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Abstract

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JEL Classification: F16, F66, J6

Keywords: Trade Shocks, jobs, Labor Mobility, adjustment costs, worker welfare

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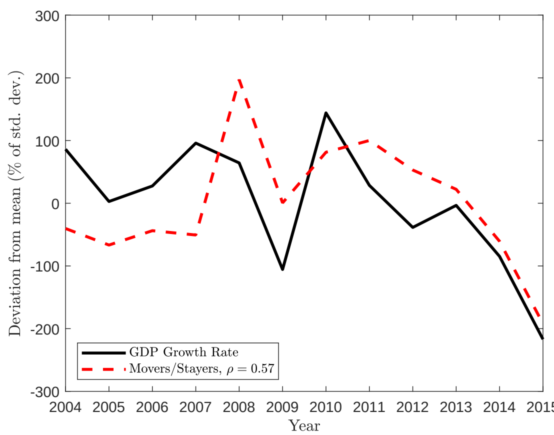
1 Introduction

How do trade shocks impact workers? What can be done to facilitate labor adjustment in response to trade shocks? Answering these questions requires an understanding of how trade shocks impact not only wages but also labor mobility. If workers can easily move across occupations, sectors, or regions, they would readily pursue alternative job options following a trade shock. For this reason, there is now a substantial body of research studying the drivers and implications of imperfect labor mobility in the international trade context.[1]

Empirically, labor mobility is pro-cyclical: it tends to increase with positive shocks and decrease with negative shocks. As an illustration of this empirical pattern, Figure 1 depicts a positive correlation between labor mobility and GDP growth using data from Brazil.

Figure 1: GDP growth rate and labor mobility, 2004-2015



Notes: The figure shows the ratio between the number of movers (workers who switch occupation, sector, or region) and the number of stayers. We use the same definitions of occupation, sector, and region as those adopted in the main analysis of the paper. See Section 2 for data sources and the precise definitions of each variable. Both series are in terms of deviations from the mean and multiplied by the inverse of their standard deviations for comparison.

There is a simple and intuitive explanation for this empirical regularity: the number of job options available to workers tends to increase with positive shocks and decrease with negative shocks. If we conceptualize workers' mobility decision based on the choice of a *job*, rather than the choice of occupations, sectors, or regions–all of which have empirically invariant quantities–, then their choice set should expand or shrink with shocks. Naturally, the size of the choice set impacts the probability of moving when there are switching costs, but existing discrete choice models cannot account for this relationship.[2] In this paper, we develop a novel general equilibrium discrete choice model that features endogenous expansion and contraction of the workers' choice set. This new channel has important

[1] This literature includes contributions from Artuç, Chaudhuri, and McLaren (2010), Dix-Carneiro (2014), Traiberman (2019), and Caliendo, Dvorkin, and Parro (2019), among others.

[2] In standard discrete choice models, shocks impact labor mobility as they can push the system out of steady state, but there is no reason for positive (negative) shocks to increase (reduce) labor mobility consistently. This paper, on the other hand, formulates a precise relationship between different dimensions of labor mobility and trade shocks.

theoretical and empirical implications: since trade shocks impact the size of the choice set, this new mechanism matters for workers' mobility and welfare; and also fundamentally changes how we should conceptualize the impact of trade shocks on workers.

The new channel we emphasize matters to workers for two main reasons. First, if a worker can choose her job out of more options, it is more likely that the best one delivers higher welfare. Second, even when she is hit by a negative labor demand shock in the future, it is more likely that she will be able to find another job without having to move to a different region or sector. Thus, a region-sector pair (henceforth referred to as a *labor market*) receiving a positive trade shock will attract more workers not just because it provides a higher wage, but also because of the larger number of job options that are created there.

In addition, a labor market with a positive trade shock will experience larger internal churning, i.e., more job switching within the labor market, which has been largely overlooked in the literature. Our model clearly shows how trade shocks affect workers' welfare through labor mobility between and within labor markets. The model delivers a structural equation of trade-induced changes in workers' welfare, which can be conveniently estimated in a reduced-form way. We then quantify the magnitude of the welfare effect of a trade shock through the full simulation of the model.

Our framework is motivated by reduced-form evidence on the effects of export shocks on labor market outcomes, which draws on rich employer-employee panel data combined with customs records on export transactions from Brazil during 2003-2015. To account for the endogeneity of exports, we construct an instrument at the labor market level, exploiting exogenous variation in sectoral import demand directed to the labor market. The IV estimates reveal a positive causal effect of exports on residual wages, employment, worker inflows, and job turnover rates within the corresponding labor market. The effect on the internal job turnover rates is the key motivation for our framework.

Building on this reduced-form evidence, we develop a new dynamic general equilibrium model of labor mobility with an endogenous number of choices. Different labor markets offer different wages and different numbers of job options. A worker chooses the job which gives her the highest utility, where the number of job options in each labor market is endogenously determined. This is a distinctive feature of our framework compared to previous discrete choice models which effectively assume that the number of choices is exogenously fixed and the same across labor markets.

Our general equilibrium framework shows that a positive trade shock to a certain labor market– e.g., a decline in the cost of exporting,–increases the number of job options in the market. Workers attach idiosyncratic utility to each job option, and thus more options lead to a higher utility from the best job. In addition, workers in a growing labor market face a lower risk of having to pay a higher switching cost in the future, as we assume that a job switch requiring a change of labor market incurs a higher switching cost than a job switch within a labor market. The prospect of a job switch generates an option value in workers' welfare. We further decompose this option value into the option value from alternative job options within the current labor market and the option value from having alternative job options in other markets.

Our model shows that the change in relative welfare of workers across labor markets is a function of only the probability of moving and the labor supply elasticity. This is a powerful result that greatly simplifies the analysis of the welfare impacts of trade shocks. The relative welfare result does not depend on the underlying moving cost structure, observed changes in future wages, or moving probabilities across jobs within a labor market.

We then structurally estimate the model using the employer-employee panel data from Brazil. In the first stage of the estimation, we pin down the common value attached to each labor market and the moving cost between labor markets using a gravity-like equation. The implied probability of moving between labor markets is then calculated with the estimated value of each market and the estimated moving cost. In the second stage, we pin down the labor supply elasticity of our model. We derive an estimable equation describing the relationship between a change in the transformed value of the labor market and a change in wages, with the labor supply elasticity governing the responsiveness of the former to the latter. We exploit variation in residual wages induced by the same instrument we used earlier for motivating evidence: the exogenous change in import demand directed to the labor market. By plugging in these estimates into the relative welfare result previously mentioned, we find that, during the sample period, a 10% rise in exports increases the lifetime welfare of a median formal sector worker by 2.99% of her annual wage.

Finally, we turn to the full general equilibrium simulation of our model to quantify the effect of a trade shock on workers' welfare and various labor market outcomes. Our benchmark trade shock is a 20% permanent decline of trade costs in the manufacturing sector from Brazil to each of its trading partners. This positive export shock to the Brazilian manufacturing sector reallocates labor from the agriculture sector toward the manufacturing and service sectors. On average, the lifetime welfare of workers increases due to this shock, with the amount of increase equivalent to a 32.45% one-time and temporary increase of the annual wage. Workers in the manufacturing sector experience 15.13% larger welfare increases than workers in the agriculture sector.

We highlight the importance of our job options channel by comparing our benchmark model to an alternative specification without this channel. We show that the benchmark model generates a larger labor reallocation between labor markets compared to the alternative model, primarily by increasing the likelihood that workers find a better job in response to a positive shock. In addition, our channel is able to capture trade-induced within-market mobility, which does not exist in the alternative specification. As a result, the welfare effect from the alternative model is 67.7% smaller than in our benchmark model with the job options channel.

Lastly, we explore the role of labor mobility frictions in welfare gains from trade by quantifying the effect of policies mitigating the degree of mobility frictions faced by workers. This is an important policy question for which the existing literature provides virtually no guidance.[3] We show that 20% lower mobility frictions across regions and sectors increase welfare gains from the same benchmark trade shock by 14.8%. Our model can be also used to quantify the effect of a policy targeting either sector-level or region-level mobility frictions separately. We show that the welfare-enhancing effect is greater when the policy targets mobility frictions across regions than across sectors.[4]

Related literature. This paper contributes to the literature of dynamic labor mobility and trade by introducing a novel channel through which mobility and welfare are affected by trade: *an endogenous number of choices*. Earlier works in the literature including Artuç, Chaudhuri, and McLaren (2010, ACM, henceforth), Dix-Carneiro (2014), and Traiberman (2019) study the dynamic transmission of trade shocks via labor mobility based on discrete choice models with a fixed number of choice options.[5] We follow the convention of this literature when modeling workers' idiosyncratic shock but endogenize the number of choice options based on a general equilibrium framework.[6] Other papers have also extended different aspects of the discrete choice framework: e.g., Dickstein and Morales (2018) and Morales, Sheu, and Zahler (2019) use a moment inequality approach for large-dimensional choice sets; and Porcher (2020) incorporated rational inattention.[7]

By endogenizing the number of job options and bringing it to the welfare analysis of trade shocks, we combine the main strength of dynamic models of labor mobility with that of the reduced-form literature on local labor market effects of trade, including influential contributions by Topalova (2010), Kovak (2013), Autor, Dorn, and Hanson (2013), McLaren and Hakobyan (2016), and Dix-Carneiro and Kovak (2017, 2019). This literature builds on the existence of frictions to spatial labor mobility to establish a causal relationship between trade shocks and employment changes in local labor markets, but does not speak to the magnitude of welfare changes.[8] Our paper answers this welfare question

[3]See, e.g., International Monetary Fund, World Bank, and World Trade Organization (2017) and Catao and Obstfeld (2019). Recently, Porcher (2020) studies the effect of lower mobility frictions related to information dissemination by focusing on the episode of expanded internet access in Brazil.

[4]Bergman, Chetty, DeLuca, Hendren, Katz, and Palmer (2020) offer experimental evidence on a successful housing mobility intervention that provided services (customized search assistance, landlord engagement, and short-term financial assistance) to reduce barriers to moving to high-upward-mobility neighborhoods. Similar policy interventions might facilitate regional labor mobility following a trade shock.

[5]McLaren (2017) offers an authoritative review of this literature.

[6]A related literature develops trade models featuring labor market search frictions–e.g., Davidson, Martin, and Matusz (1999), Coşar, Guner, and Tybout (2016), Helpman, Itskhoki, Muendler, and Redding (2017), and Ritter (2015). In standard search models explored in this literature, the number of jobs matters as it affects employment probabilities. In our model, however, workers are matched to multiple jobs as they are allowed to compare those jobs before choosing the best one. Therefore, our framework can account for the welfare effects generated by changes in both employment probabilities and the number of job options. Fajgelbaum (2020) studies job-to-job transitions by allowing for on-the-job search. Our paper models the value of jobs from a worker's perspective instead of a firm's perspective, and studies mobility frictions of various layers.

[7]In the related labor literature, Dupuy and Galichon (2014) study a discrete choice model with a continuum of choices.

[8]A related body of literature uses reduced-form methods to examine the effects of trade shocks on labor market outcomes at the industry, firm or worker level, including Revenga (1992), Goldberg and Pavcnik (2005, 2007), Verhoogen (2008), Brambilla, Lederman, and Porto (2012), Amiti and Davis (2012), Bertrand (2004), Hummels, Jørgensen, and Xiang (2014), Autor, Dorn, Hanson, and Song (2014), and Frías, Kaplan, Verhoogen, and Alfaro-Serrano (2018). Harrison, McLaren, and McMillan (2011) provide an overview of the literature on trade and inequality.

with our new model framework. We estimate the relative welfare effect using an instrumental-variable (IV) strategy analogous to that used in the reduced-form literature. We then quantify the magnitude of the welfare effect with the general equilibrium feature of the model. In addition, we show that we can apply the same IV strategy for the key structural parameter of the model, the labor supply elasticity, for a transparent identification in a dynamic setting.[9]

In a recent contribution, Caliendo, Dvorkin, and Parro (2019, CDP, henceforth) embed the workers' dynamic discrete choice problem with a fixed number of choices into a multi-country, multi-region, and multi-sector general equilibrium trade model. As important departures from CDP, we endogenize the number of job options available to workers in each labor market, and structurally estimate the welfare effects and other important primitives of the model. These structural estimates allow us to perform interesting policy simulations. For example, our model can reveal how much regional and sectoral components of the moving cost separately contribute to labor market frictions, and can be used to exactly quantify welfare implications of a specific reduction in each component calculated as a percentage change. Thus, it can shed light on which dimension—region or sector—policies should target with a priority in order to enhance the welfare gains from trade.

Interestingly, the welfare equation we derive relates closely to conventional welfare equations such as Arkolakis, Costinot, and Rodriguez-Clare (2012), which is consistent with various classes of trade models with homogeneous workers. In our model, workers' welfare depends on their actual mobility, but we still maintain the same spirit by deriving a parsimonious expression for changes in relative welfare as a function of only labor mobility and the labor supply elasticity.

Roadmap. The paper is organized as follows. In Section 2 we provide reduced-form evidence on the effect of export shocks on labor market outcomes. Motivated by this evidence, in Section 3 we introduce a new dynamic discrete choice model of labor mobility with trade and an endogenous choice set. In Section 4, we estimate the key parameters of the model and evaluate trade-induced welfare changes. In Section 5, we quantify our model and perform counterfactual exercises. In Section 6, we provide theoretical results assessing the ability of our model to match untargeted patterns in the data, and examine the model's fit and robustness in Section 7. Section 8 concludes.

2 Data and Reduced-form Evidence

In this section, we provide reduced-form evidence that motivates our model and describe the identification strategy that will be also used to estimate the structural parameters as well as the effect of trade shocks on various components of workers' welfare in Section 4.

[9] Galle, Rodriguez-Clare, and Yi (2017) develop a static multi-sector gravity model with heterogeneous workers to quantify the aggregate and group-level welfare effects of trade, and estimate a key structural parameter using the China shock as in Autor, Dorn, and Hanson (2013). Also in a static setting, Adão, Arkolakis, and Esposito (2019) exploit the same source of variation to estimate the labor allocation elasticity.

2.1 Data Sources

The empirical analysis in this paper combines and examines several sources of panel data from Brazil spanning the period 2003-2015. We provide a brief description of each data source in this section and give further details in Appendix A.3.

The main source of data is *Relação Anual de Informações Sociais* (RAIS), a labor census gathering longitudinal data on the universe of workers and firms in formal sectors of Brazil. RAIS is a high-quality administrative census of formal employers and employees, collected every year by the Brazilian Ministry of Labor. These records are used by the government to administer several government benefits programs. Workers are required to be in RAIS in order to receive payments of these programs, and firms face fines for failure to report.

RAIS covers virtually all formal workers in Brazil and provides yearly information on their demographics (age, gender, and schooling), job characteristics (detailed 6-digit occupation, wage, hours worked), as well as hiring and termination dates. For each job, the RAIS annual record reports average yearly earnings, as well as the monthly wage in December. We use the information on the December wage, so as to ensure that all labor market outcomes are measured at the same time and avoid potential mismeasurement for workers that did not work a full year. RAIS also includes information on a number of establishment-level characteristics, notably the number of employees, geographical location (municipality), and industry code (according to the 5-digit level of the Brazilian National Classification of Economic Activities). Unique identifiers (tax identification numbers) for workers and establishments make it possible to follow them over time. The establishment identifier contains 12 digits, and the first 8 digits make it possible to uniquely identify the firm.

We use the detailed classification of occupations to identify those who switch jobs. The Brazilian Classification of Occupations changed in 2002 (CBO-2) and has been reported consistently since 2003. Although the RAIS data are available for earlier years, we restrict the analysis to the post-2003 period to ensure that this important variable is defined in a consistent way throughout the sample period. There are 2637 occupation codes at the 6-digit level during this period. We use the information on the establishment's location (municipality) and industry, as well as worker-level data on gender, age, education, and December wage. While the RAIS data cover segments of the public sector, we restrict the analysis to the private sector. We focus on workers aged 16 to 64 years old.

Using RAIS, we define a *labor market* as a pair of a region and a sector. For sectors, we aggregate industries up to three broad sectors: agriculture, manufacturing, and services.[10] For regions, we use the "microregion" concept of the Brazilian Statistical Agency (IBGE) to define regional boundaries as in Dix-Carneiro and Kovak (2017). This definition groups economically integrated contiguous municipalities with similar geographic and productive attributes. We consider a set of 558 consistently

[10] We combine agricultural and natural resources industries into the agriculture sector.

defined microregions, grouping the 5571 municipalities in the data. To ensure a consistent definition of microregions over time, when necessary we merge microregions whose boundaries changed over the period of analysis.[11]

We merge RAIS with customs records on export transactions by microregion, industry, and destination each year from the administrative data collected by *Secretaria do Comércio Externo* (SECEX) of the Ministry of Development, Industry and Foreign Trade. These data are defined at the level of the municipality, detailed product category, and destination market, and are available since 1997. For consistency with the RAIS data and our definition of the labor market, we restrict the analysis to the post-2003 period and aggregate the customs records to the microregion-sector level. We further use yearly data on the industry-level imports of each of 189 Brazilian destinations reported in the customs data (sourced from all countries except Brazil) to construct the instrument and link them to the information on industry-level imports from the UN COMTRADE. We use the data of the year 2003 only for initial weights in our instrument and in the regressions, and the main analyses draw on yearly changes of variables of interest during 2004-2015 as specified in the next subsection.

2.2 Econometric Model

We now describe the econometric strategy for examining the effects of export shocks on labor markets. Our baseline specification is:

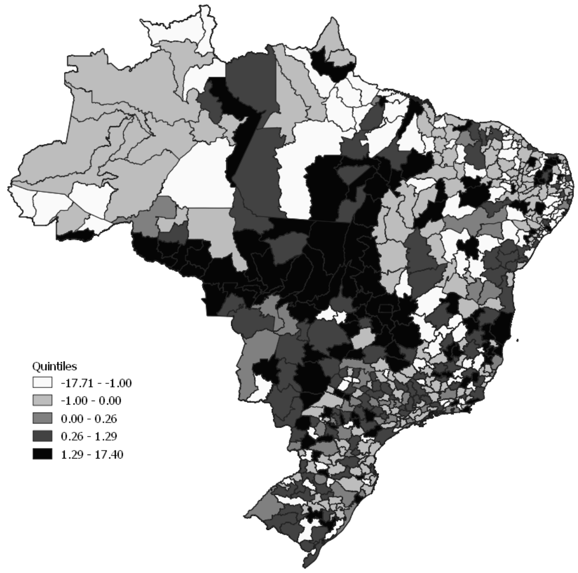
$$\Delta Y_t^k = \psi \Delta Z_t^k + \kappa_t + \epsilon_t^k, \tag{1}$$

where Y_t^k denotes the log of the outcome variable of interest in the region-sector pair, i.e., labor market, k in year t; Z_t^k is the log of export revenue originated in k; κ_t denotes a year fixed effect; and ϵ_t^k is the error term. The Δ operator denotes the linear change of a variable between year t and year $t-1$ throughout the paper.

Figure 2 depicts the change in export revenue observed in different microregions over the period of our main analysis, 2004-2015. It reveals that there exists substantial heterogeneity in the direction and the magnitude of the change in exports across space, which is convenient for identification. Notice also that because some labor markets were initially more export-oriented than others, they differ in the extent to which they are exposed to a given percentage change in export revenue. This heterogeneity is illustrated in Figure A1, which depicts the distribution of export revenue per worker across microregions in 2003. While the figures show only spatial heterogeneity, initial exports per worker also vary across sectors within each microregion. The same percentage change in exports would therefore be expected to have a stronger impact on labor market outcomes in labor markets where exports per worker were higher, to begin with. To account for this, in the regression analysis each observation is

[11] See Appendix A.3 for more details.

Figure 2: Change in export revenue, 2004-2015



Notes: Figure depicts the change in log of (1+export revenue) in Brazilian microregions during 2004-2015.

weighted by the export revenue per worker observed in the corresponding labor market in 2003.

An important concern is that changes in exports are potentially endogenous to changes in labor market outcomes. For example, lower wages or growing job turnover might lead to an increase in export activity. They could also reflect the role of omitted variables, such as underlying changes in infrastructure or technology. To address this issue, we adopt an instrumental variables approach. To identify a source of variation at the microregion-sector level, our strategy relies on variation over time in sectoral import demand directed to the region. This strategy builds on the fact that changes in external demand in a particular destination country have heterogeneous implications across labor markets; they matter more for labor markets that initially shipped a larger share of their exports to that destination. Our instrument is therefore defined as the log change of the trade-weighted sectoral imports of the initial set of destination countries of the labor market (sourced from all countries other than Brazil), where the weights are the export shares of each destination within each labor market in 2003. We formally define our instrument as follows:

$$\Delta \bar{Z}_t^k = \Delta \log \left(\sum_n \zeta_{n,2003}^k IM_{n,t}^{s_k} \right), \tag{2}$$

where $IM_{n,t}^{s_k}$ denotes destination n's total imports (excluding imports from Brazil) in sector s_k associated with labor market k in year t, and $\zeta_{n,2003}^k$ is the share of exports of labor market k to destination n in the labor market k's total export in 2003. Since different labor markets tend to serve different destinations, they vary in the degree to which they are exposed to changes in sectoral import demand from different countries. This heterogeneity is illustrated in Figure A2 in the Appendix, which depicts the main export destination of each microregion in 2003.[12] The intuition behind this instrumental variables approach follows closely that adopted in the local labor markets literature, including Topalova (2010), Kovak (2013), Autor, Dorn, and Hanson (2013). It is also closely related to earlier works using trade-weighted relative prices or import demand as the source of variation in industry-level imports or exports, such as Revenga (1992) and Bertrand (2004); as well as to more recent works exploiting similar sources of variation at the firm-level, including Brambilla, Lederman, and Porto (2012), Bastos, Silva, and Verhoogen (2018) and di Giovanni, Levchenko, and Mejean (2018).

2.3 Summary Statistics and Results

As described in Section 2.1, the overall sample consists of 3 sectors, 558 microregions, and 804 labor markets (microregion-sector pairs with non-zero observations) yielding a total of 3815 observations over the period of the main regression analyses, 2004-2015.[13] Table 1 reports means and standard deviations of the yearly changes of the variables used throughout the paper by sector (across microregions and over time). The table shows that employment growth was relatively higher in services, followed by manufacturing, and agriculture during the sample period. These differential dynamics across sectors are also reflected in the number of entrants and exiters, as well as in the number of job switchers within the labor market—defined as the number of workers who switched 6-digit detailed occupation within the labor market. As detailed in the Appendix, we purge individual-level wages from the effects of age, gender, and education, and take the average at the labor market level. Changes in these average purged wages are relatively more similar across sectors. Exports originated from firms whose main activity is within tradable sectors show significant variation, in line with the growth in external demand directed to Brazilian labor markets.

Table 1: Summary statistics, 2004-2015

	ΔEmployment	ΔWage	Δ # leaving	Δ # entering	Δ # switching jobs	ΔExports	$\Delta\bar{Z}$
Agriculture	0.0259	0.0319	0.155	0.165	0.0550	0.0213	0.0989
	(0.161)	(0.0924)	(0.821)	(0.880)	(0.453)	(1.444)	(0.229)
Manufacturing	0.0414	0.0223	0.124	0.140	0.0687	0.0548	0.0520
	(0.110)	(0.0563)	(0.596)	(0.731)	(0.393)	(0.647)	(0.129)
Services	0.0583	0.0182	0.0590	0.0828	0.0651	0.0004	0.0667
	(0.0323)	(0.0171)	(0.157)	(0.164)	(0.170)	(2.274)	(0.276)
Total	0.0362	0.0258	0.133	0.148	0.0634	0.0402	0.0702
	(0.131)	(0.0714)	(0.681)	(0.778)	(0.411)	(1.105)	(0.181)

Notes: Table reports summary statistics on the unrestricted estimation sample. Means are reported in plain text, and standard deviations are in parentheses.

[12]Notice that this pattern also varies across sectors within each microregion. This heterogeneity is exploited in the analysis, but is not reported in the figure to avoid visual cluttering.

[13]For consistency, we restrict the labor markets in our sample to be identical to those used in the structural estimation later, which requires us to impose additional restrictions. See section 4 for details.

