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JEL Classification: F15, F4, H20, H40

Keywords: Globalization, Labour taxation, size of public sector, composition of public expenditures, biases in fiscal policies, Heterogeneous Firms, selection

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Empirical evidence shows that countries with larger public sectors also have larger trade shares, also for the manufacturing sector, and a larger share of firms that export. We reconcile these links between the public and private sector in an analytically tractable general equilibrium model. We derive two-way causal relations between size (and composition) of the public sector and openness and industry structure of the private sector. First, an increase in public sector size or a shift in public expenditures toward public employment (away from transfers) increases openness of the private sector (trade share and fraction of firms exporting), and is also associated with higher average productivity, lower average unit labour costs, and improved wage competitiveness and terms of trade. These outcomes are driven by endogenous entry and selection of firms. A quantitative exercise reveals substantial quantitative differences in the effects from public sector reforms between the open and closed economy. Second, international spillovers imply that non-cooperative policies have an upward bias in the overall size of the public sector, but a downward bias in transfers as a share of public expenditures. Trade liberalization and the degree of firm heterogeneity magnify these biases and thereby shape the size and composition of the public sector.

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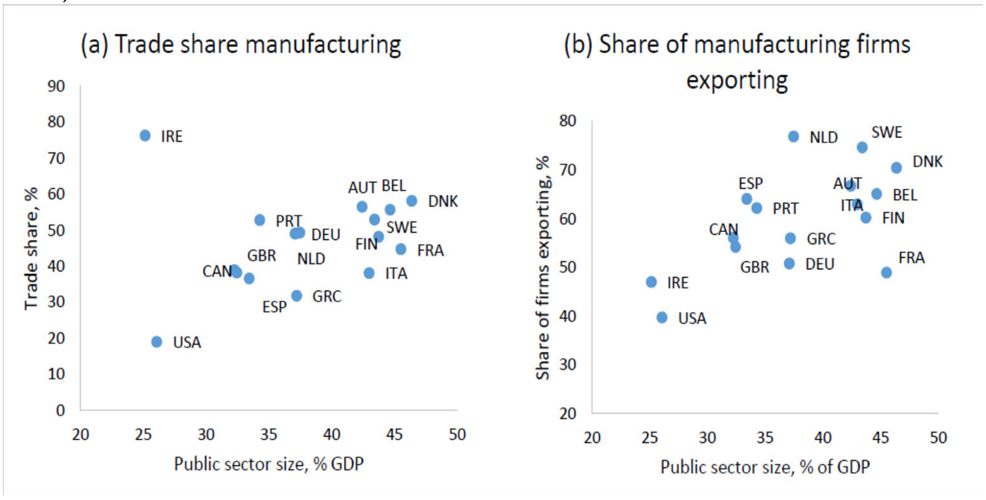
1 Introduction

It is an empirical fact that more open economies tend to have larger public sectors, see e.g. Rodrik (1998). The direction of causality is clearly open for discussion, but the literature has mainly focused on how openness via various channels increases the demand for public sector activities. Without taking a stand on this, it remains a puzzle how countries with large public sectors and thus high taxes can be among the richest countries and rank high in competitiveness rankings[1]. This is not easily reconciled with standard arguments on the distortionary effects of taxation leading to lower labour supply, higher wages and thus worsened competitiveness. A puzzle which becomes even more striking when considering the private sector explicitly. Cross-country evidence shows that a large public sector is not only associated with a higher export ratio for the private manufacturing[2] sector (Figure 1.a), but also a large share of firms being exporters (Figure 1.b).

Figure 1: Trade share, share of firms exporting and public sector size, OECD countries



Note: The trade share is measured as direct domestic value added content of gross export relative to total value added for manufacturing. The share of manufacturing firms is the share of firms with 10-250 employees exporting in percent of the total number of firms, and the public sector size is measured by total tax revenue as a share of GDP. Data is an average over the period 2013-2017 to remove business effects. Data not available for all OECD countries. Data from www.oecd-ilibrary.com.

Such evidence is only indicative, but points to the need for a detailed

[1] In e.g. the World Economic Forum competitiveness index for 2019, Sweden ranks 8, Denmark 9 and Finland 10 out of 141 countries, see http://www3.weforum.org.

[2] Our theoretical framework models the private sector as manufacturing, and therefore we consider trade openness and the share of firms exporting within manufacturing.

analysis of the interdependencies between the private and public sector in open economies. An issue which has not been systematically analysed in the literature. The present paper contributes by exploring the links from size and composition of the public sector to the industry structure, that is, the openness of and allocations across heterogeneous firms within the private sector. Moreover, it also explores the reverse link from openness of and the underlying degree of firm heterogeneity within the private sector to optimal size and composition of the public sector and international policy spillovers.

We set up a static (two-country) general equilibrium model which accounts for the public sector and allocation and selection among heterogeneous firms. The public sector is modelled in accordance with stylized facts for OECD countries. The two key expenditure items are transfers to individuals outside the labour force and public consumption, of which public employment is the dominant part.[3] The primary tax base is the (direct and indirect) taxation of labour income, accounting for the predominant part of total tax revenue for OECD countries.[4] Households are modelled in a standard way with endogenous labour supply, and the income tax rate distorts labour supply. Firm heterogeneity and trade are modelled as in the seminal Melitz (2003) monopolistic competition framework. Firm heterogeneity is required to match the firm-level heterogeneity in export behaviour in Figure 1.b. The modelling of firm heterogeneity emanating from productivity differences is motivated by the large and persistent differences in productivity levels across firms (Syverson (2011)), the strong evidence of selection into exporting based on productivity (Melitz and Redding (2014)), and the quantitative importance of allocation and selection among the heterogeneous firms in accounting for cross-country productivity differences (Bartelsman et al. (2013)).

The paper takes an analytical approach to combine standard arguments on the distortionary effects of taxation and the determination of the industry structure. The general equilibrium outcomes turn out to differ significantly from partial equilibrium arguments. Strikingly, an increase in the distortionary labour-income tax rate to finance a larger public sector is associated with a lower relative wage, i.e. an improvement in wage competitiveness, a decline in average unit labour costs, an improvement in the terms of trade,

[3] For OECD countries, in 2015 the average share of expenditures going to public consumption was 45%, and the share going to social benefits and transfers was 38%. Of the consumption expenditure, about 53% is wage expenditures. Data source: www.oecd-ilibrary.org.

[4] In the range of 80-90%, see www.oecd-ilibrary.org. Corporate taxation is often discussed, since it is an area where there are clear signs of a race-to-the-bottom, see Devereux and Loretz (2013).

and an increase in the fraction of exporting firms. A key transmission mechanism is the one-sector home market effect[5], as expansion of the public sector shrinks the private sector through resource absorption and a distortion of aggregate labour supply: A shrinking private sector makes the foreign market relatively more attractive and releases domestic intra-industry reallocations within the private sector that mirror those from trade liberalization in the seminal Melitz (2003) framework; i.e. tougher selection, only firms with high productivity survive, higher average productivity, a larger fraction of firms exporting, and more trade openness (consistent with Figure 1).

A quantitative exercise of harmonization of fiscal policies across the Eurozone shows the quantitative importance of the open economy mechanisms for policy evaluations. We calibrate a J-country extension to the 65 countries in the OECD database for trade in value added (TiVA 2018) and consider a harmonization of fiscal policies towards the mean, that is, the average value applying across Eurozone countries. With (without) selection among the heterogeneous firms, the open economy mechanisms mute the effects on real wages (on average across the Eurozone countries) by 23% (33%) relative to a similar closed economy setting.

The non-cooperative tax rate and thus public sector size exceed the cooperative outcome in our framework. This bias is known in the literature. However, and new to the literature, the expenditure share of transfers in the non-cooperative case falls short of the cooperative level (race-to-the-bottom). In short, the non-cooperative outcome has an upward bias in the tax rate and thus the overall relative size of the public sector, but a downward bias in the expenditure share on transfers. We show - under additional assumptions about functional forms - that the non-cooperative policy bias increases with trade liberalization and the degree of underlying firm heterogeneity. While the former finding appears in the literature, the latter is - to the best of our knowledge - new. Epifani and Gancia (2009) show a positive relation between trade openness (liberalization) and public consumption consistent with a terms-of-trade externality channel. Our theoretical predictions are consistent with the empirical evidence in Epifani and Gancia (2009).

We analyse optimal policies for the case of heterogeneous countries numerically and find that it is to the advantage (pecuniary externality) of a country with a strong preference for public activities that its trading partners have lower preferences for public consumption. The sizes of public sectors are strategic substitutes, and globalization tends to create divergence rather than convergence in public sector sizes across countries.

[5] Jung and Felbermayr (2015) explore in a heterogeneous firms framework the one-sector home market effect for exogenous variation in market size.

The remainder of the paper is organized as follows: Section 2 relates the analysis to the literature. The basic structure of the model is laid out in Section 3, and Section 4 characterizes the general equilibrium outcome. Section 5 analyses the effects of unilateral policy changes for industry structure and competitiveness. Moreover, it conducts a quantitative evaluation of fiscal harmonization within the Eurozone. Optimal fiscal policies and biases in non-coordinated policies are analysed in Section 6, which also analyses the effects of increased globalization on optimal policies. Section 7 provides a brief perspective of the results of this paper in relation to the globalization debate and concludes. Technical material and proofs are provided in a number of appendices.

2 Related literature

The paper builds on a large literature on trade, open macroeconomics and public economics; for space reasons we only comment on a selected but essential related literature. This analysis is focussing on the structural effects of fiscal policy, and hence the large literature on fiscal stabilization policies is not covered.

Policy spillovers through terms-of-trade play a key role for optimal trade policies. Felbermayr et al. (2013) show in a Melitz (2003) framework with Pareto distributed productivities, i.e. the same trade framework as applied in the present paper, that an ad valorem import tariff releases an anti-selection effect and a terms-of-trade improvement. The unilaterally optimal tariff rate and the Nash tariff rate are both positive due to a terms-of-trade effect and increase with reductions in real trade costs (not tariff barriers) and with the degree of firm heterogeneity (in the neighbourhood of a symmetric equilibrium).[6] In a similar framework, Sørensen (2020) shows that export promotion (which reduces firm-level fixed/sunk costs of exporting) intensifies selection in the country doing export promotion and for its trade partner and thereby entails a positive spillover on the trade partner. As changes in trade openness are the propagation mechanism for the open economy effects from fiscal policies, the paper relates to the (quantitative) gains from the trade literature, see e.g. Arkolakis et al. (2012). Indeed, in our quantitative exercise we extend the well-known formula for real wage gains from trade (liberalization) in Arkolakis et al. (2012) to account for endogenous labour supply.

The issue of cross-country interdependencies in fiscal policy has previously been widely studied. It is a robust finding that countries acting non-

[6] Costinot et al. (2020) consider trade taxes depending on both origin and firm-level productivity in a setting with two-dimensional firm heterogeneity.

cooperatively tend to choose too high levels of public activities and thus taxes. The reason is that countries perceive that they can affect the terms-of-trade to their advantage due to a home bias in public spending. This effect is not present in the cooperative case, and therefore there is an upward bias in public expenditures (see e.g. Epifani and Gancia (2009), Andersen et al. (1996), Devereux (1991), Chari and Kehoe (1990), Turnovsky (1988), and van der Ploeg (1987, 1988))[7]. This literature relies on one crucial assumption regarding trade flows, namely that the specialization structure is exogenous. Epifani and Gancia (2009) present an Armington trade model with variable trade (Iceberg costs), but the specific structure implies that the trade share depends on trade costs only, and thus is independent of fiscal policies. Andersen and Sørensen (2012) present a Ricardian trade model with a continuum of goods in which the industry and thus trade structure are endogenously determined. In this framework, an increase in the size of the public sector (and thus a smaller private sector) allows the country to specialize in a smaller set of goods and thereby to a larger extent exploit its comparative advantages.

The present paper presents a setting which endogenizes not only the production and trade structures, but also entry and exit of firms, as well as the number of producing and exporting firms. These crucial aspects of the private sector are affected by fiscal policies, and the framework thus captures essential links between the public and private sector. Importantly, we show that the upward bias in non-cooperative policies depends not only on the terms-of-trade effect, but also on intra-industry reallocations and selection across heterogeneous firms. A mechanism often highlighted in policy debates thus contributes to the non-cooperative bias. Moreover, the analysis shows the important difference between taxes financing public consumption and transfers and thus the importance of both size and composition of the public sector.

A closely related paper is Larch and Lechthaler (2013), which - inspired by the Buy National clauses in US and Chinese stimulus packages following the financial crisis - considers demand switching effects in a Krugman (1980)/Melitz (2003) setting. They find that a buy-national strategy of the public sector reduces the price of imports, which benefits private consumption, i.e. a positive terms-of-trade spillover. However, welfare is lower under the buy-national strategy, as the government enjoys less love-of-variety by consuming domestic varieties only. Similar to Larch and Lechthaler (2013),

[7] A downward bias may arise if public activities tend to increase labour supply, e.g. via day care, see Andersen (2007). Molana and Montagna (2006) show a positive spillover from unemployment benefits in a setting with aggregate economies of scale and imperfect competition (unions) and thus a downward bias.

we consider the composition of public expenditures, although we distinguish between public employment and transfers and not between buy-national and buy-international in the product market. The demand switching effect from public consumption differs in the two models, since an increase in public consumption in our framework reduces demand for domestic varieties (also relative to foreign varieties), as private consumption with an endogenous home bias due to trade frictions is lowered. In Larch and Lechthaler (2013), on the other hand, an increase in public consumption under the buy-national strategy shifts demand from foreign varieties towards domestic varieties. Hence, demand switching releases different industry dynamics in the two frameworks. Which mechanism is more important depends on the (marginal) empirical composition of public consumption.[8]

Based on a calibration exercise in a search-matching framework with heterogeneous firms, another related paper, Felbermayr et al. (2012), argues that smaller and more open economies have more generous unemployment benefit replacement rates, as a larger share of the burden from the generous benefits is borne by foreign trade partners (similar to the burden of lower labour supply due to taxation in our framework). Contrary to Larch and Lechthaler (2013) and Felbermayr et al. (2012), we obtain analytical results regarding spillovers and policy biases. Fiscal policies endogenously change the size of the private sector (demand for goods and residual supply of labour for the private sector), and accordingly the present paper is related to papers on market size effects (exogenous variation) in one-sector-heterogeneous-firms trade models such as Jung and Felbermayr (2015) and Felbermayr and Jung (2018).

Obviously, our paper relates to the empirical literature on the link between trade openness and size of the public sector in part inspired by the compensation hypothesis, Rodrik (1998) or the terms-of-trade mechanisms highlighted above.[9] Interestingly, Epifani and Gancia (2009) find that the positive correlation between trade openness and public sector size only applies to countries exporting differentiated products, i.e. countries for which the abovementioned terms-of-trade effect is present. The finding of a significant positive relation between trade openness and size of the public sector appears in some studies, while other studies based on other samples, control variables etc. cannot confirm the positive relation (see e.g. Farhad and Jetter (2019) and their references).

[8] According to Epifani and Gancia (2009), the import share in government consumption is a slim 1%.

[9] The efficiency hypothesis, based on Alesina and Wacziarg (1998), also suggests a positive relation between trade openness and size of the public sector, albeit the link is through country size.

3 The Model

We set up an analytically tractable static two-country general equilibrium model. The public sector is modelled in accordance with the stylized facts that the expenditure side mainly consists of expenditures on public consumption/employment and transfers to people outside the labour force, and that the dominant source of financing is (distortionary) taxation of labour income, cf. the Introduction. The modelling of firms and trade builds on the seminal heterogeneous-firms framework in Melitz (2003). Specifically, the Melitz (2003) framework is extended to include endogenous labour supply[10] and fiscal policies.[11] The model allows for parametric asymmetries: While structures are identical, all parameters (except for the elasticity of substitution between products) may differ across countries. The two countries are denoted Home and Foreign, respectively, and in the following Foreign variables have superscript *.

Households

Home is populated by a continuum of households with measure n. An exogenous fraction of the households, $\mu \in (0,1)$, is for various reasons not active in the labour market and receives tax-financed transfers from the public sector.[12] The remaining fraction, $1-\mu$, is active on the labour market and supplies an endogenous number of work hours[13]. Household preferences are described by the utility function

$$U_i\left(C_i, L_i, G\right) = u\left(\frac{1}{\chi}C_i^{\chi} - \frac{1}{1+\gamma^{-1}}L_i^{1+\gamma^{-1}}\right) + v(G) \text{ for all } i, \tag{1}$$

where subscript i refers to the household, C_i is consumption of a composite good (defined below), L_i is labour supply, G is a publicly provided

[10] Antrás et al. (2017) introduce a progressive and distortionary labour income tax-transfer scheme into a heterogeneous workers (firms) framework and explore how distortionary taxation affects gains from trade and selection of workers into exporting of labour services. However, they consider neither policy spillovers, optimal taxation, nor public consumption.

[11] Cooke (2016) includes endogenous labour supply in a heterogeneous firms framework and focuses on optimal monetary policy and the role of policy spillovers and industry dynamics.

[12] Households differ in labour market status but are otherwise identical. See e.g. Helpman et al. (2017) for a framework with heterogeneous firms and heterogeneous workers.

[13] This could be thought of as a compressed overlapping-generations structure where agents work as young and are retired as old. However, an explicit OLG set-up is avoided, since it would complicate the analysis considerably, and it is not essential to the endogenous determination of the production, trade and industry structure.

good/service, and it is assumed that $\gamma > 0$ and $\chi \in (0,1]$.[14] The budget constraint of household i reads $E_i = PC_i = WL_i - tWL_i$ if i is on the labour market and $E_i = PC_i = TR$ otherwise, where E_i is expenditures, P is the price index (defined below), W is the wage rate, t the proportional labour income tax rate, and TR is transfers to those not on the labour market.

The composite consumption good, C, is a CES aggregate (Dixit-Stiglitz preferences) defined by $C = \left(\int_{\omega\in\Omega} c(\omega)^{\frac{\sigma-1}{\sigma}} d\omega\right)^{\frac{\sigma}{\sigma-1}}$, where $c(\omega)$ is consumption of variety ω, Ω is the endogenous set of domestic and foreign varieties available to the households, and $\sigma \in (1,\infty)$ is the constant elasticity of substitution between any two varieties. Aggregate demand for each variety is

$$c(\omega) = E(P)^{\sigma-1} p(\omega)^{-\sigma} \text{ for all } \omega \in \Omega, \tag{2}$$

where E denotes aggregate private expenditures, $p(\omega)$ is the price of variety ω, and P is the dual price index given by $P = \left(\int_{\omega\in\Omega} p(\omega)^{1-\sigma} d\omega\right)^{\frac{1}{1-\sigma}}$.

Households participating in the labour market supply labour to maximize utility, taking the wage, the price index, public consumption and the tax rate as given. The resulting aggregate labour supply, L^s, is

$$L^s = n(1-\mu)\left(\frac{W(1-t)}{P}\right)^{\eta} \tag{3}$$

where the labour supply elasticity is $\eta \equiv \chi/(1-\chi+\gamma^{-1}) > 0$.

Public sector

The public sector offers a public good/service, G, as well as a transfer, TR, to households not in the labour market. The public good/service is produced by means of labour, and the production function can without loss of generality[15] be assumed linear $G = L^g$, where L^g denotes public employment. Public activities are financed by a proportional tax on labour income, and the public sector runs a balanced budget (static model).

To analyse the importance of public consumption and transfers (composition), it is convenient to specify the budget allocation in terms of the

[14] To ensure interior solutions for the optimal provision of public consumption and transfers, we assume that $u'(\cdot) > 0$, $u''(\cdot) < 0$, $\lim_{(\cdot)\to 0} u'(\cdot) = \infty$ and $v'(\cdot) > 0$, $v''(\cdot) < 0$, $\lim_{G\to 0} v'(\cdot) = \infty$.

[15] Utility from the public service is $v(G) = v(L^g)$, where $v(\cdot)$ satisfies standard concavity assumptions. Assume instead a concave production function, $G = f(L^G)$, in which case utility is $v(f(L^G))$, or define the utility function as $\widetilde{v}(L^G)$. Hence, the utility specified over public consumption implicitly includes the production function for the public service/good.

share of revenue (tWL) appropriated for transfers (s) and public consumption/employment $(1-s)$, respectively, i.e.

$$WL^g = (1-s)\,tWL \tag{4}$$
$$n\mu TR = stWL. \tag{5}$$

where L denotes total employment. Hence, the public budget reads

$$tWL = WL^g + n\mu TR. \tag{6}$$

Imposing the public budget constraint implies that aggregate expenditures on private goods read $E = n\mu TR + WL\,(1-t) = WL\,(1-t\,(1-s))$.

Firms

Firms operate in a single representative monopolistic industry, and each produces with labour as the only input. Each firm produces a single and unique variety, ω, implying a one-to-one correspondence between varieties and firms; firms can accordingly be labelled by ω. Prior to entry, a firm pays sunk costs[16], WF_E, to develop a firm-specific variety. In this R&D process, a constant firm-specific marginal productivity of labour, $\varphi\,(\omega)$, is determined. This marginal productivity is a random draw from a known distribution, $H\,(\varphi)$, with support $\varphi \in [\varphi_{\min}, \infty)$. After the realization of the marginal productivity, the firm decides whether to produce and which markets to serve. In order to produce, the firm has to pay a fixed cost, WF_D, and to export there is an additional fixed export cost, WF_X. Trade is also subject to iceberg trade costs, implying that per unit arriving on foreign shore a firm must ship $\tau \geq 1$ units. There is free entry into the industry, and firms enter until expected profits prior to entry are driven to zero, i.e. no aggregate profits in equilibrium. In the following, we supress ω in the notation, as firms only differ due to differences in marginal productivity of labour.

Elasticity of demand is uniform across firms/varieties, cf. (2), and across markets, cf. the only imposed parametric symmetry across countries, namely $\sigma = \sigma^*$. Accordingly, firms set prices as a constant markup upon marginal costs (including iceberg trade costs), i.e.

$$p_D\,(\varphi) = \frac{\sigma}{\sigma-1}\frac{W}{\varphi} \text{ and } p_X\,(\varphi) = \frac{\sigma}{\sigma-1}\frac{W}{\varphi}\tau = \tau p_D\,(\varphi) \tag{7}$$

[16] All costs are in terms of labour units in the country where the firm is located; i.e. the present framework does not allow for foreign sourcing.

where p_D (p_X) denotes the price in the domestic (export) market. Using the demand function (2) and the prices (7), profits can be written

$$\pi(\varphi) = \max\left[W^{1-\sigma}\varphi^{\sigma-1}\left(B + B^* I_X \tau^{1-\sigma}\right) - WF_D - I_X WF_X, 0\right] \quad (8)$$

where I_X is an indicator variable taking the value 1 if the firm exports and zero otherwise, and

$$B \equiv EP^{\sigma-1}\left(\frac{\sigma}{\sigma-1}\right)^{-\sigma}\frac{1}{\sigma-1} \quad (9)$$

$$B^* \equiv E^*(P^*)^{\sigma-1}\left(\frac{\sigma}{\sigma-1}\right)^{-\sigma}\frac{1}{\sigma-1} \quad (10)$$

denote, respectively, the Home and the Foreign demand component, which are exogenous to the individual firm but endogenously determined in equilibrium. The demand component determines the location of the residual demand curve a firm faces in a given market. It increases with market size (nominal expenditure level) and with price index (inverse measure of competitive pressure). The relative demand, $\frac{B}{B^*} = \frac{E}{E^*}\left(\frac{P}{P^*}\right)^{\sigma-1}$, comprising the relative expenditure level E/E^* as well as the (CPI based) real (effective) exchange rate[17] P/P^*, plays a key role for the international transmission mechanism of policy changes, see below.

Equation (8) shows that firm-market profits increase in firm-level productivity, and therefore there is a productivity threshold for firms to profit from supplying a given market. Let φ_D, which we refer to as the exit threshold, denote the productivity threshold for which domestic firms break even from supplying the domestic market only. Similarly, let φ_X, which we refer to as the export threshold, denote the productivity threshold for which domestic firms break even on foreign (export) market activities conditional on being active on the domestic market. We restrict the parameter space such that exporting firms always supply the domestic market ($\varphi_X > \varphi_D$). This restriction ensures that active firms - in line with empirical evidence - partition into exporters and non-exporters (supplying the domestic market only). The productivity thresholds are defined by

$$W^{1-\sigma}(\varphi_D)^{\sigma-1} B \equiv WF_D \quad (11)$$

$$W^{1-\sigma}(\varphi_X)^{\sigma-1} B^* \tau^{1-\sigma} \equiv WF_X. \quad (12)$$

For later reference define relative unit labour costs as

$$RULC \equiv \frac{W/TP}{W^*/TP^*}$$

[17] In the present setting with a focus on structural equilibrium, we ignore monetary issues, and the nominal exchange rate is normalized to unity.

where TP is average productivity defined as the sales weighted average of marginal production efficiency across firms, see Appendix C.

