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JEL Classification: C72, D62, D63, Z13

Keywords: Value of a language, Communicative benefits, Measure, axioms, Characterization

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The measurement of the value of a language*

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February 3, 2021

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1 Introduction

The latest version of the Ethnologue database (www.ethnologue.com) contains more than seven thousand distinct languages spoken all over the world. As there are only a few hundred nations, it follows that a large number of them, if not most, are multilingual. The distribution of linguistic skills in multi-lingual societies is crucial to explain opportunities and challenges, both at the individual and societal level. Moreover, the linguistic landscape is not static and could be altered by the presence of economic and cultural incentives to acquire languages in addition to one's mother tongue.

Economists and social scientists alike have long been concerned with studying the impact of acquiring foreign languages on economic outcomes (see, for instance, Ginsburgh and Weber (2020) and the literature cited therein). From the individual perspective, each agent must evaluate the benefits of learning other languages and weigh them against the cost of language acquisition. Proficiency in languages has important consequences on earnings. Job opportunities are more often open to applicants who speak several languages, though not all languages are identical in that respect.[1] Another important aspect of language acquisition for individuals is the challenges of migration. Once in the new country, or even prior to that, the migrant will have to learn (or at least improve the knowledge of) the local language to get a job, and thus be faced with a learning decision. The importance of linguistic skills for migrants' labor-markets is confirmed by the literature on patterns of language acquisition by immigrants in various countries (see Chiswick and Miller (2014) for a survey of this growing brand of the literature).

Linguistic policies in multilingual societies, such as the selection of official languages (e.g., Pool, 1991) or the choice of the language of school instruction (e.g., Ginsburgh and Weber, 2020), are of great importance and might have profound economic and societal implications. A decision on which official documents, collective goods or public services are offered in each language, and

[1]Dustmann and Fabbri (2003) showed that, in the UK, language proficiency has a positive effect on employment probabilities, and lack of English fluency leads to earning losses. Bleakley and Chin (2004) found a significant positive effect of English proficiency on wages among adults who immigrated to the United States as children. Ginsburgh and Prieto-Rodriguez (2007) showed that a second language (in most cases, English) raises wages in the range of five to fifteen percent in Austria, Finland, France, Germany, Greece, Italy, Portugal and Spain. Albouy (2008) found substantial wage differentials and, therefore, incentives, for a French-speaking Canadian to learn English, while the reverse is not true.

the subsidization (partial or full) of the acquisition of some languages may impact the patterns of language acquisition and, consequently, the economic development of the society.

Both the individual decision of learning a new language and the implementation of a linguistic policy require a proper cost-benefit analysis, which relies on the measurement of the benefits associated to a given language (or set of languages). The analysis of the latter, which is our proposal to assess the value of a language, is the objective of this paper.

The formal approach to the benefits of language learning was initially developed in the seminal paper by Selten and Pool (1991), which subsumes both private monetary rewards and 'pure communicative' benefits of exposure and access to different cultures. The more people an individual can speak with, the more advantageous the learning of other languages may seem. As Lazear (1999) points out, "the incentives are greater for each individual to learn the majority language when only a few persons in the country speak his or her native language." The benefits could be related to expanded employment opportunities and higher monetary returns, but being immersed into a different culture and gaining unfiltered access to its history, arts, and literature in the original language could be viewed as important by some. To make the Selten and Pool approach operational, Church and King (1993) examined a model where the communicative benefits are simply represented by the number of people an individual can communicate with.[2]

The intuitive appeal of the notion of communicative benefits calls for the search for its fundamentals. In this paper we axiomatically analyze the problem of evaluating the communicative benefits of languages. In principle, this analysis can be carried out from the individual perspective (i.e., how much benefit an agent obtains from knowing a set of languages) or from the social one (i.e., how much benefit society as a whole derives from the individual knowledge of each language). In the main body of the paper we adopt this second approach, while offering a discussion on how the results can be adapted to the individual learning decisions.

In our model, a linguistic landscape of a society is described by a matrix with dichotomous entries, depending on whether the corresponding individual (row) speaks the corresponding language (column) or not. The aim is to derive the communicative benefits of each of the languages in society. Instead of assuming a specific functional form, as it is done in the existing literature, we approach the problem by introducing several axioms that formalize appealing

[2]See also Ginsburgh et al., (2007), Gabszewicz et al., (2011) and Athanasiou et al., (2016).

principles from a normative perspective. Our first two axioms refer to the principle of *impartiality*, one of the most basic principles in the theory of justice (e.g., Moreno-Ternero and Roemer, 2006). A *monotonicity* axiom and an *invariance* axiom reflect how the communicative benefits should react to certain changes of the linguistic landscape. Lastly, we introduce a *consistency* axiom, another notion with a long tradition of use in normative economics (e.g., Thomson, 2012). Our main result states that the combination of these five axioms characterizes a family of communicative benefit functions, assigning to each language its value by means of a two-step procedure: First, it identifies the groups of agents that can communicate in that language. Second, each group is assigned an aggregate communicative size-dependent value, which is evenly divided among the languages of communication of this group.

We believe our work could be useful in a variety of policy implications. For instance, in a multilingual society, public authorities might be interested into promoting multilingualism upon subsidizing the acquisition of one (or some) specific language(s). Which should be the chosen language(s)? We believe this decision could be driven by the communicative benefit functions derived in this paper, which allows to rank all the existing languages. In a cost-benefit framework, one could actually use the cost per unit of (communicative) benefit gained, akin to what the so-called cost-per-QALY-gained concept conveys in the economic evaluation of health care programs (e.g., Neumann et al., 2014).[3]

Moreover, our analysis has potential empirical applications. Our family of communicative benefit functions could be used, for instance, to measure the value of different languages in the (pre and post-Brexit) European Union (e.g., Ginsburgh et al., 2017), the choice of official languages (e.g., in South Africa, see Ginsburgh and Weber, 2011), as well as in multilingual countries where the linguistic policies were linked to economic development (Easterly and Levine, 1997) or devastating conflicts (Castañeda-Dower et al., 2017). Our measures might also be relevant to study the welfare effect of language barriers in communication (e.g., Giovannoni and Xiong, 2019) or the effects of communicative benefits in models of language dynamics (e.g., Abrams and Strogatz, 2003).

The rest of the paper is organized as follows. In Section 2, we introduce the model and definitions. In Section 3, we present our characterization result. In Section 4, we conclude. Some proofs are relegated to the Appendix.

[3]See Hougaard et al. (2013) for an axiomatic characterization of QALYs as a measure of health outcomes.

