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Abstract

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JEL Classification: D43, D47, G14, L13

Keywords: High-frequency trading, intermediation, market design, Price volatility

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Intermediation and Price Volatility

Thomas Gehrig*and Klaus Ritzberger†

October 2020

Abstract

This paper analyses the role of intermediaries in providing immediacy in fast markets. Fast markets are modelled as contests with the possibility of multiple winners where the probability of casting the best quote depends on prior technology investments. Depending on the market design, equilibrium pricing by intermediaries involves a trade-off, between monopolistic price distortion and excess volatility. Since equilibrium at the pricing stage generates an externality, investments into faster trading technologies are necessarily asymmetric in equilibrium, akin to markets with vertical product differentiation. Further, equilibrium is not necessarily efficient, since it is possible that a high-cost intermediary ends up investing excessively and thus trades more frequently than low-cost rivals.

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1 Introduction

Intermediation constitutes a significant sector in advanced economies. For the U.S. Spulber (1996a, p. 137) estimates that the intermediation sector contributes about 25 percent to GDP. And it has grown since: In 2019 wholesale trade accounted for about 6 percent, retail trade for 5.5 percent, and financial intermediation for 21 percent of U.S. GDP. Moreover, technological developments

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progressively speed up the intermediation of goods and services. This in particular affects financial intermediation, which has experienced unprecedented acceleration in recent decades.

On stock markets investments into fast intermediation technology has skyrocketed in recent years with intermediaries investing heavily, if not excessively, in high speed cables or ultra-fast radio transmission, and even co-locating their high-performance computers with the servers of the exchanges. While intermediaries have always been in the business of speeding up trade (Demsetz, 1968), recent investments have been accelerating at an increasing rate. What do these technology investments imply for market quality? Does a time-scale of nanoseconds really improve the efficiency of exchange from a social perspective?

This paper provides a dynamic framework for addressing these questions. The model allows, at a first stage, for intermediaries investing in a fast trading technology that increases their chances to provide the first quotes in the market. At a second stage, impatient buyers and sellers may trade at the fast quotes before the remaining traders (and slower intermediaries) trade in the slow market, at the third stage. Access to the fast market is modelled as a contest (Tullock, 1980), though with a positive probability of multiple winners. This reflects the idea that there is both a positive chance that customers will transact with a monopolistic dealer, who happens to be faster than her rivals, as well as the residual possibility that customers see multiple quotes, from which they select the best. The closer the market design is to continuous trading, the higher are the chances that an intermediary enjoys a monopoly position. On the other hand, batch auctions at high frequency make it more likely, from the intermediary's perspective, that competing quotes are present. The odds for the matching between customers and intermediaries are determined by the prior technology investments. On the side of the intermediary the prospect of a short—or even flickery—monopoly position is a strong incentive to invest in the fast trading technology. The more she invests the higher are the chances to execute profitable trades. The main discipline stems from the trading speed of competing dealers, who by their investments will affect the likelihood of simultaneous quotes at the decision stage for the customers, in which case the latter can select the best quote on offer. From an intermediary's viewpoint this competitive pressure translates into uncertainty about the presence of rivals.

It turns out that this structure has strong implications for equilibrium pricing. At the pricing stage an equilibrium in pure strategies does not exist; intermediaries will randomize and, thus, generate excess price volatility. In equilibrium they need to balance the incentive to undercut the rival and the prospect of earning rents in the case that they are the only contestant in the market. Moreover, at the investment stage only asymmetric equilibria arise, with one intermediary investing more resources in the fast technology than others. These equilibria are structurally similar to those in models of vertical product differentiation (Shaked and Sutton, 1982; 1983; Gehrig, 1996). A number of asymmetric equilibria can arise, including inefficient equilibria with a high-cost intermediary investing more resources into the fast technology than the competitors.

The idea that dealers and brokers provide immediacy dates back to the work of Demsetz on the foundation of transactions costs. According to him "The ask-bid spread is the markup that is paid for predictable immediacy of exchange in organized markets..." (Demsetz, 1968, p. 35-36). With the advent of computerized trading, technology has contributed greatly to serve the desire for immediacy ever more effectively. But how about pricing and "transaction costs", which lie at the heart of the article? One may feel tempted to argue that competition constrains market power more effectively in fast markets. Yet, the present analysis establishes the opposite. In fast markets the problem of market power is even more pronounced and effectively drives equilibrium price volatility. In contrast to Demsetz's statement deterministic bid-ask spreads will not arise and, accordingly, his statement needs to be refined. Only an average of bid-ask quotes over a certain time interval could meaningfully be considered as a measure of transaction costs in fast markets.

The predictions of the present model compare well with empirical findings in fast markets with algorithmic trading. In a sample of 42 countries Boehmer et al. (forthcoming) find robust evidence about an increase in short-term volatility from 2001-2011 caused by algorithmic trading, a crucial element of fast markets. They also document a large increase in "messages" sent in fast markets, which includes sending and cancelling quotes in ultra-short intervals. Such behaviour is difficult to reconcile with pure strategy equilibrium, especially since most of these quotes never execute. While Boehmer et al.an agnostic view,[1] the present paper provides an explanation for this seemingly puzzling evidence. Caivano (2015) presents similar observations on an increase in short-term price volatility for the Italian Stock Market between 2011-2013, caused by high-frequency traders. Similarly, Jovanovic and Menkveld (2010) find that the advent of fast traders for Dutch stocks, after admitting those stocks for trading on Chi-X (a

[1] "Although we document higher volatility induced by greater AT, we are agnostic about the exact causes of such high volatility." (Boehmer et al., forthcoming, p. 3, WP version)

platform for high frequency traders), on April 16, 2007, triggered an 69% increase in volatility, most of which cannot be related to low frequency volatility.

1.1 Relations to the literature

The idea of intermediaries speeding up exchange has also been modelled by Stahl (1988), Yanelle (1989), Gehrig (1993), and Spulber (1996b) in stage games of slow markets, and even such models have the property that off equilibrium many subgames are characterized by mixed strategy equilibrium.[2] Randomized pricing in equilibrium has been established early in the search literature (Butters, 1977; Burdett and Judd, 1983; Janssen and Rasmusen, 2002). While that literature builds on models with unknown consumer characteristics for any given object, the present model is characterized by an unknown number of competitors on the fast, primary market. While often only one intermediary is active in any of the fast auctions, occasionally several are active in the same auction forcing price discipline. In this sense the article also contributes to the literature on contests with an unknown number of contestants (Münster, 2006; Ewerhart and Quartieri, 2016).

Closest to the present analysis is Jovanovic and Menkveld (2019) who characterize the equilibrium price distribution of a symmetric equilibrium in the bidding game. By contrast, the present paper shows that the underlying technology race leads to an asymmetric industry structure, which renders void the analysis of symmetric equilibria at the market. Budish, Cramton and Shim (2015) argue that continuous trading may give rise to excessive price distortions and related arbitrage trading. Therefore they suggest (fast) batch auctions as an alternative market design. The pesent paper may be viewed as complementing their analysis by emphasising that (fast) batch auctions, while curbing mispricing, are actually a source of (short-term) price volatility. Accordingly, efficient market design needs to balance price distortions caused by monopolistic pricing in continuous trading with price volatility in (fast) batch auctions. Both phenomena are ultimately linked to market power that arises in fast trading for very short periods during the trading process. Neither phenomenon arises in slow markes such as the historcial daily batch auctions of the early days. Hence, this paper contributes to the literature on market quality in fast markets, but unlike Biais et al. (2015) and Brogaard et al. (2014) it sidesteps price

[2] Matching and bargaining games on the other hand are typically characterized by multiple equilibria (e.g. Rubinstein,Wolinsky, 1987), from which the unique stationary one mostly gets selected.

discovery, the process of information aggregation in price determination, and concentrates on transactions costs in the absence of informational asymmetries. The present results suggest that in addition to the standard measures of market quality such as spreads, liquidity, and price discovery (Hendershott et al., 2011) also short-term volatility should be considered.

The paper is organized as follows: Section 2 introduces the model and section 3 presents the results in a backwards fashion, beginning with the slow, secondary market and ending with the investment stage. Section 4 concludes. Lengthier proofs are relegated to an appendix.

2 The Model

Consider an economy with two commodities, say, money (the numeraire) and a real good or asset. There are two types of agents, a large number of regular consumers or investors and two intermediaries. The consumers/investors, who represent the real side of the economy, care about both commodities. Yet, they are passive and take prices as given. Thus, they are represented by aggregate demand and supply functions, $D : \mathbb{R}_+ \to \mathbb{R}_+$ and $S : \mathbb{R}_+ \to \mathbb{R}_+$, respectively. There is heterogeneity among consumers, though. Some value immediacy and trade as soon as possible; others are more patient and willing to wait. Early buyers or sellers are desparate to trade immediately. Hence, they discount any potential gains from trade in later periods. It is this heterogeneity that intermediaries may exploit.

More specifically, there are two trading periods, $t = 1, 2$. In the first period either some buyers or some sellers or none of them arrive early. Buyers arrive with probability $\delta > 0$, sellers with probability $\gamma > 0$, and none of them with probability $1 - \delta - \gamma > 0$, in which case no activity takes place in $t = 1$. Early buyers are represented by a sub-demand function $D_1 : \mathbb{R}_+ \to \mathbb{R}_+$ and early suppliers by a sub-supply function $S_1 : \mathbb{R}_+ \to \mathbb{R}_+$. In the second period, $t = 2$, the remaining buyers and sellers arrive and trade on a competitive market.[3] Early traders do not retrade in $t = 2$, but leave after the first period. Hence, demand in $t = 2$ is given by $D_2 = D - D_1$ and supply in the second period by $S_2 = S - S_1$. The sub-demand and sub-supply functions satisfy the following assumptions.

