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JEL Classification: E43, E44, G11, G14

Keywords: (endogenous) interest rates, Informational efficiency, capital allocation efficiency, Rational Expectations, Unconventional Monetary Policy

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Matthijs Breugem   Adrian Buss   Joel Peress

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Interest rates play an essential role in financial markets. Foremost, they determine the rates at which investors discount future cash flows. But they also convey valuable information about the economic outlook. In recent years, however, market participants have expressed concerns that unconventional monetary policy ("quantitative easing") and an excessive demand for safe assets ("global saving glut") have distorted long-term interest rates and, with them, the prices of other assets, to the point that the prices of many assets have lost their predictive power and capital is misallocated.[1]

The purpose of this paper is to provide novel theoretical and empirical insights into the link between long-term interest rates and informational efficiency—the ability of financial markets to aggregate and disseminate private information—as well as real efficiency—their ability to allocate capital. We start by briefly examining the data, with a focus on the U.S. stock market. Indeed, we find that stock-price informativeness positively correlates with long-term interest rates, as illustrated in Figure 1 below. Moreover, consistent with this relation, price informativeness tends to increase in the supply of Treasury bonds and to decrease in the demand for Treasury bonds, a finding that lends initial empirical support to claims that policies like quantitative easing might reduce the discriminatory power of asset prices.

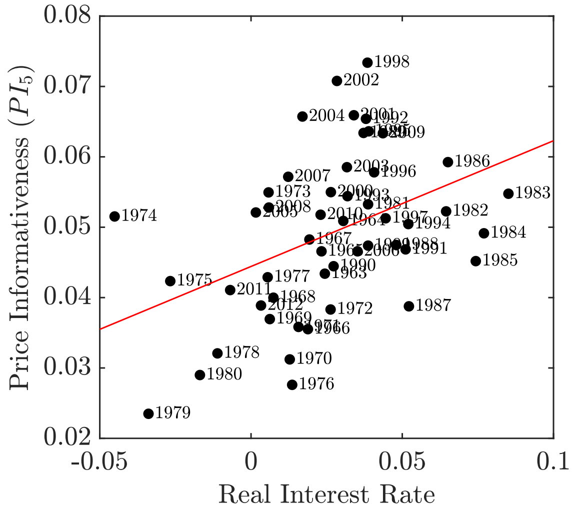

Figure 1. Stock-price informativeness and the real interest rate

Notes: The figure plots stock-price informativeness against the long-term real interest rate. Stock-price informativeness is measured as in Bai, Philippon and Savov (2016) and captures the extent to which firms' current stock prices reflect their future (five-year-ahead) cash flows. The data come from the United States and span the period from 1962 to 2017.

[1]Among others, both Jerome Powell, the chairman of the Federal Reserve, and Mario Draghi, the former chairman of the ECB, have raised such concerns (Draghi 2015, Powell 2017).

The remainder of the paper is dedicated to understanding the theoretical underpinnings of these empirical patterns. For that purpose, we develop a novel noisy rational expectations equilibrium (REE) model of the stock market. The model differs from traditional REE models, such as those of Grossman and Stiglitz (1980), Hellwig (1980), and Verrecchia (1982), along one key dimension: we relax the traditional assumption that the bond is in perfectly elastic supply, with the interest rate given exogenously (which rules out any learning from the interest rate). Instead, we assume that the bond is in fixed supply. As a consequence, the equilibrium interest rate now plays a *dual role*: it determines the discount rate and reveals information to investors.

Formally, this change manifests in the following three features that distinguish our model from other REE models of the stock market. First, the interest rate is endogenously determined. Second, investors learn not only from their private signals and the stock price but also from the interest rate. Third, investors make intertemporal consumption choices. Otherwise, the model is kept parsimonious to illustrate the economic mechanisms in the clearest possible way. That is, we consider a two-period model with a continuum of risk-averse investors who receive private signals about the fundamental. They trade a risk-free bond and a risky stock in competitive markets. Noise traders operating in both the stock and the bond market, prevent asset prices from being perfectly revealing. Finally, to illustrate the implications for allocative efficiency, we endogenize output and explicitly model the investment decision of the firm underlying the stock.

Primarily, we use the model to study *how* and *what type of information* investors learn from the bond market. Our key finding can be summarized as follows: The interest rate (or, more precisely, the resultant bond-market signal) reveals information about noise traders' demand in the *stock* market that, in turn, allows investors to extract more precise information about fundamentals from stock prices. Put differently, the bond market conveys information about discount rates that, in turn, makes stock prices more informative about cash flows.[2]

The underlying intuition simply derives from budget constraints and market clearing; accordingly, assume first that investors only consume at the terminal date.[3] Investors' budget constraints then imply that their aggregate (dollar) demand for the bond and the stock

[2] In line with the literature, we interpret noise-trader shocks as discount rate news because these shocks affect (expected) returns without affecting fundamentals.

[3] In this case, we are able to characterize the equilibrium in closed form, even though prices are nonlinear functions of the state variables. In contrast, the model with intertemporal consumption choices does not yield an analytical solution and is solved numerically.

must equal their aggregate wealth. In addition, market clearing in the stock market requires that investors' aggregate stock demand equals the stock supply minus noise traders' demand. Consequently, conditional on prices and aggregate wealth, any changes in noise traders' stock demand must be accompanied by changes in investors' aggregate bond demand. Under the traditional assumption of a bond in perfectly elastic supply, such changes in aggregate bond demand do not affect the interest rate; quantities, not prices, adjust. In contrast, with a fixed bond supply, the interest rate adjusts. As a result, the bond market provides a signal that allows investors to form *conditional* beliefs about noise traders' stock demand, with the signal error originating from noise traders' bond demand.[4]

The economic intuition extends to more complex settings, such as when investors consume early, trade multiple risky assets, or hold money or when aggregate wealth is stochastic. For instance, allowing for intertemporal consumption choices adds noise to the bond-market signal, but leaves the basic inference problem unchanged.

Notably, our model further implies that the precision of the bond-market signal *increases* in the rate of interest. Intuitively, because the signal derives from investors' budget constraints, noise traders' bond demand enters the signal scaled by its price, or, equivalently, divided by the interest rate. Hence, a higher interest rate attenuates the signal's error. Put differently, a higher interest rate makes the bond-market-clearing condition less sensitive to variations in noise traders' bond demand (while keeping fixed the sensitivity to noise traders' stock demand); hence, the signal-to-noise ratio improves.[5] A more precise bond market signal, in turn, allows investors to extract more information about fundamentals from stock prices and results in stock-price informativeness increasing in the interest rate.

After establishing the economic mechanism through which investors learn from the interest rate, we use our model to discuss how variations in the bond supply (or, equivalently, in the bond demand) affect informational and allocative efficiency as well as equilibrium asset prices. Our main results can be summarized as follows. First, because the interest rate increases in the bond supply, stock-price informativeness also increases in the bond supply (or, conversely, declines with bond demand), an effect that can be entirely attributed to learning from the bond-market signal. Second, the higher stock-price informativeness allows the firm to better differentiate between high-productivity and low-productivity states and,

[4]Strikingly, this mechanism implies that, even under a totally uninformative prior about noise traders' stock demand (i.e., with infinite prior variance), the stock price provides information about the fundamental (because the variance of the noise traders' demand *conditional* on the bond signal is finite).

[5]Indeed, more generally, the ratio of the prices of the two assets determines the precision of the signal as it governs the (relative) sensitivity of the bond-market-clearing condition to the two prices.

hence, to make more efficient investment decisions. Consequently, allocative efficiency in the economy also increases in the bond supply. Third, because of a decline in risk (thanks to higher stock-price informativeness), the bond supply negatively correlates with the stock's expected excess return and return volatility.

We also consider two extensions of our main framework featuring *additional signals*. Both not only confirm the main economic mechanism but also deliver new insights. The first extension, which allows for multiple risky assets, shows that the bond-market signal induces a negative correlation between stocks' excess returns (which declines in the bond supply), despite fundamentals and noise trading being independent across the two stocks. The second extension, which includes money, demonstrates that, similar to the rate of interest, the rate of inflation provides information about noise traders' stock demand (i.e., discount rate news). Moreover, the precision of the money market signal is increasing in the rate of inflation, and, thus, a larger money supply leads to improved stock-price informativeness and improved allocative efficiency.

Overall, these results highlight that the supply of (and demand for) bonds has important implications for price informativeness, allocative efficiency, output, and asset prices. In particular, variations in the bond supply influence the stock market and the real economy not only through their traditional impact on discount rates but also through their impact on the information environment. Indeed, these findings support critics who argue that, by purchasing government bonds through QE programs, central banks degrade informational and allocative efficiency.[6]

Our theoretical analyses also generate a rich set of novel predictions that are consistent with broad features of the data. For instance, our model predicts that stock-price informativeness increases in the real interest rate (and in bond and money supplies), in line with our empirical investigation. Related, the model predicts that allocative efficiency should be high (low) in high (low) interest-rate environments. This prediction is consistent with the empirical evidence presented by Gopinath et al. (2017), who document a simultaneous decline in the real interest rate and capital allocation efficiency in southern European countries. Moreover, in the model, periods of low interest rates are associated with an increase

[6] For example, in July 2018, the former chairman of the Federal Reserve, Ben Bernanke, warned that, because of QE-induced "distortions" in financial markets, a yield curve inversion need not point to a recession. Similar claims have been made about the effect of asset-purchase programmes by the European Central Bank and the Bank of Japan. Moreover, worries that low interest rates might distort stock prices and lead to a misallocation of capital have been frequently voiced, for instance by Richard Fisher, head of the Federal Reserve Bank of Dallas, Mario Draghi, then chairman of the ECB, and Jerome Powell, the chairman of the Federal Reserve (Fisher 2013, Draghi 2015, Powell 2017).

in the market price of risk, in the mean and variance of excess returns, and in stock-return comovement. Combined with the cyclicality of interest rates observed in the data, these results imply that the level and price of risk, as well as the volatility and comovement of stock returns, are all countercyclical, as in the data. More work is needed to ascertain whether these associations are actually causal or mere correlations.

The paper spans several strands of the literature. First and foremost, it builds on the extensive noisy REE literature pioneered by Grossman and Stiglitz (1980) and Hellwig (1980). Our main contribution to this literature is to endogenize the rate of interest. We show that the interest rate contains valuable information about a stock's noisy demand (or, equivalently, supply) and work out how investors use this information to update their beliefs about a stock's payoff. We are not aware of any other work in which *both* stock prices and the interest rate reveal information.[7] That price informativeness and investors' posterior precision endogenously vary with the rate of interest and, hence, with the business cycle further distinguishes our model from other noisy REE models. Likewise, this property connects our work to that of Kacperczyk, van Nieuwerburgh and Veldkamp (2016), who analyze how investors' knowledge depends on the state of the economy. But the mechanism they highlight is markedly different from ours in that this dependence on the state of the economy stems from variations in risk and in the price of risk (Kacperczyk, van Nieuwerburgh and Veldkamp 2016) versus variations in the interest rate (our model).