2 The model

Let $\mathcal{N}$ be the universal set of agents and $\mathcal{L}$ the universal set of languages. Both sets can be finite or infinite. A particular situation is a triple (N, L, A), where $N \subseteq \mathcal{N}$ is a finite set of agents, $L \subseteq \mathcal{L}$ is a finite set of languages, and A is a $0-1$ matrix that summarizes the multilingual reality of the society N over the set of languages L. Formally, $a_{il} = 1$ if individual $i \in N$ speaks language $l \in L$, and $a_{il} = 0$ otherwise. We thus assume that there is no distinction between speaking a language well or not, or between native and non-native languages. Let $\mathcal{S}$ be the set of possible situations. We define, for each situation $(N, L, A) \in \mathcal{S}$, the set of speakers of a given language $l \in L$ by $N_A(l)$; i.e., $N_A(l) = \{i \in N \,|\, a_{il} = 1\}$. Given a pair of situations $(N, L, A), (N, L', A') \in \mathcal{S}$, with $L \cap L' = \emptyset$, we define the union of them as a new situation $(N, L \cup L', A \cup A')$ in the natural way. We denote, for each situation $(N, L, A) \in \mathcal{S}$ and for each $L' \subset L$, the situation restricted to L' by $(N, L', A|_{L'})$, where $A|_{L'}$ is the resulting matrix from A after dismissing all the columns from $L \setminus L'$.

Given a situation $(N, L, A) \in \mathcal{S}$, we define a communicative benefit function $\phi_{(N,L,A)} : L \to \mathbb{R}_+$ that associates, for each language $l \in L$, a non-negative real number indicating the communicative benefits of this language in this situation.[4] We define $\phi \equiv \bigcup_{(N,L,A)\in\mathcal{S}} \phi_{(N,L,A)}$. For normalizing purposes, we assume that there exists a situation $(N, L, A) \in \mathcal{S}$ and a language $l \in L$ such that $\phi_{(N,L,A)}(l) = 0$. Let $\Phi : \mathcal{S} \to \mathbb{R}_+$ be such that $\Phi(N, L, A) = \sum_{l\in L} \phi_{(N,L,A)}(l)$. This function Φ indicates the total communicative benefits of all languages in a society.[5]

Axioms

Our goal is to derive communicative benefit functions axiomatically. For that matter, we impose some axioms that we find compelling.

Our first axiom, *Anonymity*, is a standard formalization of the principle of impartiality, which refers to the fact that the identity of each agent should not matter in the evaluation. To define it formally, let $\Pi^{\mathcal{N}}$ be the class of bijections from $\mathcal{N}$ into itself. For each $(N, L, A) \in \mathcal{S}$, and each $\pi \in \Pi^{\mathcal{N}}$, let $\pi(N, L, A) = (\pi(N), L, (a_{\pi(i)l})_{(i,l)\in N\times L})$, where $\pi(N) = \{i \in \mathcal{N} : \pi^{-1}(i) \in N\}$.

[4] We denote by $\mathbb{N}$ the set of natural numbers, by $\mathbb{R}$ the set of real numbers, and by $\mathbb{R}_+$ the set of non-negative real numbers.

[5] Note that we assume an unweighted aggregation of each language's communicative benefits, thus reflecting an impartiality judgement to be properly formalized next.

Anonymity: For each $(N, L, A) \in \mathcal{S}$, each $\pi \in \Pi^{\mathcal{N}}$ and each $l \in L$, $\phi_{(N,L,A)}(l) = \phi_{\pi(N,L,A)}(l)$.

Anonymity requires that ethically irrelevant information as the name of each of the agents should be excluded from the evaluation process. The next axiom, *Equal Treatment of Equal Languages*, expressed in very weak terms, establishes a similar idea for languages.

Equal Treatment of Equal Languages: For each $(N, L, A) \in \mathcal{S}$ and each pair $l, l' \in L$ such that $a_{il} = a_{il'}$ for each $i \in N$, $\phi_{(N,L,A)}(l) = \phi_{(N,L,A)}(l')$.

Equal Treatment of Equal Languages implies that if we have a situation in which two languages have the same set of speakers, then their communicative benefits are the same. Observe that this property is weaker than the classical Neutrality property, which says that a permutation of languages permutes analogously the languages' communicative benefits.[6]

To introduce the next axiom, we define the following concept of inclusion of situations: for each pair $(N, L, A), (N, L', A') \in \mathcal{S}$, we say that $(N, L, A) \subseteq (N, L', A')$ whenever for each $l \in L$, there exists $l' \in L'$ such that $a'_{il'} = 1$ for each $i \in N$ with $a_{il} = 1$.[7] Then, the axiom of *Inclusion Monotonicity* says the following:

Inclusion Monotonicity: For each pair $(N, L, A), (N, L', A') \in \mathcal{S}$ such that $(N, L, A) \subseteq (N, L', A')$, $\Phi(N, L', A') \geq \Phi(N, L, A)$.

The *Inclusion Monotonicity* axiom incorporates two ideas of weak monotonicity (adding languages is weakly good for the total communicative benefits of society and adding speakers to a language is also weakly good for the same purpose) and an idea of supermodularity (it is not worse for the total communicative benefits of society to have a common language than a set of languages whose union of speakers are the same as the ones of the common language).

The next axiom is called *Irrelevance of Non Speakers.* It says that the communicative benefits of a new language in a situation should not depend on the individuals that do not speak that

[6] The formal definition of Neutrality is: For each $(N, L, A) \in \mathcal{S}$, each possible bijection μ from $\mathcal{L}$ to itself, and each $l \in L$, $\phi_{(N,L,A)}(l) = \phi_{\mu(N,L,A)}(\mu(l))$, where $\mu(N, L, A) = (N, \mu(L), (a_{i\mu(l)})_{(i,l)\in N\times L})$, with $\mu(L) = \{l \in \mathcal{L} : \mu^{-1}(l) \in L\}$. Note that, although Neutrality implies *Equal Treatment of Equal Languages*, the opposite is not true: *Equal Treatment of Equal Languages* cannot relate the communicative benefits of a language in two different situations in which that language has the same set of speakers, while Neutrality can.

[7] Note that it is possible that $(N, L, A) \subseteq (N, L', A')$ with $L \not\subseteq L'$ and even with $L' \subset L$.

language.