Free entry and equilibrium industry structure

With free entry into the industry, firms enter until expected profit prior to entry equals zero, i.e.

$$\int_{\varphi_{\min}}^{\infty} \pi(\varphi)\, dH(\varphi) = WF_E \tag{13}$$

For analytical tractability and in line with much of the literature on heterogeneous firms in international trade (see e.g. Helpman et al. (2004) and Chaney (2008)), we assume productivities to be Pareto distributed, i.e. $H(\varphi) = 1 - (\varphi/\varphi_{\min})^{-k}$ for $\varphi \geq \varphi_{\min}$. It is further assumed that $k > \max(2, \sigma - 1, \sigma) = \max(2, \sigma)$ to ensure a finite variance of the distribution of marginal productivity, finite expected profits from entry, and a finite sales weighted average productivity. The Pareto distribution generates a size distribution of firms in line with the empirically observed distributions (see e.g. Luttmer (2007) and Simon and Bonini (1958)). From the productivity threshold conditions, (11) and (12), the free entry condition (13) and the similar conditions in Foreign, the productivity thresholds can be written as

$$\varphi_D = \varphi_A \left(1 + \left(\frac{F_D}{F_X}\right)^{\frac{k}{\sigma-1}-1} \tau^{-k} \left(\frac{B}{B^*}\right)^{-\frac{k}{\sigma-1}}\right)^{\frac{1}{k}} \tag{14}$$

$$\varphi_X = \varphi_A \left(1 + \left(\frac{F_X}{F_D}\right)^{\frac{k}{\sigma-1}-1} \tau^{k} \left(\frac{B}{B^*}\right)^{\frac{k}{\sigma-1}}\right)^{\frac{1}{k}} \left(\frac{F_D}{F_X}\right)^{-\frac{1}{k}} \tag{15}$$

where $\varphi_A \equiv \varphi_{\min} \left(\frac{\sigma-1}{k-(\sigma-1)} \frac{F_D}{F_E}\right)^{\frac{1}{k}} > 0$ is the exit threshold in autarky.

Balanced trade, labour market equilibrium, and the number of firms

In equilibrium, trade balances due to the static nature of the model. The balanced trade condition reads

$$N \int_{\varphi_X}^{\infty} E^* (P^*)^{\sigma-1} (p_X(\varphi))^{1-\sigma} \frac{dH(\varphi)}{1 - H(\varphi_D)} = N^* \int_{\varphi_X^*}^{\infty} E(P)^{\sigma-1} (p_X^*(\varphi))^{1-\sigma} \frac{dH^*(\varphi)}{1 - H^*(\varphi_D^*)},$$

where N is the number of (producing) firms. The labour market is competitive, and the equilibrium condition reads $L^s = L^{d,private} + L^g$, where $L^{d,private} = N \frac{k\sigma}{k-(\sigma-1)} F_D \left(\frac{\varphi_D}{\varphi_A}\right)^k$ is labour demand in the private sector.

The equilibrium number of firms - conditional on the relative demand component - is proportional to labour supply.[18] A positive interdependence between labour supply and the number of firms/varieties arises; higher labour supply leads to more varieties, which again leads to higher labour supply due to a higher real wage. Specifically, the elasticity of labour supply with respect to the mass of varieties equals $\varepsilon \equiv \frac{1}{\sigma-1}\eta$, where the first part, $\frac{1}{\sigma-1}$, captures the love-of-variety link from varieties to the real wage, and the second part, η, is the elasticity of labour supply with respect to the real wage. If this elasticity exceeds one, implausible effects arise[19], and therefore $\varepsilon < 1$ is assumed in the following. Empirically, this condition is likely to be satisfied[20].

Equilibrium

Equilibrium for given fiscal policies (t, s, t^*, s^*) has households maximizing utility (1) and consumption determined by (2) and labour supply by (3). Firms maximize profits (8) and are actively producing and exporting given threshold productivities determined by (14) and (15) and with prices determined by (7). The labour market clears, trade is balanced, and the public budget (6) is in balance.[21]

4 General equilibrium

The characterization of the general equilibrium is facilitated by two key model properties. First, the equilibrium solution has a recursive structure allowing all endogenous variables to be specified in terms of relative demand ($\frac{B}{B^*}$) as well as exogenous variables and parameters. Solving for relative demand thus leads to solutions for the other endogenous variables. Moreover, the domestic economy is affected by foreign fiscal policy through relative demand only. The share of private expenditures on domestically produced goods in total domestic private expenditures is a sufficient statistic for how foreign variables affect the domestic economy, see Arkolakis et al. (2012). Second,

[18] This is a well-known result from monopolistic competition models with CES preferences. This result applies in the present setting with public employment, as public employment is a constant share (share equals $t\,(1-s)$) of total employment.

[19] For example, an exogenous increase in labour supply, e.g. due to a reduction in the tax rate on labour income, results in autarky in a lower equilibrium level of labour supply for $\varepsilon > 1$.

[20] Chetty et al. (2011) report a steady state (Hicksian) macro elasticity of labour supply of 0.5 based on meta analyses, i.e. $\eta = 0.5$, Melitz and Redding (2015) set $\sigma = 4$, and Bas et al. (2017) obtain an estimate of σ equal to 5, implying $\varepsilon < 1$.

[21] See Appendix B for derivation.

the expression for the equilibrium value of relative demand can be partitioned into two parts - a fiscal policy part and a relative demand part - implying the following equilibrium relationship (see Appendix B):

$$\Phi(\frac{B}{B^*}) = \Gamma(t, s, t^*, s^*),$$

where

$$\Phi_{\frac{B}{B^*}}(\cdot) > 0, \ \Gamma_t(\cdot) < 0, \ \Gamma_s(\cdot) > 0, \ \Gamma_{t^*}(\cdot) > 0, \ \Gamma_{s^*}(\cdot) < 0.$$

Proposition 1 *There exists a unique equilibrium value for relative demand* $(\frac{B}{B^*})$ *, which in turn determines all other endogenous variables. Specifically, relative demand is decreasing (increasing) in the Home tax rate* t *(Foreign tax rate:* t^**) and increasing (decreasing) in the share of Home public expenditures going to transfers* s *(Foreign:* s^**);*

$$\begin{array}{ll} d\frac{B}{B^*}/dt < 0 & d\frac{B}{B^*}/ds > 0 \\ d\frac{B}{B^*}/dt^* > 0 & d\frac{B}{B^*}/ds^* < 0. \end{array}$$

Proof. *See Appendix B.* ■

The fiscal policy effect on relative demand comprises several effects. A higher tax rate (t) at Home directly reduces private disposable income, and this is reinforced by a decline in labour supply due to the standard distortion effect. This reduces relative private expenditure (E/E^*), and although the real exchange rate (P/P^*) appreciates, the net effect is unambiguously a decrease in relative demand. This effect is smaller, the larger the share of public expenditures going to transfers (s) and the less the elasticity of labour supply (lower η).[22] Similar reasoning explains why an increase in the transfer share (s) at Home raises disposable income and expenditure and therefore relative demand. The effects of foreign policy changes are oppositely signed to domestic policy changes.[23] These effects of fiscal policy on relative demand have a number of implications for the remaining endogenous variables to be laid out below.

[22] In fact, in the limiting case where full redistribution, i.e. $s = 1$, and inelastic labour supply, $\eta = 0$, apply simultaneously, the tax rate becomes irrelevant.

[23] In the special case of symmetric fiscal policies ($t = t^*$ and $s = s^*$), relative demand is affected by fiscal policy when labour supply elasticities differ between countries. The effects of fiscal policy changes - in the case of symmetric fiscal policies - are qualitatively identical to those of a unilateral change in the country with the larger labour supply elasticity. The remainder of the section considers unilateral policy changes, but these findings easily extend to the case of symmetric policies and policy changes. See Appendix B for details.

5 Fiscal policies: Industry structure, competitiveness and real wage

Importantly, fiscal policies affect the industry structure - a channel often disregarded both in the open macroeconomic and the public economics literature although it is important and often highlighted in policy discussions, cf. the Introduction. Proposition 2 sums up the effects of fiscal policies on the industry structure.

Proposition 2 *An increase in the tax rate (t) decreases the export threshold (φ_X) and increases the exit threshold (φ_D), sales weighted average productivity (TP), trade openness (O), the fraction of firms exporting (s_x), and fat-tailedness of the firm size distribution. The opposite occurs for an increase in the share of public expenditures going to transfers (s). The trade partner observes the opposite qualitative effects from the policy changes.*
Proof. *See Appendix C.* ■

Note for reference, in autarky, fiscal policy variables (s, t) have no effect on productivity thresholds, sorting of firms, and average productivity. The effects reported above are thus entirely open-economy effects.[24] To see the intuition underlying these results, consider an increase in the tax rate in Home, which increases the relative attractiveness of the export market due to a reduction in relative demand cf. Proposition 1[25]. Accordingly, the increased attractiveness of the export market releases effects on industry structure similar to those from trade liberalization in the seminal Melitz (2003) paper. The selection into exporting softens (falling export productivity threshold, φ_X), while the selection into survival toughens (increasing exit productivity threshold, φ_D). This shifts the entire productivity distribution among active firms, $H'(\varphi) / \Pr(\varphi \geq \varphi_D)$, upwards, which in turn increases sales-weighted[26] average productivity (TP) across firms[27]. Moreover, the

[24] It is the constant markup feature of the present framework which implies that fiscal policy has no impact on selection in autarky. For non-constant elasticities of demand as in Melitz and Ottaviano (2008), markups depend on market size, and fiscal policy may affect selection through its impact on market size.

[25] Due to the CES preferences, markups are constant, and therefore the thresholds are not affected by absolute (B and B^*) but by relative levels of demand components (B/B^*). An increase in the demand components, holding the relative demand component fixed, is simply reflected in a proportional increase in the number of firms (varieties), while the industry structure is otherwise unaffected.

[26] The same findings hold trivially for non-weighted average productivity.

[27] This finding is closely related to the finding on the relation between market size and average productivity in Felbermayr and Jung (2018). We have $TP = \left(\int_{\varphi_D}^{\infty} \frac{r(\varphi)}{\bar{r}} \varphi \frac{dH(\varphi)}{1-H(\varphi_D)} \right)$,

effects on selection increase average firm size and the fraction of firms exporting, $s_X \equiv \Pr(\varphi \geq \varphi_X | \varphi \geq \varphi_D) = (\varphi_X/\varphi_D)^{-k}$. While it is in line with common beliefs and partial equilibrium reasoning that a higher domestic tax rate makes it more difficult for firms to break-even in the domestic market, the finding that it becomes easier for domestic firms to break-even on the export market relies on general equilibrium mechanisms[28]. Increased fat-tailedness of the firm size (sales) distribution follows as increased relative demand in the export market benefits exporters (the larger firms) relative to non-exporters.[29] Below, we explore the opposite causality, i.e. that increased fat-tailedness of the firm size distribution (due lower k and thus more underlying firm heterogeneity) increases the 'optimal' size of the public sector.

An alternative measure of aggregate (average) productivity is the real wage equal to the real value added per unit of labour in the private sector. The real wage effect of an increase in the domestic tax rate is complicated, since it involves both the relative demand effect and a market size effect, and the latter dominates. Hence, the real wage is declining in the domestic tax rate (t) and increasing in the expenditures share of transfers (s)[30].

Here trade openness[31], O, is defined as the ratio of total import expenditures to total private expenditures. Imposing balanced trade, it follows that trade openness can be written $O = 1 - (\varphi_D/\varphi_A)^{-k} \in (0, 1)$. Hence, a higher tax rate increases trade openness of the private sector. Intuitively, the higher tax rate reduces the size of the private sector, and for trade to remain balanced, the smaller private sector must become more open, while the private sector of the trade partner becomes less open.

Hence, a larger public sector does not shelter the private sector to foreign competition. The positive relation between the size of the public sector (tax rate: t) and the number of firms exporting and trade openness, O, fits the empirical evidence discussed in the Introduction (see Figure 1). Hence, while

where $r(\varphi)$ is firm-level sales and $\bar{r} = \int_{\varphi_D}^{\infty} r(\varphi) \frac{dH(\varphi)}{1-H(\varphi_D)}$ is average revenue.

[28] Numerical analysis of a J-country extension shows that countries with larger public consumption (higher t and lower s) - ceteris paribus - have more open private sectors and a larger fraction of exporting firms. This replicates the empirical findings in Figure 1.

[29] More formally, we follow di Giovanni et al. (2011) and show that $d\Pr\left(\frac{r(\varphi)}{r_{\min}} > \frac{r}{r_{\min}} \middle| \varphi > \varphi_D\right) / d\frac{B^*}{B}$ is positive for exporters and zero for non-exporters. See Appendix C for details.

[30] Formally, we have that $\frac{d\frac{W}{P}}{dt} < 0$, $\frac{d\frac{W}{P}}{ds} > 0$, $\frac{d\frac{W}{P}}{dt^*} < 0$, and $\frac{d\frac{W}{P}}{ds^*} > 0$. See Appendix D.

[31] Due to balanced trade, O also equals the ratio of the value of export to the value of output in the private (manufacturing) sector. The more conventional openness measure ($\hat{O}$) given by the import-to-GDP ratio reads $\hat{O} = O(1 - (1 - s)t) < O$. In empirical work, openness is often measured as $\frac{1}{2}$(imports+exports)/GDP. In the present static model with balanced trade, this is identical to the import-to-GDP ratio.

trade openness (lower trade costs) may affect optimal fiscal policies, cf. below and e.g. Rodrik (1998), the present analysis highlights that fiscal policies also affect trade openness. This points to an important endogeneity which is often neglected in the discussion of how openness affects the public sector.

5.1 Fiscal policies and competitiveness

The effects of fiscal policy for competitiveness are central to policy discussions. This discussion features various measures of competitiveness, including wage competitiveness, relative unit labour costs, terms of trade, and the real exchange rate. Proposition 3 sums up the effects of fiscal policies on these measures of competitiveness.

Proposition 3 *An increase in the Home tax rate* (t) *increases the terms-of-trade and the real exchange rate* (P/P^*)*, while it decreases the relative wage* (W/W^*) *and relative unit labour costs* ($RULC$)*. The opposite occurs for an increase in the domestic share of public expenditures going to transfers* (s)*. The qualitative effects in Foreign are opposite to those in Home.*
Proof. *See Appendix* C. ■

To interpret these findings, consider an increase in the domestic tax rate. Surprisingly and counter to the usual perception that income taxes exert an upward pressure on relative wages, our model predicts the opposite and thus an improvement in wage competitiveness. The underlying mechanism is a one-sector home market effect, which implies that the relative wage increases in relative market size (in terms of labour), see Jung and Felbermayr (2015). Intuitively, the reduction in the size of the private market due to a higher tax rate makes entry in Foreign relatively more attractive. Accordingly, a reduction in the relative wage is required to restore the attractiveness of entry in Home[32]. Note that the literature on fiscal policy assuming an exogenous number of firms predicts the relative wage to be increasing in the domestic tax rate.[33] The present analysis highlights the importance of adjustments in the number of firms as important for how fiscal policies affect wage competitiveness.

Relative unit labour costs (RULC), here defined as sales-weighted marginal unit costs, decrease in the tax rate due to a combination of improved

[32]The same finding applies without selection and therefore exogenous productivity distribution. See Appendix E.

[33]See e.g. Alesina and Perotti (1997) for an Armington trade structure (exogenous production structure and exogenous number of firms) and Andersen and Sørensen (2012) for an analysis similar to the present under a Ricardian trade structure

wage competitiveness and higher (lower) average productivity domestically (abroad), cf. Proposition 2. This finding runs counter to the conventional reasoning on the link between fiscal policies and competitiveness measures.

Terms-of-trade (TOT), the ratio of the average export price to the average import price (both measured at the domestic border), increases in the tax rate. The improvement in wage competitiveness is dominated by an increase in the average productivity of foreign exporters and a decrease in average productivity of domestic exporters, cf. the above findings on selection into exporting. This mirrors common views on how fiscal policy affects the terms-of-trade.

The real exchange rate (P/P^*) is increasing in the tax rate. This occurs despite a reduction in the relative nominal wage and an increase in average productivity (TP) due to a counteracting and dominating market size effect released by the contraction of the domestic private sector, which matters for the ideal price indices due to love-of-variety in preferences.

The new qualitative insights on how fiscal policies affect competitiveness (wage competitiveness, relative unit labour costs and the terms-of-trade) underscore the importance of accounting explicitly for industry dynamics. Entry, exit and selection of firms as well as endogenous aggregate productivity play an important role in the transmission of fiscal policies in open economies.

5.2 Quantifying the importance of an endogenous industry structure: Fiscal harmonization within the Eurozone

As is evident from the analysis above, the endogeneity of the industry structure, including the number of producing firms, has wide implications for the effects of fiscal policy. To illustrate this, the following presents a quantification of the effects of fiscal harmonization within the Eurozone. Besides illustrating the role of an endogenous industry structure, this analysis is of interest in its own right given both the academic debate on the need for fiscal harmonization within a currency union, starting with the seminal contribution by Mundell (1961), and the vivid (European) economic policy debate on the issue.

We calibrate a J-country extension of the model to the 64 countries[34] in the OECD database for trade in value added (TiVA 2018) and conduct the counterfactual policy experiment of fiscal harmonization within the Eurozone towards the initial weighted average across countries within the Eurozone[35].

[34] And a residual 65th country being 'rest of the world'.

[35] See Appendix G for details.

We apply the exact hat algebra approach, which is widely applied within quantitative trade models, see Dekle et al. (2008) or Costinot and Rodriguez-Clare (2014). The convenient property of this approach is that we only need to impose values on σ, η and k, while the effects from all other parameters (trade costs, entry costs, fixed costs, productivity levels, population, work force participation etc.) are captured by empirical observable trade flows. Chetty et al. (2011) report a steady state (Hicksian) macro elasticity of labour supply of 0.5 based on meta analyses, i.e. we let $\eta = 0.5$. Further, we follow Melitz and Redding (2015) by setting $\sigma = 4$ and $k = 4.25$.[36]

Let $\widehat{X} \equiv X^{new}/X^{old}$, i.e. the hat notation is the ratio of a variable in the new equilibrium measured relative to the old (initial) equilibrium. We summarize the effects by considering the real wage, a crucial determinant for welfare, but similar results hold if considering e.g. GDP effects[37]. Specifically, we have for country i that the real wage effect can be written[38]

$$\widehat{\left(\frac{W_i}{P_i}\right)} = \left(\widehat{1 - O_i}\right)^{-\frac{1}{k_i}\frac{\sigma-1}{\sigma-1-\eta_i}} \left(\widehat{\chi}_i\right)^{\frac{1}{\sigma-1-\eta_i}} \left(\widehat{1 - t_i}\right)^{\frac{\eta_i}{\sigma-1-\eta_i}},$$

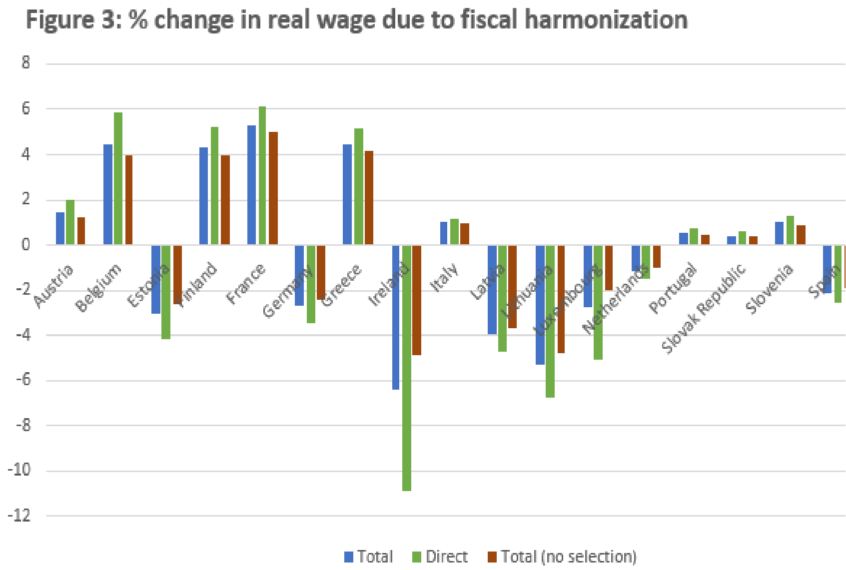
where $\chi_i \equiv 1 - (1 - s_i)\, t_i$ and $\sigma - 1 - \eta_i > 0$ follow from the constraint $\varepsilon_i < 1$. Hence, the total effect is decomposed into a direct effect from the policy change of the country captured by $\widehat{\chi}_i$ and $\widehat{1 - t_i}$, and an indirect effect running through trade openness, $\widehat{1 - O_i}$. Obviously, the indirect effect is only present in the open economy. Figure 3 depicts the counterfactual effects from fiscal harmonization on the real wage for the 19 member countries of

[36] In the quantitative exercise, these parameters are assumed identical across countries.

[37] Real GDP (Y_i) is given as $Y_i = \frac{W_i L_i}{P_i} = n_i (1 - \mu_i) \left(\frac{W_i}{P_i}\right)^{1+\eta_i} (1 - t_i)^{\eta_i} \Rightarrow \widehat{Y}_i = \left(\widehat{\left(\frac{W_i}{P_i}\right)}\right)^{1+\eta_i} \left(\widehat{1 - t_i}\right)^{\eta_i}$.

[38] As in Arkolakis et al. (2012), the real wage - welfare in their framework - is affected by foreign variables through changes in trade openness only. The exponent is slightly different in the present framework, which happens due to an elastic labour supply, i.e. $\eta_i > 0$. In fact, the endogenous labour supply amplifies the gain from increased trade openness.

the Eurozone.



Effect on real wages across the Eurozone from fiscal harmonization towards the existing average of the Eurozone. Blue bars: Total effect. Green bars: Di

Interestingly, fiscal harmonization raises real wages in countries with relatively large public sectors like Finland and France, while countries with a relatively small public sector, like Germany and Ireland, experience declining real wages. In general, countries expanding the public sector observe a drop in the real wage and vice versa. The direct effect (green bars) appears due to a scale effect, as a larger private sector supports more firms, and this increases the real wage due to love-of-variety in preferences. However, and importantly, the open economy adjustment (the differences between direct and total effects) mutes the impact on the real wage, reflecting that gains and costs are shared with the trading partners. This is particularly important for the most open economies such as Netherlands and Ireland. On average across the Eurozone countries, the open economy adjustment mutes the direct policy effect (the closed economy effect) by 23%, and by 33% if selection is absent (i.e. if all firms produce and export).[39] Hence, the open economy adjustment would be larger in the absence of the selection effects. While not shown in Figure 3, the open economy adjustment is weaker when firms are less heterogeneous (higher k).[40]

[39] Effects on 3rd countries, i.e. countries outside the Eurozone, are generally small. However, as 'average fiscal policy' of the Eurozone remains constant, this is hardly surprising.

[40] Proposition 5 below suggests that less heterogeneity (higher k) reduces the policy bias, which occurs as the trade partner takes on a lower share of the costs from an expansion of the public sector. Indeed for $k = 7$ in the quantitative exercise, the open economy mechanisms mute the direct effect by only 15% (as opposed to 23% for $k = 4.25$).

Our findings show that an assessment of the effects of fiscal harmonization building on a closed economy framework and thus ignoring the open economy adjustment effects would significantly over- (under-) estimate the welfare loss for countries that should contract (expand) their public sectors in the harmonization process. Importantly, our findings also show that the discrepancy between assessments of the counterfactual policy reform in closed and open economy frameworks increases with trade openness (expectedly) as well as with the degree of firm heterogeneity.

6 Optimal fiscal policies and policy biases

The mechanisms explored above are crucial for the optimal setting of fiscal policies. To analyse this more explicitly, assume a Utilitarian social welfare function. The average utility across households can be written as a function of openness, and domestic fiscal policies (t, s)[41]

$$\mathcal{W} = \int_0^n \frac{1}{n} U_i \left(C_i, L_i, G\right) di = \mathcal{W}\left(O, t, s\right), \tag{16}$$

where average utility increases in openness, which in turn captures fiscal spillovers. For details see Appendix D, which contains derivations and proofs for this section. We first consider non-cooperative policy biases, and then turn to the effects of product market integration.

6.1 Non-cooperative policy biases

Trade links make fiscal policies interdependent, and the first question is how non-cooperative policies compare to cooperative policies. It is well known that the sign of non-cooperative bias depends on the sign of policy spillovers; the direct effect of foreign policy choices on domestic welfare, see Cooper and John (1988). There is a precise analytical answer to the non-cooperative (Nash) policy bias summarized in the following proposition:

Proposition 4 *The tax rate (t) in non-cooperative equilibrium exceeds the tax rate in the cooperative equilibrium, while the expenditure share of transfers*

[41] Evaluating the welfare function, see Equation (1), in the equilibrium for given policy parameters, we obtain $\mathcal{W}\left(\varphi_D, t, s\right) = (1-\mu)\, u\left(\frac{1}{\eta}\frac{1}{1+\gamma^{-1}}\left(\frac{W(1-t)}{P}\right)^{\eta\left(1+\gamma^{-1}\right)}\right)$ $+\mu u\left(\frac{1}{\chi}\left(s\frac{t}{1-t}\frac{1-\mu}{\mu}\right)^{\chi}\left(\frac{W(1-t)}{P}\right)^{\eta\left(1+\gamma^{-1}\right)}\right) + v\left((1-s)\, t\,(1-\mu)\left(\frac{W(1-t)}{P}\right)^{\eta}\right)$, where $\frac{W}{P} =$ $\left(\frac{n(1-\mu)}{\sigma F_D}\left(1-t\left(1-s\right)\right)\right)^{\frac{1}{\sigma-1}\frac{1}{1-\varepsilon}}\left(1-t\right)^{\frac{\varepsilon}{1-\varepsilon}}\left(\frac{\sigma-1}{\sigma}\varphi_D\right)^{\frac{1}{1-\varepsilon}}$ and $\varphi_D = (1-O)^{-\frac{1}{k}}\varphi_A$.