Table 2 reports the first stage estimates relating changes in export revenue originated from each labor market to changes in external demand directed to the labor market, as defined in equation (2). The econometric results reveal that our instrument provides a suitable source of variation for examining the impact of plausibly exogenous export shocks on labor market outcomes. The coefficient of interest is 0.797 indicating that a 10% increase in external demand directed to the labor market leads to a 7.97% increase in exports. This relationship is precisely estimated, with a Kleibergen-Paap rk Wald F-stat of 26.10, which is indicative of a strong instrument.[14]

Table 2: First stage estimates

Dependent variable:	ΔExports
$\Delta\bar{Z}$	0.797
	(0.104)
F-stat	26.10
Observations	3815
Year effects	Y

Notes: Table reports first stage estimates. For each dependent and independent variable, we take log before computing the first differences. Observations are weighted by exports per worker in the labor market in 2003. Standard errors clustered by microregion and year are presented in parentheses.

We proceed by examining the causal effects of export shocks on various labor market outcomes. Table 3 presents the instrumental-variables estimates of (1), using the strategy discussed above. The first two columns show that a 10% increase in exports leads to a 2.4% increase in employment and 3.3% increase in average residual wages. The estimates in Columns (3)-(5) reveal that export shocks have important implications for gross worker flows between and within labor markets. A positive export shock leads to a decrease in the number of workers leaving the labor market (gross out-flows); an increase in the number of workers entering the labor market (gross in-flows); and a significant increase in the number of workers switching 6-digit detailed occupation within the labor market. The last result implies that a positive export shock increases internal churning, which is the key motivation of our model framework where workers can move between jobs in response to a trade shock.

3 Model

In this section, we first introduce a new dynamic labor mobility model with an endogenous number of job options. We define a labor market by a pair of a region and a sector consistently with the reduced-form analysis. Each labor market offers a different number of job options as well as a different wage. We then introduce international trade to this framework to examine how wages and the number of job options, and eventually between- and within-market labor mobility, are impacted by trade shocks.

[14]Since our specification requires taking log of exports, we use only observations with positive exports. All results (including the structural estimates in subsequent sections) are robust to alternative specifications such as adding one to exports before taking log or using inverse hyperbolic sine, changing by less than 10% for every variable of interest.

Table 3: IV estimates on the impact of export shocks on labor markets

Dependent variable:	ΔEmployment	ΔWage	Δ # leaving	Δ # entering	Δ # switching jobs
	(1)	(2)	(3)	(4)	(5)
Δ Exports	0.240	0.326	-0.996	0.372	0.495
	(0.040)	(0.036)	(0.142)	(0.083)	(0.114)
Observations	3815	3815	3815	3815	3815
Year effects	Y	Y	Y	Y	Y

Notes: Table reports IV estimation results of equation (1) for the baseline estimation sample, using (2) as the instrument. For all variables in change, we take log before computing the first differences. All specifications are weighted by exports per worker in the labor market in 2003. Standard errors clustered by microregion and year are in parentheses.

Workers' mobility decisions generate the dynamics of the model, while the trade part of the model determines static economic environments period by period.

3.1 Labor Mobility Model with the Endogenous Number of Job Options

Consider an economy with a continuum of workers with mass L_t at time t.[15] Each worker is in a discrete state $k \in \{1, 2, .., K\}$ which is a region-sector labor market index.[16] The mass of workers in labor market k at time t is denoted as L_t^k with $\sum_k L_t^k = L_t$. We denote r_k as the region of the labor market k and s_k as the sector of the labor market k. The total number of regions in this economy is R, and that of sectors is S, both of which we assume to be fixed over time. In most papers in the literature on the dynamic discrete choice model of labor mobility, workers are assumed to choose a labor market directly.[17] On the other hand, we model worker's problem as a choice of a *job*.

Jobs. Before formally defining *job*s, we must discuss several conceptual building blocks of the model. Production takes place by workers performing a set of *tasks*. In each period, producers organize the implementation of tasks. Tasks are indexed with $\tau \in [0, T_t^k]$, where T_t^k is the total mass of tasks implemented for production. We assume that workers are not directly assigned to tasks, but they are rather attached to *positions* which are indexed with ι, where $\iota \in [0, 1]$. For simplicity, we imagine that "positions" are very detailed stylized occupations, without any hierarchy. For example, "weaving textile technician" and "spinning textile technician" are two different positions. Workers in these two different positions perform different day to day activities, i.e., different tasks.

To implement tasks for production, producers assign tasks to positions using a one-to-one and onto mapping (bijection) which is identical across producers within a labor market. As we discuss in depth later on, producers decide the optimal mass of tasks T_t^k, which can change over time as economic

[15]We characterize the labor mobility model for an individual country and thus omit country index without loss of generality. The country index will be introduced when we characterize the multi-country trade part of the model.
[16]We denote a labor market with a single index k instead of a pair of region and sector indices. This notation is particularly convenient when we estimate the model, because not all region-sector pairs are populated in the data.
[17]For example, in ACM, workers choose a sector, and the number of sectors they can choose from is exogenously fixed. In CDP, workers choose a region-sector pair from the fixed number of region-sector pairs.

conditions change. The mapping is thus not static, but we can be agnostic about the specifics of the mapping function.[18] Although the continuum and the mass of "tasks" can change over time and across labor markets, the continuum of "positions" is fixed with a mass equal to one.

Workers are equally productive in performing any tasks in any position *ex-ante*, but performing a task is costly for them. After a worker learns about the task assignment for her position, she draws a random cost shock specific to her and associated with her specific task at hand. We denote these task costs with $\varepsilon \in \mathbb{R}$ and assume that they are iid random, drawn from a Gumbel distribution with mean zero and the scale parameter $\nu > 0$. We further assume that this cost shock is a non-pecuniary cost that affects her utility directly.

After defining these necessary components, we are ready to define the concept of *job* within our framework. A worker's *job* can be characterized by a combination of the following four elements: labor market, position, task, and random cost. Following the same order, we denote the specific combination corresponding to the job j as $(k_j, \iota_j, \tau_j, \varepsilon_j)$. The nature of a worker's job partially changes over time, as she draws fresh random task costs in every period. However, the other aspects of the job show persistence due to the switching costs which we will formally introduce shortly. Naturally, workers are not forced to perform the assigned task. Since random task costs change, workers might prefer to change their jobs to perform a different task, even if the corresponding wages were unchanged. After sampling different tasks as well as the associated task costs, workers can switch positions, and hence change tasks based on the bijective mapping of positions to tasks.[19]

The sampling process works as follows. First, the worker learns the cost, ε_j, associated with the currently assigned task of her current job j. After that, she samples $\widetilde{N}_t^k \in \mathbb{N}$ additional tasks associated with other positions in labor markets k, where $\xi T_t^k < \widetilde{N}_t^k \leq \xi T_t^k + 1$, and $\xi > 0$ is a constant sampling rate. There is no cost for sampling tasks apart from this threshold imposed by the simple sampling rate. Without loss of generality, sampling tasks from a given labor market k is equivalent to sampling jobs, since positions are mapped to tasks from a bijective mapping, and each task has a cost specific to the worker. In other words, within a labor market k, different tasks will lead to different jobs. Henceforth, we will assume that workers are actually sampling jobs, based on the process described above, to find the best one for them. More concretely, sampling jobs means sampling positions, tasks, and task costs from different labor markets, and the number of sampled job options workers can compare in labor market k at time t is given by $\widetilde{N}_t^k$. The random task cost ε_j is worker- and time-specific, but we omit both agent and time indices for the sake of notation simplicity. If the worker decides to change jobs, she has to pay fixed utility cost $\delta = \widetilde{\delta} + \nu \log(\xi)$, which is tied to changing positions. If the worker is also changing her labor market, there is an additional fixed cost

[18] The only condition that our framework requires is the existence of a bijection between tasks and positions. It is straightforward to prove that there exists a bijection between two closed and connected intervals in $\mathbb{R}^+$.

[19] This setup is consistent with the way we identify job switchers in data by workers switching detailed occupations.

associated with the between-market move, which we will elaborate later on.

Significance of job options. There are multiple job options workers can compare from each labor market, and workers choose one job in every period. Which labor market a worker belongs to is not a direct choice, but rather an outcome of her choice of a job. If a worker chooses the job j, then she belongs to the labor market k_j. This assumption is more realistic and introduces an important dimension through which aggregate shocks affect labor markets. The status of an economy impacts not only relative wages across markets but also the number of job options workers can choose from. The mass of tasks operated by producers responds to the shock, and the set of jobs expands (contracts) as the number of tasks increases (decreases). If the set of jobs expands, workers are able to sample more job options, and thus it becomes more likely to find a better job with a larger expected value.

The sampled job options of our model are analogous to the vacancy concept in the search literature. When economic conditions improve, producers expand the task space by increasing T_t^k. Then, the number of sampled job options increases, as workers are assumed to sample jobs at a constant rate. This relationship is determined at the general equilibrium which we will discuss after characterizing international trade. Our model brings the endogenous number of choices into an otherwise standard and easily tractable discrete choice model. Different from the search framework, the parameters of the model can be easily estimated using only labor market data, since the model provides simple equations that can be identified using only information on labor mobility.

Instead of choosing a job from job options available in each labor market, workers have an option to move to the residual labor market. Empirically, unemployment, home employment, and working in the informal sector all belong to this choice.[20] Each worker compares the option to move to the residual labor market with all the other job options available in each of formal labor markets.[21] We denote the choice to be in the residual labor market by $j = I$. The residual labor market is assumed to offer only one job option, and thus we do not distinguish a job from a labor market for the residual labor market, i.e., $k_I = I$ without loss of generality. We also assume that there is no job switching cost, δ, if a worker decides to move into the residual labor market.[22]

Timeline of events. Each period t consists of two stages in the following order: (1) production stage, and (2) preparation and mobility stage. In the production stage, workers start the period t attached to a job j, perform the task τ_j associated with j for production, and receive a market-level real wage $w_t^{k_j}$. Next, the preparation and mobility stage begins with producers determining the mass of tasks T_t^k and assigning tasks to positions for the next period. Workers then sample jobs for the next period and learn about the random iid costs shocks associated with each of them, the current job, and

[20] As explained in the previous section, the RAIS database covers only formal sectors. We introduce the residual market to account for the informal labor market that is not covered in the data that we will use to quantify the model.

[21] To be consistent with the way the residual market is empirically defined later, we will use the terms "formal" and "non-residual" interchangeably to denote the baseline labor markets.

[22] We could also assume that workers sample job options from the residual labor market and pay δ with minor modifications. This alternative approach does not impact the general structure of the framework or the empirical strategy.

the residual labor market. Conditional on these idiosyncratic draws and wages, workers choose the best available job and pay both the iid task cost associated with the new job as well as the moving cost, if applicable, immediately after choosing the job for the next period. Then, the period t ends.

At the preparation and mobility stage of period t, a worker can sample $\widetilde{N}_t^k$ job options from labor market k. We assume that workers have full information about the idiosyncratic cost shock and the moving cost for the current period when they choose a job. They do not know the exact future values of the idiosyncratic component but form rational expectations. Therefore, each worker compares the expected utility from $\sum_k \widetilde{N}_t^k$ alternative job options which include the option to move to the residual labor market with the expected utility from the current job at every period.[23]

Switching a job and switching a labor market are both subject to frictions. When a worker moves from a job j to a different job j', she pays the moving cost δ even when switching within the same labor market. If the switch involves a move to a different labor market, i.e., $k_j \neq k_{j'}$, then she pays an additional moving cost $C_t(k_j, k_{j'}) \geq 0$, where we assume $C_t(k,k) = 0$ for any k.

Based on the moving cost structure, we derive the present discounted utility of the agent h with job j after the production stage but before the preparation and mobility stage of period t as

$$U_t^{j,h} = w_t^{k_j} + \max_{j' \in \Omega_t^h} \left\{ \beta E_t V_{t+1}^{k_{j'}} - C_t(k_j, k_{j'}) - \mathbf{1}_{[j' \neq j \wedge j' \neq I]} \delta - \varepsilon_{j'} \right\}, \tag{3}$$

where Ω_t^h is the set of sampled job options from each formal labor market by agent h at t, the current job j, and the option to move to the residual market. By taking an expectation over the idiosyncratic component, we define the expected present discounted value for all workers in labor market k_j before the production and mobility stage of period t as $V_t^{k_j} \equiv E_\varepsilon U_t^{j,h}$. Finally, the systematic component of the utility for workers in the residual market is assumed to be equal to a fraction $\eta > 0$ of the average real wage of all formal labor markets. We assume that the fraction is exogenous and time-invariant.

3.2 Equilibrium Labor Mobility and Option Values

We can express the equilibrium probability of moving across labor markets from our model using an expression that closely resembles the standard McFadden (1973) equation.[24] First, we define

$$N_t^k \equiv \frac{\widetilde{N}_t^k}{\xi \exp(\widetilde{\delta}/\nu)}, \tag{4}$$

which is simply the number of sampled job options divided by a function of job switching costs and the shape parameter ν of the distribution of task cost shocks. Intuitively, it is the number of sampled job

[23] We effectively assume that $\widetilde{N}_t^I = 1$ in every t.

[24] Although our model can derive a standard McFadden equation, it is not isomorphic to standard discrete choice models. See section 6 and the auxiliary model in Appendix A.7 for details.

options discounted with the relevant moving cost. This conversion helps us substitute out unobserved variables, and thus greatly simplify the probability expression. The change in N_t^k can be identified directly from data as we show in the coming sections.[25]

Using the assumption of Gumbel distribution for the idiosyncratic task cost shock, the probability that a worker moves from a labor market k to a labor market l is derived as

$$m_t^{kl} = \frac{\mathbf{1}_{[l=k]}\left(1+N_t^l\right)\vartheta_{0,t}^l + \mathbf{1}_{[l\neq k\wedge l\neq I]}\vartheta_{1,t}^l \exp\left(-\frac{C_t(k,l)}{\nu}\right) + \mathbf{1}_{[l=I]}\vartheta_{I,t}^l}{\vartheta_{0,t}^k + \vartheta_{1,t}^k + \vartheta_{2,t}^k + \vartheta_{I,t}^k}, \tag{5}$$

where we define

$$\vartheta_{0,t}^k \equiv \exp\left(\frac{\beta}{\nu}E_t V_{t+1}^k\right) \tag{6}$$

$$\vartheta_{1,t}^k \equiv N_t^k \exp\left(\frac{\beta E_t V_{t+1}^k}{\nu}\right) \tag{7}$$

$$\vartheta_{2,t}^k \equiv \sum_{l'\neq k} N_t^{l'} \exp\left(\frac{\beta E_t V_{t+1}^{l'} - C_t(k,l')}{\nu}\right) \tag{8}$$

$$\vartheta_{I,t}^k \equiv \exp\left(\frac{\beta E_t V_{t+1}^I - C_t(k,I)}{\nu}\right) \tag{9}$$

for notational simplicity. For every t, $L_{t+1}^l = \sum_k m_t^{kl} L_t^k$ holds. From the expression of m_t^{kl}, the role of the number of job options becomes clear. Workers are more likely to move to a labor market where they can sample more jobs, conditional on the expected net value of a labor market. Since workers choose the job which gives them the highest present discounted utility, the expectation of the maximum of the idiosyncratic component, $-\varepsilon$, increases with the number of options, which makes it more likely for workers to move into a labor market with more options. Similarly, if formal labor markets offer more job options, then workers are less likely to move to the residual labor market. In addition, equation (5) implies that the shape parameter of the distribution of worker's idiosyncratic cost shock, ν, is essentially the inverse of the elasticity of labor mobility with respect to bilateral mobility frictions.[26]

We define additional moving probabilities for notational convenience. We denote the probability of moving from market k to l conditional on changing jobs but staying in a formal labor market by

$$\widetilde{m}_t^{kl} = \frac{\vartheta_{1,t}^l \exp\left(-\frac{C_t(k,l)}{\nu}\right)}{\vartheta_{1,t}^k + \vartheta_{2,t}^k} \tag{10}$$

for $l \neq I$, where $\vartheta_{1,t}^l$, $\vartheta_{1,t}^k$, and $\vartheta_{2,t}^k$ are as defined above. Note that this conditional probability equation is identical to the standard McFadden (1973) equation in its simplest form, which is widely used in

[25] Note that as the sampling rate goes to infinity, N_t^k is equal to $T_t^k/\exp(\widetilde{\delta}/\nu)$ at the limit, hence directly proportional to the number of tasks created by producers. See Appendix A.6 for details.

[26] In the rest of the paper, we will call this parameter as the labor supply elasticity following the literature.

discrete choice models.[27] In addition, we denote the probability of workers in labor market k staying in the same job, thus in the same labor market k, by $\mu^k_{0,t}$; the probability of changing jobs but staying in k by $\mu^k_{1,t}$; and the probability of moving from k to any other non-residual labor market $l \neq k$ (thus also changing jobs) by $\mu^k_{2,t}$. Lastly, we denote the probability of moving from labor market k to the residual labor market by $\mu^k_{I,t}$, which is effectively equal to m^{kI}_t.[28] The job options channel allows for internal—i.e., within-market—churning between jobs, whose probability is measured by $\mu^k_{1,t}$ for labor market k at time t. Since we assume that the residual labor market I offers a single job option, $\mu^I_{0,t} = \mu^I_{I,t}$ should hold.

With the new notations, we can re-write the labor-market-specific value as

$$V^k_t = w^k_t + \beta E_t V^k_{t+1} - \nu \log(\mu^k_{0,t}), \tag{11}$$

where $-\nu \log(\mu^k_{0,t})$ is an option value of moving to a different job. This option value can be decomposed into internal and external option values. The external option value is defined as the option value from alternative job options in a different labor market. By netting out the effect from switching to a different job in the same labor market, the external option value is derived as

$$-\nu \log(\mu^k_{0,t}) + \nu \log(\mu^k_{0,t} + \mu^k_{2,t} + \mu^k_{I,t}). \tag{12}$$

The internal option value is the option value from alternative job options within the same labor market, which is given as the difference between the total and the external option values,

$$-\nu \log(\mu^k_{0,t} + \mu^k_{2,t} + \mu^k_{I,t}). \tag{13}$$

3.3 Relative Welfare and the Number of Job Options

Although the model features various determinants of mobility and welfare, we can derive simple formulae describing the relationship between changes in welfare as well as the number of job options, and changes in an underlying policy, conditional on two assumptions. First, we assume that the change in an arbitrary policy variable z does not change the moving costs.[29] Second, any change in z of period t is realized after workers receive their wage. Under these assumptions, the change in the relative welfare of workers in labor market k compared to workers in labor market l can be derived as

$$\Delta_z\left(V^k_t - V^l_t\right) = \Delta_z\left[\nu\left(\log \widetilde{m}^{lk}_t - \log \widetilde{m}^{kk}_t\right) - \nu\left(\log\left(1-\mu^k_{0,t}-\mu^k_{I,t}\right) - \log\left(1-\mu^l_{0,t}-\mu^l_{I,t}\right)\right)\right], \tag{14}$$

[27] This conditional probability equation will allow us to identify structural parameters using labor mobility data.

[28] By construction, $\mu^k_{0,t} + \mu^k_{1,t} + \mu^k_{2,t} + \mu^k_{I,t} = 1$ holds for any t,k. Using equations (6)-(9), each μ can be expressed as $\mu^k_{0,t} = \frac{\vartheta^k_{0,t}}{\vartheta^k_{0,t}+\vartheta^k_{1,t}+\vartheta^k_{2,t}+\vartheta^k_{I,t}}$, $\mu^k_{1,t} = \frac{\vartheta^k_{1,t}}{\vartheta^k_{0,t}+\vartheta^k_{1,t}+\vartheta^k_{2,t}+\vartheta^k_{I,t}}$, $\mu^k_{2,t} = \frac{\vartheta^k_{2,t}}{\vartheta^k_{0,t}+\vartheta^k_{1,t}+\vartheta^k_{2,t}+\vartheta^k_{I,t}}$, and $\mu^k_{I,t} = \frac{\vartheta^k_{I,t}}{\vartheta^k_{0,t}+\vartheta^k_{1,t}+\vartheta^k_{2,t}+\vartheta^k_{I,t}}$.

[29] We will relax this assumption in counterfactual simulations where we quantify the changes in welfare, instead of relative welfare, using the full general equilibrium structure of the model.

where Δ_z denotes the linear change induced by a change of z. Next, our model shows that the change in the number of job options in each labor market driven by a change in z is

$$\Delta_z \log N_t^k = \Delta_z \left(\log \mu_{1,t}^k - \log \mu_{0,t}^k \right). \tag{15}$$

Equations (14) and (15) suggest that the labor supply elasticity and changes in moving probabilities are the sufficient statistics for changes in the relative welfare and the number of job options.

3.4 International Trade with Love for Variety of Tasks

The dynamic labor mobility model we introduce can be used to quantify the effect of various labor demand shocks on workers' welfare. In this paper, we focus on the effect of international trade. We introduce trade to our model as a static problem, where trade endogenously affects both wages and the number of job options, and eventually dynamic labor mobility as well as welfare.

The international trade part of our model is based on Eaton and Kortum (2002, EK, hereafter), where trade is driven by the standard Ricardian force. We assume that there are $\bar{N}$ countries ($n = 1, \ldots, \bar{N}$), but only country 1 has more than one region. For other countries $n \neq 1$, there is only one region which is the country itself. We now introduce a country subscript n to all relevant variables, and the labor mobility model described previously applies only to country $n = 1$ for which we study the effect of trade shocks on dynamic labor mobility and welfare in quantification.[30]

There is a continuum of products on $[0, 1]$ in each of S sectors. Consumers have identical nested CES preferences with a common elasticity of substitution $\sigma > 0$ across products and a Cobb-Douglas aggregation across sectors with the expenditure share ϕ^s for each sector s. To characterize trade between countries, while we have multiple regions only in country 1, we assume that there is a national aggregator for each sector in country 1. The aggregator of each sector can source each product of that sector from the lowest cost region within the country at no trade cost. The national price of each product is thus equalized across consumption locations within the country. Then, this aggregator trades with partner countries. If no region in country 1 is the lowest cost supplier of a certain product for country 1, after taking into account cross-country differences in production cost and bilateral trade costs, then country 1 imports that product, and no region in country 1 produces that product at the equilibrium. To simplify our analysis, we assume that there is no production in the residual labor market of country 1 and further assume that there are only formal sectors in other countries $n' \neq 1$.[31]

Labor, fixed factor, and composite intermediate inputs are used for production in country 1.

[30] Our model is able to allow for multiple regions and dynamic labor mobility for all countries. For quantification, however, we would need detailed labor mobility data from each country. Therefore, we restrict the analysis of the effect of trade shocks on internal labor mobility only to one country, which will be Brazil empirically. The general equilibrium feature of the trade part applies to all of $\bar{N}$ countries.

[31] Combined with the previous assumption on the systematic component of the utility of workers in the residual labor market, we effectively allow for neither production nor consumption in the residual market.

Products can be used either as intermediate inputs or for final consumption. We assume a Cobb-Douglas production function for each product ω from labor market k of country 1 at time t

$$Q_{1,t}^k(\omega) = \varphi_1^k(\omega)(\widetilde{L}_{1,t}^k)^{\gamma_l}(M_{1,t}^k)^{\gamma_m}(B_1^k)^{\gamma_b},$$

where $M_{1,t}^k$ is a composite intermediate input, and B_1^k is a time-invariant fixed factor used in labor market k of country 1. To focus on the dynamics from the worker side, we simplify the production by assuming that all producers in country 1 consider the aggregate price index $P_{1,t}$ as the price of composite intermediate inputs, instead of explicitly modeling an input-output structure.

