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JEL Classification: D40, D83, L10

Keywords: social media influencers, consumer search, product differentiation

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1 Introduction

Influencers on social media, such as Instagram, Twitter and Facebook, provide recommendations to their followers suggesting which products to buy. Influencers typically focus on one product market, such as cosmetics and personal care products, food and cooking, fashion and life-style or computer games. Although it is difficult to get objective data on the size of the industry, it is clear that influencer marketing is booming. In a 2018 article,[1] the New York Times estimated the industry to reach a turnover of 10 billion USD in 2020,[2] with the most successful influencers individually earning up to 1 million USD per post.[3] The market for influencers is so large that there are even intermediary firms specializing in advising firms which influencers to get involved in.[4]

This development raises important questions. How do influencers affect consumer decisions? Influencers typically only provide a recommendation or only show how they have enjoyed using a certain product. Why do consumers often follow a recommendation even if they know that the influencer gets paid (handsomely) for the posting? Are these recommendations truthful or are influencers just recommending what the highest paying firm wants them to post? Finally, do influencers have an incentive to make an effort to be informed? Thus, we are interested in whether influencers affect market outcomes and whether regulators should worry about the influence of influencers on market outcomes and the possibly false information they provide? Countries like Germany, the UK and the United States have initiated disclosure rules stating that social media posts should clearly mention if a post was paid for.[5]

This paper addresses these questions by focusing on how influencers affect consumer search. (Macro) Influencers, i.e., individuals with hundreds of thousands of followers, typically are people who are thought of as being able to know what will be trending or what many people care about when purchasing a product. People follow influencers and pay attention to their opinion as their preferences are correlated with the preferences of the influencer they choose to follow. As individuals have their own preferences and act according to these, after inspecting the good they may disagree with the recommendation of the influencer and not buy. A post by an influencer may, however, affect the order in which people search. We will show that this subtle effect on the search order has important implications for how firms price their product and where consumers buy.

[1] See, https://www.nytimes.com/2018/07/15/technology/online-stars-brands.html

[2] The estimate seems to be based on https://mediakix.com/blog/influencer-marketing-industry-ad-spend-chart/#gs.HbV2Xino where a range between 5 and 10 billion USD is given.

[3] See, e.g., https://www.webfx.com/influencer-marketing-pricing.html.

[4] See, e.g., https://mediakix.com/influencer-marketing-resources/influencer-marketing-platform-comparison

[5] Ershov and Mitchell (2020) study the effects of these regulations on the number of paid posts and their content.

We build on the seminal work by (Wolinsky, 1986), and model a market of monopolistic competition where consumers engage in sequential search to discover the products firms offer and the prices they charge. In our baseline model, the only change we make to that framework is that an influencer is sampling some of the firms' products and recommends one of them. Firms and followers observe the influencer's post. Depending on whether or not they are recommended, firms decide on their pricing; consumers decide where to start searching and how to continue the search process if they are not satisfied with the outcome of their first search.

Our first result shows that if influencers honestly recommend the firm they like best, consumers follow that recommendation and start searching at the recommended firm, despite this firm setting higher prices than its competitors (and consumers expecting them to do so). If upon visiting the firm, the consumers find out they do not have a high match value, they continue to search among the non-recommended firms. Being recommended is good news for a firm as consumers rationally believe the chances this firm produces a product they like are higher, boosting the firm's demand. In response, the firm sets higher prices, but overall welfare and consumer surplus is higher as consumers more easily find the firm that in expectation is delivering higher value.

We next show that the results of the baseline model continues to hold if firms offer financial contracts and compete for the influencer recommending them. Even if influencers are completely money-driven and recommend the firm from whom they expect to generate most revenue, influencers continue to be truthful and recommend the firm they value most and consumers value the recommendation in that they first search the recommended firm. Consumer surplus and overall welfare is higher even if the recommended firm charges higher prices. There are two parts to the underlying mechanism. First, competing firms offer financial contracts to the influencer that include a positive commission for every consumer that buys. The key idea here is that a commission leverages the influencer's information as she only accepts the contract if she expects followers to actually purchase the product. Second, as with positive commissions in a symmetric equilibrium, the influencer makes more revenue by recommending a firm that she expects to sell more products, the influencer has a natural incentive to recommend the product she likes best as this is the best indicator her followers like the product. It is clear that this result gets reinforced if we endow the influencer with an intrinsic motivation to be honest.

The above results take for granted that the influencer samples two or more products. We finally investigate the influencer's incentives to acquire information so she not only gives a truthful, but also an informative recommendation. We show that provided that the influencer's search cost is not too high, the influencer has an incentive to acquire information and give informative recommendations. The reason is simple: The more informative the recommendation, the more likely the follower will stop searching after having visited the recommended firm at the first visit. As a more informative recommendation, generates a higher number of expected sales, the influencer is willing to make an effort to get informed.

Even if our paper and its results are cast in terms of social media influencers, the results are equally relevant to advisors that give advice at a more "individual" level, i.e., if an advisor gives different advice to different individuals. What is important for our results to apply is that the advisor does not know the precise ranking of different products of an individual and that the individual can inspect the product herself to learn more whether the recommended product suits her. Although these conditions do not apply to pure credence goods, we believe that advice in some financial and health markets fits the two mentioned features. An advisor may know certain characteristics (age, gender, income, family situation) of an individual she advises and base the advice on these observed characteristics, but there typically remains enough room for individual preferences to differ from those of the average individual with the same characteristics. Moreover, through information that is available via other sources (e.g. online information about the effects of medicines or financial products) individuals may find out more information about whether the product that is recommended suits her.

The paper is related to several strands of literature. Dating back to early contributions by Owen (1977) and Pauly (1979) there is a large literature on (financial or health) advice and the way commissions and kickbacks that are paid by firms may shape the advice that is given by intermediaries. For more recent contributions, see, e.g., Inderst and Ottaviani (2012a) and Inderst and Ottaviani (2012b). The typical market feature that is considered in that literature is one of credence goods: consumers cannot assess the value of a product (or competing products) and completely rely on the information that is provided by the intermediary or expert.[6] An important question in that literature is whether advisors should be forced by regulation to disclose their financial relations with the industry in order to alleviate their biases. Inderst and Ottaviani (2012a,b) argue that an expert advisor may well provide honest advice if she does not only care about the commissions and kickbacks, but also to some extent about her advice being appropriate for the individual in question. Our contribution to this literature is that when consumers can inspect the product themselves by paying a search cost, the advisor gives honest and informed advice even if she is only interested in monetary payoffs.

A recent paper by Teh and Wright (Forthcoming) studies similar issues as ours, but arrives at very different conclusions. They assume that the advisor knows exactly the match values of a consumer with each and every firm. The only thing the advisor does not know is an individual shift parameter, which impacts whether or not a consumer

[6]There is also quite a large literature on information intermediaries focusing on the vertically differentiated products (see, e.g., Biglaiser (1993) and Lizzeri (1999).

wants to buy any product. As, like in our paper, the advisor has an incentive to recommend honestly, this assumption of knowing exact and personal match values implies that in any symmetric equilibrium consumers never search beyond the first firm on their recommendation list, giving the firm that is recommended monopoly power and making consumers worse off relative to a market without influencers. In contrast, in our model the influencer does not have perfect knowledge of the preferences of her followers and only gets a signal that correlates with individual preferences. This implies that some consumers will search on after visiting the recommended firm, giving recommended firms much less market power. In addition, the many firms that are not recommended continue charging their "normal" prices imposing a further competitive constraint on the recommended firms. As explained above, this makes that consumers are better off in markets with influencers as the latter help them to concentrate their search efforts on products they are likely to like.