(s) in the non-cooperative equilibrium falls short of the cooperative equilibrium.[42]
Proof. *See Appendix D.* ■

A higher domestic tax rate has a negative spillover effect on the trading partner via the change in industry structure and terms of trade, see above. Hence, non-cooperative policymaking does not imply a race-to-the-bottom mechanism for the overall size of the public sector. Relative to the literature discussed in Section 2, this result is here generalized to allow for an endogenous industry structure and also parametric asymmetries across countries. However, and *new* to the literature, there is a downward non-cooperative bias in the expenditure share for transfers, and thus a non-cooperative race-to-the-bottom for this part of the public sector. While non-cooperative policymaking does not produce a downward bias in the overall size of the public sector, its composition is twisted away from transfers and towards public consumption.

6.2 Policy biases and international integration

Are these policy biases becoming larger or smaller when countries integrate as a result of e.g. lower trade frictions? And related, how do biases depend on the heterogeneity across firms? To address these more specific questions, more structure must be imposed.[43] We now assume countries to be symmetric in all dimensions/parameters except potentially fiscal policies. Moreover, we choose the widely used approach of log-utilities[44], i.e. $u(\cdot) = \log(\cdot)$ and $v(\cdot) = \Psi \log(\cdot)$, with $\Psi > 0$. The log-utilities imply that the sensitivity of the real wage to policy parameters matters for optimal policy, while the actual level of the real wage does not. The latter property implies that cooperative

[42] While these results are conditional on the other policy parameter, i.e. upward (downward) bias in tax rate (expenditure share on transfers) conditional on expenditure share on transfers (tax rate), we show that they apply unconditionally to symmetric countries and log-utility in Section 7.2.

[43] In the general case, the relation between optimal policy (policy biases) and trade openness depends on higher order derivatives, e.g. 3rd order derivatives, of the utility functions $u(\cdot)$ and $v(\cdot)$, which are hard to interpret economically.

[44] In this case, welfare reads $\mathcal{W} = \mu\chi \log s + \mu\chi \log t - \mu\chi \log(1-t) + \Psi \log(1-s) + \Psi \log t + \left(1 + \gamma^{-1} + \Psi\right) \eta \log\left(\frac{W(1-t)}{P}\right) + \digamma$, where $\digamma \equiv (1-\mu)\log\left(\frac{1}{\eta}\frac{1}{1+\gamma^{-1}}\right) + \mu \log \frac{1}{\chi} + \mu\chi \log\left(\frac{1-\mu}{\mu}\right) + \Psi \log(1-\mu)$ is constant and thus invariant to the policy parameters.

policies become invariant to trade openness.[45] Similarly, cooperative policies are invariant to the degree of firm heterogeneity.

We are now able to show:

Proposition 5 *For log utility,* $u(\cdot) = \log(\cdot)$ *and* $v(\cdot) = \Psi \log(\cdot)$*, with* $\Psi > 0$*, trade liberalization (lower* τ *and/or* F_X*) and more firm heterogeneity*[46] *(lower* k*) magnify policy biases. In particular, both drivers increase (decrease) the non-cooperative tax rate (share of public expenditures devoted to transfers) relative to the cooperative outcome.*
Proof. *See Appendix D.* ■

Non-cooperative policy biases increase with both trade liberalization and firm heterogeneity. Intuitively, the more open the economy is, the larger the fraction of the loss from a shrinking private sector borne by the trade partner. Conditional on trade openness the degree of firm heterogeneity matters for non-cooperative policy bias. This finding is - to the best of our knowledge - *new* to the literature. With the Pareto distribution, more firm heterogeneity implies less concentration of the distribution of firms around the exit threshold. Accordingly, larger changes in relative demand and thus the exit thresholds are needed to restore equilibrium, and this magnifies the spillovers.

Epifani and Gancia (2009) show analytically that trade openness increases the non-cooperative equilibrium values of public consumption relative to GDP as well as the transfer-to-GDP ratio. While they find empirical support for the first prediction, which is also present in our framework[47], they find

[45] Moreover, optimal policies in general become invariant to productivity levels and other non-policy variables affecting the level of the real wage without affecting the sensitivity of the real wage.

[46] In empirical analyses it is not straightforward to measure trade liberalization in the form of lower fixed and/or variable trade costs. This is particularly true for macro models that aggregate across industries. Hence, empirical analyses usually rely on aggregate trade flows as a proxy for aggregate measures of trade liberalization, see e.g. Epifani and Gancia (2009). The findings on trade liberalization in Proposition 5 also hold if trade liberalization is replaced by the empirical observable 'trade openness' defined as import of goods to total demand of goods in the private sector. For the degree of firm heterogeneity, which empirically may be captured by the shape of the productivity distribution, there is the challenge that the degree of firm heterogeneity affects trade openness. Hence, in an empirical analysis it could potentially be difficult to test this finding while controlling for trade openness. However, the finding regarding firm heterogeneity applies both conditionally on trade openness and unconditionally. Hence, even if controlling for trade openness, the finding still holds.

[47] The relation between Nash-policies (non-cooperative) and trade liberalization follows the relation between trade liberalization and policy biases, as the cooperative polices are invariant to trade liberalization in the present setting.

no support for the latter, as the empirical relation between trade openness and the transfer-to-GDP ratio is statistically insignificant.[48] Interestingly, in our framework the relation between trade openness and the transfer-to-GDP ratio, ts, is ambiguous.

6.3 Country asymmetries: Strategic substitutability and divergence in public sector size

Country asymmetries are prevalent, and in context of the present study, differences in public sector sizes are important. Pertinent questions in policy discussions and in the literature are whether globalization triggers a convergence process, making it more difficult/costly to maintain extended public sectors and thus welfare states. This is an important issue, since the differences in public sector sizes and structure reveal asymmetry in underlying policy preferences (social welfare function).

Analyses of optimal policy for asymmetric countries are too complex for analytical results, and we thus rely on numerical analyses for the case of log-utilities; see Appendix F for parameter values and details. In the following we consider heterogeneity in public sectors originating from heterogeneity in the taste for public consumption and in the non-taste dimension via country (population) size.

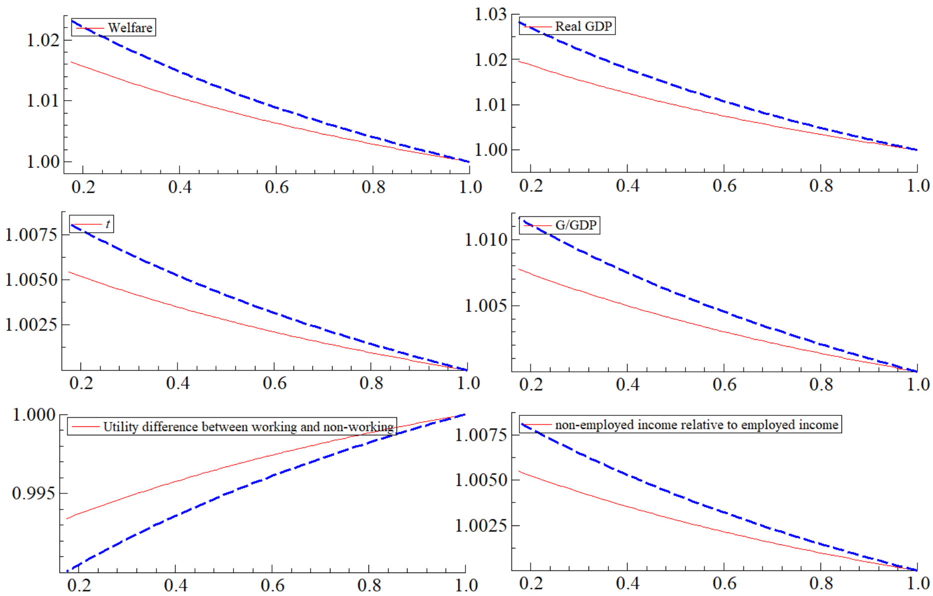
Differences in public sector size may originate from differences in preferences over public consumption (captured by the utility component: $v(G) = \Psi \log(G)$, and thus the parameter Ψ).[49] Differences in preferences release spillover effects or so-called pecuniary externalities between trading partners. The following measures preference heterogeneity by the ratio Ψ^*/Ψ; the lower the ratio, the less weight is attached to public consumption by the trading partners than in the home country. The following takes outset in the Nordic countries (large public sectors); $\Psi^*/\Psi = 1$ corresponds to a symmetric situation where all countries share the preferences in the Nordic countries, while $\Psi^*/\Psi < 1$ implies a lower weight to public activities than in the Nordics (the empirically relevant case). Such preferences affect the Nordics as follows: The lower the taste for public consumption (and thus the size of the public sector) the trade partner has, the higher is welfare, the real wage, and real GDP, and the lower is inequality between working and non-working households (both in terms of income and utility) in the Nordics.

[48] In one specification they find a significant effect. However, the effect is small and negative in that case, while their model predicts a positive effect.

[49] Social preferences for (utility) equality between working and non-working household are determined by the log utility specification and are thus identical across countries.

Moreover, the Nordics also expand the size of the public sector in this case. This finding is important in interpreting the fact that the Nordics with large public sectors have strong economic performance indicators. Moreover, globalization strengthens the spillover effects on the Nordics. Figure 4 illustrates these findings. The horizontal axis measures Ψ^*/Ψ, all values are normalized by their value in the symmetric equilibrium ($\Psi^*/\Psi = 1$), and the blue (red) line is for low (high) trade barriers.

Figure 4: Asymmetric preferences for public consumption



Values are measured relative to symmetric equilibrium value. First axis measures Ψ^*/Ψ. The blue (dashed) line is for lower iceberg trade costs.

Furthermore and importantly, the analysis shows that there is strategic substitutability in the choice of the tax rate and thus the size of the public sector. As the trade partner shrinks the public sector due to a weaker taste for public consumption ($\Delta\Psi^* < 0$), the Nordic response (for unchanged preferences) expands its public sector.

Asymmetries apply in other dimensions than preferences, and they are considered in more detail in Appendix F. For identical preferences for public consumption, i.e. for $\Psi^*/\Psi = 1$, exogenous asymmetries cause asymmetries in openness, and the more open economy has the largest public sector. Intuitively, there is some cost shifting to the trade partner, which increases with openness. A specific example is country size[50]: The smaller country

[50]Similarly, the country with lower labour force participation, lower productivity level ($\varphi_{\min}$), and less firm heterogeneity (higher k) has the larger public sector.

has the more open private sector and therefore adopts the larger public sector. This finding fits the empirical evidence on the relation between country size and size of public sector introduced by Alesina and Wacziarg (1998).[51] Our numerical analyses show that globalization magnifies such endogenous differences in public sectors.[52] Hence, the present analysis suggests that globalization may lead to divergence rather than convergence in public sector size across countries.

7 Conclusion

The interaction between the private and public sector (fiscal policy) is a pertinent policy question, especially in open economies. Public sector activities - consumption and transfers - and the associated taxes are taken to deteriorate competitiveness. Empirical studies have explored how openness affects the public sector (see e.g. Rodrik (1998)), but openness is an endogenous variable, as is fiscal policy, and to capture the mechanisms and interactions, a general equilibrium setting is necessary. The response of the private sector, that is, the selection of firms (producing/non-producing, exporting/non-exporting) and the implied reallocation of resources are important for how the economy adjusts to a change in taxes and public sector expenditures.

This paper builds an analytically manageable general equilibrium model capturing fiscal policies, heterogeneous firms, product market integration (globalization), and cross-country interdependencies. This setting allows analyses of how fiscal policy affects the industry and trade structure of the private sector and thus openness, and also how increased openness (lower trade costs) and firm heterogeneity affect the transmission of fiscal policy as well as optimal fiscal policies; i.e. it allows for analyses of two-way causal effects between the private and public sector. A larger public sector - a higher tax rate - does have standard crowding-out effects. Labour supply is distorted and thus reduced as a consequence of a higher tax rate. This matches common views on the effects of fiscal policy in open economies. However, the endogenous response of the industry and trade structure is important. The higher tax and lower labour supply reduce disposable income and therefore domestic demand, which in turn decreases employment. It becomes harder

[51] In the present setting, the effect of population size on public sector size runs via trade openness and not via economies of scale in the public sector as proposed by Alesina and Wacziarg (1998).

[52] While globalization increases openness and thus the upward bias and size of the public sectors in both countries, the more open economy with the larger public sector observes a larger increase in public sector size (measured as the difference in either tax-revenue-to-GDP ratios or public-consumption-to-GDP ratios).

for domestic firms to break-even, and only the most productive survive. Simultaneously, exporting becomes more attractive, and a larger share of firms becomes exporters. In general equilibrium this is needed to ensure balanced trade. As a consequence, average productivity across active firms increases, relative wages fall (wage competitiveness improves), and relative unit labour costs fall. These effects are different from what is often asserted to be the implications of public sector activities and distorting taxation, and they turn out to depend critically on what taxes are financing. The effects mentioned above are muted, the more taxes finance transfers rather than public consumption. Importantly, our theoretical setting is consistent with a positive relation between size of the public sector and openness of the private sector, measured either as exports relative to value-added or as share of firms exporting.

There is widespread discussion whether globalization (here lower trade costs) leads to a retrenchment of the public sector. The present paper has several insights of importance to this debate. First, as is well known, there is a non-cooperative bias in policymaking causing tax rates, and thus the overall size of the public sector, to exceed the cooperative level. We have generalized this result but also shown that for an important expenditure item, transfers, the non-cooperative bias is negatively signed. That is, in the non-cooperative case, transfers as a share of public expenditures fall short of the level in the cooperative case. Second, the consequences of harmonization of policies, as discussed in the European Monetary Union, have very different implications for the participating countries, and fiscal harmonization has effects which critically depend on trade openness and the endogenous adjustment of the industry structure, and tend to benefit those countries with the relatively largest public sectors. Finally, differences in political preferences matter, and strikingly there are positive spillovers to countries with the relatively strongest preferences for public sector activities, and these effects are stronger, the more tightly integrated economies are. Hence, the widespread view that openness and integration lead to a retrenchment of the public sector and a convergence to more lean welfare models is not supported by the present analysis.

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A Productivity thresholds

Using the definition of profits in (8), equation (13) can be written

$$\int_{\varphi_D}^{\infty}\left[W^{1-\sigma}\varphi^{\sigma-1}B-WF_D\right]dH(\varphi)+\int_{\varphi_X}^{\infty}\left[W^{1-\sigma}\varphi^{\sigma-1}B^{*}\tau^{1-\sigma}-WF_X\right]dH(\varphi)=WF_E$$

Using $W^{1-\sigma}(\varphi_D)^{\sigma-1}B = WF_D$ and $W^{1-\sigma}(\varphi_X)^{\sigma-1}B^{*}\tau^{1-\sigma} = WF_X$ (the threshold conditions), we get

$$\int_{\varphi_D}^{\infty}\left[\frac{\varphi^{\sigma-1}}{(\varphi_D)^{\sigma-1}}\frac{F_D}{F_E}-\frac{F_D}{F_E}\right]dH(\varphi)+\int_{\varphi_X}^{\infty}\left[\frac{\varphi^{\sigma-1}}{(\varphi_X)^{\sigma-1}}\frac{F_X}{F_E}-\frac{F_X}{F_E}\right]dH(\varphi)=1$$

which by use of the Pareto distribution of productivities can be written

$$\frac{F_D}{F_E}\frac{k}{k-(\sigma-1)}\left(\frac{\varphi_D}{\varphi_{\min}}\right)^{-k}-\frac{F_D}{F_E}\left(\frac{\varphi_D}{\varphi_{\min}}\right)^{-k}+\frac{F_X}{F_E}\frac{k}{k-(\sigma-1)}\left(\frac{\varphi_X}{\varphi_{\min}}\right)^{-k}-\frac{F_X}{F_E}\left(\frac{\varphi_X}{\varphi_{\min}}\right)^{-k}=1$$

From (11) and (12) follows

$$\left[\frac{\varphi_D}{\varphi_X}\right]^{\sigma-1}=\frac{F_D}{F_X}\frac{B^{*}}{B}\tau^{1-\sigma} \tag{17}$$

and hence we obtain

$$\left(\frac{\varphi_D}{\varphi_{\min}}\right)^{k}=\frac{\sigma-1}{k-(\sigma-1)}\frac{F_D}{F_E}\left(1+\frac{F_X}{F_D}\left(\frac{B}{B^{*}}\frac{F_X}{F_D}\right)^{-\frac{k}{\sigma-1}}\tau^{-k}\right)$$

which gives (14). Equation (15) follows from inserting the exit threshold into (17).

B General equilibrium

Balanced trade

Balanced trade requires that aggregate imports (I) equal aggregate exports (X), where

$$I=N^{*}\int_{\varphi_X^{*}}^{\infty}E\,(P)^{\sigma-1}\left(\frac{\sigma}{\sigma-1}\frac{W^{*}\tau^{*}}{\varphi}\right)^{1-\sigma}\frac{dH^{*}(\varphi)}{1-H^{*}(\varphi_D^{*})}$$

$$X=N\int_{\varphi_X}^{\infty}E^{*}\,(P^{*})^{\sigma-1}\left(\frac{\sigma}{\sigma-1}\frac{W\tau}{\varphi}\right)^{1-\sigma}\frac{dH(\varphi)}{1-H(\varphi_D)}.$$

Using the demand components, (9) and (10), the balanced trade condition $(I = X)$ implies

$$N^{*}B\left(W^{*}\tau^{*}\right)^{1-\sigma}\int_{\varphi_X^{*}}^{\infty}(\varphi)^{\sigma-1}\frac{dH^{*}(\varphi)}{1-H^{*}(\varphi_D^{*})}=NB^{*}(W\tau)^{1-\sigma}\int_{\varphi_X}^{\infty}(\varphi)^{\sigma-1}\frac{dH(\varphi)}{1-H(\varphi_D)}$$

Imposing the Pareto distributions, this expression can be written

$$\left(\frac{W}{W^{*}}\frac{\tau}{\tau^{*}}\right)^{\sigma-1}=\frac{k^{*}-(\sigma-1)}{k-(\sigma-1)}\frac{k}{k^{*}}\frac{N}{N^{*}}\frac{B^{*}}{B}\frac{(\varphi_D)^{k}}{(\varphi_D^{*})^{k^{*}}}\frac{(\varphi_X)^{\sigma-1-k}}{(\varphi_X^{*})^{\sigma-1-k^{*}}} \tag{18}$$

The metric of trade openness is given as

$$O\equiv\frac{I}{E}=\frac{N^{*}\int_{\varphi_X^{*}}^{\infty}E(P)^{\sigma-1}\left(\frac{\sigma}{\sigma-1}\frac{W^{*}\tau^{*}}{\varphi}\right)^{1-\sigma}\frac{dH^{*}(\varphi)}{1-H^{*}(\varphi_D^{*})}}{N\int_{\varphi_D}^{\infty}E(P)^{\sigma-1}\left(\frac{\sigma}{\sigma-1}\frac{W}{\varphi}\right)^{1-\sigma}\frac{dH(\varphi)}{1-H(\varphi_D)}+N^{*}\int_{\varphi_X^{*}}^{\infty}E(P)^{\sigma-1}\left(\frac{\sigma}{\sigma-1}\frac{W^{*}\tau^{*}}{\varphi}\right)^{1-\sigma}\frac{dH^{*}(\varphi)}{1-H^{*}(\varphi_D^{*})}}$$

Using the balanced trade condition (18) and thereafter the thresholds, we have

$$\begin{aligned}O &= \left(1+\frac{B}{B^{*}}(\tau)^{\sigma-1}\left(\frac{\varphi_D}{\varphi_X}\right)^{\sigma-1-k}\right)^{-1}=\left(1+\left(\frac{B}{B^{*}}\right)^{\frac{k}{\sigma-1}}\tau^{k}\left(\frac{F_X}{F_D}\right)^{\frac{k}{\sigma-1}-1}\right)^{-1}\\ &= 1-\left(\frac{\varphi_D}{\varphi_A}\right)^{-k}\in(0,1)\end{aligned}$$

and similarly for Foreign $O^{*}=\left(1+\left(\frac{B}{B^{*}}\right)^{-\frac{k^{*}}{\sigma-1}}(\tau^{*})^{k^{*}}\left(\frac{F_X^{*}}{F_D^{*}}\right)^{\frac{k^{*}}{\sigma-1}-1}\right)^{-1}=1-\left(\frac{\varphi_D^{*}}{\varphi_A^{*}}\right)^{-k^{*}}\in(0,1)$.

Labour demand

Aggregate labour demand is the sum of labour demand from private firms and the public sector, and it is given as

$$
\begin{aligned}
L^{d} &= L^{d,private} + L^{g} \qquad (19)\\
&= \frac{NF_{E}}{1-H\left(\varphi_{D}\right)} + NF_{D} + NF_{X}\frac{1-H\left(\varphi_{X}\right)}{1-H\left(\varphi_{D}\right)}\\
&\quad + N\int_{\varphi_{D}}^{\infty}\frac{1}{\varphi}E\left(P\right)^{\sigma-1}\left(p_{D}\left(\varphi\right)\right)^{-\sigma}\frac{dH\left(\varphi\right)}{1-H\left(\varphi_{D}\right)}\\
&\quad + N\int_{\varphi_{X}}^{\infty}\frac{\tau}{\varphi}E^{*}\left(P^{*}\right)^{\sigma-1}\left(p_{X}\left(\varphi\right)\right)^{-\sigma}\frac{dH\left(\varphi\right)}{1-H\left(\varphi_{D}\right)} + L^{g}\\
&= N\frac{k\sigma}{k-(\sigma-1)}F_{D}\left(1+\left(\frac{F_{D}}{F_{X}}\right)^{\frac{k}{\sigma-1}-1}\tau^{-k}\left(\frac{B}{B^{*}}\right)^{-\frac{k}{\sigma-1}}\right) + \left(1-s\right)tL
\end{aligned}
$$

where it has been used that

$$
L^{g} = \left(1-s\right)tL^{s}
$$

and

$$
\begin{aligned}
& N\int_{\varphi_{D}}^{\infty}\frac{1}{\varphi}E\left(P\right)^{\sigma-1}\left(p_{D}\left(\varphi\right)\right)^{-\sigma}\frac{dH\left(\varphi\right)}{1-H\left(\varphi_{D}\right)} + N\int_{\varphi_{X}}^{\infty}\frac{\tau}{\varphi}E^{*}\left(P^{*}\right)^{\sigma-1}\left(p_{X}\left(\varphi\right)\right)^{-\sigma}\frac{dH\left(\varphi\right)}{1-H\left(\varphi_{D}\right)}\\
= \; & NB(\sigma-1)W^{-\sigma}\left(\varphi_{D}\right)^{k}k\int_{\varphi_{D}}^{\infty}\varphi^{\sigma-1}\varphi^{-k-1}d\varphi\\
& + NB^{*}(\sigma-1)\tau^{1-\sigma}W^{*-\sigma}\left(\varphi_{D}\right)^{k}k\int_{\varphi_{X}}^{\infty}\varphi^{\sigma-1}\varphi^{-k-1}d\varphi
\end{aligned}
$$

and

$$
\frac{NF_{E}}{1-H\left(\varphi_{D}\right)} = N\frac{F_{E}}{\left(\frac{\varphi_{D}}{\varphi_{\min}}\right)^{-k}} = N\left(1+\left(\frac{F_{D}}{F_{X}}\right)^{\frac{k}{\sigma-1}-1}\tau^{-k}\left(\frac{B}{B^{*}}\right)^{-\frac{k}{\sigma-1}}\right)F_{D}\frac{\sigma-1}{k-(\sigma-1)}
$$

Equilibrium employment and real wages

Aggregate labour demand is, cf. (19)

$$
L^{d} = N\frac{k\sigma}{k-(\sigma-1)}F_{D}\left(1+\left(\frac{F_{D}}{F_{X}}\right)^{\frac{k}{\sigma-1}-1}\tau^{-k}\left(\frac{B}{B^{*}}\right)^{-\frac{k}{\sigma-1}}\right) + \left(1-s\right)tL^{s} \qquad (20)
$$

Using that $L^d = L^s$, we have from (20) that

$$L^d = N\frac{k\sigma}{k-(\sigma-1)}F_D\left(1+\left(\frac{F_D}{F_X}\right)^{\frac{k}{\sigma-1}-1}\tau^{-k}\left(\frac{B}{B^*}\right)^{-\frac{k}{\sigma-1}}\right)(1-t(1-s))^{-1}$$

From (14) it follows that

$$\frac{\varphi_D}{\varphi_{\min}} = \left(\frac{k-(\sigma-1)}{(\sigma-1)}\frac{F_E}{F_D}\right)^{-\frac{1}{k}}\left(1+\left(\frac{F_D}{F_X}\right)^{\frac{k}{\sigma-1}-1}\tau^{-k}\left(\frac{B}{B^*}\right)^{-\frac{k}{\sigma-1}}\right)^{\frac{1}{k}}$$

and labour demand can thus be written

$$L^d = N\left(\frac{\varphi_D}{\varphi_{\min}}\right)^k\frac{k\sigma}{(\sigma-1)}\frac{F_E}{1-t(1-s)}$$

Equating labour demand and supply, see equation (3), to solve for the real wage yields

$$\left(\frac{W}{P}\right)^\eta = \frac{1}{1-\mu}\frac{1}{n}(1-t)^{-\eta}N\frac{k\sigma}{\sigma-1}\left(\frac{\varphi_D}{\varphi_{\min}}\right)^k\frac{F_E}{(1-t(1-s))}$$

and hence

$$\frac{W}{P} = \left(\frac{N}{n}\frac{1}{1-\mu}\frac{k\sigma}{\sigma-1}\left(\frac{\varphi_D}{\varphi_{\min}}\right)^k\frac{F_E}{1-t(1-s)}\right)^{\frac{1}{\eta}}\frac{1}{1-t} \tag{21}$$

$$L = N\frac{k\sigma}{\sigma-1}\left(\frac{\varphi_D}{\varphi_{\min}}\right)^k\frac{F_E}{1-t(1-s)} \tag{22}$$

Inserting the demand component (9) into the exit threshold condition (11) implies

$$(W)^{1-\sigma}(\varphi_D)^{\sigma-1}E(P)^{\sigma-1}\left(\frac{\sigma}{\sigma-1}\right)^{-\sigma}\frac{1}{\sigma-1} = WF_D$$

Using that $E = WL(1-(1-s)t)$, it follows that

$$\left(\frac{W}{P}\right)^{1-\sigma}(\varphi_D)^{\sigma-1}L(1-(1-s)t)\left(\frac{\sigma}{\sigma-1}\right)^{-\sigma}\frac{1}{\sigma-1} = F_D$$

Next, inserting the expression for L (22) and $\frac{W}{P}$ (21) as well as the exit threshold, we obtain[53]

$$F_D = (1-t)^{\sigma-1}\left(\frac{N}{n}\frac{1}{1-\mu}\frac{k\sigma}{(\sigma-1)}\left(\frac{\varphi_D}{\varphi_{\min}}\right)^k\frac{F_E}{1-t(1-s)}\right)^{-\frac{1}{\varepsilon}}$$
$$(\varphi_D)^{\sigma-1}N\frac{k\sigma}{\sigma-1}\left(\frac{\varphi_D}{\varphi_{\min}}\right)^kF_E\left(\frac{\sigma}{\sigma-1}\right)^{-\sigma}\frac{1}{\sigma-1}$$

[53]Recall that $\varepsilon \equiv \frac{1}{\sigma-1}\frac{\chi}{1-\chi+\gamma^{-1}}$.

