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## CONTINGENT CONTRACTS IN BANKING: INSURANCE OR RISK MAGNIFICATION?

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## Abstract

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JEL Classification: D41, E4, G2

Keywords: Financial intermediation - Macroeconomics risks - State-contingent contracts - Banking regulation



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# Contingent Contracts in Banking: Insurance or Risk Magnification?*

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**Abstract**

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# 1 Introduction

*Motivation and Contribution*

The literature and the last financial crisis provide ample evidence that the costs of banking crises, in terms of GDP losses, may be very large. One reason for this is that traditional contractual arrangements in banking expose the banks to the risks associated with macroeconomic shocks, which may, in turn, lead to a crisis. Such shocks can be exogenous or can arise when many banks undertake investments with correlated risks and thereby increase economy-wide aggregate risk.

One way to solve this problem might be to induce banks to make deposit and loan contracts contingent on macroeconomic events, such as GDP growth, or other contractible macroeconomic indicators that are highly correlated with the financial health of the banking sector, such as the average bank equity of competing banks. In this paper, we examine whether the banking system can be insured against crises by the private sector through contingent loan and deposit contracts.[1] Our analysis is both a positive and normative exercise. On the positive side, we examine what happens when banks compete with contingent deposit and loan contracts. On the normative side, we explore how different regulatory approaches towards insolvent banks affect the scope of private insurance against banking crises with contingent deposit and loan contracts. The contribution of the paper is to identify the properties of double-sided Bertrand competition with contingent contracts on both sides. This opens up both insurance possibilities and the opposite—the amplification of risk—or even the generation of new risks, depending on the regulatory approach towards banking crises.

*Model and Results*

To explore risk creation and magnification, we consider a simple model in which banks alleviate agency problems in financial contracting. Banks compete for funds and offer credit contracts to potential borrowers. We allow for macroeconomic shocks affecting the average productivity of investment projects. There are two overlapping generations of agents. We focus on the behavior of the first generation while the second generation

[1]How contracts can be made dependent on macroeconomic risk by defining and maintaining standardized macroeconomic indices has been examined and discussed extensively in the context of sovereign GDP-linked bonds by Shiller (Shiller (2003) and recently by Benford, Ostry and Shiller (2018). In our concluding section, we comment on the range of possible macroeconomic indicators in our model.

may be burdened by bailout costs. While the shift of bailout costs to the second generation is a convenient device to insulate the current generation of agents from such costs, qualitatively similar results can be obtained when bailouts are financed by lump sum taxation of the current generation, as we will show.

We distinguish two scenarios. Suppose first that bank behavior is governed by expected profit maximization and failing banks go bankrupt. Then, financial intermediation with contingent contracts yields an efficient risk allocation. If macroeconomic shocks are small, depositors and entrepreneurs are offered non-contingent deposit and loan contracts. All macroeconomic risk is borne by entrepreneurs. The inside funds of entrepreneurs act as a buffer for macroeconomic risks. If macroeconomic shocks are large, banks write state-contingent contracts for depositors and debtors. Part of the macroeconomic risk is shifted to consumers, since entrepreneurs cannot bear the entire risk. Consumers and entrepreneurs together insure the banking sector, and banking crises are avoided.

Second, new phenomena occur if bank deposits are guaranteed by governments. Suppose that there are with bailout, competing banks try to make profits (positive intermediation margin) in the good state of the world and make losses (negative intermediation margin) in another state of the world. In the good state—with high productivity of investment projects—, they request high loan interest rates from entrepreneurs. To motivate entrepreneurs to invest rather than to save, banks request very low repayments in the bad state—with low productivity of investment projects. Deposit rates are non-contingent, since deposits are "insured" by bailouts.

Moreover, competition among banks for the creation of a profitable state pushes deposit rates up to the maximal amount entrepreneurs can repay in that state. As a result, banks create a state of the world with high repayment obligations to depositors, but with very low pay-back requirements for entrepreneurs. This creates large risks for future generations, even if the underlying productivity risk is small or zero. This is called the "risk-magnification effect". We also show that when banks can choose between contingent or non-contingent contracts, they will always offer non-contingent deposit contracts, but contingent loan contracts, to maximize risk creation.

As a consequence of the risk-magnification effect, the present generation obtains higher interest rates on savings than in a situation with bank failures. This induces over-investment among the current generation, at the expense of future generations. Specifically, output from investment projects over two generations is higher under bank failure than under bailout when the second generation has to bail out the first.

Finally, under a bailout regime, contingent contracts create considerably more risks and bail out risks than non-contingent contracts. The latter creates bank failures and bail out when productivity sufficiently large but no failures when these shocks are smaller.

Overall, the analysis reveals that deposit and loan contracts contingent on macroeconomic shocks has a twofold potential. Either the contracts are helpful and insure efficiently against a banking crisis or they are noxious and either generate new risks or magnify existing risks. The latter phenomena would not occur with non-contingent contracts. Thus, contingent contracts are neither an all-purpose panacea nor do they guarantee a lower likelihood and severity of banking crises, as the creation of additional risk will set in if other policy conditions are not met.[2]

We focus on a simple model to illustrate the mechanisms and their implications for investment and aggregate risk. In Section 10, we discuss several implications and extensions of the results and the model, respectively. In particular, we argue that the proposed channel has implications that are empirically distinguishable from other channels and that risk magnification is likely to be a robust phenomenon in more elaborated models. Moreover, it is hard or impossible to counteract risk magnification incentives with standard explicit deposit insurance schemes.

The paper is organized as follows. After the relation to the literature and an illustrative Example, we introduce the model in Section 4. In Section 5 we derive the equilibrium in the intermediation market without the presence of macroeconomic shocks. In Section 6, we introduce temporary productivity shocks, state-contingent deposit and loan contracts, and regulatory schemes. Sections 7 and 8 describe the equilibrium under bank failure and bailout commitment respectively. In Section 9, we draw comparisons, including a comparison to non-contingent contract settings. Section 10 discusses the implications and possible extensions of the model and Section 11 concludes.

[2]It is thus essential to have orderly default procedures for large banks, such that failing banks themselves do not threaten the stability of the banking system, as became evident when Lehman Brothers collapsed. Such default procedures would make insurance against systemic crises viable and would render the use of default procedures unnecessary in the first place. Thus, a joint policy package of orderly default procedures and contingent contracts is a promising way to reduce—or even eradicate—the fragility of the banking system.

## 2 An Example

A simple example—without equilibrium considerations–may illustrate risk amplification. Suppose a bank grants a loan of amount $I$ to an entrepreneur when the amount can be measured in real terms (amount of a capital good) or nominal terms (amount of money). The entrepreneur has some equity $e$ to invest in his firm. The project outcome is risky and depends on whether state $h$ or $l$ occurs. The outcomes are given by

- $q^h(e+I)$ if state $h$ occurs,
- $q^l(e+I)$ if state $l$ occurs,

with $q^h, q^l \in \mathbb{R}_+$ and $q^h > q^l$ denoting the productivity level. The good state $h$ occurs with probability $p$ and the bad state $l$ occurs with probability $1-p$. The bank can obtain funding from a risk-neutral investor who requires an expected return $\bar{r}$, with $\bar{r} > 0$.

Suppose that $q^l(e+I) > I(1+\bar{r}+\Delta)$, with $\Delta$ covering the cost of the bank per unit of the loan. Then, we consider the loan contract, denoted by $C(r_c)$, and the deposit contract, denoted by $D(r_d)$, with

$$r_c = \bar{r} + \Delta, r_d = \bar{r}.$$

The loan and deposit contract are both debt contracts and stipulate repayments obligatory $1+r_c$ and $1+r_d$, respectively for one unit of capital rent to the borrower (firm and bank respectively). As a consequence, the bank makes zero profit in both states. This situation ensures that the bank does not default in the bad (and good) state. All risks are absorbed by the entrepreneur. Indeed, we show that such contractual arrangements occur if banks maximize expected profits and are not rescued, and the equity of entrepreneurs is not too small.[3]

Suppose that the bank is rescued in the bad state and debt holders are bailed out. The bank can offer the following loan contract $C(r_c^h, r_c^l)$ and a debt contract $D(r_d)$ with $r_d = \bar{r}$. The state-contingent loan rates $r_c^h$ and $r_c^l$ are determined by

[3] The same logic applies for $q^l(e+I) < I(1+\bar{r}+\Delta)$, with the additional feature that repayments to banks and investors are state-contingent, such that neither the entrepreneur nor the bank defaults.

$$
\begin{aligned}
r_c^l &= \bar{r} - \delta + \Delta \\
r_c^h &= \frac{p\Delta + \delta + p(\bar{r} - \delta)}{p} = \bar{r} - \delta + \Delta + \frac{\delta}{p},
\end{aligned}
$$

when $\delta$ is selected such that $q^h(e+I) = I(1+r_c^h)$. Due to our assumption, $q^h(e+I) > q^l(e+I) > I(1+\bar{r}+\Delta)$, we have $\delta > 0$.

We note that

$$
pr_c^h + (1-p)r_c^l = \bar{r} + \Delta.
$$

Hence, the entrepreneur faces the same expected repayment as in the case of bank failure. Since $r_c^l < \bar{r} + \Delta$, the bank defaults in the bad state, but debt holders are bailed out and receive the same expected repayment as in the case of bank failure. The bank makes zero profit in the bad state and $(1+r_c^h)I - I(1+\bar{r}+\Delta)$ in the good state.

With the definition of $r_c^h$, we calculate expected profits as

$$
I(p\bar{r} - p\delta + p\Delta + \delta - p\bar{r} - p\Delta) = I\delta(1-p).
$$

Hence, expected profits are higher than in the case of bank failure. Using contingent loan contracts is beneficial for the bank, as it can create a state of the world in which profits are quite high and a state of the world in which profits are negative. The latter does not matter for the bank, though, since debt-holders are bailed out. For the government, however, the bank's contingent loan contracts creates large risks which do not occur in the first scenario with bank failures.

# 3 Relation to the Literature

A large literature, dating back at least to Fisher's debt deflation theory, has examined the problems that arise when debt contracts are not contingent on shocks. The formal recent literature with Merton (1977) and Kareken and Wallace (1978), and the large literature following (see e.g. Efing et al. (2015) for comprehensive evidence) have shown that governmental approaches towards failing banks affects the banks' risk-taking incentives. In particular, limites liability of bank shareholders and bailout can generate excessive risk-taking. We observe two new phenomena: risk amplifications and creation as consequences of double-sided banking competition with contingent deposit and loan contracts, which is complementary of this literature.

Our paper is related to the current discussion on regulatory issues regarding financial intermediaries. First, the idea that financial contracts could and should be conditioned on macroeconomic indicators has been around for some time (see e.g. Hellwig (1998) and Shiller (2003)) and it has been pointed out by Hellwig (1995, 1998) that it is unclear why the terms of the deposit contracts are not made contingent on aggregate events, such as fluctuations in the gross domestic product. Our model indicates that bailouts of banks in a crisis will not induce contingent deposit contracts, even if they become feasible, but will lead to contingent loan contracts with very large differences in state-dependent repayments. As a consequence, the possibility to write deposit and loan contracts contingent on macroeconomic shocks triggers a risk-magnification effect in the following sense: Even if the underlying productivity risk is small or zero, competition among banks, with contingent contracts and under a bailout approach, yields large macroeconomic risks for future generations.

Second, there are empirical parallels to our results. Inflation-indexed[4] or Forex-related loan and deposit contracts[5] used in Latin America and Southeast Asia may have contributed to macroeconomic instability. This suggests that contracts contingent on macroeconomic factors may trigger banking instability, or at least contribute to them. Our argument is that financial intermediation with deposit and loan contracts contingent on macroeconomic shocks can imply banking instability when such schemes are offered competitively under bailout schemes.