The role of influencers is also studied from a network perspective (see, e.g., (Fainmesser and Galeotti, 2016; Galeotti, 2010; Chen et al., 2018; Fainmesser and Galeotti, 2020) and Chen et al. (2018)). This literature builds on the literature on word-of-mouth communication in networks (see, e.g., Campbell (2013)) modeling influencers as consumers that have many more network connections than others. Connections matter as consumption is characterized by network effects: the utility of buying the good is increasing in the number of neighbours consuming the good. Knowing the network structure, firms in this literature can increase demand by targeting these influencers and offering them better deals. The papers characterize the resulting price discrimination and how it is influenced by the network structure. This literature takes it that everyone is an influencer, some more so than others. This may well capture some aspects of the impact of what are known to be micro- or nano-influencers, i.e., individuals that have a few thousands of followers or even less. These individuals may have to get the product themselves, probably at a reduced price, and others are directly (and positively) affected by their consumption. Many of the more macro-influencers, i.e., individuals with millions of followers, seem to play a different role, however, as there is a clear asymmetry between them influencing others (and knowing that) and they themselves not being influenced by the decisions that their followers take. These macro-influencers are regarded by others as "experts", knowing better what products consumers like or what the fashion will be, independent of whether or not they consume the product themselves.

The paper is also related to many of the recent articles on consumer search, especially in three different directions: (i) how search order affects firms' pricing strategies, (ii) quality and service provision and search, and (iii) observational learning in search markets. The typical result in the search literature on how search order affects prices is that the firm that is searched first charges lower prices than their competitors (see, e.g. (Armstrong et al., 2009; Armstrong, 2017; Haan and Moraga-González, 2011).[7] This is in contrast to one of the main results in our paper, where the recommended firm is searched first, but sets higher prices than competitors. The main reason for these different results is that in the previous literature, the search order is determined exogenously, for example by the order in which firms are listed on a search platform, or determined by advertising budgets, but unrelated to a firm's expected quality. In our framework, the reason for searching the recommended firm first is that it is expected to have higher quality and for the average consumer these expectations are realized. Knowing this, the recommended firm has an incentive to charge higher prices. This observation also links the paper to the search literature on quality and service provision (see, e.g. Shin (2007) and (Janssen and Ke, 2020)). An important result in that literature is that despite the cost of doing so firms may provide service to consumers making them value the consumption of the good more, even though other firms may free-ride on this service provision. Like in our paper, despite the higher prices at a service providing firm, the expectation of service provision may affect the search order. In these papers, higher prices are, however, completely driven by the cost difference of service provision and the mechanism we focus on in the current paper where recommendations affect consumers perceived valuations is absent. (Garcia and Shelegia, 2018) study how observational learning affects search and pricing in markets where consumers' valuations are correlated and consumers observe the purchase of a single predecessor. Like in our paper, search is affected by observations consumers make (or, in our setting recommendations they get) about other individuals that have somewhat similar preferences. Our paper studies, however, the effect of how one individual's (the influencer) action affects search and pricing and this action is unaffected by the prices firms charge. In such an environment, observational learning and the implications it has on pricing are not relevant.

Finally, the paper relates to the literature on referral marketing[8] (see, e.g., Schmitt et al. (2011), Mayzlin et al. (2014), Pei and Mayzlin (2019)) that studies how a firm can significantly increase referrals from word-of-mouth communication. That literature is mostly empirical, however, and does not address the mechanism by which influencers' recommendations affect search and the associated welfare effects on markets.

The rest of the paper is organized as follows. Sections 2 and 3 deal with the baseline model and the main results related to pricing and welfare. Section 4 extends the model to address the issue of paid recommendations and why consumers should still trust the influencer's recommendation. Section 5 then extends the model to demonstrate how the influencer is willing to make a costly effort to learn her match value for different

[7]A notable exception is Casner (2020) who, in the context of a platform with vertical differentiation, observes that a firm that is searched first may charge higher prices.

[8]There is some literature on referrals in economics (e.g., Garicano and Santos (2004) but that focuses more on a different problem, namely what are the incentives of a professional to refer a potential client to another professional who is better equipped to help the client if he could also keep the client for himself

firms, while Section 6 concludes with a discussion how our analysis may be relevant to advice at a more individual level. All withheld proofs can be found in Appendix A.

2 Baseline Model

The market is comprised of a unit mass of firms,[9] a mass of L consumers, and an influencer. We denote by v_i a representative consumer's value for firm i and by $\hat{v}_i$ the influencer's value for firm i. These share a a log-concave joint density and are strictly affiliated: $g(v_i, \hat{v}_i)g(v'_i, \hat{v}'_i) > g(v'_i, \hat{v}_i)g(v_i, \hat{v}'_i)$ for $v_i < v'_i$ and $\hat{v}_i < \hat{v}'_i$ in the support. The marginal distribution is denoted $G(v_i)$ with density $g(v_i)$. Throughout the paper, we illustrate our results using the joint density function $g(v_i, \hat{v}_i) = \alpha(2v_i - 1)(2\hat{v}_i - 1) + 1$, where the parameter α, lying in the unit interval, measures the degree of affiliation. This family of joint density functions has the nice property that for any $\alpha \in [0, 1]$ the marginal distributions of v_i and $\hat{v}_i$ are uniformly distributed on the interval $[0, 1]$, while for $\alpha = 0$ consumers' and the influencer's values are independent of each other, while the degree of affiliation is increasing in α.

The influencer examines $k \geq 2$ of the products in the market and recommends one of them. In the baseline model, we assume that the influencer honestly recommends the firm in the sample that provides her the highest match value. We introduce a contracting stage in Section 4 where we show why the influencer would do so even if he recommends the firm that pays him most. In section 5 we endogenize the influencer's decision regarding how many products to examine.

Firms learn whether or not they will be recommended and depending on the influencer's decision they set their prices p_R and p_i, for the recommended and non recommended firms respectively, to maximize expected profits. For notational simplicity, we normalize firms' cost to be equal to 0. The influencer then issues the recommendation to consumers, indicating the firm they recommend along with the price charged by this firm.

Consumers are initially uninformed of their match values with firms and can only learn them through costly sequential search. Each search comes at a search cost $s > 0$. Consumers have perfect recall when searching. The timing of the interaction in the baseline model is as follows. First, Nature determines the values of all agents. The influencer randomly observes her values for k firms and chooses which of these k firms to recommend. Consumers are unaware of their values until the moment they visit a firm. Second, firms observe the influencer's recommendation and set their prices. Third, consumers observe the influencer's recommendation and the related price and

[9] The continuum of firms is a common assumption in the search literature and is used not to have to worry about returning consumers, which is known to give rise to technical complications (see, e.g., Anderson and Renault (1999)). Nevertheless, if the number of firms is large, but finite, we expect our qualitative results to continue to hold.

commence their search.