Solving for N gives us

$$N=\frac{\left((1-t)^{\sigma-1}\left(\frac{1}{n}\frac{1}{1-\mu}\frac{1}{1-t(1-s)}\right)^{-\frac{1}{\varepsilon}}(\varphi_D)^{\sigma-1}\frac{1}{F_D}\left(\frac{\sigma}{\sigma-1}\right)^{-\sigma}\frac{1}{\sigma-1}\right)^{\frac{\varepsilon}{1-\varepsilon}}}{\frac{k\sigma}{\sigma-1}\left(\frac{\varphi_D}{\varphi_{\min}}\right)^{k}F_E} \tag{23}$$

which in turn implies

$$\frac{N}{N^*} = \frac{k^*F_E^*\left(\frac{\varphi_D^*}{\varphi_{\min}^*}\right)^{k^*}}{kF_E\left(\frac{\varphi_D}{\varphi_{\min}}\right)^{k}} \frac{\left((1-t)^{\sigma-1}\left(\frac{1}{n}\frac{1}{1-\mu}\frac{1}{1-t(1-s)}\right)^{-\frac{1}{\varepsilon}}(\varphi_D)^{\sigma-1}\frac{1}{F_D}\left(\frac{\sigma}{\sigma-1}\right)^{-\sigma}\frac{1}{\sigma-1}\right)^{\frac{\varepsilon}{1-\varepsilon}}}{\left((1-t^*)^{\sigma-1}\left(\frac{1}{n^*}\frac{1}{1-\mu^*}\frac{1}{1-t^*(1-s^*)}\right)^{-\frac{1}{\varepsilon^*}}(\varphi_D^*)^{\sigma-1}\frac{1}{F_D^*}\left(\frac{\sigma}{\sigma-1}\right)^{-\sigma}\frac{1}{\sigma-1}\right)^{\frac{\varepsilon^*}{1-\varepsilon^*}}}$$

From the exit threshold conditions for Home and Foreign, we have

$$\frac{W^*}{W}=\left(\left(\frac{\varphi_D^*}{\varphi_D}\right)^{\sigma-1}\frac{F_D}{F_D^*}\frac{B^*}{B}\right)^{\frac{1}{\sigma}}$$

Inserting this into the balanced trade condition, we obtain

$$\frac{N}{N^*}=\left(\left(\frac{\varphi_D^*}{\varphi_D}\right)^{\sigma-1}\frac{F_D}{F_D^*}\frac{B^*}{B}\right)^{-\frac{\sigma-1}{\sigma}}\left(\frac{\tau}{\tau^*}\right)^{\sigma-1}\frac{k-(\sigma-1)}{k^*-(\sigma-1)}\frac{k^*}{k}\frac{B}{B^*}\frac{(\varphi_X^*)^{\sigma-1-k^*}}{(\varphi_X)^{\sigma-1-k}}\frac{(\varphi_D^*)^{k^*}}{(\varphi_D)^{k}}$$

Equating the two terms for $\frac{N}{N^*}$, we get

$$\frac{(\varphi_{\min}^*)^{-k^*}}{(\varphi_{\min})^{-k}}\frac{F_E^*}{F_E}\frac{k^*-(\sigma-1)}{k-(\sigma-1)}\left(\frac{\tau^*}{\tau}\right)^{\sigma-1}\left(\frac{F_D}{F_D^*}\right)^{\frac{\sigma-1}{\sigma}} \frac{\left((1-t)^{\sigma-1}\left(\frac{1}{n}\frac{1}{1-\mu}\frac{1}{1-t(1-s)}\right)^{-\frac{1}{\varepsilon}}\frac{\left(\frac{\sigma}{\sigma-1}\right)^{-\sigma}\frac{1}{\sigma-1}}{F_D}\right)^{\frac{\varepsilon}{1-\varepsilon}}}{\left((1-t^*)^{\sigma-1}\left(\frac{1}{n^*}\frac{1}{1-\mu^*}\frac{1}{1-t^*(1-s^*)}\right)^{-\frac{1}{\varepsilon^*}}\frac{\left(\frac{\sigma}{\sigma-1}\right)^{-\sigma}\frac{1}{\sigma-1}}{F_D^*}\right)^{\frac{\varepsilon^*}{1-\varepsilon^*}}}$$

$$= \left(\frac{B}{B^*}\right)^{\frac{\sigma-1}{\sigma}+1}\left(\frac{\varphi_D}{\varphi_D^*}\right)^{\frac{(\sigma-1)(\sigma-1)}{\sigma}}\frac{(\varphi_X^*)^{\sigma-1-k^*}}{(\varphi_X)^{\sigma-1-k}}\frac{(\varphi_D^*)^{\frac{(\sigma-1)\varepsilon^*}{1-\varepsilon^*}}}{(\varphi_D)^{\frac{(\sigma-1)\varepsilon}{1-\varepsilon}}}$$

Finally, inserting the thresholds - functions of $\frac{B}{B^*}$. We thus have that the equilibrium condition can be written as

$$\Phi(\frac{B}{B^*}) = \Gamma(t, s, t^*, s^*)$$

where

$$\Phi(\frac{B}{B^*}) \equiv \left(\frac{B}{B^*}\right)^{\frac{\sigma-1}{\sigma}+1} \left(\frac{\varphi_D}{\varphi_D^*}\right)^{\frac{(\sigma-1)(\sigma-1)}{\sigma}} \frac{(\varphi_X^*)^{\sigma-1-k^*}}{(\varphi_X)^{\sigma-1-k}} \frac{(\varphi_D^*)^{\frac{(\sigma-1)\varepsilon^*}{1-\varepsilon^*}}}{(\varphi_D)^{\frac{(\sigma-1)\varepsilon}{1-\varepsilon}}}$$

and

$$\Gamma(t, s, t^*, s^*) \equiv \frac{(\varphi_{\min}^*)^{-k^*}}{(\varphi_{\min})^{-k}} \frac{F_E^*}{F_E} \frac{k^* - (\sigma-1)}{k - (\sigma-1)} \left(\frac{\tau^*}{\tau}\right)^{\sigma-1} \left(\frac{F_D}{F_D^*}\right)^{\frac{\sigma-1}{\sigma}} \frac{\left(\frac{\left(\frac{\sigma}{\sigma-1}\right)^{-\sigma}\frac{1}{\sigma-1}}{F_D}\right)^{\frac{\varepsilon}{1-\varepsilon}}}{\left(\frac{\left(\frac{\sigma}{\sigma-1}\right)^{-\sigma}\frac{1}{\sigma-1}}{F_D^*}\right)^{\frac{\varepsilon^*}{1-\varepsilon^*}}}$$
$$\frac{\left((1-t)^{(\sigma-1)}\left(n\left(1-\mu\right)\left(1-t\left(1-s\right)\right)\right)^{\frac{1}{\varepsilon}}\right)^{\frac{\varepsilon}{1-\varepsilon}}}{\left((1-t^*)^{(\sigma-1)}\left(n^*\left(1-\mu^*\right)\left(1-t^*\left(1-s^*\right)\right)\right)^{\frac{1}{\varepsilon^*}}\right)^{\frac{\varepsilon^*}{1-\varepsilon^*}}}$$

Properties of the Φ and Γ functions

For the Φ function, we have

$$\begin{aligned}
&\frac{d\Phi(\frac{B}{B^*})}{d\frac{B}{B^*}} \frac{\frac{B}{B^*}}{\Phi(\frac{B}{B^*})} \\
=\ & \frac{\sigma-1}{\sigma} + 1 + \frac{(\sigma-1)(\sigma-1)}{\sigma}\left(\frac{d\varphi_D}{d\frac{B}{B^*}}\frac{\frac{B}{B^*}}{\varphi_D} - \frac{d\varphi_D^*}{d\frac{B}{B^*}}\frac{\frac{B}{B^*}}{\varphi_D^*}\right) + (\sigma-1-k^*)\frac{d\varphi_X^*}{d\frac{B}{B^*}}\frac{\frac{B}{B^*}}{\varphi_X^*} \\
& -(\sigma-1-k)\frac{d\varphi_X}{d\frac{B}{B^*}}\frac{\frac{B}{B^*}}{\varphi_X} + \frac{(\sigma-1)}{\frac{1}{\varepsilon^*}-1}\frac{d\varphi_D^*}{d\frac{B}{B^*}}\frac{\frac{B}{B^*}}{\varphi_D^*} - \frac{\sigma-1}{\frac{1}{\varepsilon}-1}\frac{d\varphi_D}{d\frac{B}{B^*}}\frac{\frac{B}{B^*}}{\varphi_D} \\
=\ & \frac{\sigma-1}{\sigma} + 1 - \frac{\sigma-1}{\sigma}(O+O^*) - \frac{(\sigma-1-k^*)}{\sigma-1}(1-O^*) - (\sigma-1-k)\frac{1}{\sigma-1}(1-O) \\
& + \frac{1}{\frac{1}{\varepsilon^*}-1}O^* + \frac{1}{\frac{1}{\varepsilon}-1}O \\
=\ & \frac{1}{\sigma} + \left(\frac{k^*}{\sigma-1} - \frac{1}{\sigma}\right)(1-O^*) + \left(\frac{k}{\sigma-1} - \frac{1}{\sigma}\right)(1-O) + \frac{\varepsilon^*}{1-\varepsilon^*}O^* + \frac{\varepsilon}{1-\varepsilon}O
\end{aligned}$$

where it was used that

$$
\begin{aligned}
\frac{d\varphi_D}{d\frac{B}{B^*}}\frac{\frac{B}{B^*}}{\varphi_D} &= -\frac{1}{\sigma-1}O \text{ and } \frac{d\varphi_X}{d\frac{B}{B^*}}\frac{\frac{B}{B^*}}{\varphi_X} = \frac{1}{\sigma-1}(1-O) \\
\frac{d\varphi_D^*}{d\frac{B}{B^*}}\frac{\frac{B}{B^*}}{\varphi_D^*} &= \frac{1}{\sigma-1}O^* \text{ and } \frac{d\varphi_X^*}{d\frac{B}{B^*}}\frac{\frac{B}{B^*}}{\varphi_X^*} = -\frac{1}{\sigma-1}(1-O^*)
\end{aligned}
$$

A sufficient condition for

$$
\frac{d\Phi(\frac{B}{B^*})}{d\frac{B}{B^*}}\frac{\frac{B}{B^*}}{\Phi(\frac{B}{B^*})} > 0
$$

is $\varepsilon < 1$ and $\varepsilon^* < 1$, which is assumed to hold cf. the main text. Note further that $\lim_{\frac{B}{B^*}\to 0}\Phi(\frac{B}{B^*}) = 0$ and $\lim_{\frac{B}{B^*}\to\infty}\Phi(\frac{B}{B^*}) = \infty$.

Since Γ and Φ are continuous in all parameters of the model, it follows that there exists a unique and stable equilibrium value for $\frac{B}{B^*}$ (and hence all other endogenous variables). Note that $\Gamma(t,s,t^*,s^*) > 0$ and $\Phi(1) = 1$.

For the $\Gamma(t,s,t^*,s^*)$ function, we have

$$
\begin{aligned}
\frac{d\Gamma(t,s,t^*,s^*)}{dt}\frac{1}{\Gamma(t,s,t^*,s^*)} &= -\frac{\varepsilon}{1-\varepsilon}\left(\frac{\sigma-1}{1-t}+\frac{1}{\varepsilon}\frac{(1-s)}{1-t(1-s)}\right) < 0 \\
\frac{d\Gamma(t,s,t^*,s^*)}{ds}\frac{1}{\Gamma(t,s,t^*,s^*)} &= \frac{1}{1-\varepsilon}\frac{t}{1-t(1-s)} > 0 \\
\frac{d\Gamma(t,s,t^*,s^*)}{dn(1-\mu)}\frac{1}{\Gamma(t,s,t^*,s^*)} &= \frac{1}{1-\varepsilon}\frac{1}{n(1-\mu)} > 0 \\
\frac{d\Gamma(t,s,t^*,s^*)}{d\varphi_{\min}}\frac{1}{\Gamma(t,s,t^*,s^*)} &= \frac{k}{\varphi_{\min}} > 0 \\
\frac{d\Gamma(t,s,t^*,s^*)}{dt^*}\frac{1}{\Gamma(t,s,t^*,s^*)} &= \frac{\varepsilon^*}{1-\varepsilon^*}\left(\frac{\sigma-1}{1-t^*}+\frac{1}{\varepsilon^*}\frac{(1-s^*)}{1-t^*(1-s^*)}\right) > 0 \\
\frac{d\Gamma(t,s,t^*,s^*)}{ds^*}\frac{1}{\Gamma(t,s,t^*,s^*)} &= -\frac{1}{1-\varepsilon^*}\frac{t^*}{1-t^*(1-s^*)} < 0 \\
\frac{d\Gamma(t,s,t^*,s^*)}{dn^*(1-\mu^*)}\frac{1}{\Gamma(t,s,t^*,s^*)} &= -\frac{1}{1-\varepsilon^*}\frac{1}{n^*(1-\mu^*)} < 0 \\
\frac{d\Gamma(t,s,t^*,s^*)}{d\varphi_{\min}^*}\frac{1}{\Gamma(t,s,t^*,s^*)} &= -\frac{k^*}{\varphi_{\min}^*} < 0
\end{aligned}
$$

The inequality for the transfer share (s) assumes a positive tax rate ($t > 0$) and similar for the Foreign transfer share. The inequality for the tax rate (t) turns into equality in the limit where labour supply is inelastic, i.e. $\eta = 0$, and the transfer share equals unity.

Note that the absolute effects of changes in the policy parameters are inversely related to the inverse of the labour supply elasticity

$$\begin{aligned}
\frac{d\left|\frac{d\Gamma(t,s,t^*,s^*)}{dt}\frac{1}{\Gamma(t,s,t^*,s^*)}\right|}{d\frac{1-\chi+\gamma^{-1}}{\chi}} &= -\frac{(\sigma-1)}{\left(\frac{1}{\varepsilon}-1\right)^2}\left(\frac{\sigma-1}{1-t}+\frac{1-s}{1-t(1-s)}\right)<0 \\
\frac{d\left|\frac{d\Gamma(t,s,t^*,s^*)}{ds}\frac{1}{\Gamma(t,s,t^*,s^*)}\right|}{d\frac{1-\chi+\gamma^{-1}}{\chi}} &= -\frac{(\sigma-1)}{\left(\frac{1}{\varepsilon}-1\right)^2}\frac{t}{1-t(1-s)}<0
\end{aligned}$$

recalling that $\varepsilon \equiv \frac{1}{\sigma-1}\frac{\chi}{1-\chi+\gamma^{-1}} = \frac{\eta}{\sigma-1}$, and this implies that the absolute effects of changes in the policy parameters increase in the labour supply elasticity.

It follows that

$$\begin{aligned}
\frac{d\frac{B}{B^*}}{dt}\frac{Z}{\frac{B}{B^*}} &= -\frac{\varepsilon}{1-\varepsilon}\left(\frac{\sigma-1}{1-t}+\frac{(1-s)}{1-(1-s)\,t}\right)-\frac{(1-s)}{1-(1-s)\,t}<0 \\
\frac{d\frac{B}{B^*}}{ds}\frac{Z}{\frac{B}{B^*}} &= \frac{1}{1-\varepsilon}\frac{t}{1-(1-s)\,t}>0 \\
\frac{d\frac{B}{B^*}}{dt^*}\frac{Z}{\frac{B}{B^*}} &= \frac{\varepsilon^*}{1-\varepsilon^*}\left(\frac{\sigma-1}{1-t^*}+\frac{(1-s^*)}{1-(1-s^*)\,t^*}\right)+\frac{(1-s^*)}{1-(1-s^*)\,t^*}>0 \\
\frac{d\frac{B}{B^*}}{ds^*}\frac{Z}{\frac{B}{B^*}} &= -\frac{1}{1-\varepsilon^*}\frac{t^*}{1-(1-s^*)\,t^*}<0
\end{aligned}$$

where

$$\begin{aligned}
Z &\equiv \frac{d\Phi(\frac{B}{B^*})}{d\frac{B}{B^*}}\frac{\frac{B}{B^*}}{\Phi(\frac{B}{B^*})}=\frac{1}{\sigma}+\left(\frac{k^*}{\sigma-1}-\frac{1}{\sigma}\right)(1-O^*)+\left(\frac{k}{\sigma-1}-\frac{1}{\sigma}\right)(1-O) \\
&\quad +\frac{\varepsilon^*}{1-\varepsilon^*}O^*+\frac{\varepsilon}{1-\varepsilon}O \\
&> 0
\end{aligned}$$

Decomposing the relative demand component into relative demand and the real exchange rate, i.e exploiting $\frac{B}{B^*}=\frac{E}{E^*}\left(\frac{P}{P^*}\right)^{\sigma-1}$, yields[54]

$$\begin{aligned}
\frac{d\frac{E}{E^*}}{\frac{E}{E^*}} &= \frac{\sigma-1}{\sigma}\left(\frac{d\varphi_D}{\varphi_D}-\frac{d\varphi_D^*}{\varphi_D^*}\right)+\frac{1}{\sigma}\frac{d\frac{B}{B^*}}{\frac{B}{B^*}}+(\sigma-1)\frac{\varepsilon}{1-\varepsilon}\left(\frac{d\varphi_D}{\varphi_D}-\frac{dt}{1-t}\right)\\
&\quad-(\sigma-1)\frac{\varepsilon^*}{1-\varepsilon^*}\left(\frac{d\varphi_D^*}{\varphi_D^*}-\frac{dt^*}{1-t^*}\right)+\frac{1}{1-\varepsilon}\frac{-(1-s)\,dt+tds}{1-t\,(1-s)}\\
&\quad-\frac{1}{1-\varepsilon^*}\frac{-(1-s^*)\,dt^*+tds^*}{1-t^*\,(1-s^*)}\\
&= -\frac{\frac{k^*}{\sigma-1}(1-O^*)+\frac{k}{\sigma-1}(1-O)}{Z}\Psi.
\end{aligned}$$

and

$$\begin{aligned}
\frac{d\frac{P}{P^*}}{\frac{P}{P^*}} &= \frac{\sigma-1}{\sigma}\left(\frac{d\varphi_D}{\varphi_D}-\frac{d\varphi_D^*}{\varphi_D^*}\right)+\frac{1}{\sigma}\frac{d\frac{B}{B^*}}{\frac{B}{B^*}}+\frac{1}{1-\varepsilon^*}\frac{d\varphi_D^*}{\varphi_D^*}-\frac{1}{1-\varepsilon}\frac{d\varphi_D}{\varphi_D}-\frac{\varepsilon^*}{1-\varepsilon^*}\frac{dt^*}{1-t^*}\\
&\quad+\frac{\varepsilon}{1-\varepsilon}\frac{dt}{1-t}+\frac{1}{\sigma-1}\frac{1}{1-\varepsilon^*}\frac{-(1-s^*)\,dt^*+t^*ds^*}{1-t^*\,(1-s^*)}-\frac{1}{\sigma-1}\frac{1}{1-\varepsilon}\frac{-(1-s)\,dt+tds}{1-t\,(1-s)}\\
&= \frac{1}{\sigma-1}\frac{\left(\frac{k}{\sigma-1}-1\right)(1-O)+\left(\frac{k^*}{\sigma-1}-1\right)(1-O^*)+(1-O-O^*)}{Z}\Psi.
\end{aligned}$$

where $\Psi=\left(\frac{\varepsilon}{1-\varepsilon}\frac{\sigma-1}{1-t}+\frac{1}{1-\varepsilon}\frac{(1-s)}{1-(1-s)t}\right)dt-\left(\frac{\varepsilon^*}{1-\varepsilon^*}\frac{\sigma-1}{1-t^*}+\frac{1}{1-\varepsilon^*}\frac{(1-s^*)}{1-(1-s^*)t^*}\right)dt^*-\frac{1}{1-\varepsilon}\frac{t}{1-(1-s)t}ds+\frac{1}{1-\varepsilon^*}\frac{t^*}{1-(1-s^*)t^*}ds^*$. Hence, $\frac{d\frac{B}{B^*}}{\frac{B}{B^*}}=\frac{d\frac{E}{E^*}}{\frac{E}{E^*}}+(\sigma-1)\frac{d\frac{P}{P^*}}{\frac{P}{P^*}}=-\frac{\Psi}{Z}$.