[3] Competitive equilibrium is a fairly good approximation to Nash equilibrium on markets that are organized as limit order books, like most stock markets; see Ritzberger (2016).

A1 (*Demand and Supply*) The functions D_1, S_1, $D_2 \equiv D - D_1$, and $S_2 \equiv S - S_1$ are (i) twice continuously differentiable, (ii) the demand functions are strictly decreasing and the supply functions are strictly increasing, (iii) the reciprocal of the sub-demand function D_1, that is, $1/D_1(\cdot)$, is convex, (iv) the reciprocal of the sub-supply function S_1, that is, $1/S_1(\cdot)$, is convex, (v) the residual demand function $D(\cdot) - S_2(\cdot)$ is convex, and (vi) the residual supply function $S(\cdot) - D_2(\cdot)$ is concave.

Save for assuming a law of demand, A1 is a very mild assumption that is satisfied by almost all demand and supply specifications used in practise. After all, taking the reciprocal of a function is a strongly convex transformation. The role of A1 is to ensure that the intermediaries face well behaved optimization problems when active (see Proposition 1 below).

Unlike regular agents, *intermediaries* care only about money, not about the good or asset. Their only motive to hold the asset is to resell it with a profit. Furthermore, intermediaries are risk neutral and possess "deep pockets," i.e., they hold a large endowment of money, but no initial endowment of the asset. The intermediaries are engaged in the following three-stage game. Before any market opens, at $t = 0$, each intermediary i may invest an amount $y_i \geq 0$ of money into a technology that makes i "fast," at a cost $c_i(y_i) \geq 0$. To become "fast" here means that intermediary i will be able to trade with early buyers or sellers at the primary market, in period $t = 1$. A "slow" intermediary is forced to wait for $t = 2$ when the competitive secondary market opens; hence, effectively she has no motive to trade. That is, intermediaries serve the demand for immediacy of the early traders. Their investments in period $t = 0$ determine the probability of being able to trade at $t = 1$, either as a monopolist or in competition with the other intermediary. Their cost functions $c_i : \mathbb{R}_+ \to \mathbb{R}_+$ satisfy the following familiar assumption.

A2 (*Cost functions*) The cost functions $c_i : \mathbb{R}_+ \to \mathbb{R}_+$ are twice continuously differentiable, strictly increasing, convex, involve no fixed costs, i.e. $c_i(0) = 0$, and satisfy $c_i'(0) = 0$, for $i = 1, 2$.

Once an intermediary i has learned that she can trade at $t = 1$, she chooses bid and ask prices, $b_i \geq 0$ and $a_i \geq 0$, at which she commits to buy resp. sell the asset at the primary market. Early buyers will buy from the intermediary who asks the lowest price, and early sellers will sell to the intermediary who bids the highest price—if there are early buyers or sellers. The intermediaries' choice of bids and asks, however, has to be performed in ignorance of whether or not there is a competitor. This is our model of "speed:" Since being "fast" does not leave any time to screen the market for competitors, price competition takes place under uncertainty about who competes.

To model the probability of winning a ticket to the early market at $t = 1$, we borrow from the literature on contests or "all-pay auctions" (Tullock, 1980; Dixit, 1987; for surveys of this vast literature see Corchón, 2007; or Corchón and Serena, 2018). The most popular model in this literature takes as the primitive an "effectivity function" $f_i : \mathbb{R}_+ \to \mathbb{R}_+$ for each contestant i, which is continuous, strictly increasing, and concave, and determines the success probability of i as the ratio $f_i / \sum_j f_j$. This implicitly assumes exactly one winner, though. Since we want to allow for ties, we adopt a generalized model where the prior probability that i wins, but the opponent $3 - i$ does not, is given by

$$\mu_i (y_1, y_2) = \frac{f_i (y_i)}{f_0 (y_1 + y_2) + f_1 (y_1) + f_2 (y_2)} \quad \text{for } i = 1, 2 \tag{1}$$

Accordingly, the (prior) probability that *both* intermediaries are active at the early market in period $t = 1$ is given by

$$\mu_0 (y_1, y_2) = \frac{f_0 (y_1 + y_2)}{f_0 (y_1 + y_2) + f_1 (y_1) + f_2 (y_2)} \tag{2}$$

where $f_i : \mathbb{R}_+ \to \mathbb{R}_+$ is twice continuously differentiable, strictly increasing, and concave, for $i = 0, 1, 2$. The probability of no winner at all is zero, to exclude uninteresting cases. It follows that the *conditional* probability of i not facing a competitor, given that intermediary i finds herself at the early market ($t = 1$), is

$$\pi_i (y_1, y_2) = \frac{f_i (y_i)}{f_0 (y_1 + y) + f_i (y_i)} \quad \text{for } i = 1, 2 \tag{3}$$

It is with this probability π_i that intermediary i expects to be a monopolist, given that she is among the winners. Indeed, once i has learned that she is "fast," π_i times the expected monopoly profit constitutes a lower bound on i's equilibrium payoff at the early market. This is because she can always bet on the event that she is a monopolist and set the monopolistic bid and ask prices.

Two problems about $\mu = (\mu_0, \mu_1, \mu_2) : \mathbb{R}^2_+ \to [0, 1]^3$ need to be addressed, though. First, there is the well-known indeterminacy of μ at the origin $y = (y_1, y_2) = (0, 0)$. Strictly speaking, if $f_i (0) = 0$ for $i = 0, 1, 2$, then at $y = 0 \in \mathbb{R}^2_+$ the value may be anything between one point and the whole unit interval.

Luckily, in our context this does not matter much, so we only insist that μ is a function—hence, single-valued even at the origin—and that it is consistent with the interpretation of "speed," i.e. $\mu_i(0,0) = 0$ for $i = 1, 2$.[4] Second, since the μ_is for $i = 0, 1, 2$ are probabilities, it is natural to insist that they do not depend on whether investments are measured in dollars or in euro. Formally this amounts to the requirement that the functions μ_i for $i = 0, 1, 2$ are homogeneous of degree zero in the vector $y = (y_1, y_2) \in \mathbb{R}^2_+$.

A3 (*Success probabilities*) (i) The functions $f_i : \mathbb{R}_+ \to \mathbb{R}_+$ are twice continuously differentiable, strictly increasing, concave, and satisfy $f_i(0) = 0$, for $i = 0, 1, 2$. (ii) The functions $\mu_i : \mathbb{R}^2_+ \to [0,1]$ from (1) and (2) are homogeneous of degree zero in $y \in \mathbb{R}^2_+$, for $i = 0, 1, 2$. (iii) $\mu_i(0,0) = 0$ for $i = 1, 2$.

To summarize the time-structure of the interaction: Initially, at $t = 0$, intermediaries choose investments into technology that potentially makes them "fast." Then each intermediary learns whether or not she has succeeded, but not whether the opponent did. A the second stage, in period $t = 1$, successful intermediaries set bid and ask prices. Then stochastically early buyers arrive, or early sellers, and trade with the successful intermediaries; or no sellers nor buyers arrive and the primary market remains inactive. Finally, at $t = 2$, a competitive secondary market opens at which successful intermediaries may retrade and the remaining, late investors execute their trades.

The solution concept for the game among the intermediaries is Nash equilibrium (Nash, 1950). We will not attempt, however, to identify all Nash equilibria, but content ourselves with finding one by solving the model backwards.

3 Results

The game among the intermediaries has three stages, two of which are pricing stages. In period $t = 1$ only buyers or sellers with a preference for immediacy interact with fast intermediaries at the primary market; in period $t = 2$ all the residual trades take place at a competitive secondary market.

[4] The downside of this is, of course, that μ cannot be continuous at the origin.

3.1 Secondary Market

At the last stage a competitive, secondary market opens and all remaining buyers and sellers arrive and trade at a competitive price. That is, if at the second stage early buyers arrived, then at the third stage aggregate supply at price $p \geq 0$ is $S(p) = S_1(p) + S_2(p)$ and aggregate demand is $D_1(a) + D_2(p)$, where a denotes the lowest ask price among the intermediaries who were active at the second stage. This is because the cheapest intermediary went short at the second stage and now, at the third stage, has to purchase what she sold. If at the second stage early sellers arrived, then at the third stage aggregate demand at price $p \geq 0$ is $D(p) = D_1(p) + D_2(p)$ and aggregate supply is $S_1(b) + S_2(p)$, where b denotes the highest bid price among intermediaries who were active at the second stage. This is because the winning intermediary now, at the third stage, resells what she bought at the second stage. As a consequence, if early buyers arrived at the second stage, the competitive price at the third stage will be given as the solution $P_D = P_D(a)$ of the market clearing equation

$$D_1(a) + D_2(P_D) = S_1(P_D) + S_2(P_D) \tag{4}$$

Therefore, the winning intermediary, who set the lowest ask a, earns a profit of

$$U(a) = [a - P_D(a)] D_1(a) \tag{5}$$

Likewise, if early sellers arrived at the second stage, the competitive prive at the third stage will be given as the solution $P_S = P_S(b)$ of the market clearing equation

$$D_1(P_S) + D_2(P_S) = S_1(b) + S_2(P_S) \tag{6}$$

Hence, the winning intermediary, who set the highest bid b, earns a profit of

$$V(b) = [P_S(b) - b] S_1(b) \tag{7}$$

According to A1 all demand functions are continuous and strictly decreasing and all supply functions continuous and strictly increasing. Therefore, the solutions to equations (4) and (6) are unique. Finally, if neither early buyers nor early sellers arrived at the second stage, the competitive price is the solution p^* to the equation $D_1(p^*) + D_2(p^*) = S_1(p^*) + S_2(p^*)$, which is also unique.

