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Abstract

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JEL Classification: D40, D83, L13

Keywords: Dynamic pricing, capacity constraints, asymmetric information, Disclosure, industrial espionage

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Dynamic Pricing with Uncertain Capacities*

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February 19, 2021

Abstract

In markets, such as those for airline tickets and hotel accommodations, firms sell time-dated products and have private information about unsold capacities. We show that competition under private information explains observed phenomena, such as increased price dispersion and higher expected prices towards the deadline, without making specific assumptions about demand. We also show that private information severely limits the market power of firms and that information exchange about capacity negatively affects consumers. Finally, we inquire into the incentives to unilaterally disclose information or to engage in espionage about rival's capacity and show that these activities are particularly harmful for consumers.

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1 Introduction

In markets such as those for hotel accommodations, airline flights, shipping, generated electricity, or any other time-dated product, firms sell a fixed capacity with a deadline. Studying these markets, most of the revenue management literature has focused on how a monopoly firm should price her product when (an uncertain) demand is gradually unfolding over time.[1] Starting with Dudey (1992) there is also a smaller literature dealing with competing firms. Where in most markets firms are uncertain about their competitors' unsold capacity, this literature assumes capacity is commonly known. Our paper is the first to provide an equilibrium analysis of dynamic competitive pricing of time-dated products where information about unsold capacity is private.[2]

The contribution of our paper is twofold. From a positive perspective, we provide a new, supply-side, explanation that is able to reconcile a number of empirical observations. First, prices tend to increase and become more dispersed as the deadline approaches (see, e.g., McAfee and Te Velde (2007) and Clark and Vincent (2012)). Second, there is no clear, monotonic relation between price dispersion and whether demand is peak or off-peak (see, e.g., Puller et al. (2009)). This second observation is hard to reconcile with other supply-side explanations (e.g. Dana (1999) and related literature). Our focus on competition with uncertainty about rivals' unsold capacities is able to explain these patterns without having to resort to assuming heterogeneous price elasticities on the demand side. Our explanation emphasizes that firms want to limit information revelation in periods long before the deadline in order to generate more profits in periods closer to the deadline.

From a normative perspective, we show that firms have much less market power under private information than when capacities were commonly known. In particular, expected prices and profits are lower when uncertainty about unsold rivals' capacity is not resolved before firms choose prices. Conversely, if firms collectively exchange information about capacity, they are able to significantly increase their profits even without collusion on prices. Thus, our analysis addresses the competition policy issue of evaluating the welfare implications of information exchange about capacity in industries

[1] See, e.g., Talluri and Van Ryzin (2006) for an overview of early literature and e.g., Board and Skrzypacz (2016),Gershkov et al. (2018), Hörner and Samuelson (2011), Dilme and Li (2019) for important recent contributions, where optimal contracts are determined depending on whether or not a firm can commit and where buyers are forward-looking.

[2] A paper by Lin and Sibdari (2009) proposes a heuristic pricing policy based on the Nash equilibrium prices derived from the game with complete information, but they do not consider the strategic equilibrium pricing decisions under private information.

with time-dated products and argues that consumers are significantly worse off.

These results motivate the subsequent analysis into the unilateral incentives of firms to disclose information regarding their unsold capacities or gather information about rival's capacities. Airlines frequently announce (disclose) how many seats they have left at the current price they offer. Alternatively, firms may learn the unsold capacity of their rivals through industrial espionage, for example, by scraping online data sources.[3] Interestingly, we find that if the cost of disclosing or engaging in espionage is not too large, only one firm decides to disclose or acquire information, and it is the firm whose information is not revealed that benefits most. Moreover, *ex ante* market power of firms is higher than when firms have to act not knowing the private information of their rivals and even higher than when both firms exchange information!

We now explain these results in more detail. To focus on the strategic analysis of information asymmetry, we consider the most basic dynamic competitive market where two firms compete in two periods with unit demand in both periods. A firm can be either constrained or unconstrained. A constrained firm's capacity is just enough to serve demand in one period, whereas an unconstrained firm can serve consumers in both periods. A firm's (unsold) capacity at the beginning of the game is her private information. Firms set prices in both periods and consumers buy from the lowest-priced firm in the period where they are active in the market.

To be able to put the results under asymmetric information into perspective, we first revisit the results if capacities were commonly known. Obviously, if there is common knowledge that both firms are constrained, monopoly prices result, while marginal cost pricing results if both are unconstrained. When one firm is constrained while the other is not, Dudey (1992) shows that firms set monopoly prices in all periods. The unconstrained firm lets the rival sell out at the monopoly price and then covers the remaining demand at the monopoly price itself (see, also, Martínez-de-Albéniz and Talluri (2011)).[4] Clearly, this result cannot explain the empirical evidence cited above.

[3] In Canada, WestJet seems to have been engaged in industrial espionage when it acquired "access to a special reservation website for the employees and retirees of Air Canada" The Globe and Mail reported in July 2004. The article reports on a law suit initiated by Air Canada and continues that the "website contains confidential information about the number of passengers booked on all flights at Air Canada and its subsidiary Zip for up to 352 days in the future". In May 2006, CBC News reports that a settlement was reached in which "WestJet apologized to Air Canada" and that "the lawsuit centered on allegations that WestJet management used the password of a former Air Canada employee to access a website maintained by Air Canada to download "detailed and commercially sensitive" information."

[4] We show that, in addition, there exists a continuum of equilibria in weakly dominated strategies where the constrained firm sells first at an arbitrary price between marginal cost and consumers' willingness-to-pay.

Consider then the impact of private information. Given some market outcome in the first period, both firms know which firm sold one unit and at which price. This allows the non-selling firm to form a posterior belief about the residual capacity of the selling firm (either one or zero units), and, therefore, estimate the probability that in the second period she is a monopolist in the market as the other firm has sold out. Following the analysis in Janssen and Rasmusen (2002) price dispersion is an essential feature in the second-period as the non-selling firm is uncertain about whether the competitor can actively compete. The expected equilibrium prices and profits will be fully determined by this probability, as it is the only source of market power.

Turning to the first period, we characterize the unique pooling equilibrium where both firms charge the same price. This price is exactly the opportunity cost for a constrained firm of selling in the first period, given her prior belief about her rival type. In addition, there exists a continuum of semi-separating equilibria where constrained and unconstrained firms randomize their pricing decisions in both periods. In these equilibria, firms make more profit in the first period than in the unique pooling equilibrium, but their second-period profits are lower as the updated belief about the rival being unconstrained is larger. The effect on second-period profits is so strong that the pooling equilibrium Pareto-dominates all other equilibria, i.e., firms suffer from revealing information through first-period pricing. Thus, there is little price dispersion in periods further away from the deadline as firms want to hide their private information to be able to benefit later with higher expected prices and more dispersion. Also, price dispersion is highest in markets with most uncertainty and lowest when firms are pretty certain that there is either peak or off-peak demand. As the pooling equilibrium yields lower profits than in the "Dudey" equilibrium, the perfect information model overestimates the market power of firms. Importantly, the logic of the Dudey result breaks down under private information: unconstrained firms will not wait until their rival is sold out as they do not know whether this will ever happen.

Consider next the incentives for firms to disclose or acquire private information. As firms have to set up the technology, we take these activities as long-term decisions that are made before the uncertainty unfolds. It is clear that if the technology cost is large, no firm will want to disclose or engage in industrial espionage. For smaller cost, let us first discuss disclosure. There exists an asymmetric equilibrium, where one only firm discloses. In this equilibrium, firms are able to obtain "Dudey" pay-offs if the disclosing firm is constrained as for this to be true it is sufficient that it is common knowledge that at least one firm is constrained. If the disclosing firm is unconstrained, however, she has to let the non-disclosing firm sell first in order to get positive expected profits in the second period implying that the non-disclosing unconstrained firm makes more profit than the disclosing unconstrained firm: the former gets an "extraordinary" rent by selling in the first period with the option to sell again later. Interestingly, the profit of the non-disclosing firm is identical to that when information is revealed prior to prices being set. Thus, the asymmetric equilibrium exists and is unique for any positive disclosure cost that is small enough and in this equilibrium the disclosing firm makes less profit than the non-disclosing firm. Importantly, both firms are better off when one firm discloses, while consumers are always worse off.

Industrial espionage is in many ways the reverse of voluntary disclosure as the one that bears the cost gets informed about the rival. The important difference is that unlike the public nature of disclosure, espionage cannot be observed so that it is the *expectation* of industrial espionage that drives competitors' behavior rather than the act itself. Given the disclosure results described above, it is clear that an asymmetric equilibrium in pure strategies also exists under industrial espionage, but not if the cost is relatively small. In that case, there exists an equilibrium with the second firm randomizing her decision to check out the rival's capacity. Interestingly, when the cost of espionage approaches zero and the second firm almost surely engages in espionage, this asymmetric equilibrium with one firm mixing does *not* converge to the "Dudey" equilibrium.[5] One interesting auxiliary result of this analysis is that even if both firms know their rival still has unsold capacity in the last period, they do not engage in marginal cost pricing as firms may (second-order) believe that their competitor believes that they are sold out. In other words, Bertrand competition requires that firms have *common knowledge* about unsold capacity levels of all firms.

As indicated at the start of the paper, to the best of our knowledge there does not exist a paper providing an equilibrium analysis of dynamic pricing of time-dated products where capacities are private information. Two recent papers (Somogyi and Vergote (2020), Montez and Schutz (2020)) study the implications of private information about capacities in a static pricing environment. Somogyi and Vergote (2020) find that capacity-constrained firms price less aggressively than unconstrained firms as they prefer to focus on any left-over demand in case their rival is also capacity-constrained. Although we find that a similar consideration is relevant in some of our equilibria, the

[5]There is, however, no "discontinuity" when the spying cost approaches 0 as the asymmetric equilibrium with industrial espionage converges to one of the weakly dominated equilibria of the complete information game.

dynamic game allows for much richer equilibrium patterns, allowing us to explain the stylized facts and to derive welfare conclusion about information sharing. Montez and Schutz (2020) consider the classical question of Kreps and Scheinkman (1983) and allow for endogenous capacity choices that are unobserved by competitors. They show that the fact that capacity choices are typically not known to competitors drastically affects the prediction that Cournot type of behaviour should be expected in markets where firms produce in advance. Our model does not allow for endogenous capacity choice, yet has dynamic pricing instead, and shares the message that not knowing capacity choices of rivals negatively affects firms' market power.

The paper is also clearly related to the extensive literature on disclosure (see, e.g., Dranove and Jin (2010) for an overview of the quality literature) and the much smaller literature on industrial espionage (see, e.g., Solan and Yariv (2004), Nasheri (2005) and Barrachina et al. (2014)). These topics, of course, only cover part of our paper, and none of these papers investigate the incentives to disclose or spy on capacities.

The paper is also related to the literature on information exchange (see, e.g., Gal-Or (1985, 1986), Shapiro (1986), Vives (1984, 1990) for early contributions) and the competition policy issues related to information exchange (see, e.g., the Horizontal Guideliness by the EU Commission (2011) or the OECD (2010) report). It is common wisdom that information exchange can have both a positive or negative impact on the competitiveness of the market, depending on the type of information exchanged. We show that in markets for time-dated products information exchange about unsold capacity is unambiguously anti-competitive and that, in particular, with information exchange firms can achieve collusive outcomes even if they choose prices unilaterally.

