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Speculative and Precautionary Demand for Liquidity in Competitive Banking Markets

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Abstract

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JEL Classification: D11, D86, E21, E22, G21, L22

Keywords: expenditure needs, Investment opportunities, liquidity insurance, Penalty rates, competitive bank business models

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Speculative and Precautionary Demand for Liquidity in Competitive Banking Markets*

Diemo Dietrich† Thomas Gehrig‡

April 26, 2022

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1 Introduction

Already Adam Smith (1776) considered the ability of society to create, and utilize, liquidity as key for efficiently directing savings to investments. But what precisely is liquidity in this context? In this paper, we place uncertainty about the timing of future productive investment opportunities into the focus of attention. Such uncertainty generates a preference for liquidity as it induces a desire in investors to be flexible and able to withdraw from previous investments in order to take advantage of more lucrative opportunities as and when they arrive. While establishing a direct link between liquidity and the efficiency of capital allocation, this perspective has not gained much attention. We aim to bridge that gap.

Our first objective is to explore how banks can meet a preference for liquidity based upon *preserving investors' flexibility to seize future investment opportunities*. We focus on banks because the creation of liquidity is widely considered a fundamental function that they provide for society.[1] The key questions in this regard are: What does an efficient creation of liquidity for those investors look like? What intermediation arrangements will arise? To what extent do credit frictions matter?

Our second objective is to better understand bank liquidity creation as an *equilibrium outcome in the presence of various motives for liquidity demand*. We do so because banks create liquidity through a variety of on- and off-balance sheet activities (Berger and Bouwman, 2009), which suggests that a preference for liquidity can arise for various reasons. This view gives rise to a range of new questions. Above all, how does the *co-existence* of diverse motives for liquidity preferences affect market outcomes? How does the co-existence affect banks' business models? Will liquidity preferences arising for different reasons be served by different, specialist banks or will multi-product banks emerge?

Against this backdrop, we analyze bank liquidity creation when liquidity preferences based upon preserving flexibility for future investment opportunities co-exist with liquidity preferences based upon taking precautions against sudden consumption needs. The latter was pioneered by Bryant (1980) and Diamond and Dybvig (1983) and has since been a standard explanation of liquidity

[1] Since Jones and Ostroy (1976, 1984) a similar real option value has been ascribed to safe and liquid assets such as outside money, while disregarding additional efficiency gains attainable through financial intermediation.

demand in modern banking theory.[2] Starting with each motive in isolation, we establish that the optimal contract for taking precautions against consumption risks indeed differs from contracts that optimally preserve flexibility. The former is known to look like a deposit contract that pays an insurance benefit upon early withdrawal, while the latter turn out to resemble either a share in a bank's equity combined with a credit line, or a long-term savings contract with a penalty rate applicable when the account is closed before expiry.

Next, we study equilibrium bank liquidity creation if the two motives for liquidity preferences co-exist. Typically, the motives will interfere with each other such that market outcomes obtained from just a single motive as, e.g., in the Diamond and Dybvig (1983) model, will no longer hold. Only in a well defined, limited parameter range will the two motives not interfere with each other, provided standard financial frictions apply. Outside this range, limits on bank liquidity creation arise. For example, there are conditions under which the degree of maturity transformation associated with taking precautions against sudden consumption needs is inefficiently high. We also identify conditions under which competitive banks are not able to discriminate motives for liquidity preferences at all; instead, pooling is the only equilibrium outcome and liquidity can only be created by non-specialist banks. Finally, we establish conditions for which pure-strategy competitive equilibria do not exist and bank liquidity creation is inherently unstable. These findings contrast sharply with the case in which there are no significant financial friction.[3] There, the co-existence of motives generates scope for synergies. Banks that serve both motives simultaneously can take advantage of such synergies.

It is important to note that considering the co-existence of various motives for liquidity preferences in an economy opens a wider perspective than the mere co-existence of deposit taking and lending within a single bank. In this sense Kashyap et al. (2002) argue that banks can economize on costly reserve holdings through diversifying liquidity outflows from both sides of their balance sheet.

[2]This idea has been followed up by a long list of research including Calomiris and Kahn (1991), Dewatripont and Tirole (1994), Hellwig (1994), Diamond and Rajan (2001), Bhattacharya and Gale (2009), and Kahn and Wagner (2021), to name only a few. Holmström and Tirole (1998), and similarly Bolton et al. (2011), give another explanation for liquidity demand. There, for existing capital investments in production to maintain profitability, firms may have to secure follow-up funding at some point after the actual investment.

[3]As standard, market incompleteness is a prerequisite for any preference for liquidity.

By contrast, our interest lies in the competitive relations between the underlying liquidity motives themselves. For example, without financial frictions, banks take advantage of hitherto unnoticed synergies. Specifically, the credit lines granted to those who want to preserve flexibility generate excess returns for banks. Those excess returns ease a bank's constraints on its liquidity creation for all investors, including for those who take precautions against sudden consumption needs. With financial frictions, while credit lines and deposit taking do not create synergies, the co-existence of various motives for liquidity preferences affects equilibrium bank liquidity creation in non-trivial ways, and can even cause a market breakdown.

Our analysis has a number of implications for economic policy and financial regulation. Firstly, and importantly, the nature of liquidity matters.[4] This holds for market outcomes, but also for any potential policy intervention. Given the possible synergies, and limitations, that arise from a co-existence of diverse motives for liquidity preferences, any potential regulatory action will have to take a broad view on overall bank liquidity creation rather than a narrow focus on single lines of business. Secondly, financial frictions are key determinants for a possible interdependence of different liquidity services. Thus, under co-existence of motives, maturity transformation performed by banks may even be excessive relative to when a preference for liquidity exists for only a single reason. Moreover, to the extent that coordination problems affect the stability of banks, such risk is larger if associated with liquidity preferences arising from taking precautions than from preserving flexibility. Accordingly, banks are less fragile if they create liquidity for preserving flexibility than for precautionary reasons. Indeed, fragility of the latter type of banks can be even more pronounced due to the co-existence of both motives. Thirdly, while in equilibrium both bank business models earn zero profits, serving the desire for preserving flexibility requires lower levels of reserves and earns higher returns on assets than serving the need for taking precautions. This can have implications for, e.g., internal incentive schemes as managers in different business lines of a bank should be assessed according to different performance criteria. Finally, the scope for creating liquidity for those with a liquidity preference for precautionary reasons is fading if returns on long-term projects decline. This can imply that pure-strategy equilibria do not exist and equilibrium outcomes are thus

[4] This parallels the thrust of Gehrig and Jackson (1998) in a trading context.

effectively indeterminate. If such a decline of returns on long-term projects can be related to an economy-wide (exogenous) fall in the level of interest rates, hitherto unnoticed – and unintended – consequences emerge from a zero-interest rate environment.

The paper is organized as follows: Section 2 provides an overview and Section 3 the details of the model. Section 4 considers the two types of liquidity preferences in isolation, which serves as a benchmark for the following analysis. Section 5 provides the analysis of equilibrium outcomes in the presence of frictions. Section 6 contains the key insights for the frictionless economy. Section 7 discusses some implications of our analysis and briefly reviews the main features of our setup. Section 8 concludes the paper. All technical proofs are relegated to the Appendix.

2 Overview of the model

Our framework is borrowed from the canonical banking model introduced by Diamond and Dybvig (1983) where investors desire liquidity because they wish to take precautions against sudden consumption needs. Such liquidity preferences have been already widely studied – albeit only in isolation. We introduce other investors who value flexibility to seize lucrative investment opportunities even after committing funds to illiquid long-term projects. Henceforth, those who value *flexibility* are called F-investors and those who want to take *precautions* are called P-investors.

The model is laid out in Section 3. All investors start with an endowment which can be either stored or invested in a long-term project delivering safe returns after two periods. Each investor is risk-averse with relative risk aversion larger one. After the first period, some P-investors learn that they need to consume immediately, and some F-investors learn about a novel short-term investment opportunity. Such investment opportunities arise spontaneously and only to a (small) fraction of F-investors; they are scalable and more profitable than existing long-term investments. Competitive banks create liquidity for investors. However, there are frictions which, in their combination, form an obstacle for banks to do so. The first friction is that the realization of individual liquidity events is private information. This is a standard assumption of models in the Diamond and Dybvig (1983)-tradition. The second friction is that the returns of investment opportunities are not contractible.

This credit friction is standard in a corporate finance context. The third friction is that the individual motive for liquidity preference is private information. This friction gives rise to a self-selection problem, which is well-known for insurance markets with asymmetric information.

In Section 4, we study each preference for liquidity in *isolation*. Without financial frictions, F-investors deposit their endowments in a bank. In the first period, banks hold only storage. When the short-term investment opportunities arrive, banks use their stored reserves and grant them as loans to those F-investors who acquire profitable investment opportunities. After the second period, banks collect these loans and redistribute their earnings to all F-investors. This way, each investor benefits from the returns on later investment opportunities, irrespective whether or not the investor can make such investment themselves.

With frictions, such efficient risk sharing is not feasible.[5] A banking arrangement implements the best possible risk-sharing if it facilitates a back-loaded insurance. Specifically, the optimal arrangement entails a penalty rate for early withdrawal, i.e. a payoff lower than the initial deposit, and a long-run interest rate above the rate of return on the long-run project. With this arrangement, investors who are lucky in finding a better investment opportunity can withdraw from the bank and take advantage of such opportunity as and when needed. However, as they will keep all the returns on those investments to themselves, optimal risk-sharing implies they do not withdraw from the bank as much as they initially deposited there. Instead, as the other investors will rely on the returns banks generate with the long-term project, banks make long-term investments that exceed the initial deposits of future unlucky investors. That way, the latter will benefit from higher per-capita returns on those investments. By contrast, considering P-investors in isolation, it is well-known that they are best served with a banking arrangement that facilitates a front-loaded insurance, to wit a deposit contract that provides a payoff higher than the initial deposit to those who withdraw early, and a long-run interest rate below the rate of return on the long-run project.[6] Accordingly, F-investors' deposits are less liquid than P-investors' deposits in that the discount one has to accept for accessing their bank deposits before the final date is higher for the former than for the latter.

[5] Only the credit friction creates a binding constraint when looking at motives for liquidity preferences in isolation.

[6] This feature is well-known since Diamond and Dybvig (1983).

In order to study the effects of the *co-existence* of the two motives in Section 5, we consider equilibria in the spirit of Rothschild and Stiglitz (1976).[7] Equilibrium outcomes depend on the probability with which F-investors acquire a profitable future investment opportunity. Provided this probability is small, a competitive banking industry emerges where two different bank business models exist side by side, each offering one contract aiming at one type of investors. Such equilibrium generates the same allocations that would obtain if the two motives could be treated in isolation.

If the probability to access a profitable investment opportunity is large, however, the option value associated with investment opportunities gains in importance. In this case, the previous contracts for the different investor types interfere with each other: The incentives for F-investors would be such that they are better off pretending to be P-investors and thus taking advantage in the rather likely event of being able to reinvest in the profitable new investment opportunity. This is because pretending to be P-investors would allow F-investors to reinvest more as P-investors get a higher payoff upon early withdrawal than F-investors. Depending on parameters, various types of equilibrium outcomes are possible. Separating equilibria can emerge where banks create too much liquidity for P-investors, or pooling equilibria might occur in which banks offer identical contracts to all investors. Finally, pure-strategy equilibria may not exist altogether if, for example, one motive is sufficiently over-represented. Allocations in pooling equilibria resemble those that obtain in models with additional trading opportunities for consumers at the middle date (e.g. Jacklin, 1987; Farhi et al., 2009). We obtain those allocations without any trading opportunities for consumers. Importantly, non-existence of equilibrium or over-insurance do not obtain in models with trading opportunities.

When standard frictions are absent, equilibrium allocations are straight forward but in stark difference to any other study of the Diamond and Dybvig (1983) model, as we show in Section 6. There, pooling the endowments of all investors allows banks to realize efficiency gains that are unattainable when the two motives for liquidity preferences do not co-exist. These efficiency gains are the result of economies of scope. Specifically, banks hold storage in the first period, until the

[7] In the Appendix, we also allow for menus of cross-subsidizing contracts in the spirit of Miyazaki (1977), Wilson (1977) and Spence (1978) and show that our results continue to hold qualitatively.

investment opportunities arrive and uncertainty about consumption needs is resolved. Using their stored reserves, banks provide P-investors with the means to meet their early consumption needs and grant the remainder as loans to F-investors who acquire profitable investment opportunities. After the second period, banks collect these loans and redistribute their earnings to patient P-investors and all F-investors. Accordingly, banks do not invest in the illiquid long-term project. In other words, maturity transformation never obtains if investors who value flexibility co-exist with investors who want to take precautions unless there are financial frictions. Although the precise institutional arrangement is indeterminate in the absence of frictions, the following contractual arrangement can implement this allocation. In the first period, banks issue demand deposits to P-investors and equity shares to F-investors, both backed entirely by stored goods. In the second period, the banks' assets comprise only single-period loans to F-investors, their liabilities are the revolving demand deposits held by patient P-investors.

Note that our investment opportunity differs from the perspective in Holmström and Tirole (1998). While those authors also stress the liquidity implications of a limited pledgeability of future returns, our framework emphasizes the sudden occurrence of new investment opportunities as an alternative motive for early fund withdrawals. In Holmström and Tirole (1998), credit frictions are not absolute, though.[8] Therefore, banks can offer lines of credit to firms. In their model firms pay for credit lines, which is similar to the penalty rate that long-term F-investors pay in our optimal F-investor deposit contract. Unlike credit lines, however, such a deposit contract also provides for higher long-term returns.[9]

Credit frictions have been also identified to generate a demand for liquid, marketable financial assets, such as volatile bubbles (Martin and Ventura, 2012) and fiat money (Dietrich et al., 2020). In our model, credit frictions generate the maturity transformation banks typically engage in. A range of reasons has been identified for the credit friction utilized in the present paper. For example, only

[8]In Donaldson et al. (2018), warehouses serve as financial intermediaries that provide liquidity services to producers by overcoming credit frictions more effectively than direct lenders.

[9]This is in line with empirical evidence which suggests only imperfect substitutability for corporations between bank deposits and lines of credit (e.g. Acharya et al., 2007; Campello et al., 2011; Acharya et al., 2013). Loosely related are also He and Kondor (2016) and Parlatore (2019), who argue that private liquidity management by individual non-financial firms is generally associated with inefficiencies.

the investor has the specific skills needed to successfully manage and complete the project (Hart and Moore, 1994), their consumption is not observable (Wallace, 1988), or penalties like future exclusion from financial markets are ineffective for enforcing loans (Kehoe and Levine, 1993).

3 Setup

Consider an economy populated by investors and banks. There are three dates $t \in \{0,1,2\}$, with one good at each date. The good can be consumed or used for production in one of three technologies.

Technologies The technologies are: storage, long-term production and short-term production. Each technology features constant returns to scale. Storage is one-for-one and can be used at dates $t \in \{0,1\}$. Long-term production has to be initiated at $t=0$, and takes two periods until $t=2$ to produce the good. Per-unit-returns are $R > 1$. Unless indicated otherwise, long-term production cannot be prematurely liquidated at date $t=1$. Henceforth, we refer to long-term production also as R-technology. Short-term production opportunities arise at date $t=1$, to produce $Q > R$ per unit of investment after one period at date $t = 2$. Accordingly, short-term production is called Q-technology.[10]

Investors There is a continuum of investors, each endowed with one unit of the good at $t=0$. All investors have access to the R-technology at date $t=0$, and to storage at dates $t=0$ and $t=1$. There are two types of investors. One type of investors values consumption c only at date $t=2$. As of date $t=0$, there is a probability $\mu \in]0,1[$ that an investor of this type gets lucky at date $t=1$ as she will gain access to the Q-technology. With probability $1-\mu$ she will remain without access to the Q-technology. Getting lucky is uncorrelated across investors of this type. A long-term commitment to the illiquid R-technology is not optimal for them as these investors want to preserve their *flexibility* in case they get lucky at date $t = 1$. Henceforth, we call investors who seek to preserve their flexibility *F-investors*.

[10]Constant returns to scale for storage and the R-technology is a standard assumption in this class of models, even in macroeconomic applications (e.g. Bencivenga and Smith, 1991; Ennis and Keister, 2003; Fecht et al., 2008).

An investor of the other type does not know at date $t=0$ when she needs to consume. Specifically, with probability $\lambda \in]0,1[$ she will value only consumption c at date $t=1$, whereas with probability $1-\lambda$, she will value consumption only at date $t=2$. Getting impatient, i.e. having to consume early, is uncorrelated across investors of this type. For them, a long-term commitment to the illiquid R-technology is not optimal because these investors need to take *precautions* against sudden expenditure needs. They are henceforth called *P-investors*.

The investors' *Bernoulli* utility function u is independent from their type, twice continuously differentiable, and satisfies $u'(c)>0$, $u''(c)<0$, $\lim_{c\to 0} u'(c)=\infty$, and $\lim_{c\to\infty} u'(c)=0$. To simplify the exposition, we divide investors into groups, each of mass one. In every group there are either F-investors or P-investors, with γ and $1-\gamma$ as the shares of P-investor groups and F-investor groups, respectively, in the total population. As the probabilities μ and λ are deterministic and common knowledge at date $t=0$, the law of large numbers applies, i.e. a share μ in a group of F-investors is lucky, and a share λ in a group of P-investors is impatient.

Banks There is a continuum of penniless banks. They have access to storage and to the R-technology, but not to the Q-technology. Banks are perfectly competitive (Bertrand competition) and maximize expected profits. At date $t=0$, investors can exchange their endowments for contracts offered by banks. A *contract* $\mathscr{D}=(r_1,r_2)$ is a sequence of payments $\{r_t\}_{t\in\{1,2\}}$ a bank makes to investors at $t=1$ and $t=2$, respectively. A *business model* $\mathscr{M}=(r_1,r_2,y)$ consists of a contract $\mathscr{D}$ and a portfolio share held in storage $y\in[0,1]$, and is *sustainable* if designed to earn non-negative profits.[11]

Frictions Financial contracts are potentially plagued with three types of frictions. Firstly, at date $t=0$, the ex-ante motive for the liquidity preference is private information. Accordingly, investors are free to choose between all contracts banks offer. Secondly, at date $t=1$, the realized consumption need is private information, i.e. only the individual P-investors learns whether they get impatient and need to consume immediately, or patient and can wait until date $t=2$. Similarly, access to the Q-

[11] In line with the literature, we do not allow for re-depositing after a withdrawal. Re-depositing is particularly relevant in a dynamic banking context such as Bhattacharya and Padilla (1996) and Dietrich and Gehrig (2021).

technology is private information as only the individual F-investor learns at date $t=1$ whether they are lucky and can invest in the profitable new opportunity or not. Therefore, contracts cannot be made contingent on the ex-post realization of liquidity needs of investors. Thirdly, while storage and the R-technology are available to investors and banks alike, and are thus fully contractible, the Q-technology is specific to F-investors who are not able to credibly pledge the returns they realize with this superior technology at date $t=2$. In what follows we consider two different scenarios, one without any frictions and the other with all three frictions in place.