As for the comparative statics, implicit differentiation of equations (4) and

(6) yields

$$P_D'(a) = \frac{D_1'(a)}{S'(P_D) - D_2'(P_D)} < 0 \text{ and } P_S'(b) = \frac{S_1'(b)}{D'(P_S) - S_2'(P_S)} < 0 \quad (8)$$

That is, secondary market prices are strictly decreasing in the best bid and ask from the primary market. Furthermore, by definition $P_D(p^*) = p^*$ and $P_S(p^*) = p^*$.

3.2 Primary Market

At the second stage in $t = 1$ each intermediary learns whether she has managed to be "fast," but not whether or not the opponent has. A fast intermediary sets an ask price $a_i \geq 0$ and a bid price $b_i \geq 0$, in ignorance about whether, and which prices the opponent has chosen. Then with probability $\delta \in (0, 1)$ "early" buyers arrive, represented by $D_1 : \mathbb{R}_+ \rightarrow \mathbb{R}_+$. With probability $\gamma \in (0, 1)$ "early" sellers arrive, represented by $S_1 : \mathbb{R}_+ \rightarrow \mathbb{R}_+$. With probability $1 - \delta - \gamma \geq 0$ neither early buyers nor early sellers arrive, and the fast intermediaries have to remain inactive. If early buyers or sellers arrive, they trade at the highest bid and lowest ask with the fast intermediaries.

3.2.1 The Monopoly Problem

Consider first the problem of a monopolist intermediary. The expected profit of a monopolistic intermediary is

$$M(a, b) = \delta U(a) + \gamma V(b) \quad (9)$$

where a and b denote ask and bid prices, respectively.

Proposition 1. *Under assumption A1 the expected monopoly profit function $M(a, b)$ is strictly quasi-concave in a and b.*

Proof. See Appendix.

Proposition 1 establishes that there are unique monopoly bid and ask prices

$$a_M \in \arg\max_{a \geq 0} U(a) \text{ and } b_M \in \arg\max_{b \geq 0} V(b)$$

satisfying $a_M \geq p^* \geq b_M$, because $U(p^*) = V(p^*) = 0$. In fact, $M^* \equiv$

$M(a_M, b_M) > 0$ and $a_M > p^* > b_M$, because

$$\begin{aligned}
U'(p^*) &= \left(1 - P_D'(p^*)\right) D_0(p^*) + \left(p^* - P_D(p^*)\right) D_0(p^*) \\
&= \left(1 - P_D'(p^*)\right) D_0(p^*) > 0 \text{ and} \\
V'(p^*) &= \left(P_S'(p^*) - 1\right) S_0(p^*) + \left(P_S(p^*) - p^*\right) S_0(p^*) \\
&= \left(P_S'(p^*) - 1\right) S_0(p^*) < 0
\end{aligned}$$

Hence, a monopolistic intermediary makes strictly positive profit $M^* > 0$ and thereby distorts the competitive price at the secondary market. In particular, if at the primary market early buyers arrive, the secondary market price is distorted downwards; if early sellers arrive, it is distorted upwards, by (8).

3.2.2 Equilibrium

If $\pi_i = 1$ for some intermediary i, then the previous subsection establishes existence of equilibrium. For, if $\pi_i = 1$, then necessarily $\mu_{3-i} = \mu_0 = 0$, and i knows with certainty that she faces no competitor. Consequently, she will set a_M and b_M and obtain an expected profit of $M^* > 0$. Secondary market prices will be distorted as described above (see (8)).

Yet, unless the opponent does not invest at all, an intermediary who finds herself active at the primary market cannot be sure to be a monopolist. In particular, if $\pi_i < 1$, then i must take into account the possibility that she is faced with a competitor. In this case it turns out that there exists no pure strategy equilibrium.

Proposition 2. *(Non-Existence of Pure Strategy Equilibrium) If $0 < \pi_i < 1$ for $i = 1, 2$, then there exists no pure strategy equilibrium.*

Proof. Proceeding indirectly, suppose that there is a pure strategy equilibrium at which intermediary $i = 1, 2$ sets the prices a_i and b_i. Equilibrium profits must be at least $\pi_i M^* > 0$ by hypothesis, for $i = 1, 2$, because intermediary i can always bet on being a monopolist. Therefore, either $a_i > p^*$ or $p^* > b_i$ for at least one $i = 1, 2$, say, $a_1 > p^*$. But then $i = 1$ earns zero when early buyers arrive, because $i = 2$ can earn $\lim_{\varepsilon \searrow 0} U(a_1 - \varepsilon) = U(a_1)$ by slightly undercutting 1's ask price. Likewise, if $p^* > b_1$, intermediary $i = 1$ earns zero when early sellers arrive, because $i = 2$ can earn $\lim_{\varepsilon \searrow 0} V(b_1 + \varepsilon) = V(b_1)$ by slightly overbidding 1's bid price. Therefore, $i = 1$ earns zero irrespective of whether early buyers or early sellers arrive—a contradiction. □

The reason for why there are no pure-strategy equilibria is that with at least two competitors price competition drives profits to zero. Yet, zero profits are incompatible with equilibrium, because by $0 < \pi_i < 1$ each intermediary can bank on becoming a monopolist.

Since this is a game with continuum action spaces and discountinuous payoff functions, Glicksberg's (1952) existence theorem for mixed strategy equilibrium does not apply. Because both undercutting and overbidding is relevant for the game at hand, upper semi-continuity of payoffs fails and the existence theorem by Dasgupta and Maskin (1986) is not applicable either, nor is its generalization by Reny (1999). On the other hand, the game is simple enough that a mixed strategy equilibrium can be constructed explicitly.

Proposition 3. *(Existence of Mixed Strategy Equilibrium) If $0 < \pi_i < 1$ for $i = 1, 2$, then there exists a mixed-strategy equilibrium.*

Proof. Let $\pi_0 = \max_{i=1,2} \pi_i \in (0, 1)$ and consider first the competition for early buyers. Let $a_0 \in (p^*, a_M)$ solve the equation $U(a_0) = \pi_0 U(a_M)$. This exists and is unique, because U is continuous, $U(a_M) > \pi_0 U(a_M)$, $U(p^*) = 0 < \pi_0 U(a_M)$, and U is strictly quasi-concave by Proposition 1, hence increasing to the left of a_M. For each $i = 1, 2$ define the cumulative distribution function $F_{3-i} : \mathbb{R}_+ \to [0, 1]$ by

$$F_{3-i}(a) = \frac{1}{1-\pi_i}\left(1 - \pi_0 \frac{U(a_M)}{U(a)}\right) \text{ for all } a \in [a_0, a_M)$$

$F_{3-i}(a) = 0$ for all $a \in [0, a_0)$, and $F_{3-i}(a) = 1$ for all $a \geq a_M$. Note that F_{3-i} may have an atom at a_M with mass $\frac{\pi_0 - \pi_i}{1-\pi_i}$ if $\pi_0 > \pi_i$. Then $F_{3-i}(a_0) = 0$, $\lim_{a \nearrow a_M} F_{3-i}(a) = \frac{1-\pi_0}{1-\pi_i} \leq 1$, and

$$F'_{3-i}(a) = \frac{\pi_0 U(a_M) U'(a)}{(1-\pi_i)(U(a))^2} > 0 \text{ for all } a \in [a_0, a_M)$$

i.e., F_{3-i} is a distribution function. If intermediary $3 - i$ randomizes according to F_{3-i} and i sets the ask price $a \in [a_0, a_M)$, she obtains

$$\pi_i U(a) + (1 - \pi_i)[1 - F_{3-i}(a)] U(a) = \pi_0 U(a_M)$$

if i sets an ask $a \in [0, a_0)$, she obtains $U(a) < U(a_0) = \pi_0 U(a_M)$; if she sets an ask $a > a_M$, she obtains zero; and if she sets $a = a_M$, she obtains at most $\pi_0 U(a_M)$. More precisely, if $a_1 = a_2 = a_M$ realizes, then at the tie buyers will endogenously decide to buy from the intermediary for whom $\pi_i = \pi_0$, because this is the player who puts an atom at $a = a_M$. Therefore, every $a \in [a_0, a_M]$ is a best reply for i and the mixed strategy combination $(F_1(a), F_2(a))$ constitutes a Nash equilibrium.

The argument for the case of early sellers is analogous, with intermediary $3-i$ mixing according to the cumulative distribution function

$$G_{3-i}(b) = \frac{\pi_0 V(b_M)}{(1-\pi_i) V(b)} - \frac{\pi_i}{1-\pi_i}$$

supported on $[b_M, b_0]$, where $b_0 \in [b_M, p^*]$ is the unique solution to $V(b_0) = \pi_0 V(b_M)$, for $i = 1, 2$. Again, the intermediary with the larger probability to be a monpolist may put an atom at b_M (which is now the lower end of the support). $\square$

The mixed equilibrium creates an externality. Equilibrium profits are determined by the *largest* (conditional) probability to be a monpolist, i.e., they are $(\max_{i=1,2}\pi_i) \cdot M^*$ for both $i = 1, 2$. (In fact, the intermediary with the largest probability to be a monopolist puts positive mass on the monopolistic prices.) Hence, even the "weaker" intermediary profits from the investment of the "stronger" intermediary. The intuition for this is that the "stronger" player cannot put probability mass on prices that would make her worse off than with her expected profit when she is a monopolist; as a consequence, she cannot hold the "weaker" player down to his lower bound.

