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JEL Classification: O31, O38

Keywords: skills · technological change · task · complexity · wage inequality

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Who Produces the Robots?*

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Abstract

To assess how disruptive automation and digitization could be, we develop a three-industry model involving routine and non-routine production of consumption goods or services, as well as capital good production. Workers exhibit different skill levels and only high-skilled workers can perform non-routine tasks in production. We compare an industrial economy in which the production of capital goods (machines) requires routine tasks with a future economy, the robotic economy, in which the production of capital goods (robots) requires non-routine tasks. We show that in an industrial economy, technological progress in capital production has an equalizing effect on wages and leads to integrated labor markets, whereas in a robotic economy, it can lead to a disintegration of labor markets, with falling real wages for low-skilled workers and increasing real wages for high-skilled workers.

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1 Introduction

How disruptive is automation for workers? Does it generate unemployment, large wage inequalities and ever-lower wages for low-skilled workers? These issues are at the center of many academic and political debates. The Covid-19 pandemic appears to provide a further push towards automation and thus may sharpen the debate further.

Several studies appear to confirm the threatening potential of automation. Frey and Osborne (2017) estimate that 47% of all jobs are threatened by computerization. A report of the McKinsey Global Institute (2017) estimates the worldwide potential for automation at the *current* technological level to be equivalent to 1.1 billion employees or $15.8 trillion in wages. For the United States, the estimate is 60 million employees or $2.7 trillion in wages. The magnitude of these numbers calls for a thorough investigation of the possible effects of technological change. Of course, such studies cannot predict what will happen. Many new jobs will be created, while technology will further advance and increase the automation potential.[1] In the past, job creation mostly equalled—or even outperformed—job destruction and displacement in the medium run (see e.g. Brynjolfsson and McAfee (2014) and Autor (2015)).[2]

In this paper, we suggest that we may not rely on policy lessons from the industrial revolution, as capital production in the future will differ from those of the past. This is motivated by the following observations. It is well-documented that since the industrial revolution, machines have assumed tedious and arduous work[3], and that the production of machines and the necessary raw material for it, e.g. steel, and parts, similarly increased the demand for workers with low or easy-to-acquire skills in factories and logistics (see Roser (2016)). During the industrial revolution, for instance, mass production, and later on working at the assembly line, required a small range of

[1]Note that the estimates of McKinsey Global Institute (2017) are based on the current technological level. Thus, the more technology is advancing, the more jobs are threatened (Autor et al., 2003; Frey and Osborne, 2017; Autor, 2015), if we neglect the countervailing forces of job creation.

[2]Even if job creation proceeds at the same speed as job destruction, it remains unclear what will happen to the wages of today's low-skilled and high-skilled workers, as the rate of job creation and destruction differs for different skill levels (Bauer and Bender, 2004).

[3]See Autor (2015) and Mokyr (1992)

skills, and switching from agriculture to working in factories was comparatively easy (see Mokyr (1992)).

With the advent of robots and automation, factory workers are rendered now obsolete.[4] In several service industries a similar development is underway, in which low-skilled or even middle-skilled work is automated, and job profiles are redefined (see Roser (2016) and Autor (2015)). However, the production of robots is much more skill-intensive than the production of machines in the past, and to acquire the skills to produce robots, larger investments are required. The design and development of robots, their algorithms, software, and their physical production, repairing and maintenance require mostly high-skilled workers and much less labor connected to manual abilities than was the case in manufacturing in the last century.[5]

To sum up, in contrast to the production of machines, future capital production will produce robots, which require mostly high-skilled workers performing non-routine tasks. The theme of the paper is that technical progress of such types of capital production may have quite different consequences for workers. While for the most part of the last two centuries[6], technical progress in capital production tended to increase the real wages of low-skilled workers, technical progress in the present and future economy may have the opposite effect and thus, may entail adverse effects for low-skilled workers—either through lower wages or displacements if these wages fall below their reservation wages.

Of course, it is uncertain how the present and future economy will evolve, but it is conceivable that it has consequences for wages that differ significantly from what economic history tells us. We develop our argument within a three-industry economy with two different production processes for consumption goods. Since the largest share of consumption today are services, we call them "services" instead of "consumption goods", but this is not essential for our arguments. The two production processes are characterized by the respective tasks they involve. Each task, in turn, is characterized

[4]Alternatively, production is shifted to low labor costs.

[5]For a description of the history of manufacturing and robotic production see Roser (2016).

[6]There are some exceptions, as the debate about the extent to which real wages in the UK have fallen in connection with the industrial revolution indicates.

by its complexity level, the “task-complexity”. Workers with a particular skill level can only perform tasks up to a particular complexity level. Thus, there is a link between the complexity of a task and the required worker skill level.

We assume that in one industry a set of routine tasks with lower task complexity is required while in a second industry, a set of non-routine tasks with higher task-complexity is needed.[7] A third industry produces capital goods, which, in turn, can only be used in the routine production process. To simplify the subsequent analysis, we use the following denominations:

(i) Non-routine service industry: This industry produces services that are based on non-routine task-complexity;

(ii) Routine service industry: This industry produces services that are based on routine task-complexity;

(iii) Capital production: This industry produces a physical capital good that is an input factor in routine services. We further distinguish two possible *production modes* for capital:

  (1) Capital production based on *routine* (low) task-complexity;

  (2) Capital production based on *non-routine* (high) task-complexity.

The motivation for this set-up is as follows. Regarding services, many industries comprise routine task-complexities such as retail and transportation. However, a significant fraction of industries involves non-routine task-complexities such as consulting, auditing, design or software development.

Typically, since the industrial revolution, capital production has involved a large share of routine task-complexities and allowed low-skilled workers to work in factories on production and assembly lines (Goldin and Katz, 1998; Autor et al., 2003). Capital that is produced in this first production mode (1), and thus is based on routine task-complexity, will be called “machines”.

[7] Cf. Autor et al. (2003)

Frey and Osborne (2017) state that *"over the past decades, industrial robots have taken on the routine tasks of most operatives in manufacturing"* (p. 260).[8] The production of such robots involves a large amount of non-routine task-complexities. Capital that is produced in this second production mode (2), and thus is based on non-routine task-complexity, will be called "robots". The two production modes for capital define two different economies: (1) capital production based on routine task-complexity with machines as output is associated with the *industrial* economy; (2) capital production based on non-routine task-complexity with robots as output is associated with the *robotic* economy. For both the industrial economy and the robotic economy, we study the consequences of productivity improvements in the capital-producing industry on labor markets.

Our main insights are threefold. First, in an industrial economy, technological progress in machine production tends to lead to an integrated labor market. In contrast, in a robotic economy, technological progress in robot production tends to lead to disintegrated labor markets. In an integrated (disintegrated) labor market, low- and high-skill workers do (do not) work in the same industry and wage differentials are equal to (larger than) their relative productivity levels if they worked in the same industry. Second, technological progress in capital production will lower wage inequality in the industrial economy and increase it in a robotic economy. So for wage inequality, it is decisive who produces capital—i.e. who produces machines or robots. Third, technical progress in robot production may lead to a decline of wages for low-skilled workers.

A significant part of the paper is devoted to the precise conditions under which the above insights holds. It turns out that all three hold if the elasticity of substitution between robots and low-skilled labor is larger than the elasticity of substitution between products of industries using routine and non-routine tasks.[9]. There is empirical support for these conditions, which we discuss in section 5.1.

[8]Frey and Osborne (2017) also note that robots also acquiring the ability to perform some non-routine tasks. Such developments could be included in our model. However, as long as the dominant development of robots is to perform routine tasks, this observation does not impair our results

[9]Furthermore, We require that the latter elasticity is above 1 in the industrial economy, for which we also provide support in Section 5.1

In Section 2, we provide a short summary of the literature on uneven technological progress. In Section 3, we introduce a simplified version of the task-complexity model developed in Gersbach and Schmassmann (2019) as a building bloc for our model. In Section 4, we introduce the capital-producing industry and we study the two production modes representing the industrial and robotic economy, respectively. In Section 5 we discuss important related issues. Section 6 concludes.

2 Relation to the Literature

The question how productivity improvements in one industry affect employment in other industries and in the economy as a whole is a long-standing issue in economics, since Ricardo (1821) described how the substitution of human labor with machines is often harmful to workers. One of the first analytical contributors is Baumol (1967), who examined the employment consequences of uneven technological progress, and Baumol et al. (1985), who further explored uneven technological progress. In the 1990ths, a large literature explored how uneven (or even) productivity improvements affect unemployment in the presence of labor market frictions (Cohen et al., 1994; Aghion and Howitt, 1994, 1998; Peretto, 2011). Overall, the literature presents delivered results. In the past few years, the theory on labor-replacing technologies has been developed significantly: Peretto and Seater (2013) and Benzell et al. (2015) built dynamic models of factor-eliminating technical change. Acemoglu and Restrepo (2016) and Hémous and Olsen (2016) developed growth models that involve automation and horizontal innovations, producing rich dynamic patterns how wages and wage differentials of low-skilled and high-skilled workers develop.

Our paper complements these approaches to automation. We examine how productivity improvements in the industry that produces capital—i.e., machines or robots, depending on the production mode—impact the wages of lower-skilled and higher-skilled workers. Brynjolfsson and McAfee (2014) document how robots can be produced with increased ease and how this will become a central element of the future economy. We focus on this aspect. We adopt a medium-run perspective and do not specify all elements of productivity improvements in the industry that produces the robots. For our qualitative results, the fact that such productivity improvements *do* take place is important, but their magnitude is not. The magnitude of such productivity improvements can be rationalized in an endogenous growth set-up.[10]

There is a large literature on wage and income inequality—what has happened over the last decades and how it can be explained. Acemoglu and Autor (2011) or Hémous and Olsen (2016) offer detailed discussions of this literature. The following finding for the US is important for our exercise: While wages of low-skilled workers have been under pressure for quite some time, also middle-skill wages, in relative terms, have been declining in the US since the mid-1980's. The hypothesis is that many low-skilled tasks have already been automated and routine tasks such as storing, processing and gathering information performed by middle-skill workers are now undergoing an automation process (Spitz-Oener, 2006; Goos et al., 2009; Autor and Dorn, 2013; Hémous and Olsen, 2016). Middle-skill workers would, in our framework, correspond to low-skilled workers since we only have two skills categories.

Finally, in the last decade, various frameworks for task-based production have been developed to examine a variety of issues (e.g. Acemoglu and Autor (2011); Gersbach and Schmassmann (2019) and Acemoglu and Restrepo (2019)). We use a simplified version of the model developed in Gersbach and Schmassmann (2019).

3 The Service Industries

In this section, we present the first building block of our model in which no capital is produced and used.[11] We will then add the capital producing industry and the two different modes of capital production and its use in Section 4.

We consider an economy with a continuum of workers of measure 1. Labor endowment of workers is L ($L > 0$). Workers are characterized by their skill level r. We also call

[10] From a long-run perspective, the response of the labor supply to changes in wages and adjustments in the capital stock would also have to be considered.

[11] For foundations of the production functions based on the complexity of tasks see Gersbach and Schmassmann (2019).

the labor of skill level r "household" r or "worker" r. There are two skill levels, $\underline{r}$ and $\bar{r}$. Thus, the set of skill levels is $\mathcal{R} = \{\underline{r}, \bar{r}\}$. We will speak of *low-skilled* and *high-skilled* workers when referring to $\underline{r}$ and $\bar{r}$ workers, respectively. There is a share $\phi_{\underline{r}}$ ($\phi_{\bar{r}}$) of low-skilled (high-skilled) workers in the economy, where $\sum_{r \in \mathcal{R}} \phi_r = 1$.

An employed worker has to fulfill a task in the production process. Tasks are characterized by their complexity, the task-complexity, which we denote by i. It indicates the degree of difficulty to successfully complete a task, i.e., the higher i, the more difficult the production process and the higher the skill requirement.

For simplicity, we assume that there are only two production tasks, a *routine* task and a *non-routine* task, with corresponding task-complexity, which we denote by i_R and i_N, respectively. Thus, the set of task-complexities is $\mathcal{I} = \{i_R, i_N\}$.[12] Following Autor et al. (2003), a routine task-complexity is representative for a production process that follows programmed rules, whereas a non-routine task-complexity represents a production process that cannot be specified by such programmed rules. We assume that $i_N > i_R$, which implies that any worker able to perform the non-routine task is also able to perform the routine task, but not the other way round.[13] Thus, two workers of different skill levels are substitutes for a particular production process if they both fulfill the skill requirement of the task in that production process.

We assume a one-to-one mapping from task-complexities to the service industries. Hence, i may represent a task-complexity or an industry, and we will use i_R and i_N to designate industries: routine service industry (or industry i_R) and non-routine service industry (or industry i_N).[14]

The consumption product space comprises two dimensions: a service industry dimen-

[12]The model could be generalized to sets of routine and non-routine tasks as long as the tasks in each set have the same task-complexity level or to encompass any number of task-complexities (see Gersbach and Schmassmann (2019)).

[13]Graetz and Feng (2015) divide the task space into *training-intensive* and *innate ability* tasks, where each dimension is further differentiated by complexity. They observe that the division in routine and non-routine is insufficient to describe automation processes, when firms are allowed to choose which tasks they want to automate, and to show endogenized job polarization. In our model, we could introduce refined subdivisions of the task-complexities (see Footnote 12). However, for our main results, the subdivision into routine and non-routine task-complexities is already insightful.

[14]In Section 4, when we introduce the third industry that produces capital, we will have a correspondence between task-complexities and industries.

sion i and a variety dimension j. Varieties are differentiated products within an industry. In each of the two industries, there is an exogenously-given measure of firms, n_{i_R} and n_{i_N}, respectively, and each firm produces a variety j. We characterize by i a representative firm and this firm's variety of either industry .

The utility of household r, U^r, is described by a nested CES-function

$$U^r\left(\left\{c^r_{i,j}\right\}_{(i,j)\in\mathcal{I}\times[0,n_i]}\right) = C^r, \tag{1}$$

where C^r is the consumption basket

$$C^r := \left[\sum_{i\in\mathcal{I}}\left[\left[\int_0^{n_i} c^r_{i,j}{}^{\frac{\sigma_v-1}{\sigma_v}} dj\right]^{\frac{\sigma_v}{\sigma_v-1}}\right]^{\frac{\sigma_I-1}{\sigma_I}}\right]^{\frac{\sigma_I}{\sigma_I-1}} .$$

In the consumption basket, $c^r_{i,j}$ is the amount of product (i,j)—variety j of industry i—consumed by the household r. The parameter σ_I describes the elasticity of substitution *between* industries and σ_v describes the elasticity of substitution between varieties *within* an industry. We assume that $\sigma_I < \sigma_v$, i.e., products *within* an industry are closer substitutes than products *between* industries. The budget constraint of household r is

$$\sum_{i\in\mathcal{I}}\int_0^{n_i} p_{i,j}c^r_{i,j}dj \le Lw^r + \Pi^r \ , \tag{2}$$

where $p_{i,j}$ denotes the price of product (i,j). The wage of household r is denoted by w^r and the profit it obtains by Π^r. Prices, wages and profits are expressed in some unit of account. For our analysis only relative prices will matter and thus one can also anchor prices and wages for instance by the wage of low-skilled workers (we will do this later). The demand of household r for a product (i,j) is

$$c^r_{i,j} = \left[\frac{p_{i,j}}{P_i}\right]^{-\sigma_v}\left[\frac{P_i}{P}\right]^{-\sigma_I} C^r \ ,$$

where $P_i := \left[\int_0^{n_i} p_{i,j}^{1-\sigma_v}dj\right]^{\frac{1}{1-\sigma_v}}$ and $P := \left[\sum_{i\in\mathcal{I}} P_i^{1-\sigma_I}\right]^{\frac{1}{1-\sigma_I}}$. P_i denotes the respective service industry price index and P the aggregate price index. Aggregate demand from households for the product (i,j) is given by

$$c_{i,j} = \sum_{r\in\mathcal{R}}\phi_r c^r_{i,j} = \left[\frac{p_{i,j}}{P_i}\right]^{-\sigma_v}\left[\frac{P_i}{P}\right]^{-\sigma_I} C \ , \tag{3}$$

where $C := \sum_{r\in\mathcal{R}} \phi_r C^r$ is total consumption. Aggregation of the budget constraints yields

$$PC = \sum_{r\in\mathcal{R}} \phi_r L w^r + \sum_{r\in\mathcal{R}} \phi_r \Pi^r \ .$$

A producing firm holds a patent to produce product (i, j). Henceforth we consider the case of the representative firm i.[15] Firm i chooses (i) the labor to employ, (ii) the price of the product, and thereby its output. Firm i produces an amount x_i of its service by hiring a measure of $l_i(r)$ workers with skill level r. Thus,

$$x_i = \sum_{r\in\mathcal{R}} \kappa_1(r)\kappa_2(i) l_i(r) \ , \tag{4}$$

where $\kappa_1(r)\kappa_2(i)$ is the production function of a firm i for a worker of skill level r. The production function consists of a skill-dependent productivity factor, $\kappa_1(r) : \mathcal{R} \to \mathbb{R}_+$, with $\kappa_1'(.) > 0$ (the higher the skill, the more productive the production), and a complexity-dependent productivity factor, $\kappa_2(i) : \mathcal{I} \to \mathbb{R}_+$.

Following the task approach to production, we assume:

Assumption 1 (Appropriate Skill Condition)

Workers of low skill level (i.e. with skill $\underline{r}$) can only perform routine tasks (i.e. tasks of complexity i_R) whereas high-skilled workers (i.e. with skill $\overline{r}$) can perform both routine (i_R) and non-routine (i_N) tasks.

This implies that the non-routine service industry can only employ high-skilled workers, and the routine service industry has to decide on how to split work between high-skilled and low-skilled workers. We introduce $\tilde{r}(i)$ as the skill-threshold of industry i, i.e. $\tilde{r}(i_R) < \underline{r} < \overline{r}$ and $\underline{r} < \tilde{r}(i_N) < \overline{r}$. Assumption 1 is captured by the last line in the maximization problem of a representative firm i:

$$\begin{aligned}
\max_{p_i, l_i(\underline{r}), l_i(\overline{r})} \quad & \sum_{r\in\mathcal{R}} \left[p_i x_i(r) - l_i(r) w^r\right] \ , \\
s.t. \quad & x_i(r) = \kappa_1(r)\kappa_2(i) l_i(r) \ , \\
& \sum_{r\in\mathcal{R}} x_i(r) = \left[\frac{p_i}{P_i}\right]^{-\sigma_v} \left[\frac{P_i}{P}\right]^{-\sigma_I} C \ , \\
& l_i(\underline{r}) = 0 \ \text{ if } i = i_N \ .
\end{aligned} \tag{5}$$

[15] We discard the subscript j whenever convenient.

Firm i chooses price and labour in order to maximize profits given the productivity of workers, the market demand for its service and skill requirements of the production process in its industry. We denote the set of skills in production hired by firm i by $\mathcal{R}_i := \{r \in \mathcal{R} \mid l_i(r) > 0\}$. From Assumption 1 it is clear that $\mathcal{R}_{i_N} = \{\bar{r}\}$. The output of firm i is $x_i := \sum_{r \in \mathcal{R}_i} x_i(r)$. Firm i's maximization problem is solved by dividing it into the following two sub-problems: (i) *Cost Minimization* and (ii) *Profit Maximization.*

(i) *Cost Minimization*

Firm i minimizes costs by choosing a subset of skills, $\mathcal{R}_i \subseteq \mathcal{R}$, to minimize the cost per unit of output, taking Assumption 1 into account,

$$\min_r \quad \frac{w^r}{\kappa_1(r)\kappa_2(i)} \qquad s.t. \quad \underline{r} \notin \mathcal{R}_i \text{ if } i = i_N \ .$$

(ii) *Profit Maximization*

Given the optimal set of skill levels in production, $\mathcal{R}_i$, firm i chooses a price to solve its profit maximization problem in (5). Without loss of generality, we can assume that all of firm i's production is performed by a single skill level, i.e., $\mathcal{R}_i = \{r\}$, that satisfies Assumption 1. Firm i's optimization problem then is

$$\begin{aligned} \max_{p_i} \qquad & p_i x_i - x_i \frac{w^r}{\kappa_1(r)\kappa_2(i)} \ , \\ s.t. \qquad & x_i = \left[\frac{p_i}{P_i}\right]^{-\sigma_v} \left[\frac{P_i}{P}\right]^{-\sigma_I} C \ . \end{aligned}$$

This yields

$$p_i = \frac{\sigma_v}{\sigma_v - 1} \frac{w^r}{\kappa_1(r)\kappa_2(i)} \ . \tag{6}$$

The price equals the constant mark-up, $\frac{\sigma_v}{\sigma_v - 1}$, times the marginal cost. Knowing the firm's price decision, the amount of produced services, x_i, and the necessary labor input for production, are also determined in equilibrium. Note that the costs per unit of output might be minimized for both skill inputs. In such cases, a firm is indifferent between the two skill levels, as the skill levels are perfect substitutes. Next, we derive the equilibrium wage scheme and establish the equilibrium. We start with its definition.