Assumptions For our analysis we make two technical assumptions.

A1 (Relative Risk Aversion)

The coefficient of relative risk aversion exceeds one, i.e., $-cu''(c)/u'(c) > 1$.

Without this assumption, many results will just be reversed for $-cu''(c)/u'(c) < 1$.

A2 (Single Crossing Condition)

In the (r_1, r_2) space, indifference curves of P-investors and of F-investors cross only once.

Single crossing is a standard assumption in mechanism design theory. It is satisfied, for example, if relative risk aversion is constant. In our context, this assumption ensures that the set of probabilities μ, for which equilibrium types obtain, is convex.

Finally, an *economy* $\mathscr{E}$ is a description of F-investors, P-investors, and technologies, i.e. $\mathscr{E} = (u, \gamma, \lambda, \mu, Q, R)$.

4 Benchmark

In this section we consider each type of investors separately, i.e. the sole motive behind investors' liquidity preferences is either preserving flexibility or taking precautions. Looking at each motive in isolation establishes a benchmark which later helps to understand the implications of their *co-existence*.

4.1 Preserving flexibility

Suppose that preserving flexibility is the only motive for liquidity demand in the economy, and consider first the case where loans to F-investors can be enforced. Let c_R and c_Q denote the consumption by an F-investor with and without investment opportunity, respectively. Similarly, x_R and y are resources per F-investor directed to the R-technology and to storage, respectively. Finally, let x_Q be the investment into the Q-technology per F-investor who has actually access to it. As all F-investors are ex-ante identical, the first-best thus solves

$$\max_{(c_R,c_Q,x_R,x_Q,y)\in\mathbb{R}_+^4\times[0,1]} \quad \mu u(c_Q)+(1-\mu)u(c_R)$$
$$\text{s.t.}\begin{cases} x_R+y & \leq & 1 \\ \mu x_Q & \leq & y \\ \mu c_Q+(1-\mu)c_R & \leq & \mu Q x_Q + R x_R + y - \mu x_Q \end{cases} \tag{1}$$

The first constraint is the resource constraint at date $t=0$; the second constraint states that investment in the Q-technology at date $t=1$ cannot be larger than what has been stored at date $t=0$; the third constraint states that total consumption at date $t=2$ is limited by the amount of goods produced by either technology and what is left of the storage at date $t=1$. By standard arguments, all constraints hold with equality, i.e. the problem becomes

$$\begin{aligned} &\max_{(c_R,c_Q,y)\in\mathbb{R}_+^2\times[0,1]} \quad \mu u(c_Q)+(1-\mu)u(c_R) \\ &\text{s.t.} \quad c_R=\tfrac{Qy+R(1-y)-\mu c_Q}{1-\mu} \end{aligned} \tag{2}$$

A solution for this problem satisfies the first-order condition

$$\mu u'(c_Q)-\mu u'\left(\frac{Qy+R(1-y)-\mu c_Q}{1-\mu}\right)=0, \tag{3}$$

which implies $c_Q = c_R = Qy + R(1-y)$, i.e. *full insurance*. The optimization problem thus simplifies further to

$$\max_{y \in [0,1]} \quad Qy + R(1-y). \tag{4}$$

Since $Q > R$, the solution to program (1) is $y = 1$ and $c_R = c_Q = Q$.

Accordingly, the first-best allocation is a corner solution. The intuition is straightforward. F-investors care only about late consumption and hence they are interested only in maximizing the amount of the good available at date $t = 2$. As all technologies are constant returns-to-scale, this is achieved if all resources end up being invested into the Q-technology, i.e. there is no investment in the R-technology at date $t = 0$.

As F-investors gain access to the Q-technology only with some probability μ, the first-best does not obtain in autarky. However, banks are able to implement the first-best, provided that at date $t = 2$ they can collect principal and interest of any loans granted to F-investors at date $t = 1$. Suppose a bank operates a business model $\mathscr{M} = (0, Q, 1)$. That is, the bank accepts endowments from F-investors in exchange for promises to pay $r_1 = 0$ and $r_2 = Q$; a possible contract that delivers this sequence of payments are shares in the bank's equity. Moreover, the bank stores all endowments from date $t = 0$ to date $t = 1$. At this date, the bank lends out the stored goods at a lending rate of Q to F-investors who wish to borrow. Provided these investors invest the loans into the Q-technology, the loan earnings collected at date $t = 2$ are used to pay the initially promised amount of Q to every F-investor at date $t = 2$. F-investors with access to the Q-technology are thus indifferent between borrowing and not borrowing at date $t = 1$, while F-investors without access are strictly better off by not borrowing. Hence, F-investors with access to the Q-technology, and only those, actually borrow from the bank at date $t = 1$.

Suppose now that, while preserving flexibility is still the only motive for liquidity demand in the economy, a bank cannot enforce loan repayments and an investor's access to the Q-technology remains her private information. Therefore, F-investors cannot be made to share the return on their investment in the Q-technology once they have gained access to it. The associated, constrained-efficient, allocation is a solution to

$$\max_{(c_R,c_Q,x_R,x_Q,y)\in\mathbb{R}^4_+\times[0,1]} \mu u(c_Q)+(1-\mu)u(c_R)$$
$$\text{s.t.}\begin{cases} x_R+y &= 1 \\ \mu x_Q &= y \\ (1-\mu)c_R &= Rx_R \\ c_Q &= Qx_Q \\ c_R &\geq x_Q \\ c_Q &\geq c_R \end{cases} \tag{5}$$

Stating the first four constraints directly with equality is innocent but simplifies the exposition.[12] The first and second constraints are as before. The third constraint states that consumption by F-investors without access to the Q-technology is equal to what is generated with the R-technology. The fourth constraint states that consumption by F-investors with access to the Q-technology is equal to what is generated with this technology. The final two lines are the incentive constraints, ensuring that unlucky F-investors have no incentive to pretend they got access to the Q-technology, and that lucky F-investors have no incentive to pretend they got no access to it.

Disregarding for now the final two constraints, a solution to this problem satisfies the first four constraints and the first-order condition

$$u'\left(\tfrac{R(1-y)}{1-\mu}\right)-\tfrac{Q}{R}u'\left(\tfrac{Qy}{\mu}\right)=0. \tag{6}$$

Let y^d denote the solution to condition (6). Then, banks can implement the solution to problem (5) with a business model $\mathscr{M}=(r_1^d,r_2^d,y^d)$, provided $r_1^d=x_Q=y^d/\mu$ and $r_2^d=Rx_R/(1-\mu)=R(1-y^d)/(1-\mu)$.

[12] In short, equality follows from non-satiation together with $Q>R>1$. The latter implies that it is neither efficient to keep any storage between date $t=1$ and $t=2$ nor to use the R-technology for the consumption by investors with access to the Q-technology.

The contract (r_1^d, r_2^d) features certain characteristics worthy further elaboration. Let c_Q^d and c_R^d be the consumption by F-investors with and without access to the Q-technology, respectively, associated with the business model (r_1^d, r_2^d, y^d). Then, condition (6) implies $c_Q^d > c_R^d$. For constant relative risk aversion equal to one, i.e. $-cu''(c)/u'(c) = 1$, we obtain $cu'(c) = u'(1)$. Hence, $Ru'(R) = Qu'(Q)$, and the first-order condition (6) requires $c_Q^d = Q$ and $c_R^d = R$. Accordingly, the bank's business model satisfies $r_1^d = 1$, $r_2^d = R$ and $y^d = \mu$, i.e. F-investors are allowed to withdraw at date $t = 1$ exactly what they have deposited in the bank at date $t = 0$. For relative risk aversion greater one, i.e. $-cu''(c)/u'(c) > 1$, we obtain $Ru'(R) > Qu'(Q)$. Therefore, condition (6) requires $R < c_R^d < c_Q^d < Q$. Accordingly, the bank's business model satisfies $r_1^d < 1$, $r_2^d = R(1-\mu r_1^d)/(1-\mu) > R$, and $y^d < \mu$. Note $r_1^d < 1$ implies that the contract entails a penalty for early withdrawals.

With relative risk aversion being greater than one, the contract (r_1^d, r_2^d) is also incentive compatible, i.e. F-investors have no incentive to misrepresent themselves. Consider F-investors without access to the Q-technology. Since $r_1^d < r_2^d$, they are better off withdrawing at date $t = 2$. Next consider those with access to the Q-technology. If they withdraw at date $= 1$, they consume Qr_1^d, while their consumption is r_2^d if they withdraw at date $t = 2$. Since $Q/R > 1$, the first-order condition (6) implies $Qr_1^d > r_2^d$, such that these investors are better off withdrawing at date $t = 1$.

Figure 1 illustrates the constrained-efficient solution to the F-investors' problem. The contract that serves best their demand for liquidity is characterized by a pair (r_1^d, r_2^d) for which the F-investors' indifference curve is tangent to the banks' intertemporal budget line $r_2 = R(1-\mu r_1)/(1-\mu)$, provided the banks' business model is targeted solely at F-investors. For relative risk aversion greater than one, this contract lies to the north-west of $(1, R)$. Lemma 1 summarizes these results.

Lemma 1 (Term Deposit Contract)

Provided the returns on the short-term Q-technology are not contractible, the optimal contract is a term deposit contract $\mathscr{D} = (r_1^d, r_2^d)$ *with a penalty* $1 - r_1^d > 0$ *for early withdrawals.*

Finally, note that if loan enforcement is perfect, then all endowments will eventually end up in the projects with the highest productivity anyway.[13] Accordingly, in the first-best, the probability of

[13] Of course, this salient allocation profile is partly due to the constant returns to scale for all technologies.

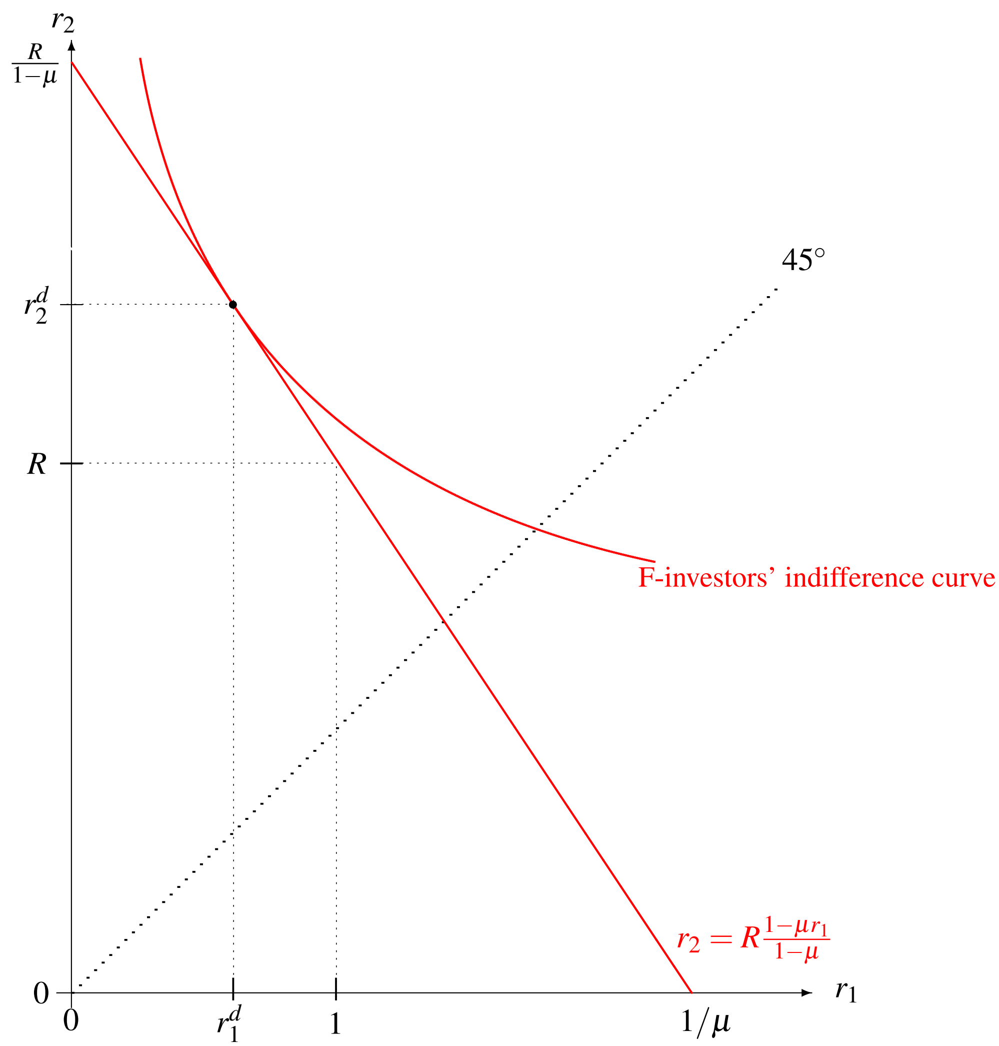

Figure 1: Constrained-efficient F-Investor Contract ($-cu''(c)/u'(c) > 1$).

acquiring a short-term investment opportunity is irrelevant for the allocation. By contrast, without loan enforcement, not all resources can be directed into the Q-technology, and the allocation very much depends on the probability of short-term investment opportunities, μ, as implied by the first-order condition (6).

4.2 Taking precautions

Next, suppose taking precautions is the only motive for liquidity demand in the economy. The P-investors' problem is well known since Diamond and Dybvig (1983). Therefore, we keep it concise in showing how a bank can solve the P-investors' problem such that the first-best allocation obtains.

Let c_1 and c_2 denote a P-investor's consumption if impatient and patient, respectively. As all P-investors are ex-ante identical, the first-best thus solves

$$\max_{(c_1,c_2,x_R,y)\in\mathbb{R}^3_+\times[0,1]} \quad \lambda u(c_1)+(1-\lambda)u(c_2)$$
$$\text{s.t.}\begin{cases} x_R+y & \leq & 1 \\ \lambda c_1 & \leq & y \\ \lambda c_1+(1-\lambda)c_2 & \leq & Rx_R+y \\ c_1 & \leq & c_2 \end{cases} \tag{7}$$

The restrictions are the feasibility constraints. The first line requires that the investment in the R-technology and the amount held in storage cannot exceed the endowment of P-investors. According to the second line, total consumption by impatient P-investors at date $t=1$ cannot be larger than the stored goods available at that date. The third line states that total consumption is limited by the total availability of stored and produced goods. The last line ensures incentive compatibility if information about early consumption needs is private. The first three constraints hold with equality. Therefore, if information about early consumption needs is not private, the solution to the P-investors' problem satisfies $c_1^\delta=y^\delta/\lambda$ and $c_2^\delta=R(1-y^\delta)/(1-\lambda))$ where y^δ solves the first-order condition

$$u'\left(\tfrac{y^\delta}{\lambda}\right)-Ru'\left(\tfrac{R(1-y^\delta)}{1-\lambda}\right)=0. \tag{8}$$

As P-investors need to consume early only with some probability λ, the first-best cannot be achieved in autarky. However, banks are able to implement the first-best by choosing the business model $\mathscr{M}=(r_1^\delta,r_2^\delta,y^\delta)$ with $r_1^\delta=y^\delta/\lambda$ and $r_2^\delta=R(1-y^\delta)/(1-\lambda)$. For relative risk aversion equal to one, the payment a bank offers to those who withdraw early is $r_1^\delta=1$. Then, from the feasibility constraint for $t=2$, we obtain $r_2^\delta=R$. For relative risk aversion greater one, the bank pays an insurance benefit at date $t=1$, i.e. more than a P-investor has deposited with the bank in the first place. Specifically, the bank pays $r_1^\delta=y^\delta/\lambda>1$ and $r_2^\delta=R(1-\lambda r_1^\delta)/(1-\lambda)\in]r_1^\delta,R[$. Accordingly,

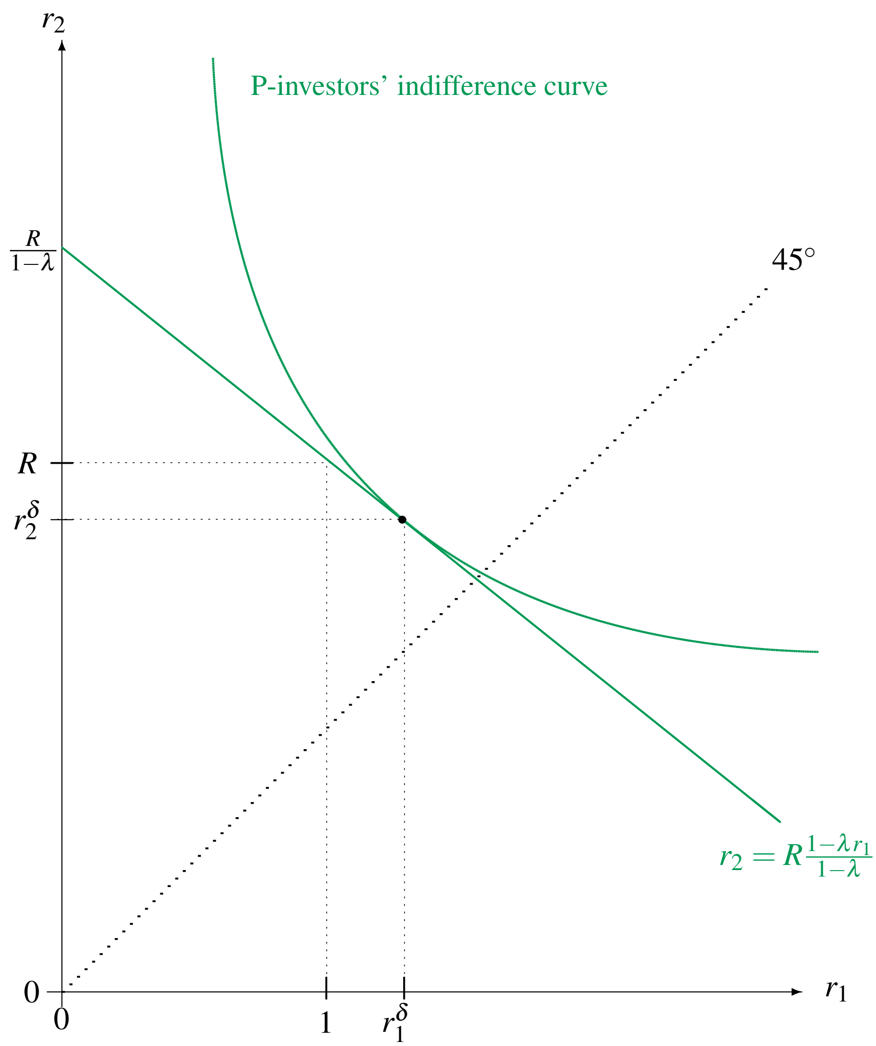

Figure 2: Efficient P-investor Contract $(-cu''(c)/u'(c) > 1)$.