Finally, there is an interesting relation between our paper and the literature on sequential auctions with budget constraints (see, e.g., Pitchik (2009), Pitchik and Schotter (1988) and Ghosh and Liu (2019)). An alternative way to interpret our model is that two players compete in two sequential auctions for two objects with a common value and that they have a privately known budget which is either equal to the value of one object or the value of two objects. The sequential auction literature differs in that it is important *how much* budget a bidder has left to compete in a later auction and not only whether a bidder can compete. This difference allows us to fully characterize the set of equilibria and derive their properties.

The rest of the paper is organized as follows. The next section presents the baseline model with private information, while Section 4 presents the results. In between, Section 3 briefly analyzes the complete information model as a benchmark. Section 5 develops the arguments pertaining to disclosure, while Section 6 deals with espionage. Section 7 concludes. Proofs that are essential for a proper understanding of the results are given in the main body of the paper; other proofs are relegated to Appendix B. Detailed discussions on equilibria under espionage are in Appendix A, while some additional material can be found in Appendices C and D.

2 The Basic Model and Solution Concept

The basic model[6] builds upon the work of Dudey (1992). We consider a homogeneous goods market where two firms compete in prices over time. The demand side is represented by myopic consumers with unit demand and we normalize their willingness to pay to 1.[7] Consumers enter the market at discrete moments of time, observe the prices that are charged in the market at that time and choose to buy at the lowest price if that is below their willingness to pay. Otherwise, the consumer leaves the market and does not come back. In case the observed prices are the same, all consumers buy from one of the firms (with each selling with equal probability).[8]

As introducing private information about capacity creates additional complexity, we focus on the simplest possible dynamic setting by considering two periods only. Half of the consumers enter in period $t = 1$ and the other half enters in period $t = 2$. We normalize the mass of consumers to two and say that one consumer enters in each period. Each firm has an initial capacity which is private information. To have an interesting model, a firm is either constrained so that she can only sell to one consumer, or she is unconstrained in which case she can cover the demand in both periods. Firms' production cost is normalized to zero. The prior probability a firm is constrained is denoted by α and is independent across firms.[9]

The game unfolds as follows. In period 1, depending on their capacity each firm i chooses a price p_{i1}, $i = 1, 2$. The consumer that entered the market in period 1 observes

[6]This section describes the model with private information only. The modeling of disclosure and/or industrial espionage is discussed in the respective sections.

[7]If risk-neutral consumers were forward looking, it cannot be the case that expected prices are decreasing over time as the first-period consumers would want to wait until the next period. We show that in most equilibria expected prices are increasing over time so that even if consumers are forward looking, they do not want to postpone their decision.

[8] Note that under an alternative tie-breaking rule where both firms sell half of their capacity in the first period if they set identical prices, the continuation game is outcome-equivalent to the one we study here, but more complicated to analyze. Details are given in Appendix C.

[9]For the case of asymmetric priors, see Footnote 19.

both prices and buys at the lowest price, provided that price is not larger than 1. Firms observe both first-period prices[10] and know from whom the consumer bought in the first period (if at all). At the beginning of the second period, they update their beliefs about whether or not the rival firm has unsold capacity left. All firms with unsold capacity set a price p_{i2}, $i = 1, 2$ in the second period. The period 2 consumer observes both prices p_{i2} and also buys at the lowest price if that is not larger than 1. Firms choose their prices so as to maximize the sum of profits in both periods.

We solve the model using Perfect Bayesian Equilibrium as solution concept (see, e.g., Fudenberg and Tirole (1991)). Refinement concepts, such as the Intuitive Criterion developed by Cho and Kreps (1987), do not impose additional restrictions.

3 Revisiting the complete information results

Before analyzing the private information game and the different ways to overcome private information through disclosure or industrial espionage, in this section we briefly revisit the results for the complete information game. Obviously, when there is common knowledge both firms are constrained, they will set monopoly prices when they have unsold capacity left. When there is common knowledge both firms are unconstrained, they will set prices equal to marginal cost in both periods. When there is common knowledge one firm is constrained, while the other is unconstrained, we have the following result.

Proposition 1. *If it is common knowledge that one firm is constrained while the other is not, then there exists a unique subgame perfect equilibrium in undominated strategies where the constrained firm sells at a price of* 1 *in the first period and the unconstrained firm sells at a price of* 1 *in the second period. In addition, there exists a continuum of subgame perfect equilibria in weakly dominated strategies, indexed by* $x \in (0, 1)$, *where the constrained firm sells at* x *in the first period and the unconstrained firm sells at* 1 *in the second period.*

[10]In many markets where firms and consumers make online transactions, it is easy for firms to observe competitor's prices: if consumers can observe both prices, it is typically also possible for firms to observe them. In addition to being more realistic in many markets, the assumption also helps to simplify the analysis as otherwise firms have to form conjectures about the first-period price the competitor chooses and what this implies for him to be sold out in the second period. In Appendix D, we show, however, that the pooling equilibrium characterized in Section 4 remains an equilibrium when first-period prices are hidden from competitors. Finally, we follow the literature in this respect and the assumption helps to understand the comparative impact of private information in this setting.

To understand why there are multiple equilibria, it is best to provide the subgame perfect equilibrium strategies and to recall the result of Blume (2003) who shows that in the Bertrand competition model with homogeneous goods, there exists an equilibrium where one firm prices at marginal cost, while the other firm randomizes uniformly over a very small interval above this price. With this result in mind, suppose that the constrained firm sets $p_1 = x$ for some $x \in (0, 1]$, while the unconstrained firm uniformly randomizes her first-period price in the interval $(x, x + \varepsilon)$ for some small $\varepsilon > 0$. The constrained firm makes an equilibrium profit of x, while the unconstrained firm makes a profit of 1 as she can sell at the monopoly price in the second period. If the constrained firm does not sell in the first period, firms engage in marginal cost pricing in the second period.

Clearly, deviating is not optimal for either firm. If the constrained firm sets a higher price, there is a large probability that she will not sell in the first period and faces Bertrand competition in the second period. Thus, this deviation results in a profit close to 0. Selling at a lower price in the first period is clearly also not profitable. If the unconstrained firm undercuts the constrained firm in the first period, she will also face Bertrand competition in the second period, making the deviation also unprofitable.

It is clear that the unconstrained firm does not have a strict incentive to set first-period prices smaller than 1. Moreover, if the constrained firm would deviate and set $p_1 \in (x, 1)$, the unconstrained firm would be better off setting a price larger than p_1. Thus, setting a price $p_1 < 1$ is a weakly dominated strategy for the unconstrained firm. Therefore, for the benchmark model of complete information, we continue focusing on the Dudey (1992) monopoly outcome as that is the unique equilibrium in undominated strategies. In Section 6 we will refer, however, to the weakly dominated equilibria to explain that equilibria of the espionage game do not converge to the Dudey (1992) monopoly outcome if the cost of espionage approaches 0. For future reference, the ex ante expected profits to the firms if information is revealed before pricing decisions are made are equal to $1 - (1 - \alpha)^2 = \alpha(2 - \alpha)$: firms make a profit of 1 in all cases except when both are unconstrained.

4 The Implications of Information Asymmetry

Before all else, it is important to understand in more detail how private information about capacity undermines Dudey's result. If the Dudey equilibrium would be an equilibrium in the game with uncertainty about capacity, the equilibrium path must be one where in the first period constrained firms choose a price of 1 and unconstrained firms choose a higher price. Given these first-period prices, in the second period the firms would price at 1 in all cases where at least one firm would set a price of 1 in the first period, whereas firms would price at marginal cost if it were observed that both firms would have set a price larger than 1 in the first period. This cannot be an equilibrium, however, as the unconstrained firm has an incentive to imitate the constrained firm in the first period. There are two benefits of doing so. First, the unconstrained firm has the chance of being able to sell in the first period at the maximum possible price, increasing his expected first-period profits. Second, the deviation triggers that the rival believes that the firm is constrained so that the rival will set a price of 1 in the second period, which the unconstrained firm then can undercut. This implies that also his expected second-period profit is higher as he will always sell in the second period at a price close to the maximum possible price. So, the unconstrained firm accrues extraordinary rents by imitating the constrained firm. Thus, the Dudey equilibrium requires a level of coordination between constrained and unconstrained firms that is impossible to achieve under private information.

The above argument is actually more general. In particular, separating equilibria do not exist as the type that is supposed to set the higher price wants to imitate the other type's first-period price. This is the content of our first result:

Proposition 2. *A separating equilibrium does not exist.*

Proof. The proof is given in the Appendix.

We will next construct a pooling equilibrium where both types choose the same first-period price. As this equilibrium plays an important role also in the next sections, we discuss its construction in some detail. We first focus on the second-period pricing game following such a pooling outcome in the first period. Let $\theta \in (0, 1)$ be the posterior probability that the rival is constrained given the first-period price the rival chose and let the price distributions in the second period be denoted by F^S and F^N for firm S (who *sold* in period 1) and firm N (who did *not sell* in period 1), respectively.[11] In the second period, it is common knowledge that firm N still has a unit to sell, whereas firm S is sold out with probability θ. Thus, as in Janssen and Rasmusen (2002), with probability θ firm N is a monopolist in period 2, while she faces a competitor with probability $1-\theta$.

[11] In a pooling equilibrium $\theta = \alpha$. At this stage, we want to keep the analysis at a more general level, however, as many of the equilibria that we will analyze later use a similar argument.

It is clear from their analysis that a Nash equilibrium in pure strategies does not exist and the same argument applies to our setting. By setting a price $p_2 \leq 1$ in the second period, firm N has an expected profit of $\pi_2^N(p) = \theta p_2 + (1-\theta)\left(1 - F^S(p_2)\right) p_2$: with probability θ she is a monopolist and always sells, while with the remaining probability the firm only sells if the competitor sets a larger price. As firm N gets an expected profit of θ when setting $p_2 = 1$, it is easy to see that to make firm N indifferent, the unconstrained firm S must randomize according to

$$F^S(p_2) = 1 - \frac{\theta(1-p_2)}{(1-\theta)p_2} \tag{1}$$

with $p_2 \in [\theta, 1)$. Similarly, the expected second-period profit of firm S equals $\left(1 - F^N(p_2)\right) p_2$. To make firm S indifferent between any $p_2 \in [\theta, 1)$ it must be that

$$F^N(p_2) = 1 - \frac{\theta}{p_2} \tag{2}$$

with a mass point of θ at 1. Thus, importantly, provided they still have unsold capacity left both firms make an expected profit of θ in the second period and to do so they randomize their pricing decisions in that period if the uncertainty concerning their rival's capacity is not fully resolved. As a consequence, in a pooling equilibrium a constrained firm is not willing to accept a price below α in the first period for her sole unit, as by setting $p_1(1) > 1$ and letting the other firm sell first she can expect at least α in the second period.

To finalize the construction of a candidate pooling equilibrium, we now argue that it must be that the first-period equilibrium price p_1^* equals α. It is clear that p_1^* cannot be smaller than α as a constrained firm would want to deviate upwards as she can get an expected profit of α in the second period. On the other hand, by having $p_1^* > \alpha$, the constrained firm would get an expected pay-off of $\pi^{*1} = \frac{1}{2}p^* + \frac{1}{2}\alpha$ and then would like to undercut p_1^*. Thus, it must be that $p^* = \alpha$.