In the production function, $\widetilde{L}_{1,t}^k$ is expressed in terms of the number of efficiency units provided by all workers in labor market k, which differs from $L_{1,t}^k$, the actual number of workers. As described in Section 3.1, producers in labor market k decide the optimal mass of tasks T_t^k and pay the marginal cost of task operation $\tilde{c} > 0$. From the producers' point of view, workers in the same labor market are equally productive regardless of the task to which they are assigned, and thus are paid the same wage as long as they are in the same labor market. Therefore, producers will choose the total mass of tasks to operate, T_t^k, as well as the total efficiency units of labor they use, $\widetilde{L}_{1,t}^k$, taking into account the fact that it is optimal to evenly allocate the labor supply $L_{1,t}^k$ they observe into different tasks.[32]

For each labor market k, we assume that $\widetilde{L}_{1,t}^k$ is a CES aggregate of all efficiency units provided by each task in labor market k. Then, the total labor aggregate is

$$\widetilde{L}_{1,t}^k = \left[\int_{\tau\in[0,T_t^k]} (\widetilde{L}_{1,t}^k(\tau))^{\frac{\tilde{\sigma}-1}{\tilde{\sigma}}} d\tau\right]^{\frac{\tilde{\sigma}}{\tilde{\sigma}-1}},$$

where $\widetilde{L}_{1,t}^k(\tau)$ is the efficiency units task τ provides in labor market k of country 1; and $\tilde{\sigma}$ is the elasticity of substitution between tasks. Under the assumption of identical productivity across tasks, $\widetilde{L}_{1,t}^k$ can be rewritten as $\widetilde{L}_{1,t}^k = L_{1,t}^k(T_t^k)^{\frac{1}{\tilde{\sigma}-1}}$. If $\tilde{\sigma} > 1$, then the number of tasks increases with the optimal demand for total labor force, conditional on the labor supply. This is the love for variety of tasks channel, which is analogous to the well-known love for variety of products in the Armington (1969) trade model.[33] Therefore, if there is a positive labor demand shock for labor market k, then the number of tasks operated there will increase, conditional on $L_{1,t}^k$. Empirically, producers may post a new type of task to operate a more differentiated production structure or open new establishments to meet the increased demand.[34] The exact magnitude of the response will be determined at the general equilibrium, as $L_{1,t}^k$ responds to the shock as well from the labor mobility part of the model.

The effect of a labor demand shock on labor mobility through the number of job options is two-

[32] Note that the idiosyncratic task cost matters for individual worker's utility but does not matter for producers.
[33] Feenstra (1994) further studies an endogenous change of the mass of varieties.
[34] Becker, Egger, Koch, and Muendler (2020) document that larger plants operate more specialized tasks.

fold. First, a positive labor demand shock to a labor market increases the number of tasks operated by producers through the love for variety of tasks channel. As a result, it increases the number of job options that workers can sample, and thus attracts more workers. Second, a positive labor demand shock in a formal labor market will decrease the probability of moving to the residual labor market. This second channel is similar to the standard search model, where the effect of a macro shock operates through the unemployment margin. In our model, a labor demand shock affects workers' welfare not only through the transition between formal and residual labor markets but also through the higher utility from being able to compare more options in order to choose the best one.

Other countries $n \neq 1$ have a simpler production function using only aggregate labor $L^k_{n,t}$ and composite intermediate inputs $M^k_{n,t}$ for each labor market k:

$$Q^k_{n,t}(\omega) = \varphi^k_n(\omega)(L^k_{n,t})^{\bar{\gamma}_{n,l}}(M^k_{n,t})^{1-\bar{\gamma}_{n,l}},$$

where $\bar{\gamma}_{n,l}$ is the value-added share of country n. Since we assume that there is only one region in country $n \neq 1$, the market index k effectively denotes a sector there. Countries $n \neq 1$ all have the same number of sectors S, which is equal to the number of unique s_k over all $k = 1, \ldots, K$ in country 1. Therefore, the sector-level trade flows between country 1 and $n \neq 1$ are well-defined. At the general equilibrium, $\sum_{k=1}^{S} L^k_{n,t} = L_n$ holds for every (n, t), where L_n is the exogenously fixed total labor endowment of country n. The input-output structure is simplified for countries $n \neq 1$ as well, where the aggregate price index is equal to the price of the composite intermediate inputs in each country.

Factor-neutral productivity for each product is randomly drawn from a Fréchet distribution and fixed over time.[35] For each country n, $\varphi^k_n(\omega)$ is randomly drawn from

$$F^k_n(\varphi) = \exp(-A^k_n \varphi^{-\theta}),$$

where k denotes a region-sector pair in country 1 and a sector in other countries. As in EK, the productivity draws are independent across space. There is no domestic trade cost between regions within country 1, but there are iceberg trade costs between countries, $d^s_{nn',t}$, for products of sector s shipped from country n to n' at time t. In our counterfactual exercises, we will introduce an exogenous trade shock by changing this iceberg trade cost parameter.

3.5 Equilibrium Trade Flows and Price Indices

For producers of country 1, there is a clear trade-off involved in the decision of $T^k_{1,t}$. Having more tasks increases the total efficiency units of labor, conditional on labor supply $L^k_{1,t}$. However, to have a more differentiated production technology, they have to pay a higher cost for training or building a new

[35] We can easily relax this time-invariance assumption to study the effect of an exogenous change in A^k_n.

plant, for example, where its marginal cost is defined by $\tilde{c}$. The Cobb-Douglas production function gives us the following unit cost function for all firms in labor market k of country 1 at time t,

$$c_{1,t}^{k} = \Upsilon_1 (\widetilde{w}_{1,t}^{k} \tilde{c}^{\frac{1}{\bar{\sigma}-1}})^{\gamma_l} (P_{1,t})^{\gamma_m} (b_{1,t}^{k})^{\gamma_b}, \tag{16}$$

where $\widetilde{w}_{1,t}^{k}$ is the nominal wage of workers, and $b_{1,t}^{k}$ is the price of the fixed factor of labor market k in country 1 at time t.[36] Υ_1 is the Cobb-Douglas constant which is a function of γ_l, γ_m, and γ_b. Similarly, the unit cost function for producers in country $n' \neq 1$ at time t is

$$c_{n',t} = \Upsilon_{n'} (\widetilde{w}_{n',t})^{\bar{\gamma}_{n',l}} (P_{n',t})^{1-\bar{\gamma}_{n',l}}, \tag{17}$$

with the assumption of perfect labor mobility between sectors for countries $n' \neq 1$.

At the equilibrium under perfect competition as in EK, between regions r and r' in country 1, region r''s expenditure share on sector s products produced in region r at time t is given by

$$\lambda_{(1,r),(1,r'),t}^{s} = \frac{A_{1,t}^{(r,s)} (c_{1,t}^{(r,s)})^{-\theta}}{\sum_{r''} A_{1,t}^{(r'',s)} (c_{1,t}^{(r'',s)})^{-\theta} + \sum_{n' \neq 1} A_{n',t}^{s} (c_{n',t} d_{n'1,t}^{s})^{-\theta}} = \frac{X_{(1,r),(1,r'),t}^{s}}{X_{(1,r'),t}^{s}}, \tag{18}$$

where $X_{(1,r),(1,r'),t}^{s}$ is the expenditure of region r' of country 1 on sector s products from region r of country 1, and $X_{(1,r'),t}^{s}$ is their total expenditure on sector s products.[37] From the assumption of no domestic trade cost between regions within country 1, $\lambda_{(1,r),(1,r'),t}^{s}$ is equalized across r'. Similarly, the equilibrium trade share of sector s from country $n \neq 1$ to region r of country 1 is determined by

$$\lambda_{n,(1,r),t}^{s} = \frac{A_{n,t}^{s} (c_{n,t} d_{n1,t}^{s})^{-\theta}}{\sum_{r''} A_{1,t}^{(r'',s)} (c_{1,t}^{(r'',s)})^{-\theta} + \sum_{n' \neq 1} A_{n',t}^{s} (c_{n',t} d_{n'1,t}^{s})^{-\theta}} = \frac{X_{n,(1,r),t}^{s}}{X_{(1,r),t}^{s}}, \tag{19}$$

which is also equalized across r due to consumer's identical preference and no domestic trade cost. The actual demand level varies by region, depending on each region's income. Lastly, the reverse trade share from region r of country 1 to a country $n \neq 1$ and the trade share between countries $n \neq 1$ and $n'' \neq 1$ are derived as

$$\lambda_{(1,r),n,t}^{s} = \frac{A_{1,t}^{(r,s)} (c_{1,t}^{(r,s)} d_{1n,t}^{s})^{-\theta}}{\sum_{r''} A_{1,t}^{(r'',s)} (c_{1,t}^{(r'',s)} d_{1n,t}^{s})^{-\theta} + \sum_{n' \neq 1} A_{n',t}^{s} (c_{n',t} d_{n'n,t}^{s})^{-\theta}} = \frac{X_{(1,r),n,t}^{s}}{X_{n,t}^{s}}. \tag{20}$$

$$\lambda_{n,n'',t}^{s} = \frac{A_{n,t}^{s} (c_{n,t} d_{nn'',t}^{s})^{-\theta}}{\sum_{r''} A_{1,t}^{(r'',s)} (c_{1,t}^{(r'',s)} d_{1n'',t}^{s})^{-\theta} + \sum_{n' \neq 1} A_{n',t}^{s} (c_{n',t} d_{n'n'',t}^{s})^{-\theta}} = \frac{X_{n,n'',t}^{s}}{X_{n'',t}^{s}}. \tag{21}$$

[36] Note that the wage is per worker, not per efficiency unit.

[37] To make a distinction between region and sector in trade flow equations, each labor market k is denoted as a region-sector pair (r_k, s_k). The bilateral expenditure is defined similarly for other country-region, region-country, and country-country pairs in the rest of the paper. The total expenditure variable is also defined similarly for other countries.

The exact price indices for sector s in country 1 and country $n \neq 1$ at time t are

$$P^s_{1,t} = \left[\Gamma(\frac{\theta+1-\sigma}{\theta})\right]^{1/(1-\sigma)} \left[\sum_{r''} A^{(r'',s)}_{1,t}(c^{(r'',s)}_{1,t})^{-\theta} + \sum_{n'\neq 1} A^s_{n',t}(c_{n',t}d^s_{n'1,t})^{-\theta}\right]^{-\frac{1}{\theta}} \tag{22}$$

$$P^s_{n,t} = \left[\Gamma(\frac{\theta+1-\sigma}{\theta})\right]^{1/(1-\sigma)} \left[\sum_{r''} A^{(r'',s)}_{1,t}(c^{(r'',s)}_{1,t}d^s_{1n,t})^{-\theta} + \sum_{n'\neq 1} A^s_{n',t}(c_{n',t}d^s_{n'n,t})^{-\theta}\right]^{-\frac{1}{\theta}}, \tag{23}$$

where we assume $\sigma < \theta + 1$ so that the price indices are well-defined. From the Cobb-Douglas assumption across sectors, the aggregate price index is given by $P_{n,t} = \prod_s (\frac{P^s_{n,t}}{\phi^s})^{\phi^s}$ for each n.

3.6 Market Clearing

The total expenditure on sector s products by all agents in region r of country 1 can be derived as

$$X^s_{(1,r),t} = \phi^s\gamma_m \sum_{s'} \left(\sum_{r'} \lambda^{s'}_{(1,r),(1,r'),t} X^{s'}_{(1,r'),t} + \sum_{n'\neq 1} \lambda^{s'}_{(1,r),n',t} X^{s'}_{n',t}\right) + \phi^s \left(\sum_{k\in\{k|r_k=r\}} \left(\widetilde{w}^k_{1,t} L^k_{1,t} + \bar{D}^k_{1,t}\right)\right), \tag{24}$$

where $\{k \mid r_k = r\}$ is the set of region-sector pairs associated with region r. We assume that country-level trade deficit $\bar{D}_{n,t}$ is exogenously fixed as the share of the total world GDP, and that country 1's trade deficit is distributed across labor markets of country 1 proportionally to each market's labor income share. The trade deficit distributed to labor market k of country 1 is denoted by $\bar{D}^k_{1,t}$. Similarly, the total expenditure on sector s products by all agents in country $n \neq 1$ at time t is

$$X^s_{n,t} = \phi^s(1-\bar{\gamma}_{n,l}) \sum_{s'} \left(\sum_{r'} \lambda^{s'}_{n,(1,r'),t} X^{s'}_{(1,r'),t} + \sum_{n'\neq 1} \lambda^{s'}_{n,n',t} X^{s'}_{n',t}\right) + \phi^s \left(\widetilde{w}_{n,t} L_{n,t} + \bar{D}_{n,t}\right). \tag{25}$$

Finally, the set of factor market clearing conditions is given by

$$\widetilde{w}^k_{1,t} L^k_{1,t} = \gamma_l \left(\sum_{r'} \lambda^{s_k}_{(1,r_k),(1,r'),t} X^{s_k}_{(1,r'),t} + \sum_{n'\neq 1} \lambda^{s_k}_{(1,r_k),n',t} X^{s_k}_{n',t}\right), \tag{26}$$

$$b^k_{1,t} B^k_1 = (1-\gamma_l-\gamma_m) \left(\sum_{r'} \lambda^{s_k}_{(1,r_k),(1,r'),t} X^{s_k}_{(1,r'),t} + \sum_{n'\neq 1} \lambda^{s_k}_{(1,r_k),n',t} X^{s_k}_{n',t}\right) \tag{27}$$

for each labor market k of country 1, and

$$\widetilde{w}_{n,t} L_n = \bar{\gamma}_{n,l} \sum_{s'} \left(\sum_{r'} \lambda^{s'}_{n,(1,r'),t} X^{s'}_{(1,r'),t} + \sum_{n'\neq 1} \lambda^{s'}_{n,n',t} X^{s'}_{n',t}\right), \tag{28}$$

for country $n \neq 1$. The equilibrium labor supply for each labor market of country 1, $L_{1,t}^{k}$, is pinned down by the labor model taking into account workers' mobility decisions in each period, whereas the labor supply L_n for other countries $n \neq 1$ is assumed to be exogenously fixed.

A temporary equilibrium at t is a vector of nominal wages $\widetilde{\mathbf{w}}_t = \left(\widetilde{w}_{1,t}^{1}, \ldots, \widetilde{w}_{1,t}^{K}, \widetilde{w}_{2,t}, \ldots, \widetilde{w}_{N,t}\right)$ and a vector of prices of the fixed factor in country 1 $\mathbf{b}_t = \left(b_{1,t}^{1}, \ldots, b_{1,t}^{K}\right)$ which satisfy (16)-(28), conditional on the labor supply $\mathbf{L}_t = \left(L_{1,t}^{1}, \ldots, L_{1,t}^{K}, L_{2,t}, \ldots, L_{N,t}\right)$ and other fundamental parameters. A sequential competitive equilibrium is the period-by-period sequence of $\mathbf{L}_t$, $\widetilde{\mathbf{w}}_t$, $\mathbf{b}_t$, and $\mathbf{m}_t = \left\{m_{1,t}^{kl}\right\}_{k=1,l=1}^{K,K}$ which solve the labor mobility model in every period, conditional on the initial labor allocation $\mathbf{L}_0$ and fundamental parameters of the model.

4 Estimation

In this section, we describe how we structurally estimate the labor part of our model using RAIS from Brazil. The main objects to be estimated are the moving probabilities, between-market moving costs, and the inverse labor supply elasticity ν which is identified by the empirical strategy described in Section 2. With these estimates, we turn to our variables of interest: changes in relative welfare, option values, and the number of job options due to trade shocks. In our reduced-form analysis, we did not study the effect of trade shocks on these variables, because they do not have direct data counterparts. Once we estimate our model, we can use the sufficient statistic result in equations (14) and (15). In other words, we can study the effect of trade shocks on the welfare-related variables in a reduced-form way, before doing full general equilibrium counterfactuals in the next section.

We restrict the sample to the labor markets where at least 100 workers move in and out respectively in every year of our sample period 2004-2015. Since the identification of moving probabilities is based on workers' mobility, values of labor markets with little labor mobility cannot be identified. Unlike ACM or CDP, we include corridors with zero mobility.[38] After this restriction, we have 804 labor markets in our sample.

4.1 Estimation of the Flows

In practice, it is not possible to obtain $\widetilde{m}_t^{kl}$ directly from data without any estimation. The bin-estimator for a moving probability, i.e., the number of switchers divided by the number of workers in origin, only works with a large sample size and a small number of choices. Since we have 804 labor markets each of which offers multiple job options, the bin-estimator is infeasible. Therefore, instead of using the bin-estimator, we estimate $\log \widetilde{m}_t^{kl}$ by imposing a parametric structure on the moving costs.

[38] For example, if less than 100 workers move out of or into labor market A, we drop A. However, if no worker moves from labor market B to labor market C, but there are more than 100 workers in total moving out of B, then we keep the labor market B and the B-C corridor in the estimation. All main results we present are robust to the mobility cutoff.

For origin-specific moving probabilities, $\mu^k_{1,t}$, $\mu^k_{2,t}$, $\mu^k_{3,t}$, and $\mu^k_{I,t}$, on the other hand, the bin-estimators are feasible since each vector refers to a broad choice unlike the detailed transition matrices.

First, we can express $\log \widetilde{m}^{kl}_t$ of our model as follows:

$$\log \widetilde{m}^{kl}_t = \widetilde{V}^l_t - \widetilde{C}_t(k,l) + \widetilde{\Gamma}^k_t - \log \widetilde{Y}^k_t, \tag{29}$$

where

$$\begin{aligned}
\widetilde{V}^l_t &\equiv E_t \frac{\beta}{\nu} V^l_{t+1} - \log \mu^l_{0,t} + \log \mu^l_{1,t} \\
\widetilde{C}_t(k,l) &\equiv \frac{C_t(k,l)}{\nu} \\
\widetilde{\Gamma}^k_t &\equiv -\log \sum_{l' \neq I} \exp\left(\widetilde{V}^{l'}_t - \widetilde{C}_t(k,l')\right) + \log \widetilde{Y}^k_t,
\end{aligned}$$

and $\widetilde{Y}^k_t$ is the number of workers who change jobs within labor market k or move out of the labor market k but stay in a formal labor market. If we define the number of workers observed in the sample moving from a labor market k to l conditional on changing jobs but staying in a formal labor market by $\widetilde{y}^{kl}_t$, then we have $\widetilde{Y}^k_t = \sum_{l \neq I} \widetilde{y}^{kl}_t$. The log-likelihood function is then equal to

$$\log \mathcal{L} = \sum_{k \neq I} \sum_{l \neq I} \widetilde{y}^{kl}_t \left[\widetilde{\Gamma}^k_t + \widetilde{V}^l_t - \widetilde{C}_t(k,l) - \log(\widetilde{Y}^k_t)\right], \tag{30}$$

using (29). If $L^k_t \to \infty$, then $\widetilde{Y}^k_t \to L^k_t(\mu^k_{1,t} + \mu^k_{2,t})$, and $\widetilde{y}^{kl}/\widetilde{Y}^k_t \to \widetilde{m}^{kl}_t$: i.e., as the sample size goes to infinity, the maximum likelihood estimator (MLE) becomes equivalent to the bin-estimator. For the sake of practicality, we use Poisson pseudo-maximum-likelihood (PPML) method instead of MLE to estimate $\widetilde{\Gamma}^k_t$, $\widetilde{V}^l_t$, and $\widetilde{C}_t(k,l)$ for each period, because we can write $\widetilde{y}^{kl}_t$ as

$$\widetilde{y}^{kl}_t = \exp\left(\widetilde{\Gamma}^k_t + \widetilde{V}^l_t - \widetilde{C}_t(k,l)\right) + \epsilon^{kl}_t, \tag{31}$$

where ϵ is a mean-zero sampling error, and thus $E_t \widetilde{y}^{kl}_t = \widetilde{m}^{kl}_t \left(\mu^k_{1,t} + \mu^k_{2,t}\right) L^k_t$. We can interpret $\widetilde{\Gamma}^k_t$ as the origin fixed effect, $\widetilde{V}^l_t$ as the destination fixed effect, and $\widetilde{C}_t(k,l)$ as bilateral frictions. Guimaraes, Figueiredo, and Woodward (2003) show that the MLE of a discrete choice problem for firm's location is identical to PPML. A similar intuition applies to our model. In Appendix A.9, we prove that the PPML and MLE for our model are equivalent, with identical point estimates.

In the estimation, we consider a simple moving cost structure as follows:

$$\widetilde{C}_t(k,l) = \widetilde{C}^1_t D^{kl} + \widetilde{C}^2_t \mathbf{1}_{s_k \neq s_l} + \widetilde{C}^3_t \mathbf{1}_{s_k \neq s_l \wedge r_k \neq r_l}, \tag{32}$$

where D^{kl} is the log of distance between labor markets k and l; $\mathbf{1}_{s_k \neq s_l}$ is an indicator function that is equal to one if labor markets k and l are associated with different sectors; and $\mathbf{1}_{s_k \neq s_l \wedge r_k \neq r_l}$ is an indicator function that is equal to one if k and l are associated with different sectors and regions.[39]

Based on PPML estimation result reported in Table A2, we find that the moving cost coefficient between regions, $\widetilde{C}_t^1$, is close to 1, where the total cost is proportional to log distance. The moving cost between sectors, $\widetilde{C}_t^2$, is around 1.6. To put these numbers in perspective based on distance, the moving cost between São Paulo and Rio de Janeiro is about 5.76, while that between São Paulo and Salvador is about 7.5. The average moving cost between region pairs is around 7.62. Therefore, the moving cost between regions is more than four times larger than the moving cost across sectors. We find that moving both regions and sectors makes the total cost slightly lower, with $\widetilde{C}_t^3$ around -0.18.[40] As reported in Table A2, the estimated coefficients are not identical but very similar between years. With the estimates of $\widetilde{\Gamma}_t^k$, $\widetilde{V}_t^l$, and $\widetilde{C}_t(k,l)$, we can simply compute the estimates for $\log \widetilde{m}_t^{kl}$ from (29).

4.2 Estimation of the Labor Supply Elasticity

The key structural parameter we need to pin down for quantification is the labor supply elasticity $1/\nu$. We estimate this parameter by embedding the reduced-form approach into our structural model. We derive a regression equation similar to Autor, Dorn, and Hanson (2013), where the reduced-form coefficient has a direct link to ν.[41] The model results depend on this key parameter, since the responsiveness of labor market outcomes depends on how elastic labor mobility is.

Once we estimate $\widetilde{V}_t^k$ as the destination fixed effect in the PPML equation, we can calculate the present discounted value of the expected wage as

$$E_t \frac{\beta}{\nu} w_{t+1}^k = \widetilde{V}_t^k + \left(\log \mu_{0,t}^k - \log \mu_{1,t}^k\right) - \beta E_t \left[\widetilde{V}_{t+1}^k - \log \mu_{1,t+1}^k\right]$$

with the bin-estimators of $\mu_{0,t}^k$ and $\mu_{1,t}^k$. Then, we can estimate ν by estimating

$$\Delta Y_t^k = \psi \Delta Z_t^k + \kappa_t + \epsilon_t^k, \tag{33}$$

where we set $Y_t^k = \widetilde{V}_t^k + \left(\log \mu_{0,t}^k - \log \mu_{1,t}^k\right) - \beta \left[\widetilde{V}_{t+1}^k - \log \mu_{1,t+1}^k\right]$ and $Z_t^k = w_{t+1}^k$. Conditional on $\beta = 0.95$, we back out the elasticity ν from the estimate of ψ.[42]

As in Section 2, we measure w_{t+1}^k from purged wages from individual characteristics and adopt

[39] Alternatively, we can have $\mathbf{1}_{r_k \neq r_l}$ instead of D^{kl}, but we use the information on physical distance to back out the region-level mobility friction following the migration literature. We impose $D^{kk} = 0$ for every k and $D^{kl} = 0$ if $r_k = r_l$. Distance is measured in kilometers.

[40] We can compare the sectoral moving cost estimate to earlier findings in the literature after multiplying the number by ν. In the next section, we will find ν is equal to 0.493 which translates into a moving cost of approximately 0.80 times annual average wage which is significantly smaller than ACM's estimates, and closer to Dix-Carneiro (2014) results.

[41] Our estimation strategy is in a similar spirit of Galle, Rodriguez-Clare, and Yi (2017) and Adão, Arkolakis, and Esposito (2019). The labor market adjustment in our model is dynamic and subject to mobility friction, while these two papers interpret the elasticity in a static setting without explicit mobility frictions.

