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DEBT AND DEFICIT CEILINGS, AND SUSTAINABILITY OF FISCAL POLICIES: AN INTERTEMPORAL ANALYSIS

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ABSTRACT

Debt and Deficit Ceilings, and Sustainability of Fiscal Policies: An Intertemporal Analysis*

This paper examines the consequences for the sustainability of fiscal policy of imposing restrictive ceilings on deficits and debt. Our theoretical framework is a generalization of the government intertemporal budget constraint which allows for time-varying interest rates, endogenous primary deficits, a finite planning horizon and future policy shifts. We show how published forecasts can be used and we derive a measure of fiscal pressure suitable for the medium term. We find that fiscal policy is not sustainable for most industrialized countries over an infinite horizon, but is sustainable in the medium term in the absence of ceilings. Imposing ceilings, however, generates unsustainability.

JEL Classification: E6, H6, H87
Keywords: fiscal policy, sustainability, fiscal criteria, stochastic discount rate, forecasts

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NON-TECHNICAL SUMMARY

This paper provides a formal theoretical framework for analysing the sustainability of fiscal policy based on the government intertemporal budget constraint and derives conditions that determine whether a given fiscal stance is sustainable. This framework generalizes the existing literature in several important respects. It allows for time-varying interest rates, for the primary deficit to be endogenous, and for a finite planning horizon suitable for medium-term policy-making for possible future policy shifts. The paper shows how published forecasts can be used and provides a measure of fiscal pressure. This analysis is then applied to the fiscal positions of the United States and the EU countries since 1970 and to their planned positions over the next decade.

There is a large volume of literature on the intertemporal budget constraint, with two different approaches that both yield inconclusive results. The first approach tests the long-run stationarity condition of the debt and/or deficit. Fiscal policy is considered sustainable if the government budget constraint holds in present-value terms. Results vary with the specification of the budget constraint. Hamilton and Flavin (1986), Smith and Zin (1988), Trehan and Walsh (1989), and Ahmed and Rogers (1995) find support for the sustainability of US fiscal policy. In contrast, Wilcox (1989), Kremers (1989), and Hakkio and Rush (1991) show that fiscal policy violates the intertemporal budget constraint. The second approach emphasizes the medium term and uses an indicator of sustainability consisting of the difference between the current tax rate, and a sustainable tax rate constant over a finite horizon that leads to a stable debt-to-GDP ratio. Blanchard *et al* (1990) find that most OECD countries are following sustainable policies in the medium term, while Buiter, Corsetti and Rubini (1993) reach the opposite conclusion.

This paper examines the sustainability of fiscal policy in the United States and in the European Union by combining these two approaches. The contribution of this paper is two-fold. First, it extends the results of Wilcox (1989) to the case where the discount rate is stochastic and time-varying, and the discounted primary deficit can be either exogenous or endogenous. It shows that a necessary and sufficient condition for sustainability is that the discounted debt-GDP ratio has a zero-mean stationary process. Second, the paper uses forecasted values for the fundamental variables. In addition to extending the sample, forecasted values allow investigation of the future fiscal stances of governments, and calculation of the fiscal pressure for each year over the next five years. Thus, in contrast to previous studies, this paper

captures the spirit of a fiscal plan by providing a profile of the evolving fiscal pressure, and quantifying the impact of ceilings on each country's fiscal profile.

The paper analyses the long-run sustainability condition by testing the hypothesis that the discounted debt-GDP ratio has a zero-mean stationary process. It shows that the market value of the *discounted* debt-GDP ratio is mean reverting for several countries, while that of the *undiscounted* ratio is non-stationary in all countries. The null hypothesis is rejected at the 5% significance level for Denmark and the Netherlands with the Augmented Dickey-Fuller test and for France with the Phillips-Perron test. In the case of Ireland, rejection occurs at 10% with the Phillips-Perron test. The effect of extending the data period by including the OECD forecasts is to slightly reduce the test statistics, reflecting the recent shift towards fiscal austerity. The United Kingdom, and to a lesser extent Belgium and Italy, now get closer to having a mean-reverting debt process.

The paper examines the medium-term sustainability or the intertemporal consistency condition by running simulations based on the finite-horizon stability condition. For each period up to 1989, it calculates the *ex-post* fiscal pressure resulting from the *ex-post* reduction in debt over the next five years and the *ex-post* cumulative discounted surplus. After 1990, it incorporates into the computations the forecast values for growth, inflation, the interest rate, debt and the deficit. For example, the 1991 measure of the fiscal stance is based on the actual values of 1992, 1993 and 1994, and the forecasts for 1995 and 1996. From 1994 onwards, the calculations use only forecasted values.

First, the paper considers the current situation and evaluates the fiscal pressure entailed in fulfilling the sustainability criterion. For this, it calculates at each point in time the change in the average tax rate required over *the next 5 years* (i.e. through the five-period horizon) to satisfy the intertemporal budget constraint. Second, this calculation is repeated imposing the condition that countries must satisfy the 60% Maastricht debt criterion. Third, the consequences of replacing this condition by a deficit limit of 0% or 3% are examined. The paper shows that most countries' fiscal stance is sustainable, but imposing 0% or 3% deficit limits, or a 60% debt limit would throw most governments' budgets onto an intertemporally inconsistent path unless a policy change that generates primary surpluses is implemented.

The sustainability of fiscal policies is yet again in the headlines, both in the United States and Europe. In the United States, Congress and the Administration have proposed budgets designed to reduce the deficit to zero by the year 2002, while European countries are trying to meet the challenging fiscal constraints of the Maastricht treaty by 1997. The treaty requires governments to run a deficit of no more than 3 percent of GDP with a maximum debt-to-GDP ratio of 60 percent. In most countries this implies a shift to austerity that is likely to entail a cost of foregone output and higher unemployment. The authorities often justify their restrictive stand by pointing to the necessity of pursuing "sustainable" fiscal policies that do not cause an excessive build up of debt and therefore put pressure on monetary policy.

In this paper we examine the likely consequences for the sustainability of fiscal policy of pursuing goals that rely on restrictive ceilings on deficits and debt. We provide a formal theoretical framework for analyzing the sustainability of fiscal policy based on the government intertemporal budget constraint and derive conditions that determine whether a given fiscal stance is sustainable. This framework generalizes the existing literature in several important respects. We allow for time-varying interest rates, for the primary deficit to be endogenous, for a finite planning horizon suitable for medium-term policy making for possible future policy shifts. We show how published forecasts can be used and we provide a measure of fiscal pressure. We then apply this analysis to the fiscal positions of the United States and the European Union countries since 1970, and to their planned positions over the next decade.

There is a large literature on the intertemporal budget constraint. The general conclusion to emerge from this is that fiscal policy is sustainable if the government budget constraint holds in present value terms. More precisely, the current debt should be offset by the sum of expected future discounted primary budget surpluses (exclusive of interest payments).

Two approaches to analyzing the sustainability of fiscal policy have been used. The first consists of testing the stationarity of the debt and/or deficit. Results vary with the specification of the budget constraint. Hamilton and Flavin (1986) reject the nonstationarity of constant-dollar undiscounted U.S. debt under the assumption of constant real interest rates. Smith and Zin (1988) obtain the same result with a similar specification for Canadian data. Wilcox (1989) allows for stochastic interest rates and finds that discounted U.S. debt is nonstationary. Other studies look for a cointegrating relationship linking the primary deficit, the stock of outstanding debt and interest payments for the United States. The results of Trehan and Walsh (1988), among others, find support for the sustainability of U.S. fiscal policy by showing that the deficit inclusive of interest payments is stationary. In contrast, Kremers (1989), and Hakkio and Rush (1991) show that in recent years fiscal policy violates the intertemporal budget constraint. More recently, using historical data that goes back to 1700s, Ahmed and Rogers (1995) find strong evidence favoring the sustainability of the U.S. fiscal policy and some support for the sustainability of the U.K. fiscal policy.

All of these tests of sustainability are based on the assumption that the processes generating deficits and debt will continue into the future. In practice, however, it may be necessary to alter fiscal policy to achieve sustainability. This suggests the need to analyze sustainability allowing for expected future changes in fiscal policy. Using a sustainability indicator Blanchard et al. (1990) find that most OECD countries are following sustainable policies in the medium-term. This indicator consists of the difference between the current tax rate, and a sustainable tax rate constant over a finite horizon that leads to a stable debt-to-GDP ratio. Using a similar approach, Buiter, Corsetti and Roubini (1993), and Roubini (1995) find that many OECD countries follow unsustainable policies.

Although this approach is forward-looking, existing studies do not use forecasted values for the fundamental variables; instead, they assume them to be constant. However, as the large fluctuations in interest rates in 1980s show, any assumption of constancy is unrealistic in the medium-term. Moreover, these studies calculate the current fiscal pressure arising from the need for solvency. Yet, in reality, governments plan their fiscal policy over a medium-term horizon, and smooth out the changes they plan to implement.