We now argue that no type of firm wants to deviate from this candidate pooling equilibrium. It is clear that neither type wants to deviate upwards as they then forego the possibility to sell in the first period and the above argument on second-period pricing implies their expected profit in that period equals α as the firm that sells is constrained with probability α.[12] In addition, it is clear that the constrained firm does not want

[12]The second-period profit is determined by the belief of the deviating non-selling player regarding the probability that the non-deviating selling player is constrained. Clearly, given the selling player

to deviate downwards. Whether an unconstrained firm wants to deviate downwards depends on how one specifies the out-of-equilibrium beliefs. If she sticks to the pooling price, she expects an overall equilibrium profit of $\frac{3}{2}\alpha$. If she undercuts the first-period price by setting $p_1 < \alpha$ and induces an out-of-equilibrium belief θ', she expects a pay-off of $p + \theta'$. Thus, it is not profitable to deviate if $\theta' \leq \frac{\alpha}{2}$. Such a belief is actually very reasonable. For example, the Intuitive Criterion (Cho and Kreps (1987)) implies that $\theta' = 0$ as only the unconstrained type can possibly benefit from undercutting.

Thus, we have proved the following:[13]

Proposition 3. *There exists a unique symmetric*[14] *pooling equilibrium, where both types of firms charge $p_1^* = \alpha$. In the second period firms' prices satisfy $F^S(p_2) = 1 - \frac{\alpha(1-p_2)}{(1-\alpha)p_2}$ and $F^N(p_2) = 1 - \frac{\alpha}{p_2}$ for $p_2 < 1$ and a mass point of α at 1.*

The ex-ante expected profit in this pooling equilibrium is equal to $\alpha^2 + (1-\alpha)(\frac{\alpha}{2} + \alpha) = \alpha\frac{3-\alpha}{2}$ and strictly smaller than $\alpha(2-\alpha)$, which, as we have seen in the previous section, is the profit under complete information. This is an important step in our overall claim that under private information, firms have less market power than under full information.

4.1 Semi-separating Equilibria

We will now argue that, in addition to the pooling equilibrium, there exists a continuum of semi-separating equilibria where firms choose different mixed strategies in the first period, depending on their capacity. As a result, firms imperfectly learn from first-period prices as different first-period prices are associated with different posterior beliefs and, consequently, different outcomes in the second period.

We let $\theta(p)$ denote the posterior belief that a firm that charged p in the first period was constrained and we let $Q(p)$ denote the probability that such a firm sells in the first period. Notice that $\theta(p)$ is determined on the equilibrium path by the probability distributions that each firm type uses. The expected profit of a constrained firm is

chose his equilibrium price in the first period, Bayes' Rule implies this belief should be equal to α.

[13] Under the alternative tie-breaking rule (see footnote 8), the equilibrium has the constrained types charge a price of 1 and expect to make $\alpha/2$ while the unconstrained types mix in $[\alpha, 1)$ and expect to make α. The expected profit of the constrained type is α and that of the unconstrained type is $3\alpha/2$.

[14] There also exist two asymmetric pooling equilibria, in which independent of their type one of the firms charges α and the other mixes in the interval $(\alpha, \alpha + \varepsilon)$. This equilibrium is outcome equivalent to the pooling equilibrium.

$$\Pi^c = Q(p)p + \int^p \theta(p')d(1 - Q(p')). \tag{3}$$

If the rival firm charges some price $p' > p$, the constrained firm will sell and exit. Otherwise, she will continue and get a profit that depends on the posterior belief about her rival's residual capacity. It can be shown that the equilibrium profit of the constrained firm is indeed the same as in the pooling equilibrium, equal to the prior belief α, which must also be the lower bound of the price distribution.

The expected profit of an unconstrained firm is

$$\Pi^u = Q(p)(p + \theta(p)) + \int^p \theta(p')d(1 - Q(p')) + Q(p)\theta(p). \tag{4}$$

The unconstrained firm's profit includes the additional value of holding an extra unit. This value is the continuation profit conditional on selling at the chosen price. In a semi-separating equilibrium, for all prices in the support, the profit of the constrained and the unconstrained firm is constant. This implies that $Q(p)\theta(p)$ must be equal to a constant, and so the posterior belief must be an increasing function of the price (since the quantity is downward-slopping). Since the profit of this additional unit must be positive, the unconstrained firm will only charge prices that sell with positive probability in the first period. As a result, the upper bound of the price distribution must have an atom.

Using these insights and the randomization conditions, we are able to characterize the equilibrium strategies. The results are summarized in the following proposition:

Proposition 4. *All semi-separating equilibria of our model are characterized by a $\overline{p} \in (\alpha, 1]$ such that both types randomize their first-period prices over the interval $[\alpha, \overline{p})$ with a mass point at $\overline{p}$.*

The constrained type's equilibrium profit equals α, while the unconstrained type's profit equals $\alpha\left(1 - \left(W_{-1}(\frac{2\alpha}{e^2\overline{p}})\right)^{-1}\right)$, which is decreasing in $\overline{p}$, where W_{-1} is the negative branch of the Lambert-W function.[15] *These equilibria are Pareto-ranked and inferior, from the firms' perspective, to the pooling equilibrium.*

Proof. The formal proof for this result can be found in the Appendix.

The result follows from three basic observations. First, no equilibrium exists in which both types of firms choose prices on distinct supports, as (similar to a separating

[15] Recall that the Lambert-W function is such that $W(xe^x) = x$.

equilibrium) the unconstrained firm would have an incentive to deviate and mimic the constrained type. Second, as discussed in the beginning of this subsection lower prices must induce lower posteriors to keep firms indifferent across different first-period prices in the interval $[\alpha, \overline{p})$. Third, the mass point at the upper bound of the distribution must induce a posterior belief exactly equal to the price. This is because the firm must be indifferent between selling at price $\overline{p}$ and losing the tiebreak to another firm selling at the same price.

Depending on $\overline{p}$, the semi-separating equilibria can reveal more or less information. The smaller $\overline{p}$, the less information is revealed, with the pooling equilibrium (which can be regarded as the limit of the semi-separating equilibria for $\overline{p}$ converging to α) revealing no information. Equilibria with a wider range of first-period prices induce a higher chance that an unconstrained firm sells in the first period, leading to a more informative second-period's posterior beliefs and firms assessing there is a higher probability of competition in the next period and thus to lower profits. That is, information revelation through market outcomes is detrimental for firms' profits since it necessarily involves a sort of miscoordination: unconstrained firms are more likely to sell early. In the proof in the appendix we show that a greater $\overline{p}$ induces lower ex-ante profits, as the impact of a lower expected posterior in the second-period profit outweighs the (stochastic) increase in first-period prices brought about by a higher upper-bound.

We now establish some results that allow us to make comparisons across equilibria. For this reason, let us refer to E1 as the equilibrium associated with $\bar{p}_1$ and E2 as the equilibrium associated with $\bar{p}_2$, with $\bar{p}_2 > \bar{p}_1$. The following comparisons can be made:

Proposition 5. *When compared with E1, E2 displays*

1. *a more informative first-period price distribution,*
2. *a higher probability that the unconstrained firm sells in the first period,*
3. *a lower expected second-period price.*

These results are illustrated in Figure 1 where we depict the ex-ante expected profit and the odds-ratio measuring the likelihood that the constrained firm sells relative to the prior in each equilibrium for $\alpha = 0.2$. The Figure shows that the odds ratio is quickly decreasing in $\overline{p}$. If $\overline{p} = \alpha$, the odds ratio is 1 as both types are equally likely to sell, but if $\overline{p} = 1$, it is approximately equal to only 0.33 so that the unconstrained firm is disproportionately much more likely to sell. This results in firms assessing it is much more likely that there is competition in the second period, resulting in lower expected second-period prices and profits. The Figure also shows that as a result the ex-ante expected profit *over both periods* is decreasing from 0.56 to 0.47 when $\bar{p}$ ranges from α to 1, amounting to a 16% decrease.

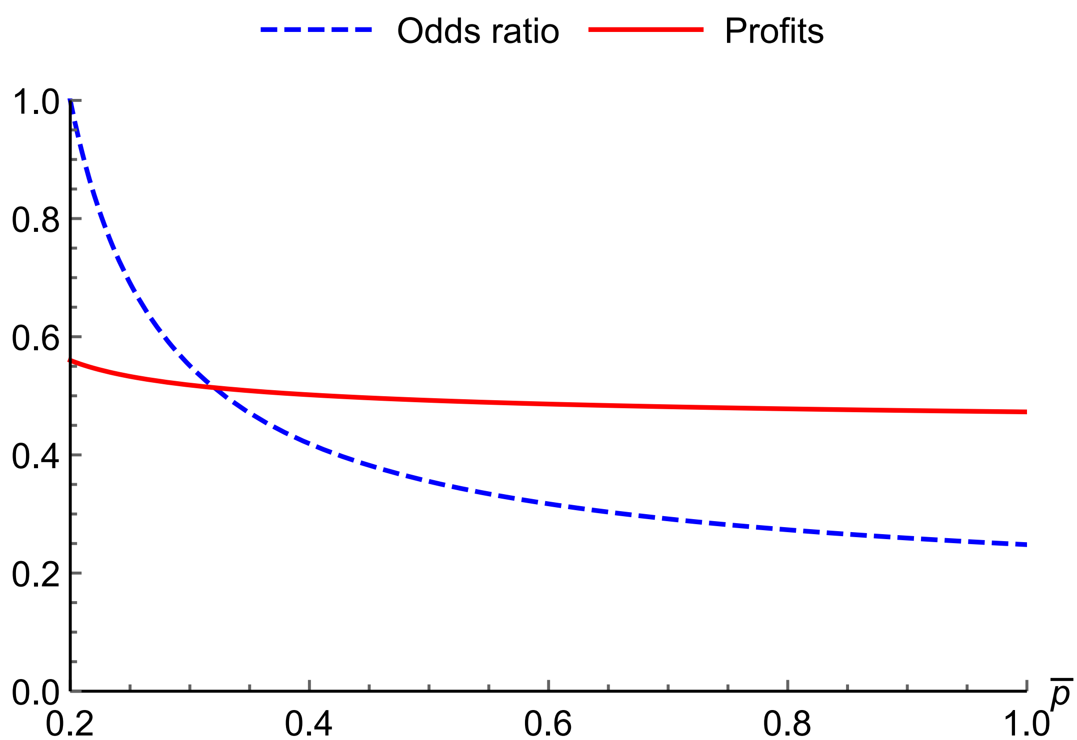

Figure 1: This plot establishes the ex ante expected profit (red) and the relative likelihood of an unconstrained firm selling in the first period (blue, dashed) in different equilibria (as a function of $\bar{p}$). Calibrated for $\alpha = 0.2$.

4.2 Empirical and Welfare Implications

Now that we have characterized all equilibria of the game with private information, we discuss the implications. First, from the above it is clear that Dudey (1992) overstates the market power of firms engaging in dynamic competitive pricing: the pooling equilibrium under private information yields already lower profits and the semi-separating equilibria result in even lower profits. Thus, the ratio of ex ante equilibrium profits under private information and under complete information equals at most $\frac{\alpha(3-\alpha)}{2\alpha(2-\alpha)}$. For small values of α this ratio is close to 75%, implying that firms experience significant losses if they cannot condition pricing behavior on their rival's capacity. This sheds an important light on the incentives for information exchange regarding firms' capacities.

Even if they price unilaterally, firms can gain significantly if they exchange information about capacities so that rivals' capacities are common knowledge. As consumers have unit demand and are served in any equilibrium, it follows that ex ante expected prices are lower and consumer surplus is higher under private information.