Definition 1 (Equilibrium) *An equilibrium is*

(i) *a set of skill levels* $\mathcal{R}_i \subseteq \mathcal{R}$ *for representative firm* i*,*

(ii) *a set of output levels,* $\{x_i(r)\}_{(i,r)\in\mathcal{I}\times\mathcal{R}_i}$*, and labor,* $\{l_i(r)\}_{(i,r)\in\mathcal{I}\times\mathcal{R}_i}$*, for representative firm* i*,*

(iii) *a set of consumption levels,* $\left\{c^r_{i,j}\right\}_{(i,j,r)\in\mathcal{I}\times[0,n_i]\times\mathcal{R}}$*, for household* r*'s consumption of each product* (i,j)*,*

(iv) *a set of service prices,* $\{p_{i,j}\}_{i\in\mathcal{I}\times[0,n_i]}$*,*

(v) *a set of wage rates,* $\{w^r\}_{r\in\mathcal{R}}$*,*

such that

(A) $\{l_i(r)\}_{r\in\mathcal{R}_i}$ *and* p_i *solve the representative firm* i*'s profit maximization problem* (5)*,* $\forall i \in \mathcal{I}$*,*

(B) $\left\{c^r_{i,j}\right\}_{i\in\mathcal{I}\times[0,n_i]}$ *maximizes the utility of the household* r *given in* (1)*, subject to this household's budget constraint in* (2)*,* $\forall r \in \mathcal{R}$*,*

(C) *markets clear for all products,*

(D) *labor markets clear.*

Before we derive the equilibrium, we relate the labor of any worker to the productivity of the high-skilled worker,

$$\tilde{l}_i(r) := l_i(r)\frac{\kappa_1(r)}{\kappa_1(\bar{r})} \ . \tag{7}$$

We note that $\tilde{l}_i(r)$ expresses labor demand of firm i, normalized in productivity across skill levels. We call labor in this normalized form "effective" labor. In equilibrium, the allocation of labor of a certain skill to firms may remain indeterminate. However, we can always infer how much effective labor a firm demands, which is independent of r.

Next, we take a closer look at the labor market clearing shown in Condition (8),

$$\sum_{r\in\mathcal{R}:r\geq\tilde{r}(\hat{\imath})}^{\bar{r}} \phi_r \frac{\kappa_1(r)}{\kappa_1(\bar{r})} L \geq \sum_{i\in\mathcal{I}:i\geq\hat{\imath}} n_i \tilde{l}_i \left(\{w^r\}_{r\in\mathcal{R}}\right), \quad \forall\, \hat{\imath}\in\mathcal{I}\,, \tag{8}$$

where the wage scheme is denoted by $\{w^r\}_{r\in\mathcal{R}}$. The wage scheme balances supply and demand for skills, such that Condition (8) is fulfilled. The left-hand side is the supply of effective labor that is able to at least manage the task-complexity of industry $\hat{\imath}$. The right-hand side is the demand by all firms for effective labor—given a wage scheme—that is able to at least manage the task-complexity $\hat{\imath}$. There is an equilibrium if Condition (8) holds for industry i_N and with equality for industry i_R.

By using (6), we can derive that a firm i_R is indifferent between producing a product with skill levels $\underline{r}$ or $\bar{r}$ if and only if the wages satisfy

$$w^{\underline{r}} = \frac{\kappa_1(\underline{r})}{\kappa_1(\bar{r})} w^{\bar{r}}\ ,$$

i.e., when the relative productivity difference between $\underline{r}$ and $\bar{r}$ is reflected in the workers' respective wages. However, in equilibrium wages may not only reflect productivity differences. We set

$$w^{\underline{r}} = \frac{\kappa_1(\underline{r})}{\kappa_1(\bar{r})}\ . \tag{9}$$

This anchors all prices and wages in our economy. In turn, we define the "scaling factor" $\omega = w^{\bar{r}}$. From the above it becomes clear that if $\omega = 1$, the wage difference between the two types of workers can be explained solely by their difference in productivity. However, if $\omega > 1$, high-skilled workers earn more than can be explained by their higher productivity. Whenever $\omega > 1$ we speak of a *Disintegrated Labor Market* (DLM) and whenever $\omega = 1$ we speak of an *Integrated Labor Market* (ILM). In a DLM Equilibrium, demand for high skills from industry i_N is larger than supply if wages only reflect relative productivities.[16] Thus, the wage of workers with high enough skills to work in the non-routine service industry is scaled by the scaling factor. Hence, these workers obtain a wage premium. The wage premium translates into higher marginal costs and

[16]Note that $\omega \geq 1$ always holds. If this did not hold, then all firms would want to employ high-skilled workers and thereby these workers' wages would increase.

higher commodity prices. In particular, firms in industry i_N set prices according to

$$p_{i_N} = \frac{\sigma_v}{\sigma_v - 1} \frac{\omega}{\kappa_1(\bar{r})\kappa_2(i_N)} \ .$$

It is convenient for the following derivations to define the scaling factor in dependence of the industries,

$$\omega_i = \begin{cases} 1 & \text{if } i = i_R \\ \omega & \text{if } i = i_N \ . \end{cases} \tag{10}$$

Clearly, $\omega_i = 1 \quad \forall i \in \mathcal{I}$ if the economy is in the ILM Equilibrium. Using aggregate prices, we can infer the households' demand, which, in turn, leads to effective labor demand by firms, denoted by $\tilde{l}_i$, that is equal to

$$\tilde{l}_i = \omega_i^{-\sigma_I} \left[\kappa_1(\bar{r})\kappa_2(i)\right]^{\sigma_I - 1} n_i^{\frac{\sigma_v - \sigma_I}{1-\sigma_v}} \left[\sum_{\hat{i}\in\mathcal{I}} \left[\frac{\omega_{\hat{i}}}{\kappa_1(\bar{r})\kappa_2(\hat{i})} \right]^{1-\sigma_I} n_{\hat{i}}^{\frac{1-\sigma_I}{1-\sigma_v}} \right]^{\frac{\sigma_I}{1-\sigma_I}} C \quad \forall\, i \in \mathcal{I} \ . \tag{11}$$

The derivation is given in Appendix A. Equation (11) times the corresponding wages yields total wages earned by workers in industry i. Wages earned are entirely spent for consumption. We aggregate total wages earned in the economy, denoted by TW and rearranging yields

$$C = \left[\sum_{i\in\mathcal{I}} \left[\frac{\omega_i}{\kappa_1(\bar{r})\kappa_2(i)} \right]^{1-\sigma_I} n_i^{\frac{1-\sigma_I}{1-\sigma_v}} \right]^{\frac{1}{\sigma_I - 1}} TW \ ,$$

where $TW := \sum_{r\in\mathcal{R}} \phi_r w^r L = \left[\phi_{\underline{r}} \frac{\kappa_1(\underline{r})}{\kappa_1(\bar{r})} + \omega \phi_{\bar{r}}\right] L$.[17] Whenever the labor market is disintegrated, there are *two* separate labor markets, one for the low-skilled and one for the high-skilled workers, respectively. Thus, in the DLM Equilibrium, the following inequalities hold with equality and $\omega > 1$, whereas in the ILM Equilibrium, they remain weak inequalities and $\omega = 1$:

$$\tilde{\phi}_{\underline{r}} L \leq n_{i_R}^{\frac{1-\sigma_I}{1-\sigma_v}} \left[\kappa_1(\bar{r})\kappa_2(i_R)\right]^{\sigma_I - 1} \left[\sum_{\hat{i}\in\mathcal{I}} \left[\frac{\omega_{\hat{i}}}{\kappa_1(\bar{r})\kappa_2(\hat{i})} \right]^{1-\sigma_I} n_{\hat{i}}^{\frac{1-\sigma_I}{1-\sigma_v}} \right]^{-1} TW \ ,$$

$$\tilde{\phi}_{\bar{r}} L \geq n_{i_N}^{\frac{1-\sigma_I}{1-\sigma_v}} \left[\kappa_1(\bar{r})\kappa_2(i_N)\right]^{\sigma_I - 1} \omega^{-\sigma_I} \left[\sum_{\hat{i}\in\mathcal{I}} \left[\frac{\omega_{\hat{i}}}{\kappa_1(\bar{r})\kappa_2(\hat{i})} \right]^{1-\sigma_I} n_{\hat{i}}^{\frac{1-\sigma_I}{1-\sigma_v}} \right]^{-1} TW \ ,$$

[17] Note that $PC = \frac{\sigma_v}{\sigma_v - 1} TW$.

where $\tilde{\phi}_r := \phi_r \frac{\kappa_1(r)}{\kappa_1(\bar{r})}$ is the effective labor supply of skill level r. We can now solve for the scaling factor ω, and thus for the high-skilled workers' wage,

$$\omega = \max\left\{1, \left[\frac{\tilde{\phi}_{\underline{r}}}{\tilde{\phi}_{\bar{r}}}\left[\frac{\kappa_2(i_N)}{\kappa_2(i_R)}\right]^{\sigma_I-1}\left[\frac{n_{i_N}}{n_{i_R}}\right]^{\frac{1-\sigma_I}{1-\sigma_v}}\right]^{\frac{1}{\sigma_I}}\right\} .$$

We next present the equilibria.

Proposition 1

There exists a unique equilibrium, either ILM or DLM, with

(i) $\omega^\star = \begin{cases} 1 & \textit{if } eq = \{ILM\} \ , \\ \left[\frac{\tilde{\phi}_{\underline{r}}}{\tilde{\phi}_{\bar{r}}}\left[\frac{\kappa_2(i_N)}{\kappa_2(i_R)}\right]^{\sigma_I-1}\left[\frac{n_{i_N}}{n_{i_R}}\right]^{\frac{1-\sigma_I}{1-\sigma_v}}\right]^{\frac{1}{\sigma_I}} & \textit{if } eq = \{DLM\} \ , \end{cases}$

(ii) $\mathcal{R}^\star_{i_N} = \{\bar{r}\}, \mathcal{R}^\star_{i_R} \begin{cases} \subseteq \mathcal{R} & \textit{if } eq = \{ILM\} \ , \\ = \{\underline{r}\} & \textit{if } eq = \{DLM\} \ , \end{cases}$

(iii) $p_i^\star = \frac{\sigma_v}{\sigma_v-1}\frac{\omega_i^\star}{\kappa_1(\bar{r})\kappa_2(i_N)} \quad \forall\, i \in \mathcal{I} \ ,$

$P_i^\star = \frac{\sigma_v}{\sigma_v-1}\frac{\omega_i^\star}{\kappa_1(\bar{r})\kappa_2(i_N)} n_i^{\frac{1}{1-\sigma_v}} \quad \forall\, i \in \mathcal{I} \ ,$

$P^\star = \frac{\sigma_v}{\sigma_v-1}\left[\sum_{i\in\mathcal{I}}\left[\frac{\omega_i^\star}{\kappa_1(\bar{r})\kappa_2(i)}\right]^{1-\sigma_I} n_i^{\frac{1-\sigma_I}{1-\sigma_v}}\right]^{\frac{1}{1-\sigma_I}} \ ,$

(iv) $TW^\star = \left[\tilde{\phi}_{\underline{r}} + \omega^\star\tilde{\phi}_{\bar{r}}\right] L \ ,$

(v) $\tilde{l}_i^\star = \frac{\omega_i^{\star\, -\sigma_I}\kappa_2(i)^{\sigma_I-1} n_{i_N}^{\frac{\sigma_v-\sigma_I}{1-\sigma_v}}}{\sum_{\hat{i}\in\mathcal{I}}\omega_{\hat{i}}^{\star\, 1-\sigma_I}\kappa_2(\hat{i})^{\sigma_I-1} n_{\hat{i}}^{\frac{1-\sigma_I}{1-\sigma_v}}} TW^\star \ ,$

(vi) $x_i^\star = \kappa_1(\bar{r})\kappa_2(i)\tilde{l}_i^\star \ ,$

(vii) $\pi_i^\star = \frac{\tilde{l}_i^\star}{\sigma_v-1} \ ,$

(viii) $C^\star = \left[\sum_{i\in\mathcal{I}}\left[\frac{\omega_i^\star}{\kappa_1(\bar{r})\kappa_2(i)}\right]^{1-\sigma_I} n_i^{\frac{1-\sigma_I}{1-\sigma_v}}\right]^{\frac{1}{\sigma_I-1}} TW^\star \ ,$

and $\quad P^\star C^\star = \frac{\sigma_v}{\sigma_v-1} TW^\star \ ,$

where $\pi_i^\star$ denotes the equilibrium profit of representative firm i.

The equilibrium is unique up to the exact allocation of skill levels to firms in industry i_R in an ILM Equilibrium.

4 Capital Production

In this section, we enrich our model by three features. First, there is a third industry that generates capital. As motivated in Section 1, we distinguish two capital production *modes*: (i) capital (machines) is produced with *routine* task-complexity—the industrial economy—and (ii) capital (robots) is produced with *non-routine* task-complexity—the robotic economy. Second, capital can substitute routine work to some extent. Third, there is technological progress in capital production.[18]

The substitution of labor through capital is called "automation", which thus encompasses machines *and* robots. However, machines and robots can *only* be used as a substitute for routine-based production.[19] We assume a constant elasticity between capital and labor in the production of routine services. The production function for the representative firm i_R is thus given by

$$x_{i_R} = \left[\left[\sum_{r \in \mathcal{R}_{i_R}} \kappa_1(r) \kappa_2(i_R) l_{i_R}(r) \right]^{\frac{\sigma_R - 1}{\sigma_R}} + k_{i_R}^{\frac{\sigma_R - 1}{\sigma_R}} \right]^{\frac{\sigma_R}{\sigma_R - 1}} , \tag{12}$$

where k_{i_R} stands for capital input of firm i_R (machines or robots) and σ_R denotes the elasticity of substitution between labor and capital.[20] The elasticity of substitution between capital and labor is larger than the elasticity of substitution between industries and 1, i.e.,

$$\sigma_R > \max\{\sigma_I, 1\} \ .$$

We will show that in an *industrial economy*, the substitution of workers in the routine service industry by machines—that can be produced by the same workers—leads to an integrated labor market when there is technological progress in capital production. In contrast, in a *robotic economy*, the substitution of workers in the routine service

[18] A first version of the model has been presented in Schmassmann (2018).

[19] We discuss this assumption in Section 5.2.

[20] As to the impact of productivity improvements on automation in an industrial economy and in a robotic economy, we will take the elasticity of substitution between capital input and labor performing routine tasks as given. Arguably, increasing automation may also impact directly the elasticities of substitution. As long as this is a second-order effect, our results are not affected by automation induced changes of elasticities.

industry by robots—that can only be produced with workers of high skill levels— leads to disintegrated labor markets.[21] We next analyze the industrial economy and the robotic economy each in turn.

4.1 Industrial Economy

In this section, we assume that the production of machines is based on a production process involving the routine task-complexity i_R. Specifically

$$k = A \sum_{r \in \mathcal{R}} \kappa_1(r)\kappa_2(i_R)l(r) \ , \tag{13}$$

where A is an exogenous parameter that captures technological progress in capital production.[22] A will be central to our comparative statics analysis.[23] For simplicity we assume that this industry is competitive. Thus, the price of capital, p_k, equals its marginal production costs, $p_k = [A\kappa_1(\bar{r})\kappa_2(i_R)]^{-1}$.[24] The derivation of the equilibrium is shown in Appendix B.1 in detail. In the following, we describe the ILM Equilibrium and the DLM Equilibrium.

Equilibria

ILM Equilibrium. There is sufficient high-skilled labor in the economy to meet high-skilled labor demand. Thus, the scaling factor remains equal to unity ($\omega = 1$). High-skilled labor may be employed in any of the three industries prevalent in the economy: capital production, routine services, and non-routine services. There exists a unique

[21] Frey and Osborne (2017) focus on the destruction effect of technology, neglecting the effect that increased productivity makes firms expand production and employment. The way this expansion effect impacts wage inequality will be essential in our analysis of the industrial economy and the robotic economy.

[22] Note that we denote capital as a factor input by k_{i_R} and capital as an output by k.

[23] Increases in A can be interpreted in two ways. First, the production of machines itself becomes more efficient, i.e., more capital can be produced with the same labor input. This is the intuitive interpretation when considering the functional form chosen. Second, machines become more effective in the production of routine services for the same price.

This second interpretation is more in line with historical records.

[24] We used the production function (13), the industry specific scaling factor (10), and the effective labor normalization (7) to derive p_k. Note that whenever $\omega > 1$, only low-skilled workers are employed in capital production.

equilibrium wage scheme and a unique equilibrium, up to the allocation of labor and skills to industries.

DLM Equilibrium. Labor markets are disintegrated, and all workers able to perform the non-routine task-complexity are employed in industry i_N. Wages balance demand and supply of skills, resulting in a scaling factor greater than unity for high-skilled labor. Thus, the scaling factor, $\omega > 1$, must be such that both labor markets clear,[25]

$$\begin{aligned}\tilde{\phi}_{\bar{r}} L =& n_{i_N} \tilde{l}_{i_N} \\ =& n_{i_N}^{\frac{1-\sigma_I}{1-\sigma_v}} \kappa_1(\bar{r})^{-1} \kappa_2(i_N)^{\sigma_I - 1} \omega^{-\sigma_I} \hat{\mathcal{M}}(\omega)^{\frac{\sigma_I}{1-\sigma_I}} C \ ,\end{aligned} \tag{14}$$

$$\begin{aligned}\tilde{\phi}_{\underline{r}} L =& n_{i_R} \tilde{l}_{i_R} + \tilde{L}_k \\ =& n_{i_R}^{\frac{1-\sigma_I}{1-\sigma_v}} \kappa_1(\bar{r})^{-1} \kappa_2(i_R)^{\sigma_I - 1} \hat{\theta}(A)^{\frac{1-\sigma_I}{1-\sigma_R}} \hat{\mathcal{M}}(\omega)^{\frac{\sigma_I}{1-\sigma_I}} C \ ,\end{aligned} \tag{15}$$

where $n_{i_N} \tilde{l}_{i_N}$, $n_{i_R} \tilde{l}_{i_R}$ and $\tilde{L}_k$ are the amounts of effective labor in the three industries, respectively, and $\hat{\mathcal{M}}(\omega) := n_{i_N}^{\frac{1-\sigma_I}{1-\sigma_v}} \kappa_2(i_N)^{\sigma_I - 1} \omega^{1-\sigma_I} + n_{i_R}^{\frac{1-\sigma_I}{1-\sigma_v}} \kappa_2(i_R)^{\sigma_I - 1} \hat{\theta}(A)^{\frac{1-\sigma_I}{1-\sigma_R}}$.

Using (14) and (15), we determine the scaling factor in equilibrium,

$$\omega^\star = \left[\frac{\tilde{\phi}_{\underline{r}}}{\tilde{\phi}_{\bar{r}}} \left[\frac{\kappa_2(i_N)}{\kappa_2(i_R)} \right]^{\sigma_I - 1} \left[\frac{n_{i_N}}{n_{i_R}} \right]^{\frac{1-\sigma_I}{1-\sigma_v}} \hat{\theta}(A)^{\frac{1-\sigma_I}{\sigma_R - 1}} \right]^{\frac{1}{\sigma_I}} \ . \tag{16}$$

Equilibria in the Industrial Economy. The principal variables of the ILM Equilibrium and the DLM Equilibrium are the following:

(i) $\omega^\star = \begin{cases} 1 & \text{if } eq = \{ILM\} \ , \\ \left[\frac{\tilde{\phi}_{\underline{r}}}{\tilde{\phi}_{\bar{r}}} \left[\frac{\kappa_2(i_N)}{\kappa_2(i_R)} \right]^{\sigma_I - 1} \left[\frac{n_{i_N}}{n_{i_R}} \right]^{\frac{1-\sigma_I}{1-\sigma_v}} \hat{\theta}(A)^{\frac{1-\sigma_I}{\sigma_R - 1}} \right]^{\frac{1}{\sigma_I}} & \text{if } eq = \{DLM\} \ , \end{cases}$

(ii) $P^\star = \frac{\sigma_v}{\sigma_v - 1} \kappa_1(\bar{r})^{-1} \hat{\mathcal{M}}(\omega^\star)^{\frac{1}{1-\sigma_I}}$,

(iii) $C^\star = \kappa_1(\bar{r}) \hat{\mathcal{M}}(\omega^\star)^{\frac{1}{\sigma_I - 1}} \tilde{L}$ and $P^\star C^\star = \frac{\sigma_v}{\sigma_v - 1} \tilde{L}$,

where $\hat{\theta}(A) := 1 + A^{\sigma_R - 1}$, and $\tilde{L} = \left[\tilde{\phi}_{\underline{r}} + \tilde{\phi}_{\bar{r}} \right] L$ denotes total effective labor.[26]

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[25] For a derivation see Appendix B.1.