In this paper we examine the sustainability of fiscal policy in the U.S. and in the EU by combining these two approaches. The contribution of this paper to the literature is twofold. First, we extend the results of Wilcox (1989) to the case where the discount rate is stochastic and time-varying, and the discounted primary deficit can be either exogenous or endogenous. We show that a necessary and sufficient condition for sustainability is that the discounted debt-GDP ratio should be a stationary zero-mean process. An advantage of working with discounted debt and discounted deficit when examining infinite horizon sustainability is that it avoids the problem identified by Bohn (1995) of needing to take explicit account of risk when allowing the discount rate to be stochastic.

Second, we use forecasted values for the fundamental variables. In addition to extending our sample, forecasted values allow investigation of the future fiscal stances of governments, and calculation of the fiscal pressure for each year over the next five years. Thus, we are able to capture the spirit of a fiscal plan by providing a profile of the evolving fiscal pressure, while quantifying the impact of ceilings on each country's fiscal profile.

Our empirical results indicate that with an infinite-horizon, there is some evidence that the fiscal stance in Denmark, the Netherlands and Ireland is sustainable, while fiscal policy in the United States, Spain, Italy, Belgium, and Portugal is not. Results improve for all countries when we include future fiscal consolidation programmes by extending the sample into the year 2000.

Our simulation analysis shows that under current policies, the paths of future policies are sustainable for all countries. However, imposing 0 or 3 percent deficit limits, and 60 percent debt limits within the next 3 to 6 years throws most governments' budgets onto an intertemporally inconsistent path unless they generate major primary surpluses.

The paper is organized as follows. In the first section, we derive the government's intertemporal budget constraint for a stochastic discount rate adjusted for inflation and real GDP growth. We discuss the implication of this constraint for the sustainability of fiscal policy under both infinite and finite horizons. In the second section, we present the empirical results on the sustainability of EU and U.S. fiscal policies. In the third section, we examine the effect of imposing deficit and debt ceilings on government budgets. We present our conclusions in the fourth section.

I. The government intertemporal budget constraint

The government intertemporal budget constraint can be written in nominal terms as

$$G_t - T_t + i_t B_{t-1} = \Delta B_t + \Delta M_t = -S_t \qquad (1)$$

where G=government expenditure, T=tax revenue, B=government debt at the end of period t, M=monetary base, S=total budget surplus, i=interest rate on government debt.[1] The correct implementation of the budget constraint requires the use of the net market value of debt. Net debt is defined as gross debt minus financial assets.

Expressing (1) in terms of ratios to nominal GDP we get:

$$g_t - \tau_t + (i_t - \pi_t - \eta_t) b_{t-1} = \Delta b_t + \Delta m_t + (\pi_t + \eta_t) m_{t-1} = -s_t \tag{2}$$

where the lower-case letters g, τ, b, m, and s denote the ratio of the corresponding upper-case variables-to-nominal GDP, $\pi_t = (P_t - P_{t-1})/P_{t-1}$ and $\eta_t = (Y_t - Y_{t-1})/Y_{t-1}$, with P and Y standing for the price level and real GDP. Equation (2) says that the interest-inclusive government deficit is financed by new bond issues, base-money creation and seignorage. Equation (2) can be rewritten as:

$$d_t + \rho_t b_{t-1} = \Delta b_t \tag{3}$$

where $d_t = g_t - \tau_t - \Delta m_t - (\pi_t + \eta_t) m_{t-1}$ is the primary government deficit expressed as a proportion of nominal GDP and $\rho_t = i_t - \pi_t - \eta_t$ is the real *ex post* interest rate adjusted for real output growth. Equation (3) is an identity which holds *ex post* in time t. Looking forward, the identity can only hold in *ex ante* terms. Thus, in period t+1,

$$b_t = E_t[(1 + \rho_{t+1})^{-1}(b_{t+1} - d_{t-1})] \tag{4}$$

where b_t is known in period t, and for the one period budget constraint to hold in expectational terms, must equal the expected discounted net debt-GDP ratio in period t+1, conditional on information at time t. In order for fiscal policy to be sustainable for one period in the future, equation (4) must hold.

[1] Strictly speaking, the left-hand-side of equation (1) also includes a term reflecting changes in general government net debt due to factors such as revaluations of financial assets, exchange-rate changes, and privatizations. The debt series used in the analysis are consistent with this definition.

The corresponding expression for n periods ahead is obtained by solving forwards and successively substituting out the future compound discounted debt-GDP ratio to give the n-period intertemporal budget constraint:

$$b_t = E_t\ \delta_{t,n} b_{t+n} - E_t \sum_{i=1}^{n} \delta_{t,i} d_{t+i} \tag{5}$$

where

$$\delta_{t,n} = \prod_{s=1}^{n} (1+\rho_{t+s})^{-1}$$

is the time-varying real discount factor n periods ahead, adjusted for real GDP growth rate.[2,3]

An alternative way of expressing the equation (5) is to introduce a sequence of discount factors $\{\alpha_s;\ s=1,2,\ldots,t+n\}$ where $\delta_{t,n}=\alpha_{t+n}/\alpha_t$, $\alpha_0=0$, and α_t, $\alpha_{t-1}, \ldots$, are known at time t. [4] This enables the equation (5) to be written as

$$\alpha_t b_t = E_t \alpha_{t+n} b_{t+n} - E_t \sum_{i=1}^{n} \alpha_{t+i} d_{t+i} \tag{6}$$

or as

$$X_t = E_t X_{t+n} - E_t \sum_{i=1}^{n} Z_{t+i} \tag{7}$$

where $X_t=\alpha_t b_t$ and $Z_t=\alpha_t d_t$ are the discounted debt-GDP and the primary deficit-GDP ratios,

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[2] Most studies of sustainability assume a constant discount factor (see Hamilton and Flavin 1986, Trehan and Walsh, 1988, Kremers, 1989, Blanchard et al., 1990). An exception is Wilcox (1989). For n periods, this assumption would amount to a constant discount factor $\delta=(1+\rho)^{-n}$.

[3] In obtaining (5) we use the fact that $\delta_{t,i}b_{t+i}=E_{t+i}[\delta_{t,i+1}(b_{t+i+1}-d_{t+i+1})]$ and $E_t[\delta_{t,i}b_{t+i}]=E_t[\delta_{t,i+1}(b_{t+i+1}-d_{t+i+1})]$.

[4] The choice of base period for calculating the discount rate is unimportant as it does not affect the test for stationarity. It can be chosen on the grounds of convenience.

respectively. It also follows that the one-period budget constraint written in discounted terms is[5]

$$Z_t = \Delta X_t \tag{8}$$

1. Sustainability of fiscal deficits with an infinite horizon

A necessary and sufficient condition for sustainability is that as n goes to infinity the discounted value of the expected debt-GDP ratio converges to zero. This is also known as the transversality condition, and implies that no Ponzi games are allowed, meaning no new debt is issued to meet interest payments. This condition can be expressed as[6]:

$$\lim_{n \to \infty} E_t \delta_{t+n} b_{t+n} = 0 \tag{9}$$

It then follows that the current debt-GDP ratio is offset by the sum of current and expected future discounted surpluses expressed as a proportion of GDP, implying that the government budget constraint holds in present value terms with

$$b_t = -\lim_{n \to \infty} E_t \sum_{i=1}^{n} \delta_{t,i} d_{t+i} \tag{10}$$

When the debt-GDP ratio is discounted back to period 1, the transversality condition and the government intertemporal budget constraint are:

$$\lim_{n \to \infty} E_t X_{t+n} = 0 \tag{11}$$

[5] It may be noted that (3) can be written in discounted terms as $b_{t-1} = (1+\rho_t)^{-1}(b_t - d_t) = (\alpha_t / \alpha_{t-1})(b_t - d_t)$. Hence, $X_{t-1} = \alpha_{t-1} b_{t-1} = \alpha_t b_t - \alpha_t d_t = X_t - Z_t$.

[6] For b and ρ constant, ρ must be positive. Ball et al. (1995) note that ρ_t is negative for 10 to 12 percent of the time with the return on government debt less than the growth rate of GDP. However, negative values of ρ_t can still be compatible with the transversality condition as long as ρ_t is predominantly positive.

It may be noted that the transversality conditions in (9) and (11) do not require that the debt-GDP

$$X_t = -\lim_{n\to\infty} E_t \sum_{i=1}^{n} Z_{t+i} \tag{12}$$

ratio goes to zero, only that it does not grow faster than the growth-adjusted real discount rate. In principle, current debt can be sustainable by any sequence of primary deficits or surpluses, provided they satisfy equation (10) (or (12)), meaning that they offset the current level of debt.

The transversality condition has been tested in a number of ways, each depending on the processes postulated for d_t and ρ_t. Hamilton and Flavin (1986), and Trehan and Walsh (1988) examine the case where d_t is strictly exogenous and ρ_t is constant. Wickens and Uctum (1993) conduct the analysis with endogenous d_t and a constant ρ_t. Wilcox considers the case with exogenous d_t but variable ρ_t.