[42] Our main results are robust to the choice of the discount factor.

the same instrument $\Delta \bar{Z}_t^k$ defined in (2) to address potential endogeneity in wage changes. In the first stage, we regress wage changes on $\Delta \bar{Z}_t^k$. Then, we use the predicted wage change as the explanatory variable in equation (33). In summary, our previous IV strategy provides a simple and clean identification for the key structural parameter of the model.

Table 4 reports the second-stage estimate of β/ν as well as the first stage result for wage changes with the same instrument $\Delta \bar{Z}_t^k$. If we assume $\beta = 0.95$, then the implied ν is 0.493. Our estimate is similar to what other papers in the literature have found: e.g., Artuç and McLaren (2015) find $\nu = 0.56$ with $\beta = 0.9$ and $\nu = 1.613$ with $\beta = 0.97$.

Table 4: Estimation results for β/ν

		A. First stage	B. Second stage for β/ν
	$\Delta \bar{Z}$	0.418	1.927
		(0.029)	(0.774)
	F-stat	134.17	
Observations		3815	3815
Year effects		Y	Y

Notes: The sample consists of 804 labor markets as before. Panel A reports the first stage result with instrument $\Delta \bar{Z}$ for wage changes. Panel B reports the second stage result for the estimate of β/ν. Standard errors clustered by microregion and year are in parentheses.

4.3 Welfare Changes from the Trade Shock

With the estimates of ν and the moving probabilities, the sufficient statistic result presented in Section 3.3 allows us to estimate the impact of trade on the welfare-related variables as well as the number of job options that are not measured directly from data. We revisit the simple regression equation (1) from Section 2 and use the welfare expression in (14), the number of job options formula in (15), and the option value formulas in (13) and (12) in change, respectively, as the dependent variable. The explanatory variable is again the log change in market-level exports, instrumented by an exogenous change in import demand directed to each labor market, $\Delta \bar{Z}_t^k$, as defined in equation (2). This exercise is to study the effect of trade shocks on workers' welfare in a reduced-form way before performing a full general equilibrium counterfactual simulation in the following section.

Table 5 reports the estimation results for each welfare-related outcome of interest. Since all dependent variables except for the number of job options are divided by ν as shown in equations (12)-(14), we use the estimated $\nu = 0.493$ to back out their implied elasticities with respect to export revenues of the labor market. The result shows that a positive export shock increases workers' welfare and the number of job options offered in the corresponding labor market.

Table 5: Export-induced changes in welfare-related variables

	Coefficients	s.e.	Implied elasticities with $\nu = 0.493$
Welfare	0.607	(0.198)	0.299
Job options	0.652	(0.143)	0.652
Internal option values	0.147	(0.021)	0.073
External option values	-0.161	(0.050)	-0.079

Notes: Each row reports the IV estimation result with 804 labor markets and 3815 observations. Year effects are included in all specifications. Standard errors clustered by microregion and year are reported in parentheses.

Another interesting result is that a positive export shock increases internal option values but decreases external option values. In existing models such as ACM and CDP, a positive export shock in your own labor market should decrease the option value, as other labor markets become relatively less valuable. This is captured by the effect on the external option value of our model. On the other hand, our model further shows that the internal option value moves towards the opposite direction, because the number of job options increases with a positive export shock. Due to this additional effect that our model is able to capture through the endogenous number of job options, a positive export shock generates extra positive effects on the total option values compared to the existing models, which will be further confirmed based on simulations in the following section.

5 Model Quantification and Counterfactual Exercises

In this section, we first quantify our model by calibrating the remaining model parameters. We then run various counterfactual simulations to understand the effect of changes in trade environment, such as changes in trade cost on labor allocation, welfare, the number of job options, option values, and inequality. Our model can also assess the impact of a trade shock under different policy environments. Specifically, we quantify the role of labor mobility frictions in transmitting a trade shock to workers. Our rich framework allows us to look at potentially different roles of sectoral versus geographical mobility frictions. This exercise is particularly relevant for trade adjustment policies, because it can shed light on which margin to target when it comes to the distribution of gains from trade.

5.1 Modified Dynamic Hat Algebra

A full general equilibrium simulation of a dynamic model of labor adjustment like ours is typically very challenging, because it involves solving for expected values at every period. Caliendo, Dvorkin, and Parro (2019) propose a convenient way of doing counterfactual exercises with this class of models. The main advantage of this method is that a quantitative counterfactual analysis does not require an estimation of many primitives that are not of interest in the counterfactual scenario. In a spirit similar to their "dynamic hat algebra" technique which requires an assumption of log utility, we propose a modification to that approach, which does not require any restriction on consumer's utility.

First, we re-write the model in terms of changes in each variable from its initial steady state value. For the main counterfactual exercise, we allow between-labor-market moving costs to vary across periods, while the switching cost between jobs is assumed to be time-invariant. We also assume that the marginal cost of task operation $\tilde{c}$ and the job sampling rate ξ are exogenously fixed.

For any variable x of the model, we denote the value of this variable at the initial steady state by x_0, i.e., $t = 0$, and define the following three operators:

$$\begin{aligned}
\ddot{x}_t &\equiv \exp\left(\frac{\beta E_{t-1}x_t - \beta x_0}{\nu}\right) \\
\dot{x}_t &\equiv \exp\left(\frac{x_t - x_0}{\nu}\right) \\
\widehat{x}_t &\equiv \frac{x_t}{x_0}.
\end{aligned}$$

Using these operators, for labor market k, the probability of staying in the same job is re-written as

$$\mu^k_{0,t} = \frac{\ddot{V}^k_{t+1}\mu^k_{0,0}}{\ddot{V}^k_{t+1}\left[\mu^k_{0,0} + \left(m^{kk}_0 - \mu^k_{0,0}\right)\widehat{N^k_t}\right] + \sum_{l'\neq k}\ddot{V}^{l'}_{t+1}\widehat{N^{l'}_t}m^{kl'}_0\left[\dot{C}_t(k,l')\right]^{-1}}, \tag{34}$$

and the Bellman equation in changes becomes

$$\log\ddot{V}^k_t = \frac{\beta}{\nu}u^k_0(\widehat{u^k_t} - 1) + \beta\log\ddot{V}^k_{t+1} - \beta\log\widehat{\mu}^k_{0,t}, \tag{35}$$

where u^k_t is the instantaneous utility of workers in labor market k at time t. If we assume $u^k_t = \log w^k_t$, then equation (35) converges to the result of CDP. Between-labor-market moving probability m^{kl}_t can be also written in terms of changes:

$$m^{kl}_t = \frac{\mathbf{1}_{l=k}\ddot{V}^k_{t+1}\left[\mu^k_{0,0} + (m^{kk}_0 - \mu^k_{0,0})\widehat{N^k_t}\right] + \mathbf{1}_{l\neq k}\ddot{V}^l_{t+1}\widehat{N^l_t}m^{kl}_0\left[\dot{C}(k,l)\right]^{-1}}{\ddot{V}^k_{t+1}\left[\mu^k_{0,0} + \left(m^{kk}_0 - \mu^k_{0,0}\right)\widehat{N^k_t}\right] + \sum_{l'\neq k}\ddot{V}^{l'}_{t+1}\widehat{N^{l'}_t}m^{kl'}_0\left[\dot{C}(k,l')\right]^{-1}}. \tag{36}$$

The trade part of the model can also be re-written in changes based on our modified dynamic hat algebra technique. Since the trade part is solved for the period-by-period static equilibrium, the hat algebra is relatively simpler only with $\widehat{x}_t$ as in the standard exact hat algebra of Dekle, Eaton, and Kortum (2008). Factor market clearing conditions, equations (26)-(28), can be re-written as follows:

$$\widehat{\tilde{w}}^k_{1,t}\widehat{L}^k_{1,t}\tilde{w}^k_{1,0}L^k_{1,0} = \gamma_l\left(\sum_{r'}\widehat{\lambda}^{s_k}_{(1,r_k),(1,r'),t}\lambda^{s_k}_{(1,r_k),(1,r'),0}X^{s_k}_{(1,r'),t} + \sum_{n'\neq 1}\widehat{\lambda}^{s_k}_{(1,r_k),n',t}\lambda^{s_k}_{(1,r_k),n',0}X^{s_k}_{n',t}\right) \tag{37}$$

$$\widehat{b}_{1,t}^{k} b_{1,0}^{k} B_{1}^{k} = (1-\gamma_l-\gamma_m)\left(\sum_{r'} \widehat{\lambda}_{(1,r_k),(1,r'),t}^{s_k} \lambda_{(1,r_k),(1,r'),0}^{s_k} X_{(1,r'),t}^{s_k} + \sum_{n'\neq 1} \widehat{\lambda}_{(1,r_k),n',t}^{s_k} \lambda_{(1,r_k),n',0}^{s_k} X_{n',t}^{s_k}\right) \quad (38)$$

$$\widehat{\widetilde{w}}_{n,t} \hat{L}_{n,t} \widetilde{w}_{n,0} L_{n,0} = \bar{\gamma}_{n,l} \sum_{s'} \left(\sum_{r'} \widehat{\lambda}_{n,(1,r'),t}^{s'} \lambda_{n,(1,r'),0}^{s'} X_{(1,r'),t}^{s'} + \sum_{n'\neq 1} \widehat{\lambda}_{n,n',t}^{s'} \lambda_{n,n',0}^{s'} X_{n',t}^{s'}\right). \quad (39)$$

The detailed derivation for the trade part in changes is provided in Appendix A.5.

5.2 Solution Algorithm

Once the model is calibrated to the base year and the initial w_0^k, L_0^k, m_0^{kl}, and $\mu_{1,0}^k$ from the steady state simulation, we simulate our model for 30 periods ($\bar{T} = 30$) to see transitional dynamics. We solve for $\left(\widehat{\widetilde{\mathbf{w}}}_1, \widehat{\widetilde{\mathbf{w}}}_{-1}, \widehat{\mathbf{b}}\right)$ by solving the system of market clearing conditions in (37)-(39), where

$$\begin{aligned}
\widehat{\widetilde{\mathbf{w}}}_1 &= \left\{\widehat{\widetilde{w}}_{1,1}^{1}, \ldots, \widehat{\widetilde{w}}_{1,\bar{T}}^{1}, \ldots, \widehat{\widetilde{w}}_{1,1}^{k}, \ldots, \widehat{\widetilde{w}}_{1,\bar{T}}^{k}, \ldots, \widehat{\widetilde{w}}_{1,1}^{K}, \ldots, \widehat{\widetilde{w}}_{1,\bar{T}}^{K}\right\} \\
\widehat{\widetilde{\mathbf{w}}}_{-1} &= \left\{\widehat{\widetilde{w}}_{2,1}, \ldots, \widehat{\widetilde{w}}_{2,\bar{T}}, \ldots, \widehat{\widetilde{w}}_{n,1}, \ldots, \widehat{\widetilde{w}}_{n,\bar{T}}, \ldots, \widehat{\widetilde{w}}_{\bar{N},1}, \ldots, \widehat{\widetilde{w}}_{\bar{N},\bar{T}}\right\} \\
\widehat{\mathbf{b}} &= \left\{\widehat{b}_1^1, \ldots, \widehat{b}_{\bar{T}}^1, \ldots, \widehat{b}_1^k, \ldots, \widehat{b}_{\bar{T}}^k, \ldots, \widehat{b}_1^K, \ldots, \widehat{b}_{\bar{T}}^K\right\}.
\end{aligned}$$

We denote $\mathbf{L}$, $\ddot{\mathbf{V}}$, $\widehat{\mathbf{N}}$, $\widehat{\mathbf{P}}$, $\mathbf{m}$, and $\boldsymbol{\mu}$ vectors similarly.

Our iterative algorithm solves for $\left(\widehat{\widetilde{\mathbf{w}}}_1, \widehat{\widetilde{\mathbf{w}}}_{-1}, \widehat{\mathbf{b}}\right)$ in the outer iteration and solves for labor dynamics within each iteration of the outer loop.[43] We first guess the initial $\left(\widehat{\widetilde{\mathbf{w}}}_1, \widehat{\widetilde{\mathbf{w}}}_{-1}, \widehat{\mathbf{b}}\right)$ and solve for $\widehat{\mathbf{P}}$ and compute $\widehat{\mathbf{N}}$ as derived in Appendix A.5. For the first iteration, we guess labor allocation $\mathbf{L}$. We then solve for labor flows $\mathbf{m}$ and $\boldsymbol{\mu}$ using equations (36) and (34). Using the labor mobility flows, the guess of $\widehat{\widetilde{\mathbf{w}}}_1$, and the corresponding $\widehat{\mathbf{N}}$, we calculate implied changes in the values $\ddot{\mathbf{V}}$ as in equation (35). At the end of the labor loop, we calculate $\mathbf{L}$ which updates the guess for $\mathbf{L}$. Using the updated labor allocation $\mathbf{L}$, the initial guess of $\left(\widehat{\widetilde{\mathbf{w}}}_1, \widehat{\widetilde{\mathbf{w}}}_{-1}, \widehat{\mathbf{b}}\right)$, and the solved $\widehat{\mathbf{P}}$, we calculate the excess demand of each factor following Equations (37)-(39). We update the guess of $\left(\widehat{\widetilde{\mathbf{w}}}_1, \widehat{\widetilde{\mathbf{w}}}_{-1}, \widehat{\mathbf{b}}\right)$ and repeat the algorithm until all excess demands of factors become close to zero at every period.

5.3 Taking the Model to the Data

We calibrate the model to the base year 2003 before solving for changes. The model is quantified for 21 countries including Brazil, plus the rest of the world, thus, $\bar{N} = 22$.[44] Taking advantage of the detailed data, RAIS, we assume that country 1 is Brazil where there are multiple regions. We use the same definition of the labor market as before, with three sectors—agricultural, manufacturing, and services—and 558 microregions. We use the average of estimates for $\widetilde{C}_t^1$, $\widetilde{C}_t^2$, and $\widetilde{C}_t^3$ reported

[43] Iteration over the outer loop is similar to Alvarez and Lucas (2007).

[44] The list of countries is provided in Appendix A.3.

in Table A2 to compute the baseline between-market mobility frictions. With this baseline mobility friction, estimated ν, labor allocation data, and the wage data for the base year, we solve for the labor allocation and values at the initial steady state as described in Appendix A.5.[45]

We calibrate the cost share of labor (γ_l for Brazil and $\bar{\gamma}_{n,l}$ for all other countries), the cost share of intermediate inputs (γ_m for Brazil) to the base year using the World Input-Output Database (WIOD) and the share of fixed factors in value-added of Brazil from CDP. We also allow these cost shares to vary by sector.[46] In addition, we calibrate labor income of each labor market of Brazil ($\widetilde{w}_{1,0}^k L_{1,0}^k$); GDP of each country other than Brazil ($\widetilde{w}_{n,0} L_{n,0}$); trade shares, λ's; and sectoral expenditure shares in preference, ϕ's, to exactly match the data of the base year. We further allow for ϕ's to vary by country in the calibrated model. We calibrate the initial employment share of the residual market to be 0.2659 to match the informal employment share of the data in 2003. Lastly, we set $\beta = 0.95$ and set the trade elasticity $\theta = 4$ following Simonovska and Waugh (2014).

Once the baseline economy is specified, we use the simulated data from the model to calibrate the elasticity of substitution between tasks, $\tilde{\sigma}$. Since this parameter is related to the love for variety of tasks, we calibrate it to match the elasticity of the number of job options to exports in Table 5.[47] First, we simulate the model with a 20% decline in trade costs from Brazil to each partner country uniformly for all sectors. We then run the same regression described in Section 4.3 with the simulated data for changes in exports and the number of job options following the shock. The calibrated value of $\tilde{\sigma}$ is 2.4429 with which the elasticity of job options in Table 5, 0.652, is matched exactly. The calibrated value of $\tilde{\sigma}$ is larger than one and thus confirms the love for variety of tasks channel.

5.4 Counterfactual Exercises

The benchmark counterfactual shock we explore is a 20% permanent decline in iceberg trade costs d in the manufacturing sector from Brazil to each of its trading partners. This shock can be interpreted as a positive export shock to all manufacturing labor markets in Brazil. For the baseline counterfactuals, all the other model parameters are fixed. We then shut down our job options channel by setting the cost of task operation to be infinite and compare this alternative model to the baseline. Lastly, we simulate the benchmark shock with different levels of between-market moving costs to understand the role of mobility frictions in the transmission of trade shocks on workers' welfare. We focus our analysis on Brazil for the effect of the counterfactual shock on outcomes of interest.

[45] As in Section 2, we use the residual wage after controlling for observable worker characteristics.

[46] These cost shares are assumed to be the same across Brazilian regions within the same sector, because the input-output data are not available at the detailed region level in Brazil.

[47] In addition to the reason related to the interpretation of this parameter, we do not match the other three elasticities in the table because they also depend on the labor supply elasticity ν.

5.4.1 Benchmark Simulation

We quantify the effect of the benchmark counterfactual shock on labor mobility and welfare in Brazil using our framework. Table 6 reports long-run labor mobility responses in Brazil to this positive manufacturing export shock, summarized by sector. All numbers are changes between the initial and the new steady states, weighted by the initial employment share of a region within each sector. The first column reports changes in the employment share of each sector within the formal labor market. The employment share increases in the manufacturing and service labor markets, while it decreases in agriculture. The reallocation toward the manufacturing sector is straightforward from the nature of the benchmark shock. The employment share of the service sector also increases, because aggregate real income increases in Brazil due to this positive export shock, and the service sector is initially less traded. The increased demand is satisfied predominantly by domestic producers in the service sector due to lower tradability, which in turn attracts workers. Unlike predictions from static models, the transition to the new steady state is not instant in our model with mobility frictions. For example, it takes 5 years for sectoral employment shares to reach about 99% of the level of the new steady state.

Table 6: Long-run changes in employment share, the number of job options, and internal mobility

	Employment share (pp)	Number of job options (%)	Internal churning (%)
Agriculture	-0.2991	0.7853	-0.4250
Manufacturing	0.1169	7.4769	5.7548
Services	0.1822	12.4017	9.3334

Notes: Table reports the average of changes in mobility-related outcomes in Brazil between the initial and the new steady states after the benchmark trade shock, weighted by the initial employment share of a labor market within each sector. The first column is in percentage points, and the other two columns are in percentage changes.

At the same time, the employment share of the residual labor market goes down by -1.26 pp at the new steady state, because demand is constant there due to the assumption of no production. In addition, formal labor markets expand the number of job options as reported in the second column of Table 6, which further attracts workers. A 20% decline in manufacturing trade cost increases the number of job options workers can compare in manufacturing markets by 7.48% in the long run. The number of job options increases even more in services where domestic producers mostly absorb the positive effect of the higher real income. The agriculture labor markets also see an increase in the number of job options, but the magnitude of the increase is much smaller than the other two sectors.

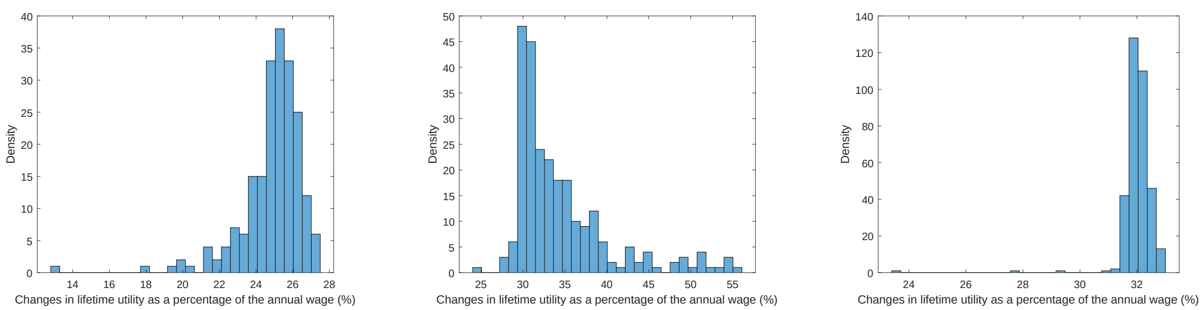
Changes in the number of job options lead to both between- and within-market labor mobility. Since workers can compare more options in manufacturing and services, it becomes more likely for them to find a better match and thus move there. This coincides with the result reported in the first column of Table 6. Changes in the number of job options also lead to internal churning. The last column of Table 6 shows that workers in the manufacturing and service sectors move significantly more between jobs without moving to a different labor market. As more job options emerge in those labor markets, it becomes more likely to find a better match even within the same market.

These labor mobility responses translate into changes in workers' welfare.[48] Our model predicts that the benchmark shock increases the present discounted value of Brazilian workers' lifetime utility in all labor markets. In aggregate, the increase of lifetime utility is equivalent to a 32.45% one-time increase in the annual wage.[49] Between sectors, workers in manufacturing labor markets gain the most (34.78% of the annual wage), and those in agriculture gain the least (24.55% of the annual wage) from the benchmark shock. The larger welfare increase in manufacturing is true even after taking into account the fact that labor reallocation would put downward pressure on real wages in the long run.

5.4.2 Mobility Frictions and the Heterogeneous Welfare Effect of a Trade Shock

Within each sector, we find that welfare gains are heterogeneous across regions as shown in Figure 3. Where is this within-sector spatial heterogeneity originated from, while all labor markets in the same sector originally face the same trade shock?

Figure 3: Within-sector distribution of changes in welfare



(a) Agriculture (b) Manufacturing (c) Service

Notes: Each plot shows the within-sector distribution of changes in average lifetime utility as a percentage of the annual wage across regions in Brazil.

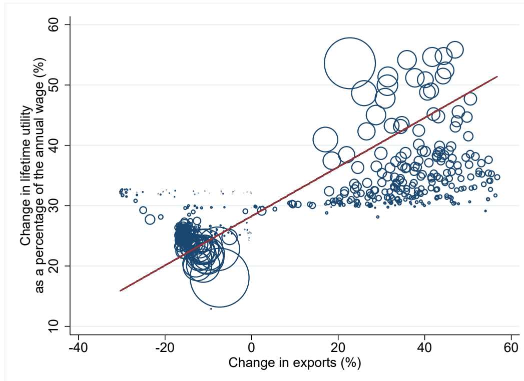
In Sections 2 and 4.3, we focused on the regional variation of *exposure* to changes in export demand following the reduced-form literature. In a similar spirit, Figure 4 shows how much of the change in welfare from our simulated model is explained by changes in exports from each Brazilian labor market induced by the counterfactual shock. Each dot is weighted by initial exports per worker in the data. There is a strong positive relationship between changes in each labor market's own exports and changes in welfare. This pattern is driven by the labor markets that are initially more open, i.e., more export-oriented. In this figure, most labor markets that experience a decrease in exports from the benchmark

[48]Welfare in our model is measured by the average value of each labor market V_t^k. Using our modified dynamic hat algebra, we recover $V_t^k - V_0^k$. We will use welfare, value, and more precisely the present discounted value of workers' lifetime utility interchangeably throughout the rest of the paper.

[49]It is also possible to annualize this number by multiplying it by $(1 - \beta)$.

shock are agricultural markets, which is expected due to the nature of the shock.

Figure 4: Changes in welfare and changes in exports



Notes: Each dot represents a labor market and is weighted by the initial export per worker in 2003. Both axes report simulated changes from the model.

While Figure 4 shows that the change in each labor market's exports is a strong predictor of the welfare changes of the market, there still remains sizable heterogeneity in welfare response across markets. We argue that how labor markets are interconnected with one another is very important to explain this heterogeneous welfare response. The interconnectedness between labor markets in our model depends on the mobility frictions between labor markets. We calculate the "remoteness" of each labor market from our baseline estimate of $C(k,l)$. For each k, the remoteness measure is defined as the average of $C(k,l)$ across all $l \neq k$, weighted by the initial employment of the destination l. This measure describes how remote a labor market is from all other labor markets of the economy.

