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JEL Classification: C78, D02, D50, D85, F60

Keywords: competitive exchange, Matching theory, networks, market integration, gainers and losers

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Gainers and Losers from Market Integration*

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We compare integration of economic, matching and networking markets. There can be losers from integration in all three cases, but their relative numbers depend on the type of market. There can be many losers from integration of pure exchange economies. There are relatively few losers from integration of networking markets. In the matching case, the relative numbers tend to lie between those of the other two cases.

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1 Introduction

When two hitherto separate economic or social systems are merged to become one, all previous outcomes are still feasible. Therefore, the integrated system can, in principle, achieve better or at least equally good outcomes. This optimistic outlook is correct from a social planner's perspective. Yet, some individuals can be worse off after integration when the outcome is determined in a competitive or strategic equilibrium.

Both theory and casual empiricism suggest that economic integration can have beneficial effects on some economic agents and detrimental effects on others. In this paper, we make some simple comparisons how integration in three well understood equilibrium models produces gainers and losers: competitive pure exchange economies, two-sided pairwise matching and strategic network formation.

We first consider pure exchange economies in the sense of Arrow-Debreu, that is economies as described in Debreu (1959). It is well understood that economic integration can harm some economic agents, in particular some consumers. To quote Samuelson (1962, p. 823): "Practical men and economic theorists have always known that trade may help some people and hurt others. Our problem is to show that trade lovers are theoretically able to compensate trade haters for the harm done them, thereby making everyone better off." Samuelson (1962), Kemp (1962), Dixit and Norman (1986), among others examine various ways to achieve such a compensation. Under certain conditions, there exists a best-case scenario where everybody benefits from economic integration. Here we are mostly concerned with the question how many losers there could be in the worst case and how this compares to integration in other markets.

Second, we consider two-sided pairwise matching studied by Gale and Shapley

(1962) and Roth and Sotomayor (1990), with stable matching as the solution concept. Crawford (1991), among others, studies the comparative statics of adding agents to matching markets. He finds that adding an agent to one side of the market weakens the competitive positions of the other agents on that side and strengthens the competitive positions of the agents on the other side.[1] In a similar vein, Sprumont (1990) obtains that every assignment game with at least two sellers and two buyers where every seller-buyer pair derives positive gains from trade, does not admit a population monotonic allocation scheme in the sense of Sprumont. Crawford's result is also consistent with our findings: Suppose a woman is added to the marriage market. Then the only agents who can lose are the women who were matched with a man before. But the number of previously matched women does not exceed the number of men plus the number of women who previously remained single. Hence the original agents who lose from entry consist of at most half of the original population as predicted by Proposition 3.

Finally, we consider strategic network formation. A number of recent contributions have treated social and economic networks as the outcome of a network formation game. The players of the game constitute the nodes of the network to be formed. In the purely noncooperative approach of Bala and Goyal (2000) adopted here, addition and deletion of links are unilateral decisions of the player from whom the respective links originate.[2] The player's strategy is a specification of the set of agents with whom he forms links. The costs of link formation are incurred only by the player who initiates the link. The formed links define the network.

Our investigation is focused on the gainers and losers from integration and,

[1]See also Theorem 2.26 in Roth and Sotomayor (1990).

[2]Pairwise stability à la Jackson and Wolinsky (1996) treats addition of a link as a bilateral decision by the two players involved, whereas severance of a link constitutes a unilateral decision.

more precisely, their relative numbers. In the pure exchange context, almost all but not all consumers can lose. In the matching context, less than half of the members of each group can be losers. In the strategic network formation setting, integration of two groups causes at most half the members plus one in one group to be losers and nobody to lose in the other group. We shall comment on these results in Section 5 and relate them to enduring debates about gainers and losers from market integration in the literature.

The next three sections are devoted to the three different integration scenarios.

2 Pure Exchange Economies

Pure exchange economies belong to the canon of contemporary microeconomic theory. Therefore, we confine their formal description to the bare minimum.

2.1 Brief Outline of the Model

A finite pure exchange economy is specified by a tuple $\mathcal{E} = (X_i, \succsim_i, \omega_i)_{i \in I}$. The economy consists of a finite set I of consumers. There exist a finite number of commodities $l = 1, \ldots, \ell$. Each consumer has consumption set $X_i = \mathbb{R}^{\ell}_{+}$. Superscripts denote commodities. We denote by x_i^l the quantity of commodity l consumed by $i \in I$. Consumer i's consumption bundles thus assume the form $x_i = (x_i^1, \ldots, x_i^{\ell})$. In our general theory, ℓ can be any finite number. In the numerical examples, $\ell = 2$. Consumer i has complete and transitive preferences on X_i, represented by the binary relation $\succsim_i$. Finally, each consumer is endowed with a commodity bundle $\omega_i \in X_i$.

The following definitions apply to the economy $\mathcal{E}$ at large:

Feasible Allocations:	A feasible allocation is a vector $x = (x_i)_{i \in I} \in \mathcal{X} \equiv \Pi_{i \in I} X_i$ such that $\sum_{i \in I} x_i = \sum_{i \in I} \omega_i$.
Pareto Optima:	A feasible allocation $x = (x_i)_{i \in I}$ is Pareto optimal if there is no feasible allocation $(x'_i)_{i \in I}$ such that $x'_i \succ_i x_i$ for some $i \in I$ and $x'_i \succsim_i x_i$ for all $i \in I$.
Weak Pareto Optima:	A feasible allocation $x = (x_i)_{i \in I}$ is weakly Pareto optimal if there is no feasible allocation $(x'_i)_{i \in I}$ such that $x'_i \succ_i x_i$ for all $i \in I$.
Competitive Equilibrium:	A competitive equilibrium is a pair (x^*, p^*) where $x^* = (x^*_i)_{i \in I}$ is a feasible allocation, $p^* \in \mathbb{R}^\ell_+$ is a price system, and for each consumer $i \in I$: $p^* x^*_i \leq p^* \omega_i$ and $x_i \in X_i, x_i \succ_i x^*_i$ implies $p^* x_i > p^* \omega_i$. x^* is called an equilibrium allocation. p^* is called an equilibrium or market clearing price system.

We shall invoke the following version of the first welfare theorem.

Proposition 1 (First Welfare Theorem)**.**

(a) *Equilibrium allocations are weakly Pareto optimal.*

(b) *If preferences are locally non-satiated, then equilibrium allocations are Pareto optimal.*

Notice that (a) holds without any assumption on preferences whereas (b) holds for locally non-satiated and transitive but not necessarily complete preferences.

By Proposition 11.C.1 of Mas-Colell, Whinston and Green (1995), a finite pure exchange economy $\mathcal{E} = (X_i, \succsim_i, \omega_i)_{i \in I}$ has a competitive equilibrium if $\sum_{i \in I} \omega_i \gg 0$

and every consumer has continuous, strictly convex and strongly monotone preferences. These sufficient conditions are not necessary. In particular, a competitive equilibrium exists if all consumers have Cobb-Douglas preferences. However, non-existence may occur if the sufficient conditions are violated. Of special interest to us is the case where each of two separate pure exchange economies has a competitive equilibrium, but there is no competitive equilibrium of the integrated economy. This means that market integration can have a destabilizing effect. Such a case will be presented in 2.4. We first consider a simple example to illustrate how integration generates gainers and losers.