Our paper further relates to three substreams of the noisy REE literature. The first studies economies with multiple assets (see, e.g., Admati (1985), Brennan and Cao (1997), Kodres and Pritsker (2002), van Nieuwerburgh and Veldkamp (2009, 2010), Biais, Bossaerts and Spatt (2010), Kacperczyk, van Nieuwerburgh and Veldkamp (2016)). Though our main model features two assets with informative prices, it distinctly differs from these models in that our second asset is *risk-free*. In particular, we show that the risk-free asset reveals information about the stock despite its payoff and noisy demand being *uncorrelated* with those of the stock. This is in sharp contrast to Admati (1985) and the work that followed, in which, absent (exogenous) cross-asset correlations, nothing is to be learned from one asset about another. In addition, in our extension to multiple stocks, bond market clearing endogenously generates a (negative) correlation between stocks' returns (which is also absent

[7] Detemple (2002) also proposes an REE model in which the interest rate is endogenous. However, in his setting, information is revealed only through the interest rate (whereas the stock price does not reveal any information), and, moreover, this information pertains to *cash flows*. Indeed, the economic mechanism and insights differ substantially; for example, the (positive) link that we highlight between the interest rate and stock-price informativeness is absent from his framework.

in Admati 1985). Second, through its emphasis on information about the stock's noisy demand (or, equivalently, its supply), our work is also related to Watanabe (2008), Ganguli and Yang (2009), Manzano and Vives (2011), Farboodi and Veldkamp (2019), and Yang and Zhu (2019). In these papers, investors receive a private and exogenous signal (which they either purchase or are endowed with) about the stock supply. In contrast, the supply signal (also referred to as order flow or discount rate information in the literature) is public and endogenous in our setup. Finally, our paper is part of the substream of the literature that seeks to generalize noisy REE models and explore their robustness to assumptions (see, e.g., Barlevy and Veronesi 2000, 2003, Peress 2004, Breon-Drish 2015, Banerjee and Green 2015, Albagli, Hellwig and Tsyvinski 2015). Our contribution is to endogenize the interest rate in an otherwise standard noisy REE model and identify what features survive or differ.

Our work also relates to the literature studying the impact of monetary policies on stock prices.[8] While this literature typically assumes information is symmetric across investors, we allow for private (asymmetric) information. Doing so makes it possible to analyze the impact of monetary policy on the informational and allocative efficiency of the stock market. Related, a large literature in macroeconomics studies the impact of financial frictions, in particular, credit constraints, on capital misallocation and real efficiency.[9] In contrast, the friction we consider (asymmetric information) operates in the stock market.

Finally, our paper relates to the literature studying the importance of an endogenous rate of interest in asset pricing models under *symmetric* information. Lowenstein and Willard (2006) highlight that, the assumption of the riskless asset being in perfectly elastic supply can yield misleading conclusions (e.g., with respect to the impact of noise traders or violations of the Law of One Price). Our work is distinctly different from their paper because of the presence of private information and our focus on price informativeness. Moreover, we find that the main conclusions of the traditional noisy REE literature are robust to endogenizing the interest rate. Instead, we illustrate that new (unexplored) mechanisms arise when the bond market clears under a fixed bond supply.

The remainder of the paper is organized as follows. Section I presents the novel empirical findings motivating our theoretical analysis. Section II introduces our main economic

[8] Lucas (1982), LeRoy (1984*a*,*b*), Svensson (1985), Danthine and Donaldson (1986), and Marshall (1992) study how changes in monetary policy affect real and nominal asset prices. Sellin (2001) surveys this topic.

[9] See, for example, Bernanke and Gertler (1989) or Kiyotaki and Moore (1997). Brunnermeier and Pedersen (2008), Rampini and Viswanathan (2010), He and Krishnamurthy (2013), Biais, Hombert and Weill (2014), and Brunnermeier and Sannikov (2014), among others, study the impact of frictions on asset prices.

framework. Section III discusses, in a tractable version of the model, the economic mechanism through which investors learn from the interest rate. In Section IV, we then study the full model and relate the characteristics of the bond market to equilibrium outcomes. Section V explores extensions with additional price signals. Finally, Section VI concludes. The appendix provides the proofs and describes the numerical solution approach.

I. Empirical Patterns in Price Informativeness

In this section, we offer novel empirical evidence on the relation between the informativeness of stock prices and characteristics of the bond market. In particular, we document patterns in price informativeness linked to long-term interest rates and to the supply of and the demand for Treasury bonds that guide the theory presented in the next sections.

I.A. Data and Estimation Procedures

Our analysis focuses on the U.S. market over the period from 1962 to 2017.

Price Informativeness: We measure the informativeness of stock prices using the proxy developed by Bai, Philippon and Savov (2016). Their proxy captures the extent to which firms' current stock prices reflect their future cash flows and directly relates to capital allocation efficiency. Specifically, in each year, we run the following cross-sectional regression of year-$t+h$ earnings on year-t stock prices:

$$\frac{E_{j,t+h}}{A_{j,t}} = a_{t,h} + b_{t,h} \log\left(\frac{M_{j,t}}{A_{j,t}}\right) + c_{t,h}\, X_{j,t} + \epsilon_{j,t,h}, \tag{1}$$

where h denotes the forecasting horizon; $E_{j,t+h}/A_{j,t}$ denotes firm j's earnings before interest and taxes (EBIT) in year $t+h$ scaled by year-t total assets; $M_{j,t}/A_{j,t}$ denotes firm j's market capitalization (i.e., stock price times the number of shares outstanding) in year t scaled by year-t total assets; and $X_{j,t}$ denotes a set of firm-level controls, namely, current earnings, $E_{j,t}/A_{j,t}$, and industry fixed effects (one-digit SIC codes).[10]

Intuitively, the coefficient $b_{t,h}$ reflects how closely current stock prices track future earnings and, hence, how much fundamental information is capitalized in stock prices. Price

[10]To align price informativeness with bond market characteristics, we sample stock prices at the end of the U.S. government's fiscal year (either June or September). For each firm, we measure accounting variables at the end of the previous fiscal year—typically December—to ensure that the information is readily available to market participants. We adjust earnings using the gross domestic product (GDP) deflator from the Bureau of Economic Analysis (BEA).

informativeness at horizon h, $PI_{t,h}$, is then measured as the coefficient estimate $b_{t,h}$ multiplied by the year-t cross-sectional standard deviation of (scaled) stock prices:

$$PI_{t,h} = b_{t,h} \times \sigma_t \left(\log \left(\frac{M_{j,t}}{A_{j,t}} \right) \right). \tag{2}$$

As discussed in Bai, Philippon and Savov (2016), $PI_{t,h}^2$ captures the variance of the predictable component of firms' payoffs, F_j, given stock prices: $\mathbb{V}\mathrm{ar}(\mathbb{E}[F_j \mid P_j])$. Hence, $PI_{t,h}$ serves as a natural proxy for forecasting price efficiency.

We obtain stock price data from the Center for Research in Security Prices (CRSP) and accounting data from Compustat. Like Bai, Philippon and Savov (2016), we focus on S&P 500 nonfinancial firms whose characteristics have remained remarkably stable over time.[11] Moreover, we concentrate on forecasting horizons (h) of 3 and 5 years, horizons that, from a capital allocation perspective, are most important (see, e.g., the time-to-build literature, in particular, Koeva 2000) and for which prices are particularly useful in predicting earnings (as reported in Bai, Philippon and Savov 2016).

Bond Market Characteristics: Our measures of bond market characteristics closely follow those used by Krishnamurthy and Vissing-Jorgensen (2012). U.S. real interest rates are obtained by deducting expected inflation from long-term nominal rates. The nominal rate on long-maturity Treasury bonds is measured as the average yield on government bonds with a maturity of 10 years or longer (up to 1999) and the 20-year Treasury constant-maturity rate (from 2000 on), both of which are obtained from the Federal Reserve's FRED database. Expected inflation is estimated using a simple random-walk model (applied to the Consumer Price Index of the BEA).[12]

To measure the supply of U.S. Treasuries, we use the U.S. government debt-to-GDP ratio, specifically the ratio of the market value of publicly held government debt to GDP. For that purpose, we adjust the book (par) value of U.S. government debt (obtained from the Treasury Bulletin) using the Treasury debt market price index provided by the Dallas Fed. Government debt and, accordingly, GDP are measured at the end of the government's

[11] In contrast, as shown in Bai, Philippon and Savov (2016), the characteristics of non-S&P-500 firms have dramatically changed over time, rendering any time-series analysis potentially misleading.

[12] The random-walk model delivers the best out-of-sample performance for predicting inflation over our sample period. Our findings are robust to the use of alternative models for expected inflation, namely, AR(1) and ARMA(1,1) models.

fiscal year (i.e., the end of June up to 1976 and the end of September from 1977 onward).[13] To explicitly study the impact of demand-driven factors, such as quantitative easing, we also include in our analysis the Federal Reserve banks' holdings of U.S. Treasury securities and, as an instrument, their holdings of mortgage-backed securities (MBS). Both are scaled by U.S. GDP and based on data from the Federal Reserve System.

Control Variables: We estimate stock market and cash flow volatility as the annualized standard deviation of daily S&P 500 returns over the past 12 months and the cross-sectional standard deviation of firms' (scaled) earnings, respectively.

Table A1 in Appendix A reports summary statistics for all variables.

I.B. Price Informativeness and Bond Market Characteristics

In the first step, we analyze the relation between the informativeness of stock prices and the real interest rate. Panel A of Figure 2, which plots five-year price informativeness, PI_5, against the real interest rate, strongly suggests a positive correlation between the two series.[14] A corresponding regression of price informativeness on the real interest rate confirms that this positive relation is statistically significant, with a slope coefficient of 0.179 (t-statistic of 2.67). In terms of economic magnitude, a one-standard-deviation (SD) increase in the real interest rate leads to a 0.42-SD increase in price informativeness.

A natural limitation of this test is that the rate of interest is endogenous; that is, it is determined in equilibrium jointly with other quantities, including price informativeness. Hence, our next analysis instead focuses on exogenous variation in Treasury supply and demand. Indeed, it seems implausible that the government chooses its debt level or that Federal Reserve Banks choose their Treasury or MBS holdings in accordance with the informativeness of stock prices.

Table 1 reports the results of our regression analyses. The dependent variable in each regression is price informativeness (typically PI_5) and the primary explanatory variables are the Treasury-bond supply and demand. The regressions in Table 1 are estimated using

[13] Our results remain unchanged when using the debt-to-GDP series prepared by Krishnamurthy and Vissing-Jorgensen (2012). In fact, the correlation between the two data series is 0.9966. We are grateful to the authors for sharing their data with us.

[14] Our time series of price informativeness ends in 2012, because we need to forecast five-year-ahead earnings, which go until 2017.