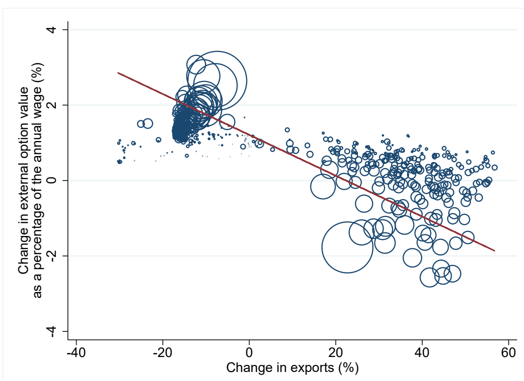
We then compute residual changes for each labor market's welfare after regressing simulated changes in welfare on simulated changes in own exports. Figure 5 plots these residual changes in welfare against the remoteness of the labor market, weighted by the initial exports per worker. Changes in welfare that are not explained by changes in own exports are negatively correlated with the remoteness of the labor market, which implies that workers in a labor market that is more remote from other markets gain significantly less from the same shock. This is because they find it more difficult to move to other labor markets and take advantage of growing job options. Therefore, the degree of mobility frictions faced by workers matters for the magnitude of welfare gains from a trade shock.

Mobility frictions also affect workers' option value, $-\nu \log(\mu^k_{0,t})$, which can be decomposed into external and internal option values to distinguish utility values of between-market mobility from those of within-market mobility. External option values are expected to be negatively affected by a positive shock to the labor market, because potential destinations of moving are doing relatively worse. Internal option values, on the other hand, are expected to be positively affected, because a positive shock to the labor market increases the number of job options. We showed these relationships in a reduced-form way in Table 5. Figure 6 confirms them with the simulation by plotting simulated external and internal option value changes separately against simulated changes in exports of the labor market.

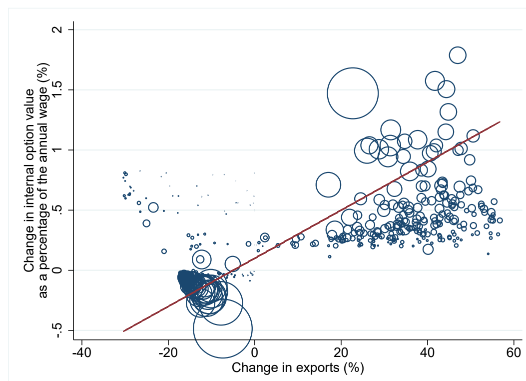
Figure 5: Residual changes in welfare and remoteness of the labor market

Notes: This figure plots residual changes in welfare of each labor market as a percentage of the annual wage against the remoteness measure. Each dot represents a labor market and is weighted by the initial export per worker in 2003.

Figure 6: Changes in external/internal option values and changes in exports



(a) External option values



(b) Internal option values

Notes: Each panel plots changes in external or internal option values as a percentage of the annual wage against changes in exports from each labor market. Each dot represents a labor market and is weighted by the initial export per worker.

To understand the heterogeneous changes in external and internal option values across labor markets, we perform a similar exercise as we did for welfare. First, we compute residual changes in external and internal option values after regressing them on changes in exports from each labor market. For option values, a relevant remoteness measure should capture how far a labor market is from other export-oriented markets that offer more options in response to the shock. Therefore, we compute a measure of the "remoteness from export" by taking an average of between-market moving cost across all destination markets, weighted by the initial exports per worker of the destination. We plot residual changes in external and internal option values against the remoteness from export measure in Figure 7.[50] These figures show that external option values increase more in the labor markets that are better-connected to initially export-oriented markets, while the relationship is exactly the opposite for

[50] Since the remoteness measure is already weighted by the initial export per worker, dots are not weighted.

internal option values. In other words, having the option of moving between labor markets increases the welfare of the workers who face relatively lower mobility frictions to export-oriented markets. For workers in a labor market that is more remote from export opportunities of other markets, having the option to move between jobs within the same labor market generates larger welfare gains.

Figure 7: Residual changes in external/internal option values and remoteness from export

(a) External option values

(b) Internal option values

Notes: Each panel plots residual changes in external or internal option values of each labor market as a percentage of the annual wage against against the remoteness from export measure of each labor market.

In summary, the welfare effect of a trade shock on each labor market depends significantly on the degree of mobility frictions faced by workers. This finding motivates policy simulations about lowering mobility frictions across regions and/or sectors, which will be discussed in Section 5.4.4.

5.4.3 The Role of the Job Options Channel

We can conveniently quantify the effect of our job options channel on welfare gains from trade by shutting down that channel with an infinite cost of creating tasks. This limiting assumption effectively leads to $\widehat{N}_t^k = 1$ for all t and k. This alternative model can be simulated with the same benchmark counterfactual shock of a 20% decline of manufacturing trade costs from Brazil to each partner country. Table 7 summarizes aggregate and sectoral average changes in present discounted lifetime utility as a percentage of the annual wage. Aggregate welfare increases in both baseline and alternative specifications, but the amount of increase is on average 67.7% smaller when we shut down our job options channel from the model. In other words, welfare gains from a positive export shock are significantly magnified through the job options channel we introduce to the model.

This magnification effect of the job options channel can be explained based on the magnitude of labor reallocation and the associated change in option values. The average percentage point change in the employment share of each labor market is 0.037% in the baseline model, while it is 0.026% without the job options channel. The baseline model also captures the increase of internal mobility, and thus

Table 7: Average changes in welfare from baseline and alternative specifications (%)

	Baseline	No job options channel
Aggregate	32.45	10.47
Agriculture	24.55	1.64
Manufacturing	34.78	12.49
Services	32.15	10.32

Notes: Table reports the average of changes in present discounted lifetime utility as a percentage of the annual wage for each model specification, weighted by the initial employment share of a labor market in 2003.

internal option values, from the positive export shock, while the alternative specification does not. These mobility results translate into a larger welfare effect for workers, as it becomes more likely to find a better match after comparing more options. We also find that this magnification effect is larger in agricultural markets which are hit by a relatively negative shock in our benchmark counterfactual scenario. When we allow the number of job options to respond endogenously to the shock, workers in agricultural markets can enjoy larger option values as other non-agricultural markets offer new job options. This partially offsets the negative effect on their relative labor demand.

5.4.4 Policy Experiment: Lowering Mobility Frictions

The degree of mobility frictions that workers face is an important determinant of the dynamic welfare consequences of any aggregate shock. To study the role of mobility frictions in transmitting trade shocks to workers' welfare, we run simulations for two alternative specifications. First, we lower the baseline $C(k,l)$ for all (k,l) such that $k \neq l$ by 20% and simulate the model with the same benchmark trade shock. Second, we lower only the moving cost by the same amount, but without the trade shock. These two scenarios allow us to calculate the net effect of having lower mobility costs on the welfare response. We also study the role of geographical and sectoral dimensions of mobility frictions separately by lowering $C(k,l)$ across only regions or sectors by the same amount 20%.

Columns (2)-(4) of Table 8 report the net effect of having lower mobility frictions on the welfare effect of the benchmark trade shock. Regardless of the adjustment margin, the welfare gains from the same benchmark trade shock are significantly larger with lower mobility frictions between markets. If mobility frictions are 20% lower than our estimates, the aggregate welfare gains from the trade shock are on average 14.8% larger. Columns (3) and (4) show that if a policymaker can lower mobility frictions across either regions or sectors, the same amount of the decrease in mobility frictions increases the welfare effect more when regions are targeted. This is a powerful result as we define labor markets with a much larger number of regions than sectors. Even with this definition of the labor market, workers turn out to face significantly larger mobility frictions across regions than across sectors, and thus a labor market policy mitigating such friction across regions can be more effective.

Table 8: Average changes in welfare with lower mobility frictions (%)

		Lower mobility frictions		
	(1) Baseline	(2) Both	(3) Regions	(4) Sectors
Aggregate	32.45	37.25	36.54	32.88
Agriculture	24.55	29.73	28.68	25.25
Manufacturing	34.78	39.49	38.79	35.19
Services	32.15	36.95	36.26	32.57

Notes: Table reports for each model specification the average of changes in present discounted lifetime utility as a percentage of the initial annual labor income, weighted by the initial employment share of a labor market in 2003.

6 Matching Untargeted Patterns in the Data

We provide additional evidence for the importance of the channels discussed herein and the model's fit in replicating these channels. Our model predicts that labor mobility should be highly correlated not only with trade shocks, but with most aggregate shocks. The pattern along which labor mobility in data changes over the business cycle is consistent with our model's prediction, but different from what other structural labor mobility models in the trade literature predict.

We first define a stylized "positive aggregate shock" z, as a shock that increases the number of job options, N_t^k, by $\varrho^k > 0$ for all k.[51], which means that after the shock, the number of job options, therefore the number of sampled tasks, increases everywhere. The amount of increase can vary by labor market. As we assumed in Section 3, this shock does not change the underlying moving cost structure and is realized after workers earn their wage at t. Also, we further assume $\mu_{0,t}^k > 0, \forall k$; and that for a given k, there exists l such that $m_t^{k,l} > 0$. Note that we do not assume that all entries in the transition matrix are non-zero. We assume that there are at least one stayer and one mover for each row. Then, we can state the following proposition about the "positive aggregate shock".

Proposition 1. *A "positive aggregate shock" as defined above increases the (i) arithmetic mean and the (ii) geometric mean of the ratio of within-labor-market movers to stayers in a job within formal labor markets. I.e.,*

(i) $\frac{\partial}{\partial z}\left[\frac{1}{K}\sum_k \frac{\mu_{1,t}^k L_t^k}{\mu_{0,t}^k L_t^k}\right] > 0,$ *and*

(ii) $\frac{\partial}{\partial z}\left[\prod_k \frac{\mu_{1,t}^k L_t^k}{\mu_{0,t}^k L_t^k}\right]^{\frac{1}{K}} > 0,$ *where* $k \neq I$.

Proofs of all propositions are provided in Appendix A.8. Proposition 1 implies that economy-wide positive shocks increase the average churning within labor markets. In fact, we can identify changes in the number of job options directly from changes in internal churning. We extend this idea

[51] More precisely, we define $\varrho^k \equiv \frac{\partial}{\partial z} N_t^k$.

to between-labor-market mobility in Proposition 2. However, a formal proof is only possible for the geometric mean, not for the arithmetic mean. Since arithmetic and geometric means are often highly correlated, positive shocks increase average mobility along all dimensions empirically, irrespective of the calculation method.

Proposition 2. *A "positive aggregate shock" as defined above increases the geometric mean of (i) the ratio of movers to a different market to stayers in a job, and (ii) the ratio of the number of all movers to the number of stayers in a job within formal labor markets. I.e.,*

(i) $\frac{\partial}{\partial z}\left[\prod_k \prod_{l\neq k} \frac{m_t^{k,l} L_t^k}{\mu_{0,t}^k L_t^k}\right]^{\frac{1}{K(K-1)}} > 0,$ *and*

(ii) $\frac{\partial}{\partial z}\left[\left(\prod_k \prod_{l\neq k} \frac{m_t^{k,l} L_t^k}{\mu_{0,t}^k L_t^k}\right)\left(\prod_k \frac{\mu_{1,t}^k L_t^k}{\mu_{0,t}^k L_t^k}\right)\right]^{\frac{1}{K^2}} > 0,$ *where* $k \neq I$ *and* $l \neq I$*.*

Proposition 2-(ii) follows directly from Propositions 2-(i) and 1-(ii). Since both the ratio of movers within a labor market and the ratio of movers across markets increase, the ratio of all movers increases. In addition to predictions on labor mobility patterns, our model has a prediction about a potential empirical bias in the estimation of moving costs if the job options channel is ignored. In Appendix A.7, we define an "auxiliary model" where we convert our model to a standard discrete choice model with the fixed number of choices, but combined with utility shifters and moving costs that are functions of the number of job options. We denote the moving cost of the auxiliary model as $\bar{C}_t(k,l)$. If we ignore the fact that the number of choices can endogenously change over time, empirically it would look like the moving costs, which are assumed to be exogenous, are changing endogenously. In other words, the moving costs estimated using ACM, or similar models without the job options channel, would fluctuate counter-cyclically over time as the number of job options is changing.

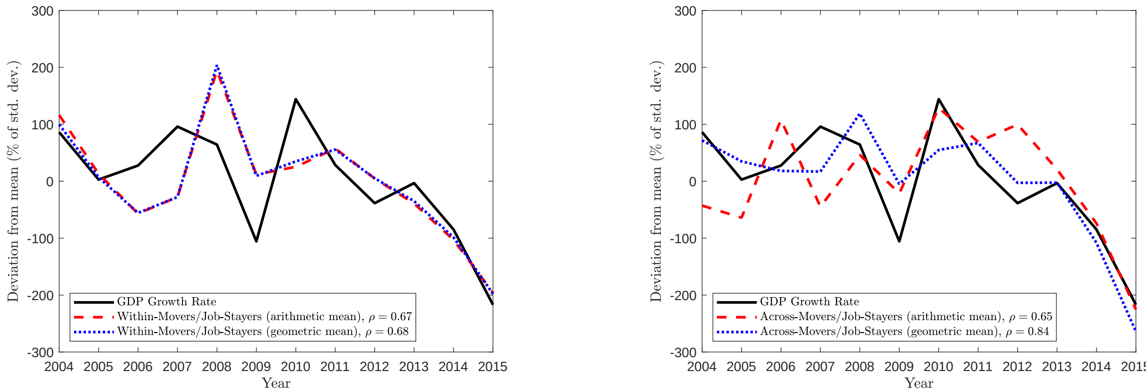
Proposition 3. *A "positive aggregate shock" as defined above reduces the average implied moving cost difference between the auxiliary and main models. I.e.,*

$$\frac{\partial}{\partial z}\frac{1}{K(K-1)}\sum_k \sum_{l\neq k}\left[\bar{C}_t(k,l) - C_t(k,l)\right] < 0.$$

Based on the propositions above, we expect to see a positive correlation between aggregate productivity and mobility within and across labor markets in data. The left panel of Figure 8 plots the GDP growth rate in Brazil between 2004 and 2015 together with arithmetic and geometric means of within-labor-market churning, i.e. $\mu_{1,t}^k/\mu_{0,t}^k$, which should be positively associated with N_t^k as implied by the model.[52] The correlation coefficient between the series is equal to 0.67 for the arithmetic and 0.68 for the geometric means, showing a positive correlation as discussed in Proposition 1.

[52] All series are re-scaled by subtracting their mean and dividing by standard deviations for comparison. The mobility between time t and time $t+1$ is reported at time $t+1$, rather than at time t as we did previously, to make the calculations consistent with the standard GDP growth rate reporting norms.

Figure 8: GDP growth rate and mobility within and across labor markets, 2004-2015



(a) Within labor markets (b) Across labor markets

Notes: Panel (a) depicts the ratio between the number of workers who change jobs within a labor market and the number of job-stayers. Panel (b) depicts the ratio between the number of workers who move to other labor markets and the number of job-stayers. All series are in terms of deviations from the mean and multiplied by the inverse of their standard deviations for comparison.

Similarly, according to Proposition 2-(i), between-market mobility should be positively correlated with aggregate shocks as well. Although the proposition makes this statement only for the geometric mean of between-market mobility, we can empirically expect to see a positive correlation with the arithmetic mean as well, since geometric and arithmetic means are often highly correlated. The right panel of Figure 8 confirms the empirical correlation between positive shocks and between-market mobility with correlations coefficients of 0.65 and 0.84 for arithmetic and geometric means.[53]

Next, the left panel of Figure 9 presents the correlation between overall mobility (between- and within-market mobility combined) and aggregate shocks, based on Proposition 2-(ii). Finally, following Proposition 3, the right panel of Figure 9 shows a positive correlation between the inverse of moving cost differences between the baseline and the auxiliary models and the GDP growth rate.[54] These results suggest that the estimated moving cost from standard dynamic labor mobility models without our job options channel captures fundamentally endogenous variations, while this cost is assumed to be exogenous in those models. Our framework shows that the endogenous response of the number of job options to an aggregate shock is the source of the variation. In summary, the figures presented in this section show the model's ability to capture important channels related to labor mobility.

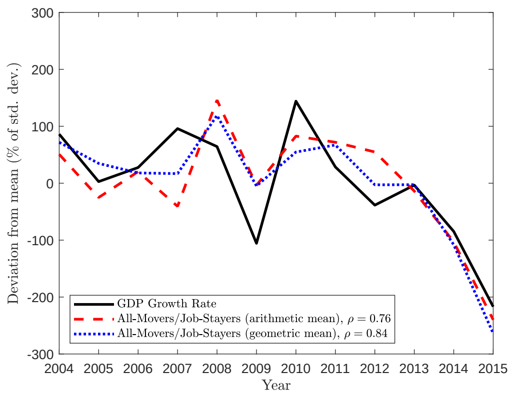
7 Model Fit and Robustness

In this section, we show that the model can match the labor flows in the data reasonably well, despite having a simple moving cost structure with only three parameters; and that the empirical results are robust to alternative job definitions from the data.

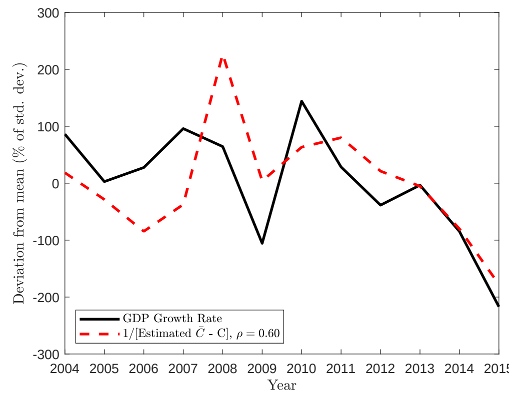
[53]We added a small error, 0.001, to the number of workers in the numerator and denominator, to prevent zeros in the calculation of the geometric mean.

[54]In addition to the evidence from Brazil, we also document a similar relationship for the U.S. in Appendix Figure A5.

Figure 9: GDP growth rate, total labor mobility and moving cost differences, 2004-2015



(a) Total labor mobility



(b) Inverse of implied moving cost differences

Notes: Panel (a) depicts the ratio between the number of workers who change jobs (within the same labor market or to another labor market) and the number of stayers. Panel (b) depicts the inverse of the difference between the moving cost implied by the auxiliary model and the actual moving cost from our estimation. All series are in terms of deviations from the mean and multiplied by the inverse of their standard deviations for comparison.

7.1 Data and Simulated Flows

In Section 4.1, we estimated $\widetilde{m}_t^{kl}$, and the three parameters of the moving cost function by PPML. We used the average of the estimated moving cost parameters over the sample period to characterize the initial steady state flows for the counterfactual simulation as explained in detail in Appendix A.5.1.1. The steady state simulation backs out values to match the labor allocations in the data, but does not target flows. In fact, steady state simulations only use labor allocation data, wage data, moving cost, and labor supply elasticity parameters. We show that despite the simple moving cost function, the model matches the labor flows in the data very well. Figure A6 panel (a) shows the average number of workers switching regions in the data versus in the simulation before we plug in a counterfactual shock. Similarly, panels (b)-(d) show comparisons between data and simulations for sector switchers, both sector and region switchers, and workers leaving the residual labor market. The simulated flows and the actual flows from data are highly correlated for all types of mobility.

7.2 Alternative Definitions of a Position

The characterization of the job in our model implies that a worker changes jobs when she changes her position which is empirically identified through her detailed occupation. Alternatively, one can attempt to empirically identify changes in positions by adding other criteria, such as changes in worker's firm (establishment) or detailed industry. We consider the following alternative empirical definitions of a position to check the robustness of our results: (i) a 6-digit detailed occupation category (our baseline definition); (ii) a combination of occupation and firm, therefore if a worker changes either occupation or firm, she effectively changes her position; and (iii) a combination of occupation, firm, and detailed industry (5-digit), therefore if a worker changes occupation, firm, or industry she changes her position.

First, we re-estimate moving costs with these alternative definitions, since the moving costs are estimated based on mobility observed from changes of positions. The results are summarized in Table A3. We find that the first two moving cost parameters from all three definitions are very similar. The additional cost of changing both sector and region is more different overall, but it is relatively insignificant for the overall moving cost level compared to the other two cost components.

Second, we re-estimate the implications of export shocks for welfare, the number of job options, and option values reported in Table 5, using alternative definitions of positions. We use the same IV strategy discussed in the earlier sections, where the first-stage results are presented in Table 4. The new results are presented in Table A4. We find that most results are very close across all definitions of a position. We can conclude that as long as the detailed occupation is a part of the definition of position, as suggested by the theoretical framework, the model produces very robust results.

8 Conclusion

We introduce a new general equilibrium framework to quantify the impacts of trade shocks on labor mobility and workers' welfare, combining the advantages of the structural and reduced-form methodologies. The key new mechanism of our model is the endogenous number of job options workers can choose from. We show that a labor market experiencing a positive trade shock attracts more workers because of the larger number of job options that are created there. More job options translate into higher welfare, as workers can choose the best job after comparing more options.

We first provide empirical evidence on the causal effects of export shocks on labor markets using rich employer-employee panel data combined with customs records on export transactions from Brazil during 2003-2015. Using changes in import demand directed to the labor market as a source of variation in exports, we document a positive causal effect of positive export shocks on employment, residual wages, and job turnover rates in the corresponding labor market.

Motivated by this reduced-form evidence, we develop a dynamic general equilibrium model of labor mobility and trade, featuring the endogenous number of job options as well as frictional labor mobility between and within labor markets. Our model characterizes the mechanism through which trade shocks affect the number of job options, labor mobility, and workers' welfare. We provide a sufficient statistic result showing that the effects of a trade shock on relative welfares are fully embedded in the gross flows between labor markets. This is a powerful result that greatly simplifies the analysis of the welfare impacts of trade shocks. We then structurally estimate the model using the same administrative data from Brazil and the same IV strategy used in our reduced-form analyses. Armed with the estimates of the structural parameters and the sufficient statistic result, we find that a 10% change in exports during the sample period increases the lifetime welfare of a median worker in the formal labor market by 2.99% of the annual wage.

Finally, we quantify the effect of trade shocks on labor allocation, the number of job options, and workers' welfare. Using the benchmark shock of a 20% decline in manufacturing trade costs from Brazil to its trading partners, we show that a positive manufacturing export shock reallocates labor toward the manufacturing and service sectors and increases internal mobility there. We show that the aggregate lifetime welfare increases in all labor markets from this shock, but workers in manufacturing on average experience 41.7% larger welfare increases compared to those in agriculture.

By comparing our baseline specification to the alternative one without the job options channel, we show that our main channel significantly magnifies welfare gains from a positive export shock. Our model can also shed light on predicting the potential benefit of a policy mitigating labor mobility frictions. We show that when workers face 20% lower mobility frictions both across regions and sectors, the welfare increase is 14.8% larger. Reduced mobility frictions across regions have a larger magnification effect compared to the reduced mobility costs across sectors.

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A.1 Appendix Tables

Table A1: Gross job flow rates by sector and year

	Agriculture		Manufacturing		Services		All sectors	
year	inflows	outflows	inflows	outflows	inflows	outflows	inflows	outflows
2004	0.084	0.086	0.066	0.063	0.054	0.076	0.068	0.075
2005	0.091	0.091	0.067	0.094	0.059	0.082	0.072	0.089
2006	0.112	0.091	0.087	0.071	0.060	0.076	0.087	0.079
2007	0.087	0.087	0.069	0.072	0.060	0.079	0.072	0.079
2008	0.083	0.089	0.067	0.074	0.062	0.075	0.071	0.079
2009	0.090	0.102	0.068	0.090	0.067	0.079	0.075	0.090
2010	0.089	0.101	0.075	0.080	0.068	0.082	0.077	0.088
2011	0.094	0.098	0.074	0.082	0.072	0.086	0.080	0.089
2012	0.107	0.094	0.074	0.078	0.072	0.081	0.084	0.084
2013	0.094	0.087	0.076	0.073	0.070	0.073	0.080	0.077
2014	0.092	0.078	0.070	0.065	0.064	0.063	0.075	0.069
Average	0.093	0.091	0.072	0.076	0.064	0.077	0.076	0.082

Notes: Table reports average gross job inflow and outflow rates for each sector and year in Brazilian microregions in 2004-2015. The entry for 2004 refers to job flow rates between 2004 and 2005, the entry for 2005 refers to job flow rates between 2005 and 2006, and so on.