In this paper we consider a more general case where ρ_t is stochastic and d_t (or Z_t) is allowed to be either exogenous, or endogenous. We show that in both cases, the necessary and sufficient condition for the intertemporal budget constraint to hold is that the discounted debt-GDP ratio should be stationary.[7] Before proceeding further, we define the concepts of exogeneity and endogeneity employed in this paper.

Z_t is said to be *exogenous* with respect to X_t if it is generated by a process

$$Z_t = \mu + e_t = \mu + \theta(L)\epsilon_t, \qquad \theta_0 = 1 \tag{13}$$

where ϵ_t is iid$(0,\sigma^2)$ and $\theta(L)$ is a polynomial in the lag operator L either with one root on the unit circle and the rest outside (when Z_t is I(1)), or with all roots outside the unit circle (when Z_t is I(0)).

[7]An advantage of using the discounted variables X_t and Z_t is that when evaluating the transversality condition (11) we can work directly with these stochastic processes. This avoids the need to take explicit account of the fact that ρ_t is stochastic and as a result would generate a risk premium arising from conditional covariation with the undiscounted primary deficit (see Bohn, 1995).

Z_t is said to be *endogenous* with respect to X_t if there is a negative feedback from discounted debt to the discounted primary deficit, i.e., if it can be expressed by a functional form such as:

$$Z_t = \mu - \alpha X_{t-1} + e_t = \mu - \alpha X_{t-1} + \theta(L)\epsilon_t \tag{14}$$

with $\alpha>0$. Equation (14) can be viewed as having been derived from a complete econometric model with e_t summarizing the effect of all of the other variables in the model, including any dynamics.[8]

PROPOSITION. *Whether Z_t is an exogenous or endogenous process, a necessary and sufficient condition for the transversality condition given by (11) to be satisfied is that, if X_t is structurally stable, then it should be a zero-mean stationary process.*

PROOF (see Technical Appendix).

This proposition implies that if the process generating Z_t is structurally stable, it must also be a zero-mean stationary process. It also implies that the primary deficit must have a zero mean. If fiscal policy is currently unsustainable, then it will need to change in the future.

An assumption underlying this conclusion is that the time horizon is infinite. However, both the EU and the United States governments are facing self-imposed deadlines that are less than 10 years away. Thus, for practical purposes, we need to consider a finite-horizon version of the sustainability problem, which permits a change in future policy. We turn to this issue in the next section.

2. Sustainability of fiscal policy with a finite horizon

In the medium term current fiscal policy can be said to be sustainable, or intertemporally consistent, if it is able to achieve a given target level of the debt-GDP ratio. Unlike the infinite

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[8] One justification for this negative feedback arises if government debt generates higher tax revenues which reduce the primary deficit. If government debt is regarded as net wealth, the additional spending increases both indirect and, through higher income, direct tax revenues.

horizon case, this level may be non-zero. Denoting the desired level of the debt-GDP ratio at the end of the planning period by b*, from equation (4) the government intertemporal budget constraint becomes

$$b_t - E_t \delta_{t,n} b^*_{t+n} = -E_t \sum_{i=1}^{n} \delta_{t,i} d_{t+i} \tag{15}$$

Equation (15) can be interpreted as follows. The left-hand-side (LHS) is the difference between the current debt-GDP ratio and the expected value of the desired discounted debt-GDP ratio n periods ahead. In other words, it is the desired change in the discounted debt-GDP ratio. The right-hand-side (RHS) is the flow of discounted future primary deficit/surplus-GDP ratios, expected at time t.

Equation (15) determines the flow of future primary balances required to achieve the debt-GDP objective, on the basis of the current fiscal stance. If equation (15) is satisfied then the current fiscal stance can be said to be sustainable, or intertemporally consistent. If it is not satisfied, then either the debt objective or the fiscal stance must be altered. If, for example, a government wishes to reduce its debt burden then the LHS will be positive. To be sustainable, fiscal policy is then required to produce a corresponding sequence of discounted surpluses. An example of this case is the U.K. experience of the early 1980s.

A positive LHS may also be caused by a low discount ratio (or a high discount rate) due to high real interest rates or low growth. This makes the present value of the future debt-GDP ratio lower than the current value. In this case, even though there may be no change in the actual level of debt (or in the debt-GDP ratio), pursuing a tight fiscal policy will still be necessary. This example illustrates a need to adopt an active policy in the face of cyclical changes.

A change in the fiscal stance can be accomplished by changing taxes or government expenditures or by money financing.[9] Blanchard et al. (1990) have considered the average tax rate τ^* required to satisfy the government intertemporal budget constraint, and used it to derive a

[9] Strictly speaking, the primary deficit does not have to be generated by a structurally stable process since the expectation operator allows an anticipated future change of fiscal policy to ensure that the government intertemporal budget constraint is satisfied.

measure of fiscal pressure under the assumption of a constant discount ratio. This measure is defined as the discrepancy between the present tax rate and the sustainable tax rate. Their assumption of a constant discount rate is restrictive and can be relaxed, for it is possible to calculate the actual tax rate at every period in time and compare it with the sustainable tax rate at every period in time.

Thus, if the discounted surplus is insufficient to permit the desired reduction in discounted debt, then at each period in time an increase in the average tax rate of

$$\tau^{*}-\tau=[E_t\sum_{i=1}^{n}\delta_{t,i}]^{-1}[b_t-E_t\delta_{t,n}b_{t+n}^{*}+E_t\sum_{i=1}^{n}\delta_{t,i}d_{t+i}] \tag{16}$$

is required to satisfy the intertemporal budget constraint through to the n-period horizon. The change in the average tax rate will vary over time as new information becomes available on future primary deficits/surpluses and the future discount rate. The size of the tax change required is a measure of fiscal pressure. An alternative to a tax change would, of course, be to change government expenditures.

The intuitive appeal of this fiscal indicator is that it is a simple measure that can be easily implemented because it is based on observable variables or forecasts available at time t. Since the terminal condition in the medium-term will not usually require the complete annihilation of government debt, the indicator provides a useful medium-term policy tool connecting the desired evolution of debt with expected future deficits.

II. Sustainability of U.S. and EU fiscal policies: solvency tests

In this section, after describing the data used in the analysis, we examine the trends in the debt-GDP ratio defined in various ways. We then conduct a stability test of the discounted value of government debt.

1. Data

There are two major issues that must be addressed when using government debt data: whether to measure debt at market value or at face value (at par), and how to measure the discount rate. The public debate on fiscal policy is based on official budget measures expressed in nominal terms. However, as several authors pointed out (Eisner and Pieper, 1986, and Hamilton and Flavin, 1986) this measure misstates the true fiscal stance of the government. From a conventional accounting perspective, we need to consider not only the inflation rate but also the market value of the debt and the deficit. The correct implementation of the government intertemporal budget constraint requires the use of the discounted net market value of debt. However, the available debt figures are usually expressed at par, since the market value of debt for the whole sample is available for just a few countries.[10] We obtain an estimate of the market value of debt b_t by multiplying the face value by the implied market price $1/(1+\rho_t)$, where ρ_t is the yield on government debt. Thus, we calculate the market value of the debt-GDP ratio as $b_t = (1+\rho_t)^{-1} b^N_t$, where b^N_t is the government debt-GDP ratio at par.

The measurement of the yield on government debt is complicated by its heterogenous composition. Government debt is commonly sold at a discount and redeemed at par, i.e., at face value. At any point in time, the new debt issued will be redeemed at different dates in the future, (i.e., will contain different maturities) and have different coupons. The appropriate measure of the rate of return on government debt in any period is a weighted average of the one period holding-returns for each of the maturities in existence, the weights being the share of any issue in the total market value of outstanding debt. Since these weights are in general unknown, it is necessary to estimate the average rate of return in a different way. In this paper, we adopt the most common way that consists of dividing total net debt interest payments by the end of last period's (or the beginning of this period's) face value of net debt.[11]

Our sample covers the period 1965 to 1994.[12] To capture any structural change that

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[10] Eisner and Pieper (1984) calculate the market value of the U.S. government debt by quantifying the market value of tangible assets owned by the government.

[11] A problem in implementing this approach, which is usually overlooked, is that the published data on net interest payments will be an underestimate of the figure required because it is based only on coupon payments. Thus, if all debt were zero coupon (e.g., Treasury Bills), the published value of interest payments would be zero. In a sustainability analysis, what is required is the implied interest payment. For zero coupon bonds this would be the holding period return multiplied by the previous period's market value of that issue.

[12] Due to unavailability of government debt series, the starting dates are 1970 for France, and Italy, 1976 for Spain, 1977 for Ireland, and 1980 for Austria.

might occur following the recent shift toward fiscal austerity, we also extend the sample period to the year 2000 and include the forecast values of discounted and undiscounted debt.