Third, at a more general level, our investigation indicates that new contractual opportunities, i.e. the possibility to make deposit and loan contracts contingent on macroeconomic shocks, may increase both aggregate credit and financial instability. Our exercise complements the insights of Shin (2009) that securitization enables credit expansion through higher leverage of the financial system as a whole, while the impact on financial stability is ambiguous.

Fourth, the actions taken in the banking system change entirely if there is a credible commitment by the regulator to let insolvent banks go bankrupt. Then, productivity shocks are fully absorbed by the fluctuation of income or risk-neutral agents who enter contingent loan contracts. This insures the banking system and risk-averse depositors. Banks never default in equilibrium. The risk allocation is efficient. This can be

[4]See Chen (2018).

[5]See Claessens and Horen (2014) and Ivashina et al. (2015) for assessments.

viewed as an efficient private deposit insurance scheme.[6] Historically, countries have implemented public deposit insurance schemes. In the US, for instance, federal deposit insurance started under the (Glass-Steagall) Banking Act of 1933 which created the Federal Deposit Insurance Corporation (FDIC) that is in charge of insuring deposits at commercial banks. A thorough discussion of this scheme can be found in Pennacchi (2009). Calomiris and Jaremski (2016) provide a comprehensive historical account of the economic and political theories of deposit insurance. They conclude that public deposit insurance tends to increase systemic risk rather than reduce it. We complement this literature and show that public deposit insurance generates the risk-magnification effect.

# 4 The Model

We consider a two-period model ($t = 1, 2$) and one generation of agents living for two periods. A generation consists of a continuum of agents, indexed by [0,1], and living for two periods. There are two classes of agents. A fraction $\eta$ ($0 < \eta < 1$) of individuals consists of potential entrepreneurs and a fraction $1-\eta$, of the population are consumers. Potential entrepreneurs and consumers differ insofar that only the former have access to investment technologies. There is one physical perishable good that can be used for consumption or investment. Each individual receives an endowment $e$ of the good in period $t = 1$ when young and nothing in period $t = 2$ when old.

Each entrepreneur has access to a production project that converts time 1 goods into time 2 goods. The required funds for an production project are $F := e + I$. Hence, an entrepreneur must borrow $I$ units of the good to undertake the investment project. The class of entrepreneurs is not homogeneous. We assume that entrepreneurs are indexed by a quality parameter $q$ uniformly distributed on $[\overline{q} - 1, \overline{q}]$, $1 < \overline{q} < 2$, in the population of entrepreneurs. If an entrepreneur of type $q$ obtains additional resources $I$ and decides to invest, he realizes gross investment returns in the next period of

$$q(I + e). \tag{1}$$

$\overline{q}$ is the aggregate indicator of the productivity of investment projects in period 1. If $\overline{q}$ is

[6] Gersbach (2013) provides a survey of insurance tools against banking crises.

uncertain, the generation under consideration faces macroeconomic risk. For simplicity, we assume that potential entrepreneurs are risk-neutral and are only concerned with consumption in their old age, i.e., they do not consume when young.

Consumers consume in both periods. They have utility functions $u(c^1)+\beta u(c^2)$ defined over consumption in the two periods, with $\beta$ $(0 < \beta < 1)$ denoting the discount factor. The variables $c^1, c^2$ are the consumption of the consumers when young and old, respectively. Consumers are risk-averse and the utility function satisfies standard assumptions. If an agent can transfer wealth between the two periods at a riskless real interest rate, denoted by $r$, the solution of the agent's intertemporal consumption problem generates the continuous saving function, denoted by $s\{r\}$. We assume that substitution effect (weakly) dominates the income effect, i.e., savings are an increasing function of the interest rate.

The rationale for the above banking model and the nature of contracts that are used are given in Gersbach and Uhlig (2006), who abstract from macroeconomic shocks. A detailed description of this banking model is given in Appendix B. In the main text, we summarize the two core assumptions and the two properties of this banking model that will be used in the current investigation.

- **Assumption 1:** Banks cannot observe quality $q$ before entering into loan contracts.
- **Assumption 2:** Banks can secure the liquidation value.
- **Property 1:** An entrepreneur with quality $q$ will invest if he obtains a loan.
- **Property 2:** The contracts banks are offering are debt contracts (deposit and loan contracts).

With these two assumptions and the simplification to assume zero monitoring costs for banks, we concentrate on the consequences when banks compete with deposit and loan contracts contingent on macroeconomic shocks.

On purpose, we assume that banks have zero equity capital, which puts the difference between insurance and risk magnification through contingent contracts in the starkest possible way.[7] Moreover, for all our arguments, it will suffice that two banks compete.[8] Hence, we assume that there are two banks, indexed by $j$ ($j = 1, 2$), which finance entrepreneurs.

First, we discuss the nature of contracts offered by banks indexed by $j = 1, 2$. Bank $j$ can sign deposit contracts $D(r_j^d)$, where $1 + r_j^d$ is the repayment offered for one unit of resources. Loan contracts of bank $j$ are denoted by $C(r_j^c)$, where $1 + r_j^c$ is the repayment required from entrepreneurs for one unit of funds. If macroeconomic risk is present, we allow contracts to be conditioned on the realization of $\overline{q}$, or equivalently, on the resulting GDP in period 1. In such cases, state-contingent deposit or loan contracts can be written.

Note that the availability of production technologies from period 1 to 2 allows depositors and entrepreneurs to trade among themselves.[9] A new generation is only affected by the preceding generation when banks have accumulated losses and the next generation is taxed to bail out banks. This is the focus in Section 8, while in the preceding sections, it is sufficient to focus on one generation.

Finally, we assume that banks are owned by entrepreneurs who are risk-neutral.[10] We consider two scenarios: either failing banks go bankrupt or failing banks are rescued.

We assume that banks maximize expected profits. With bailout, expected profit maximization is identical to limited liability and thus identical to return on equity maximization under limited liability, as shareholders have zero returns in the event of losses. Hence, when we will consider bailouts, we will cover limited liability. When banks are not bailed out, banks face losses in the event of default.[11]

[7] When banks have a positive but not large amount of equity capital, insurance in the case of bank failure works as presented in the paper. When banks are bailed out, the incentives to generate risk remains, but the risk generated for future generations is reduced by the equity buffer. Details are available upon request.

[8] As we focus on Bertrand Competition, an extension to more than two banks is straightforward.

[9] In this model, intergenerational trade does not improve autarky for all generations.

[10] If banks are owned by depositors, the objective function of banks is more subtle and one has to add a risk premium, since different bank strategies are associated with different risks and insurance opportunities for risk-averse depositors.

[11] In practice, there may be pecuniary and non-pecuniary penalties associated with default. The non-pecuniary utility loss for banks may occur because career opportunities decline and/or reputation is being destroyed. This may be actively promoted by bank regulators if they punish the bank managers' failure.

# 5 Equilibrium without Macroeconomic Shocks

## 5.1 The Game

To illustrate the working of the model, we first consider an economy without shocks. This will help to understand how entrepreneurs self-select saving or investing and how investments and savings are balanced in equilibrium. Moreover, this lays the foundation for the analysis of the impact of macroeconomic shocks and contingent loan and deposit contracts. Deposit and loan contracts will have a duration of one period as no transformation of maturities needs to take place. We examine the four-stage intermediation game for the generation under consideration.

**Period 1**

1. Banks offer deposit contracts to consumers and entrepreneurs.
2. Banks offer loan contracts to entrepreneurs subject to the available resources.
3. Consumers and entrepreneurs decide which contracts to accept. Resources are exchanged. Entrepreneurs start production, which is subject to macroeconomic risk.

**Period 2**

4. Production ends. Entrepreneurs pay back to banks. Banks pay back depositors.

The game is a multi-stage game with observed actions. At each stage, actions are chosen simultaneously and players know the actions of all previous stages when they enter the next stage. As there is a continuum of consumers and entrepreneurs, they are assumed to be contract-takers. Banks are the only strategic players that set deposit and loan interest rates.

Entrepreneurs are contract-takers and thus make loan application decisions with the assumption that they will not be rationed at banks that offer the highest deposit rate.[12]

[12] As only those banks will obtain deposits and will be active, it is intuitive that entrepreneurs seeking loans only apply at banks that offer the highest deposit rate. We could relax the assumption by modeling entrepreneurs as contract-takers at any bank, which, however, complicates the analysis considerably.

If entrepreneurs seeking loans were rejected, they would choose to save $s$ at the banks that offer the highest deposit rate. In all equilibria in this paper, the entrepreneurs applying for loans are not rationed and thus their expectations are correct.[13] It is useful to start with the loan application decision of an entrepreneur with quality $q$, given that he observes $r_j^d$, $r_j^c$ of banks.

If an entrepreneur obtains a loan, he also has an incentive to invest. The reason is the assumption, detailed in Appendix B, according to which monitoring technologies are efficient enough at reducing the private benefits of entrepreneurs if they do not invest. If an entrepreneur applies for a loan at the bank offering the lowest loan rate, because of limited liability, his terminal wealth or consumption $W(q)$ will amount to

$$W(q) = \max\{q(e+I) - I(1+\min_j\{r_j^c\}), 0\}. \tag{2}$$

We note that an entrepreneur defaults if $q(e+I) < I\left(1+\min_j\{r_j^c\}\right)$. Since banks can secure the liquidation value, the entrepreneur has zero wealth in such cases. If he does not apply, he obtains $e(1+\max_j\{r_j^d\})$ by saving his endowments. Thus, there exists a critical quality parameter, denoted by $q^*$, and given by

$$q^* = 1 + \frac{I\min_j\{r_j^c\} + e\max_j\{r_j^d\}}{e+I}, \tag{3}$$

which motivates entrepreneurs with $q \geq q^*$ to apply for loans and entrepreneurs with $q < q^*$ to save. Equation (3) determines the marginal entrepreneur.

Note that we have assumed that banks can ensure investment and can verify output conditional on investment. Thus, they are not concerned about low-quality entrepreneurs since such entrepreneurs would have less consumption than with saving endowments and thus will not apply for a loan. Still, entrepreneurs may default if they cannot repay the loan. In such cases, banks obtain the output of the entrepreneur as the liquidation value.

Banks are assumed to maximize expected profits. The assumption is justified in detail in Section 4. Hence, conditional on granting a credit to an entrepreneur with quality

[13] If a bank does not have enough deposits to lend to all candidate borrowers, loans are rationed. In such a case, we assume that the loan applicants at the said bank are rationed with the same probability, such that loan volume and deposits are balanced. Other rationing schemes might be considered, where rejected entrepreneurs go to another bank to apply for loans. In an extended version of the model, we show that the results are robust for different rationing schemes. The main argument is that more sophisticated rationing schemes tend to lower the profits of banks that deviate from an equilibrium. Details are available upon request.

level $q$ and receiving funds from savers, profits per credit of a bank $j$ amount to

$$G_j = \min\{q(e+I), I(1+r_j^c)\} - I(1+r_j^d). \tag{4}$$

If the entrepreneur does not default, profits per credit are

$$G_j = I(1+r_j^c) - I(1+r_j^d) = I(r_j^c - r_j^d) = I\Delta_j. \tag{5}$$

The difference $\Delta_j$ is the intermediation margin of bank $j$. In order to derive the intermediation equilibrium, we make the following technical assumptions:

**Assumption 1**

$(1-\eta)\, s\,\{\bar{q}-1\} < \eta\, I.$

**Assumption 2**

$(1-\eta)s\{0\} + \eta\, e\big(1-(\overline{q}-1)\big) < \eta\,(\overline{q}-1)\, I.$

The first assumption implies that savings are never sufficient to fund all entrepreneurs. The reasons are as follows. A bank that serves depositors and borrowers makes no losses if $r_j^c \geq r_j^d$ and if the entrepreneur does not default. If all entrepreneurs are funded, we have $(\bar{q}-1)(e+I) \geq e(1+r_j^d) + I(1+r_j^c)$ which, together with $r_j^c \geq r_j^d$, yields $1 + r_j^d \leq \bar{q} - 1$. Hence, in order to fund all entrepreneurs, $r_j^d$ cannot exceed $\bar{q}-1$ as long as banks do not make losses, since we have assumed that the savings of consumers are weakly increasing in the deposit rate, Assumption 1 is a sufficient condition for savings to be lower than the funds needed to finance all entrepreneurs.

