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Abstract

Land is back. The increase in wealth in the second half of 20th century arose from housing and land. It should be taxed. We introduce land and housing structures in Judd's standard setup: first best optimal taxation is achieved with a property tax on land and requires no tax on capital. With positive taxes on housing rents, a first best is still possible but with subsidies to rental housing investments, and either with differential land tax rates or with a tax on imputed rents. It can be taxed. Even absent land taxes, one can tax it indirectly and reach a Ramsey-second best still with no tax on capital and positive housing rent taxes in the steady-state. This result extends to the dynamics under restrictions on parameters.

JEL Classification: D63, R14

Keywords: Capital, wealth, housing, land, Optimal tax, First best, Second best

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Short abstract

Land is back. The increase in wealth in the second half of 20th century arose from housing and land. *It should be taxed.* We introduce land and housing structures in Judd's standard setup: first best optimal taxation is achieved with a property tax on land and requires no tax on capital. With positive taxes on housing rents, a first best is still possible but with subsidies to rental housing investments, and either with differential land tax rates or with a tax on imputed rents. *It can be taxed.* Even absent land taxes, one can tax it indirectly and reach a Ramsey-second best still with no tax on capital and positive housing rent taxes in the steady-state. This result extends to the dynamics under restrictions on parameters.

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Longer abstract

Land is back. The increase in wealth-to-income ratios in the second half of 20th century has recently received much attention with recommendations of uniform wealth taxation. It however appears that, in many major Western countries analyzed, housing and specifically its land component are responsible for the trends of higher wealth relative to income.

It should be taxed. To analyze optimal taxation, we introduce land and housing structures in Judd's standard setup. We find that the first best is achieved with a positive property tax on land and no tax on capital, in the steady-state and on a path converging to it. Taxing housing rents is not optimal, even with additional taxes on imputed rents. With positive taxes on housing rents, a first best is still possible but with subsidies to rental housing investments and either with differential land tax rates or with a tax on imputed rents. From the two distortions introduced by the tax on rent, the one on land use and the other on the dynamic of structures, the simulation reveals that the latter is much more critical quantitatively. The upshot is that a rent tax supplemented by a structure subsidy does almost as well as a land tax in improving social welfare. These results are discussed with respect to the traditional Henry George (1879) tax scheme and we provide a formula for taxing land which accounts for the depreciation of elastic housing structures.

It can be taxed. Even absent land taxes, one can tax it indirectly and reach a Ramsey-second best still with no tax on capital and positive housing rent taxes in the steady-state. This result extends to the dynamics under restrictions on parameters. For that, we use the so called implementability condition in Atkeson et al. (1999). Besides, following Straub and Werning (2020) approach of convergence of dynamic multipliers, we show that Judd's proposition of no second-best capital taxation at the limit steady-state extends, in the presence of land, to a larger range of parameters.

1 Introduction

Uniform taxation of wealth is coming back as a response to rising public debts and increase in inequality in most Western countries. This paper enters this debate from the striking observation that the housing component of national private wealth explains the spectacular rise of wealth relative to national income in several countries. The historical rise of housing wealth is in major part due to the rise in the share of land: Figure 1 represents the historical evolution of wealth-to-income ratios in five countries, using Piketty's decomposition into housing, agricultural land, net foreign assets and other domestic asset[1].

The drop in total wealth in the beginning of the 20th century as compared to the 19th century is mostly due to the secular decline in agricultural land in all panels of Figure 1 and was sped up by the decline in physical capital after WWI in France and the UK. The levels rose again in the second half of the 20th century and partially caught up their values of the previous century, but this is due to housing becoming a major component of wealth. It actually explains most of the recent upward trends in France, the UK and Germany (panels a, b, d), the rise in land values explaining most of the trends in the first two countries. This can be seen in inspecting the evolution of all series below the shaded area, which represents all capital except housing (agricultural land, net foreign assets and other capital)[2]. These wealth-to-income ratios would have permanently remained low after WWI if one excluded housing wealth from total wealth and instead of a U-curve, one would have observed a L-curve with a flat level of capital in the second half of the 20th century. In Canada and the US (panels c, e), there is neither a long run trend nor a recent increase of this ratio.

Given the heterogeneity in sources of wealth and their diverging trends, a unique wealth tax could be sub-optimal, especially given their differences in supply elasticities. We assess the merits of differentiated taxation of wealth and compare land taxation, housing rents taxation and physical capital taxes. For that, we extend a Judd-type model of capitalists and workers, a well-suited framework to study redistributive capital taxation, in introducing land, housing structures, housing consumption and housing rental market. Indeed, the Judd-Chamley framework (Judd, 1985; Chamley, 1986) represents a turning point in the literature

[1] In the data in Figure 1, the measurement of housing wealth is the sum of two elements, structures and developed land with constructs, and Online Appendix F describes the main method as well as a comparison with alternative methods used in national accounts.

[2] Further, a large part of the rising importance of housing in wealth is due to the land component, as can be seen on at the end of the sample period when a decomposition is possible. Rents play a limited role of rents in these developments, as discussed in Appendix. In Bonnet et al. (2014), we provided a detailed discussion of the magnitude of discrepancy between housing prices and rents and how this changes the evaluation of wealth increases relative to national income.

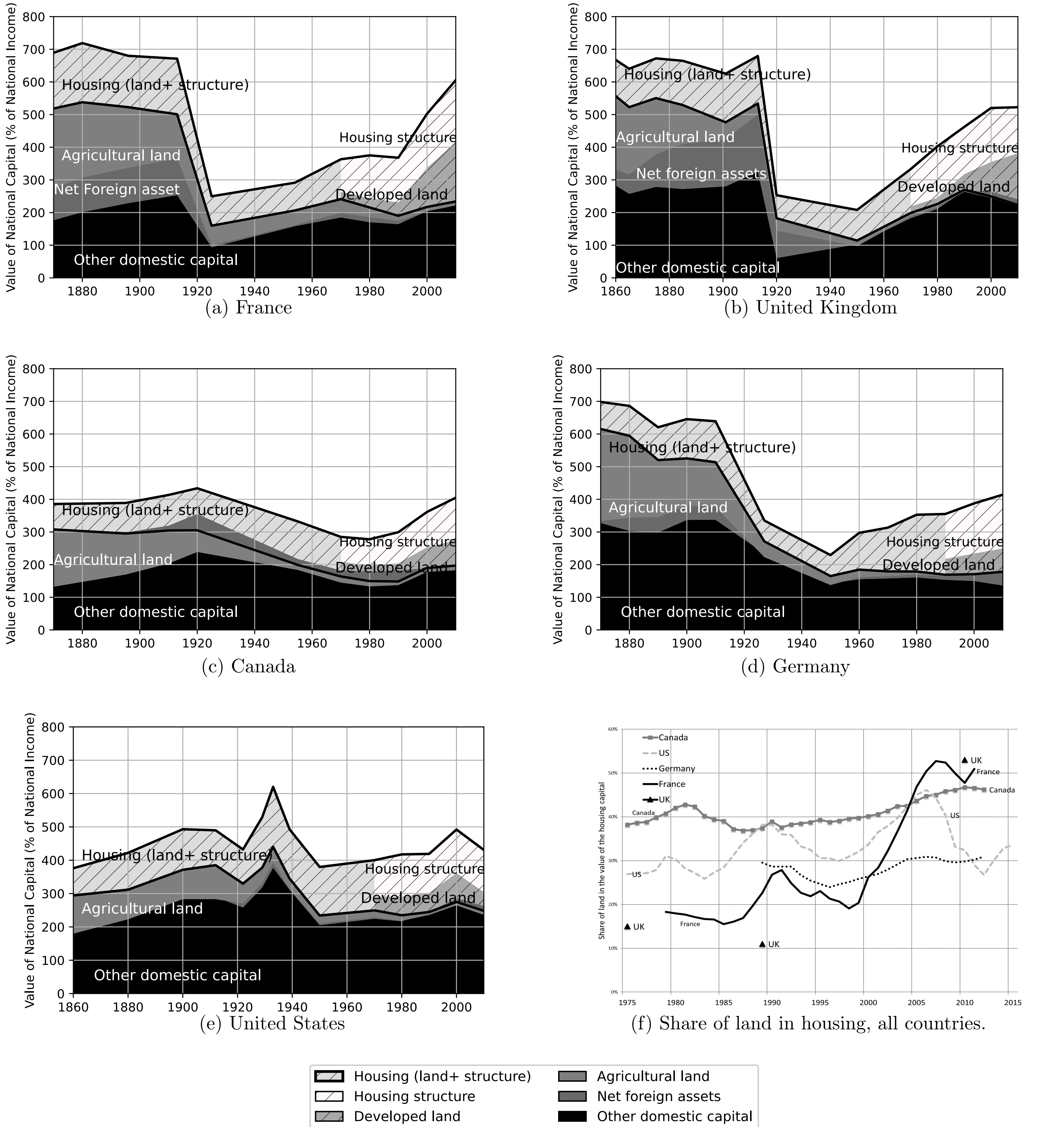
discussing capital and wealth taxation issues. The range of applicability of these results was subsequently better understood (Lansing, 1999; Atkeson et al., 1999; Benhabib and Szőke, 2019; Chari et al., 2016, 2020; Reinhorn, 2019; Straub and Werning, 2020) while leaving open the issue of how to fund income support given the convergence in the long run to a zero-tax rate on capital. Our analysis will address these concerns.

We extend the literature which has explored partial menus of taxes: e.g. tax on labor and capital income as in Chamley-Judd, labor and dividends as in Abel (2007), or taxes on consumption, labor and pure rents as in Diamond-Mirlees. In this paper, we explore a new corner, focusing on taxes on capital, land and housing, thus contributing to the literature by enlarging the set of menus. We perform this analysis both in first-best and second-best settings, under full commitment.

A uniform land tax is theoretically enough to achieve the first best. We discuss in which cases and at which level land taxes can be achieved, and notably propose a formula for a property tax of land to reach the social planner's objective. The tax rate has to be set so as to reduce the inequalities of welfare between capitalists and workers-tenants. This tax will allow to compensate the wage-earners for the fact that they have no property right on capital and land. When land can indeed be taxed at the first-best level, taxing productive capital is not necessary. The quantitative and theoretical importance of a fixed factor, land, is reminiscent of the so-called Georgist view, subsequently endorsed by many prominent economists (see the literature review section). Henry George's manifesto (George, 1879) *Progress and Poverty* argued within "the single tax movement" that a tax on land rent would allow to redistribute the return of the common heritage to benefit all individuals[3].

[3] The original Henry George's idea was to confiscate land rents and use it to finance various spending including income assistance while keeping initial property rights. Arnott and Stiglitz (1979) seminal paper showed the robustness of this idea in a different context (with a local public good and rent gradient), that was subsequently endorsed by many economists (see Section 6).

Figure 1: The role of land and housing in the secular variations in the wealth-to-income ratio. **France, US, Canada, UK and Germany**.



(a) France (b) United Kingdom

(c) Canada (d) Germany

(e) United States (f) Share of land in housing, all countries.

Figure's notes. Sources panels a) to e): Capital in the 21st Century, Figures 4.6 & Chart 3.2- see `piketty.pse.ens.fr/capital21c` and author's calculation. In Canada, net foreign assets are negative, explaining the small part above agricultural rent that is covered by housing wealth. Sources panel f): France, Canada and Germany: Piketty (`http://piketty.pse.ens.fr/en/capital21c`). United States: Davis and Heathcote (2007) (`https://www.lincolninst.edu/subcenters/land-values/price-and-quantity.asp`), last consultation November 2015. United Kingdom: Knoll et al. (2017), with 3 data points only. The decomposition methodology between land and structures is described in detail in Online Appendix section E.1)

However, taxing land raises implementation issues and is seldom put in practice[4]. Furthermore, land can hardly be distinguished from housing structures that are themselves elastic, and in order to reach the first-best, static and dynamic distortions must be addressed. Therefore, we consider richer schemes that may and actually do preserve the first best. An indirect land taxation via the taxation of housing rent is possible, but it cannot be implemented alone to reach the first best. The rent tax would distort the allocation of land across types of agents through a classical tax-wedge effect; adding a tax on imputed rents (taxing homeowners and landlords) corrects for the land allocation distortion but affects investments in residential structures which is then sub-optimal. A tax on rent therefore requires the addition of i) a tax on land differentiated across the use of land - less on rental land and more on home-occupied land, and ii) a specific subsidy on investments on rented structures. In the absence of this specific combination of tax/subsidies structures, the first best cannot be achieved. However, from the two distortions introduced by the tax on rent, the one on land use and the other on the dynamic of structures, the simulation reveals that the latter is much more critical quantitatively. The upshot is that a rent tax supplemented by a structure subsidy does almost as well as a land tax in improving social welfare. This discussion illustrates the non-triviality of a tax scheme attempting to overcome the non-feasibility of an optimal uniform land tax. This set of instruments provides a rare example of the usefulness of Lipsey-Lancaster (1956) approach, combining three distortive instruments to mimic the impact of a non- distortive instrument.

Regarding the Judd economy, most of the research has been devoted to studying the steady-state capital tax, and little was known about the dynamic of taxes. We then explore how these results in the steady-state extend on dynamic convergent paths to the steady-state. We obtain new results regarding the dynamic of capital taxes in the Judd economy without and with land, as well as results about the rent tax when it can be introduced. Adopting a second-best Ramsey logic, where the social planner acts under the rationality constraints of agents, in the spirit of Judd (1985) and the subsequent literature cited above, we next assume that a land tax is not available but we allow for a rent tax. An implementability constraint as in Atkeson et al. (1999) defines the set of feasible allocations. The zero tax to capital applies in the steady-state. Further restrictions are needed to study the dynamics (separability of housing and consumption, CRRA and inverse of intertemporal elasticity of substitution below a threshold). No taxation on capital is required along the dynamics except initially, where a positive tax is needed in the Rawlsian case. We next qualify Judd's result of no taxation

[4] A well-known exception is the Pittsburgh experience - see Oates and Schwab (1997).

of productive capital in the limit, following the vein of Straub and Werning (2020) where dynamic multipliers are defined each period. The existence of a fixed factor, land, extends the range of parameters under which a steady-state with no capital tax is socially desirable at the limit, reinforcing the scope of the original Chamley-Judd's results. The bottom line is that taxing rents is less distortive in the dynamics and in the steady-state than taxing capital, and it is even second best optimal in the absence of land tax.

This theory exercise has practical applications for policy. We indeed propose explicit formulas for optimal land taxes, subsidies to housing structure investments, and a second-best optimal rent tax that follows an inverse elasticity rule à la Ramsey. We apply these taxes in a simulation exercise. It shows that land taxes and other first best schemes based on taxes on rents and imputed rents combined with subsidies to structures do slightly better than only a tax on actual rents still compensated by a subsidy on rental structures. This last scheme itself does much better than taxes on rents, themselves doing much better than taxes on capital or capital and rents which reduce welfare. The take-away message is that a wealth tax taxing uniformly all three kinds of wealth, land, structures and capital at the same rate is not recommended, as it does not exploit the tax-elasticity heterogeneity of different types of wealth.

Our paper is organized as follows. **Section 2** introduces the optimal taxation framework in a model of workers and capitalists à la Judd augmented with housing (land and structure). **Section 3** provides welfare analysis in the first best with and without a land tax. **Section 4** explores second best Ramsey results in a simplified case with no structures. **Section 5** simulates the model and **Section 6** positions the paper in the literature. **Section 7** concludes. Proofs and extensions are in Appendices.

2 A model of optimal taxation with housing and physical capital

This Section develops a framework to understand how returns to wealth, productive capital and land can be redistributed. Judd (1985)'s model is particularly well suited to discuss redistributive optimal taxation of capital because its structure yields the highest incentives to tax capital for redistributive purposes: i) there are agents with no savings - that is, no access to credit markets - and consuming only their labor income and the possible transfers; and ii) there are agents with the ability to transfer wealth across periods owning a combination of productive capital. We assume as a starting point that the distribution of urban land and capital are perfectly correlated, mainly for clarity and simplicity reasons but also to focus on efficiency issues and discuss later how to extend the results to different types of capitalists.

We use Judd's redistribution framework of a two-class economy with a homogeneous class of workers and capitalists producing one composite good. We extend it with housing, as a combination of land and structures. The agents with no access to savings market are both workers and tenants who consume a rental housing service. The agents with access to capital market own physical capital, invest and replace depreciation, own land, consume part of it for themselves and obtain rents from workers/tenants for the rest; they also invest in structures.

The model is thus a model of a representative working class with no assets and a representative capitalist class holding all assets, land and physical capital, in the spirit of Judd (1985). We discuss the quantitative relevance of these assumptions in the calibration section and in Appendix E.2 with further references. We will consider throughout this Section, for expositional reasons, the case where the mass of workers and capitalists are the same. In Online Appendix B we discuss the extension to the general case of individual agents of different mass.

We start from the social planner's program referred to as the first-best situation. Next, we look at how a decentralized equilibrium with appropriate taxes reaches the first best of the social planner. We then study the case in which the land tax cannot be implemented but yet how a first best can be achieved with additional instruments. We later briefly discuss how the results are preserved in the presence of intra-class heterogeneity, land market transactions and finally a second best analysis.

2.1 Setup

Time is discrete, indexed by t. In each period, agents consume a representative consumption good and a composite housing service. The housing service associates both land and housing structures. There are two types of agents, a representative capitalist and a representative worker.

2.1.1 Capitalists

The class owning all assets is called generically capitalists. Following the traditional exposition and notations, capitalists consume an amount C_t and a housing service denoted by H_t. The utility function is $\sum_{t=0}^{\infty} \beta^t U(C_t, H_t)$ where $0 < \beta < 1$ is a discount factor and we denote the marginal utility as respectively $U'_C(t)$ and $U'_H(t)$ when inputs are at time t. The preferences for each good are supposed to satisfy the Inada conditions, that is, the first units of consumption (respectively housing service) have an infinite marginal utility for all consumption baskets. Inada conditions also impose that the limit of marginal utility will go to 0 at infinity which will insure transversality conditions

Capitalists own both a capital good K_t and the total quantity of land $\bar{\mathcal{L}}$, assumed to be fixed; they also have the property of housing structures. Capitalists allocate part of land $\mathcal{L}_t$ to their own consumption, and devote the rest to the rental housing sector, in quantity $\mathcal{L}_t - \bar{\mathcal{L}}$. Capital is used to produce a composite good with the mass of labor of size 1, thanks to a constant returns to scale production technology denoted $F(K_t, 1) = f(K_t)$. Capitalists inherit a stock of capital K_0, which depreciates at rate δ; they invest a quantity I_t so that

$$K_{t+1} = K_t(1-\delta) + I_t.$$

Following the traditional exposition and notations, capitalists consume an amount C_t. The production of housing service is denoted by

$$H_t = H(\mathcal{L}_t, S_t)$$

where S_t is the housing structure of the housing units consumed. This production of service follows Lancaster (1966) and combines different inputs (land and structures) that enters into utility as a bundle. This is similar to Becker's home production theory, where in Becker (1965), this is a combination of time and resources. Unless specified, the function $H(.,.)$ will be assumed to be constant-returns to scale and subject to Inada conditions. It means that camping (land but not structure) or piles of bricks alone (structures but no land) provides no utility.

We denote by $H'_{\mathcal{L}}(t)$ and $H'_S(t)$ the marginal product of housing service of each argument at time t. S_t is another capital good and its law of motions is similar to that of investment with depreciation rate δ_S:

$$S_{t+1} = S_t(1-\delta_S) + I_t^S$$

Similarly, they invest in the structures for rented units denoted by s_t and therefore:

$$s_{t+1} = s_t(1-\delta_s) + I_t^s$$

In what follows and to simplify exposition without loss of generality, we assume $\delta_s = \delta_S$.

The produced good $f(K_t)$ can be transformed into consumption C_t, c_t, into new capital

K_{t+1} and into new structures S_{t+1} or s_{t+1}. We assume for simplicity that the marginal rate of transformation between these different components is 1.

2.1.2 Workers

Workers earn wages in offering their labor to the capitalists in fixed quantity. The consumption of the composite good is denoted c_t and the housing services h_t. The housing service is supplied according to the production function similar to H_t, denoted h_t with

$$h_t = h(l_t, s_t)$$

where $l_t = \bar{\mathcal{L}} - \mathcal{L}_t$. As before, we denote by $h'_s(t)$ and $h'_l(t)$ the marginal product of each input. Formally, $h'_{\mathcal{L}}(t) = -h'_l < 0$[5]. Finally, workers' utility is function of one period's consumption of goods and housing service, $u(c_t, h_t)$.

2.2 Social planner's program

2.2.1 Social planner's objective

The social planner wants to maximize a weighted average of the utility of each agent, where the weight of capitalists/landowners is given by $1 \geq \gamma \geq 0$ and the weight of the workers is normalized to 1, following the notations in Straub and Werning (2020), as follows:

$$\sum_t \beta^t u(c_t, h(\bar{\mathcal{L}} - \mathcal{L}_t, s_t)) + \gamma U(C_t, H(\mathcal{L}_t, S_t))$$

over a converging path of consumption and capital accumulation. In the above function, a weight γ set to zero implies that the social planner only cares about those without property rights on land and capital (we will call this situation Rawlsian). A weight equal to 1 is the utilitarian case. We will mostly consider intermediate cases with $0 < \gamma < 1$ with low values of γ, consistently with the literature. We prove in Online Appendix B.2 that all subsequent results hold if we depart from the case of equal masses[6].

[5] For this housing service s_t, as for that of the capitalist defined above, we will think of their production as constant-returns to scale function with Inada conditions applying there too. This specification is sometimes used in the macroeconomic literature. See for instance the recent paper by Garriga et al. (2019).

[6] We do not consider values of γ outside the range (0,1) but a value below zero would correspond to a punitive role of taxation to reduce the well being of the richer class. Values above 1 correspond to a recognition by the social planner of the specific role of capitalists and may be used to alleviate participation constraints that high lump sum taxes may raise.

The social planner resource constraint is

$$f(K_t) + (1-\delta)K_t + (1-\delta_S)(S_t + s_t) = c_t + C_t + S_{t+1} + s_{t+1} + K_{t+1} \tag{1}$$

Hence, taxes affecting the decentralized equilibrium will be purely redistributive and have the purpose of raising the consumption of goods and housing of the worker.

2.2.2 Social planner's optimization

We assume that $U(.,.)$ and $u(.,.)$ are increasing and concave in each argument. The budget constraint per period of the social planner is concave as well as its objective function. The maximization problem of the social planner can then be replaced by the following Lagrangian:

$$\begin{aligned}\max_{C_t,c_t,S_{t+1},s_{t+1},\mathcal{L}_t,K_{t+1}} & \sum_t \beta^t \left\{u(c_t, h(\bar{\mathcal{L}} - \mathcal{L}_t, s_t)) + \gamma U(C_t, H(\mathcal{L}_t, S_t))\right\} \\ & + \sum_t \beta^t \lambda_t \left\{f(K_t) + (1-\delta)K_t + (1-\delta_S)(S_t + s_t)\right. \\ & \left. -c_t - C_t - S_{t+1} - s_{t+1} - K_{t+1}\right\}\end{aligned}$$

subject to three transversality conditions on each stock:

$$\beta^t U_C'(t) M_{t+1} \to 0 \tag{2}$$

for $M_{t+1} = K_{t+1}, S_{t+1}, s_{t+1}$. R_{t+1}^{Kgross} is the gross return to capital producing returns next period and defined as:

$$R_{t+1}^{Kgross} = f'(K_{t+1}) + 1 - \delta$$

We obtain the following first order conditions in the steady-state[7]:

$$\begin{aligned} \partial C_t, c_t \quad & \lambda = \gamma U_C' = u_c' && (3)\\ \partial \mathcal{L}_t \quad & \gamma U_H' H_{\mathcal{L}}' = u_h' h_l' && (4)\\ Euler \quad S_{t+1} \quad & \beta^{-1} = \frac{U_H' H_S'}{U_C'} + 1 - \delta_S && (5)\\ Euler \quad s_{t+1} \quad & \beta^{-1} = \frac{u_h' h_S'}{u_c'} + 1 - \delta_S && (6)\\ Euler \quad K_{t+1} \quad & \beta^{-1} = f'(K) + 1 - \delta = R^{Kgross} && (7)\end{aligned}$$

[7]See Online Appendix A.2 for detailed calculations and for the first order conditions out of the steady-state

The first two equalities are the result of intraperiod optimization. The first one states that the planner wants to equalize the marginal utilities of consumption across the two types of agents, up to the social weight γ, and the second one has a similar interpretation in terms of the marginal utility of the housing service with respect to land. They also imply that the marginal rate of substitution between land as a producer of housing service and consumption are equal across agents: $\frac{U'_H H'_{\mathcal{L}}}{U'_C} = \frac{u'_h h'_l}{u'_c}$, that leads also to determine the ratio of marginal utilities across agents and fix it to γ:

$$\frac{u'_h h'_l}{U'_H H'_{\mathcal{L}}} = \frac{u'_c}{U'_c} = \gamma \tag{8}$$

In addition, there are three intertemporal first-order conditions, the Euler equations of the problem. The Euler equation on capital states that the net return on capital at the social optimum has to be equal to the discount rate of agents. The above equations on structures have similar interpretations: one invests on structures up to the point that the net return of structures given by the marginal rate of substitution ($U'_H H'_S / U'_C$ or $u'_h h'_s / u'_c$) net of depreciation will be equal to the discount rate. It stems from these three Euler equations that the net rate of return in investing in productive capital or in the two types of structures must be the same. Last, comparing equations (5), (6) and (7), one can easily see that the return on capital and structures is identical up to differences to depreciation rates. These three equations together with equation (8) and the resource constraint (1) define five conditions for six endogenous variables: $c, C, K, S, s, \mathcal{L}$. The conditions are independent of the social weight γ. Hence, they define an efficient allocation set of dimension 1 and correspond to what we hereafter refer to as the "first best". The particular solution chosen by the social planner solution depends of γ and is calculated with equations (3) and (4).

Finally, the marginal rate of transformation between space (land) and structures must be equal across agents regardless of the weight given to the capitalist:

$$\frac{H'_S}{H'_{\mathcal{L}}} = \frac{h'_s}{h'_l} \tag{9}$$

and combining equations (8) and (9), the marginal rate of substitution between consumption and structures are equalized across agents:

$$\frac{U'_C}{U'_H H'_S} = \frac{u'_c}{u'_h h'_s}. \tag{10}$$

2.3 The decentralized equilibrium with taxes

2.3.1 Setup

We discuss the distortions generated by several tax schemes. First, we investigate the simplest form of capital taxation relying on a single tax on land $\tau_{\bar{\mathcal{L}}}$ on productive capital τ_K . In the welfare discussion, we also consider a tax on housing rents paid by landlords τ_H.