[54] Exploiting that

$$\begin{aligned}
\frac{E}{E^*} &= \frac{W}{W^*}\frac{L}{L^*}\frac{1-(1-s)\,t}{1-(1-s^*)\,t^*}=\left(\frac{\varphi_D}{\varphi_D^*}\right)^{\frac{\sigma-1}{\sigma}}\left(\frac{F_D^*}{F_D}\right)^{\frac{1}{\sigma}}\left(\frac{B}{B^*}\right)^{\frac{1}{\sigma}}\\
&\quad\frac{\left(\left(\frac{1}{n}\frac{1}{1-\mu}\right)^{-\frac{1}{\varepsilon}}\frac{1}{F_D}\left(\frac{\sigma}{\sigma-1}\right)^{-\sigma}\frac{1}{\sigma-1}\right)^{\frac{\varepsilon}{1-\varepsilon}}\left((1-t)\,\varphi_D\right)^{(\sigma-1)\frac{\varepsilon}{1-\varepsilon}}\left(1-t\,(1-s)\right)^{\frac{1}{1-\varepsilon}}}{\left(\left(\frac{1}{n^*}\frac{1}{1-\mu^*}\right)^{-\frac{1}{\varepsilon^*}}\frac{1}{F_D^*}\left(\frac{\sigma}{\sigma-1}\right)^{-\sigma}\frac{1}{\sigma-1}\right)^{\frac{\varepsilon^*}{1-\varepsilon^*}}\left((1-t^*)\,\varphi_D^*\right)^{(\sigma-1)\frac{\varepsilon^*}{1-\varepsilon^*}}\left(1-t^*\,(1-s^*)\right)^{\frac{1}{1-\varepsilon^*}}}
\end{aligned}$$

$$\begin{aligned}
\frac{P}{P^*} &= \frac{\frac{W^*}{P^*}}{\frac{W}{P}}\frac{W}{W^*}=\left(\frac{\varphi_D}{\varphi_D^*}\right)^{\frac{\sigma-1}{\sigma}}\left(\frac{F_D^*}{F_D}\right)^{\frac{1}{\sigma}}\left(\frac{B}{B^*}\right)^{\frac{1}{\sigma}}\\
&\quad\frac{\left(n^*\,(1-\mu^*)\,(1-t^*\,(1-s^*))\,\frac{1}{F_D^*}\left(\frac{\sigma}{\sigma-1}\right)^{-\sigma}\frac{1}{\sigma-1}\right)^{\frac{1}{\sigma-1}\frac{1}{1-\varepsilon^*}}(1-t^*)^{\frac{\varepsilon^*}{1-\varepsilon^*}}}{\left(n\,(1-\mu)\,(1-t\,(1-s))\,\frac{1}{F_D}\left(\frac{\sigma}{\sigma-1}\right)^{-\sigma}\frac{1}{\sigma-1}\right)^{\frac{1}{\sigma-1}\frac{1}{1-\varepsilon}}(1-t)^{\frac{\varepsilon}{1-\varepsilon}}}\frac{(\varphi_D^*)^{\frac{1}{1-\varepsilon^*}}}{(\varphi_D)^{\frac{1}{1-\varepsilon}}}
\end{aligned}$$

C Effects of fiscal policy

C.1 Thresholds

From equations (14) and (15), it follows that

$$\begin{aligned}
\frac{d\varphi_D}{dz}\frac{1}{\varphi_D} &= \frac{d\varphi_D}{d\frac{B}{B^*}}\frac{\frac{B}{B^*}}{\varphi_D}\frac{d\frac{B}{B^*}}{dz}\frac{1}{\frac{B}{B^*}} = -\frac{k}{\sigma-1}\frac{1}{k}\frac{\left(\frac{F_D}{F_X}\right)^{\frac{k}{\sigma-1}-1}\tau^{-k}\left(\frac{B}{B^*}\right)^{-\frac{k}{\sigma-1}}}{1+\left(\frac{F_D}{F_X}\right)^{\frac{k}{\sigma-1}-1}\tau^{-k}\left(\frac{B}{B^*}\right)^{-\frac{k}{\sigma-1}}}\frac{d\frac{B}{B^*}}{dz}\frac{1}{\frac{B}{B^*}} \\
&= -\frac{1}{\sigma-1}O\frac{d\frac{B}{B^*}}{dz}\frac{1}{\frac{B}{B^*}}
\end{aligned}$$

and

$$\begin{aligned}
\frac{d\varphi_X}{dz}\frac{1}{\varphi_X} &= \frac{d\varphi_X}{d\frac{B}{B^*}}\frac{\frac{B}{B^*}}{\varphi_X}\frac{d\frac{B}{B^*}}{dz}\frac{1}{\frac{B}{B^*}} = \frac{1}{k}\frac{k}{\sigma-1}\frac{\left(\frac{F_X}{F_D}\right)^{\frac{k}{\sigma-1}-1}\tau^{k}\left(\frac{B}{B^*}\right)^{\frac{k}{\sigma-1}}}{1+\left(\frac{F_X}{F_D}\right)^{\frac{k}{\sigma-1}-1}\tau^{k}\left(\frac{B}{B^*}\right)^{\frac{k}{\sigma-1}}}\frac{d\frac{B}{B^*}}{dz}\frac{1}{\frac{B}{B^*}} \\
&= \frac{1}{\sigma-1}(1-O)\frac{d\frac{B}{B^*}}{dz}\frac{1}{\frac{B}{B^*}}
\end{aligned}$$

and due to the symmetric structures of the countries, we get that

$$\frac{d\varphi_D^*}{dz}\frac{1}{\varphi_D^*} = \frac{1}{\sigma-1}O^*\frac{d\frac{B}{B^*}}{dz}\frac{1}{\frac{B}{B^*}} \text{ and } \frac{d\varphi_X^*}{dz}\frac{1}{\varphi_X^*} = \frac{1}{\sigma-1}(1-O^*)\frac{d\frac{B}{B^*}}{dz}\frac{1}{\frac{B}{B^*}}.$$

C.2 Average productivity

Average productivity is defined as the sales weighted average marginal production efficiency across firms. It reads

$$TP = \frac{\int_{\varphi_D}^{\infty} A_{TP}\left(\frac{1}{\varphi}\right)^{1-\sigma}\varphi\frac{dH(\varphi)}{1-H(\varphi_D)} + \int_{\varphi_X}^{\infty} A_{TP}^*\left(\frac{\tau}{\varphi}\right)^{1-\sigma}\varphi\frac{dH(\varphi)}{1-H(\varphi_D)}}{\int_{\varphi_D}^{\infty} A_{TP}\left(\frac{1}{\varphi}\right)^{1-\sigma}\frac{dH(\varphi)}{1-H(\varphi_D)} + \int_{\varphi_X}^{\infty} A_{TP}^*\left(\frac{\tau}{\varphi}\right)^{1-\sigma}\frac{dH(\varphi)}{1-H(\varphi_D)}}$$

where $A_{TP} = E(P)^{\sigma-1}\left(\frac{\sigma}{\sigma-1}W\right)^{1-\sigma} = \sigma(W)^{1-\sigma}B$ and $A_{TP}^* = E^*(P^*)^{\sigma-1}\left(\frac{\sigma}{\sigma-1}W\right)^{1-\sigma} = \sigma(W)^{1-\sigma}B^*$. Hence, (and using the Pareto distribution) we get

$$TP = \frac{k-(\sigma-1)}{k-\sigma}\frac{A_{TP}+A_{TP}^*\tau^{1-\sigma}\left(\frac{\varphi_X}{\varphi_D}\right)^{\sigma-k}}{A_{TP}+A_{TP}^*\tau^{1-\sigma}\left(\frac{\varphi_X}{\varphi_D}\right)^{\sigma-1-k}}\varphi_D,$$

which by use of (11), (12), (14) and (15) implies

$$TP = \frac{k-(\sigma-1)}{k-\sigma}\frac{1+\tau^{1-k}\left(\frac{F_X}{F_D}\right)^{\frac{\sigma-k}{\sigma-1}}\left(\frac{B}{B^*}\right)^{\frac{1-k}{\sigma-1}}}{1+\tau^{-k}\left(\frac{F_X}{F_D}\right)^{\frac{\sigma-1-k}{\sigma-1}}\left(\frac{B}{B^*}\right)^{-\frac{k}{\sigma-1}}}\varphi_D.$$

It follows that

$$\begin{aligned}&\frac{dTP}{dz}\frac{1}{TP}\\ = \quad &\frac{d\varphi_D}{dz}\frac{1}{\varphi_D}+\left(\frac{\frac{1-k}{\sigma-1}\tau^{1-k}\left(\frac{F_X}{F_D}\right)^{1+\frac{1-k}{\sigma-1}}\left(\frac{B}{B^*}\right)^{\frac{1-k}{\sigma-1}}}{1+\tau^{1-k}\left(\frac{F_X}{F_D}\right)^{1+\frac{1-k}{\sigma-1}}\left(\frac{B}{B^*}\right)^{\frac{1-k}{\sigma-1}}}+\frac{\frac{k}{\sigma-1}\tau^{-k}\left(\frac{F_X}{F_D}\right)^{1-\frac{k}{\sigma-1}}\left(\frac{B}{B^*}\right)^{\frac{-k}{\sigma-1}}}{1+\tau^{-k}\left(\frac{F_X}{F_D}\right)^{1-\frac{k}{\sigma-1}}\left(\frac{B}{B^*}\right)^{\frac{-k}{\sigma-1}}}\right)\frac{d\frac{B}{B^*}}{dz}\frac{1}{\frac{B}{B^*}}.\end{aligned}$$

Using that

$$\frac{d\varphi_D}{dz}\frac{1}{\varphi_D} = -\frac{1}{\sigma-1}\frac{\left(\frac{B}{B^*}\right)^{-\frac{k}{\sigma-1}}\left(\frac{F_D}{F_X}\right)^{\frac{k}{\sigma-1}-1}\tau^{-k}}{1+\left(\frac{B}{B^*}\right)^{-\frac{k}{\sigma-1}}\left(\frac{F_D}{F_X}\right)^{\frac{k}{\sigma-1}-1}\tau^{-k}}\frac{d\left(\frac{B}{B^*}\right)}{dz}\frac{1}{\frac{B}{B^*}}$$

it follows that

$$\begin{aligned}\frac{dTP}{dz}\frac{1}{TP} &= \frac{k-1}{\sigma-1}\left(-\frac{\tau^{1-k}\left(\frac{F_X}{F_D}\right)^{1+\frac{1-k}{\sigma-1}}\left(\frac{B}{B^*}\right)^{\frac{1-k}{\sigma-1}}}{1+\tau^{1-k}\left(\frac{F_X}{F_D}\right)^{1+\frac{1-k}{\sigma-1}}\left(\frac{B}{B^*}\right)^{\frac{1-k}{\sigma-1}}}+\frac{\tau^{-k}\left(\frac{F_X}{F_D}\right)^{1-\frac{k}{\sigma-1}}\left(\frac{B}{B^*}\right)^{\frac{-k}{\sigma-1}}}{1+\tau^{-k}\left(\frac{F_X}{F_D}\right)^{1-\frac{k}{\sigma-1}}\left(\frac{B}{B^*}\right)^{\frac{-k}{\sigma-1}}}\right)\frac{d\frac{B}{B^*}}{dz}\frac{1}{\frac{B}{B^*}}\\ &= \frac{k-1}{\sigma-1}\frac{\tau^{-k}\left(\frac{F_X}{F_D}\right)^{1-\frac{k}{\sigma-1}}\left(\frac{B}{B^*}\right)^{\frac{-k}{\sigma-1}}\left(1-\left[\tau^{\sigma-1}\frac{F_X}{F_D}\frac{B}{B^*}\right]^{\frac{1}{\sigma-1}}\right)}{\left(1+\tau^{1-k}\left(\frac{F_X}{F_D}\right)^{1+\frac{1-k}{\sigma-1}}\left(\frac{B}{B^*}\right)^{\frac{1-k}{\sigma-1}}\right)\left(1+\tau^{-k}\left(\frac{F_X}{F_D}\right)^{1-\frac{k}{\sigma-1}}\left(\frac{B}{B^*}\right)^{\frac{-k}{\sigma-1}}\right)}\frac{d\frac{B}{B^*}}{dz}\frac{1}{\frac{B}{B^*}}\\ &= \frac{k-1}{\sigma-1}\frac{O\left(1-\left[\tau^{\sigma-1}\frac{F_X}{F_D}\frac{B}{B^*}\right]^{\frac{1}{\sigma-1}}\right)}{\left(1+\tau^{1-k}\left(\frac{F_X}{F_D}\right)^{1+\frac{1-k}{\sigma-1}}\left(\frac{B}{B^*}\right)^{\frac{1-k}{\sigma-1}}\right)}\frac{d\frac{B}{B^*}}{dz}\frac{1}{\frac{B}{B^*}} = -\frac{k-1}{\sigma-1}\frac{O\left(\frac{\varphi_X}{\varphi_D}-1\right)}{\left(1+\frac{\varphi_X}{\varphi_D}\frac{O}{1-O}\right)}\frac{d\frac{B}{B^*}}{dz}\frac{1}{\frac{B}{B^*}}\end{aligned}$$

and hence $sign\left(\frac{dTP}{dz}\right) = sign\left(-\frac{d\frac{B}{B^*}}{dz}\right)$ since $\varphi_X > \varphi_D$.

C.3 Fraction of firms exporting

The fraction of firms exporting is $s_X = \left(\frac{\varphi_X}{\varphi_D}\right)^{-k}$, and it follows that $\frac{ds_X}{dz}\frac{1}{s_X} = -k\left(\frac{d\varphi_X}{dz}\frac{1}{\varphi_X} - \frac{d\varphi_D}{dz}\frac{1}{\varphi_D}\right) = -\frac{k}{\sigma-1}\frac{d\frac{B}{B^*}}{dz}\frac{1}{\frac{B}{B^*}}$,and the symmetric structure implies $\frac{ds_X^*}{dz}\frac{1}{s_X^*} = \frac{k}{\sigma-1}\frac{d\frac{B}{B^*}}{dz}\frac{1}{\frac{B}{B^*}}$ for $z = t, t^*, s, s^*$.

C.4 Fat-tailedness of size distribution

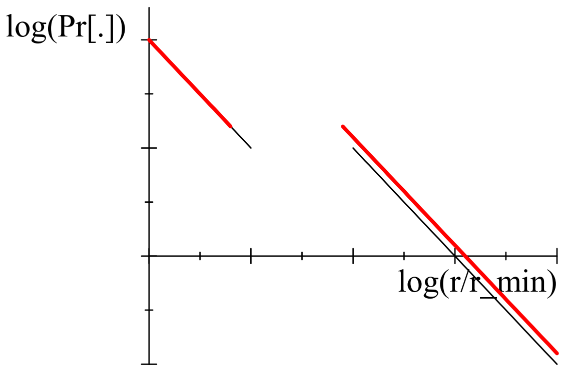
We consider the size distribution, where size is measured by sales. We follow di Giovanni et al. (2011). To focus on the fat-tailedness of the distribution and not the location of the distribution we only consider firms with positive size and we normalize size by the minimum size of active firms, i.e. we consider $\frac{r(\varphi)}{r_{\min}}$, where $r^{\min} = \sigma W F_D$ is minimum sales and $r(\varphi)$ is sales of a firm with efficiency φ. We find that

$$\begin{aligned}
\Pr\left(\frac{r(\varphi)}{r_{\min}} > \frac{r}{r_{\min}}\middle|\, \varphi > \varphi_D\right) &= \Pr\left(\frac{\sigma W^{1-\sigma}\varphi^{\sigma-1}\left(B + I_X B^* \tau^{1-\sigma}\right)}{r_{\min}} > \frac{r}{r_{\min}}\middle|\, \varphi > \varphi_D\right) \\
&= \Pr\left(\varphi > \varphi_D\left(\frac{r}{r_{\min}}\frac{1}{1 + I_X\frac{B^*}{B}\tau^{1-\sigma}}\right)^{\frac{1}{\sigma-1}}\middle|\, \varphi > \varphi_D\right) \\
&= \left(\frac{r}{r_{\min}}\right)^{-\frac{k}{\sigma-1}}\left(1 + I_X\frac{B^*}{B}\tau^{1-\sigma}\right)^{\frac{k}{\sigma-1}},
\end{aligned}$$

where I_X is a dummy variable taking the value 1 if exporting, i.e. for $\varphi > \varphi_X$, and zero otherwise. It follows that

$$\frac{d\Pr\left(\frac{r(\varphi)}{r_{\min}} > \frac{r}{r_{\min}}\middle|\, \varphi > \varphi_D\right)}{d\frac{B^*}{B}} = \Pr\left(\frac{r(\varphi)}{r_{\min}} > \frac{r}{r_{\min}}\middle|\, \varphi > \varphi_D\right)\frac{k}{\sigma-1}\frac{I_X\tau^{1-\sigma}}{1 + I_X\frac{B^*}{B}\tau^{1-\sigma}}.$$

The figure below illustrates the effect on the size distribution in Home of a change in relative sales. The thick (red) line is for high $\frac{B^*}{B}$, whereas the thin (black) line is for low $\frac{B^*}{B}$.



The size distribution is more fat-tailed for higher $\frac{B^*}{B}$, and this determines how fiscal policies affect the fat-tailedness of the size distribution.

C.5 Trade openness

From above, we have that $O = \left(1 + \left(\frac{B}{B^*}\right)^{\frac{k}{\sigma-1}} \tau^k \left(\frac{F_X}{F_D}\right)^{\frac{k}{\sigma-1}-1}\right)^{-1}$. It follows that $\frac{dO}{dz} = -\frac{k}{\sigma-1} \frac{\left(\frac{B}{B^*}\right)^{\frac{k}{\sigma-1}} \tau^k \left(\frac{F_X}{F_D}\right)^{\frac{k}{\sigma-1}-1}}{1+\left(\frac{B}{B^*}\right)^{\frac{k}{\sigma-1}} \tau^k \left(\frac{F_X}{F_D}\right)^{\frac{k}{\sigma-1}-1}} \frac{d\left(\frac{B}{B^*}\right)}{dz} \frac{1}{\frac{B}{B^*}} = -\frac{k}{\sigma-1} O\left(1-O\right) \frac{d\left(\frac{B}{B^*}\right)}{dz} \frac{1}{\frac{B}{B^*}}$.

C.6 Wage competitiveness

From the exit threshold conditions, (11), for Home and Foreign we have $\frac{W}{W^*} = \left(\frac{\varphi_D}{\varphi_D^*}\right)^{\frac{\sigma-1}{\sigma}} \left(\frac{B}{B^*}\right)^{\frac{1}{\sigma}} \left(\frac{F_D}{F_D^*}\right)^{-\frac{1}{\sigma}}$, and it follows that

$$\begin{aligned}
\frac{d\frac{W}{W^*}}{dz}\frac{1}{\frac{W}{W^*}} &= \frac{1}{\sigma}\frac{d\frac{B}{B^*}}{dz}\frac{1}{\frac{B}{B^*}} + \frac{\sigma-1}{\sigma}\left(\frac{d\varphi_D}{dz}\frac{1}{\varphi_D} - \frac{d\varphi_D^*}{dz}\frac{1}{\varphi_D^*}\right) \\
&= \frac{1}{\sigma}\frac{d\frac{B}{B^*}}{dz}\frac{1}{\frac{B}{B^*}} + \frac{\sigma-1}{\sigma}\left(-\frac{O}{\sigma-1}\frac{d\left(\frac{B}{B^*}\right)}{dz}\frac{1}{\frac{B}{B^*}} - \frac{O^*}{\sigma-1}\frac{d\left(\frac{B}{B^*}\right)}{dz}\frac{1}{\frac{B}{B^*}}\right),
\end{aligned}$$

where the latter line inserted $\frac{d\varphi_D}{dz}\frac{1}{\varphi_D}$ and $\frac{d\varphi_D^*}{dz}\frac{1}{\varphi_D^*}$. This implies

$$\frac{d\frac{W}{W^*}}{dz}\frac{1}{\frac{W}{W^*}} = \frac{1-O-O^*}{\sigma}\frac{d\left(\frac{B}{B^*}\right)}{dz}\frac{1}{\frac{B}{B^*}},$$

where $1-O-O^* = \frac{\tau^{\sigma-1}\left(\frac{\varphi_X}{\varphi_D}\right)^{k-(\sigma-1)}(\tau^*)^{\sigma-1}\left(\frac{\varphi_X^*}{\varphi_D^*}\right)^{k^*-(\sigma-1)}-1}{\left(\tau^{\sigma-1}\frac{B}{B^*}\left(\frac{\varphi_X}{\varphi_D}\right)^{k-(\sigma-1)}+1\right)\left((\tau^*)^{\sigma-1}\frac{B^*}{B}\left(\frac{\varphi_X^*}{\varphi_D^*}\right)^{k^*-(\sigma-1)}+1\right)} > 0$ as $\tau \geq 1, \tau^* \geq 1, \frac{\varphi_X}{\varphi_D} > 1$ and $\frac{\varphi_X^*}{\varphi_D^*} > 1$.

C.7 Relative unit labour costs

We have that $RULC \equiv \frac{\frac{W}{TP}}{\frac{W^*}{TP^*}} = \frac{W}{W^*}\frac{TP^*}{TP}$ implying that $\frac{\partial RULC}{\partial z}\frac{1}{RULC} = \frac{d\left(\frac{W}{W^*}\right)}{dz}\frac{1}{\frac{W}{W^*}} - \frac{d\left(\frac{TP}{TP^*}\right)}{dz}\frac{1}{\frac{TP}{TP^*}}$ and $sign\left(\frac{dRULC}{dz}\right) = sign\left(\frac{d\left(\frac{B}{B^*}\right)}{dz}\right)$ follows as $sign\left(\frac{d\left(\frac{W}{W^*}\right)}{dz}\right) = sign\left(-\frac{d\left(\frac{TP}{TP^*}\right)}{dz}\frac{1}{\frac{TP}{TP^*}}\right) = sign\left(\frac{d\left(\frac{B}{B^*}\right)}{dz}\right)$ cf. above.

C.8 The terms-of-trade (TOT)

The terms-of-trade is the average export price divided by the average import price evaluated at domestic border. Hence, we have that

$$TOT = \frac{\frac{\int_{\varphi_X}^{\infty}\frac{p_X(\varphi)}{\tau}\tau E^*(P^*)^{\sigma-1}(p_X(\varphi))^{-\sigma}\frac{dH(\varphi)}{1-H(\varphi_X)}}{\int_{\varphi_X}^{\infty}\tau E^*(P^*)^{\sigma-1}(p_X(\varphi))^{-\sigma}\frac{dH(\varphi)}{1-H(\varphi_X)}}}{\frac{\int_{\varphi_X^*}^{\infty}p_X^*(\varphi)E(P)^{\sigma-1}\left(p_X^*(\varphi)\right)^{-\sigma}\frac{dH^*(\varphi)}{1-H^*\left(\varphi_X^*\right)}}{\int_{\varphi_X^*}^{\infty}E(P)^{\sigma-1}\left(p_X^*(\varphi)\right)^{-\sigma}\frac{dH^*(\varphi)}{1-H^*\left(\varphi_X^*\right)}}} = \frac{1}{\tau}\frac{\frac{\int_{\varphi_X}^{\infty}(p_X(\varphi))^{1-\sigma}\frac{dH(\varphi)}{1-H(\varphi_X)}}{\int_{\varphi_X}^{\infty}(p_X(\varphi))^{-\sigma}\frac{dH(\varphi)}{1-H(\varphi_X)}}}{\frac{\int_{\varphi_X^*}^{\infty}\left(p_X^*(\varphi)\right)^{1-\sigma}\frac{dH^*(\varphi)}{1-H^*\left(\varphi_X^*\right)}}{\int_{\varphi_X^*}^{\infty}\left(p_X^*(\varphi)\right)^{-\sigma}\frac{dH^*(\varphi)}{1-H^*\left(\varphi_X^*\right)}}}$$

$$= \frac{1}{\tau^*}\frac{W}{W^*}\frac{\frac{\int_{\varphi_X}^{\infty}(\varphi)^{\sigma-1}\frac{dH(\varphi)}{1-H(\varphi_X)}}{\int_{\varphi_X}^{\infty}(\varphi)^{\sigma}\frac{dH(\varphi)}{1-H(\varphi_X)}}}{\frac{\int_{\varphi_X^*}^{\infty}(\varphi)^{\sigma-1}\frac{dH^*(\varphi)}{1-H^*\left(\varphi_X^*\right)}}{\int_{\varphi_X^*}^{\infty}(\varphi)^{\sigma}\frac{dH^*(\varphi)}{1-H^*\left(\varphi_X^*\right)}}} = \frac{1}{\tau^*}\frac{W}{W^*}\frac{\frac{k-\sigma}{k-(\sigma-1)}\frac{1}{\varphi_X}}{\frac{k^*-\sigma}{k^*-(\sigma-1)}\frac{1}{\varphi_X^*}} = \frac{1}{\tau^*}\frac{k-\sigma}{k^*-\sigma}\frac{k^*-(\sigma-1)}{k-(\sigma-1)}\frac{W}{W^*}\frac{\varphi_X^*}{\varphi_X}$$

$$\begin{aligned}
\frac{dTOT}{dz}\frac{1}{TOT} &= \frac{d\frac{W}{W^*}}{dz}\frac{1}{\frac{W}{W^*}} + \frac{d\varphi_X^*}{dz}\frac{1}{\varphi_X^*} - \frac{d\varphi_X}{dz}\frac{1}{\varphi_X} \\
&= \frac{1-O-O^*}{\sigma}\frac{d\left(\frac{B}{B^*}\right)}{dz}\frac{1}{\frac{B}{B^*}} - \frac{1}{\sigma-1}(1-O^*)\frac{d\frac{B}{B^*}}{dz}\frac{1}{\frac{B}{B^*}} - \frac{1}{\sigma-1}(1-O)\frac{d\frac{B}{B^*}}{dz}\frac{1}{\frac{B}{B^*}} \\
&= -\frac{1}{\sigma}\left[\frac{(1-O)+(1-O^*)}{\sigma-1}+1\right]\frac{d\frac{B}{B^*}}{dz}\frac{1}{\frac{B}{B^*}}
\end{aligned}$$

$$sign\frac{dTOT}{dz}\frac{1}{TOT} = sign\left(-\frac{d\frac{B}{B^*}}{dz}\frac{1}{\frac{B}{B^*}}\right)$$

C.9 Real exchange rate

From Appendix B we have

$$\frac{d\frac{P}{P^*}}{\frac{P}{P^*}} = \frac{1}{\sigma-1}\frac{\left(\frac{k}{\sigma-1}-1\right)(1-O)+\left(\frac{k^*}{\sigma-1}-1\right)(1-O^*)+(1-O-O^*)}{Z}\Psi.$$

where $\Psi = \left(\frac{\varepsilon}{1-\varepsilon}\frac{\sigma-1}{1-t}+\frac{1}{1-\varepsilon}\frac{(1-s)}{1-(1-s)t}\right)dt - \left(\frac{\varepsilon^*}{1-\varepsilon^*}\frac{\sigma-1}{1-t^*}+\frac{1}{1-\varepsilon^*}\frac{(1-s^*)}{1-(1-s^*)t^*}\right)dt^* - \frac{1}{1-\varepsilon}\frac{t}{1-(1-s)t}ds + \frac{1}{1-\varepsilon^*}\frac{t^*}{1-(1-s^*)t^*}ds^*$