Assumption 2 states that investments exceed savings when both deposit and loan interest rates are zero. Indeed, at such interest rates, it follows from equation (3) that $q^* = 1$. Therefore, entrepreneurs with $q \in [1, \overline{q}]$ apply for loans, while entrepreneurs with $q \in [\overline{q}-1, 1)$ save their endowments. As no entrepreneur would apply for loans at interest rate $r_j^c > \bar{q}$ and that $\bar{q} > 1$, Assumption 2 implies that savings and investments can be balanced at positive interest rates. A subgame perfect Nash Equilibrium among banks is a tuple

$$\Big\{ \big\{r_j^{d*}\big\}_{j=1,2},\ \big\{r_j^{c*}\big\}_{j=1,2} \Big\}$$

so that

- entrepreneurs make optimal credit application and saving decisions, as contract-takers, i.e., entrepreneurs with $q \geq q^*$, apply for loans and entrepreneurs with $q < q^*$ save, where $q^*$ is given by Equation (3);
- no bank has an incentive to offer different deposit or loan interest rates.

Therefore, the strategy spaces of banks are deposit and loan rates.[14] To complete the description of the game, we assume that failing banks are not bailed out.[15]

## 5.2 The Equilibrium

The following proposition is proved in Appendix A.

**Proposition 1**

*There exists a unique equilibrium of the intermediation game with*

*(i)*

$$r^* = r_j^{c*} = r_j^{d*} \qquad \textit{for } j = 1, 2,$$

*(ii)* $r^*$ *is determined by*

$$(1-\eta)\, s\ \{r^*\} + \eta e\Big(1 + r^* - (\bar{q} - 1)\Big) = \eta\Big(\bar{q} - (1 + r^*)\Big)\, I,$$

*(iii)*

$$q^* = 1 + r^*,$$

*(iv) bank profits are zero.*

We note that Condition (ii) displays the balance of the savings (left-hand side) and investments (right-hand side). The critical quality of the entrepreneur who is indifferent between saving and investment is equal to interest rate factor $1 + r^*$. Hence, all entrepreneurs with quality levels $q \in [1 + r^*, \bar{q}]$ will apply for loans.

Hence, the intermediation game yields the competitive outcome in which savings and investments are balanced and in which there is a common interest rate for loans and

[14] In addition banks apply the rationing scheme described in footnote 5.1

[15] We note, however, that the equilibrium below continues to exist if failing banks are bailed out, but uniqueness is not necessarily guaranteed.

deposits.[16] We can conclude from Proposition 1 that intermediation margins are zero in equilibrium, and savings and investments are balanced.

Note that in our model the incentive of banks to corner one side of the market, in order to obtain monopoly rents on the other side, does not destroy the perfect competition outcome.[17] Suppose a bank offers a deposit rate slightly above $r^*$ in order to attract all depositors. If this bank raises $r^c$ in order to exploit its monopoly power among entrepreneurs, a portion of them will switch market sides. This, however, causes large excess resources for the deviating bank, inducing a loss greater than the excess returns from the remaining entrepreneurs. The excess resources resulting from market side switching is only one of several arguments why Walrasian outcomes can arise. For our purpose, it is important that competitive outcomes occur.

In equilibrium, all entrepreneurs with projects whose quality levels $q$ are equal to or above $1 + r^*$ will obtain funds and invest.

Aggregate income in period 2, denoted by $Y$, is given by

$$Y = \eta(I + e) \cdot \left\{ \frac{(\overline{q})^2 - (1 + r^*)^2}{2} \right\} - e. \tag{6}$$

Formally, aggregate income is given by

$$\begin{aligned} Y \;=\;& \underbrace{(1-\eta)s\{r^*\}(1+r^*)}_{\text{wealth of investors at the end}} + \underbrace{\eta\big((1+r^*) - (\overline{q}-1)\big)(1+r^*)e}_{\text{wealth of not investing entrepreneurs at the end}} \\ & \underbrace{+\eta \int_{1+r^*}^{\overline{q}} \big(q(e+I) - I(1+r^*)\big)dq}_{\text{wealth of investing entrepreneurs at the end}} - \underbrace{e}_{\text{total wealth at the beginning}} . \end{aligned} \tag{7}$$

Using the equation in point (ii) in Proposition 1, yields the expression of $Y$ in Equation (6). Aggregate income in period 2 is the output generated by investments in period 1 minus the initial wealth at the beginning of period 1. Note that banks do not need to

[16] Note that we have assumed $\overline{q} \leq 2$. If $\overline{q} > 2$, the pool of entrepreneurs has such high quality that loan demand is very high and equilibria with positive intermediation margins may exist.

[17] See Stahl (1988) and Yanelle (1989 and 1997) for seminal contributions on the theory of two-sided intermediation, and Gehrig (1997) for an extension to differentiated bank services.

put up equity to perform their intermediary function as they can fully diversify their lending activities.

# 6 Temporary Productivity Shocks, Contracts, and Regulation Schemes

In this section, we consider the possibility of aggregate productivity shocks. In period 1, $\bar{q}$ is assumed to be $\bar{q}^h$ with probability $p$ (good state) or $\bar{q}^l$ with probability $1-p$ (bad state) with $0 \leq p \leq 1$. The distribution of the entrepreneurs' qualities varies accordingly. We assume $\bar{q}^l < \bar{q}^h$ and $z = \bar{q}^h - \bar{q}^l$ denotes the size of the shock, which is given. The expected productivity of the entrepreneurs with the highest quality is $\bar{q}^e = p \cdot \bar{q}^h + (1-p)\bar{q}^l$. At the time when financial contracts are written, macroeconomic events are not known.

We maintain the assumptions that savings and investment can be balanced at positive interest rates for any of the following constellations. In particular, we assume that the boundary conditions in Assumptions 1 and 2 from the last section hold for both shock scenarios $\overline{q}^l$ and $\overline{q}^h$.

Equilibria of the intermediation game will now crucially depend on the regulator's approach to bank default and thus when one or both banks are unable to repay depositors. We distinguish two polar cases: bailout and failure. If the regulator commits to failure, banks that are unable to pay back depositors go bankrupt. If the regulator commits to bailout, he will rescue banks and pay back depositors. There are, of course, intermediate scenarios where the regulator taxes the current generation to bail out banks. This is discussed in Subsection 8.2.

With bailout, we assume that banks expect losses to be recovered precisely so that they will have zero profits in the future. If banks incur no losses in period $t$, they will anticipate zero profits due to Bertrand Competition. The assumption ensures that we can define an equilibrium of the financial intermediation game for a particular period.

The focus of our paper is to compare two regulatory schemes for banking crises when banks compete with contingent deposit and loan contracts.[18]

[18]The regulatory schemes could be endogenized in the following way. Suppose that the current generation can determine the regulatory approach toward banking crises. If the costs to establish a

With stochastic aggregate productivity shocks, banks can offer state-contingent contracts in period $t-1$. We use $C(r_j^{ch}, r_j^{cl})$ to denote the credit contract offered by bank $j$. $r_j^{ch}$ and $r_j^{cl}$ denote the interest rates demanded from borrowers in the good state and in the bad state respectively. Similarly, $D(r_j^{dh}, r_j^{dl})$ denotes deposit contracts with deposit rates $r_j^{dh}$ and $r_j^{dl}$, depending on the realization of macroeconomic shocks. We maintain the assumption that banks are risk-neutral.[19]

Since consumers are risk-averse, they prefer a riskless interest rate over a lottery $\{r_j^{dh}, r_j^{dl}\}$ with the same expected interest rate. We assume that the consumers' intertemporal preferences and their attitudes towards risk generate the saving function, now denoted by $s\{r_j^{dh}, r_j^{dl}\}$, which is assumed to be strictly increasing in each of its arguments.

The expected deposit rate is denoted by $r_j^{de} = pr_j^{dh} + (1-p)r_j^{dl}$. Similarly, the expected interest rate on loans is given by $r_j^{ce} = pr_j^{ch} + (1-p)r_j^{cl}$. To simplify notation, we use the following convention. An entrepreneur is characterized by his quality in the good state, $q \in [\overline{q}^h - 1, \overline{q}^h]$, or by his quality in the bad state, $q - z \in [\overline{q}^l - 1, \overline{q}^l]$ (with $z = \overline{q}^h - \overline{q}^e$), or by his expected quality, denoted by $q^e$, and given by

$$q^e = p \cdot q + (1-p)(q-z). \tag{8}$$

The critical entrepreneur is denoted by $q^{e*}$ and depends on $(r_j^{ch}, r_j^{cl}, r_j^{dh}, r_j^{dl})$. An entrepreneur with an expected quality $q^e$ and associated quality $q$ in the good state faces the following choices.

Applying for a credit yields expected wealth

$$\begin{aligned}\mathbb{E}[W(q)] = p\Big\{&\max\{q(I+e) - I(1+r_j^{ch}), 0\}\Big\} \\ &+ (1-p)\Big\{\max\left\{(q-z)(I+e) - I(1+r_j^{cl}), 0\right\}\Big\}.\end{aligned} \tag{9}$$

Saving funds yields expected wealth

$$e\Big(p(1+r_j^{dh}) + (1-p)(1+r_j^{dl})\Big) = e(1+r_j^{de}).$$

new banking system after the failure of the existing one are negligible, the current generation will always choose failure when faced with the case of a banking crisis. If the costs are prohibitively high and the current and future generations are taxed in the same way to pay for the set-up costs of new banks, existing banks will be saved.

[19] Since entrepreneurs as owners of banks are risk-neutral, the assumption follows naturally.

Potential entrepreneurs are risk-neutral. Thus, the comparison of the expected wealth from investing and saving determines the critical quality level above which entrepreneurs choose to invest. We note that in the bad state, the project returns may be insufficient to pay back the loan. In the following section, we examine the intermediation game in period $t-1$, depending on the size of the shock.

# 7 Bank Failure

We first investigate the equilibria when insolvent banks go bankrupt.

## 7.1 Small Productivity Shocks

We first consider the case where shocks are so small that funded and investing entrepreneurs are always able to pay back. The upper limit for small shocks such that this assumption holds is given in the next proposition. In this case, the expected quality of the critical entrepreneur satisfies

$$q^{e*} = 1 + \frac{I \min\{r_j^{ce}\} + e \max\{r_j^{de}\}}{e+I}, \tag{10}$$

such that entrepreneurs with $q^e \geq q^{e*}$ apply for loans, while entrepreneurs with $q^e < q^{e*}$ save their endowments.[20] Note that $q^{e*}$ is associated with a critical quality value in the good state, denoted by $q^*$ and defined by

$$q^{e*} = p\, q^* + (1-p)(q^* - z).$$

We first derive the equilibrium when the regulator commits to failure. In the case of failure, depositors know that banks can never pay back a promised deposit rate if the lending rate is lower in the same state of the world. Hence, we restrict our analysis to $r_j^{dh} \leq r_j^{ch}$ and $r_j^{dl} \leq r_j^{cl}$, as banks have no incentive to offer deposit rates $r_j^{ch} < r_j^{dh}$, since it would not be credible.