In next sub-sections, we also consider a tax on imputed rents for landlords living in their own property (denoted by τ_{HI}), and finally turn to a living tax on the housing consumption of both capitalists and workers (τ_{liv}). Finally, we discuss how these taxes might be combined with a tax or a subsidy on housing investments, i.e on structures for capitalists (τ_S) or workers (τ_s). For convenience and to avoid repetitions, all taxes are introduced simultaneously in Online Appendix A.3, but we start here only with taxes on capital, land and rent taxes. Capitalists do not work, so their income is the sum of the net return on physical capital, net rents and other taxes that may affect them. They pay a market wage w_t. Markets are perfectly competitive, and we define the wage w_t as:

$$w_t = f(K_t) - f'(K_t)K_t \tag{11}$$

The net, after-tax return on capital is

$$R_t^{Knet} = (1 - \tau_{K,t})R_t^{Kgross} \tag{12}$$

where taxes on capital at time t are $\tau_{K,t}$.

We also use the notation R_t^{Hgross} for the gross rent on land so that $R_t^{Hnet} = R_t^{Hgross}(1 - \tau_{H,t})$ and capitalists therefore receive a rent income $h_t R_t^{Hnet}$. Let $T_t^K = \tau_{K,t}R_t^{Kgross}K$ be the tax revenue from capital, $T_t^H = \tau_{H,t}R_t^{Hgross}h_t$ the tax revenue from rents. The income from taxation of capitalist's land is $T_t^{\mathcal{L}} = \tau_{\bar{\mathcal{L}},t}\bar{\mathcal{L}}$. The sum of these components are the total taxes T_t that will be transferred to the worker, with $T_t = T_t^K + T_t^H + T_t^{\mathcal{L}}$.

2.3.2 Decentralized agents' program

The objective function of each agent (capitalist and workers) is concave as well as their resource constraints so that we can directly proceed with their respective Lagrangians. The capitalist optimizes over an infinite horizon, whereas the program of the worker is a static one, consuming the current disposable income and transfers into consumption and housing services. In the absence of government bonds and thus other assets than housing and capital, the capitalist solves:

$$\max_{C_t,H_t,\mathcal{L}_t,K_{t+1},s_{t+1},S_{t+1}} \sum_t \beta^t U(C_t, H(\mathcal{L}_t, S_t))$$
$$+\sum_t \beta^t \lambda_t^K \left\{R_t^{Knet} K_t + R^{Hnet} h(\bar{\mathcal{L}} - \mathcal{L}_t, s_t)\right.$$
$$+(1-\delta_S)S_t + (1-\delta_S)s_t - T_t^{\mathcal{L}}$$
$$\left.-C_t - S_{t+1} - s_{t+1} - K_{t+1}\right\}$$

subject again to three transversality conditions on each stock:

$$\beta^t U_C'(t) M_{t+1} \to 0 \tag{13}$$

for $M_{t+1} = K_{t+1}, S_{t+1}, s_{t+1}$, and the program of the worker is, noting that, as tenant, it cannot choose separately land and structures:

$$\max_{c_t,h_t} u(c_t, h_t)$$
$$\text{subject to } c_t + h_t R_t^{Hgross} = w_t + T_t$$

The first order conditions out of the steady-state are reported in the general case in Online Appendix A.3.1. In the steady-state and after re-arrangement of the different terms detailed in the same Online Appendix, one obtains:

Intraperiod allocations

$$\frac{u_h' h_l'}{U_H' H_{\mathcal{L}}'} (1-\tau_H) = \frac{u_c'}{U_C'} \tag{14}$$

Intertemporal allocations (Euler)

$$\partial S \quad \beta^{-1} = 1 - \delta_S + \frac{U_H' H_S'}{U_C'} \tag{15}$$

$$\partial s \quad \beta^{-1} = 1 - \delta_s + (1-\tau_H)\frac{u_h' h_s'}{u_c'} \tag{16}$$

$$\partial K \quad \beta^{-1} = R^{Kgross}(1-\tau_K) \tag{17}$$

This equation (14) comes from market clearing conditions on housing services at the equilibrium rental rate. It states that the opportunity cost of getting more housing services is the forgone utility of consumption for each agent. The total equilibrium land allocation also satisfies $\bar{\mathcal{L}} = \mathcal{L}_t + l_t$.

3 Achieving the first best

3.1 Achieving the social planner's objective with a unique tax on land

Let us start in Proposition 1 with a first set of claims that are not the contribution of this paper, but rather a simple exposition of the first-best benchmark with a pure and uniform land tax, to which alternative policies can be gauged. We also derive there an explicit formula for the optimal land tax that gives the idea of the trade-offs of the social planner, before proceeding with our main results in Proposition 2 in the next sub-sections.

Proposition 1 (Optimal land tax): *With a land tax, a first best can be achieved in the steady-state with the combination of*

1. *a zero tax on returns on capital:* $\tau_K = 0$,
2. *a zero tax on housing rents:* $\tau_H = 0$,
3. *a single tax on land fully redistributed to workers, with revenue* $T^{\mathcal{L}} = \tau_{\bar{\mathcal{L}}}\bar{\mathcal{L}}$.
4. *In the particular (utilitarian) case* $\gamma = 1$ *where the utilities of capitalist and worker are separable and identical, the tax revenue is used to redistribute rents and to share total production net of depreciation, that is equalize consumption levels:*

$$\tau_{\bar{\mathcal{L}}}\bar{\mathcal{L}} = hR^{Hgross} - w + \frac{1}{2}\left[f(K) - \delta K - \delta_S(s+S)\right] \tag{18}$$

5. *In the general case* $0 < \gamma < 1$, *there is no such explicit formula, but a similar formula exists in special cases.*
   - *With an exponential negative sub-utility functions (CARA)* $\ln u(c) = -\nu c$ *and* $\ln U(C) = -\nu C$, *and for admissible values of* $\gamma > 0$,

$$\tau_{\bar{\mathcal{L}}}\bar{\mathcal{L}} = hR^{Hgross} - w + \frac{1}{2}\left[f(K) - \delta K - \delta_S(s+S)\right] - \frac{\ln\gamma}{2\nu} \tag{19}$$

- *With instead a CCRA function for the sub-utility of consumption for both the worker and capitalist* $U(C) = \frac{C^{1-\sigma_C}}{1-\sigma_C}$ *and* $u(c) = \frac{c^{1-\sigma_C}}{1-\sigma_C}$*, the optimal land tax for any social welfare weight for the capitalist is given by*

$$\tau_{\bar{\mathcal{L}}}\bar{\mathcal{L}} = hR^{Hgross} - w + \frac{f(K) - \delta K - \delta_S(s+S)}{1+\gamma^{1/\sigma_C}} \tag{20}$$

With $\gamma = 0$*, the optimal land tax exhausts the revenue of the capitalist.*

Proof: See Online Appendix A.4.3.■

In the utilitarian case of 4) above, the simple formula for the optimal property tax implies an equal sharing rule of total resources of the economy net of depreciation. The social planner uses the tax on land to compensate workers for the returns on capital and land of the capitalists and equalize their consumption. The land tax actually redistributes the sum of returns on housing and returns to capital. To see this, the formula in equation (18) can be rewritten as $\tau_{\bar{\mathcal{L}}}\bar{\mathcal{L}} = \left[hR^{Hgross} - \frac{1}{2}\delta_S(s+S)\right] + \left[K.R^{Kgross} - \frac{1}{2}\left(f(K) + \delta K\right)\right]$[8]. The other two formulas apply to $0 < \gamma < 1$ in the CARA and CRRA (point 5 above) cases. In all cases, the tax payers do treat the tax rate on land as constant and are considered as atomistic for our purpose so that the tax on land is lump sum. A similar logic of compensation for unequal property rights applies. As expected, the optimal land tax levied increases as γ goes down.

We end up this subsection by interpreting our results in light with Henry George's proposal to fully confiscate what he called the rent of undeveloped land[9]. He indeed argued that property rights (the "shell") would be preserved, but to fully taxing "the kernel of land property". Our Proposition 1 extends his logic of fully taxing the product of rents but George's proposal is limited by the existence of incentive problems based on the role of structures, that we address in next sub-section. To better understand this discussion, we need to define the rent from rented land and exclude the rental part from accumulated structure. The natural

[8] If capitalists receive the total amount to be paid without knowing the calculation, their incentives are unaffected but this may be a limit to this uniform land tax scheme, as detailed and addressed in next Sub-section. Note also for completeness that the optimal tax may be negative if the capital share in production is small and the capitalist pays large replacement costs of capital and structures. This is due to the fact that the capitalist and the worker are given an equal demographic weight. In Online Appendix B we show that the introduction of demographic weights never affect the first order conditions of the decentralized equilibrium and of the social planner, only the resource constraints and the level of income and consumption of individual agents.

[9] George (1879), (VIII.2.12) wrote: "*I do not propose either to purchase or to confiscate private property in land. The first would be unjust; the second, needless. Let the individuals who now hold it still retain, if they want to, possession of what they are pleased to call their land. Let them continue to call it their land. Let them buy and sell, and bequeath and devise it. We may safely leave them the shell, if we take the kernel. It is not necessary to confiscate land; it is only necessary to confiscate rent.*"

rental price for rented land is therefore $R^{Hgross}h'_l(s,l)l = R^{Hgross}h(l,s) - R^{Hgross}h'_s(l,s)s$ using the CRS assumption. Similarly, the imputed rent on occupied land by capitalists is given by $R^{Hgross}H'_{\mathcal{L}}(\mathcal{L},S)\mathcal{L} = R^{Hgross}H(\mathcal{L},S) - R^{Hgross}H'_s(\mathcal{L},S)S$. The generalized Henry George's tax T^{HG} in our model is the sum of the two previous terms with a confiscatory tax rate is 100%:

$$T^{HG} = R^{Hgross}\left[h'_l(s,l) + H'_{\mathcal{L}}(\mathcal{L},S)\mathcal{L}\right]$$

It would clearly distort the efficiency of land allocation and the dynamic accumulation of both s and S, which is not the case of formulas (18)-(20). It then illustrates that taxing land with structures in place is not fully obvious, if we want to respect efficiency.

3.2 Policy constraints on land taxes and rent taxes

We are going to show in the rest of the paper that the uniform tax on land is not necessary. It is important to find alternatives, since many constraints, including political ones (Bird and Slack, 2004) prevent a uniform first-best land tax from being implemented, particularly if it is too high (see below). Instead we will show that a first best can still be achieved by: i) differentiated tax on land, in particular a lower land tax on rented housing thus possibly alleviating the feasibility constraint ; ii) and a tax on rents complemented by subsidies on rental structures. We will also explore the distance between the first best and a world where the tax on land cannot be set at its optimal value.

Among the constraints, as put explicitly by Chari et al. (2020), it may not be possible to confiscate a fraction of capital owned by capitalists once and for all. As discussed above, a one-time expropriation of physical capital is not necessary if the social planner could tax land, especially depending on its use. A land tax plays a similar role to the one-time taxation of initial physical capital K_0 as its base is inelastic. Further, the full expropriation also means that landlords might not necessarily want to hold on their rented land neither to reinvest in structures. Indeed, the optimal land tax in Proposition 1 equation (18) represents a perfect sharing of resources or equivalently a total redistribution of property rights. It may therefore be limited by participation or incentive constraints, and not all first best solutions can be reached: some values of γ may not be implementable. Other limits to a land tax are the absence of land register[10].

[10] Only 50 countries out of 200 have one according to van der Molen (2003). The other constraints are more political in nature. First it appears that the property tax is the most "hated" tax, as coined in Cabral and Hoxby (2012) in the US, and in Sweden, see Nordblom et al. (2006). California is famous for the cap on the property tax (proposition 13, June 6, 1978).

It is sometimes claimed that taxes on imputed rents might restore efficiency, but they may also be difficult to implement, because they are not observed directly contrary to actual rents, but only estimated imperfectly or even fixed arbitrarily leading to resistance from tax payers.

Overall, we are now going to study the efficiency aspects of a positive tax on rents, and see under which conditions and with which additional tax instruments one can restore efficiency, in a Lipsey-Lancaster's approach (Lipsey and Lancaster, 1956) to the decentralisation of the first best.

3.2.1 New tax instruments in the presence of positive tax on rents

New instruments can be studied, such as a positive tax on rents τ_H. It will introduce a wedge in the allocation of land and intertemporal distortions in housing investments. We may introduce further instruments correcting for these distortions. In particular, we introduce a tax on the housing service of owner-occupiers, denoted by τ_{HI}, which is hereafter called a tax on imputed rents.

We also introduce a net tax on structures which can be differentiated across usages: the structures S_t or the structures s_t. At this stage, we do not restrict the sign of the taxes: they can be negative to allow for subsidies if needed, e.g. to favor investment and possibly correct for distortions from other taxes. To simplify and to treat both taxes/subsidies symmetrically, we introduce the notations $\tau_{S,t}$ and $\tau_{s,t}$ the tax rates per unit invested, the product of the tax equals: $T_t^{s,S} = \tau_{S,t}\left[S_{t+1} - S_t(1-\delta_S)\right] + \tau_{s,t}\left[s_{t+1} - s_t(1-\delta_S)\right]$. Online Appendix A.3 derives the first order conditions of agents in the general case. With the taxes considered in this sub-section, these conditions are:

Intraperiod allocations

$$\frac{u'_h h'_l}{U'_H H'_{\mathcal{L}}}\left(1 - \tau_H + \tau_{HI}\frac{H'_{\mathcal{L}}}{h'_l} + \frac{\Delta\tau_{\mathcal{L}}}{R^{Hgross} h'_l}\right) = \frac{u'_c}{U'_C} \tag{21}$$

Intertemporal allocations (Euler)

$$\partial S \quad \beta^{-1} = 1 - \delta_S + \frac{1}{1+\tau_S}\frac{U'_H H'_S}{U'_C} - \frac{\tau_{HI}}{1+\tau_S} R^{Hgross} H'_S \tag{22}$$

$$\partial s \quad \beta^{-1} = 1 - \delta_s + \frac{1-\tau_H}{1+\tau_s}\frac{u'_h h'_s}{u'_c} \tag{23}$$

3.2.2 Restoring first best efficiency even with positive tax on rents

Let us right away summarize our result into a single proposition.

Proposition 2 (Restoring the first best with distortive instruments):
With a positive tax on rents, the social planner can restore first best efficient outcomes:

1. *The distortion of the intraperiod allocation of land $\mathcal{L}$ induced by the tax on rents can be compensated:*

   (a) *either by a positive differential tax on land $\Delta\tau_{\mathcal{L}} = \tau_{\mathcal{L}} - \tau_l$, with:*

$$\Delta\tau_{\mathcal{L}} = \tau_H R^{Hgross} h'_l \tag{24}$$

   (b) *or, a tax on imputed rents τ_{HI} such that*

$$\tau_{HI} = \tau_H \frac{h'_l}{H'_{\mathcal{L}}} \tag{25}$$

   (c) *or, a combination of both instruments as long as*

$$1 - \tau_H + \tau_{HI} \frac{H'_{\mathcal{L}}}{h'_l} + \frac{\Delta\tau_{\mathcal{L}}}{R^{Hgross} h'_l} = 1 \tag{26}$$

2. *The distortion on rental structures s induced by the tax on rents can be compensated by subsidizing rental structures at the same rate, with the rule $\tau_s = -\tau_H < 0$.*

3. *There is no distortion on owner-occupiers' structures S if the social planner only uses a differential land tax and not a tax on imputed rents as discussed in part 1 of this proposition. However, in the case of a tax on imputed rents, a subsidy S is needed to restore the first best.*

Proof: See Online Appendix A.4.4 for details of the proofs. ■

This proposition first states that a common way to restore efficiency of the intraperiod allocation of land discussed in the first part of Proposition 2 is to introduce a tax on imputed rents τ_{HI}, that is on the "rent equivalent" that homeowners serve to themselves. This solution however distort the intertemporal allocation of resources, except in the special case in which structures S are inelastic.

An alternative scheme is the introduction of a differentiated tax on land depending on land use, denoted respectively by $\tau_{\mathcal{L}}$ for the tax on capitalists land and τ_l the tax on landlords land, and denoting their difference by $\Delta\tau_{\mathcal{L}} = \tau_{\mathcal{L}} - \tau_l$. In this case, the social planner has one extra degree of freedom to reach its desired, first best allocation, and this is true for any social weight. This is what the third part of the Proposition establishes.

These two alternative options remove the static distortion of the tax on rents, either by taxing more the land used by landlords than the tax on rented land, and the differential tax on land evaluated at its marginal benefits for housing services must be equal to the amount of tax on rents; or by taxing the housing services of homeowner at a different level as tenants, and more, in the plausible case $H'_{\mathcal{L}} < h'_l$.

An intertemporal distortion also arises from the rent tax itself: it distorts the provision of rental service by deviating away from the optimal Euler equation (6). A subsidy to these structures is therefore necessary, as established in part 2 of the Proposition.

Overall, the proposition links the subsidies on structures and the tax on rents. This is similar to the analysis of physical capital taxation in Abel (2007) who introduced tax credit on investment that allows for investment spending. Taxing imputed rents rather than a differentiated land tax is however less appealing because it imposes a fourth instrument (the subsidy to owner-occupiers' structures).

A last remark is that if contrary to the prescription of part 1.b) of Proposition 2, the social planner sets for simplicity an equal rate for the actual and imputed rents taxes, that is if $\tau_H = \tau_{HI}$, efficiency only arises if the housing service is uniformly linear in land. If the service is not linear in land, the equality $\tau_H = \tau_{HI}$ implies, in the absence of the differentiated land tax, a deviation of land use $\mathcal{L}$ from the first best. The direction of the variation from the first best depends on comparing the marginal product of land in housing services. In particular, if tenants live in smaller housing units, e.g. if $H'_{\mathcal{L}} < h'_l$, the quantity of land allocated to tenants is too small relative to the first best, under separability of housing and consumption in utility.

3.2.3 Further results

Signing the distortions of rent taxes When subsidies to structures are not possible, it is easy to see that the tax on imputed rents reduces the demand for investments in homeowner structures S in equation (22), although this may not be a primary concern for social planner interested only in workers (a zero value for γ). However, in equation (23) the tax on rents also reduces the demand for structures of tenants to a sub-optimal level: landlords under-invest in the walls and other landlord-provided equipment, leading to a lower quality of the housing service. We summarize this discussion in Appendix A.4.5.

Other taxes on consumption and living taxes First, the living tax can be defined as a tax paid by both workers and capitalists, on the rental value of their respective housing units, that is proportional respectively to h and H. In our framework, this tax is not particularly compelling since it immediately reduces the living standard of the worker and requires higher taxes to reach the desired level of consumption. We explore this in Appendix A.4.6.

Similarly, one can introduce a consumption tax on both workers and capitalists. Although the same objection applies (it taxes uniformly poor and rich agents before redistributing its product), studying this tax is important as it exists in most countries and there are good arguments in favor of it, such as an alternative way to tax installed capital, see Coleman II (2000) and Chamley (1986). We show in Online Appendix A.4.7 that a first best solution improving the situation of the worker tenant with respect to the laissez-faire can be reached with a combination of the following distorting instruments: a uniform consumption tax for all types of agents, τ_c, a living tax for the worker only at the same rate as the uniform consumption tax for the worker solely, $\tau_{liv} = \tau_c$, and tax on imputed rent for the capitalist, τ_{HI}, satisfying $\tau_{HI} = \tau_c \frac{h'_l}{H'_{\mathcal{L}}}$. The logic is easy to understand: with a consumption tax, the intraperiod allocation requires a tax on the housing service of workers, itself matched by a tax on the housing service of land-owners: taxes on actual rents received by landlords are no longer necessary. This is again an application of Lipsey-Lancaster's idea that combinations of distortive taxes can nevertheless provide a first best optimum[11].

3.3 Results along a dynamic path

Interestingly, the results of this Section can be generalized to a dynamic path, out of the steady-state. Appendix A.5 proves this. It starts from the first order conditions out of the steady-state to formally establish a set of results that we simply summarize here:

1. Proposition 1, parts 1 and 2 in dynamics holds: the first best can be achieved with $\tau_{K,t} = 0$, after $t = 1$ and $\tau_{H,t} = 0$ as soon as t=0. Taxing capital in the initial period, a well known result e.g. Chamley (1986), is possible and a substitute to land taxation, but would then lead to commitment problems.

2. Proposition 2, part 1 holds: when a tax on rents however exists, it is possible to reach the first best intraperiod land allocation and alleviate the consequences of the tax wedge with either a differentiated tax on land (on rental housing and owner-occupied housing)

[11] We do not emphasize further this result because it may be difficult to implement the scheme in practice, due to imperfect observability of the value of the service of housing for landlords.

or a tax on imputed rents that is not necessarily equal to the tax on actual rent - there is a correcting term.

3. Proposition 2 parts 2 and 3 in dynamics hold: if the tax on rents is constant over time after the current period, a constant subsidy to rental structures s is first best for the structures s, with $\tau_s = -\tau_H$. Further, no subsidy for owner-occupiers structures is necessary in the absence of imputed rent taxes since the related Euler equation on those structures is not distorted.

In addition, one shows the dynamic equations for subsidies on structures if rent taxes are not constant over time and when imputed rent taxes are positive; see Appendix equations (A.85) and (A.86). If for instance the government announces an increase in rent taxes next period, it is optimal to introduce a subsidy on structures today.

3.4 Heterogeneous capitalists

In an extension with heterogeneous capital, we have the following results: in the first best, in the absence of taxes on rents, land taxes are positive, and uniform across capitalists as long as there are perfect housing markets. With different tax rates on rents, it is possible to differentiate the tax on land, and obtain a tax scheme in a spirit similar to those of the early propositions (Propositions 1 and 2), except that the land tax differentiation is not in land use (rental vs. own-occupation) but across capitalists.

It is beyond the scope of this paper to analyze in greater details every aspect of this discussion and this is left for future analysis, in particular to study different alternative tax schemes or more frictional situations. We simply provisionally conclude that the principle of a land tax, differentiated by sub-class of capitalists, still holds in cases of imperfect land market but is difficult to implement with fluid land markets and in the absence of non-linearity in returns to land.

3.5 Concluding comments

Our results show that the first best is possible even with taxes on rents, but only if one creates tax/subsidies combinations. These combinations of subsidies exist in some countries where landlords receive subsidies to develop new rental units, sometimes targeted towards housing for low income households, e.g. the Low Income Housing Tax Credit (LIHTC) program in the US, or in France with renting housing tax credits introduced since the 90's. Our analysis sheds light on the actual practice of several countries which use a cocktail of tax instruments in the housing sphere. Differential land taxes are generally absent, there are tax credits which subsidize investments but also land purchases, which takes us away from the first best. Our analysis shows that there is a room from improvement for the actual housing tax policies.

4 Achieving the Ramsey-second best

The previous section shows how the capital tax and the rent tax are distortive. The ability of the social planner to redistribute is limited by the elastic response of investments in physical capital as well as that of residential investments and land allocation. In this section, we study second-best optimal policies in the absence of land tax, and investigate their dynamic paths. Atkeson et al. (1999) and Chari et al. (2020) have developed a convenient framework to address these problems that we use and extend in what follows.

4.1 The implementability viewpoint

4.1.1 Generalization to Judd's economy

Let us first introduce Atkeson et al. (1999) and Chari et al. (2020) approach to a setup without housing, to set the benchmark. The original idea is to calculate the set of feasible consumption of the representative agent consistent with its optimal behavior over an infinite horizon, and use this as a constraint (instead of each-period's constraint) in the second best problem of the social planner. Taking the aggregate resource constraint of the economy into account allows to determine the second best optimal quantities. Next, one recovers a set of prices and taxes decentralizing the second best optimum, satisfying the implementability constraint and the aggregate resource constraint of the economy.

We use their approach to a different setup of the Judd economy with two-agents. The implementability constraint only applies to the capitalist who transfers wealth to the next period. The consumption profile is such that, given initial wealth K_0, one has the following implementability constraint (see Appendix C.1 for step-by-step calculation):

$$\sum_{t=0}^{+\infty} \beta^t U'_C(t) C_t = U'(C_0) R_0^{Knet} K_0 \tag{27}$$

adopting the more compact notation $U'_C(t)$ and $U'_C(t+1)$ for $U'_C(C_t)$ and $U'_C(t+1)$ respectively and more generally for all derivatives, when there is a need to specify the time period (by default this applies to period t).

The second best program is therefore, using the notations of previous sections:

$$\max_{K_{t+1},C_t,c_t} \sum_{t=0}^{+\infty} \beta^t \left[u(c_t) + \gamma U(C_t)\right]$$
$$+\nu \left[U'_C(0) R_0^{Knet} K_0 - \sum_{t=0}^{+\infty} \beta^t U'_C(t) C_t\right]$$
$$+\sum_{t=0}^{+\infty} \lambda_t \left[f(K_t) + (1-\delta)K_t \; - c_t - C_t - K_{t+1}\right]$$

where ν is the multiplier on the feasibility constraint and λ_t are the multiplier on the resource constraint faced by the social planner. A few steps are described in Appendix C.1. It is easy to show the following: when the problem converges[12], one obtains two first order conditions that are met simultaneously. The first one involves the gross return on capital:

$$\lambda_t = \lambda_{t+1}\left[f'_K(K_{t+1}) + (1-\delta)\right] \tag{28}$$

The second one involves the marginal utility of consumption.

$$\beta^t \left(\gamma U'_C - \nu U''_{CC} C_t - \nu U'_C\right) = \lambda_t; \text{ for } t \geq 1 \tag{29}$$

At time 0, equation (29) contains an additional term. Therefore, plugging equation (29) only after $t = 1$ into equation (28), one obtains:

$$R_{t+1}^{Kgross} = \frac{\lambda_t}{\lambda_{t+1}} = R_{t+1}^{Knet} = \frac{U'_C(t)}{\beta U'_C(t+1)}$$

and therefore a zero taxation of capital for $t \geq 2$, under the necessary condition that $C_t U''_{CC}/U'_C$ is constant over time. This holds for all CRRA utility functions. To sum up formally:

Lemma 1 *(Extension of Atkeson et al. (1999)): When the steady-state exists, and if the utility function of consumption is CRRA: i) it is not optimal to tax capital from period 2 and thereafter; and ii) with $\gamma = 0$, the tax rate on capital is strictly between 0 and 1 regardless of any initial lump-sum tax on capital in period 0.*

[12] A necessary condition for that is that all constraints are concave, implying a condition on the third derivative of $U(C)$.

See Appendix C.1 for the full development of the calculations. This Lemma generalizes Atkeson et al. (1999) result originally obtained in a Chamley representative agent economy to a Judd economy.