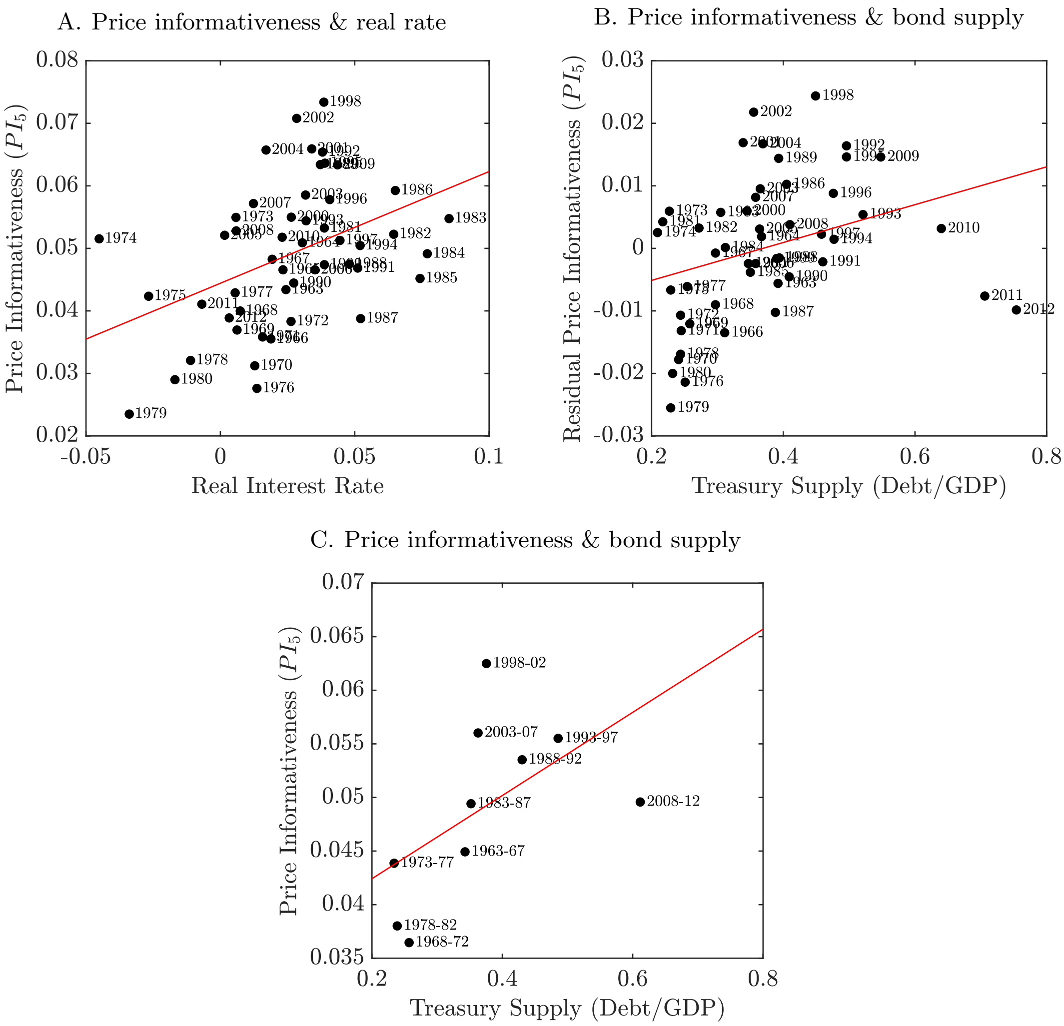

FIGURE 2. EMPIRICAL PATTERNS IN STOCK-PRICE INFORMATIVENESS

Notes: The panels plot stock price informativeness against the real interest rate (Panel A) and the debt-to-GDP ratio (Panels B and C). The sample consists of annual observations from 1963 to 2012. Residual price informativeness (Panel B) is measured as the residuals of a univariate regression of price informativeness on the Federal Reserve Banks' MBS holdings. In Panel C, price informativeness and the debt-to-GDP ratio are averaged over (nonoverlapping) five-year periods. The solid line in all graphs represents the fitted values of a univariate regression of the y-axis variables on the x-axis variables.

ordinary least squares, with standard errors adjusted for serial correlation using the Newey-West procedure with five lags.[15]

The baseline regression in Column 1 shows a significant positive relation between price informativeness and bond supply (t-statistic of 3.18). Changes in bond supply have an economically sizeable effect on price informativeness; for example, all else equal, a one-SD

[15] Our choice of lags is based on two considerations. First, price informativeness is measured by overlapping regressions, with a maximum overlap of five years for earnings in the case of PI_5. Second, the optimal lag-selection-procedure of Newey and West (1994) recommends lags between three and five years. Our results are robust to alternative specifications.

	Base	1963-2009	5-year periods	FED: Treasury	Lagged variables	Volatility Controls		PI_3
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Debt/GDP	0.060	0.074	0.079	0.048	0.066	0.063	0.061	0.033
	[3.13]	[4.97]	[3.24]	[3.40]	[4.09]	[3.21]	[3.34]	[1.98]
FED Hold./GDP	-0.331		-0.452	-0.364	-0.421	-0.369	-0.359	-0.248
	[-2.52]		[-2.27]	[-3.38]	[-5.45]	[-2.55]	[-3.21]	[-2.36]
S&P500 Vola.						0.028		0.037
						[0.81]		[1.03]
Cashflow Vola.							0.664	0.506
							[3.12]	[2.45]
R^2	0.211	0.336	0.600	0.228	0.260	0.226	0.350	0.235
Observations	50	46	10	50	50	50	50	50

TABLE 1—IMPACT OF BOND SUPPLY AND DEMAND ON STOCK-PRICE INFORMATIVENESS

Notes: The table reports results of regressions relating stock price informativeness to Treasury-bond supply and demand. The dependent variable is 5-year price informativeness, PI_5, (except in Column 8 which is based on 3-year price informativeness, PI_3). *Debt/GDP* is the ratio of the market value of Treasury debt held by the public to U.S. GDP. *FED Hold./GDP* is the ratio of the Federal Reserve banks' holdings of MBS (or Treasury in Column 4) divided by U.S. GDP. *S&P500 Vola.* and *Cashflow Vola.* are measures of volatility of, respectively, the S&P500 returns and firms' earnings. Regressions are estimated using OLS and standard errors are adjusted for serial correlation using the Newey-West procedure with five lags. We report t-statistics in brackets.

increase in the debt-to-GDP ratio (from its mean value of 0.3830 to 0.4940) increases price informativeness by 15% (0.64 SD). Based on our model, we estimate that this effect translates into increases in allocative efficiency of about 30% and in GDP of about 0.4%—a sizeable fraction of average annual US real GDP growth of 2% over the last decades.[16] Panel B of Figure 2 illustrates this positive relation. It plots the residual price informativeness (i.e., the residuals of a univariate regression of price informativeness on Treasury demand) against the Treasury supply.

Consistent with a positive correlation between price informativeness and Treasury *supply*, Column 1 also documents a strong negative correlation between price informativeness and bond *demand*, measured by the FED's MBS holdings (t-statistic of -2.29). All else equal, an increase in the FED's MBS holdings from its mean of 0.005 to 0.06 (the mean following QE) lowers price informativeness by more than 35%, or a 1.61 SD.

The remainder of Table 2 confirms that our findings hold up to a series of robustness checks. Column 2 focuses on the period from 1962 to 2009, over which Treasury demand

[16] Specifically, we define allocative efficiency as the additional output resulting from more informative asset prices and establish that it is proportional to squared price informativeness. Therefore, a one-SD increase in the debt-to-GDP ratio improves allocative efficiency by 30% (thanks to a 15% increase in price informativeness). Based on standard estimates from the literature (i.e., a depreciation rate of 10% and (quadratic) adjustment costs of 5% (Bloom 2009)), this translates into effects of about 0.4% of GDP (to be precise, of S&P500 firms' aggregate earnings). Confer Footnote 33 for more details.

was constant and so does not need to be controlled for.[17] Column 3 (also illustrated in Panel C) exploits only low-frequency variations in the series; that is, it reports the results of a regression of (nonoverlapping) five-year averages of the variables (i.e., a total of 10 data points). Column 4 uses the FED's Treasury holdings (instead of their MBS holdings) to control for Treasury demand. Column 5 lags bond supply and demand. Columns 6 and 7 control for stock market and cash flow volatility, respectively. Finally, Column 8 uses the price-informativeness measure, PI_3, based on a three-year forecasting horizon.

Taken together, the regressions in Table 1 provide robust empirical evidence that price informativeness positively correlates with Treasury supply and negatively correlates with Treasury demand. These results pose a substantial challenge to traditional information choice models and motivate our subsequent theoretical analysis.

II. An REE Model with Bond Market Clearing

In this section, we introduce our main economic framework. The framework differs from traditional competitive rational expectation equilibrium (REE) models, such as those of Grossman and Stiglitz (1980), Hellwig (1980), and Verrecchia (1982), along three key (related) dimensions. First, the bond market clears, and the rate of interest is endogenously determined. Second, investors learn not only from their private signals and the stock price but also from the interest rate. Third, agents consume not only in the final period but also in the trading period. Moreover, to illustrate the implications for allocative efficiency, we endogenize firms' real-investment decisions and, thus, output. In the following, we describe the details of the model.

Information Structure and Timing

We consider a two-period model. Figure 3 illustrates the sequence of events. In period 1, investors observe their private signals, the stock price, and the interest rate. Based on this information, they set up their portfolio and choose period-1 ("initial") consumption. In addition, a representative firm chooses its real investment, conditional on asset prices. Finally, asset prices clear financial markets. In period 2, productivity and output are realized, and investors consume the proceeds from their investments ("terminal" consumption).

[17] Among others, Gorton, Lewellen and Metrick (2012) document that Treasury demand for "safe" (information-insensitive) debt was constant during this period.

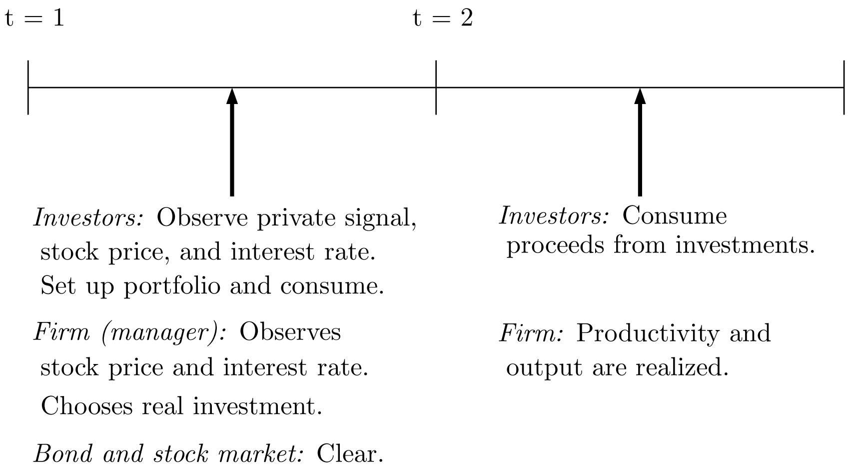

FIGURE 3. SEQUENCE OF EVENTS

Notes: The figure illustrates the sequence of the events.

Investment Opportunities

Two financial securities are traded in competitive markets: a riskless asset (the "bond") and a risky asset (the "stock"). The bond has a payoff of one in period 2, with a gross rate of interest R_f, or, equivalently, a price of $1/R_f$.[18] The stock is a claim to the representative firm's endogenous output F (the "fundamental"), which is only observable in period 2. Its price is denoted by P. The firm also makes a deterministic payout of F_1 in period 1. The stock and the bond are in finite supply, denoted by $\bar{X}^{\mathcal{S}}$ and $\bar{X}^{\mathcal{B}}$, respectively.

Output

Output is produced by a representative firm that employs a linear ("ZK") production technology and is endowed with assets in place K_1. Its fundamental value, v, is modeled as in standard q-theory:

$$v(z,I) \equiv \underbrace{(K_1 - I)}_{\equiv F_1} + \underbrace{(1+z)\big((1-\delta)\,K_1 + I\big) - \frac{\kappa}{2\,K_1}\,I^2}_{\equiv F}. \tag{3}$$

[18]In our setting, the consumption good serves as the numéraire, and, hence, all prices (and payoffs) are denominated in units of the good. This contrasts with traditional REE models, such as those of Grossman and Stiglitz (1980), Hellwig (1980), and Verrecchia (1982), in which the exogenous riskless bond serves as the numéraire.