Throughout the paper we focus on symmetric Perfect Bayesian Equilibria where firms choose their strategies to maximize expected profits given their information and consumers choose an optimal sequential search strategy. The Prékopa-Leindler inequality ensures the existence of equilibrium (see Lemma A.1).

We will now discuss some of the features of the baseline model. First, although formally we consider a market with one influencer who is followed by all consumers, the results continue to hold if there is a finite number of influencers where different groups of consumers follow different influencers with consumers following the influencer that fits their preferences best. The current analysis does not cover the case, however, where consumers get competing recommendations.

Second, one way to conceive of the correlation between consumer and influencer match values is that a consumer's value for firm i is composed of a common factor θ_i that is shared among all consumers (that follow the influencer) and an idiosyncratic factor η_{ij} that differs between consumers so that the value of consumer j for firm i is $v_{ij} = \theta_i + \eta_{ij}$. The influencer's valuation can be conceived in a similar way so that the correlation between the influencer's and consumers' valuations arises in a natural way. We will use this formulation in the concluding section when discussing the role of individual advice.

Third, we assume that consumers have the same search cost whether or not they follow the recommendation. One may argue, however, that social media influencers typically make it easy for followers to follow upon the recommendation, for example by inserting a link to the firm's website in their post. As we show that consumers follow the recommendation even if the search cost of doing so is not smaller than the cost of searching other firms, our results continue to hold if the cost of following the recommendation is smaller.

Fourth, we recognize that recommendations may come in different forms and that our model pertains to influencers recommending a particular product, and that in other cases an influencer places more general ads. Most recommendations on a platform like Instagram seem to have the form we model here.[10]

Fifth, in many instances the influencer may not mention the price of the product along with her recommendation. The main advantage of the influencer "advertising" the recommended firm's price is that it commits the firm to charging that price. Our way of modeling has the advantage that it gets rid of uninteresting equilibria where consumers do not visit a recommended firm as they expect it charges a very high price and the

[10]See, e.g., https://www.instagram.com/p/BYYr9JMgNMt/ for cameras, https://www.instagram.com/p/CFxQz4sFSo-/ for gaming controllers, https://www.instagram.com/p/CJ4JJQylI_W/ for hair products, https://www.instagram.com/p/B7mr6DAp4XU/ for tire cleaners or https://www.instagram.com/p/CLE7HNGBST5/ for protein powder.

recommended firm charges such a price as anyway no consumer is going to visit. Without pre-commitment of prices, our analysis continues to hold, however, and in particular it captures all equilibria where consumers first search the recommended firm. An alternative way to achieve this would be to assume that following a recommendation comes at no cost (or a cost close to 0) so that following the recommendation is a "no regret" option.

Sixth, changing the timing of the model to account for the fact that firms have already chosen prices before a firm is recommended and allow all firms to adapt their prices given the information who is recommended, does not change the results as the non-recommended firms would not want to change their price choices.

Finally, the fact that we assume k to be known to firms and consumers is of no importance to the results. The only thing that matters is that firms and consumers believe that $k \leq 2$ so that the recommendation is somewhat informative. In Section 5 we explicitly analyze the incentives of the influencer to acquire information.

3 Search and Pricing

We now show that consumers will follow the recommendation despite the fact that the recommended firm charges a higher price than the other firms. Moreover, consumers are on average better off and total welfare is also increasing because of the presence of the influencer. To do so, we first characterize the optimal search strategy of consumers.

The optimal search strategy for consumers is to follow Pandora's rule (Weitzman, 1979). Firm i's *reservation price* r_i is the highest price at which a consumer is willing to first inspect the firm rather than take an outside option of zero outright. Pandora's rule dictates that at each decision node, a consumer takes the best option among previously inspected firms if that has a higher net value than the net value $r_i - p_i$ of all uninspected firms; otherwise he should continue searching the firm offering the highest uninspected net value. Standard considerations imply that the reservation prices for the recommended and non-recommended firms, denoted by r_R and r, are implicitly defined by

$$\int_{r_R}^{\bar{v}} (1 - G(v|R))\, dv = s = \int_{r}^{\bar{v}} (1 - G(v))\, dv,$$

where $G(v_i|R)$ denotes a consumer's posterior over his match value with the recommended firm, with posterior density

$$g(v_i|R) = \frac{\Pr(R|v_i)g(v_i)}{\Pr(R)}. \tag{1}$$

Thus, the search-order depends in part on the relationship between r_R and r. As there are a large number of firms, the match values across firms are independent following the recommendation. Letting K denote the set of sampled firms and $\hat{v}_{-i} \equiv \max\{\hat{v}_j\}_{j \in K \setminus \{i\}}$, the chance that $i \in K$ is recommended when v_i is the consumer's match value is $\Pr(R|v_i) = E[1 - G(\hat{v}_{-i}|v_i)]$ where the expectation is taken over $\hat{v}_{-i}$. Since match values are strictly affiliated $1 - G(\hat{v}_{-i}|v_i)$ is strictly increasing in v_i (Milgrom, 1981). Consequently, the ratio $g(v_i|R)/g(v_i)$ is also strictly increasing, implying the posterior $G(v_i|R)$ has likelihood ratio dominance - and thus, hazard rate and first-order stochastic dominance (Shaked and Shanthikumar, 2007) - over the prior $G(v_i)$. So, we have the following Lemma:

Lemma 1. *For any search cost s the recommended firm has a larger reservation price than the other firms, i.e., $r_R > r$.*

Since the influencer reveals the recommended firm's price p_R, charging too high a price will dissuade consumers from following the recommendation. Denoting the price consumers conjecture non-recommended firms will charge by p, consumers first search the recommended firm if $p_R \leq r_R - r + p$, where the RHS is larger than p. As the recommended firm faces expected profit $p_R(1 - G(r - p + p_R|R)) > 0$ when $p_R \leq r_R - r + p$ and zero profit otherwise, it is clear that $p_R \leq r_R - r + p$ and that consumers first search the recommended firm, even though p_R may be larger than p.[11]

Concentrating now on the optimal recommended price, it easily follows that there are two candidates: either the recommended firm charges the interior optimal price $p'_R < r_R - r + p$, which standard considerations (Wolinsky, 1986; Anderson and Renault, 1999), reveal to be equal to

$$p'_R = \frac{1 - G(r - p + p'_R|R)}{g(r - p + p'_R|R)}, \tag{2}$$

or he charges the upper bound $p''_R = r_R - r + p$ that still draws customers. Thus, the recommended firm charges $p_R \equiv \min\{p'_R, p''_R\}$.

As after visiting a non-recommended firm a consumer never strictly prefers to then visit the recommended firm nor to make a purchase at a previously inspected firm, upon being visited, offering value v_i firm i charging p_i makes a sale if and only if $v_i - p_i \geq r - p$. From standard calculations (Wolinsky, 1986; Anderson and Renault, 1999), the equilibrium price for non-recommended firms equals

$$p = \frac{1 - G(r)}{g(r)}. \tag{3}$$

[11]In an alternative set-up where the influencer does not announce the price of the recommended firm and the firm is not committed to the price he charges there is a third possibility, namely that the only equilibrium is such that consumers do not follow the recommendation as they rationally expect that $p_R > r_R - r + p$. In that uninteresting case the influencer is ineffective in influencing consumer behavior and the market outcome is unaffected.