4.1.2 Introduction of land

Denote again by $l_t = \overline{\mathcal{L}} - \mathcal{L}_t$ the land used by tenants. We remove structures to simplify, and therefore pose $H_t = \mathcal{L}_t$ and $h_t = l_t$ to simplify the comparison with the case with structures. As discussed in Appendix C.2, the implementablity constraint of the capitalist and the worker both include terms related to housing:

$$\sum_{t=0}^{+\infty} \beta^t U_C'(t) C_t - \sum_{t=0}^{+\infty} \beta^t U_H'(t) h_t = U_C'(0) R_0^{Knet} K_0 \tag{30}$$

and the Ramsey problem is stated as

$$\begin{aligned} \max_{K_{t+1}, C_t, c_t, h_t} \sum_{t=0}^{+\infty} \beta^t \left[u(t) + \gamma U(t) \right] \\ +\nu \left[U_C'(0) A_0 + \sum_{t=0}^{+\infty} \beta^t U_H'(t) h_t - \sum_{t=0}^{+\infty} \beta^t U_C'(t) C_t \right] \\ + \sum_{t=0}^{+\infty} \lambda_t \left[f(K_t) + (1-\delta) K_t - c_t - C_t - K_{t+1} \right] \end{aligned}$$

where ν is the multiplier on the implementablity constraint, λ_t are the multipliers on the resource constraint faced by the social planner[13].

In the particular case of a separable utility in C_t, H_t, one obtains two first order conditions that are met simultaneously and that are identical to those in the previous analysis with no land, in equations (28) and (29). For a zero taxation of capital to hold out of the steady-state at a given $t \geq 2$, it is necessary to have $C_t U_{CC}'' / U_C'$ to be constant over time. This again holds for all CRRA utility functions. See Appendix C.2 for the full development of the calculations in the case with land.

Proposition 3 *If the solution to the Ramsey problem in Judd with land converges to a steady-state,*

[13] No implementability constraint for the worker is included in the Ramsey program due to Walras law. It can easily be shown that one recovers prices, taxes and transfers, once determined the optimal sequences of quantities, K, C, c, h

1. *the tax rate on capital income is zero in the steady-state,*
2. *if preferences are additively separable in consumption and housing as land, and if the subutility function of consumption of the capitalist is CRRA but not log*[14]*,*
   (a) *it is not optimal to tax capital from period 2 and thereafter,*
   (b) *in the Rawlsian case, the tax rate on capital in period 1 is strictly positive and strictly less than 1.*
3. *if preferences are additively separable in consumption and housing as land, the tax wedge on the rental housing market are not constant over time and therefore requires a time-varying second best instrument on rents (tax or subsidy). In the Rawlsian case, this optimal rent tax is always positive at each period* $t = 1, 2...$ *and not constant.*

Note that, if we added structures to land in this problem, a specific difference with the feasibility constraint of Atkeson et al. (1999) arises. The term C_t in equation (30) is augmented with terms on investment in structures, so that the CRRA assumption on utility delivers the result of no taxation of capital only on a dynamic path where structures are proportional to consumption. Although this may qualitatively be plausible, we cannot offer a general result here. Alternatively, a model where the housing service is made of structures only allows for simpler solutions and one can obtain a similar result of absence of taxation of capital after period 2, but one also looses land as a convenient tax base.

4.2 More on the convergence of dynamic multipliers (Straub-Werning 2020)

To study this problem, we also abstract away from structures since they add two dynamic (Euler) equations to a system that is already complex to solve dynamically, and we still assume that $H_t = \mathcal{L}_t$ and $h_t = l_t$.

4.2.1 Definition and issues

The optimal taxation problem in the absence of the first best solutions is complex. It involves convergence problems emphasized in Straub and Werning (2020). We will however attempt to provide some partial results in line with those already discussed.

[14] Our analysis is valid for all CRRA functions in a neighborhood of the log case, but does not converge to the solution obtained in the log utility case: the implementability approach cannot be applied here as such and needs a specific treatment left outside this paper. See Lansing (1999) for the study of this case for the Judd economy without land.

We study a Ramsey problem where the planner has two distortive tax instruments available: a tax rate on capital, and a tax on housing rents. With the revenues, the government finances redistribution from capitalists to workers. We exclude from the outset a lump-sum tax on capitalists, because we want to rule out confiscation of capital. The social decision maker maximizes social welfare under the following constraints: the resource constraint of the economy for each period, the first order conditions of the capitalist (Euler, intraperiod allocation between consumption and housing, transversality) and the FOC of the worker. An important dimension here is the value of the intertemporal elasticity of substitution (IES) in consumption.

4.2.2 Restatement of Judd's result without housing

In the original Judd's model in the absence of housing, the social planner's program reads:

$$\underset{c_t, C_t, K_{t+1}}{Max} \sum_{t=0}^{\infty} \beta^t \left[u(c_t) + \gamma U(C_t) \right]$$

$$s.t. \quad f(K_t) + (1-\delta)K_t - c_t - C_t - K_{t+1} = 0 \quad \text{multiplier} \quad \lambda_t \beta^t$$

$$\beta U'_C(t+1)(C_{t+1} + K_{t+2}) - U'_C(t)K_{t+1} = 0 \quad \text{multiplier} \quad \mu_t \beta^t$$

with a transversality constraint $\beta^t U'_C(t)K_{t+1} \to 0$ and an initial stock of capital K_0 given. The multipliers of the resource constraints and Euler equation are the sequence $\lambda_t \beta^t$ and $\mu_t \beta^t$ respectively. As discussed in Straub and Werning (2020), in their framework this is necessary and sufficient to have $\sigma_C = -CU''_{CC}/U'_C$ below unity to insure that μ converges. We will show that a similar condition holds here, although the cutoff point is no longer 1 but a larger value.

Judd's result in the version given by Straub and Werning (2020) can be summarized in the next Lemma, proved in Online Appendix D.1[15]:

Lemma 2 *(Straub and Werning, 2020): Suppose quantities and multipliers converge to an interior steady-state, i-e., C_t, c_t and K_t converge to positive values and μ_t converges. Then the tax on capital is zero in the limit.*

[15]Note that we have treated each agent in the social planner program as representative. We make all proofs with a mass of capitalists that can be treated as a parameter m. This mass will simply be isomorphic to the social weight of capitalists and therefore ex post does not matter.

We provide next an intermediate result that complements Judd's statement as it appears in the above Lemma. We focus to a case already considered by Straub-Werning and assume that the capitalist's utility derived from consumption is iso-elastic, that is, there exist σ_C, the inverse of which being the intertemporal elasticity of substitution (IES) with $\sigma_C > 0$ and $\sigma_C \neq 1$ such that $U(C) = \frac{C^{1-\sigma_C}}{1-\sigma_C}$.

A useful notation captures the distance to the social planner's first best in terms of the ratio of marginal utilities: let the quantity $\tilde{\gamma}_c$ captures this, with $\tilde{\gamma}_c = \gamma \frac{U'_C}{u'_c}$. This ratio of marginal utility weighted by social welfare weights, 1 for the worker and γ for the capitalist, should be equal to 1 in the first best. In a second best analysis, the consumption of capitalists should be higher than the consumption of workers and then typically $\tilde{\gamma}_c < 1$, and the lower $\tilde{\gamma}_c$, the farther the second best is from the first best. Even if $\tilde{\gamma}_c$ is related to one of the primitives of the model, γ, it is clearly endogenous.

Lemma 3: *Suppose quantities converge to an interior steady-state. Then the multiplier μ_t converges to $\mu = \frac{\gamma(1-\tilde{\gamma}_c)}{\tilde{\gamma}_c(1-\sigma_C)}$ and is positive iff $(1-\tilde{\gamma}_c)(1-\sigma_C) > 0$. More specifically, if $\tilde{\gamma}_c < 1$ then the convergence of multipliers (to a positive number) occurs if and only if $\sigma_C < 1$.*

The convergence of the Euler condition multiplier arises when three conditions are met: $\sigma_C < 1$ (intertemporal elasticity of substitution larger than 1), the capitalist consumes more than the worker and the social welfare weight γ of the capitalist is lower than that of the worker. The condition $\tilde{\gamma}_c < 1$ is always satisfied at the second best for a redistributive social planner (with $\gamma < 1$). See Online Appendix D.1 for details.

We are now ready to characterize the second best optimum with housing but we will do it within the frame of the above Lemma. In particular, we will first restrict our attention to the case of $\sigma_C < 1$ before proving additional results in the extension to the case $\sigma_C > 1$.

4.2.3 Second best taxation with housing

We specify the preferences of both agents to be additively separable, with $U(C_t, H_t) = U(C_t) + V(H_t)$ and $u(c_t, \bar{H} - H_t) = u(c_t) + v(h_t)$. We still assume $U(C) = \frac{C^{1-\sigma_C}}{1-\sigma_C}$ and $u(c) = \frac{c^{1-\sigma_c}}{1-\sigma_c}$. At this stage, $V(.)$ and $v(.)$ remain general instead, we simply denote by $\sigma_H(H) = -HV''_{HH}/V'_H$ and $\sigma_h(h) = -hv''_{hh}/v'_h$ the (non-constant) coefficients of relative risk aversion of the utility for housing of respectively homeowners and tenants.

The separability assumption allows us to simplify the proofs of the next propositions.

Consider the planner program with housing:

$$\max_{c_t, C_t, H_t, K_{t+1} \tau_{H,t}} \sum_{t=0}^{\infty} \beta^t \{u(c_t) + v(h_t) + \gamma [U(C_t) + V(H_t)]\}$$

$$s.t. \qquad f(K_t) + (1-\delta)K_t - c_t - C_t - K_{t+1} = 0, \quad \text{multiplier} \quad \lambda_t \beta^t$$

$$\beta U_C'(t+1) \left[C_{t+1} + K_{t+2} - R_{t+1}^{Hgross}(1 - \tau_{H,t+1})h_{t+1} \right] - U_C'(t)K_{t+1} = 0, \quad \text{multiplier} \quad \mu_t \beta^t$$

$$R_t^{Hgross} u_c'(t) - v_h'(t) = 0 \quad \text{multiplier} \quad \eta_{1t}\beta^t$$

$$R_t^{Hgross}(1 - \tau_{H,t})U_C'(t) - V_H'(t) = 0 \quad \text{multiplier} \quad \eta_{2t}\beta^t$$

$$\tau_{H,t}\Phi_t = 0 \quad \text{multiplier} \quad \Phi_t \beta^t \geq 0$$

$$\beta^t U_C'(t) K_{t+1} \to 0$$

We introduce another indicator of distance to the social planner's first best, in the housing dimension this time: $\tilde{\gamma}_h = \gamma \frac{V_H'(H_t)}{v_h'(h_t)}$. As for consumption, we expect $\tilde{\gamma}_h$ to be lower than 1 at the second best optimum. The main result of this section is in the next Proposition.

Proposition 4 (Optimal capital tax in the Ramsey problem with housing): *Assume that the following instruments are available to the decision maker: a tax on capital, a lump sum benefit to workers and a tax on rents. Consider an economy where the preferences of both the capitalist and workers are separable, with a CCRA sub-additive utility of consumption. Suppose that quantities converges to an interior steady-state and that $\sigma_C < 1$, $\tilde{\gamma}_c < 1$ and $\tilde{\gamma}_h < 1$, then the optimal tax on capital is 0 and the optimal tax on rents is positive in the limit. Consequently the stock of capital in the second best remains equal to the stock of capital in the first best.*

The proposition amounts to proving that both the numerator and denominator of the multiplier of Euler's equation in the social planner's program are positive, as displayed in Online Appendix D, equation (D.22). A value $\sigma_C < 1$ is sufficient for this multiplier to be positive at the steady-state. We offer the following interpretation of the result. In a static setting, Diamond and Mirrlees (1971a,b) obtains a very robust result in optimal taxation: they show that it is better not to distort production and therefore not to tax intermediate goods. However, depending on the context, it may be second best optimal to tax consumption. According to Chari et al. (2016), these results cannot be directly applied to a dynamic setting because of variations in labor productivity in time, that depends on the capital stock. Nevertheless, the intuition conveyed by the Diamond-Mirlees papers still holds in our context, specifically in the steady-state. Housing is a consumption good and under some conditions it can be optimal to tax it, while it is not optimal to tax capital because it is an intermediate good. In addition, the fact that developed land is in fixed supply in our model leads to simpler proofs.

Introducing the notation ϵ_s for the supply elasticity of rental housing land with respect to net rent, that is formally defined in Appendix equation (D.25), another proposition delivers the optimal tax formula which is an inverse elasticity rule à la Ramsey:

Proposition 5 (Ramsey-optimal rent tax): *The optimal rent tax in the steady-state is given by*

$$\frac{\tau_H}{1-\tau_H} = \frac{1-\tilde{\gamma}_c}{\epsilon_s}$$

The supply elasticity with respect to net rent should be equal in absolute terms to the demand elasticity with respect to gross rent at the equilibrium of the rental market.

Note also that in the simplest case of the same constant relative risk aversion function for the sub-utility of housing as for consumption ($\sigma_C = \sigma_h = \sigma_H$), one obtains in addition that $\epsilon_s = \frac{1}{\sigma_H}$, and the formula is $\frac{\tau_H}{1-\tau_H} = \sigma_H(1-\tilde{\gamma}_c)$.

Assume $\sigma_h, \sigma_H > 0$. In this case, one does not require $\sigma_C > 1$ to obtain a convergent program: σ_C only need to be lower than a cutoff elasticity, defined from Online Appendix equation (D.22). In that equation, the numerator is positive and the denominator is positive if and only if:

$$\sigma_C < \sigma_C^* = \frac{1}{1 - \frac{hR^{Hnet}}{c}\left(\frac{1}{\sigma_h} + \frac{h}{H}\frac{\sigma_H}{\sigma_h}\right)} \geq 1 \qquad (31)$$

Proposition 6 (Range of convergence of Ramsey solutions with housing): *Even if $\sigma_C \geq 1$, as long as $\sigma_C < \sigma_C^*$ where $\sigma_C^* \geq 1$, the convergence of multipliers still holds, as in Lemma 3. Instead, consistent with Straub and Werning (2020), in the absence of housing consumption that is when the share of housing $\frac{hR^{Hnet}}{c}$ is close to zero, one finds $\sigma_C^* = 1$ in the limit case in which preferences are such that the housing consumption share of workers tends to zero relative to the consumption of the composite goods.*

Our second best optimal taxation results in Propositions 5 and 6 holds for a large set of parameters, in particular with an IES smaller than 1, within a range that can be large. The intuition for the extension in the range is the following: when $\sigma_C > 1$, the substitution effect of taxation is smaller than the income effect of taxation. In the absence of housing, capital taxation would normally increase savings, leading to the divergence of multipliers as soon as $\sigma_C < 1$. Adding up land and a new tax instrument related to land, we can partially relax this effect and the set of parameters leading to interior solutions of second best taxation is therefore larger. Our result can be brought closer to the interpretation of Chari et al. (2016) according to which the result of Straub and Werning (2020) of heavy taxation of capital when IES is lower than 1 is to some extent an illustration of an incomplete set of fiscal instruments. They show that with a moderate consumption taxation in the second period, we do not need to tax capital anymore.

5 Simulation exercise

5.1 Functional forms and parameters choices

We now turn our model into a quantitative exercise. We first simulate the steady-state, illustrate the role of structures, and then simulate the dynamics in a simpler case.

We assume a Cobb-Douglas function for utility and the production of housing services in line with recent empirical evidence provided in Combes et al. (2017); Epple et al. (2010); Sommer and Sullivan (2018), and for the production function of the composite good. Welfare is defined as the sum of the utility of the workers and of the capitalists. We assume that there is a mass m of capitalists and keep as a normalisation a mass 1 of workers, so that the social welfare function is $u(c, h) + \gamma m u(C, H)$[16].

Table 1 summarizes all functional forms and their parameters, as well as their sources. We take the parameters from the literature. The benchmark value for risk aversion σ is set to 2.5 as in Sommer and Sullivan (2018). We follow Straub and Werning (2020) to set the share of capital in production, $\alpha_K = 1/3$ and the depreciation of capital, $\delta = 0.1$. The depreciation rate for housing is taken from Sommer and Sullivan (2018) and is set to 0.015, and is consistent with most recent estimates in national accounts. The share of land in the production of housing services $\alpha_{\mathcal{L}}$ is set to 0.35 following Combes et al. (2017) with robustness checks discussed below. The total quantity of land $\bar{\mathcal{L}}$ is normalized to 1. The weight m is taken equal to 1/10: the top 10% owns most of non-housing assets (70% in France, 80% in

[16] All variables, C, s, S and $\mathcal{L}$ correspond to the consumption or investment of *one* capitalist. To get total consumption or investment, one must multiply the quantity by m. K corresponds instead to the total quantity of capital. c and h corresponds to the consumption of one worker, which is equal to the total consumption of workers given the normalization.

the US as reported in Appendix Table E.1). Denoting by $\Omega = mC + c + R^{Hgross}(mH + h)$ the total income of the economy, which is GDP net of investments, we match the stock of capital and housing wealth as a fraction of Ω, to be consistent with those reported for France or the UK in Figure 1. This leads to a ratio α (the share of housing in consumption) equal to 0.24[17]. Finally, the discount rate is set to $\beta = 0.95$ so as to match capital/income ratios. This value appears reasonable as it is the one used in Straub and Werning (2020).

The steady-state values are reported in Table 2 with positive taxation to match the targeted statistics: the tax on rents is 30%, we add a tax on imputed rents of 10% and a tax on returns to capital $\hat{\tau}_K$ of 30%[18]. To preserve the efficient level of capital, we add the subsidy on capital investments discussed in Appendix equation (A.25). We then report the equilibrium in the absence of any tax as a benchmark for the comparative static exercise that follows. In that last column, consumption inequality between capitalists and workers is quite large: the consumption of goods and housing of a capitalist is almost 5 times larger. Capitalists also occupy $32\% = m \times 3.2$ of the land while only representing 9.1% of the population.

[17]There is a wide range of estimates in the literature for the share of housing in consumption. Combes et al. (2019) find a share of housing (including taxes) of 0.3 from micro data. On the other hand, national accounts gathered in Piketty and Zucman (2013) report a ratio between net rents (including imputed rents) and national income of 0.07. OECD (2021) reports that rents and imputed rents represent about 18% of household's disposable income. Our implied value of 0.24 is in the middle of the range, and the exact value matters little for the qualitative results of this section from various unreported simulations.

[18]This corresponds to a tax τ_K as defined in the text that is determined by $\tau_K[1+f'(K)-\delta] = \hat{\tau}_K[f'(K)-\delta]$. At the efficient level of capital this implies $\tau_K = \hat{\tau}_K(1-\beta) = 1.5\%$ and an investment subisdy of $-\tau_I = .1$. The latter is a bit large compared to national accounts data where this is typically closer to 5% but this sets a benchmark of efficient capital to test rent and land taxes against.

Name	Symbol	Value	Source
Panel a) Households			
Utility	$u(c,h)$, $U(C,H)$	$\frac{(c^{\alpha}h^{1-\alpha})^{1-\sigma}}{1-\sigma}$, $\frac{(C^{\alpha}H^{1-\alpha})^{1-\sigma}}{1-\sigma}$	Sommer and Sullivan (2018)
Risk aversion	σ	2.5	Sommer and Sullivan (2018)
Share housing / income	$1-\alpha$	$0.24([0.3; 0.075])$	Combes et al. (2019) Piketty and Saez (2013)
Discount aactor	β	$0.95([0.95; 0.985])$	Straub and Werning (2020) Sommer and Sullivan (2018)
Panel b) Production			
Housing production	$h(l,s), H(\mathcal{L},S)$	$l^{a_{\mathcal{L}}}s^{1-a_{\mathcal{L}}}, \mathcal{L}^{a_{\mathcal{L}}}S^{1-a_{\mathcal{L}}}$	Combes et al. (2017)
Land share	$a_{\mathcal{L}}$	$0.35([0.35; 0.5])$	Combes et al. (2017)
Structure depreciation	δ_s,δ_S	0.015	Sommer and Sullivan (2018)
Production	$f(K)$	K^{a_K}	Straub and Werning (2020)
Capital share	α_K	1/3	Straub and Werning (2020)
Capital depreciation	δ	0.1	Straub and Werning (2020)
Panel c) Welfare			
Social welfare weights	γ	$[0,1]$	–
Demographic weights	m	0.1	see Table E.1

Table 1: Summary of the calibration parameters

Name	Symbol	Value (From calibration)	Value (Benchmark for comp. stat.)
Taxes			
Tax on rents	τ_H	0.3	0
Tax on imputed rents	τ_{HI}	0.1	0
Tax on returns to capital	$\hat{\tau}_K$	0.30	0
Subsidy on investments	$-\tau_I$	0.10	0
Consumption of land and goods			
Capitalists consumption (per capitalist)	C	2.87	3.56
Workers consumption	c	0.83	0.75
Housing of capitalist (per capitalist)	H	5.5	7.08
Housing of workers (per worker)	h	1.28	1.49
Land consumed per capitalist	$\mathcal{L}$	3.0	3.22
Land consumed per worker	l	0.70	0.68
Production and stocks			
Total capital	K	3.2	3.2
Total rental structures	ms	1.8	2.3
Total owned structures	mS	0.76	1.1
Wealth components			
Capital to nat. income	K/Ω	2.2	2.2
Price of housing P	$R^{Hnet}/(1-\beta)$	2.88	3.18
Housing wealth / nat. income	$P(mH+h)/\Omega$	3.52	4.80
Income components			
Net rents / nat. income	$\left[R^{Hnet}(mH+h) - m\delta_s(s+S)\right]/\Omega$	0.15	0.21
Wages / nat. income	w/Ω	0.66	0.68
Welfare			
Welfare (Rawlsian)	$u(c,h)$	-0.75	-0.80
Welfare (Utilitarian)	$u(c,h)+mU(C,H)$	-0.77	-0.81

Table 2: Steady-state equilibrium values in the calibrated economy with taxes, and steady-state equilibrium values in a benchmark without taxes

5.2 Comparison of different tax schemes

A first exercise, reported in Figure 2 is to compare the first best taxation on land from Proposition 1 to alternative tax profiles explored in Proposition 2 as well as suboptimal taxation schemes. We systematically compare results for two extreme values of γ (Rawlsian case 0 and utilitarist case 1).

The horizontal axis represents the redistribution rate as fraction of total income Ω of each tax scheme. As clear from the Figure, the social planner wishes positive taxes to insure redistribution towards workers, since welfare increases with redistribution for non-distortive taxes (in green). The first best is obtained from a pure land tax (scheme i, in the legend). Another first best tax scheme is to tax rents, compensated by a subsidy on structures of tenants and a differential tax on land. This is represented by the curve with stars (ii, in the legend). Here, the tax base is therefore smaller, requiring higher taxes. The first best scheme with a taxation on both rents and imputed rents compensated by a subsidy on both types of structures also requires higher tax rates (iii, squares). These three first best taxes are superposed although they vary by tax rate[19]. The curve of the land tax increases faster than all other curves, but with this calibration, stops exactly when it exhausts the total revenue of the capitalist[20].

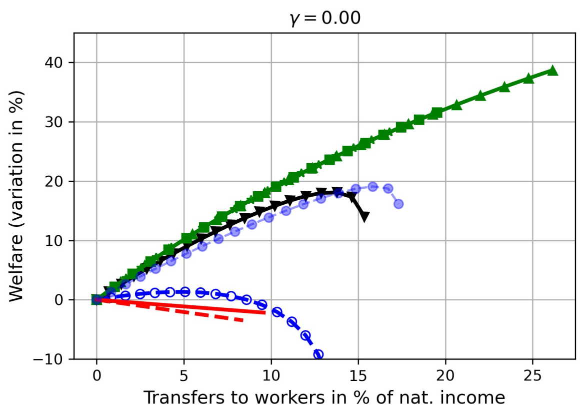
The most effective scheme, away from the first best, is to combine a tax on rents and a subsidy to rental housing structures (black inverted triangles, iv in the legend): it does well initially but the remaining distortion on the intratemporal allocation of land accumulates as taxes grow. In contrast, the other tax schemes are ineffective or even detrimental: the rent tax alone only improves welfare by a few percent at most and the others tax policies reduce welfare from the first dollar levied. It is interesting to note that the relative inefficiency of the tax on rents arises from the response of structures. This can be seen by comparing the performance of the rent tax when structures are endogenous (bright blue curve, scheme v) and when we shut down the variation of the structures (transparent dotted blue line with plain circles, scheme viii) in treating them as parameters at the steady-state value in the absence of taxation. This counterfactual exercise removes the negative welfare impact of taxes on

[19] In Appendix Figure E.2 we also express taxes in terms of their respective tax rate. Not surprisingly, the first best tax on land dominates first best taxation schemes with no tax on land because a low tax rate is enough, the tax base being large and inelastic.

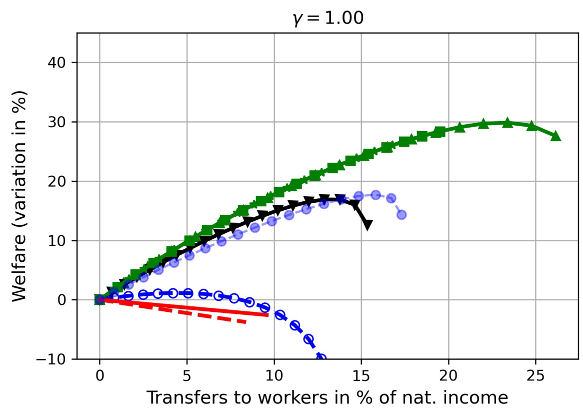
[20] Remember that the optimal land tax in Proposition 1, equation (20) with $\gamma = 0$ implied that optimal tax is also the one leading to starvation of the capitalist, while with positive γ, the optimal tax is below the starvation point, hence the decreasing part after that optimal tax for $\gamma = 1$.

structures[21].

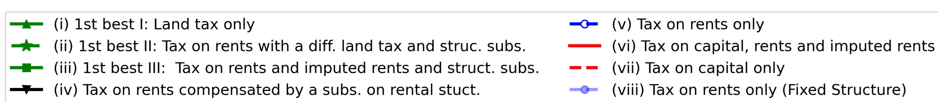
Figure 2: Variation in the social welfare function $u(c_t)+\gamma mU(C_t)$ with various tax schemes, as a proportion of the transfers to workers. Endogenous structures (except viii) and benchmark parameters.



(a) Welfare and transfers, $\gamma = 0$



(b) Welfare and transfers, $\gamma = 1$



Figure's note. Endogenous structures (except viii) and benchmark parameters. Variation in the social welfare function $u(c)+\gamma mU(C)$ vs tax revenue in the decentralized equilibrium, for different values of γ, the social welfare weight (respectively 0 and 1) with a mass of capitalist of 0.1.
Comparison between the first best policies:
(i) an homogeneous tax on land $\hat{\tau}_{\mathcal{L}}$ redistributed to workers [plain green line with triangles up] which stops when it exhausts the revenue of the capitalist,
(ii) a positive tax on rents $\tau_H > 0$, a differentiated tax on land ($\Delta\tau_{\mathcal{L}} = \tau_H R^{Hgross} h'_l$) and a subsidy to housing structures of tenants ($\tau_s = -\tau_H$) [discontinued green line with stars]
(iii) a tax on rent ($\tau_H > 0$), imputed rent ($\tau_{HI} = \tau_H \frac{h'_l}{H'_{\mathcal{L}}}$) combined with subsidies on structures ($\tau_s = \tau_S = -\tau_H$) [plain green line with squares],
and second best policies and other distortive policies:
(iv) a tax on rents compensated by a subsidy on residential investments $\tau_s < 0$ [plain black line with triangles down],
(v) a tax on rents $\tau_H > 0$ alone [discontinued blue line, empty circles];
(vi) a tax on capital equalized to the tax on rents and imputed rents $\tau_K = \tau_{HI} = \tau_H$ [plain red line];
(vii) a pure tax on capital ($\tau_K > 0$) [dashed red line].
(viii) The last exercise simulate the impact of a tax on rents only $\tau_H > 0$, when structures are exogenous [blue and dotted line, plain circles]
X-axis expresses taxes as the respective total tax revenue as a function of national income Ω.