Second, if we measure equilibrium price dispersion by the interval of prices that can be charged along the equilibrium path, then price dispersion is larger in the second period than in the first period. This is obviously true in the pooling equilibrium, since there is no price dispersion in the first period, but it also holds true in every semi-separating equilibrium. In a semi-separating equilibrium, the interval of possible equilibrium prices in the first period is equal to $[\alpha, \overline{p}]$, while in the second period it is $[\theta(p_1), 1]$, where $\theta(p_1)$ is the posterior belief that the firm that sold in the first period is constrained given that it sold at p_1. The interval of second-period prices follows from the discussion of the second-period pricing above. As in any semi-separating equilibrium $\overline{p} \leq 1$, the result on price dispersion follows if $\theta(p_1) \leq \alpha$. But as the expected pay-off of a constrained firm must be equal to α in any semi-separating equilibrium, and $\theta(p_1)$ is also the expected second-period profit, it must be that $\theta(p_1) < \alpha$ as otherwise a constrained firm will prefer setting $p_1 > \alpha$. The same results holds true for other measures of dispersion, such as variance, both in the pooling and semi-separating equilibria.

Figure 2 presents the first- and second-period price distributions for the case where $\alpha = 0.2$ and $\overline{p} = 0.35$.[16] The Figure clearly shows not only that the interval of prices that can be charged along the equilibrium path is larger in the second period than in the first, but also that first-period prices are more concentrated if other measures of price dispersion are used. In fact, one can also show that a property the figure displays, namely that the first- and second-period price distributions intersect only once, is always true so that the second-period price distributions have more mass both to the left and to the right of this intersection point. Thus, based on our model, an explanation for the observed increase in price dispersion towards the deadline is that in the first period(s) of competition firms try not to signal their capacity levels by choosing similar prices independent of their types. The pressure to sell (just) before the deadline, together with the remaining uncertainty regarding firms' types results in more price dispersion in later periods as suggested by the empirical evidence discussed in the Introduction.

Third, our model explains that flights that are expected to be peak do not have more

[16]More precisely, for every $p_1 \in [\alpha, \overline{p}]$ there is a different price distribution that is characterised by $\theta(p_1)$. When we talk in this paragraph about the second-period price distribution, we mean the average second-period price distribution, where the average is taken over all possible $\theta(p_1)$'s.

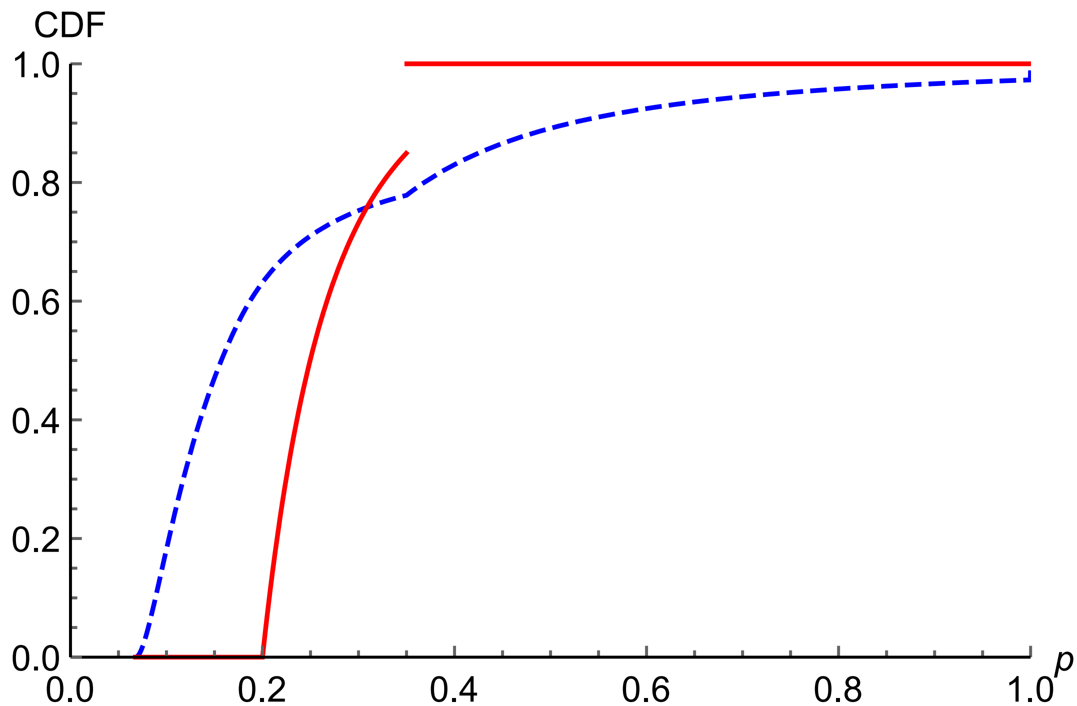

Figure 2: First- (red, solid) and second-period (blue, dashed) price distributions. Calibrated for $\alpha = 0.2$ and $\overline{p} = 0.35$.

dispersion than those that are expected to be off-peak (Puller et al. (2015)) as follows. Whether or not a flight is expected to be peak is captured by the prior probability α a firm is constrained: the higher α, the more likely it is the flight is peak. If α is large, then there is no reason for firms to strongly compete and prices in both periods are close to the monopoly price with little price dispersion. Conversely, if α is small, then firms compete severely in both periods and all prices are close to marginal cost: even if the second-period distribution ranges from α to 1 almost all probability mass is close to α. Instead, when α is intermediate, the second period price distribution displays high variance and the first-period price lies at the lower bound of this distribution. Thus, our model predicts that there is no clear monotonic relationship between price dispersion and peak and off-peak flights. Figure 3 depicts the overall price dispersion across periods as a function of α in the pooling equilibrium.

Fourth, empirical evidence also suggests that transaction prices, *i.e.*, the prices at which consumers buy the good, tend to be higher towards the deadline. This is obviously true in the pooling equilibrium as the first-period price equals α, while second-period prices are distributed over the interval $[\alpha, 1]$. For semi-separating equilibria with $\overline{p}$ close to α the same holds true. However, other semi-separating equilibria exhibit

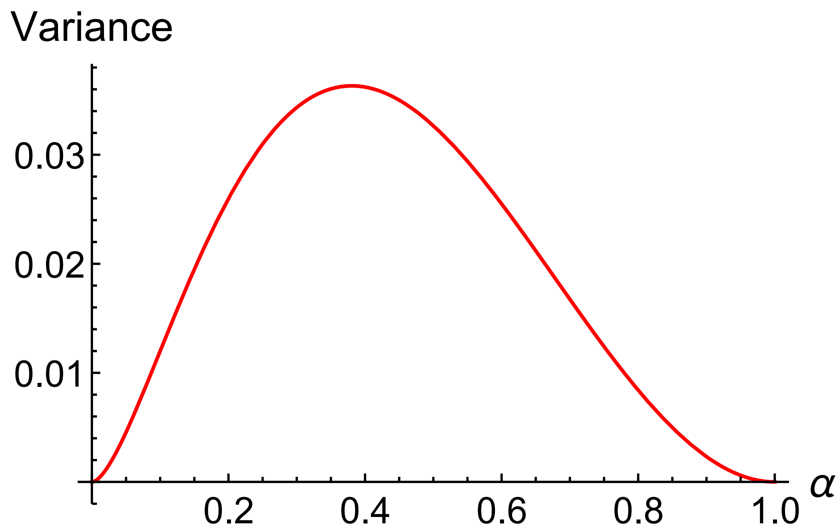

Figure 3: The variance of transaction prices in both periods in the pooling equilibrium for different levels of α.

the opposite feature. Thus, the empirical evidence is consistent with our model, but not strictly implied by it. However, as firms' profits are inversely related to $\overline{p}$ one may argue that first-period prices are lower as firms are likely able to coordinate on the equilibria that are most profitable for them. Interestingly, there is an intimate connection between firms' profits and the evolution of transaction prices over time. A simple statistic summarizing the evolution of transaction prices is $\mathbf{E}[p_2 \mid p_1]$, *i.e.*, the expected second-period price given the first-period's price. It turns out that the expected second-period price is $(2-\theta(p_1))\theta(p_1)$,[17] which depends on the posterior belief. In the pooling equilibrium, this posterior is equal to α and so the expected price is $(2-\alpha)\alpha > \alpha$. That is, prices follow a sub-martingale and we know that the pooling equilibrium is the best possible equilibrium from the firms' perspective. Instead, in the worst semi-separating equilibrium, where $\overline{p} = 1$, each price in $[\alpha, 1)$ is associated with a posterior $\theta(p_1) \leq \frac{1}{2}p_1$ and so $(2-\theta(p_1))\theta(p_1) < p_1$. In addition, since $\theta(1) = 1$, we have that $\mathbf{E}[p_2 \mid 1] = 1$. As a result, for all $p_1 \in [\alpha, 1]$, $\mathbf{E}[p_2 \mid p_1] \leq p_1$ so that prices follow a super-martingale in the worst possible equilibrium from the firms' perspective.[18] This

[17] This can be seen as follows. First, with probability $\theta(p_1)$, only one firm has unsold capacity left in the second period and in that case the expected transaction price is simply the expected price given that the CDF of prices is given by (2), *i.e.*, $\theta(p_1)(1-\ln(\theta(p_1)))$. With the remaining probability $1-\theta(p_1)$ there are two firms with unsold capacity in the second period. Using (1) and (2) the CDF of the minimum price in that case is given by $1 - \frac{\theta(1-p_2)}{(1-\theta)p_2}\frac{\theta}{p_2}$. The second-period transaction price follows by adding these different expressions.

[18] Across semi-separating equilibria, if the first-period price is competitive (below the mass point), the continuation price is expected to fall further. Instead, if the market price is not competitive (at

is illustrated in Figure 4, depicting expected first- and second-period transaction prices as a function of $\overline{p}$ for $\alpha = 0.2$. The figure shows that in the pooling equilibrium the expected first-period price is close to two times the expected second-period price, but that this ratio changes quickly when $\overline{p}$ increases and that when $\overline{p}$ is close to 1 expected second-period transaction prices can be in the order of four times larger than first-period prices!

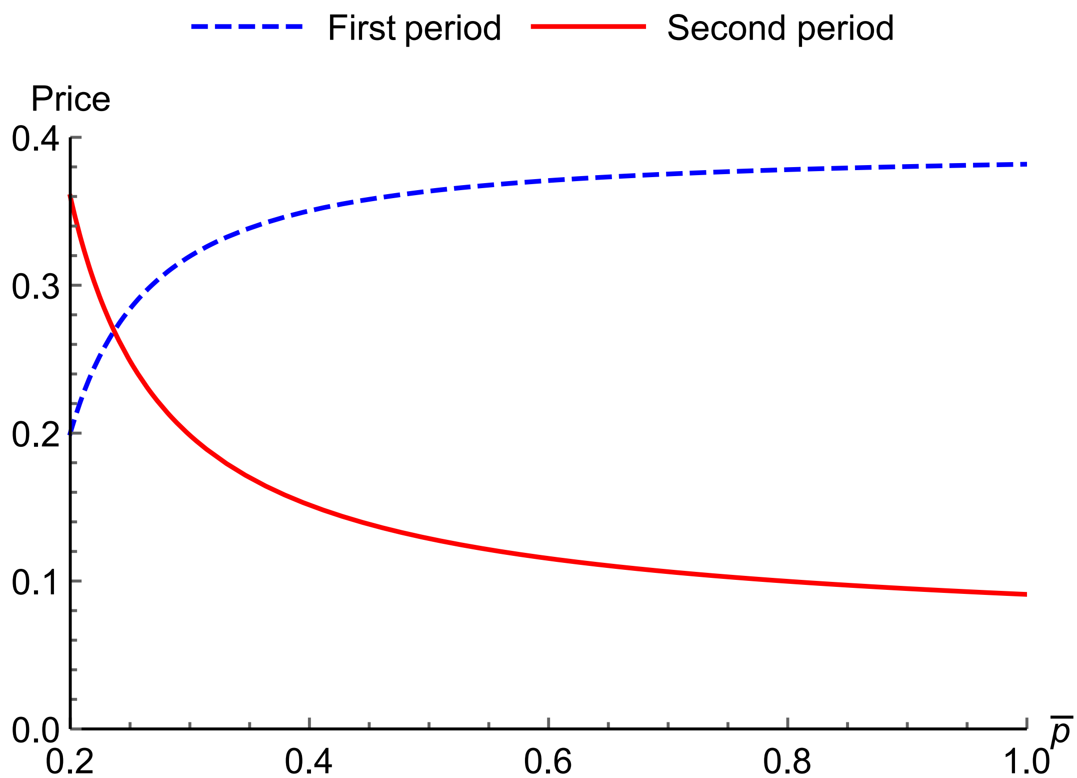

Figure 4: This plot shows the ex ante expected first- (blue, dashed) and second-period (red, solid) transaction prices in different equilibria (as a function of $\bar{p}$). Calibrated for $\alpha = 0.2$.