P-investors receive a subsidized rate for early withdrawal, rather than a penalty as it was the case with F-investors.

Note that with this deposit contract, P-investors do not have incentives to misrepresent themselves if information about early consumption needs is private. This is because impatient P-investors have no choice but to withdraw at date $t=1$, while patient P-investors are strictly better off by waiting until date $t=2$ since $r_2^\delta > r_1^\delta$.[14]

Figure 2 illustrates the solution to the P-investors' problem. The contract that serves best their precautionary motive is characterized by a pair (r_1^δ, r_2^δ) for which the P-investors' indifference curve

[14] A bank-run equilibrium is ruled out here as production cannot be liquidated at date $t=1$.

is tangent to the banks' intertemporal budget constraint $r_2 = R(1-\lambda r_1)/(1-\lambda)$, provided the banks' business model is targeted solely at P-investors. For relative risk aversion greater than one, this contract point lies to the north-west of the 45° line, where P-investors would get full insurance, and to the south-east of $(1,R)$, which implies $r_1^\delta > 1$.

5 Co-existence: The case with frictions

In this section we study the implications of the co-existence of F-investors and P-investors, provided their identity and the individual liquidity event are private information and banks cannot enforce loan repayments. We first define our equilibrium concept, followed by possible equilibrium outcomes.

5.1 Equilibrium concept

We consider competitive deposit markets and focus on equilibria in the spirit of Rothschild and Stiglitz (1976). Specifically, we consider pure-strategy equilibria, where each bank is limited to offering one deposit contract, and investors choose from all contracts offered by banks to maximize their expected utility but cannot randomize their choice. It is useful to begin with a definition of incentive compatible contracts.

Definition 1 (Incentive Compatible Contracts)

Let $\mathscr{D}^{\mathrm{F}} = (r_1^{\mathrm{F}}, r_2^{\mathrm{F}})$ *be the contract for F-investors, and* $\mathscr{D}^{\mathrm{P}} = (r_1^{\mathrm{P}}, r_2^{\mathrm{P}})$ *the contract for P-investors. An* incentive compatible *menu of contracts* $\left\{\mathscr{D}^{\mathrm{F}},\mathscr{D}^{\mathrm{P}}\right\}$ *satisfies*

$$\mu u(Qr_1^{\mathrm{F}})+(1-\mu)u(r_2^{\mathrm{F}})\geq \mu u(Qr_1^{\mathrm{P}})+(1-\mu)u(r_2^{\mathrm{P}}) \tag{9}$$

$$\lambda u(r_1^{\mathrm{P}})+(1-\lambda)u(r_2^{\mathrm{P}})\geq \lambda u(r_1^{\mathrm{F}})+(1-\lambda)u(r_2^{\mathrm{F}}) \tag{10}$$

$$Qr_1^{\mathrm{F}}\geq r_2^{\mathrm{F}} \tag{11}$$

$$r_1^{\mathrm{F}}\leq r_2^{\mathrm{F}} \tag{12}$$

$$r_1^{\mathrm{P}}\leq r_2^{\mathrm{P}} \tag{13}$$

$$\mu u(Qr_1^{\mathrm{F}})+(1-\mu)u(r_2^{\mathrm{F}})\geq \sup\{\mu u\left(Qy+R(1-y)\right)+(1-\mu)u\left(R(1-y)+y\right):y\in[0,1]\} \tag{14}$$

$$\lambda u(r_1^{\mathrm{P}})+(1-\lambda)u(r_2^{\mathrm{P}})\geq \sup\{\lambda u\left(y\right)+(1-\lambda)u\left(R(1-y)+y\right):y\in[0,1]\} \tag{15}$$

Condition (9) requires that F-investors prefer the contract intended for F-investors over the contract intended for P-investors, with strict inequality for $\left(r_1^{\mathrm{F}},r_2^{\mathrm{F}}\right)\succ_{\mathrm{F}}\left(r_1^{\mathrm{P}},r_2^{\mathrm{P}}\right)$. Condition (10) requires that P-investors prefer the contract intended for P-investors, with strict inequality for $\left(r_1^{\mathrm{P}},r_2^{\mathrm{P}}\right)\succ_{\mathrm{P}}\left(r_1^{\mathrm{F}},r_2^{\mathrm{F}}\right)$. These two incentive constraints need to be satisfied at date $t=0$. For contracts to be incentive compatible, there are also incentive constraints to be observed at date $t=1$ when investors have learnt about their status. Specifically, condition (11) requires that F-investors with access to the Q-technology must not be better off by pretending to have no access; condition (12) that F-investors without access to the Q-technology must not be better off by pretending to have access; and condition (13) that patient P-investors must not be better off by pretending to be impatient. Finally, contracts must be such that depositing with banks makes investors better off than autarky. This holds provided the expected utility associated with their contract is at least as large as the expected utility an investor achieves in autarky, i.e. if contracts satisfy the participation constraints (14) and (15).

We can now define a banking equilibrium.

Definition 2 (Banking Equilibrium)

A perfect-competition, pure-strategy banking equilibrium *is an incentive compatible menu of con-*

tracts $\{\mathscr{D}^{\mathrm{F}}, \mathscr{D}^{\mathrm{P}}\}$ *such that the associated business models* $\{\mathscr{M}^{\mathrm{F}}, \mathscr{M}^{\mathrm{P}}\}$ *are sustainable, while no bank can profitably enter the market with another contract* $\mathscr{D}' \notin \{\mathscr{D}^{\mathrm{F}}, \mathscr{D}^{\mathrm{P}}\}$.

A bank's business models is *sustainable* if the bank does not make a loss and would thus be strictly better off exiting the market. A business model $\mathscr{M}^{\mathrm{F}} = (r_1^{\mathrm{F}}, r_2^{\mathrm{F}}, y^{\mathrm{F}})$ of offering contracts only to F-investors is sustainable if $\mu r_1^{\mathrm{F}} \leq y^{\mathrm{F}}$ and $(1-\mu) r_2^{\mathrm{F}} \leq R(1-y^{\mathrm{F}})$; a business model $\mathscr{M}^{\mathrm{P}} = (r_1^{\mathrm{P}}, r_2^{\mathrm{P}}, y^{\mathrm{P}})$ of offering contracts only to P-investors is sustainable if $\lambda r_1^{\mathrm{P}} \leq y^{\mathrm{P}}$ and $(1-\lambda) r_2^{\mathrm{P}} \leq R(1-y^{\mathrm{P}})$; and a business model $\mathscr{M}^{\mathrm{Pool}} = (r_1^{\mathrm{Pool}}, r_2^{\mathrm{Pool}}, y^{\mathrm{Pool}})$ of offering the same contract, a pooling contract, to F-investors and to P-investors alike, i.e. $\mathscr{D}^{\mathrm{F}} = \mathscr{D}^{\mathrm{P}} = (r_1^{\mathrm{Pool}}, r_2^{\mathrm{Pool}})$, is sustainable if $(\gamma\lambda + (1-\gamma)\mu) r_1^{\mathrm{Pool}} \leq y^{\mathrm{Pool}}$ and $(1 - (\gamma\lambda + (1-\gamma)\mu)) r_2^{\mathrm{Pool}} \leq R(1-y^{\mathrm{Pool}})$. Provided either of these inequalities is strict, the respective business model is associated with strictly positive profits.

In equilibrium, there is *no profitable market entry* by banks with contracts other than $\mathscr{D}^{\mathrm{F}}$ and $\mathscr{D}^{\mathrm{P}}$. Therefore, operating banks make zero profits. A business model associated with a contract only for F-investors thus satisfies $(1-\mu) r_2^{\mathrm{F}} = R(1-\mu r_1^{\mathrm{F}})$; a business model associated with a contract only for P-investors satisfies $(1-\lambda) r_2^{\mathrm{P}} = R(1-\lambda r_1^{\mathrm{P}})$; and a business model associated with one contract for both investor types, satisfies $(1 - (\gamma\lambda + (1-\gamma)\mu)) r_2^{\mathrm{Pool}} = R(1 - (\gamma\lambda + (1-\gamma)\mu) r_1^{\mathrm{Pool}})$.

A separating equilibrium is *credit-constrained* if the credit friction constitutes the only constraint that is actually binding. Hence, the contract $\mathscr{D}^{\mathrm{F}}$ maximizes the expected utility of F-investors subject only to the zero-profit condition $(1-\mu) r_2 = R(1-\mu r_1)$ and the contract $\mathscr{D}^{\mathrm{P}}$ maximizes the expected utility of P-investors subject only to the zero-profit condition $(1-\lambda) r_2 = R(1-\lambda r_1)$ (see the F-investors' problem (5) and the P-investors' problem (7)). That is, while the credit friction precludes that lucky F-investors can share their returns on the Q-technology, private information about an investor's type or about an investor's realized liquidity event does not imply that any of the incentive constraints (9) through (13) are binding. Such equilibrium is necessarily separating.

A separating equilibrium is called *incentive-constrained* if, in addition to the credit friction, at least one of the incentive constraints arising from the private information about the investor type, (9) or (10), is binding. For example, if (9) is binding, then banks cannot profitably stay in, or enter, the market with a business model $(r_1^{\mathrm{P}}, r_2^{\mathrm{P}}, y^{\mathrm{P}}) = (r_1^{\delta}, r_2^{\delta}, y^{\delta})$ because they would attract not only all P-investors but also all F-investors, which renders such business model unviable. However, as in a credit-constrained equilibrium, banks' contract offers to the F-investors satisfy the zero-profit condition $(1-\mu)r_2 = R(1-\mu r_1)$, while banks' contract offers to the P-investors satisfy the zero-profit condition $(1-\lambda)r_2 = R(1-\lambda r_1)$.

In a *pooling equilibrium* F-investors and P-investors obtain one and the same contract, i.e. $\mathscr{D}^{\mathrm{F}} = \mathscr{D}^{\mathrm{P}}$, and this contract satisfies the joint zero-profit constraint that obtains if banks pool the resources of all investors, i.e. $(1-(\gamma\lambda+(1-\gamma)\mu))r_2 = R(1-(\gamma\lambda+(1-\gamma)\mu)r_1)$. Banks offering separating contracts cannot profitably enter the market in such pooling equilibria. This is because either F-investors and P-investors would both prefer the pooling contract over the separating contracts, or P-investors prefer the F-investors' contract, F-investors prefer the P-investors' contract, or both.

5.2 A special case

For the special case of constant relative risk aversion equal to one, the equilibrium outcome is straightforward, since the optimal deposit contracts are identical for both liquidity motives. In this case, no insurance benefit is offered for impatient P-investors, nor is there any compensation for F-investors for not getting access to the higher-yielding Q-technology.

Lemma 2 (Logarithmic Utility)

Suppose $-cu''(c)/u'(c) = 1$. Then, the banking equilibrium is a menu of contracts $\{\mathscr{D}^{\mathrm{I}}, \mathscr{D}^{\mathrm{C}}\}$ with $\mathscr{D}^{\mathrm{I}} = \mathscr{D}^{\mathrm{C}} = (1, R)$.

Proof: See Appendix A. □

If relative risk-aversion is equal to one, the optimum contracts for F-investors and P-investors satisfy $(r_1^d, r_2^d) = (1, R)$ and $(r_1^{\delta}, r_2^{\delta}) = (1, R)$. Even though the contracts are identical, the underlying business models can be different as the contract for P-investors requires a business model with reserves $y^{\delta} = \lambda$ and for F-investors $y^d = \mu$. As the F-investors' demand for liquidity is thus best met with a contract that also best meets the P-investors' demand for liquidity, there is no incentive for F-investors or P-investors to misrepresent themselves. Moreover, being on every banks' budget line, contract $(1,R)$ is an allocation any bank can offer, regardless of the respective shares of impatient depositors, μ and λ. Therefore, the equilibrium is constrained-efficient.

In what follows, we first consider economies where the success probability μ is not too high and investment opportunities thus rare. Whilst bearing in mind the stylized nature of the model, examples could be linked to economies with a low research intensity, or innovation trajectory. Following this, we consider more dynamic economies where investment opportunities are frequent as the probability μ is comparatively high.

5.3 Rare investment opportunities

It is easy to observe that for $\mu \leq \lambda$ there will always be a credit-constrained separating equilibrium. To see why, recall the zero-profit conditions for banks. For banks meeting the F-investors' demand for flexibility, it reads $r_2 = R(1-\mu r_1)/(1-\mu)$, and for banks catering to the P-investors' need to take precautions it reads $r_2 = R(1-\lambda r_1)/(1-\lambda)$. The zero-profit lines are linear in a (r_1, r_2) space and go through $(1,R)$ for both bank types, regardless of the value for λ and μ. We have also established that, considering each investor type in isolation, the credit-constrained provision of liquidity to F-investors is characterized by a point on the respective zero-profit line to the north-west of $(1,R)$, while the P-investors' needs to take precautions are best met in a point to the south-east of $(1,R)$ on the respective zero-profit line (see Figures 1 and 2). Finally, a bank's zero-profit line is steeper for a larger μ and λ, respectively.

If the proportion of impatient P-investors is not smaller than the proportion of lucky F-investors, $\mu \leq \lambda$, the zero-profit line for P-investors is steeper than the respective zero-profit line for F-investors. Since the efficient P-investors' contract (r_1^δ, r_2^δ) is to the south-west of $(1,R)$, it lies below the zero-profit line associated with the F-investors' problem, i.e. inside the set of feasible contracts for F-investors. Therefore, F-investors prefer their own credit-constrained contract (r_1^d, r_2^d) over the efficient contract for P-investors. Intuitively, from an F-investor's perspective, the insurance benefit of a contract for P-investors to those withdrawing early is small relative to what one has to give up when remaining patient. This makes the P-investors' contract sufficiently unattractive to F-investors. A similar argument can be made for the incentives of P-investors. The contract intended for F-investors is unattractive to P-investors because, as P-investors are more likely to withdraw early, the penalty associated with an F-investors' contract is particularly costly for P-investors.

Credit-constrained separation equilibria not only exist for $\mu \leq \lambda$, but even for $\mu > \lambda$ up to a critical level $\bar{\mu} < 1$, above which these equilibria do not exist.

Proposition 1 (Credit-constrained Separation)

Consider economies $\mathscr{E} = (u, \gamma, \lambda, \mu, Q, R)$, where u satisfies Assumption 1 *and* Assumption 2*. Then, for every $R > 1$, $Q > R$, and $\lambda \in]0,1[$, there is $\bar{\mu} \in]\lambda, 1[$ such that a credit-constrained separation equilibrium exists* if and only if $\mu \leq \bar{\mu}$*. In credit-constrained separation equilibria, the marginal rate of substitution between r_1 and r_2 is lower for F-investors than for P-investors, i.e. $-\frac{\mu}{1-\mu}\frac{u'(Qr_1^{\mathrm{F}})}{u'(r_2^{\mathrm{F}})}Q > -\frac{\lambda}{1-\lambda}\frac{u'(r_1^{\mathrm{P}})}{u'(r_2^{\mathrm{P}})}$.*

Proof: See Appendix B. □

Figure 3 illustrates equilibria that involve credit-constrained separation. F-investors strictly prefer the solution (r_1^d, r_2^d) to their problem (5) over the solution (r_1^δ, r_2^δ) to the P-investors' problem (7), as (r_1^δ, r_2^δ) lies below the F-investors' indifference curve going through their own contract (r_1^d, r_2^d). Similarly, P-investors strictly prefer (r_1^δ, r_2^δ) over (r_1^d, r_2^d). Under Assumptions 1 and 2, these preference relations imply that the indifference curve of F-investors is flatter than the indifference curve of P-investors, as can be seen at the intersection of both curves. It is not possible for any bank to profitably enter the market by offering a contract designated either exclusively to F-investors or exclusively to P-investors, because F-investors as well as P-investors already enjoy the best allocation possible given the credit friction. Also, a bank cannot profitably enter the market with a pooling contract. This is because the zero-profit constraint associated with pooling, $r_2 = R\frac{1-(\gamma\lambda+(1-\gamma)\mu)r_1}{1-(\gamma\lambda+(1-\gamma)\mu)}$, does not facilitate any contracts that are Pareto-improvements to the separating contracts (r_1^δ, r_2^δ) and (r_1^d, r_2^d).