It is not difficult to see that in equilibrium, the recommended price is strictly larger than the non-recommended price. The next Proposition states this result. If the boundary solution is relevant, it immediately follows from lemma that $p''_R = r_R - r + p > p$. But this also holds if the recommended price is equal to p'_R. The main reason is that the recommended firm faces a higher demand and because demand is "behaving normally" it reacts by setting higher prices. More technically, a consequence of the hazard rate dominance is that

$$\frac{1 - G(r|R)}{g(r|R)} > \frac{1 - G(r)}{g(r)}.$$

which is exactly saying that the marginal profit of the recommended firm in the interior solution evaluated at $\tilde{p}_R = p$ is positive.

Proposition 1. *In equilibrium, consumers commence their search at the recommended firm, while this firm charges a higher price than the firms that are not recommended, i.e,* $p_R > p$*. In addition, the presence of the influencer increases total welfare and industry profits.*

Total welfare increases as consumers are (at least) weakly better off and the influencer and industry profits are also higher as the non-recommended firms are equally well off, while the recommended firm is strictly better off. For consumers to *strictly* benefit from the presence of the influencer, the firm must charge the interior optimum with $r_R - p_R > r - p$, so that the price increase does not dominate the increased anticipated match value. Intuitively, for this to be the case the expected demand curve facing the recommended firm must not be too inelastic relative to the demand facing other firms.

Figure 1 depicts how the price p_R of the recommended firm and consumer surplus depend on the number k of firms the influencer samples and on the degree affiliation α. Both α and k can be seen as measures of how informative the recommendation is for consumers, either through the direct affiliation between values or because of the larger sample size. One can clearly see that the more informative the influencer's recommendation the higher the price the recommended firm will charge as he clearly wants to reap the benefits of the recommendation. If $\alpha = 0$, we are in the Wolinsky equilibrium where the recommended firm and the non-recommended firms charge the same price. As the price that is charged by the non-recommended firms is independent of α, the price increase can also be re-interpreted as the price differential between the recommended and non-recommended firms. Despite the higher recommended price, consumers still want to follow the recommendation and are still better off because of the way their first search is directed to the firm that is more likely to deliver a good match.[12] One can also see that the effects are quantitatively substantial: comparing relatively uninformative outcomes with informative equilibrium outcomes shows that

[12] The latter result does not always need to be true, however. In particular, if the informativeness of the recommendation is so strong that all parties grow arbitrarily certain that the consumers' value is in a small neighborhood of $\bar{v}$, then the recommended firm can extract all of the surplus. For example, if consumers'

the recommended price can be in the order of 10 per cent higher, while consumer surplus may increase even by 25 per cent. The reason is that even if the price to be paid is higher, consumers are likely to get a much better match value on their first search.

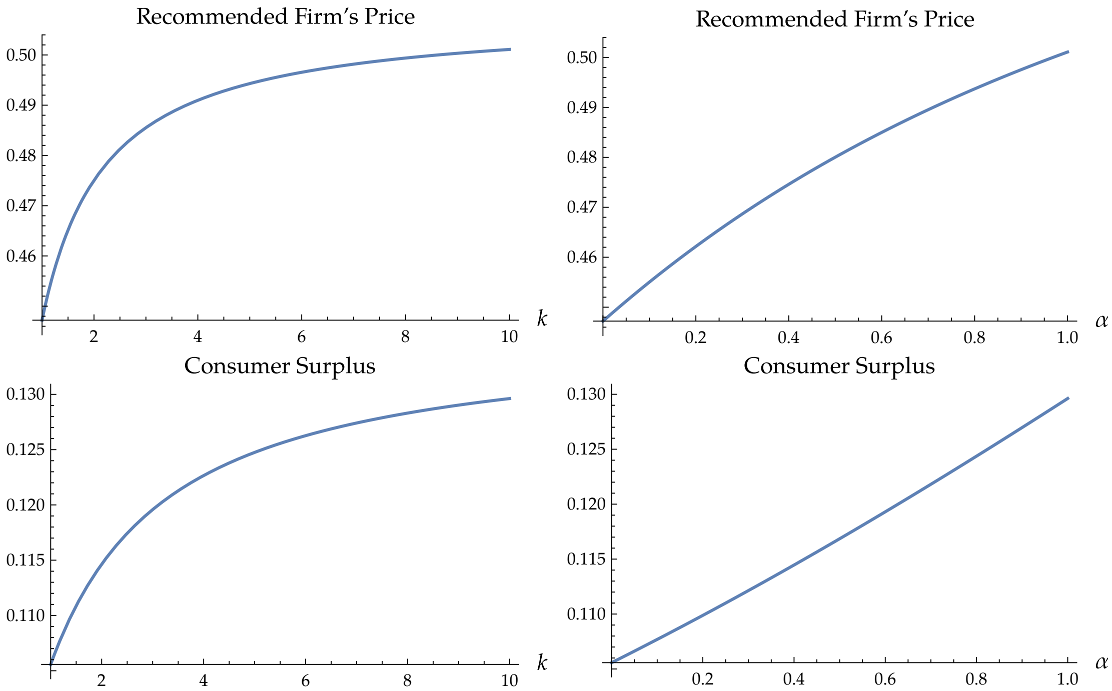

Figure 1: The different figures plot the price charged by the recommended firm and consumer surplus when varying the number k of firms sampled by the influencer (for $\alpha = 1$) and the degree of affiliation α between consumers and the influencer (for $k = 10$). In all figures $s = 0.1$.

4 Paying for recommendations

The welfare gains in the previous section rely on the influencer providing an honest recommendation. In reality, influencers often receive financial compensation from the firm they recommend. Especially in relation to macro influencers with millions of followers, firms often compete with one another to get an explicit endorsement of an infuencer. To study how this affects the incentives of the influencer, the reasons of consumers to (not) trust the recommendation and market outcomes more widely, we now consider that the k firms that are reviewed by the influencer compete with each other for the influencer to recommend their product. Although influencers may also care for their reputation or have an intrinsic motivation for being honest, we abstract in this section from these considerations to understand the role of paying for

and influencer's values are perfectly correlated and uniformly distributed on $[0, 1]$, then letting the sample size k grow large, the expected demand facing the recommended firm becomes increasingly inelastic as it becomes increasingly likely that consumers' value is in a small neighborhood of 1, and the price eventually approaches p_R''. In this case, there is a finite sample size beyond which consumers are worse off when k increases beyond that.

recommendations and consider that influencers are only purely financially motivated. We show that even if that is the case, the influencer recommends the product she thinks is best.

The timing of the interaction in this section proceeds as follows. After Nature determines the values of all agents, the influencer begins by sampling $k \geq 2$ firms. Firms that are sampled simultaneously offer a contract. A contract comprises a nonnegative lump sum payment and a commission rate on sales revenue generated by the followers of the influencer.[13] After examining the contracts and the products, the influencer is able to predict which price a firm will set if it is recommended and what her revenue will be from recommending the firm. The influencer accepts the contract generating most revenue. If multiple contracts are equally desirable, one is randomly selected, each with equal probability. Firms then set prices, the influencer issues the recommendation, and consumers commence their search. Consumers know the structure of the game and know the influencer recommends what generates most revenue for her, but do not know the details of the contract between the influencer and the recommended firm.