Table A2: Estimated moving cost parameters

Year	$\widetilde{C}_t^1$ (log distance)	s.e.	$\widetilde{C}_t^2$ (sector)	s.e.	$\widetilde{C}_t^3$ (both)	s.e.
2003	1.0136	(0.0012)	1.7988	(0.0076)	-0.2289	(0.0151)
2004	0.9947	(0.0012)	1.7015	(0.0079)	-0.1984	(0.0153)
2005	0.9768	(0.0013)	1.3623	(0.0075)	0.1478	(0.0155)
2006	0.9756	(0.0012)	1.6622	(0.0077)	-0.1918	(0.0146)
2007	1.0079	(0.0011)	1.8289	(0.0077)	-0.4176	(0.0144)
2008	0.9809	(0.0011)	1.6994	(0.0072)	-0.2260	(0.0134)
2009	0.9791	(0.0010)	1.5765	(0.0066)	-0.1882	(0.0124)
2010	0.9679	(0.0010)	1.5914	(0.0068)	-0.1573	(0.0126)
2011	0.9658	(0.0011)	1.5705	(0.0079)	-0.1044	(0.0146)
2012	0.9526	(0.0012)	1.5962	(0.0083)	-0.1050	(0.0152)
2013	0.9607	(0.0010)	1.6092	(0.0073)	-0.1470	(0.0133)
2014	0.9704	(0.0010)	1.7361	(0.0072)	-0.1832	(0.0131)
Average	0.9781		1.644		-0.1667	

Notes: Table reports moving cost estimates, divided by ν, for each year.

Table A3: Moving costs with alternative definitions for job switching

	Alternative definitions		
	(i)	(ii)	(iii)
$\widetilde{C}_t^1$	0.9781	1.0192	1.0481
$\widetilde{C}_t^2$	1.6444	1.8806	2.0534
$\widetilde{C}_t^3$	-0.1667	-0.3905	-0.5418

Notes: Table reports average of yearly estimates of moving cost with alternative definitions of job switching. (i): Position is detailed occupation, i.e. the baseline definition. (ii): Position is a combination of occupation and firm. (iii): Position is a combination of occupation, firm and industry, where industry is defined at the 5-digit level. The numbers in Column (i) match the numbers in the last row of Table A2.

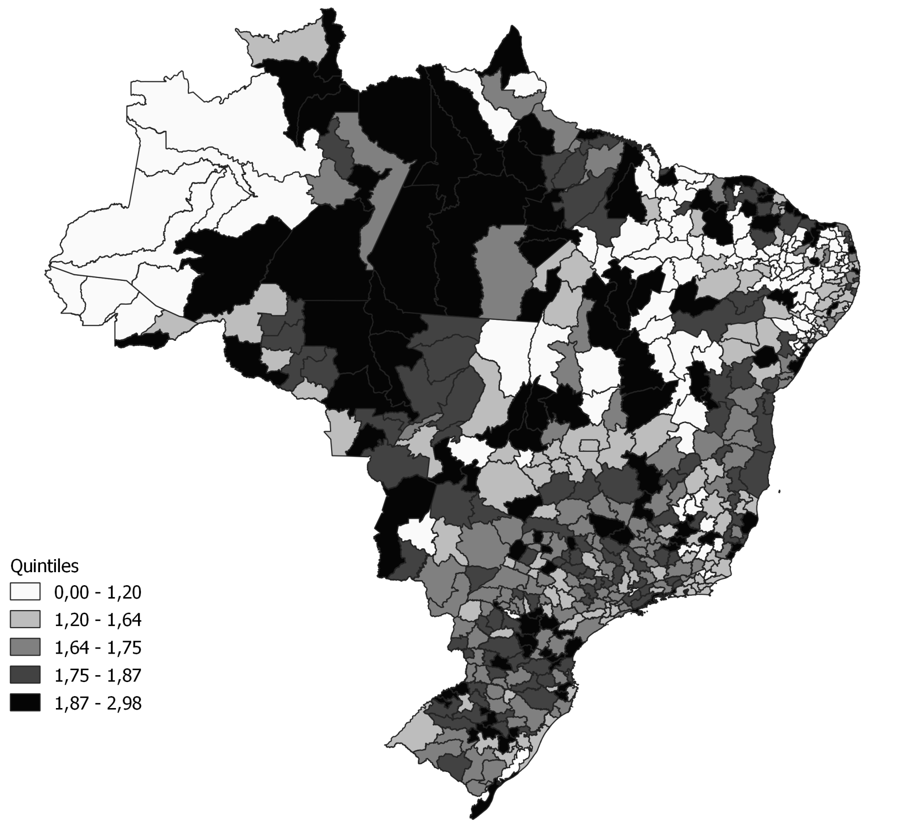
Table A4: Main empirical results with alternative definitions for job switching

	Alternative definitions		
	(i)	(ii)	(iii)
Welfare	0.607	0.639	0.765
	(0.198)	(0.207)	(0.230)
Number of job options	0.653	0.632	0.578
	(0.142)	(0.139)	(0.172)
Internal option values	0.147	0.149	0.155
	(0.021)	(0.023)	(0.029)
External option values	-0.161	-0.154	-0.154
	(0.050)	(0.051)	(0.056)

Notes: Table reports main estimation results reported in Table 5 with alternative definitions of job switching. (i): Position is detailed occupation, i.e. the baseline definition. (ii): Position is a combination of occupation and firm. (iii): Position is a combination of occupation, firm and industry, where industry is defined at the 5-digit level. Standard errors clustered by microregion and year are reported in parentheses.

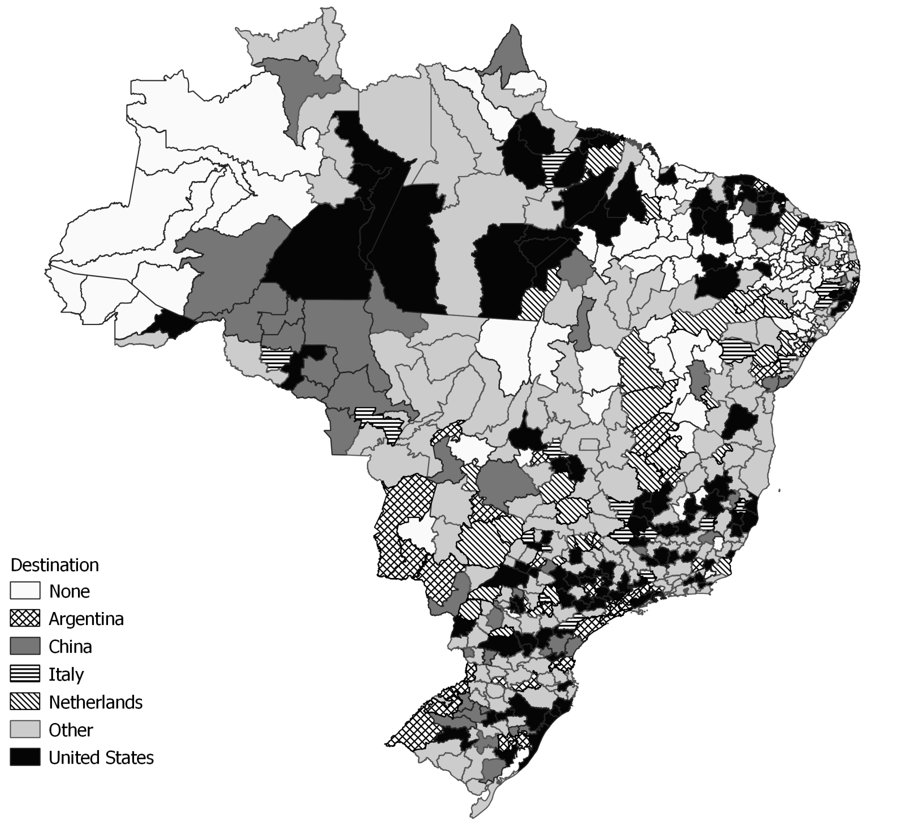
A.2 Appendix Figures

Figure A1: Export revenue per worker, 2003



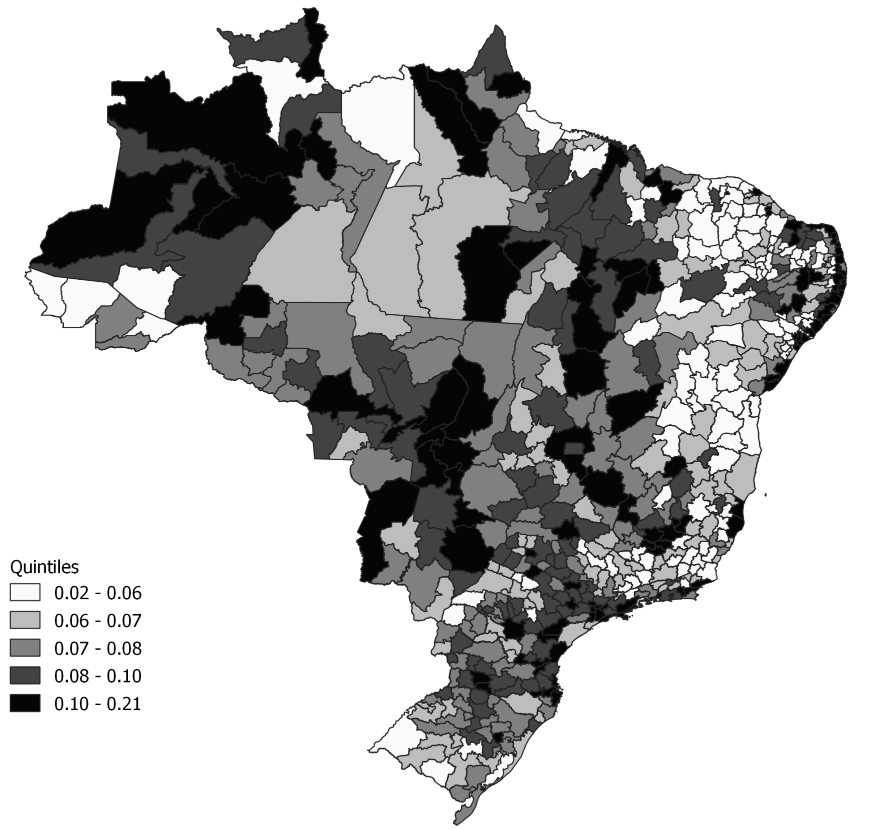
Notes: Figure depicts the log of (1+exports) per worker in Brazilian microregions in 2003.

Figure A2: Top export destinations, 2003



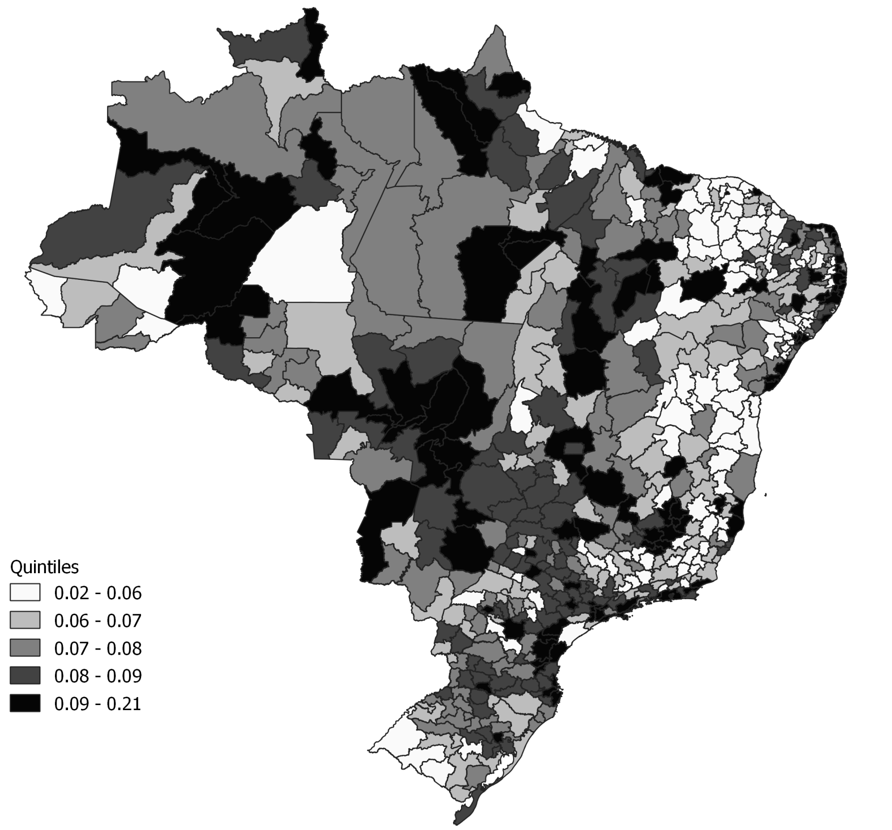
Notes: Figure depicts the top export destination of each Brazilian microregion in 2003.

Figure A3: Gross outflow rates by microregion, 2004-2015



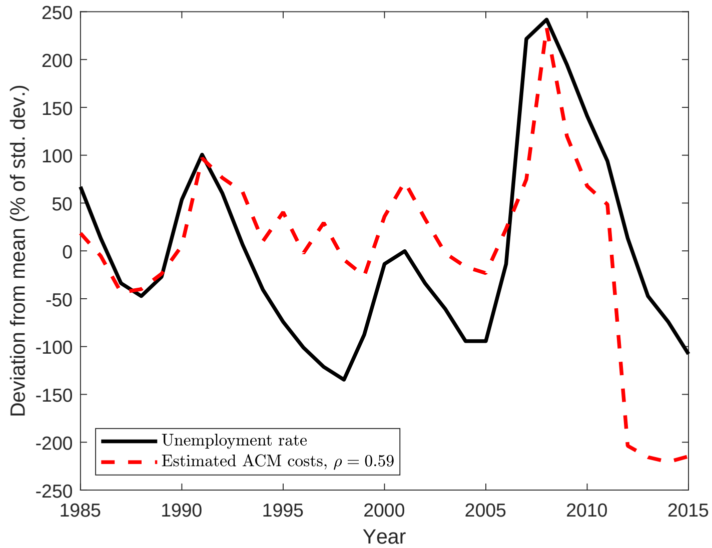
Notes: Figure depicts the average gross job outflow rates observed in each microregion in 2004-2015.

Figure A4: Gross inflow rates by microregion, 2004-2015



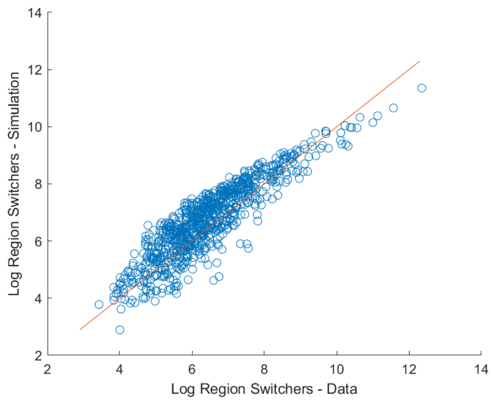
Notes: Figure depicts the average gross job inflow rates observed in each microregion in 2004-2015.

Figure A5: Additional evidence on model fit from the USA

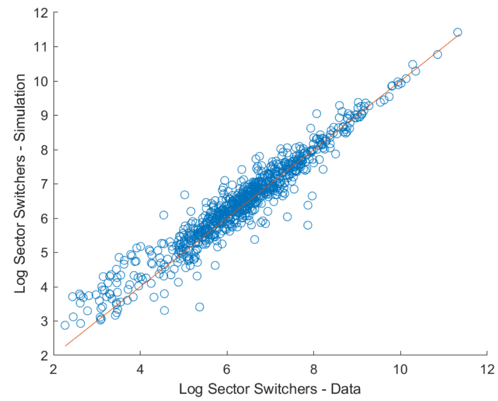


Notes: Figure depicts the correlation between the ACM moving cost and unemployment rate in USA between 1985 and 2009. The correlation coefficient between the two series is equal to 0.59.

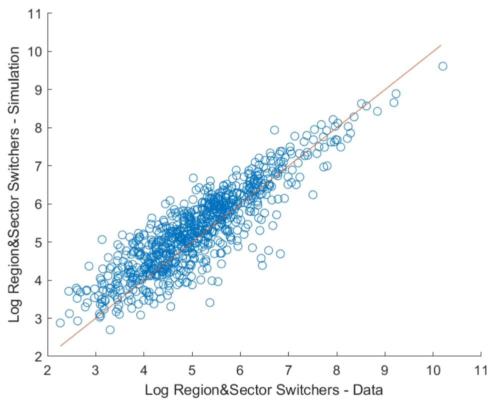
Figure A6: Simulated flows and average flows in the data



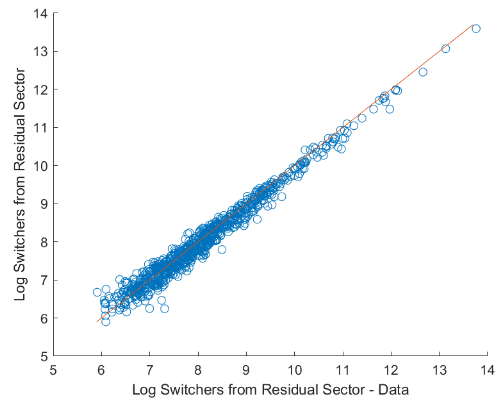
(a) Number of workers changing regions



(b) Number of workers changing sectors



(c) Number of workers changing regions and sectors



(d) Number of workers leaving residual sector

Notes: Each panel shows the number of people moving in a specific corridor in data scatter-plotted against the simulated number of people at the initial steady state. The red line in each panel is the 45-degree line.

Appendices for Online Publication

A.3 Data Sources and Description

Here we provide further details about the data sets used in the empirical analysis and in the quantification of the model.

Employer-employee panel data: *Relação Anual de Informações Sociais* (RAIS) is a labor census gathering longitudinal data on the universe of formal workers and firms in Brazil. We use data for the period 2003-2015. RAIS is a high-quality administrative census of formal employees and employers collected every year by the Brazilian Ministry of Labor. These records are used by the government to administer several government benefits programs. Workers are required to be in RAIS in order to receive payments of these programs and firms face fines for failure to report, until they do report. These requirements ensure that RAIS is an accurate and complete census of the formal sector in Brazil (Dix-Carneiro and Kovak, 2017).

RAIS covers virtually all formal workers and provides yearly information on demographics (age, gender, and schooling), job characteristics (detailed 6-digit occupation, wage, hours worked), as well as hiring and termination dates. For each job, the RAIS annual record reports average yearly earnings and the monthly wage in December. We use the information on the December wage, so as to ensure that all labor market outcomes are measured at the same time and avoid potential mismeasurement for workers that did not work a full year. RAIS further includes information on a number of establishment-level characteristics, notably number of employees, geographical location (municipality), and industry code, defined according to the 5-digit level of the Brazilian National Classification of Economic Activities (CNAE).

Unique identifiers for workers and firms are consistently defined across years and therefore make it possible to follow them over time. The unique identifier for a worker is the number associated with her registration in *Programa de Inserção Social* (PIS). The unique identifier for an establishment (*Cadastro Nacional de Pessoa Jurídica*) (CNPJ) is an identification number issued to Brazilian companies by *Secretaria Especial da Receita Federal* of Brazil. It consists of a 12 digit number, of which the first 8 digits uniquely identify the firm and the remaining four digits identify the establishment. Therefore, it is possible to identify and track multi-establishment firms. While the RAIS database covers also segments of the public sector, we restrict the analysis to the private sector. The industry classification contains 572 industries at the 5-digit level, of which 42 are in agriculture and natural resources, 286 are in manufacturing, and the remainder are in services.

The information on the level of education of the worker is reported in 11 categories (to which we assign the corresponding typical years of education in parenthesis): illiterate (corresponding to 0 years of education); primary school dropout (indicating from 1 to 3 years of education), primary school graduate (4 years of education), middle school dropout (5 to 7 years of education), middle school graduate (8 years of education), high school dropout (9 to 10 years of education), high school graduate (11 years of education), college dropout (12 to 14 years of education), college graduate (15 years of education), Masters (18 years of education) and PhD (22.5 years of education). To compute average years of education for school dropouts, we consider the mid-point of the interval of years of education assigned to the category reported in RAIS.

We use the detailed classification of occupations as a measure of a position, which then is used to identify the workers who switch jobs. The Brazilian Classification of Occupations changed in 2002 (CBO-2) and has been reported consistently since 2003. Although the RAIS data are available for earlier years, we restrict the analysis to the post-2003 period in order to ensure that this important variable is defined in a consistent way throughout the period of analysis. CBO-2 aims to portray the reality of the professions of the Brazilian labor market. It was established with a legal basis in Administrative Rule no. 397 of October 10, 2002. There are 2637 unique occupation codes at the 6-digit level during this period. The description of each 6-digit code is available at http://portalfat.mte.gov.br/wp-content/uploads/2016/04/CBO2002_Liv3.pdf.

We use the information on the establishment's location (municipality) and industry, and worker-level data on gender, age, education and December wage. We focus on workers aged 16 to 64 years old. As in Dix-Carneiro and Kovak (2017), we use the "microregion" concept of the Brazilian Statistical Agency (IBGE) to define regional boundaries. This definition groups together economically integrated contiguous municipalities with similar geographic and productive attributes. The documentation supporting this definition is available at https://biblioteca.ibge.gov.br/index.php/biblioteca-catalogo?id=22269&view=detalhes. We consider a set of 558 consistently defined microregions, grouping the 5571 municipalities in the data. To ensure a consistent definition of microregions over time, when necessary we merge microregions whose boundaries changed over the period of analysis.

Customs records: We also use customs data on export transactions by microregion, industry, and destination each year. These customs records are administrative data collected by *Secretaria do Comércio Externo* (SECEX) of the Ministry of Development, Industry and Foreign Trade. These data are available since 1997 and contain information on FOB export values and quantities. They were made available to us by SECEX at the level of the municipality, detailed product category, and destination market. The customs records were originally collected by SECEX at the firm-product-destination level. To aggregate up to the municipality level, SECEX attributed each firm-level export transaction to the municipality where the headquarters of the exporting firm are located. The product classification is *Nomenclatura Comum do MERCOSUL* (NCM), at the 8-digit level. For consistency with RAIS, we restricted the analysis to the post-2003 period, and aggregated up to microregion-sector (i.e., labor market) level. To aggregate exports from the NCM 8-digit level to the 5-digit level of the

CNAE, we used a concordance made available to us by SECEX.

Industry-level imports of Brazil's destinations: To construct an instrument for exports, we further use yearly data on the industry-level imports of each of Brazil's export destinations. To capture changes in sectoral import demand that are plausibly exogenous to microregions in Brazil, we consider the imports of these countries sourced from all countries other than Brazil–i.e., we exclude imports sourced from Brazil from total imports of each country in a given industry-year. There is a total of 189 destinations reported in the customs data, to which we link the information on sectoral import demand from the UN COMTRADE database.

List of countries for the full quantification: Brazil, Australia, Belgium, Canada, China, France, Germany, India, Indonesia, Italy, Japan, Korea, Mexico, Netherlands, Russia, Spain, Sweden, Switzerland, Turkey, United Kingdom, USA, and the constructed rest of the world. (22 countries in total)

A.4 Variable Definitions in the Econometric Analysis

This section describes in detail the variables used in the econometric analysis.

Δ Employment: log change in the number of employees in microregion-sector k between years $t-1$ and t;

Δ Wage: log change in the average wage in the microregion-sector k between years $t-1$ and t. For each year, we purge individual-level wages from the effects of age, gender, and education, and take the average at the microregion-sector level;

Δ # leaving: log change in the number of workers leaving microregion-sector k (i.e., incumbent workers changing microregion or sector) between years $t-1$ and t;

Δ # entering: log change in the number of workers entering microregion-sector pair k between year $t-1$ and t from other microregion or sector (thus excluding new entrants to the formal labor force);

Δ # switching jobs: log change in the number of workers that switch jobs (i.e., switch detailed 6-digit occupations based on CBO-2), while staying in the same microregion-sector between years $t-1$ and t;

Δ Exports: log change in the value of the exports originated in the microregion-sector between years $t-1$ and t;

Δ $\bar{Z}$: log change in the value of import demand directed to the microregion-sector between years $t-1$ and t, as defined in equation (2) in text.