Last, the detrimental effect of capital taxation can be seen from curves vii and viii, either

[21] All tax schemes with fixed structure are explored in Appendix Figure E.4.

without or with a rent tax[22].

5.3 Dynamics with fixed structures

The entire set of first order conditions out of the steady-state are in Appendix A.3.1. In this example, structures are assumed to be fixed to the steady-state value calculated previously: $s = s^\star$ and $S = S^\star$. Using *Dynare* (Adjemian et al., 2011), we then simulate the impact of a shock reducing the capital stock by 20% at time t=1 in five difference scenarii (with no tax, with a first best tax on land, $\tau_{\mathcal{L}}$, with a tax on rents, τ_H, with a tax on capital, τ_K, or with a tax on rents, imputed rents and capital). This is represented in Figure 3.

For comparability, the tax rates are set in each scenarios such that their respective tax product represents 10% of national income Ω in the steady-state. Starting from different steady-state levels, the dynamics of convergence to the final steady-state are rather similar. Interestingly, while the tax on capital reduces the steady-state value of society's welfare, it is not affecting the speed of convergence.

6 Literature review

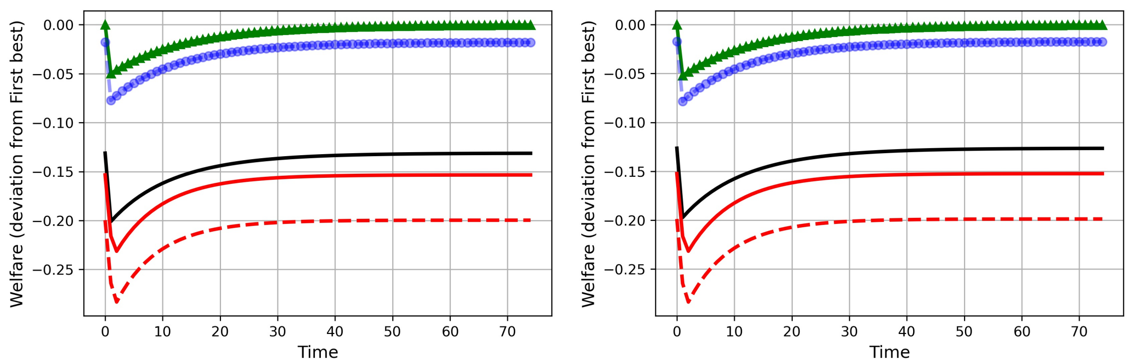
Our paper touches different topics that we all discuss in this Section. First, it is a paper on wealth and capital accumulation. The rise of the capital/income ratio and its implications on inequalities derived in Piketty (2014) have generated many contributions that responded to various challenges of the main thesis[23]. A particular line of discussion was linked to the role of housing[24] and jointly, the recognition that earnings of capital relative to 10% of national income had not evolved as much as the wealth-to-income ratio. We documented in detail in

[22] Note that if capital must be taxed as a political constraint, it is then better to also tax rents and imputed rents, to reduce the distortion in the returns to investments across types of capital (tenant structures and capital) and in land allocation. See for instance Skinner (1996) on this point. As additional remarks, we report in the top panel of Figure E.2 the tax rate in horizontal axis instead of the share of national income. We also show that a greater share of land raises the land use distortion associated to the combined rent tax and structures subsidies. We explore this in Figure E.3 where one can compare the gap with first best in that Figure where a 50% land share is chosen, as opposed to Figure 2 where it is 30%.

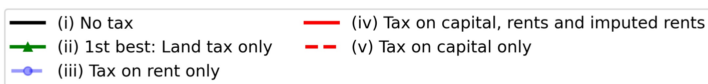
[23] Krusell and Smith Jr (2015), Rognlie (2015), Acemoglu and Robinson (2015), Stiglitz (2015a,b), Mankiw and Summers (2015), Weil (2015), Auerbach and Hassett (2015), Jones (2015), Kopczuk (2015), Hildenbrand (2016), among others.

[24] To our knowledge, our working paper Bonnet et al. (2014) was the first paper having centered on the role of housing in rising wealth and its implications for the validity of a theory of explosive accumulation of wealth, followed by Rognlie (2014). Another early article discussing housing is Husson (2014). It is also important to note that Piketty and Zucman (2014) not only acknowledged the role of housing, but also decompose the effects of prices versus real quantities in the development of K/Y.

Figure 3: Variation in the social welfare function $u(c_t) + \gamma m U(C_t)$ for different tax schemes. Exogenous structures and benchmark parameters. Dynamics of the economy following a 20% depreciation shock on physical capital.



(a) Welfare and transfers, $\gamma = 0$ (b) Welfare and transfers, $\gamma = 1$



Figure's notes. Exogenous structures and benchmark parameters. Variation in the social welfare function $u(c) + \gamma m U(C)$ after a shock at time 1 in the decentralized equilibrium when structures are fixed, Comparison between
(i) the situation with no Tax,
(ii) the first best policy, a homogeneous tax on land $\hat{\tau_{\mathcal{L}}}$ (plain green line with triangles up)
(iii) a tax on rents τ_H alone (discontinued blue line, plain circles);
(iv) a tax on capital, rents and imputed rents
(v) A tax on capital only.

our previous work (Bonnet et al., 2014) for France with the apparent inconsistency of the data with the "first law of capitalism": the positive co-movement between the capital share and the capital-to-income ratio is not visible in the data. This suggested that an important ingredient was either missing or at least deserved more emphasis. This missing piece that we discussed in that paper is the heterogeneity of capital, that naturally appears when one explicitly distinguishes wealth from capital.

Second, our paper is about the heterogeneity of capital and in particular the role of land as a fixed factor generating rents. The question of optimal land and housing taxation used to be central. A seminal contribution can be viewed in Henry George (1879)'s book, *"Progress and Poverty"*, advocating for a single tax on land. This contribution was adapted to the urban economics framework with the famous Henry-George Theorem developed in Arnott and Stiglitz (1979) which states that, at an optimal city size, a land rent tax is the only tax needed to finance local public goods. In Arnott and Stiglitz (1979), land is the area between the city center and the boundary of the city, that is, land is occupied by the population of the city[25].

These analyses were recently reformulated in a dynamic and macroeconomic setting by Mattauch et al. (2013). The authors derive an optimal public investment formula in terms of the land rent. An important assumption in their work, and a difference with the urban literature, is that residential land does not enter the utility function. In the past, many prominent economists shared the view that land should be taxed. Vickrey (1996) wrote "*removing almost all business taxes, including property taxes on improvements, excepting only taxes reflecting the marginal social cost of public services rendered to specific activities, and replacing them with taxes on site values, would substantially improve the economic efficiency of the jurisdiction.*"; a quote from Milton Friedman supports the land tax: "*In my opinion, the least bad tax is the property tax on the unimproved value of land, the Henry George argument of many, many years ago.*"; while a manifesto of economists (William Vickrey, Jacques Thisse, Tibor Scitovsky, James Tobin, Richard Musgrave, Franco Modigliani, Zvi Griliches, William Baumol, Robert Solow among others) wrote a Letter to Gorbatchev in 1990 *"It is important that the rent of land be retained as a source of government revenue. While the governments of developed nations with market economies collect some of the rent of land in taxes, they do not collect nearly as much as they could, and they therefore make unnecessarily great use of taxes that impede their economies – taxes on such things as incomes, sales and the value of capital."*. The specific role of land in the optimal taxation literature is discussed in Stiglitz (2015a): when land is only a productive input, its taxation would increase the consumption of workers. In his setting there is only one consumption good and land does not provide a housing service. The model leaves aside residential land to focus on land consumed by firms. Eerola and Määttänen (2013) on the other hand, address specifically the question of housing taxation. They develop a model with a representative agent that derives utility from non-housing consumption, leisure and housing which is only composed by its structure and has no land component[26].

[25]Our model will follow their interpretation of land in the absence of structures - that is undeveloped but inhabited, and we add the case of endogenous structures. Our calculations of the optimal tax can be thought of as an extension of Henry George and Arnott and Stiglitz (1979) to the case of structures but without arguably spatial gradients nor public goods.

[26]This key assumption leads to the conclusion that "in the first-best, the tax treatment of business and housing capital should always be the same", and that in "the second-best, in contrast, the optimal tax treatment of housing capital depends on the elasticities of substitution between non housing consumption, housing, and leisure". The model is also silent on the role of residential land. Our optimal taxation of land and rents in the second best rejoins these recent approaches but accounts for the fact that land is consumed by households through housing services and thus enters the utility function.

Third, this paper is part of a very dynamic literature on capital taxation for redistributive reasons[27]. In the early work of Diamond and Mirrlees (1971b), capital as an intermediate good should not be taxed under linear tax schedules, a perspective recently reinvested in Chari et al. (2020) in a dynamic setting. Atkinson and Stiglitz (1976) favor non-linear labor taxation instead. In the Ramsey framework, Chamley (1986) argues that future capital investments should not be taxed in the long-run, within the range of convergent paths, as well as Judd (1985) in a second-best convergent path with workers without assets. Under the assumption of uninsurable shocks, Aiyagari (1995) corrects the overaccumulation of capital for precautionary motives with a capital tax instead. With asymmetries of information on individual's characteristics, Golosov et al. (2003) establish a similar result under non-linear tax schemes. A subsequently central mechanism to the literature pioneered by this article is that wealth raises leisure and reduces work incentives.

Judd's paper was subsequently revisited by several authors. Lansing (1999)'s finding is that under log preferences, optimal capital taxes could be positive forever at some interior steady-state. Reinhorn (2019) shows that it is the only way in which an interior steady-state can violate the zero tax result in Judd's framework. Atkeson et al. (1999) calculate optimal taxes out of the steady-state using an implementability constraint that we also use, following their lead. Instead of having additional taxes on consumption and prices of goods, we focus on an alternative set of taxes on land and housing. We also show that their result of no capital taxation on the dynamic path from the third period originally in the Chamley setting also holds in the Judd economy with and without land. Note that this is not a particular case of their heterogeneous agents model, given that workers in Judd do not optimize intertemporally.

Again, out of the steady-state, Straub and Werning (2020) discuss non-convergent paths in Judd's analysis. In particular, with low elasticities of intertemporal substitution, savings is too important and taxing capital must lead to long-term expropriation, while with higher elasticities, taxes tend to zero, but the convergence is very slow. In a more general setting Bassetto and Benhabib (2006) and Benhabib and Szőke (2019) establish that with more heterogeneity in wealth, a positive capital tax rate can prevail in the steady-state, without considering knife-edge preferences. The last authors consider CRRA preferences and constant elasticity of substitution (CES) production functions and an uneven distribution of wealth. They show that in the framework of neo-classical growth model, agents in the bottom or middle part of the distribution may prefer the redistributive effects associated to a tax on capital forever even if it entails large inefficiency costs. It would be interesting to see whether

[27]See for instance Ndiaye (2017), and for a most recent survey of specific dynamic issues, Stantcheva (2020)

our results convey to this more general setting. In this line, the closest paper to ours is Borri and Reichlin (2020) who share our aim to study optimal wealth taxation when wealth derives from business capital and homeownership in a world where property rights are evenly distributed. They also concentrate on steady-states motivated by the need to focus on long-term phenomena. They find that Chamley-Judd's zero steady-state tax on business capital survives, whereas housing wealth is taxed at a non zero rate[28].

7 Conclusion

Once upon a time, land was a central piece of the classical analysis. But it has been moved at the periphery of modern theory of prices. Several recent evolutions however point towards the need to reinstate land: housing represents between 20 and 30% of consumer expenditures and at least 40% of household wealth. In most places, prices of developed land surge, in particular in metropolitan areas. Reintroducing land and land price leads to the natural conclusion that land tax must be positive and large. This idea does not need to be confined to urban economics where it already plays a key role, as beautifully advocated for the funding of local public goods with the so-called Henry-George theorem. The importance of land taxes goes beyond the microeconomics of cities and naturally embraces the issue of redistribution at the macro level with implications on long-term growth.

We document that capital is heterogeneous and the recent inflation of housing wealth matters more than before: since a large part of housing reflects the underlying fixed factor (land). Our main set of conclusions is normative. In terms of optimal taxation, it is crucial to distinguish between produced capital and housing (physical capital and structures) and land. Indeed, taxing land (or property) enables to make transfers from landowners to workers/tenants and to increase the income of the latter. However, taxing housing structures through a tax on rents has distortive effects. These distortions may be indeed important, but can be alleviated with subsidies to rental housing investments. Further, these results hold not only in the steady-state but also in any transition path after an initial period where initial capital stock holds.

The analysis led us to conclude that long-run trends in wealth may eventually become good news for policy makers if they are able to implement non-distortive redistribution. Our

[28]The main difference is that they consider land in a two-sector model where the housing sector needs new land in addition to labor and capital, whereas the other sector does not require land as input. They assume new land is public property, and the government can fully tax the land sales: with taxation of labor and differentiated wealth tax they plead for progressive taxation of housing.

analysis indicates that differentiating wealth taxes by type of wealth allows for a lighter taxation on physical capital than on housing structures of homeowners, themselves lighter than developed land. If Judd's model used as a benchmark in this paper appears as a natural framework to reintroduce land and housing in dynamic public finance, it is not the only one. On the one hand, it is an extreme view of the world since all sources of wealth belong to the capitalist hands. On the other hand, we acknowledge that the analysis must be pursued to understand better the concentration of capital in some hands and not others, and how intergenerational forces affect the dynamics of wealth accumulation through capital and land. A related issue is to fully integrate capitalists' heterogeneity according to the various types of wealth in the analysis, something left for subsequent papers.

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Land is back, it should be taxed, it can be taxed
ONLINE APPENDIX

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A Online Appendix: Theory Appendix: Social planner's allocation and decentralized equilibrium with structures

A.1 Notations

In this Appendix, we report in the most general case the first order conditions of the social planner and of the decentralized equilibrium as in the text. Given the Inada conditions assumed in the text, all partial derivatives are strictly between 0 and infinity.

When we derive dynamic equation out of the steady-state, utility functions and marginal utility do not report all inputs, but the time index is reported in parenthesis when relevant, e.g. $U'_C(t+1)$ is the marginal utility of consumption at time $t+1$ for inputs C_{t+1} and H_{t+1}, etc.

A.2 Social planner's first order conditions

We have:

$$\partial C_t, c_t \quad \lambda_t = \gamma U'_C(t) = u'_c(t) \tag{A.1}$$

$$\partial \mathcal{L}_t \quad \gamma U'_H(t) H'_{\mathcal{L}}(t) = u'_h h'_l(t) \tag{A.2}$$

$$\partial S_{t+1} \quad \lambda_t = \beta\lambda_{t+1}(1-\delta_S) + \gamma\beta U'_H(t+1) H'_S(t+1) \tag{A.3}$$

$$\partial s_{t+1} \quad \lambda_t = \beta\lambda_{t+1}(1-\delta_S) + \beta u'_h(t+1) h'_s(t+1) \tag{A.4}$$

$$\partial K_{t+1} \quad \lambda_t = \beta\lambda_{t+1} R^{Kgross}_{t+1} \tag{A.5}$$

This notably implies the following steady-state relations:

$$\partial C_t, c_t \quad \lambda = \gamma U'_C = u'_c \tag{A.6}$$

$$\partial \mathcal{L}_t \quad \gamma U'_H H'_{\mathcal{L}} = u'_h h'_l \tag{A.7}$$

$$Euler \quad S_{t+1} \quad \beta^{-1} = \frac{U'_H H'_S}{U'_C} + 1 - \delta_S \tag{A.8}$$

$$Euler \quad s_{t+1} \quad \beta^{-1} = \frac{u'_h h'_S}{u'_c} + 1 - \delta_S \tag{A.9}$$

$$Euler \quad K_{t+1} \quad \beta^{-1} = R^{Kgross} = f'(K) + 1 - \delta \tag{A.10}$$

A.3 Decentralized equilibrium

We start from the most general tax structure, to save on calculations later on. We therefore consider all settings considered in the main text as particular cases. As compared to the text, we also add a tax on investments in physical capital τ_I treated symmetrically with the taxes/subsidies on structures to show its substitutability with the tax on returns.

We consider $\tau_{S,t}$ and $\tau_{s,t}$ as (net) taxes (a negative value is a subsidy) per unit invested, and the product of the tax is equal to:

$$T_t^{s,S} = \tau_{S,t} \left[S_{t+1} - S_t(1-\delta_S)\right] + \tau_{s,t} \left[s_{t+1} - s_t(1-\delta_s)\right] \tag{A.11}$$

By symmetry, we similarly introduce a tax on net investment,

$$T_t^I = \tau_{I,t} \left[K_{t+1} - K_t(1-\delta)\right]$$

and net returns on capital are taxed too:

$$T_t^K = \tau_{K,t} \left[f'(K_t) + 1 - \delta\right]$$

and use the notation for the net return on capital as:

$$R_t^{Knet} = R_t^{Kgross}(1 - \tau_{K,t})$$

In the steady-state, the tax basis is therefore positive and respectively $\delta_S S$ and $\delta_s s$, and taxes on land are potentially differentiated, with $\tau_{\mathcal{L}}$ the tax on the land occupied by landlords, and τ_l the tax on land developed for tenants, and therefore a tax revenue on land equal to:

$$T_t^{\mathcal{L}} = \tau_{\mathcal{L}}\mathcal{L} + \tau_l(\bar{\mathcal{L}} - \mathcal{L})$$

opening the door for a correction in the allocation of land by landlords. When the tax rates are identical, we denote them (in the text or in this Appendix) as $\tau_{\bar{\mathcal{L}}} = \tau_{\mathcal{L}} = \tau_l$.

Rents are also taxed at a rate $\tau_{H,t}$. A tax on imputed rents for the landlord may be possible. It is introduced as $\tau_{HI,t}$. Both lead to a revenue:

$$T_t^H = \tau_{H,t} R_t^{Hgross} h_t + \tau_{HI,t} R_t^{Hgross} H_t$$

and we use the notation for the net return on housing investments as:

$$R_t^{Hnet} = R_t^{Hgross}(1 - \tau_{H,t})$$

Finally, we allow for a living tax, paid by all residents (homeowners and tenants). The tax revenue is calculated at the rent or implicit rent (gross), and the tax rate is denoted by $\tau_{liv,t}$ and leads to revenue:

$$T_t^{liv} = \tau_{liv,t} R_t^{Hgross}(H_t + h_t)$$

Overall, all taxes sum up to deliver the total amount taxed T_t:

$$T_t = T_t^{liv} + T_t^H + T_t^K + T_t^I + T_t^{s,S}$$

while the total net transfer is instead:

$$Transfert_t = \tau_{liv,t} R_t^{Hgross}(H_t) + T_t^H + T_t^K + T_t^I + T_t^{s,S}$$

It is interesting to note that the living tax for the landlord is calculated in the same way as for the imputed rent. In what follows, we keep track of these two perfect substitute taxes and the sum $\tau_{liv} + \tau_{HI}$ appear in all first order condition of the capitalist, as the sum of the living tax and the imputed rent tax for the landlord.

Then, the program of the capitalist is, using $F(K_t) - w_t = K_t f'_(K_t)$,

$$\max_{C_t, H_t, \mathcal{L}_t, K_{t+1}, s_{t+1}, S_{t+1}} \sum_t \beta^t \{U(C_t, H(\mathcal{L}_t, S_t))\}$$

$$+\sum_t \beta^t \lambda_t^K \Big\{ R_t^{Knet} K_t + \tau_{I,t}(1-\delta) K_t + R_t^{Hgross}(1-\tau_{H,t}) h_t(\bar{\mathcal{L}} - \mathcal{L}_t, s_t))$$

$$-(\tau_{liv,t} + \tau_{HI,t}) R_t^{Hgross} H_t$$

$$+(1-\delta_S)(1+\tau_{S,t}) S_t + (1-\delta_s)(1+\tau_{s,t}) s_t - T_t^{\mathcal{L}}$$

$$-C_t - S_{t+1}(1+\tau_{S,t}) - s_{t+1}(1+\tau_{s,t}) - K_{t+1}(1+\tau_{I,t})\}$$

subject to subject to $\beta^t U'_C(t) M_{t+1} \to 0$, for $M_t = K_t, S_t, s_t$.

The program of the worker is:

$$\max_{c_t, h_t} u(c_t, h_t)$$

$$\text{subject to } c_t + h_t R_t^{Hgross}(1+\tau_{liv,t}) = w_t + T_t$$

A.3.1 First order conditions

We have, denoting by λ_t^w the multiplier of the constraint of the worker, and λ_t^K the multiplier of the constraint of the capitalist, and $\Delta\tau_{\mathcal{L},t} = \tau_{\mathcal{L},t} - \tau_{l,t}$:

$$
\begin{aligned}
&\text{Worker:} \quad \text{intraperiod} \\
&\partial c_t \qquad u'_c = \lambda^w_t && \text{(A.12)} \\
&\partial h_t \quad u'_h(t) = \lambda^w_t R^{Hgross}_t(t)(1+\tau_{liv,t}) && \text{(A.13)} \\
&\Leftrightarrow u'_h(t) = u'_c R^{Hgross}_t(1+\tau_{liv,t}) && \text{(A.14)} \\
&\text{Capitalist:} \quad \text{intraperiod} \\
&\partial C_t \qquad U'_C(t) = \lambda^K_t && \text{(A.15)} \\
&\partial \mathcal{L}_t \quad U'_H(t)H'_{\mathcal{L}}(t) = \lambda^K_t\left(R^{Kgross}_t(1-\tau_{H,t})h'_l(t) + \Delta\tau_{\mathcal{L}} + (\tau_{liv,t}+\tau_{HI,t})R^{Hgross}_t H'_{\mathcal{L}}(t)\right) && \text{(A.16)} \\
&\text{Capitalist:} \quad \text{intertemporal (Euler)} \\
&\partial S_{t+1} \quad \lambda^K_t(1+\tau_{S,t}) = \beta\lambda^K_{t+1}(1-\delta_S)(1+\tau_{S,t+1}) + \beta U'_H(t+1)H'_S(t+1) \\
&\qquad - \lambda^K_{t+1}\beta(\tau_{liv,t+1}+\tau_{HI,t+1})R^{Hgross}_{t+1}H'_S(t+1) && \text{(A.17)} \\
&\partial s_{t+1} \quad \lambda^K_t(1+\tau_{s,t}) = \beta\lambda^K_{t+1}(1-\delta_s)(1+\tau_{s,t+1}) + \beta\lambda^K_{t+1}R^{Hgross}_{t+1}(1-\tau_{H,t+1})h'_s(t+1) && \text{(A.18)} \\
&\partial K_{t+1} \quad \lambda^K_t(1+\tau_{I,t}) = \beta\lambda^K_{t+1}\left[R^{Knet}_{t+1}+(1-\delta)\tau_{I,t+1}\right] && \text{(A.19)}
\end{aligned}
$$

or in the steady-state,

$$
\begin{aligned}
&\text{Worker:} \quad \text{intraperiod} \\
&u'_h = u'_c R^{Hgross}(1+\tau_{liv}) && \text{(A.20)} \\
&\text{Capitalist:} \quad \text{intraperiod} \\
&\partial C_t \qquad U'_C = \lambda^K && \text{(A.21)} \\
&\partial \mathcal{L}_t \quad U'_H H'_{\mathcal{L}} = U'_C\left(R^{Hgross}(1-\tau_H)h'_l + \Delta\tau_{\mathcal{L}} + (\tau_{liv}+\tau_{HI})R^{Hgross}H'_{\mathcal{L}}\right) && \text{(A.22)} \\
&\text{Capitalist:} \quad \text{intertemporal (Euler)} \\
&\partial S \quad U'_C(1+\tau_S) = \beta U'_C(1-\delta_S)(1+\tau_S) + \beta U'_H H'_S \\
&\qquad - U'_C\beta(\tau_{liv}+\tau_{HI})R^{Hgross}H'_S && \text{(A.23)} \\
&\partial s \quad U'_C(1+\tau_s) = \beta U'_C(1-\delta_s)(1+\tau_s) + \beta U'_C R^{Hgross}(1-\tau_H)h'_s && \text{(A.24)} \\
&\partial K \quad U'_C(1+\tau_I) = \beta U'_C\left[R^{Knet}+(1-\delta)\tau_I\right] && \text{(A.25)}
\end{aligned}
$$

Re-arranging the terms, using in particular $R^{Hgross} = \frac{u'_h}{u'_c}(1+\tau_{liv})^{-1}$, one obtains:

$$\text{Intraperiod allocations}$$

$$\frac{u'_h h'_l}{U'_H H'_{\mathcal{L}}}\left(\frac{1-\tau_H}{1+\tau_{liv}} + \frac{\tau_{liv}+\tau_{HI}}{1+\tau_{liv}}\frac{H'_{\mathcal{L}}}{h'_l} + \frac{\Delta\tau_{\mathcal{L}}}{(1+\tau_{liv})R^{Hgross}h'_l}\right) = \frac{u'_c}{U'_C} \tag{A.26}$$

$$\text{Intertemporal allocations (Euler)}$$

$$\partial S \quad \beta^{-1} = 1-\delta_S + \frac{1}{1+\tau_S}\frac{U'_H H'_S}{U'_C} - \frac{\tau_{liv}+\tau_{HI}}{1+\tau_S}R^{Hgross}H'_S \tag{A.27}$$

$$\partial s \quad \beta^{-1} = 1-\delta_s + \frac{1-\tau_H}{1+\tau_s}\frac{1}{1+\tau_{liv}}\frac{u'_h h'_s}{u'_c} \tag{A.28}$$

$$\partial K \quad \beta^{-1} = \frac{R^{Kgross}(1-\tau_K)+(1-\delta)\tau_I}{1+\tau_I} \tag{A.29}$$

The system can be compared to the social planner's allocation, characterized by the following equation in the steady-state. We have:

$$\text{Intraperiod allocations}$$

$$\frac{u'_h h'_l}{U'_H H'_{\mathcal{L}}} = \frac{u'_c}{U'_C} = \gamma \tag{A.30}$$

$$\text{Intertemporal allocations (Euler)}$$

$$\partial S \quad \beta^{-1} = 1-\delta_S + \frac{U'_H H'_S}{U'_C} \tag{A.31}$$

$$\partial s \quad \beta^{-1} = 1-\delta_S + \frac{u'_h h'_s}{u'_c} \tag{A.32}$$

$$\partial K \quad \beta^{-1} = R^{Kgross} \tag{A.33}$$

The system described in Proposition 1, is:

$$\text{Intraperiod allocations}$$
$$\frac{u'_h h'_l}{U'_H H'_{\mathcal{L}}}(1-\tau_H) = \frac{u'_c}{U'_C} \tag{A.34}$$
$$\text{Intertemporal allocations (Euler)}$$
$$\partial S \quad \beta^{-1} = 1 - \delta_S + \frac{U'_H H'_S}{U'_C} \tag{A.35}$$
$$\partial s \quad \beta^{-1} = 1 - \delta_S + (1-\tau_H)\frac{u'_h h'_s}{u'_c} \tag{A.36}$$
$$\partial K \quad \beta^{-1} = R^{Knet} \tag{A.37}$$

The system described in Proposition 2, *with the same tax en rents and returns on capital, a differential tax on land by land use and taxes or subsidies on residential structures s and S,* but without tax on imputed rents, is given by:

$$\text{Intraperiod allocations}$$
$$\frac{u'_h h'_l}{U'_H H'_{\mathcal{L}}}\left(1-\tau_H + \frac{\Delta\tau_{\mathcal{L}}}{R^{Hgross} h'_l}\right) = \frac{u'_c}{U'_C} \tag{A.38}$$
$$\text{Intertemporal allocations (Euler)}$$
$$\partial S \quad \beta^{-1} = 1 - \delta_S + \frac{1}{1+\tau_S}\frac{U'_H H'_S}{U'_C} \tag{A.39}$$
$$\partial s \quad \beta^{-1} = 1 - \delta_s + \frac{1-\tau_H}{1+\tau_s}\frac{u'_h h'_s}{u'_c} \tag{A.40}$$
$$\partial K \quad \beta^{-1} = R^{Knet} \tag{A.41}$$

A.4 Sufficient conditions for first best efficiency at the steady-state

A.4.1 Physical capital

Comparison of social planner Euler equation on capital (A.33) and the general case of the Euler decentralized equilibrium (A.29) implies that the tax on returns on capital and the tax on investments are redundant and individually distortive. In other words, if one taxes returns on capital with a positive tax τ_K, one would need to subsidy its investment with a negative tax rate τ_I hence the perfect substitutability between the two tax instruments. If there is no subsidy nor a tax on private investment, $\tau_I = 0$, then the optimal taxation result applies:

$\tau_K = 0$ reaches the first best.