2.2 A First Example

We consider the separate economies of Aland and Eland and then the economy after integration of the two. The economy of Aland consists of three consumers, Anna, Bart, and Carl, labeled a, b and c, respectively. Let $I_A = \{a, b, c\}$ denote the set of these consumers. There are two commodities ($\ell = 2$). Each $i \in I_A$ has an endowment $\omega_i = (1, 1)$ and consumption set $X_i = \mathbb{R}^2_+$. Preferences are given by the utility functions

$$\begin{array}{rcll} U_a(x_a^1, x_a^2) & = & x_a^1 (x_a^2)^3 & \text{for Anna;} \\ U_b(x_b^1, x_b^2) & = & x_b^1 x_b^2 & \text{for Bart;} \\ U_c(x_c^1, x_c^2) & = & \min\{x_c^1, x_c^2\} & \text{for Carl.} \end{array}$$

Up to price normalization, this economy has a unique competitive equilibrium (x_A^*, p_A^*) given as $p_A^* = (1, 5/3)$, $x_A^* = (x_a^*, x_b^*, x_c^*)$, $x_a^* = (2/3, 6/5)$, $x_b^* = (4/3, 4/5)$, $x_c^* = (1, 1)$, $U_a(x_a^*) = 144/125$, $U_b(x_b^*) = 16/15$, $U_c(x_c^*) = 1$.

The economy of Eland consists of two consumers, Dennis and Esther, labeled d, and e, respectively. Let $I_E = \{d, e\}$ denote the set of these consumers. Each

$i \in I_E$ has an endowment $\omega_i = (1,1)$ and consumption set $X_i = \mathbb{R}^2_+$. Preferences are given by the utility functions

$$\begin{aligned} U_d(x_d^1, x_d^2) &= (x_d^1)^3 x_d^2 \quad \text{for Dennis;} \\ U_e(x_e^1, x_e^2) &= x_e^1 x_e^2 \quad \text{for Esther.} \end{aligned}$$

Up to price normalization, this economy has a unique competitive equilibrium (x_E^*, p_E^*) given as $p_E^* = (1, 3/5)$, $x_E^* = (x_d^*, x_e^*)$, $x_d^* = (6/5, 2/3)$, $x_e^* = (4/5, 4/3)$, $U_d(x_d^*) = 144/125, , U_e(x_e^*) = 16/15$.

In the integrated economy with consumers in $I = I_A \cup I_E$, the unique equilibrium (up to price normalization) is $(\hat{x}, \hat{p})$ with $\hat{p} = (1,1)$, $\hat{x} = (\hat{x}_a, \hat{x}_b, \hat{x}_c, \hat{x}_d, \hat{x}_e)$, $\hat{x}_a = (1/2, 3/2)$, $\hat{x}_b = (1,1)$, $\hat{x}_c = (1,1)$, $\hat{x}_d = (3/2, 1/2)$, $\hat{x}_e = (1,1)$, $U_a(\hat{x}_a) = 27/16$, $U_b(\hat{x}_b) = 1$, $U_c(\hat{x}_c) = 1$, $U_d(\hat{x}_d) = 27/16$, $U_e(\hat{x}_e) = 1$. It follows that Anna and Dennis are gainers from integration, Bart and Esther are losers, and Carl is unaffected by integration. In this particular example, Bart and Esther actively trade in the respective equilibria before integration and do not trade after integration. More commonly, a consumer might trade before and after market integration, but nevertheless lose from integration.

Further notice that the allocation $x^* = (x_i^*)_{i \in I}$ is of course feasible in the integrated economiy, but not weakly Pareto optimal. It is strictly dominated by some feasible allocation x'. For instance, $x'_b = x'_e = (16/15 - \varepsilon, 15/16 - \varepsilon)$, $x'_a = (2/3 + \varepsilon, 6/5 + \varepsilon)$, $x'_c = (1 + \varepsilon, 1 + \varepsilon)$, $x'_d = (6/5 + \varepsilon, 2/3 + \varepsilon)$ with sufficiently small $\varepsilon > 0$ will do. However, x^* is not dominated by $\hat{x}$.

2.3 More General Analysis

We consider two countries whose economies are initially separated and compare original equilibrium outcomes with equilibrium allocations after market integration. As a consequence of the first welfare theorem, we obtain

Proposition 2.

(a) *Not all consumers can be losers from market integration.*

(b) *In case preferences are locally non-satiated, if there exists a loser, then there also exists a gainer from market integration.*

PROOF. Let the two countries A and B have respective sets of consumers I_A and I_B. Set $I = I_A \cup I_B$. Before integration, let (p_A^*, x_A^*) be a competitive equilibrium for A's economy, with $x_A^* = (x_i^*)_{i \in I_A}$, and (p_B^*, x_B^*) be a competitive equilibrium for B's economy, with $x_B^* = (x_i^*)_{i \in I_B}$. Furthermore, consider a competitive equilibrium $(\hat{x}, \hat{p})$ of the integrated economy, with $\hat{x} = (\hat{x}_i)_{i \in I}$. Finally, define $x^* = (x_i^*)_{i \in I}$, a feasible allocation of the integrated economy.

(a) Suppose all consumers are losers from market integration, that is, $x_i^* \succ_i \hat{x}_i$ for all $i \in I$. Then $\hat{x}$ is not weakly Pareto optimal, contradicting part (a) of Proposition 1. Hence to the contrary, not all consumers can be losers.

(b) Suppose that there are some losers from market integration and no gainers. That means $x_i^* \succ_i \hat{x}_i$ for some $i \in I$ and $x_i^* \succeq_i \hat{x}_i$ for all $i \in I$. Therefore, $\hat{x}$ is not Pareto optimal. But if preferences are locally non-satiated, then by part (b) of Proposition 1, the equilibrium allocation $\hat{x}$ is Pareto optimal. Thus a contradiction results. Hence to the contrary, if there are some losers, there has to exist a gainer as well. ∎

Remarks

1.) The proposition does not rule out the possibility that almost all consumers lose. Consider, for example, consumer populations $I_A = \{1, \ldots, n\}$ and $I_B = \{n+1\}$ with $n \geq 2$. Let the commodity space be $\mathbb{R}^2$ and each consumer i's consumption set be $X_i = \mathbb{R}^2_+$. All consumers have Cobb-Douglas preferences represented by $u_i(x_i^1, x_i^2) = x_i^1 x_i^2$ for $(x_i^1, x_i^2) \in \mathbb{R}^2_+$. Consumer 1 has endowment bundle $\omega_1 = (1, 1)$ whereas consumers $i = 2, \ldots, n+1$ have endowment bundle $\omega_i = (1, 2)$. Then consumers 1 and $n+1$ gain from economic integration and all other consumers lose. This example exhibits the maximal number of losers in each of the groups I_A and I_B. Namely, invoking core inclusion (Proposition 18.B.1 of Mas-Colell et al. (1995)) instead of the first welfare theorem, one can show

Corollary 1. (a) *I_A and I_B each contain a non-loser from integration.*

(b) *If $K \geq 2$ originally separate economies are merged, then each of these economies contains a non-loser from integration.*

This result explains that there are no losers from economic integration when the consumption sector of each country is represented by a representative consumer, as is the case in many macroeconomic and international trade models. It takes heterogeneous consumers in a country to observe losers from market integration or trade liberalization.