Irrelevance of Non Speakers: For each $(N, L, A), (N, L, A'), (N, \{l\}, A'') \in \mathcal{S}$, with $l \notin L$, such that $a_{ik} = a'_{ik}$ for each $i \in N_{A''}(l)$ and $k \in L$,

$$\phi_{(N, L\cup\{l\}, A\cup A'')}(l) = \phi_{(N, L\cup\{l\}, A'\cup A'')}(l).$$

The final axiom is called *Null Agent Consistency.* It says that the addition of an individual that does not speak any language has no impact on the total communicative benefits of the situation. This is the unique axiom of our set of properties that is referred to a variable population context and implies that the communicative benefit function is an absolute function, and not a relative one.

Null Agent Consistency: For each pair $(N, L, A), (N \cup \{i\}, L, A') \in \mathcal{S}$ such that $i \notin N$, and for each $l \in L$, $a'_{il} = 0$ and $a'_{jl} = a_{jl}$ for each $j \in N$,

$$\Phi(N \cup \{i\}, L, A') = \Phi(N, L, A).$$

A family of communicative benefit functions

We now describe a specific family of communicative benefit functions. Each of them assigns a value to each language l in a situation (N, L, A) by means of the following procedure. First, it focuses only on the groups of agents that can communicate between them in this language: these are the subgroups $M \subseteq N$ such that $\min_{i\in M}\{a_{il}\} = 1$. Second, each group M has a total communicative value that depends on its size, $\omega(|M|)$, and this value is divided evenly between the languages in which the group can communicate: as the total number of languages in which M can communicate is $\sum_{l'\in L} \min_{i\in M}\{a_{il'}\}$, the communicative benefits of language l for the subgroup M are $\min_{i\in M}\{a_{il}\} \cdot \frac{\omega(|M|)}{\sum_{l'\in L} \min_{i\in M}\{a_{il'}\}}$. Finally, the communicative benefits of language l are the sum of these values for all coalitions that can communicate in this language: as $\min_{i\in M}\{a_{il}\} = 0$ for the languages in which the group cannot communicate, we can express the total value of language l by the formula included in the following definition.

Definition 1 *A communicative benefit function ϕ belongs to the class $\mathcal{F}$ if there exists a mapping $\omega : \mathbb{N} \to \mathbb{R}_+$ such that for each $(N, L, A) \in \mathcal{S}$ and each $l \in L$,*[8]

$$\phi_{(N,L,A)}(l) = \sum_{M \subseteq N,\, M \neq \emptyset} \left(\omega(|M|) \cdot \frac{\min\limits_{i \in M}\{a_{il}\}}{\sum\limits_{l' \in L} \min\limits_{i \in M}\{a_{il'}\}} \right).$$

The members of this class of communicative benefit functions differ on the weights of each subgroup size, represented by the mapping ω, for which the unique restriction is that they should be non–negative. For each mapping ω, we shall denote by ϕ^ω the communicative benefit function within class $\mathcal{F}$ associated with ω. Observe that, if $\phi = \phi^\omega$, then for each $(N, L, A) \in \mathcal{S}$,

$$\Phi(N, L, A) = \sum_{l \in L} \phi^\omega_{(N,L,A)}(l) = \sum_{M \subseteq N,\, M \neq \emptyset} \left(\omega(|M|) \cdot \max_{l \in L} \{\min_{i \in M}\{a_{il}\}\} \right).$$

3 The characterization result

The main result of the paper states that the set of axioms introduced above is only satisfied by the class of communicative benefit functions $\mathcal{F}$.

Theorem 1 *A communicative benefit function ϕ satisfies Anonymity, Equal Treatment of Equal Languages, Inclusion Monotonicity, Irrelevance of Non Speakers and Null Agent Consistency if and only if it belongs to the class $\mathcal{F}$.*

In order to prove Theorem 1, we need two lemmata providing several implications of the axioms in the statement of the theorem. To present the first lemma, we need to introduce some notation. We say that a situation is *nested* if there exists a language that is spoken by all agents that speak any other language. Formally, $(N, L, A) \in \mathcal{S}$ is nested if there exists $l \in L$ with $a_{il} = 1$ for each $i \in N$ such that $a_{il'} = 1$ for some $l' \in L$.[9] We denote the set of *nested situations* as $\mathcal{S}^* \subseteq \mathcal{S}$. If $(N, L, A) \in \mathcal{S}^*$, we denote $l^*_{(N,L,A)}$ a language that is spoken by the maximal set of agents in this situation.[10] Then, the first lemma states that, under *Anonymity*, *Inclusion Monotonicity* and *Null Agent Consistency*, the total communicate benefits of all languages in any nested situation only depend on the number of speakers of the maximal language.

[8] The quotients with the indeterminate form $\frac{0}{0}$ should be replaced by 0.

[9] This is reminiscent of the concept of nested graphs (e.g., Koning et al., 2014; Joshi et al., 2020).

[10] Observe that $l^*_{(N,L,A)}$ may not be unique.

Lemma 1 *Let ϕ be a communicative benefit function that satisfies Anonymity, Inclusion Monotonicity and Null Agent Consistency. Then, there exists a mapping $\Omega : \mathbb{N} \cup \{0\} \rightarrow \mathbb{R}_+$ such that, for each $(N, L, A) \in \mathcal{S}^*$,*

$$\Phi(N, L, A) = \Omega(|N_A(l^*_{(N,L,A)})|).$$

The proof of Lemma **??** is relegated to the Appendix.

The second lemma states that, under *Irrelevance of Non Speakers*, the communicative benefits of any language in any situation coincide with the communicative benefits that this language has in a particular nested situation in which it is a maximal language.

Lemma 2 *Let ϕ be a communicative benefit function that satisfies Irrelevance of Non Speakers. For each $(N, L, A) \in \mathcal{S}$, and each $l \in L$, let A' be such that, for each $l' \in L$,*

$$a'_{il'} = \begin{cases} a_{il'} & \textit{if } i \in N_A(l) \\ 0 & \textit{otherwise.} \end{cases}$$

Then,

$$\phi_{(N,L,A)}(l) = \phi_{(N,L,A')}(l).$$

The proof of Lemma **??** is obtained by applying *Irrelevance of Non Speakers* to the situations $(N, L \setminus \{l\}, A|_{L\setminus\{l\}})$, $(N, L \setminus \{l\}, A'|_{L\setminus\{l\}})$ and $(N, \{l\}, A|_{\{l\}})$.

With Lemmas **??** and **??**, we can proceed to prove Theorem 1.

Proof of Theorem ??