Provided funds are received and credit is granted to the entrepreneurs, expected profits

[20] Note that $\min\{r_j^{ce}\}$ is restricted to the set of banks that offer the highest deposit rate, as entrepreneurs seeking loans will only apply at these banks.

per credit of bank $j$, when there is no bailout, amount to

$$\begin{aligned}\mathbb{E}[G_j] &= pI(r_j^{ch} - r_j^{dh}) + (1-p)I(r_j^{cl} - r_j^{dl}) \\ &= I(r_j^{ce} - r_j^{de}).\end{aligned} \tag{11}$$

The critical entrepreneur in equilibrium is denoted by $q_f^{e*}$. We obtain

**Proposition 2**

*Suppose that the regulator commits to failure. Then, there exists a unique equilibrium of the intermediation game if*

$$z \leq \frac{e(1+r^f)}{p(e+I)},$$

*where $r^f$ is determined by*

$$(1-\eta)\ s\{r^f, r^f\} + \eta e\Big(1 + r^f - (\overline{q}^e - 1)\Big) = \eta\Big(\overline{q}^e - (1+r^f)\Big)I,$$

*with $\overline{q}^e = p\overline{q}^h + (1-p)\overline{q}^e$. The equilibrium is given by*

*(i)*

$$r_j^{ch} = r_j^{cl} = r_j^{dh} = r_j^{dl} = r^f, \qquad j = 1, 2.$$

*(ii)*

$$q_f^{e*} = 1 + r^f.$$

*At the equilibrium interest rates, the critical entrepreneur does not default in the bad state:*

$$(q_f^{e*} - zp)(e+I) \geq I(1+r^f). \tag{12}$$

*Banks make zero profits in both states of the world.*

The proof is given in Appendix A. We first note that no entrepreneur defaults at the equilibrium interest rates as long as the size of the macroeconomic shock fulfills the condition of the proposition $\left(z \leq \frac{e(1+r^f)}{p(e+I)}\right)$. Second, we observe that the equilibrium interest rates, the critical entrepreneur, and the upper bound of the shock are fully determined by the exogenous variables.

Proposition 2 implies that financial intermediation, with a commitment to bankruptcy of insolvent banks by the regulator, yields the following intra-generational allocation of risks for the generation under consideration. Risk-neutral entrepreneurs can bear the entire macroeconomic risk, since they can repay the same interest rate in both states as the macroeconomic shock is below the critical size. The productivity shock is fully absorbed by the fluctuation of the entrepreneurs' income, which insures the banking system. Banks never default in equilibrium. The risk allocation ensuing from Proposition 2 is efficient in the sense that the consumption allocation is Pareto efficient. As risk-neutral entrepreneurs bear the entire risk and insure risk-averse consumers,[21] any other allocation of consumption in the first and second period would make at least one class of agents (consumers, saving entrepreneurs or investing entrepreneurs) worse off.[22]

Finally, we observe that Proposition 1 can be viewed as a special case of Proposition 2 when we set $p = 1$. In this case, all expressions of Proposition 2 collapse into those of Proposition 1.

## 7.2 Large Productivity Shocks

We complete our analysis with the examination of the case in which the shock is large. If the shock is sufficiently large, this makes complete insurance of depositors impossible in the failure regime. The essential condition is that the wealth of entrepreneurs is insufficient to insure depositors, i.e., $z > \frac{e(1+r^f)}{p(e+I)}$, where $r^f$ is determined by the next Proposition 2.

**Proposition 3**
*Suppose that the regulator commits to failure and that $z > \frac{e(1+r^f)}{p(e+I)}$. Then there exists an equilibrium of the intermediation game with:*

*(i)*

$$r^h = r_j^{ch} = r_j^{dh}, \qquad j = 1, 2.$$

*(ii)*

$$r^l = r_j^{cl} = r_j^{dl}, \qquad j = 1, 2.$$

*(iii)*

$$r^h = r^h(r^l) := \frac{I(1 + r^l) + (e + I)\ \left\{zp - 1 - (1 - p)r^l\right\}}{p(e + I)}.$$

[21] This can be viewed as a private deposit insurance scheme.
[22] Formal details are available upon request.

*(iv) $r^l$ is smaller than $r^h$ and is determined by*

$$(1-\eta)\cdot s\left\{r^h(r^l),\ r^l\right\}+\eta e\Big(q_f^{e*}-(\bar{q}^e-1)\Big)=\eta(\bar{q}^e-q_f^{e*})\cdot I, \quad \textit{with}$$

*(v)*

$$q_f^{e*}=1+pr^h+(1-p)r^l=\frac{I\,(1+r^l)}{e+I}+zp.$$

*Banks make zero profits in both states of the world.*

The proof is given in Appendix A.[23]

Several remarks are in order. First, the three endogenous variables, interest rates $\{r^h, r^l\}$ and the critical entrepreneur $q_f^{e*}$ are determined by three conditions: savings/investment balance, indifference of critical entrepreneur between savings and investment and the condition that the critical entrepreneur and all other entrepreneurs just do not default in the bad state, i.e., they pay back what they owe. As shown in the proof, this yields the three conditions (iii), (iv) and (v). Condition (iv) is the savings and investment balance since quality levels above $q_f^{e*}$ apply for loans. Condition (iii) is the value of the interest rate in the good state such that an entrepreneur with $q_f^{e*}$ is indeed indifferent between investing and saving. Condition (v) expresses that the output of the critical entrepreneur in the bad state is equal to his repayment obligation in that same state.

Second, deviations by an individual bank that either cause default or positive profits for the critical entrepreneur in the bad state are not profitable.

Third, from the proof, we observe that $r^h > r^l$ for sufficiently large productivity shocks and that $r^h - r^l$ is monotonically increasing in the size of the shock. Larger shocks require larger spreads, as otherwise the critical entrepreneur would default or savings and investment would not balance.

Fourth, as $r^h > r^l$ for sufficiently large productivity shocks, banks offer state-contingent deposit and loan contracts. Part of the macroeconomic risk is shifted to depositors. No aggregate risk is shifted to future generations.

Fifth, there is room for further improvements in risk allocation by repackaging deposit contracts into two securities. Risk-neutral entrepreneurs who save could hold very

[23] Establishing uniqueness is extremely cumbersome. Details on how to prove that other equilibria do not exist are available upon request.

risky contracts, and could bear the entire macroeconomic risk. Risk-averse consumers could be offered less risky or even risk-free contracts. This contract arrangement would further improve intra-generational risk allocation, as the risk would be shifted entirely to risk-neutral agents.

The allocation described in Proposition 3 and the repackaging of state-contingent deposit contracts into safe contracts for consumers and highly risky contracts for risk-neutral entrepreneurs is identical to the Arrow-Debreu solution of the model and thus would be Pareto efficient, which we show next.

We allow for differentiation between deposit rates for entrepreneurs and consumers. We denote the return on deposits for the entrepreneurs by $r_{jE}^{dh}$ and $r_{jE}^{dl}$ and for comsumers by $r_{jC}^{dh}$ and $r_{jC}^{dl}$ respectively, with $j \in \{1, 2\}$. We obtain the following corollary as extension of Proposition 3.

**Corollary 1**

*Suppose that the regulator commits to failure and that $z > \frac{e(1+r^f)}{p(e+I)}$. Then there exists an equilibrium of the intermediation game with maximal insurance of depositors. Either $r_E^{dl} = 0$ and $r_c dh > r_c^{dl}$ are householders also bear some macroeconomics risk or $r_E^{dl} > 0$ and $r_c dh > r_c dl$ and householders are fully insured against macroeconomic risk.*

The proof, which case will occur for which parameters as well as the explicit deposit rates for consumers and entrepreneurs in both cases are given in Appendix A.

The corollary shows that it is possible to repackage state-contingent deposit contracts such that entrepreneurs bare the maximal risk while the risk for households is reduced as much as possible. This allocation corresponds to the Arrow-Debreu solution and is thus Pareto-efficient.[24]

# 8 Bailouts

We next explore the equilibria with bailouts and thus assume that the regulator commits to bailouts. We distinguish two cases: bailout by a future generation and bailout by the current generation.

[24]Details are available upon request.

## 8.1 Bailout by a future generation

We introduce a (overlapping) second generation into the model that is taxed to pay for the bailout. In this case, banks might be tempted to request a particularly high interest rate on loans in the good state and a low interest rate in the bad state. It is instructive to first show that for this reason, the efficient risk allocation, as expressed in Proposition 2, can no longer be an equilibrium.

**Proposition 4**
*Suppose that the regulator commits to bailouts. Then, the intra-generational risk allocation under failure is not an equilibrium.*

The proof is given in Appendix A. The intuition is as follows: A bank can profitably deviate by offering a slightly higher deposit rate, a slightly higher loan rate in the good state, and an appropriately chosen lower loan rate in the bad state. Then, the bank will attract all deposits and all entrepreneurs while savings and loans remain balanced. By raising the loan rate in the good state more than the deposit rate, the bank makes positive profits in this state. Losses in the bad state do not matter, as the bank is bailed out.

In the next proposition, we establish the equilibrium of the game. The critical entrepreneur who is indifferent between saving and applying for a loan in the case of bailouts is denoted by $q_w^{e*}$.

**Proposition 5**
*Suppose* $(\overline{q}^e - 1 - p)e + (\overline{q}^e - 2p)I \leq 0$ *and that the regulator commits to bailouts. Then, there exists a unique equilibrium with*

*(i)*

$$r^w = r_j^{ch} = r_j^{dh} = r_j^{dl}, \qquad j = 1, 2,$$

*(ii)*

$$r_j^{cl} = -1, \qquad j = 1, 2.$$

*(iii)* $r^w$ *is determined by*

$$(1-\eta)s\{r^w,\ r^w\} + \eta e\Big(q_w^{e*} - (\overline{q}^e - 1)\Big) = \eta(\overline{q}^e - q_w^{e*})I,$$

*with*

$$q_w^{e*} = 1 + \frac{I\Big(pr^w - (1-p)\Big) + er^w}{e+I}.$$

*Banks make zero profits in the good state and aggregate losses*

$$(1-\eta)s\{r^w, r^w\}(1+r^w) + e\eta(q_w^{e*} - (\bar{q}^e - 1))(1+r^w)$$

*in the bad state.*

The proof is given in Appendix A. The intuition for this result is as follows. Under bailout, banks wish to create a profitable state, i.e., a state of the world where $r_j^{ch} - r_j^{dh}$ is large, while being unconcerned about losses in the other state. In the good state, competition drives profits to zero and we have $r_j^{ch} = r_j^{dh}$. To be able to demand high interest rates from entrepreneurs in one state of the world, banks do not require any repayment in the bad state. This motivates entrepreneurs to apply for loans. Moreover, deposit rates are non-contingent. If one bank offered a lowered deposit rate in the bad state, all depositors would switch to the other bank, since their expected returns would be higher, as they are bailed out in the bad state.

The condition in Proposition 5 is fulfilled as long as the expected upper level of the productivity is not too high and the probability of the good state is not too low.[25] The condition on $\bar{q}^e$ in the proposition essentially requires that the expected productivity of entrepreneurs is not too large, i.e. $\bar{q}^e$ is not too large. Otherwise, banks could profitably deviate by offering higher deposit rates and loan rates in the good state.