Specifically, with period-1 productivity being normalized to one, initial-period output, F_1, is simply given by assets-in-place K_1 less investment I. Period-2 output, F, is given by the product of period-2 productivity, $1+z$, and available capital (assets-in-place K_1 depreciated at rate δ, plus investment I), minus quadratic adjustment costs $(\kappa/2K_1)\, I^2$ (with $\kappa \geq 0$).[19] Period-2 net productivity, z, is random and normally distributed with mean zero and precision τ_z: $z \sim \mathcal{N}(0, 1/\tau_z)$.

For simplicity, we assume that the firm (manager) has no private information about productivity, z, but learns about productivity from stock and bond prices.[20] This creates a *feedback effect* from financial markets to real investment decisions.[21]

Investors

There exists a continuum of atomless investors with unit mass. At the beginning of period 1, each investor i receives a private signal about productivity: $S_i = z + \varepsilon_i$, where ε_i is *i.i.d.* normally distributed with mean zero and precision τ_ε. Investors have constant absolute risk aversion (CARA) preferences over initial and terminal consumption, $C_{i,1}$ and $C_{i,2}$:

$$\text{(4)} \qquad \mathcal{U}_i(C_{i,1}, C_{i,2}) = -\frac{1}{\rho} \exp\bigl(-\rho\, C_{i,1}\bigr) + \beta\, \mathbb{E}\left[-\frac{1}{\rho} \exp\bigl(-\rho\, C_{i,2}\bigr) \,\middle|\, \mathcal{F}_i\right],$$

where ρ denotes absolute risk aversion; $\beta \in (0, 1]$ denotes the rate of time preference; and $\mathcal{F}_i = \{S_i, P, R_f\}$ describes investor i's time-1 information set.

While initial wealth plays no role in traditional settings with CARA preferences and an exogenous interest rate, clearing the bond market requires defining investors' initial wealth. Specifically, we assume that investor i is endowed with initial wealth $W_{i,1}$ in the form of $X^{\mathcal{S}}_{i,0}$ shares of the stock and $X^{\mathcal{B}}_{i,0}$ units of a bond maturing in period 1.

[19] As is standard in such models, investment, I, can be positive (representing capital expenditures) or negative (representing an asset sale).

[20] In particular, in our single-stock economy, the firm represents the entire productive sector, so z can be interpreted as *aggregate* productivity, about which the manager has plausibly no private information.

[21] Bond, Edmans and Goldstein (2012) survey the literature on feedback effects. For more recent contributions, see Foucault and Frésard (2014), Edmans, Goldstein and Jiang (2015), Goldstein and Yang (2017), and Dessaint et al. (2018).

Noise Traders

Noise (liquidity) traders operate in *both* the bond and the stock market. Their behavior is not explicitly modeled; instead, their demand for the stock and the bond is given by exogenous random variables $u^{\mathcal{S}} \sim \mathcal{N}(0, 1/\tau_{u^{\mathcal{S}}})$ and $u^{\mathcal{B}} \sim \mathcal{N}(0, 1/\tau_{u^{\mathcal{B}}})$, where z, $u^{\mathcal{S}}$, and $u^{\mathcal{B}}$ are uncorrelated.[22] In particular, note that, in addition to the usual stock market noise, we assume a noisy bond demand; this assumption prevents the bond and stock prices from being jointly perfectly revealing.

Equilibrium Definition

Investor i aims to maximize expected utility (4) subject to the following budget constraints:

$$C_{i,1} + X_i^{\mathcal{S}} P + X_i^{\mathcal{B}} R_f^{-1} = W_{i,1}, \qquad \text{and} \qquad C_{i,2} = X_i^{\mathcal{S}} F + X_i^{\mathcal{B}}, \tag{5}$$

where $X_i^{\mathcal{S}}$ and $X_i^{\mathcal{B}}$ denote the number of shares of the stock and the bond held by the investor, respectively. The objective of the manager is to maximize the expected firm value.

Accordingly, a rational expectations equilibrium is defined by consumption choices $\{C_{i,1}, C_{i,2}\}$, portfolio choices $\{X_i^{\mathcal{S}}, X_i^{\mathcal{B}}\}$, a real investment choice I, and asset prices $\{P, R_f\}$ such that

1. $\{C_{i,1}, C_{i,2}\}$ and $\{X_i^{\mathcal{S}}, X_i^{\mathcal{B}}\}$ maximize investor i's expected utility (4) subject to the budget constraints in (5), taking prices P and R_f as given,
2. I maximizes the expected firm value $\mathbb{E}\left[v(z, I) \mid R_f, P\right]$,
3. the investors' and the manager's expectations are rational,
4. aggregate demand equals aggregate supply in the bond and the stock markets:[23]

$$\int_0^1 X_i^{\mathcal{S}}\, di + u^{\mathcal{S}} = \bar{X}^{\mathcal{S}}, \quad \text{and} \quad \int_0^1 X_i^{\mathcal{B}}\, di + u^{\mathcal{B}} = \bar{X}^{\mathcal{B}}. \tag{6}$$

[22] This correlation structure highlights that, in equilibrium, the bond price reveals information about the stock even though its payoff and demand are uncorrelated with those of the stock.

[23] By Walras' law, market clearing in the bond and the stock market guarantees market clearing in the goods market in period 1.

It is important to highlight that, in equilibrium, *both* asset prices play a dual role: each price not only clears its respective market but also aggregates and transmits investors' private information.

III. The Economic Mechanism

We now first illustrate *how* and *what type of information* investors learn from the interest rate. To do so, we use a simplified version of our model that provides the key economic intuition and allows for simple closed-form solutions.

III.A. Setting

The setting differs from the framework described in the preceding section along one key dimension: investors consume exclusively on the terminal date.

Moreover, to facilitate the exposition of the economic mechanism, we make the following two simplifying assumptions that reduce the framework to a Hellwig (1980) model but with bond market clearing. First, we abstract from real investment and treat the stock's payouts, F_1 and F, as exogenous; specifically, we assume that F is normally distributed with mean μ_F and precision τ_F. Second, we assume that the aggregation of investors' stock endowments coincides with the period-1 residual stock supply (i.e., $\int X^{\mathcal{S}}_{i,0}\, di = \bar{X}^{\mathcal{S}} - u^{\mathcal{S}}$), that the cross-sectional variance of those endowments is infinite (i.e., $\mathbb{V}\mathrm{ar}(X^{\mathcal{S}}_{i,0} \,|\, u^{\mathcal{S}}) = \infty$), and that investors have no endowments in bonds (i.e., $X^{\mathcal{B}}_{i,0} = 0$).[24]

III.B. Equilibrium

Because of learning from the interest rate, equilibrium asset prices are *nonlinear* functions of the state variables, in stark contrast to traditional frameworks. However, by conjecturing the functional form of the market-clearing conditions (which remain linear), instead of stipulating the functional form of the interest rate and the stock price (which are not linear), we are still able to characterize the equilibrium in closed form, as stated in the following theorem:

[24]The assumptions imply, respectively, that the stock price cancels out of the bond-market clearing condition and that (individual) stock endowments contain no information about noise traders' stock demand. As a result, the equilibrium price ratio, $R_f P$, is characterized by a simple expression comparable to that in Hellwig (1980). The results discussed in this section also hold in the case of arbitrary endowments (see Appendix B.A.2), but with a price ratio characterized by a more complicated (cubic) equation.

Theorem 1. *There exists a unique (conditionally linear) rational expectations equilibrium. The equilibrium asset prices are given by*

$$R_f = \frac{\bar{X}^{\mathcal{B}} - u^{\mathcal{B}}}{F_1\left(\bar{X}^{\mathcal{S}} - u^{\mathcal{S}}\right)}; \quad \textit{and} \tag{7}$$

$$R_f\, P = \left(\frac{\tau_F}{\tau}\,\mu_F + \frac{\tau_\epsilon\,\tau_{u^{\mathcal{S}}|R_f}}{\rho\,\tau}\,\mu_{u^{\mathcal{S}}|R_f}\right) + \frac{\tau_\epsilon\left(\rho^2 + \tau_\epsilon\,\tau_{u^{\mathcal{S}}|R_f}\right)}{\tau\,\rho^2}\left(F - \frac{\rho}{\tau_\epsilon}\,u^{\mathcal{S}}\right), \tag{8}$$

with $\tau_{u^{\mathcal{S}}|R_f}$, $\mu_{u^{\mathcal{S}}|R_f}$, *and* τ *defined in Equations* (11), (12), *and* (13) *below.*

Investor i*'s optimal stock and bond holdings equal*

$$X_i^{\mathcal{S}} = \frac{\mathbb{E}[F\,|\,\mathcal{F}_i] - P\,R_f}{\rho\,\mathbb{V}ar(F\,|\,\mathcal{F}_i)} \quad \textit{and} \quad X_i^{\mathcal{B}} = R_f\left(W_{i,1} - X_i^{\mathcal{S}}\,P\right). \tag{9}$$

The optimal demand for the stock, $X_i^{\mathcal{S}}$, follows the standard mean-variance portfolio rule. The optimal bond investment is simply given by initial wealth minus stock holdings.

The equilibrium interest rate, R_f, is a function of realized stock and bond demands and, thus, is stochastic.[25] As expected, it is increasing in the "residual" bond supply, $\bar{X}^{\mathcal{B}} - u^{\mathcal{B}}$; specifically, a larger residual supply requires a lower bond price for the market to clear and, hence, a higher interest rate. Moreover, since wealth must be entirely saved, the interest rate declines in aggregate wealth and, hence, in the residual stock supply $(\bar{X}^{\mathcal{S}} - u^{\mathcal{S}})$ and in the initial stock payout (F_1).

The equilibrium price ratio, $R_f\,P$, has the familiar structure of, for example, that of Hellwig (1980) and Verrecchia (1982), with one critical difference: the ratio features the mean and precision of the noisy stock demand, $\mu_{u^{\mathcal{S}}|R_f}$ and $\tau_{u^{\mathcal{S}}|R_f}$, *conditional* on the interest rate, instead of its prior mean and precision. This difference arises because investors use the information revealed by the bond market to update their beliefs about the noise traders' stock demand. In other words, they receive discount rate news. Specifically, in the absence of initial consumption, investors' period-1 budget constraints and market clearing imply

[25] The gross interest rate, R_f, can be negative in this illustrative framework—if noise traders' demand for the bond or for the stock exceeds the respective asset's supply. This does not, however, lead to arbitrage opportunities. Indeed, negative rates are caused by the fact that investors have a preference over terminal consumption only (i.e., do not consume in period 1), and, thus, in contrast to our main economic framework, the interest rate is entirely determined by budget considerations and not by marginal utilities.

that the payout from the stock is entirely invested in the bond:

$$(10) \qquad \left(\bar{X}^{\mathcal{S}} - u^{\mathcal{S}}\right) F_1 = \left(\bar{X}^{\mathcal{B}} - u^{\mathcal{B}}\right) R_f^{-1} \quad \Leftrightarrow \quad \frac{\bar{X}^{\mathcal{S}} R_f F_1 - \bar{X}^{\mathcal{B}}}{R_f F_1} = u^{\mathcal{S}} - \frac{u^{\mathcal{B}}}{R_f F_1}.$$

Consequently, the bond market provides a signal about the (unobservable) stock demand, $u^{\mathcal{S}}$, with bond demand, $u^{\mathcal{B}}$, acting as noise. The following lemma describes the resultant conditional distribution of the noisy stock demand.