A.4.2 Intraperiod housing/consumption allocation

Comparison of the social planner's equation reflecting intraperiod allocations, equation (A.30) and the corresponding decentralized equation (A.26) implies that in the decentralized equilibrium, the marginal rate of substitution of goods consumption and land consumption have to be equalized to that of the first best, which is obtained when:

$$\frac{1-\tau_H}{1+\tau_{liv,t}} + \frac{\tau_{liv}+\tau_{HI}}{1+\tau_{liv,t}}\frac{H'_{\mathcal{L}}}{h'_l} + \frac{\Delta\tau_{\mathcal{L}}}{(1+\tau_{liv})R^{Hgross}h'_l} = 1 \tag{A.42}$$

A.4.3 Proof of Proposition 1, parts 4 and 5: a formula for the optimal land tax

In the first best, one can now calculate the optimal tax levied on land to reach the social optimum desired by the social planner. We study the case with $\tau_I = \tau_K = \tau_S = \tau_{liv} = \tau_H = 0$, one has that s, S, K reach their first best level.

We assume here separable utility functions $U(C_t, H_t) = U(C_t) + V(H_t)$, $u(c_t, h_t) = u(C_t) + v(H_t)$ and further identical utility functions. To insure $\frac{u'_c}{U'_C} = \gamma$, the social planner must then fix an optimal level of transfers using the tax $\tau_{\mathcal{L}}$:

$$c + hR^{Hgross} = f(K) - Kf'(K) + \tau_{\mathcal{L}}\bar{\mathcal{L}} \tag{A.43}$$

$$C - hR^{Hgross} = Kf'(K) - \delta K - \delta_S(s+S) - \tau_{\mathcal{L}}\bar{\mathcal{L}} \tag{A.44}$$

with $\beta^{-1} = R^{Kgross} = f'(K) + 1 - \delta$ thus $K(\delta, \beta)$. In all cases:

$$c + C = f(K) - \delta K - \delta_S(s+S)$$

In the case $\gamma = 1$, the consumption are equalized, leading to:

$$c = C = \frac{f(K) - \delta K - \delta_S(s+S)}{2}$$

and, by difference of the two resource constraints (A.43) and (A.44):

$$2hR^{Hgross} = f(K) - 2Kf'(K) + \delta K + \delta_S(s+S) + 2\tau_{\bar{\mathcal{L}}}\bar{\mathcal{L}}$$

or

$$\tau_{\bar{\mathcal{L}}}\bar{\mathcal{L}} = hR^{Hgross} + Kf'(K) - \frac{1}{2}\left[f(K) + \delta K + \delta_S(s+S)\right] \tag{A.45}$$

Using $Kf'(K)+w = f(K)$, one obtains equation (18) in the text, In the general case with positive γ and CARA utility function, if $\ln u(c) = -\nu c$ and $\ln U(C) = -\nu C$, therefore:

$$c - C = -\frac{\ln\gamma}{\nu}$$

In that case, by difference of the resource constraints, for separable utility functions, identical and exponential negative functions (CARA) in the general case $\gamma \leq 1$:

$$\tau_{\bar{\mathcal{L}}}\bar{\mathcal{L}} = hR^{Hgross} + Kf'(K) - \frac{1}{2}\left[f(K) + \delta K + \delta_S(s+S) - \frac{\ln\gamma}{\nu}\right] \tag{A.46}$$

which is equation (19) in the text after using $f(K) = Kf'(K) + w$. The product of the land tax must grow as γ (share of the capitalist) goes down.

Still in the general case of $\gamma \leq 1$, equation (20) in the text is obtained in the case of CRRA sub-utility functions $U(C) = \frac{1}{1-\sigma}C^{1-\sigma_C}$. At the first best, $\gamma U'_C = \gamma C^{-\sigma_C} = c^{-\sigma_C}$. Indeed, we have $\gamma\,\frac{C^{-\sigma_C}}{c^{-\sigma_C}} = 1$ or $\frac{C}{c} = \gamma^{1/\sigma_C}$.

Then

$$c + C = c\left(1 + \frac{C}{c}\right) = c\left[1 + \gamma^{1/\sigma_C}\right] \tag{A.47}$$

Now, summing up equations (A.43) and (A.44), one obtains

$$c + C = f(K) - \delta K - \delta_S(s+S) \tag{A.48}$$

Using (A.47) and (A.48) one gets,

$$c = \frac{f(K) - \delta K - \delta_S(s+S)}{(1+\gamma)^{1/\sigma_C}} \tag{A.49}$$

Now using equation (A.43), one obtains

$$\tau_{\mathcal{L}}\overline{\mathcal{L}} = hR^{Hgross} + Kf'(K) - f(K) + c \tag{A.50}$$

Replacing c by its expression in (A.49) in (A.50) and again using the usual decomposition $f(K) = Kf'(K) + w$ one obtains the result in equation (20) in the text.

A.4.4 Proofs of Proposition 2

On the role of land taxes From the intra period allocation of land, in equation (A.42), on has that:

- in the absence of a living tax ($\tau_{liv} = 0$) and of imputed rent tax ($\tau_{HI} = 0$), the tax on rents can and must be compensated by a differential tax on land such that

$$\Delta\tau_{\mathcal{L}} = \frac{\tau_H}{R^{Hgross} h_l'}$$

  - In particular, the landlord occupying its land must be taxed at a higher rate than the landlord renting its land to tenants, to restore the intraperiod allocation of income of tenants and landlords.
  - Again, in the absence of a tax on rents, the first best is reached with a non-differentiated tax on land:

$$\Delta\tau_{\mathcal{L}} = 0$$

    which does not exclude a tax on land itself. See infra.

- In the absence of a differential tax on land ($\Delta\tau_{\mathcal{L}} = 0$), the tax on rent could potentially be compensated by a negative living tax (that is, a subsidy) on landlords implicit rents, but this is not possible with a positive tax and in any event, this would distort the choice of structures S - see below.

On the role of taxation/subventions of structures Still without living tax, comparisons of the decentralized equilibrium Euler equations on structures, equations (A.39) and (A.40) to the social planner counterparts (A.31) and (A.32), one obtains the optimal value of the tax/subsidy on S when

$$\tau_S = 0$$

Instead, with a positive tax on imputed rents, the optimal subsidy of structures is given after basic calculation and noting that $U_C' R^{Hgross}/U_H' = H_{\mathcal{L}}'/h_l'$ by the simple formula:

$$\tau_S = -\tau_{HI} \frac{H_{\mathcal{L}}'}{h_l'}$$

The equivalent result on the structures of tenants s financed by landlords, the optimal value of the tax/subsidy on s is reached when

$$\tau_H = -\tau_s \tag{A.51}$$

that is, when a subsidy on investment in structures exactly compensate for the tax on rents.

A.4.5 Inefficiencies in the absence of subsidies to structures

Appendix Lemma A1 (Distortions with structures): *There are several deviations from the first best with taxes on rents with imputed rents.*

1. *A single tax on actual rents also distorts the investment in housing structures as well as the intraperiod allocation of space.*

2. *When the social planner tries to restore the intraperiod allocations of housing between homeowners and tenants with positive imputed rents, this further distorts the investment in structures S of the capitalist.*

To see this, pose $\mathbf{s} = (s/l)$ the structure quantity per unit of land. The rent tax including imputed rent entails a new equilibrium with underprovision of $\mathbf{s}$ with respect to the first best Pose $\mathbf{S} = \frac{S}{\mathcal{L}}$ the capitalist-structure quantity per unit of land. We further assume that the housing production function is Cobb-Douglas. Then, the rent tax including imputed rent with compensation for the land distortion considered in Proposition 2 leads to a new equilibrium with underprovision of $\mathbf{S}$ with respect to the first best.

On the suboptimal investment in s.

Let us defined $\mathbf{s} = \frac{s}{l}$ the structure quantity per unit of land. The rent tax including imputed rent entails a new equilibrium with underprovision of $\mathbf{s}$ with respect to the first best. Introducing a small tax τ_H, the new equilibrium must respect the Euler equation with respect to tenant structure equation (23) is defined by:

$$\beta^{-1} = 1 - \delta_s + (1 - \tau_H) R^{Hgross} h'_s(s, l)$$

Using the constant return to scale property of the function h such that leads to $h(s, l) = lg(\mathbf{s})$ where $g(\mathbf{s})$ is an increasing concave function of land per structure. The Euler equation with respect to tenant structure now writes

$$\beta^{-1} = 1 - \delta_s + (1 - \tau_H) R^{Hgross} g'(\mathbf{s})$$

Fully differentiating it and rearranging leads to

$$\frac{d\mathbf{s}}{d\tau_H} = \frac{g'\left(1-(1-\tau_H)\frac{dR^{Hgross}}{d\tau_H R^{Hgross}}\right)}{(1-\tau_H)R^{Hgross}g''} \tag{A.52}$$

The denominator of the RHS of (A.52) is negative given the concavity of g. Now we can prove that the numerator is positive. Because the housing market is competitive, it is a standard result that we we have full shifting of the tax, that is $dR^{Hgross} - dR^{Hnet} = d\tau_H R^{Hgross}$ or

$$\frac{dR^{Hgross}}{d\tau_H R^{Hgross}} < 1$$

implying that the numerator of (A.52) is positive. It follows that $\frac{d\mathbf{s}}{d\tau_H} < 0$.■

On the suboptimal investment in S.

Similarly, denote by $\mathbf{S} = \frac{S}{\mathcal{L}}$ the capitalist-structure quantity per unit of land. We further assume that the housing production function is Cobb-Douglas. With the rent tax profiles including imputed rents tax considered in Proposition 2, part 1b,

$$\tau_{HI} = \tau_H \frac{h'_l}{H'_{\mathcal{L}}} \tag{A.53}$$

Therefore condition (26 becomes

$$\frac{U'_H}{U'_C} = \frac{u'_h h'_l}{u'_c H'_{\mathcal{L}}} = R^{Hgross}\frac{h'_l}{H'_{\mathcal{L}}} \tag{A.54}$$

We show that a new equilibrium is reached with underprovision of $\mathbf{S}$ with respect to the first best. Introducing a small tax τ_{HI}, the new equilibrium must respect equation (22) rewritten here for convenience:

$$\beta^{-1} = 1-\delta_s + H'_S\left(\frac{U'_H}{U'_C} - \tau_{HI}R^{Hgross}\right) \tag{A.55}$$

Combining equations (A.53) and (A.54) into (A.55) we obtain

$$\beta^{-1} = 1-\delta_s + (1-\tau_H)R^{Hgross}H'_S\frac{h'_l}{H'_{\mathcal{L}}} \tag{A.56}$$

As in the previous proof on s, using that H is constant return to scale leads to rewrite equation

(A.55) as

$$\beta^{-1} = 1 - \delta_s + (1 - \tau_H) R^{Hgross} G'_S \frac{h'_l}{H'_{\mathcal{L}}} \tag{A.57}$$

with $H(S, \mathcal{L}) = \mathcal{L} G(\mathbf{S})$

and $G(\mathbf{S})$ is a concave function. Under the assumption that both h and H are Cobb-Douglas with the same elasticity ν, $h(s, l) = l^{\nu} s^{(1-\nu)} = l g(\mathbf{s})$ and $H(S, \mathcal{L}) = \mathcal{L} G(\mathbf{S})$ with $g(x) = G(x) = x^{1-\nu}$, one has then

$$h'_l = \nu (s/l)^{1-\nu} = \nu \mathbf{s}^{1-\nu}$$

$$H'_L = \nu (S/\mathcal{L})^{1-\nu} = \nu \mathbf{S}^{1-\nu}$$
$$G' = (1 - \nu) \mathbf{S}^{-\nu}$$

Using these expressions in (A.56) leads to

$$\beta^{-1} = 1 - \delta_s + (1 - \tau_H) R^{Hgross} (1 - \nu) \frac{\mathbf{s}^{1-\nu}}{\mathbf{S}} \tag{A.58}$$

Fully differentiate (A.58) gives after a few steps, one has

$$-R^{Hgross} \mathbf{s} d\tau_H + (1 - \tau_H) \mathbf{s} dR^{Hgross} - (1 - \tau_H) R^{Hgross} \frac{\mathbf{s}}{\mathbf{S}} d\mathbf{S} + (1 - \tau_H)(1 - \nu) R^{Hgross} d\mathbf{s} = 0$$

and then finally

$$\frac{d\mathbf{S}}{d\tau_H} = -\frac{\mathbf{S}\left(1 - (1 - \tau_H)\frac{dR^{Hgross}}{d\tau_H R^{Hgross}}\right)}{(1 - \tau_H) R^{Hgross}} + (1 - \nu) \frac{\mathbf{S}}{\mathbf{s}} \frac{d\mathbf{s}}{d\tau_H} \tag{A.59}$$

We already know that the first term of the RHS of (A.59) is negative as well as that the second term is negative (see the proofs for the underprovivsion of $\mathbf{s}$). Hence,

$$\frac{d\mathbf{S}}{d\tau_H} < 0$$

■

A.4.6 Living tax with no other tax

We document here the discussion of the living tax in subsection 3.2.3 in the particular case in which the tax on imputed rents is removed: $\tau_{HI} = 0$. In that case, the decentralized system becomes:

$$\text{Intraperiod allocations}$$

$$\frac{u'_h h'_l}{U'_H H'_{\mathcal{L}}} \left(\frac{1}{1+\tau_{liv,t}} + \frac{\tau_{liv}}{1+\tau_{liv,t}} \frac{H'_{\mathcal{L}}}{h'_l} \right) = \frac{u'_c}{U'_C} \tag{A.60}$$

$$\text{Intertemporal allocations (Euler)}$$

$$\partial S \quad \beta^{-1} = 1 - \delta_S + \frac{U'_H H'_S}{U'_C} - \tau_{liv} R^{Hgross} H'_S \tag{A.61}$$

$$\partial s \quad \beta^{-1} = 1 - \delta_s + \frac{1}{1+\tau_{liv}} \frac{u'_h h'_s}{u'_c} \tag{A.62}$$

It is easy to show that the living tax is distortive on S and reduces its return. It is also easy to see from equation (A.62) that the living tax reduces the returns on investment in s. In both cases, both marginal rates of substitution $\frac{U'_H H'_S}{U'_C}$ and $\frac{u'_h h'_s}{u'_c}$ must increase. Last, from equation (A.60), a positive living tax increases $\frac{u'_h h'_l}{U'_H H'_{\mathcal{L}}} / \frac{u'_c}{U'_C}$ relative to the first best.

A.4.7 Consumption tax in subsection 3.2.3

We also look at the polar case of subsection 3.2.3 of a consumption tax and a living tax with a tax on imputed rents.

Appendix Lemma A2 **(Restoring the first best)** *A first best solution improving the situation of the worker tenant with respect to the laissez-faire can be reached with a combination of the following distorting instruments: A consumption tax τ_c, a living tax $\tau_{liv} = \tau_c$ and tax on imputed rent for the capitalist τ_{HI}, satisfying*

$$\tau_{HI} = \tau_c \frac{h'_l}{H'_{\mathcal{L}}}. \tag{A.63}$$

Proof: For the worker, this tax system leaves the MRS of housing for consumption is unchanged:

$$\frac{u'_h}{u'_c} = R^{Hgross}. \tag{A.64}$$

The budget equation for the capitalist is now $R_t^{KGross}K_t + (1-\delta)K_t + R_t^{Hgross}h_t(\overline{\mathcal{L}} - \mathcal{L}_t, s_t) +$

$(1-\delta_s)(s_t + S_t) = C_t^M(1+\tau_c) + s_{t+1} + S_{t+1} + K_{t+1} + \tau_{HI}R_t^{Hgross}H_t(\mathcal{L}_t, S_t)$ The first order conditions are

$$\partial C_t : \lambda_t^K(1+\tau_c) = U_C'(t) \tag{A.65}$$
$$\partial \mathcal{L}_t : U_{\mathcal{L}}'H_{\mathcal{L}}'(t) - \lambda_t^K\tau_{HI}R_t^{Hgross}H_{\mathcal{L}}'(t) = \lambda_t^K R^{Hgross}h_l'(t) \tag{A.66}$$

$$\partial S_{t+1} : \lambda_t^K = \beta\lambda_{t+1}^K\ (1-\delta_s - \tau_{HI}R_{t+1}^{Hgross}H_s'(t+1)) + \beta U_H'(t+1)H_s'(t+1) \tag{A.67}$$
$$\partial s_{t+1} : \lambda_t^K = \beta\lambda_{t+1}^K\ (1-\delta_s) + \beta\lambda_{t+1}^K R_{t+1}^{Hgross}h_s'(t+1) \tag{A.68}$$

At the steady-state we have:

$$\partial C : \lambda^K = \frac{U_C'}{1+\tau_c} \tag{A.69}$$
$$\partial s : \beta^{-1} = \ (1-\delta_s) + R^{Hgross}h_s'$$

Therefore, the Euler's condition on structures is as in the laisser-faire. Regarding the intraperiod allocation, starting from (A.66) and using (A.69), we get

$$\frac{U_{\mathcal{L}}'}{U_C'}H_{\mathcal{L}}'(1+\tau_c) - \tau_{HI}R^{Hgross}H_{\mathcal{L}}' = R^{Hgross}h_l'$$

Using once (A.64) we obtain

$$\frac{U_{\mathcal{L}}'}{U_C'}H_{\mathcal{L}}' + H_{\mathcal{L}}'(\frac{U_{\mathcal{L}}'}{U_C'}\tau_c - \tau_{HI}\frac{u_h'}{u_c'}) = R^{Hgross}h_l'$$

Stating $\tau_{HI} = \frac{U_{\mathcal{L}}'}{U_C'}\frac{u_c'}{u_h'}\tau_c$, we get back to the first-best intraperiod condition

$$\frac{U_{\mathcal{L}}'}{U_C'}H_{\mathcal{L}}' = \frac{u_l'}{u_c'}h_l'$$

which finally gives

$$\tau_{HI} = \tau_c\frac{h_l'}{H_{\mathcal{L}}'}$$

Now looking at the Euler equation regarding S (A.67), it writes at the steady-state, using again (A.69),

$$\beta^{-1} = (1 - \delta_s - \tau_{HI} R^{Hgross} H_s^{'}) + (1 + \tau_c)\frac{U_H'}{U_C'} H_s'$$

$$\beta^{-1} = (1 - \delta_s) + \frac{U_H'}{U_C'} H_s' + H_s'(\tau_c \frac{U_H'}{U_C'} - \tau_{HI} R^{Hgross})$$

The last term vanishes: this allows to conclude that the Euler condition for S is similar to the first best. ■

A.5 Extension of the results out of the steady-state

In this sub-section we compare the first order conditions of the decentralized equilibrium out of the steady-state, that is equations (A.12) to (A.19) to those of the social planner, that is (A.1) to (A.5).

Can we reach a first best along the dynamics? For that, it is necessary to have:

$$\lambda_t = \lambda_t^w = \gamma \lambda_t^K = u_c' = \gamma U_C'$$

One can therefore rewrite the SP optimal path as:

$$\partial C_t, c_t \quad \gamma U_C'(t) = u_c'(t) \tag{A.70}$$

$$\partial \mathcal{L}_t \quad \gamma U_H'(t) H_{\mathcal{L}}'(t) = u_h' h_l'(t) \tag{A.71}$$

$$\partial S_{t+1} \quad U_C'(t) = \beta U_C'(t+1)(1 - \delta_S) + \beta U_H'(t+1) H_S'(t+1) \tag{A.72}$$

$$\partial s_{t+1} \quad U_C'(t) = \beta U_C'(t+1)(1 - \delta_S) + (\beta/\gamma)\, u_h'(t+1) h_s'(t+1) \tag{A.73}$$

$$\partial K_{t+1} \quad U_C'(t) = \beta U_C'(t+1) R_{t+1}^{Kgross} \tag{A.74}$$

and using an expression for the gross rent,

$$R_t^{Hgross}(t) = \frac{u_h'(t)/u_c'(t)}{1 + \tau_{liv,t}}$$

Capitalist: intraperiod

$$\partial \mathcal{L}_t \quad U'_H(t)H'_{\mathcal{L}}(t) = U'_C(t)\left(\frac{u'_h(t)/u'_c(t)}{1+\tau_{liv,t}}(1-\tau_{H,t})h'_l(t) + \Delta\tau_{\mathcal{L}}(t) + (\tau_{liv,t}+\tau_{HI,t})\frac{u'_h(t)/u'_c(t)}{1+\tau_{liv,t}}H'_{\mathcal{L}}(t)\right) \tag{A.75}$$

Capitalist: intertemporal (Euler)

$$\begin{aligned}\partial S_{t+1} \quad U'_C(t)(1+\tau_{S,t}) &= \beta U'_C(t+1)(1-\delta_S)(1+\tau_{S,t+1}) + \beta U'_H(t+1)H'_S(t+1)\\ &\quad - U'_C(t+1)\beta(\tau_{liv,t+1}+\tau_{HI,t+1})\frac{u'_h(t+1)/u'_c(t+1)}{1+\tau_{liv,t+1}}H'_S(t+1)\end{aligned} \tag{A.76}$$

$$\partial s_{t+1} \quad U'_C(t)(1+\tau_{s,t}) = \beta U'_C(t+1)(1-\delta_s)(1+\tau_{s,t+1}) + \beta U'_C(t+1)\frac{u'_h(t+1)/u'_c(t+1)}{1+\tau_{liv,t+1}}(1-\tau_{H,t+1})h'_s(t+1) \tag{A.77}$$

$$\partial K_{t+1} \quad U'_C(t)(1+\tau_{I,t}) = \beta U'_C(t+1)\left[R^{Knet}_{t+1} + (1-\delta)\tau_{I,t+1}\right] \tag{A.78}$$

We can now prove the validity of each bloc of Section 2 in sequence.

A.5.1 Extension of Proposition 1 out of the steady-state

There is no tax on capital after $t \geq 1$ if no tax on investment, $\tau_{I,t} = \tau_{I,t+1} = 0$. This comes from the comparison of equations (A.74) and equation (A.78). This extends the result of Proposition 1, part 1, away from the steady-state. In the first period ($t = 0$), capital is already installed and can therefore be taxed.

There is also no tax on rents if no other taxes on imputed rents and living tax (part 2 of Proposition 1). Replacing the gross rent of the worker into the net rent of the capitalist intraperiod condition, one has:

$$U'_H(t)H'_{\mathcal{L}}(t) = U'_C(t)\left(\frac{u'_h(t)/u'_c(t)}{(1+\tau_{liv,t})}(1-\tau_{H,t})h'_l(t) + \Delta\tau_{\mathcal{L}}(t) + (\tau_{liv,t}+\tau_{HI,t})\frac{u'_h(t)/u'_c(t)}{1+\tau_{liv,t}}H'_{\mathcal{L}}(t)\right)$$

that can then be compared to the equivalent social planner's condition

$$\frac{U'_H(t)H'_{\mathcal{L}}(t)}{U'_C(t)} = \frac{u'_h(t)h'_l(t)}{u'_c(t)}\left(= R^{Hgross}_t(t)h'_l(t)(1+\tau_{liv,t})\right)$$

and after replacement,

$$1 = \left(\frac{1-\tau_{H,t}}{1+\tau_{liv,t}} + \frac{\Delta\tau_{\mathcal{L}}(t)}{R^{Hgross}_t(t)h'_l(t)(1+\tau_{liv,t})} + \frac{\tau_{liv,t}+\tau_{HI,t}}{1+\tau_{liv,t}}\frac{H'_{\mathcal{L}}(t)}{h'_l(t)}\right) \tag{A.79}$$

So, under the conditions of Proposition 1, the absence of a living tax and of an imputed tax on rents, one has from the equation above that:

$$\tau_{H,t} = 0$$

for all $t \geq 1$ which extends the result of Proposition 1, part 2, away from the steady-state.

A.5.2 Extension of Proposition 2 out of the steady-state

Part 1 of Proposition 2 in the text states that, if there is a positive tax on rent and a tax on imputed rent, they can also be compensated, by either a differentiated tax on land, e.g.

$$\Delta\tau_{\mathcal{L}}(t) = R_t^{Hgross}(t) \times h_l'(t)\tau_{H,t}$$

or by a tax on imputed rents, e.g.

$$\tau_{HI,t} = \tau_{H,t}\frac{h_l'(t)}{H_{\mathcal{L}}'(t)}$$

The proof comes from equation (A.79) which, absent the living tax, the condition above becomes:

$$\Delta\tau_{\mathcal{L}}(t) = R_t^{Hgross}(t) \times h_l'(t)\left(\tau_{H,t} - \tau_{HI,t}\frac{H_{\mathcal{L}}'(t)}{h_l'(t)}\right)$$

and each special case discussed above, $\Delta\tau_{\mathcal{L}}(t) = 0$ or $\tau_{HI,t} = 0$ delivers the results. This extends the result of Lemma 1 away from the steady-state.