[26] Total wages, TW, are related to total effective labor through $TW^\star = \hat{\mathcal{N}}(\omega)^{-1} \hat{\mathcal{M}}(\omega) \tilde{L}$, where $\hat{\mathcal{N}}(\omega) = n_{i_N}^{\frac{1-\sigma_I}{1-\sigma_v}} \kappa_2(i_N)^{\sigma_I - 1} \omega^{\star - \sigma_I} + n_{i_R}^{\frac{1-\sigma_I}{1-\sigma_v}} \kappa_2(i_R)^{\sigma_I - 1} \hat{\theta}(A)^{\frac{1-\sigma_I}{1-\sigma_R}}$.

Technological Progress in Capital Production

We now relate technological progress in capital production (exogenous increase in the productivity parameter A) to wage inequality and aggregate consumption in an industrial economy.

ILM Equilibrium. In an ILM Equilibrium, the wage scheme does not change with technological progress in capital production, A. Consequently, A has primarily an income effect, thus aggregate consumption increases. Nevertheless, relative industry price indices change, i.e., $\frac{P_{i_R}}{P_{i_N}}$ falls and consumption allocations shift. Thereby, technological progress might tighten (loosen)—increase (decrease) the relative demand for high-skilled labor. Whether or not the labor market tightens and thereby drifts towards a DLM Equilibrium depends on σ_I. (i) If $\sigma_I > 1$, households consume more of the services from industry i_R, as prices in this industry fall with technological progress. (ii) If, however, $\sigma_I < 1$, households shift their consumption towards services of industry i_N, and the integrated labor market tightens until it finally disintegrates. Recall that high-skilled labor is also employed in the routine service industry, as well as in the capital-producing industry. In an ILM Equilibrium, all workers benefit equally from the efficiency gains, i.e., lower prices in industry i_R.

DLM Equilibrium. In a DLM Equilibrium, the scaling factor is a function of technological progress, as can be seen in (16). The following lemma presents the derivative of the scaling factor with respect to technological progress in capital production.

Lemma 1

The elasticity of the scaling factor with respect to A, $\sigma_{A,\omega}$, is

$$\sigma_{A,\omega} = \frac{\partial \omega}{\partial A}\frac{A}{\omega} = \frac{1-\sigma_I}{\sigma_I}\frac{A^{\sigma_R - 1}}{\theta(A)} \begin{cases} < 0 & \text{if } \sigma_I > 1 \ , \\ = 0 & \text{if } \sigma_I = 1 \ , \\ > 0 & \text{if } \sigma_I < 1 \ . \end{cases} \tag{17}$$

Technological progress decreases the skill premium if the elasticity of substitution between industries is greater than one ($\sigma_I > 1$). In this case, low-skilled labor is substituted in the routine service industry by capital that is produced in the capital-producing industry through increasingly productive low-skilled labor, i.e., low-skilled labor is substituted and reemployed in the capital-producing industry, where it is more productive than before. Thus, wage inequality decreases, as long as $\sigma_I > 1$.

We define the real wage of workers with skill $r \in \mathcal{R}$ as

$$\tilde{w}^r = \frac{w^r}{P} \; . \tag{18}$$

For high-skilled workers we obtain

$$\frac{\partial \tilde{w}^{\bar{r}}}{\partial A} = \frac{\sigma_v - 1}{\sigma_v} \frac{\partial C}{\partial A} \frac{\omega}{TW} \sigma_I^{-1} > 0 \; .$$

Real wages for high-skilled labor always increase with technological progress in capital production. High-skilled labor benefits from a positive income effect, as prices of the routine services lower. Whereas for the low-skilled,

$$\frac{\partial \tilde{w}^{\underline{r}}}{\partial A} = \frac{\sigma_v - 1}{\sigma_v} \frac{\partial C}{\partial A} \frac{\frac{\kappa_1(\underline{r})}{\kappa_1(\bar{r})}}{TW} \left[1 + \frac{\sigma_I - 1}{\sigma_I} \frac{\omega \tilde{\phi}_{\bar{r}}}{\tilde{\phi}_{\underline{r}}} \right]$$

may be positive or negative. We can distinguish three cases on the real wage dynamics of low-skilled labor, where we use $s_{w^{\bar{r}}} := \frac{\omega \tilde{\phi}_{\bar{r}}}{\omega \tilde{\phi}_{\bar{r}} + \tilde{\phi}_{\underline{r}}}$ and $s_{w^{\underline{r}}} := \frac{\tilde{\phi}_{\underline{r}}}{\omega \tilde{\phi}_{\bar{r}} + \tilde{\phi}_{\underline{r}}}$ to denote the share of total wages earned by the *high*-skilled and *low*-skilled, respectively:

(i) $\sigma_I > 1$: The income effect and the substitution effect are aligned. Both effects lead to increasing consumption of services from industry i_R. Thus, real wages of low-skilled workers increase (and the increase is higher than the increase of the real wages of high-skilled labor. See the upcoming Proposition 2).

(ii) $\sigma_I \in [s_{w^{\bar{r}}}, 1]$: The substitution effect leads households to shift their consumption towards services from industry i_N. The income effect leads households to consume more from both industries. The demand for routine services increases—the income effect remains stronger than (or equal to) the substitution effect—, leading to an increase in real wages for low-skilled labor, albeit not as high as the increase in real wages for high-skilled labor (see Proposition 2).

(iii) $\sigma_I \in (0, s_{w^{\bar{r}}})$: The substitution effect dominates the income effect and households shift their consumption towards services from industry i_N. The increased demand for i_N-services increases the scaling factor ω and thereby lowers the real wage of low-skilled workers.

We further decompose the substitution and income effects in Appendix B.2. From the analysis we can derive the following lemma.

Lemma 2

The real wage of the high-skilled labor always increases in A. The real wage of low-skilled labor increases if $\sigma_I > s_{w^{\bar{r}}}$.

We can infer the following proposition from Lemmas 1 and 2.

Proposition 2

The real wage of high-skilled labor always increases less (more) in A than the real wage of low-skilled labor if $\sigma_I > 1$ ($\sigma_I < 1$).

In Appendix B.3, we analyze the implications of $\sigma_I < 1$ on the wage dynamics in detail.

The change in real wages plus the change in real profits must equal the change in consumption. Thus

$$\frac{\partial C}{\partial A} = \frac{\sigma_v}{\sigma_v - 1}\left[\phi_{\underline{r}}\frac{\partial \tilde{w}^{\underline{r}}}{\partial A} + \phi_{\bar{r}}\frac{\partial \tilde{w}^{\bar{r}}}{\partial A}\right] L \ .$$

For the sake of simplicity, we next assume that each worker obtains the profits that emerge from his own work.[27] Under this assumption, the real wage of a household r, $\tilde{w}^r$, times the factor $\frac{\sigma_v}{\sigma_v - 1}$ must equal household r's total consumption, C^r,

$$C^r = \frac{\sigma_v}{\sigma_v - 1}\tilde{w}^r \ . \tag{19}$$

Then the change in consumption from technological progress is shared between low-skilled and high-skilled labor according to

$$\phi_{\bar{r}} L \frac{\sigma_v}{\sigma_v - 1}\frac{\partial \tilde{w}^{\bar{r}}}{\partial A} = \frac{\partial C}{\partial A} s_{w^{\bar{r}}} \sigma_I^{-1} \ , \tag{20}$$

$$\phi_{\underline{r}} L \frac{\sigma_v}{\sigma_v - 1}\frac{\partial \tilde{w}^{\underline{r}}}{\partial A} = \frac{\partial C}{\partial A}\left[1 - s_{w^{\bar{r}}} \sigma_I^{-1}\right] \ . \tag{21}$$

Total high-skilled labor obtains $\sigma_I^{-1} s_{w^{\bar{r}}}$ and total low-skilled labor obtains $1 - \sigma_I^{-1} s_{w^{\bar{r}}}$ from total consumption.

[27] A rationale for this assumption is that each worker runs his own firm and pays himself a wage, being his own employee, and earns profits from his own firm.

In our model the benefits of a marginal expansion of varieties do not have to be the same for the two industries. Analyzing again (16) reveals that if $\sigma_I > 1$, then

$$\frac{\partial \omega}{\partial \frac{n_{i_N}}{n_{i_R}}} = \frac{1}{\sigma_v - 1} \frac{\sigma_I - 1}{\sigma_I} \omega \frac{n_{i_R}}{n_{i_N}} > 0 \ ,$$

i.e., an increase in the ratio of non-routine varieties to routine varieties increases the scaling factor, as demand for high-skilled workers increases. Thus, relative wages depend on where—in which industry—developments of new varieties occur more often. Marginal social returns from increasing either variety are

$$\frac{\partial C}{\partial n_{i_R}} = C \frac{1}{\sigma_v - 1} s_{w^{\underline{r}}} n_{i_R}^{-1} > 0 \ ,$$
$$\frac{\partial C}{\partial n_{i_N}} = C \frac{1}{\sigma_v - 1} s_{w^{\bar{r}}} n_{i_N}^{-1} > 0 \ .$$

We observe that $\frac{\partial C}{\partial n_{i_R}} > \frac{\partial C}{\partial n_{i_N}}$ whenever $\frac{\tilde{\phi}_{\underline{r}}}{n_{i_R}} > \frac{\omega \tilde{\phi}_{\bar{r}}}{n_{i_N}}$, i.e., when production costs per variety are higher in industry i_R compared to industry i_N.

Summary of an Industrial Economy

While in the ILM Equilibrium, technological progress in capital production benefits workers irrespective of their skill level, the DLM Equilibrium features different wage developments in response to such technological progress. For the consumption decisions of households, we must know how profits are distributed in the economy. We continue assuming that income through profits is proportional to households' wage income, (19). The effects in the two equilibria are summarized in Appendix B.4.

ILM Equilibrium. The scaling factor stays equal to unity and the aggregate price index unambiguously decreases due to technological progress. Thereby, real wages increase for every worker in the economy. Because of lower prices, demand for service i_R increases, leading to higher production in this industry. Depending on whether industries are substitutes ($\sigma_I > 1$) or complements ($\sigma_I < 1$), consumers shift part of their consumption away from or towards services of industry i_N. This pattern is the same for low-skilled and high-skilled households.

DLM Equilibrium. The scaling factor ($\omega > 1$) falls in A whenever industries are substitutes ($\sigma_I > 1$). An increase in A leads to lower prices for service i_R. Thus, if services (industries) are substitutes, households shift their consumption towards service i_R. Thereby, relative demand for services i_N decreases and accordingly also the labor demand of industry i_N. As only industry i_N employs high-skilled labor, less demand for labor of this industry directly decreases the scaling factor. Furthermore, as long as every high-skilled worker is used in the production of services i_N (always in a DLM Equilibrium), output in this industry cannot vary, i.e., stays constant.[28] So, the scaling factor adjusts in order to balance demand for high-skilled labor given the constant output level.

The aggregate price index falls if $\sigma_I > s_{w^{\bar{r}}}$ and thus, the real wage of the low-skilled increases. If now $\sigma_I < s_{w^{\bar{r}}}$, the scaling factor increases sufficiently, due to higher demand for high-skilled labor, to overcompensate the efficiency gains from lower prices in industry i_R on the aggregate price level. Intuitively, when $\sigma_I < 1$, households wish to consume more of services i_N. The production of this service, however, is restricted by the supply of high-skilled labor, leading to an increase in the scaling factor to balance demand and supply. Now, whenever σ_I is lower than total wages of the high-skilled workers relative to total wages of all workers, the increased demand of all households for services i_N, due to technological progress, results in an increase of the scaling factor, and thereby also of the industry price index P_{i_N}, which makes the low-skilled worse off. Because the services are strong complements, the efficiency gain in production of service i_R (with the help of machines) does not suffice to compensate the higher prices for service i_N for the low-skilled. In contrast, the high-skilled benefit from lower prices for service i_R and higher wages.

The low-skilled consume less of service i_R if $\sigma_I < \frac{\omega\tilde{\phi}_{\bar{r}}}{2\omega\tilde{\phi}_{\bar{r}}+\tilde{\phi}_{\underline{r}}}$, i.e., if their demand is strongly inelastic. Whenever this is the case, they consume less of both services.[29]

[28]Thus, the changes in consumption decisions of the low-skilled and the high-skilled must mirror each other,

$$\phi_{\bar{r}}\frac{\partial C^{\bar{r}}_{i_N}}{\partial A} = -\,\phi_{\underline{r}}\frac{\partial C^{\underline{r}}_{i_N}}{\partial A}\ .$$

The proof is given in Appendix D.7, showing that this equivalence must indeed hold.

[29]The derivation of this result is given in Appendix D.6. Note that we can assume that $\sigma_I = \frac{\omega\tilde{\phi}_{\bar{r}}}{2\omega\tilde{\phi}_{\bar{r}}+\tilde{\phi}_{\underline{r}}}$. This is equal to $\frac{\sigma_I-1}{\sigma_I} = -\frac{\omega\tilde{\phi}_{\bar{r}}+\tilde{\phi}_{\underline{r}}}{\omega\tilde{\phi}_{\bar{r}}}$. We can use this second equality in Equation (B.5) to observe that the substitution effect for the low-skilled is exactly as strong to both mirror the

4.2 Robotic Economy

In the previous section, we assumed that low-skilled and high-skilled workers can be used in the capital-producing industry. Suppose now, the economy has developed to a more advanced, more automated state. The capital-producing industry now produces capital that we call robots, which can be used as a substitute for low-skilled labor in industry i_R, just as machines in the previous section. However, their design and production requires high-skilled labor. The economy captures the ongoing automation processes exemplified in Brynjolfsson and McAfee (2014).

We assume that the production of robots requires task-complexity i_N that characterizes the high complexity of robot development, design and production. Thus, only labor that is able to perform non-routine tasks can be used to produce robots. In any other aspect, the robotic economy is equivalent to the industrial economy. The production function of capital production now is (depending on i_N rather than on i_R as in (13)),

$$k = A\kappa_1(\bar{r})\kappa_2(i_N)l(\bar{r}) \ ,$$

where A is again an exogenous technological parameter. A detailed derivation of the equilibrium is given in Appendix C.1. In the following, we again analyze the two cases of an integrated and a disintegrated labor market.

We make the following assumption throughout the section.

Assumption 2 (Elasticity Order)
The elasticity of substitution between capital and labor is larger than the elasticity of substitution between industries and 1, i.e.,

$$\sigma_R > \max\{\sigma_I, 1\} \ .$$

The assumption means that there is a significant substitutability between labor and capital for routine tasks and thus to perform tasks with low complexity. In Subsection 5.1, we discuss that the empirical evidence supports this assumption.

substitution effect of the high-skilled *and* to reverse the income effect of the low-skilled.

Equilibria

ILM Equilibrium. Integrated labor markets imply that $\omega = 1$ and $\tilde{L} = TW$. The demand for high-skilled labor from the service industry i_N and from the capital-producing industry is smaller than the high-skilled labor supply. Thus, high-skilled labor is also employed in industry i_R.

DLM Equilibrium. Labor markets are disintegrated, i.e., $\omega > 1$. High-skilled labor is no longer employed in industry i_R. Firms in industry i_N and in the capital-producing industry are willing to pay a wage premium for the scarce skill levels. There are two separate markets, i.e., $\tilde{\phi}_{\bar{r}} L = \; n_{i_N} \tilde{l}_{i_N} + \tilde{L}_k$ and $\tilde{\phi}_{\underline{r}} L = \; n_{i_R} \tilde{l}_{i_R,j}$. The scaling factor ω then clears the labor markets,

$$\begin{aligned}
\tilde{\phi}_{\bar{r}} L =& n_{i_N}^{\frac{1-\sigma_I}{1-\sigma_v}} \kappa_1(\bar{r})^{-1} \kappa_2(i_N)^{\sigma_I - 1} \omega^{-\sigma_I} \tilde{\mathcal{M}}(\omega)^{\frac{\sigma_I}{1-\sigma_I}} C + \\
& n_{i_R}^{\frac{1-\sigma_I}{1-\sigma_v}} \kappa_1(\bar{r})^{-1} \kappa_2(i_R)^{\sigma_I - 1} \tilde{\theta}(A,\omega)^{\frac{\sigma_I - \sigma_R}{\sigma_R - 1}} \omega^{-1} \mu(A,\omega) \tilde{\mathcal{M}}(\omega)^{\frac{\sigma_I}{1-\sigma_I}} C \ , \\
\tilde{\phi}_{\underline{r}} L =& n_{i_R}^{\frac{1-\sigma_I}{1-\sigma_v}} \kappa_1(\bar{r})^{-1} \kappa_2(i_R)^{\sigma_I - 1} \tilde{\theta}(A,\omega)^{\frac{\sigma_I - \sigma_R}{\sigma_R - 1}} \tilde{\mathcal{M}}(\omega)^{\frac{\sigma_I}{1-\sigma_I}} C \ ,
\end{aligned}$$

where $\tilde{\mathcal{M}}(\omega) := n_{i_N}^{\frac{1-\sigma_I}{1-\sigma_v}} \kappa_2(i_N)^{\sigma_I-1} \omega^{1-\sigma_I} + n_{i_R}^{\frac{1-\sigma_I}{1-\sigma_v}} \kappa_2(i_R)^{\sigma_I-1} \tilde{\theta}(A,\omega)^{\frac{1-\sigma_I}{1-\sigma_R}}$. We take the ratio of the two equalities above and define

$$\mathcal{F} := \tilde{X} \tilde{\theta}(A,\omega)^{\frac{\sigma_I - \sigma_R}{1-\sigma_R}} \omega^{-\sigma_I} - \frac{\tilde{\phi}_{\bar{r}}}{\tilde{\phi}_{\underline{r}}} [1 - z] = 0 \ , \tag{22}$$

where $\tilde{X} := \left[\frac{n_{i_N}}{n_{i_R}}\right]^{\frac{1-\sigma_I}{1-\sigma_v}} \left[\frac{\kappa_2(i_N)}{\kappa_2(i_R)}\right]^{\sigma_I - 1}$ and

$$z := \frac{\tilde{\phi}_{\underline{r}}}{\omega \tilde{\phi}_{\bar{r}}} \mu(A,\omega) \tag{23}$$

denotes the fraction of the high-skilled labor force demanded through industry i_R's need for capital. Using the Implicit Function Theorem we determine the dynamics of the scaling factor. We take the partial derivative of (22) with respect to ω and, together with Assumption 2, obtain

$$\frac{\partial \mathcal{F}}{\partial \omega} = \tilde{X} \tilde{\theta}(A,\omega)^{\frac{\sigma_I - \sigma_R}{1-\sigma_R}} \omega^{-\sigma_I - 1} \left[[\sigma_I - \sigma_R] \frac{\mu(A,\omega)}{\tilde{\theta}(A,\omega)} - \sigma_I \right] - \sigma_R \omega^{-2} \mu(A,\omega) < 0 \ .$$

Thus, for $\omega \geq 1$, there is a unique solution $\omega^\star$, as $\frac{\partial \mathcal{F}}{\partial \omega}$ is strictly negative.

Equilibria in the Robotic Economy. The principal variables of the ILM Equilibrium and the DLM Equilibrium are the following:

(i) $\omega^\star = \begin{cases} 1 & \text{if } eq = \{ILM\} \ , \\ \mathcal{F}|_{\omega^\star} = 0 & \text{if } eq = \{DLM\} \ , \end{cases}$

(ii) $P^\star = \frac{\sigma_v}{\sigma_v - 1} \kappa_1(\bar{r})^{-1} \tilde{\mathcal{M}}(\omega^\star)^{\frac{1}{1-\sigma_I}}$,

(iii) $C^\star = \kappa_1(\bar{r}) \tilde{\mathcal{M}}(\omega^\star)^{\frac{1}{\sigma_I - 1}} \tilde{L}$ and $P^\star C^\star = \frac{\sigma_v}{\sigma_v - 1} \tilde{L}$,

where $\tilde{\theta}(A, \omega^\star) := 1 + \mu(A, \omega^\star)$, where $\mu(A, \omega^\star) := \left[\frac{A}{\omega^\star} \frac{\kappa_2(i_N)}{\kappa_2(i_R)} \right]^{\sigma_R - 1}$.[30]

The term $\mu(A, \omega)$ is the cost of capital when optimally used as an input relative to one unit of effective labor. The ratio $\frac{\mu(A,\omega)}{\tilde{\theta}(A,\omega)}$ denotes the share of total costs in the routine service industry allocated to the capital input (robots), and therefore indirectly paid to non-routine labor.