A.5 Solution Method

In this section, we explain in detail the modified dynamic hat algebra technique we proposed to solve the model for quantitative counterfactual exercises in Section 5. We re-write the model in terms of each variable's deviation from its level at the initial steady state. We begin by defining the following operators for any variable x:

$$\begin{aligned}
\ddot{x}_t &\equiv \exp\left(\frac{\beta E_{t-1}x_t - \beta x_0}{\nu}\right), \\
\dot{x}_t &\equiv \exp\left(\frac{x_t - x_0}{\nu}\right), \\
\widehat{x}_t &\equiv \frac{x_t}{x_0},
\end{aligned}$$

where x_t denotes the value of the variable x at period t, and x_0 denotes the value of the variable x at the initial steady state. In other words, x_t is the value of the variable x, t periods after the initial steady state. Note that all three operators we define are different forms of a deviation from the initial steady state. With these operators, we can re-write the labor part of the model, the trade part of the model, and factor market clearing conditions for the general equilibrium.

A.5.1 Workers' Optimization Problem in Changes

A.5.1.1 Flow equations for the baseline steady state

We use the following expression to solve for the labor allocation of the initial steady state as explained in detail in Section A.5.3 below.

$$m_0^{kl} = \frac{\mathbf{1}_{[l=k]}\vartheta_0^l + \mathbf{1}_{[l\neq I]}\vartheta_1^l \exp\left(-\frac{C(k,l)}{\nu}\right) + \mathbf{1}_{[l=I]}\vartheta_I^k}{\vartheta_0^k + \sum_{[l'\neq I]}\vartheta_1^{l'} \exp\left(-\frac{C(k,l')}{\nu}\right) + \vartheta_I^k}, \tag{A1}$$

where we define

$$\begin{aligned}
\vartheta_0^k &\equiv \exp\left(\frac{\beta V_0^k}{\nu}\right), \\
\vartheta_1^k &\equiv \exp\left(\frac{\beta V_0^k}{\nu}\right)\frac{\mu_{1,0}^k}{\mu_{0,0}^k}\widehat{N}_0^k, \\
\vartheta_2^k &\equiv \sum_{l'\neq k}\vartheta_1^{l'} \exp\left(-\frac{C(k,l')}{\nu}\right) \\
\vartheta_I^k &\equiv \exp\left(\frac{\beta V_0^I - C(k,I)}{\nu}\right).
\end{aligned}$$

Note that $\widehat{N}_0^k = 1$ at the initial steady state. The following expression also holds for the origin-specific moving probabilities (but not needed for the solution of the baseline steady state):

$$\mu^k_{x,0} = \frac{\vartheta^k_x}{\vartheta^k_0 + \vartheta^k_1 + \vartheta^k_2 + \vartheta^k_I}, \tag{A2}$$

where $x \in \{0, 1, 2, I\}$.

A.5.1.2 Flow equations for transition

We use the following expression for moving probabilities between labor markets in transition:

$$m^{kl}_t = \frac{\mathbf{1}_{l=k}\ddot{V}^k_{t+1}\left[\mu^k_{0,0} + (m^{kk}_0 - \mu^k_{0,0})\widehat{N^k_t}\right] + \mathbf{1}_{l\neq k}\ddot{V}^l_{t+1}\widehat{N^l_t}m^{kl}_0\left[\dot{C}(k,l)\right]^{-1}}{\ddot{V}^k_{t+1}\left[\mu^k_{0,0} + \left(m^{kk}_0 - \mu^k_{0,0}\right)\widehat{N^k_t}\right] + \sum_{l'\neq k}\ddot{V}^{l'}_{t+1}\widehat{N^{l'}_t}m^{kl'}_0\left[\dot{C}(k,l')\right]^{-1}}, \tag{A3}$$

where the residual sector (I) is one of the choices indexed with l and l' and $\widehat{N^I_t} = 1$ by the assumption of the model. We can also re-write the origin-specific moving probabilities as follows:

$$\mu^k_{0,t} = \frac{\ddot{V}^k_{t+1}\mu^k_{0,0}}{\ddot{V}^k_{t+1}\left[\mu^k_{0,0} + \left(m^{kk}_0 - \mu^k_{0,0}\right)\widehat{N^k_t}\right] + \sum_{l'\neq k}\ddot{V}^{l'}_{t+1}\widehat{N^{l'}_t}m^{kl'}_0\left[\dot{C}(k,l')\right]^{-1}}. \tag{A4}$$

A.5.1.3 Bellman equation

Define $u^k_t = w^k_t$ as the instantaneous utility in k at t as a function of wage. Then, the Bellman equation can be re-written as:

$$\log \ddot{V}^k_t = \frac{\beta}{\nu}u^k_0(\widehat{u^k_t} - 1) + \beta \log \ddot{V}^k_{t+1} - \beta \log \widehat{\mu}^k_{0,t}. \tag{A5}$$

Note that if $u^k_t = \log w^k_t$ the expression above simplifies slightly and becomes equivalent to the dynamic hat algebra from CDP.

A.5.1.4 Labor allocation

In transition, the following relationship must hold with Equation (A3) at every t and k.

$$L^k_{t+1} = \sum_l m^{lk}_t L^l_t. \tag{A6}$$

A.5.1.5 The number of sampled jobs

The step function we described in the paper leads to

$$\widehat{N^k_t} = \widehat{T^k_t}, \tag{A7}$$

at the limit as sampling ratio $\xi \to \infty$. We formally prove this result in Section A.6. The change in the mass of tasks $\widehat{T_t^k}$ is defined from the trade part of the model and derived in the next subsection.

A.5.2 Trade and Market Clearing in Changes

Since the trade part of the model solves for the period-by-period equilibrium, the dynamic hat algebra is much simpler with only the operator $\widehat{x}$, compared to the labor part of the model.

A.5.2.1 Unit cost

For given changes in nominal wages $\widehat{\widetilde{w}}$, changes in aggregate price indices, and changes in the price of the fixed factor in each labor market of Brazil, the market-level unit cost for country 1 and the country-level unit cost for all the other countries $n' \neq 1$ can be derived as follows:

$$\begin{aligned} \widehat{c}^k_{1,t} &= (\widehat{\widetilde{w}}^k_{1,t})^{\gamma_l}(\widehat{P}_{1,t})^{\gamma_m}(\widehat{b}^k_{1,t})^{\gamma_b}, && \text{(A8)} \\ \widehat{c}_{n',t} &= (\widehat{\widetilde{w}}_{n',t})^{\bar{\gamma}_{n',l}}(\widehat{P}_{n',t})^{1-\bar{\gamma}_{n',l}}, && \text{(A9)} \end{aligned}$$

by assuming that the Cobb-Douglas cost shares and the marginal cost of offering tasks are both time-invariant. As will be explained in detail in the next subsection, changes in factor prices and price indices at the initial iteration are random guesses, while they are the informed guesses afterward.

A.5.2.2 Price indices and the number of job options

We solve for $\widehat{P}^s_{n,t}$ from the following system of equations. We have NS changes in prices to solve for in each period, and the following two equations yield the same number of equations. Alvarez and Lucas (2007) show that this system of equations in each period is a contraction mapping and that there exists a unique solution for the system from the fixed point theorem. We assume that the trade elasticity θ is time-invariant. For country 1,

$$\widehat{P}^s_{1,t} = \left[\sum_{r''} \lambda^s_{(1,r''),(1,r),0} \hat{T}^{(r'',s)}_{1,t} (\hat{c}^{(r'',s)}_{1,t})^{-\theta} + \sum_{n' \neq 1} \lambda^s_{n',(1,r),0} \hat{T}^s_{n',t} (\hat{c}_{n',t} \hat{d}^s_{n'1,t})^{-\theta}\right]^{-\frac{1}{\theta}}, \tag{A10}$$

which should hold for any r, because both $\lambda^s_{(1,r''),(1,r),0}$ and $\lambda^s_{n',(1,r),0}$ are equalized across r, repsectively. For country $n \neq 1$,

$$\widehat{P}^s_{n,t} = \left[\sum_{r''} \lambda^s_{(1,r''),n,0} \widehat{A}^{(r'',s)}_{1,t} (\widehat{c}^{(r'',s)}_{1,t} \widehat{d}^s_{1n,t})^{-\theta} + \sum_{n' \neq 1} \lambda^s_{n',n,0} \widehat{A}^s_{n',t} (\widehat{c}_{n',t} \widehat{d}^s_{n'n,t})^{-\theta}\right]^{-\frac{1}{\theta}}. \tag{A11}$$

These sectoral price indices are then used to compute changes in aggregate price indices. We have

$\widehat{P}_{n,t} = \prod_s \widehat{P}^{s\ \phi^s}_{n,t}$, assuming that the Cobb-Douglas expenditure share ϕ^s in preference is fixed over time.

Once we solve for sectoral and aggregate price indices, we can compute changes in the number of tasks offered by producers from the cost minimization problem and the Cobb-Douglas production function:

$$\widehat{T}^k_{1,t} = \left[\left(\widehat{L}^k_{1,t} \right)^{\gamma_l} \frac{\widehat{P}^{s_k}_{1,t}}{\left(\widehat{P}_{1,t} \right)^{\gamma_m}} \right]^{\frac{\tilde{\sigma}-1}{(\tilde{\sigma}-1)(1-\gamma_m)-\gamma_l}}. \tag{A12}$$

We assume that the elasticity of substitution between tasks, $\tilde{\sigma}$, and the cost share parameters of the Cobb-Douglas production function are both time-invariant. From the trade part's perspective, changes in the labor allocation are given from the labor part of the model. As discussed previously, $\widehat{N}^k_t = \widehat{T}^k_{1,t}$ as the sampling rate goes to infinity.

Note that the unit cost is used to solve for price indices. After solving for price indices, we calculate the unit cost again using (A8) and (A9). Note that the trade flows we need are only for the initial year $t = 0$. Trade flows after the initial period are solved from the model as below.

A.5.2.3 Trade flows

Since the bilateral trade shares are derived in multiplicative forms, the derivation of the trade shares in changes is straightforward. For $n \neq 1$ and $n'' \neq 1$,

$$\widehat{\lambda}^s_{(1,r),(1,r'),t} = \widehat{A}^{(r,s)}_{1,t} \left(\frac{\widehat{c}^{(r,s)}_{1,t}}{\widehat{P}^s_{1,t}} \right)^{-\theta} \tag{A13}$$

$$\widehat{\lambda}^s_{n,(1,r),t} = \widehat{A}^s_{n,t} \left(\frac{\widehat{c}_{n,t} \widehat{d}^s_{n1,t}}{\widehat{P}^s_{1,t}} \right)^{-\theta} \tag{A14}$$

$$\widehat{\lambda}^s_{(1,r),n,t} = \widehat{A}^{(r,s)}_{1,t} \left(\frac{\widehat{c}^{(r,s)}_{1,t} \widehat{d}^s_{1n,t}}{\widehat{P}^s_{n,t}} \right)^{-\theta} \tag{A15}$$

$$\widehat{\lambda}^s_{n,n'',t} = \widehat{A}^s_{n,t} \left(\frac{\widehat{c}_{n,t} \widehat{d}^s_{nn'',t}}{\widehat{P}^s_{n'',t}} \right)^{-\theta} \tag{A16}$$

Note that any region r' has the same $\hat{\lambda}^s_{(1,r),(1,r'),t}$ and that any region r has the same $\hat{\lambda}^s_{n,(1,r),t}$ due to the assumption of the identical preference and no domestic trade cost.

A.5.2.4 Expenditures

Expenditures can be solved from the following system of equations. We have $S(R+\bar{N}-1)$ expenditures and the same number of equations in each t. In other words, we need to solve for S expenditures for each of the R regions in country 1 and S expenditures for each of the remaining $\bar{N}-1$ countries. Since the closed form of the change in the expenditure equation is complicated, it is more convenient to solve for $X^s_{(1,r),t}$ and $X^s_{n,t}$ instead of $\widehat{X}^s_{(1,r),t}$ and $\widehat{X}^s_{n,t}$, using the relationship of $x_t = \widehat{x}_t x_0$ from the definition of the hat operator.

For region r of country 1,

$$\begin{aligned} X^s_{(1,r),t} &= \phi^s \gamma_m \sum_{s'} \left(\sum_{r'} \widehat{\lambda}^{s'}_{(1,r),(1,r'),t} \lambda^{s'}_{(1,r),(1,r'),0} X^{s'}_{(1,r'),t} + \sum_{n' \neq 1} \widehat{\lambda}^{s'}_{(1,r),n',t} \lambda^{s'}_{(1,r),n',0} X^{s'}_{n',t} \right) \\ &\quad + \phi^s \left(\sum_{k \in \{k | R_k = r\}} \left(\widehat{\widetilde{w}}^k_{1,t} \widehat{L}^k_{1,t} \widetilde{w}^k_{1,0} L^k_{1,0} + \bar{D}^k_{1,t} \right) \right), \end{aligned} \tag{A17}$$

and for country $n \neq 1$,

$$\begin{aligned} X^s_{n,t} &= \phi^s (1 - \bar{\gamma}_{n,l}) \sum_{s'} \left(\sum_{r'} \widehat{\lambda}^{s'}_{n,(1,r'),t} \lambda^{s'}_{n,(1,r'),0} X^{s'}_{(1,r'),t} + \sum_{n' \neq 1} \widehat{\lambda}^{s'}_{n,n',t} \lambda^{s'}_{n,n',0} X^{s'}_{n',t} \right) \\ &\quad + \phi^s \left(\widehat{\widetilde{w}}_{n,t} \widehat{\bar{L}}_{n,t} \widetilde{w}_{n,0} \bar{L}_{n,0} + \bar{D}_{n,t} \right). \end{aligned} \tag{A18}$$

We assume that the cost share parameter, γ's, in the Cobb-Douglas production function and the sectoral expenditure share parameter, ϕ's, in the Cobb-Douglas utility function are both time-invariant. $\bar{D}^k_{1,t}$ denotes the share of country 1's deficit allocated to labor market k at period t, and $\bar{D}_{n,t}$ denotes country n's total deficit. Following Caliendo and Parro (2015), we assume that each country's aggregate deficit is exogenously fixed as the share of the total world GDP. We further assume that country 1's aggregate deficit is allocated across labor markets, proportionally to each market's labor income.

A.5.2.5 Market clearing

We have the following set of factor market clearing conditions for each period t.

$$\widehat{\widetilde{w}}^k_{1,t} \widehat{L}^k_{1,t} \widetilde{w}^k_{1,0} L^k_{1,0} = \gamma_l \left(\sum_{r'} \widehat{\lambda}^{s_k}_{(1,r_k),(1,r'),t} \lambda^{s_k}_{(1,r_k),(1,r'),0} X^{s_k}_{(1,r'),t} + \sum_{n' \neq 1} \widehat{\lambda}^{s_k}_{(1,r_k),n',t} \lambda^{s_k}_{(1,r_k),n',0} X^{s_k}_{n',t} \right) \tag{A19}$$

$$\widehat{b}^k_{1,t} b^k_{1,0} B^k_1 = (1 - \gamma_l - \gamma_m) \left(\sum_{r'} \widehat{\lambda}^{s_k}_{(1,r_k),(1,r'),t} \lambda^{s_k}_{(1,r_k),(1,r'),0} X^{s_k}_{(1,r'),t} + \sum_{n' \neq 1} \widehat{\lambda}^{s_k}_{(1,r_k),n',t} \lambda^{s_k}_{(1,r_k),n',0} X^{s_k}_{n',t} \right) \tag{A20}$$

$$\widehat{\widetilde{w}}_{n,t} \hat{L}_{n,t} \widetilde{w}_{n,0} L_{n,0} = \bar{\gamma}_{n,l} \sum_{s'} \left(\sum_{r'} \widehat{\lambda}^{s'}_{n,(1,r'),t} \lambda^{s'}_{n,(1,r'),0} X^{s'}_{(1,r'),t} + \sum_{n' \neq 1} \widehat{\lambda}^{s'}_{n,n',t} \lambda^{s'}_{n,n',0} X^{s'}_{n',t} \right). \tag{A21}$$

A.5.3 Baseline Steady State

Our goal here is to calculate m_0^{kl} that will give us the baseline steady state consistent with the estimated parameters of the model. If we have an infinite number of workers, we could directly take m_0^{kl} from data by dividing the number of movers by the number of workers initially in the origin market, i.e., the bin-estimator. In the data which are a collection of sample workers, we do not observe the true m_0^{kl}, but a realization of that from a finite sample. As discussed in the main paper, using the finite sample realization of this moving probability is problematic because of the large number of choices we have.

The calculated m_0^{kl} will feed into the transition solution as the initial steady state. We take L_0^k, $\mu_{0,0}^k$, $\mu_{1,0}^k$ and $\mu_{I,0}^k$ from data.[55] Assume that $\Theta = \{\beta, C, \nu\}$ is the set of parameters. We set $\widehat{N_0^k} = 1$ and $\widehat{w_0^k} = 1$ for the steady state. We follow these steps below to solve for the initial steady state.

- Using (A1) and (A6), solve for V_0^k that gives L_0^k subject to Θ.
- We calculate m_0^{kl} using (A1) if k is a formal sector. We take m_0^{Il} from data when the origin is the residual sector, subject to a simple re-weighting to make sure that inflows and outflows from the residual sector are equal at the steady state. Otherwise population growth breaks the balance in the data.
- Plug V_0^k into (A1) and (A2) to recover m_0^{kl} to use in the transition solution.

A.5.4 Transition

Again, we take L_0^k and w_0^k from data, where the latter is the market-level average of the individual worker's wages purged from their demographic characteristics as explained before. We use m_0^{kl} and $\mu_{1,0}^k$ from the steady state solution. $\widehat{\mathbf{N}}$ is a vector of $\widehat{N_t^k}$'s, $\mathbf{L}$ is a vector of L_t^k, $\mathbf{m}$ is a vector of m_t^{kl}, μ is a vector of μ_t^k, $\mathbf{w}$ is a vector of w_t^k, $\mathbf{b}$ is a vector of b_t^k, and $\ddot{\mathbf{V}}$ is a vector of $\ddot{V}_t^k$ for $t \in \{1, 2, 3, .., T\}$, and all possible combinations of k and l as defined in the main text of the paper. For example, $\widehat{\mathbf{N}} = \{\widehat{N_1^1}, \widehat{N_2^1}, ..., \widehat{N_T^1}, \widehat{N_1^2}, ..., \widehat{N_T^K}\}$.

The following steps describe how we solve for the general equilibrium in transition.

1. Consider an initial guess for $\widehat{\mathbf{w}}$, $\widehat{\mathbf{b}}$, $\mathbf{L}$, $\ddot{\mathbf{V}}$ and $\widehat{\mathbf{N}}$.
2. Solve for $\widehat{\mathbf{P}}$ using (A10) and (A11) with a separate loop that iterates over $\widehat{\mathbf{P}}$.
3. Compute $\widehat{\mathbf{N}}$ using (A12) and (A7).
4. Compute changes in real wages with the guess of $\widehat{\mathbf{w}}$ and the solved $\widehat{\mathbf{P}}$ from Step 2.

[55] As explained in the main paper as well, using the simple bin-estimators for these origin-specific moving probabilities is less problematic because there are only four possible choices.

5. Calculate implied flows, $\mathbf{m}$ and μ using (A3) and (A4).

6. Calculate implied $\ddot{\mathbf{V}}$ via (A5) using $\widehat{\mathbf{w}}$ and $\widehat{\mathbf{N}}$.

7. Calculate implied $\mathbf{L}$ using (A6) with $\mathbf{m}$ from previous step.

8. Update $\widehat{\mathbf{w}}$ and $\widehat{\mathbf{b}}$ to minimize the excess demand of factors which can be calculated by (A19), (A20), and (A21). If the there is an excess factor demand, the algorithm increases the factor prices, otherwise reduces them.

9. Update $\widehat{\mathbf{P}}$ using the updated $\widehat{\mathbf{w}}$ by solving (A10) and (A11) inside a separate loop.

10. Update the guesses of $\mathbf{L}$ and $\ddot{\mathbf{V}}$ using implied numbers.

11. Go to Step 3 and continue iterations until all excess factor demands are sufficiently close to zero.

A.6 Sampling Rate and Asymptotics

In this section, we analytically prove that $\widehat{N_t^k} = \widehat{T_t^k}$ holds at the limit as the sampling rate $\xi \to \infty$, where $N_t^k \equiv \frac{\widetilde{N}_t^k}{\xi \exp(\widetilde{\delta}/\nu)}$ as defined in the main part of the paper.

Consider the number of sampled jobs $\widetilde{N}_t^k = \varpi(T_t^k)$, where $\varpi(.)$ is a step function $\varpi : \mathbb{R}^+ \to \mathbb{N}^+$ such that

$$\xi T_t^k < \varpi(T_t^k) \leq \xi T_t^k + 1. \tag{A22}$$

Then, the following two hold at the limit:

$$\begin{aligned}
\lim_{\xi\to\infty} \frac{\xi T_t^k}{\xi \exp(\widetilde{\delta}/\nu)} &= \frac{T_t^k}{\exp(\widetilde{\delta}/\nu)}, \\
\lim_{\xi\to\infty} \frac{\xi T_t^k + 1}{\xi \exp(\widetilde{\delta}/\nu)} &= \frac{T_t^k}{\exp(\widetilde{\delta}/\nu)}.
\end{aligned}$$

Therefore,

$$\lim_{\xi\to\infty} N_t^k = \frac{T_t^k}{\exp(\widetilde{\delta}/\nu)}, \tag{A23}$$

which implies

$$\lim_{\xi\to\infty} \widehat{N}_t^k = \widehat{T}_t^k. \tag{A24}$$

Note that $\nu > 0$, $\widetilde{\delta} > 0$, and $T_t^k > 0$ are finite real numbers. Therefore, N_t^k is also positive and finite.

A.7 Auxiliary Model

In Section 6 of the paper, we show the importance of our job options channel in matching patterns in aggregate data. In this section, we elaborate on the *auxiliary model* we defined earlier to study the implications of ignoring the job options channel in labor mobility models. The auxiliary model illustrates the differences between our model and standard discrete choice models with the exogenously fixed number of choices, because it is isomorphic to our baseline model but the equations are similar to standard discrete choice models with bilateral moving costs except for two key differences: (i) each choice includes an endogenous utility shifter based on the number of job options; and (ii) the implied moving costs change endogenously with the number of job options. In this section, we first provide the basic setup of the auxiliary model and show that it is isomorphic to our baseline model. We then extend the basic auxiliary model to allow for arbitrary choice sets.

A.7.1 Basic Setup of the Auxiliary Model

Consider the following dynamic labor mobility model with a constant number of choices. The economy has L agents, and each agent is attached to a region-sector labor market k, where $k \in \{1, 2, .., K\}$. The number of agents in labor market k is denoted as L_t^k. An agent h attached to labor market k will receive instantaneous utility $\bar{u}_t^h$ at time t defined as

$$\bar{u}_t^h = w_t^k + \log\left(1 + N_t^k\right), \tag{A25}$$

where N_t^k is as defined in the main model section. Note that the second term in (A25) corresponds to the difference (i) mentioned in the beginning of this section.

For $k \neq l$, workers pay moving cost

$$\bar{C}_t(k, l) = C_t(k, l) + \log\left(1 + N_t^l\right) - \log\left(N_t^k\right), \tag{A26}$$

where $\bar{C}_t(k, l)$ is the implied moving cost with $\bar{C}_t(k, k) = 0$, and $C_t(k, l)$ is the structural moving cost parameter of our baseline model herein. This equation describes the difference (ii) mentioned above. The expected value variables, V_t^k, are equal (i.e. identical values) in both main and auxiliary models. The auxiliary model above is isomorphic to the model described in Section 3 and produce identical mobility results, as long as one accurately takes N_t^k into account. We are omitting the formal proof here, but it is a straightforward algebra to derive moving cost and welfare equations using this auxiliary model and show that they are identical to those of the baseline model. Formal proofs are available upon request.

A.7.2 Extending the Auxiliary Model to Allow for Arbitrary Choice Sets

It is possible to present alternative auxiliary models that are isomorphic to the baseline model, with different and arbitrary choice sets. Assume that Θ_k is the set of all jobs in the labor market k. We can split Θ_k into arbitrary choice sets, and the new auxiliary model would still be isomorphic to the main baseline model.