D Optimal policies

D.1 The real wage

Inserting the mass of firms (23) into (21) to obtain

$$\frac{W}{P} = \left(n(1-\mu)(1-t(1-s))(\varphi_D)^{\sigma-1}\frac{1}{F_D}\left(\frac{\sigma}{\sigma-1}\right)^{-\sigma}\frac{1}{\sigma-1}(1-t)^{\eta}\right)^{\frac{1}{\sigma-1}\frac{1}{1-\varepsilon}}$$

it follows that

$$\frac{d\frac{W}{P}}{d\left(\frac{B}{B^*}\right)}\frac{\left(\frac{B}{B^*}\right)}{\frac{W}{P}} = \frac{1}{1-\varepsilon}\frac{d\varphi_D}{d\left(\frac{B}{B^*}\right)}\frac{\left(\frac{B}{B^*}\right)}{\varphi_D} = -\frac{1}{1-\varepsilon}\frac{1}{\sigma-1}O$$

$$\begin{aligned}
\frac{d\frac{W}{P}}{dt}\frac{1}{\frac{W}{P}} &= -\frac{1}{1-\varepsilon}\frac{1}{\sigma-1}O\frac{d\left(\frac{B}{B^*}\right)}{dt}\frac{1}{\left(\frac{B}{B^*}\right)} - \left(\frac{\eta}{\sigma-1}\frac{1}{1-\varepsilon}\frac{1}{1-t}+\frac{1}{\sigma-1}\frac{1}{1-\varepsilon}\frac{(1-s)}{1-t(1-s)}\right) \\
&= \left(\frac{1}{1-\varepsilon}\frac{O}{Z}-1\right)\left(\frac{\varepsilon}{1-\varepsilon}\frac{1}{1-t}+\frac{1}{\sigma-1}\frac{1}{1-\varepsilon}\frac{(1-s)}{1-(1-s)t}\right) \\
&= -\left(\frac{\left(1-\frac{1}{\sigma}\right)(1-O^*-O)+\left(\frac{k^*}{\sigma-1}-1\right)(1-O^*)+\frac{k}{\sigma-1}(1-O)+\frac{\varepsilon^*}{1-\varepsilon^*}O^*}{Z}\right) \\
&\quad \left(\frac{\varepsilon}{1-\varepsilon}\frac{1}{1-t}+\frac{1}{\sigma-1}\frac{1}{1-\varepsilon}\frac{(1-s)}{1-(1-s)t}\right)
\end{aligned}$$

D.2 Welfare function

From the welfare function (16) and the utility function (1), it follows that

$$\begin{aligned}
\mathcal{W} &= (1-\mu)\,U\left(C_i, L_i, G\right) + \mu U\left(TR/P, 0, G\right) \\
&= (1-\mu)\,u\left(\frac{1}{\chi}\left(\frac{W\,(1-t)}{P}L_i\right)^{\chi} - \frac{1}{1+\gamma^{-1}}\left(L_i\right)^{\left(1+\gamma^{-1}\right)}\right) \\
&\quad +\mu u\left(\frac{1}{\chi}\left(st\frac{W}{P}L_i\frac{1-\mu}{\mu}\right)^{\chi}\right) + v\left(\left(1-s\right)tL_i\left(1-\mu\right)\right) \\
&= (1-\mu)\,u\left(\left(\frac{1}{\chi} - \frac{1}{1+\gamma^{-1}}\right)\left(\frac{W\,(1-t)}{P}\right)^{\frac{1+\gamma^{-1}}{1-\chi+\gamma^{-1}}\chi}\right) \\
&\quad +\mu u\left(\frac{1}{\chi}\left(s\frac{t}{1-t}\frac{1-\mu}{\mu}\right)^{\chi}\left(\frac{W\,(1-t)}{P}\right)^{\frac{1+\gamma^{-1}}{1-\chi+\gamma^{-1}}\chi}\right) \\
&\quad +v\left(\left(1-s\right)t\left(1-\mu\right)\left(\frac{W\,(1-t)}{P}\right)^{\frac{\chi}{1-\chi+\gamma^{-1}}}\right)
\end{aligned}$$

Imposing that $v\left(\right) = \Psi\log()$ and $u\left(\right) = \log\left(\right)$, it follows that the welfare becomes

$$\begin{aligned}
\mathcal{W} &= (1-\mu)\log\left(\frac{1}{\chi} - \frac{1}{1+\gamma^{-1}}\right) + (1-\mu)\,\frac{\chi\left(1+\gamma^{-1}\right)}{1-\chi+\gamma^{-1}}\log\left(\frac{W\,(1-t)}{P}\right) \\
&\quad +\mu\log\frac{1}{\chi} + \mu\chi\log\left(s\frac{t}{1-t}\frac{1-\mu}{\mu}\right) + \mu\frac{\chi\left(1+\gamma^{-1}\right)}{1-\chi+\gamma^{-1}}\log\left(\frac{W\,(1-t)}{P}\right) \\
&\quad +\Psi\left(\log\left(1-s\right) + \log t + \log\left(1-\mu\right) + \frac{\chi}{1-\chi+\gamma^{-1}}\log\left(\frac{W\,(1-t)}{P}\right)\right) \\
&= \digamma + \mu\chi\log s + \mu\chi\log t - \mu\chi\log\left(1-t\right) + \frac{\left(1+\gamma^{-1}+\Psi\right)\chi}{1-\chi+\gamma^{-1}}\log\left(\frac{W}{P}\right) \\
&\quad +\Psi\log\left(1-s\right) + \Psi\log t + \frac{\left(1+\gamma^{-1}+\Psi\right)\chi}{1-\chi+\gamma^{-1}}\log\left(1-t\right)
\end{aligned}$$

where $\digamma \equiv (1-\mu)\ln\left(\frac{1}{\chi} - \frac{1}{1+\gamma^{-1}}\right) + \mu\ln\left(\frac{1}{\chi}\right) + \mu\chi\ln\left(\frac{1-\mu}{\mu}\right) + \Psi\ln\left(1-\mu\right)$ is constant and thus invariant to policy parameters. The first-order conditions for optimal policies read

$$\frac{d\mathcal{W}}{dt} = \frac{\mu\chi}{t} + \frac{\mu\chi}{1-t} + \frac{\Psi}{t} + \frac{\left(1+\gamma^{-1}+\Psi\right)\chi}{1-\chi+\gamma^{-1}}\left(\frac{d\frac{W}{P}}{dt}\frac{1}{\frac{W}{P}} - \frac{1}{1-t}\right)$$

$$\frac{d\mathcal{W}}{ds} = \frac{\mu\chi}{s} - \frac{\Psi}{1-s} + \frac{(1+\gamma^{-1}+\Psi)\,\chi}{1-\chi+\gamma^{-1}} \frac{d\frac{W}{P}}{ds} \frac{1}{\frac{W}{P}}$$

where

$$\begin{aligned}
\frac{d\frac{W}{P}}{ds}\frac{1}{\frac{W}{P}} &= \frac{\frac{1}{\sigma-1}}{1-\varepsilon}\left(\frac{t}{1-t+ts} - O\frac{d\frac{B}{B^*}}{ds}\frac{1}{\frac{B}{B^*}}\right) \\
\frac{d\frac{W}{P}}{dt}\frac{1}{\frac{W}{P}} &= \frac{\frac{1}{\sigma-1}}{1-\varepsilon}\left(-\frac{\chi}{1-\chi+\gamma^{-1}}\frac{1}{1-t} - \frac{1-s}{1-t+ts} - O\frac{d\frac{B}{B^*}}{dt}\frac{1}{\frac{B}{B^*}}\right).
\end{aligned}$$

D.3 Cooperative case: Symmetric countries

Due to the property of the log, neither the optimal tax rate nor the optimal expenditure fraction for transfers depend on openness in the cooperative case. Optimal policies are determined by

$$\frac{d\mathcal{W}}{ds} = \mu\chi\frac{1}{s} - \Psi\frac{1}{1-s} + \left(1+\gamma^{-1}+\Psi\right)\frac{\varepsilon}{1-\varepsilon}\frac{t}{1-t+ts} = 0$$

$$\begin{aligned}
\frac{d\mathcal{W}}{dt} &= \mu\chi\frac{1}{t} + \mu\chi\frac{1}{1-t} + \Psi\frac{1}{t} - \frac{(1+\gamma^{-1}+\Psi)\frac{\chi}{1-\chi+\gamma^{-1}}}{1-\varepsilon}\frac{1}{1-t} \\
&\quad - \left(1+\gamma^{-1}+\Psi\right)\frac{\varepsilon}{1-\varepsilon}\frac{1-s}{1-t+ts} \\
&= 0.
\end{aligned}$$

Note that these optimal values do not depend on trade costs and the shape parameter of the Pareto distribution. In fact we have that[55]

$$t^{coop} = t^{coop}(\mu,\Psi,\chi,\gamma,\sigma) \text{ and } s^{coop} = s^{coop}(\mu,\Psi,\chi,\gamma,\sigma).$$

D.4 Non-cooperative case: Symmetric countries

In the non-cooperative case, we have that openness affects optimal policies due to the spillovers. We have that

$$\begin{aligned}
\frac{d\left(\frac{B}{B^*}\right)}{dt} &= \frac{\frac{\partial\Gamma}{\partial t}}{\frac{\partial\Phi(\frac{B}{B^*},\tau,F_X)}{\partial\frac{B}{B^*}}} = -\frac{\frac{1}{1-t}\frac{\chi}{1-\chi+\gamma^{-1}} + \frac{1-s}{1-t(1-s)}}{\left(2\left(\frac{k}{\sigma-1}-\frac{1}{\sigma}\right)(1-O)+\frac{1}{\sigma}\right)(1-\varepsilon)+2\varepsilon O} \\
\frac{d\left(\frac{B}{B^*}\right)}{ds} &= \frac{\frac{\partial\Gamma}{\partial s}}{\frac{\partial\Phi(\frac{B}{B^*},\tau,F_X)}{\partial\frac{B}{B^*}}} = \frac{\frac{t}{1-t(1-s)}}{\left(2\left(\frac{k}{\sigma-1}-\frac{1}{\sigma}\right)(1-O)+\frac{1}{\sigma}\right)(1-\varepsilon)+2\varepsilon O}
\end{aligned}$$

[55] Recall that $\varepsilon \equiv \frac{1}{\sigma-1}\frac{\chi}{1-\chi+\gamma^{-1}}$.

It follows that

$$\frac{\partial \left.\frac{d\left(\frac{B}{B^*}\right)}{dt}\right|_O}{\partial k} = -\left.\frac{d\left(\frac{B}{B^*}\right)}{dt}\right|_O \frac{2\frac{1}{\sigma-1}(1-O)}{2\left(\frac{k}{\sigma-1}-\frac{1}{\sigma}\right)(1-O)+\frac{1}{\sigma}+2\frac{\varepsilon}{1-\varepsilon}O} > 0$$

$$\frac{\partial \left.\frac{d\left(\frac{B}{B^*}\right)}{ds}\right|_O}{\partial k} = -\left.\frac{d\left(\frac{B}{B^*}\right)}{ds}\right|_O \frac{2\frac{1}{\sigma-1}(1-O)}{2\left(\frac{k}{\sigma-1}-\frac{1}{\sigma}\right)(1-O)+\frac{1}{\sigma}+2\frac{\varepsilon}{1-\varepsilon}O} < 0.$$

Hence, the model predicts conditional on trade openness that the magnitude of the spillovers decreases in shape parameter of the efficiency distribution (k) and/or the shape parameter of the sales distributions $\left(\frac{k}{\sigma-1}\right)$. Hence, when firms are more heterogeneous, the spillovers become larger and so do the differences between the non-cooperative and cooperative equilibria.

The first-order conditions evaluated in a symmetric equilibrium read

$$\begin{aligned}
\frac{d\mathcal{W}}{dt} &= \frac{\mu\chi}{t} + \frac{\mu\chi}{1-t} + \frac{\Psi}{t} - \frac{\left(\left(1+\gamma^{-1}\right)+\Psi\right)\chi}{1-\chi+\gamma^{-1}}\frac{1}{1-t} \\
&\quad - \left(\left(1+\gamma^{-1}\right)+\Psi\right)\frac{\varepsilon}{1-\varepsilon}\left(\frac{1-s}{1-t+ts} + \frac{\chi}{1-\chi+\gamma^{-1}}\frac{1}{1-t}\right) \\
&\quad \left(1 - \frac{O}{\left(2\left(\frac{k}{\sigma-1}-\frac{1}{\sigma}\right)(1-O)+\frac{1}{\sigma}\right)(1-\varepsilon)+2\varepsilon O}\right) \\
&= \frac{\mu\chi}{t} + \frac{\mu\chi}{1-t} + \frac{\Psi}{t} - \frac{\left(1+\gamma^{-1}+\Psi\right)\chi}{1-\chi+\gamma^{-1}}\frac{1}{1-t} \\
&\quad - \left(\frac{1-s}{1-t+ts} + \frac{\chi}{1-\chi+\gamma^{-1}}\frac{1}{1-t}\right) QH(O,k) \\
&= 0
\end{aligned}$$

and

$$\begin{aligned}
\frac{d\mathcal{W}}{ds} &= \frac{\mu\chi}{s} - \frac{\Psi}{1-s} \\
&\quad + \left(\left(1+\gamma^{-1}\right)+\Psi\right)\frac{\varepsilon}{1-\varepsilon}\frac{t}{1-t+ts} \\
&\quad \left(1 - \frac{O}{\left(2\left(\frac{k}{\sigma-1}-\frac{1}{\sigma}\right)(1-O)+\frac{1}{\sigma}\right)(1-\varepsilon)+2O\varepsilon}\right) \\
&= \frac{\mu\chi}{s} - \frac{\Psi}{1-s} + \frac{t}{1-t+ts} QH(O,k) = 0
\end{aligned}$$

where

$$
\begin{aligned}
Q &= \left(1+\gamma^{-1}+\Psi\right)\frac{\varepsilon}{1-\varepsilon}>0 \\
H(O,k) &= 1-\frac{O}{\left(2\left(\frac{k}{\sigma-1}-\frac{1}{\sigma}\right)(1-O)+\frac{1}{\sigma}\right)(1-\varepsilon)+2\varepsilon O} \\
H(O,k) &> 0 \\
\frac{\partial H(O,k)}{\partial O} &= -\frac{(1-\varepsilon)\left(2\left(\frac{k}{\sigma-1}-\frac{1}{\sigma}\right)+\frac{1}{\sigma}\right)}{\left(\left(2\left(\frac{k}{\sigma-1}-\frac{1}{\sigma}\right)(1-O)+\frac{1}{\sigma}\right)(1-\varepsilon)+2\varepsilon O\right)^2} \\
&= -(1-\varepsilon)\left(2\left(\frac{k}{\sigma-1}-\frac{1}{\sigma}\right)+\frac{1}{\sigma}\right)\left(\frac{1-H(O,k)}{O}\right)^2<0 \\
\frac{\partial H(O,k)}{\partial k} &= 2\left(1-H(O,k)\right)^2\left(\frac{1}{\sigma-1}\right)\left(\frac{1-O}{O}\right)(1-\varepsilon)>0 \text{ for } O>0 \\
\frac{dH(O,k)}{dk} &= \frac{\partial H(O,k)}{\partial k}+\frac{\partial H(O,k)}{\partial O}\frac{dO}{dk} \\
&= \frac{\partial H(O,k)}{\partial k}-O^2\ln\left(\left(\frac{F_X}{F_D}\right)^{\frac{1}{\sigma-1}}\tau\right)\frac{\partial H(O,k)}{\partial O}>0 \text{ for } O>0
\end{aligned}
$$

Note that $H(O,k)>0$ and $\frac{d\mathcal{W}}{ds}=0\Rightarrow\mu\chi\frac{1}{s}-\Psi\frac{1}{1-s}<0$.

We rewrite the first-order conditions to obtain

$$
\begin{aligned}
\frac{d\mathcal{W}}{ds} &= 0\Rightarrow\frac{ts}{1-t(1-s)}QH(O,k)=\Psi\frac{s}{1-s}-\mu\chi \\
\frac{d\mathcal{W}}{dt} &= 0\Rightarrow\mu\chi\frac{1}{1-t}+\Psi-\frac{\left(1+\gamma^{-1}+\Psi\right)\chi}{1-\chi+\gamma^{-1}}\frac{t}{1-t} \\
&= \left(\frac{t(1-s)}{1-t(1-s)}+\frac{\chi}{1-\chi+\gamma^{-1}}\frac{t}{1-t}\right)QH(O,k)
\end{aligned}
$$

Inserting the former into the latter implies that

$$
t=\frac{\mu}{s(1-\mu)+\mu}-\frac{1}{s(1-\mu)+\mu}\frac{s}{1-s}\frac{\Psi}{\left(1+\gamma^{-1}\right)}
$$

and from the derivative of this we obtain

$$
ds=-\frac{\left(1+\gamma^{-1}\right)\left(\mu+(1-\mu)s\right)}{\left(\Psi\frac{1}{(1-s)^2}+(1-\mu)\left(1+\gamma^{-1}\right)t\right)}dt.
$$

From the total derivative of $\frac{d\mathcal{W}}{ds}=0$, it follows that

$$\begin{aligned}
& \frac{ts}{1-t\left(1-s\right)}Q\frac{\partial H\left(O,k\right)}{\partial O}dO+\frac{s}{\left(1-t\left(1-s\right)\right)^{2}}QH\left(O,k\right)dt \\
& +\frac{t\left(1-t\right)}{\left(1-t\left(1-s\right)\right)^{2}}QH\left(O,k\right)ds \\
= & \ \Psi\frac{1}{\left(1-s\right)^{2}}ds
\end{aligned}$$

Insert ds to get

$$\begin{aligned}
\frac{dt}{dO} &= -\frac{\frac{ts}{1-t(1-s)}Q\frac{\partial H(O,k)}{\partial O}}{\frac{s}{(1-t(1-s))^{2}}QH\left(O,k\right)-\frac{(1+\gamma^{-1})(\mu+(1-\mu)s)}{\left(\Psi\frac{1}{(1-s)^{2}}+(1-\mu)(1+\gamma^{-1})t\right)}\left(\frac{t(1-t)}{(1-t(1-s))^{2}}QH\left(O,k\right)-\Psi\frac{1}{(1-s)^{2}}\right)} \\
&= \frac{-tsQ\frac{\partial H(O,k)}{\partial O}}{\frac{s}{1-t(1-s)}QH\left(O,k\right)+\frac{(1+\gamma^{-1})(\mu+(1-\mu)s)\left(\Psi\frac{s}{(1-s)^{2}}+\mu\chi\frac{1-t}{s}\right)}{\Psi\frac{1}{(1-s)^{2}}+(1-\mu)(1+\gamma^{-1})t}}>0
\end{aligned}$$

where $\frac{ts}{1-t(1-s)}QH\left(O,k\right)=\Psi\frac{s}{1-s}-\mu\chi$ is used from the first to the latter equality. It follows from ds that

$$\frac{ds}{dO}=-\frac{\left(1+\gamma^{-1}\right)\left(\mu+\left(1-\mu\right)s\right)}{\left(\Psi\frac{1}{(1-s)^{2}}+\left(1-\mu\right)\left(1+\gamma^{-1}\right)t\right)}\frac{dt}{dO}<0.$$

We could next redo the analysis with respect to k, which like O only enters through $H\left(O,k\right)$. Note from above that while $\frac{\partial H(O,k)}{\partial O}<0$, we have that $\frac{\partial H(O,k)}{\partial k}>0$ and $\frac{dH(O,k)}{dk}>0$. It follows that we have the opposite results for k than for O, i.e.

$$\frac{dt}{dk}<0 \text{ and } \frac{ds}{dk}>0.$$

For the ratio of public employed to total employed, we have that

$$\frac{L^{g}}{L^{s}}=\left(1-s\right)t$$

and thus that

$$\frac{d\frac{L^{g}}{L^{s}}}{dO}=\left(1-s\right)\frac{dt}{dO}-t\frac{ds}{dO}>0.$$

Transfers relative to GDP read ts, and it follows that

$$\begin{aligned}\frac{d(ts)}{dO} &= s\frac{dt}{dO}+t\frac{ds}{dO}\\ &= \left(s-t\frac{(1+\gamma^{-1})(\mu+(1-\mu)s)}{\Psi\frac{1}{(1-s)^2}+(1-\mu)(1+\gamma^{-1})t}\right)\frac{dt}{dO}\\ &= \left(\frac{\Psi\frac{s}{(1-s)^2}-(1+\gamma^{-1})\mu t}{\Psi\frac{1}{(1-s)^2}+(1-\mu)(1+\gamma^{-1})t}\right)\frac{dt}{dO}\\ &= \left(\frac{(1+\gamma^{-1})}{(1-s)}\frac{\mu-2\mu t(1-s)-st}{\Psi\frac{1}{(1-s)^2}+(1-\mu)(1+\gamma^{-1})t}\right)\frac{dt}{dO}\lesseqgtr 0\end{aligned}$$

as $\mu-2\mu t(1-s)-st\lesseqgtr 0$.

D.5 Asymmetric countries: Numerical analysis

When the countries are asymmetric, the first-order conditions read

$$\begin{aligned}\frac{d\mathcal{W}}{dt} &= \frac{\mu\chi}{t}+\frac{\mu\chi}{1-t}+\frac{\Psi}{t}-Q\left(\frac{(1-s)}{1-(1-s)t}+\frac{\sigma-1}{1-t}+O\frac{d\frac{B}{B^*}}{dt}\frac{1}{\frac{B}{B^*}}\right)=0\\ \frac{d\mathcal{W}}{ds} &= \frac{\mu\chi}{s}-\frac{\Psi}{1-s}+Q\left(\frac{t}{1-(1-s)t}-O\frac{d\frac{B}{B^*}}{ds}\frac{1}{\frac{B}{B^*}}\right)=0\\ \frac{d\mathcal{W}^*}{dt^*} &= \frac{\mu^*\chi^*}{t^*}+\frac{\mu^*\chi^*}{1-t^*}+\frac{\Psi^*}{t^*}-Q^*\left(\frac{1-s^*}{1-(1-s^*)t^*}+\frac{\sigma-1}{1-t^*}-O^*\frac{d\frac{B}{B^*}}{dt^*}\frac{1}{\frac{B}{B^*}}\right)=0\\ \frac{d\mathcal{W}^*}{ds^*} &= \frac{\mu^*\chi^*}{s^*}-\frac{\Psi^*}{1-s^*}+Q^*\left(\frac{t^*}{1-(1-s^*)t^*}+O^*\frac{d\frac{B}{B^*}}{ds^*}\frac{1}{\frac{B}{B^*}}\right)=0\end{aligned}$$

where

$$Q=\left(1+\gamma^{-1}+\Psi\right)\frac{\varepsilon}{1-\varepsilon} \text{ and } Q^*=\left(1+(\gamma^*)^{-1}+\Psi^*\right)\frac{\varepsilon^*}{1-\varepsilon^*}.$$

Numerical procedure: Solve five equations - the equilibrium condition determining $\frac{B}{B^*}$ and the four first-order conditions for the policy variables taking into account that O, O^* and $\frac{d\frac{B}{B^*}}{dz}\frac{1}{\frac{B}{B^*}}$ for $z=t,s,t^*,s^*$ are functions of $\frac{B}{B^*}$ - in the five unknowns t,s,t^*,s^* and $\frac{B}{B^*}$. After solving for these variables, the other variables follow straightforwardly.