Proposition 5 is extreme since banks are able to write contracts with entrepreneurs, demanding negative interest rates in one state of the world. If we restrict the set of contracts to non-negative interest rates, our results are qualitatively the same, but the potential losses for future generations decrease. In the bad state, banks will demand $r_j^{cl} = 0$, which is the lowest possible interest rate. [26]

[25] If the condition in Proposition 5 is not fulfilled, the results qualitatively remain the same. Banks will still demand less repayment from entrepreneurs in the bad state.

[26] All other interest rates as well as the critical entrepreneur who is indifferent between investing and saving adjust accordingly. The formulae follow from Proposition 5 when we replace $r_j^{cl} = -1$ by $r_j^{cl} = 0$.

An important implication of Proposition 5 is that bailing out banks in the bad state is accompanied by bailing out firms as well. Entrepreneurs pay no interest on their loan (if $r_j^{cl} = 0$) or do not have to pay anything (if $r_j^{cl} = -1$) and hence can still make profits in the bad state. There are various cases where bailout guarantees for banks and hidden subsidies to entrepreneurs have contributed to the emergence of banking crises (see e.g. Krugman (1999) for the Asian crisis). Our analysis suggests that this will naturally arise when banks compete with contingent contracts under a bailout regime, even if moral hazard of entrepreneurs has been eliminated since banks offer large spreads in contingent loan interest rates.

Proposition 5 holds independently of the size of the shock, provided $\overline{q}^e$ fulfills the aforementioned condition. Thus, even if the macroeconomic risk is small, future generations face large aggregate risks.

Moreover, Proposition 5 even holds if the distinction between the good state and the bad state is caused by a sunspot variable, but is not reflected by real quality differences of projects of entrepreneurs. This case occurs if there are sunspot random variables with the probability distribution $(p, 1-p)$, upon which banks write contingent deposit and loan contracts, but $\bar{q}^l = \bar{q}^h$.

Proposition 5 shows that banks generate risk for future generations, in this case even if there is no underlying productivity risk. Hence, we use the term "risk-magnification effect" rather than the well-known "risk-shifting effect" to describe the equilibrium outcome in Proposition 5, as risk is generated independently of the magnitude of the underlying risk. We summarize these observations which follow from the proof of Proposition 5 in the following corollary:

**Corollary 2**
*Suppose $(\overline{q^e} - 1 - p)e + (\overline{q^e} - 2p)I \leq 0$ and that the regulator commits to bailouts.*

*(i) If macroeconomic events are sunspot, i.e., $z = 0$, Proposition 5 continues to hold. In particular, financial intermediation generates real risk for future generations.*

*(ii) In the bad state, future generations face losses equal to the savings of the last generation (if $r_j^{cl} = -1$) or equal to interest rate payments (if $r_j^{cl} = 0$).*

The logic and the proof of the results in Proposition 5 also reveal that when banks can offer contingent or non-contingent contracts, they will choose the contracts in Proposition 5, since nothing prevents banks in the proof of Proposition 5 to choose contracts with $r_j^{ch} = r_j^{cl}$ and $r_j^{dh} = r_j^{dl}$ and thus non-contingent contracts. We summarize this observation in the following corollary.

**Corollary 3**
*Suppose that the assumptions of Proposition 5 hold. If banks can choose between contingent and non-contingent contracts, they choose*

*(i) non-contingent deposit contracts with deposit rates according to Proposition 5,*

*(ii) contingent loan contracts with loan interest rates according to Proposition 5.*

Hence, if contingent contracts are available, such contracts will be chosen and will create large risks for future generations.

## 8.2 Bailout financed by the current generation

In Proposition 5 we assumed that the bailout was financed by taxation of a second generation. Now we consider what happens if the current generation is subject to a lump-sum tax in the bad state to finance the bailout. In particular, each entrepreneur and consumer is taxed by an amount $T$. The amount $T$ will be determined in equilibrium, but is taken as given for individual consumers. We consider the case when interest rates cannot be negative. We denote by $s\{r^w, r^w, T\}$ the savings functions of consumers if they expect a deposit rate $r^w$ in both states of the world and lump-sum taxation $T$ in the bad state. We obtain the following equilibrium:

**Proposition 6**
*Suppose $(\overline{q}^e - 1 - p)e + (\overline{q}^e - 2p)I \leq 0$ and that the regulator commits to bailouts that are paid by the current generation. Then, if $s\{r^w,\ r^w, T\}(1 + r^w) \geq T$, $e(1 + r^w) \geq T$ and $(q_w^{e*} - z)(e + I) \geq T$, where $T$ denotes the aggregate losses (equal to per capita lump sum taxes) in the bad state, an equilibrium is characterized by*

*(i)*

$$r^w = r_j^{ch} = r_j^{dh} = r_j^{dl}, \qquad j = 1, 2,$$

*(ii)*

$$r_j^{cl} = 0, \qquad j = 1, 2.$$

*(iii)* $r^w$ *is determined by*

$$(1-\eta)s\{r^w,\ r^w,T\}+\eta e\Big(q_w^{e*}-(\bar{q}^e-1)\Big)=\eta(\bar{q}^e-q_w^{e*})I,$$

*with*

$$q_w^{e*}=1+\frac{pr^wI+er^w}{e+I}.$$

*Banks make zero profits in the good state and aggregate losses are*

$$\begin{aligned}T &= (1-\eta)s\{r^w,r^w,T\}(1+r^w)+e\eta(q_w^{e*}-(\bar{q}^e-1))(1+r^w)-2(\bar{q}^e-q_w^{e*})I\\ &= r^w\big\{(1-\eta)s\{r^w,r^w,T\}+e\eta(q_w^{e*}-(\bar{q}^e-1))\big\}\end{aligned}$$

*in the bad state. Per capita lump sum taxes are equal to the aggregate losses in the bad state.*

The proof is given in Appendix A.

Several remarks are in order. First, all of the preceding considerations only hold if every citizen (consumers and entrepreneurs) is able to pay taxes. Second, if the current generation finances the bailout, risk-magnification continues to hold as entrepreneurs have to pay zero interest rate in the bad state. Third, uniqueness of equilibria is not generally guaranteed as it depends on how savings of consumers react to lump-sum taxation in the bad state. Savings can increase or decrease in $T$.

Since entrepreneurs pay the same amount of taxes whether they save or invest and thus their decision is ceteris paribus unaffected by taxation in the bad state, aggregate investment can increase or decrease compared to bail out by future generations depending on how savings of households react to taxation.

We provide an example to illustrate the conclusions for logarithmic utility functions. Suppose

$$u(c^1,c^2)=\ln c^1+\beta\ln c^2,$$

with $\beta(0<\beta<1)$ being the discount factor. Given interest rates $r^w$ on deposits and lump sum taxation of $T$ in the bad state, the problem of the consumer is given by

$$
\begin{aligned}
&\max_{c^1,c^2_h,c^2_l}[\ln c^1 + \beta(p\ln c^2_h + (1-p)\ln c^2_l)] \\
&\text{s.t.} \begin{cases} c^2_h &= (e-c^1)(1+r^w) \\ c^2_l &= (e-c^1)(1+r^w) - T. \end{cases}
\end{aligned} \tag{13}
$$

The solution of the optimization (13) is obtained by finding the maximum of the function $f:\mathbb{R}\to\mathbb{R}$ defined by

$$
f(c_1) = \ln c^1 + \beta(p\ln[(e-c^1)(1+r^w)] + (1-p)\ln[(e-c^1)(1+r^w)-T]).
$$

The first-order condition is given by

$$
\begin{aligned}
\frac{\partial f}{\partial c_1}(c_1) &= \frac{1}{c_1} - \frac{\beta p}{e-c_1} - \frac{\beta(1-p)(1+r^w)}{(e-c_1)(1+r^w)-T} \\
&= \frac{1}{c_1} - \frac{\beta(1+r^w)(e-c_1)-\beta pT}{(e-c_1)[(e-c_1)(1+r^w)-T]} \\
&= \frac{(e-c_1)[(e-c_1)(1+r^w)-T] - c_1[\beta(1+r^w)(e-c_1)-\beta pT]}{c_1(e-c_1)[(e-c_1)(1+r^w)-T]} \\
&= \frac{(e-c_1)(1+r^w)[e-(1+\beta)c_1] - T[e-(1-p\beta)c_1]}{c_1(e-c_1)[(e-c_1)(1+r^w)-T]} \\
&= \frac{(e-c_1)(1+r^w)[e-(1+\beta)c_1] - T[e-(1+\beta-\beta(1+p))c_1]}{c_1(e-c_1)[(e-c_1)(1+r^w)-T]} \\
&= \frac{[(e-c_1)(1+r^w)-T][e-(1+\beta)c_1]}{c_1(e-c_1)[(e-c_1)(1+r^w)-T]} - \frac{Tc_1(1+p)\beta}{c_1(e-c_1)[(e-c_1)(1+r^w)-T]} \\
&= \frac{[e-(1+\beta)c_1]}{c_1(e-c_1)} - \frac{Tc_1(1+p)\beta}{c_1(e-c_1)[(e-c_1)(1+r^w)-T]} \\
&= 0.
\end{aligned}
$$

The above equality translates into

$$
\begin{aligned}
[e-(1+\beta)c_1] = &\frac{Tc_1(1+p)\beta}{[(e-c_1)(1+r^w)-T]} \\
&\iff [e-(1+\beta)c_1][(e-c_1)(1+r^w)-T] = Tc_1(1+p)\beta \\
&\iff (1+\beta)(1+r^w)c_1^2 + [-(1+r^w)e(2+\beta) + T(1-p\beta)]c_1 + e[e(1+r^w)-T] = 0.
\end{aligned}
$$

The above equality has two solutions since

$$
\Delta = T^2(1-p\beta)^2 + (1+r^w)^2e^2\beta^2 + 2\beta eT(1+r^w)(1+p(2+\beta)) > 0,
$$

which are given by

$$
x_{1,2} = \frac{[(1+r^w)e(2+\beta) - T(1-p\beta)] \pm \sqrt{\Delta}}{2(1+\beta)(1+r^w)}.
$$

For $T = 0$, the solution has to be identical to the standard solution under certainty, i.e. $c_1 = \frac{e}{1+\beta}$. Hence, only the solution with the minus before $\sqrt{\Delta}$ is economically meaningful.

Therefore, the savings function $s(r^w, T) = e - c_1$ is given by

$$s(r^w, T) = \frac{[(1 + r^w)e\beta) + T(1 - p\beta)] + \sqrt{\Delta}}{2(1 + \beta)(1 + r^w)}. \tag{14}$$

We observe that savings are increasing in $T$ and thus aggregate investment will be increasing when the current generation bails out itself.

# 9 Comparison

In this section, we compare the situation under failure and bailouts when banks can offer contingent contracts. Then we compare the results to the outcomes when banks can only offer non-contingent deposit and loan contracts. We start with the comparison for contingent contracts when potential bailouts are financed by the next generation.

**Proposition 7**

*Suppose that the interest rate elasticity of savings is strictly positive. The comparison between bailout and failure in the case of small productivity shocks yields*

*(i)* $r^w > r^f$,

*(ii)* $q_w^{e*} < q_f^{e*}$.

The proof is given in Appendix A. Proposition 7 implies that under the bailout regime, the current generation invests more compared to the bank failure regime, and depositors receive more attractive interest rates. Since entrepreneurs do not need to pay back in one state of the world under bailout, a larger percentage of entrepreneurs choose to invest rather than save compared to the failure regime.