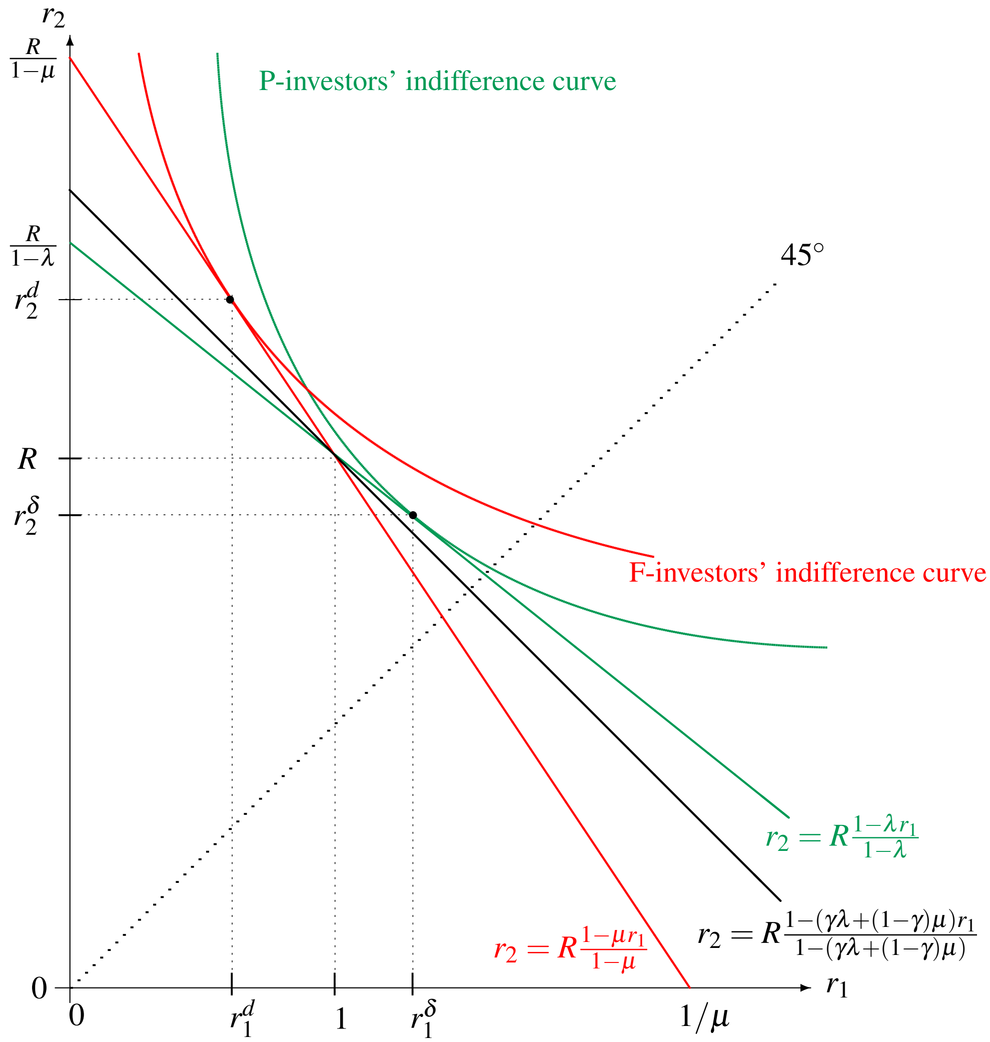

Figure 3: Credit-constrained Separation.

To sum up, provided the share μ of lucky F-investors is not too large, a banking equilibrium efficiently provides for the liquidity needs of P-investors and F-investors subject only to the credit constraint. P-investors are insured against the risk of the need to consume early, while F-investors are insured against the risk of missing a better investment opportunity. Both motives require some liquidity management, but optimal contracts stipulate different solutions. While the insurance payment is front-loaded in the contract with P-investors, and back-loaded in the contract with F-investors, nobody has an incentive to hide their own motive for their liquidity preference.

Corollary 1 (Bank Reserves)

For $\mu \leq \lambda$ the reserve holdings of the P-investor bank are larger than the F-investor bank, i.e. $y^{\mathrm{P}} >$

y^{F}. *Accordingly, expected returns for a bank specializing on F-investors exceed those of a bank focusing on P-investors.*

Business models $(r_1^{\mathrm{F}}, r_2^{\mathrm{F}}, y^{\mathrm{F}})$ associated with term deposits for F-investors thus require lower reserve holdings than business models $(r_1^{\mathrm{P}}, r_2^{\mathrm{P}}, y^{\mathrm{P}})$ associated with demand deposits for P-investors. This is because F-investors require reserves below their probability of getting lucky μ, i.e. $y^{\mathrm{F}} = y^d < \mu$, while P-investors require reserves in excess of their probability to consume early λ, i.e. $y^{\mathrm{P}} = y^{\delta} > \lambda$. Therefore, $y^{\mathrm{F}} < y^{\mathrm{P}}$ for $\mu \leq \lambda$. The differences in bank portfolios have direct implications for the returns on bank assets. As those are determined by $y + R(1-y)$ the returns on assets are higher for a bank with F-investors than for a bank with P-investors for $y^d < y^{\delta}$. To the extent that the different business models are offered in-house by a single (universal) bank, the Corollary implies the return on assets be applied differently across services.[15]

5.4 Frequent investment opportunities

Let us now consider economies with a relatively high probability of getting access to highly productive investment opportunities, i.e. $\mu > \bar{\mu}$. How will equilibrium outcomes be affected under such conditions? It turns out that the outcomes can vary substantially, depending on the specific characteristic of the economy at hand: there can be separating equilibria with inflated insurance for P-investors, or pooling equilibria, or it can even be that no pure-strategy equilibria exist altogether.

Incentive-constrained separation with inflated insurance for P-investors Suppose the P-investors' marginal rate of substitution between r_1 and r_2 exceeds the rate for F-investors for all realizations of (r_1, r_2), yet the probability μ of accessing the Q-technology is sufficiently large such that F-investors prefer the efficient contract $(r_1^{\delta}, r_2^{\delta})$ for P-investors over the credit-constrained contract (r_1^d, r_2^d) for F-investors. Therefore, separation constrained solely by the credit friction

[15] This implication is in contradiction to real-world conduct as presented in Pennacchi and Santos (2021), according to which management compensation is based on total return on equity, aggregated across all product lines.

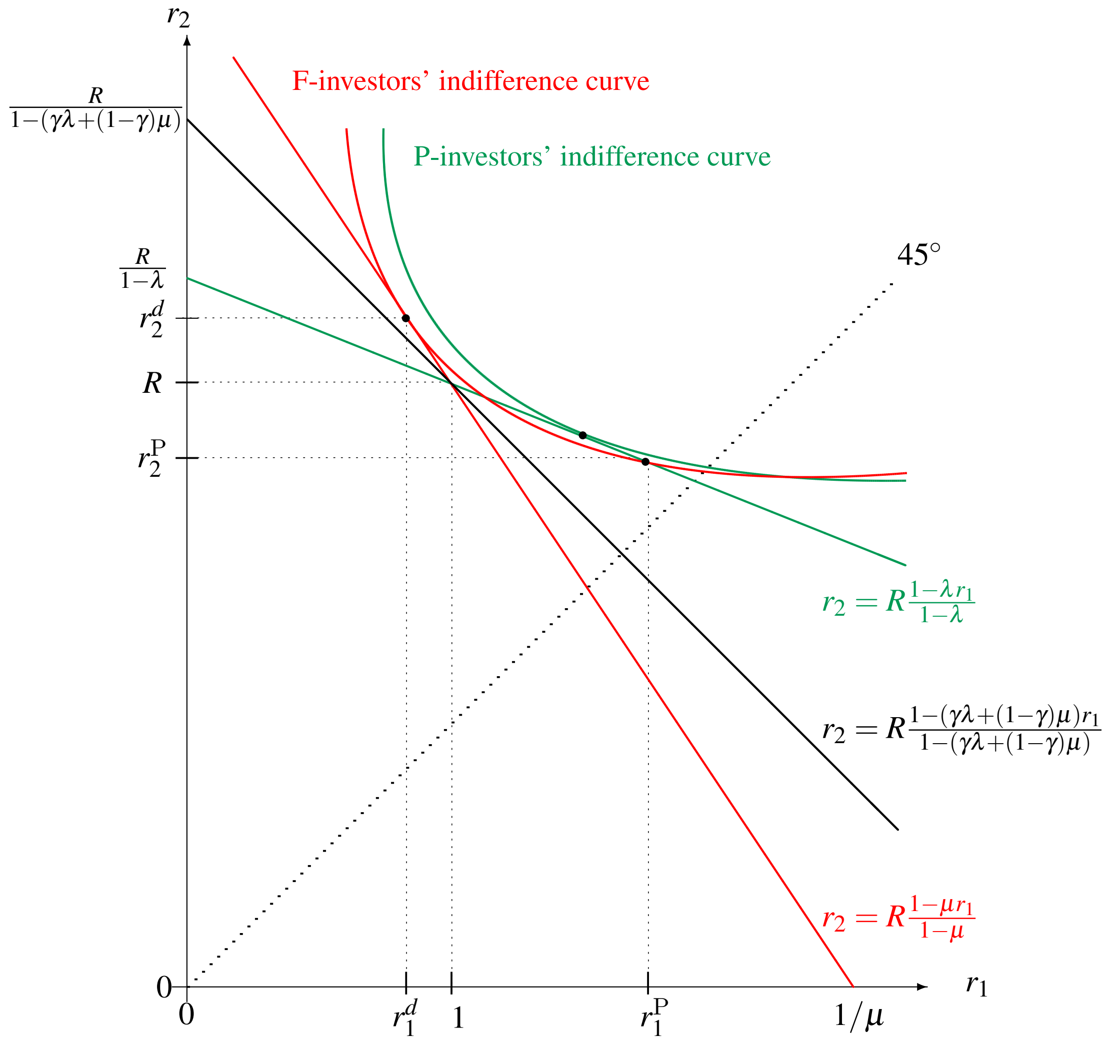

Figure 4: Incentive-constrained Separation with Inflated Insurance for P-investors.

breaks down, as the incentive constraint for F-investors (9) is violated for $(r_1^{\mathrm{F}}, r_2^{\mathrm{F}}) = (r_1^d, r_2^d)$ and $(r_1^{\mathrm{P}}, r_2^{\mathrm{P}}) = (r_1^\delta, r_2^\delta)$.[16]

Figure 4 illustrates a possible scenario for this case. If both contracts are on the same indifference curve for F-investors, they weakly prefer their own contract, (r_1^d, r_2^d), over the contracts offered to P-investors, $(r_1^{\mathrm{P}}, r_2^{\mathrm{P}})$. Banks make zero-profits with P-investors if $(r_1^{\mathrm{P}}, r_2^{\mathrm{P}})$ is on the respective zero-profit line. In Figure 4, there are thus two potential contracts, characterized by the intersection of the F-investors' indifference curve and the P-investors' zero-profit line. One contract is to the north-west

[16]If the P-investors' incentive constraint (10) is violated but not the F-investors' incentive constraint (9), then the P-investors' marginal rate of substitution between r_1 and r_2 cannot exceed the respective rate for F-investors.

of (r_1^δ, r_2^δ), and the other to the south-east. F-investors are indifferent between these two. However, as long as the contract to the south-east of (r_1^δ, r_2^δ) satisfies $r_1^{\mathrm{P}} < r_2^{\mathrm{P}}$, P-investors strictly prefer this one because their marginal rate of substitution between r_1 and r_2 exceeds the rate for F-investors.[17]

Such equilibrium thus implies an even larger insurance benefit to P-investors relative to the case where F-investors (who aim for flexibility) are absent. Given the incentive constraint of F-investors, $(r_1^{\mathrm{P}}, r_2^{\mathrm{P}})$ is the best separating contract P-investors can get. Also, a bank cannot profitably enter the market with a pooling contract as a pooling business model does not facilitate contracts that would generate zero profits and be a Pareto-improvement to the two separating contracts, (r_1^d, r_2^d) and $(r_1^{\mathrm{P}}, r_2^{\mathrm{P}})$.

The following Proposition generalizes these insights.

Proposition 2 (Incentive-constrained Separation with Inflated Insurance for P-investors)
Consider economies $\mathscr{E} = (u, \gamma, \lambda, \bar{\mu}, Q, R)$ for which $\bar{\mu}$ is such that $(r_1^\delta, r_2^\delta) \sim_{\mathrm{F}} (r_1^d, r_2^d)$ and $(r_1^\delta, r_2^\delta) \succ_{\mathrm{P}} (r_1^d, r_2^d)$. Under Assumption 2*, for each such economy $\mathscr{E}$ there exist $\eta(\mathscr{E}) > 0$ such that there are economies $\mathscr{E}' = (u, \gamma, \lambda, \hat{\mu}, Q, R)$ with $\hat{\mu} \in]\bar{\mu}, \bar{\mu} + \eta(\mathscr{E})[$ where a* separating *equilibrium obtains in which the F-investors' contract $(r_1^{\mathrm{F}}, r_2^{\mathrm{F}})$ satisfies $r_1^{\mathrm{F}} = r_1^d$ and $r_2^{\mathrm{F}} = r_2^d$, and the P-investors' contract $(r_1^{\mathrm{P}}, r_2^{\mathrm{P}})$ satisfies $r_1^{\mathrm{P}} > r_1^\delta$ and $r_2^{\mathrm{P}} < r_2^\delta$.*

Proof: See Appendix C. □

The next corollary states an interesting feature of the limits to inflated insurance for P-investors.

Corollary 2 (Populations dominated by P-investors)
The set of probabilities μ of accessing the Q-technology, for which equilibria with inflated liquidity insurance for P-investors obtain, converges to the empty set if the share of P-investors in the population γ approaches one.

Proof: See Appendix D. □

[17] If $r_1^{\mathrm{P}} > r_2^{\mathrm{P}}$, patient P-investors are better off pretending to be impatient and withdraw at date $t = 1$, which renders this contract incentive incompatible.

Intuitively, neither the indifference curves nor the zero-profit lines associated with separating contracts depend on the composition of the population, but the slope of the pooling zero-profit line, $r_2 = R\frac{1-(\gamma\lambda+(1-\gamma)\mu)r_1}{1-(\gamma\lambda+(1-\gamma)\mu)}$, does (see Figure 4). As γ goes to one, it converges to the zero-profit line for P-investors, $r_2 = R\frac{1-\lambda r_1}{1-\lambda}$. Therefore, the pooling zero-profit line eventually intersects a set of contracts enclosed by the P-investors' indifference curve going through $(r_1^{\mathrm{P}}, r_2^{\mathrm{P}})$ and the zero-profit line associated with P-investors. Their indifference curves being steeper than the F-investors' indifference curves, a set of pooling contract becomes thus available that are Pareto-improvements to the separating contracts $(r_1^{\mathrm{P}}, r_2^{\mathrm{P}})$ and (r_1^d, r_2^d). Therefore, for any given μ for which inflated insurance for P-investors is an equilibrium provided $\gamma = 0$, there is a $\bar{\gamma} < 1$ such that separating contracts with inflated insurance cannot be an equilibrium for all $\gamma \in]\bar{\gamma}, 1[$.

Pooling Suppose the F-investors' marginal rate of substitution between r_1 and r_2 exceeds the respective rate for P-investors. Then, separation cannot exist in equilibrium. To see how, consider first two contracts between which F-investors are just indifferent. Of these two contracts, let one contract satisfy the zero-profit condition associated with P-investors, $r_2 = R(1-\lambda r_1)/(1-\lambda)$, and the other the zero-profit condition associated with F-investors, $r_2 = R(1-\mu r_1)/(1-\mu)$. Among these two contracts, P-investors then strictly prefer the contract intended for F-investors if and only if the marginal rate of substitution between r_1 and r_2 is higher for F-investors than for P-investors. Conversely, if we consider two contracts between which P-investors are just indifferent, again one contract satisfying the zero profits with P-investors, $r_2 = R(1-\lambda r_1)/(1-\lambda)$, the other zero profits with F-investors, $r_2 = R(1-\mu r_1)/(1-\mu)$, then F-investors will prefer the contract intended for P-investors.

While equilibria with separating contracts are, therefore, not possible, equilibria in which banks offer *pooling contracts* may still exist. Such pooling contracts specify identical payment schedules, $\mathscr{D}^{\mathrm{F}} = \mathscr{D}^{\mathrm{P}} = (r_1^{\mathrm{Pool}}, r_2^{\mathrm{Pool}})$, to all investors. Figure 5 illustrates this. Competitive banks with business models associated with pooling contracts offer payments satisfying $r_2 = R\frac{1-(\gamma\lambda+(1-\gamma)\mu)r_1}{1-(\gamma\lambda+(1-\gamma)\mu)}$, i.e. they are located on the pooling zero-profit line. Consider any contract on that line other than $(1, R)$, for example as in Point A. Given that the F-investors' marginal rate of substitution between r_1 and r_2 exceeds the respective rate for P-investors, there is a contract B such that F-investors are just indifferent between A and B, while P-investors strictly prefer B. Hence, a bank could profitably enter the market by offering contract B, pulling away P-investors from banks offering the pooling contract A. Left with only F-investors as clientele, contract A is no longer sustainable. Therefore, contract A cannot be an equilibrium. In turn, contract B as part of a separating equilibrium is not sustainable either, given the condition for the marginal rates of substitution between r_1 and r_2. A similar argument can be made for pooling contracts to the north-west of $(1,R)$, ruling out pooling contracts on that upper branch of the pooling zero-profit line.

Next, consider the only remaining contract, $(1,R)$. Any contract on the P-investors' zero-profit line to the south-east of $(1,R)$ would not only make P-investors better off but also F-investors, and any contract on the F-investors' zero-profit line to the north-west of $(1,R)$ would not only make F-investors better off but also P-investors. Therefore, there are no separating contract offers which can break a pooling contract $(1,R)$. Indeed, as long as the slope of the pooling zero-profit line is between the slope of the indifference curve of the P-investors and the slope of the indifference curve of the F-investors, there are no other contracts on the pooling zero-profit line that would be Pareto-improvements to $(1,R)$ and thus attract both types of investors.

The following proposition formalizes these insights.

Proposition 3 (Pooling)

Consider economies $\mathscr{E} = (u, \gamma, \lambda, \mu, Q, R)$ *with* $\frac{\mu}{1-\mu}\frac{u'(Qr_1)}{u'(r_2)}Q > \frac{\lambda}{1-\lambda}\frac{u'(r_1)}{u'(r_2)}$ *for all* $(r_1, r_2) \in \mathbb{R}^2_+$. *If* $\frac{\mu}{1-\mu}u'(Q)Q > \frac{\gamma\lambda+(1-\gamma)\mu}{1-(\gamma\lambda+(1-\gamma)\mu)}u'(R)R > \frac{\lambda}{1-\lambda}u'(1)$ *the only equilibrium is a pooling equilibrium. The contract is determined as* $(r_1^{\text{Pool}}, r_2^{\text{Pool}}) = (1,R)$.