Finally, there is a novel empirical implication of our model that, to the best of our knowledge, has not been tested yet, namely, that prices are positively correlated over time, even conditional on type, *i.e.,* higher first-period prices induce higher beliefs and, as a result, higher second-period prices. This implication follows from the fact that, for a given $\overline{p}$, $\theta(p_1)$ is increasing in p_1: higher prices are more likely to be set by constrained firms as they do not benefit from selling in the first period as when they do, they cannot sell again in the second period. As $\theta(p_1)$ is the lower-bound of

the mass point) the continuation price is expected to raise even further.

the distribution of second-period prices, a higher $\theta(p_1)$ means a first-order stochastic dominance shift of second-period prices.

5 Disclosure

In this section we allow firms to disclose their private information about capacity and investigate whether such disclosure may increase market power and thus, be anti-competitive. As disclosure requires investing in technology to be able to do so, disclosure is a long-term decision that is taken before firms learn their type. If a firm discloses the rival knows the capacity of the disclosing firm precisely.

It is clear that when the disclosure cost is too high, neither firm wants to disclose and the equilibrium analysis of the previous section remains valid. Therefore, in this section we mainly focus on disclosure costs being small enough so that at least one firm will want to deviate from the equilibrium of the previous section and disclose. To this end, we start the analysis by assuming that one firm discloses her private information, while the other does not, and consider the impact disclosure has on the pricing strategies of firms. We then evaluate the overall profit the disclosing and non-disclosing firm make. We conclude that when the disclosing cost is small enough there only exist asymmetric equilibria where one firm discloses.

Consider first the situation where the disclosing firm is constrained. The continuation game is essentially identical to the complete information game in which one firm is constrained and the other is unconstrained. As we described earlier, equilibrium requires the disclosing firm to sell in the first period at a price p^* and her rival to sell in the second period at a price $p = 1$. Unlike the complete information game, equilibria where the first-period price is smaller than α do not exist as due to her ignorance about the rival's capacity, the disclosing firm has an expected profit of at least α. Similar to the complete information game, there is a continuum of equilibria in weakly dominated strategies and the only equilibrium in undominated strategies is the Dudey equilibrium with $p^* = 1$. It is on this equilibrium that we will focus.

The more interesting situation is where the disclosing firm is unconstrained. *If* she sells in the first period, it is common knowledge that both firms have capacity left in the second period and Bertrand competition results in marginal cost pricing. But this would leave the non-disclosing firm without profit in any period, which cannot be an equilibrium outcome. It follows that in equilibrium both types of the non-disclosing firm sell in the first period and set the same price. Using the second-period results of the pooling equilibrium discussed in the previous section reveals that both firms make an expected profit of α in the second period and the unconstrained non-disclosing firm can expect an overall profit of 2α. This outcome can be sustained with the following strategies. Following Blume (2003) the disclosing firm uniformly randomizes over the interval $(\alpha, \alpha+\varepsilon)$ in the first period, while both types of the non-disclosing firm charge α. In the second period, the disclosing firm has a posterior belief of α that his rival is constrained and both firms choose prices according to (1) and (2) as in the second period of the private information game.[19]

Combining the two cases, it is easy to see that the disclosing firm makes an expected ex ante profits of $\alpha p^* + (1-\alpha)\alpha - d$ where d is the disclosure cost: with probability α she is constrained and makes a profit of $p^* \in [\alpha, 1]$ in the first period, whereas with the remaining probability she is unconstrained and makes an expected profit of α in the second period. On the other hand, the non-disclosing but informed rival makes an expected ex ante profits of $\alpha \cdot 1 + (1-\alpha)(\alpha^2 + (1-\alpha)\cdot 2\alpha) = \alpha(3-3\alpha+\alpha^2)$: if the disclosing firm is constrained he makes a profit of 1 in the second period, while if the disclosing firm is unconstrained his overall profits are either α or 2α, depending on whether or not he himself is constrained.

Interestingly, the non-disclosing firm's profit exceeds the maximal pay-off she can get in the complete information game and, hence, she would never want to deviate. The reason is that she makes more profit if the rival is unconstrained. In that case, the disclosing firm lets him sell in the first period and also creates the opportunity for the non-disclosing firm to sell in the second period as the disclosing firm is uninformed about the rival's capacity. In addition, the non-disclosing firm earns more profits than the disclosing firm, regardless of the equilibrium that is being played. Therefore, equilibria exist in which only one firm discloses if the disclosure cost is sufficiently small ($d < \frac{\alpha(1-\alpha)}{2}$) so that the disclosing firm makes more profit than in the game under private information.

We summarize the above discussion in the Proposition below.

Proposition 6. *If $2d < \alpha(1-\alpha)$, then the disclosure game has a unique equilibrium in undominated strategies in which one firm discloses and obtains $\alpha(2-\alpha) - d$, while her rival obtains $\alpha(3-3\alpha+\alpha^2)$. If $d > \alpha(1-\alpha)(1-\zeta)$, where $\zeta \in (0, \frac{1}{2})$*[20] *all equilibria*

[19] This environment is strategically equivalent to a market with asymmetric priors. In such a case, the firm with a lower prior sells in the first period at a price equal to her prior.

[20] In the proof of Proposition 4 in the appendix, we show that the lowest possible equilibrium pay-off

involve no disclosure and are outcome equivalent to the incomplete information game. For intermediate values, asymmetric disclosure and no disclosure equilibria co-exist.

The operating profits of disclosing, given by $\alpha(2-\alpha)$, are strictly larger than the maximal pay-off in the private information game analyzed in the previous section. Thus, for small enough disclosure cost, it is optimal for one firm to disclose. The multiplicity of equilibria arises for intermediate disclosure cost and stems from the multiplicity of equilibria studied in the previous section. As the private information game has multiple semi-separating equilibria with different pay-offs, the decision whether or not to disclose depends on a firm's expectations regarding which equilibrium they will play if she does not disclose.

Notice that all of these equilibria are less competitive than the incomplete information benchmark. More importantly, the disclosure equilibrium induces higher prices than in the complete information outcome! Therefore, allowing *voluntary* information disclosure in this market is anti-competitive, and even more so than mutual information exchange, and accordingly severely harms consumers. From a competition policy perspective, our analysis indicates that there may be good reasons to forbid firms to reveal their capacities in a dynamic pricing setting where capacity constraints are important. The disclosing firm mainly gains in all cases where she is capacity constrained, while the additional loss of not having private information when she is unconstrained is insufficient to dominate this gain. The non-disclosing firm is mainly free-riding on the disclosure decision of her rival and benefits especially if she is unconstrained.

6 Industrial Espionage

We now consider the incentives of firms to spy on the rival and to secretly learn his capacity. Industrial espionage is, in a sense, the reverse of voluntary disclosure as the one who incurs the spying cost c is the one who learns the capacity of the rival. If there exists equilibria with industrial espionage, then the firm expecting to be spied upon also expects the rival to know her capacity. As in the previous section, we model espionage as a long-term endeavor that is decided upon before pricing decisions are made and it delivers certainty about whether or not the rival is constrained. Importantly, and unlike

of a semi-separating equilibrium under private information is given by $\alpha^2+(1-\alpha)\alpha(1+\zeta)$, where $\zeta=-\left(W_{-1}\left(-\frac{2\alpha}{e^2}\right)\right)^{-1}$ and W_{-1} is the negative branch of the Lambert function. As the highest possible equilibrium pay-off of a disclosing firm equals $\alpha+(1-\alpha)\alpha-d$ there cannot exist a disclosure equilibrium if $\alpha+(1-\alpha)\alpha-d<\alpha^2+(1-\alpha)\alpha(1+\zeta)$, which gives the condition stated in the Proposition.

the previous section, the act of spying cannot be observed so that it is the expectation of being spied upon rather than the act itself that affects a firm's behavior.

It is not difficult to see that for any $c > 0$, there cannot be an equilibrium where both firms spy for sure. If they would do so, the equilibrium outcome of the complete information game would result with the firms setting price equal to marginal cost if and only if both turn out to be unconstrained. But then one of the firms can obtain the same operating profits without incurring the spying cost, by setting a first-period price of 1 if constrained and a larger price if unconstrained. If the rival is constrained, a firm can then anyway obtain a profit of 1 (possibly in the second period), while if the rival is unconstrained she also obtains a profit of 1 if constrained. This result should not come as a surprise: there is something secretive about spying, and one may think that a firm may not want to act in such a way that the rival expects her to engage in industrial espionage for sure.

Similar to the previous section, when the spying cost is large an equilibrium without industrial espionage exists, whereas if the spying cost is smaller, but not too small, an asymmetric equilibrium exists where one firm spies and the other does not. We provide the details of the threshold values where these equilibria exist in Appendix A. Equilibrium play in this equilibrium mimics that of the asymmetric equilibrium in the disclosure game - one firm is informed about her rival's type and the other knows this. It follows from Section 5 that the ex-ante expected candidate equilibrium profits are equal to $\alpha(3 - 3\alpha + \alpha^2) - c$ and $\alpha(2 - \alpha)$ for the spying (informed) and non-spying (uninformed) firm, respectively. Since the non-spying firm has a lower equilibrium payoff, she may be tempted to secretly spy on her rival and use this information in the second period to increase her profit. This puts a lower bound on the spying cost for this pure strategy asymmetric equilibrium to exist.[21]

Important new considerations apply when the spying costis small and we now turn our attention to constructing an equilibrium in this range where one firm spies for sure, while the other firm spies with some probability $0 < \beta < 1$. We refer to the latter firm as the mixing firm. If this mixing firm is constrained, this event is common knowledge and this firm will sell at a price not lower than the prior.