E No-selection

Consider the model without selection; i.e. $\varphi_{\min} > \varphi_X$ and all firms are sufficiently productive to export. The zero profit conditions on the Home market and the corresponding conditions on the Foreign market become non-binding in this case and are thus ignored. The balanced trade condition reads $N^* B (W^* \tau^*)^{1-\sigma} \Lambda^* = NB^* (W\tau)^{1-\sigma} \Lambda$, where $\Lambda \equiv \int_{\varphi_{\min}}^{\infty} \varphi^{\sigma-1} dH(\varphi)$ and trade openness is given by

$$O = \frac{N^* B (W^* \tau^*)^{1-\sigma} \Lambda^*}{N^* B (W^* \tau^*)^{1-\sigma} \Lambda^* + NB (W)^{1-\sigma} \Lambda} = \frac{1}{1 + \frac{B}{B^*} (\tau)^{\sigma-1}}.$$

The free entry condition in Home reads $W^{-\sigma} (B + B^* \tau^{1-\sigma}) = \frac{F_E + F_D + F_X}{\Lambda}$ and the relative wage is thus given by

$$\frac{W}{W^*} = \left(\frac{\frac{B}{B^*} + \tau^{1-\sigma}}{1 + \frac{B}{B^*} (\tau^*)^{1-\sigma}} \frac{\Lambda}{\Lambda^*} \frac{F_E^* + F_D^* + F_X^*}{F_E + F_D + F_X} \right)^{\frac{1}{\sigma}}.$$

It follows that

$$\frac{d\frac{W}{W^*}}{d\frac{B}{B^*}} \frac{\frac{B}{B^*}}{\frac{W}{W^*}} = \frac{1}{\sigma} \left(\frac{\frac{B}{B^*}}{\frac{B}{B^*} + \tau^{1-\sigma}} - \frac{\frac{B}{B^*} (\tau^*)^{1-\sigma}}{1 + \frac{B}{B^*} (\tau^*)^{1-\sigma}} \right) = \frac{1}{\sigma} (1 - O - O^*) > 0,$$

which conditional on openness is identical to the effect in the case with selection. The real wage becomes

$$\begin{aligned} \frac{W}{P} &= \frac{W}{\frac{\sigma}{\sigma-1} \left(N (W)^{1-\sigma} \Lambda + N^* (\tau^* W^*)^{1-\sigma} \Lambda^* \right)^{\frac{1}{1-\sigma}}} \\ &= \frac{\sigma - 1}{\sigma} N^{\frac{1}{\sigma-1}} \Lambda^{\frac{1}{\sigma-1}} \left(1 + \frac{B^*}{B} (\tau)^{1-\sigma} \right)^{\frac{1}{\sigma-1}}. \end{aligned}$$

Labour demand reads

$$\begin{aligned} L^d &= L^{d,privat} + L^g \\ &= N (F_E + F_D + F_X) + NE (P)^{\sigma-1} \int_{\varphi_{\min}}^{\infty} \frac{1}{\varphi} (p_D(\varphi))^{-\sigma} dH(\varphi) \\ &\quad + NE^* (P^*)^{\sigma-1} \int_{\varphi_{\min}}^{\infty} \frac{\tau}{\varphi} (p_X(\varphi))^{-\sigma} dH(\varphi) + L^g \\ &= N (F_E + F_D + F_X) + (W)^{-\sigma} N (\sigma - 1) \left(B + B^* (\tau)^{1-\sigma} \right) \Lambda + L^g \\ &= (F_E + F_D + F_X) N\sigma + (1 - s) tL, \end{aligned}$$

where the free-entry condition has been imposed. Labour supply reads

$$
\begin{aligned}
L^s &= n(1-\mu)\left(\frac{W(1-t)}{P}\right)^{\frac{\chi}{1-\chi+\gamma^{-1}}} \\
&= n(1-\mu)\left(\frac{\sigma-1}{\sigma}\Lambda^{\frac{1}{\sigma-1}}\left(1+\frac{B^*}{B}(\tau)^{1-\sigma}\right)^{\frac{1}{\sigma-1}}(1-t)\right)^{\frac{\chi}{1-\chi+\gamma^{-1}}} N^{\frac{1}{\sigma-1}\frac{\chi}{1-\chi+\gamma^{-1}}},
\end{aligned}
$$

where the latter line has inserted the real wage. Labour market equilibrium implies that

$$
\begin{aligned}
N &= \left(\frac{\sigma-1}{\sigma}\Lambda^{\frac{1}{\sigma-1}}\left(1+\frac{B^*}{B}(\tau)^{1-\sigma}\right)^{\frac{1}{\sigma-1}}(1-t)\right)^{\frac{(\sigma-1)\varepsilon}{1-\varepsilon}}\left(\frac{n(1-\mu)(1-(1-s)t)}{(F_E+F_D+F_X)\sigma}\right)^{\frac{1}{1-\varepsilon}} \\
\frac{W}{P} &= \left(\frac{\sigma-1}{\sigma}\right)^{\frac{1}{1-\varepsilon}}(1-t)^{\frac{\varepsilon}{1-\varepsilon}}\left(\Lambda\frac{n(1-\mu)(1-(1-s)t)}{(F_E+F_D+F_X)\sigma}\left(1+\frac{B^*}{B}(\tau)^{1-\sigma}\right)\right)^{\frac{1}{1-\varepsilon}\frac{1}{\sigma-1}}.
\end{aligned}
$$

Note that

$$
\frac{d\frac{W}{P}}{d\frac{B}{B^*}}\frac{\frac{B}{B^*}}{\frac{W}{P}} = -\frac{1}{1-\varepsilon}\frac{1}{\sigma-1}\frac{\frac{B^*}{B}(\tau)^{1-\sigma}}{1+\frac{B^*}{B}(\tau)^{1-\sigma}} = -\frac{1}{1-\varepsilon}\frac{1}{\sigma-1}O,
$$

which is the same effect conditional on openness as in the case with selection. Insert the relative wage as well as the masses of varieties into the balanced trade condition to obtain the following condition determining the relative demand component

$$
\Phi_2\left(\frac{B}{B^*}\right) = \Gamma_2(t,s,t^*,s^*),
$$

where

$$
\begin{aligned}
\Phi_2\left(\frac{B}{B^*}\right) &\equiv \frac{B}{B^*}\left(\frac{\frac{B}{B^*}+\tau^{1-\sigma}}{1+\frac{B}{B^*}(\tau^*)^{1-\sigma}}\right)^{\frac{\sigma-1}{\sigma}}\frac{\left(1+\frac{B}{B^*}(\tau^*)^{1-\sigma}\right)^{\frac{\varepsilon^*}{1-\varepsilon^*}}}{\left(1+\frac{B^*}{B}(\tau)^{1-\sigma}\right)^{\frac{\varepsilon}{1-\varepsilon}}} \\
\Gamma_2(t,s,t^*,s^*) &\equiv \xi\frac{(1-t)^{\frac{(\sigma-1)\varepsilon}{1-\varepsilon}}(1-(1-s)t)^{\frac{1}{1-\varepsilon}}}{(1-t^*)^{\frac{(\sigma-1)\varepsilon^*}{1-\varepsilon^*}}(1-(1-s^*)t^*)^{\frac{1}{1-\varepsilon^*}}} \\
\xi &\equiv \frac{\left(\frac{\sigma-1}{\sigma}\Lambda^{\frac{1}{\sigma-1}}\right)^{\frac{(\sigma-1)\varepsilon}{1-\varepsilon}}\left(\frac{n(1-\mu)}{(F_E+F_D+F_X)\sigma}\right)^{\frac{1}{1-\varepsilon}}}{\left(\frac{\sigma-1}{\sigma}(\Lambda^*)^{\frac{1}{\sigma-1}}\right)^{\frac{(\sigma-1)\varepsilon^*}{1-\varepsilon^*}}\left(\frac{n^*(1-\mu^*)}{(F_E^*+F_D^*+F_X^*)\sigma}\right)^{\frac{1}{1-\varepsilon^*}}}\left(\frac{\Lambda}{\Lambda^*}\frac{F_E^*+F_D^*+F_X^*}{F_E+F_D+F_X}\right)^{\frac{1-\sigma}{\sigma}}\left(\frac{\tau}{\tau^*}\right)^{1-\sigma} \\
&> 0
\end{aligned}
$$

and

$$\begin{aligned}
\frac{\partial \Phi_2}{\partial \frac{B}{B^*}} \frac{\frac{B}{B^*}}{\Phi_2} &= 1 + \frac{\sigma - 1}{\sigma} \left(\frac{\frac{B}{B^*}}{\frac{B}{B^*} + \tau^{1-\sigma}} - \frac{\frac{B}{B^*} (\tau^*)^{1-\sigma}}{1 + \frac{B}{B^*} (\tau^*)^{1-\sigma}} \right) \\
&\quad + \frac{\varepsilon^*}{1 - \varepsilon^*} \frac{\frac{B}{B^*} (\tau^*)^{1-\sigma}}{1 + \frac{B}{B^*} (\tau^*)^{1-\sigma}} + \frac{\varepsilon}{1 - \varepsilon} \frac{\frac{B^*}{B} (\tau)^{1-\sigma}}{1 + \frac{B^*}{B} (\tau)^{1-\sigma}} \\
&= \frac{1}{\sigma} + \frac{\sigma - 1}{\sigma} (1 - O + 1 - O^*) + \frac{\varepsilon^*}{1 - \varepsilon^*} O^* + \frac{\varepsilon}{1 - \varepsilon} O \equiv Z_2 > 0 \\
\frac{\partial \Gamma_2}{\partial t} \frac{1}{\Gamma_2} &= -\frac{\varepsilon}{1 - \varepsilon} \frac{\sigma - 1}{1 - t} - \frac{1}{1 - \varepsilon} \frac{(1 - s)}{1 - (1 - s) t} < 0 \\
\frac{\partial \Gamma_2}{\partial s} \frac{1}{\Gamma_2} &= \frac{1}{1 - \varepsilon} \frac{t}{1 - (1 - s) t} > 0 \\
\frac{\partial \Gamma_2}{\partial t^*} \frac{1}{\Gamma_2} &= \frac{\varepsilon^*}{1 - \varepsilon^*} \frac{\sigma - 1}{1 - t^*} + \frac{1}{1 - \varepsilon^*} \frac{(1 - s^*)}{1 - (1 - s^*) t^*} > 0 \\
\frac{\partial \Gamma_2}{\partial s^*} \frac{1}{\Gamma_2} &= -\frac{1}{1 - \varepsilon^*} \frac{t^*}{1 - (1 - s^*) t^*} < 0
\end{aligned}$$

Hence, $\frac{\partial \Gamma_2}{\partial y} \frac{1}{\Gamma_2} = \frac{\partial \Gamma}{\partial y} \frac{1}{\Gamma}$ for $y = t, t^*, s, s^*$. It follows that the effects of fiscal policy on the relative demand component are identical to those with selection, except that $Z_2 \neq Z$. Conditional on openness, we have that $Z_2 < Z$ as $k > \sigma - 1$ and $k^* > (\sigma - 1)$. Hence, conditional on openness, the impact of fiscal policies on the relative demand component $(\frac{B}{B^*})$ is smaller with selection than without selection as

$$\frac{d \frac{B}{B^*}}{dz} \frac{1}{\frac{B}{B^*}} = \frac{\frac{\partial \Gamma_2}{\partial z} \frac{1}{\Gamma_2}}{\frac{\partial \Phi_2}{\partial \frac{B}{B^*}} \frac{\frac{B}{B^*}}{\Phi_2}} = \frac{\frac{\partial \Gamma_2}{\partial z} \frac{1}{\Gamma_2}}{Z_2}.$$

It follows that spillovers and biases (conditional on openness) are smaller with selection than without selection.

F Parameter choices and numerical analyses for asymmetric countries

We follow Melitz and Redding (2015) by setting $\sigma = 4$ and $k = 4.25$. Parameters in the utility function capturing marginal utility from consumption (χ) and labour (γ) are chosen to fit the steady state (Hicksian) macro elasticity of labour supply of 0.5 from Chetty et al. (2011), i.e. $\eta = \chi / (1 - \chi + \gamma^{-1}) = 0.5$. We let $\gamma = 0.5$ and $\chi = 1$ to match this elasticity. We set the fraction of households outside the labour market (μ) and the preferences for public consumption (Ψ) to align the model predictions with the OECD average of general government revenue as a share of GDP of 38.1% in 2015, equal to t in our framework, and the OECD average of social benefits in government expenditures in 2015 of 41.1%, equal to s in our model.[56] This implies that $\Psi = 0.5$ and $\mu = 0.4725$.[57] We further assume in the baseline case that $\tau = 1.2$, $F_X = F_D = F_E = n = 1$ and $\varphi_{\min} = 20$.

See Appendix D.5 for the numerical procedure. The table below shows the direction in which equilibrium values of unilateral optimal fiscal variables, welfare and openness of the private sector respond to changes in country parameters.[58]

	Δt	Δt^*	Δs	Δs^*	$\Delta \mathcal{W}$	$\Delta \mathcal{W}^*$	ΔO	ΔO^*
Δn	$-$	$+$	$+$	$-$	$+$	$+$	$-$	$+$
Δn^*	$+$	$-$	$-$	$+$	$+$	$+$	$+$	$-$
$\Delta \varphi_{\min}$	$-$	$+$	$+$	$-$	$+$	$+$	$-$	$+$
$\Delta \varphi^*_{\min}$	$+$	$-$	$-$	$+$	$+$	$+$	$+$	$-$
$\Delta \Psi$	$+$	$-$	$-$	$+$		$-$	$+$	$-$
$\Delta \Psi^*$	$-$	$+$	$+$	$-$	$-$		$-$	$+$
$\Delta(1-\mu)$	$-$	$+$	$-$	$-$	$+$	$+$	$-$	$+$
$\Delta(1-\mu^*)$	$+$	$-$	$-$	$-$	$+$	$+$	$+$	$-$

Albeit not explored in the table, numerical analyses show as expected that the spillover running through changes in the size of the private sector is larger when trade costs are lower.

G J-country quantitative version (exogenous policy)

In this appendix we extend the model to a J-country model. Countries have identical structures, but we allow for parametric asymmetries for all parameters except σ. Firm-level profits of a firm located in country i with productivity φ read

[56] See OECD (2017).

[57] We have considered other combinations of parameter values that yield similar qualitative predictions.

[58] The effect of Ψ (Ψ^*) on domestic (foreign) welfare $\mathcal{W}$ ($\mathcal{W}^*$) is ignored, as it includes a direct utility effect. A "+" indicates a positive effect, and a "$-$" indicates a negative effect.

$$\pi_i(\varphi) = \max\left[W_i^{1-\sigma}\varphi^{\sigma-1}\sum_j B_j I_j \tau_{ij}^{1-\sigma} - W_i \sum_j I_j F_{ij}, 0\right]$$

where $B_j \equiv E_j P_j^{\sigma-1}\left(\frac{\sigma}{\sigma-1}\right)^{-\sigma}\frac{1}{\sigma-1}$ is the demand component in country j, W_i is the wage in country i, τ_{ij} is the iceberg trade costs when exporting from country i to country j, F_{ij} is the fixed costs (in country i labour units) of serving country j for a firm in country i, and I_j is a dummy variable taking value 1 if serving country j and zero otherwise. To ease notation let $\Phi_{ij} \equiv (F_{ij})^{1-\frac{k_i}{\sigma-1}}(\tau_{ij})^{-k_i}$, which is a trade costs aggregate for exports from country i to country j. The zero-profit conditions read $W_i^{1-\sigma}(\varphi_{ij})^{\sigma-1} B_j \tau_{ij}^{1-\sigma} \equiv W_i F_{ij}$ for all (i, j), while the free-entry conditions read $\int_{\varphi_{\min,i}}^{\infty} \pi_i(\varphi)\, dH_i(\varphi) = W_i F_{\text{Ei}}$ for all i, where F_{Ei} is the entry costs (in local labour units) in country i, and $H_i(\varphi)$ is the underlying productivity distribution in country i, and φ_{ij} is the productivity threshold for a firm in country i to serve country j. It follows from the zero-profit condition that $B_j \tau_{ij}^{1-\sigma} \equiv \frac{W_i F_{ij}}{W_i^{1-\sigma}(\varphi_{ij})^{\sigma-1}}$ and taking the ratio for country i and country j yields $\varphi_{ij} \equiv \varphi_{ii}\left(\frac{B_i}{B_j}\frac{F_{ij}}{F_{ii}}\right)^{\frac{1}{\sigma-1}}\frac{\tau_{ij}}{\tau_{ii}}$, which in turn implies that the free entry condition may be rewritten as

$$\sum_j F_{ij}\left(\left[\int_{\varphi_{ij}}^{\infty}\left(\frac{\varphi}{\varphi_{ij}}\right)^{\sigma-1} - 1\right] dH_i(\varphi)\right) = \frac{(\sigma-1)}{k_i-(\sigma-1)}\sum_j F_{ij}\left(\frac{\varphi_{ij}}{\varphi_{\min,i}}\right)^{-k_i} = F_{\text{Ei}}$$

where k_i is the shape parameter of the Pareto distribution for productivities in country i. Using $\varphi_{ij} \equiv \varphi_{ii}\left(\frac{B_i}{B_j}\frac{F_{ij}}{F_{ii}}\right)^{\frac{1}{\sigma-1}}\frac{\tau_{ij}}{\tau_{ii}}$, it follows that

$$\begin{aligned}
\varphi_{ii} &= \varphi_{\min,i}\left(\frac{(\sigma-1)}{k_i-(\sigma-1)}\sum_j \frac{F_{ij}}{F_{\text{Ei}}}\left(\frac{B_i}{B_j}\frac{F_{ij}}{F_{ii}}\right)^{-\frac{k_i}{\sigma-1}}\left(\frac{\tau_{ij}}{\tau_{ii}}\right)^{-k_i}\right)^{\frac{1}{k_i}} \\
&= \varphi_{\min,i}\left(\frac{(\sigma-1)}{k_i-(\sigma-1)}\frac{F_{ii}}{F_{\text{Ei}}}\sum_j \frac{\Phi_{ij}}{\Phi_{ii}}\left(\frac{B_i}{B_j}\right)^{-\frac{k_i}{\sigma-1}}\right)^{\frac{1}{k_i}} \\
&= \varphi_{A,i}\left(\sum_j \frac{\Phi_{ij}}{\Phi_{ii}}\left(\frac{B_i}{B_j}\right)^{-\frac{k_i}{\sigma-1}}\right)^{\frac{1}{k_i}}
\end{aligned}$$

where $\varphi_{A,i} = \varphi_{\min,i}\left(\frac{(\sigma-1)}{k_i-(\sigma-1)}\frac{F_{ii}}{F_{\text{Ei}}}\right)^{\frac{1}{k_i}}$ is the autarky threshold.

Imports and exports read

$$I_i = \sum_{j \neq i} N_j \int_{\varphi_{ji}}^{\infty} E_i (P_i)^{\sigma - 1} \left(\frac{\sigma}{\sigma - 1} \frac{W_j \tau_{ji}}{\varphi} \right)^{1-\sigma} \frac{dH_j(\varphi)}{1 - H_j(\varphi_{jj})}$$

and

$$X_i = N_i \sum_{j \neq i} \int_{\varphi_{ij}}^{\infty} E_j (P_j)^{\sigma - 1} \left(\frac{\sigma}{\sigma - 1} \frac{W_i \tau_{ij}}{\varphi} \right)^{1-\sigma} \frac{dH_i(\varphi)}{1 - H_i(\varphi_{ii})},$$

where N_i is the number of active firms (exporters and non-exporters) in country i. Trade balance implies

$$N_i \sum_{j \neq i} \int_{\varphi_{ij}}^{\infty} B_j \left(\frac{W_i \tau_{ij}}{\varphi} \right)^{1-\sigma} \frac{dH_i(\varphi)}{1 - H_i(\varphi_{ii})} = \sum_{j \neq i} N_j \int_{\varphi_{ji}}^{\infty} B_i \left(\frac{W_j \tau_{ji}}{\varphi} \right)^{1-\sigma} \frac{dH_j(\varphi)}{1 - H_j(\varphi_{jj})}$$

which can be rewritten as

$$W_i N_i \frac{k_i}{k_i - (\sigma - 1)} \sum_{j \neq i} F_{ij} \left(\frac{\varphi_{ij}}{\varphi_{ii}} \right)^{-k_i} = \sum_{j \neq i} W_j N_j F_{ji} \frac{k_j}{k_j - (\sigma - 1)} \left(\frac{\varphi_{ji}}{\varphi_{jj}} \right)^{-k_j}$$

or

$$W_i = \frac{\sum_{j \neq i} W_j N_j F_{ji} \frac{k_j}{k_j - (\sigma-1)} \left(\frac{\varphi_{ji}}{\varphi_{jj}} \right)^{-k_j}}{N_i \frac{k_i}{k_i - (\sigma-1)} \sum_{j \neq i} F_{ij} \left(\frac{\varphi_{ij}}{\varphi_{ii}} \right)^{-k_i}},$$

i.e. J equations (one per country). Normalize $W_1 = 1$. Openness reads

$$
\begin{aligned}
& O_i \\
= \ & \frac{I_i}{E_i} \\
= \ & \frac{\sum_{j\neq i} N_j \int_{\varphi_{ji}}^{\infty} E_i (P_i)^{\sigma-1} \left(\frac{\sigma}{\sigma-1}\frac{W_j\tau_{ji}}{\varphi}\right)^{1-\sigma} \frac{dH_j(\varphi)}{1-H_j\left(\varphi_{jj}\right)}}{\sum_{j\neq i} N_j \int_{\varphi_{ji}}^{\infty} E_i (P_i)^{\sigma-1} \left(\frac{\sigma}{\sigma-1}\frac{W_j\tau_{ji}}{\varphi}\right)^{1-\sigma} \frac{dH_j(\varphi)}{1-H_j\left(\varphi_{jj}\right)} + N_i \int_{\varphi_{ii}}^{\infty} E_i (P_i)^{\sigma-1} \left(\frac{\sigma}{\sigma-1}\frac{W_i\tau_{ii}}{\varphi}\right)^{1-\sigma} \frac{dH_i(\varphi)}{1-H_i(\varphi_{ii})}} \\
= \ & \frac{\sum_{j\neq i} N_j \int_{\varphi_{ji}}^{\infty} \left(\frac{W_j\tau_{ji}}{\varphi}\right)^{1-\sigma} \frac{dH_j(\varphi)}{1-H_j\left(\varphi_{jj}\right)}}{\sum_{j\neq i} N_j \int_{\varphi_{ji}}^{\infty} \left(\frac{W_j\tau_{ji}}{\varphi}\right)^{1-\sigma} \frac{dH_j(\varphi)}{1-H_j\left(\varphi_{jj}\right)} + N_i \int_{\varphi_{ii}}^{\infty} \left(\frac{W_i\tau_{ii}}{\varphi}\right)^{1-\sigma} \frac{dH_i(\varphi)}{1-H_i(\varphi_{ii})}} \\
= \ & \left(1 + \frac{N_i \int_{\varphi_{ii}}^{\infty} \left(\frac{W_i\tau_{ii}}{\varphi}\right)^{1-\sigma} \frac{dH_i(\varphi)}{1-H_i(\varphi_{ii})}}{\sum_{j\neq i} N_j \int_{\varphi_{ji}}^{\infty} \left(\frac{W_j\tau_{ji}}{\varphi}\right)^{1-\sigma} \frac{dH_j(\varphi)}{1-H_j\left(\varphi_{jj}\right)}}\right)^{-1} \\
= \ & \left(1 + \frac{B_i \int_{\varphi_{ii}}^{\infty} \left(\frac{\tau_{ii}}{\varphi}\right)^{1-\sigma} \frac{dH_i(\varphi)}{1-H_i(\varphi_{ii})}}{\sum_{j\neq i} \int_{\varphi_{ij}}^{\infty} B_j \left(\frac{\tau_{ij}}{\varphi}\right)^{1-\sigma} \frac{dH_i(\varphi)}{1-H_i(\varphi_{ii})}}\right)^{-1} \\
= \ & \left(1 + \frac{F_{ii}}{\sum_{j\neq i} F_{ij} \left(\frac{\varphi_{ij}}{\varphi_{ii}}\right)^{-k_i}}\right)^{-1} = \frac{\sum_{j\neq i} F_{ij} \left(\frac{\varphi_{ij}}{\varphi_{ii}}\right)^{-k_i}}{\sum_{j\neq i} F_{ij} \left(\frac{\varphi_{ij}}{\varphi_{ii}}\right)^{-k_i} + F_{ii}} = \frac{\sum_{j\neq i} \Phi_{ij} B_j^{\frac{k_i}{\sigma-1}}}{\sum_{j} \Phi_{ij} B_j^{\frac{k_i}{\sigma-1}}} \\
= \ & 1 - \left(\frac{\varphi_{ii}}{\varphi_{\min,i}}\right)^{-k_i} \frac{\sigma-1}{k_i-(\sigma-1)} \frac{F_{ii}}{F_{\mathrm{Ei}}} = 1 - \left(\frac{\varphi_{ii}}{\varphi_{A,i}}\right)^{-k_i}
\end{aligned}
$$

where $\Phi_{ij} \equiv (F_{ij})^{1-\frac{k_i}{\sigma-1}} (\tau_{ij})^{-k_i}$ is a composite trade barrier from country i to country j. We can thus write the exit thresholds as

$$\varphi_{ii} = \varphi_{A,i} \left(\frac{1}{1-O_i}\right)^{\frac{1}{k_i}},$$

which shows a strict link between selection and openness.