The following example illustrates the construction in the proof with linear demand and supply functions.

Example 1. For $\lambda, \mu \in (0,1)$ let $D_1(p) = \lambda(1-p)$, $D_2(p) = (1-\lambda)(1-p)$, $S_1(p) = \mu p$, and $S_2(p) = (1-\mu)p$. Then

$$P_D(a) = \frac{1-\lambda a}{2-\lambda} \text{ and } P_S(b) = \frac{1-\mu b}{2-\mu}$$

with $P'_D(a) = -\lambda/(2-\lambda)$ and $P'_S(b) = -\mu/(2-\mu)$. It follows that

$$U(a) = \lambda\frac{3a-2a^2-1}{2-\lambda} \text{ and } V(b) = \mu\frac{b-2b^2}{2-\mu}$$

and $a_M = 3/4 > p^* = 1/2 > b_M = 1/4$. Hence, $U(a_M) = \lambda/(16-8\lambda)$ and $V(b_M) = \mu/(16-8\mu)$. Solving $U(a_0) = \pi_0 U(a_M)$ yields $a_0 = \frac{3}{4} - \frac{1}{4}\sqrt{1-\pi_0}$.

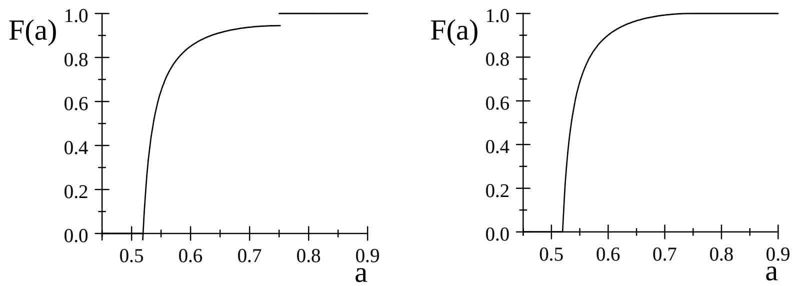

Figure 1: The distribution functions for the intermediary with the larger π_i (left) and the lower π_i (right).

Substituting into the equation for F_{3-i} yields

$$F_1(a) = \frac{8\left(3a - 2a^2 - 1\right) - \pi_0}{8(1-\pi_2)(3a - 2a^2 - 1)} \text{ and } F_2(a) = \frac{8\left(3a - 2a^2 - 1\right) - \pi_0}{8(1-\pi_1)(3a - 2a^2 - 1)}$$

for all $a \in [3/4 - \sqrt{1-\pi_0}/4, 3/4)$. Figure 1 depicts the cumulative distribution functions (for the demand side) in the mixed equilibrium; the left part for the intermediary with the larger (conditional) probability to be a monopolist, and the right part for the one with the smaller. Note that the highest density occurs at $a_0 = 3/4 - \sqrt{1-\pi_0}$, the lower end of the support.

The insight encapsulated in Propositions 2 and 3 is akin to what Butters (1977) finds in the context of advertising and to what Burdett and Judd (1983) find for search markets (see also Janssen and Rasmusen, 2002). Uncertainty about the opponents together with discontinuous payoff functions gives rise to mixed strategy equilibria.

In the present context of intermediation this effect has an important implication. Primary market prices will be volatile, because intermediaries mix over non-degenerate price intervals. This is *excess volatility* in the sense that it does not come from fundamentals, but is generated by the intermediaries' market activity. So, intermediation produces a trade-off: Either there is a monoplistic intermediary (when $\pi_i = 1$) who distorts secondary market prices, or primary market prices fluctuate due to randomized strategies employed by intermediaries (when $\pi_i < 1$).

3.3 Investment Stage

How the trade-off between excess volatility and price distortion gets resolved depends on what investments in period $t = 0$ will be in equilibrium. Thus the next step of the analysis concerns how the optimal investments will balance the costs versus the benefits from fast trading.

Before turning to that, it is useful however to deduce an implication of assumption A3(ii). In particular, in analogy to Skaperdas (1996) and Clark and Riis (1998) it can be shown that the scale-invariance from A3(ii) has strong implications for the functional forms of the effectivity functions f_i.

Lemma 1. *The functions $\mu_1(y)$, $\mu_2(y)$, and $\mu_0(y)$ from (1) and (2) are homogeneous of degree zero if and only if $f_i(y_i) = a_i y_i^r$, for $i = 1, 2$, and $f_0(y) = a_0(y_1 + y_2)^r$, where $a_0, a_1, a_2 > 0$ are positive constants and $0 < r \leq 1$.*

Proof. See Appendix.

The previous lemma pins down the functional forms for the relevant probabilities that enter the expected profits from primary market activity. This will play a role in this section. Note that w.l.o.g. the coefficient $a_0 > 0$ can be normalized to $a_0 = 1$, which we will henceforth do.

On the primary market the stronger intermediary cannot hold down her competitor to the weak intermediary's security level. This generates an externality, since the weaker intermediary can expect the same fraction of the monopoly profit M^* as the stronger, who has a higher conditional probability to be the monopolist. Therefore, depending on investment levels $y \in \mathbb{R}_+^2$ at the initial stage, two distinct payoff functions are relevant for each player: If i ends up with $\pi_i(y) > \pi_{3-i}(y) \Leftrightarrow f_i(y_i) > f_{3-i}(y_{3-i})$, then her expected profit net of costs will be

$$B_i^s(y) = \mu_i(y) M^* - c_i(y_i) \tag{10}$$

and if i ends up with $\pi_i(y) < \pi_{3-i}(y) \Leftrightarrow f_i(y_i) < f(y_{3-i})$, then her expected profit net of costs is

$$B_i^w(y) = [\mu_0(y) + \mu_i(y)] \pi_{3-i}(y) M^* - c_i(y_i) \tag{11}$$

where μ_i is defined in (1) for $i = 1, 2$ and in (2) for $i = 0$, and π_i is defined in (3) for $i = 1, 2$. Both of these functions are well behaved, as the next result shows.

Proposition 4. *(a) The expected profit function B_i^s for the stronger intermediary is strictly concave in $y_i \geq 0$, for $i = 1, 2$.*

(b) The expected profit function B_i^w for the weaker intermediary is strictly quasi-concave in $y_i \geq 0$, for $i = 1, 2$.
(c) The difference $\Lambda_i(y_{3-i}) = \max_{y_i \geq 0} B_i^s(y_1, y_2) - \max_{y_i \geq 0} B_i^w(y_1, y_2)$ is strictly decreasing in $y_{3-i} > 0$, for $i = 1, 2$.

Proof. See Appendix.

Proposition 4 pins down the structure of best replies at the investment stage. By (a) and (b) together with the maximum theorem the maximizers of both B_i^s and B_i^w are continuous functions of the opponent's investment.[5] Of course, these are not the true best replies, because ex-ante intermediary i maximizes the expected profit function

$$B_i(y) = [\mu_0(y) + \mu_i(y)] \max\{\pi_1(y), \pi_2(y)\} M^* - c_i(y_i) \quad \text{for } i = 1, 2 \tag{12}$$

with respect to her own investment y_i. Since $B_i(y) = \max\{B_i^s(y), B_i^w(y)\}$ by (3), i's true best replies will be either the maximizer of B_i^s or the maximizer of B_i^w, depending on which yields higher profit B_i. Proposition 4(c) identifies when a switch occurs: For low investments by the opponent intermediary i invests a large amount corresponding to the maximizer of B_i^s; against high investment by the opponent, however, it pays to invest less and free-ride on the externality, corresponding to the maximizer of B_i^w. In between a switch occurs, but at most one, as the difference between the respective maxima is strictly decreasing. At this switch point y_{3-i}^* intermediary i has two best replies, the maximizer of B_i^s and the maximizer of B_i^w. That is, the true best replies are discontinuous at exactly one point y_{3-i}^*.

This has no implications for existence of equilibrium, of course. Because B_i as defined in (12) is continuous and by A2 the domain can be taken to be compact, Glicksberg's (1952) theorem guarantees the existence of an equilibrium, though possibly a mixed one. But the best-reply structure creates a stunning variety of equilibria at the investment stage. Depending on parameter values, many different types of equilibria can exist. Effectively, there is only one kind of equilibrium that can be ruled out: a symmetric pure strategy equilibrium, when symmetry is defined by $f_1(y_1) = f_2(y_2)$.

Proposition 5. *(Non-Existence of Symmetric Equilibrium) Under assumptions A2 and A3 there is no symmetric pure strategy equilibrium, with $f_1(y_1) =$*

[5] Strictly speaking this requires a compact domain. But that easily follows from A2; see the proof of Proposition 4(c).

$f_2(y_2)$.