Lemma 1. *The distribution of the noisy stock demand, $u^{\mathcal{S}}$, conditional on the equilibrium interest rate, R_f, is characterized by*

$$(11) \qquad \mu_{u^{\mathcal{S}}|R_f} \equiv \mathbb{E}\left[u^{\mathcal{S}} \mid R_f\right] = \frac{\tau_{u^{\mathcal{B}}} R_f^2 F_1^2}{\tau_{u^{\mathcal{S}}|R_f}} \frac{\bar{X}^{\mathcal{S}} R_f F_1 - \bar{X}^{\mathcal{B}}}{R_f F_1}; \text{ and}$$

$$(12) \qquad \tau_{u^{\mathcal{S}}|R_f} \equiv \mathbb{V}ar\left(u^{\mathcal{S}} \mid R_f\right)^{-1} = \tau_{u^{\mathcal{S}}} + R_f^2 F_1^2 \tau_{u^{\mathcal{B}}}.$$

Intuitively, investors combine their prior beliefs with the signal provided by the bond market to update their beliefs about the noisy stock demand. The conditional mean, $\mu_{u^{\mathcal{S}}|R_f}$, is simply the precision-weighted average of the prior mean (equal to zero) and the bond signal in (10). Similarly, the conditional precision, $\tau_{u^{\mathcal{S}}|R_f}$, is the sum of the prior precision ($\tau_{u^{\mathcal{S}}}$) and the precision of the bond market signal ($R_f^2 F_1^2 \tau_{u^{\mathcal{B}}}$).

Notably, the conditional precision, $\tau_{u^{\mathcal{S}}|R_f}$, is increasing in the interest rate R_f, as is illustrated in Figure 4. Intuitively, because the signal derives from investors' budget constraints (which tie together the noisy demands for the stock and the bond), the (dollar) value of these demands is what matters. Indeed, noise traders' bond demand enters the signal scaled by its price, or, equivalently, divided by the interest rate (see the first equation in (10)). Hence, a higher interest rate attenuates the signal's error and, thus, implies a higher signal-to-noise ratio for the bond market signal. In other words, with dampened bond noise, the interest rate is a more accurate signal of the stock's demand.

III.C. Equilibrium Price Informativeness

We can now turn to the precision of investors' conditional beliefs:

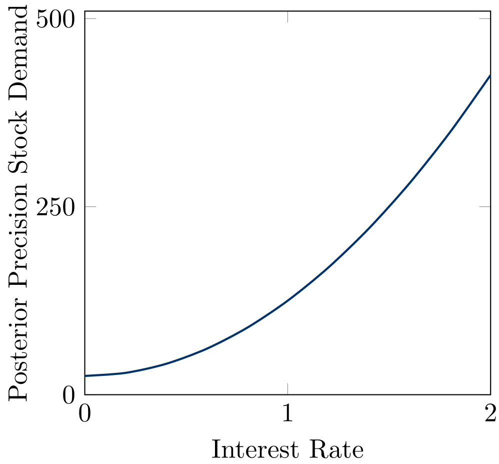

FIGURE 4. POSTERIOR PRECISION OF STOCK DEMAND (ABSENT INIT. CONSUMPTION)

Notes: The figure plots investors' posterior precision for the stock's noisy demand, $\tau_{u^{\mathcal{S}}|R_f}$, as a function of the interest rate R_f. The graph is based on the following parameter values: $\rho = 4$, $\mu_F = 1$, $\tau_F = 2.5^2$, $F_1 = 1$, $\tau_\epsilon = 0.75^2$, $\bar{X}^{\mathcal{S}} = 1$, $\tau_{u^{\mathcal{S}}} = 5^2$, $\bar{X}^{\mathcal{B}} = 1$, and $\tau_{u^{\mathcal{B}}} = 10^2$and assumes that investors consume only at the terminal date.

Lemma 2. *The precision of investor i's conditional beliefs about the payoff F is given by*

$$\tau \equiv \mathbb{V}ar(F \mid \mathcal{F}_i)^{-1} = \tau_F + \tau_\varepsilon + \frac{\tau_\varepsilon^2}{\rho^2}\, \tau_{u^{\mathcal{S}}|R_f}, \tag{13}$$

where $(\tau_\varepsilon/\rho)^2\, \tau_{u^{\mathcal{S}}|R_f}$ *represents the informativeness of the stock price.*

The posterior precision, τ, has the same form as in Hellwig (1980) and comprises three components: (1) the precision of the investors' prior beliefs τ_F, (2) the precision of their private signal τ_ε, and (3) the precision of the stock price signal $(\tau_\varepsilon/\rho)^2\, \tau_{u^{\mathcal{S}}|R_f}$, which is driven by the precision of the stock demand $\tau_{u^{\mathcal{S}}|R_f}$ and the signal-to-noise ratio of the stock price signal (τ_ε/ρ). Consistent with Hellwig (1980), the posterior precision is increasing in all three precisions and in investors' risk tolerance. However, as with the equilibrium price function, price informativeness differs from the expression in Hellwig (1980) in that it features investors' *conditional* precision of the stock demand, $\tau_{u^{\mathcal{S}}|R_f}$, rather than investors' prior precision.

This observation has three important implications, which are illustrated in Panel A of Figure 5, and which distinguish our model from traditional noisy REE models. First, investors' posterior precision is higher than in Hellwig (1980), thanks to the information on the noisy stock demand obtained from the bond market. Second, price informativeness

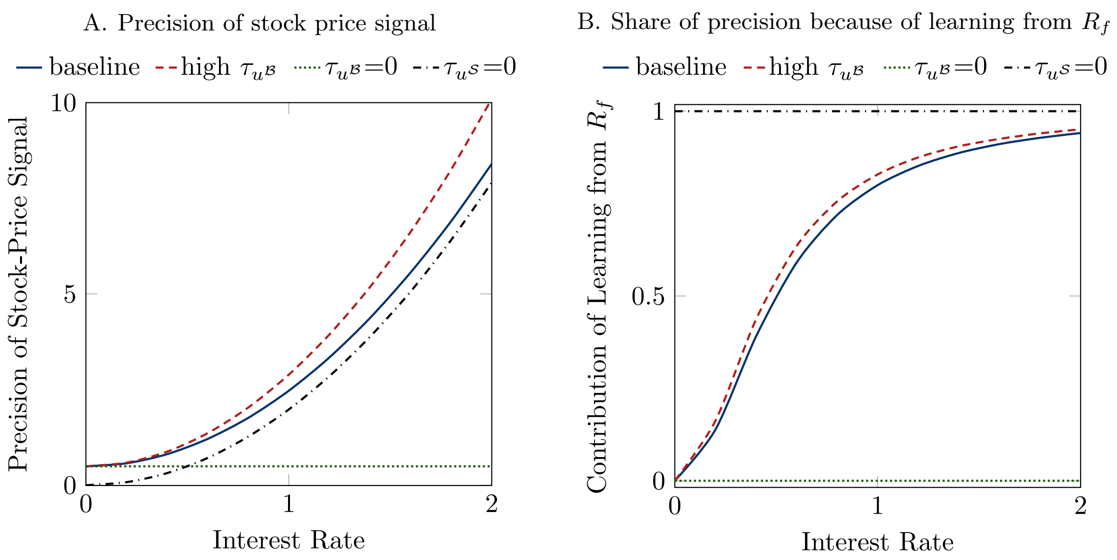

FIGURE 5. PRECISION OF THE STOCK PRICE SIGNAL (ABSENT INIT. CONSUMPTION)

Notes: The figure plots the precision of the stock price signal (i.e., its informativeness), $(\tau_\varepsilon/\rho)^2\,\tau_{u^{\mathcal{S}}|R_f}$ (Panel A), and the share of the stock price signal's precision that can be attributed to investors learning from the interest rate (Panel B) as functions of the interest rate R_f for different levels of the prior precision of the bond demand, $\tau_{u^{\mathcal{B}}}$. The graphs are based on the following baseline parameter values: $\rho = 4$, $\mu_F = 1$, $\tau_F = 2.5^2$, $F_1 = 1$, $\tau_\epsilon = 0.75^2$, $\bar{X}^{\mathcal{S}} = 1$, $\tau_{u^{\mathcal{S}}} = 5^2$, $\bar{X}^{\mathcal{B}} = 1$, and $\tau_{u^{\mathcal{B}}} = 10^2$and assumes that investors consume only at the terminal date. High $\tau_{u^{\mathcal{B}}}$ describes an economy with a higher precision of the bond demand; $\tau_{u^{\mathcal{B}}} = 0$ describes an economy in which investors do not learn from the rate of interest and $\tau_{u^{\mathcal{S}}} = 0$ describes an economy in which the (prior) stock demand is completely uninformative.

depends on (specifically, increases in) the rate of interest, R_f, because investors can extract more information from the stock's price about its payoff (thanks to their more precise information about the noisy stock demand). Third, price informativeness and investors' posterior precision are stochastic and, hence, *ex ante* unknown—a feature that could, in a model with endogenous information choice, deliver new insights into investors' demand for information.

As expected, the impact of learning from the bond market signal is stronger, the more precise are priors about the bond demand (i.e., for a higher $\tau_{u^{\mathcal{B}}}$). Accordingly, the share of the stock price signal's precision that can be attributed to learning from the interest rate (relative to the overall precision of the stock price signal) increases in the rate of interest and the precision of the bond demand (Panel B).

Figure 5 also illustrates two interesting limiting cases. First, if the variance of noise trading in the bond market is infinite ($\tau_{u^{\mathcal{B}}} = 0$), then the precision of the stock signal does not vary with the rate of interest (Panel A) because the bond signal cannot be used to form more precise (conditional) beliefs about the stock's demand; that is, $\tau_{u^{\mathcal{S}}|R_f} = \tau_{u^{\mathcal{S}}}$.[26] Accordingly, all learning can be attributed to the stock price (Panel B). Second, the stock signal provides information about the stock's payoff even if the variance of noise trading in the stock market is infinite ($\tau_{u^{\mathcal{S}}} = 0$) because, conditional on the interest rate, that variance is finite ($\tau_{u^{\mathcal{S}}|R_f} > 0$). This situation cannot arise in Hellwig (1980) and implies that all learning stems from the interest rate (Panel B).

IV. Rational Expectations Equilibrium with an Endogenous Interest Rate

We now turn to the "full" model with initial consumption, which is considerably less tractable and, hence, is solved numerically. Most importantly, we demonstrate that the key insights from the illustrative model continue to hold, namely, that the bond market reveals discount rate news and that prices are more informative when the interest rate is higher. In addition, we explicitly relate the supply of the bond to informational and allocative efficiency, and to asset prices.

IV.A. Learning from the Interest Rate

The key intuition for learning from the bond market again derives from investors' budget constraints and market clearing. That is, in equilibrium, the aggregate demand for the stock and the bond plus aggregate consumption must equal aggregate wealth or formally:

$$\left(\bar{X}^{\mathcal{S}} - u^{\mathcal{S}}\right) P + \left(\bar{X}^{\mathcal{B}} - u^{\mathcal{B}}\right) R_f^{-1} + \underbrace{\int_0^1 C_{i,1}\, di}_{\equiv \bar{C}_1} = \underbrace{\int_0^1 W_{i,1}\, di}_{\equiv \bar{W}_1}$$

$$\Leftrightarrow \qquad \frac{\bar{X}^{\mathcal{S}} R_f P + \bar{X}^{\mathcal{B}} - R_f \bar{W}_1}{R_f P} = u^{\mathcal{S}} + \frac{u^{\mathcal{B}}}{R_f P} - \frac{\bar{C}_1}{P}. \tag{14}$$

Hence, as in our illustrative framework, the bond market provides a signal about the noisy stock demand $u^{\mathcal{S}}$ (i.e., discount rate news), which, in turn, allows investors to more precisely infer the fundamental. Note, however, that the signal is now perturbed not only by noise traders' bond demand $u^{\mathcal{B}}$ but also by investors' period-1 aggregate consumption $\bar{C}_1$ (which is a function of the state variables and, hence, stochastic).