Our main result in this section is that if firms offer positive commissions, then the influencer recommends honestly as we assumed in the previous section. The argument is relatively straightforward. As firms are ex ante identical and compete to be recommended, they offer contracts where the fixed and variable components are identical. Among these contracts that are offered in an equilibrium, the influencer's payment is increasing in the number of sales he expects to generate and this is increasing in his own match value.

Proposition 2. *In any symmetric equilibrium where firms offer contracts with positive commissions, the influencer recommends the seller with the highest match value.*

As the influencer is honestly recommending the product with the highest match value even if he does not have an intrinsic motivation for being honest, it is clear that adding such a motivation to the analysis, or adding a similar reputational concern would not affect the result qualitatively. What is affected is the equilibrium payment the influencer receives. The more he cares about providing an honest recommendation, the more firms realize their payment is less important for being recommended and competition is therefore less severe, resulting in lower commission fees.

As it continues to be true that the recommended firm will charge $p_R \leq r_R - r + p$, the welfare results of the previous section remain valid. Comparing markets with and without financial contracting, it is obvious that the influencer benefits, the recommended firm's profit decreases and the non-recommended firms are unaffected. As the recommended firm typically will charge a higher price because of the positive commission,

[13] https://later.com/blog/affiliate-marketing-for-influencers/ as -unlike in other settings- it is not more difficult to enforce than a commission on revenues as the influencer and firm agree on the price that is charged as it is part of the influencer's announcement.

consumers are also worse off.

One may wonder whether other equilibria exist where firms do not offer positive commissions, but instead only offer a flat fee that is independent of sales. We will now argue that in any equilibrium satisfying a reasonable refinement this is not the case. To make the argument we should show that starting at a conjectured equilibrium where firms do not offer commissions, some firm would expect to strictly benefit by deviating to another contract that includes a commission. This depends on the firm's beliefs about the influencer's match value conditional on her accepting the alternative contract. If a firm believes only influencers with a high enough match value would accept the alternative contract, then the deviation would be gainful. Below, we show that such a belief follows from considerations that are akin to the Intuitive Criterion.[14] Accordingly, we show that any equilibrium that satisfies this Intuitive Criterion type of reasoning has to have positive commissions.

To this end, let us define x_i to be a contract lying off the equilibrium path for firm i and x^*_{-i} to be a vector of contracts on the equilibrium path for the other firms. Moreover, let the influencer's vector of match values be given by $\hat{\mathbf{v}}$ and, for a given strategy by consumers, let $U^*(x^*_{-i}, \hat{\mathbf{v}})$ denote the highest expected payoff to the influencer from accepting one of the equilibrium contracts in x^*_{-i} and $U(\tilde{p}, x_i, \hat{\mathbf{v}})$ the influencer's expected payoff from accepting the contract x_i when the firm subsequently charges $\tilde{p}$.[15] In the spirit of the Intuitive Criterion, we will say that accepting contract x_i is **equilibrium dominated*** for $(x^*_{-i}, \hat{\mathbf{v}})$ if

$$U^*(x^*_{-i}, \hat{\mathbf{v}}) > \sup_{\tilde{p}} U(\tilde{p}, x_i, \hat{\mathbf{v}}). \tag{4}$$

Definition 1 (Intuitive Criterion Type Reasoning)**.** *An equilibrium satisfies Intuitive Criterion type reasoning if at the information set following the influencer accepting a contract x_i lying off the equilibrium path, firm i assigns probability zero to any $(x^*_{-i}, \hat{\mathbf{v}})$ for which accepting contract x_i is equilibrium dominated*, provided that it is not dominated for all $(x^*_{-i}, \hat{\mathbf{v}})$.*

Thus, a deviating firm should believe that his contract is only accepted by the influencer if it has the chance of offering the influencer a higher profit than she would receive by accepting one of the contracts offered in equilibrium. Using this definition, the following proposition argues that in any equilibrium where firms' beliefs are reasonable in the above sense, firms offer contracts with a positive commission.

[14]Note Cho and Kreps (1987) developed the Intuitive Criterion in the context of a Sender-Receiver game where the receiver infers the private information of the Sender on the basis of the latter's deviation. Here, we do not have a traditional Sender-Receiver game and it is the uninformed firm that deviates and makes an inference about the information of the influencer conditional on the latter accepting the deviation contract. As we will indicate below, the type of reasoning we follow is very much in the spirit of the Intuitive Criterion.

[15]Note that this pay-off implicitly also depends on consumer beliefs about the match value of the recommended product if it is sold at a price that is different from the equilibrium price they expect.

Proposition 3. *Any equilibrium satisfying Intuitive Criterion type reasoning where consumer's follow the recommendation has the recommended firm offering a contract with a positive commission.*

The key idea that is used in the proof is that a commission leverages the influencer's information as with positive commissions she only accepts a contract if she expects enough followers to actually purchase the product. In this sense, the incentives of firms and influencer are alligned and firms are happy to offer a positive commission if she expects more sales.

5 Sampling Sellers

So far, we have taken for granted that the influencer inspects at least two firms and demonstrated in this context that an influencer provides honest recommendations even if she only has a financial interest in providing recommendations. In this section, we inquire into the incentives of the influencer to acquire information and to provide not only an honest, but also an *informed* recommendation. To do so, we think of the influencer as an agent who also has a search cost, which we denote by c to distinguish it from the consumer search cost s, and sequentially inspects the products different firms offer.[16] Consumers and firms only observe the recommendation the influencer provides, but not how many products she has inspected, *i.e.*, the actual search process is privately observed by the influencer only. The timing of the interaction is similar to the previous section; the only difference is that instead of the influencer sampling an exogenous number of firms, she engages in optimal sequential search. To do so, all firms offer a contract to the influencer before the influencer starts searching.

It is clear that equilibria exist where the influencer does not acquire information. As the influencer cannot credibly convince firms and consumers that they do provide information, there always exist equilibria where firms and consumers believe that influencers do not make an effort and randomly choose their recommended firm. Given these beliefs, the influencer does not have an incentive to acquire information.

More interesting is the question whether there exist equilibria where the influencer does make an effort and acquire information. Whether or not a recommendation is informative depends on the search strategy followed by the influencer. Typically, for a given value of c a search strategy is a cut-off strategy saying to continue searching if, and only if, the highest match value that is observed up to a particular moment during the search process is smaller than a certain cut-off value, denoted by $v^*(c)$. We will say

[16] As influencers are typically more professionally engaged than individual consumers, their search cost may be significantly smaller. Moreover, influencers may receive free review copies lowering their search cost even lower. In our model, it is trivial to se that all firms would have an incentive to offer free review copies so we do not model this option explicitly.

that the recommendation is informative if the influencer follows a search strategy that is such that $v^*(c) > \underline{v}$.

The main question is whether informative equilibria exist, or whether the influencer has an incentive to deviate and not make a search effort. Another question is how these informative equilibria, if they exist, compare in welfare terms to the non-informative equilibria. The next proposition argues that informative equilibria exist if the influencer's search cost is not too large, while when they exist they are Pareto superior to uninformative equilibria.