Proof: See Appendix E. $\square$

Note that a payment schedule $(1,R)$ also obtains in economies without banks but with asset markets. There, all investors choose their own portfolio allocation between storage and the R-technology at date $t=0$, and then trade storage for R-projects in an asset market at date $t=1$ depending on their liquidity needs. For the asset market equilibrium to be arbitrage-free, equilibrium requires that the asset price equals one as only then storage and R-technology generate the same return between dates

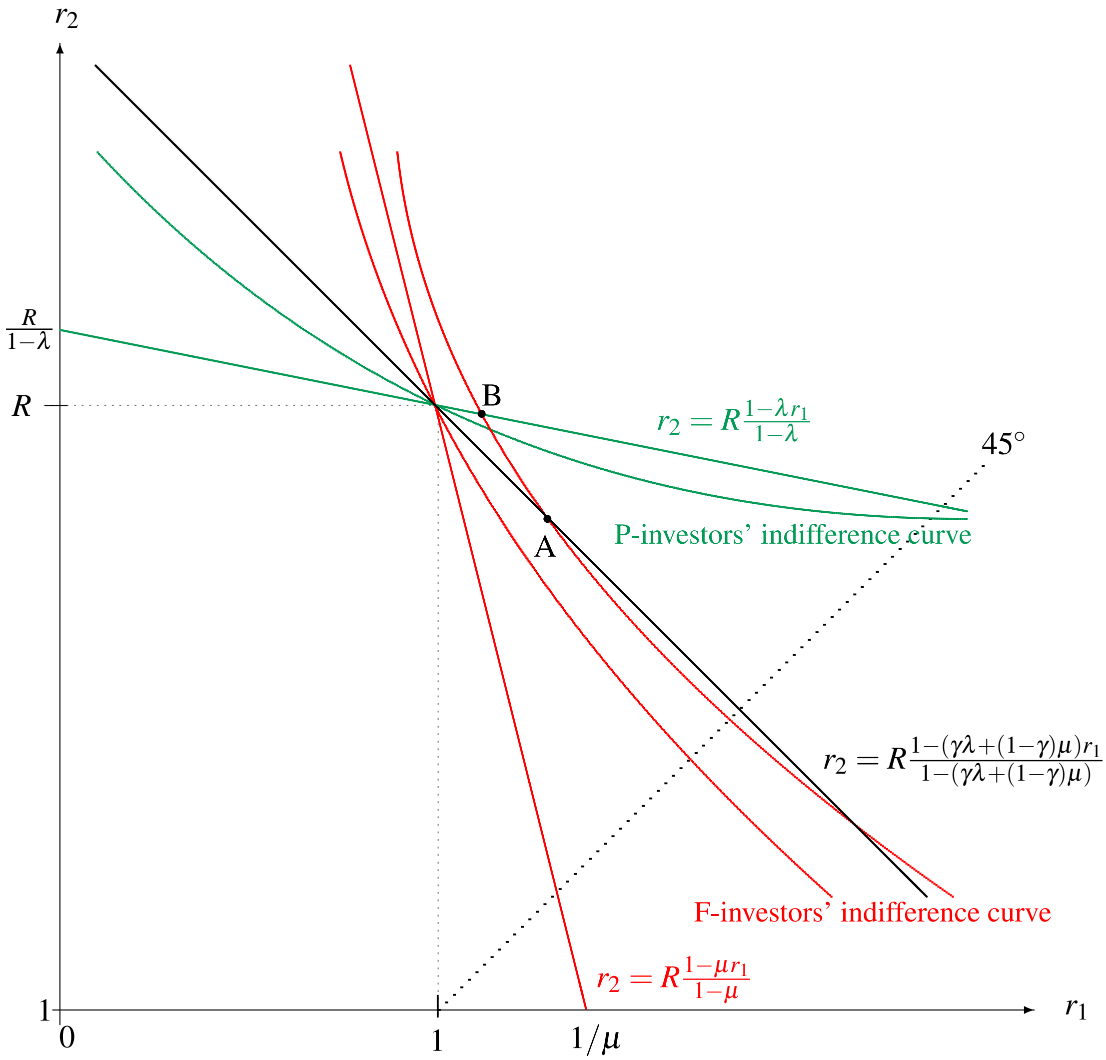

Figure 5: Pooling.

$$\left(\frac{\mu}{1-\mu}u'(Q)Q > \frac{\gamma\lambda+(1-\gamma)\mu}{1-(\gamma\lambda+(1-\gamma)\mu)}u'(R)R > \frac{\lambda}{1-\lambda}u'(1)\right).$$

$t=0$ and $t=1$. With asset prices equal to one, impatient P-investors will sell their R-projects and consume one unit, and patient P-investors will use all their storage to buy R-projects and consume R units of the good. As for F-investors, those with access to the Q-technology will sell their holdings of R-projects and invest one unit in the new opportunity. F-investors without access use their storage to buy additional R-projects.

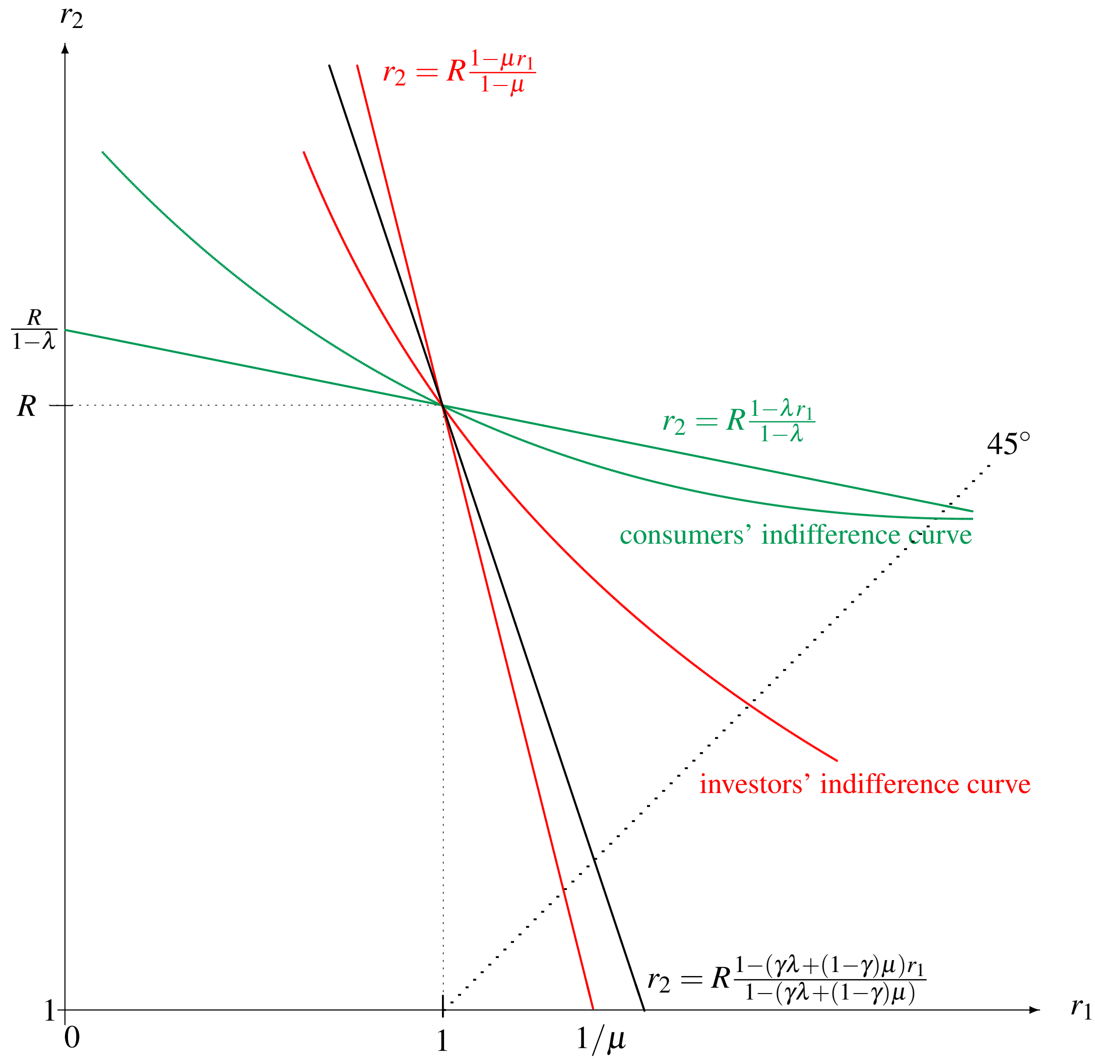

Figure 6: Non-existence of Equilibrium in Pure Strategies $\left(\frac{\gamma\lambda+(1-\gamma)\mu}{1-(\gamma\lambda+(1-\gamma)\mu)}u'(R)R > \frac{\mu}{1-\mu}u'(Q)Q > \frac{\lambda}{1-\lambda}u'(1)\right)$.

Non-existence of pure-strategy equilibria To conclude the analysis of possible equilibria, economies can also be such that there is no contract that cannot be dominated by another contract.[18]

Proposition 4 (Non-existence of Equilibrium)

Consider economies $\mathscr{E} = (u, \gamma, \lambda, \mu, Q, R)$ *with* $\mu > \lambda$ *and* $\frac{\mu}{1-\mu}\frac{u'(Qr_1)}{u'(r_2)}Q > \frac{\lambda}{1-\lambda}\frac{u'(r_1)}{u'(r_2)}$ *for all* $(r_1, r_2) \in$

[18]While mixed strategy equilibria will exist when randomization across contracts is allowed for ((Dasgupta and Maskin, 1986), we do not pursue this possibility in this paper. By their very nature mixed strategy equilibria will induce added strategic uncertainty, and, hence, instability in market outcomes (see Gehrig and Ritzberger, 2020).

$\mathbb{R}^2_+$. *There is no equilibrium in pure strategies, provided the following condition* $\frac{\mu}{1-\mu}u'(Q)Q > \frac{\gamma\lambda+(1-\gamma)\mu}{1-(\gamma\lambda+(1-\gamma)\mu)}u'(R)R > \frac{\lambda}{1-\lambda}u'(1)$ *is violated.*

Proof: See Appendix F. $\square$

Under the conditions of this Proposition there is no viable contract that is not dominated by another contract. Figure 6 illustrates such case. Since the F-investors' marginal rate of substitution between r_1 and r_2 exceeds the respective rate for P-investors, neither separating contracts nor pooling contracts other than $(1,R)$ are feasible in equilibrium by the arguments already made above. However, a pooling contract $(1,R)$ cannot be an equilibrium either. To see why, suppose banks were offering a pooling contract $(1,R)$. Then, another bank could profitably enter the market by offering another pooling contract, for there are Pareto-improvements to $(1,R)$ along the pooling zero-profit line — to the north-west of $(1,R)$ in Figure 6. As argued before, those contracts cannot be an equilibrium either given the marginal rates of substitution between r_1 and r_2.

A pure-strategy choice of equilibrium contracts, i.e. one which does not apply lotteries over contracts, fails to exist here. Therefore, there is no stable market outcome. Interestingly, pure-strategy equilibria do not exist, if the population is highly unbalanced in either direction, i.e. if the proportion of P-investors, γ, is either very close to zero or to unity. The value of γ determines only the slope of the pooling zero-profit line. It converges to the F-investors' zero-profit line for $\gamma \to 0$ and to the P-investors' zero-profit line for $\gamma \to 1$. Corollary 3 summarizes the implications for the limiting cases.

Corollary 3 (Unbalanced Populations)

Consider economies $\mathscr{E} = (u,\gamma,\lambda,\mu,Q,R)$ *with* $\mu > \lambda$ *and* $\frac{\mu}{1-\mu}\frac{u'(Qr_1)}{u'(r_2)}Q > \frac{\lambda}{1-\lambda}\frac{u'(r_1)}{u'(r_2)}$ *for all* $(r_1,r_2) \in \mathbb{R}^2_+$. *There is no equilibrium in pure strategies if the proportion of P-investors in the population,* γ, *is either very large or very low.*

Proof: See Appendix G. $\square$

6 Co-existence: The case without frictions

In this section we study the implications of the co-existence of F-investors and P-investors, and thus of two different motives for liquidity demand, if for whatever reason, type and loans to F-investors can be fully enforced. It is well-known that intermediaries can deploy technologies to efficiently enforce loan contracts (Diamond, 1984). However, how the ability to enforce loan contracts effects the maturity transformation by banks in the framework of Diamond and Dybvig (1983) has not been explored. We begin with describing optimal allocations if contracts are enforceable by banks, followed by possible ways how banks can implement those allocations.

6.1 Allocation

Consider the allocation that is optimal under the economy-wide feasibility constraints. Storage, the long-term R-technology, and the short-term Q-technology are all constant returns to scale. Hence, the Q technology dominates the R-technology in terms of producing consumption goods available at date $t=2$. On the other hand, storage dominates in terms of providing both, early consumption goods and funds for investment in the Q-technology. Accordingly, it is optimal to store all endowments from all investors between dates $t=0$ and $t=1$, and then use this storage to fund the Q-technology and the consumption by impatient P-investors. The returns on the Q-technology will then fund the consumption by F-investors and by patient P-investors.

While the optimal allocation of funds between storage, long-term production, and short-term production is determinate, Pareto-optimal allocations of consumption are indeterminate. We focus on the allocation that provides P-investors with the same consumption profile as if F-investors would not exist. At date $t=0$, all endowments are put into storage until date $t=1$. Once the future investment opportunities arrive and uncertainty about consumption needs is resolved, this storage is partly used to provide for impatient P-investors, $c_1^\delta = y^\delta/\lambda$, with y^δ satisfying the first-order condition (8). The remainder of the stored endowments, $1-\gamma y^\delta > 1-\gamma$, is invested in the Q-technology. At date $t=2$, the Q-technology will produce $Q(1-\gamma y^\delta)$, which will be distributed to patient P-investors and all F-investors. Each patient P-investor gets $c_2^\delta = R(1-y^\delta)/(1-\lambda)$, leaving for each F-investor an amount of $Q+\frac{\gamma}{1-\gamma}(Q-R)(1-y^{\delta}) > Q$. Therefore, P-investors receive the consumption plan that corresponds to the first-best in case of isolation, and F-investors will be able to consume more than they could by providing for themselves. The reason is that by pooling the endowments of all investors, the comparatively unproductive investment in the R-technology can be avoided, in which P-investors would have to invest if they were left to their own devices. Instead, all goods for consumption at date $t=2$ are produced with the comparatively more productive Q-technology.

6.2 Implementation

Provided banks know the individual motive for their customers' liquidity preference and can fully enforce loan repayments, a competitive banking sector can implement the optimal allocation. To see how, suppose all investors deposit their endowments in banks at date $t=0$. P-investors do so in exchange for a demand deposit contract which allows them to withdraw $r_1^{\mathrm{P}}=c_1^{\delta}$ if they get impatient and $r_2^{\mathrm{P}}=c_2^{\delta}$ if they remain patient. F-investors are granted credit lines to be drawn at a gross interest rate equal to Q at date $t = 1$ and receive shares in the bank's equity which allows them to share the value of the bank's assets net of payments to P-investors at date $t=2$, i.e. $r_1^{\mathrm{F}}=0$ and $r_2^{\mathrm{F}}=Q+\frac{\gamma}{1-\gamma}(Q-R)(1-\lambda r_1^{\mathrm{P}})>Q$. At the middle date $t=1$, lucky F-investors draw on their credit lines, borrowing all of the banks' remaining storage $1-\gamma\lambda r_1^{\mathrm{P}}$. At the final date $t=2$, F-investors settle their debt and pay $Q(1-\gamma\lambda r_1^{\mathrm{P}})$ to banks. With these earnings, banks pay patient P-investors r_2^{P} and F-investors $r_2^{\mathrm{F}}=Q+\frac{\gamma}{1-\gamma}(Q-R)(1-\lambda c_1^{\mathrm{P}})$. Accordingly, we conclude:

Lemma 3 (Economies of Scope)
The co-existence of a precaution-driven and a flexibility-driven demand for liquidity entails efficiency gains from combining liquidity creation through credit lines with liquidity creation through deposit-taking. Provided banks can distinguish investors by their type and fully enforce loans to F-investors, banks can realize such economies of scope without engaging in maturity transformation.

Under such ideal conditions, the business of accepting deposits and simultaneously granting lines of credit is a result of economies of scope.[19] Interestingly, banks would not have to engage in any maturity transformation at all to reap these economies of scope. At date $t=0$, banks issue demand deposits to P-investors and equity shares to F-investors, both backed entirely by stored goods. From date $t=1$ onward, the banks' assets comprise the loans to F-investors and their liabilities are the demand deposits still held by patient P-investors, with F-investors holding the residual claims on the banks' asset returns.

7 Implications and discussion

Bank runs The present paper adds to the vast literature on bank runs and fragility of the banking system.[20] While term deposits are, in principle, prone to the same type of coordination failure as demand deposits, an important difference is that the thresholds for coordination failures are higher with term deposits.

Without going into the details of a fully fledged model of bank runs as a result of coordination failures, our analysis lends itself to some preliminary conclusions. Suppose that the R-technology can be liquidated at date $t=1$ for a per-unit scrap value equal to one; that depositors withdrawing at date $t=1$ are served sequentially; and that neither the bank nor the banking supervisory authority can precommit to suspend convertibility if a bank run is underway (see, e.g., Ennis and Keister, 2009). Suppose next that a depositor believes that the share of depositors actually withdrawing from their own bank at date $t=1$ is at least ν. Then, if $r_1 > (1-\nu r_1)R/(1-\nu)$, or $\nu > \tilde{\nu} := (R-r_1)/(r_1(R-1))$, an investor is better off withdrawing at date $t=1$ irrespective of her own liquidity event. Accordingly, $\tilde{\nu}$ can be seen as a measure of a bank's susceptibility to bank runs.[21] A lower $\tilde{\nu}$ indicates a higher susceptibility, and banks are not prone to runs at all if $\tilde{\nu} > 1$.

[19]Note, these economies arise here in absence of any incentive problems at the bank level. In Calomiris and Kahn (1991) and Diamond and Rajan (2001), for example, demand deposits are considered to provide incentives for banks to create value on behalf of their customers.