2.) It is possible that all consumers benefit from market integration. For example, assume that all consumers have the endowment bundle $(1, 1)$, all consumers within I_A have identical Cobb-Douglas preferences, all consumers within I_B have identical Cobb-Douglas preferences, but preferences in I_A differ from those in I_B.

3.) It is also possible that the welfare of consumers in country B, say, is unaffected by economic integration, but merely their presence in the integrated economy affects the consumers in country A. For example, let A's economy consist of consumers 1 and 2, both with endowment bundle $(1,1)$. Consumer 1 has lexicographic preferences: $(x_1^1, x_1^2) \succ_1 (y_1^1, y_1^2)$ if $x_1^1 > y_1^1$ or $x_1^1 = y_1^1$ & $x_1^2 > y_1^2$. Consumer 2 only cares for commodity 2, hence has utility representation $U_2(x_2^1, x_2^2) = x_2^2$. Up to price normalization, A's economy has a unique competitive equilibrium (x_A^*, p_A^*) with $p_A^* = (1,1)$, $x_A^* = (x_1^*, x_2^*) = ((2,0),(0,2))$. Country B consists of consumer 3 with endowment bundle $\omega_3 = (1,1)$ and utility function $U_3(x_3^1, x_3^2) = x_3^1 + 2x_3^2$. Up to price normalization, this economy has a unique competitive equilibrium (x_B^*, p_B^*) with $p_B^* = (1,2)$ and $x_B^* = x_3^* = \omega_3 = (1,1)$. Up to price normalization, the integrated economy has a unique competitive equilibrium $(\hat{x}, \hat{p})$ with $\hat{p} = (1,2)$ and $\hat{x} = (\hat{x}_1, \hat{x}_2, \hat{x}_3) = ((3,0),(0,1.5),(0,1.5))$.

4.) Like in the proof of Proposition 2, let $x^* = (x_i^*)_{i \in I}$ be a competitive equilibrium allocation achieved before integration. If after integration, consumers re-trade from endowments x_i^*, $i \in I$, then nobody loses from integration. This follows from individual rationality of competitive equilibrium allocations. In contrast, losers can occur if trade in the integrated economy takes place from initial endowments $\omega_i, i \in I$.

5.) Chambers and Hayashi (2019) also consider integration of pairs of pure exchange economies with $\ell \geq 2$ commodities. They examine the response of allocation mechanisms (given by a social choice function) to such a merger. They find that there is no Pareto efficient allocation rule that always encourages economic integration. Up to one third of the consumers involved may be hurt if in addition, the rule satisfies equal treatment of equals in welfare terms. Several qualifying comments are indicated:

5.1. Chambers and Hayashi (2019) postulate on the outset that all consumers have strictly positive endowment bundles and preference relations drawn from the set $\mathcal{R}$ of strictly convex, strongly monotone and weak orders. Yet the analysis demonstrating their main results, Theorems 1 and 2, rests on an example with three consumers having Cobb-Douglas preferences — which are not strictly convex and strongly monotone. But their conclusions still hold if $\mathcal{R}$ is replaced by any set of preferences that contains the particular Cobb-Douglas preferences. Moreover, their analysis can be redone with an example of three consumers having preferences in $\mathcal{R}$. Take $\ell = 2$ and three consumers with utility representation $u(x^1, x^2) = \sqrt{x^1} + \sqrt{x^2}$. Let country A's economy consist of consumers 1 and 2 with respective endowment bundles $\omega_1 = (5, 1)$ and $\omega_2 = (1, 5)$. Let country B's economy consist of consumer 3 with endowment bundle $\omega_3 = (30, 3)$. Then consumers 2 and 3 gain from market integration while consumer 1 loses. The extension to arbitrary efficient social choice functions obtains as in their paper.

5.2. Our above example (in Remark 3).) with $n - 2$ losers and 2 gainers from integration can also be used to perform the Chambers & Hayashi analysis and yields

Corollary 2. *For any $n \geq 2$, the fraction of losers from integration is $(n-1)/(n+1) = 1 - 2/(n+1)$ in some instances.*

Following Chambers and Hayashi, this result can be extended to any efficient social choice function.[3]

[3]Indeed, their Theorem 2 is an immediate corollary of the proof of their Theorem 1.

5.3. While Chambers and Hayashi (2019) are primarily concerned with the possibility of losers, our main focus lies on comparing the upper bounds on the number of losers from integration in three different market scenarios.

6.) Kumar et al. (2020) show that in a housing market à la Shapley and Scarf (1974), there can be at most $N - K$ losers when K separate markets are integrated and the entire population I has size $N = |I|$. They further show the upper bound is attained in some example. These results parallel our findings in Corollary 1(b) and the surrounding discussion. Notice that the housing market model constitutes an instance of an exchange economy with discrete objects. Both proofs of the upper bound result stem from the fact that the pertinent outcomes of the integrated market are core allocations.[4] Kumar et al. (2020) focus on the unique core allocation, which is the outcome of the top trading cycle algorithm. We resort to the core inclusion theorem.

2.4 Destabilizing Effect of Integration

This example relies on non-convex preferences of the consumers in country A. Let country A consist of two consumers 1 and 2, each with endowment bundle $\omega_i = (1, 1)$ and utility function $U_i(x_i^1, x_i^2) = (x_i^1)^2 + (x_i^2)^2$. Up to price normalization, this economy has two equilibria with price system $p_A^* = (1, 1)$ where one consumer chooses consumption bundle $(2, 0)$ and the other consumer chooses consumption bundle $(0, 2)$.

Let country B consist of consumer 3 with endowment bundle $\omega_3 = (1, 1)$ and

[4]Since in the housing market, individuals are matched with discrete objects, the model is often viewed as a particular matching model.

utility function $U_3(x_3^1, x_3^2) = x_3^1 + 2x_3^2$. Up to price normalization, this economy has a unique competitive equilibrium (p_B^*, x_B^*) with $p_B^* = (1,2)$ and $x_B^* = x_3^* = \omega_3 = (1,1)$.