Labour demand in country i reads

$$\begin{aligned}
L_i^d &= L_i^{d,privat} + L_i^g \\
&= \frac{N_i F_{\text{Ei}}}{1 - H_i(\varphi_{ii})} + N_i \sum_j F_{ij} \frac{1 - H_i(\varphi_{ij})}{1 - H_i(\varphi_{ii})} \\
&\quad + N_i \sum_j \int_{\varphi_{ij}}^{\infty} \frac{\tau_{ij}}{\varphi} E_j (P_j)^{\sigma-1} (\tau_{ij} p_i(\varphi))^{-\sigma} \frac{dH_i(\varphi)}{1 - H_i(\varphi_{ii})} + L_i^g \\
&= N_i \frac{k_i \sigma}{k_i - (\sigma - 1)} \sum_j F_{ij} \left(\frac{\varphi_{ij}}{\varphi_{ii}} \right)^{-k_i} + (1 - s_i) t_i L_i^s \\
&= N_i \frac{k_i \sigma}{k_i - (\sigma - 1)} \sum_j F_{ij} \left(\frac{B_i}{B_j} \frac{F_{ij}}{F_{ii}} \right)^{-\frac{k_i}{\sigma-1}} \left(\frac{\tau_{ij}}{\tau_{ii}} \right)^{-k_i} + (1 - s_i) t_i L_i^s
\end{aligned}$$

and labour market equilibrium implies for country i that

$$\begin{aligned}
L_i &= \frac{N_i}{1 - (1 - s_i) t_i} \frac{k_i \sigma}{k_i - (\sigma - 1)} \sum_j F_{ij} \left(\frac{\varphi_{ij}}{\varphi_{ii}} \right)^{-k_i} = \frac{N_i}{1 - (1 - s_i) t_i} \frac{k_i \sigma}{\sigma - 1} \left(\frac{\varphi_{ii}}{\varphi_{\min,i}} \right)^{k_i} F_{\text{Ei}} \\
&= \frac{N_i}{1 - (1 - s_i) t_i} \frac{k_i \sigma}{k_i - (\sigma - 1)} \frac{F_{ii}}{1 - O_i} \\
\frac{W_i}{P_i} &= \left(\frac{1}{n_i (1 - \mu_i)} \frac{N_i}{1 - (1 - s_i) t_i} \frac{k_i \sigma}{k_i - (\sigma - 1)} \sum_j F_{ij} \left(\frac{\varphi_{ij}}{\varphi_{ii}} \right)^{-k_i} \right)^{\frac{1}{\eta_i}} \frac{1}{1 - t_i} \\
&= \left(\frac{1}{n_i (1 - \mu_i)} \frac{N_i}{1 - (1 - s_i) t_i} \frac{k_i \sigma}{\sigma - 1} \left(\frac{\varphi_{ii}}{\varphi_{\min,i}} \right)^{k_i} F_{\text{Ei}} \right)^{\frac{1}{\eta_i}} \frac{1}{1 - t_i} \\
&= \left(\frac{1}{n_i (1 - \mu_i)} \frac{N_i}{1 - (1 - s_i) t_i} \frac{k_i \sigma}{k_i - (\sigma - 1)} \frac{F_{ii}}{1 - O_i} \right)^{\frac{1}{\eta_i}} \frac{1}{1 - t_i}
\end{aligned}$$

To find the number of firms, start with the exit threshold condition and insert the demand component

$$W_i^{1-\sigma} (\varphi_{ii})^{\sigma-1} E_i P_i^{\sigma-1} \left(\frac{\sigma}{\sigma - 1} \right)^{-\sigma} \frac{1}{\sigma - 1} \tau_{ii}^{1-\sigma} = W_i F_{ii}$$

and then insert the private budget constraint $E_i = W_i L_i (1 - (1 - s_i) t_i)$ to obtain

$$\left(\frac{W_i}{P_i} \right)^{1-\sigma} (\varphi_{ii})^{\sigma-1} L_i (1 - (1 - s_i) t_i) \left(\frac{\sigma}{\sigma - 1} \right)^{-\sigma} \frac{1}{\sigma - 1} \tau_{ii}^{1-\sigma} = F_{ii}.$$

Next, insert the labour market equilibrium conditions for $\frac{W_i}{P_i}$ and L_i to obtain the number of firms

$$N_i = \frac{k_i - (\sigma - 1)}{k_i \sigma} (1 - O_i) \left(\frac{\left(\frac{\sigma}{\sigma-1}\right)^{\sigma} (\sigma - 1) \tau_{ii}^{\sigma-1}}{\left((1 - t_i) \varphi_{ii}\right)^{\sigma-1}} \right)^{\frac{\eta_i}{1-\sigma+\eta_i}} \left(\frac{F_{ii}}{n_i (1 - \mu_i) (1 - (1 - s_i) t_i)} \right)^{\frac{\sigma-1}{1-\sigma+\eta_i}}.$$

G.1 Quantitative exercise

Here we consider the effects of exogenous changes in fiscal policies using the method known as exact hat algebra, see e.g. Dekle et al. (2008) or Costinot and Rodríguez-Clare (2014). In the following, $\hat{X} \equiv \frac{X'}{X}$, where X' is the value after the change (in fiscal policies) and X is the value in the initial equilibrium. Furthermore, to shorten notation let $\chi_i \equiv 1 - (1 - s_i) t_i$. From the zero profit conditions for country i regarding market i and market j, it follows that

$$\hat{B}_j = \frac{\hat{W}_i^{\sigma}}{\left(\hat{\varphi}_{ij}\right)^{\sigma-1}} \text{ and } \hat{B}_i = \frac{\hat{W}_i^{\sigma}}{\left(\hat{\varphi}_{ii}\right)^{\sigma-1}}$$

which in turn yields

$$\hat{\varphi}_{ij} = \left(\frac{\hat{B}_i}{\hat{B}_j} \right)^{\frac{1}{\sigma-1}} (\hat{\varphi}_{ii}) = \left(\frac{\hat{W}_i}{\hat{W}_j} \right)^{\frac{\sigma}{\sigma-1}} \hat{\varphi}_{jj}.$$

Turning to the free-entry condition, we have

$$\hat{\varphi}_{ii} = \left(\sum_j \left(\frac{\hat{B}_i}{\hat{B}_j} \right)^{-\frac{k_i}{\sigma-1}} \frac{F_{ij} \left(\frac{\varphi_{ij}}{\varphi_{ii}} \right)^{-k_i}}{\sum_j \left(\frac{\varphi_{ij}}{\varphi_{ii}} \right)^{-k_i} F_{ij}} \right)^{\frac{1}{k_i}} = \left(\sum_j \left(\frac{\hat{B}_i}{\hat{B}_j} \right)^{-\frac{k_i}{\sigma-1}} va_{ij} \right)^{\frac{1}{k_i}},$$

where $va_{ij} = \frac{VA_{ij}}{VA_i} = \frac{F_{ij}\left(\frac{\varphi_{ij}}{\varphi_{ii}}\right)^{-k_i}}{\sum_j F_{ij}\left(\frac{\varphi_{ij}}{\varphi_{ii}}\right)^{-k_i}}$ is the share of value added in country i (in the private sector) sold in country j.

Trade balance for country i implies

$$\begin{aligned} \hat{W}_i \hat{N}_i \sum_{j \neq i} \left(\frac{\hat{\varphi}_{ij}}{\hat{\varphi}_{ii}} \right)^{-k_i} \frac{F_{ij} \left(\frac{\varphi_{ij}}{\varphi_{ii}} \right)^{-k_i}}{\sum_{j \neq i} F_{ij} \left(\frac{\varphi_{ij}}{\varphi_{ii}} \right)^{-k_i}} &= \sum_{j \neq i} \hat{W}_j \hat{N}_j \left(\frac{\hat{\varphi}_{ji}}{\hat{\varphi}_{jj}} \right)^{-k_j} \frac{W_j N_j F_{ji} \frac{k_j}{k_j - (\sigma-1)} \left(\frac{\varphi_{ji}}{\varphi_{jj}} \right)^{-k_j}}{\sum_{j \neq i} W_j N_j F_{ji} \frac{k_j}{k_j - (\sigma-1)} \left(\frac{\varphi_{ji}}{\varphi_{jj}} \right)^{-k_j}} \\ \hat{W}_i \hat{N}_i \sum_{j \neq i} \left(\frac{\hat{\varphi}_{ij}}{\hat{\varphi}_{ii}} \right)^{-k_i} s_{ij}^{x} &= \sum_{j \neq i} \hat{W}_j \hat{N}_j \left(\frac{\hat{\varphi}_{ji}}{\hat{\varphi}_{jj}} \right)^{-k_j} s_{ij}^{I}, \end{aligned}$$

where $s^I_{ij} = \frac{I_{ij}}{I_i} = \frac{W_j N_j F_{ji} \frac{k_j}{k_j-(\sigma-1)} \left(\frac{\varphi_{ji}}{\varphi_{jj}}\right)^{-k_j}}{\sum_{j\neq i} W_j N_j F_{ji} \frac{k_j}{k_j-(\sigma-1)} \left(\frac{\varphi_{ji}}{\varphi_{jj}}\right)^{-k_j}}$ is the share of country i imports from country j, and $s^x_{ij} = \frac{X_{ij}}{X_i} = \frac{F_{ij}\left(\frac{\varphi_{ij}}{\varphi_{ii}}\right)^{-k_i}}{\sum_{j\neq i} F_{ij}\left(\frac{\varphi_{ij}}{\varphi_{ii}}\right)^{-k_i}}$ is the share of exports of country i going to country j. Note that $va_{ii} = 1 - O_i$ and $va_{ij} = O_i s^x_{ij}$, where s^x_{ij} is the share of country i exports going to country j.

Labour market equilibrium for country i implies that

$$\begin{aligned} \hat{L}_i &= \frac{\hat{N}_i \left(\hat{\varphi}_{ii}\right)^{k_i}}{\widehat{(\chi_i)}} \\ \frac{\hat{W}_i}{\hat{P}_i} &= \left(\frac{\hat{N}_i \left(\hat{\varphi}_{ii}\right)^{k_i}}{\widehat{\chi}_i}\right)^{\frac{1}{\eta_i}} \frac{1}{\widehat{(1-t_i)}} \end{aligned}$$

and from the exit threshold condition, combined with the definition of the demand component and the labour market equilibrium it follows that

$$\hat{N}_i = \left(\hat{\varphi}_{ii}\right)^{-k_i - \frac{(\sigma-1)\eta_i}{1-\sigma+\eta_i}} \left(\widehat{\chi}_i\right)^{\frac{1-\sigma}{1-\sigma+\eta_i}} \left(\widehat{1-t_i}\right)^{\frac{(1-\sigma)\eta_i}{1-\sigma+\eta_i}} .$$

Inserting this into the equation for the real wage implies

$$\begin{aligned} \frac{\hat{W}_i}{\hat{P}_i} &= \left(\hat{\varphi}_{ii}\right)^{-\frac{(\sigma-1)}{1-\sigma+\eta_i}} \left(\widehat{\chi}_i\right)^{-\frac{1}{1-\sigma+\eta_i}} \left(\widehat{1-t_i}\right)^{-\frac{\eta_i}{1-\sigma+\eta_i}} \\ &= \left(\widehat{1-O_i}\right)^{\frac{1}{k_i}\frac{\sigma-1}{1-\sigma+\eta_i}} \left(\widehat{\chi}_i\right)^{-\frac{1}{1-\sigma+\eta_i}} \left(\widehat{1-t_i}\right)^{-\frac{\eta_i}{1-\sigma+\eta_i}} . \end{aligned}$$

For trade openness we obtain

$$\widehat{1-O_i} = \left(\widehat{\varphi_{ii}}\right)^{-k_i} \Rightarrow \widehat{O_i} = \frac{1}{O_i}\left(1 - \left(\widehat{\varphi_{ii}}\right)^{-k_i}(1-O_i)\right).$$

G.1.1 Numerical procedure

First, find $\{O_i\}$ for $i = 1, 2, .., J$ and $\left\{s^x_{ij}, s^I_{ij}\right\}$ for $i = 1, 2, .., J$ and $j \neq i$ in the data. Second, compute va_{ij} for all $i = 1, 2.., J$ and $j = i = 1, 2.., J$. Third, pick values for parameters σ and $\{k_i, \eta_i\}$ for $i = 1, 2, .., J$ from the literature. Fourth, assume exogenous changes in fiscal policies, i.e. $\widehat{\chi}_i$ and $\widehat{1-t_i}$ for $i = 1, 2, ..J$. Fifth, exploit the recursive structure of the model. Substitute the other equations into the balanced trade equation for country

i to obtain

$$\left(\sum_j \left(\frac{\hat{B}_i}{\hat{B}_j}\right)^{-\frac{k_i}{\sigma-1}} va_{ij}\right)^{\frac{\sigma-1}{k_i}\left(\frac{1}{\sigma}-\frac{\eta_i}{1-\sigma+\eta_i}\right)-1}$$

$$\left(\widehat{\chi}_i\right)^{\frac{1-\sigma}{1-\sigma+\eta_i}}\left(\widehat{1-t_i}\right)^{\frac{(1-\sigma)\eta_i}{1-\sigma+\eta_i}}\sum_{j\neq i}\left(\frac{\hat{B}_i}{\hat{B}_j}\right)^{-\frac{k_i}{\sigma-1}} s_{ij}^x$$

$$= \sum_{j\neq i}\left(\frac{\hat{B}_j}{\hat{B}_i}\right)^{\frac{1}{\sigma}-\frac{k_j}{\sigma-1}}\left(\sum_i \left(\frac{\hat{B}_j}{\hat{B}_i}\right)^{-\frac{k_j}{\sigma-1}} va_{ji}\right)^{\frac{\sigma-1}{k_j}\left(\frac{1}{\sigma}-\frac{\eta_j}{1-\sigma+\eta_j}\right)-1}$$

$$\left(\widehat{\chi}_j\right)^{\frac{1-\sigma}{1-\sigma+\eta_j}}\left(\widehat{1-t_j}\right)^{\frac{(1-\sigma)\eta_j}{1-\sigma+\eta_j}} s_{ij}^I,$$

which gives us $J-1$ independent equations in $J-1$ unknowns ($\hat{B}_1 = 1$ due to normalization - only relative $\hat{B}$'s matter - and one of the balanced trade conditions is redundant due to Walras' law). Finally, after solving these $J-1$ equations for $\hat{B}_i$ for $i = 2, 3, .., J$ the other variables follow easily.

G.1.2 Data and parameter values

In our theoretical framework, trade is in value added (no global value chains) and only within the private sector. We take this into account when 'calibrating' the model. We apply the OECD TiVA (trade in value added) database.[59] We consider the year 2015, which is the most recent year in the database, and use data for the 64 countries in the database and a residual 'rest of the world' country, i.e. a total of 65 countries ($J = 65$). We use 'origin of value added in final demand' and consider the aggregate across all sectors excluding the public sector, i.e. 'D84T98 - Public admin, education and health; social and personal services'. From these data we compute the value added shares, export shares and import shares. Next, we impose the shares on the model, cf. Appendix G.1. Empirically, trade is not balanced, and accordingly 'openness' is computed as the average of 'export openness' and 'import openness'. Value added shares are in the quantitative exercise computed as '1 minus openness' for the domestic share and as 'export share times openness' for other shares.

[59] We use the 2018 edition. See: https://www.oecd.org/sti/ind/measuring-trade-in-value-added.htm

Parameter values for $\sigma, \{\eta_j, k_j\}_{j=1}^{J}$ are picked from the literature. In particular, we follow Melitz and Redding (2015) and assume that $\sigma = 4$ and $k_j = 4.25$ for all j. The labour supply elasticities are also assumed identical across countries, i.e. $\eta_j = \eta$ for all j, and following Chetty et al. (2011) we set $\eta = 0.5$.

G.1.3 Quantitative experiment

The quantitative experiment is a harmonization of fiscal policies for the 19 Eurozone (EMU) countries. Data is extracted from EUROSTAT and to be consistent with the trade data the year 2015 is chosen.[60] The size of the public sector, t is our model, is captured by 'total general government expenditure' as a percentage of GDP.[61] The share of public expenditure devoted to transfers, s in our model, is 'social protection' relative to 'total general government expenditure'. The experiment is that each Eurozone country changes fiscal policy to that of the average of the Eurozone (i.e. a weighted average across the 19 countries), where $t_{EMU} = 0.484$ and $s_{EMU} = 0.415$.

G.2 No selection

Now consider the case with no selection. Countries have identical structures, but we allow for parametric asymmetries for all parameters except σ. Firm-level profits of a firm located in country i with productivity φ read

$$\pi_i(\varphi) = W_i^{1-\sigma} \varphi^{\sigma-1} \sum_j B_j \tau_{ij}^{1-\sigma} - W_i \sum_j F_{ij}$$

where $B_j \equiv E_j P_j^{\sigma-1} \left(\frac{\sigma}{\sigma-1}\right)^{-\sigma} \frac{1}{\sigma-1}$ is the demand component in country j, W_i is the wage in country i, τ_{ij} is the iceberg trade costs when exporting from country i to country j, F_{ij} is the fixed costs (in country i labour units) of serving country j for a firm in country i. To ease notation, let $\Phi_{ij} \equiv (F_{ij})^{1-\frac{k_i}{\sigma-1}} (\tau_{ij})^{-k_i}$, which is a trade costs aggregate for exports from country i to country j. The free-entry conditions read $\int_{\varphi_{\min,i}}^{\infty} \pi_i(\varphi)\, dH_i(\varphi) = W_i F_{\mathrm{Ei}}$ for all i, where F_{Ei} is the entry costs (in local labour units) in country i, and $H_i(\varphi)$ is the underlying productivity distribution in country i, and φ_{ij} is the productivity threshold for a firm in country i to serve country j. The free

[60] It is COFOG99 data.

[61] Alternatively, we could have used total (tax) revenue (income) or some average of the two. This matters mostly for countries with a significant deviation from a balanced public budget.

entry condition may be rewritten as

$$W_i^{-\sigma}\tilde{\varphi}_i \sum_j B_j \tau_{ij}^{1-\sigma} = F_{\text{Ei}} + \sum_j F_{ij}$$

where $\tilde{\varphi}_i \equiv \int_{\varphi_{\min,i}}^{\infty} \varphi^{\sigma-1} dH_i(\varphi)$. Imports and exports read

$$\begin{aligned} I_i &= \sum_{j\neq i} N_j \int_{\varphi_{\min,j}}^{\infty} E_i (P_i)^{\sigma-1} \left(\frac{\sigma}{\sigma-1}\frac{W_j \tau_{ji}}{\varphi}\right)^{1-\sigma} dH_j(\varphi) \\ X_i &= N_i \sum_{j\neq i} \int_{\varphi_{\min,i}}^{\infty} E_j (P_j)^{\sigma-1} \left(\frac{\sigma}{\sigma-1}\frac{W_i \tau_{ij}}{\varphi}\right)^{1-\sigma} dH_i(\varphi), \end{aligned}$$

where N_i is the number of firms in country i. Trade balance implies

$$N_i \sum_{j\neq i} B_j (W_i \tau_{ij})^{1-\sigma} \tilde{\varphi}_i = B_i \sum_{j\neq i} N_j (W_j \tau_{ji})^{1-\sigma} \tilde{\varphi}_j.$$

and openness reads

$$\frac{B_i (\tau_{ii})^{1-\sigma}}{\sum_j B_j (\tau_{ij})^{1-\sigma}} = 1 - O_i$$

Labour demand in country i reads

$$\begin{aligned} L_i^d &= L_i^{d,privat} + L_i^g \\ &= N_i \left(F_{\text{Ei}} + \sum_j F_{ij} + \sum_j \int_{\varphi_{\min,i}}^{\infty} \frac{\tau_{ij}}{\varphi} E_j (P_j)^{\sigma-1} (\tau_{ij} p_i(\varphi))^{-\sigma} dH_i(\varphi)\right) + L_i^g \\ &= N_i \left(F_{\text{Ei}} + \sum_j F_{ij} + \tilde{\varphi}_i (\sigma-1)(W_i)^{-\sigma} \sum_j B_j (\tau_{ij})^{1-\sigma}\right) + (1-s_i) t_i L_i^s \\ &= N_i \sigma \left(F_{\text{Ei}} + \sum_j F_{ij}\right) + (1-s_i) t_i L_i^s \end{aligned}$$

and labour market equilibrium implies for country i that

$$\begin{aligned} L_i &= \frac{N_i \sigma \left(F_{\text{Ei}} + \sum_j F_{ij}\right)}{1-(1-s_i) t_i} \\ \frac{W_i}{P_i} &= \left(\frac{F_{\text{Ei}} + \sum_j F_{ij}}{1-(1-s_i) t_i} \frac{N_i \sigma}{n_i (1-\mu_i)}\right)^{\frac{1}{\eta_i}} \frac{1}{1-t_i}. \end{aligned}$$

From the definition of the demand component, it follows that

$$\begin{aligned}
B_j &= E_j P_j^{\sigma-1}\left(\frac{\sigma}{\sigma-1}\right)^{-\sigma}\frac{1}{\sigma-1} = W_j L_j\left(1-\left(1-s_j\right)t_j\right)P_j^{\sigma-1}\left(\frac{\sigma}{\sigma-1}\right)^{-\sigma}\frac{1}{\sigma-1} \\
&= \left(\frac{W_j}{P_j}\right)^{1-\sigma}\left(W_j\right)^{\sigma} L_j\left(1-\left(1-s_j\right)t_j\right)\left(\frac{\sigma}{\sigma-1}\right)^{-\sigma}\frac{1}{\sigma-1} \\
&= \left(\frac{W_j}{P_j}\right)^{1-\sigma}\left(W_j\right)^{\sigma} N_j\sigma\left(F_{Ej}+\sum_i F_{ji}\right)\left(\frac{\sigma}{\sigma-1}\right)^{-\sigma}\frac{1}{\sigma-1}
\end{aligned}$$

Applying the exact hat algebra approach (see Dekle et al. (2008)), we have that

$$\widehat{N_i}\left(\widehat{W_i}\right)^{1-\sigma}\sum_{j\neq i}\widehat{B_j}\frac{B_j\left(\tau_{ij}\right)^{1-\sigma}}{\sum_{j\neq i}B_j\left(\tau_{ij}\right)^{1-\sigma}} = \widehat{B_i}\sum_{j\neq i}\widehat{N_j}\left(\widehat{W_j}\right)^{1-\sigma}\frac{N_j\left(W_j\tau_{ji}\right)^{1-\sigma}\tilde{\varphi}_j}{\sum_{j\neq i}N_j\left(W_j\tau_{ji}\right)^{1-\sigma}\tilde{\varphi}_j}$$

$$\begin{aligned}
\widehat{B_i} &= \widehat{N}_i\left(\widehat{W_i}\right)^{\sigma}\left(\frac{\widehat{W_i}}{\widehat{P_i}}\right)^{1-\sigma} \\
\frac{\widehat{W_i}}{\widehat{P_i}} &= \left(\frac{\widehat{N_i}}{1-\widehat{\left(1-s_i\right)t_i}}\right)^{\frac{1}{\eta_i}}\frac{1}{\widehat{1-t_i}} \\
1 &= \left(\widehat{W_i}\right)^{-\sigma}\sum_j\widehat{B_j}\frac{B_j\tau_{ij}^{1-\sigma}}{\sum_j B_j\tau_{ij}^{1-\sigma}}
\end{aligned}$$

Using that $s_{ij}^{x} = \frac{X_{ij}}{X_i} = \frac{B_j(\tau_{ij})^{1-\sigma}}{\sum_{j\neq i}B_j(\tau_{ij})^{1-\sigma}}$, $s_{ij}^{I} = \frac{I_{ij}}{I_i} = \frac{N_jW_j\tau_{ji}\tilde{\varphi}_j}{\sum_{j\neq i}N_jW_j\tau_{ji}\tilde{\varphi}_j}$ and $va_{ij} = \frac{B_j\tau_{ij}^{1-\sigma}}{\sum_j B_j\tau_{ij}^{1-\sigma}}$ to obtain

$$\begin{aligned}
\widehat{N_i}\left(\widehat{W_i}\right)^{1-\sigma}\sum_{j\neq i}\widehat{B_j}s_{ij}^{x} &= \widehat{B_i}\sum_{j\neq i}\widehat{N_j}\left(\widehat{W_j}\right)^{1-\sigma}s_{ij}^{I} \\
\widehat{W_i} &= \left(\sum_j\widehat{B_j}va_{ij}\right)^{\frac{1}{\sigma}}
\end{aligned}$$

Next, combine to obtain

$$\begin{aligned}
\widehat{B_i} &= \widehat{N}_i \left(\widehat{W_i}\right)^{\sigma} \left(\frac{\widehat{W_i}}{\widehat{P_i}}\right)^{1-\sigma} = \left(\widehat{N_i}\right)^{\frac{\eta_i-(\sigma-1)}{\eta_i}} \left(\sum_j \widehat{B_j} va_{ij}\right) \left(1-\widehat{(1-s_i)\,t_i}\right)^{\frac{\sigma-1}{\eta_i}} \left(\widehat{1-t_i}\right)^{(\sigma-1)} \\
\frac{\widehat{W_i}}{\widehat{P_i}} &= \left(\frac{\widehat{N_i}}{1-\widehat{(1-s_i)\,t_i}}\right)^{\frac{1}{\eta_i}} \frac{1}{\widehat{1-t_i}} \\
&= \left(\sum_j \frac{\widehat{B_j}}{\widehat{B_i}} va_{ij}\right)^{-\frac{1}{\eta_i-(\sigma-1)}} \left(1-\widehat{(1-s_i)\,t_i}\right)^{-\frac{1}{\eta_i-(\sigma-1)}} \left(\widehat{1-t_i}\right)^{-\frac{\eta_i}{\eta_i-(\sigma-1)}} \\
&= \left(\widehat{1-O_i}\right)^{\frac{1}{\eta_i-(\sigma-1)}} \left(1-\widehat{(1-s_i)\,t_i}\right)^{-\frac{1}{\eta_i-(\sigma-1)}} \left(\widehat{1-t_i}\right)^{-\frac{\eta_i}{\eta_i-(\sigma-1)}}
\end{aligned}$$

as

$$\widehat{1-O_i} = \frac{1}{\sum_j \frac{\widehat{B_j}}{\widehat{B_i}} va_{ij}}$$

Insert into the balanced trade condition to obtain

$$\begin{aligned}
&\left(\sum_j \frac{\widehat{B_j}}{\widehat{B_i}} va_{ij}\right)^{\frac{\eta_i}{(\sigma-1)-\eta_i}+\frac{1-\sigma}{\sigma}} \left(1-\widehat{(1-s_i)\,t_i}\right)^{\frac{\sigma-1}{(\sigma-1)-\eta_i}} \left(\widehat{1-t_i}\right)^{\frac{(\sigma-1)\eta_i}{(\sigma-1)-\eta_i}} \sum_{j\neq i} \frac{\widehat{B_j}}{\widehat{B_i}} s^x_{ij} \\
= &\sum_{j\neq i} \left(\sum_i \frac{\widehat{B_i}}{\widehat{B_j}} va_{ji}\right)^{\frac{\eta_j}{(\sigma-1)-\eta_j}+\frac{1-\sigma}{\sigma}} \left(1-\widehat{(1-s_j)\,t_j}\right)^{\frac{\sigma-1}{(\sigma-1)-\eta_j}} \left(\widehat{1-t_j}\right)^{\frac{(\sigma-1)\eta_j}{(\sigma-1)-\eta_j}} \left(\frac{\widehat{B_j}}{\widehat{B_i}}\right)^{\frac{1-\sigma}{\sigma}} s^I_{ij}.
\end{aligned}$$

Note that this equation and the equation for the real wage are similar to the case with selection for $k_i = \sigma - 1$ for all i. Next, follow the procedure outlined in the case with selection.