Parts 2 and 3 of Proposition 2 are related to the Euler equations of the decentralized equilibrium. Comparing them with those of the social planners one obtains:

$$\begin{aligned} DEC \quad U_C'(t)(1+\tau_{S,t}) &= \beta U_C'(t+1)(1-\delta_S)(1+\tau_{S,t+1}) + \beta U_H'(t+1)H_S'(t+1) \\ &\quad - U_C'(t+1)\beta(\tau_{liv,t+1}+\tau_{HI,t+1})\frac{u_h'(t)/u_c'(t)}{1+\tau_{liv,t}}H_S'(t+1) \end{aligned} \tag{A.80}$$

$$SP \quad U_C'(t) = \beta U_C'(t+1)(1-\delta_S) + \beta U_H'(t+1)H_S'(t+1) \tag{A.81}$$

and

$$DEC \quad U_C'(t)(1+\tau_{s,t}) = \beta U_C'(t+1)(1-\delta_s)(1+\tau_{s,t+1}) + \beta U_C'(t+1)\frac{u_h'(t+1)/u_c'(t+1)}{(1+\tau_{liv,t})}(1-\tau_{H,t+1})h_s'(t+1) \tag{A.82}$$

$$SP \quad U_C'(t) = \beta U_C'(t+1)(1-\delta_S) + (\beta/\gamma)\, u_h'(t+1)h_s'(t+1) \tag{A.83}$$

In that case, substracting these equations 2 by 2, a profile of subsidies to structure such that

$$\left[\frac{U'_H(t+1)H'_S(t+1)}{U'_C(t+1)}\right]\tau_{S,t} = (1-\delta_S)(\tau_{S,t+1}-\tau_{S,t}) - (\tau_{liv,t+1}+\tau_{HI,t+1})\frac{u'_h(t)/u'_c(t)}{1+\tau_{liv,t}}H'_S(t+1)$$
$$\left[\frac{u'_h(t+1)h'_s(t+1)}{\gamma U'_C(t+1)}\right](1+\tau_{s,t}) = (1-\delta_s)(\tau_{s,t+1}-\tau_{s,t}) + \frac{u'_h(t+1)/u'_c(t+1)}{1+\tau_{liv,t}}(1-\tau_{H,t+1})h'_s(t+1) \tag{A.84}$$

insures the first best along the transition. After one more step, using again the SP conditions: for the first equation,

$$\frac{U'_H}{U'_C} = \frac{u'_h}{u'_c}\frac{h'_l}{H'_{\mathcal{L}}}$$

and $\gamma U'_C = u'_c$ and assume away the living tax so as to get a simpler expression, one has then:

$$\tau_{S,t} = (1-\delta_S)\left(\frac{\tau_{S,t+1}-\tau_{S,t}}{R^{Hgross}_{t+1}(t)\frac{h'_l(t+1)}{H'_{\mathcal{L}(t+1)}}H'_S(t+1)}\right) - \tau_{HI,t+1}\frac{H'_{\mathcal{L}}}{h'_l}(t+1) \tag{A.85}$$

$$\tau_{s,t} = (1-\delta_s)\frac{\tau_{s,t+1}-\tau_{s,t}}{R^{Hgross}_{t+1}(t)h'_s(t+1)} - \tau_{H,t+1} \tag{A.86}$$

To extend the result of Proposition 2, one can search for a pattern of stable subsidies. Equation (A.86) delivers a stable subsidy when the tax on rents is constant, and therefore one obtains

$$\tau_{s,t} = -\tau_H$$

for all periods, which generalizes the result of Proposition 2 away fron the steady-state. For the first equation, this is even simpler: $\tau_{HI,t} = 0$ insures $\tau_{S,t} = 0$.

B Online Appendix: Extension with demographic weights

In this Appendix, we verify that the first order conditions in the decentralized equilibrium and of the Pareto-optimum are independent of the assumption of a unique representative capitalist. We assume that there is a mass m of capitalists and keep as a normalization a mass 1 of workers. Only the resource condition will have m as an parameter, that shifts the consumption and income of capitalists.

B.1 Decentralized program with demographic weight

The program of the capitalist is written, using C_t, H_t for its per capita consumption, as well as S_t, s_t, K_t for the structures. The total marginal product of one additional unit of capital K_t is $f'(K_t)$. That amount is split in each of the capitalists, so each capitalist receives for its marginal investment in K_t the value $f'(K_t)/m$ and pays for its own individual depreciation. It follows that one can define the gross return to capital for each capitalist as:

$$\frac{R_t^{Kgross}}{m} = \frac{f'(K_t) + (1-\delta)}{m}$$

and

$$\frac{R_t^{Knet}}{m} = \left[\frac{f'(K_t) + 1 - \delta}{m}\right](1 - \tau_{K,t})$$

The program of the capitalist is:

$$\begin{aligned}
&\max_{C_t, H_t, \mathcal{L}_t, K_{t+1}, s_{t+1}, S_{t+1}} \sum_t \beta^t \left\{U\left[C_t, H\left(\mathcal{L}_t, S_t\right)\right]\right\} \\
&+ \beta^t \lambda_t^K \left\{\frac{R_t^{Knet} K_t}{m} + \tau_{I,t}(1-\delta)\frac{K_t}{m} + \frac{R_t^{Hgross}}{m}(1 - \tau_{H,t}) h_t(\bar{\mathcal{L}} - m\mathcal{L}_t, m s_t)\right. \\
&- (\tau_{liv,t} + \tau_{HI,t}) R_t^{Hgross} H_t(\mathcal{L}_t, S_t) \\
&+ (1-\delta_S)(1+\tau_{S,t}) S_t + (1-\delta_s)(1+\tau_{s,t}) s_t - T_t^{\mathcal{L}} \\
&\left. - C_t - S_{t+1}(1+\tau_{S,t}) - s_{t+1}(1+\tau_{s,t}) - \frac{K_{t+1}}{m}(1+\tau_{I,t})\right\}
\end{aligned}$$

leading to first order conditions:

On C_t:

$$U'_C(t) = \lambda_t^K$$

On K_{t+1}:

$$\lambda_t^K (1 + \tau_{I,t})\frac{1}{m} = \beta\lambda_{t+1}^K \frac{R_{t+1}^{Knet}}{m} + \tau_{I,t+1}(1-\delta)\frac{1}{m}$$

On $\mathcal{L}_t$:

$$U'_H H'_{\mathcal{L}}(t)/\lambda_t^K = \frac{R_t^{Hgross}}{m}(1 - \tau_{H,t}) m h'_{\mathcal{L}}(t) + (\tau_{liv,t} + \tau_{HI,t}) R_t^{Hgross} H'_{\mathcal{L}}(t) + \tau_{\mathcal{L}} - \tau_l$$

On S_{t+1}:

$$\beta U'_H H'_S(t+1) = \lambda_t^K (1+\tau_{S,t}) + \beta\lambda_{t+1}^K \left\{(\tau_{liv,t+1} + \tau_{HI,t+1}) R_{t+1}^{Hgross} H_S(t+1) - (1-\delta_S)(1+\tau_{S,t+1})\right\}$$

On s_{t+1}:

$$\lambda_t^K (1+\tau_{s,t}) = \beta\lambda_{t+1}^K \left\{\frac{R_{t+1}^{Hgross}}{m}(1 - \tau_{H,t+1}) h'_s(t+1) m + (1-\delta_s)(1+\tau_{s,t+1})\right\}$$

subject to subject to $\beta^t U'_C(t) M_{t+1} \to 0$, for $M_t = K_t, S_t, s_t$ and where $\bar{\mathcal{L}}$ is the total land owned by all capitalist. It is easy to verify that m vanishes letting the first order conditions of the decentralized equilibrium unchanged.

B.2 Social planner with demographic weights

The system can be compared to the social planner's allocation. Its program is now:

$$\max_{C_t, c_t, \mathcal{L}_t, S_{t+1}, s_{t+1}, K_{t+1}} \sum_t \beta^t \left\{ u\left[c_t, h\left(\overline{\mathcal{L}} - m\mathcal{L}_t, m s_t\right)\right] + m\gamma U\left[C_t, H\left(\mathcal{L}_t, S_t\right)\right]\right\}$$
$$+ \beta^t \lambda_t \left\{ f(K_t) + (1-\delta)K_t + (1-\delta_S)m(S_t + s_t) \right.$$
$$\left. - c_t - mC_t - mS_{t+1} - ms_{t+1} - K_{t+1} \right\}$$

subject to three transversality conditions on each stock:

$$\beta^t U'_C(t) M_{t+1} \to 0 \tag{B.1}$$

for $M_t = K_t, S_t, s_t$. We then obtain the first order conditions for each period:

$$\partial C_t \quad m\lambda_t = \gamma m U'_C(t) \tag{B.2}$$
$$\partial c_t \quad \lambda_t = u'_c(t) \tag{B.3}$$
$$\partial \mathcal{L}_t \quad m\gamma U'_H(t) H'_{\mathcal{L}}(t) = m u'_h(t) h'_l(t) \tag{B.4}$$
$$\partial S_{t+1} \quad m\lambda_t = m\beta\lambda_{t+1}(1-\delta_S) + m\gamma\beta U'_H(t+1) H'_S(t+1) \tag{B.5}$$
$$\partial s_{t+1} \quad m\lambda_t = m\beta\lambda_{t+1}(1-\delta_S) + m\beta u'_h(t+1) h'_s(t+1) \tag{B.6}$$
$$\partial K_{t+1} \quad \lambda_t = \beta\lambda_{t+1} R^{Kgross}_{t+1} \tag{B.7}$$

This notably implies the following steady-state relations, m disappears and one is left with:

$$\partial C_t, c_t \quad \lambda = \gamma U'_C = u'_c \tag{B.8}$$
$$\partial \mathcal{L}_t \quad \gamma U'_H H'_{\mathcal{L}} = u'_h h'_l \tag{B.9}$$
$$\textit{Euler} \quad S_{t+1} \quad \beta^{-1} = \frac{U'_H H'_S}{U'_C} + 1 - \delta_S \tag{B.10}$$
$$\textit{Euler} \quad s_{t+1} \quad \beta^{-1} = \frac{u'_h h'_S}{u'_c} + 1 - \delta_S \tag{B.11}$$
$$\textit{Euler} \quad K_{t+1} \quad \beta^{-1} = R^{Kgross} = f'(K) + 1 - \delta \tag{B.12}$$

C Online Appendix: Proofs of sub-Section 4.1

C.1 Proof of Lemma 1: The implementability condition in the Judd model

We start from $R_t^{Knet} = (1-\tau_t)R_t^{Knet}$

$$\begin{aligned} C_t + K_{t+1} &= R_t^{Knet} K_t & \text{(C.1)} \\ K_{t+1} &\geq 0 & \text{(C.2)} \end{aligned}$$

The capitalist can eat capital and so he can transfer wealth from period to period, there is a store value of capital, but the capitalist cannot borrow. There is limited possibility of transferring wealth. $C_0 + K_1 = R_0^{Knet} K_0$; then $C_1 + K_2 = R_1^{Knet} K_1$ or $K_1 = \frac{C_1^K + K_2}{R_1^{Knet}}$ $C_0 + \frac{C_1}{R_1^{Knet}} + \frac{K_2}{R_1^{Knet}} = R_0^{Knet} K_0$ and so on, leading to:

$$\sum_{t=0}^{T} \frac{C_t}{\prod\limits_{j=1}^{t} R_j^{Knet}} + \frac{K_{T+1}}{\prod\limits_{j=1}^{T} R_j^{Knet}} = R_0^{Knet} K_0 \tag{C.3}$$

Now the first order conditions of the capitalist program are

$$\begin{aligned} \text{for } t &\geq 0, R_{t+1}^{Knet} = \frac{1}{\beta} \frac{U_C'(t)}{U_C'(t+1)} & \text{(C.4)} \\ \lim_{t \to +\infty} \beta^t U_C'(t) K_{t+1} &= 0 & \text{(C.5)} \end{aligned}$$

Now using (C.4)

$$\prod_{j=1}^{t} R_j^{Knet} = \frac{1}{\beta^t} \frac{U_C'(0)}{U_C'(t)} \tag{C.6}$$

Plugging this expression into (C.3) gives

$$\sum_{t=0}^{T} \beta^t U_C'(t) C_t + \beta^T U_C'(T) K_{T+1} = U_C'(0) R_0^{Knet} K_0$$

Using the transversality condition at the limit

$$\lim_{t \to +\infty} \sum_{t=0}^{T} \beta^t U_C'(t) C_t = U_C'(0) R_0^{Knet} K_0 \tag{C.7}$$

Let a sequence $\{c_t, C_t, K_t\}$ satisfying the resource constraint for each period $f(K_t) + (1-\delta)K_t - c_t - C_t + K_{t+1} = 0$ and the implementability condition (C.7). Define $\omega_t = F(K_t) - F'(K_t)K_t$ and then $c_t - \omega_t$ where $= T_t$ the transfer to the workers. Starting from the level of capital K_t we deduce the gross return $R^{Kgross} = F_k'(K_t) + 1 - \delta$

We deduce the net return from the Euler equation $R_{t+1}^{Knet} = \frac{1}{\beta} \frac{U_C'(t)}{U_C'(t+1)}$ and deduce τ_t from $R_t^{Knet} =$

$(1-\tau_t)R_t^{Kgross}$. This works for τ_t $t \geq 1$. For $t=0$, this does not apply in the absence of an Euler equation for that period but the implementability condition gives R_0^{Knet} and we deduce τ_0.

So the Ramsey problem can be set up as maximizing the Lagrangien

$$\begin{aligned}\max_{C_t, c_t, K_{t+1}} \sum_{t=0}^{+\infty} \beta^t \left[u(c_t) + \gamma U(C_t)\right] \\ +\nu \left[U_C'(0) R_0^{net} K_0 - \sum_{t=0}^{+\infty} \beta^t U_C'(t) C_t\right] \\ +\sum_{t=0}^{+\infty} \lambda_t \left[f(K_t) + (1-\delta)K_t \; - c_t - C_t - K_{t+1}\right]\end{aligned}$$

As in Atkeson et al. (1999), we define the auxiliary function

$$W(C_t, \nu) = \gamma U(C_t) - \nu U_C'(t) C_t$$

which allows to redefine the objective

$$\begin{aligned}\max_{C_t, c_t, K_{t+1}} \sum_{t=0}^{+\infty} \beta^t \left[u(c_t) + W(C_t, \nu)\right] + \nu U_C'(0) R_0^{net} K_0 \\ +\sum_{t=0}^{+\infty} \lambda_t \left[f(K_t) + (1-\delta)K_t \; - c_t - C_t - K_{t+1}\right]\end{aligned}$$

and calculates the first order conditions:

$$u_c'(t) = \frac{\lambda_t}{\beta^t} \tag{C.8}$$

$$W_C'(C_t, \nu) = \frac{\lambda_t}{\beta^t}; \text{ for } t \geq 1 \tag{C.9}$$

$$W_C'(C_0, \nu) = \lambda_0 - \nu U_{cc}''(0) R_0^{net} K_0 \tag{C.10}$$

$$\lambda_t = \lambda_{t+1} \left[f_K'(K_{t+1}) + (1-\delta)\right] \tag{C.11}$$

The necessary optimality conditions are

$$W_C'(C_t, \nu) = u_c'(t) \tag{C.12}$$

$$W_C'(C_t, \nu) = \beta W_C'(C_{t+1}, \nu) \left[f_K'(K_{t+1}) + (1-\delta)\right] \tag{C.13}$$

$$W'_C(C_0,\nu) = \beta W'_C(C_1,\nu)\left[f'_K(K_1) + (1-\delta)\right] - \nu U''_{cc}(0) R_0^{net} K_0 \tag{C.14}$$

This leads to the result in Lemma 1 in the main text, that we formally proved below.

Proof of Lemma 1, part i): Starting from

$$W'_C(C_t,\nu) = \gamma U'_C - \nu(U''_{CC}C_t + U'_C)$$

and noting that

$$\frac{W'_C(C_t,\nu)}{U'_C} = \gamma - \nu\left(\frac{U''_{CC}C_t}{U'_C} + 1\right) \tag{C.15}$$

which is constant with CCRA. Then (C.13) becomes

$$U'_C(t) = \beta U'_C(t+1)\left[f'_K(K_{t+1}) + (1-\delta)\right]$$

for all $t \geq 1$. Using Euler equation (C.4) this implies that

$$R_{t+1}^{Knet} = f'_K(K_{t+1}) + (1-\delta))$$

and then no tax on capital from $t = 2$. Part ii) of Lemma 1 is a special case of Proposition 3 without land and the proof is relegated there to save space. ■

C.2 Proof of Proposition 3: Implementability in Judd economy with no structures - only land

The aggregate resource constraint is:

$$F(K_t) + K_t(1-\delta) = C_t + c_t + K_{t+1}$$

The capitalist resource constraint is:

$$R_t^{Hnet} h_t + R_t^{Knet} K_t = C_t + K_{t+1}$$

Denote $l_t = \overline{\mathcal{L}} - \mathcal{L}_t$ as before. Pose $H_t = \mathcal{L}_t$ and $h_t = l_t$ so simplify the comparison with the case with structures, so that $H'_{\mathcal{L}} = 1$, $h'_l = 1$ and $h_s = 0, H_S = 0$.

In the general case, the intratemporal constraints are:

$$R_t^{Hnet} = \frac{U'_H H'_{\mathcal{L}}}{U'_C h'_l}(t) = \frac{U'_H}{U'_C}(t)$$
$$R_t^{Hgross} = \frac{u'_h}{u'_c}(t)$$

Worker's budget constraint

It is given by

$$c_t + R_t^{Hgross} h_t = w_t + T_t = y_t \tag{C.16}$$

with

$$w_t = f(K_t) - f'(K_t)K_t \tag{C.17}$$

with y_t the disposable income.

Capitalist constraints

$$R_t^{Hnet} h_t + R_t^{Knet} K_t = C_t + K_{t+1} \tag{C.18}$$

$$K_{t+1} \geq 0 \tag{C.19}$$

$$0 \leq \mathcal{L}_t \leq \overline{\mathcal{L}} \tag{C.20}$$

We have

$$R_0^{Hnet} h_0 + R_0^{Knet} K_0 = C_0 + K_1$$

$$R_1^{Hnet} h_1 + R_1^{Knet} K_1 = C_1 + K_2$$

which implies $K_1 = \frac{C_1 + K_2 - R_1^{Hnet} h_1}{R_1^{net}}$, so

$$C_0 - R_0^{Hnet} h_0 + \frac{C_1 - R_1^{Hnet} h_1}{R_1^{net}} + \frac{K_2}{R_1^{net}} = R_0^{Knet} K_0$$

and the substitution of capital next period can be repeated until time T, leading to

$$\sum_{t=0}^{T} \frac{C_t - R_t^{Hnet} h_t}{\prod_{j=0}^{t} R_j^{Knet}} + \frac{K_{T+1}}{\prod_{j=0}^{T} R_j^{Knet}} = R_0^{Knet} K_0$$

where by convention,

$$\prod_{j=0}^{0} R_j^{Knet} = 1$$

The Euler equation

$$\partial K_{t+1} \quad U_C'(t) = \beta U_C'(t+1) R_{t+1}^{Knet}$$

leads, for all $t \geq 1$, to

$$U'_C(0) = \beta U'_C(1) R_1^{Knet} = ... = \beta^t \prod_{j=0}^{t} R_j^{Knet} U'_C(t)$$
$$\Leftrightarrow \prod_{j=0}^{t} R_j^{Knet} = \frac{1}{\beta^t} \frac{U'_C(0)}{U'_C(t)}$$

Getting back to

$$\sum_{t=0}^{T} \frac{C_t - R_t^{Hnet} h_t}{\frac{1}{\beta^t} \frac{U'_C(0)}{U'_C(t)}} + \frac{K_{T+1}}{\frac{1}{\beta^t} \frac{U'_C(0)}{U'_C(t)}} = R_0^{Hnet} h_0 + R_0^{Knet} K_0$$

leading to:

$$\sum_{t=0}^{T} \beta^t U'_C(t) \left[C_t - R_t^{Hnet} h_t\right] + \sum_{t=0}^{T} \beta^t U'_C(t) K_{T+1} = U'_C(0) R_0^{Knet} K_0$$

Using the transversality condition, and replacing the net rent by the ratio of marginal utilities, we have the implementability condition:

$$\sum_{t=0}^{+\infty} \beta^t U'_C(t) C_t - \sum_{t=0}^{+\infty} \beta^t U'_H(t) h_t = U'_C(0) A_0$$

with

$$A_0 = R_0^{Knet} K_0$$

So the Ramsey problem is solved in maximizing the Lagrangien

$$\max_{C_t, c_t, h_t, K_{t+1}} \sum_{t=0}^{+\infty} \beta^t \left[u(t) + \gamma U(t)\right]$$
$$+\nu \left[U'_C(0) A_0 + \sum_{t=0}^{+\infty} \beta^t U'_H(t) h_t - \sum_{t=0}^{+\infty} \beta^t U'_C(t) C_t\right]$$
$$+ \sum_{t=0}^{+\infty} \lambda_t \left[f(K_t) + (1-\delta) K_t - c_t - C_t - K_{t+1}\right]$$

To prove the first part of Proposition 3, it is sufficient to derive the first order condition on C_t on the one hand, and K_{t+1} on the other hand, and apply the steady-state condition, assuming it exists, which gives:

$$(1 - \tau_K) \left[f'(K^*) + 1 - \delta K^*\right] = R^{Knet*} = 1/\beta$$

To prove the second part of Proposition 3, we need further assumptions. Assuming separability

$U(C_t, H_t) = U(C_t) + V(H_t)$, $u(c_t, h_t) = u(c_t) + v(h_t)$, we define

$$\begin{aligned} W(C_t, \nu) &= \gamma U(C_t) - \nu U'_C(t) C_t \\ \Gamma(H_t, \nu) &= \gamma V(H_t) + \nu U'_H h_t \end{aligned}$$

which allows to redefine the objective

$$\begin{aligned} &\max_{C_t, c_t, h_t, K_{t+1}} \sum_{t=0}^{+\infty} \beta^t \left[u(c_t) + v(h_t) + W(C_t, \nu) + \Gamma(h_t, \nu) \right] \\ &+ \nu U'(C_0) R_0 K_0 \\ &+ \sum_{t=0}^{+\infty} \lambda_t \left[f(K_t) + (1-\delta) K_t \; - c_t - C_t - K_{t+1} \right] \end{aligned}$$

The first order conditions are therefore:

$$u'_c(c_t) = \frac{\lambda_t}{\beta^t} \tag{C.21}$$

$$\beta^t W'_C(C_t, \nu) = \lambda_t; \text{ for } t \geq 1 \tag{C.22}$$

$$W'_C(C_0, \nu) = \lambda_0 - \nu U''_{cc}(0) A_0 \tag{C.23}$$

$$\lambda_t = \lambda_{t+1} \left[f'_K(K_{t+1}) + 1 - \delta \right] \tag{C.24}$$

and the last first order condition over the housing allocation:

$$\Gamma'_h(h_t, \nu) = -v'_h(t) \tag{C.25}$$

Using equations (C.21), (C.22) and (C.24), it is easy to see that the proof of Lemma 1 of a strictly positive tax on capital for $t > 1$ established for the case of Judd without land still applies and this proved Propostion 3, part 2a. To prove part 2b of Proposition 3, we can also derive the expression for the optimal tax rate of capital in period 1. So far, we established that the tax is zero after period 2. Now, using equations (C.22), (C.23), (C.24) for $t > 1$, one obtain, after using period 0 Euler equation $\frac{1}{R_1^{Knet}} = \frac{\beta U'_C(C_1)}{U'_C(C_0)}$ to remove marginal utilities in period 1, and for CRRA utility $\frac{U^{"}_{cc}(C_0) C_0}{U'_C(C_0)} = -\sigma_C$:

$$\left(1 - \frac{1}{R_1^{Knet}}\right) R_1^{Kgross} = \frac{\nu \sigma_C}{a} \frac{R_0^{Knet} K_0}{C_0} \tag{C.26}$$

A few steps lead to a formula for the optimal capital tax in period 1:

$$\tau_{K_1} = \frac{\sigma_C R_0^{Knet} K_0 + (-a/\nu)\, C_0 (R_1^{Kgross} - 1)}{\sigma_C R_0^{Knet} K_0 + (-a/\nu)\, C_0 R_1^{Kgross}}$$

The Rawlsian case is when $\gamma = 0$ or $a = -\nu(1-\sigma_C)$ and therefore,

$$\tau_{K_1} \quad = \quad \frac{\sigma_C R_0^{Knet} K_0 + (1-\sigma_C) C_0 (R_1^{Kgross} - 1)}{\sigma_C R_0^{Knet} K_0 + (1-\sigma_C) C_0 R_1^{Kgross}}$$

proving part 2b in Proposition 3.

Part 3) of Proposition 3[1] on second best taxes is as follows. For $t \geq 1$, using equations (C.21), (C.22) and (C.25) we deduce that for $t \geq 1$,

$$-\frac{\Gamma'_h(h_t,\nu)}{W'_C(C_t,\nu)} = \frac{v'_h(t)}{u'_c(t)} \tag{C.27}$$

Computing the terms in the LHS of (C.27), one obtain after a few steps, using $\sigma_H(H_t) = \frac{V''_H H_t}{V'_H}$, and $\sigma_C(C_t) = \frac{U''_{CC} C_t}{U'_C}$,

$$-\frac{\Gamma'_h(h_t,\nu)}{W'_C(C_t,\nu)} = \frac{v'_h}{u'_c(c_t)} \Leftrightarrow \frac{V'_H}{U'_C}(t)\left[\frac{\gamma - \nu\left[1 + \sigma_H(H_t)\frac{h_t}{H_t}\right]}{\gamma - \nu\left[1 - \sigma_C(C_t)\right]}\right] = \frac{v'_h}{u'_c}(t)$$

violating the first best condition

$$\frac{V'_H}{U'_C}(t) = \frac{v'_h}{u'_c}(t)$$

Since $\frac{V'_H}{U'_C}(t) = R^{Hnet}(t)$ and $\frac{v'_h}{u'_c}(t) = R^{Hgross}(t)$, one gets after simple steps that

$$\tau_H(t) = 1 - \frac{\gamma - \nu\left[1 - \sigma_C(C_t)\right]}{\gamma - \nu\left[1 + \sigma_H(H_t)\frac{h_t}{H_t}\right]}$$

which implies that $\tau_H(t) \lesseqgtr 0$: it cannot be signed in general. In the Rawlsian case, this simplifies to

$$\tau_H(t) = 1 - \frac{\nu\left[1 - \sigma_C(C_t)\right]}{\nu\left[1 + \sigma_H(H_t)\frac{h_t}{H_t}\right]}$$

This is strictly positive but, even in the isoelastic case, the second best tax on rents is not constant given that the ratio h_t/H_t is susceptible to vary.