All data are annual and are taken from the OECD Economic Outlook (1994, 1995). The same publication also gives forecasts of the GDP growth rate, the inflation rate and government debt until 1997 and forecast of these variables (except net debt) for the year 2000, based on a medium term base scenario.[13] Forecasts for the intervening years 1998 and 1999 are constructed using a linear interpolation. Forecasts between the year 2000 and 2005 are kept constant. Implicitly, this amounts to assuming that the dynamic path is converging toward a steady state at the start of EMU. The forecasts for the primary deficit and net interest payments are from unpublished OECD sources. The series for the high-powered-money until 1995 are obtained from the International Financial Statistics (line 14). Between 1995 and the year 2000 we assume that the base money grows at the same rate as nominal GDP.

Data for net debt after 1997 are not available, therefore we generate a net debt series until 2000 by assuming that net debt grows at the same rate as gross debt. This assumption implies that the ratio of government financial assets-GDP is constant after 1997, or that nominal financial assets owned by government grow at the same rate as nominal GDP. Hence, for $t>1997$, we use the formula $\Delta b^N_t = \Delta b^G_t$, where b^G is gross debt.

2. Stylized facts

The rise in government debt has been a major concern for policy makers for the last decade. The overall picture, however, reflects a general tendency for fiscal austerity in the coming years. Inspection of the data reveals that in several countries the slippage of the early 1990s, which was preceded by major consolidation efforts, is expected to give way to fiscal austerity toward the year 2000 (Chart 1).

During the latter half of the 1980s fiscal austerity in most countries lead to a decline or stabilization of the net (undiscounted) debt-GDP ratio. By contrast, in the early 1990s, following

[13]The figures for government debt refer to the debt of the general government sector, defined as the aggregate of the central and local government sectors and the social security sector. We should note that due to data unavailability, we conducted the analysis for Ireland and Portugal with gross debt.

the German unification and the European recession, debt-GDP ratios increased rapidly. The U.S. debt-GDP figure has behaved similarly; in the second half of the 1980s, fiscal austerity and rapid growth lead to a decline in this ratio, and the subsequent slowdown in the economy caused it to increase.

As noted above, the transversality condition requires discounted debt to converge to zero. A short-run indicator is that discounted debt should be falling over the forecast period. Extending the series by including forecasted values to the year 2000, the data indicate that several countries have a declining discounted debt-GDP ratio, such as France, Denmark, the Netherlands, Belgium and Ireland.[14] More formal tests based on unit root tests are reported in the next section.

3. Stationarity tests

In this section we conduct ADF and Phillips-Perron tests for the undiscounted debt (Table 1, first four columns) and for the discounted debt at market value (Table 1, last four columns). We also conducted the tests for the discounted debt at face value, but we do not report the results since they are very similar. The tests were performed on actual data until 1994, and on a data set extended to include the forecasts up to 2000.

The general conclusion to emerge from these tests is that the market value of the debt-GDP is mean reverting in some countries, and there is a general improvement in fiscal stances toward the end of the century. The null hypothesis of a unit root cannot be rejected for the *undiscounted* debt-GDP ratio but can be rejected for several countries when debt is *discounted.* The null hypothesis is rejected at the 5 percent significance level for the Netherlands and Denmark with the ADF test, and for France with the Phillips-Perron test. In the case of Ireland, the rejection occurs at 10 percent with the Phillips-Perron test. The effect of extending the data period by including the OECD forecasts is slightly to reduce the test statistics, reflecting the recent shift toward fiscal austerity. The United Kingdom, and to a lesser extent Italy and Belgium now get closer to having a mean reverting debt process.

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[14]Chart 1 reveals two other points of interest. First, in all countries the market value of debt is less than the face value. This is consistent with the finding of Eisner and Pieper (1989) who show that by 1980, the market value of the U.S. government debt was $66 billion less than its par value. Second, the discounted market value of debt lies below the undiscounted value, except in high nominal growth countries.

III. Sustainability of U.S. and EU fiscal policies: a simulation analysis

We now consider the sustainability of the current fiscal stance of the United States and the EU countries over a 5-year horizon to the year 2000. Our aim is to determine whether the fiscal deficit over the next several years is compatible with the evolution of government debt. We show that most countries' fiscal stance is sustainable, but imposing 0 or 3 percent deficit limits, or a 60 percent debt limit would throw most governments' budgets onto an intertemporally inconsistent path unless a policy change that generates primary surpluses is implemented.

For each period up to 1989 we calculate the *ex post* fiscal pressure resulting from the *ex post* reduction in debt over the next five years ($b_t - \delta_{t,n} b_{t+n}$) and the *ex post* cumulative discounted surplus $-\sum \hat{\delta}_{t,i} b_{t+i}$. After 1990, we incorporate into the computations the forecast values for growth, inflation, the interest rate, debt and the deficit. For example, the 1991 measure of the fiscal stance is based on the actual values of 1992, 1993 and 1994, and the forecasts for 1995 and 1996. From 1994 onwards, the calculations use only forecasted values.

First, we consider the current situation and evaluate the fiscal pressure entailed in fulfilling the sustainability criterion. This involves using equation (16) to calculate at each point in time the change in the average tax rate required over *the next 5 years* (i.e., through the 5-period horizon) to satisfy the intertemporal budget constraint. Second, we repeat this calculation imposing the condition that countries have to satisfy the 60 percent Maastricht debt criterion. Third, we examine the consequences of replacing this condition by a deficit limit of 0 or 3 percent.

1. Where do countries stand with respect to the sustainability criterion?

When the government debt and deficit are discounted with implicit rates, all countries appear to satisfy the intertemporal budget constraint. Chart 2 displays our measure of fiscal pressure. This is based on using the market value of debt and a seignorage-adjusted primary deficit. The horizontal axis denotes years (from 1980 to 2000) and the vertical axis shows the fiscal pressure as a percentage of GDP. In each panel, the area above 0 represents positive fiscal pressure, and indicates the need for a tightening of fiscal policy (by, for example), a higher tax rate to satisfy the intertemporal budget constraint. In other words, positive fiscal pressure indicates that to achieve the desired change in government debt, larger future surpluses are required. Conversely, the area below 0 represents a negative fiscal pressure, showing that the intertemporal budget constraint is not binding and that the government can even follow an expansionary policy.

Chart 2 indicates that by the year 2000, almost no country needs to raise the average tax rate because the fiscal pressure is non positive. In fact, under current policies, several countries even have the option of fiscal expansion ranging from 0.1 to 4 percent of GDP without violating the intertemporal budget constraint. The United States and Spain are the only exceptions. They have been pursuing unsustainable policies that have severely reduced their room for maneuver during the last decade. However, the required fiscal adjustment is modest. Both countries need to raise their tax rate by 0.2 to 0.3 percent by the year 2000.

More insight into these results can be obtained by examining the behavior of the two components of equation (16) depicted in Chart 2. These components are the desired change in the debt-GDP ratio (the left-hand-side of equation (15)), and the cumulative discounted surpluses (the right-hand-side of equation (15)). These are shown in the chart in appendix A. The left panel is the desired change in the debt-GDP ratio. A negative slope violates the sustainability condition because it indicates that the discounted value of debt five years ahead is larger than the current outstanding stock of debt. The right panel represents the cumulated discounted primary surpluses. The area under the curve is the cumulated future primary surplus (deficit) if it is positive (negative).

For most countries the late 1980s is a period of rising desired debt accumulation (left panel). The desired discounted debt is higher than the current stock of debt and the desired debt accumulates at an increasing rate. In most cases this trend is reversed by the early 1990s when the rate at which desired debt accumulation slows. By 1995, all countries are back to a sustainable path of government finances with a lower desired discounted debt-GDP ratio than the current one. The United States reached the sustainable path in 1993 and has been on a stable path since then. The trend reversal is more dramatic in high-debt countries such as Italy, Belgium and Ireland due to the maintenance of a high rate of desired debt reduction throughout the 1990s.

The right panel of the chart in appendix A shows how these countries finance their desired debt accumulation. The second-half of the 1990s reflects a general tendency for fiscal consolidation with cumulated primary surpluses in all countries. The most virtuous countries are, again, the most indebted ones. Ireland, and Belgium have been running discounted cumulated primary surpluses since the mid-1980s, and Italy since 1989. The United States had periodic small discounted cumulated primary deficits until 1992, after which it achieved a balance. Thus, fiscal consolidation in most countries and a reduction in the desired debt accumulation account for the lack of fiscal pressure displayed in Chart 2.

To analyze the effect of inflation on deficit financing we also performed simulations with the primary deficit unadjusted for seignorage. We found that although the inflation-tax turns out to be insignificant for several countries, it still constitutes a nonnegligible source of finance for others. Not surprisingly, seignorage revenues are highest in countries with an expected inflation rate of greater than 3 percent at the end of the century. Between 1996 and 2000, Spain and Portugal are expected to finance respectively 2 and 5 percent of their deficits through seignorage revenues, while the United States and Ireland will use around 1½ percent of inflation financing.