Technological Progress in Capital Production

We now analyze the effects of technological progress in capital production, A, on wages and on aggregate consumption in our robotic economy.

ILM Equilibrium. The wage scheme is unaffected by technological progress. Technological progress leads to lower prices of robots, and benefits all workers in the economy equally. Though, the following result shows that the bounty of increased productivity in capital production is only temporarily given to all workers.

Lemma 3

Technological progress moves the economy towards a DLM Equilibrium, i.e.,

$$\frac{\partial \tilde{L}^d_{i_n}}{\partial A} > 0 \ .$$

$\tilde{L}^d_{i_n}$ denotes the total demand for effective labor able to do non-routine work. The derivation is given in Appendix D.1. Lemma 3 implies that, starting from an ILM Equilibrium, technological progress eventually results in a DLM Equilibrium.

[30] Note that $\tilde{\mathcal{M}}(\omega) \neq \hat{\mathcal{M}}(\omega)$ of the industrial economy.

DLM Equilibrium. In order to analyze wage dynamics, we first derive the partial derivative of $\mathcal{F}$ with respect to technological progress A,

$$\frac{\partial \mathcal{F}}{\partial A} = \frac{\mu(A,\omega)}{A}\left[\tilde{X}\left[\sigma_R - \sigma_I\right]\tilde{\theta}(A,\omega)^{\frac{\sigma_I - 1}{1-\sigma_R}}\omega^{-\sigma_I} + \left[\sigma_R - 1\right]\omega^{-1}\right] > 0 \ ,$$

which is strictly greater zero for $\sigma_R > \max\{1, \sigma_I\}$ (Assumption 2). In equilibrium it must always hold that (see Equation (23))

$$1 > z \ . \tag{24}$$

The demand for effective high-skilled labor originating from industry i_R cannot be greater than total effective high-skilled labor supply, as otherwise the scaling factor ω rises sufficiently to restore (24).[31] In the following lemma, we analyze the dynamics of the scaling factor with respect to exogenous variations in A.

Lemma 4

The elasticity of the scaling factor ω with respect to A, $\sigma_{A,\omega}$, is positive, i.e.,

$$\sigma_{A,\omega} := \frac{\partial \omega}{\partial A}\frac{A}{\omega} = -\frac{\frac{\partial \mathcal{F}}{\partial A}}{\frac{\partial \mathcal{F}}{\partial \omega}}\frac{A}{\omega} = \frac{1}{1 + \frac{z + [1-z]\sigma_I}{z[\sigma_R - 1] + [1-z][\sigma_R - \sigma_I]\frac{\mu(A,\omega)}{\tilde{\theta}(A,\omega)}}} > 0 \ . \tag{25}$$

In a robotic economy, technological progress in capital production leads to an increase in the scaling factor—even if $\sigma_I < 1$. Intuitively, high-skilled labor produces robots at decreasing costs, and the robots can be used as a substitute for low-skilled labor. [32]

We now examine the real wage changes. For the low-skilled the derivative of the real wage $\tilde{w}^{\underline{r}}$, as defined in (18), with respect to technology is

$$\begin{aligned}\frac{\partial \tilde{w}^{\underline{r}}}{\partial A} = \frac{\sigma_v - 1}{\sigma_v}\kappa_1(\underline{r})\tilde{\mathcal{M}}(\omega)^{\frac{1}{\sigma_I - 1} - 1}\Bigg[& n_{i_R}^{\frac{1-\sigma_I}{1-\sigma_v}}\kappa_2(i_R)^{\sigma_I - 1}\tilde{\theta}(A,\omega)^{\frac{\sigma_R - \sigma_I}{1-\sigma_R}}\frac{\mu(A,\omega)}{A} - \\ & \left[n_{i_N}^{\frac{1-\sigma_I}{1-\sigma_v}}\kappa_2(i_N)^{\sigma_I - 1}\omega^{-\sigma_I} + n_{i_R}^{\frac{1-\sigma_I}{1-\sigma_v}}\kappa_2(i_R)^{\sigma_I - 1}\tilde{\theta}(A,\omega)^{\frac{\sigma_R - \sigma_I}{1-\sigma_R}}\frac{\mu(A,\omega)}{\omega}\right]\frac{\partial \omega}{\partial A}\Bigg] \ .\end{aligned} \tag{26}$$

[31] For the limiting case, when $A \to \infty$, (24) holds with equality if $\sigma_I > 1$. This result will be derived in Lemma 6.

[32] Note that $\frac{\partial \mathcal{F}}{\partial \frac{\phi_{\bar{r}}}{\phi_{\underline{r}}}} = -1$. Assuming $\sigma_I > 1$, we obtain $\frac{\partial \mathcal{F}}{\partial \frac{n_{i_N}}{n_{i_R}}} = \frac{\sigma_I - 1}{\sigma_v - 1}\tilde{X}\tilde{\theta}(A,\omega)^{\frac{\sigma_I - \sigma_R}{1-\sigma_R}}\omega^{-\sigma_I}\frac{n_{i_R}}{n_{i_N}} > 0$. Then the scaling factor increases in the scarcity of high-skilled labor: $\frac{\partial \omega}{\partial \frac{\phi_{\underline{r}}}{\phi_{\bar{r}}}} = -\frac{\partial \mathcal{F}}{\partial \frac{\phi_{\underline{r}}}{\phi_{\bar{r}}}}\frac{\partial \omega}{\partial \mathcal{F}} > 0$, and the scaling factor increases in the ratio of non-routine varieties to routine varieties $\frac{\partial \omega}{\partial \frac{n_{i_N}}{n_{i_R}}} = -\frac{\partial \mathcal{F}}{\partial \frac{n_{i_N}}{n_{i_R}}}\frac{\partial \omega}{\partial \mathcal{F}} > 0$.

The derivation is given in Appendix D.2. The derivative can be positive or negative, depending on parameters and on the elasticity of the scaling factor with respect to technological progress. Note that the dynamics of the real wages of low-skilled labor equal the inverse dynamics of the aggregate price index P.

Lemma 5

A marginal increase in technological progress decreases (increases) the real wage of low-skilled labor $\tilde{w}^r$, if $\sigma_{A,\omega} > z$ ($< z$).

The proof is given in Appendix D.3. Observe that z also denotes the share of the high-skilled labor's total wages, $\omega\tilde{\phi}_{\bar{r}}$, that is paid through the demand for robots. An increase in ω leads to a one-to-one increase of the prices in industry i_N. If $z < \sigma_{A,\omega}$ then the price increases in industry i_N are not sufficiently counteracted by the lower prices in industry i_R caused by an increase in A and thus real wages of the low-skilled decrease.[33] The following condition demonstrates whether or not $\sigma_{A,\omega}$ is greater or smaller than z. The real wage of the low-skilled decreases in A if the following condition is fulfilled:

$$[\sigma_R - \sigma_I] \frac{1 + \omega \frac{\tilde{\phi}_{\bar{r}}}{\tilde{\phi}_{\underline{r}}}}{1 + z\omega \frac{\tilde{\phi}_{\bar{r}}}{\tilde{\phi}_{\underline{r}}}} > \frac{1}{1-z} \ . \tag{27}$$

The derivation is shown in Appendix D.4. If Condition (27) holds, the real wage of low-skilled workers decreases.[34]

We next examine whether the efficiency increase through technological progress in capital production overcompensates the demand for non-routine skill in production of robots. Taking derivatives, it can be shown that $\frac{\partial z}{\partial A} > 0$ if the elasticity of substitution between the industries, σ_I, is large enough.

[33] Suppose that $\sigma_{A,\omega} = 0.5$ and $\sigma_I = 1$. Then a 1% ($= g_A$) growth in A leads to a 0.5%($= g_\omega$) increase in ω. If now $z < 0.5$, fewer high-skilled workers are used in the production of robots than in the production of service i_N. In such a case, the productivity gains in capital production lead to prices for routine services that are not lowered enough compared to the increased prices for non-routine services to make the low-skilled better off due to technological progress. This effect can also be shown by merely analyzing the dynamics within the high-skilled labor force. Both technological progress and the scaling factor affect the cost-productivity ratio of the high-skilled. Then $z\,[g_\omega - g_A] + [1-z]g_\omega > 0$ (and equals 0 if $z = 0.5$), i.e., on average, prices increase despite the productivity increase.

[34] In Appendix C.2, we provide numerical examples for which the real wage of the low-skilled decreases temporarily.

Lemma 6

The share of non-routine labor demanded to produce the robots increases in A (i.e., $\frac{\partial z}{\partial A} > 0$) if $\frac{\sigma_R - 1}{\sigma_R} > \sigma_{A,\omega}$, which is true whenever

$$\sigma_I > \frac{\mu(A,\omega)}{\mu(A,\omega) + \frac{\sigma_R - 1}{\sigma_R}} \ . \tag{28}$$

This always holds if $\sigma_I > 1$.

The derivation is shown in Appendix D.5. Inequality (28) depends on $\mu(A,\omega)$ which always increases in A. The scaling factor cannot increase faster than technological progress, i.e., $\sigma_{A,\omega} < 1$ (see also Appendix D.5 for the derivation). The more advanced A, the more restrictive (28) becomes. However, (28) always holds for $\sigma_I > 1$, independent of the level of A. Intuitively, if the elasticity of substitution between industries, σ_I, is greater than one, households shift their consumption towards routine services in relative terms, because prices in this industry always decline, no matter how much of the production is performed by high-skilled labor. Thus, technological progress in capital production leads to more efficient robot production, and the resulting price decrease in this industry is *not* outweighed by the scaling factor increase.[35]

If industries are complements ($\sigma_I < 1$), the income effect lets households consume more from both industries. The substitution effect lets households shift their consumption towards services i_N. If this substitution effect is strong enough, the scaling factor rises sufficiently to dominate the productivity increase, i.e., less high-skilled labor is employed to produce the robots which are needed in the routine service industry.

For the high-skilled real wages always increase,

$$\frac{\partial \tilde{w}^{\bar{r}}}{\partial A} = \frac{\sigma_v - 1}{\sigma_v} \kappa_1(\bar{r}) \tilde{\mathcal{M}}(\omega)^{\frac{1}{\sigma_I - 1}} \left[\frac{1}{\sigma_I - 1} \tilde{\mathcal{M}}(\omega)^{-1} \frac{\partial \tilde{\mathcal{M}}(\omega)}{\partial A} + \frac{\partial \omega}{\partial A} \right] > 0.$$

This leads to the following lemma:

Lemma 7

The real wage of the high-skilled always increases in A. The real wage of the low-skilled might increase or decrease in A.

[35] In Appendix C.3 we analyze the limits of technological progress, i.e., $A \to \infty$, and the substitution elasticity between robots and labor, i.e., $\sigma_R \to \infty$.

Because of the scaling factor in the wage of high-skilled labor and the scaling factor's positive elasticity with respect to A (Lemma 4), the following proposition must hold under Assumption 2.

Proposition 3

The real wage of the high-skilled always increases more in A than the real wage of the low-skilled.

Thus, Assumption 2 is crucial for the results obtained. We will argue in Section 5.1 that σ_R is likely to be relatively high, compared to σ_I.

Summary of a Robotic Economy

Again, we assume that profits are distributed proportionally to real wages, thus (19) denotes household r's consumption, and that Assumption 2 holds. The effects in the two equilibria are summarized in Appendix C.4.

ILM Equilibrium. The scaling factor remains unaffected by technological progress and the price index falls, thereby raising real wages of all workers in the economy equally. Demand for services i_R increase because of decreasing prices. If $\sigma_I > 1$ ($\sigma_I < 1$), the lower prices in the i_R-industry induce households to shift their consumption away from (towards) the i_N-industry.

DLM Equilibrium. The scaling factor always rises in response to technological progress in capital production, thus, the wage of a high-skilled worker always increases more than the wage of a low-skilled worker. If Condition (27) holds, the real wage of the low-skilled even falls in response to technological progress. More services of the i_R-industry are demanded, as their prices fall in the wake of the productivity increase. For the i_N-industry, there are countervailing effects: Technological progress leads to higher demand of services from industry i_R because of lower prices and exerts upward pressure on the scaling factor, because high-skilled workers produce the robots needed in industry i_R. In parallel, the services from industry i_N become more expensive to purchase because of higher production costs (scaling factor). Thus, even if industries are complements ($\sigma_I < 1$), the diametrically opposed directions of the price dynamics in the two industries induce households to further shift their consumption towards services of industry i_R. In Appendix C.4 we provide the critical boundary values.

5 Comparison

We next put the results into perspective. For this purpose, we note that the parameters of the industrial and robotic economies can be quite different, and we will account for some of these differences in our discussion. Note also that the underlying structures of the two economies (the industrial and the robotic economy) differ. We will nonetheless compare their respective reaction to technological change in capital production.

5.1 Elasticity of Substitution

Between Industries

The elasticity of substitution between industries, σ_I, plays a core role in our framework. σ_I is presented as a preference parameter (see (1)). Typically, the literature has found that the elasticity of substitution between manufacturing and services is close to one, one or slightly above one (see Herrendorf et al. (2013) and Atalay (2017)). Hence, since we are considering two subsectors of the service industry and thus less coarsely defined industries, the elasticity of substitution between these two subsectors is expected to be at least one.

We note that we could also model a single consumption good produced by a competitive firm which produces a final good by assembling intermediate inputs from the industries. In such a context, σ_I—as part of the production function—reflects a technological parameter. We observe, however, that with this interpretation, σ_I differs from estimates usually used in labor market contexts to measure the elasticity of substitution between high-skilled and low-skilled workers, based on the relative supply and wage differentials of college graduates and high-school graduates (Katz and Murphy, 1992; Acemoglu and Autor, 2011). In these studies it is assumed that there is an aggregate CES-production function. The estimated elasticities lie between 1.4 and 2 (Freeman, 1986; Heckman et al., 1998; Acemoglu and Autor, 2011). In our context, any two workers are perfectly substitutable under the condition that both are able to do the task they are hired for and given that labor markets are integrated. In the aggregate, however, labor markets might disintegrate. As long as labor markets are *integrated*, the elasticity of substitution between industries does *not* affect the labor market. The equilibrium in such an economy is indeterminate with respect to the exact allocation of workers to firms.[36]

Between Capital and Labor

DeCanio (2016) estimates that the elasticity of substitution between robots and humans is greater than 1.9. Similarly to our approach, he uses a production function with three factors, robotic capital, ordinary capital and humans, to analyze the conditions under which the expansion of the robotic input leads to a decline in the human wage. However, he does not distinguish different skill levels. Thus, his estimate can be used as a lower bound for the substitutability of routine labor and robots in our model. Furthermore, DeCanio (2016) analyzes the short-run dynamics of substitution. The medium-term elasticity of substitution is likely to be higher, as technologies get refined and the adoption of new technologies is more likely to happen. Hémous and Olsen (2016) use a value of 4 for the elasticity of substitution between capital and low-skilled labor in their model. Hence, the estimates for σ_R, the above considerations, and the estimates of σ_I suggest that Assumption 2 is supported by the data, although the empirical estimates vary a great deal.[37]

5.2 Technological Progress

We consider two stylized economies, the industrial economy and the robotic economy. The former equipped with a capital-producing industry that uses routine labor input to

[36] A natural arising allocation would nevertheless match workers with high-skills to tasks of high task-complexity and workers of low skills to tasks of low task-complexity.

[37] Most estimates of the elasticities of substitution between capital and labor in general for the US are below 1 (see Knoblach et al. (2016)), but estimates differ widely. However, these estimates are not applicable to our context since we consider the elasticity of substitution between robots and low-skilled workers.

produce machines and the latter equipped with a capital-producing industry that uses non-routine labor input to produce robots. In the ILM Equilibrium, both economies show the same wage scheme and all households in the economy benefit equally from technological progress in capital production.

However, in the DLM Equilibrium the differences are stark. Table 1 summarizes the occurring effects of technological progress in capital production first in an industrial economy and second in a robotic economy. Table 1 only shows the case of the DLM Equilibria. It presents the main variables of the model and illustrates whether or not they are affected by technological progress in capital production. Unaffected variables, i.e., those that stay constant, are marked with a 'c'. Variables that increase are marked with a '$+$' and variables that decrease are marked with a '$-$'.[38]

[38] We are interested in the qualitative effects, i.e., the focus lies on the direction rather than on the magnitude of the effects, where the latter would have to be analyzed in an endogenous growth framework.

Table 1: Effects of Rising Productivity in Capital Production.

Variable		DLM Equilibrium − *Industrial* Economy			DLM Equilibrium − *Robotic* Economy		
		Aggregate	$\underline{r}$	$\overline{r}$	Aggregate	$\underline{r}$	$\overline{r}$
1. Wage Premium	ω	$-$ $(\sigma_I > 1)$ $+$ $(\sigma_I < 1)$			$+$		
2. Price Index	P	$-$ $\left(\sigma_I > \frac{\omega\tilde{\phi}_{\overline{r}}}{\omega\tilde{\phi}_{\overline{r}}+\tilde{\phi}_{\underline{r}}}\right)$ $+$ $\left(\sigma_I < \frac{\omega\tilde{\phi}_{\overline{r}}}{\omega\tilde{\phi}_{\overline{r}}+\tilde{\phi}_{\underline{r}}}\right)$			$-$ Cond. (27) $+$ (otherwise)		
3. Real Wage	$\tilde{w}^r$		$-$ $\left(\sigma_I < \frac{\omega\tilde{\phi}_{\overline{r}}}{\omega\tilde{\phi}_{\overline{r}}+\tilde{\phi}_{\underline{r}}}\right)$ $+$ $\left(\sigma_I > \frac{\omega\tilde{\phi}_{\overline{r}}}{\omega\tilde{\phi}_{\overline{r}}+\tilde{\phi}_{\underline{r}}}\right)$	$+$		$-$ Cond. (27) $+$ (otherwise)	$+$
4. Service i_R	$n_{i_R}x_{i_R}$	$+$			$+$		
Service i_N	$n_{i_N}x_{i_N}$	c			$-$ $(\sigma_I > \kappa_{A,\omega})$ $+$ $(\sigma_I < \kappa_{A,\omega})$		
5. Consumption i_R	$c^r_{i_R}$		$-$ $\left(\sigma_I < \frac{\omega\tilde{\phi}_{\overline{r}}}{2\omega\tilde{\phi}_{\overline{r}}+\tilde{\phi}_{\underline{r}}}\right)$ $+$ $\left(\sigma_I > \frac{\omega\tilde{\phi}_{\overline{r}}}{2\omega\tilde{\phi}_{\overline{r}}+\tilde{\phi}_{\underline{r}}}\right)$	$+$		$-$ $(\sigma_I < \kappa_{i_R,R})$ $+$ $(\sigma_I > \kappa_{i_R,R})$	$+$
Consumption i_N	$c^r_{i_N}$		$-$ $(\sigma_I < 1)$ $+$ $(\sigma_I > 1)$	$-$ $(\sigma_I > 1)$ $+$ $(\sigma_I < 1)$		$-$ $(\sigma_I > \kappa_{i_N,R})$ $+$ $(\sigma_I < \kappa_{i_N,R})$	$-$ $(\sigma_I > 1)$ $+$ $(\sigma_I < 1)$

where $\kappa_{A,\omega} := \frac{\mu(A,\omega)}{\mu(A,\omega)+\frac{\sigma_R-1}{\sigma_R}}(<1)$, and $\kappa_{i_N,R} := \frac{\mu(A,\omega)\left[1-\sigma_{A,\omega}z^{-1}\right]}{\mu(A,\omega)+\sigma_{A,\omega}}(<1)$, and $\kappa_{i_R,R} := \frac{\sigma_{A,\omega}-z}{1-z}\frac{1+\mu(A,\omega)}{\sigma_{A,\omega}+\mu(A,\omega)}$, and $\mu(A,\omega) := \left[\frac{A}{\omega}\frac{\kappa_2(i_N)}{\kappa_2(i_R)}\right]^{\sigma_R-1}$.

An important comparison is the constellation $\sigma_I > 1$ in the industrial economy and $\sigma_R > \sigma_I \geq 1$ in the robotic economy. The constellation satisfies Assumption 2 and is, as we have argued, supported by the data. Then, we can apply Propositions 2 and 3 to obtain the following theorem.