It is straightforward to check that all communicative benefit functions within class $\mathcal{F}$ satisfy the axioms in the statement. Conversely, let ϕ be a communicative benefit function that satisfies these axioms. By Lemma **??**, there exists a mapping $\Omega : \mathbb{N} \cup \{0\} \rightarrow \mathbb{R}_+$ such that for each $(N, L, A) \in \mathcal{S}^*$, $\Phi(N, L, A) = \Omega(|N_A(l^*_{(N,L,A)})|)$. We now construct iteratively from Ω the mapping $\hat{\omega} : \mathbb{N} \cup \{0\} \rightarrow \mathbb{R}$ as follows:

$$\hat{\omega}(x) = \begin{cases} \Omega(x) & \text{if } x = 0 \\ \Omega(x) - \sum_{y=0}^{y=x-1} \left[\binom{x}{y} \cdot \hat{\omega}(y) \right] & \text{otherwise.} \end{cases}$$

Observe that, then, for each $x \in \mathbb{N}$,

$$\Omega(x) = \sum_{y=0}^{y=x} \left[\binom{x}{y} \cdot \hat{\omega}(y) \right].$$

Note that there are two differences between $\hat{\omega}$ and the ω mapping introduced in the definition of the communicative benefit functions within class $\mathcal{F}$. On the one hand, the domain of $\hat{\omega}$ does not include only the natural numbers, but also 0. On the other hand, it is not clear with this construction whether the range of $\hat{\omega}$ includes only non–negative real numbers, as it occurs with ω.

The rest of the proof is decomposed into four steps. The first one establishes that the communicative benefits of a language can be computed applying a similar formula to the one from Definition **??**, but with two differences: on the one hand, it calculates the communicative value of each group M by $\hat{\omega}(|M|)$ instead of $\omega(|M|)$ and, on the other hand, it also considers that the empty group could have a value $\hat{\omega}(0)$. The two intermediate steps are dedicated to prove that the range of $\hat{\omega}$ is $\mathbb{R}_+$ and that $\hat{\omega}(0) = 0$. Finally, the last step wraps up the previous three steps to finish the proof.

Step 1: *For each* $(N, L, A) \in \mathcal{S}$ *and each* $l \in L$,

$$\phi_{(N,L,A)}(l) = \sum_{M \subseteq N} \left(\hat{\omega}(|M|) \cdot \frac{\min\limits_{i \in M}\{a_{il}\}}{\sum\limits_{l' \in L} \min\limits_{i \in M}\{a_{il'}\}} \right).$$

Let $(N, L, A) \in \mathcal{S}$ and $l \in L$. Assume that we have already proved that

$$\phi_{(\hat{N},\hat{L},\hat{A})}(\hat{l}) = \sum_{M \subseteq \hat{N}} \left(\hat{\omega}(|M|) \cdot \frac{\min\limits_{i \in M}\{\hat{a}_{i\hat{l}}\}}{\sum\limits_{l' \in \hat{L}} \min\limits_{i \in M}\{\hat{a}_{il'}\}} \right) \text{ for each } (\hat{N}, \hat{L}, \hat{A}) \in \mathcal{S} \text{ and each } \hat{l} \in \hat{L} \text{ with } |\hat{N}_{\hat{A}}(\hat{l})| < |N_A(l)|$$

and we are going to prove that $\phi_{(N,L,A)}(l)$ follows the formula of the statement of Step 1.[11]

Let A' be such that, for each $l' \in L$,

$$a'_{il'} = \begin{cases} a_{il'} & \text{for each } i \in N_A(l) \\ 0 & \text{otherwise.} \end{cases}$$

[11]In order to avoid redundancy, we do not explicitly provide the proof of the base case (i.e., when $|N_A(l)| = 0$) because its proof is analogous to the upcoming one.

By Lemma ??, $\phi_{(N,L,A)}(l) = \phi_{(N,L,A')}(l)$. Thus, it suffices to show that

$$\phi_{(N,L,A')}(l) = \sum_{M\subseteq N} \left(\hat{\omega}(|M|) \cdot \frac{\min\limits_{i\in M}\{a'_{il}\}}{\sum\limits_{l'\in L} \min\limits_{i\in M}\{a'_{il'}\}} \right).$$

Observe that $(N, L, A') \in \mathcal{S}^*$ and that $l^*_{(N,L,A')} = l$. Then, we can apply Lemma ?? to obtain

$$\Phi(N, L, A') = \Omega(|N_{A'}(l)|) = \sum_{y=0}^{y=|N_{A'}(l)|} \left[\binom{|N_{A'}(l)|}{y} \cdot \hat{\omega}(y) \right].$$

Note that $\Phi(N, L, A') = \sum\limits_{\hat{l}:N_{A'}(\hat{l})\subset N_{A'}(l)} \phi_{(N,L,A')}(\hat{l}) + \sum\limits_{\hat{l}:N_{A'}(\hat{l})=N_{A'}(l)} \phi_{(N,L,A')}(\hat{l})$ and, thus,

$$\sum_{\hat{l}:N_{A'}(\hat{l})=N_{A'}(l)} \phi_{(N,L,A')}(\hat{l}) = \sum_{y=0}^{y=|N_{A'}(l)|} \left[\binom{|N_{A'}(l)|}{y} \cdot \hat{\omega}(y) \right] - \sum_{\hat{l}:N_{A'}(\hat{l})\subset N_{A'}(l)} \phi_{(N,L,A')}(\hat{l}). \qquad (1)$$

Let $\bar{l} \in L$ be such that $N_{A'}(\bar{l}) \subset N_{A'}(l)$. By the induction hypothesis,

$$\phi_{(N,L,A')}(\bar{l}) = \sum_{M\subseteq N} \left(\hat{\omega}(|M|) \cdot \frac{\min\limits_{i\in M}\{a'_{i\bar{l}}\}}{\sum\limits_{l'\in L} \min\limits_{i\in M}\{a'_{il'}\}} \right).$$

As, for each $M \not\subset N_{A'}(l)$, there exists $i \in M$ such that $a'_{i\bar{l}} = 0$, it follows that

$$\phi_{(N,L,A')}(\bar{l}) = \sum_{M\subset N_{A'}(l)} \left(\hat{\omega}(|M|) \cdot \frac{\min\limits_{i\in M}\{a'_{i\bar{l}}\}}{\sum\limits_{l'\in L} \min\limits_{i\in M}\{a'_{il'}\}} \right).$$