[20]See e.g. Allen and Gale (2004), Bucher et al. (2018), Cooper and Ross (1998), Ennis and Keister (2006), Matutes and Vives (1996) and Rochet and Vives (2004).

[21]He and Manela (2016) refer to this measure, i.e. the mass of depositors it takes to run down the bank, as *bank liquidity*.

In pooling equilibria with a contract $(r_1^{\text{Pool}}, r_2^{\text{Pool}}) = (1, R)$ for all investors, we obtain $\tilde{v} = 1$ such that banks are not prone to bank runs. In credit-constrained separating equilibria, banks providing liquidity to F-investors are not prone to bank runs either since $r_1^{\text{F}} < 1$ and, therefore, $\tilde{v} > 1$. However, banks that provide liquidity services to P-investors are prone to bank runs since $r_1^{\text{P}} > 1$ and, therefore, $\tilde{v} < 1$.[22] Interestingly, by this measure, these banks can be considered even more prone to bank runs in equilibria with incentive-constrained separation than in equilibria with credit-constrained separation because $r_1^{\text{P}} > r_1^{\delta}$ (see Proposition 2). Therefore, the co-existence of various motives for liquidity demand can put further strain on the stability of banks, but it is banks providing liquidity insurance to P-investors which are affected.

Low interest rate environment Since the long-term production generates safe returns R, they can be expected to be linked to the return on long-term government debt. How would equilibrium be affected in a low interest rate environment, i.e. if the long-term rate R converges to one? It is readily verified that in such an environment the demand deposit contract converges to a contract merely repaying P-investors their initial endowment, i.e. $\lim_{R\to 1}(r_1^{\delta}, r_2^{\delta}) = (1, 1)$. In other words, taking precautions looses relevance as a motive for liquidity demand, while preserving flexibility remains active as long as $Q > 1$.

Interestingly, a low interest rate environment can contribute to instability as equilibria in pure strategies may cease to exist when the returns on the long-term production fall. The following example illustrates this. Suppose that the initial return with long-term production is $R = R_0$, and that for this value a pooling equilibrium just obtains, i.e. there is a small $\varepsilon > 0$ such that

$$\frac{\mu}{1-\mu}u'(Q)Q - \varepsilon = \frac{\gamma\lambda + (1-\gamma)\mu}{1-(\gamma\lambda + (1-\gamma)\mu)}u'(R_0)R_0 > \frac{\lambda}{1-\lambda}u'(1).$$

Suppose next that the return on the long-term technology, R, falls to one. For relative risk aversion larger one we obtain $\frac{\mathrm{d}}{\mathrm{d}R}(u'(R)R) < 0$, with $\lim_{R\to 1} u'(R)R = u'(1) > u'(Q)Q$. Therefore, a fall of R

[22] Taking a global games perspective as put forward by Goldstein and Pauzner (2005), the threshold for bad fundamental news needed to trigger a run would thus be significantly higher for term deposit contracts that serve the F-investors, compared to demand deposit contracts that serve the P-investors.

to one will lead to a violation of $\frac{\mu}{1-\mu}u'(Q)Q > \frac{\gamma\lambda+(1-\gamma)\mu}{1-(\gamma\lambda+(1-\gamma)\mu)}u'(R)R$ for sufficiently small ε, i.e. a pooling equilibrium, which exists and is the only equilibrium for $R = R_0$, ultimately fails to exist as $R \to 1$. In other words, a decrease in the long-term interest rate, as measured by R, increases the range of unstable outcomes. Clearly, this type of instability will not arise in a world with only a single motive for liquidity demand.

However, by the argument developed above, run-related concerns for systemic risk would be of declining relevance in a low interest rate environment. Hence, the focus in business models shifts from "front-loaded" demand deposits to "back-loaded" term deposits. This regime shift should also be reflected in the supervisory and regulatory framework for banks.

Bank regulation While banking regulation tends to maintain common (minimum) standards for all banking institutions alike[23] our analysis reveals that a more case-based approach may be socially preferable. In equilibria with credit-constrained separation, serving investors with a desire to preserve flexibility requires a different bank business model than addressing the precautionary motive of other investors (see Corollary 1). Moreover, our discussion of bank runs suggests that while the liquidity demand by P-investors exposes banks to coordination failures, the liquidity demand by F-investors may not. Therefore, the motive of the liquidity demand matters for the design of regulations aiming at bank stability.[24] Regulation should particularly focus on banks catering to the precautionary motive, while other banks serving a desire to preserve flexibility would seem to require less stringent regulation.

Furthermore, our analysis so far can provide no reason why banks catering to both liquidity preferences are better (or worse) than banks specializing on one type of liquidity preference. But this is due to our equilibrium concept as it restricts each bank to offer only contracts that do not generate losses. A case for, or against, in-house pooling of business lines can be made if we instead consider competitive equilibria where each individual bank can offer a *menu of cross-subsidizing contracts* (in the spirit of Miyazaki, 1977; Wilson, 1977; Spence, 1978). Then, should cross-subsidizing contracts

[23] See Basel Committee for Banking Supervision (2013) on liquidity regulation, for example.

[24] In our model insensitive regulation also interferes in the competitive relation between different business models as long as it is binding somewhere.

prevail in equilibrium over banks offering only loss-free contracts, the two different motives could be expected to be served together. However, as we show in Appendix H, banks offering menus of cross-subsidizing contracts never prevail in equilibrium. Importantly, the possibility to offer menus of cross-subsidizing contracts can render equilibria with single-contract offers impossible such that no equilibrium exists at all. Accordingly, this suggests a reason for regulators to stop the practice of cross-subsidizing lines of business within a bank. This does not necessarily require the separation of ownership into several banking units but it does require to treat, and manage, separate business models separately from an organizational point of view. Accordingly, allowing bank shareholders to insist on the same, highest rate of return across all divisions within a multi-product bank, may be counter-productive.

Institutional indeterminacy Although we have made reference to demand deposits, term deposits, and even equity shares, as the contractual means that serve the interests of a specific type of investors, it should be clarified that the use of these terms is primarily to keep the presentation simple. We associate differences in contracts only with differences in the sequence of payments. Other, undoubtedly important features of contracts, such as being negotiable or tradable, and constraints often associated with certain contracts, such as sequential service, are not the focus of attention in this paper.

For example, that banks can refinance themselves entirely with equity in absence of frictions (see Section 6) is only one of many contractual solutions. What matters here is that banks do not engage in maturity transformation with either institutional arrangement. It is commonly understood that without frictions, little can be said about the specific institutional arrangement which maintains an allocation. This also includes whether banks set themselves up as universal banks, integrating several business lines via an internal capital market, or as separate entities which use an interbank loan market. Either arrangement will achieve the Pareto-optimal allocation in the absence of frictions. With an interbank market, both banks store the endowments of their customers in the first period. After the first period, the bank specializing on P-investors grants the remainder of their stored goods $1-\lambda c_1^{\delta}$ as a loan to the other bank for one period. If the interest rate on such interbank loans is R, the bank specializing on P-investors will pay its patient P-investors c_2^δ. The banks specializing in F-investors can grant its own stored funds, along with the funds borrowed from the other bank, as a loan to its lucky F-investors and thus implements the efficient allocation.

Diversity of motives versus heterogeneity in risk preferences Some of our results are robust with regards to other specifications of heterogeneous liquidity preferences, further strengthening one of our main arguments that their co-existence matters for market outcomes. For example, upon inspection of Propositions 1 through 3, the same type of equilibrium phenomena would arise in a world in which all investors want to take precautions against sudden consumption needs but differ substantially with respect to their risk preferences: some have relative risk aversion of $-cu_1''(c)/u_1' < 1$ and others have relative risk aversion of $-cu_2''(c)/u_2' > 1$. Liquidity shocks for the more risk-tolerant investors arise with probability $\lambda_1 = \mu$ and for the more risk-averse investors with probability $\lambda_2 = \lambda$, where μ and λ are as in the main analysis above. In this case, the optimal contract for the more risk-tolerant investor is given by (r_1^1, r_2^1) with a penalty payment for early withdrawal $r_1^1 < 1 < R < r_2^1$, similar to the constrained-efficient F-Investor contract (Figure 1). The contract for the more risk-averse investors (r_1^2, r_2^2) is front-loaded $1 < r_1^2 < r_2^2 < R$ and resembles the efficient P-Investor contract (Figure 2).

With such modification, our previous equilibrium analysis of the co-existence of different motives is identical to one with heterogeneity in risk preferences only. However, and crucially, such a claim requires that the set of efficient contracts for one type of investors requires a penalty rate ($r_1^1 < 1$), while the other set of efficient contracts requires an insurance benefit ($r_1^2 > 1$). Therefore, the similarity in results obtains only if there is one group of highly risk-tolerant investors (with $-cu_1''(c)/u_1' < 1$) and another group of quite risk-averse investors (with $-cu_1''(c)/u_1' > 1$). In other words, heterogeneity in risk-preferences alone will not be enough to generate all our phenomena within a Diamond and Dybvig (1983) model. Specifically, they do not obtain if either $-cu''(c)/u' > 1$ for all investors or $-cu_1''(c)/u_1' < 1$ for all investors.

Another important difference between the co-existence of different risk-preferences and the co-existence of different motives refers to the case without financial frictions. In the absence of investors who desire to preserve their flexibility for future, better investment opportunities, the potential for economies of scope no longer exists. Therefore, with only P-investors in place, welfare will be much reduced.

8 Concluding remarks

Our analysis reveals that the nature of liquidity demand crucially matters for competitive market outcomes. This does not only hold for the different motives in isolation but in particular for the co-existence of motives at any point in time. In a simple framework we have shown that the co-existence of need-based liquidity demand with an option-based motive has the potential to benefit from economies of scope in a frictionless world. Likewise, in the presence of frictions, the precise nature of these frictions as well as their interplay will affect the nature of market outcomes, and, therefore, potential policy implications. For example, focusing on one motive and one friction only is likely to direct the policy debate towards bank runs, even for constellations, when their occurrence is not likely because maturity transformation does not take place in equilibrium. But also, as we show, constellations may arise, where the severity of the bank-run problem is underestimated because of the ignorance of other motives.

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Appendix

A Proof of Lemma 2

For $-cu''(c)/u'(c) = 1$, we obtain $r_1^\delta = r_1^d = 1$. Hence, both incentive constraints hold with equality. The P-investors' participation constraints are satisfied with strict inequality for all $\lambda \in]0,1[$ because $u(1) \geq u(y)$ and $u(R) \geq u(R(1-y)+y)$ for all $y \in [0,1]$ since $R > 1$, with $u(1) > u(y)$ for $y < 1$ and $u(R) > u(R(1-y)+y)$ for $y > 0$. The F-investors' participation constraints are satisfied with strict inequality for all $\mu \in]0,1[$ because $u(Q) \geq u(Qy+R(1-y))$ since $Q > R$ and $u(R) \geq u(R(1-y)+y)$ for all $y \in [0,1]$ since $R > 1$, with $u(Q) > u(Qy+R(1-y))$ for $y < 1$ and $u(R) > u(R(1-y)+y)$ for $y > 0$.

B Proof of Proposition 1

The proof is by establishing six claims consecutively.

Claim 1: (r_1^d, r_2^d) and (r_1^δ, r_2^δ) satisfy the participation constraints for F-investors and P-investors, respectively.

The participation constraints are satisfied with strict inequality:

- For P-investors:

$$\begin{aligned} &\lambda u\left(r_1^\delta\right) + (1-\lambda)u\left(\tfrac{R(1-\lambda r_1^\delta)}{1-\lambda}\right) \\ &\quad > \lambda u(1) + (1-\lambda)u(R) \\ &\qquad > \sup\{\lambda u(y) + (1-\lambda)u(R(1-y)+y) \mid y \in [0,1]\} \end{aligned} \tag{16}$$

  The first inequality obtains since $r_1^\delta \in \arg\max\{\lambda u(r_1) + (1-\lambda)u\left(\frac{R(1-\lambda r_1)}{1-\lambda}\right) \mid r_1 \in [0, \lambda^{-1}]\}$. The second inequality obtains since $R > 1$ implies for all $y \in [0,1]$ that $u(1) \geq u(y)$ and $u(R) \geq u(R(1-y)+y)$, with $u(1) > u(y)$ for $y < 1$ and $u(R) > u(R(1-y)+y)$ for $y > 0$.

- For F-investors:

$$\begin{aligned}
&\lambda u\left(Qr_1^d\right)+(1-\lambda)u\left(\tfrac{R(1-\lambda r_1^d)}{1-\lambda}\right)\\
&\qquad > \lambda u(Q)+(1-\lambda)u(R)\\
&\qquad\qquad > \sup\{\mu u(Qy+R(1-y))+(1-\mu)u(R(1-y)+y)\,|\,y\in[0,1]\}
\end{aligned}\tag{17}$$

The first inequality obtains since $r_1^d \in \arg\max\{\lambda u(Qr_1)+(1-\lambda)u\left(\frac{R(1-\lambda r_1)}{1-\lambda}\right) \mid r_1 \in [0,\lambda^{-1}]\}$. The second inequality obtains since $Q>R>1$ implies for all $y\in[0,1]$ that $u(Q)\geq u(Qy+R(1-y))$ and $u(R)\geq u(R(1-y)+y)$, with $u(Q)>u(Qy+R(1-y))$ for $y<1$ and $u(R)>u(R(1-y)+y)$ for $y>0$.

Claim 2: $(r_1^d,r_2^d)\succ_{\mathrm{I}}(r_1^\delta,r_2^\delta)$ and $(r_1^\delta,r_2^\delta)\succ_{\mathrm{C}}(r_1^d,r_2^d)$ for all $\lambda\geq\mu$.

1. For $\mu=\lambda$, the incentive constraints are satisfied for all investors:

   - F-investors: $\lambda u\left(Qr_1^d\right)+(1-\lambda)u\left(\frac{R(1-\lambda r_1^d)}{1-\lambda}\right)\geq\lambda u\left(Qr_1^\delta\right)+(1-\lambda)u\left(\frac{R(1-\lambda r_1^\delta)}{1-\lambda}\right)$ for all $r_1^\delta\in[0,\lambda^{-1}]$, with strict inequality if $-\frac{u''(c)}{u'(c)}c\neq1$, since $r_1^d\in\arg\max\{\lambda u(Qr_1)+(1-\lambda)u\left(\frac{R(1-\lambda r_1)}{1-\lambda}\right)\,\middle|\,r_1\in[0,\lambda^{-1}]\}$.
   - P-investors: $\lambda u(r_1^\delta)+(1-\lambda)u\left(\frac{R(1-\lambda r_1^\delta)}{1-\lambda}\right)\geq\lambda u(r_1^d)+(1-\lambda)u\left(\frac{R(1-\lambda r_1^d)}{1-\lambda}\right)$ for all $r_1^d\in[0,\lambda^{-1}]$, with strict inequality if $-\frac{u''(c)}{u'(c)}c\neq1$, since $r_1^\delta\in\arg\max\{\lambda u(r_1)+(1-\lambda)u\left(\frac{R(1-\lambda r_1)}{1-\lambda}\right)\,\middle|\,r_1\in[0,\lambda^{-1}]\}$.

   Therefore, $(r_1^d,r_2^d)\succ_{\mathrm{F}}(r_1^\delta,r_2^\delta)$ and $(r_1^\delta,r_2^\delta)\succ_{\mathrm{P}}(r_1^d,r_2^d)$ for $\lambda=\mu$ and $-\frac{u''(c)}{u'(c)}c>1$.

2. For $\lambda>\mu$, it suffices to consider the incentive constraints for F-investors and P-investors, respectively, letting λ increase for a given μ, starting from $\lambda=\mu$.

   - F-investors: The LHS of condition (9) is not affected by changes in λ. Hence, the total effect on the differential of expected utilities is positive as the RHS of condition (9) changes according to

$$\frac{\mathrm{d}}{\mathrm{d}\lambda}\left(\mu u(Qr_1^\delta)+(1-\mu)u(r_2^\delta)\right)=\mu Qu'(Qr_1^\delta)\frac{\mathrm{d}r_1^\delta}{\mathrm{d}\lambda}+(1-\mu)u'(r_2^\delta)\frac{\mathrm{d}r_2^\delta}{\mathrm{d}\lambda}<0\tag{18}$$

as $\frac{\mathrm{d}r_1^\delta}{\mathrm{d}\lambda}, \frac{\mathrm{d}r_2^\delta}{\mathrm{d}\lambda} < 0$. The latter follows from applying the implicit function theorem to the P-investors' first-order condition (8). If written as

$$u'\left(r_1^\delta\right) - Ru'\left(\frac{R(1-\lambda r_1^\delta)}{1-\lambda}\right) = 0$$

we have

$$\frac{\mathrm{d}r_1^\delta}{\mathrm{d}\lambda} = -\frac{R^2 u''(r_2^\delta)\frac{r_1^\delta - 1}{(1-\lambda)^2}}{u''(r_1^\delta) + R^2 u''(r_2^\delta)\frac{\lambda}{1-\lambda}} < 0,$$

and if written as

$$u'\left(\frac{R-(1-\lambda)r_2^\delta}{\lambda R}\right) - Ru'(r_2^\delta) = 0$$

we have

$$\frac{\mathrm{d}r_2^\delta}{\mathrm{d}\lambda} = -\frac{-u''(r_1^\delta)\frac{R-r_2^\delta}{\lambda^2 R}}{-u''(r_1^\delta)\frac{1-\lambda}{\lambda R} - Ru''(r_2^\delta)} < 0.$$

- P-investors: By the Envelope theorem, the LHS in condition (10) changes in response to increases in λ by $u(r_1^\delta) - u(r_2^\delta)$. The RHS in condition (10) changes in response to increases in λ by $u(r_1^d) - u(r_2^d)$. Hence, the total effect on the differential of expected utilities is $\left(u(r_1^\delta) - u(r_1^d)\right) - \left(u(r_2^\delta) - u(r_2^d)\right)$ which is positive since $r_1^\delta > r_1^d$ and $r_2^\delta < r_2^d$.

Claim 3: There is $\tilde{\mu} > \lambda$ such that $(r_1^\delta, r_2^\delta) \succ_{\mathrm{F}} (r_1^d, r_2^d)$ for all $\mu \in]\tilde{\mu}, 1[$ and $(r_1^d, r_2^d) \succ_{\mathrm{F}} (r_1^\delta, r_2^\delta)$ for all $\mu \in]0, \tilde{\mu}[$.