Proof. Proceeding indirectly, suppose there is a pure strategy equilibrium $y^* \in \mathbb{R}^2_{++}$ with $f_1(y_1^*) = f_2(y_2^*)$. Then by (3) it must holds that $\pi_1(y^*) = \pi_2(y^*)$. Hence, at the point $y^* \in \mathbb{R}^2_{++}$ the expected profit function B_i is not differentiable. Still, that slightly increasing i's investment from y_i^* is unprofitable implies that

$$\frac{(f_0' + f_i') f_{3-i}}{(f_0 + f_1 + f_2)^2} \cdot \frac{f_i}{f_0 + f_i} M^* + \frac{f_0 f_i' - f_0' f_i}{(f_0 + f_1 + f_2)(f_0 + f_i)} M^* \leq c_i'$$

and that slightly decreasing i's investment from y_i^* is unprofitable implies that

$$\frac{(f_0' + f_i') f_{3-i}}{(f_0 + f_1 + f_2)^2} \cdot \frac{f_{3-i}}{f_0 + f_{3-i}} M^* - \frac{(f_0 + f_i) f_{3-i} f_0'}{(f_0 + f_1 + f_2)(f_0 + f_{3-i})^2} M^* \geq c_i'$$

Evaluating these two inequalities at $f_1 = f_2$ yields

$$\begin{aligned} \frac{(f_0' + f_i') f_1^2 M^*}{(f_0 + 2f_1)^2 (f_0 + f_1)} + \frac{f_0 f_i' M^*}{(f_0 + 2f_1)(f_0 + f_1)} - \frac{f_0' f_1 M^*}{(f_0 + 2f_1)(f_0 + f_1)} &\leq \\ \leq c_i' \leq \frac{(f_0' + f_i') f_1^2 M^*}{(f_0 + 2f_1)^2 (f_0 + f_1)} - \frac{f_0' f_1 M^*}{(f_0 + 2f_1)(f_0 + f_1)} \end{aligned}$$

Yet, these two inequalities imply that

$$\frac{f_0 f_i' M^*}{(f_0 + 2f_1)(f_0 + f_1)} \leq 0$$

in contradiction to assumption A3(i), according to which $f_i' > 0$.

It remains to show that inactivity, $y = 0 \in \mathbb{R}^2_+$, cannot be an equilibrium either. To do so, proceed again indirectly and assume that it is. Then by A3(iii) both intermediaries expect zero profits from the primary market. But if, say, $i = 1$ deviated to $y_1 = \varepsilon > 0$, by Lemma 1 her profit would be

$$B_1(\varepsilon, 0) = \mu_1(\varepsilon, 0) M^* - c_1(\varepsilon) = \frac{a_1 M^*}{1 + a_1} - c_1(\varepsilon)$$

Since $a_1 M^* / (1 + a_1) > 0$ is a positive constant and $c_1'(0) = 0$ by A2, there is some $\varepsilon > 0$ small enough such that this profit is positive—a contradiction. □

Intuitively, that a symmetric pure strategy equilibrium fails to exist is because of the externality at the equilibrium of the primary market. Deviating

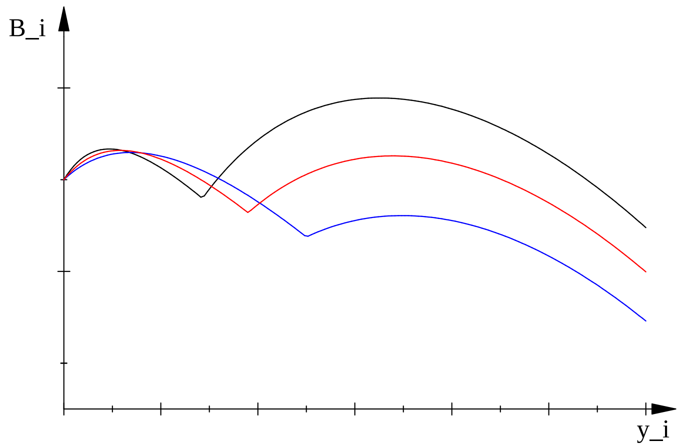

Figure 2: Expected profit for low (black), medium (red), and high (blue) investment of the opponent.

upwards from symmetric investments puts you into the stronger position. Deviating downwards allows you to costlessly reap the benefits of the other's higher investment.

While a symmetric equilibrium fails to exist, pure strategy equilibria with asymmetric investments may still exist. To illustrate the potential existence of *asymmetric* pure strategy equilibria, even when the firms' characteristics are symmetric, we offer the following example.

Example 2. Let $c_i(y_i) = y_i^2/2$, $a_i = 2$, for $i = 1, 2$, $r = 1$, and $M^* = 1$. Since this example is entirely symmetric, $f_1(y_1) = f_2(y_2)$ holds if and only if $y_1 = y_2$. The expected profit functions are

$$B_i(y_1, y_2) = \frac{3y_i + y_{3-i}}{3y_1 + 3y_2} \max\left\{\frac{2y_i}{3y_i + y_{3-i}}, \frac{2y_{3-i}}{3y_{3-i} + y_i}\right\} - \frac{1}{2}y_i^2$$

For three different investment levels of the opponent these functions are depicted in Figure 2. When the opponent invests very little, it pays to invest more so as to turn $f_i(y_i) > f_{3-i}(y_{3-i})$. When the opponent invests a lot, it pays to give in and invest less so as to turn $f_i(y_i) < f_{3-i}(y_{3-i})$. In between sits an investment level by the opponent where i is indifferent between giving in by investing little and trying to overtake by high investment. In the present example this point

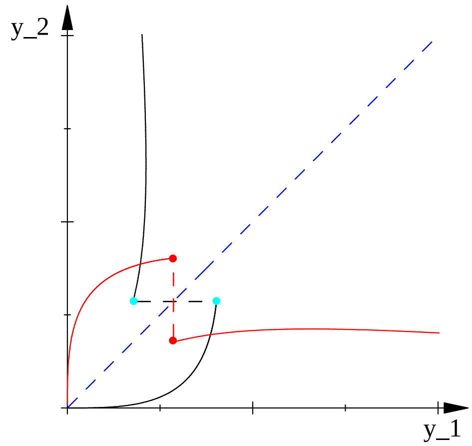

Figure 3: Best replies with discontinuities at the vertical and horizontal dashed lines (black for 1 and red for 2).

occurs at $y^*_{3-i} = 0.286$. At this point i's best replies jump from high to low investment.

This does not mean, though, that pure equilibria always fail to exist. As seen in Figure 3, asymmetric pure strategy equilibria may still exist, because the externality from the primary market compensates the intermediary who invests less.

The example illustrates that with entirely symmetric firm characteristics there may still be two asymmetric pure strategy equilibria. With asymmetric firm characteristics there is even the possibility that the less cost-efficient intermediary ends up in the stronger position. That a competitor with higher costs than the opponent may "win the race" is illustrated in the next example.

Example 3. Let $c_1(y_1) = 5y_1^{6/5}/6$ and $c_2(y_2) = 2y_2^{3/2}/3$, hence, $i = 1$ has higher costs than $i = 2$ in the relevant range. Further, set $a_1 = 2$, $a_2 = 1$, $r = 4/5$, and $M^* = 1$. Then 1's best replies are discontinuous at $y_2^* = 0.2489$, and 2's best replies are discontinuous at $y_1^* = 0.04348$. As Figure 4 shows, this results in a unique asymmetric pure strategy equilibrium, $(\hat{y}_1, \hat{y}_2) \approx (0.17, 0.09)$, where the less cost-efficient intermediary ends up in the stronger position. (The dashed blue ray in Figure 4 indicates where $f_1(y_1) = f_2(y_2)$.) This is because $i = 1$ commands more impact on the probability to become a monopolist, as $a_1 = 2 > a_2 = 1$.

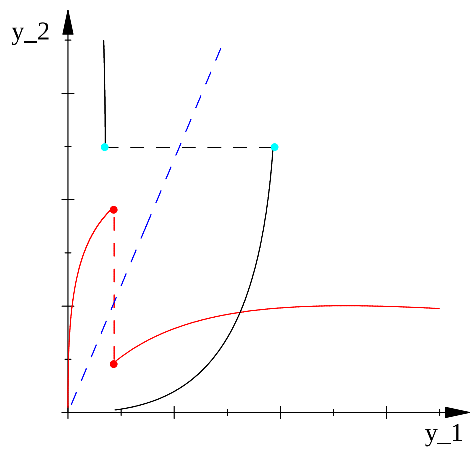

Figure 4: Unique asymmetric pure strategy equilibrium (1's best replies in black, 2's in red).

In this example it can still be argued that player 1 is actually more efficient, because even though she has higher costs she also has more impact on the probabilities. Yet, examples can be produced with $a_1 = a_2$ and $c_1(y) > c_2(y)$ for all $y \geq 0$ where intermediary 1 ends up in the strong position in an equilibrium. Specifically, with $a_i = 2$ for $i = 1, 2$, $r = M^* = 1$, $c_1(y) = y^2$, and $c_2(y) = 3y^2/4$ for all $y \geq 0$ an example is obtained that has *two* pure strategy equilibria; in one of those player 1 is in the stronger position, $\pi_1 > \pi_2$.

Existence of pure strategy equilibria is a delicate matter, though. This is because the expected profit functions from (12) are twin-peaked (see Figure 2), which gives rise to discontinuous best replies. Indeed, if the model is too far from the standard contest model with a single winner, pure strategy equilibria may not exist at all. Specifically, if the a_i's for $i = 1, 2$ are small enough, so the function $f_0(y_1 + y_2)$ dominates the probabilities, then no pure strategy equilibrium may exist, as the next example shows.