Now consider the integrated economy. Because of monotonicity of preferences, an equilibrium price system p has to satisfy $p = (p_1, p_2) \gg (0,0)$. Let us take commodity 1 as numéraire so that $p_1 = 1$. We can distinguish several cases depending on the size of p_2. If $p_2 < 1$, then consumers 1 and 2 each demand more than 2 units of good 2; hence there is positive excess demand for good 2. If $p_2 = 1$ then there are three subcases: In case both consumers in A demand two units of good 2, there is positive excess demand for good 2. In case one of the consumers in A demands 2 units of good 1 and the other demands 2 units of good 2, there is still positive excess demand for good 2, since consumer 3 will demand 2 units of that good. In case both consumers in A demand two units of good 1, then there is excess demand for good 1. If $p_2 > 1$, then consumers 1 and 2 each demand more than 2 units of good 1 and there is positive excess demand for good 1. Thus there are no relative prices at which the market in the integrated economy is cleared. This shows that market integration can have a destabilizing effect.

We should add that market integration can have a stabilizing effect as well. A trivial example obtains when every consumer i has endowment bundle $\omega_i = (1,1)$ and utility representation $U_i(x_i^1, x_i^2) = (x_i^1)^2 + (x_i^2)^2$ and each country's economy consists of an odd number of consumers. Another example is given by $\ell = 2$, $I_A = \{1\}$, $I_B = \{2\}$, $U_1(x_1^1, x_1^2) = x_1^1 + \sqrt{x_1^2}$, $\omega_1 = (1,0)$, $U_2(x_2^1, x_2^2) = \sqrt{x_2^1} + x_2^2$, $\omega_2 = (0,1)$.

3 Pairwise Two-sided Matching

Our focus lies on two-sided pairwise matching as in the seminal contribution of Gale and Shapley (1962). In the marriage market interpretation, the population consists of men and women. A matching selects heterosexual couples such that each individual is matched with exactly one partner of the other sex or remains unmatched. Stability requires that no matched person prefers to be single and no pair consisting of a man and a woman prefers being a couple to the status quo. This presupposes that individuals have preferences over partners, including having no partner. In the standard setting of two-sided matching, in principle, a man can be matched with any woman and vice versa — if one disregards preferences.

We observe that a stable matching exists in any matching market where all individuals have complete and transitive preferences (Gale and Shapley 1962). We will establish as a general result that with strict preferences, the number of losers from integration cannot exceed the number of gainers. More precisely, if two or more separate groups are merged into one, then within each group, at most half of the members are losers from integration.

3.1 Basic Model

Some of the notation and terminology is adopted from Roth and Sotomayor (1990). There is a finite population I that is partitioned into a non-empty male subpopulation $M = \{m_1, \ldots, m_k\}$ and a non-empty female subpopulation $W = \{w_1, \ldots, w_\ell\}$. A bipartite graph or bigraph based on the given partition is an undirected graph whose edges are of the form $\{m, w\}$ for some $m \in M$ and some $w \in W$.

Matching. A **matching** is a bigraph $\mathcal{M}$ based on the given partition such that no two of its edges share an element: $\{m, w\} \cap \{m', w'\} = \emptyset$ if $\{m, w\}$, $\{m', w'\} \in$

$\mathcal{M}$, $\{m, w\} \neq \{m', w'\}$. Elements of $\mathcal{M}$ are called matches. A matching $\mathcal{M}$ can be identified with a bijection $\mu : I \to I$ with the properties that (a) $\mu^{-1} = \mu$; (b) $\mu(m) \in W \cup \{m\}$ for all $m \in M$; (c) $\mu(w) \in M \cup \{w\}$ for all $w \in W$.

Preferences. Individuals have **preferences for partners**, including the possibility of not having a partner. Each $m \in M$ has complete and transitive preferences on $W \cup \{m\}$, represented by an ordered list $P(m)$. For example

$$P(m) = w_1, w_2, m, w_3, \ldots, w_\ell$$

means that m's first choice is to be married to woman w_1, his second choice is to be married to woman w_2, and his third choice is to remain single. Women w_3 to w_ℓ are not acceptable to m. He would rather be single than be married to one of them. An abbreviated list contains only the acceptable women:

$$P(m) = w_1, w_2.$$

Similarly, each $w \in W$ has complete and transitive preferences on $M \cup \{w\}$, represented by an ordered list $P(w)$. If a person is indifferent between several possible mates, then those are put in square brackets in the preference list:

$$P(w) = m_2, [m_3, m_4, w], m_1$$

means that m_2 is w's first choice; to be married to m_3, to be married to m_4 and being single tie for second choice while m_1 is her third choice. The abbreviated list is $P(w) = m_2, [m_3, m_4, w]$.

$\mathbf{P} = (P(m_1), \ldots, P(m_k), P(w_1), \ldots, P(w_\ell))$ denotes a preference profile, that is a collection of lists, one for each individual. Then a particular marriage market is specified by a triple $(M, W; \mathbf{P})$. The notation $w >_m w'$ means that m prefers woman w to woman w' and $w \geq_m w'$ means that m likes w at least as much as w'. $m >_w m'$ and $m \geq_w m'$ are defined in an analogous way. Woman w is

acceptable to man m if he likes her at least as much as remaining single, i.e., $w \geq_m m$. Analogously, m is acceptable to w if $m \geq_w w$. An individual is said to have **strict preferences** if he or she is not indifferent between any two acceptable alternatives.

Stable Matching. Formally, we consider matchings that are stable according to the following

Definition 1. *A matching $\mathcal{M}$ (or, equivalently, μ) in the marriage market $(M, W; \mathbf{P})$ is stable if:*

1. *For any $m \in M$, $w \in W$ who are matched in $\mathcal{M}$, neither partner wants to go single. That is, $\{m, w\} \in \mathcal{M}$ (or, equivalently $m = \mu(w)$) implies $w \geq_m m$ and $m \geq_w w$.*

2. *There is no pair $(m, w) \in M \times W$ who can get married to each other and who prefer this marriage to the status quo. That is, $\mu(m) \geq_m w$ or $\mu(w) \geq_w m$.*

In particular, a stable matching μ is **individually rational**, that is $\mu(i) \geq_i i$ for all $i \in I$. If a pair $\{m, w\}$ renders a potential matching unstable, we say that $\{m, w\}$ **blocks** or destabilizes the particular matching. That is, $\{m, w\}$ blocks the matching μ if $w >_m \mu(m)$ and $m >_w \mu(w)$. This is the **strong version of blocking**. In the weak version of blocking, the pair $\{m, w\}$ blocks the matching μ if $w >_m \mu(m)$, $m \geq_w \mu(w)$ or if $w \geq_m \mu(m)$, $m >_w \mu(w)$. The two notions of blocking coincide in the case of strict preferences. Gale and Shapley (1962) show existence of stable matchings in the case of strict preferences. Their method of proof can be extended to the case of preferences with possible ties. See Theorem 2.8 in Roth and Sotomayor (1990). If the weak version of blocking is employed and ties are possible, then the corresponding set of "strongly stable" matchings may be empty, as a simple example in Irving (1994) demonstrates.