Proposition 4. *For influencer search costs c below some threshold value $c_0 > 0$, there exists an informative equilibrium where the influencer recommends the first product that is searched and has a match value that is larger than $v^*(c)$, consumers follow the recommendation and sellers offer a positive commission rate. Relative to an uninformative equilibrium, consumers and influencers are better off in an informative equilibrium, while firms are indifferent.*

The reason why the influencer does not want to shirk and not inquire information is essentially that an informative recommendation increases the expected number of sales relative to an uninformative equilibrium and thereby increases her revenues in any equilibrium with positive commission rates. If the influencer's search cost is small enough, the cost of getting informed is smaller than the marginal increase in revenues, making it individually optimal for the influencer to acquire information even if consumers and firms do not observe the search effort.

Figure 2 illustrates the Proposition. It displays the recommended firm's price, the influencer's expected payoff, and the consumer surplus in the case where the joint density function of match values is given by $g(v, \hat{v}) = (2v - 1)(2\hat{v} - 1) + 1$ and the consumer's search cost is set at 0.1. As all firms compete to be recommended, their profit equals 0. If the influencer's search cost is too high, the influencer will not make a search effort and always recommend the first sampled firm. Thus, in all the plots, the horizontal dashed line represents the outcomes in the uninformative equilibrium (which exists for all values of c). The Figure clearly shows that consumers and influencer are much better off in an informative equilibrium, and that the effect is stronger, the lower the influencer's search cost (or the higher her cut-off value v^*), despite the recommended firm setting higher prices. The Figure also shows that the difference between informative and uninformative equilibia can be quite significant with effects for consumer surplus and the price of the recommended firm in the same order as we have seen in Section 3,while the effect on the influencer's pay-off being potentially even stronger and in the order of 50 per cent.

There is one feature of the Figure that requires more explanation, which is the wedge between the price of the recommended firm and consumer surplus between the two equilibria at the cutoff value c_0. To understand this wedge, we have to explain

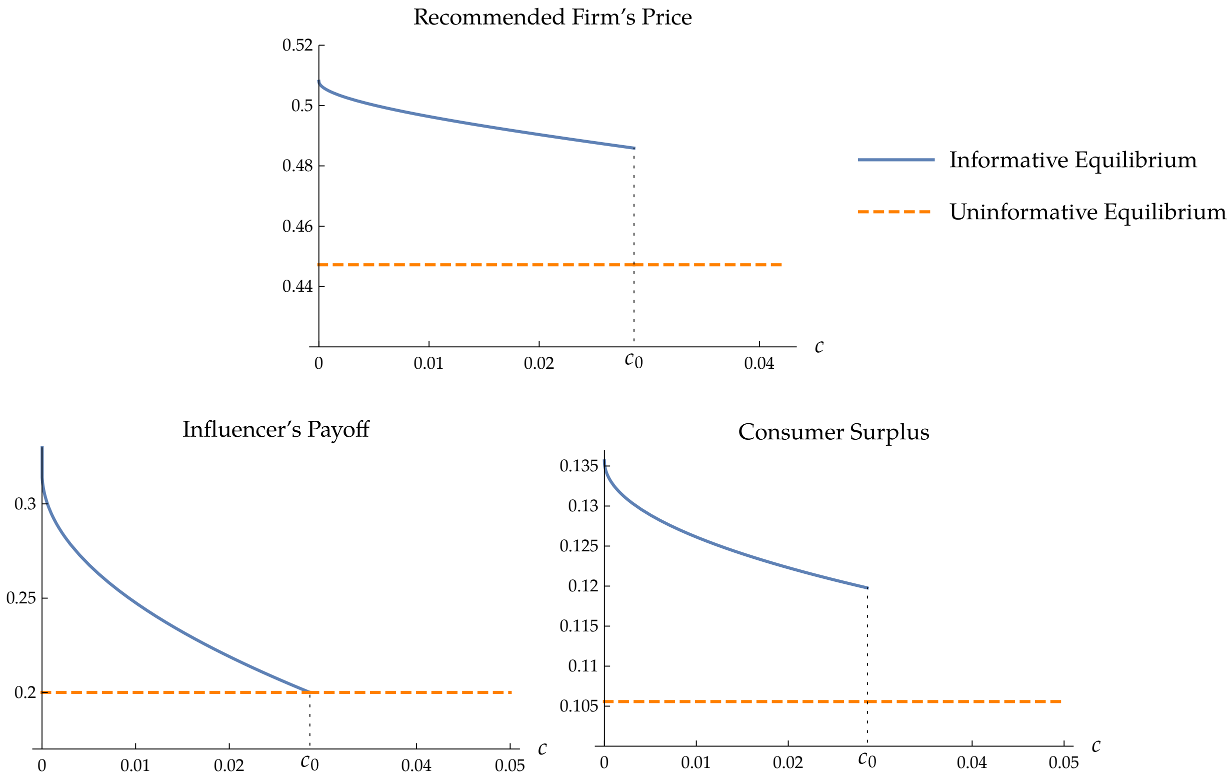

Figure 2: The different figures plot the price charged by the recommended firm, the influencer's expected payoff, and consumer surplus as a function of the influencer's search cost c and the consumers' search cost set at 0.1 for both the informative and uninformative equilibrium.

the equilibrum construction in some more detail. To this end, let $\pi(\tilde{v}^*)$ be a firm's expected revenue when the influencer's match value $\hat{v}$ is known to exceed the cutoff $\tilde{v}^*$. Also, define $\underline{\pi}(\tilde{v}^*)$ to be the influencer's expected payoff of accepting the contract offered by a firm where her value is precisely equal to the cut-off $\tilde{v}^*$. Because firms engage in Bertrand-type competition to get their product recommended, it follows that $\underline{\pi}(\tilde{v}^*) < \pi(\tilde{v}^*)$. The cutoff $\tilde{v}^*$ is defined as the match value where the influencer is indifferent between continuing searching for a firm with a higher match value and accepting the contract of the firm offering her an expected revenue associated with the match value $\tilde{v}^*$, i.e.,

$$\underline{\pi}(\tilde{v}^*) = (1 - G(\tilde{v}^*))\pi(\tilde{v}^*) + G(\tilde{v}^*)\underline{\pi}(\tilde{v}^*) - c. \tag{5}$$

Given the equilibrium contracts and a search cost c, the maximal pay-off the influencer can get by searching is given by $\pi(v^*) - c/(1 - G(v^*))$. To characterize an equilibrium, two types of deviations are crucial to consider. First, the influencer should not be better off by not searching and simply recommend a randomly selected firm. Second, firms should not have an incentive to offer a different contract. Unlike the influencer's pay-off a firm's deviation pay-off is, however, not easy to define as it depends on a firm's beliefs about how the influencer will react to this deviation (will she inspect the product, and if so will she recommend it?), which in turn depends on the price the firm will choose (which in turn depends on the firm's belief about the match value of the influencer). For every equilibrium, one can define a set of contracts that are such that the influencer is willing to inspect them and accept. The cut-off value c_0 is defined in such a way that for ever $c \leq c_0$ there is no contract in this set that also gives the deviating firm an incentive to deviate. The proof shows that at c_0 the informative equilibrium still has an interior solution $v^* > \underline{v}$ so that at c_0 the influencer's threshold match value does not continuously transition to $\underline{v}$. This explains the wedge at this point between the price of the recommended firm and consumer surplus in an informative equilibrium and an uninformative equilibrium. The proof also shows that at c_0 the influencer does not want to deviate by accepting an equilibrium contract and not search.

