \Delta, p_0^* + \Delta)$, in which case both firms have positive demand.
(iv) $p_0^* > \underline{V}$ implies incomplete coverage of the captive market. As we already know $p_0^* < \overline{V}$, if $p_1^* < p_0^* - \Delta$, the entrant could profitably increase prices without losing demand. If $p_1^* \geq p_0^* + \Delta$, the entrant would have zero demand. Thus $p_1^* \in (p_0^* - \Delta, p_0^* + \Delta)$. Clearly, either (a) or (b) must hold. ■

[36]Here and elsewhere, we stay agnostic about the behavior of consumers who are indifferent between the two firms: As they have mass zero, we can neglect them.
[37]As there are no mass points by assumption, we ignore the possibility of ties.