3.2 Desegregation of Marriage Markets

Segregation of various sorts, be it racial, religious or political, restricts the marriage market. As an actual example for the latter, consider the case of South and North Korea or former West and East Germany. How would desegregation affect the marriage market? We are going to study this question first in a simple example. More specifically, we consider the case where the population I is segregated into two sub-populations $I_1 = M_1 \cup W_1$ and $I_2 = M_2 \cup W_2$, with $M_i = M \cap I_i$ and $W_i = W \cap I_i$. Marriages are only possible within I_1 and within I_2.

Example 1 (Effect of desegregation)**.** Let $k = \ell = 8$ and

$P(m) = w_1, w_2, \ldots, w_k$ for all $m \in M$;

$P(w) = m_1, m_2, \ldots, m_k$ for all $w \in W$.

Suppose the population $I = M \cup W$ is segregated into two parts $I_1 = M_1 \cup W_1$ and $I_2 = M_2 \cup W_2$ where $M_1 = \{m_1, m_2, m_3, m_4\}, M_2 = \{m_5, m_6, m_7, m_8\}, W_1 = \{w_2, w_3, w_4, w_5\}, W_2 = \{w_1, w_6, w_7, w_8\}$. Marriages are only possible within I_1 and within I_2. Let $\mathbf{P_1}$ denote the restriction of the preference profile $\mathbf{P}$ to members of I_1 and $\mathbf{P_2}$ denote the restriction of the preference profile $\mathbf{P}$ to members of I_2. Then:

Under segregation, there exists a unique stable matching μ given by

$\mu(m_1) = w_2, \mu(m_2) = w_3, \mu(m_3) = w_4, \mu(m_4) = w_5$ for members of I_1,

$\mu(m_5) = w_1, \mu(m_6) = w_6, \mu(m_7) = w_7, \mu(m_8) = w_8$ for members of I_2.

After desegregation, there exists a unique stable matching μ' given by

$\mu'(m_i) = w_i$ for $i = 1, \ldots, 8$.

Men m_1, m_2, m_3, m_4 and woman w_1 gain from desegregation whereas male m_5 and females w_2, w_3, w_4, w_5 lose. $\square\square$

In the example, there are equal numbers of losers and gainers in I, and also in each of the subpopulations I_1 and I_2. This observation generalizes as follows:

Proposition 3. *Suppose the population is segregated into $K \geq 2$ non-empty subpopulations $I_1, \ldots, I_K$. Let μ be any matching under segregation and μ' be a stable matching after desegregation. Then*

$$|\{i \in I_k : \mu'(i) <_i \mu(i)\}| \leq |I_k|/2 \textit{ for } k = 1, \ldots, K.$$

PROOF. Suppose that $|\{i \in I_k : \mu'(i) <_i \mu(i)\}| > |I_k|/2$ for some $k \in \{1, \ldots, K\}$. $\mu'(i) <_i \mu(i)$ implies $\mu(i) \neq i$ for $i \in I_k$. That is, all members of $J_k \equiv \{i \in I_k : \mu'(i) <_i \mu(i)\}$ were matched under μ. Since $|J_k| > |I_k|/2$, there exist two members of J_k who were matched with each other under μ. Therefore, they can block μ', contradicting the assumed stability of μ'. Hence to the contrary, the assertion of the proposition has to hold. ■

This results has been first reported as Proposition 8 in our working paper version Gersbach and Haller (2015). The proposition says that in each group I_k, there are weakly more gainers than losers. As a corollary of our result, we obtain a result by Ortega (2018): In the grand economy, there are weakly more winners than losers, too. It is possible that, indeed, half of the people in each group are losers. It is also possible that there are no losers at all.

Ortega (2019) measures individual gains from integration as the difference in spouse ranking before and after integration. For instance, an agent's gain is +3 if the agent obtains his 5th best partner before integration and his 2nd best partner after integration; the agent's gain is -2 if he gets his best partner before integration and his third best partner after integration. Ortega finds that the sum of individual gains from integration can be quite negative. Aue et al. (2020) show that in pairwise two-sided matching, none of the agents on one side of the market may gain from integration. In their Example 1 (in our terminology), two of the three men lose from integration and one is unaffected by integration where the male-optimal stable matchings are chosen before and after integration. But the choice of stable matchings matters in the example: If the matchings before integration are female-optimal and the matching after integration is male-optimal, then there are no gainers or losers.

4 Strategic Network Formation

To our knowledge, integration of networking markets has not been investigated before. Our analysis of strategic network formation games is going to be two-fold. First we will demonstrate, by means of an elaborate example with a heterogeneous player population, that integration can cause isolation of a previously central player. Second, we study the effects of integrating two groups where all individuals are homogeneous and linkage costs are small. We show that from an ex ante point of view, integration always proves desirable with a homogeneous player population (and small linkage costs). While ex post losers cannot be ruled out, we find striking restrictions on their numbers. As a rule, the upper bound on the number of losers from integration of two networking groups is less than the corresponding upper bound when two matching markets are integrated.

4.1 Basic Model

Our basic model is the two-way flow connections model à la Galeotti, Goyal, and Kamphorst (2006) that incorporates cost and value heterogeneity. We adopt the notation of Haller, Kamphorst and Sarangi (2007) for the case of perfectly reliable links.

Let $n \geq 3$. $N = \{1, \ldots, n\}$ denotes the set of players with generic elements i, j, k. N also constitutes the set of nodes of the network to be formed. For ordered pairs $(i, j) \in N \times N$, the shorthand notation ij or i,j is used and for non-ordered pairs $\{i, j\} \subset N$ the shorthand $[ij]$ is used. The model is specified by two families of parameters, indexed by ij, with $i \neq j$:

- Cost parameters $c_{ij} > 0$.
- Value parameters $V_{ij} > 0$.

In case $c_{ij} \neq c_{kl}$ ($V_{ij} \neq V_{kl}$) for some $ij \neq kl$, the model exhibits **cost (value) heterogeneity**; otherwise, it exhibits **cost (value) homogeneity**.

We only consider pure strategies. A pure strategy for player i is a vector $g_i = (g_{i1}, \ldots, g_{ii-1}, g_{ii+1}, \ldots, g_{in}) \in \{0, 1\}^{N \setminus \{i\}}$. The set of all pure strategies of agent i is denoted by $\mathcal{G}_i$. It consists of 2^{n-1} elements. The joint strategy space is given by $\mathcal{G} = \mathcal{G}_1 \times \cdots \times \mathcal{G}_n$.