Example 4. Let $a_i = 3/5$ for $i = 1, 2$, $r = 1$, $M^* = 2$, and $c_i(y) = 8y^{9/8}/9$ for all $y \geq 0$ and $i = 1, 2$. Then, because the a_i's are small (relative to $a_0 = 1$), the maximizer of B_i^w is zero investment. That is, because the large effect of f_0 on the probability to win drives up the externality to such an extent that whenever the opponent invests enough, it is optimal not to invest at all. This can be seen

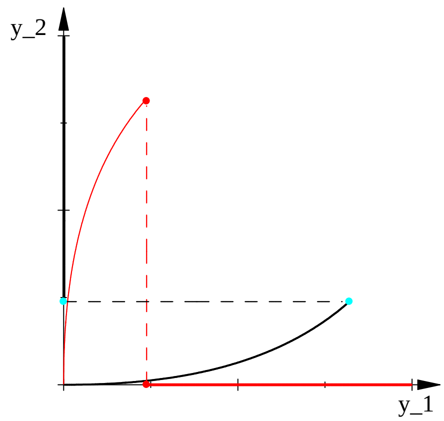

Figure 5: Nonexistence of pure strategy equilibrium (1's best replies in black, 2's in red)

by evaluating the partial derivative of $B_i^w(y)$ w.r.t. y_i at $y_i = 0$, which gives

$$\left.\frac{\partial B_i^w(y)}{\partial y_i}\right|_{y_i=0} = \frac{M^* a_{3-i}\left(\left(1+a_{3-i}\right)a_i - 1\right)}{\left(1+a_{3-i}\right)^3 y_{3-i}} = -\frac{13}{512 \cdot y_{3-i}} < 0$$

Since B_i^w is strictly concave by Proposition 4(a), it is maximized at $y_i = 0$.[6] A numerical computation shows that the discontinuity of i's best replies occurs at $y_{3-i}^* = 0.04762$. In this case no pure strategy equilibrium exists, as illustrated by Figure 5 that depicts the best reply structure for this example.

To summarize what can be said about the investment stage: The industry structure resulting from equilibrium investments is necessarily asymmetric. This may involve the less cost-efficient intermediary investing more and ending up in the stronger position. There may be multiple pure strategy equilibria, like the two in Example 2, or there may be a unique pure strategy equilibrium, like in Example 3. Finally, if the probability assignments are dominated by the term f_0, then there may be no pure strategy equilibrium at all, and all equilibria involve randomized investment decisions.

[6] Note that $a_i \geq 1$ for $i = 1, 2$ would imply that $\left.\partial B_i^w / \partial y_i\right|_{y_i=0} > 0$. Hence, a_i's that are at least 1 would imply that the maximizer of B_i^w is interior.

4 Conclusion

The traditional role of intermediaries to speed up trading and provide immediacy forces intermediaries into a technology race culminating in ultra-fast trading technologies. One consequence of fast trading is that at the pricing stage "most of the time" prices are quoted by a single intermediary only, who at that instant enjoys (very) temporary market power. Price discipline is provided by competition and the possibility that occasionally two (or more) intermediaries are quoting prices simultaneously. However, the flickery monopoly position is profitable enough to generate sizeable incentives to invest in fast trading technologies and to try and outcompete rivals.

The paper establishes that the conflicting interests in exploiting market power and attempting to undercut rivals necessarily results in mixed pricing in equilibrium. The Bertrand paradigm with pure strategy equilibrium does not apply to fast markets. As a consequence, equilibrium prices will be excessively volatile, as characterized by the distributions of quotes in the mixed strategy equilibrium. Fast markets will necessarily be characterized by higher price volatility than slow markets, where pure strategy equilibria may be feasible at zero (instantaneous) volatility.

Moreover, given the properties of the price equilibrium, symmetric equilibria at the investment stage can be ruled out. By necessity, in equilibrium one of the intermediaries will enjoy a temporary advantage in soliciting trades. The trading advantage can be attained by sufficiently high technology investments, in certain cases even if the marginal costs of technological improvements exceed those of rivals. While the technology race is modeled as one stage, incentives to invest and leapfrog the leader are enhanced by declining costs of technology advancement. Consequently, there is a strong potential for excessive technology investments.

While all results have been derived within a duopoly setting, the mechanisms identified here most likely also apply to general oligopolies. However, technical challenges arise such a segmented price competition and "overlapping duopoly" (Armstrong and Vickers, 2019). In this sense, deriving the mixed strategy equilibrium at the pricing stage and generalizing contests with more than one winner for three or more competitors remains a fruitful agenda for future research.

The current analysis suggests that market orders will increasingly face monopolistic quotes as trading speed accelerates in ultra-fast markets. This suggests that customers may be better served by offering alternative order types that allow them to solicit competing quotes, rather than the best quote at a given point in time. One may think about replacing market orders by the "best quote within a pre-specified period", or the best quote out of a minimum number of (independent) quotes. Such alternative order types may still preserve the benefits from speed while at the same time increasing the competitiveness at the pricing stage and reducing rents, and, hence, transaction costs.[7]

Appendix: Proofs

This appendix contains the lengthier proofs. For the reader's convenience the statements are included.

Proposition 1. *Under assumption A1 the expected monopoly profit function $M(a,b)$ is strictly quasi-concave in a and b.*

Proof. Since $M(\cdot,\cdot)$ is additively separable in bids and asks, it is enough to show that $U(a)$ is strictly quasi-concave in a and that $V(b)$ is strictly quasi-concave in b. Eq. (4), which defines P_D, can be rewritten as

$$D_1(a) = S_1(P_D) + S_2(P_D) - D_2(P_D)$$

The right-hand side of this equation, the residual supply function $S - D_2$, is a strictly increasing and continuous function of P_D and therefore has an inverse, denoted F_D, which is also strictly increasing, i.e. $F_D' > 0$. Rewriting (6), which defines P_S, as

$$S_1(b) = D_1(P_S) + D_2(P_S) - S_2(P_S)$$

shows that the right-hand side, the residual demand function $D - S_2$, is continuous and strictly decreasing and therefore has an inverse, denoted F_S, which is also strictly decreasing, i.e. $F_S' < 0$. It follows that

$$P_D = F_D(D_1(a)) \text{ and } P_S = F_S(S_1(b))$$

Now consider the profit function $U(a)$. Its first-order derivative is

$$U'(a) = [a - F_D(D_1(a))]\, D_1'(a) + [1 - F_D'(D_1(a))\, D_1'(a)]\, D_1(a)$$

[7]Of course, such alternative order types will also significantly affect market regulation, since e.g. the concept of best bid and offer prices needs to be adjusted accordingly.

and its second-order derivative is

$$\begin{aligned}U''(a) &= 2D_1'(a) - 2F_D'(D_1(a))(D_1'(a))^2 + [a - F_D(D_1(a))]D_1''(a) \\ &- \left[F_D''(D_1(a))(D_1'(a))^2 + F_D'(D_1(a))D_1''(a)\right]D_1(a)\end{aligned}$$

If there is no interior extremum, then U is strictly monotone, hence, strictly quasi-concave. If there is an interior extremum of U, then it must satisfy the f.o.c.

$$[a - F_D]D_1' + [1 - F_D'D_1']D_1 = 0$$

At a point where $U'(a) = 0$ the second-order derivative evaluates to

$$U'' = 2D_1' - 2F_D'(D_1')^2 - F_D''(D_1')^2 D_1 - \frac{D_1''D_1}{D_1'}$$

Assumption A1 then implies that $2D_1' \leq D_1''D_1/D_1'$ by A1(iii), and that F_D is convex, i.e. $F_D'' \geq 0$, by A1(vi). Therefore,

$$\begin{aligned}U'' &= 2D_1' - 2F_D'(D_1')^2 - F_D''(D_1')^2 D_1 - \frac{D_1''D_1}{D_1'} \\ &\leq -(D_1')^2[2F_D' + F_D''D_1] < 0\end{aligned}$$

Hence, any interior critical point is an isolated maximum. Thus, if there is an interior critical point, then U is single-peaked, hence strictly quasi-concave.

Next, consider the profit function $V(b)$. Its first-order derivative is

$$V'(b) = [F_S(S_1(b)) - b]S_1'(b) + [F_S'(S_1(b))S_1'(b) - 1]S_1(b)$$

and its second-order derivative is

$$\begin{aligned}V''(b) &= 2[F_S'(S_1(b))S_1'(b) - 1]S_1'(b) + [F_S(S_1(b)) - b]S_1''(b) \\ &+ \left[F_S''(S_1(b))(S_1'(b))^2 + F_S'(S_1(b))S_1''(b)\right]S_1(b)\end{aligned}$$

Again, if there is no interior critical point, then V is strictly monotone, hence, strictly quasi-concave. If there is an interior critical point, it must satisfy the f.o.c.

$$F_S - b = [1 - F_S'S_1']\frac{S_1}{S_1'}$$

At such a point the second-order derivative evaluates to

$$V'' = 2F_S' \left(S_1'\right)^2 - 2S_1' + F_S'' \left(S_1'\right)^2 + \frac{S_1'' S_1}{S_1'}$$

Assumption A1 then implies that $S_1'' S_1 / S_1' \leq 2S_1'$ by A1(iv), and that F_S is concave, i.e. $F_S'' \leq 0$, by A1(v). Therefore, it follows from $F_S' < 0$ that

$$\begin{aligned} V'' &= 2F_S' \left(S_1'\right)^2 - 2S_1' + F_S'' \left(S_1'\right)^2 + \frac{S_1'' S_1}{S_1'} \\ &\leq \left(S_1'\right)^2 \left[2F_S' + F_S'' S_1\right] < 0 \end{aligned}$$

As any interior extremum is a maximum, V is strictly quasi-concave. □

Lemma 1. *The functions $\mu_1(y)$, $\mu_2(y)$, and $\mu_0(y)$ from (1) and (2) are homogeneous of degree zero if and only if $f_i(y_i) = a_i y_i^r$, for $i = 1, 2$, and $f_0(y) = a_0 (y_1 + y_2)^r$, where $a_0, a_1, a_2 > 0$ are positive constants and $0 < r \leq 1$.*