Theorem 1 *Suppose $\sigma_I > 1$ in an industrial economy and $\sigma_R > \sigma_I \geq 1$ in a robotic economy. Then, technological progress in capital production will lower wage inequality in the industrial economy and increase it in a robotic economy.*

Theorem 1 contains our main results of the paper. It shows that it is decisive for the wage inequality who produces capital. In an industrial economy technological progress benefits all types of workers and wage inequality declines. In a robotic economy wage inequality widens and low-skill workers may even experience an absolute decline of their real wage.

There is also some tentative empirical support for our assumptions and results, e.g. Acemoglu and Restrepo (2017), who find large negative and robust effects of robots on employment and wages. In their assessment these effects, however, not only apply to routine or low-skilled labor. DeCanio (2016), who analyzes the elasticity between labor and robots, notes that the[39] *"Expansion of AIs' skill sets (which in the terminology of the paper entails increases in the elasticity of substitution between AIs and humans) is likely to depress wages over time. This will increase measured inequality unless the returns to robotic assets are broadly spread across the population"* (p. 289). Frey and Osborne (2017) note that more and more non-routine tasks can be executed by robots. Thus, more research has to be done to further understand the substitution possibilities in production of increasingly sophisticated robots and their implications for the labor market and for wage inequality.

The industrial economy resembles the economy in the 20^{st} century[40], and the robotic economy could resemble the economy of tomorrow. We recall that the robotic econ-

[39] AI: Artificial Intelligence. DeCanio (2016) refers to systems equipped with AI, the technology that can match or surpass human capabilities in his definition, as robots.

[40] Of course, a detailed comparison has to take into account the sectoral shifts, in particular the decline of the agriculture sector and the shift from the manufacturing to the service sector.

omy has much stronger separating forces compared to the industrial economy. These separating forces unambiguously lead to higher wage inequality when the production of robots becomes more efficient.

We have addressed technological progress by assuming efficiency gains in capital production. However, if one generalizes the model to encompass more than two task-complexities and an entire skill distribution as in Gersbach and Schmassmann (2019), other means of technological progress can be studied. This would arguably reinforce the results of this paper, for instance, (i) if the task-complexity level required for capital production increases and thereby the amount of labor that is able to do the task decreases, (ii) if robots evolve and are able to substitute more and more task-complexities, or (iii) if emerging new services require higher and higher task-complexities.

5.3 A Growth Perspective

We have derived our results by comparing two static economies in which we performed comparative exercises with respect to technological progress in the capital producing industry. It is straightforward to embed the economies in an infinite horizon neoclassical growth model by letting households decide about savings and investments in the capital stock, where the economy operates either in the industrial or in the robotic mode and technological progress takes place in capital production.[41] Ceteris paribus, the results from Theorem 1 can be applied in each period. For the overall impact on wages one has to add the capital accumulation effect which affects wages in addition to technological progress.[42]

The growth perspective suggests that we might consider a transition from the industrial economy to a robotic economy. The transition itself is less easily described from the results obtained so far, as well as how this transition affects wages. Most likely, the shift from routine labor input to non-routine labor input in capital production and

[41] As long as technological progress is exogenous, embedding the static economy in the dynamic setting is straightforward. Endogenizing growth will require to add one of the known processes from endogenous growth theories in the model.

[42] Capital accumulation cannot undo the impact of technological progress on wage inequality.

the simultaneous rise in productivity in this industry lead to a gradual shift from the industrial economy towards the robotic economy, with rising wage inequality, in accordance with the empirical observations. A way to model this transition would be to implement a rising task-complexity level in the production process of capital, that might depend on some innovation process, from low to high task-complexities. Reasonably, this would be embedded in a growth model with a large number of task-complexities (see Gersbach and Schmassmann (2019)).

If we allow for non-homothetic preferences—an empirically robust fact that is able to explain structural change in a growth scenario (Boppart, 2014)—it turns out that the separating forces at play in an industrial and robotic economy are reinforced.[43]

6 Conclusion

The fundamental assumption of our model is that the skill-task-assignment is determined by the minimum skill requirement of every production process.[44] Thus, if not every skill level is usable in every production process, the assignment of skills to tasks co-determines labor market dynamics.

We show that in an industrial economy, where routine labor is substituted by capital that can be produced by routine labor, technological progress in capital production has equaling effects on the wage scheme. In contrast, the robotic economy, where routine labor is substituted by capital that can only be produced by non-routine labor, technological progress in capital production reveals strong tendencies towards a diverging wage scheme.

There are numerous extensions that can be pursued. Apart from an endogenous growth perspective discussed in the last section, technical progress could be considered including changes in the task-complexities in production processes. In such a framework, accounting for changes in job-profiles of workers—when jobs are defined as sets of

[43]Cf. Schmassmann (2018) and details are available upon request.

[44]The minimum skill requirement can be rationalized by a minimum-quality constraint imposed on the production processes in a technological environment of O-ring production (Gersbach and Schmassmann, 2019).

tasks, each task having its respective task-complexity—would allow the study of polarization across age groups and the effects of continuing education. Furthermore, the skill of a worker and the complexity of a task could both be divided into several dimensions, assuming that a worker has to fulfill all requirements of the task (or the tasks of the job). Finally, the reaction of workers and the society as a whole to a robotic economy—with education and training or income support—may be one of the most important policy themes that the robotic economy calls for.

Appendix

A Derivation of Equation 11

We impose service market clearing and equate (3) to (4). We obtain

$$\sum_{r\in\mathcal{R}} \kappa_1(r)\kappa_2(i)l_i(r) = \left[\frac{p_{i,j}}{P_i}\right]^{-\sigma_v} \left[\frac{P_i}{P}\right]^{-\sigma_I} C \ . \tag{A.1}$$

Expressing (A.1) in effective labor terms using (7) yields

$$\kappa_1(\bar{r})\kappa_2(i)\tilde{l}_i = \left[\frac{p_{i,j}}{P_i}\right]^{-\sigma_v} \left[\frac{P_i}{P}\right]^{-\sigma_I} C \ . \tag{A.2}$$

Next we use firms' optimal price decision (6) and the industry specific scaling factor (10) to obtain prices

$$\begin{aligned}
p_{i,j} =& \frac{\sigma_v}{\sigma_v - 1}\frac{\omega_i}{\kappa_1(\bar{r})\kappa_2(i)} \quad \forall\,(i,j) \in \mathcal{I} \times n_i \ , \\
P_i =& \frac{\sigma_v}{\sigma_v - 1}\frac{\omega_i}{\kappa_1(\bar{r})\kappa_2(i)} n_i{}^{\frac{1}{1-\sigma_v}} \quad \forall\, i \in \mathcal{I} \ , \\
P =& \frac{\sigma_v}{\sigma_v - 1}\left[\sum_{i\in\mathcal{I}}\left[\frac{\omega_i}{\kappa_1(\bar{r})\kappa_2(i)}\right]^{1-\sigma_I} n_i{}^{\frac{1-\sigma_I}{1-\sigma_v}}\right]^{\frac{1}{1-\sigma_I}} \ .
\end{aligned}$$

Using the prices in (A.2) and rearranging yields Equation (11).

B Industrial Economy

B.1 Derivation of the Equilibrium in the Industrial Economy

The optimal choice of a firm in the routine service industry between capital and labor, obtained by equating the relative marginal products to relative marginal cost of the two inputs—capital and effective labor—is reflected in the ratio

$$\frac{\tilde{l}_{i_R}}{k_{i_R}} = p_k^{\sigma_R} \left[\kappa_1(\bar{r})\kappa_2(i_R)\right]^{\sigma_R - 1} \ .$$

Using optimal relative inputs given the factor prices, we can compute the marginal costs of representative firm i_R. The marginal costs, denoted by mc, are equal to costs per unit of output and average costs, because the production function is linear, i.e.,

$$mc_{i_R} = [\kappa_1(\bar{r})\kappa_2(i_R)]^{-1}\,\hat{\theta}(A)^{\frac{1}{1-\sigma_R}} \ ,$$

where $\hat{\theta}(A) = 1 + A^{\sigma_R - 1}$. We aggregate prices[45] and obtain the ideal price index

$$P = \frac{\sigma_v}{\sigma_v - 1}\kappa_1(\bar{r})^{-1}\hat{\mathcal{M}}(\omega)^{\frac{1}{1-\sigma_I}} \ ,$$

where $\hat{\mathcal{M}}(\omega) = n_{i_N}^{\frac{1-\sigma_I}{1-\sigma_v}}\kappa_2(i_N)^{\sigma_I - 1}\omega^{1-\sigma_I} + n_{i_R}^{\frac{1-\sigma_I}{1-\sigma_v}}\kappa_2(i_R)^{\sigma_I-1}\hat{\theta}(A)^{\frac{1-\sigma_I}{1-\sigma_R}}$.

The effective labor demand of a firm of industry i_R incorporates the effective labor needed to produce k_{i_R} as an input, i.e., to produce x_{i_R} effective labor of the amount $\tilde{l}_{i_R}\hat{\theta}(A)^{\frac{\sigma_R}{\sigma_R-1}}$ is needed, and thus

$$x_{i_R} = \kappa_1(\bar{r})\kappa_2(i_R)\tilde{l}_{i_R}\hat{\theta}(A)^{\frac{\sigma_R}{\sigma_R-1}} \ .$$

The aggregate household demand faced by the representative firm of each industry is

$$\begin{aligned} c_{i_R} &= n_{i_R}^{\frac{\sigma_v-\sigma_I}{1-\sigma_v}}\kappa_2(i_R)^{\sigma_I}\hat{\theta}(A)^{\frac{\sigma_I}{\sigma_R-1}}\hat{\mathcal{M}}(\omega)^{\frac{\sigma_I}{1-\sigma_I}}C \ , \\ c_{i_N} &= n_{i_N}^{\frac{\sigma_v-\sigma_I}{1-\sigma_v}}\kappa_2(i_N)^{\sigma_I}\omega^{-\sigma_I}\hat{\mathcal{M}}(\omega)^{\frac{\sigma_I}{1-\sigma_I}}C \ . \end{aligned} \tag{B.1}$$

Service market clearing implies

$$\tilde{l}_{i_R} = n_{i_R}^{\frac{\sigma_v-\sigma_I}{1-\sigma_v}}\kappa_1(\bar{r})^{-1}\kappa_2(i_R)^{\sigma_I-1}\hat{\theta}(A)^{\frac{\sigma_I-\sigma_R}{\sigma_R-1}}\hat{\mathcal{M}}(\omega)^{\frac{\sigma_I}{1-\sigma_I}}C \ , \tag{B.2}$$

$$\tilde{l}_{i_N} = n_{i_N}^{\frac{\sigma_v-\sigma_I}{1-\sigma_v}}\kappa_1(\bar{r})^{-1}\kappa_2(i_N)^{\sigma_I-1}\omega^{-\sigma_v}\hat{\mathcal{M}}(\omega)^{\frac{\sigma_I}{1-\sigma_I}}C \ . \tag{B.3}$$

Total effective labor in capital production we denote by $\tilde{L}_k$. Effective labor in an industry with routine-based production always equals wages.[46] Therefore we know that

[45] Symmetry of the production technology across firms within an industry leads to the following prices and price aggregators:

$$\begin{aligned} p_{i_R,j} &= \frac{\sigma_v}{\sigma_v-1}[\kappa_1(\bar{r})\kappa_2(i_R)]^{-1}\,\hat{\theta}(A)^{\frac{1}{1-\sigma_R}} \quad \text{and} \quad p_{i_N,j} = \frac{\sigma_v}{\sigma_v-1}[\kappa_1(\bar{r})\kappa_2(i_N)]^{-1}\,\omega \ , \\ P_{i_R} &= \frac{\sigma_v}{\sigma_v-1}[\kappa_1(\bar{r})\kappa_2(i_R)]^{-1}\,\hat{\theta}(A)^{\frac{1}{1-\sigma_R}}n_{i_R}^{\frac{1}{1-\sigma_v}} \quad \text{and} \quad P_{i_N} = \frac{\sigma_v}{\sigma_v-1}[\kappa_1(\bar{r})\kappa_2(i_N)]^{-1}\,\omega n_{i_N}^{\frac{1}{1-\sigma_v}} \ . \end{aligned}$$

[46] Because we normalized $\omega_{i_R} = 1$.

$n_{i_R} p_k k_{i_R} = n_{i_R} A^{\sigma_R - 1} \tilde{l}_{i_R} = \tilde{L}_k$, i.e., revenues equal total costs and also total effective labor. Labor market clearing yields

$$\tilde{L} = \tilde{L}_k + n_{i_R}\tilde{l}_{i_R} + n_{i_N}\tilde{l}_{i_N} = n_{i_R}\tilde{l}_{i_R}\hat{\theta}(A) + n_{i_N}\tilde{l}_{i_N} \ ,$$

where $\tilde{L} = \left[\tilde{\phi}_{\underline{r}} + \tilde{\phi}_{\bar{r}}\right] L$ denotes total effective labor. Total wages paid are

$$TW = \tilde{L}_k + n_{i_R}\tilde{l}_{i_R} + \omega n_{i_N}\tilde{l}_{i_N} = n_{i_R}\tilde{l}_{i_R}\hat{\theta}(A) + \omega n_{i_N}\tilde{l}_{i_N} \ .$$

Total wages paid and total consumption are linked through the following equation:

$$C = \kappa_1(\bar{r})\hat{\mathcal{M}}(\omega)^{\frac{1}{\sigma_I - 1}} TW \ . \tag{B.4}$$

B.2 Substitution and Income Effects

We rewrite Equations (20) and (21) to isolate the income and substitution effects,

$$\begin{aligned} \phi_{\bar{r}} L \frac{\sigma_v}{\sigma_v - 1}\frac{\partial \tilde{w}^{\bar{r}}}{\partial A} &= \frac{\partial C}{\partial A}\left[\frac{1-\sigma_I}{\sigma_I} s_{w^{\bar{r}}} + s_{w^{\bar{r}}}\right] \ , \\ \phi_{\underline{r}} L \frac{\sigma_v}{\sigma_v - 1}\frac{\partial \tilde{w}^{\underline{r}}}{\partial A} &= \frac{\partial C}{\partial A}\left[\frac{\sigma_I - 1}{\sigma_I} s_{w^{\bar{r}}} + s_{w^{\underline{r}}}\right] \ . \end{aligned} \tag{B.5}$$

The first term in the bracket denotes the substitution effect and the second term the income effect respectively. We now can isolate the income effect by assuming that $\sigma_I = 1$ (Cobb-Douglas Utility), i.e., there is no substitution effect. Then each of the group obtains a share of consumption gains according to each groups' share in total wages. This share in total wages stays constant if $\sigma_I = 1$.[47] If now σ_I is *not* equal to unity, then the substitution effect requires that due to technological progress, either more of service i_R is consumed (if $\sigma_I > 1$), which lowers demand for high-skilled labor and puts downward pressure on the scaling factor, or more of service i_N is consumed (if $\sigma_I < 1$), which raises demand for high-skilled labor and puts upward pressure on the scaling factor. The substitution effect shifts the wage shares earned by the two groups through changes in the scaling factor. Thereby, the income effect is either shifted towards high-skilled (if $\sigma_I < 1$) or low-skilled labor (if $\sigma_I > 1$).

[47] In Lemma 1, we show that in such case, the scaling factor stays constant.

B.3 Wage Dynamics with an Inelastic Elasticity

From Proposition 2 we can infer the two following corollaries:

Corollary 1

The real wage of low-skilled labor more likely decreases in A if $\sigma_I < 1$, the higher the current scaling factor.

Corollary 1 follows from the derivative $\frac{\partial}{\partial \omega} s_{w^{\bar{r}}} > 0$ and states that $s_{w^{\bar{r}}}$ increases whenever $\sigma_I < 1$, because in such case the scaling factor must increase (Lemma 1). This shift in the strength of the income effect and substitution effect implies the following corollary:

Corollary 2

If $\sigma_I < 1$ then the real wage of low-skilled labor eventually decreases when A grows large.

B.4 Effects

Table 2 summarizes the occurring effects in the ILM Equilibrium and the DLM Equilibrium. It presents the main variables of the model and illustrates whether or not they are affected by technological progress in capital production. Unaffected variables, i.e., those that stay constant, are marked with a 'c'. Variables that increase are marked with a '+' and variables that decrease are marked with a '−'. The effects of rising productivity in capital production within an ILM (DLM) Equilibrium are presented on the left (right) side of Table 2. Derivations are presented in Appendix D.6.

Table 2: Effects of Rising Productivity in Capital Production—Industrial Economy

Variable		ILM Aggregate	ILM $\underline{r}$	ILM $\overline{r}$	DLM Aggregate	DLM $\underline{r}$	DLM $\overline{r}$
1. Wage Premium	ω	c			$-$ $(\sigma_I > 1)$ $+$ $(\sigma_I < 1)$		
2. Price Index	P	$-$			$-$ $\left(\sigma_I > \frac{\omega\tilde{\phi}_{\bar{r}}}{\omega\tilde{\phi}_{\bar{r}}+\tilde{\phi}_{\underline{r}}}\right)$ $+$ $\left(\sigma_I < \frac{\omega\tilde{\phi}_{\bar{r}}}{\omega\tilde{\phi}_{\bar{r}}+\tilde{\phi}_{\underline{r}}}\right)$		
3. Real Wage	$\tilde{w}^r$		$+$	$+$		$-$ $\left(\sigma_I < \frac{\omega\tilde{\phi}_{\bar{r}}}{\omega\tilde{\phi}_{\bar{r}}+\tilde{\phi}_{\underline{r}}}\right)$ $+$ $\left(\sigma_I > \frac{\omega\tilde{\phi}_{\bar{r}}}{\omega\tilde{\phi}_{\bar{r}}+\tilde{\phi}_{\underline{r}}}\right)$	$+$
4. Service i_R	$n_{i_R}x_{i_R}$	$+$			$+$		
Service i_N	$n_{i_N}x_{i_N}$	$-$ $(\sigma_I > 1)$ $+$ $(\sigma_I < 1)$			c		
5. Consumption i_R	$c^r_{i_R}$		$+$	$+$		$-$ $\left(\sigma_I < \frac{\omega\tilde{\phi}_{\bar{r}}}{2\omega\tilde{\phi}_{\bar{r}}+\tilde{\phi}_{\underline{r}}}\right)$ $+$ $\left(\sigma_I > \frac{\omega\tilde{\phi}_{\bar{r}}}{2\omega\tilde{\phi}_{\bar{r}}+\tilde{\phi}_{\underline{r}}}\right)$	$+$
Consumption i_N	$c^r_{i_N}$		$-$ $(\sigma_I > 1)$ $+$ $(\sigma_I < 1)$	$-$ $(\sigma_I > 1)$ $+$ $(\sigma_I < 1)$		$-$ $(\sigma_I < 1)$ $+$ $(\sigma_I > 1)$	$-$ $(\sigma_I > 1)$ $+$ $(\sigma_I < 1)$

C Robotic Economy

C.1 Derivation of the Equilibrium in the Robotic Economy

Firms choose the labor they employ, given Assumption 1. The capital-producing industry is assumed to be competitive and the price of robots equals marginal costs: $p_k = \omega \left[A\kappa_1(\bar{r})\kappa_2(i_N)\right]^{-1}$. Note that the price of robots linearly depends on the scaling factor ω. Optimal relative inputs chosen by the representative firm i_R are

$$\frac{\tilde{l}_{i_R}}{k_{i_R}} = \left[\frac{\omega}{A}\right]^{\sigma_R} \kappa_1(\bar{r})^{-1}\kappa_2(i_N)^{-\sigma_R}\kappa_2(i_R)^{\sigma_R-1} \ ,$$

where we transformed labor into effective labor. Marginal costs, denoted by mc, of a representative firm i_R then are

$$mc_{i_R} = \left[\kappa_1(\bar{r})\kappa_2(i_R)\right]^{-1} \tilde{\theta}(A,\omega)^{\frac{1}{1-\sigma_R}} \ ,$$

where $\tilde{\theta}(A,\omega) = 1 + \mu(A,\omega)$.[48] The term $\mu(A,\omega) := \left[\frac{A}{\omega}\frac{\kappa_2(i_N)}{\kappa_2(i_R)}\right]^{\sigma_R-1}$ denotes the cost of capital optimally used as an input relative to one unit of effective labor. The higher the productivity of the robots, i.e., the higher A, the more of this input firm i_R would like to use. The contrary holds true for ω. Then the aggregate price index is

$$P = \frac{\sigma_v}{\sigma_v - 1}\kappa_1(\bar{r})^{-1}\tilde{\mathcal{M}}(\omega)^{\frac{1}{1-\sigma_I}} \ ,$$

where $\tilde{\mathcal{M}}(\omega) = n_{i_N}^{\frac{1-\sigma_I}{1-\sigma_v}}\kappa_2(i_N)^{\sigma_I-1}\omega^{1-\sigma_I} + n_{i_R}^{\frac{1-\sigma_I}{1-\sigma_v}}\kappa_2(i_R)^{\sigma_I-1}\tilde{\theta}(A,\omega)^{\frac{1-\sigma_I}{1-\sigma_R}}$.[49] The production function in the service industry i_R, with optimal relative input choice, is

$$x_{i_R} = \kappa_1(\bar{r})\kappa_2(i_R)\tilde{l}_{i_R}\tilde{\theta}(A,\omega)^{\frac{\sigma_R}{\sigma_R-1}} \ .$$

[48] Note the difference to $\hat{\theta}(A) = 1 + A^{\sigma_R-1}$, which we defined for the industrial economy.