There is a one-to-one correspondence between the set of joint strategies $\mathcal{G}$ and the set of all directed graphs or networks with vertex set N. Namely, to a strategy profile $g = (g_1, \ldots, g_n) \in \mathcal{G}$ corresponds the graph $(N, E(g))$ with edge or node set $E(g) = \{(i, j) \in N \times N | \, i \neq j, \, g_{ij} = 1\}$. In the sequel, we shall identify a joint strategy g and the corresponding graph and use the terminology **directed graph** or **directed network** g. Since our aim is to model network formation, $g_{ij} = 1$ is interpreted as a direct link between i and j is initiated by player i (edge ij is formed by i) whereas $g_{ij} = 0$ means that i does not initiate the link (ij is not formed). Regardless of what player i does, player j can set $g_{ji} = 1$, i.e., initiate a link with i, or set $g_{ji} = 0$, i.e., not initiate a link with i.

Benefits. A link between agents i and j potentially allows for **two-way (symmetric) flow of information**. Accordingly, the benefits from network g are derived from its closure $\overline{g} \in \mathcal{G}$, defined by $\overline{g}_{ij} := \max\{g_{ij}, g_{ji}\}$ for $i \neq j$. Moreover, a player receives information from others not only through direct links, but also via indirect links. To be precise, information flows from player j to player i, if i and j are linked by means of a path in $\overline{g}$ from i to j. A **path** of length m in $f \in \mathcal{G}$ from player i to player $j \neq i$, is a finite sequence $i_0, i_1, \ldots, i_m$ of pairwise distinct players such that $i_0 = i$, $i_m = j$, and $f_{i_k i_{k+1}} = 1$ for $k = 0, \ldots, m-1$. Let us denote

$$N_i(f) = \{j \in N |\, j \neq i, \text{ there exists a path in } f \text{ from } i \text{ to } j\},$$

the set of other players whom player i can access or "observe" in the network f. Information received from player j is worth V_{ij} to player i. Therefore, player i's benefit from a network g with perfectly reliable links and two-way flow of information is (as in Galeotti, Goyal, and Kamphorst (2006)):

$$B_i(g) = B_i(\overline{g}) = \sum_{j \in N_i(\overline{g})} V_{ij}.$$

Notice that $\overline{g}$ belongs to the set $\mathcal{H} = \{h \in \mathcal{G} | h_{ij} = h_{ji} \text{ for } i \neq j\}$. In turn, there is a one-to-one correspondence between the elements of $\mathcal{H}$ and the non-directed networks (graphs) with node set N. Namely, for $h \in \mathcal{H}$ and $i \neq j$, $[ij]$ is an edge of the corresponding non-directed network if and only if $h_{ij} = h_{ji} = 1$.

Costs. Player i incurs the cost c_{ij} when she initiates the direct link ij, i.e., if

$g_{ij} = 1$. Hence when the network g is formed, i incurs the total costs

$$C_i(g) = \sum_{j \neq i} g_{ij} c_{ij}.$$

Payoffs. Player i's payoff from the strategy profile g is the net benefit

$$\Pi_i(g) = B_i(g) - C_i(g). \tag{1}$$

Nash Networks. Given a network $g \in \mathcal{G}$, let g_{-i} denote the network that remains when all of agent i's links have been removed so that $g_{-i} \in \mathcal{G}_{-i} \equiv \prod_{j \neq i} \mathcal{G}_i$. Clearly $g = g_i \oplus g_{-i}$ where the symbol $\oplus$ indicates that g is formed by the union of links in g_i and g_{-i}. A strategy g_i is a **best response** of agent i to g_{-i} if

$$\Pi_i(g_i \oplus g_{-i}) \geq \Pi_i(g'_i \oplus g_{-i}) \;\text{ for all }\; g'_i \in \mathcal{G}_i.$$

Let $BR_i(g_{-i})$ denote the set of agent i's best responses to g_{-i}. A network $g = (g_1, \ldots, g_n)$ is said to be a **Nash network** if $g_i \in BR_i(g_{-i})$ for each i, that is if g is a Nash equilibrium of the strategic game with normal form $(N, (\mathcal{G}_i)_{i \in N}, (\Pi_i)_{i \in N})$.

Efficient Networks. Let $W : \mathcal{G} \to \mathbb{R}$ be defined as $W(g) = \sum_{i=1}^{n} \Pi_i(g)$. A network $\widehat{g}$ is **efficient (in the narrow traditional sense)** if $W(\widehat{g}) \geq W(g)$ for all $g \in \mathcal{G}$. Efficiency is a major performance criterion for network designers or planners and plays a prominent role in the traditional network literature. It is most attractive for cost-benefit analysis when payoffs are monetary and side-payments between players are feasible. Efficiency constitutes an important benchmark for network performance even when network formation is decentralized and structured as a strategic game. In economics, the term "efficiency" is often used in a broader sense, as a synonym for Pareto optimality.

Some Graph-theoretic Concepts. We now introduce some definitions of a more graph-theoretic nature. The network with no links is called the **empty network** and will be denoted e. A network g is said to be **connected** if there is a path in $\overline{g}$ between any two agents i and j. A connected network g is **minimally connected**, if it is no longer connected after the deletion of *any* link.

4.2 Isolation Caused by Integration

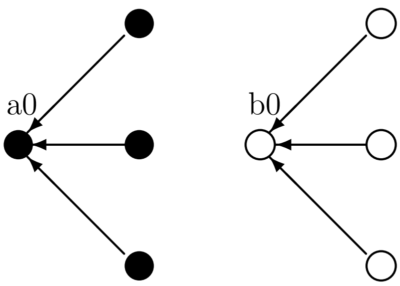
Let $A = \{a0, a1, \ldots, am\}$ with $m \geq 1$ and $B = \{b0, b1, \ldots, bn\}$ with $n \geq 2$ be two groups of players. We assume $V_{ja0} = 0$ for all $j \neq a0$ and $V_{ij} = 1$ otherwise.

Costs can take three possible values: $m+n+2$, c_A, c_B with $m-2 < c_A < m-1$ and $c_B < 1$. Costs are realized as follows: $c_{ak,a0} = c_A$ for $k = 1, \ldots, m$; $c_{ak,b0} = c_B$ for $k = 1, \ldots, m$; $c_{b\ell,b0} = c_B$ for $\ell = 1, \ldots, n$; and $c_{ij} = m + n + 2$ otherwise.

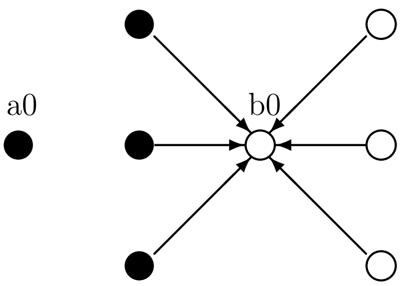
Thus in equilibrium, $a0$ will not form any links and value-wise is worthless by himself. Still, his serving as an intermediary can benefit the other players and himself. Indeed, when network formation is confined to group A, then the empty network is Nash and the only non-empty Nash network is the periphery-sponsored star S_A with $a0$ as center. Similarly, when network formation is confined to group B, the periphery-sponsored star S_B with $b0$ at the center is the only Nash network — while the empty network is not Nash.