From the F-investors' first-order condition (6), we obtain $\lim_{\mu \to 1} y^d = \lim_{\mu \to 1} r_1^d = 1$. The LHS of condition (9) converges to $u(Q)$ and the RHS to $u(Qr_1^\delta) > u(Q)$ since $r_1^\delta > 1$. By the intermediate value theorem, there is thus $\bar{\mu} > \lambda$ such that $(r_1^\delta, r_2^\delta) \succ_{\mathrm{F}} (r_1^d, r_2^d)$ for all $\mu \in]\bar{\mu}, 1[$. Since the utility differential $Z_{\mathrm{I}} = (\mu u(Qr_1^d) + (1-\mu)u(r_2^d)) - (\mu u(Qr_1^\delta) + (1-\mu)u(r_2^\delta))$ is monotone in μ with $\mathrm{d}Z_{\mathrm{I}}/\mathrm{d}\mu = \left(u(Qr_1^d) - u(Qr_1^\delta)\right) - \left(u(r_2^d - u(r_2^\delta))\right) < 0$, the claim is established.

Claim 4: If Q is large, and λ small, there is $\hat{\mu} \in]\lambda, 1[$ such that $(r_1^d, r_2^d) \succ_{\mathrm{P}} (r_1^\delta, r_2^\delta)$ for all $\mu \in]\hat{\mu}, 1[$ and $(r_1^\delta, r_2^\delta) \succ_{\mathrm{P}} (r_1^d, r_2^d)$ for all $\mu \in]0, \hat{\mu}[$.

From the F-investors' first-order condition (6), we obtain $\lim_{\mu\to 1} y^d = \lim_{\mu\to 1} r_1^d = 1$. Therefore, $(r_1^d, r_2^d) \succ_{\mathrm{P}} (r_1^\delta, r_2^\delta)$ holds for $\mu \to 1$ provided

$$\lambda u(1) + (1-\lambda)u(r_2^d) > \lambda u(r_1^\delta) + (1-\lambda)u(r_2^\delta). \tag{19}$$

The P-investors' contract (r_1^δ, r_2^δ) does not depend on μ or Q. The first-order condition (6), determining the F-investors' contract (r_1^d, r_2^d), implies $\mathrm{d}r_2^d/\mathrm{d}Q > 0$ for all μ if $-cu''(c)/u'(c) > 1$. Hence, condition (19) is more likely to hold if Q is large or λ is small.

The utility differential $Z_{\mathrm{P}} = (\lambda u(r_1^\delta) + (1-\lambda)\,u(r_2^\delta)) - (\lambda u(r_1^d) + (1-\lambda)\,u(r_2^d))$ is monotone in μ with

$$\frac{\mathrm{d}Z_{\mathrm{P}}}{\mathrm{d}\mu} = -\left(\lambda u'(r_1^d)\frac{\mathrm{d}r_1^d}{\mathrm{d}\mu} + (1-\lambda)u'(r_2^d)\frac{\mathrm{d}r_2^d}{\mathrm{d}\mu}\right) < 0 \tag{20}$$

as $\frac{\mathrm{d}r_1^d}{\mathrm{d}\mu}, \frac{\mathrm{d}r_2^d}{\mathrm{d}\mu} > 0$. The latter follows from applying the implicit function theorem to the F-investors' first-order condition (6). If written as

$$Qu'\left(r_1^d\right) - Ru'\left(\frac{R(1-\mu r_1^\delta)}{1-\mu}\right) = 0$$

we have

$$\frac{\mathrm{d}r_1^d}{\mathrm{d}\mu} = -\frac{-R^2 u''(r_2^d)\frac{1-r_1^d}{(1-\mu)^2}}{Q^2 u''(r_1^d) + R^2 u''(r_2^d)\frac{\mu}{1-\mu}} > 0,$$

and if written as

$$Qu'\left(Q\frac{R-(1-\mu)r_2^d}{\mu R}\right) - Ru'(r_2^d) = 0$$

we have

$$\frac{\mathrm{d}r_2^d}{\mathrm{d}\mu} = -\frac{u''(Qr_1^d)Q^2\frac{r_2^d - R}{\mu^2 R}}{-u''(Qr_1^d)Q^2\frac{1-\mu}{\mu R} - Ru''(r_2^d)} > 0.$$

By the intermediate value theorem, there is thus $\hat{\mu} \in]\lambda, 1[$ such that $(r_1^d, r_2^d) \succ_{\mathrm{P}} (r_1^\delta, r_2^\delta)$ for all $\mu \in]\hat{\mu}, 1[$ and $(r_1^\delta, r_2^\delta) \succ_{\mathrm{P}} (r_1^d, r_2^d)$ for all $\mu \in]0, \hat{\mu}[$ if (19) holds and the claim is established. If (19) does not hold, then $(r_1^\delta, r_2^\delta) \succ_{\mathrm{P}} (r_1^d, r_2^d)$ for all $\mu \in]0, 1[$.

Claim 5 For $\mu \in]0, \bar{\mu}]$, there is no pooling contract which is a Pareto-improvement to $\left((r_1^d, r_2^d), (r_1^\delta, r_2^\delta)\right)$.

The slope of the zero-profit constraint for the pooling contract is between the slopes of the two zero-profit constraints associated with a separating equilibrium. A necessary condition for a pooling contract, which lies on the pooling zero-profit line, to make P-investors better off than (r_1^δ, r_2^δ) is thus that $r_1 < 1$, while a necessary condition for a pooling contract to make F-investors better off than (r_1^d, r_2^d) is that $r_1 > 1$. As these two condition rule each other out, there is no Pareto-improvement through pooling.

By claims 1 through 5, there is $\bar{\mu} = \min\left\{\mu \in]\lambda, 1[\,\middle|\, (r_1^\delta, r_2^\delta) \succsim_{\mathrm{F}} (r_1^d, r_2^d) \wedge (r_1^d, r_2^d) \succsim_{\mathrm{P}} (r_1^\delta, r_2^\delta)\right\}$ such that $(r_1^d, r_2^d) \succsim_{\mathrm{F}} (r_1^\delta, r_2^\delta)$ and $(r_1^\delta, r_2^\delta) \succsim_{\mathrm{P}} (r_1^d, r_2^d)$ if and only if $\mu \in]0, \bar{\mu}[$.

Claim 6 $-\frac{\mu}{1-\mu}\frac{u'(Qr_1^d)}{u'(r_2^d)}Q > -\frac{\lambda}{1-\lambda}\frac{u'(r_1^d)}{u'(r_2^d)}$ and $-\frac{\mu}{1-\mu}\frac{u'(Qr_1^\delta)}{u'(r_2^\delta)}Q > -\frac{\lambda}{1-\lambda}\frac{u'(r_1^\delta)}{u'(r_2^\delta)}$ obtains for all $\mu \in]0, \bar{\mu}]$.

The proof is by contradiction. Suppose μ is such that an equilibrium with credit-constrained separation exists, i.e. $(r_1^d, r_2^d) \succsim_{\mathrm{F}} (r_1^\delta, r_2^\delta)$ and $(r_1^\delta, r_2^\delta) \succsim_{\mathrm{P}} (r_1^d, r_2^d)$. If either $-\frac{\mu}{1-\mu}\frac{u'(Qr_1^d)}{u'(r_2^d)}Q > -\frac{\lambda}{1-\lambda}\frac{u'(r_1^d)}{u'(r_2^d)}$ or $-\frac{\mu}{1-\mu}\frac{u'(Qr_1^\delta)}{u'(r_2^\delta)}Q > -\frac{\lambda}{1-\lambda}\frac{u'(r_1^\delta)}{u'(r_2^\delta)}$ would not hold, Assumption 2 implies that either $(r_1^d, r_2^d) \succ_{\mathrm{P}} (r_1^\delta, r_2^\delta)$, or $(r_1^\delta, r_2^\delta) \succ_{\mathrm{F}} (r_1^d, r_2^d)$, or both, would necessarily hold.

C Proof of Proposition 2

For any given (r_1, r_2), the slope of the F-investors' indifference curve is

$$\frac{\mathrm{d}r_2}{\mathrm{d}r_1} = -\frac{\mu}{1-\mu}\frac{u'(Qr_1)}{u'(r_2)}Q$$

and the slope of the P-investors' indifference curve is

$$\frac{\mathrm{d}r_2}{\mathrm{d}r_1} = -\frac{\lambda}{1-\lambda}\frac{u'(r_1)}{u'(r_2)}.$$

By Assumption 2, if $(r_1^\delta, r_2^\delta) \sim_{\mathrm{F}} (r_1^d, r_2^d)$ and $(r_1^\delta, r_2^\delta) \succ_{\mathrm{P}} (r_1^d, r_2^d)$, the P-investors' indifference curve is steeper than the F-investors' indifference curve at $r_1 = r_1^\delta$ and $r_2 = r_2^\delta$, i. e.

$$-\frac{\bar{\mu}}{1-\bar{\mu}} \frac{u'(Qr_1^\delta)}{u'(r_2^\delta)} Q > -\frac{\lambda}{1-\lambda} \frac{u'(r_1^\delta)}{u'(r_2^\delta)}. \tag{21}$$

which together with (8) implies

$$-\frac{\bar{\mu}}{1-\bar{\mu}} \frac{u'(Qr_1^\delta)}{u'(r_2^\delta)} Q > -\frac{\lambda}{1-\lambda} R. \tag{22}$$

Let Z be defined by

$$Z := (\mu u(Qr_1^d) + (1-\mu)\, u(r_2^d)) - (\mu u(Qr_1) + (1-\mu)\, u(r_2))$$

with $r_1^d = y^d/\mu$, $r_2^d = R(1-y^d)/(1-\mu)$, and y^d solves (6). By definition, $\mu = \bar{\mu}$ implies $Z = 0$ for $r_1 = r_1^\delta = y^\delta/\lambda$ and $r_2 = r_2^\delta = R(1-y^\delta)/(1-\lambda)$, with y^δ solving (8). Concavity of u thus implies that there is (r_1', r_2') with $r_1' < r_1^\delta = y^\delta/\lambda$ and $r_2' > r_2^\delta = R(1-y^\delta)/(1-\lambda)$, which are also feasible as they satisfy $r_2' = \frac{R(1-\lambda r_1')}{1-\lambda}$, and for which $Z = 0$ also holds. However, since (r_1^δ, r_2^δ) maximizes the P-investors' expected utility subject only to their zero-profit constraint, $(r_1^\delta, r_2^\delta) \succ_{\mathrm{P}} (r_1', r_2')$. Hence, in response to a marginal increase in μ, starting from $\bar{\mu}$, P-investors strictly prefer a marginal adjustment to a contract (r_1^δ, r_2^δ) over a marginal adjustment to a contract (r_1', r_2'). Therefore, banks offering marginal adjustment to a contract (r_1^δ, r_2^δ) will prevent other banks offering marginal adjustments to (r_1', r_2') from entering the market, even though both satisfy the F-investors' incentive constraint $Z = 0$.

Applying the implicit function theorem to $Z = 0$ we obtain $\mathrm{d}r_2/\mathrm{d}\mu = -(\mathrm{d}Z/\mathrm{d}\mu)/(\mathrm{d}Z/\mathrm{d}r_2)$ with

$$\frac{\mathrm{d}Z}{\mathrm{d}\mu} = u(Qr_1^d) - u(Qr_1) + u(r_2) - u(r_2^d), \tag{23}$$

$$\frac{\mathrm{d}Z}{\mathrm{d}r_2} = \mu \frac{Q}{R} u'(Qr_1) \frac{1-\lambda}{\lambda} - (1-\mu) u'(r_2). \tag{24}$$

Equation (23) follows by taking into account the Envelope theorem, according to which the effects of changes in y^d, induced by changes in μ, have no effect as the first-order condition (6) applies. Equation (24) follows by taking into account the zero-profit constraints, according to which $r_1 = (R-(1-\lambda)r_2)(\lambda R)^{-1}$. Evaluating (23) at $r_1 = r_1^\delta$ and $r_2 = r_2^\delta$ yields $\mathrm{d}Z/\mathrm{d}\mu < 0$ because $r_2^d > r_2^\delta$ and $r_1^d < r_1^\delta$. Evaluating (24) at $r_1 = r_1^\delta$ and $r_2 = r_2^\delta$ yields $\mathrm{d}Z/\mathrm{d}r_2 < 0$ because of (22). Hence, $\mathrm{d}r_2/\mathrm{d}\mu < 0$ and $\mathrm{d}r_1/\mathrm{d}\mu = -((1-\lambda)/\lambda R)(\mathrm{d}r_2/\mathrm{d}\mu) > 0$. By continuity, the result also applies to all $\mu > \bar{\mu}$ in some neighborhood of $\bar{\mu}$. Therefore, the Proposition obtains.

D Proof of Corollary 2

A necessary condition for incentive-constrained separation equilibria to exist with inflated insurance for P-investors is $-\frac{\mu}{1-\mu}\frac{u'(Qr_1)}{u'(r_2)}Q > -\frac{\lambda}{1-\lambda}\frac{u'(r_1)}{u'(r_2)}$ for all $(r_1, r_1) \in \mathbb{R}_+^2$. Consider two contracts (r_1^A, r_2^A) and (r_1^B, r_2^B) such that $(r_1^A, r_2^A) \sim_{\mathrm{F}} (r_1^B, r_2^B) \sim_{\mathrm{F}} (r_1^d, r_2^d)$, $r_2^A = R(1-\lambda r_1^A)/(1-\lambda)$, and $r_2^B = R(1-\lambda r_1^B)/(1-\lambda)$. As

$$\lim_{\gamma \to 1} R\frac{1-(\gamma\lambda+(1-\gamma)\mu)r_1}{1-(\gamma\lambda+(1-\gamma)\mu)} = R\frac{1-\lambda r_1}{1-\lambda}.$$

pooling contracts $(r_1^{\text{Pool}}, r_2^{\text{Pool}})$ exist with $r_1^{\text{Pool}} \in]r_1^A, r_1^B[$ and $r_2^{\text{Pool}} = R\frac{1-(\gamma\lambda+(1-\gamma)\mu)r_1^{\text{Pool}}}{1-(\gamma\lambda+(1-\gamma)\mu)}$ such that $(r_1^{\text{Pool}}, r_2^{\text{Pool}}) \succsim_{\mathrm{F}} (r_1^d, r_2^d)$ and $(r_1^{\text{Pool}}, r_2^{\text{Pool}}) \succsim_{\mathrm{P}} (r_1^A, r_2^A)$ as well as $(r_1^{\text{Pool}}, r_2^{\text{Pool}}) \succsim_{\mathrm{P}} (r_1^B, r_2^B)$.

E Proof of Proposition 3

The proof proceeds in five steps.

Step 1: Under the condition of Proposition 3, when $-\frac{\mu}{1-\mu}\frac{u'(Qr_1)}{u'(r_2)}Q < -\frac{\lambda}{1-\lambda}\frac{u'(r_1)}{u'(r_2)}$ for all (r_1, r_2), we have $(r_1^\delta, r_2^\delta) \succ_{\mathrm{F}} (r_1^d, r_2^d)$ and $(r_1^\delta, r_2^\delta) \succ_{\mathrm{P}} (r_1^d, r_2^d)$. Accordingly, credit-constrained separation cannot be an equilibrium.

Step 2: Pooling equilibria satisfy the pooling zero-profit constraint, $r_2 = R\frac{1-(\gamma\lambda+(1-\gamma)\mu)r_1}{1-(\gamma\lambda+(1-\gamma)\mu)}$. The associated zero-profit line intersect with the zero-profit lines associated with contracts intended for each of the investor types only in $(1,R)$.

Step 3: If $\frac{\mu}{1-\mu}u'(Q)Q > \frac{\gamma\lambda+(1-\gamma)\mu}{1-(\gamma\lambda+(1-\gamma)\mu)}u'(R)R > \frac{\lambda}{1-\lambda}u'(1)$ the slope of the pooling zero-profit line lies between the P-investors' and the F-investors' marginal rates of substitution between r_1 and r_2 at $(1,R)$. In this case, for any contract $(\tilde{r}_1,\tilde{r}_2)$ on the pooling zero-profit line with $\tilde{r}_1 < 1$ and $\tilde{r}_2 > R$, there exists a contract $(\hat{r}_1,\hat{r}_2)$ on the zero-profit line for F-investors (i.e. with slope $\mu/(1-\mu)$) that is equivalent for P-investors to $(\tilde{r}_1,\tilde{r}_2)$. Given the conditions on preferences $(\hat{r}_1,\hat{r}_2) \succ_{\mathrm{F}} (\tilde{r}_1,\tilde{r}_2)$. Hence $(\tilde{r}_1,\tilde{r}_2)$ cannot constitute an equilibrium contract.

Step 4: Analogously, if $\frac{\mu}{1-\mu}u'(Q)Q > \frac{\gamma\lambda+(1-\gamma)\mu}{1-(\gamma\lambda+(1-\gamma)\mu)}u'(R)R > \frac{\lambda}{1-\lambda}u'(1)$, then for any contract $(\tilde{r}_1,\tilde{r}_2)$ on the pooling zero-profit line with $\tilde{r}_1 > 1$ and $\tilde{r}_2 < R$, there exists a contract $(\hat{r}_1,\hat{r}_2)$ on the zero-profit line for P-investors (i.e. with slope $\lambda/(1-\lambda)$) that is equivalent for F-investors to $(\tilde{r}_1,\tilde{r}_2)$. Given the conditions on preferences $(\hat{r}_1,\hat{r}_2) \succ_{\mathrm{P}} (\tilde{r}_1,\tilde{r}_2)$. Hence $(\tilde{r}_1,\tilde{r}_2)$ cannot constitute an equilibrium contract either.

Step 5: Accordingly, contract $(1,R)$ is the only contract that is feasible and not dominated by any other contract. This proves the claim of the Proposition.

F Proof of Proposition 4

The proof is similar to the Proof of Proposition 3. However, since in this case the condition $\frac{\mu}{1-\mu}u'(Q)Q > \frac{\gamma\lambda+(1-\gamma)\mu}{1-(\gamma\lambda+(1-\gamma)\mu)}u'(R)R > \frac{\lambda}{1-\lambda}u'(1)$ is violated, the only potential pooling contract $(1,R)$ is dominated by either (r_1^d, r_2^d) for F-investors or by (r_1^δ, r_2^δ) for P-investors. Hence, no equilibrium contract obtains in this case. This proves the Proposition.