When the mixing firm is unconstrained, instead, equilibrium play dictates that the rival (spying) firm sells in the first period. Continuation play will then depend on the

[21] Note that the non-disclosing (informed) firm in the previous section, would never want to deviate as she makes the most profit, and by disclosing she does not get more information itself. As the roles are reversed, this does not hold for the non-spying (uninformed) firm.

capacity of the rival and the information gathered by the mixing firm. What is of special interest in the second period is that if the mixing firm spied, she charges the monopoly price if her rival is constrained and charges lower, but still strictly positive, prices otherwise. If both firms spy and are unconstrained, both firms know that there is enough unsold capacity in the market to result in severe competition, *i.e.*, marginal cost pricing in both periods. However, the firm that is supposed to always spy does not know that the mixing firm (spied and) knows that she is unconstrained. As both the constrained and unconstrained spying firm sell at the same first-period price (and learn nothing about the rival), an unconstrained spying firm believes with probability β that the mixing rival still has his posterior of her being constrained equal to the prior α, and thus is setting positive prices, therefore the spying unconstrained firm reacts correspondingly by charging positive prices as well. *Thus, even if both firms are unconstrained and know that the rival is unconstrained, they may escape marginal cost pricing as it is not common knowledge that both are unconstrained.*

In the appendix we show the following proposition holds, which essentially says that the candidate equilibrium we outlined above is indeed an equilibrium:

Proposition 7. *If $0 < c < \alpha(1-\alpha)^2$, then there exists an asymmetric equilibrium where one firm engages in industrial espionage for sure, while the other randomizes between spying and not spying. The randomizing firm has the same ex ante expected profit as under complete information, while the spying firm's profits is smaller. As the cost of spying approaches 0, the spying probability of the mixing firm approaches 1, but the ex ante expected equilibrium profit of the firm that always engages in industrial espionage converges to α, which is smaller than that under the Dudey outcome.*

Notice that as explained in Section 3 there is a continuum of equilibria of the complete information game. Thus, even though the asymmetric equilibrium we constructed here does not converge to the "Dudey outcome", it does converge to one of the equilibria of the complete information game. In this equilibrium the firm that always engages in industrial espionage has an expected profit of $\alpha + \frac{c}{1-\alpha}$, which is increasing in the cost of spying. The reason is that the rival firm's probability of spying decreases and that because of this the spying firm makes a higher expected profit in the second period, which also translates into a high first-period price in case the rival is unconstrained.

In this equilibrium, if c approaches 0 the ex ante expected pay-off of the spying firm converges to α, which is considerably below the pay-off of the mixing firm and also the ex-ante expected pay-off of the complete information game, namely $\alpha + (1-\alpha)\alpha$.

Interestingly, it is also lower than a firm's ex ante expected profit in any of the equilibria under private information. Calculating the average pay-off of a firm in this equilibrium as $\frac{1}{2}\alpha + \frac{1}{2}(1-\alpha)\alpha$, it is clear that this is strictly smaller than firms' ex ante expected profit in the pooling equilibrium under private information, $\alpha\frac{3-\alpha}{2}$. Thus, firms may prefer the interaction under pure private information so that no firm spies. Conversely, consumer may actually be better off with firms spying on each other.

An open question remains as to whether other equilibria supporting different outcomes exist. In particular, a symmetric equilibrium may exist whereby both firms choose to spy with some positive probability. In such an equilibrium, firms will mix in overlapping intervals depending on both their type and their information. As the cost of spying converges to zero, it may well be that these price distributions collapse to the prices in the Dudey outcome.

7 Discussion and Conclusion

This is the first paper that performs an equilibrium analysis of dynamic competitive pricing in markets for time-dated products (such as hotel rooms, airline flights, shipping, generated electricity), where firms have private information about their unsold capacities. To make such an analysis feasible, we focus on the simplest possible model with two firms and consumers arriving in two periods only. Despite its simplicity, the equilibrium analysis is intricate and the results yield interesting insight into existing empirical observations. We show that the existence of private information considerably restricts the market power that firms have in such situations and that information exchange of private information regarding unsold capacity is anti-competitive and detrimental to consumer welfare. We also show that the model can explain observed pricing patterns. In particular, our model provides an explanation for increasing prices and price dispersion as the deadline approaches. We extend the analysis by considering the private incentives to voluntary disclose private information or to engage in industrial espionage. Surprisingly, from a consumer welfare perspective, one-sided voluntary disclosure is even worse than mutual information exchange as the disclosing firm is able to get the full information pay-offs, but it is especially the non-disclosing firm that benefits from the additional information regarding the rival's capacity while maintaining uncertainty regarding his own capacity. This raises the question whether voluntary disclosure of unsold capacity should be forbidden from a competition policy perspective. Industrial espionage is in some sense similar to ”reverse voluntary disclosure”, with the difference being that it is a secretive act that creates private rather than public information. One important result we obtain in this context is that if the spying cost is sufficiently small an equilibrium exists where consumers benefit from industrial espionage compared to the private information case.

From a policy perspective, our work is also related to the recent literature studying potential channels in which (pricing) algorithms may impact competition (Assad et al. (2020), Harrington (2018) and Calvano et al. (2019, 2020)). As indicated above, one part of our analysis shows that it is not only essential for firms coordinating their pricing that they are informed about their rivals' capacity but that that it is known by their rivals that they know. When it is commonly known that competitors use similar algorithmic tools, one possible implication of our research is that the use of these tools may lead to a substantial increase in prices and a reduction in consumer surplus.

Our analysis points at many angles for future research. Obvious extensions include studying markets with more than two firms, and/or more than two periods. This should be done in such a way that there remains a possibility that in later periods firms may enjoy monopoly power. For example, if the strategic interaction of more than two firms is performed in a two-period model, then one should include the possibility that a firm is already sold out before the pricing game starts as otherwise, it is common knowledge that there will be competition in all periods. Another obvious angle for future research is to investigate our conjecture at the end of Section 6 and to inquire into the existence of a symmetric equilibrium where both firms randomize their decision to engage in industrial espionage and to study whether the equilibrium converges to the Dudey equilibrium outcome if the cost of industrial espionage becomes negligible. Less obvious, but equally important, would be to study more realistic demand behaviours. In our model, demand in every period is perfectly known to both firms and consumers cannot postpone or prepone their purchasing decisions. One obvious consideration is that the pricing behaviours in our paper point to the fact that consumers cannot gain anything from postponing their purchasing decisions as expected prices and price dispersion are increasing towards the deadline. In a monopoly context, more realistic demand behaviours have been studied by (Hörner and Samuelson (2011) and Board and Skrzypacz (2016)) and it would be interesting to see to what extent these behaviours can be incorporated into a setting where there is competition between firms with privately known capacities.

A Equilibria when the Cost of Espionage is not Small

In this Appendix, we provide more details of two equilibria of the game with industrial espionage discussed in Section 6. In particular, we establish two propositions dealing with large and intermediate spying costs, where the next two subsections first investigate for which values of c one can sustain the equilibrium outcomes of the previous two sections when firms have the possibility to spy on each other.

A.1 Equilibria without Spying

It is clear that for large enough spying cost, firms will not engage in industrial espionage. To determine boundary costs where spying is not profitable, we compare the candidate equilibria under spying with the pooling equilibria under private information. This keeps the analysis focused without qualitatively affecting the results. In order to establish the cutoff value of c such that no spying constitutes an equilibrium, we need to derive the optimal strategy of a firm that decides to spy her rival when is not expected to do so. If the deviating firm discovers that her rival is unconstrained, she has no profitable deviation and will charge α in both periods (if she has available capacity in the second period), obtaining $\frac{3}{2}\alpha$ if unconstrained and α if constrained. If, on the other hand, she discovers the rival is constrained the optimal pricing depends on the value of α. Clearly, she could guarantee herself monopoly profits in the second period by letting the rival sell for sure in the first period (by deviating to a higher price). If she herself is, however, unconstrained, she may stick to the equilibrium (pooling) pricing strategy, selling with probability 0.5 in the first period, yielding an overall expected profit of 2α, or, if she does not sell, which also happens with probability 0.5, make monopoly profits in the second period. clearly, the overall profit of $\frac{1+2\alpha}{2}$ is larger than 1 if $\alpha > 0.5$. The associated profit of spying is therefore

$$(1-\alpha)\alpha\frac{3-\alpha}{2} + \alpha\left[\alpha \cdot 1 + (1-\alpha)\cdot \max\{1, \frac{1+2\alpha}{2}\}\right] - c,$$

where the first term reflects the profit if the rival is unconstrained, and the second - if the rival turns out to be constrained.

Thus, it is not optimal to deviate if

$$c > \alpha(1-\alpha)\left[\max\left\{1, \frac{1+2\alpha}{2}\right\} - \frac{\alpha}{2}\right] \equiv c_N$$

yielding the following result:

Proposition 8. *If $c > c_N$, there exists an equilibrium where firms choose not to spy and price as in the pooling equilibrium of Proposition 3.*

In case that the continuation equilibrium is semi-separating, the value of c_N would be modified slightly since (i) the equilibrium profit of a non-spying unconstrained firm falls, and (ii) the equilibrium profit of an unconstrained firm who learns that her rival is unconstrained also falls. It can be shown that the net benefit from spying increases as $\overline{p}$ increases. In the limit case in which $\overline{p} = 1$, the corresponding cutoff value is $(1-\alpha)\alpha$.

A.2 Asymmetric Equilibria for Intermediate Spying Cost

Next, we consider whether, and if so under what conditions, an asymmetric equilibrium exists where one firm spies and the other does not. The equilibrium play in such an equilibrium mimics the one under disclosure with the roles reversed. It follows from Section 5 that the ex-ante expected candidate equilibrium profits are equal to $\alpha(3-3\alpha+\alpha^2)-c$ and $\alpha(2-\alpha)$ for the spying and non-spying firm, respectively.

To see for which values of c these strategies constitute an equilibrium, we should verify whether the spying and non-spying firms have incentives to deviate. If a firm is supposed to spy and she does not, she will optimally choose a first-period's price of α, yielding α if constrained and (for out-of-equilibrium beliefs satisfying the Intuitive Criterion) $\alpha(2-\alpha)$ if unconstrained. It follows that the firm that is expected to spy will do so if $c \leq \alpha(1-\alpha)$. Similarly, if the firm that is supposed not to spy deviates and spies, she may tailor her price to the information. Given equilibrium play, this ability is useless if she has only one unit, as she would have obtained monopoly profits regardless. If, instead, the deviating firm is unconstrained she will allow her rival to sell in the first period and reap the monopoly rents in the second period, increasing her expected pay-off by $\alpha(1-\alpha)^2$. It then follows that

Proposition 9. *If $\alpha(1-\alpha)^2 \leq c \leq \alpha(1-\alpha)$, an asymmetric equilibrium exists where one firm spies for sure and the other does not. The operating profit of the spying firm* *is larger than that of the non-spying firm, but the difference is smaller than the spying cost.*

Interestingly, as $c_N < \alpha(1-\alpha)$, there exists a range of c values such that the asymmetric equilibrium characterized in this section co-exist with the equilibrium where no firm spies. It is clear that industrial espionage by one firm increases the market power of both firms and that it is therefore anti-competitive and bad for consumer welfare. Also of interest is that the non-spying firm always makes a profit of $\alpha(2-\alpha)$, which is the ex ante equilibrium profit if the uncertainty is revealed before prices are chosen. A firm does not need to know her rival's capacity: it is enough that the rival knows her capacity! Finally, as the equilibrium only exists for $c \geq \alpha(1-\alpha)^2$ it is clear that in this equilibrium the profit $\alpha(3-3\alpha+\alpha^2)-c$ of the spying firm is smaller than the profit $\alpha(2-\alpha)$ of the non-spying firm.