After the two groups are integrated, the star S_B persists as an equilibrium subnetwork. Moreover, in equilibrium at least one link ak,$b0$ is formed, say $a1$,$b0$. First of all, there cannot be a Nash equilibrium where all links in S_A and S_B plus the link $a1$,$b0$ exist. Namely, if link $a2$,$b0$ does not exist, then player $a2$ is better off severing link $a2$,$a0$ and forming link $a2$,$b0$ and if link $a2$,$b0$ does exist, then player $a2$ is better off severing link $a2$,$a0$ and keeping link $a2$,$b0$. Hence in Nash equilibrium, some link ak,$b0$, say $a1$,$b0$, and all links in S_B, but not all links in S_A are formed. Let link $a\ell$,$a0$ be one of the links in S_A that are no longer formed. Then the link $a\ell$,$b0$ exists in equilibrium. Now let $h \neq 0, \ell$. If both ah,$a0$ and ah,$b0$ exist, then ah's payoff is at most $m + n - 1 - (c_A + c_B)$. When ah severs the link ah,$a0$, then the payoff is at least $n + 1 - c_B = n - 1 - c_B + 2 = m + n - 1 - c_B + 2 - m = m + n - 1 - c_B - (m - 2) > m + n - 1 - (c_B + c_A)$. Hence ah prefers severing that link. If only ah,$a0$ exists, then ah is better off severing that link and forming ah,$b0$. If only ah,$b0$ exists, then ah does not want to change his strategy. It follows that in Nash equilibrium, all links in S_B and all links ak,$b0$ exist and none of the links in S_A. Consequently, $a0$ is isolated now.

There are two possible scenarios. First, the empty network A may be group A's equilibrium network. Then all players benefit from integration except $a0$ who remains isolated. Second, the star S_A may be group A's equilibrium network. Then again all players benefit from integration except $a0$ who loses the privileged intermediary status and becomes isolated. The case $m = n = 3$ is depicted in the following diagram.



Before Integration



After Integration

4.3 Effects of Integration in Homogeneous Populations

Next let us consider the homogeneous model with $V_{ij} = V$ and $c_{ij} = c$ for all ij. We focus on the low cost case where $c < V$. Without loss of generality, we may proceed with $V = 1$ and $c < 1$. For that case, Bala and Goyal (2000, p. 1202) show that the Nash networks are all the minimally connected networks.[5] Starting with two separate groups A and B with $|A| = m > 1$ and $|B| = n \geq 1$, take any Nash network g_A in A and any Nash network g_B in B. Then the aggregate welfare in group A is $m(m-1)-(m-1)c$ and the per capita welfare is $m-1-(1-1/m)c$. The latter is also a player's ex ante expected payoff if all Nash networks have an equal chance of being formed. As a consequence, we obtain the first assertion of the next proposition.

Proposition 4. *In the homogeneous model of network formation with $V = 1$ and $c < 1$, the following assertions hold:*

(a) *Ex ante, all players benefit from integration.*

(b) *Ex post, a player may lose.*

(c) *Ex post, at least one player gains.*

(d) *Ex post, all players may gain.*

PROOF. (a) The ex ante expected payoff is increasing in the number of players. (b) Suppose $B = \{b\}$. The periphery-sponsored star S_A with center $a \in A$ is a Nash network in A. The center-sponsored star $S_{A\cup B}$ with center a is a Nash network after integration. Thus a remains the intermediary, but instead of getting

[5] Haller, Kamphorst and Sarangi (2007) show that Nash networks do not exist in some heterogeneous models.

a free ride, he shoulders all the costs in the larger network. Player a obtains payoffs $\Pi_a(S_A) = m - 1$ and $\Pi_a(S_{A\cup B}) = m - mc$. Hence in case $c > 1/m$, he is worse off after integration.

(c) A minimally connected network of $m + n$ players has $m + n - 1$ links. Hence ex post, at least one player does not form a link and gains from integration. Namely, the player's benefits increase without a cost increase.

(d) If one labels $i = 1, \ldots, m$ the players in A and $i = m + 1, \ldots, m + n$ the players in B, then the network consisting of the links i,i+1 for $i = 1, \ldots, m+n-1$ is minimally connected and hence a Nash network after integration. In that network, each player forms at most one link and, therefore, gains from integration. ■

Proposition 5. *In every Nash equilibrium after integration of the homogeneous model of network formation with $V = 1$ and $c < 1$, there are either no losers in group A and at most $1 + (n - 1)/m$ losers in group B or no losers in group B and at most $1 + (m - 1)/n$ losers in group A.*

PROOF. A member of A enjoys the added benefit n from integration. In order to be a loser, the player has to form more than n/c links. Similarly, a loser in B has to form more than m/c links. Let L_A denote the number of losers in A and L_B denote the number of losers in B. Then all losers combined form more than $L_A \cdot \frac{n}{c} + L_B \cdot \frac{m}{c}$ links. Since the total number of links is $m + n - 1$ in equilibrium,

$$L_A \cdot \frac{n}{c} + L_B \cdot \frac{m}{c} < m + n - 1$$

has to hold. This implies

$$L_A \cdot n + L_B \cdot m < m + n - 1$$

because of $c < 1$. If there is at least one user in each group, then $L_A \cdot n + L_B \cdot m \geq$

$m+n$, a contradiction. Thus there is no loser in one of the groups, say group A. In the latter case, $L_B \cdot m < m+n-1$ or $L_B < 1+(n-1)/m$. In case there are no losers in B, then $L_A < 1+(m-1)/n$. ■

The proposition implies that if the two groups have equal even numbers of members, then at most 25% of all players lose from integration and all belong to one group. For instance, $L_A < 1+(m-1)/n$ and $n \geq 2$ imply $L_A < 1+(m-1)/2 = (m+1)/2$ and $L_A \leq m/2 = |A|/2$.

Inspection of the proof shows that the assertion of the proposition can be slightly sharpened by considering non-gainers instead of losers. Hence if the two groups have equal even numbers of members, then at least 75% of all players gain from integration and one of the groups consists only of gainers. Further inspection shows that a member of A is either a gainer or a loser if n/c is not a natural number — and a member of B is either a gainer or a loser if m/c is not a natural number. Therefore,

Corollary 3. *For generic cost parameters, including irrational c, ex post every player is either a gainer or a loser from integration.*

We also obtain

Corollary 4. *If $c < k \cdot \min\{m,n\}/(n+m-1)$, then there are less than k losers.*

Ramifications

Suppose that we start with $K > 2$ mutually disjoint and non-empty groups of players G_k, $k = 1, \ldots, K$. Each group G_k has size $n_k = |G_k| \geq 1$. We assume $n_k > 1$ for at least one k. Then the assertion and proof of Proposition 4 generalize. The rationale behind Proposition 5 generalizes as well. In particular, the ex post losers from integration belong all to just one group.