[49] Note that $\tilde{\mathcal{M}}(\omega) \neq \hat{\mathcal{M}}(\omega)$. Prices and industry price aggregators are

$$p_{i_R} = \frac{\sigma_v}{\sigma_v-1}\left[\kappa_1(\bar{r})\kappa_2(i_R)\right]^{-1}\tilde{\theta}(A,\omega)^{\frac{1}{1-\sigma_R}} \quad \text{and} \quad p_{i_N} = \frac{\sigma_v}{\sigma_v-1}\left[\kappa_1(\bar{r})\kappa_2(i_N)\right]^{-1}\omega \ ,$$

$$P_{i_R} = \frac{\sigma_v}{\sigma_v-1}\left[\kappa_1(\bar{r})\kappa_2(i_R)\right]^{-1} n_{i_R}^{\frac{1}{1-\sigma_v}}\tilde{\theta}(A,\omega)^{\frac{1}{1-\sigma_R}} \quad \text{and} \quad P_{i_N} = \frac{\sigma_v}{\sigma_v-1}\left[\kappa_1(\bar{r})\kappa_2(i_N)\right]^{-1} n_{i_N}^{\frac{1}{1-\sigma_v}}\omega \ .$$

Total household demand faced by the representative firms of industries i_R and i_N is

$$\begin{aligned} c_{i_R} =& n_{i_R}^{\frac{\sigma_v-\sigma_I}{1-\sigma_v}} \kappa_2(i_R)^{\sigma_I} \tilde{\theta}(A,\omega)^{\frac{\sigma_I}{\sigma_R-1}} \tilde{\mathcal{M}}(\omega)^{\frac{\sigma_I}{1-\sigma_I}} C \ , \\ c_{i_N} =& n_{i_N}^{\frac{\sigma_v-\sigma_I}{1-\sigma_v}} \kappa_2(i_N)^{\sigma_I} \omega^{-\sigma_I} \tilde{\mathcal{M}}(\omega)^{\frac{\sigma_I}{1-\sigma_I}} C \ . \end{aligned} \tag{C.1}$$

Industry i_R's demand for capital, k_{i_R}, results in a demand for high-skilled labor, denoted by $\tilde{L}_k$. As the capital-producing industry is competitive, all revenues are paid to workers, i.e., $n_{i_R} p_k k_{i_R} = \omega \tilde{L}_k$, and thus

$$\omega \tilde{L}_k = n_{i_R} \tilde{l}_{i_R} \mu(A,\omega) \ .$$

The rationale for this equality is the following: Per unit of effective routine labor input, industry i_R demands capital at cost $\mu(A,\omega)$. In capital production all revenues are paid to the sole factor input, which is non-routine labor in the robotic economy.

We impose market clearing and solve for the effective labor demand of firm i_R,

$$\tilde{l}_{i_R} = [\kappa_1(\bar{r})\kappa_2(i_R)]^{-1} \, n_{i_R}^{\frac{\sigma_v-\sigma_I}{1-\sigma_v}} \kappa_2(i_R)^{\sigma_I} \tilde{\theta}(A,\omega)^{\frac{\sigma_I-\sigma_R}{\sigma_R-1}} \tilde{\mathcal{M}}(\omega)^{\frac{\sigma_I}{1-\sigma_I}} C \ . \tag{C.2}$$

Analogously, we apply market clearing to the service industry i_N and a firm i_N's demand for effective labor then is

$$\tilde{l}_{i_N} = [\kappa_1(\bar{r})\kappa_2(i_N)]^{-1} \, n_{i_N}^{\frac{\sigma_v-\sigma_I}{1-\sigma_v}} \kappa_2(i_N)^{\sigma_I} \omega^{-\sigma_I} \tilde{\mathcal{M}}(\omega)^{\frac{\sigma_I}{1-\sigma_I}} C \ . \tag{C.3}$$

Labor market clearing implies that

$$\tilde{L} = \tilde{L}_k + n_{i_R} \tilde{l}_{i_R} + n_{i_N} \tilde{l}_{i_N} \ ,$$

and total wages paid are

$$TW = \omega \tilde{L}_k + n_{i_R} \tilde{l}_{i_R} + \omega n_{i_N} \tilde{l}_{i_N} = \ n_{i_R} \tilde{l}_{i_R} \tilde{\theta}(A,\omega) + \omega n_{i_N} \tilde{l}_{i_N} \ .$$

Aggregate consumption then is a function of total wages paid to workers,

$$C = \kappa_1(\bar{r}) \tilde{\mathcal{M}}(\omega)^{\frac{1}{\sigma_I-1}} TW \ . \tag{C.4}$$

C.2 Numerical Examples for Real Wage Decreases of the Low-skilled

To analyze the conditions under which real wages for the low-skilled decrease, it is helpful to study the process of labor market separation. When the labor market is just integrated ($\omega = 1$), then, $z = \frac{\tilde{\phi}_{\underline{r}}}{\tilde{\phi}_{\bar{r}}} \mu(A,1)$, with $\mu(A,1) := \left[A\frac{\kappa_2(i_N)}{\kappa_2(i_R)}\right]^{\sigma_R - 1}$. Thus, keeping all other parameters constant, there exists a technological level at which the labor market disintegration starts, i.e., where $\omega = 1$ and a marginal increase in A separates the market. This technological level is denoted by A^s, where the superscript s stands for "separation". A^s solves the following equation

$$\mathcal{F}^s = \tilde{X}\left[1 + \mu(A^s,1)\right]^{\frac{\sigma_I - \sigma_R}{1-\sigma_R}} - \frac{\tilde{\phi}_{\bar{r}}}{\tilde{\phi}_{\underline{r}}} + \mu(A^s,1) = 0 \ .$$

$\mathcal{F}^s$ is strictly increasing in A^s, and there is a unique solution.

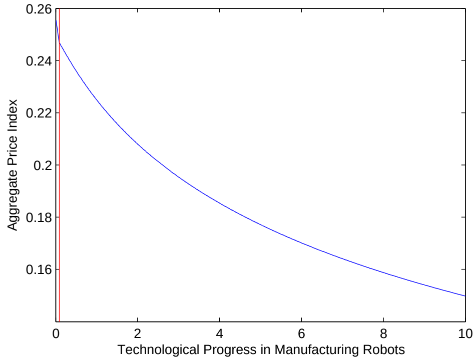
We now analyze Condition (27) in more detail at the technological level A^s. At the point of labor market separation, the right-hand side of (27) is minimized. If we can show that there exist parameters that result in

$$[\sigma_R - \sigma_I]\,\frac{1 + \frac{\tilde{\phi}_{\bar{r}}}{\tilde{\phi}_{\underline{r}}}}{1 + \mu(A^s,1)} > \frac{1}{1 - \frac{\tilde{\phi}_{\underline{r}}}{\tilde{\phi}_{\bar{r}}}\mu(A^s,1)} \ , \tag{C.5}$$

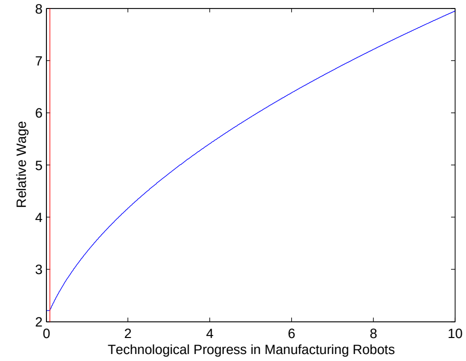
we know that there are some stretches along the evolution of A where routine labor looses even in real terms.[50] If A grows sufficiently large, real wages of the low-skilled must increase, as the right-hand side of (27) grows to infinity, whereas the left-hand side is always finite, given finite σ_R (see also Lemma 8). Inequality (C.5) indeed holds if parameters are such that $\sigma_R - \sigma_I \gg 1$ (σ_R large), and $\frac{\tilde{\phi}_{\underline{r}}}{\tilde{\phi}_{\bar{r}}}\mu(A^s,1)$ small, then the real wage of routine workers decreases right after labor market separation. Intuitively, a marginal increase in the technological factor has a higher effect on the replacement of low-skilled labor, the higher σ_R and the smaller the level of A. The higher A, the smaller the replacement effect of low-skilled labor due to a marginal increase in A and thus the efficiency gain through technology (lower prices) surpasses the effect of replacement.

[50] This was already the case in Lemma 9 under strong assumptions.

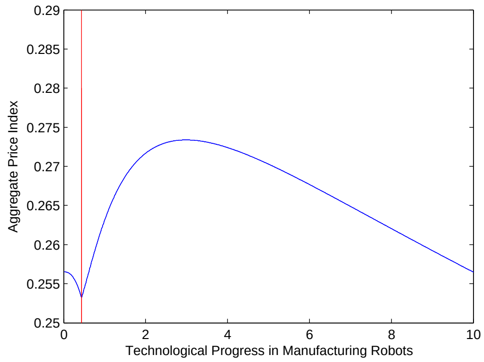
Figure 1: Aggregate Price Index and Relative Wages



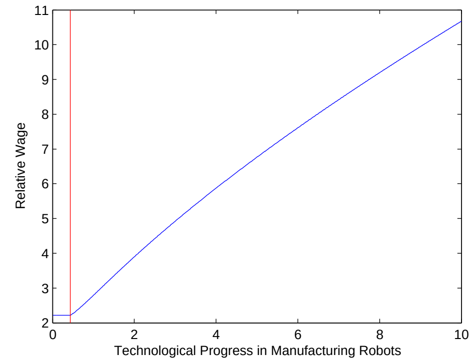
(a) Price index ($\sigma_R = 2$)



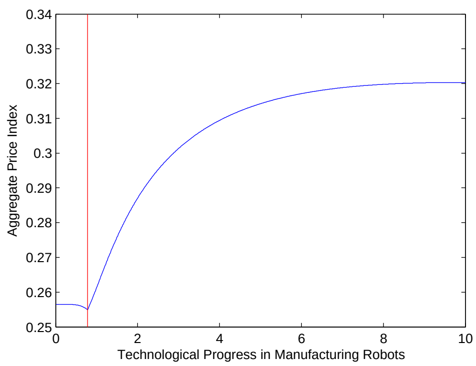
(b) Relative wage ($\sigma_R = 2$)



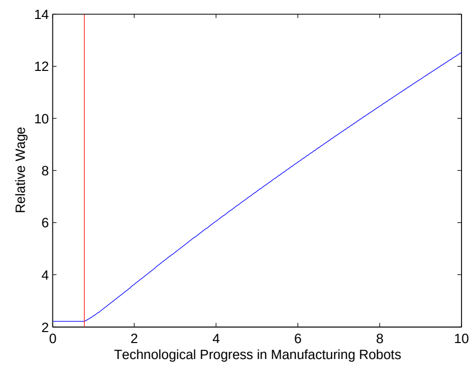
(c) Price index ($\sigma_R = 3.5$)



(d) Relative wage ($\sigma_R = 3.5$)



(e) Price index ($\sigma_R = 6$)



(f) Relative wage ($\sigma_R = 6$)

Figure (1) demonstrates the evolution of the price index and relative wages, $\frac{w^{\bar{r}}}{w^{\underline{r}}}$, with respect to different parameter choices of σ_R ($\sigma_R = 2$, $\sigma_R = 3.5$, $\sigma_R = 6$) and for an exogenous evolution of productivity parameter A.[51] Figure (1a) demonstrates a situation where low-skilled workers benefit from technological progress right from the

[51] The remaining parameters are $i_R = 0.5$, $i_N = 1$, $n_{i_R} = 2$, $n_{i_N} = 1$, $\bar{r} = 0.9$, $\lambda = 2$, $\sigma_I = 1.6$, $\sigma_v = 2.3$, $\tilde{\phi}_{\bar{r}} = 0.4$, and $\tilde{\phi}_{\underline{r}} = 0.6$.

start of labor market separation. In this case, the productivity effect dominates the replacement effect. However, evidence in the literature suggests rather high replacement effects due to robots (Acemoglu and Restrepo, 2017). Thus, in line with our assumption of high substitution elasticities among input factors, a high parameter choice of σ_R seems appropriate. Parameter estimates in the literature range from the lower-bound estimate 1.9 (DeCanio, 2016) up to 4 (Hémous and Olsen, 2016).[52] Figures (1c), (1d), (1e) and (1f) present the dynamics when σ_R is in a higher range. In these cases, the real wage of low-skilled workers falls first and raises again with increasing technological advancement. For high substitution elasticity, σ_R, technology needs to improve much more than with low σ_R after labor market separation, until low-skilled workers benefit again from the technological evolution. Note that the technological level of labor market separation, A^s, increases in higher values of σ_R. This is because non-routine labor must become sufficiently productive, such that firms increasingly produce with robots, when robots and low-skilled labor are close substitutes.

- Figure (1a) and (1b) with $\sigma_R = 2$, and $A^s \approxeq 0.088$,
- Figure (1c) and (1d) with $\sigma_R = 3.5$, and $A^s \approxeq 0.437$,
- Figure (1e) and (1f) with $\sigma_R = 6$, and $A^s \approxeq 0.779$.

C.3 Limits

In the following, we assume that $\sigma_I > 1$. Then, z (as defined in (23)) always increases in A, i.e., the routine service industry expands relative to the non-routine service industry in demanding non-routine labor for the production of robots, and the right-hand side of (27) strictly increases in A. In contrast, the left-hand side of (27) can increase or decrease. Therefore, real wages of the low-skilled can increase or decrease as a reaction to technological progress.

Lemma 8

(i) Assume σ_R is bounded. Then if A is large enough, the real wage of low-skilled labor always increases.

[52] In Section 5.1 we discuss this parameter.

(ii) When A is large, the elasticity of the scaling factor with respect to A, $\sigma_{A,\omega}$, converges to $\frac{\sigma_R - 1}{\sigma_R}$.

The proof is given in Appendix D.8. We now study the case of perfect substitutability between labor and robots in industry i_R.

Lemma 9

Assume $\sigma_R \to \infty$, then the real wage of the low-skilled workers decreases in A.

The proof is given in Appendix D.9. Intuitively, when firms of industry i_R are indifferent between producing with labor or robots, they choose the cheaper input factor for the production. The cost of effective labor is always 1. Thus, whenever the price of robots, p_k, falls below this threshold, firms of industry i_R solely want to produce with robots. Real wages of the low-skilled must then fall and the scaling factor rises to keep up with the technological advancement of robots. Thus, ω must be such that the price of robots equals the price of labor, i.e., $p_k = 1$.[53]

C.4 Effects

Table 3 summarizes the effects of technological progress in capital production in an ILM Equilibrium and in a DLM Equilibrium. The dynamics of aggregate variables are indicated, as well as the dynamics of variables at the household level. The remaining proofs of these effects are given in Appendix D.10, in particular the effects of A on consumption in the DLM Equilibrium.[54] Again we assume that profits are distributed proportional to real wages, and thus (19) denotes a household r's consumption.

Note that in Table 3, $\kappa_{A,\omega}$, the critical value of σ_I, indicating regime change, is always less than unity. $\kappa_{A,\omega}$ is equal to the term in Lemma 6, i.e., higher demand for high-skilled labor to produce robots mechanically diminishes high-skilled labor demand in industry i_N, and thus directly infers less output in this industry. The derivation of $\kappa_{A,\omega}$ is presented in Appendix D.5. The high-skilled essentially receive a higher real

[53] Note that we abstain from analyzing $A \to \infty$ in conjunction with $\sigma_R \to \infty$.

[54] An increase of a variable in response to a rise in A is indicated by a '+', and a decrease is indicated by a '−'. A variable that remains constant is simply presented with a 'c'. If an increase or a decrease can occur, then a condition under which the increase/decrease occurs is presented.

wage through lower prices for services i_R and through the scaling factor they earn. The aggregate price level might increase, however, the increase in the scaling factor overcompensates high-skilled workers for a potentially higher aggregate price index. Low-skilled workers may have a decreasing or an increasing real wage, depending on (27). A low-skilled household consumes less of service i_R if $\sigma_I < \kappa_{i_R,R}$. The value of $\kappa_{i_R,R}$ is always smaller than unity (see Table 3 and Appendix D.10). This essentially means that if a household endowed with a low skill level consumes more of services i_N in response to technological progress, services must be stronger complements than if a household of a high skill level consumes more of services i_N. In other words, high-skilled households always consume more of service i_N when A increases if the two services are complements. For low-skilled households, this must not be the case. The reason is that for low-skilled households, the service i_R becomes cheaper through technological progress, while the service i_N becomes more expensive through the increase in the scaling factor. Whenever a low-skilled worker's real wage decreases, he consumes less of i_N-services. Now, if (27) holds, there are values for σ_I for which the low-skilled decrease their consumption in both services. This is the case when $\kappa_{i_R,R} > \sigma_I > 0 > \kappa_{i_N,R}$, i.e., whenever the real wage of low-skilled decreases and $\sigma_I \in (0, \kappa_{i_R,R})$.

Table 3: Effects of Rising Productivity in Capital Production—Robotic Economy

Variable		ILM Equilibrium			DLM Equilibrium		
		Aggregate	$\underline{r}$	$\overline{r}$	Aggregate	$\underline{r}$	$\overline{r}$
1. Wage Premium	ω	c			$+$		
2. Price Index	P	$-$			$-/+$ (27)		
3. Real Wage	$\tilde{w}^r$		$+$	$+$		$-/+$ (27)	$+$
4. Service i_R	$n_{i_R} x_{i_R}$	$+$			$+$		
Service i_N	$n_{i_N} x_{i_N}$	$-$ $(\sigma_I > 1)$ $+$ $(\sigma_I < 1)$			$-$ $(\sigma_I > \kappa_{A,\omega})$ $+$ $(\sigma_I < \kappa_{A,\omega})$		
5. Consumption i_R	$c^r_{i_R}$		$+$	$+$		$-$ $(\sigma_I < \kappa_{i_R,R})^\dagger$ $+$ $(\sigma_I > \kappa_{i_R,R})$	$+$
Consumption i_N	$c^r_{i_N}$		$-$ $(\sigma_I > 1)$ $+$ $(\sigma_I < 1)$	$-$ $(\sigma_I > 1)$ $+$ $(\sigma_I < 1)$		$-$ $(\sigma_I > \kappa_{i_N,R})^\star$ $+$ $(\sigma_I < \kappa_{i_N,R})$	$-$ $(\sigma_I > 1)$ $+$ $(\sigma_I < 1)$

where $\kappa_{A,\omega} := \frac{\mu(A,\omega)}{\mu(A,\omega)+\frac{\sigma_R-1}{\sigma_R}} (< 1)$, and $\kappa_{i_N,R} := \frac{\mu(A,\omega)\left[1-\sigma_{A,\omega} z^{-1}\right]}{\mu(A,\omega)+\sigma_{A,\omega}} (< 1)$, and $\kappa_{i_R,R} := \frac{\sigma_{A,\omega}-z}{1-z} \frac{1+\mu(A,\omega)}{\sigma_{A,\omega}+\mu(A,\omega)}$, and $\mu(A,\omega) := \left[\frac{A}{\omega} \frac{\kappa_2(i_N)}{\kappa_2(i_R)}\right]^{\sigma_R-1}$.

† If (27) holds, then consumption of service i_R by the low-skilled may or may not decrease. Otherwise, it always increases.

* If (27) holds, then consumption of service i_N by the low-skilled always decreases. However, the contrary is not necessarily true.

D Proofs

D.1 Proof of Lemma 3

Here, we derive that $\frac{\partial \tilde{L}^d_{i_N}}{\partial A} > 0$ for $\sigma_R > \max\{1, \sigma_I\}$. We use effective labor demand (C.2) and (C.3), and $\tilde{L}^d_{i_N} = n_{i_N}\tilde{l}_{i_N} + n_{i_R}\tilde{l}_{i_R}\mu(A,1)$, and $C = \kappa_1(\bar{r})\tilde{\mathcal{M}}(1)^{\frac{1}{\sigma_I - 1}}\tilde{L}$.