Observe now that, for each $\hat{l} \in L$ such that $N_{A'}(\hat{l}) = N_{A'}(l)$, $\min\limits_{i\in M}\{a'_{i\hat{l}}\} = 1$ for each $M \subset N_{A'}(l)$. Let $\alpha_l = |\{\hat{l} \in L \mid N_{A'}(\hat{l}) = N_{A'}(l)\}|$. Thus,

$$\sum_{\bar{l}:N_{A'}(\bar{l})\subset N_{A'}(l)} \phi_{(N,L,A')}(\bar{l}) = \sum_{M\subset N_{A'}(l)} \left[\omega(|M|) \cdot \left(\sum_{l'\in L} \frac{\min\limits_{i\in M}\{a'_{il'}\} - \alpha_l}{\sum\limits_{l'\in L} \min\limits_{i\in M}\{a'_{il'}\}} \right) \right].$$

Then,

$$\sum_{\bar{l}:N_{A'}(\bar{l})\subset N_{A'}(l)} \phi_{(N,L,A')}(\bar{l}) = \sum_{M\subset N_{A'}(l)} \left[\omega(|M|) \cdot \left(1 - \frac{\alpha_l}{\sum\limits_{l'\in L} \min\limits_{i\in M}\{a'_{il'}\}} \right) \right].$$

Note that $\binom{|N_{A'}(l)|}{y} = |\{M \subseteq N_{A'}(l) \,:\, |M| = y\}|$ for each $y \in \{0, \ldots, |N_{A'}(l)|\}$. It follows by (1) that

$$\sum_{\hat{l}:N_{A'}(\hat{l})=N_{A'}(l)} \phi_{(N,L,A')}(\hat{l}) = \sum_{M\subseteq N_{A'}(l)} \left(\hat{\omega}(|M|) \cdot \frac{\alpha_l}{\sum\limits_{l'\in L} \min\limits_{i\in M}\{a'_{il'}\}} \right).$$

By *Equal Treatment of Equal Languages*, $\alpha_l \cdot \phi_{(N,L,A')}(l) = \sum\limits_{\hat{l}:N_{A'}(\hat{l})=N_{A'}(l)} \phi_{(N,L,A')}(\hat{l})$. Therefore,

$$\phi_{(N,L,A')}(l) = \sum_{M \subseteq N_{A'}(l)} \left(\hat{\omega}(|M|) \cdot \frac{1}{\sum\limits_{l' \in L} \min\limits_{i \in M} \{a'_{il'}\}} \right).$$

As $\min\limits_{i \in M}\{a'_{il}\} = 1$ for each $M \subseteq N_{A'}(l)$, and $\min\limits_{i \in M}\{a'_{il}\} = 0$ for each $M \nsubseteq N_{A'}(l)$, we have

$$\phi_{(N,L,A')}(l) = \sum_{M \subseteq N} \left(\hat{\omega}(|M|) \cdot \frac{\min\limits_{i \in M}\{a'_{il}\}}{\sum\limits_{l' \in L} \min\limits_{i \in M} \{a'_{il'}\}} \right).$$

Finally, observe that, by construction of A', for each $M \subseteq N$:

- $\min\limits_{i \in M}\{a'_{il}\} = 1$ if and only if $\min\limits_{i \in M}\{a_{il}\} = 1$,
- If $\min\limits_{i \in M}\{a'_{il}\} = 1$, then $\sum\limits_{l' \in L} \min\limits_{i \in M}\{a'_{il'}\} = \sum\limits_{l' \in L} \min\limits_{i \in M}\{a_{il'}\}$.

Therefore, for each $M \subseteq N$,

$$\frac{\min\limits_{i \in M}\{a'_{il}\}}{\sum\limits_{l' \in L} \min\limits_{i \in M} \{a'_{il'}\}} = \frac{\min\limits_{i \in M}\{a_{il}\}}{\sum\limits_{l' \in L} \min\limits_{i \in M} \{a_{il'}\}}.$$

Consequently,

$$\phi_{(N,L,A')}(l) = \sum_{M \subseteq N} \left(\hat{\omega}(|M|) \cdot \frac{\min\limits_{i \in M}\{a_{il}\}}{\sum\limits_{l' \in L} \min\limits_{i \in M} \{a_{il'}\}} \right),$$

as desired.

Step 2: $\hat{\omega}(x) \geq 0$ *for each* $x \in \mathbb{N} \cup \{0\}$.

Let $x \in \mathbb{N} \cup \{0\}$. Let $(N, \{l\}, A) \in \mathcal{S}$ be such that $|N_l(A)| = x$. By Lemma **??**, $\Phi(N, \{l\}, A) = \Omega(x)$. As there is only one language in this situation, $\Phi(N, \{l\}, A) = \phi_{(N,\{l\},A)}(l)$ and, thus, $\phi_{(N,L,A)}(l) = \Omega(x)$.

If $x = 0$, then $\Omega(x) = \hat{\omega}(x)$ and, thus, $\phi_{(N,\{l\},A)}(l) = \hat{\omega}(x)$. As, by definition, the range of ϕ is $\mathbb{R}_+$, we obtain that $\omega(0) \in \mathbb{R}_+$.

If $x \geq 1$, then $\Omega(x) = \sum\limits_{y=0}^{y=x} \left[\binom{x}{y} \cdot \hat{\omega}(y) \right]$. Thus, $\Phi(N, \{l\}, A) = \sum\limits_{y=0}^{y=x} \left[\binom{x}{y} \cdot \hat{\omega}(y) \right]$. Let $(N, L, A') \in \mathcal{S}$ be such that

- $|L| = x$,

- $|N_{A'}(\bar{l})| = x - 1$ for each $\bar{l} \in L$,
- $N_{A'}(\bar{l}) \neq N_{A'}(\bar{l}')$ for each pair $\bar{l}, \bar{l}' \in L$, and
- $\bigcup\limits_{\bar{l} \in L} N_{A'}(\bar{l}) = N_A(l)$.