2. Imposing the 60 percent debt ceiling of the Maastricht treaty

The Maastricht debt limit is imposed on gross government debt at par in 1999.[15] We need to translate this restriction into a limit on the net market value of debt. From the assumptions that $b^G_t - b^G_{t-1} = b^N_t - b^N_{t-1}$ and $b_t = (1+\rho_t)^{-1} b^N_t$, we can express the net market value of debt in terms of gross par value of debt[16] $b_t=(1+\rho_t)^{-1}[(1+\rho_{t-1})b_{t-1}+b^G_t-b^G_{t-1}]$. Thus, for t=1999, and $b^G_{1999}=0.6$ percent, the net debt-GDP ratio at par becomes $b_{1999}=(1+\rho_{1999})^{-1}[(1+\rho_{1998})b_{1998}+0.6-b^G_{1998}]$.

Imposing this debt requirement for the year 1999 would make several countries' fiscal stance intertemporally inconsistent and would require fiscal tightening. Not surprisingly, the 60

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[15] Strictly speaking, the debt limit imposed on gross public debt defined under the Maastricht Treaty differs somewhat from the corresponding OECD figures (for more details see OECD, 1994, p.17). However, since these series start only in 1990 and they are not complete for all countries, we used the OECD series for the empirical analysis. For consistency, we also used the same OECD series for the simulation analysis.

[16] This expression is obtained as follows: From the first assumption we get $b^N_t=b^G_t-b^G_{t-1}+b^N_{t-1}$. Replacing b^G_t in the second assumption we get $b_t=(1+\rho_{t-1})^{-1}(b^G_t-b^G_{t-1}+b^N_{t-1})$. The second assumption also implies that $b_{t-1}=(1+\rho_{t-1})^{-1}b^N_{t-1}$. Solving for b^N_{t-1} in terms of b_{t-1} and replacing it in the previous equation, we get an expression for the net market value of debt as a function of par value of gross debt.

percent debt requirement does not affect countries where the constraint is nonbinding (Germany, France, the United Kingdom, Denmark, Austria) since this assumption is incorporated in the forecasts of the OECD (Chart 3). It increases fiscal pressure moderately in countries where the constraint would be binding because their debt-GDP ratio is slightly above this ceiling (the United States --if it had to satisfy the criterion--, Spain, Portugal, Ireland and the Netherlands). However, in countries with a debt-GDP ratio above 100 percent, the debt criterion is difficult to achieve politically because it requires sizeable sacrifices (Italy and Belgium).

Fiscal pressure is, in general, mean reverting even in high-debt countries. By 1999, the tax rate required for sustainability falls below the current level, indicating that the constraint becomes nonbinding. The two exceptions are again the United States and Spain. In both countries fiscal pressure remains small but positive and appears to continue beyond the year 2000.

To satisfy both the intertemporal budget and debt constraints over the course of 4 to 5 years, the medium-debt countries need to generate cumulated primary surpluses ranging from 6 percent to 12 percent of GDP. The Netherlands, Spain, and Portugal are required to raise their tax revenues or reduce expenditures by a total of 5 to 6 percent of their respective GDPs within 7 years (starting from 1994 until the year 2000), amounting to an average of roughly 1 percent a year. Clearly, the later the starting date, the larger is the required annual rise in the tax ratio.

Ireland, a medium-debt country since the end of the 1980s, needs a more radical fiscal retrenchment, despite a cumulated discounted surplus of 70 percent between 1994 and 2000 (right panel, chart in appendix A). This will represent an increase in debt pressure of about 2 percent.

If the United States had to comply with the Maastricht debt criterion, it would fall into the category of medium-debt countries, and would be required to increase tax revenues (or reduce expenditures) by a cumulated 1 percent of GDP over 7 years. Although small, this figure represents an increase of 16 percent over what it would have had to generate if its aim were simply to satisfy its intertemporal budget constraint over the same period.

Italy and Belgium constitute a third category where fiscal pressure on otherwise solvent economies becomes very severe. Between 1994 and 2000, both countries have been and are planning to generate primary surpluses of about 25 percent per year. However, to satisfy the debt criterion and be solvent by 1999, Belgium must further raise its primary surplus by a cumulated 55 percent of GDP, or an average of 8 percent of GDP per year, bringing the annual primary surplus-GDP ratio to 32 percent. The additional sacrifice is slightly lower for Italy, 35 percent (5 percent per year), but still politically difficult since it raises the total annual primary surplus-GDP figure to 30 percent.

This analysis shows the economic and political infeasibility of the strict application of the Maastricht treaty's debt criterion for governments with initial high indebtedness. In contrast, the fiscal pressure entailed in satisfying the intertemporal budget constraint without imposing the debt criterion is generally viable and still imposes a fiscal discipline.

3. Imposing deficit ceilings

In this section, we show that, like debt ceilings, deficit ceilings make the fiscal stance of countries intertemporally inconsistent unless tax ratios are raised significantly. However, unlike the debt ceiling, the deficit ceiling creates a positive and persistent fiscal pressure on all countries, which lasts beyond the date these limits are imposed.

The deficit limit in the Maastricht Treaty is defined inclusive of interest payments on debt. To impose a deficit ceiling, we calculate a debt-GDP ratio consistent with the constraint. From equation (2) we can express the outstanding stock of current debt in terms of the deficit *inclusive of interest payments* h_t as

$$b_t = h_t + [1 - (\eta_t + \pi_t)] b_{t-1} \tag{17}$$

A ceiling on total deficits can be expressed by imposing a limiting value on h_t. We consider the effects on the intertemporal budget constraint of two types of deficit limits: a 3 percent limit by 1999, as required by the Maastricht Treaty, and a 0 percent limit as planned by the U.S. government by 2002. In both cases a deficit limit imposed by the end of the century forces all governments to adopt even more austere fiscal programmes even though they follow otherwise sustainable fiscal policies.

To satisfy the balanced budget condition by the year 2002 and simultaneously be solvent, between 1996 and 2000, the United States would have to maintain a positive growth rate of the tax-GDP ratio of about 7 percent per year. This amounts to a cumulated surplus of 18 percent of current GDP over the course of 4 years, which is 17 percent more than what is required to satisfy solvency. Moreover, the rate of increase continues to be positive beyond the year 2000.

Although the fiscal retrenchment in Germany, and France is less severe, both countries would need to increase the growth rate of their tax-GDP ratio by about 5 percent per year for 3 years until 1998, and keep the inflow of revenues at an annual constant rate of 2 percent of GDP, thereafter. While in the second half of the 1990s the U.K. budget was not under any fiscal pressure, it would now need to generate a primary surplus of an average of 1 percent of GDP per year for 5 years until 1998, and keep it constant at 1/2 percent of GDP afterwards.

Austria and the Netherlands were also previously among the countries that did not have any fiscal pressure. Now, between 1996-1998, they would both be required to raise their tax revenue respectively by 1 and 1/2 percent of GDP on average. Ireland would have to generate an additional 3 percent annual primary surplus between 1994-98. All three countries would need to keep their tax revenue at a constant 1 to 3 percent of GDP after 1998.

A third group of European countries would be required to make more sizeable sacrifices to simultaneously satisfy the deficit constraint and the solvency condition. Until 1998, Belgium and Portugal have to raise the annual growth of tax revenues to about 6 percent for two and four years, respectively. For both Italy and Spain the increase would need to be at least 10percent for three years. After 1998, all four countries would need to keep their higher tax ratios at a constant 1.5 to 4 percent.

On average, between 1994 and 2000, the deficit ceilings create an excess fiscal pressure of 11 percent, pointing to the need for further fiscal consolidation. It is interesting to note that this rise in the average primary surplus is required even though the OECD forecasts incorporate the assumption that, by the year 2000, almost all of the countries satisfy the 3 percent deficit constraint. This analysis shows the importance of pursuing fiscal retrenchment *consistent* with the cyclical economic activity, instead of imposing arbitrary deficit limits. Taken in isolation, these limits give no information about the true fiscal profile of the economy.

IV. Conclusion

In this paper we have derived conditions suitable for determining whether or not fiscal policies are sustainable in the long run, in the medium term, and in the presence of debt and deficit ceilings. We have applied these conditions to the fiscal stances of the United States and the EU countries since 1970 and their planned positions over the next decade. On the basis of infinite horizon-tests the broad conclusion is that many countries do not have a sustainable policy. However, there is some evidence that the government discounted net debt is mean-reverting for a few countries, implying that their fiscal policies are sustainable. The evidence in favor of sustainability is strengthened for most countries when data are extended to incorporate future fiscal consolidation plans. This reflects the general shift towards fiscal austerity in recent years.