For the case $t = 0$, using equations (C.21), (C.23) and (C.25) we deduce that for $t = 0$

$$-\frac{\Gamma'_h(h_t,\nu)}{W'_C(C_0,\nu) + \nu U''_{cc}(C_0) R_0^{Knet} K_0} = \frac{v'_h}{u'_c}(0) \tag{C.28}$$

[1]This is also part (ii) of Lemma 1, therefore this proof includes that of Lemma 1 ii).

Then (C.28) is equivalent to

$$\frac{V_H'}{U_C'}(0)\left[\frac{\gamma-\nu\left[1+\sigma_H(H_0)\frac{h_0}{H_0}\right]}{\gamma-\nu\left[1-\sigma_C(C_0)(1-R_0^{Knet}K_0/C_0)\right]}\right] \quad = \quad \frac{v_h'}{u_c'}(0)$$

Using again the intratemporal Focs of both agents to get

$$R^{Hnet}(0)\left[\frac{\gamma-\nu\left[1+\sigma_H(H_0)\frac{h_0}{H_0}\right]}{\gamma-\nu\left[1-\sigma_C(C_0)(1-R_0^{Knet}K_0/C_0)\right]}\right]=R^{Hgross}(0)$$

The intratemporal MRS between workers and capitalists are different and a tax in zero $\tau_H(0)\neq 0$ is also needed. In the Rawlsian case, one simplifies the expression to

$$\tau_H(0) \quad = \quad 1-\frac{1-\sigma_C(C_0)(1-R_0^{Knet}K_0/C_0)}{1+\sigma_H(H_0)\frac{h_0}{H_0}}$$

which even in the isoelastic case is not signed, because $1-\frac{R_0^{Knet}K_0}{C_0}<0$ whatever R_0^{Knet}. ■

D Online Appendix to Sub-Section 4.2

D.1 Proof of Lemmas 2 and 3: convergence of multipliers, model without housing

For the sake of the completeness of the proof, we restate the optimization problem as follows, allowing for a weight $m<1$ of capitalists - the text is the special case $m=1$.

$$\max_{K_{t+1},c_t,C_t}\sum_{t=0}^{\infty}\beta^t\left[u(c_t)+\gamma m U(C_t)\right]$$

$$f(K_t)+(1-\delta)K_t-c_t-mC_t-K_{t+1}=0\ ;\text{multiplier }\lambda_t\beta^t$$

$$\beta U_C'(t+1)(mC_{t+1}+K_{t+2})-U_C'(t)K_{t+1}=0\ ;\text{multiplier }\mu_t\beta^t \tag{D.1}$$

$$K_0\text{ given}$$

$$\beta^t U_C'(t)K_{t+1}\to 0\text{ transversality condition}$$

where the constraint of the capitalist has been introduced in the Euler equation. The first order conditions are:

$$\beta^t u'_c(t) = \beta^t \lambda_t \tag{D.2}$$

$$m\gamma U'_C(t) + \mu_{t-1}\left[U''_{CC}(t)(mC_t + K_{t+1}) + mU'_C(t)\right] = m\lambda_t + \mu_t U''_{CC}(t)K_{t+1} \tag{D.3}$$

$$\beta^{t+1}\lambda_{t+1}\left[f'(K_{t+1}) + 1 - \delta\right] + \beta^t \mu_{t-1}U'_C(t) = \beta^t \mu_t U'_C(t) + \beta^t \lambda_t \tag{D.4}$$

To prove Lemma 2, just assume that the two multipliers converge and the result is that capital must converge to the value insuring the first best level, $R^{Kgross}(K) = 1/\beta$. Further, we deduce from (D.2)

$$u'_c(t) = \lambda_t$$

Then (D.3) becomes

$$m\gamma U'_C(t) - mu'_c(t) + \mu_{t-1}\left[U''_{CC}(t)(mC_t + K_{t+1}) + mU'_C(t)\right] = \mu_t U''_{CC}(t)K_{t+1} \tag{D.5}$$

or dividing by $U''_{CC}(t)K_{t+1}$

$$\mu_t = \mu_{t-1}\left(\frac{mC_t + K_{t+1}}{K_{t+1}} + \frac{mU'_C(t)}{U''_{CC}(t)K_{t+1}}\right) + \frac{m\gamma U'_C(t)}{U''_{CC}(t)K_{t+1}} - \frac{mu'_c(t)}{U''_{CC}(t)K_{t+1}}$$

Using the power specification of the utility function,

$$U(C) = \frac{C^{1-\sigma_C}}{1-\sigma_C}$$

implying

$$\sigma_C = -\frac{U''}{U'_C}C$$

we obtain after trivial steps

$$\mu_t - \mu_{t-1} = \frac{mC_t}{\sigma_C K_{t+1}}\left[\mu_{t-1}(\sigma_C - 1) - \gamma + \frac{u'_c(t)}{U'_C(t)}\right] \tag{D.6}$$

Consider (D.6) when $\mu_t - \mu_{t-1} = 0$ at the limit. We check that the multiplier is positive. Seady sarting from

$$\mu(\sigma_C - 1) - \gamma + \frac{u'_c}{U'_C} = 0$$

with μ being the limit of μ_t in infinity and defining

$$\frac{\gamma U'_C}{u'_c} = \tilde{\gamma}_c$$

we finally obtain

$$\mu = \frac{\gamma(1-\tilde{\gamma}_c)}{\tilde{\gamma}_c(1-\sigma_C)} > 0 \tag{D.7}$$

proving Lemma 3. ■ Note that, when multiplier μ converges, it is easily shown that $RKgross = 1/beta$ (see for instance, step 3 in Appendix D.2 below in the more general case of land).

D.2 Proof of Propositions 4 to 6: With housing and a rent tax

The Ramsey problem we consider is the following, with separability in utility between consumption and housing and posing $h_t = \overline{H} - mH_t$:

$$\max_{c_t, C_t, H_t, K_{t+1}, \tau_{H,t}} \sum_{t=0}^{\infty} \beta^t \left[u(c_t) + v_h(h_t)\right] + \gamma m \left[U(C_t) + V(H_t)\right]$$

such that:

$$f(K_t) + (1-\delta)K_t - c_t - mC_t - K_{t+1} = \text{ 0 multiplier } \lambda_t \beta^t \geq 0 \tag{D.8}$$

$$\beta U'_C(t+1)\left[mC_{t+1} + K_{t+2} - R^{Hgross}_{t+1}(1-\tau_{H,t+1})h_{t+1}\right] - U'_C(t)K_{t+1} = 0 \quad \text{multiplier } \mu_t \beta^t \tag{D.9}$$

$$R^{Hgross}_t u'_c(c_t) - u'_h(t) = 0, \text{ multiplier } \eta_{1t}\beta^t \tag{D.10}$$

$$R^{Hgross}_t(1-\tau_{H,t})U'_C(t) - V'_H(H_t) = 0, \text{ multiplier } \eta_{2t}\beta^t \tag{D.11}$$

$$\tau_{H,t} \geq 0, \text{ multiplier } \beta^t \phi_t \geq 0 \tag{D.12}$$

$$\beta^t U'_C(t)K_{t+1} \to 0 \tag{D.13}$$

$$K_0 \text{ given}$$

In the program, we have inserted from the start the budget constraint of the capitalist in the Euler equation. The Walras Law implies that we can omit to write down the budget constraint of the worker. In the following we make a small abuse of notations in omitting the subscripts for the subutility of consumption and housing both for the worker and capitalist. The first order conditions are, simplifying by β^t:

- With respect to c_t

$$\lambda_t = u'_c(t) + \eta_{1t} R^{Hgross}_t u''_{cc}(c_t) \tag{D.14}$$

- With respect to C_t

$$m\gamma U_C'(t) + \mu_{t-1}\left\{U''(t)\left[mC_t + K_{t+1} - R_t^{Hnet}h_t\right] + mU_C'(t)\right\} \\ + \eta_{2t}U_{CC}''(t)R_t^{Hnet} = m\lambda_t + \mu_t U_{CC}''(t)K_{t+1} \tag{D.15}$$

- With respect to K_{t+1}

$$\beta\lambda_{t+1}\left[f'(K_{t+1}) + 1 - \delta\right] + \mu_{t-1}U_C'(t) = \mu_t U_C'(t)\lambda_t \tag{D.16}$$

- With respect to H_t

$$-mu_h'(t) + m\gamma U_H'(t) + m\mu_{t-1}\beta U_C'(t)R_t^{Hnet} + m\eta_{1t}u_{hh}''(t) - \eta_{2t}U_{HH}''(t) = 0 \tag{D.17}$$

- With respect to $\tau_{H,t}$

$$U_C'(t)R_t^{Hgross}h_t\mu_{t-1} - R_t^{Hgross}U_C'(t)\eta_{2t} + \phi\beta^{-t} = 0$$

or

$$h_t\mu_{t-1} - \eta_{2t} + \phi_t = 0 \tag{D.18}$$

- with in addition, the usual Kuhn and Tucker condition.

$$\phi_t\tau_{H,t} = 0$$

Proof of Proposition 4 (Optimal capital tax in the Ramsey problem with housing), Section 4.2.3:

The proof consists of exhibiting sufficient conditions for the Euler equation multiplier to be still positive. We express the other multipliers in terms of the Euler equation multiplier. The proof is in three steps.

Step 1: Expression of other multipliers in function of the Euler multiplier Since Proposition 3, we know that it is optimal to have a wedge on the rental market for any t, therefore $\phi_t = 0$. Then from (D.18), still using $h_t = \bar{H} - mH_t$

$$\eta_{2t} = h_t\mu_{t-1} \tag{D.19}$$

If quantities and μ_t converge, then η_{2t} converges. Plugging (D.19) in (D.17) gives

$$-mu_h'(t) + m\gamma U_H'(t) + m\mu_{t-1}\beta U_C'(t)R_t^{Hnet} + m\eta_{1t}u_{hh}''(t) - h_t\mu_{t-1}U_{HH}''(t) = 0$$

leading to

$$\eta_{1t} = \frac{-mu_h'(t) + m\gamma U_H'(t) + \mu_{t-1}\left[mR_t^{Hnet}U_C'(t) - h_t U_{HH}''(t)\right]}{-mu_{hh}''(t)}$$

Using (D.11) we get

$$\eta_{1t} = \frac{-mu_h'(t) + m\gamma U_H'(t) + \mu_t(mU_H'(t) - h_t U_{HH}''(t))}{-mu_{hh}''(t)}$$

Factorising $U_H'(t)$ and defining

$$\frac{\gamma U_H'(t))}{u_h'(t)} = \widetilde{\gamma}_h \tag{D.20}$$

$$\eta_{1t} = U_H'(t)\frac{-\frac{\gamma m}{\widetilde{\gamma}_h} + \gamma m + \mu_{t-1}\left(m - h_t\frac{U_{HH}''(t)}{U_H'(t)}\right)}{-mu_{hh}''(t)}$$

Finally denoting $\sigma_H(H_t) = -H_t\frac{U''(t)}{U_H'(t)} > 0$ we get

$$\eta_{1t} = \frac{U_H'(t)}{-mu_{hh}''(t)}\left[-\frac{\gamma m}{\widetilde{\gamma}_h}(1-\widetilde{\gamma}_h) + \mu_{t-1}\left(m + \frac{h_t}{H_t}\sigma_H(H_t)\right)\right] \tag{D.21}$$

If quantities and μ_t converge, then η_{1t} converges. From (D.14) we deduce that if η_{1t} converges, then λ_t converges.

Step 2 Expression of the Euler multiplier μ_t. Now we look at equation (D.15)

$$\begin{aligned}&m\gamma U_C'(t) + \mu_{t-1}\left\{U_{CC}''(t)\left[mC_t + K_{t+1} - R_t^{Hnet}h_t\right] + mU_C'(t)\right\} + \eta_{2t}U_{CC}''(t)R_t^{Hnet}\\ &= m\lambda_t + \mu_t U_{CC}''(t)K_{t+1}\end{aligned}$$

Using the expression (D.14) for λ_t and the expression (D.19) for η_{2t}, the above expression becomes

$$\begin{aligned}&m\gamma U_C'(t) + \mu_{t-1}\left\{U_{CC}''(t)\left[mC_t + K_{t+1} - R_t^{Hnet}h_t\right] + mU_C'(t)\right\} + \mu_{t-1}U_{CC}''(t)R_t^{Hnet}h_t\\ &= mu'(t) + \mu_t U_{CC}''(t)K_{t+1} + m\eta_{1t}\ R_t^{Hgross}u_{cc}''(c_t)\end{aligned}$$

Simplifying and dividing by $U_{CC}''(t)K_{t+1}$ and noticing that only the last term of the LHS is different from the expression obtained without housing (see D.5) one gets using (D.6)

$$\mu_t - \mu_{t-1} = \frac{mC_t}{K_{t+1}}\left[\mu_{t-1}(\sigma_C - 1) - \gamma + \frac{u_c'(t)}{U_C'(t)}\right] - \frac{m\eta_{1t}R_t^{Hgross}u_{cc}''(t)}{U_{CC}''(t)K_{t+1}}$$

Now replacing η_{1t} by its expression in (D.21)

$$\mu_t - \mu_{t-1} = \frac{mC_t}{\sigma_C K_{t+1}}\left[\mu_{t-1}(\sigma_C - 1) - \gamma + \frac{u_c'(t)}{U_C'(t)}\right] - \frac{R_t^{Hgross}u_{cc}''(t)U_H'(t)}{U_{CC}''(t)K_{t+1}u_{hh}''(t)}\left[\frac{\gamma m}{\widetilde{\gamma}_h}(1-\widetilde{\gamma}_h) - \mu_{t-1}\left(m + \frac{h_t}{H_t}\sigma_H(H_t)\right)\right]$$

or

$$\mu_t - \mu_{t-1} = \frac{mC_t}{\sigma_C K_{t+1}} \left[\mu_{t-1}(\sigma_C - 1) - \gamma + \frac{u_c'(t)}{U_C'(t)} \right] - \frac{R_t^{Hgross} u_{cc}''(t) U_H'(t)}{K_{t+1} u_{hh}''(t)} \frac{C_t U_C'(t)}{C_t U_{CC}''(t) U_C'(t)} \left[\frac{\gamma m}{\widetilde{\gamma}_h}(1 - \widetilde{\gamma}_h) - \mu_{t-1}\left(1 + \frac{h_t}{mH_t}\sigma_H(H_t)\right) \right]$$

In case of an isoelastic utility function on consumption, one gets introducing σ_C

$$\mu_t - \mu_{t-1} = \frac{mC_t}{\sigma_C K_{t+1}} \left[\mu_{t-1}(\sigma_C - 1) - \gamma + \frac{u_c'(t)}{U_C'(t)} \right] + \frac{C_t R_t^{Hgross} u_{cc}''(t) U_H'(t)}{\sigma_C K_{t+1} u_{hh}''(t) U_C'(t)} \left[\frac{\gamma m}{\widetilde{\gamma}_h}(1 - \widetilde{\gamma}_h) - \mu_{t-1}\left(m + \frac{h_t}{H_t}\sigma_H(H_t)\right) \right]$$

Putting $\frac{mC_t}{\sigma_C K_{t+1}}$ in factor and using (D.10)

$$\mu_t - \mu_{t-1} = \frac{mC_t}{\sigma_C K_{t+1}} \left[\mu_{t-1}(\sigma_C - 1) - \gamma + \frac{u_c'(t)}{U_C'(t)} + \frac{u_h'(t) u_{cc}''(t) U_H'(t)}{u_{hh}''(t) u_c'(t) U_C'(t)} \left(\frac{\gamma}{\widetilde{\gamma}_h}(1 - \widetilde{\gamma}_h) - \mu_{t-1}\left(1 + \frac{h_t}{mH_t}\sigma_H(H_t)\right) \right) \right]$$

Introducing σ_c and $\sigma_h(h_t) = -h_t \frac{U''(h_t)}{U'(h_t)}$

$$\mu_t - \mu_{t-1} = \frac{mC_t}{\sigma_C K_{t+1}} \left[(\mu_{t-1}(\sigma_C - 1) - \gamma + \frac{u_c'(t)}{U_C'(t)} + \frac{\sigma_c h_t U_H'(t)}{\sigma_h(h_t) c_t U_C'(C_t)} \left(\frac{\gamma}{\widetilde{\gamma}_h}(1 - \widetilde{\gamma}_h) - \mu_{t-1}\left(1 + \frac{h_t}{mH_t}\sigma_H(H_t)\right) \right) \right]$$

And using (D.11)

$$\mu_t - \mu_{t-1} = \frac{mC_t}{\sigma_C K_{t+1}} \left[\mu_{t-1}(\sigma_C - 1) - \gamma + \frac{u_c'(t)}{U_C'(t)} + \frac{\sigma_c h_t R_t^{Hnet}}{\sigma_h(h_t) c_t} \left(\frac{\gamma}{\widetilde{\gamma}_h}(1 - \widetilde{\gamma}_h) - \mu_{t-1}\left(1 + \frac{h_t}{mH_t}\sigma_H(H_t)\right) \right) \right]$$

Consider an interior steady-state. If the multipliers converge, at the limit $\mu_{t+1} - \mu_t = 0$. We are looking at the value of the limit value of the multiplier μ and we establish at which sufficient conditions it is positive

$$\mu(\sigma_C - 1) - \gamma + \frac{u_c'}{U_C'} + \frac{\sigma_c h R_t^{Hnet}}{\sigma_h c} \left[\frac{\gamma}{\widetilde{\gamma}_h}(1 - \widetilde{\gamma}_h) - \mu\left(1 + \frac{h_t}{mH_t}\sigma_H\right) \right] = 0$$

or

$$\mu(\sigma_C - 1) - \gamma \frac{\widetilde{\gamma}_c - 1}{\widetilde{\gamma}_c} + \frac{\sigma_c h R_t^{Hnet}}{\sigma_h c} \left[\frac{\gamma}{\widetilde{\gamma}_h}(1 - \widetilde{\gamma}_h) - \mu\left(1 + \frac{h_t}{mH_t}\sigma_H\right) \right] = 0$$

$$\gamma \frac{1 - \widetilde{\gamma}_c}{\widetilde{\gamma}_c} + \frac{\sigma_c h R_t^{Hnet}}{\sigma_h c} \frac{\gamma}{\widetilde{\gamma}_h}(1 - \widetilde{\gamma}_h) = \mu \left[1 - \sigma_C + \frac{\sigma_c h R_t^{Hnet}}{\sigma_h c} \left(1 + \frac{h}{mH}\sigma_H\right) \right]$$

$$\mu = \gamma \frac{\frac{1 - \widetilde{\gamma}_c}{\widetilde{\gamma}_c} + \frac{\sigma_c h R_t^{Hnet}}{\sigma_h c} \frac{1 - \widetilde{\gamma}_h}{\widetilde{\gamma}_h}}{1 - \sigma_C + \sigma_c \left[\frac{h R_t^{Hnet}}{c} \left(\frac{1}{\sigma_h} + \frac{h}{mH} \frac{\sigma_H}{\sigma_h} \right) \right]} \tag{D.22}$$

or in the case where $\sigma_C = \sigma_c$

$$\mu = \gamma \frac{\frac{1-\tilde{\gamma}_C}{\tilde{\gamma}_C} + \frac{\sigma_C h R_t^{Hnet}}{\sigma_h c}\frac{1-\tilde{\gamma}_h}{\tilde{\gamma}_h}}{1-\sigma_C+\sigma_C\left[\frac{hR_t^{Hnet}}{c}\left(\frac{1}{\sigma_h}+\frac{h}{mH}\frac{\sigma_H}{\sigma_h}\right)\right]} \tag{D.23}$$

We can then state the following Appendix Lemma generalizing Lemma 3 in the text.

Appendix Lemma A3. *Consider separable preferences conditions with CCRA subadditive utility of consumption. Suppose that there exists an interior steady-state, $K, c, C, h, H > 0$. Assume $\sigma_C = \sigma_c < 1$, $\tilde{\gamma}_c < 1$ and $\tilde{\gamma}_h < 1$. Then the multipliers μ_t converge to a positive value.*

Step 3 Dynamic accumulation of capital at the steady state Now we move to equation (D.16)

$$\beta^{t+1}\lambda_{t+1}\left[f'(K_{t+1})+1-\delta\right]+\beta^t\mu_{t-1}U_C'(t)=\beta^t\mu_tU_C'(t)+\beta^t\lambda_t$$

which becomes

$$\frac{\lambda_{t+1}}{\lambda_t}\left[f'(K_{t+1})+1-\delta\right]=\frac{1}{\beta}+\frac{\mu_t-\mu_{t-1}}{\beta\lambda_t}U_C'(t)$$

If there is an interior steady-state, we have shown that the multipliers μ_t converge with the stated conditions and step 1 shows that if μ_t converges, then λ_t also converges. We deduce that $f'(K_{t+1}) + 1 - \delta) = \frac{1}{\beta}$ and then $R^{Kgross} = \frac{1}{\beta}$.

The Euler equation of the capitalist at the steady-state implies that $\tau_K = 0$ and then the stock of capital remains the same as in the first best. ■

Proposition 3 shows that tax wedges should be introduced in the rental market. A simple surplus analysis shows that a rental subsidy combined with a tax on the worker dominates a rental tax combined with a subsidy to the worker at the steady-state.

Taking stock of the all previous steps, we can state Proposition 5 in the text.

Proof of Proposition 5 in the paper, Section 4.2.3: *Under the assumptions stated in Lemma 2, the optimal rental tax at the steady-state is given by*

$$\frac{\tau_H}{1-\tau_H}=\frac{1-\tilde{\gamma}_c}{\epsilon_s}$$

where ϵ_s the elasticity of the rental housing supply with respect to the net rent, being computed at the housing equilibrium.

Proof: We consider the stationary state where capital and wage rate are at their first best values by virtue of Proposition 2. Let us denote them respectively K^* and w^*. The Ramsey problem we have to solved is simpler than the original one given by conditions (D.8) to (D.11), since we do not have any more to consider explicitly condition (D.9), the Euler equation. The separability utility is specifically helpful here because the marginal utility of consumption of the capitalist does not depend on his consumption of housing. The conditions (D.10) and (D.11) still hold and it is therefore useful to express the problem using the indirect utilities functions of the capitalist and the worker. Let us denote them respectively $\hat{V}(\tau_H)$ and $\hat{v}(\tau_H)$. At the steady-state, the decision maker maximizes the weighted sum of indirect utility functions

$$\begin{aligned} Max_{\tau_H, 0<\tau_H<1, T^H} W = \hat{v}(w^* + T^H - R^{Hgross}(\tau_H)h, h) \\ + \gamma m \hat{V}\left(\frac{1-\beta}{\beta}\frac{K^*}{m} + (1-\tau_H)R^{Hgross}(\tau_H)(\overline{H}/m - H), H\right) \end{aligned}$$

under the budget constraint

$$T^H = \tau_H R^{Hgross}(\tau_H) h(R^{Hgross}(\tau_H)), \text{ multiplier } \lambda$$

It is somewhat simpler to work with the ad valorem tax $\widetilde{\tau_H}$ and the net-of-tax rent R^{Hnet}

$$R^{Hnet}(1+\widetilde{\tau_H}) = R^{Hgross}$$

than to work with τ_H defined by

$$R^{Hnet} = R^{Hgross}(1-\tau_H)$$

The two tax rates are related by

$$\widetilde{\tau_H} = \frac{\tau_H}{1-\tau_H} \tag{D.24}$$

We define the elasticity of the rental supply with respect to the net-of-tax rent R^{Hnet}

$$\epsilon_s = \frac{(\overline{H} - mH)' R^{Hnet}}{\overline{H} - mH}$$

or with $h(R^{Hnet}) = \overline{H} - mH(R^{Hnet}) = m\left(\frac{\overline{H}}{m} - H\right)$

$$\epsilon_s = \frac{h'(R^{Hnet})R^{Hnet}}{hR^{Hnet}} \tag{D.25}$$

Now the objective function reads

$$W(T^H, \widetilde{\tau_H}) = \hat{v}(w^* + T^H - R^{Hnet}(1+\widetilde{\tau_H})h, h) + \gamma m \hat{V}\left(\frac{1-\beta}{\beta}\frac{K^*}{m} + R^{Hnet}\frac{h(R^{Hnet})}{m}, H\right)$$

under the resource constraint

$$T^H = \widetilde{\tau_H} R^{Hnet}(\widetilde{\tau_H}) h(R^{Hnet}(\widetilde{\tau_H}))$$

Using the envelope theorem, the first order conditions with respect to T and $\widetilde{\tau_H}$ are:

$$\hat{v}'_c(c) = \lambda$$

$$\hat{v}'_c(c)\left[-R^{Hnet}h - (1+\widetilde{\tau_H})R'^{Hnet}h\right] + \gamma m \hat{V}'_C.(.)\left(R'^{Hnet}\frac{h}{m}\right) + \lambda\left[R^{Hnet}h + \widetilde{\tau_H}(R'^{Hnet}h + R^{Hnet}h'R'^{Hnet})\right] = 0$$

Eliminating λ and regrouping terms we obtain

$$\hat{v}'_c(c)\left[-(1+\widetilde{\tau_H})R'^{Hnet}h + \widetilde{\tau_H}(R'^{Hnet}h + R^{Hnet}h'R'^{Hnet})\right] + \gamma \hat{V}'_C(C)(R'^{Hnet}h) = 0$$

Dividing by $\hat{v}'_c(c)$ and using the definition of $\widetilde{\gamma}_c$

$$-(1+\widetilde{\tau_H})R'^{Hnet}h + \widetilde{\tau_H}(R'^{Hnet}h + R^{Hnet}h'R'^{Hnet}) + \widetilde{\gamma}_c(R'^{Hnet}h) = 0$$

Dividing by $R'^{Hnet}h$ we obtain

$$-1 - \widetilde{\tau_H} + \widetilde{\tau_H}(1+\epsilon_s)) + \widetilde{\gamma}_c = 0$$

or

$$\widetilde{\tau_H} = \frac{1-\widetilde{\gamma}_c}{\epsilon_s}$$

and using (D.24) we get

$$\frac{\tau_H}{1-\tau_H} = \frac{1-\widetilde{\gamma}_c}{\epsilon_s}$$

■

E Online Appendix: Further empirical facts leading to parameter choices, simulation

E.1 Trends in the land component of wealth

An observation derived from Figure 1 in introduction is the specific role of developed land in the trends. Housing prices can be decomposed into a land component and construction costs (or alternatively the price of “structures”). The land component is almost a fixed factor: it is not easily reproducible or at least relatively inelastic in the short run due to geographical or legal barriers; while housing structures are the outcome of current or past residential investments and are more elastic. Therefore, before proceeding with a normative analysis, it is worth separating out the role of each respective component of housing.