By Step 1,

$$\phi_{(N,L,A')}(\bar{l}) = \sum_{M \subseteq N} \left(\hat{\omega}(|M|) \cdot \frac{\min\limits_{i \in M}\{a'_{i\bar{l}}\}}{\sum\limits_{l' \in L} \min\limits_{i \in M}\{a'_{il'}\}} \right) \text{ for each } \bar{l} \in L.$$

As, for each $M \not\subset N_A(l)$, $\min\limits_{i \in M}\{a'_{i\bar{l}}\} = 0$ for each $\bar{l} \in L$, we have

$$\phi_{(N,L,A')}(\bar{l}) = \sum_{M \subset N_A(l)} \left(\hat{\omega}(|M|) \cdot \frac{\min\limits_{i \in M}\{a'_{i\bar{l}}\}}{\sum\limits_{l' \in L} \min\limits_{i \in M}\{a'_{il'}\}} \right) \text{ for each } \bar{l} \in L.$$

Thus,

$$\begin{aligned} \Phi(N, L, A') = \sum_{\bar{l} \in L} \phi_{(N,L,A')}(\bar{l}) &= \sum_{\bar{l} \in L} \sum_{M \subset N_A(l)} \left(\hat{\omega}(|M|) \cdot \frac{\min\limits_{i \in M}\{a'_{i\bar{l}}\}}{\sum\limits_{l' \in L} \min\limits_{i \in M}\{a'_{il'}\}} \right) \\ &= \sum_{M \subset N_A(l)} \hat{\omega}(|M|) \cdot \left(\sum_{\bar{l} \in L} \frac{\min\limits_{i \in M}\{a'_{i\bar{l}}\}}{\sum\limits_{l' \in L} \min\limits_{i \in M}\{a'_{il'}\}} \right). \end{aligned}$$

Now, for each $M \subset N_A(l)$, there is $\bar{l} \in L$ such that $M \subseteq N_{A'}(\bar{l})$. Then, $\left(\sum\limits_{\bar{l} \in L} \frac{\min\limits_{i \in M}\{a'_{i\bar{l}}\}}{\sum\limits_{l' \in L} \min\limits_{i \in M}\{a'_{il'}\}} \right) = 1$, for each $M \subset N_A(l)$. Therefore,

$$\Phi(N, L, A') = \sum_{M \subset N_A(l)} \hat{\omega}(|M|).$$

Finally, observe that there are exactly $\binom{x-1}{|M|}$ subsets of $N_A(l)$ with size $|M|$. Therefore,

$$\Phi(N, L, A') = \sum_{y=0}^{y=x-1} \left[\binom{x}{y} \cdot \hat{\omega}(y) \right].$$

We have then deduced that $\Phi(N, \{l\}, A) = \Phi(N, L, A') + \hat{\omega}(x)$. As $(N, L, A') \subseteq (N, \{l\}, A)$, it follows, by *Inclusion Monotonicity*, that $\Phi(N, \{l\}, A) \geq \Phi(N, L, A')$. Therefore, $\hat{\omega}(x) \geq 0$, as desired.

Step 3: $\hat{\omega}(0) = 0$.

One of the assumptions of the model is that there exists $(\bar{N}, \bar{L}, \bar{A}) \in \mathcal{S}$ and a language $\bar{l} \in L$ such that $\phi_{(\bar{N},\bar{L},\bar{A})}(\bar{l}) = 0$. Then, by Step 1,

$$\phi_{(\bar{N},\bar{L},\bar{A})}(\bar{l}) = 0 = \sum_{M \subseteq \bar{N}} \left(\hat{\omega}(|M|) \cdot \frac{\min_{i \in M}\{\bar{a}_{i\bar{l}}\}}{\sum_{l' \in \bar{L}} \min_{i \in M}\{\bar{a}_{il'}\}} \right).$$

As, by Step 2, $\hat{\omega}(x) \geq 0$ for each $x \in \mathbb{N} \cup \{0\}$, a necessary and sufficient condition for this equality to hold is that $\hat{\omega}(0) = \ldots = \hat{\omega}(|\bar{N}_{\bar{A}}(\bar{l})|) = 0$, which concludes the proof.

Step 4: *Conclusion.*

By Step 1, we have that for each $(N, L, A) \in \mathcal{S}$ and each $l \in L$,

$$\phi_{(N,L,A)}(l) = \sum_{M \subseteq N} \left(\hat{\omega}(|M|) \cdot \frac{\min_{i \in M}\{a_{il}\}}{\sum_{l' \in L} \min_{i \in M}\{a_{il'}\}} \right).$$

By Step 3, $\hat{\omega}(0) = 0$ and, therefore, we have that for each $(N, L, A) \in \mathcal{S}$ and each $l \in L$,

$$\phi_{(N,L,A)}(l) = \sum_{M \subseteq N, M \neq \emptyset} \left(\hat{\omega}(|M|) \cdot \frac{\min_{i \in M}\{a_{il}\}}{\sum_{l' \in L} \min_{i \in M}\{a_{il'}\}} \right).$$

Let ω be the restriction of $\hat{\omega}$ to the domain $\mathbb{N}$. That is, $\omega : \mathbb{N} \to \mathbb{R}$ is such that $\omega(x) = \hat{\omega}(x)$ for all $x \in \mathbb{N}$. By Step 2, the range of ω is $\mathbb{R}_+$. Therefore, we have shown that there exists a mapping $\omega : \mathbb{N} \to \mathbb{R}_+$ such that for each $(N, L, A) \in \mathcal{S}$ and each $l \in L$,

$$\phi_{(N,L,A)}(l) = \sum_{M \subseteq N, M \neq \emptyset} \left(\omega(|M|) \cdot \frac{\min_{i \in M}\{a_{il}\}}{\sum_{l' \in L} \min_{i \in M}\{a_{il'}\}} \right).$$

Thus, ϕ belongs to class $\mathcal{F}$. ■

Independence of the axioms

As the next proposition states, Theorem 1 is a tight result. The proof is relegated to the Appendix.

Proposition 1 *Anonymity, Equal Treatment of Equal Languages, Inclusion Monotonicity, Irrelevance of Non Speakers and Null Agent Consistency are independent axioms.*

4 Concluding remarks

In this paper we explore the axiomatic approach to the problem of measuring communicative benefits. We show that the combination of intuitive axioms reflecting the celebrated principles of impartiality, monotonicity, invariance and consistency characterizes a family of two-stage communicative benefit functions. First, the groups of agents that can communicate in a given language are identified. Second, each member of the family associates each group a value that depends on its size, and this value is divided evenly among the languages in which the group can communicate. As such, our work fully aligns with the tradition of axiomatic work that can be traced back to the seminal contributions of Nash (1950), Arrow (1951) and Shapley (1953). The aim is to provide a list of requirements (axioms) formalizing ethical or operational principles that a rule should satisfy. The ideal is to derive the set of rules that fulfill such axioms. During the ensuing seven decades countless authors have applied the axiomatic approach to a variety of problems and measures ranging from conventional concepts such as taxation (e.g., Young, 1988), income inequality (e.g., Bossert, 1990), or claims problems (e.g., Ju et al., 2007) to more sophisticated ones (and somewhat unconventional), such as polarization (e.g., Esteban and Ray, 1994) or resilience (e.g., Asheim et al., 2020).