6 Discussion and Conclusion

In this paper we have argued that social influencers play a beneficial role in directing consumers' search efforts towards products they are likely to like best. This conclusion holds even if influencers are paid by firms for their recommendation and influencers do not have an intrinsic motivation for providing honest recommendations. What is important for this result to hold is that consumers can walk away from the recommendation if after having discovered their own match value for the good, they think they have better options. Thus, our results apply to search markets, not in pure credence goods markets where consumers do not have an option to verify whether the product is a good match for them.

To compare the role an influencer plays in our paper to the role of an expert advisor in a credence goods market, in the spirit of Inderst and Ottaviani (2012a,b) and extending our analysis, one could imagine that a consumer j's match value with firm i is comprised of three independent components: $v_{ij} = \theta_i + \gamma_i + \epsilon_{ij}$, where θ_i represents a common component that can be identified by the consumer upon inspection, γ_i represents the "credence good" component that the consumer cannot observe due to a lack of expertise and ϵ_{ij} captures the purely private component, only observable to the consumer. Whether or not our results in this paper apply to this extended setting depends on whether the expert can clearly identify what the consumer will and will not observe when inspecting the good. If the expert can clearly separate θ_i from γ_i, then as one might expect from Inderst and Ottaviani (2012a,b), the credence good component is disregarded when making a recommendation if the expert does not have an intrinsic concern for getting the recommendation right. However, if the expert cannot clearly identify what the consumers may be able to find out when inspecting the good, e.g., she only observes $\theta_i + \gamma_i$, and not the individual components, then our results apply and the expert recommends honestly which product is best for the consumer.

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A Appendix: Proofs

Before we prove the propositions, we state and prove a technical lemma that turns out to be useful in the proofs. To save on notation, this appendix withholds the firm-specific subscript for match values.

Lemma A.1. *If $g(v, \hat{v})$ is log-concave, then $g(v|R)$ and $g(v|\hat{v} \geq v^*)$ are log-concave in v.*

Proof. Upon sampling k firms and recommending one with the highest match value, the distribution of the influencer's match value with the recommended firm becomes $G(\hat{v})^k$ with density $kg(\hat{v})G(\hat{v})^{k-1}$.

$$g(v|R) = k \int g(v, \hat{v}) G(\hat{v})^{k-1} \mathrm{d}\hat{v} \tag{6}$$

is thus log-concave in v since the well-known Prékopa-Leindler inequality establishes that the product and marginals of log-concave functions are log-concave. For this same reason

$$g(v|\hat{v} \geq v^*) = (1 - G(v^*))^{-1} \int_{\hat{v}^*}^{\bar{v}} g(v, \hat{v}) \mathrm{d}\hat{v} \tag{7}$$

is likewise log-concave in v. $\square$

Proof of Proposition 2. Supposing all firms offer fixed fee a and commission rate $\theta > 0$, the influencer's expected payment from accepting a contract from a firm with whom her match value is $\hat{v}$ is $a + \theta \cdot L \cdot p_r \cdot P(v \geq r - p + p_r | \hat{v})$. By Theorem 5 in Milgrom and Weber (1982), the chance of a purchase is $P(v \geq r - p + p_R | \hat{v})$ and thus the influencer's expected payment is increasing in the match value $\hat{v}$. Hence, honestly recommending the firm with the highest match value maximizes the influencer's expected payoff. □

Proof of Proposition 3. Toward a contradiction, suppose consumers follow the recommendation in an equilibrium without commissions. Since competition for the influencer's services is Bertrand, it is clear that the equilibrium lump sum almost surely equals the expected profit upon being recommended. As the influencer breaks indifference uniformly, the expected profit is simply $\pi_0 \equiv \frac{[1-G(r)]^2}{g(r)}$. Let $\mathcal{P}$ denote the non-empty set of prices at which consumers follow the recommendation in the conjectured equilibrium which are determined by consumer's equilibrium beliefs.

To show that this cannot be supported as an equilibrium satisfying Intuitive Criterion type reasoning, suppose a firm deviates to an alternative contract that cuts the fixed fee down to $(1 - \theta) \cdot \pi_0$, but now issues a commission rate of $\theta > 0$. Accepting this new contract is equilibrium dominated* for any match value satisfying

$$\pi_0 > (1 - \theta)\pi_0 + \theta\pi^\dagger(\hat{v}), \quad \text{with} \quad \pi^\dagger(\hat{v}) \equiv \sup_{\tilde{p} \in \mathcal{P}} \tilde{p}(1 - G(r - p + \tilde{p}|\hat{v}),$$

or put simply $\pi_0 > \pi^\dagger(\hat{v})$. As $\pi^\dagger(\hat{v})$ is continuous, strictly increasing, and $\pi^\dagger(\underline{v}) < \pi_0 < \pi^\dagger(\bar{v})$,[17] there exists an interior $\hat{v}_\theta$ such that this new contract is equilibrium dominated* if and only if $\hat{v} < \hat{v}_\theta$. Hence, the equilibrium price charged at this node $p_\theta \in \mathcal{P}$ is optimal with respect to some beliefs for the firm μ taking full support in $[\hat{v}_\theta, \bar{v}]$. Notice that[18]

$$\int p_\theta(1 - G(r - p + p_\theta|\hat{v}) \mathrm{d}\mu(\hat{v}) \geq \pi^\dagger(\hat{v}_\theta) \tag{8}$$

so that $p_\theta(1 - G(r - p + p_\theta|\bar{v})) > \pi_0$. But because $p_\theta(1 - G(r - p + p_\theta|\hat{v})$ is continuous in $\hat{v}$, this strict inequality must hold for a set of match values with positive measure. Thus, the influencer's equilibrium strategy must involve accepting this contract when $\hat{v}^*_\theta \leq \hat{v}$ for some $\hat{v}^*_\theta < \bar{v}$. It then follows that deviating to this contract yields the firm an expected profit of

$$\max_{\tilde{p}} \left[(1 - G(r - p + \tilde{p}|\hat{v} \geq \hat{v}^*_\theta) - \pi_0\right] (1 - \theta) \left(1 - G(\hat{v}^*_\theta)\right) > 0, \tag{9}$$

[17] Letting $\underline{p}$ be optimal price when $\hat{v} = \underline{v}$ in the event consumers follow the recommendation regardless of the price: $\pi^\dagger(\underline{v}) \leq \underline{p}(1 - G(r - p + \underline{p}|\underline{v})) < \underline{p}(1 - G(r - p + \underline{p})) \leq \pi_0$. An analogous proof gives $\pi^\dagger(\bar{v}) > \pi_0$.

[18] For any price $\tilde{p}$: $\int \tilde{p}(1 - G(r - p + \tilde{p}|\hat{v})\mathrm{d}\mu(\hat{v}) \geq \tilde{p}(1 - G(r - p + \tilde{p}|\hat{v}_\theta)$; hence, taking the supremum of both sides over $\tilde{p} \in \mathcal{P}$ preserves the inequality. Further, the supremum of the left side must correspond to the maximum since a best response must be well-defined in equilibrium.

contradicting the existence of the conjectured equilibrium. □

Proof of Proposition 4. For the proof, we shall construct an equilibrium in which all firms offer identical contracts to the influencer who then sequentially inspects products, stopping her search when her match value with a given firm exceeds some threshold. To simplify notation, let $p_{\tilde{v}^*}$ be the optimal price and $\pi(\tilde{v}^*)$ the ensuing expected profit when the influencer's match value is known by both the firm and consumers to exceed the cutoff, i.e., $\hat{v} \geq \tilde{v}^*$, and consumers break indifference by following the recommendation. Define $\underline{\pi}(\tilde{v}^*) \equiv p_{\tilde{v}^*}(1 - G(r - p + p_{\tilde{v}^*}|\tilde{v}^*))L$ to be the influencer's expected payoff when her value is precisely at the cutoff.