In the medium-term analysis we have shown that in the absence of ceilings most countries have sustainable fiscal policies. However, imposing 0 or 3 percent deficit limits or 60 percent debt limit in the next 3 to 5 years throws most government budgets onto an intertemporally inconsistent path. The recovery in Europe is currently being adversely affected by government attempts to conform to arbitrary fiscal criteria such as rigid fiscal ceilings which do not take into account cyclical factors. Our results suggest that satisfying the intertemporal budget constraint provides a sufficient fiscal discipline for governments. Introducing arbitrary additional constraints could lead to fiscal policy becoming politically infeasible.

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Technical Appendix

Proof of Proposition

1. Z_t is exogenous

It is assumed that Z_t is determined by

$$Z_t = \mu + e_t = \mu + \theta(L)\epsilon_t \tag{1a}$$

with $\theta_0=1$ and ϵ_t is a zero-mean i.i.d. process (equation 12). Two cases can be distinguished: $Z_t \sim I(0)$ or $Z_t \sim I(1)$.

Case 1: Z_t is I(0)

In this case the roots of $\theta(L)$ lie outside the unit circle. From equation (6):

$$\lim_{n\to\infty} E_t X_{t+n} = X_t + \lim_{n\to\infty} E_t \sum_{i=1}^{n} Z_{t+i} = X_t + \lim_{n\to\infty} n\mu + \lim_{n\to\infty} E_t \sum_{i=1}^{n} e_{t+i} \tag{2a}$$

Using the methodology by Hansen and Sargent (1980):

$$\begin{aligned}\lim_{n\to\infty} E_t \sum_{i=1}^{n} e_{t+i} &= \left([\theta(L)-\theta_0]L^{-1} + [\theta(L)-(\theta_0+\theta_1 L)]L^{-2} + [\theta(L)-(\theta_0+\theta_1 L+\theta_2 L^2)]L^{-3} + \ldots\right)\epsilon_t \\ &= \left(\theta(L)L^{-1}[1+L^{-1}+L^{-2}+\ldots] - L^{-1}[\theta_0+\theta_1+\theta_2+\ldots] - L^{-2}[\theta_0+\theta_1+\theta_2+\ldots] - \ldots\right)\epsilon_t \\ &= \left(\theta(L)L^{-1}[1+L^{-1}+L^{-2}+\ldots] - \theta_0[L^{-1}+L^{-2}+\ldots] - \theta_1[L^{-1}+L^{-2}+\ldots] - \ldots\right)\epsilon_t \\ &= \frac{[\theta(1)-\theta(L)]\ \epsilon_t}{(1-L)}\end{aligned}$$

Substituting this into equation (2a), we obtain

$$\lim_{n\to\infty} E_t X_{t+n} = X_t + \lim_{n\to\infty} n\mu + \frac{[\theta(1)-\theta(L)]\ \epsilon_t}{(1-L)} \tag{3a}$$

Thus, if

$$X_t = -\lim_{n\to\infty} n\mu - \frac{[\theta(1)-\theta(L)]\epsilon_t}{(1-L)} \tag{4a}$$

then the expected value of the discounted debt converges to zero as $n\to\infty$. It follows that we require

$\mu=0$, which means in effect that the discounted primary deficit should be a mean-zero process. We then obtain $\Delta X_t = Z_t - \theta(1)\epsilon_t$. But from (7) we know that $\Delta X_t = Z_t$, hence we require $\theta(1)\epsilon_t=0$. This is satisfied if either $Z_t=0$ (i.e., the discounted deficit -or the primary deficit itself- is zero at each period in time), or more realistically, if $X_t = \theta^*(L)\epsilon_t$, where $\theta^*(L)=\theta(L)/(1-L)$. In other words, if Z_t is I(0), then for the transversality condition to be satisfied the process generating the discounted debt must be a zero-mean stationary process.

Case 2: Z_t is I(1)

Suppose, for example that

$$\Delta Z_t = \mu + e_t = \mu + \theta(L)\epsilon_t, \quad \theta_0 = 1 \tag{5a}$$

where again the roots of $\theta(L)$ lie outside theunit circle. It follows that

$$\lim_{n\to\infty} E_t X_{t+n} = X_t + \lim_{n\to\infty} E_t \sum_{i=1}^{n} Z_{t+i} = X_t + \lim_{n\to\infty} \sum_{i=1}^{n} (i\mu + Z_t + E_t \sum_{j=0}^{i-1} e_{t+i}) \tag{6a}$$

Clearly, the transversality condition will not be satisfied if Z_t is I(1).

2. Z_t is endogenous

It is assumed that Z_t is determined by

$$Z_t = \mu - \alpha X_{t-1} + e_t \tag{7a}$$

where $\alpha>0$ and $e_t=\theta(L)\epsilon_t$ is stationary. Taking first differences and substituting $Z_t=\Delta X_t$ we obtain

$$\begin{aligned} Z_t &= (1-\alpha)Z_{t-1} + \Delta e_t \\ &= (1-L)\theta(L)[1-(1-\alpha)L]^{-1}\epsilon_t = \phi(L)\epsilon_t \end{aligned} \tag{8a}$$

Given our assumptions, Z_t will be stationary zero-mean process, and will be equivalent to (1a) with $\mu=0$. It follows that previous results apply, namely, the case where Z_t is exogenous and I(0). Thus, for Z_t satisfying (7a) -or equivalently, (8a)- the transversality condition will hold and X_t will be stationarity.

Since $\Delta X_t=Z_t$, it can be shown from (8a) that in general $X_t=X_0+\theta(L)[1-(1-\alpha)L]^{-1}\epsilon_t$. However, to satisfy the transversality condition we require $X_0=0$, or that X_t be a zero-mean process.

To summarize, whether Z_t is exogenous or endogenous, if they are structurally stable, we require that both X_t and Z_t have zero-mean stationary processes for the transversality condition to be satisfied.

Table 1: Integrability Tests for the market value of net debt†

	Undiscounted debt-GDP ratio b_t					Discounted debt-GDP ratio X_t			
	ADF		P-P			ADF		P-P	
	1994	2000	1994	2000		1994	2000	1994	2000
The U.S.	-1.3	-1.0	-1.1	-0.9		0.1	-0.8	1.3	-0.2
Germany	0.1	-0.5	0.5	-0.3		-1.3	-1.8	-1.5	-2.1
France	1.7	-0.2	2.8	0.5		-2.9	-3.4	-4.3	-4.8
The U.K.	-1.8	-2.0	-1.6	-1.8		-1.7	-2.5	-1.0	-1.6
Austria	-0.6	-1.2	-0.6	-1.2		-1.5	-1.8	-1.6	-1.9
Denmark	-2.1	-2.8	-0.6	-1.2		-3.0	-3.5	-0.9	-1.3
Netherl.	-0.8	-1.5	0.7	-0.2		-3.0	-3.2	-2.2	-2.3
Ireland*	-1.9	-1.5	-1.8	-1.2		-2.3	-2.7	-2.3	-2.9
Spain	-0.1	-1.1	0.2	-0.7		-1.0	-1.9	-0.3	-1.4
Italy	1.3	-1.2	1.2	-1.3		-1.2	-2.1	0.2	-1.7
Belgium	-0.9	-1.6	-0.2	-1.2		-2.0	-2.1	-1.6	-1.9
Portugal*	-0.3	-1.1	-0.2	-1.1		1.1	-0.7	1.8	-0.3

†ADF is the augmented Dickey-Fuller test with the regression equation:
$\Delta x_t = \mu + (\alpha - 1)x_{t-1} + \Sigma \beta_i \Delta x_{t-i} + u_t$.
The null hypothesis is H_0: $\mu=0$ and $\alpha=1$. Critical values for the 5% and 10% confidence interval are -2.9 and -2.6, respectively.