[26] As a result, the equilibrium price ratio, $R_f P$, coincides with that in Hellwig (1980). However, the interest rate remains stochastic, so that the equilibrium is not identical to Hellwig's.

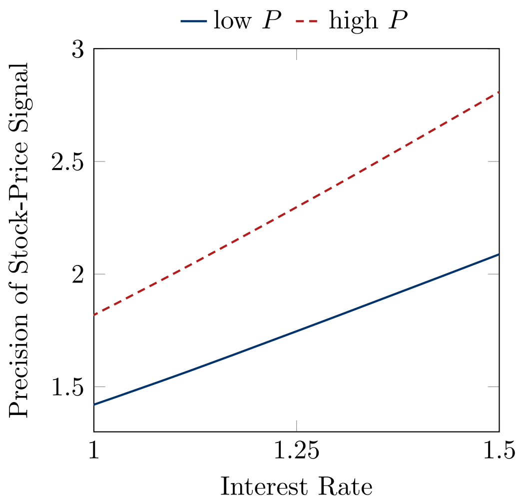

Figure 6. Precision of the Stock Price Signal

Notes: The figure plots the precision of the stock price signal as a function of the interest rate, R_f—for two levels of the stock price P. The precision of the stock price signal is measured as the difference between an (uninformed) investor's posterior precision, conditional on public prices, and her prior precision: $\mathbb{V}\text{ar}(z|P, R_f)^{-1} - \tau_z$. The graph is based on the following parameter values: $\beta = 0.95$, $\rho = 4$, $\tau_z = 2.5^2$, $\tau_\epsilon = 0.75^2$, $\bar{X}^{\mathcal{S}} = 1$, $\tau_{u^{\mathcal{S}}} = 5^2$, $\bar{X}^{\mathcal{B}} = 1$, $\tau_{u^{\mathcal{B}}} = 10^2$, $W_{i,1} = 3$, $K_1 = 1$, $\kappa = 5$, and $\delta = 0$.

Notably, the precision of the bond market signal (14) continues to depend on the rate of interest. That is, as before, a higher interest rate attenuates the noise originating from the bond demand and, thus, improves the signal's precision. Note, however, that the stock price now plays a similar attenuating role, a consequence of allowing for arbitrary stock endowments (unlike in the illustrative model). Specifically, the price ratio, $R_f P$, determines the overall intensity at which the noise from the bond demand perturbs the bond signal.[27] Figure 6 illustrates these properties. It confirms that the attenuating effect on the noisy bond demand from an increase in the interest rate or the stock price is not offset by any (variations in the) noise originating from aggregate consumption $\bar{C}_1$.

Note also that, because of investors' *intertemporal* consumption choices, aggregate consumption, $\bar{C}_1$, depends on expected trading profits, which are a nonlinear function of the state variables.[28] As a result, the bond-market-clearing condition (14) is no longer linear in the state variables. Accordingly, the model must be solved numerically. For that purpose, we extend the numerical solution approach presented in Breugem and Buss (2019) to allow for learning from the interest rate, intertemporal consumption choices, and endogenous out-

[27] Appendix B.A.2 presents the solution to the illustrative model for arbitrary initial endowments. It makes apparent that, in the case of arbitrary stock endowments, the price ratio, $R_f P$, determines the precision of the bond market signal.

[28] In particular, expected trading profits typically depend on the aggregate squared Sharpe ratio, $\int \left(\mathbb{E}\left[F \mid \mathcal{F}_i\right] - R_f P\right)^2 di$, which is a nonlinear function of the state variables.

put. The algorithm relies on discretizing the state space, which, in turn, allows to explicitly compute investors' posterior beliefs and to exactly solve the first-order and market-clearing conditions. Notably, the algorithm allows for arbitrary price and demand functions; that is, one does not need to parameterize these functions in any form. See Appendix C for additional details.

In the following, we rely on a specific set of parameter values (displayed in the figure captions) to illustrate the predictions of our model. We confirm that the patterns exhibited in the figures obtain for a wide range of parameter values. Indeed, in *all* our numerical analyses, the effect of variations in bond supply on informational and allocative efficiency and asset prices is as illustrated below. At the end of the section, we provide a brief comparative statics analysis to illustrate how the effects vary quantitatively with the main parameters.

IV.B. Bond Supply, Informational Efficiency, and Asset Prices

We now study how variations in the bond supply affect informational and allocative efficiency and asset prices. Intuitively, variations in the bond supply (or demand) can be linked to government and central bank policies through their influence on the mean and precision of the residual bond supply, $\bar{X}^{\mathcal{B}} - u^{\mathcal{B}}$. For instance, an elevated bond demand due to quantitative easing would lower the (residual) supply of bonds available to investors. Likewise, policies designed to stabilize long-term interest rates (e.g., by offsetting fluctuations in the liquidity-motivated demand for bonds) or to improve the transparency of central-bank communications would increase the precision of bond market noise.[29]

IV.B.1 Price Informativeness

We start our analysis with the impact of the bond supply on the informativeness of the stock price. We define price informativeness, PI, as the square root of the unconditional variance of the predictable component of the payoff, F, conditional on prices:

$$PI^2 = \mathbb{V}\text{ar}\big(\mathbb{E}\left[F \mid R_f, P\right]\big) = \mathbb{V}\text{ar}\left(F\right) - \mathbb{E}\left[\mathbb{V}\text{ar}\left(F \mid R_f, P\right)\right]. \tag{15}$$

[29] For instance, emerging countries engage in quantitative easing even though their rates are well above zero, in order to offset variations in the demand for long-term bonds and, thus, in long-term interest rates (as they might do for exchange rates); see, e.g., The Economist (2020).

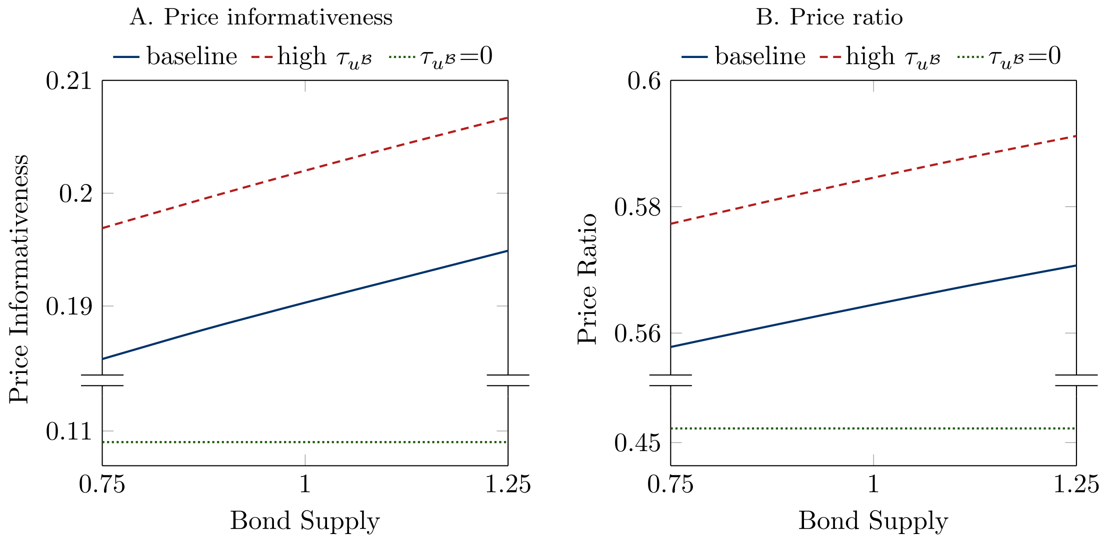

FIGURE 7. PRICE INFORMATIVENESS AND PRICE RATIO

Notes: The figure plots price informativeness (Panel A) and the expected price ratio (Panel B) as functions of the bond supply $\bar{X}^{\mathcal{B}}$. Price informativeness, PI, is calculated as in (15). The expected price ratio is calculated as the unconditional expectation of the ratio of the stock and bond price averaged over all realizations of the state variables. The graphs are based on the following baseline parameter values: $\beta = 0.95$, $\rho = 4$, $\tau_z = 2.5^2$, $\tau_\epsilon = 0.75^2$, $\bar{X}^{\mathcal{S}} = 1$, $\tau_{u^{\mathcal{S}}} = 5^2$, $\tau_{u^{\mathcal{B}}} = 10^2$, $W_{i,1} = 3$, $K_1 = 1$, $\kappa = 5$, and $\delta = 0$. High $\tau_{u^{\mathcal{B}}}$ describes an economy with a higher prior precision of the bond demand, and $\tau_{u^{\mathcal{B}}} = 0$ describes an economy in which investors do not learn from the bond-market signal.

This is the natural one-stock counterpart to the price informativeness measure employed in our empirical analyses. The higher PI, the more information prices contain.[30]

As Figure 7 illustrates, both stock-price informativeness and the price ratio, $R_f P$, are increasing in the bond supply, $\bar{X}^{\mathcal{B}}$. Indeed, they reinforce each other in equilibrium: the higher the price ratio, the more precise is information; conversely, the more precise information, the higher is the price ratio. Specifically, an increase in the interest rate leads to an increase in the price ratio, $R_f P$, regardless of whether information is private. In our setup, this increase improves the signal-to-noise ratio of the bond market signal (as discussed in the preceding section) and, thus, stock-price informativeness.[31] In turn, this improvement, by reducing risk and the associated stock price discount, pushes up the stock price and, hence, the price ratio. This leads to a further improvement in informativeness, generating

[30] In Section V.A, we employ a multiple-stock extension of the model to demonstrate that our theoretical results are robust to using the *cross-sectional variance* of the predictable component of firms' payoffs, as in the empirical measure (2).

[31] Our numerical analyses show that any noise originating from aggregate consumption only plays a secondary role here. Indeed, this increase in price informativeness in the supply of the bond shows up in *all* parametrizations of the model that we explore.

the concomitant increases in price informativeness and the price ratio (in the bond supply) illustrated in Figure 7.[32]

As before, an increase in the prior precision of the bond demand, $\tau_{u^{\mathcal{B}}}$, increases the precision of the bond signal and, hence, strengthens the impact of the bond market channel. Thus, price informativeness and the price ratio go up further (Panels A and B). Only if the variance of the noisy bond demand is infinite ($\tau_{u^{\mathcal{B}}} = 0$), there is no learning from the bond and price informativeness is independent of the bond supply (as in traditional REE models, such as that of Hellwig 1980).