To the original Piketty series, we added a decomposition of housing into land and structures when available, typically after the 1970's. Following the methodology in Davis and Heathcote (2007), one uses a perpetual inventory method similar to the one used to recover capital stocks. This decomposes the evolution of each component (land and structure). It is further illustrated in panel f) which presents the developed land leverage (the share of land in the value of housing wealth). In all countries, the share of land in housing has been rising over the last 4 decades. Nowadays, it represents about 50% of the housing wealth in Canada, France and the UK. We can therefore conclude that occupied land is the main factor responsible for the rise of housing wealth and therefore in the wealth-to-income ratios. This is especially visible in the two countries where the wealth-to-income ratio increased most since the 1950's, France and the UK.

Our analysis rejoins that of other works on the overall trends of land. In particular, similar conclusions on the importance of land price dynamics for a sample of 14 OECD countries were found in Knoll et al. (2017). Most of the appreciation in housing prices comes from an appreciation of land prices while construction costs only went through a moderate increase[2]. Housing structures are endogenous and likely to increase with land prices. As land becomes scarce, it becomes more intensely used. That's the essence of the Muth extension of Alonso's model Muth (1969).

The reasons behind the rise of residential land value are beyond the scope of this paper and have been discussed convincingly in a growing stream of literature having notably documented the large values of urban land (Albouy (2016); Albouy et al. (2018)) which might be partially explained by land use regulation (Albouy and Ehrlich (2018); Glaeser et al. (2005)).

E.2 Distribution and composition of wealth for the calibration exercise

Table E.1 represents the share of assets in different part of the wealth distribution in France and the US from different sources, Garbinti et al. (2020, 2018); Saez and Zucman (2016). Our focus is on the share of non-housing assets in the top 10% of the wealth distribution that justifies our choice of $m = 1$. It is easy to verify from the table that the share of non-housing wealth of the top 10% in France is 71.4%, and that the share of non-housing wealth of the top 10% in the US is 79.3%.

[2]We do not take a stance on the reasons for the increasing importance of land. It might be caused by the lack of innovation in the transportation sector (Knoll et al., 2017), a concentration of individuals within the national territory in major agglomerations, it could be financial shocks and innovations as in Garriga et al. (2019) who study the decorrelation of US housing prices and rents due to financial shocks. It can finally be due to land regulation interacting with growing housing demand. The causes of these dynamics are left for further research but our empirical findings rejoin those of Knoll et al. (2017), Geerolf (2018), Grossmann and Steger (2016) and Borri and Reichlin (2016). Finally, even if our focus is on the capital stock, it is worth noting that similar remarks can apply to trends in capital income: Cette et al. (2019) recently emphasized that the decline in labor share could partially be explained by the rise of real estate income.

	Total wealth	Housing	Financial assets	Business Assets	Others (incl Pensions)
Panel a) Net wealth in France in 2014 Garbinti et al. (2020)					
Bottom 90%	45%	30%	6%	2%	6.8%
90-99	32%	14%	11%	5%	2%
Top 1%	23%	5%	17%	1%	0.2%
Total	100%	48%	34%	8%	9%
Panel b) Net wealth in the US in 2012 Saez and Zucman (2016)					
Bottom 90%	22.8%	5%	-1%	3%	16%
90-99	35.4%	8%	11%	3%	14%
Top 1%	41.8%	3.4%	28%	5%	6%
Total	100%	16%	37%	10%	36%

Table E.1: Distribution and composition of wealth, France and the US.

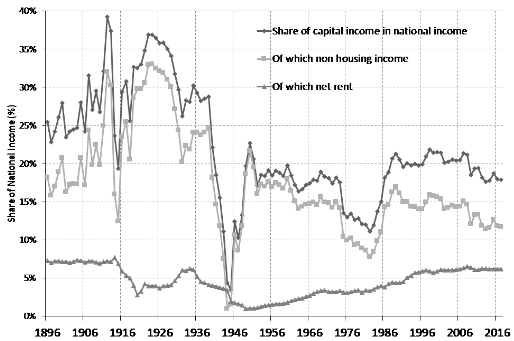
Housing property is more widespread than financial asset, but on the other hand, land ownership is more concentrated: According to Wolff (2017) in the U.S., the wealthiest 10% owned 82% of undeveloped land which is a buffer stock of developed land. The concentration of land is even greater in England where half of land is owned by less than 1% of its population, e.g. Shrubsole (2019).

E.3 Share of housing in consumption

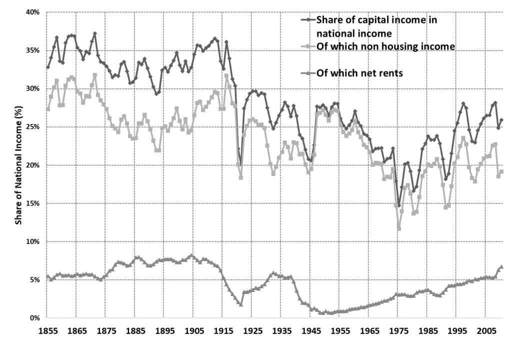
Figure E.1 represents the share of rents and imputed rents in our sample as a fraction of National Income. It is used to determine parameter α in the simulation. It conveys two main messages, depending on the time horizon considered. First, over the very long-run, one does not observe a secular *increase* in the share of physical capital income relative to national income as illustrated in panel a) and b) for France and United Kingdom respectively. Second, returns to capital with and without housing capital diverge. There is notably a decline in France and in the UK, over the second half of the samples, in the share of non-housing capital (corporate capital income, share of self-employment net income, foreign capital income, net govt. interest payments), and a relative stability or mild increase in Canada and the US, Germany facing instead a rise in those returns but over a much shorter period of analysis. There is a relative stability over the 20th century of the share of net rents in national income, but the returns rose in relative value quite a lot since WWII[3], which compensates for the decline of the returns to physical capital in the UK and France. This increase in net rents over the second half of the XXth century appears in most of the countries in our sample with the exception of Germany. The relative rise of rents over the last decades has been documented for several other OECD countries in Cette et al. (2019).

[3]The income from housing capital is the sum of paid rents and of the implicit income of homeowners (56% of the population). It is back to its 7% value of the beginning of the XXst century. Note that, on panels a) in Figure E.1, we use the most recent statistics for rental income as described in the note below the Figure.

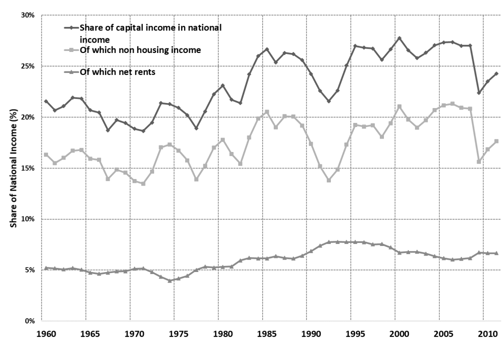
Figure E.1: Decomposition of the share of capital income in national income: the role of housing rents in **France, US, Canada, UK and Germany**.



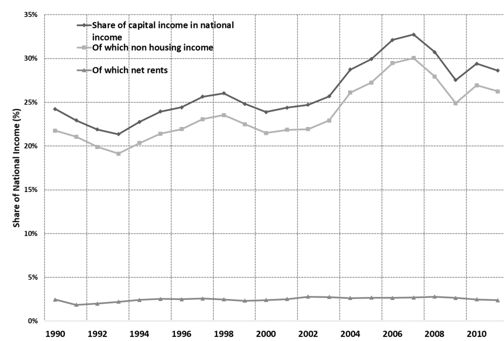
(a) France



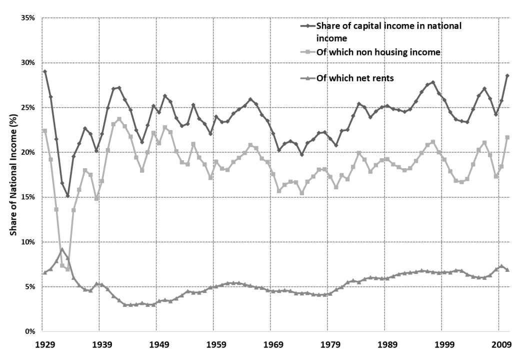
(b) United Kingdom



(c) Canada



(d) Germany



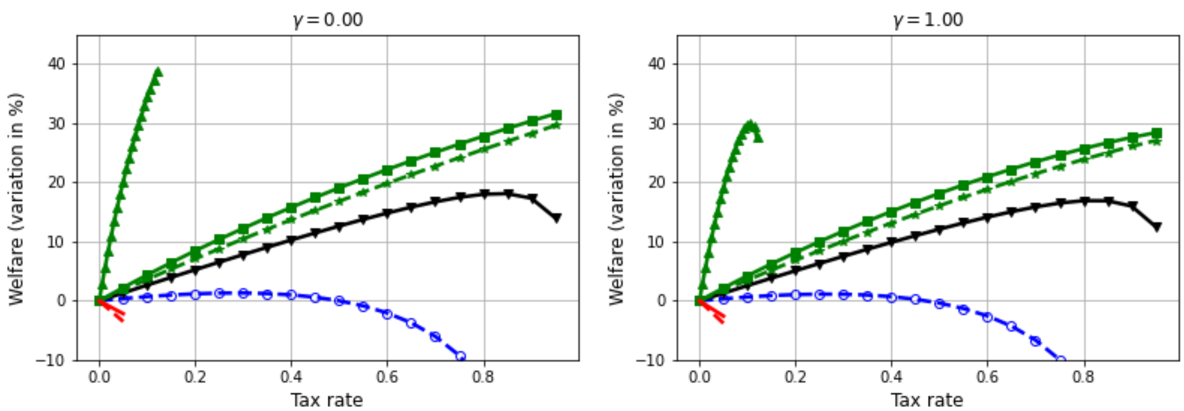
(e) United States

The upper series are the addition of implicit and monetary rents (lowest series) and non housing capital income (intermediate series)
Sources panel a) INSEE, National Account - Base 2014 from Friggit (2018) which is an update from Piketty and Zucman (2014) - `http://piketty.pse.ens.fr/en/capital21c`)
Source panel b)-e) Piketty and Zucman (2014)

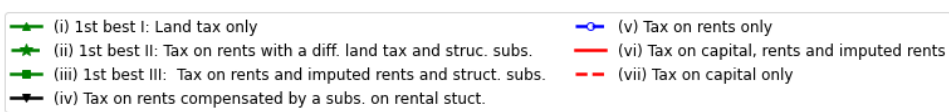
E.4 Supplementary simulation graphs with endogenous structures

In Figure E.2 we report in the top panels of the graph the change in welfare relative to the tax rate of each scheme. In particular, the tax on land is expressed relative to price of properties defined as $P = R^{Hnet}/(1-\beta)$, so that the tax rate of land relative to the price is $\hat{\tau}_{\bar{\mathcal{L}}} = T^{\mathcal{L}}(1-\beta)/R^{Hnet}$.

Figure E.2: Variation in the social welfare function $u(c_t)+\gamma mU(C_t)$ with various tax schemes, as % of their tax base. . Endogenous structures.



(a) Welfare and tax rate, $\gamma = 0$ (b) Welfare and tax rate, $\gamma = 1$

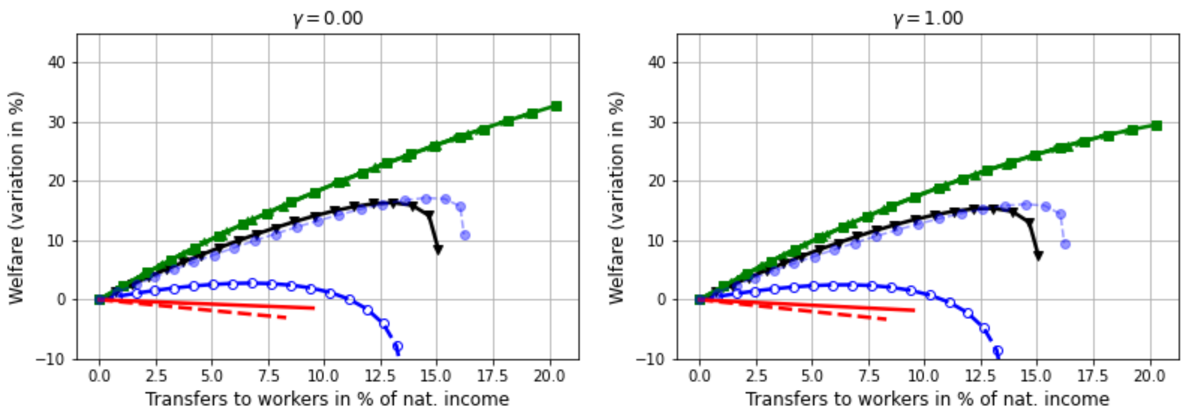


Figure's note. Endogenous structures, benchmark parameters. Variation in the social welfare function $u(c) + \gamma mU(C)$ and the tax revenue in the decentralized equilibrium, for different values of γ, the social welfare weight (respectively 0 and 1) with a mass of capitalist of 0.1.
Comparison between first best policies:
(i) an homogeneous tax on land $\hat{\tau_{\mathcal{L}}}$ redistributed to workers [plain green line with triangles up]
(ii) with a positive tax on rents $\tau_H > 0$, a differentiated tax on land ($\Delta\tau_{\mathcal{L}} = \tau_H R^{Hgross} h_l'$) and a subsidy to housing structures of tenants ($\tau_s = -\tau_H$) [discontinued green line with stars]
(iii) a tax on rent ($\tau_H > 0$), imputed rent ($\tau_{HI} = \tau_H \frac{h_l'}{H_{\mathcal{L}}'}$) combined with subsidies on structures ($\tau_s = \tau_S = -\tau_H$) [plain green line with squares],

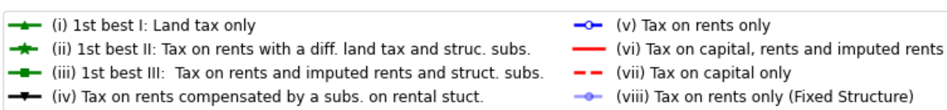
and second best policies and other distortive policies:
(iv) a tax on rents compensated by a subsidy on residential investments $\tau_s < 0$ [plain black line with triangles down],
(v) a tax on rents $\tau_H > 0$ alone [discontinued blue line];
(vi) a tax on capital equalized to the tax on rents and imputed rents $\tau_K = \tau_{HI} = \tau_H$ [plain red line];
(vii) a pure tax on capital ($\tau_K > 0$) [dashed red line].
x-axis is the respective tax rates; for ii), iii), iv) and vi) the rate on rents is reported.

E.5 Robustness check with a land share set to 0.5 in the production of housing

Figure E.3: Variation in the social welfare function $u(c_t)+\gamma mU(C_t)$ with various tax schemes, as a proportion of the transfers to workers. Robustness check when the land share is 50%



(a) Welfare and transfers, $\gamma = 0$ (b) Welfare and transfers, $\gamma = 1$



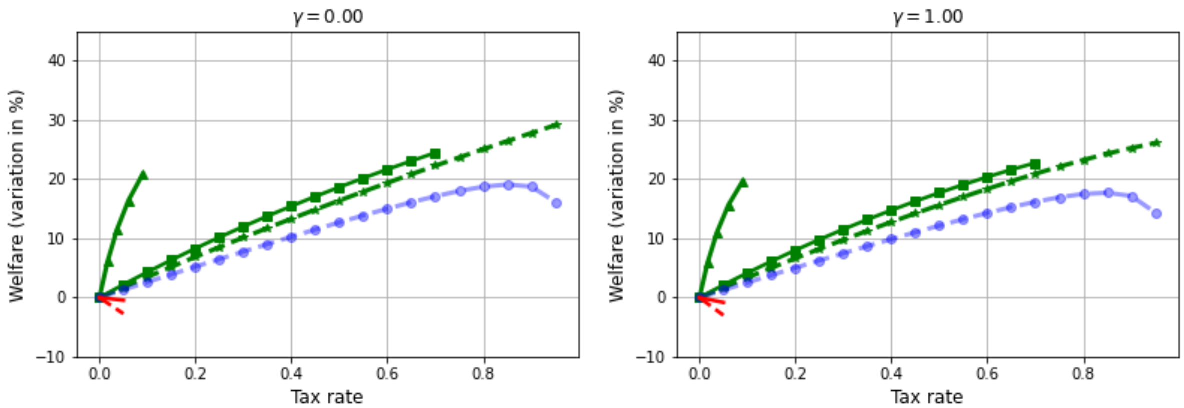
Figure's note. Endogenous structures with a larger share of land (0.5) in housing. Variation in the social welfare function $u(c) + \gamma mU(C)$ vs tax revenue in the decentralized equilibrium, for different values of γ, the social welfare weight (respectively 0 and 1) with a mass of capitalist of 0.1.
Comparison between first best policies:
(i) an homogeneous tax on land $\hat{\tau_{\mathcal{L}}}$ redistributed to workers [plain green line with triangles up],
(ii) a positive tax on rents $\tau_H > 0$, a differentiated tax on land ($\Delta\tau_{\mathcal{L}} = \tau_H R^{Hgross} h'_l$) and a subsidy to housing structures of tenants ($\tau_s = -\tau_H$) [discontinued green line with stars]
(iii) a tax on rents ($\tau_H > 0$), imputed rents ($\tau_{HI} = \tau_H \frac{h'_l}{H'_{\mathcal{L}}}$) combined with subsidies on structures ($\tau_s = \tau_S = -\tau_H$) [plain green line with squares],
and
and second best policies and other distortive policies:
(iv) a tax on rents compensated by a subsidy on residential investments $\tau_s < 0$ [plain black line with triangles down], *
(v) a tax on rents $\tau_H > 0$ alone [discontinued blue line, empty circles];
(vi) a tax on capital equalized to the tax on rents and imputed rents $\tau_K = \tau_{HI} = \tau_H$ [plain red line];
(vii) a pure tax on capital ($\tau_K > 0$) [dashed red line].
(viii) The last exercise simulate the impact of a tax on rents only $\tau_H > 0$, when structures are exogenous [blue and transparent dotted line, plain circles]
X-axis expresses taxes as the respective total tax revenue as a function of national income Ω.

E.6 Different tax schemes on land and capital when structures are fixed

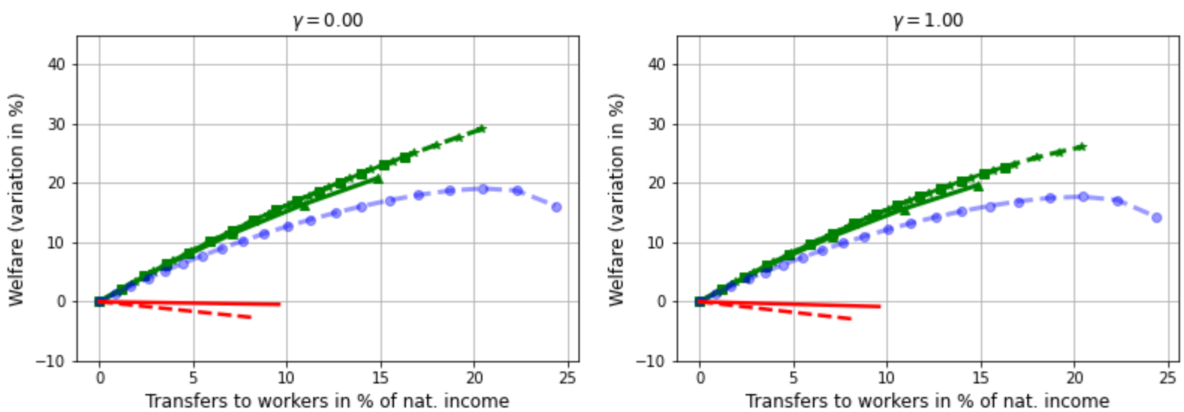
We now compare the role of housing taxation with and without endogeneity of structures. When exogeneous, structure are assumed to be fixed to the steady-state value calculated previously: $s = s^{\star}$ and $S = S^{\star}$.

For completeness, Figure E.4 reports the welfare variation under all tax schemes explored in the text but when structures are fixed. One can observe that the three first best schemes dominate the simple tax on rents even when structures are fixed as distorsions subsist on the intra-period allocation when rent taxation is not compensated by a differentiated tax on land or imputed rent taxation.

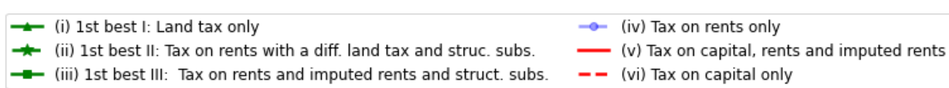
Figure E.4: Variation of the social welfare function $u(c_t) + \gamma m U(C_t)$ with various tax schemes. Exogenous structures set at their steady-state values without taxes.).



(a) Welfare and tax rate, $\gamma = 0$ (b) Welfare and tax rate, $\gamma = 1$



(c) Welfare and transfers, $\gamma = 0$ (d) Welfare and transfers, $\gamma = 1$



Figure's notes. Exogenous structures at $S = S^*, s = s^*$, benchmark parameters. Variation in the social welfare function $u(c) + \gamma m U(C)$ and the tax revenue in the decentralized equilibrium when structures are fixed, for different values of γ, the social welfare weight (respectively 0 and 1).
Comparison between first best policies:
(i) an homogeneous tax on land $\hat{\tau_{\mathcal{L}}}$ redistributed to workers [plain green line with triangles up]
(ii) a positive tax on rents $\tau_H > 0$, a differentiated tax on land ($\Delta\tau_{\mathcal{L}} = \tau_H R^{Hgross} h_l'$) and a subsidy to housing structures ($\tau_s = -\tau_H$) [discontinued green line with stars]
(iii) a tax on rents ($\tau_H > 0$), imputed rents ($\tau_{HI} = \tau_H \frac{h_l'}{H_{\mathcal{L}}'}$) combined with subsidies on structures ($\tau_s = \tau_S = -\tau_H$) [plain green line with squares],
and second best policies and other distortive policies:
(iv) a tax on rents $\tau_H > 0$ alone [discontinued blue line];
(v) a tax on capital equalized to the tax on rents and imputed rents $\tau_K = \tau_{HI} = \tau_H$ [plain red line];
(vi) a pure tax on capital ($\tau_K > 0$) [dashed red line].
Top panels: x-axis is the respective tax rates. The rate on rents is reported in ii), iii) and v);
Bottom panels: x-axis is is the total transfers to workers in % of national income Ω.

F Online Appendix: The measurement of housing capital by statistical agencies: the example of France (INSEE)

Piketty and Zucman's measurement of capital "*follows the most recent international guidelines as set forth in the 2008 System of National Accounts*" Piketty and Zucman (2013). They use series of statistical agencies to measure their capital/income ratio. The measurement of capital, and in particular housing capital, follows a particular methodology which is summarized and highlight to justify the adjustments proposed here.

According to OECD (2001), the framework used in several countries is the perpetual inventory methodology (PIM) briefly described by Piketty and Zucman: "*The goal of the perpetual inventory method (PIM) is to approximate the current market value of a number of capital assets when it cannot be directly observed. The general idea is that this value can be approximated by cumulating past investment flows and making suitable price adjustments.*" Piketty and Zucman (2013).

Our main interest will be in the "*suitable price adjustments*" for the housing market, namely the price index used to evaluate the value of the volume of capital. The perpetual inventory methodology used by the French institute of statistics (INSEE) to provide an estimate of the national housing capital stock is described as follows.

F.1 Measurement of the stock (volume) of housing capital in a reference year

"*In France, housing capital is estimated through a first step of estimating the total stock and value of housing in a reference year (1988). INSEE then follows over time the evolution of the number of buildings from aggregate housing investments, deflated the housing construction index; and the evolution of land with constructs using the evolution of the surface area covered by housing units and the development of the surface area covered by houses. To get the year- by-year value of housing capital stock, the above- described volume is multiplied by the price index of existing housing. Furthermore, new buildings were also evaluated at the price of existing housing units. Hence, housing capital follows year-to-year evolutions of housing prices, by contraction.*"

The assessment of the housing capital starts indeed from an initial survey-based assessment in the reference year. Two main surveys of 1988 (French Housing Survey and Survey on building land[4] were used to assess the housing capital stock divided between buildings and their underlying land.

From the surveys we calculate:

$$K_{1988}^{Housing} = K_{1988}^{Dwelling} + K_{1988}^{land}$$

[4]In 1988, two surveys are available: the housing survey "Enquête Logement", (EL) and information gathered by tax authority on land price (from the IMO file from the Direction Générale des Impôts which provides for the last time in 1988 an evaluation of the price of building land) Baron (2008).

where the housing capital which is broken down between dwellings and land.

F.2 The time evolution of the volume of housing capital

As we saw, housing capital is divided between land and dwellings, the evolutions of which are followed separately.

For dwellings, the stock in the following years is computed iteratively from the initial year[5] taking into account depreciation (δ)[6] and yearly capital increments (Gross Fixed Capital Formation, or GFCF)) deflated by construction costs index (CCI)[7]:

$$Vol(K_{n+1}^{Dwelling}) = (1-\delta)Vol(K_n^{Dwelling}) + \frac{GFCF_{n+1}}{CCI_{n+1}}$$

For land, the statistical agency similarly follows the changes in developed land on the national territory with respect to the reference year using an index of the surface developed (S):

$$Vol(K_{n+1}^{land}) = S_{n+1} \times K_{1988}^{land}$$

Finally, the volume of housing capital is just the addition of both series for each year at the price of the reference year, here 1988[8]:

$$Vol(K_{n+1}^{Housing}) = Vol(K_{n+1}^{Dwelling}) + Vol(K_{n+1}^{land})$$

F.3 Pricing of the evolution of housing capital and decomposition

The volume of capital is then obtained multiplying its volume by the house price index[9] (HP):

$$K_{n+1}^{Housing} = HP_{n+1} \times Vol(K_{n+1}^{Housing})$$

This value (used in *Capital in the 21st Century*) is then broken down into land and dwellings. This step appears to be the most important to understand our reasoning since it shows that housing capital is evaluated at the market price of year n+1. The value of the structure alone can be recovered by multiplying the volume of dwelling with the construction price index:

$$K_{n+1}^{Dwelling} = Vol(K_{n+1}^{Dwelling}) \times CCI_{n+1}$$

[5]For each of these years after 1988, the value of the stock of housing will be calculated step by step.Baron (2008).

[6]From the CCF rate (that is, the depreciation rate) we compute net capital.Baron (2008)

[7]To evaluate the net capital of 1988 at 2000s prices, we deflate using the construction cost index as a price index for buildings Baron (2008).

[8]E.g., the volume of housing capital at the end of 1989 (at 1988 prices) is found adding the net capital flow of end 1989 evaluated at 1988 prices Baron (2008).

[9]"The housing patrimony at the end of 1989 is obtained multiplying by the price index for the whole France " Baron (2008).

The developed land capital is the residual:

$$K_{n+1}^{land} = K_{n+1}^{Housing} - K_{n+1}^{Dwelling}$$

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