*Gross debt (net debt figures are not available)

Chart 1: Government Net Debt as a % GDP

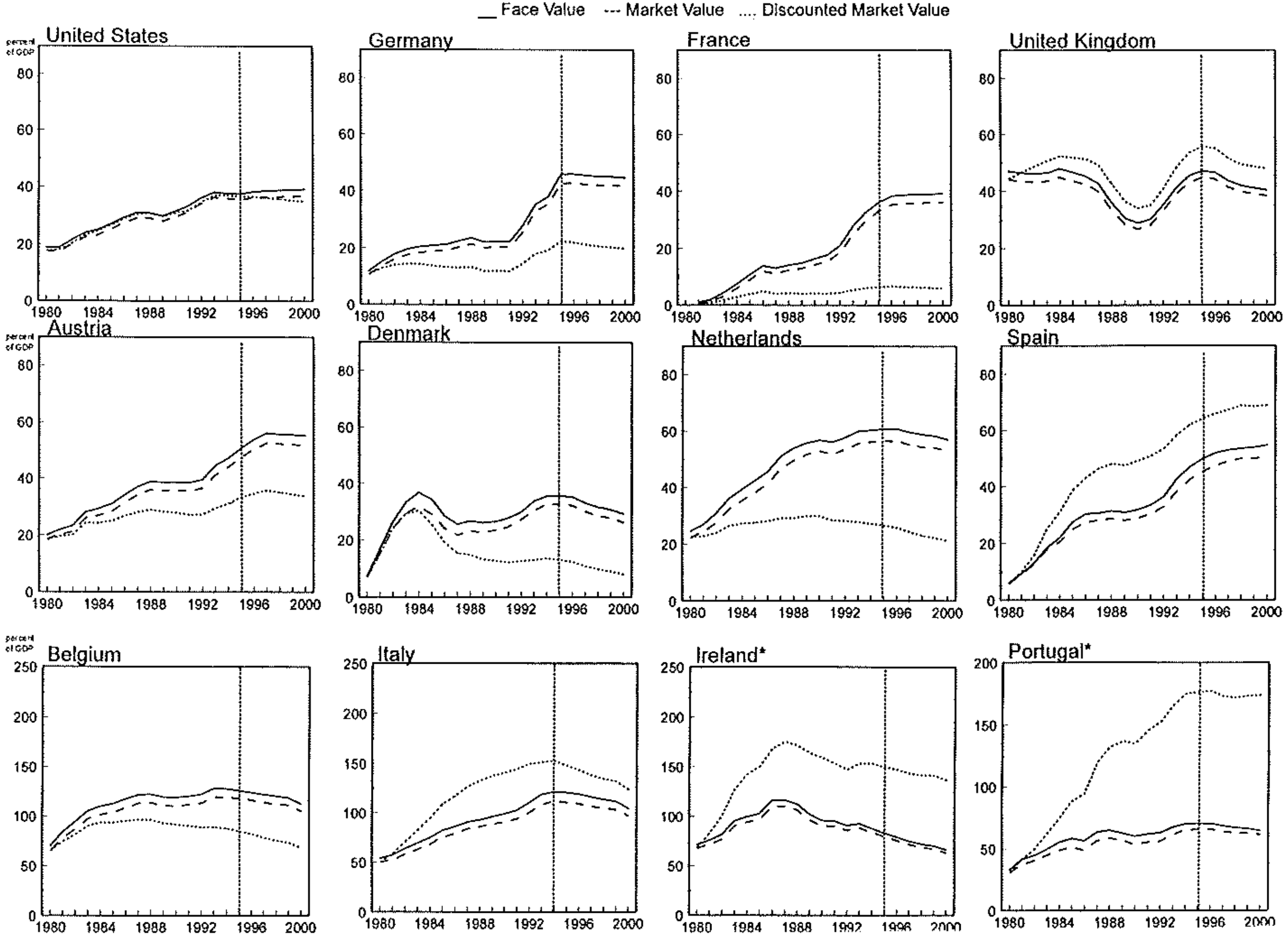
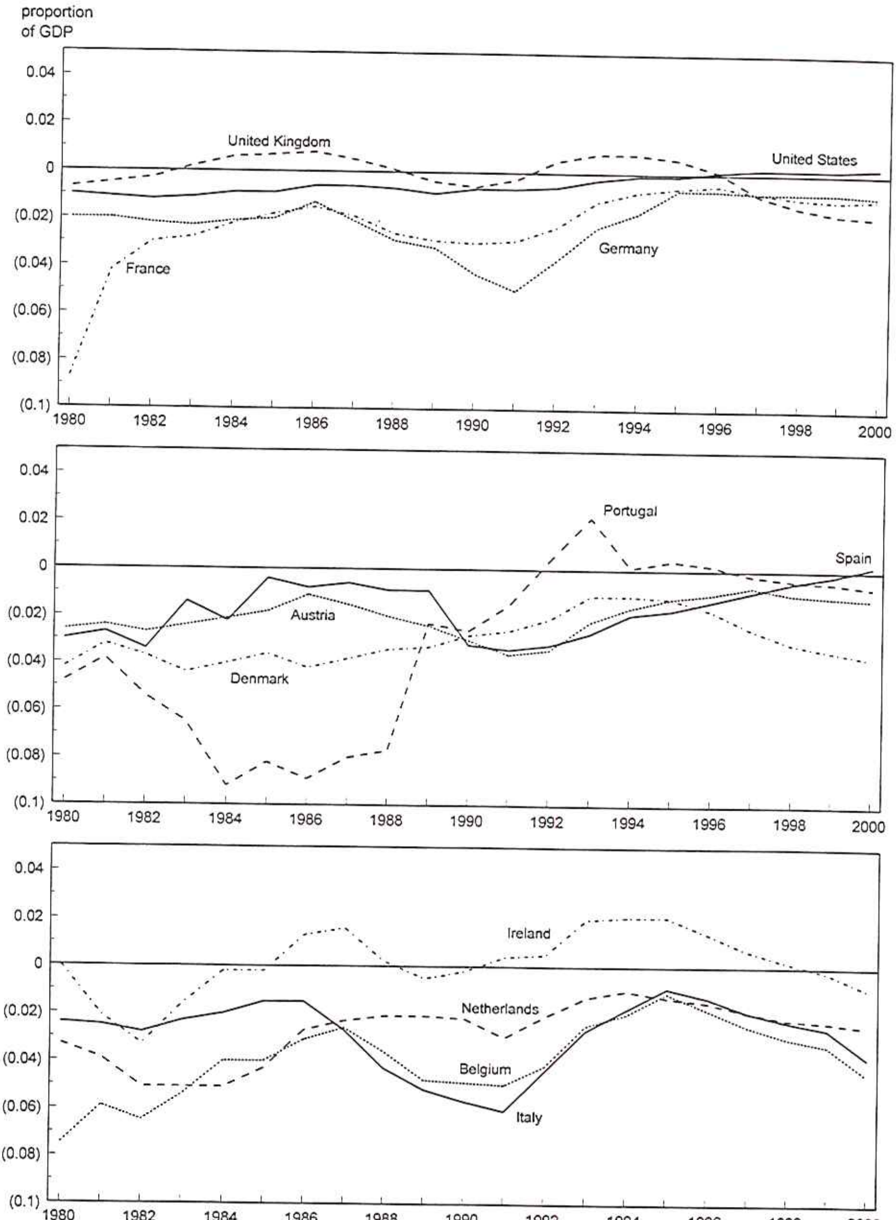
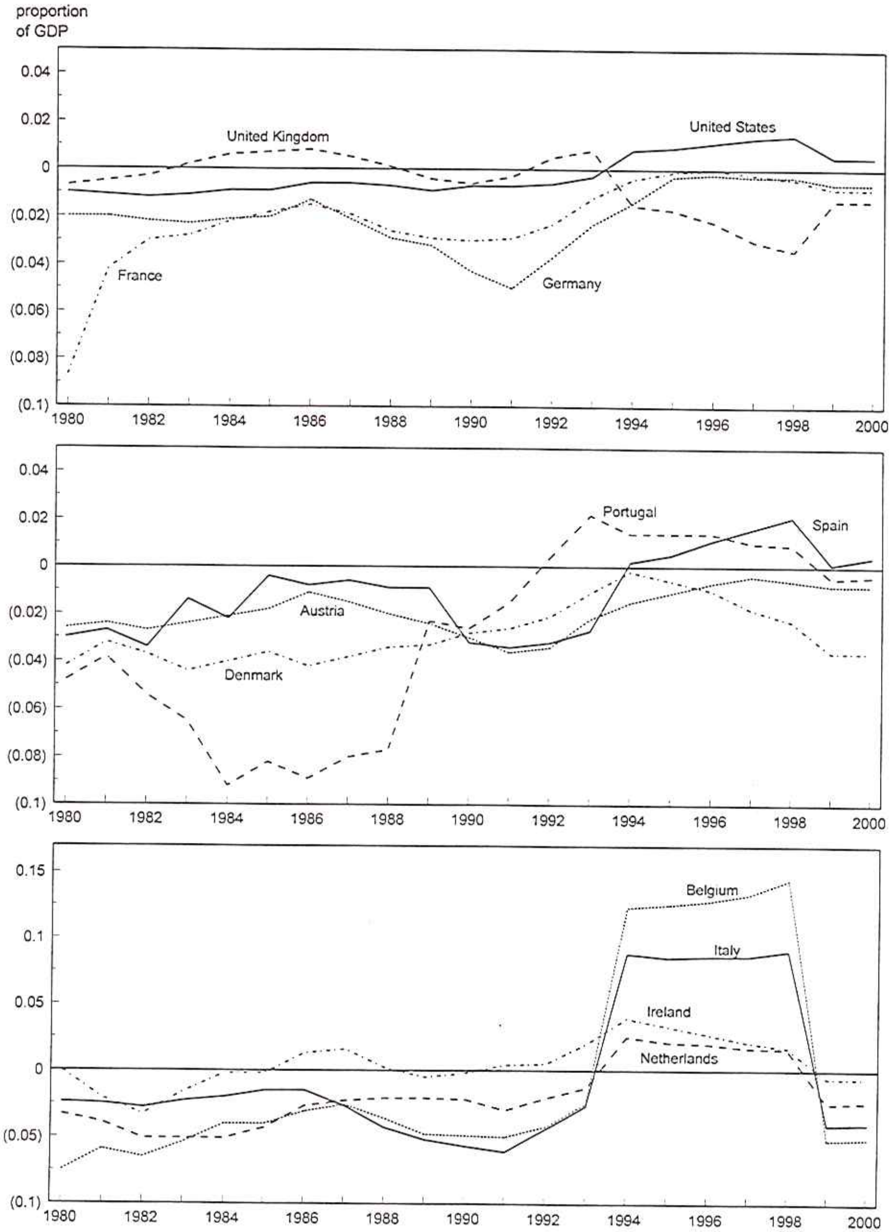