Proposition 7 can be interpreted as a lending boom under bailout, as aggregate credit expands. The result complements the seminal work on lending booms (Dell' Ariccia and Marquez (2008)) who show that lending standards may endogenously decline, which, in turn, may increase aggregate surplus, but also the risk of financial instability. In our model, intertemporal aggregate output is lower in the bailout regime. Specifically, aggregate output over two generations is higher under bank failure than under bailout when the second generation has to bailout the first generation.[27] This is obvious in the simplest case when the interest elasticity of savings is zero. Then we have $q_w^{e*} = q_f^{e*}$ and, hence, expected aggregate output in the first generation is the same in both regimes. In the next period, however, savings and investment are lower in the bailout regime than in the failure regime, when the bad state has occurred in the generation before. In the good state, output is identical in both regimes. Hence, expected aggregate output over two generations is smaller in the bailout regime than in the failure regime.[28]

Moreover, both households and entrepreneurs benefit from bailout. Households and savings entrepreneurs benefit from higher and safe interest rates on their savings. Investing entrepreneurs benefit as their expecting repayments is lower.

Finally we obtain that risk magnification under contingent deposit and loan contracts and bailouts leads to significantly different outcomes than those that would occur under non-contingent deposit and loan contracts and bailout. With non-contingent deposit and loan contracts, for small shocks ($z \leq \frac{e(1+r^f)}{p(e+I)}$), Proposition 2 holds under a bailout. The equilibrium derived in Proposition 2 is also an equilibrium with the restriction that contracts have to be non-contingent and bailout since contracts turns out to be non-contingent and no bank can profitably deviate. For larger productivity shocks, non-contingent contracts also lead to bank failures, but the losses in equilibrium are considerable smaller than with contingent contracts since entrepreneurs face a repayment obligation with a positive loan interest rate in good and bad times.[29]

[27]To prevent the decline in aggregate output, the regulator could fix deposit rates at the level $r^f$ from the outside. Such an ex ante deposit rate ceiling would not, however, eliminate the risk magnification incentive of banks, since banks would still like to create a profitable and an unprofitable state of the world on the loan side.

[28]The general proof is tedious. Two effects occur. First, entrepreneurs of low quality (i.e. entrepreneurs with $q_w^{e*} < q^{e*} \leq q_f^{e*}$) invest in the first generation under the bailout regime, but not under the failure regime. Second, bailout reduces investment of entrepreneurs with higher quality levels than $q_f^{e*}$ in the second generation. Accordingly, aggregate output over two generations is higher under the failure regime than under the bailout regime. Details are available upon request.

[29]Details are available upon request.

# 10 Implications and Extensions

We have limited ourselves to a simple model to identify risk magnification. In this section, we outline several implications for empirical analysis and for policy, and we sketch possible extensions.

Regarding empirical analysis, we have to recognize that the model neglects the costs of intermediation, which would have to be added to the intermediation margins. Moreover, in practice, many more elements enter the determination of intermediation margins. Nevertheless, the results imply two empirical predictions. First, the results suggest that intermediation margins are strongly procyclical, large in good times and low or even negative in bad times. Second, high risks are not created by banks shifting into particularly risky activities, but by changing intermediation margins for bad times. Hence, systemic risk and subsequent banking crises can occur even if balance sheets of banks change very little or not at all with respect to their assets, and if asset prices move very little.[30]

Regarding policy, the most obvious issue is whether risk magnification can be avoided by pricing deposit insurance fairly. While a comprehensive formal analysis is beyond the scope of this paper, we observe that risk premia based on asset risk or even on systemic risk created by the banking system cannot avoid risk magnification. The risk premia would be added to the intermediation margins, but the incentive to create a profitable and a non-profitable state with high and low or even negative intermediation margins would continue to exist. The only way to avoid risk magnification would be to punish banks by the same amount as they expect to gain when banks start to differentiate ex-ante intermediation margins for good and bad states of the world. This would require detailed observations of contracting and is far away from existing deposit insurance schemes (see Calomiris and Jaremski (2016)).

The current framework should allow a number of useful extensions. For instance, it may be useful to consider a wider range of macroeconomic shocks and security design issues in the spirit of Repullo and Suarez (1998). In particular, one could condition contracts on other contractible macroeconomic events that are highly correlated with the financial health of the banking sector. For instance, one might try to use an index

[30]The formula in Proposition 5 offers relationships between the parameters of the model and the size of the risk that is created by the bank. This might also be exploited in empirical analysis.

that measures the average default rate of entrepreneurs, or the level of aggregate bank capital that would occur if the good state were assumed. In our model, all these variants of macroeconomic indicators would yield the same results.

It is also useful to consider contingent bailout schemes. For example, one may conjecture that with small shocks, the regulator is expected to stay out, while with large macroeconomic shocks, the regulator is expected to step in. Such contingent government bailout schemes would preserve the incentives of banks to generate profitable states of the world. Hence, contingent bailout would, at best, alleviate the risk-magnification effect.

Furthermore, competition is essential for the results to hold. Among other things, the fact that two banks are competing makes sure that banks make zero or negative profit depending on the state of the world. If one would consider a monopolistic bank, different equilibria could occur, involving, for example, a credit rate that exceeds the deposit rates such that banks make a positive profit and risk magnification may be reduced. This option is left for further research.

# 11 Conclusions

We developed a model to study the interdependence of contingent contracts and public guarantees on bank stability, saving and investment. Our analysis indicates that the combination of allowing banks to fail, along with contingent deposit and loan contracts, tends to yield an efficient intra-generational risk allocation. However, if there is a chance for a bailout, large macroeconomic risks are created, even if the underlying risk is small or zero. Such phenomena do not occur with non-contingent deposit and loan contracts, and thus this is a serious issue for banking regulation.[31] .

Numerous extensions can and should be pursued in future research, as outlined in the last section. The present paper is, hopefully, a basis for this programme.

[31] See Dewatripont and Tirole (1994), Hellwig (1998), and Allen and Santomero (1998) for early but deep assessments and Vives (2016) and Freixas, Laeven and Peydro (2015) for more recent accounts.

# 12 Appendix A

**Proof of Proposition 1:**

We first show the existence of the equilibrium. The boundary condition of Assumption 2 ensures that at least one solution exists for the equality in (ii). Indeed, for sufficiently high interest rates, investments are zero, and hence the left-hand side of the equation in point (ii) of the proposition for $r^*$ is greater than the right-hand side. For $r^* = 0$, the boundary condition ensures that the right-hand side is greater than the left-hand side. Moreover, both sides are continuous and the left-hand side of the implicit equation for $r^*$ in Proposition 1 is monotonically increasing in $r^*$. In contrast, the right-hand side is decreasing in $r^*$. Hence, the intermediate value theorem establishes that at least one solution exists and it is unique.

Loan application decisions of entrepreneurs are optimal, given $r^d = r^c = r^*$. Profits of banks per credit contract are zero (see Equation (5)).

Changing one interest rate, while leaving the other at $r^*$, is never profitable for a bank; Consider a change of $r_j^d$. Profits are either negative if $r_j^d > r^*$, or a deviating bank obtains no resources if $r_j^d < r^*$. Consider a change of $r_j^c$. Profits are negative if $r_j^c < r^*$ since the interest rate margin is negative, or the deviating bank does not obtain loan applications if $r_j^c > r^*$, as entrepreneurs seeking loans go to competing banks offering better terms. If entrepreneurs were rejected at competing banks, they would save according to our rationing schemes.

Suppose, however, that bank $j$ offers slightly better conditions for depositors, $r_j^d = r^* + \epsilon$, with some $\epsilon > 0$, and tries to exploit its monopolistic power on the lending side, i.e., the bank changes both interest rates. Since bank $j$ would obtain all deposits, entrepreneurs can only receive loans at this bank. Hence, profits of the deviating bank, denoted by $\pi_j$, amount to

$$\begin{aligned}\pi_j = \eta(\bar{q} - q^*)I(1 + r_j^c) - \eta e\Big(q^* - (\bar{q} - 1)\Big)(1 + r^* + \epsilon) \\ - (1 - \eta)s\left\{r^* + \epsilon\right\}(1 + r^* + \epsilon),\end{aligned} \tag{15}$$

where

$$q^* = 1 + \frac{Ir_j^c + e(r^* + \epsilon)}{e + I}$$

and

$$r_j^c > r^* + \epsilon.$$

As $q^* > 1 + r^*$, bank $j$ has excess resources since fewer entrepreneurs than in the equilibrium apply for loans. The amount of excess resources is

$$(1-\eta)s\left\{r^* + \epsilon\right\} + \eta e\Big(q^* - (\bar{q} - 1)\Big) - \eta(\bar{q} - q^*)I,$$

which, however, can neither be invested nor used in the next period, since the good is perishable. We obtain

$$\begin{aligned}
\frac{\partial \pi_j}{\partial r_j^c} &= \eta\left\{(\bar{q} - q^*)I - \frac{I}{e+I}I(1 + r_j^c)\right\} - \eta e\frac{I}{e+I}(1 + r^* + \epsilon) \\
&= \frac{\eta I}{e+I}\Big\{(\bar{q} - 1)(e + I) - 2Ir_j^c - I - e(1 + 2r^* + 2\epsilon)\Big\} \\
&= \frac{\eta I}{e+I}\Big\{(\bar{q} - 2)(e + I) - 2Ir_j^c - e(2r^* + 2\epsilon)\Big\} \\
&\leq \frac{\eta I}{e+I}\Big\{(\bar{q} - 2)(e + I)\Big\}.
\end{aligned}$$

Therefore, $\frac{\partial \pi_j}{\partial r_j^c}$ is negative since $\overline{q} \leq 2$.

Hence, profits are negative for $r_j^c = r^* + \epsilon$ because of excess resources and because profits are decreasing for $r_j^c \geq r^* + \epsilon$ with the loan interest rate. Thus, bank $j$ makes losses by offering $r_j^d = r^* + \epsilon$ and a lending rate $r_j^c \geq r^* + \epsilon$. Finally, it is obvious that setting $r_j^d = r^* + \varepsilon$ and $r_j^c < r^* + \varepsilon$ is not profitable because profits are negative.

Uniqueness follows through similar observations. First, if both banks chose the interest rates $\tilde{r}^c = \tilde{r}^d < r^*$, loan demand would exceed savings, and both banks would make zero profits. By setting $r_j^d = \tilde{r}^d + \epsilon$ and $r_j^c = \tilde{r}^c + 2\epsilon < r^*$, bank $j$ would generate positive profits. Second, if both banks chose $\tilde{r}^c = \tilde{r}^d > r^*$, both would make losses due to the excess resources, and a bank $j$ would be better off by choosing $\tilde{r}_j^c = \tilde{r}_j^d = r^*$ and making zero profit. Finally, no interest rate constellation with $r^d < r^c$ can be an equilibrium. A bank can profitably deviate by setting $r^d + \delta_1$, $(\delta_1 > 0)$ and $r^c - \delta_2$, $(\delta_2 > 0)$, where $\delta_1$ and $\delta_2$ are arbitrarily small and can be selected such that no excess resources are generated.

**Proof of Proposition 2:**

We observe that, given $r_j^{ce}$ and $r_j^{de}$, and hence a given critical entrepreneur $q_f^{e*}$ and a given profit per credit, banks can offer risk-averse depositors the highest utility by setting $r_j^{dh} = r_j^{dl}$. Therefore, Bertrand Competition will lead to $r_j^{dh} = r_j^{dl} = r_j^{de}$. Moreover, banks are forced to offer $r_j^{ce} = r_j^{de}$. Raising $r_j^{de}$ slightly and increasing $r_j^{ce}$ to obtain monopoly profits from entrepreneurs is not profitable for the same reasons as outlined in Proposition 1. $r_j^{dh} = r_j^{dl} = r_j^{de} = r_j^{ce}$ and the repayment conditions $r_j^{dh} \leq r_j^{ch}$ and $r_j^{dl} \leq r_j^{cl}$ imply $r_j^{ch} = r_j^{cl} = r_j^{dh} = r_j^{dl}$.