Without loss of generality, we omit the time index just for now. We will re-introduce it shortly. Let us index the new choice sets with ς. The new choice set Θ_ς satisfies $\Theta_\varsigma \subset \Theta_{k_\varsigma}$, where k_ς denotes the labor market of jobs in Θ_ς. Also, $\Theta_\varsigma \bigcap \Theta_{\varsigma'} = \emptyset$, if $\varsigma \neq \varsigma'$. For these sets to be well-defined, we assume

$$\bigcup_{\varsigma | l = k_\varsigma} \Theta_\varsigma = \Theta_l,$$

for every labor market l. In other words, every labor market has a collection of arbitrary choice sets for job options, the union of which covers the set of all job options in the labor market. Let N_ς denote the measure of job options in Θ_ς. From the definition of Θ_ς, the following is straightforward:

$$\sum_{\varsigma | l = k_\varsigma} N_\varsigma = N_l, \tag{A27}$$

for every labor market l.

We plug the time index back to the relevant variables from here. An agent h, attached to a job in Θ_ς of labor market k_ς, receives instantaneous utility $\bar{u}_t^h$ at time t defined as

$$\bar{u}_t^h = w_t^{k_\varsigma} + \log\left(1 + N_t^\varsigma\right). \tag{A28}$$

If a worker moves from a job in Θ_ς from another job in $\Theta_{\varsigma'}$, she has to pay the moving cost

$$\bar{C}_t(\varsigma, \varsigma') = C_t(k_\varsigma, k_{\varsigma'}) + \log\left(1 + N_t^\varsigma\right) - \log\left(N_t^{\varsigma'}\right), \tag{A29}$$

where $\bar{C}_t(\varsigma, \varsigma')$ is the implied moving cost with $\bar{C}_t(\varsigma, \varsigma) = 0$. The expected value variables are again equal (i.e. identical values) between the baseline model and this extended auxiliary model. This extended auxiliary model is also isomorphic to the model described in Section 3, and will produce identical mobility results as long as one calculates N_t^ς correctly. While alternative definitions and arbitrary grouping of labor markets can affect the estimated moving cost variables and wages empirically, theoretically our model is invariant to the arbitrary grouping of jobs within a labor market. The formal proof for the isomorphism between the baseline model and this extended auxiliary model is again a straightforward algebra, which is available upon request.

A.8 Proofs of Propositions

We provide the proofs of the three propositions presented in Section 6. Note that a stylized "positive aggregate shock" z, is defined as a shock that increases the number of job options, N_t^k, by $\varrho^k > 0$ for all k. Precisely, $\varrho^k \equiv \frac{\partial}{\partial z} N_t^k$. We also assumed $\mu_{0,t}^k > 0, \forall k$; and that for a given k, there exists l such that $m_t^{k,l} > 0$. Lastly, the underlying moving cost structure does not respond to this shock, and the shock is realized after workers receive their wage for the period.

A.8.1 Proof of Proposition 1

Let us define $\Xi_t^k \equiv \left(\vartheta_{0,t}^k + \vartheta_{1,t}^k + \vartheta_{2,t}^k + \vartheta_{3,t}^k\right)^{-1}$. We can omit the multiplicative term $1/K$ since it does not affect the sign of the derivative. Similarly, we also omit L_t^k terms for simplicity, since this variable denotes labor allocation in the beginning of t before the shock is realized. For the arithmetic mean,

$$
\begin{aligned}
\frac{\partial}{\partial z} \sum_k \frac{\mu_{1,t}^k}{\mu_{0,t}^k} &= \frac{\partial}{\partial z} \left(\sum_k \frac{N_t^k \exp(\beta E_t V_{t+1}^k/\nu)\Xi^k}{\exp(\beta E_t V_{t+1}^k/\nu)\Xi^k} \right) > 0, \\
&= \frac{\partial}{\partial z} \left(\sum_k N_t^k \right) > 0,
\end{aligned}
$$

since $\frac{\partial}{\partial z} N_t^k = \varrho^k > 0$, $\forall k$ by definition.

For the geometric mean, we can omit the power term $1/K$ and L_t^k terms and follow the same steps

$$
\frac{\partial}{\partial z} \prod_k \frac{\mu_{1,t}^k}{\mu_{0,t}^k} = \frac{\partial}{\partial z} \left(\prod_k N_t^k \right) > 0,
$$

since $\frac{\partial}{\partial z} N_t^k = \varrho^k > 0$, $\forall k$ as above. Note that this proposition also directly follows from the equation for the number of job options in equation (15). □

A.8.2 Proof of Proposition 2

Let us again define $\Xi_t^k \equiv \left(\vartheta_{0,t}^k + \vartheta_{1,t}^k + \vartheta_{2,t}^k + \vartheta_{3,t}^k\right)^{-1}$. For (i), we can omit the power term $1/K(K-1)$ since it does not affect the sign of the derivative. Similarly, we also omit L_t^k terms for simplicity for the same reason discussed in the previous proof.

$$
\begin{aligned}
\frac{\partial}{\partial z} \prod_k \prod_{l \neq k} \frac{m_t^{k,l}}{\mu_{0,t}^k} &= \frac{\partial}{\partial z} \prod_k \prod_{l \neq k} \frac{N_t^l \exp\left(\beta E_t V_{t+1}^l/\nu\right) \exp\left(-C^{k,l}/\nu\right) \Xi^k}{\exp\left(\beta E_t V_{t+1}^k/\nu\right) \Xi^k}, \\
&= \frac{\partial}{\partial z} \prod_k \prod_{l \neq k} \frac{N_t^l \exp\left(\beta E_t V_{t+1}^l/\nu\right) \exp\left(-C^{k,l}/\nu\right)}{\exp\left(\beta E_t V_{t+1}^k/\nu\right)}, \\
&= \frac{\partial}{\partial z} \left(\prod_k N_t^k \right)^{K-1} \left(\prod_k \prod_{l \neq k} \exp(-C^{k,l}/\nu) \right) > 0,
\end{aligned}
$$

since $\prod_k \prod_{l \neq k} \exp(-C^{k,l}/\nu) > 0$ and does not respond to z, $N_t^k > 0$, $\forall k$, and $\frac{\partial}{\partial z} N_t^k = \varrho^k > 0$, $\forall k$ by the definition of z.

The part (ii) follows directly from the proofs of Proposition 1-(ii) and Proposition 2-(i). We can again omit the power terms and the L_t^k terms. The derivative of the term in the first parenthesis of the bracket is positive from Proposition 1-(ii), and that of the term in the second parenthesis of the bracket is positive from Proposition 2-(i). Each term itself is also positive from the assumption. Therefore, the derivative of the product is also positive. □

A.8.3 Proof of Proposition 3

The proof of Proposition 3 follows from the definition of the moving cost of the auxiliary model and the assumption that the shock z does not alter the underlying structural moving cost of our baseline model.

$$
\begin{aligned}
\frac{\partial}{\partial z} \frac{1}{K(K-1)} \sum_k \sum_{l \neq k} \left[\bar{C}_t(k,l) - C_t(k,l)\right] &= \frac{1}{K(K-1)} \sum_k \sum_{l \neq k} \frac{\partial}{\partial z} \left[\log\left(N_t^l + 1\right) - \log N_t^k\right], \\
&= \frac{1}{K} \sum_k \frac{\partial}{\partial z} \log\left(\frac{N_t^k + 1}{N_t^k}\right), \\
&= \frac{1}{K} \sum_k \frac{N_t^k}{N_t^k + 1} \frac{\partial}{\partial z} \left(1 + \frac{1}{N_t^k}\right), \\
&= \frac{1}{K} \sum_k \frac{N_t^k}{N_t^k + 1} \left(0 - \frac{1}{\left(N_t^k\right)^2} \varrho^k\right) < 0.
\end{aligned}
$$

□

A.9 Estimation of Switching Probabilities, Values, and Moving Costs

Our main objective for the empirical analysis is to estimate the bilateral moving probabilities between labor markets conditional on switching jobs and staying in a formal labor market, $\widetilde{m}_t^{kl}$. For each year, we have 804×804 of these probabilities. We implement the estimation, by putting a simple structure based on the model, as in equation (10). The regression equation has $804 + 3 - 1 = 806$ parameters to pin down. The parameters are 804 destination fixed effects plus 3 moving costs parameters in equation (32), minus the reference labor market fixed effect which can be set to zero. Therefore, we also recover the moving cost parameters in the process of estimating moving probabilities. In the estimation, we use the number of people who move between labor markets, conditional on changing jobs and staying in a formal labor market, i.e. $\widetilde{y}_t^{kl}$. Each person who switches jobs is a realization of this probability, thus an observation. The realized error is only due to sampling.

We use PPML rather than MLE for practicality because, for our model, PPML and MLE are identical. Here we show that the PPML orthogonality conditions are equivalent to the MLE first-order conditions for the estimation of values and moving costs of our model (subject to normalization with ν). Instead of using the moving cost in equation (32) which has three parameters, we use a slightly simpler moving cost structure as below for proof in order to make the exposition simpler. The logic we follow for the proof directly applies to the moving cost with three parameters as well. We also omit the time subscripts for the proof without loss of generality.

We denote the number of workers moving from labor market k to labor market l (and thus changing jobs) but staying in a formal labor market with $\widetilde{y}^{kl}$; the expected value of labor market k (i.e., the destination fixed effect) with $\widetilde{V}^k$; the origin fixed effect with $\widetilde{\Gamma}^k$; and the moving cost for the move from labor market k to labor market l with $\widetilde{C}(k,l)$. As mentioned above, we consider the following moving cost structure for the proof without loss of generality:

$$\widetilde{C}(k,l) = \widetilde{C_1} D^{kl} + \widetilde{C_2} \mathbf{1}_{s_k \neq s_k}, \tag{A30}$$

where $\widetilde{C_1}$ is the distance coefficient (divided by ν); D^{kl} is the log of distance between k and l; $\widetilde{C_2}$ is the sector switching cost (divided by ν); and $\mathbf{1}_{s_l \neq s_k}$ is an indicator function that is equal to one if l and k are associated with different sectors. Note that we impose $D^{ll} = 0$ for every l.

A.9.1 Maximum Likelihood Estimation (First-Order Conditions)

Note that for the proof, all summations and products are taken over formal labor markets. As described in Section 4.1 of the paper, the likelihood function of our case is

$$\mathcal{L} = \prod_k \prod_l \left(\widetilde{m}^{kl}\right)^{\widetilde{y}^{kl}}, \tag{A31}$$

or alternatively, by using logarithm,

$$\log \mathcal{L} = \sum_k \sum_l \widetilde{y}^{kl} \log(\widetilde{m}^{kl}). \tag{A32}$$

Note that the moving probability $\widetilde{m}^{kl}$ can be expressed as

$$\begin{aligned}
\widetilde{m}^{kl} &= \frac{\exp\left(\widetilde{V}^k - \widetilde{C_1} D^{kl} - \widetilde{C_2} \mathbf{1}_{s_k \neq s_l}\right)}{\sum_{l'} \exp\left(\widetilde{V}^{l'} - \widetilde{C_1} D^{kl'} - \widetilde{C_2} \mathbf{1}_{s_k \neq s_{l'}}\right)}, \\
&= \exp\left(\widetilde{\Gamma}^k + \widetilde{V}^l - \widetilde{C_1} D^{kl} - \widetilde{C_2} \mathbf{1}_{s_k \neq s_l} - \log(\widetilde{Y}^k)\right),
\end{aligned}$$

where

$$\widetilde{\Gamma}^k \equiv -\log\left[\sum_{l'} \exp\left(\widetilde{V}^{l'} - \widetilde{C_1} D^{kl'} - \widetilde{C_2} \mathbf{1}_{s_k \neq s_{l'}}\right)\right] + \log(\widetilde{Y}^k) \tag{A33}$$

and $\widetilde{Y}^k = \sum_l \widetilde{y}^{kl}$. The log-likelihood function in equation (A32) then can be written as

$$\log \mathcal{L} = \sum_k \sum_l \widetilde{y}^{kl} \left[\widetilde{\Gamma}^k + \widetilde{V}^l - \widetilde{C_1} D^{kl} - \widetilde{C_2} \mathbf{1}_{s_k \neq s_l} - \log(\widetilde{Y}^k) \right].$$

The goal is to find $\widetilde{V}^l$, $\widetilde{C_1}$, and $\widetilde{C_2}$ coefficients that maximize the log likelihood function. Note that we will use the following partial derivatives

$$\begin{aligned}
\partial\widetilde{\Gamma}^k / \partial\widetilde{V}^l &= -\widetilde{m}^{kl} \\
\partial\widetilde{\Gamma}^k / \partial\widetilde{C_1} &= \sum_{l \neq k} D^{kl} \widetilde{m}^{kl} \\
\partial\widetilde{\Gamma}^k / \partial\widetilde{C_2} &= \sum_{l \neq k} \mathbf{1}_{s_k \neq s_l} \widetilde{m}^{kl}
\end{aligned}$$

and the previously derived $\widetilde{m}^{kl} = \exp\left(\widetilde{\Gamma}^k + \widetilde{V}^l - \widetilde{C_1} D^{kl} - \widetilde{C_2} \mathbf{1}_{s_k \neq s_l} - \log(\widetilde{Y}^k) \right)$ in the proofs below.

A. The first-order condition with respect to values:

We take the derivative of the log likelihood function with respect to $\widetilde{V}^l$ to find the first order condition as follows:

$$\frac{d \log \mathcal{L}}{d\widetilde{V}^l} = \frac{\partial \log \mathcal{L}}{\partial \widetilde{V}^l} + \sum_k \frac{\partial \log \mathcal{L}}{\partial \widetilde{\Gamma}^k} \frac{\partial \widetilde{\Gamma}^k}{\partial \widetilde{V}^l} = 0. \tag{A34}$$

Note that we first use the notation for derivatives, not for partial derivatives, because $\widetilde{\Gamma}^k$ is not the parameter we directly estimate but a function which is fully recovered by the other parameters and observations. By rearranging terms of this first-order condition,

$$\begin{aligned}
0 &= \frac{\partial \log \mathcal{L}}{\partial \widetilde{V}^l} - \sum_k \frac{\partial \log \mathcal{L}}{\partial \widetilde{\Gamma}^k} \widetilde{m}^{kl} \\
&= \sum_k \widetilde{y}^{kl} - \sum_k \left(\sum_l \widetilde{y}^{kl} \right) \widetilde{m}^{kl} \\
&= \sum_k \widetilde{y}^{kl} - \sum_k \widetilde{Y}^k \widetilde{m}^{kl} \\
&= \sum_k \widetilde{y}^{kl} - \sum_k \exp\left[\widetilde{\Gamma}^k + \widetilde{V}^l - \widetilde{C_1} D^{kl} - \widetilde{C_2} \mathbf{1}_{s_k \neq s_l} \right] \\
&= \sum_k \left(\widetilde{y}^{kl} - \exp\left[\widetilde{\Gamma}^k + \widetilde{V}^l - \widetilde{C_1} D^{kl} - \widetilde{C_2} \mathbf{1}_{s_k \neq s_l} \right] \right).
\end{aligned}$$

Thus, the first-order condition associated with values is

$$\sum_k \left(\widetilde{y}^{kl} - \exp\left[\widetilde{\Gamma}^k + \widetilde{V}^l - \widetilde{C_1} D^{kl} - \widetilde{C_2} \mathbf{1}_{I_k \neq I_k} \right] \right) = 0, \tag{A35}$$

for every $l = 1, \ldots, K$.

B. The first-order condition with respect to the distance coefficient:

Next, we take the derivative of the log likelihood function with respect to the distance coefficient $\widetilde{C_1}$ as follows:

$$\frac{d \log \mathcal{L}}{d\widetilde{C_1}} = \frac{\partial \log \mathcal{L}}{\partial \widetilde{C_1}} + \sum_k \frac{\partial \log \mathcal{L}}{\partial \widetilde{\Gamma}^k} \frac{\partial \widetilde{\Gamma}^k}{\partial \widetilde{C_1}} = 0, \tag{A36}$$

which can be rearranged as

$$\begin{aligned}
0 &= -\sum_k \sum_{l \neq k} D^{kl} \widetilde{y}^{kl} + \sum_k \left(\sum_l \widetilde{y}^{kl} \right) \sum_{l \neq k} D^{kl} \widetilde{m}^{kl} \\
&= -\sum_k \sum_{l \neq k} D^{kl} \widetilde{y}^{kl} + \sum_k \widetilde{Y}^k \sum_{l \neq k} D^{kl} \widetilde{m}^{kl} \\
&= -\sum_k \sum_{l \neq k} D^{kl} \widetilde{y}^{kl} + \sum_k \sum_{l \neq k} D^{kl} \exp \left[\widetilde{\Gamma}^k + \widetilde{V}^l - \widetilde{C_1} D^{kl} - \widetilde{C_2} \mathbf{1}_{s_k \neq s_l} \right].
\end{aligned}$$

Thus, lastly by reversing the sign, the first order condition associated with $\widetilde{C_1}$ is

$$\sum_k \sum_{l \neq k} D^{kl} \left(\widetilde{y}^{kl} - \exp \left[\widetilde{\Gamma}^k + \widetilde{V}^l - \widetilde{C_1} D^{kl} - \widetilde{C_2} \mathbf{1}_{s_k \neq s_k} \right] \right) = 0. \tag{A37}$$

C. The first-order condition with respect to the sector switching coefficient:

Lastly, we take the derivative of the log likelihood function with respect to the coefficient associated with sector switching, $\widetilde{C_2}$, as follows:

$$\frac{d \log \mathcal{L}}{d\widetilde{C_2}} = \frac{\partial \log \mathcal{L}}{\partial \widetilde{C_2}} + \sum_k \frac{\partial \log \mathcal{L}}{\partial \widetilde{\Gamma}^k} \frac{\partial \widetilde{\Gamma}^k}{\partial \widetilde{C_2}} = 0, \tag{A38}$$

which can be rearranged as

$$\begin{aligned}
0 &= -\sum_k \sum_{l \neq k} \mathbf{1}_{s_k \neq s_l} \widetilde{y}^{kl} + \sum_k \left(\sum_l \widetilde{y}^{kl} \right) \sum_{l \neq k} \mathbf{1}_{s_k \neq s_l} \widetilde{m}^{kl} \\
&= -\sum_k \sum_{l \neq k} \mathbf{1}_{s_k \neq s_l} \widetilde{y}^{kl} + \sum_k \widetilde{Y}^k \sum_{l \neq k} \mathbf{1}_{s_k \neq s_l} \widetilde{m}^{kl} \\
&= -\sum_k \sum_{l \neq k} \mathbf{1}_{s_k \neq s_l} \widetilde{y}^{kl} + \sum_k \sum_{l \neq k} \mathbf{1}_{s_k \neq s_l} \exp \left[\widetilde{\Gamma}^k + \widetilde{V}^l - \widetilde{C_1} D^{kl} - \widetilde{C_2} \mathbf{1}_{s_k \neq s_l} \right].
\end{aligned}$$

Again, by reversing the sign, the first order condition associated with $\widetilde{C_2}$ is

$$\sum_k \sum_{l \neq k} \mathbf{1}_{s_k \neq s_l} \left(\widetilde{y}^{kl} - \exp \left[\widetilde{\Gamma}^k + \widetilde{V}^l - \widetilde{C_1} D^{kl} - \widetilde{C_2} \mathbf{1}_{s_k \neq s_l} \right] \right) = 0. \tag{A39}$$

A.9.2 PPML (Orthogonality Conditions)

Now we turn to the PPML regression equation. We will show that the orthogonality conditions implied by the PPML regression equation are identical to the ML first-order conditions. PPML can be preferable to ML for the following reason. Since it is straightforward to take analytical derivatives of the orthogonality conditions, PPML is of very low cost computationally. We will prove that PPML and ML estimators are identical for our model. Again, we omit the time subscripts and use the simplified moving cost structure, and summations are all taken over formal labor markets.

The PPML regression equation is

$$\widetilde{y}^{kl} = \exp \left[\widetilde{\Gamma}^k + \widetilde{V}^l - \widetilde{C_1} D^{kl} - \widetilde{C_2} \mathbf{1}_{s_k \neq s_l} \right] + \epsilon^{kl}, \tag{A40}$$

which can be written in matrix form as follows:

$$\widetilde{y} = \exp [\mathbb{X}\Psi] + \epsilon, \tag{A41}$$

where $\widetilde{y}$ is a vector with elements $\widetilde{y}^{kl}$; $\mathbb{X}$ is a matrix of destination and origin dummies as well as switching cost variables (i.e., log distance and the indicator function for sector switching); Ψ is the vector of coefficients.

The orthogonality condition of the PPML regression is

$$0 = \mathbb{X}' \left(\widetilde{y} - \exp [\mathbb{X}\Psi] \right), \tag{A42}$$

which implies a vector of equations. We can now group the rows (i.e. equations) of the orthogonality condition matrix above into four categories:

I. Equations associated with the origin coefficients

$$\sum_l \left(\widetilde{y}^{kl} - \exp \left[\widetilde{\Gamma}^k + \widetilde{V}^l - \widetilde{C_1} D^{kl} - \widetilde{C_2} \mathbf{1}_{s_k \neq s_l} \right] \right) = 0, \forall k, \tag{A43}$$

II. Equations associated with the destination coefficients

$$\sum_k \left(\widetilde{y}^{kl} - \exp \left[\widetilde{\Gamma}^k + \widetilde{V}^l - \widetilde{C_1} D^{kl} - \widetilde{C_2} \mathbf{1}_{s_k \neq s_l} \right] \right) = 0, \forall l, \tag{A44}$$

III. Equation associated with the distance coefficient

$$\sum_k \sum_{l \neq k} D^{kl} \left(\widetilde{y}^{kl} - \exp\left[\widetilde{\Gamma}^k + \widetilde{V}^l - \widetilde{C_1} D^{kl} - \widetilde{C_2} \mathbf{1}_{s_k \neq s_l} \right] \right) = 0, \tag{A45}$$

IV. Equation associated with the sector switching cost coefficient

$$\sum_k \sum_{l \neq k} \mathbf{1}_{s_k \neq s_l} \left(\widetilde{y}^{kl} - \exp\left[\widetilde{\Gamma}^k + \widetilde{V}^l - \widetilde{C_1} D^{kl} - \widetilde{C_2} \mathbf{1}_{s_k \neq s_l} \right] \right) = 0. \tag{A46}$$

Note that equation (A35) is same as (A44); equation (A37) is same as (A45); and equation (A39) is same as (A46). To conclude the proof, we only need to show the first-order condition (A33) of the ML estimation is same as the equation (A43) of the PPML regression.

Consider equation (A43) from above and arrange the terms as follows:

$$\begin{aligned}
0 &= \sum_l \left[\widetilde{y}^{kl} - \exp\left(\widetilde{\Gamma}^k\right) \exp\left(\widetilde{V}^l - \widetilde{C_1} D^{kl} - \widetilde{C_2} \mathbf{1}_{s_k \neq s_l} \right) \right] \\
\sum_l \widetilde{y}^{kl} &= \exp\left(\widetilde{\Gamma}^k\right) \sum_l \exp\left(\widetilde{V}^l - \widetilde{C_1} D^{kl} - \widetilde{C_2} \mathbf{1}_{s_k \neq s_l} \right) \\
\widetilde{Y}^k &= \exp\left(\widetilde{\Gamma}^k\right) \sum_l \exp\left(\widetilde{V}^l - \widetilde{C_1} D^{kl} - \widetilde{C_2} \mathbf{1}_{s_k \neq s_l} \right) \\
\exp\left(\widetilde{\Gamma}^k\right) &= \frac{\widetilde{Y}^k}{\sum_l \exp\left(\widetilde{V}^l - \widetilde{C_1} D^{kl} - \widetilde{C_2} \mathbf{1}_{s_k \neq s_l} \right)}
\end{aligned}$$

Thus, we get

$$\widetilde{\Gamma}^k = \log(\widetilde{Y}^k) - \log\left(\sum_l \exp(\widetilde{V}^l - \widetilde{C_1} D^{kl} - \widetilde{C_2} \mathbf{1}_{s_k \neq s_l}) \right), \tag{A47}$$

which is identical to the first-order condition (A33) in the ML estimation. Therefore, for our model, solving the first-order conditions of the ML estimation is equivalent to solving the orthogonality conditions in PPML.