Chart 2: Fiscal Pressure With No Debt Constraints*



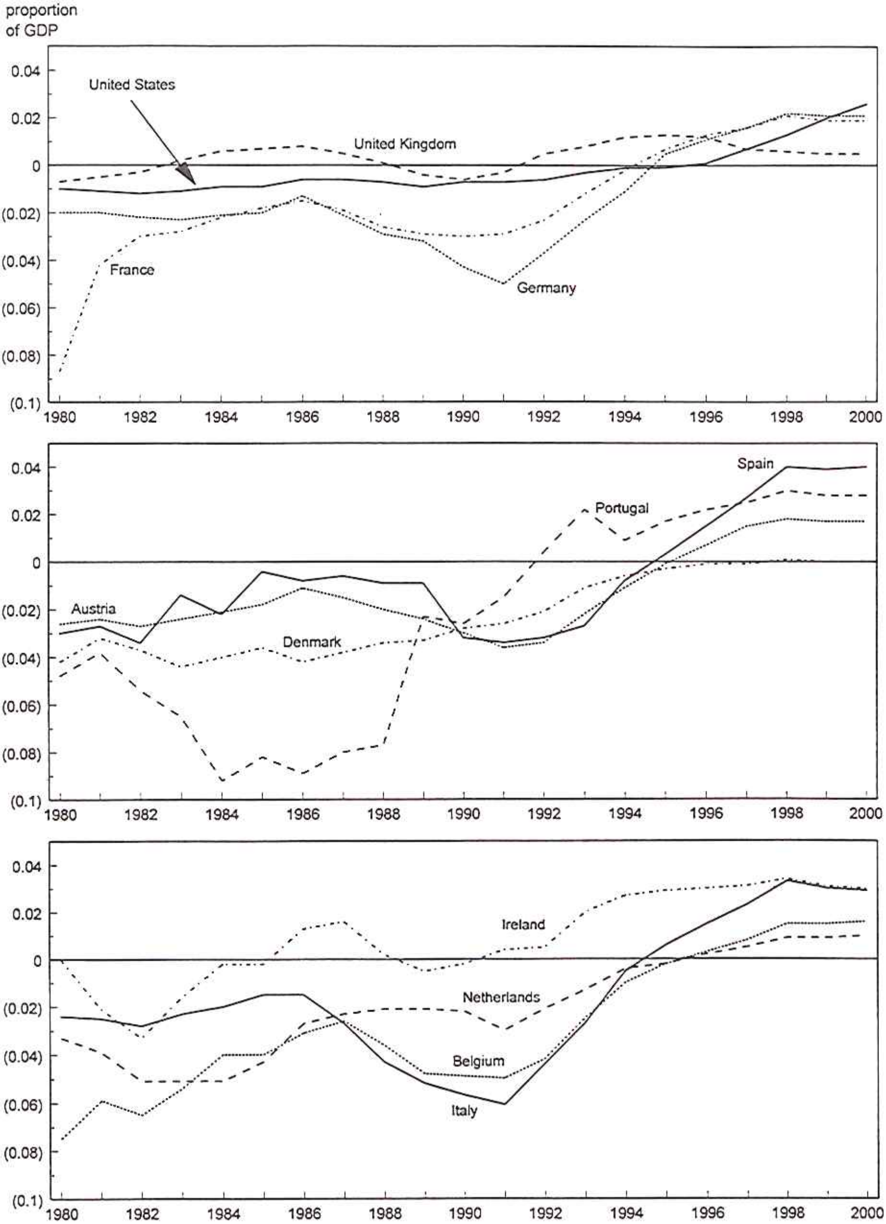
* The chart shows at each point in time the required change in the average tax (or spending) rate *over the next five years* to satisfy the intertemporal budget constraint (equation 15). A positive fiscal pressure (the area above 0) indicates that the intertemporal budget constraint is not satisfied, and future primary surpluses should be generated.

Chart 3: Fiscal Pressure With 60% Debt Limit in 1999*



* See note in Chart 2 for further details.

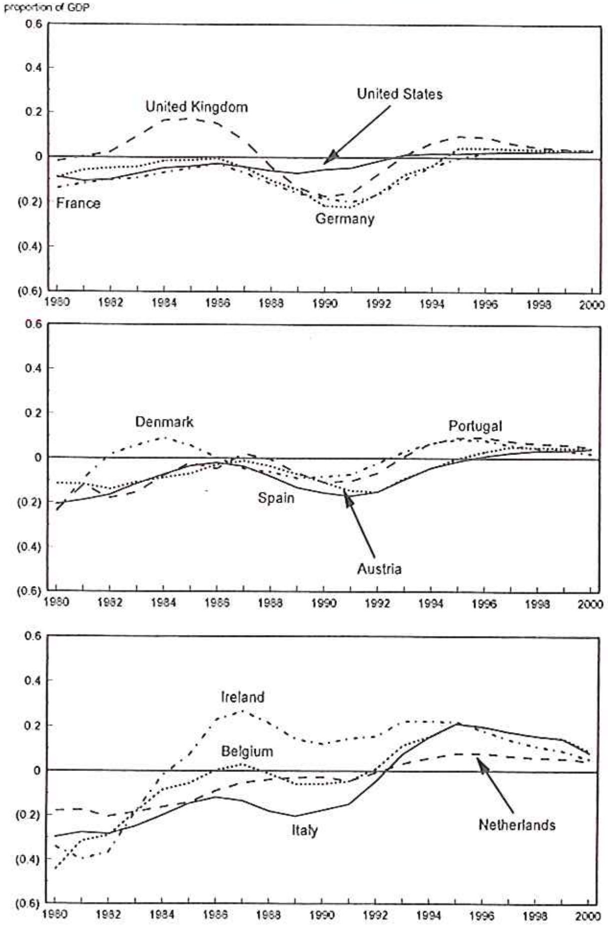
Chart 4: Fiscal Pressure With Deficit Limits of 3% in 1999 for the EU and 0% in 2002 for the US*



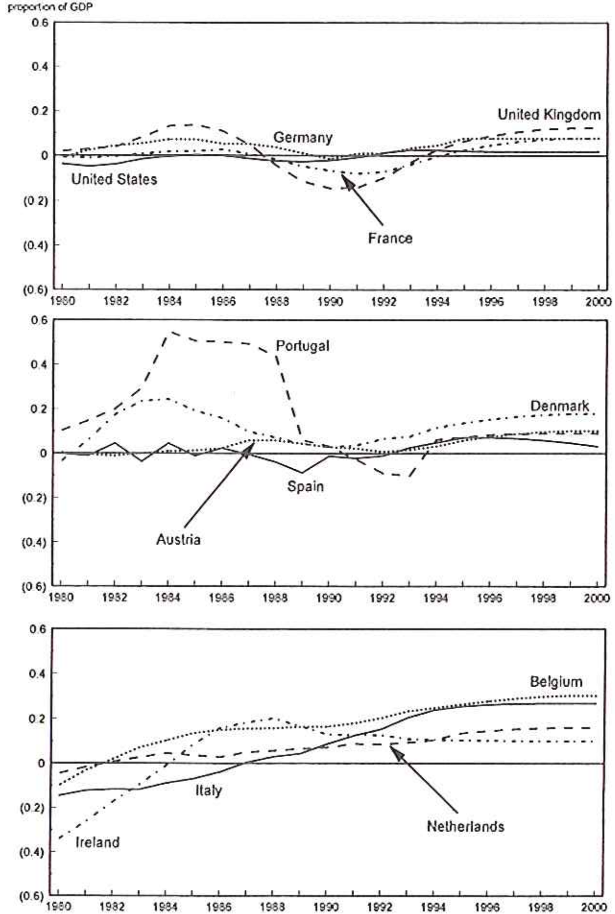
* See note in Chart 2 for further details.

Appendix A*

Desired Debt Accumulation



Cumulated Future Primary Balances



* The chart shows the left-hand-side (left panel) and the right-hand-side (right panel) of equation (14). Left panel: a negative slope violates the sustainability condition because it indicates that the discounted value of debt five years ahead is larger than the current outstanding stock of debt. Right panel: the area under the curve is the cumulated future