This equilibrium interest rate is denoted by $r^f$ and determined by the saving and investment balance. A solution indeed exists. Finally, we need to verify that banks are able to pay back in both states of the world, since otherwise, their deposit rates would not be credible. In the bad state, the repayment condition for the entrepreneur with the lowest quality is given by

$$(q^* - z)(e + I) = (q_f^{e*} - zp)(e + I) \geq I\,(1 + r^f).$$

Using $q_f^{e*} = 1 + r^f$, this is implied by our assumption

$$z \leq \frac{e(1 + r^f)}{p(e + I)}.$$

**Proof of Proposition 3:**

a) We construct a candicate for an equilibrium in the following way. In the bad state, we determine the interest rate $r^l$ by the requirement that the critical entrepreneur can just pay back. We must have

$$(q^* - z)(e + I) = I\,(1 + r^l). \tag{16}$$

Using

$$q^e = pq + (1 - p)(q - z)$$

and thus

$$q_f^{e*} = p\, q^* + (1-p)(q^* - z)$$

leads to

$$q^* - z = q_f^{e*} - zp.$$

We obtain

$$(q_f^{e*} - zp)(e+I) = I\,(1+r^l). \tag{17}$$

Inserting $q_f^{e*} = 1 + pr^h + (1-p)r^l$, which follows from Equation (10) by assuming that expected loan rates are equal to expected deposit rates, yields

$$r^h = \frac{I\,(1+r^l) + (e+I)\left\{zp - 1 - (1-p)\,r^l\right\}}{p\,(e+I)}$$

which corresponds to (iii). (v) follows by solving Equation (17) for $q_f^{e*}$. Conditions (iii), (iv) and (v) determine $\{r^h, r^l\}$ and $q_f^{e*}$.[32]

b) For sufficiently large productivity shocks, we establish that $r^h > r^l$:

Using (iii), $r^h > r^l$ is equivalent to

$$p(e+I)r^l < p(e+I)\,r^h = I(1+r^l) + (e+I)\left\{zp - 1 - (1-p)\,r^l\right\},$$

which can be simplified to (18)

$$er^l < I + (e+I)(zp-1).$$

For a given $r^l$, $q_f^{e*}$ is increasing in $z$. In order to fulfill the savings/investment balance in (iv), an increase in $z$ leads to a decline in $r^l$. Hence, for sufficiently high $z$, Equation (18) is fulfilled. Therefore, $r^h > r^l$ for sufficiently large productivity shocks.

c) Expected profits of banks are zero. Suppose bank $j$ offers deposit interest rates $r^h$ and $r^l + \epsilon$, for some small $\epsilon > 0$. Since bank $j$ obtains all deposits, it could change the individually optimal interest rates on loans. In order to avoid an excess resource problem, bank $j$ needs to ensure that enough entrepreneurs want to apply for credits. Therefore, $q^e$ should not rise above $q_f^{e*} = 1 + pr^h + (1-p)r^l$.

[32] Although we have assumed $s\{r^h, r^l\}$ is strictly increasing in $r^h$ and $r^l$, uniqueness is not guaranteed. A sufficient condition for uniqueness is that $r^h(r^l)$ in (iii) is non-decreasing in $r^l$, which requires $\frac{1-p}{p} \leq \frac{I}{e}$.

If the deviating bank wishes to achieve $q^e = q_f^{e*}$, i.e.

$$q_f^{e*} = 1 + \frac{I\, r^{ce} + e\Big(pr^h + (1-p)(r^l + \epsilon)\Big)}{e + I} = 1 + pr^h + (1-p)r^l,$$

we obtain

$$r^{ce} = pr^h + (1-p)r^l - \frac{e\varepsilon(1-p)}{I} < pr^h + (1-p)r^l < r^{de} = pr^h + (1-p)(r^l + \varepsilon).$$

Accordingly, expected profits per credit amount to

$$\begin{aligned}
\mathbb{E}[G_j] &= p(r_j^{ch} - r_j^{dh})I + (1-p)(r_j^{cl} - r_j^{dl})I \\
&= I\,(r^{ce} - r^{de}) \\
&\leq 0.
\end{aligned}$$

Hence, the deviation does not benefit bank $j$. Similar reasoning for any other potential deviation establishes that $\Big\{\{r_j^{dh} = r_j^{ch} = r^h\}_{j=1,2},\ \{r_j^{dl} = r_j^{cl} = r^l\}_{j=1,2}\Big\}$ is an equilibrium.

**Proof of Corollary 1:**

a) We allow differentiation between deposit rates for consumers and entrepreneurs. That means, we have to determine six contingent rates $r_C^{dl}, r_C^{dh}, r_E^{dl}, r_E^{dh}, r^{cl}, r^{ch}$. We explore whether an equilibrium exists if $r_E^{dl} = 0$ such that entrepreneurs bare the maximal risk.

   The interest rate $r^{cl}$ is determined by the requirement that the critical entrepreneur can pay back, i.e., we must have:

$$(q^* - z)(e + I) = I(1 + r^{cl})$$

   Following the same line as part a) of the Proof of Proposition 3 we obtain:

$$r^{ch} = r^{ch}(r^{cl}) := \frac{I(1 + r^{cl}) + (e+I)\ \big\{zp - 1 - (1-p)r^{cl}\big\}}{p(e+I)}.$$

b) As proven in section c) of the Proof of Proposition 3, competition guarantees that banks make zero profits. This implies that:

$$\eta(\bar{q} - q_f^{e*})I(1 + r^{ch}) = (q_f^{e*} - (\bar{q} - 1))\eta e(1 + r_E^{dh}) + (1-\eta)s\{r_C^{dh}, r_C^{dl}\}(1 + r_C^{dh})$$

in the good state. Together with the investment and savings balance, this yields:

$$\eta(\bar{q}-q_f^{e*})Ir^{ch} = (q_f^{e*}-(\bar{q}-1))\eta e r_E^{dh} + (1-\eta)s\{r_C^{dh}, r_C^{dl}\}r_C^{dh}.$$

Furthermore, indifference between saving or investing of the critical entrepreneur means:

$$ep(1+r_E^{dh}) + (1-p)e = (e+I)q_f^{e*} - pI(1+r^{ch}) - I(1-p)(1+r^{cl}).$$

Using the expression of $r^{ch}$ in terms of $r^{cl}$ and solving for $r_E^{dh}$ yields:

$$r_E^{dh} = \frac{e^2(q_f^{e*}-1) + eI(2q_f^{e*}-2+p-pz) + I^2(q_f^{e*}-2+p-r^{cl}-pz)}{e(e+I)p}$$

Using the zero profit condition in the bad state and $r_E^{dl}=0$ we know that

$$r^{cl} = \frac{(1-\eta)s\{r_C^{dh}, r_C^{dl}\}r_C^{dl}}{\eta(\bar{q}-q_f^{e*})I}$$

This means that $r_E^{dh}, r^{ch}$ can be expressed in terms of $r_C^{dl}$.

c) Using the zero profit condition in the good state, $r_C^{dh}$ is given by:

$$r_C^{dh} = \frac{\eta(I(q_f^{e*}-\bar{q})r^{ch} + e(1-\bar{q}+q_f^{e*})r_E^{dh})}{(\eta-1)s\{r_C^{dh}, r_C^{dl}\}}$$

This means that $r_C^{dh}$ is completely determined by $r_C^{dl}$ which is defined by the saving-investment balance:

$$(1-\eta)\cdot s\left\{r_C^{dh}(r_C^{dl}),\ r_C^{dl}\right\} + \eta e\Big(q_f^{e*}-(\bar{q}^e-1)\Big) = \eta(\bar{q}^e-q_f^{e*})\cdot I$$

with

$$q_f^{e*} = \frac{I(1+r^{cl})}{e+I} + zp.$$

d) The above values are only an equilibrium if: $r_C^{dh} \geq r_C^{dl}$. Otherwise, with $r_C^{dl} > r_C^{dh}$, entrepreneurs overinsure depositors, with risk-averse depositors bearing unnecessary risk by having lower returns in the good state than in the bad state. The condition translates into

$$r_C^{dl} \geq -\frac{\eta(\bar{q}-q_f^{e*})(e^2(-1+\bar{q}-q_f^{e*})(q_f^{e*}-1)}{(\eta-1)(I(3\bar{q}-1-3q_f^{e*}) + e(\bar{q}-q_f^{e*})s\{r_C^{dh}, r_C^{dl}\}}$$
$$+\frac{eI(1+2\bar{q}(q_f^{e*}-1) - eq_f^{e*^2} + pz + I^2(1+\bar{q}(q_f^{e*}-1) - q_f^{e*^2}+pz))}{(\eta-1)(I(3\bar{q}-1-3q_f^{e*}) + e(\bar{q}-q_f^{e*})s\{r_C^{dh}, r_C^{dl}\}}.$$

In this case, the saving entrepreneurs need to bare the full risk to decrease the risk for the households and thus $r_E^{dl} = 0$.

e) Suppose next that the condition in Step $d$) is not fulfilled. In this case, the minimal interest rate $r_E^{dl} = 0$ is not necessary to insure depositors. We thus determine the maximal deposit rate for entrepreneurs in the bad state, such that depositors are still fully insured by setting $r_C^d = r_C^{dh} = r_C^{dl}$ in equilibrium.

$r_C^d$ is still determined by the investment saving balance:

$$(1-\eta) \cdot s\left\{r_C^{dh}(r_C^{dl}),\ r_C^{dl}\right\} + \eta e\Big(q_f^{e*} - (\bar{q}^e - 1)\Big) = \eta(\bar{q}^e - q_f^{e*}) \cdot I$$

The zero profit conditions in the good and bad state give:

$$\eta(\bar{q} - q_f^{e*})Ir^{ch} = (q_f^{e*} - (\bar{q} - 1))\eta e r_E^{dh} + (1-\eta)s\{r_C^d, r_C^d\}r_C^d.$$

$$\eta(\bar{q} - q_f^{e*})Ir^{cl} = (q_f^{e*} - (\bar{q} - 1))\eta e r_E^{dl} + (1-\eta)s\{r_C^d, r_C^d\}r_C^d.$$

$r^{ch}$ is still given in terms of $r^{cl}$ by:

$$r^{ch} = r^{ch}(r^{cl}) := \frac{I(1+r^{cl}) + (e+I)\ \left\{zp - 1 - (1-p)r^{cl}\right\}}{p(e+I)}.$$

Equating this expression with the one from the zero profit condition we obtain:

$$r_E^{dh} = \frac{p(\eta-1)r_C^d s\{r_C^d\}(e+I) + I\eta(\bar{q} - q_f^{e*})(I(p-2)r^{cl} + e(p-1)r^{cl} - e + pz(e+I))}{e(e+I)\eta p(q_f^{e*} - (\bar{q} - 1))}.$$

Indifference of the critical entrepreneur yields:

$$r_E^{dh} = \frac{e^2(-1 + q_f^{e*} + (p-1)r_E^{dl}) + I^2(-1 + q_f^{e*} + r^{cl} - pz) + eI(-1 + eq_f^{e*} + (p-1)r_E^{dl} - pz)}{e(e+I)p}.$$

Equating these two expressions yields $r^{cl}$ in terms of $r_E^{dl}$. Substituting this in the zero profit condition in the bad state, we obtain:

$$\begin{aligned} r_E^{dl} = {} & \frac{1}{eI\eta(q_f^{e*} - (\bar{q} - 1))}\Big[e^2\eta(q_f^{e*} - 1)(q_f^{e*} - \bar{q} + (q_f^{e*} - \bar{q})^2) + e(\bar{q} - q_f^{e*})((\eta - 1)r_C^d s\{r_C^d\} \\ & + I\eta(1 + 2\bar{q}(q_f^{e*} - 1) - 2q_f^{e*} - 2q_f^{e*} + pz)) + I\{(\eta - 1)(1 + \bar{q} - q_f^{e*})r_C^d s\{r_C^d\} \\ & + I\eta(\bar{q} - q_f^{e*})(1 + \bar{q}(q_f^{e*} - 1) - q_f^{e*^2} + pz)\}\Big]. \end{aligned}$$

Since $r_C^d$ is determined by the investment saving balance and all other parameters can be related to $r_C^d$, the equilibrium is uniquely determined.