Proof. Since the if-part is straightforward, we will only demonstrate the only-if part. Homogeneity of degree zero of μ_1, μ_2, and μ_0 is equivalent to

$$\begin{aligned} \frac{f_1(\lambda y_1)}{f_1(y_1)} &= \frac{f_0(\lambda y_1 + \lambda y_2) + f_1(\lambda y_1) + f_2(\lambda y_2)}{f_0(y_1 + y_2) + f_1(y_1) + f_2(y_2)} \\ \frac{f_2(\lambda y_2)}{f_2(y_2)} &= \frac{f_0(\lambda y_1 + \lambda y_2) + f_1(\lambda y_1) + f_2(\lambda y_2)}{f_0(y_1 + y_2) + f_1(y_1) + f_2(y_2)} \\ \frac{f_0(\lambda y_1 + \lambda y_2)}{f_0(y_1 + y_2)} &= \frac{f_0(\lambda y_1 + \lambda y_2) + f_1(\lambda y_1) + f_2(\lambda y_2)}{f_0(y_1 + y_2) + f_1(y_1) + f_2(y_2)} \end{aligned}$$

which in turn is equivalent to

$$\frac{f_1(\lambda y_1)}{f_1(y_1)} = \frac{f_0(\lambda y_1 + \lambda y_2)}{f_0(y_1 + y_2)} \text{ and } \frac{f_2(\lambda y_2)}{f_2(y_2)} = \frac{f_0(\lambda y_1 + \lambda y_2)}{f_0(y_1 + y_2)}$$

for all $\lambda > 0$ and $y \in \mathbb{R}^2_{++}$. The first shows that $f_0(\lambda y_1 + \lambda y_2) / f_0(y_1 + y_2)$ is independent of y_2, and the second equation shows that $f_0(\lambda y_1 + \lambda y_2) / f_0(y_1 + y_2)$ is independent of y_1. Therefore, this ratio is a function of λ alone, say, $g(\lambda) = f_0(\lambda y_1 + \lambda y_2) / f_0(y_1 + y_2)$. Since also

$$\frac{f_1(\lambda y_1)}{f_1(y_1)} = g(\lambda) = \frac{f_2(\lambda y_2)}{f_2(y_2)}$$

the left-hand side of this equation is independent of y_1, and the right-hand side is independent of y_2. Therefore, we can write

$$\frac{f_i(\lambda y_i)}{f_i(y_i)} = \frac{f_i(\lambda)}{f_i(1)} \Leftrightarrow f_i(\lambda y_i) = \frac{f_i(\lambda) f_i(y_i)}{f_i(1)}$$

for $i = 1, 2$. Defining $F_i(y_i) = f_i(y_i) / f_i(1)$, the functional equation is $F_i(\lambda y_i) = F_i(\lambda) F_i(y_i)$. This is the fourth of Cauchy's functional equations for which unique nonzero solutions are known (see e.g. Efthimiou, 2010, pp. 86). Since f_i is strictly increasing, the unique solution is $f_i(y_i) = a_i y_i^{r_i}$ for constants $r_i > 0$ and $a_i = f_i(1) > 0$, for $i = 1, 2$.

That $f_1(\lambda y_1) / f_1(y_1) = f_2(\lambda y_2) / f_2(y_2)$ implies $r_1 = r_2 \equiv r$, and that

$$\frac{f_0(\lambda y_1 + \lambda y_2)}{f_0(y_1 + y_2)} = \frac{f_i(\lambda y_i)}{f_i(y_i)} = \lambda^r \text{ for } i = 1, 2$$

yields that $f_0(y_1 + y_2)$ is homogeneous of degree $r > 0$. Therefore, $f_0(y_1 + y_2) = a_0(y_1 + y_2)^r$. That $r \leq 1$ follows from concavity, A1(i). □

Proposition 4. *(a) The expected profit function B_i^s for the stronger intermediary is strictly concave in $y_i \geq 0$, for $i = 1, 2$.*
(b) The expected profit function B_i^w for the weaker intermediary is strictly quasiconcave in $y_i \geq 0$, for $i = 1, 2$.
(c) The difference $\Lambda_i(y_{3-i}) = \max_{y_i \geq 0} B_i^s(y_1, y_2) - \max_{y_i \geq 0} B_i^w(y_1, y_2)$ is strictly decreasing in $y_{3-i} > 0$, for $i = 1, 2$.

Proof. (a) By Lemma 1 homogeneity of degree zero of $\mu_i(y)$ in y implies $f_i' = r f_i / y_i$ and $f_i'' = r(r-1) f_i / y_i^2$ for any $y = (y_1, y_2) \gg 0$. W.l.o.g. assume that $f_1(y_1) > f_2(y_2)$. By (1) and (3) the profit function of the stronger intermediary $i = 1$ can be written as $B_1^s(y) = \mu_1(y) M^* - c_1(y_1)$. The first-order partial derivative of μ_1 w.r.t. $y_1 > 0$ is

$$\begin{aligned}\frac{\partial \mu_1}{\partial y_1} &= \frac{(f_0 + f_2) f_1' - f_1 f_0'}{(f_0 + f_1 + f_2)^2} = \frac{r(y_1 + y_2)(f_0 + f_2) f_1 - r y_1 f_1 f_0}{y_1 (y_1 + y_2)(f_0 + f_1 + f_2)^2} \\ &= \frac{r(y_2 f_0 + (y_1 + y_2) f_2) f_1}{y_1 (y_1 + y_2)(f_0 + f_1 + f_2)^2} > 0\end{aligned}$$

and its second-order partial derivative w.r.t. $y_1 > 0$ is

$$\begin{aligned}\frac{\partial^2 \mu_1}{\partial y_1^2} &= \frac{(f_0 + f_2) f_1'' - f_0'' f_1}{(f_0 + f_1 + f_2)^2} - \frac{2 (f_0' + f_1') [(f_0 + f_2) f_1' - f_1 f_0']}{(f_0 + f_1 + f_2)^3} \\ &= \frac{r (r-1) \left[(y_1 + y_2)^2 (f_0 + f_2) f_1 - y_1^2 f_0 f_1 \right]}{y_1^2 (y_1 + y_2)^2 (f_0 + f_1 + f_2)^2} - \frac{2 (f_0' + f_1')}{f_0 + f_1 + f_2} \cdot \frac{\partial \mu_1}{\partial y_1} \\ &= \frac{r (r-1) \left[y_1^2 f_2 + y_2 (2y_1 + y_2) (f_0 + f_2) \right] f_1}{y_1^2 (y_1 + y_2)^2 (f_0 + f_1 + f_2)^2} - \frac{2 (f_0' + f_1')}{f_0 + f_1 + f_2} \cdot \frac{\partial \mu_1}{\partial y_1} < 0\end{aligned}$$

Hence, μ_1 is a strictly increasing and strictly concave function of y_1. As B_1 is a sum of a strictly concave and a concave function ($-c_1$), it is strictly concave.

(b) Demonstrating strict quasi-concavity of the profit function B_2^w of the "weak" intermediary $i = 2$ is a bit more involved. To do so, fix $y_1 > 0$ and change variables by defining $x = y_2/y_1 \in \left(0, (a_1/a_2)^{1/r}\right)$, where the upper bound makes sure that $f_2(y_2) < f_1(y_1)$. Then, exploiting homogeneity, rewrite B_2^w as a function of x alone by

$$B_2(x) = \frac{a_1 [(1+x)^r + a_2 x^r]}{[(1+x)^r + a_1 + a_2 x^r] [(1+x)^r + a_1]} M^* - c_2(y_1 x) \tag{13}$$

Now consider the function $F : \left(0, (a_1/a_2)^{1/r}\right) \to [0,1]$ defined by

$$F(x) = \frac{a_1 [(1+x)^r + a_2 x^r]}{[(1+x)^r + a_1 + a_2 x^r] [(1+x)^r + a_1]}$$

which is the first term in B_2. We claim that $F'(x) \geq 0$ implies $F''(x) < 0$ for all $x > 0$.

To see this, first decompose $F(x)$ into the product of $\pi_1(x)$ and $1 - \mu(x)$, that is, write $F(x) = [1 - \mu(x)] \pi_1(x)$, where

$$\mu(x) = \mu_1(x) = \frac{a_1}{(1+x)^r + a_1 + a_2 x^r} \text{ and } \pi_1(x) = \frac{a_1}{(1+x)^r + a_1}$$

Second, define the auxiliary functions $g_k, h_k : \left[0, (a_1/a_2)^{1/r}\right] \to [0,1]$ for $k = 1, 2$ by

$$g_1(x) = \frac{r \left((1+x)^{r-1} + a_2 x^{r-1} \right)}{(1+x)^r + a_1 + a_2 x^r} > 0 \text{ and } h_1(x) = \frac{r (1+x)^{r-1}}{(1+x)^r + a_1} > 0$$

$$g_2(x) = -r(1-r)\frac{(1+x)^{r-2} + a_2 x^{r-2}}{(1+x)^r + a_1 + a_2 x^r} \leq 0 \text{ and}$$
$$h_2(x) = -r(1-r)\frac{(1+x)^{r-2}}{(1+x)^r + a_1} \leq 0$$