IV.B.2 Real Investment and Allocative Efficiency

The firm's optimal investment, I, is characterized by the standard q-theory investment condition (Tobin 1969):

$$\frac{I}{K_1} = \frac{\mathbb{E}\left[z \,|\, P, R_f\right]}{\kappa}. \tag{16}$$

Importantly, the investment rate, I/K_1, is driven by the manager's *conditional* expectation of productivity z, given asset prices. This creates a feedback from financial markets to real investment decisions whereby the stock's price (aggregating investors' private information) not only reflects but also affects the firm's value. A natural measure of allocative efficiency is the "surplus output" that is expected in excess of the output produced by an uninformed manager (who optimally invests zero):[33]

$$\mathcal{E} = \mathbb{E}\big[\mathbb{E}\left[v(z, I) \,|\, P, R_f\right]\big] - \mathbb{E}\left[v(z, 0)\right]. \tag{17}$$

Panel A of Figure 8 shows that allocative efficiency is increasing in the bond supply. Intuitively, the more precise the manager's information, the more efficient the firm's invest-

[32] Appendix B.A.2 explicitly describes this two-way relation between the signal precision and the price ratio (which manifests itself in a cubic equation for the price ratio) in the illustrative model with arbitrary endowments. It demonstrates that stock-price informativeness is unambiguously increasing in the price ratio.

[33] Note that surplus output $\mathcal{E}$ is proportional to squared price informativeness: $\mathcal{E} = (K_1/2\kappa)\, PI^2$. Moreover, because output in the absence of learning is equal to $(2-\delta)\, K_1$ (and, hence, unrelated to price informativeness), variations in allocative efficiency can be directly traced back to variations in price informativeness: $d\mathcal{E}/\mathcal{E} = 2\, dPI/PI$. Consequently, the implied change in output $O \equiv \mathbb{E}\big[\mathbb{E}\left[v(z, I) \,|\, P, R_f\right]\big]$ is given by:

$$\frac{dO}{O} = \frac{d\mathcal{E}}{O} = \frac{\mathcal{E}}{O}\,\frac{d\mathcal{E}}{\mathcal{E}} = \frac{\mathcal{E}/\mathbb{E}\left[v(z,0)\right]}{1+\mathcal{E}/\mathbb{E}\left[v(z,0)\right]}\,\frac{d\mathcal{E}}{\mathcal{E}} \quad \text{with} \quad \frac{\mathcal{E}}{\mathbb{E}\left[v(z,0)\right]} = \frac{1}{2\,\kappa\,(2-\delta)}\, PI^2,$$

which supports the quantitative assessment in Footnote 16.

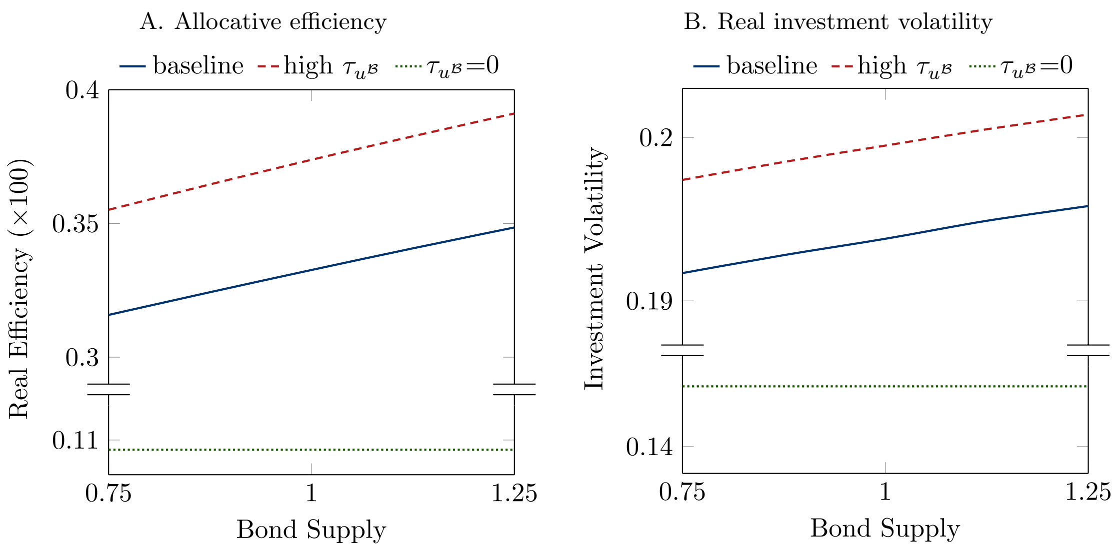

FIGURE 8. ALLOCATIVE EFFICIENCY

Notes: The figure plots allocative efficiency $\mathcal{E}$ (Panel A) and the unconditional real investment volatility $\sqrt{\mathbb{V}\text{ar}(I)}$ (Panel B) as functions of the bond supply $\bar{X}^{\mathcal{B}}$. The graph is based on the following baseline parameter values: $\beta = 0.95$, $\rho = 4$, $\tau_z = 2.5^2$, $\tau_\epsilon = 0.75^2$, $\bar{X}^{\mathcal{S}} = 1$, $\tau_{u^{\mathcal{S}}} = 5^2$, $\tau_{u^{\mathcal{B}}} = 10^2$, $W_{i,1} = 3$, $K_1 = 1$, $\kappa = 5$, and $\delta = 0$. High $\tau_{u^{\mathcal{B}}}$ describes an economy with a higher prior precision of the bond demand, and $\tau_{u^{\mathcal{B}}} = 0$ describes an economy in which investors do not learn from the bond-market signal.

ment. In particular, the increase in stock-price informativeness (thanks to a larger bond supply) allows the manager to improve her forecast of the productivity shock z and, hence, her investment. Notably, the higher allocative efficiency does not result from a higher *level* of investment. Instead, the positive effect of the bond supply on allocative efficiency results from more *efficient* investment decisions; that is, the manager can better differentiate between high-productivity states (in which she should invest more) and low-productivity states (in which she should invest less). The effect also manifests in a higher volatility of real investment, as illustrated in Panel B. Again, the effects are stronger, the more informative the bond demand (high $\tau_{u^{\mathcal{B}}}$).

IV.B.3 Consumption Choices

Variations in the bond supply also affect investors' consumption choices, as Figure 9 illustrates. Multiple effects shape those choices.

First, standard consumption-smoothing effects (unrelated to bond market learning) are at play. On the one hand, a higher rate of interest increases the price of period-1 consumption relative to period-2 consumption and, thus, shifts consumption from period 1 to period 2 (substitution effect). On the other hand, a higher interest rate makes investors "richer"

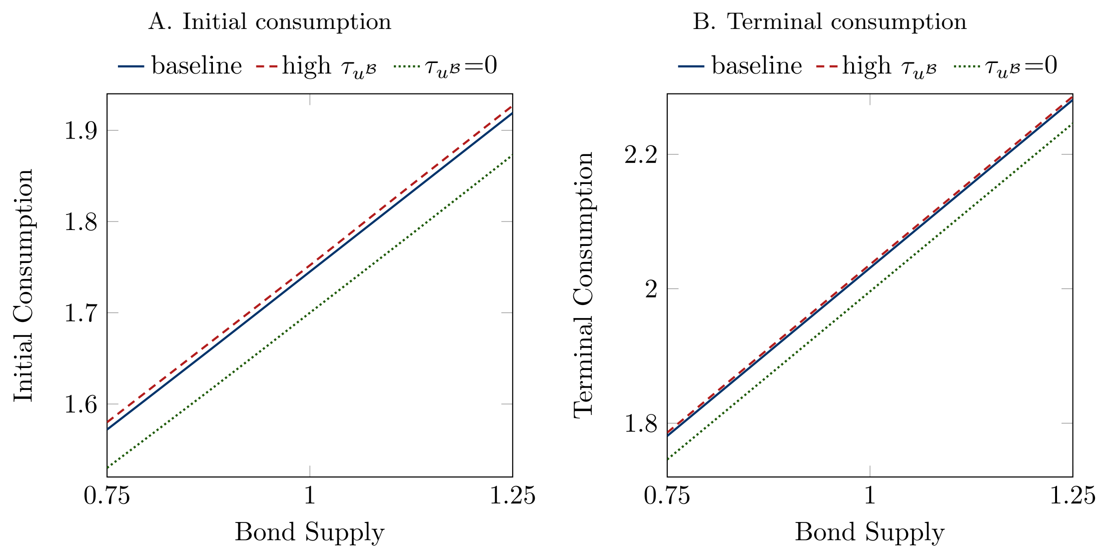

FIGURE 9. CONSUMPTION

Notes: The figure plots initial consumption (Panel A) and terminal consumption (Panel B) as functions of the bond supply $\bar{X}^{\mathcal{B}}$. We report the unconditional expectation for both quantities averaged over all realizations of the state variables. The graphs are based on the following baseline parameter values: $\beta = 0.95$, $\rho = 4$, $\tau_z = 2.5^2$, $\tau_\epsilon = 0.75^2$, $\bar{X}^{\mathcal{S}} = 1$, $\tau_{u^{\mathcal{S}}} = 5^2$, $\tau_{u^{\mathcal{B}}} = 10^2$, $W_{i,1} = 3$, $K_1 = 1$, $\kappa = 5$, and $\delta = 0$. High $\tau_{u^{\mathcal{B}}}$ describes an economy with a higher prior precision of the bond demand, and $\tau_{u^{\mathcal{B}}} = 0$ describes an economy in which investors do not learn from the bond-market signal.

and increases consumption in both periods (income effect). Both effects push up consumption in period 2 but operate in opposite directions for period-1 consumption. For usual levels of (absolute) risk aversion, the income effect dominates, and, hence, consumption in period 1 increases as well (as can be seen in Panel A in the case of an uninformative bond demand: $\tau_{u^{\mathcal{B}}} = 0$).

Bond market learning amplifies these consumption-smoothing effects. Specifically, a higher bond supply improves stock-price informativeness, which, in turn, reduces uncertainty and, hence, tempers investors' precautionary-savings motives. As a result, the bond price drops (interest rate increases), which strengthens consumption-smoothing effects and, hence, further pushes up consumption in both periods. In addition, the improvement in allocative efficiency increases expected output and, thus, consumption in both periods.