B Proofs of Propositions

Proof of Proposition 2. Suppose an unconstrained and a constrained firm set different prices in the first period, with $p_{i1}(1) = p_1$ and $p_{i1}(2) = p_2$. There can be two cases: $p_1 < p_2$ or $p_1 > p_2$. In the first case, $p_1 < p_2$, the constrained firm is going to sell in the first period against an unconstrained rival for sure. If an unconstrained firm sets p_2 in the first period, she has an expected profit of $\alpha + \frac{1-\alpha}{2}p_2$. However, if she imitates the other type's strategy, when selling in the first period, she "fools" both types of rivals and undercuts slightly in the second period. This gives her in expectation $1 + \frac{2-\alpha}{2}p_1$. This deviation is not profitable as long as $p_2 \geq \frac{2-\alpha}{1-\alpha}p_1 + 2$, which is clearly not possible, given that the valuation of the consumer is $v = 1$.
Now, suppose that $p_2 < p_1$. By choosing p_2, the unconstrained type generates positive profits only in the first period so that $\pi^2(p_2) = \frac{1+\alpha}{2}p_2$. In the same way, a constrained firm can make profits only in the case where the rival is also constrained - $\pi^1(p_1) = \frac{\alpha}{2}(p_1+1)$. Deviating to p_2 yields $\pi^1(p_2) = \pi^2(p_2) = \frac{1+\alpha}{2}p_2$. As both types should not be willing to imitate each other in equilibrium results in contradiction:

$$\pi^1(p_1) = \frac{\alpha}{2}(p_1+1) \geq \pi^1(p_2) = \pi^2(p_2) = \frac{1+\alpha}{2}p_2 \geq \pi^2(p_1) = \frac{\alpha}{2}(p_1+2)$$

This condition cannot be satisfied unless $\alpha = 0$ and $p_2 = 0$, but if $\alpha = 0$ there will be only one type in the market and we cannot talk about separating equilibria.

Proof of Proposition 4. The first claim is that there cannot exist an equilibrium in which, following some equilibrium price, the posterior belief drops to zero. To see this define $Q(p)$ to be the probability that a given firm who charges price p sells in the first period and $\theta(p)$ denote the posterior following such a price. The profit of a constrained firm is

$$\Pi^c(p) = \int_{p'<p} \theta(p')d(1-Q(p')) + Q(p)p, \tag{5}$$

while the profit of an unconstrained firm is $\Pi^u(p) = \Pi^c(p) + Q(p)\theta(p)$. Suppose then that there exists a price p such that only unconstrained firms charge it. It must be the case that $\Pi^u(p) = \Pi^c(p) \leq \Pi^c(p')$ for every p' in the support of the constrained firm, since $\theta(p)$ is by assumption equal to zero. Let p^* denote the lowest price such that $\theta(p^*) > 0$. Clearly $\Pi^u(p^*) = \Pi^c(p) + Q(p^*)\theta(p^*) \geq \Pi^c(p) + \alpha\theta(p^*) > \Pi^u(p)$, a contradiction.

Similarly, there cannot exist an interval with positive measure (p^*, p^{**}) such that only constrained firms charge it. To see this, notice that

$$\begin{aligned}\Pi^c(p^{**}) &= \int_{p'^{**})} \theta(p')d(1-Q(p') + \int_{p'^*,p^{**})} d(1-Q(p'^{**})p^{**} \\ &> \int_{p'<p)} \theta(p')d(1-Q(p') + Q(p)p,\end{aligned}$$

for every $p \in (p^*, p^{**})$. From this observation it also follows that there must exist a mass point at the upper bound of the price distribution. For otherwise, $\lim_{p\to\bar{p}} Q(p)\theta(p) = 0$, while $Q(p)\theta(p) > 0$ for the lowest price in the support.

The second claim is that, in any equilibrium, the expected profits of the constrained type equal the prior belief. That is, $\Pi^c = \alpha$, and hence, $Q(\alpha) = 1$. To establish this fact, we rearrange the profits as follows:

$$\Pi^c(\bar{p}) = \bar{p}Q(\bar{p}) + \int^{\bar{p}} \theta(p')d(1-Q(p'))dp' \tag{6}$$

$$= \bar{p}Q(\bar{p}) + \theta(\bar{p})Q(\bar{p}) + \int^{\bar{p}} -Q'(p')\theta(p)dp', \tag{7}$$

where the last step follows from the uniform tie-breaking rule. On the other hand,

$$\Pi^c(\bar{p}^-) = 2\bar{p}Q(\bar{p}) + \int^{\bar{p}} -Q'(p')\theta(p')dp'. \tag{8}$$

Thus, $\theta(\bar{p}) = \bar{p}$. Because the prior must equal the posterior, we have

$$\alpha = 2Q(\bar{p})\theta(\bar{p}) + \int^{\bar{p}} -Q'(p')\theta(p')dp'^{c}(\bar{p}). \tag{9}$$

Hence, the profit of a constrained firm equals α. Then, by construction, α is also the lower bound of the price distribution.

In addition, we know that the unconstrained firm must be willing to randomize over the whole support. Her profit can be written as

$$\Pi^{u}(p) = Q(p)(p + \theta(p)) + \int^{p} -Q'(p')\theta(p')dp'^{c}(p) + Q(p)\theta(p). \tag{10}$$

Since $\Pi^{c}(p)$ is constant in the support, $\theta(p)Q(p) = \theta(\alpha)$. But then notice that

$$\lim_{p\to\bar{p}} Q(p)\theta(p) = 2Q(\bar{p}) \lim_{p\to\bar{p}} \theta(p) = Q(\bar{p})\bar{p}.$$

Thus, $\lim_{p\to\bar{p}} \theta(p) = \bar{p}/2$, and $\Pi^{u}(p) = \alpha + \theta(\alpha)$.

It suffices then to characterize the functions $Q(p)$ and $\theta(p)$. To this end, notice that the constrained firm must be willing to charge both $\bar{p}$ and every price $p \in (\alpha, \bar{p})$. Thus,

$$Q'(p)(p - \theta(p)) + Q(p) = 0. \tag{11}$$

If we interpret $\theta(p)$ as the continuation value for a constrained firm who loses to another firm on the margin at p, this is the classical optimal markup expression. Unconstrained firms must also randomize. Hence,

$$Q'(p)\theta(p) + \theta'(p)Q(p) = 0. \tag{12}$$

Rewriting,

$$-\frac{Q(p)}{Q'(p)} = \frac{\theta(p)}{\theta'(p)}. \tag{13}$$

it follows that

$$p - \theta(p) - \frac{\theta(p)}{\theta'(p)} = 0. \tag{14}$$

This equation admits as a solution,

$$\theta(p) = \frac{-p}{W(-\exp(-c)p)}, \tag{15}$$

where $W(x)$ is the Lambert function and the boundary condition $\lim_{p\to\bar{p}}\theta(p) = \frac{\bar{p}}{2}$. This equation can only be satisfied by the negative branch of the Lambert function, which we denote by $W_{-1}(x)$. Solving for c we have $c = 2 + \log\left(\frac{\bar{p}}{2}\right)$. This function is increasing and convex and strictly positive in $(\alpha, \bar{p})$.

Since $Q(p)\theta(p) = \theta(\alpha)$, we have

$$Q(p) = \theta(\alpha)\frac{-W_{-1}\left(-\frac{2p}{e^2\bar{p}}\right)}{p}. \tag{16}$$

To conclude the proof we need to verify that this is indeed an equilibrium. There is one equilibrium condition we have not imposed. Namely, that the ex-ante expected posterior equals the prior (Bayesian plausibility). That is,

$$\int_{\alpha}^{\bar{p}} -Q'(p)\theta(p)dp + 2Q(\bar{p})\theta(\bar{p}) = \alpha. \tag{17}$$

Notice that $\theta(p)$ must be consistent with the distributions of both types of firms. Let $f^c(p)$ and $f^u(p)$ denote the density of the price distribution of the constrained and the unconstrained firm respectively, and let μ^c and μ^u denote their respective mass points. It then holds that

$$\theta(p) = \frac{\alpha f^c(p)}{\alpha f^c(p) + (1-\alpha)f^u(p)},$$

for every $p < \bar{p}$, and $\theta(\bar{p}) = \frac{\alpha\mu^c}{\alpha\mu^c + (1-\alpha)\mu^u}$. By definition, $-Q'^c(p) + (1-\alpha)f^u(p)$ and $2Q(\bar{p}) = \alpha\mu^c + (1-\alpha)\mu^u$. Hence,

$$\int_{\alpha}^{\bar{p}} \alpha f^u(p)dp + \alpha\mu^c = \alpha \tag{18}$$

Hence, for every $\bar{p} \in [\alpha, 1]$, there exists an equilibrium with

$$\theta^*(p;\bar{p}) = \frac{-p}{W_{-1}\left(-\frac{2p}{e^2\bar{p}}\right)}$$

and

$$Q^*(p;\bar{p}) = \frac{\alpha}{p}\frac{W_{-1}\left(-\frac{2p}{e^2\bar{p}}\right)}{W_{-1}\left(-\frac{2\alpha}{e^2\bar{p}}\right)}.$$

Furthermore, we have that $\Pi^c = \alpha$ and

$$\Pi^u = \alpha - \frac{\alpha}{W_{-1}\left(-\frac{2\alpha}{e^2\bar{p}}\right)}.$$

That is, all equilibria are pay-off equivalent for the constrained type but the unconstrained type obtains higher pay-off in those equilibria with lower upper bound in the price distribution (and, thus, less information revelation). Among these equilibria the best is a pooling equilibrium in which $\alpha = \bar{p}$ and $\Pi^u = \frac{3}{2}\alpha$, and the worst has $\bar{p} = 1$ and $\Pi^u = \alpha(1+\xi)$, with

$$\xi = \frac{-1}{W_{-1}\left(-\frac{2\alpha}{e^2}\right)} \in (0, 1/2).$$

Proof of Proposition 5. We first establish (i). By definition, the expected posterior about the type of each firm is the prior in both E1 and E2. The distribution of posteriors in equilibrium i is

$$F_i(\theta) = 1 + \frac{\alpha}{\theta W_{-1}(-\frac{2\alpha}{e^2\bar{p}_i})},$$

for $\theta < \bar{p}_i/2$, $F_i(p) = F_i(\bar{p}_i/2)$ for $p \in (\bar{p}_i/2, \bar{p}_i)$ and $F_i(\bar{p}_i) = 1$ otherwise. It is easy to see that $F_1(\theta) < F_2(\theta)$ for all $\theta < \bar{p}_1$ and $F_1(\theta) \geq F_2(\theta)$ for all $\theta \geq \bar{p}_1$. This establishes that the distribution of posteriors in E2 is more disperse than under E1 and since they have the same mean, F_2 is a mean-preserving spread of F_1. This trivially guarantees that $F2$ is more informative under the disperse order than $F1$.

We now show (ii) and (iii). The expected second-period profit is $\Pi(\theta) = \theta + (1-\theta)\theta = (2-\theta)\theta$, which is increasing and concave in θ. Let θ_1 and θ_2 the random variables associated with a given firm's posterior in both equilibria. By Rotschild-Stiglitz, $\theta_2 = \theta_1 + \varepsilon$ for some ε with zero mean. Let $G_i(\theta)$ denote the distribution of the minimum of two draws from $F_i(\theta)$. Notice then that,

$$\begin{aligned}\int \Pi(\theta_2)dG_2(\theta_2) &\leq \int\int \Pi(\theta_1+\varepsilon)dG_1(\theta_1)dH(\varepsilon)\\ &\leq \int \Pi(\theta_1)dG_1(\theta_1).\end{aligned}$$

The first step follows from the fact that the minimum of θ_2 is not higher than the minimum of θ_1 plus a random draw of ε, establishing (ii); while the last step follows from Jensen's inequality, proving (iii).