Then the first- and second-order derivatives of μ and π_1 w.r.t. x can be written as

$$\begin{aligned}
\mu' &= -\frac{a_1 r\left[(1+x)^{r-1} + a_2 x^{r-1}\right]}{\left[(1+x)^r + a_1 + a_2 x^r\right]^2} = -\mu g_1 < 0 \\
\pi_1' &= -\frac{a_1 r (1+x)^{r-1}}{\left[(1+x)^r + a_1\right]^2} = -\pi_1 h_1 < 0 \\
\mu'' &= 2\mu\left(\frac{r\left[(1+x)^{r-1} + a_2 x^{r-1}\right]}{(1+x)^r + a_1 + a_2 x^r}\right)^2 \\
&\quad + \mu r(1-r)\frac{(1+x)^{r-2} + a_2 x^{r-2}}{(1+x)^r + a_1 + a_2 x^r} = 2\mu g_1^2 - \mu g_2 > 0 \\
\pi_1'' &= 2\pi_1\left(\frac{r(1+x)^{r-1}}{(1+x)^r + a_1}\right)^2 + \pi_1 r(1-r)\frac{(1+x)^{r-2}}{(1+x)^r + a_1} = 2\pi_1 h_1^2 - \pi_1 h_2 > 0
\end{aligned}$$

With this notation the first-order derivative of F is

$$\begin{aligned}
F' &= -\mu'\pi_1 + (1-\mu)\pi_1' = \mu\pi_1 g_1 - (1-\mu)\pi_1 h_1 \\
&= \mu\pi_1(g_1 + h_1) - \pi_1 h_1
\end{aligned}$$

and its second-order derivative is

$$\begin{aligned}
F'' &= -\mu''\pi_1 - 2\mu'\pi_1' + (1-\mu)\pi_1'' \\
&= -2\mu\pi_1 g_1^2 + \mu\pi_1 g_2 - 2\mu\pi_1 g_1 h_1 + 2(1-\mu)\pi_1 h_2^2 - (1-\mu)\pi_1 h_2 \\
&= \mu\pi_1 g_2 - (1-\mu)\pi_1 h_2 + 2\pi_1 h_1^2 - 2\mu\pi_1\left(g_1^2 + g_1 h_1 + h_1^2\right) \\
&= \mu\pi_1 g_2 + \frac{1-r}{1+x}(1-\mu)\pi_1 h_1 + 2\pi_1 h_1^2 - 2\mu\pi_1\left(g_1^2 + g_1 h_1 + h_1^2\right)
\end{aligned}$$

where the last line follows from $h_2 = -(1-r)h_1/(1+x)$. Because for $x > 0$ it holds that $1 + x > x \Leftrightarrow 1/x > 1/(1+x)$, it follows that

$$g_2 = -r(1-r)\frac{(1+x)^{r-1}(1+x)^{-1} + a_2 x^{r-1} \cdot x^{-1}}{(1+x)^r + a_1} \leq -\frac{1-r}{1+x} g_1$$

Therefore,

$$
\begin{aligned}
F'' &\leq -\frac{1-r}{1+x}\mu\pi_1 g_1 + \frac{1-r}{1+x}(1-\mu)\pi_1 h_1 + 2\pi_1 h_1^2 - 2\mu\pi_1\left(g_1^2 + g_1 h_1 + h_1^2\right) \\
&= -\frac{1-r}{1+x}\left[\mu\pi_1 g_1 - (1-\mu)\pi_1 h_1\right] + 2\pi_1 h_1^2 - 2\mu\pi_1\left(g_1^2 + g_1 h_1 + h_1^2\right) \\
&= 2\pi_1\left[(1-\mu) h_1^2 - \mu g_1 (g_1 + h_1)\right] - \frac{1-r}{1+x}F' \\
&= 2\left[(1-\mu)\pi_1 h_1^2 - \pi_1 g_1 h_1 - g_1\left(\mu\pi_1 (g_1 + h_1) - \pi_1 h_1\right)\right] - \frac{1-r}{1+x}F' \\
&= 2\pi_1 h_1\left[(1-\mu) h_1 - g_1\right] - \left(2g_1 + \frac{1-r}{1+x}\right)F'
\end{aligned}
$$

The term in the square brackets of the last line is negative, because $h_1 < g_1$ is equivalent to $-a_2 x^{r-1}(1+x)^{r-1} < a_1 a_2 x^{r-1}$, which is always true; and $0 \leq \mu \leq 1$ then implies that $(1-\mu) h_1 < g_1$. Thus, if $F' \geq 0$, then $F'' < 0$.

With this claim at hand, consider the function $B_2(x)$, as defined in (13). If there is no interior critical point, then B_2 is strictly monotone, hence strictly quasi-concave. If there is, then at an interior extremum it must hold that $B_2' = F'M^* - c_2' y_1 = 0$, hence in particular $F' = c_2' y_1 / M > 0$. The second-order derivative at an extremum is

$$B_2'' = F''M^* - y_1^2 c_2''$$

which is strictly negative by the above claim and A2 (convexity of the cost function $c_2(\cdot)$). Therefore, any interior extremum is a maximum, showing that the expected profit function B_2 of the weak intermediary is strictly quasi-concave.

(c) First, observe that as $\partial B_i^s / \partial y_i \to_{y_i \to 0} +\infty$ if $r < 1$ and $\lim_{y_i \to 0} \partial B_i^s / \partial y_i = a_i M / \left((1 + a_{3-i}) y_{3-i}\right) > 0$ if $r = 1$, the maximizer for B_i^s is always interior. This is not necessarily so for B_i^w, which may be maximized at $y_i = 0$ if $a_{3-i} \leq 1$, because $\lim_{y_i \to 0} \partial B_i^w / \partial y_i = r(1 + a_{3-i})^{-3} a_{3-i}(a_{3-i} - 1) M / y_{3-i}$. (If $a_{3-i} > 1$, then also the maximizer of $B_i^w(\cdot)$ is always interior.)

Because $c_i(\cdot)$ is strictly increasing by A2, there exists some $\bar{y} > 0$ such that $M^* - c_i(y_i) \leq 0$ for all $y_i \geq \bar{y}$ and for $i = 1, 2$. Hence, no intermediary will ever invest more than $\bar{y}$. Therefore, action spaces at the investment stage can be taken to be the compact intervals $[0, \bar{y}]$. It follows from (a) and (b) above and the maximum theorem (Aliprantis and Border, 1999, p. 570) that both $\max_{y_i} B_i^s(y)$ and $\max_{y_i} B_i^w(y)$, as well as the associated maximizers, are continuous functions of y_{3-i}.

To ease notation, fix $y_{3-i} > 0$ and denote $z_{is} = \arg\max_{y_i} B_i^s(y)$ and $z_{iw} = \arg\max_{y_i} B_i^w(y)$. Then consider the difference $\Lambda_i(y_{3-i}) = \max_{y_i} B_i^s(y) - \max_{y_i} B_i^w(y)$. Differentiating w.r.t. y_{3-i} and applying the envelope theorem yields

$$\begin{aligned}\Lambda_i'(y_{3-i}) = {} & \frac{\partial \mu_i(y_{3-i}, z_{is})}{\partial y_{3-i}} M^* \\ & - \left[\frac{\partial \mu_0(y_{3-i}, z_{iw})}{\partial y_{3-i}} + \frac{\partial \mu_i(y_{3-i}, z_{iw})}{\partial y_{3-i}}\right] \pi_{3-i}(y_{3-i}, z_{iw}) M^* \\ & - [\mu_0(y_{3-i}, z_{iw}) + \mu_i(y_{3-i}, z_{iw})] \frac{\partial \pi_{3-i}(y_{3-i}, z_{iw})}{\partial y_{3-i}} M^*\end{aligned}$$

At the respective maxima the f.o.c.'s

$$\begin{aligned}\frac{\partial \mu_i(y_{3-i}, z_{is})}{\partial z_{is}} M^* = c_i'(z_{is}) \quad \text{and} \\ \left[\frac{\partial \mu_0(y_{3-i}, z_{iw})}{\partial z_{iw}} + \frac{\partial \mu_i(y_{3-i}, z_{iw})}{\partial z_{iw}}\right] \pi_{3-i}(y_{3-i}, z_{iw}) M^* + \\ + [\mu_0(y_{3-i}, z_{iw}) + \mu_i(y_{3-i}, z_{iw})] \frac{\partial \pi_{3-i}(y_{3-i}, z_{iw})}{\partial z_{iw}} M^* \leq c_i'(z_{iw})\end{aligned}$$

have to hold, where the second f.o.c. holds with inequality because of a possible corner solution. By A3(ii) the μ_is are homogeneous of degree zero, for $i = 0, 1, 2$, and so is π_{3-i} by Lemma 1. Hence, by Euler's theorem

$$\begin{aligned}\frac{\partial \mu^j(y_{3-i}, y_i)}{\partial y_{3-i}} &= -\frac{y_i}{y_{3-i}} \frac{\partial \mu^j(y_{3-i}, y_i)}{\partial y_i} \quad \text{for } j = 0, i \text{ and } y_i \in \{z_{is}, z_{iw}\} \\ \frac{\partial \pi_{3-i}(y_{3-i}, z_{iw})}{\partial y_{3-i}} &= -\frac{z_{iw}}{y_{3-i}} \frac{\partial \pi_{3-i}(y_{3-i}, z_{iw})}{\partial z_{iw}}\end{aligned}$$

Substituting the right-hand sides of these equations, evaluated at z_{is} and z_{iw}, into the expression for $\Lambda_i'(y_{3-i})$ and invoking the f.o.c.'s yields

$$\Lambda_i'(y_{3-i}) \leq \frac{1}{y_{3-i}} [z_{iw} c_i'(z_{iw}) - z_{is} c_i'(z_{is})] < 0$$

where the last inequality follows from $0 \leq z_{iw} < z_{is}$ and convexity of the cost function (A2). Hence, Λ_i is strictly decreasing in y_{3-i}. □

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