Our axioms characterize a whole family of functions, giving freedom to consider arbitrary weights for group sizes. One might be interested into being more accurate and shrink the family to a unique or just several functions, which could be obtained adding axioms with implications on the weights. For instance, one could consider the basic axiom stating that a canonical society with a unique individual offers null communicative benefits.[12] This would amount to impose the condition $\omega(1) = 0$ in the definition of our family (thus, shrinking it). Likewise, one could also consider additional axioms that would end up reflecting into the monotonicity of the ω mapping.

As we have mentioned in the Introduction, we have decided to develop the model measuring the social benefits of each language, but a similar approach can be done to measure the benefits each individual obtains from each set of languages. This would imply the construction of a function $\varphi_{(N,L,A)}$ that associates for each agent $i \in N$ the communicative benefits this agent obtains from situation (N, L, A). A family of these functions, sharing the spirit of our characterized

[12]Formally: For any $(N, L, A) \in \mathcal{S}$ such that $|N| = 1$, $\Phi(N, L, A) = 0$.

family at Section 3, would be the following:

$$\varphi^{\omega}_{(N,L,A)}(i) = \sum_{i \in M \subseteq N} \frac{\omega(|M|)}{|M|} \max_{l \in L}\{\min_{j \in M}\{a_{jl}\}\}.$$

In words, first the groups with which i can communicate are identified (these are the subgroups $M \subseteq N$, with $i \in M$, such that $\max_{l \in L}\{\min_{j \in M}\{a_{jl}\}\} = 1$). Second, the total communicative value of each of these groups, defined by $\omega(|M|)$ as in Definition 1, is divided evenly among its members. It can be shown (with similar arguments to the ones used at the proof of Theorem **??**) that a suitable adaptation of the axioms presented above would also characterize this family.[13]

While our model is distinctive and generates an innovative and novel result, we would like to point out some links to classical contributions in the literature. First, the communicative benefit function we derive for each language is reminiscent to the Shapley value for cooperative games (e.g., Shapley, 1953).[14] In that sense, we align with the tradition of deriving *Shapley value functions* for a diverse range of problems, including airport problems (e.g., Littlechild and Owen, 1973), telecommunication problems (e.g., van den Nouweland et al., 1996), museum-pass problems (e.g., Ginsburgh and Zang, 2003) or broadcasting problems (e.g., Bergantiños and Moreno-Ternero, 2020). Second, the input of our problem coincides with that of *approval voting* (e.g., Brams and Fishburn, 1978). In that case, each voter casts her vote for as many candidates she wishes; each positive vote is counted in favour of the candidate. The votes are then added by candidate, and the winner is the one who gets the largest number of votes. All other candidates can also be ranked, according to the number of votes they obtain. An alternative aggregation procedure, akin to the one we obtain here, is *Cumulative Voting* (e.g., Glasser, 1959; Sawyer and MacRae, 1962), which allows voters to distribute points among candidates in any arbitrary way. An interesting case is the one in which every agent is endowed with a

[13]Observe that $\sum_{i \in N} \varphi^{\omega}_{(N,L,A)}(i) = \sum_{l \in L} \phi^{\omega}_{(N,L,A)}(l)$ for each ω.

[14]More precisely, for each situation (N, L, A) and each mapping $\omega : \mathbb{N} \to \mathbb{R}_+$, we can describe a TU-game (L, v^{ω}), where v^{ω} is the characteristic function that assigns to each subset of languages $K \subseteq L$ the amount

$$v^{\omega}(K) = \sum_{M \subseteq N, M \neq \emptyset} \omega(|M|) \max_{l \in K}\{\min_{j \in M}\{a_{jl}\}\},$$

to be interpreted as the communicative benefits K generates to the situation (N, L, A) according to ω. The Shapley value of the TU-game (L, v^{ω}) yields precisely $\phi^{\omega}_{(N,L,A)}$. Analogously, we could describe a TU-game (N, v^{ω}), for agents instead of languages, whose Shapley value rationalizes the φ^{ω} function introduced above.

fixed number of votes that are evenly divided among all candidates for whom she approves.[15] In our setting, if we interpret that each group of individuals (instead of only indivivuals alone) *approve* the languages in which they can communicate, our family of communicative benefit functions coincides with the Cumulative Voting score of each language (when all coalitions of the same size receives the same number of votes).

Finally, our work can be extended in plausible ways. One refers to the case in which intermediate levels of language proficiency related to *partial learning* (e.g., Blume, 2000; Chen at al., 2020) are considered. Another refers to the case in which linguistic distances (e.g., Dyen et al., 1992) might play a role in determining the communication of a group of individuals. These lines are left for further research.

Appendix

Proof of Lemma ??

Let $x \in \mathbb{N} \cup \{0\}$ and $(N, \{l\}, A) \in \mathcal{S}$ be such that $|N_A(l)| = x$. Let $\Omega(x) = \Phi(N, \{l\}, A)$.

We first show that for any $(\hat{N}, \{\hat{l}\}, \hat{A}) \in \mathcal{S}$ such that $|\hat{N}_{\hat{A}}(\hat{l})| = x$, $\Phi(\hat{N}, \{\hat{l}\}, \hat{A}) = \Omega(x)$. To do so, we distinguish two cases:

Case 1: $|N| = |\hat{N}|$.

Let $\pi \in \Pi^{\mathcal{N}}$ be such that $\pi(N_A(l)) = \hat{N}_{\hat{A}}(\hat{l})$ and $\pi(N) = \hat{N}$. Then, $\pi(N, \{l\}, A) = (\hat{N}, \{\hat{l}\}, \hat{A})$. By *Anonymity*, $\phi_{(N,\{l\},A)}(l) = \phi_{(\hat{N},\{\hat{l}\},\hat{A})}(\hat{l})$. Thus, $\Phi(N, \{l\}, A) = \Phi(\hat{N}, \{\hat{l}\}, \hat{A})$.

Case 2: $|N| \neq |\hat{N}|$.

Assume without loss of generality that $|N| > |\hat{N}|$. Let $(\bar{N}, \{\hat{l}\}, \bar{A}) \in \mathcal{S}$ be such that $\hat{N} \subset \bar{N}$, $|\bar{N}| = |N|$, $\bar{a}_{i\hat{l}} = 0$ for each $i \in \bar{N} \setminus \hat{N}$, and $\bar{a}_{j\hat{l}} = \hat{a}_{j\hat{l}}$ for each $j \in \hat{N}$. As $|N_A(l)| = |\bar{N}_{\bar{A}}(\hat{l})|$ and $|\bar{N}| = |N|$, we can deduce from Case 1 that $\Phi(N, \{l\}, A) = \Phi(\bar{N}, \{\hat{l}\}, \bar{A})$. Therefore, by iterated application of *Null Agent Consistency*, $\Phi(\hat{N}, \{\hat{l}\}, \hat{A}) = \Phi(\bar{N}, \{\hat{l}\}, \bar{A})$ and, thus, $\Phi(N, \{l\}, A) = \Phi(\hat{N}, \{\hat{l}\}, \hat{A})$.