G Proof of Corollary 3

A necessary condition for pooling is

$$\frac{\mu}{1-\mu}u'(Q)Q > \frac{\gamma\lambda+(1-\gamma)\mu}{1-(\gamma\lambda+(1-\gamma)\mu)}u'(R)R > \frac{\lambda}{1-\lambda}u'(1).$$

However,

$$\lim_{\gamma\to 0}\frac{\gamma\lambda+(1-\gamma)\mu}{1-(\gamma\lambda+(1-\gamma)\mu)}u'(R)R = \frac{\mu}{1-\mu}u'(R)R > \frac{\mu}{1-\mu}u'(Q)Q$$

for $-cu''(c)/u'(c) > 1$. Therefore, for $\gamma \to 0$, $\gamma > 0$, Pareto-improving contracts to $(1,R)$ exist with $r_1 < 1$ and $r_2 = R\frac{1-(\gamma\lambda+(1-\gamma)\mu)r_1}{1-(\gamma\lambda+(1-\gamma)\mu)} > R$.

Similarly,

$$\lim_{\gamma\to 1}\frac{\gamma\lambda+(1-\gamma)\mu}{1-(\gamma\lambda+(1-\gamma)\mu)}u'(R)R = \frac{\lambda}{1-\lambda}u'(R)R < \frac{\lambda}{1-\lambda}u'(1)$$

for $-cu''(c)/u'(c) > 1$. Therefore, for $\gamma \to 1$, $\gamma < 1$, Pareto-improving contracts to $(1,R)$ exist with $r_1 > 1$ and $r_2 = R\frac{1-(\gamma\lambda+(1-\gamma)\mu)r_1}{1-(\gamma\lambda+(1-\gamma)\mu)} < R$.

H Equilibria with Cross-subsidizing Contracts

In this Appendix we consider competitive equilibria when banks can offer a menu of potentially cross-subsidizing contracts (in the spirit of Miyazaki, 1977; Wilson, 1977; Spence, 1978). Our starting point are the Rothschild and Stiglitz (1976)-type equilibria as obtained in the main text, and then allowing banks to enter the market with menus of cross-subsidizing contracts. If such market entry is profitable, it destroys the initial Rothschild and Stiglitz (1976)-type equilibrium. We also check if such menus of cross-subsidizing contracts can prevail over further potential market entries with single-contract offers. We restrict attention to environments in which credit frictions preclude loans to (lucky) F-investors and there is private information about investor type and individual liquidity event.

H.1 Credit-constrained Separation

Suppose credit-constrained contracts obtain in equilibrium if banks are restricted to contracts without cross-subsidization (see Proposition 1). Since both contracts are already the best possible for either type of investors, no Pareto-improvements are possible through menus of cross-subsidizing contracts.

H.2 Incentive-constrained Separation

Suppose incentive-constrained separation with inflated insurance for P-investors obtains in equilibrium if banks are restricted to contracts without cross-subsidization (see Proposition 2). The following shows that while menus of cross-subsidizing contracts will break incentive-constrained separation with inflated insurance for P-investors for large enough γ, such menus do not constitute an equilibrium themselves.

Lemma 4 *Consider an economy where banks are prevented to offer cross-subsidizing menus of contracts and a competitive banking equilibrium with incentive-constrained separation for which inflated insurance for P-investors obtains. Suppose banks are now permitted to offer cross-subsidizing menus of contracts:*

1. *There is $\underline{\gamma} > 0$ such that for all $\gamma < \underline{\gamma}$ the incentive-constrained separation with inflated insurance for P-investors remains the only equilibrium.*

2. *There is $\overline{\gamma} < 1$ such that for all $\gamma > \overline{\gamma}$ menus of cross-subsidizing contracts exist that will crowd out incentive-constrained separation with inflated insurance for P-investors.*

Proof. In incentive-constrained separation equilibria with inflated insurance for P-investors, F-investors get the best contract banks can offer subject to the respective zero-profit constraint. Hence, a necessary condition for any equilibrium with menus of cross-subsidizing contracts is that contracts for F-investors generate losses and contracts for P-investors generate profits. Accordingly, the bank's optimization problem with cross-subsidizing contracts can be written as

$$\max_{(r_1^{\mathrm{P}},r_2^{\mathrm{P}},r_1^{\mathrm{F}},r_2^{\mathrm{F}},y)\in\mathbb{R}_+^4\times[0,1]} \quad \lambda u(r_1^{\mathrm{P}}) + (1-\lambda)\,u(r_2^{\mathrm{P}}) \tag{25}$$

$$\text{s.t.} \begin{cases} \gamma\lambda r_1^{\mathrm{P}} + (1-\gamma)\,\mu r_1^{\mathrm{F}} - y \;\leq\; 0 \\ \gamma(1-\lambda)\,r_2^{\mathrm{P}} + (1-\gamma)\,(1-\mu)\,r_2^{\mathrm{F}} - \big(R(1-y)+y-\big(\gamma\lambda r_1^{\mathrm{P}} + (1-\gamma)\,\mu r_1^{\mathrm{F}}\big)\big) \;\leq\; 0 \\ \mu u(Qr_1^{\mathrm{P}}) + (1-\mu)\,u(r_2^{\mathrm{P}}) - \big(\mu u(Qr_1^{\mathrm{F}}) + (1-\mu)\,u(r_2^{\mathrm{F}})\big) \;\leq\; 0 \end{cases}$$

The first constraint is a bank's liquidity constraint, and the second constraint requires that a bank's business model is sustainable, not for individual lines of business but across business lines; both constraints hold with equality. The third constraint is the incentive compatibility constraint for F-investors. Solving the first constraint for r_1^{F} and the second constraint for r_2^{F}, we can rewrite the problem as a Lagrangian

$$\begin{aligned} \mathscr{L}\left(r_1^{\mathrm{P}},r_2^{\mathrm{P}},y,\alpha\right) &= \lambda u\left(r_1^{\mathrm{P}}\right) + (1-\lambda)\,u\left(r_2^{\mathrm{P}}\right) \\ &\quad + \alpha\left(\mu u\left(Q\tfrac{y-\gamma\lambda r_1^{\mathrm{P}}}{(1-\gamma)\mu}\right) + (1-\mu)\,u\left(\tfrac{R(1-y)-\gamma(1-\lambda)r_2^{\mathrm{P}}}{(1-\gamma)(1-\mu)}\right) - \mu u\left(Qr_1^{\mathrm{P}}\right) - (1-\mu)\,u\left(r_2^{\mathrm{P}}\right)\right) \end{aligned}$$

with $\alpha \geq 0$ as Langrangian multiplier for the incentive compatibility constraint. Let $(\bar{r}_1^{\mathrm{P}},\bar{r}_2^{\mathrm{P}})$ denote the (inflated insurance) contract for P-investors that obtains if banks were not permitted to offer menus of cross-subsidizing contracts. Let

$$\begin{aligned} E_1 &:= \mu u(Qr_1^{\mathrm{F}}) + (1-\mu)\,u(r_2^{\mathrm{F}}) - \mu u(Qr_1^{\mathrm{P}}) - (1-\mu)\,u(r_2^{\mathrm{P}}), \\ E_2 &:= \lambda u(r_1^{\mathrm{P}}) + (1-\lambda)\,u(r_2^{\mathrm{P}}) - \lambda u(\bar{r}_1^{\mathrm{P}}) - (1-\lambda)\,u(\bar{r}_2^{\mathrm{P}}), \\ E_3 &:= \frac{Q}{R}u'(Qr_1^{\mathrm{F}}) - u'(r_2^{\mathrm{F}}). \end{aligned}$$

Accordingly, $E_1 = 0$ if $(r_1^{\mathrm{C}},r_2^{\mathrm{C}}) \sim_{\mathrm{I}} (r_1^{\mathrm{I}},r_2^{\mathrm{I}})$, $E_2 = 0$ if $(r_1^{\mathrm{C}},r_2^{\mathrm{C}}) \sim_{\mathrm{C}} (\bar{r}_1^{\mathrm{C}},\bar{r}_2^{\mathrm{C}})$, and $E_3 = 0$ if $(r_1^{\mathrm{I}},r_2^{\mathrm{I}})$ satisfy the first-order condition for above Lagrangian. Then, $E_1 = 0$, $E_2 = 0$, and $E_3 = 0$ define r_1^{P},

r_2^{P} and r_2^{F} as implicit functions of r_1^{F} according to the implicit function theorem with

$$\begin{aligned}
\frac{\mathrm{d}r_1^{\mathrm{P}}}{\mathrm{d}r_1^{\mathrm{F}}} &= \frac{\frac{\partial E_1}{\partial r_2^{\mathrm{P}}}\frac{\partial E_2}{\partial r_1^{\mathrm{F}}} - \frac{\partial E_2}{\partial r_2^{\mathrm{P}}}\frac{\partial E_1}{\partial r_1^{\mathrm{F}}}}{\frac{\partial E_1}{\partial r_1^{\mathrm{P}}}\frac{\partial E_2}{\partial r_2^{\mathrm{P}}} - \frac{\partial E_1}{\partial r_2^{\mathrm{P}}}\frac{\partial E_2}{\partial r_1^{\mathrm{P}}}} < 0, \\
\frac{\mathrm{d}r_2^{\mathrm{P}}}{\mathrm{d}r_1^{\mathrm{F}}} &= \frac{\frac{\partial E_2}{\partial r_1^{\mathrm{P}}}\frac{\partial E_1}{\partial r_1^{\mathrm{F}}} - \frac{\partial E_1}{\partial r_1^{\mathrm{P}}}\frac{\partial E_2}{\partial r_1^{\mathrm{F}}}}{\frac{\partial E_1}{\partial r_1^{\mathrm{P}}}\frac{\partial E_2}{\partial r_2^{\mathrm{P}}} - \frac{\partial E_1}{\partial r_2^{\mathrm{P}}}\frac{\partial E_2}{\partial r_1^{\mathrm{P}}}} > 0, \\
\frac{\mathrm{d}r_2^{\mathrm{F}}}{\mathrm{d}r_1^{\mathrm{F}}} &= \frac{\frac{Q^2}{R}u''\left(Qr_1^{\mathrm{F}}\right)}{u''\left(r_2^{\mathrm{F}}\right)} > 0.
\end{aligned}$$

Next, consider a bank's profit

$$\Pi = R\,(1-y) - \gamma(1-\lambda)\,r_2^{\mathrm{P}} - (1-\gamma)\,(1-\mu)\,r_2^{\mathrm{F}}$$

which is zero under incentive-constrained separation with inflated insurance for P-investors without cross-subsidization. Any contract $\left(r_1^{\mathrm{F}}, r_2^{\mathrm{F}}\right)$ satisfying $E_3 = 0$ makes the smallest possible loss in its business with F-investors. Therefore, overall profits Π associated with a menu of contracts $\left((r_1^{\mathrm{P}}, r_2^{\mathrm{P}}), (r_1^{\mathrm{F}}, r_2^{\mathrm{F}})\right)$ that satisfy $E_1 = 0$ and $E_3 = 0$ are maximal if $E_2 = 0$ also holds. Profits then satisfy

$$\frac{\mathrm{d}\Pi}{\mathrm{d}r_1^{\mathrm{F}}} = -R\left(\gamma\lambda\frac{\mathrm{d}r_1^{\mathrm{P}}}{\mathrm{d}r_1^{\mathrm{F}}} + (1-\gamma)\,\mu\right) - \gamma(1-\lambda)\,\frac{\mathrm{d}r_2^{\mathrm{P}}}{\mathrm{d}r_1^{\mathrm{F}}} - (1-\gamma)\,(1-\mu)\,\frac{Q^2}{R}\frac{u''\left(Qr_1^{\mathrm{F}}\right)}{u''\left(r_2^{\mathrm{F}}\right)}$$

Therefore, if $\mathrm{d}\Pi/\mathrm{d}r_1^{\mathrm{F}} < 0$, then $\Pi < 0$ for any $\left((r_1^{\mathrm{P}}, r_2^{\mathrm{P}}), (r_1^{\mathrm{F}}, r_2^{\mathrm{F}})\right)$ that would constitute a Pareto-improvement over the initial incentive-constrained separation with inflated insurance for P-investors. A sufficient condition for $\mathrm{d}\Pi/\mathrm{d}r_1^{\mathrm{F}} < 0$ is

$$\gamma\lambda\frac{\mathrm{d}r_1^{\mathrm{P}}}{\mathrm{d}r_1^{\mathrm{F}}} + (1-\gamma)\,\mu > 0$$

which holds for $\gamma \to 0$. For $\gamma \to 1$ we obtain

$$\frac{\mathrm{d}\Pi}{\mathrm{d}r_1^{\mathrm{F}}} = -R\lambda \frac{\mathrm{d}r_1^{\mathrm{P}}}{\mathrm{d}r_1^{\mathrm{F}}} - (1-\lambda)\frac{\mathrm{d}r_2^{\mathrm{P}}}{\mathrm{d}r_1^{I}}$$

which is positive provided $\mathrm{d}r_2^{\mathrm{P}}/\mathrm{d}r_1^{\mathrm{P}} > -\lambda R/(1-\lambda)$, which is true in some neighborhood of $\bar{r}_1^{\mathrm{P}}$ as the zero-profit line associated with its business with P-investors is steeper than the P-investors' indifference curve at $\bar{r}_1^{\mathrm{P}}$. Hence, for $\gamma \to 0$, there are no cross-subsidizing contract offers which can break the inflated insurance outcome. For $\gamma \to 1$, there are cross-subsidizing contract offers that break the inflated insurance outcome; this is because there are then menus of contracts which are Pareto-improvements over the initial separating contracts with inflated insurance while allowing banks to generate positive profits. Finally, as Π is continuous, by the intermediate value theorem, the Lemma obtains. ■

Proposition 1 *Consider an economy where banks are prevented to offer cross-subsidizing menus of contracts and a competitive banking equilibrium with incentive-constrained separation for which inflated insurance for P-investors obtains. Suppose banks are now permitted to offer cross-subsidizing menus of contracts:*

1. *There is no equilibrium with cross-subsidization.*

2. *An incentive-constrained separation with inflated insurance for P-investors is not an equilibrium for $\gamma > \bar{\gamma}$.*

Proof. The proof is in three steps.

Claim 1 Suppose a competitive banking equilibrium with cross-subsidization exists. In such equilibrium, the incentive compatibility constraint

$$\mu u(Qr_1^{\mathrm{F}}) + (1-\mu)u(r_2^{\mathrm{F}}) \geq \mu u(Qr_1^{\mathrm{P}}) + (1-\mu)u(r_2^{\mathrm{P}}) \tag{26}$$

is binding.

A necessary condition for a competitive banking equilibrium with cross-subsidization is $\left(r_1^{\mathrm{F}}, r_2^{\mathrm{F}}\right) \succ_{\mathrm{F}} \left(r_1^{\mathrm{d}}, r_2^{\mathrm{d}}\right)$. Hence, the line of business with F-investors incurs a loss to the bank since $\left(r_1^{\mathrm{F}}, r_2^{\mathrm{F}}\right)$ is the best a bank can offer to investors while making zero overall profit.

Perfect competition requires overall profit of banks to be zero, i.e.

$$r_2^{\mathrm{P}} = \frac{R(1-\left((1-\gamma)\,\mu r_1^{\mathrm{F}} + \gamma\lambda r_1^{\mathrm{P}}\right)) - (1-\gamma)\,(1-\mu)\,r_2^{\mathrm{F}}}{\gamma(1-\lambda)} \tag{27}$$

Suppose the incentive constraint were slack. Then, as the the line of business with P-investors is profit-making, a bank offering only contracts to P-investors could enter the market with an offer to P-investors $(\bar{r}_1^{\mathrm{P}}, \bar{r}_2^{\mathrm{P}}) \succ_{\mathrm{P}} (r_1^{\mathrm{P}}, r_2^{\mathrm{P}})$, allowing to also make strictly positive profits without attracting any investors away from the incumbent banks.

Claim 2 There is no offer of cross-subsidizing contracts $\left(r_1^{\mathrm{P}}, r_2^{\mathrm{P}}\right)$ and $\left(r_1^{\mathrm{F}}, r_2^{\mathrm{F}}\right)$ that satisfies condition (26) with equality and Eq. (27), while preventing banks from market entry with an alternative contract offer $\left(\tilde{r}_1^{\mathrm{P}}, \tilde{r}_2^{\mathrm{P}}\right)$ for P-investors and no offer for investors, such that $\left(r_1^{\mathrm{F}}, r_2^{\mathrm{F}}\right) \sim_{\mathrm{F}} \left(\tilde{r}_1^{\mathrm{P}}, \tilde{r}_2^{\mathrm{P}}\right)$ and $\left(\tilde{r}_1^{\mathrm{P}}, \tilde{r}_2^{\mathrm{P}}\right) \succ_{\mathrm{P}} \left(r_1^{\mathrm{P}}, r_2^{\mathrm{P}}\right)$.

If cross-subsidization is not allowed, the original competitive banking equilibrium with inflated P-investor insurance implies that the MRS for investors is flatter than the MRS for P-investors.

Claim 3 The initial incentive-constrained separation with inflated insurance for P-investors is an equilibrium if and only if there is no cross-subsidizing contract offer $\left(r_1^{\mathrm{P}}, r_2^{\mathrm{P}}\right)$ and $\left(r_1^{\mathrm{F}}, r_2^{\mathrm{F}}\right)$, satisfying condition (26) with equality and Eq. (27), that would constitute a Pareto-improvement.

The last claim is self-evident. Together with Lemma 4, the proposition obtains. ■

H.3 Pooling and Non-existence

Suppose either pooling obtains in equilibrium or no pure-strategy equilibrium exists if banks are restricted to contracts without cross-subsidization (see Propositions 3 and 4). Those cases require that the marginal rate of transformation between r_1 and r_2 is higher for F-investors than for P-investors. The two incentive compatibility constraints (9) and (10) are thus mutually exclusive. Therefore, any menu of two different contracts that satisfies one of these constraint violates the other, regardless of those contracts generating profits or losses. If any, only identical contracts for both investor types are incentive compatible. Therefore, menus of different, cross-subsidizing contracts do not emerge in equilibrium.