Proof of Proposition 7. We start the analysis in the second period of the price competition and consider the case where the spying firm has successfully sold a unit in the first period and believes that the rival is informed with probability β and uninformed with probability $(1-\beta)$.[22] Applying the insights gained in Section 4, the spying firm expects that the mixing firm, when she does not spy and is uninformed, randomizes over some interval $[\hat{p}, 1]$ with some cumulative distribution function $G^{MU}(p)$ which has a mass point at the upper bound. Let us denote the mass by ω. As a best response, the spying firm must also randomize on the interval $[\hat{p}, 1)$. However, an informed mixing firm (that has spied and knows the rival's type) chooses different prices, depending on whether the rival is constrained. If the rival is constrained, then the mixing firm naturally chooses the monopoly price as the spying firm has sold in the first period. If, however, the spying firm is unconstrained, then the mixing firm knows that and will undercut this interval in order to maximize profits. As there cannot be an equilibrium with the informed mixing firm choosing a second-period price for sure (as the spying firm will have an incentive to undercut), the spying firm should not only mix over $[\hat{p}, 1]$ but also over a lower interval. Thus, the mixing strategy of spying firm will be represented by some cumulative distribution function $G^S(p)$ over the interval $[p_L, 1]$, for some $p_L < \hat{p}$. In addition, when informed the mixing firm will randomize over the interval $[p_L, \hat{p}]$ with $G^{MI}(p)$.

In what follows, we derive the unknowns $(\omega, \hat{p}, p_L)$, and some properties of the functions $G^S(p)$, $G^{MI}(p)$ and $G^{MU}(p)$. First, by undercutting the mass point of ω at 1, the spying firm has an expected profit of $(1-\beta)\omega$. Thus, $G^{MI}(p)$ and $G^{MU}(p)$ are characterized such that any price in the interval $[p_L, 1]$ yields the spying firm the same profit. Second, as by pricing at p_L firms know for sure they will sell, it must be that $p_L = (1-\beta)\omega$. Third, as by setting $\hat{p}$ the spying firm makes a second-period profit of $(1-\beta)\hat{p}$ it should be that $\hat{p} = \omega$. Fourth, by not spying and setting a price equal to 1 an unconstrained mixing firm expects to get a second-period profit of α. As the non-spying firm expects to get the same profit over the whole domain $[\hat{p}, 1)$, it should be that $(\alpha + (1-\alpha)(1-G^S(\hat{p})))\hat{p} = \alpha$. Fifth, as by spying and setting a price equal to p_L in

[22] The latter implicitly assumes that the first-period pricing strategy of the mixing firm is such that no information regarding the mixing firm's type is revealed. However, as the mixing firm is not supposed to sell in the first period anyway, the pricing strategy in this period is pay-off irrelevant and we may as well assume that both types choose the same first-period pricing strategy.

case the rival is unconstrained and choosing a price of 1 if the rival is constrained, the mixing firm gets an ex-ante expected pay-off of $\alpha+(1-\alpha)\alpha+(1-\alpha)^2(1-\beta)\omega-c$ when she actually spies, while the ex ante expected pay-off of not spying equals $\alpha+(1-\alpha)\alpha$ it should be that $(1-\alpha)^2(1-\beta)\omega-c=0$. Sixth, as when spying the mixing firm should be indifferent between setting any price in the interval $[p_L,\hat{p}]$, it should be that $(1-\beta)\omega=(1-G^S(\hat{p}))\hat{p}=(1-G^S(\hat{p}))\omega$ so that $G(\hat{p})=\beta$.

Combining the last three points, yields expressions for β and ω in terms of exogenous parameters. It follows that for any $c>0$, $\omega>\alpha>0$ and from the fifth point that β converges to 1 and ω converges to 1 if c approaches 0. This implies that in this asymmetric equilibrium the expected second-period profit of the spying firm converges to 0. In terms of first-period pricing strategies, as the mixing firm should not be willing to deviate from her strategy of pushing the rival to sell when she spies, the first-period price set by the spying firm if the rival is unconstrained should be equal to $(1-\beta)\omega$ and the interval of prices where the mixing firm uniformly randomizes over is $[(1-\beta)\omega,(1-\beta)\omega+\varepsilon]$. It follows that these prices also converge to 0 if c approaches 0 and that in the limit the spying firm will only make profit if the mixing firm is constrained. Thus, if c approaches 0 the ex ante expected pay-off of the spying firm equals α, which is considerably below the pay-off of $\alpha+(1-\alpha)\alpha$, which is the pay-off of the mixing firm and also the ex-ante expected pay-off of the complete information game. Interestingly, it is also lower than the profit under private information.

Proof of Proposition 9. In this proof, we provide more detail why deviating from the equilibrium strategies is not optimal. First, if the spying firm, *i.e.*, the firm that according to the asymmetric equilibrium is supposed to spy, deviates and decides not to spy, the non-spying firm does not observe this deviation and sticks to his first-period pricing, so that the spying firm only knows that with probability α the rival is constrained and charges 1 and with probability $1-\alpha$ randomizes uniformly above α. The deviation profits clearly depend on the continuation pricing strategy and on the beliefs of the rival. To give the candidate equilibrium maximal chance, we look for every possible pricing strategy at reasonable beliefs of the rival that create the lowest deviation pay-off. The candidate equilibrium can be sustained as an equilibrium if all of the possible pricing strategies yield a lower pay-off than the equilibrium pay-off for some reasonable belief of the rival.

If, after deviating and not spying, the firm sets a first-period price of α independent of whether or not he himself is constrained, her deviation profit is (at least) equal to

$\alpha + \alpha(1-\alpha)^2$. To see this, note that with this pricing strategy, the firm sells and the first-period profit equals α. A firm can only make profit in the second period if he is unconstrained. As α is the equilibrium first-period price if the non-spying firm is unconstrained, the unconstrained, non-spying firm has to believe that the rival plays according to the candidate equilibrium so the expected second-period profit equals α. However, for the constrained non-spying firm α is an out-of-equilibrium price, and he may well believe that his rival is unconstrained and in that case, the expected second-period profit equals 0. Thus, the deviating firm IS only makes a second-period profit of α if both firms are unconstrained, yielding an overall deviation profit of $\alpha + \alpha(1-\alpha)^2$.

All other first-period prices can generate lower pay-offs for reasonable beliefs of the non-spying firm. After a first-period price by the spying firm larger than 1, the non-spying firm may believe that his rival is unconstrained and in that case the spying firm will only make an expected profit of α (in the second period). On the other hand, after a first-period price by the spying firm that is larger than $\alpha + \varepsilon$ and smaller than 1, the non-spying firm may again believe his rival is unconstrained and in that case the spying firm will only make an expected profit of α (in the first period).

Thus, the spying firm does not want to deviate if $\alpha(3-3\alpha+\alpha^2) - c \geq \alpha + \alpha\ (1-\alpha)^2$ or $c \leq \alpha(1-\alpha)$.

Next, consider that the non-spying firm deviates and decides to spy, which is not observed by his rival. As the spying firm knows his rival's type and let the firm that was not supposed to spy sell at a price of 1 if he is constrained, the non-spying firm does not gain anything from spying if he is constrained. If, however, he is unconstrained, learning the capacity of his rival can increase his expected profit. If he learns that his rival is constrained, the non-spying firm can still push his rival to sell in the first period (as the spying rival believes a non-spying firm would do), but then set the monopoly price in the second period. In case the spying firm is unconstrained as well, the deviating non-spying firm cannot do better than choosing the equilibrium prices. Hence, the ex-ante expected deviation profit of the non-spying firm equals $\alpha \cdot 1 + (1-\alpha)(\alpha \cdot 1 + (1-\alpha) \cdot \alpha) - c = \alpha(3 - 3\alpha + \alpha^2) - c$. This is smaller than the equilibrium profit of $\alpha(2-\alpha)$ if $c \geq \alpha(1-\alpha)^2$.

C Alternative Tie-breaking Rules

In this part, we show that the equilibrium outcome of the game with hidden capacities does not depend on the specific tie-breaking rule that we assumed in the main paper. In particular, we are interested in the alternative tie-breaking rule that allocates the demand evenly among all firms that charge the lowest price instead of randomly allocating all demand to one firm.

First, as mentioned in footnote 13 of the main text observe that the pooling equilibrium outcome can be supported by an asymmetric equilibrium in which one of the firms, say firm 1, charges $p = \alpha$, while her rival randomizes in an interval $(p, p + \varepsilon)$, for $\varepsilon > 0$ sufficiently small. In this case, there are no ties in equilibrium (so the tie-breaking rule - whatever it is - does not apply) and the same equilibrium outcome emerges. In particular, a constrained firm 2 has no incentive to deviate as her expected (second-period) profit equals α and any deviation in the first period cannot yield more profit. Like in Section 4, whether an unconstrained firm 2 has an incentive to deviate depends on the out-of-equilibrium beliefs if firm 1 observe her rival sets a first-period price smaller than α. As only unconstrained firms may have such an incentive to deviate, it is natural that firm 1 believes a deviating, undercutting firm is unconstrained, resulting in zero expected second-period profits, implying that a deviation by an unconstrained firm 2 is not profitable.

Second, the pooling equilibrium in pure strategies can also be supported by any tie-breaking rule. Indeed, consider the pooling equilibrium in which both firms charge α in the first period and then randomize in the second period in $[\alpha, 1]$. Moreover, consider that given the first-period prices firm i's first-period market share equals q_i. Notice that the on-path continuation game following such a split of demand in the first period is such that with probability $1 - \alpha$ firm i is unconstrained while with probability α firm i has capacity to serve exactly $1 - q_i$ share of the market. It follows that the continuation profit of a constrained firm is $(1 - q_i)\alpha$, and this firm will randomize in the interval $[\alpha, 1]$. Thus, the overall expected pay-off of a constrained firm i is equal to α and independent of q_i. Deviating in either period cannot improve upon this expected profit. The unconstrained firm setting the lowest price in the support sells for sure and obtains α, which is also her maximum continuation profit. Thus, the overall expected pay-off of a constrained firm i equals $\alpha(1 + q_i)$. For the same reason as above, an unconstrained firm also does not want to deviate in the first period. Thus, for any tie-breaking rule the pooling equilibrium remains an equilibrium.

Finally, consider the semi-separating equilibrium strategies described in Section 4.1. If a tie occurs, she must be at the highest equilibrium price $\bar{p}$. If a firm sells quantity q_i at $\bar{p}$, she expects to make $q_i\bar{p} + (1 - q_i)\theta(\bar{p}) = \bar{p}$, regardless of q_i. Hence, the same first-period equilibrium strategies constitute an equilibrium (together with the natural adaptation of the second-period strategy).

D Hidden Prices

In certain markets it may be difficult for a firm to observe her rival's price. Since prices act as signals, it is natural to ask whether the results would significantly differ in an environment in which prices are unobserved. We now argue that the pooling equilibrium outcome we characterized in the beginning of Section 4 is the natural outcome for such markets as it is the unique equilibrium outcome.

First, just like in the case of alternative tie-breaking rules, it is easy to see that the asymmetric equilibrium that induces the pooling equilibrium outcome remains an equilibrium regardless of the information about prices. Second, it is straightforward to check that the symmetric pooling equilibrium breaks down if prices are hidden. As deviations are not observed if they do not result in another firm selling in the first period, an unconstrained firm can undercut her competitor and pretend that it simply was lucky in the first period and was selected by the tie-breaking rule. For this very same reason, a separating equilibrium cannot be supported. Third, a similar argument can be used to show that no semi-separating equilibrium can exist in this case. Recall that in a semi-separating equilibrium, the expected profit of selling the second unit in the second period must be the same, regardless of the price chosen in the first period. This requires that higher first-period prices induce higher continuation profits. But any firm that charges a marginally lower price in the first period can simply mimic the second-period behavior of a firm who charged a higher first-period price, making such a deviation profitable (as it is unobserved).

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