[15] Both Approval Voting and Cumulative Voting can be seen as members of a family of voting procedures dubbed as *Size Approval Voting*, which are characterized by Alcalde-Unzu and Vorsatz (2009).

Second, let $(\tilde{N}, \tilde{L}, \tilde{A}) \in \mathcal{S}^*$ be such that $|\tilde{N}_{\tilde{A}}(\tilde{l}^*_{(\tilde{N},\tilde{L},\tilde{A})})| = x$. By the analysis above, we know that $\Phi(\tilde{N}, \{\tilde{l}^*_{(\tilde{N},\tilde{L},\tilde{A})}\}, \tilde{A}|_{\{\tilde{l}^*_{(\tilde{N},\tilde{L},\tilde{A})}\}}) = \Omega(x)$. As $(\tilde{N}, \tilde{L}, \tilde{A}) \subseteq (\tilde{N}, \{\tilde{l}^*_{(\tilde{N},\tilde{L},\tilde{A})}\}, \tilde{A}|_{\{\tilde{l}^*_{(\tilde{N},\tilde{L},\tilde{A})}\}})$ and $(\tilde{N}, \{\tilde{l}^*_{(\tilde{N},\tilde{L},\tilde{A})}\}, \tilde{A}|_{\{\tilde{l}^*_{(\tilde{N},\tilde{L},\tilde{A})}\}}) \subseteq (\tilde{N}, \tilde{L}, \tilde{A})$, we can apply *Inclusion Monotonicity* twice to obtain that $\Phi(\tilde{N}, \tilde{L}, \tilde{A}) = \Phi(\tilde{N}, \{\tilde{l}^*_{(\tilde{N},\tilde{L},\tilde{A})}\}, \tilde{A}|_{\{\tilde{l}^*_{(\tilde{N},\tilde{L},\tilde{A})}\}})$. Thus, $\Phi(\tilde{N}, \tilde{L}, \tilde{A}) = \Omega(x)$.

Proof of Proposition ??

Consider the following communicative benefit functions:

- Let v^1 be a communicative benefit function that behaves similarly to one from class $\mathcal{F}$, but only taking into account the coalitions to which one particular agent $j \in \mathcal{N}$ belongs to. Formally, consider a mapping $\xi : 2^{\mathcal{N}} \to \mathbb{R}_+$ such that $\xi(M) = 1$ if $j \in M$ and 0 otherwise, and a mapping $\omega : \mathbb{N} \to \mathbb{R}_+$. Then, v^1 is such that, for each $(N, L, A) \in \mathcal{S}$,

$$v^1_{(N,L,A)}(l) = \sum_{M \subseteq N} \left(\omega(|M|) \cdot \xi(M) \cdot \frac{\min\limits_{i \in M}\{a_{il}\}}{\sum\limits_{l' \in L} \min\limits_{i \in M}\{a_{il'}\}} \right)$$

  for each $l \in L$. Then, v^1 satisfies *Equal Treatment of Equal Languages*, *Inclusion Monotonicity*, *Irrelevance of Non Speakers* and *Null Agent Consistency*, but not *Anonymity.*

- Let v^2 be a communicative benefit function that considers the groups of agents that can communicate in each language and the value of each group, that depends on its size, is divided among the languages in which the group can communicate, but unevenly. Formally, consider a particular language $\bar{l} \in \mathcal{L}$ and a mapping $\delta : \mathcal{L} \to \mathbb{R}_+$ be such that $\delta(\bar{l}) = 2$ and $\delta(l') = 1$ for each $l' \in \mathcal{L} \setminus \{\bar{l}\}$. Consider also a mapping $\omega : \mathbb{N} \to \mathbb{R}_+$. Then, v^2 is such that for each $(N, L, A) \in \mathcal{S}$,

$$v^2_{(N,L,A)}(l) = \sum_{M \subseteq N} \left(\omega(|M|) \cdot \frac{\min\limits_{i \in M}\{a_{il}\} \cdot \delta(l)}{\sum\limits_{l' \in L} \left(\min\limits_{i \in M}\{a_{il'}\} \cdot \delta(l') \right)} \right)$$

  for each $l \in L$. Then, v^2 satisfies *Anonymity*, *Inclusion Monotonicity*, *Irrelevance of Non Speakers* and *Null Agent Consistency*, but not *Equal Treatment of Equal Languages.*

- Let v^3 be the communicative benefit function that yields for each language a value equal to its number of speakers. Formally, for each $(N, L, A) \in \mathcal{S}$,

$$v^3_{(N,L,A)}(l) = |N_A(l)|$$

for each $l \in L$. Then, v^3 satisfies *Anonymity*, *Equal Treatment of Equal Languages*, *Irrelevance of Non Speakers* and *Null Agent Consistency*, but not *Inclusion Monotonicity.*

- Let v^4 be the communicative benefit function arising from normalizing a member within the class $\mathcal{F}$. Formally, consider a communicative benefit function ϕ within the class $\mathcal{F}$, and let v^4 be such that, for each $(N, L, A) \in \mathcal{S}$,

$$v^4_{(N,L,A)}(l) = \frac{\phi_{(N,L,A)}(l)}{\Phi(N, L, A)}$$

  for each $l \in L$. Then, v^4 satisfies *Anonymity*, *Equal Treatment of Equal Languages*, *Inclusion Monotonicity* and *Null Agent Consistency*, but not *Irrelevance of Non Speakers.*

- Let v^5 be the communicative benefit function arising from combining two members within the class $\mathcal{F}$, depending on whether the number of agents in the situation is odd or even. Formally, consider two communicative benefit functions ϕ, ϕ' within the class $\mathcal{F}$, and let v^5 be such that, for each $(N, L, A) \in \mathcal{S}$ and each $l \in L$,

$$v^5_{(N,L,A)}(l) = \begin{cases} \phi_{(N,L,A)}(l) & \text{if } |N| \text{ is odd.} \\ \phi'_{(N,L,A)}(l) & \text{if } |N| \text{ is even.} \end{cases}$$

  Then, v^5 satisfies *Anonymity*, *Equal Treatment of Equal Languages*, *Inclusion Monotonicity* and *Irrelevance of Non Speakers*, but not *Null Agent Consistency.*

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