For notational convenience we define $j_N = n_{i_N}^{\frac{1-\sigma_I}{1-\sigma_v}}\kappa_2(i_N)^{\sigma_I - 1}$, and $j_R = n_{i_R}^{\frac{1-\sigma_I}{1-\sigma_v}}\kappa_2(i_R)^{\sigma_I - 1}$, and $j_A = [\sigma_R - 1]\left[\frac{\kappa_2(i_N)}{\kappa_2(i_R)}\right]^{\sigma_R - 1} A^{\sigma_R - 2}$. Note that when $\omega = 1$, then $\tilde{\mathcal{M}}(1) = j_N + j_R\tilde{\theta}(A,1)^{\frac{1-\sigma_I}{1-\sigma_R}}$ and that $1 - \mu(A,1)\tilde{\theta}(A,1)^{-1} = \tilde{\theta}(A,1)^{-1}$.

We take the partial derivative of labor demand from the non-routine service industry with respect to the technological factor A and obtain[55]

$$
\begin{aligned}
\frac{\partial \tilde{L}^d_{i_n}}{\partial A} =& \frac{\partial}{\partial A}\left\{j_N + j_R\tilde{\theta}(A,1)^{\frac{\sigma_I - \sigma_R}{\sigma_R - 1}}\mu(A,1)\right\}\frac{\tilde{L}}{\tilde{\mathcal{M}}(1)} \\
=& j_N j_R j_A \frac{1-\sigma_I}{\sigma_R - 1}\tilde{\theta}(A,1)^{\frac{\sigma_I - \sigma_R}{\sigma_R - 1}}\frac{\tilde{L}}{\tilde{\mathcal{M}}(1)^2} + j_R j_A \frac{\sigma_I - \sigma_R}{\sigma_R - 1}\tilde{\theta}(A,1)^{\frac{\sigma_I - \sigma_R}{\sigma_R - 1} - 1}\mu(A,1)\frac{\tilde{L}}{\tilde{\mathcal{M}}(1)} + \\
& j_R^2 j_A \frac{1-\sigma_I}{\sigma_R - 1}\tilde{\theta}(A,1)^{2\frac{\sigma_I - \sigma_R}{\sigma_R - 1}}\mu(A,1)\frac{\tilde{L}}{\tilde{\mathcal{M}}(1)^2} + j_R j_A \tilde{\theta}(A,1)^{\frac{\sigma_I - \sigma_R}{\sigma_R - 1}}\frac{\tilde{L}}{\tilde{\mathcal{M}}(1)} \\
=& \frac{\tilde{L}}{\tilde{\mathcal{M}}(1)^2}\frac{j_A j_R}{\sigma_R - 1}\tilde{\theta}(A,1)^{\frac{\sigma_I - \sigma_R}{\sigma_R - 1}}\Bigg\{[1-\sigma_I]\, j_N + [\sigma_I - \sigma_R]\frac{\mu(A,1)}{\tilde{\theta}(A,1)}\tilde{\mathcal{M}}(1) + \\
& [1-\sigma_I]\, j_R\tilde{\theta}(A,1)^{\frac{\sigma_I - \sigma_R}{\sigma_R - 1}}\mu(A,1) + [\sigma_R - 1]\,\tilde{\mathcal{M}}(1)\Bigg\} \\
=& \frac{\tilde{L}}{\tilde{\mathcal{M}}(1)^2}\frac{j_A j_R}{\sigma_R - 1}\tilde{\theta}(A,1)^{\frac{\sigma_I - \sigma_R}{\sigma_R - 1}}\Bigg\{[1-\sigma_I]\, j_N + [\sigma_I - \sigma_R]\frac{\mu(A,1)}{\tilde{\theta}(A,1)} j_N + \\
& [\sigma_I - \sigma_R]\, j_R\tilde{\theta}(A,1)^{\frac{\sigma_I - \sigma_R}{\sigma_R - 1}}\mu(A,1) + \\
& [1-\sigma_I]\, j_R\tilde{\theta}(A,1)^{\frac{\sigma_I - \sigma_R}{\sigma_R - 1}}\mu(A,1) + [\sigma_R - 1]\, j_N + [\sigma_R - 1]\, j_R\tilde{\theta}(A,1)^{\frac{1-\sigma_I}{1-\sigma_R}}\Bigg\} \\
=& \frac{\tilde{L}}{\tilde{\mathcal{M}}(1)^2}\frac{j_A j_R}{\sigma_R - 1}\tilde{\theta}(A,1)^{\frac{\sigma_I - \sigma_R}{\sigma_R - 1}}\Bigg\{[\sigma_R - \sigma_I]\, j_N + [\sigma_I - \sigma_R]\frac{\mu(A,1)}{\tilde{\theta}(A,1)} j_N + \\
& [1-\sigma_R]\, j_R\tilde{\theta}(A,1)^{\frac{\sigma_R - \sigma_I}{1-\sigma_R}}\mu(A,1) + [\sigma_R - 1]\, j_R\tilde{\theta}(A,1)^{\frac{1-\sigma_I}{1-\sigma_R}}\Bigg\} \\
=& \frac{\tilde{L}}{\tilde{\mathcal{M}}(1)^2}\frac{j_A j_R}{\sigma_R - 1}\tilde{\theta}(A,1)^{\frac{\sigma_I - \sigma_R}{\sigma_R - 1}}\left\{[\sigma_R - \sigma_I]\, j_N\tilde{\theta}(A,1)^{-1} + [\sigma_R - 1]\, j_R\tilde{\theta}(A,1)^{\frac{\sigma_I - \sigma_R}{\sigma_R - 1}}\right\} > 0\ .
\end{aligned}
$$

The last inequality holds if $\sigma_R > \max\{1, \sigma_I\}$ and hence we proved Lemma 3. □

[55] The curly brackets are just normal brackets. They are solely used to facilitate the orientation.

D.2 Derivation of Equation 26

We start with $\tilde{w}^{\underline{r}} = \frac{w^{\underline{r}}}{P} = \frac{\sigma_v-1}{\sigma_v}\kappa_1(\underline{r})\tilde{\mathcal{M}}(\omega)^{\frac{1}{\sigma_I-1}}$. Using $\tilde{\mathcal{M}}(\omega) := n_{i_N}^{\frac{1-\sigma_I}{1-\sigma_v}}\kappa_2(i_N)^{\sigma_I-1}\omega^{1-\sigma_I} + n_{i_R}^{\frac{1-\sigma_I}{1-\sigma_v}}\kappa_2(i_R)^{\sigma_I-1}\tilde{\theta}(A,\omega)^{\frac{1-\sigma_I}{1-\sigma_R}}$, we take the derivative with respect to A,

$$\begin{aligned}\frac{\partial \tilde{w}^{\underline{r}}}{\partial A} = & \frac{\sigma_v-1}{\sigma_v}\kappa_1(\underline{r})\frac{1}{\sigma_I-1}\tilde{\mathcal{M}}(\omega)^{\frac{1}{\sigma_I-1}-1}\left[n_{i_N}^{\frac{1-\sigma_I}{1-\sigma_v}}\kappa_2(i_N)^{\sigma_I-1}[1-\sigma_I]\omega^{-\sigma_I}\frac{\partial\omega}{\partial A}-\right.\\ & \left. n_{i_R}^{\frac{1-\sigma_I}{1-\sigma_v}}\kappa_2(i_R)^{\sigma_I-1}[1-\sigma_I]\tilde{\theta}(A,\omega)^{\frac{\sigma_R-\sigma_I}{1-\sigma_R}}\mu(A,\omega)\left[\frac{1}{A}-\frac{1}{\omega}\frac{\partial\omega}{\partial A}\right]\right] ,\\ = & \frac{\sigma_v-1}{\sigma_v}\kappa_1(\underline{r})\tilde{\mathcal{M}}(\omega)^{\frac{1}{\sigma_I-1}-1}\left[n_{i_R}^{\frac{1-\sigma_I}{1-\sigma_v}}\kappa_2(i_R)^{\sigma_I-1}\tilde{\theta}(A,\omega)^{\frac{\sigma_R-\sigma_I}{1-\sigma_R}}\frac{\mu(A,\omega)}{A}-\right.\\ & \left.\left[n_{i_N}^{\frac{1-\sigma_I}{1-\sigma_v}}\kappa_2(i_N)^{\sigma_I-1}\omega^{-\sigma_I}+n_{i_R}^{\frac{1-\sigma_I}{1-\sigma_v}}\kappa_2(i_R)^{\sigma_I-1}\tilde{\theta}(A,\omega)^{\frac{\sigma_R-\sigma_I}{1-\sigma_R}}\frac{\mu(A,\omega)}{\omega}\right]\frac{\partial\omega}{\partial A}\right] .\end{aligned}$$

D.3 Proof of Lemma 5

For notational convenience we use $j_N = n_{i_N}^{\frac{1-\sigma_I}{1-\sigma_v}}\kappa_2(i_N)^{\sigma_I-1}$, and $j_R = n_{i_R}^{\frac{1-\sigma_I}{1-\sigma_v}}\kappa_2(i_R)^{\sigma_I-1}$, and $\mu(A,\omega) = \left[\frac{A}{\omega}\frac{\kappa_2(i_N)}{\kappa_2(i_R)}\right]^{\sigma_R-1}$. From (26) we know that

$$\begin{aligned}\frac{\partial \tilde{w}^{\underline{r}}}{\partial A} = & \frac{\sigma_v-1}{\sigma_v}\kappa_1(\underline{r})\tilde{\mathcal{M}}(\omega)^{\frac{1}{\sigma_I-1}-1}\times\\ & \left[j_R\tilde{\theta}(A,\omega)^{\frac{\sigma_R-\sigma_I}{1-\sigma_R}}\frac{\mu(A,\omega)}{A}-\left[j_N\omega^{-\sigma_I}+j_R\tilde{\theta}(A,\omega)^{\frac{\sigma_R-\sigma_I}{1-\sigma_R}}\frac{\mu(A,\omega)}{\omega}\right]\frac{\partial\omega}{\partial A}\right] .\end{aligned}$$

We divide by $j_R\tilde{\theta}(A,\omega)^{\frac{\sigma_R-\sigma_I}{1-\sigma_R}}\frac{\mu(A,\omega)}{\omega}\frac{\partial\omega}{\partial A}$ and focus on the the term in brackets, which determines the sign. Suppose now the following term is negative

$$\frac{\omega}{A}\frac{\partial A}{\partial\omega} - \tilde{X}\omega^{1-\sigma_I}\tilde{\theta}(A,\omega)^{\frac{\sigma_R-\sigma_I}{\sigma_R-1}}\mu(A,\omega)^{-1} - 1 < 0 , \tag{D.1}$$

where $\tilde{X} = \frac{j_N}{j_R}$. From $\mathcal{F}$ in (22), we know that $\tilde{X}\tilde{\theta}(A,\omega)^{\frac{\sigma_R-\sigma_I}{\sigma_R-1}}\omega^{-\sigma_I} = \frac{\tilde{\phi}_{\bar{r}}}{\tilde{\phi}_{\underline{r}}}[1-z]$. We further use (23) and (D.1) to obtain

$$\frac{\omega}{A}\frac{\partial A}{\partial\omega} - \frac{1}{z} < 0 .$$

The partial derivative of the real wage of a low-skilled worker with respect to technological change in a DLM equilibrium of a robotic economy is negative whenever

$$\sigma_{A,\omega} > z ,$$

and positive whenever the inequality is reversed. □

D.4 Proof of Condition 27

We use the Implicit Function Theorem to rewrite the condition presented in Lemma 5 and Appendix D.3, i.e.

$$\frac{\partial \mathcal{F}}{\partial A} > -z \frac{\partial \mathcal{F}}{\partial \omega} \frac{\omega}{A} \ .$$

Using again $\tilde{X} \tilde{\theta}(A, \omega)^{\frac{\sigma_R - \sigma_I}{\sigma_R - 1}} \omega^{-\sigma_I} = \frac{\tilde{\phi}_{\bar{r}}}{\tilde{\phi}_{\underline{r}}} [1 - z]$, and noting that $\tilde{\theta}(A, \omega) = 1 + z\omega \frac{\tilde{\phi}_{\bar{r}}}{\tilde{\phi}_{\underline{r}}}$, we arrive after some algebraic manipulations at

$$[\sigma_R - \sigma_I] \frac{1 + \omega \frac{\tilde{\phi}_{\bar{r}}}{\tilde{\phi}_{\underline{r}}}}{1 + z\omega \frac{\tilde{\phi}_{\bar{r}}}{\tilde{\phi}_{\underline{r}}}} > \frac{1}{1 - z} \ .$$

The condition shows under what circumstances the real wage of low-skilled workers decreases with technological progress in capital production. □

D.5 Proof of Lemma 6

We show that the partial derivative of z with respect to A, $\frac{\partial z}{\partial A}$, is greater than zero if $\sigma_{A,\omega} < \frac{\sigma_R - 1}{\sigma_R}$. Using the Implicit Function Theorem this inequality can be rewritten as

$$\frac{1 - \sigma_R}{\sigma_R} \frac{\partial \mathcal{F}}{\partial \omega} > \frac{A}{\omega} \frac{\partial \mathcal{F}}{\partial A} \ .$$

Further manipulations yield

$$\begin{aligned}
& \frac{1 - \sigma_R}{\sigma_R} \tilde{X} \tilde{\theta}(A, \omega)^{\frac{\sigma_I - \sigma_R}{1 - \sigma_R}} \omega^{-\sigma_I} \left[[\sigma_I - \sigma_R] \frac{\mu(A, \omega)}{\tilde{\theta}(A, \omega)} - \sigma_I \right] - [1 - \sigma_R] \frac{\mu(A, \omega)}{\omega} \\
& > \mu(A, \omega) \left[\tilde{X} [\sigma_R - \sigma_I] \tilde{\theta}(A, \omega)^{\frac{\sigma_I - 1}{1 - \sigma_R}} \omega^{-\sigma_I} + [\sigma_R - 1] \omega^{-1} \right] \\
\iff \quad & \frac{1 - \sigma_R}{\sigma_R} \tilde{X} \tilde{\theta}(A, \omega)^{\frac{\sigma_I - \sigma_R}{1 - \sigma_R}} \omega^{-\sigma_I} \left[[\sigma_I - \sigma_R] \frac{\mu(A, \omega)}{\tilde{\theta}(A, \omega)} - \sigma_I \right] \\
& > \mu(A, \omega) \tilde{X} [\sigma_R - \sigma_I] \tilde{\theta}(A, \omega)^{\frac{\sigma_I - 1}{1 - \sigma_R}} \omega^{-\sigma_I} \\
\iff \quad & \frac{1 - \sigma_R}{\sigma_R} \tilde{\theta}(A, \omega) \left[[\sigma_I - \sigma_R] \frac{\mu(A, \omega)}{\tilde{\theta}(A, \omega)} - \sigma_I \right] + [\sigma_I - \sigma_R] \mu(A, \omega) > 0 \\
\iff \quad & [1 - \sigma_R] \left[[\sigma_I - \sigma_R] \mu(A, \omega) - \sigma_I \tilde{\theta}(A, \omega) \right] + \sigma_R [\sigma_I - \sigma_R] \mu(A, \omega) > 0 \qquad (\star) \\
\iff \quad & \frac{\sigma_R - 1}{\sigma_R} + \frac{\sigma_I - 1}{\sigma_I} \mu(A, \omega) > 0 \ .
\end{aligned}$$

where $\tilde{\theta}(A,\omega) = 1 + \mu(A,\omega)$. Whenever $\frac{\sigma_R - 1}{\sigma_R} + \frac{\sigma_I - 1}{\sigma_I}\mu(A,\omega) > 0$ the share of high-skilled workers demanded in the production of robots increases with the productivity gains in the capital-producing industry. This is equivalent to the expression stated in Lemma 6. □

Repeating the manipulations starting at $\sigma_{A,\omega} < 1$ (instead of $\sigma_{A,\omega} < \frac{\sigma_R - 1}{\sigma_R}$), we can show that this inequality indeed must be true. Intuitively, the elasticity of the scaling factor with respect to technological progress can never exceed 1, i.e., the scaling factor cannot increase faster than the productivity gains from an increase in A. We start at the line indicated by ($\star$) in the above derivation, divide by σ_R, and replace the resulting factor $\frac{1-\sigma_R}{\sigma_R}$ with (-1) and obtain

$$\begin{aligned} 0 <& - \left[\left[\sigma_I - \sigma_R\right] \mu(A,\omega) - \sigma_I \tilde{\theta}(A,\omega) \right] + \left[\sigma_I - \sigma_R\right] \mu(A,\omega) \\ =& \sigma_I \tilde{\theta}(A,\omega) \ . \end{aligned}$$

This is essentially the same as taking the limit $\sigma_R \to \infty$. □

D.6 Derivations for Table 2

For the analysis of a single household's consumption we must know how profits are distributed in the economy. For sake of simplicity we assumed in the main text that profits are distributed proportional to the wage distribution, (19). Total income of a household must equal its consumption,

$$\begin{aligned} C^{\bar{r}} =& \kappa_1(\bar{r}) \hat{\mathcal{M}}(\omega)^{\frac{1}{\sigma_I - 1}} \omega \ , \\ C^{\underline{r}} =& \kappa_1(\underline{r}) \hat{\mathcal{M}}(\omega)^{\frac{1}{\sigma_I - 1}} \ . \end{aligned}$$

Then, household's demand (B.1) lets us obtain a single household's consumption in each industry,

$$\begin{aligned} C^{\bar{r}}_{i_N} =& n_{i_N}^{\frac{\sigma_I}{\sigma_v - 1}} \kappa_1(\bar{r}) \kappa_2(i_N)^{\sigma_I} \omega^{1-\sigma_I} \hat{\mathcal{M}}(\omega)^{-1} \ , \\ C^{\bar{r}}_{i_R} =& n_{i_R}^{\frac{\sigma_I}{\sigma_v - 1}} \kappa_1(\bar{r}) \kappa_2(i_R)^{\sigma_I} \hat{\theta}(A)^{\frac{\sigma_I}{\sigma_R - 1}} \omega \hat{\mathcal{M}}(\omega)^{-1} \ , \\ C^{\underline{r}}_{i_N} =& n_{i_N}^{\frac{\sigma_I}{\sigma_v - 1}} \kappa_1(\underline{r}) \kappa_2(i_N)^{\sigma_I} \omega^{-\sigma_I} \hat{\mathcal{M}}(\omega)^{-1} \ , \\ C^{\underline{r}}_{i_R} =& n_{i_R}^{\frac{\sigma_I}{\sigma_v - 1}} \kappa_1(\underline{r}) \kappa_2(i_R)^{\sigma_I} \hat{\theta}(A)^{\frac{\sigma_I}{\sigma_R - 1}} \hat{\mathcal{M}}(\omega)^{-1} \ . \end{aligned}$$

We can now obtain all consumption decisions of households. For notational convenience, we use $j_A = [\sigma_R - 1]\left[\frac{\kappa_2(i_N)}{\kappa_2(i_R)}\right]^{\sigma_R-1} A^{\sigma_R-2}$ and $j_N = n_{i_N}^{\frac{1-\sigma_I}{1-\sigma_v}}\kappa_2(i_N)^{\sigma_I-1}$.