An equilibrium where the influencer sequentially inspects firms and accepts a contract if her match value with the firm exceeds $\tilde{v}^*$ yields the influencer an expected payoff of

$$a + \theta\pi(\tilde{v}^*) - \frac{c}{P(\hat{v} \geq \tilde{v}^*)}, \tag{10}$$

where a is some fixed fee. As an equilibrium contract cannot deliver negative expected profit to the firm and firms compete a la Bertrand to be recommended, the contract and cutoff that maximize the influencer's expected payoff sets $a = (1 - \theta)\pi(v^*)$ with cutoff $v^* = v^*(c) \in \arg\max_{\tilde{v}^*} \pi(\tilde{v}^*) - \frac{c}{P(\hat{v} \geq \tilde{v}^*)}$. The first-order condition for an interior optimum satisfies

$$\pi(v^*) - \underline{\pi}(v^*) - \frac{c}{P(\hat{v} \geq v^*)} = 0. \tag{11}$$

As the boundaries exhibit $\pi(\underline{v}) - \underline{\pi}(\underline{v}) > 0$ and $\lim_{\tilde{v}^* \to \bar{v}} \pi(\tilde{v}^*) - \underline{\pi}(\tilde{v}^*) = 0$, there is an interior solution if the search cost is small enough, i.e., $c < \pi(\underline{v}) - \underline{\pi}(\underline{v})$. Define $u = u(c) \equiv \pi(v^*) - \frac{c}{P(\hat{v} \geq v^*)}$ as the influencer's expected pay-off.

Let us show that there is an equilibrium in which firms offer a contract so that the influencer indeed wishes to follow this cutoff v^*. In particular, we need to find a contract $x^* = (a^*, \theta^*)$ satisfying $a^* + \theta^*\underline{\pi}(v^*) = u$.

For each contract $x = (a, \theta)$ define v_x^* to be the cutoff the influencer would use if she were to inspect a firm offering this contract when her continuation value is u. Formally, v_x^* either equates $a + \theta\underline{\pi}(v_x^*) = u$ if there is an interior solution, or $v_x^* = \underline{v}$ if the left exceeds the right for all values, or $v_x^* = \bar{v}$ if the right exceeds the left for all values. For the influencer to be willing to follow the cutoff v^* when $a = (1 - \theta)\pi(v^*)$, there must exist a $\theta \in (0, 1]$ equating $(1 - \theta)\pi(v^*) + \theta\underline{\pi}(v^*) = u$. From the first-order conditions, the contract $x^* = (0, 1)$ achieves the optimal cutoff. We shall show that when the influencer's search cost is sufficiently small there is an equilibrium in which all firms offer x^*.

Notice that when firms offer x^* a decrease in the search cost strictly increases the payoff to searching as $c' < c$ implies $\max_{\tilde{v}^*} \left(\pi(\tilde{v}^*) - \frac{c'}{P(\hat{v} \geq \tilde{v}^*)}\right) \geq \pi(v^*(c)) - \frac{c'}{P(\hat{v} \geq v^*(c))} >$

$\pi(v^*(c)) - \frac{c}{P(\hat{v} \geq v^*(c))}$. As $\pi(\underline{v})$ is the influencer's expected pay-off in an uninformative equilibrium where firms and consumers optimally react to the uninformative recommendation, accepting a contract x^* without inspection yields a payoff less than or equal to $\pi(\underline{v})$. As a small enough search cost c' guarantees $\pi(v^*(c)) - \frac{c'}{P(\hat{v} \geq v^*(c))} > \pi(\underline{v})$ and the payoff to searching is continuous, there exists a cost c_0 satisfying $\pi(v^*(c_0)) - \frac{c_0}{P(\hat{v} \geq v^*(c_0))} = \pi(\underline{v})$. For all $c < c_0$, the influencer's best response to all firms offering x^* is to search. For the remainder of the proof, fix $c < c_0$.

Suppose the influencer plays a best response to all firms charging $p_{v_x^*}$ when their contract x is accepted and consumers following the recommendation. Consider the best response to a contract profile of the form (x_i, x^*_{-i}) wherein firm i offers $x_i = (a, \theta)$ and the remaining firms offer x^*. Let us show that any x_i inducing the influencer to either immediately accept i's contract or to strictly prefer to start her search with i, the firm must obtain a negative expected profit. Accepting i's contract without inspection yields the influencer the expected payoff $a + \theta p_{v_x^*}(1 - G(r - p + p_{v_x^*}))$. If accepting x_i without inspection is strictly preferred to searching, then

$$a + \theta\pi(\underline{v}) \geq a + \theta p_{v_x^*}(1 - G(r - p + p_{v_x^*})) > u > \pi(\underline{v}).$$

The first inequality is due to $\pi(\underline{v})$ being the maximal profit for an uninformative recommendation, the second inequality captures the influencer's strict preference to immediately accept x_i, and the final equality holds when $c < c_0$. Hence, subtracting $a + \theta\pi(\underline{v})$ from the above provides $0 > (1 - \theta)\pi(\underline{v}) - a$, i.e., firm i's expected profit is negative.

If instead the influencer strictly prefers to start her search with i then

$$(a + \theta\pi(v_{x_i}))(1 - G(v_{x_i})) - c + G(v_{x_i})u > u.$$

But from this and the definition of u it follows that

$$a + \theta\pi(v_{x_i}) - \frac{c}{1 - G(v_{x_i})} > u \equiv \max\left\{\tilde{a} + \tilde{\theta}\pi(\tilde{v}^*) - \frac{c}{1 - G(\tilde{v}^*)} : (1 - \tilde{\theta})\pi(\tilde{v}^*) - \tilde{a} \geq 0\right\}.$$

Therefore $(1 - \theta)\pi(v_{x_i}) - a < 0$ and so firm i has a negative expected payoff. Therefore, when the influencer best responds to the pricing strategy $p_{v_x^*}$ and all other firms offer x^*, firm i's best response is to also offer x^*.

Thus, we can construct an equilibrium where firms believe the influencer to have inspected and followed the cutoff strategy v_x^* at a firm's information set following the acceptance of contract x, so that it is optimal for them to charge $p_{v_x^*}$. Moreover, we can specify consumers to believe that the infuencer follows cutoff v^* so that following the recommendation is a best response. Notice that these beliefs are consistent with play on the equilibrium path.

In terms of welfare, consumers are strictly better off in an informative equilibrium relative to an uninformative equilibrium whenever they strictly prefer to follow the recommendation and are indifferent otherwise. The recommended firm is indifferent as it receives zero profit in both equilibria. The influencer is strictly better off in an informative equilibrium than an uninformative equilibrium since $\pi(v^*(c)) - \frac{c}{P(\hat{v} \geq v^*(c))} > \pi(\underline{v})$ holds when $c \leq c_0$. $\square$