**Proof of Proposition 4:**

Consider the risk allocation of Proposition 2. We show that a bank $j$ can deviate and be better off by offering the following interest rates:

$$\begin{aligned} r_j^{dh} = r_j^{dl} &= r^f + \epsilon, \\ r_j^{ch} &= r^f + \delta, \\ r_j^{cl} &= r^f - \frac{p\delta}{1-p}, \end{aligned}$$

where $\delta > \epsilon > 0$. Hence,

$$\begin{aligned} r_j^{de} = pr_j^{dh} + (1-p)r_j^{dl} &= p(r^f + \epsilon) + (1-p)(r^f + \epsilon) = r^f + \epsilon > r^f \\ r_j^{ce} = pr_j^{ch} + (1-p)r_j^{cl} &= p(r^f + \delta) + (1-p)(r^f - \frac{p\delta}{1-p}) = r^f. \end{aligned}$$

Bank $j$ would obtain all deposits since $r_j^{de} > r^f$. The critical entrepreneur amounts to

$$q_f^{e*} = 1 + \frac{Ir^f + e(r^f + \epsilon)}{e + I} = 1 + r^f + \frac{e\epsilon}{e + I}.$$

Hence, for sufficiently small $\epsilon$, savings and investments are almost balanced. Since $r_j^{dh} < r_j^{ch}, r_j^{dl} > r_j^{cl}$, bank $j$ will not be able to pay back depositors in the bad state. However, since banks are bailed out, their profit in the bad state will be zero. Hence, expected bank profits per credit in this case amount to

$$\mathbb{E}[G_j] = p \cdot I(\delta - \epsilon). \tag{19}$$

For a sufficiently small amount of $\epsilon$, excess resources from depositors are negligible. However, by choosing $\delta > \epsilon$ and making $\delta$ sufficiently large, expected profits will be large. Hence, the profitable deviation of bank $j$ eliminates the existence of the efficient intra-generational risk allocation equilibrium.

**Proof of Proposition 5:**

We first observe that $r^w$ is uniquely determined by the equation in (iii). The left-hand side of the implicit equation for $r^w$ in Proposition 5 is increasing in $r^w$, since

$s\{r^w,\ r^w\}$ and $q_w^{e*}$ are monotonically increasing in $r^w$. By contrast, the right-hand side is decreasing in $r^w$. The two boundary conditions ensure that a unique solution exists.

The most promising deviation of bank $j$ would be[33]

$$
\begin{aligned}
r_j^{dh} = r_j^{dl} &= r^w + \epsilon, && (20)\\
r_j^{cl} &= -1, && (21)
\end{aligned}
$$

for some small $\epsilon > 0$. The bank would obtain all resources and would try to maximize expected profits by choosing an interest rate $r_j^{ch}$ ($r_j^{ch} \geq r^w + \epsilon$). Entrepreneurs expect to obtain loans at the deviating bank $j$ only. Expected profits for banks are given by

$$
\begin{aligned}
\mathbb{E}[\pi_j] = p\Big\{&\eta(\bar{q}^e - q^*)I\Big(1 + r_j^{ch}\Big) - \eta e\Big(q^* - (\bar{q}^e - 1)\Big)(1 + r^w + \epsilon)\\
&-(1-\eta)s\{r^w + \epsilon,\ r^w + \epsilon\}(1 + r^w + \epsilon)\Big\},\\
\text{with} \qquad & q^* = 1 + \frac{I\Big(pr_j^{ch} - (1-p)\Big) + e(r^w + \epsilon)}{e + I}.
\end{aligned} \tag{22}
$$

We obtain

$$
\begin{aligned}
\frac{\partial \mathbb{E}[\pi_j]}{\partial r_j^{ch}} &= \frac{p\,\eta\,I}{e+I}\Big\{(\bar{q}^e - 1)(e + I)\\
&\qquad - I\left\{pr_j^{ch} - (1-p)\right\} - e(r^w + \epsilon) - pI(1 + r_j^{ch}) - ep(1 + r^w + \epsilon)\Big\}\\
&= \frac{p\,\eta\,I}{e+I}\Big\{(\bar{q}^e - 1)(e + I) - I(2pr_j^{ch} + 2p - 1)\\
&\qquad - e(p + r^w(1+p) + \epsilon(1+p))\Big\}\\
&\leq \frac{p\,\eta\,I}{e+I}\Big\{(\bar{q}^e - 1 - p)(e + I) + I(1-p) - 2Ir_j^{ch} - e(r^w(1+p) + \epsilon(1+p))\Big\}\\
&\leq \frac{p\,\eta\,I}{e+I}\Big\{(\bar{q}^e - 1 - p)(e + I) + I(1-p)\Big\}\\
&= \frac{p\,\eta\,I}{e+I}\Big\{(\bar{q}^e - 1 - p)e + (\bar{q}^e - 2p)I\Big\}.
\end{aligned} \tag{23}
$$

Note that we have used that $r_j^{ch}$, $r^w$ and $\epsilon$ are non-negative to obtain the inequality. The assumption of the proposition implies that the last expression is not positive. Hence, the deviation with a positive $\epsilon$ is not profitable if the assumption of the proposition

[33] It is straightforward but tedious to verify that no other potential deviation is profitable.

holds. A deviation with a negative $\epsilon$ is also not profitable. [34]

**Proof of Proposition 6:**

Note that the equilibrium considerations regarding interest rates are carried over from the proof of Proposition 5. However, if the bank defaults in the bad state, the current generation is subject to a lump sum tax to finance the bailout. The total tax amount necessarily equals the aggregated losses:

$$T = r^w\{(1-\eta)s\{r^w, r^w\} + e\eta(q_w^{e*} - (\bar{q}^e - 1))\}.$$

The equilibrium only holds if every member of the population is able to pay the tax. For the consumers this means: $s\{r^w,\ r^w, T\}(r^w) \geq T$. Note that consumers anticipate the tax and thus the probability of having to pay the tax influences the saving decision. The conditions for the entrepreneurs are determined by the incomes from savings or their income in the bad state:

$$e(r^w) \geq T$$

$$(q_w^{e*} - z)(e + I) \geq T.$$

Note that if the critical entrepreneur is able to pay the tax in the bad state, so will be all entrepreneurs with higher quality.

**Proof of Proposition 7:**

We compare the savings and investment balance in both cases. Suppose that $r^w \leq r^f$. As $0 < p < 1$, this implies that

$$q_w^{e*} < 1 + \frac{I\,r^f + e\,r^f}{e + I} = 1 + r^f = q_f^{e*}.$$

Hence, using Proposition 2, we obtain

$$(1-\eta)\,s\{r^f, r^f\} + \eta\,e\left(q_w^{e*} - (\bar{q}^e - 1)\right) < \eta\,(\bar{q}^e - q_w^{e*})\,I.$$

[34]Uniqueness can be established by first establishing that any constellation with $r_j^{cl} > -1$ cannot be an equilibrium. Second, in any equilibrium, loan and deposit rates in the good state have to be identical.

The strict inequality is reinforced when $r^f$ is lowered to $r^w$ because savings will (weakly) decline. This is, however, a contradiction to the savings and investment balance in the bailout case. Hence, we obtain $r^w > r^f$. Moreover, $r^w > r^f$ implies that $q_w^{e*} < q_f^{e*}$, as the interest elasticity of savings is strictly positive in order to balance savings and investments.

# 13 Appendix B: Financial Intermediation and Contracts

Let us briefly state the underlying agency conflicts that provide the rationale for the occurrence of financial intermediation in our paper. The depositors face the following informational asymmetries. The quality $q$ is known to entrepreneurs, but not to depositors. Moreover, depositors cannot verify whether an entrepreneur invests. To alleviate such agency problems in financial contracting, financial intermediation can act as delegated monitoring (see Diamond (1984)). Bank activities are characterized by two features: First, banks cannot observe the quality of an entrepreneur ex-ante, but can verify output conditional on investment at low or zero costs. The assumption is justified by the possibilities that banks have to secure the repayments if entrepreneurs invest. Monitoring in order to secure repayments takes different forms: inspection of firms' cash flow when customers pay, and efforts to collateralize assets if they have been created in the process of investing and selling products to customers, for instance. If the final products of an entrepreneur's project are physical goods, such as houses or machines, standard banks can secure repayment conditional on investment at very low costs.

Second, entrepreneurs can have large private benefits if they do not invest, but banks are able to reduce these benefits by monitoring. The monitoring can take many forms. For instance, standard banks can collateralize parts of the credit, or may release the funds sequentially to the entrepreneur, depending on his investment behavior. Such efforts can reduce the private benefits of entrepreneurs who do not invest.

The simplest monitoring function is given by the following discrete choice problem: If a bank $j$ offers a loan $I$ to an entrepreneur and monitors by paying a resource cost $m, m \geq 0$, it can secure a repayment of $\gamma_1 I$ with $0 < \gamma_1 \leq 1$. If $\gamma_1$ is sufficiently high, such that $q(e+I)-(1+r^c)I \geq e+(1-\gamma_1)I$, where $r^c$ is interest on loans, an entrepreneur with quality $q$ will invest if he obtains a loan and is monitored. If bank $j$ does monitor, it can only secure a repayment of $\gamma_2 I(\gamma_2 > \gamma_1)$, with $q(e+I)-(1+r^c)/ < e+(1-\gamma_2)I$. Hence, the entrepreneur does not invest if he is not monitored.

We assume that monitoring technologies are efficient enough at reducing the private benefits of entrepreneurs, such that entrepreneurs applying for loans will always invest if they are monitored. For simplicity, we also assume that monitoring outlays per credit contract are negligible for a bank. Our analysis, however, is also applicable to the case where banks can completely alleviate agency problems in contracting by investing a fixed amount per credit contract in monitoring. In this case, the interest rate spread will be positive and will cover the costs of monitoring in equilibrium.[35] For simplicity of presentation, we assume in this paper that such fixed monitoring costs are zero.

We next justify the use of debt contracts in financing entrepreneurs, either unconditional contracts or contracts conditional on macroeconomic shocks. A theoretical justification is given in Gersbach and Uhlig (2006). They abstract from monitoring, as we do in this paper. They show that banks enter into a Bertrand-like Competition for the different types of investing borrowers in such games. This makes it impossible for a lender to cross-subsidize among them. In any pure strategy equilibrium, only debt contracts will be offered.[36] As the argument can easily be extended to banks with monitoring technologies[37], we assume directly that banks compete with debt contracts.

[35] A further extension could allow banks to compete on monitoring intensity, which may increase risk magnification when banks choose a low intensity of monitoring (see e.g. Gehrig and Stenbacka (2004)).

[36] Moreover, in the optimal contract entrepreneurs must invest their endowments if they apply for loans. Otherwise, shirking would become attractive and would deter banks from lending.

[37] The monitoring technology simply allows banks to reduce the cost of shirking and increases the share of investing entrepreneurs.

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