5 Concluding Remarks

We have seen that the integration of commodity markets, matching or marriage markets, or networking markets can have both gainers and losers. Losers can be prevalent when commodity markets are integrated. The relative numbers of losers tend to be smallest when networking markets are merged. It turns out that the relative number of losers declines when social or economic interaction becomes more intense. In the case of competitive pure exchange economies, consumers interact anonymously with the market, not one-on-one. A change in the composition of the consumer population can alter the terms of trade to the disadvantage of most consumers. In the marriage or matching market, an agent is directly affected by the choice of partner, but not the identity of others. In the case of networking markets, the welfare of an agent is affected by direct links as well as indirect ones. One would expect occurrence of some losers from integration when network externalities are negative, like in the coauthor model. In contrast, the benchmark model underlying our analysis exhibits positive network externalities: A player benefits or at least is not harmed when someone else forms a link. Therefore, to have losers from market integration is not that plausible. Still, there can be some losers, though not very many. To be a loser, a player has to create many links. Then others get away with few links. In turn, the loser has an incentive to form the large number of links. In a sense, a loser gets boxed in at equilibrium.

We have limited ourselves to some simple comparisons regarding gainers and losers of integration of economic, matching and networking markets. There are entire branches in economics that deal with gainers and losers in the context of production. The famous 1941 Stolper-Samuelson result showed that for a two-goods, two-markets economy, market integration with a labor-abundant economy can yield lower real wages and thus hurt workers.[6]. Since 1941, the literature has repeatedly dealt with potential and actual negative consequences of market integration or "globalization" on subgroups and small regions within the economy, coexisting with positive aggregate effects (see Autor et al. (2013, 2015), Feenstra and Sasahara (2018), Krugman (2019)). Similarly, reducing obstacles to labor mobility and thus achieving more integration in the labor market has been the topic of much political debate and empirical analysis. Hammond and Sempere (2006) consider potential gains from free migration in addition to or in lieu of free trade. Finally, there is an extended literature on how firms (and managers) are affected by market integration—with conclusions depending on market structures and the nature of competition.[7] Our study is complementary to the literature, as we focus on comparisons of relative gainers and losers across three different types of markets.

Let us conclude with two more observations. Our analysis also applies to secession or market segregation if one reads the results in reverse. And there may be other reasons than purely economic ones why a country wants to join or leave a group like the European Union. Gainers and losers from politically motivated integration are an important area for thorough future analysis.

[6]Stolper and Samuelson (1941)

[7]See, e.g., Gersbach and Schmutzler (2014) for the integration of Cournot markets, Eaton et al. (2011) for empirical evidence of a differential impact of international trade on French firms.

References

Aue, R., Klein, T. and Ortega, J., 2020: "What Happens when Separate and Unequal School Districts Merge?" ZEW - Centre for European Economic Research Discussion Paper No. 20-032.

Autor, D. H., Dorn, D. and Hanson, G. H., 2013: "The China Syndrome: Local Labor Market Effects of Import Competition in the United States," *American Economic Review*, 103, 2121-2168.

Autor, D. H., Dorn, D. and Hanson, G. H., 2015: "Untangling Trade and Technology: Evidence from Local Labor Markets," *Economic Journal*, 125, 621-646.

Bala, V. and Goyal, S., 2000: "A Non-Cooperative Model of Network Formation," *Econometrica,* 68, 1181-1229.

Chambers, C. P. and Hayashi, T., 2019: "Can Everyone Benefit from Economic Integration?" *Journal of Public Economic Theory*, DOI: 10.1111/jpet.12420.

Crawford, V. P., 1991: "Comparative Statics in Matching Games," *Journal of Economic Theory*, 54, 389-400.

Debreu, G., 1959: *The Theory of Value: An Axiomatic Analysis of Economic Equilibrium*, Wiley: New York.

Dixit, A. and Norman, 1986: "Gains from Trade without Lump-Sum compensation," *Journal of International Economics*, 21, 111-122.

Eaton, J., Kortum, S. and Kramarz, F., 2011: "An Anatomy of International Trade: Evidence from French Firms," *Econometrica*, 79, 1453-1498.

Feenstra, R. C. and Sasahara, A., 2018: "The 'China Shock,' Exports and U.S. Employment: A Global Input-output Analysis," *Review of International Economics*, 26, 1053-1083.

Gale, D. and Shapley, L. S., 1962: "College Admissions and the Stability of Marriage", *American Mathematical Monthly*, 69, 91-14.

Galeotti, A., Goyal, S. and Kamphorst, J., 2006: "Network Formation with Heterogeneous Players," *Games and Economic Behavior*, 54, 353-372.

Gersbach, H. and Haller, H., 2015: "Matching on Bipartite Graphs," CES Ifo Working Paper No. 5575.

Gersbach, H. and Schmutzler, A., 2014: "Does Globalization Create Superstars? A Simple Theory of Managerial Wages", *European Economic Review*, 71, 34-51.

Haller, H., Kamphorst, J. and Sarangi, S., 2007: "(Non-)Existence and Scope of Nash Networks," *Economic Theory*, 31, 597-604.

Hammond, P. J. and Sempere, J., 2006: "Gains from Trade versus Gains from Migration: What Makes Them So Different?" *Journal of Public Economic Theory*, 8, 145-170.

Irving, R. W., 1994: "Stable Marriage and Indifference," *Discrete Applied Mathematics*, 48, 261-272.

Jackson, M. O., Wolinsky, A., 1996: "A Strategic Model of Economic and Social Networks," *Journal of Economic Theory*, 71, 44-74.

Kemp, M. C., 1962: "The Gain from International Trade," *The Economic Journal*, 72, 803-819,

Krugman, P., 2019: "What Did We Miss about Globalization?", *Corden Public Lecture 2019*, University of Melbourne. https://www.youtube.com/watch?v=rWQ3jCURzy0 (accessed November 4, 2019).

Kumar, R., Manocha, K., Ortega, J., 2020: "On the Integration of Shapley-Scarf Housing Markets," *arXiv preprint arXiv:2004.09075.*

Mas-Colell, A., Whinston, M. D. and Green, J. R., 1995: *Microeconomic Theory.* Oxford University Press: New York and Oxford.

Ortega, J., 2018: "Social Integration in Two-sided Matching Markets," *Journal of Mathematical Economics*, 78, 119-126.

Ortega, J., 2019: "The Losses from Integration in Matching Markets Can Be Large," *Economics Letters*, 174, 48-51.

Roth, A. E. and Sotomayor, M. A. O., 1990: *Two-Sided Matching: A Study in Game-Theoretic Modeling and Analysis.* Cambridge University Press: Cambridge, UK.

Samuelson, P. A. 1962: "The Gains from International Trade Once Again," *The Economic Journal*, 72, 820-829.

Sprumont, Y., 1990: "Population Monotonic Allocation Schemes for Cooperative Games with Transferable Utility," *Games and Economic Behavior*, 2, 378-394.

Stolper, W. F. and Samuelson, P. A., 1941: "Protection and Real Wages," *Review of Economic Studies*, 9, 58-73.