- Consumption of service i_N by a high-skilled household:

$$
\begin{aligned}
\frac{\partial c_{i_N}^{\bar{r}}}{\partial A} =& \frac{\partial}{\partial A} n_{i_N}^{-1} j_N \kappa_1(\bar{r})\kappa_2(i_N)\omega^{1-\sigma_I}\hat{\mathcal{M}}(\omega)^{-1}w^{\bar{r}} \\
=& \left[1-\sigma_I\right] n_{i_N}^{-1} j_N \kappa_1(\bar{r})\kappa_2(i_N)\omega^{1-\sigma_I}\hat{\mathcal{M}}(\omega)^{-2}\hat{\theta}(A)^{-1}A^{\sigma_R-2}w^{\bar{r}}\times \\
& \left[\frac{1-\sigma_I}{\sigma_I}\hat{\mathcal{M}}(\omega) - \frac{1-\sigma_I}{\sigma_I} j_N\omega^{1-\sigma_I} + j_R\hat{\theta}(A)^{\frac{1-\sigma_I}{1-\sigma_R}}\right] \\
=& \left[1-\sigma_I\right] n_{i_N}^{-1} j_N \kappa_1(\bar{r})\kappa_2(i_N)\omega^{1-\sigma_I}\hat{\mathcal{M}}(\omega)^{-2}\hat{\theta}(A)^{-1}A^{\sigma_R-2}w^{\bar{r}}\frac{j_R}{\sigma_I}\hat{\theta}(A)^{\frac{1-\sigma_I}{1-\sigma_R}} \ ,
\end{aligned}
$$

  which is > 0, if $\sigma_I < 1$ (and vice-versa). □

- Consumption of service i_N by a low-skilled household:

$$
\begin{aligned}
\frac{\partial c_{i_N}^{\underline{r}}}{\partial A} =& \frac{\partial}{\partial A} n_{i_N}^{-1} j_N \kappa_1(\bar{r})\kappa_2(i_N)\omega^{-\sigma_I}\hat{\mathcal{M}}(\omega)^{-1}w^{\underline{r}} \\
=& \left[1-\sigma_I\right] n_{i_N}^{-1} j_N \kappa_1(\bar{r})\kappa_2(i_N)\omega^{-\sigma_I}\hat{\mathcal{M}}(\omega)^{-2}\hat{\theta}(A)^{-1}A^{\sigma_R-2}w^{\underline{r}}\times \\
& \left[-\hat{\mathcal{M}}(\omega) - \frac{1-\sigma_I}{\sigma_I} j_N\omega^{1-\sigma_I} + j_R\hat{\theta}(A)^{\frac{1-\sigma_I}{1-\sigma_R}}\right] \\
=& \left[\sigma_I - 1\right] n_{i_N}^{-1} j_N \kappa_1(\bar{r})\kappa_2(i_N)\omega^{-\sigma_I}\hat{\mathcal{M}}(\omega)^{-2}\hat{\theta}(A)^{-1}A^{\sigma_R-2}w^{\underline{r}}\frac{j_N}{\sigma_I}\omega^{1-\sigma_I} \ ,
\end{aligned}
$$

  which is > 0, if $\sigma_I > 1$ (and vice-versa). □

- Consumption of service i_R by a high-skilled household:

$$
\begin{aligned}
\frac{\partial c_{i_R}^{\bar{r}}}{\partial A} =& \frac{\partial}{\partial A} n_{i_R}^{-1} j_R \kappa_1(\bar{r})\kappa_2(i_R)\hat{\theta}(A)^{\frac{\sigma_I}{\sigma_R-1}}\omega\hat{\mathcal{M}}(\omega)^{-1}w^{\bar{r}} \\
=& n_{i_R}^{-1} j_R \kappa_1(\bar{r})\kappa_2(i_R)\hat{\theta}(A)^{\frac{\sigma_I}{\sigma_R-1}-1}\omega\hat{\mathcal{M}}(\omega)^{-2}A^{\sigma_R-2}w^{\bar{r}}\times \\
& \left[\left[\frac{\left[1-\sigma_I\right]^2}{\sigma_I}+1\right]\hat{\mathcal{M}}(\omega) - \frac{\left[1-\sigma_I\right]^2}{\sigma_I} j_N\omega^{1-\sigma_I} + \left[1-\sigma_I\right] j_R\hat{\theta}(A)^{\frac{1-\sigma_I}{1-\sigma_R}}\right] \\
=& n_{i_R}^{-1} j_R \kappa_1(\bar{r})\kappa_2(i_R)\hat{\theta}(A)^{\frac{\sigma_I}{\sigma_R-1}-1}\omega\hat{\mathcal{M}}(\omega)^{-2}A^{\sigma_R-2}w^{\bar{r}}\left[\hat{\mathcal{M}}(\omega) + \frac{1-\sigma_I}{\sigma_I} j_R\hat{\theta}(A)^{\frac{1-\sigma_I}{1-\sigma_R}}\right] \ ,
\end{aligned}
$$

  which is > 0, always. □

- Consumption of service i_R by a low-skilled household:

$$\begin{aligned}\frac{\partial c^{\underline{r}}_{i_R}}{\partial A} =&\frac{\partial}{\partial A} n_{i_R}^{-1} j_R \kappa_1(\bar{r})\kappa_2(i_R)\hat{\theta}(A)^{\frac{\sigma_I}{\sigma_R-1}}\hat{\mathcal{M}}(\omega)^{-1} w^{\underline{r}} \\ =&n_{i_R}^{-1} j_R \kappa_1(\bar{r})\kappa_2(i_R)\hat{\theta}(A)^{\frac{\sigma_I}{\sigma_R-1}-1}\hat{\mathcal{M}}(\omega)^{-2} A^{\sigma_R-2} w^{\underline{r}} \times \\ &\left[\sigma_I \hat{\mathcal{M}}(\omega) - \frac{[1-\sigma_I]^2}{\sigma_I} j_N \omega^{1-\sigma_I} + [1-\sigma_I]\, j_R \hat{\theta}(A)^{\frac{1-\sigma_I}{1-\sigma_R}}\right] \\ =&n_{i_R}^{-1} j_R \kappa_1(\bar{r})\kappa_2(i_R)\hat{\theta}(A)^{\frac{\sigma_I}{\sigma_R-1}-1}\hat{\mathcal{M}}(\omega)^{-2} A^{\sigma_R-2} w^{\underline{r}} \left[\hat{\mathcal{M}}(\omega) - \frac{1-\sigma_I}{\sigma_I} j_N \omega^{1-\sigma_I}\right] \ ,\end{aligned}$$

which is > 0, whenever $\sigma_I > \frac{\omega\tilde{\phi}_{\bar{r}}}{2\omega\tilde{\phi}_{\bar{r}}+\tilde{\phi}_{\underline{r}}}$ (and vice-versa). □

D.7 Proof of Constant i_N-Service Production

Industry i_N adjusts its labor input in response to technological progress according to[56]

$$\begin{aligned}\frac{\partial \tilde{L}^d_{i_N}}{\partial A} =&\frac{\partial}{\partial A} j_N \omega^{-\sigma_I} \hat{\mathcal{N}}(\omega)^{-1}\tilde{L} \\ =&[\sigma_I - 1]\, j_N \omega^{-\sigma_I}\hat{\mathcal{N}}(\omega)^{-2}\hat{\theta}(A)^{-1} A^{\sigma_R-2}\tilde{L}\left[\hat{\mathcal{N}}(\omega) - j_N\omega^{-\sigma_I} + j_R \hat{\theta}(A)^{\frac{1-\sigma_I}{1-\sigma_R}}\right] \\ =&0 \ ,\end{aligned}$$

i.e., the labor input remains constant. The supply of high-skilled labor is constraint.

This result can also be shown in consumption terms. It must hold that consumption changes of the two skill groups in response to technological progress must balance each other,

$$\begin{aligned}\phi_{\bar{r}}\frac{\partial C^{\bar{r}}_{i_N}}{\partial A} =&-\phi_{\underline{r}}\frac{\partial C^{\underline{r}}_{i_N}}{\partial A} \\ \phi_{\bar{r}} L w^{\bar{r}} j_R \hat{\theta}(A)^{\frac{1-\sigma_I}{1-\sigma_R}} =&\phi_{\underline{r}} L w^{\underline{r}} j_N \omega^{-\sigma_I} \\ \tilde{\phi}_{\bar{r}} j_R \hat{\theta}(A)^{\frac{1-\sigma_I}{1-\sigma_R}} =&\tilde{\phi}_{\underline{r}} j_N \omega^{-\sigma_I} \\ \omega =& \left[\frac{\tilde{\phi}_{\underline{r}}}{\tilde{\phi}_{\bar{r}}}\left[\frac{\kappa_2(i_N)}{\kappa_2(i_R)}\right]^{\sigma_I-1}\left[\frac{n_{i_N}}{n_{i_R}}\right]^{\frac{1-\sigma_I}{1-\sigma_v}}\hat{\theta}(A)^{\frac{1-\sigma_I}{\sigma_R-1}}\right]^{\frac{1}{\sigma_I}} \ ,\end{aligned}$$

where the last equality is the equilibrium outcome of the scaling factor (16), and thereby the equivalence is shown. □

[56] $\mathcal{N}(\omega)$ is defined in Footnote 26.

D.8 Proofs of Lemma 8

(i) Whenever $\sigma_I > 1$, the share z strictly increases in A (see Lemma 6) and $\lim_{A\to\infty} z = 1$, as the routine service is provided through robots—produced with high-skilled labor—at cost approaching zero. The price of robots, p_k, declines because the elasticity of the scaling factor with respect to A is always smaller than one, $\sigma_{A,\omega} < 1$ (see Appendix D.5).

Then, the left-hand side of (27) is bounded by the assumption that σ_R is bounded, while the right-hand side eventually increases enough to become greater than the left-hand side when z converges to 1. Using Condition (27), we show statement (i) by contradiction: Suppose Condition (27) is true for large A and the parameter restrictions ($\sigma_I > 1$ and σ_R bounded). Then

$$\lim_{A\to\infty} [\sigma_R - \sigma_I] \frac{1 + \omega \frac{\tilde{\phi}_{\bar{r}}}{\tilde{\phi}_{\underline{r}}}}{1 + z\omega \frac{\tilde{\phi}_{\bar{r}}}{\tilde{\phi}_{\underline{r}}}} > \lim_{A\to\infty} \frac{1}{1-z}$$

$$[\sigma_R - \sigma_I] \frac{1 + \lim_{A\to\infty} \omega \frac{\tilde{\phi}_{\bar{r}}}{\tilde{\phi}_{\underline{r}}}}{1 + \lim_{A\to\infty} z \lim_{A\to\infty} \omega \frac{\tilde{\phi}_{\bar{r}}}{\tilde{\phi}_{\underline{r}}}} > \frac{1}{1 - \lim_{A\to\infty} z}$$

$$[\sigma_R - \sigma_I] > \infty \ ,$$

which contradicts the supposition. $\square$

(ii) We know that $\lim_{A\to\infty} z = 1$ and for the share of total costs in the routine service industry allocated to the capital input it holds that $\lim_{A\to\infty} \frac{\mu(A,\omega)}{\tilde{\theta}(A,\omega)} = 1$. We use this in Lemma 4,

$$\lim_{A\to\infty} \frac{1}{1 + \frac{z+[1-z]\sigma_I}{z[\sigma_R-1]+[1-z][\sigma_R-\sigma_I]\frac{\mu(A,\omega)}{\tilde{\theta}(A,\omega)}}} = \frac{1}{1 + \frac{\lim_{A\to\infty} z+[1-\lim_{A\to\infty} z]\sigma_I}{\lim_{A\to\infty} z[\sigma_R-1]+[1-\lim_{A\to\infty} z][\sigma_R-\sigma_I]\lim_{A\to\infty}\frac{\mu(A,\omega)}{\tilde{\theta}(A,\omega)}}}$$

$$= \frac{\sigma_R - 1}{\sigma_R} \ ,$$

which is smaller than 1 for $\sigma_R \geq 1$. $\square$

D.9 Proofs of Lemma 9

Suppose A has finite value. First, we show that ω has finite value too. Second, we show that this implies $z < 1$ and thus, that the right-hand side of (27) is bounded.

Third, assuming $\sigma_R \to \infty$ we show that the left-hand side of (27) is unbounded.

(i) Observe from (22) that ω must be of finite value, whenever A is of finite value. We show this by contradiction. Thus, suppose that $\omega \to \infty$. Then $\lim_{\omega\to\infty} z = 0$, and $\lim_{\omega\to\infty} \tilde{\theta}(A,\omega) = 1$, and

$$\begin{aligned}\lim_{\omega\to\infty} \mathcal{F} =& \lim_{\omega\to\infty} \tilde{X}\tilde{\theta}(A,\omega)^{\frac{\sigma_I-\sigma_R}{1-\sigma_R}}\omega^{-\sigma_I} - \lim_{\omega\to\infty} \frac{\tilde{\phi}_{\bar{r}}}{\tilde{\phi}_{\underline{r}}}[1-z] \\ =& -\frac{\tilde{\phi}_{\bar{r}}}{\tilde{\phi}_{\underline{r}}} < 0 \ .\end{aligned}$$

This contradicts the definition of $\mathcal{F}$. Thus, for A of finite value, also ω must be of finite value.

(ii) If both A and ω are finite, then z cannot converge to 1 because of (22), i.e., $\lim_{\sigma_R\to\infty} \tilde{X}\tilde{\theta}(A,\omega)^{\frac{\sigma_I-\sigma_R}{1-\sigma_R}}\omega^{-\sigma_I} > 0$ and thus $\lim_{\sigma_R\to\infty} z < 1$.

(iii) We reexamine (27) and obtain

$$\begin{aligned}\lim_{\sigma_R\to\infty}[\sigma_R-\sigma_I]\frac{1+\omega\frac{\tilde{\phi}_{\bar{r}}}{\tilde{\phi}_{\underline{r}}}}{1+z\omega\frac{\tilde{\phi}_{\bar{r}}}{\tilde{\phi}_{\underline{r}}}} &> \lim_{\sigma_R\to\infty}\frac{1}{1-z} \\ \lim_{\sigma_R\to\infty}[\sigma_R-\sigma_I]\frac{1+\frac{1}{z}}{2} &> \lim_{\sigma_R\to\infty}\frac{1}{1-z} \\ \infty &> \lim_{\sigma_R\to\infty}\frac{1}{1-z} \ ,\end{aligned}$$

and thus, the condition for decreasing real wages for the low-skilled is fulfilled, which was to be shown. □

D.10 Derivations for Table 3

We analyze household consumption. We assumed in the main text that profits are distributed proportional to the wage distribution, (19). Essentially, a household's total income deflated by the aggregate price index P must be equal to the household's consumption C^r,

$$\begin{aligned}C^{\bar{r}} =&\kappa_1(\bar{r})\tilde{\mathcal{M}}(\omega)^{\frac{1}{\sigma_I-1}}\omega \ , \\ C^{\underline{r}} =&\kappa_1(\underline{r})\tilde{\mathcal{M}}(\omega)^{\frac{1}{\sigma_I-1}} \ .\end{aligned}$$

Integrating over all households yields total consumption in the economy, (C.4). We use household demand (C.1) to obtain all consumption decisions of households. For notational convenience we use again $j_N = n_{i_N}^{\frac{1-\sigma_I}{1-\sigma_v}} \kappa_2(i_N)^{\sigma_I - 1}$, and $j_R = n_{i_R}^{\frac{1-\sigma_I}{1-\sigma_v}} \kappa_2(i_R)^{\sigma_I - 1}$, and $\mu(A,\omega) = \left[\frac{A}{\omega}\frac{\kappa_2(i_N)}{\kappa_2(i_R)}\right]^{\sigma_R - 1}$. Note that $\frac{j_N}{j_R} = \tilde{X}$.

- Consumption of service i_N by a high-skilled household:

$$\begin{aligned}\frac{\partial c_{i_N}^{\bar{r}}}{\partial A} =& \frac{\partial}{\partial A} n_{i_N}^{-1} j_N \kappa_1(\bar{r})\kappa_2(i_N)\omega^{1-\sigma_I}\tilde{\mathcal{M}}(\omega)^{-1} w^{\bar{r}} \\ =& \left[1-\sigma_I\right] n_{i_N}^{-1} j_N \kappa_1(\bar{r})\kappa_2(i_N)\omega^{1-\sigma_I}\tilde{\mathcal{M}}(\omega)^{-2} A^{-1} w^{\bar{r}} \times \\ & \left[\tilde{\mathcal{M}}(\omega)\sigma_{A,\omega} - j_N\omega^{1-\sigma_I}\sigma_{A,\omega} - j_R\tilde{\theta}(A,\omega)^{\frac{\sigma_R-\sigma_I}{1-\sigma_R}}\mu(A,\omega)\left[\sigma_{A,\omega}-1\right]\right] \\ =& \left[1-\sigma_I\right] n_{i_N}^{-1} j_N \kappa_1(\bar{r})\kappa_2(i_N)\omega^{1-\sigma_I}\tilde{\mathcal{M}}(\omega)^{-2} A^{-1} w^{\bar{r}} \times \\ & \left[j_R\tilde{\theta}(A,\omega)^{\frac{\sigma_R-\sigma_I}{1-\sigma_R}}\left[\sigma_{A,\omega}+\mu(A,\omega)\right]\right] ,\end{aligned}$$

  which is > 0, if $\sigma_I < 1$ (and vice-versa). □

- Consumption of service i_N by a low-skilled household:

$$\begin{aligned}\frac{\partial c_{i_N}^{\underline{r}}}{\partial A} =& \frac{\partial}{\partial A} n_{i_N}^{-1} j_N \kappa_1(\bar{r})\kappa_2(i_N)\omega^{-\sigma_I}\tilde{\mathcal{M}}(\omega)^{-1} w^{\underline{r}} \\ =& n_{i_N}^{-1} j_N \kappa_1(\bar{r})\kappa_2(i_N)\omega^{-\sigma_I}\tilde{\mathcal{M}}(\omega)^{-2} A^{-1} w^{\underline{r}} \times \\ & \left[-\sigma_I\tilde{\mathcal{M}}(\omega)\sigma_{A,\omega} + \left[\sigma_I - 1\right] j_N\omega^{1-\sigma_I}\sigma_{A,\omega} + \left[\sigma_I-1\right] j_R\tilde{\theta}(A,\omega)^{\frac{\sigma_R-\sigma_I}{1-\sigma_R}}\mu(A,\omega)\left[\sigma_{A,\omega}-1\right]\right] \\ =& n_{i_N}^{-1} j_N \kappa_1(\bar{r})\kappa_2(i_N)\omega^{-\sigma_I}\tilde{\mathcal{M}}(\omega)^{-2} A^{-1} w^{\underline{r}} \times \\ & \left[\left[1-\sigma_I\right] j_R\tilde{\theta}(A,\omega)^{\frac{\sigma_R-\sigma_I}{1-\sigma_R}}\left[\sigma_{A,\omega}+\mu(A,\omega)\right] - \tilde{\mathcal{M}}(\omega)\sigma_{A,\omega}\right] ,\end{aligned}$$

  which is > 0, if $\sigma_I < \frac{\mu(A,\omega)\left[1-\frac{\sigma_{A,\omega}}{z}\right]}{\mu(A,\omega)+\sigma_{A,\omega}} := \kappa_{i_N,R}$ (and vice-versa). Observe that $\kappa_{i_N,R} < 1$ always. Furthermore, whenever (27) holds, then $\sigma_{A,\omega} > z$ and σ_I would have to be negative, for the consumption of services i_N to increase. This is not possible, as $\sigma_I > 0$ by assumption. Therefore, consumption of services i_n by the low-skilled increases with A if $\sigma_I < \kappa_{i_N,R}$ and always decreases if their real wage decreases. □

- Consumption of service i_R by a high-skilled household:

  $c_{i_R}^{\bar{r}}$ must always increase in A. Firstly, the prices of services i_R decreases in

A. And secondly, high-skilled workers always earn higher real wages when A increases. Thus, the derivative of $c_{i_R}^{\bar{r}}$ with respect to A must always be positive. $\square$

- Consumption of service i_R by a low-skilled household:

$$\begin{aligned}\frac{\partial c_{i_R}^{\underline{r}}}{\partial A} =& \frac{\partial}{\partial A} n_{i_R}^{-1} j_R \kappa_1(\bar{r})\kappa_2(i_R)\tilde{\theta}(A,\omega)^{\frac{\sigma_I}{\sigma_R-1}} \tilde{\mathcal{M}}(\omega)^{-1} w^{\underline{r}} \\ =& n_{i_R}^{-1} j_R \kappa_1(\bar{r})\kappa_2(i_R)\tilde{\theta}(A,\omega)^{\frac{\sigma_I}{\sigma_R-1}-1} \tilde{\mathcal{M}}(\omega)^{-2} A^{-1} w^{\underline{r}} \times \\ & \Big[\sigma_I \tilde{\mathcal{M}}(\omega)\mu(A,\omega)\left[1-\sigma_{A,\omega}\right] - \left[1-\sigma_I\right]\tilde{\theta} \\ & \Big[j_N \omega^{1-\sigma_I}\sigma_{A,\omega} + j_R \tilde{\theta}(A,\omega)^{\frac{\sigma_R-\sigma_I}{1-\sigma_R}} \mu(A,\omega)\left[\sigma_{A,\omega}-1\right]\Big]\Big] \ ,\end{aligned}$$

which is > 0, if $\sigma_I > \frac{\sigma_{A,\omega}-z}{1-z}\frac{1+\mu(A,\omega)}{\sigma_{A,\omega}+\mu(A,\omega)} := \kappa_{i_R,R}$ (and vice-versa). Note that only when (27) holds, i.e., $\sigma_{A,\omega} > z$, there are potential parameter values for σ_I, in particular $\sigma_I \in (0, \kappa_{i_R,R})$, for which the consumption of services i_R decreases. Whenever the real wage of low-skilled workers increases, also their consumption of service i_R increases. $\square$

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