IV.B.4 Asset Prices and Returns

Variations in the bond supply also have important implications for equilibrium asset prices. As expected, a higher supply of the bond requires a higher rate of interest to clear the market

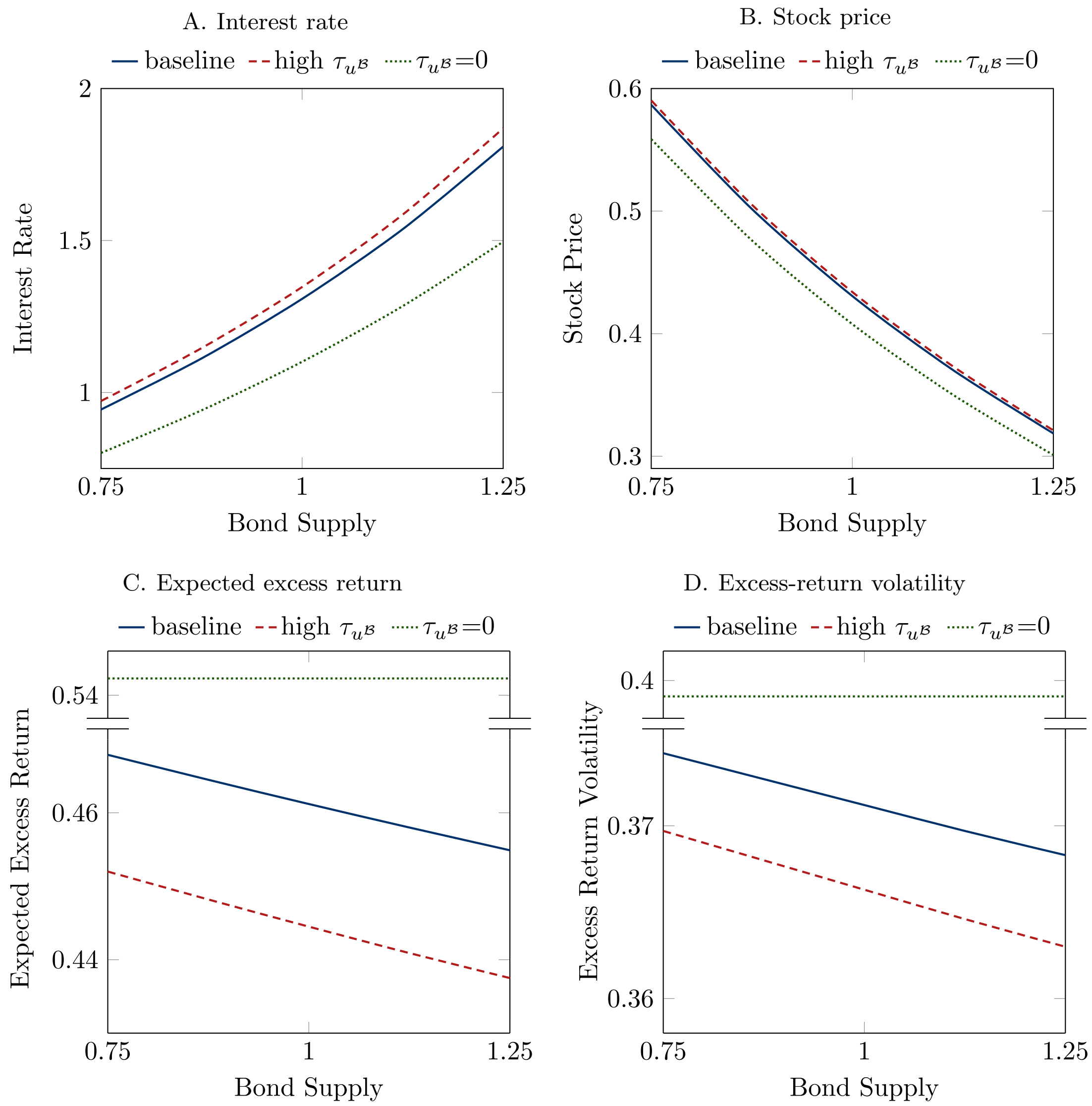

FIGURE 10. ASSET PRICES AND RETURNS

Notes: The figure plots the interest rate (Panel A), the stock price (Panel B), the stock's excess return (Panel C), and the excess-return volatility (Panel D) as functions of the bond supply $\bar{X}^{\mathcal{B}}$. We report the unconditional expectation of all quantities averaged over all realizations of the state variables. The graphs are based on the following baseline parameter values: $\beta = 0.95$, $\rho = 4$, $\tau_z = 2.5^2$, $\tau_\epsilon = 0.75^2$, $\bar{X}^{\mathcal{S}} = 1$, $\tau_{u^{\mathcal{S}}} = 5^2$, $\tau_{u^{\mathcal{B}}} = 10^2$, $W_{i,1} = 3$, $K_1 = 1$, $\kappa = 5$, and $\delta = 0$. High $\tau_{u^{\mathcal{B}}}$ describes an economy with a higher prior precision of the bond demand, and $\tau_{u^{\mathcal{B}}} = 0$ describes an economy in which investors do not learn from the bond-market signal.

(Panel A of Figure 10).[34] Moreover, the more precise the noise traders' bond demand, the

[34] It is straightforward to show that the (gross) interest rate is always positive here (in contrast to the illustrative setting without initial consumption; see Footnote 25). Intuitively, any investor's first-order condition for optimal consumption implies that the equilibrium interest rate is pinned down by the marginal rate of substitution across periods: $R_f = \frac{1}{\beta} \frac{\exp(-\rho\, C_{i,1})}{\mathbb{E}\left[\exp\left(-\rho\, C_{i,2}\right) \mid \mathcal{F}_i\right]} > 0$.

higher is the rate of interest. This increase is caused by the resultant reduction in uncertainty and, as a consequence, in investors' precautionary savings.

The stock price declines in the bond supply because of stronger discounting (Panel B). Note, however, that the simultaneous improvement in price informativeness partially offsets this decline as it reduces the risk borne by investors and, consequently, the price discount they demand. By the same account, the stock's expected excess return is decreasing in the bond supply (Panel C). Finally, the increase in price informativeness also implies that the stock's price tracks its payoff more closely, thereby reducing the excess-return volatility (Panel D). The latter two effects can be fully attributed to learning from the bond market signal, as demonstrated by the comparison with the reference case of an uninformative bond market ($\tau_{u^{\mathcal{B}}} = 0$). Accordingly, both effects are more pronounced for a higher prior precision of the noisy bond demand.

IV.C. Comparative Statics and Robustness

In this section, we provide a brief comparative statics analysis of the main parameters of the model. In addition, we demonstrate that our insights hold for preferences other than CARA; namely, they hold under constant relative risk aversion (CRRA) preferences. Figure 11 illustrates the results of these exercises. Importantly, the observation that, across all four panels, price informativeness depends on (specifically, rises in) the bond supply, confirms a central finding of the paper.

Panel A focuses on the impact of prior precisions. Consistent with the notion that signals are strategic substitutes, a reduction in the precisions of the noisy stock demand, ($\tau_{u^{\mathcal{S}}}$), of investors' private signals (τ_ε) and of the fundamental shock (τ_z) all lower price informativeness but (because of the lower level of price informativeness) the relative contribution of bond learning increases.

Panel B reports the implications of variations in investors' endowments. A decline in initial wealth ($W_{i,1}$) leads to a reduction in the demand for the bond and to a higher interest rate and price ratio, $R_f P$. The latter, in turn, improves stock price informativeness. Turning to the composition of endowed wealth, endowing each investor with the stock supply ($X^{\mathcal{S}}_{i,0} = 1$, whereas $X^{\mathcal{S}}_{i,0} = 0$ in the baseline) strengthens the impact of the bond-learning channel.[35] Indeed, the value of investors' equity holdings, and so their initial wealth, now

[35]To keep fixed the total supply of goods in the initial period, we endow investors also with $X^{\mathcal{B}}_{i,0} = 2$ shares of a bond maturing in period 1.

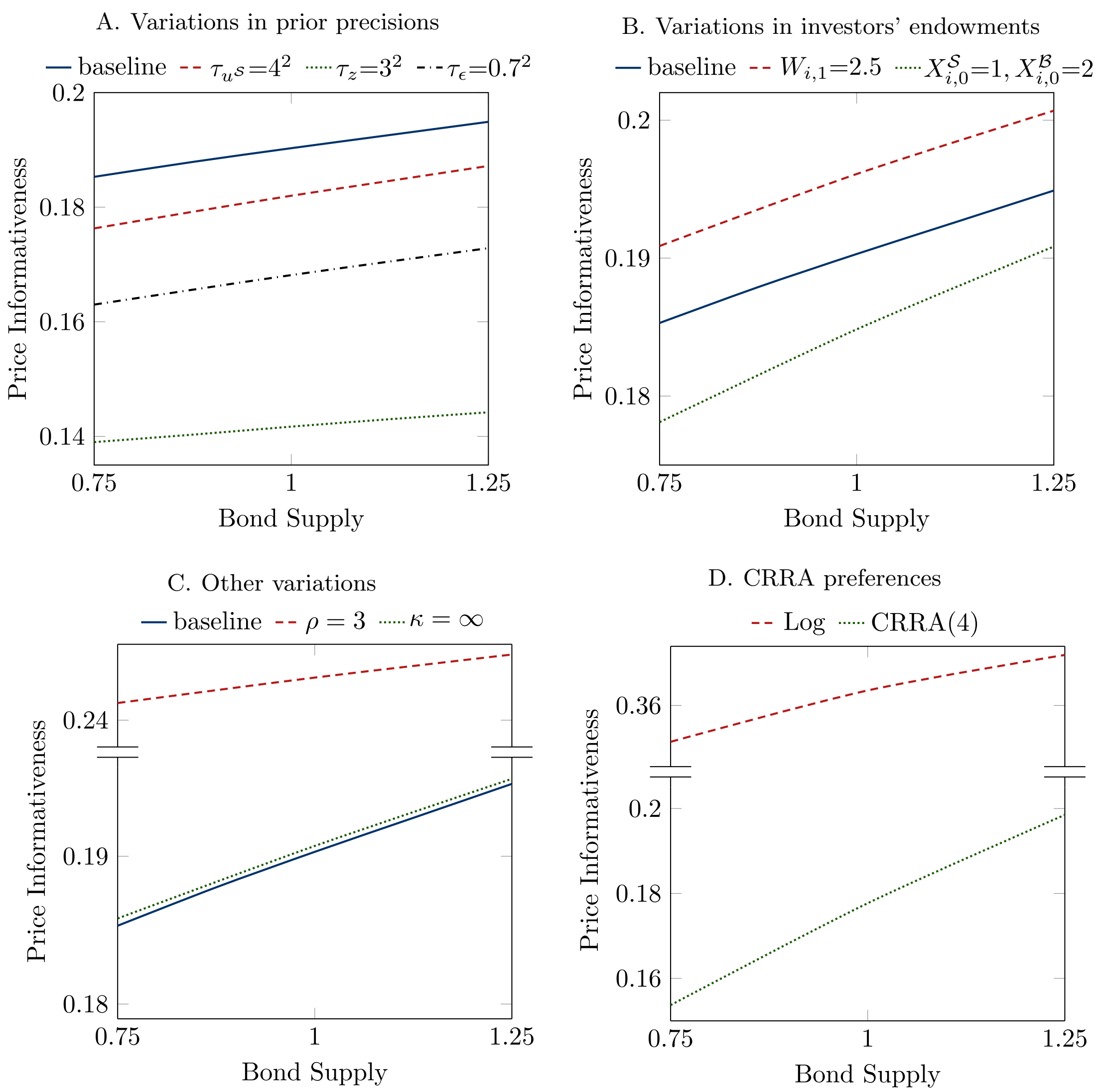

Figure 11. Comparative statics

Notes: The figure plots price informativeness as a function of the bond supply $\bar{X}^{\mathcal{B}}$ for variations of our main model setup. Price informativeness, PI, is calculated as in Equation (15). The baseline parameter values are as follows: $\beta = 0.95$, $\rho = 4$, $\tau_z = 2.5^2$, $\tau_\epsilon = 0.75^2$, $\bar{X}^{\mathcal{S}} = 1$, $\tau_{u^{\mathcal{S}}} = 5^2$, $\tau_{u^{\mathcal{B}}} = 10^2$, $W_{i,1} = 3$ (i.e., $X^{\mathcal{S}}_{i,0} = 0$ and $X^{\mathcal{B}}_{i,0} = 3$), $K_1 = 1$, $\kappa = 5$, and $\delta = 0$.

depends on the bond supply. This makes the price ratio and, hence, price informativeness more sensitive to the bond supply (i.e., the curve steepens).

Panel C displays the results of the remaining comparative statics analyses. A decline in risk aversion (ρ) shifts up the informativeness of the stock price because investors trade more aggressively on their private signals. However, the relative contribution of bond learning weakens (i.e., the curve flattens). This is because it makes the stock price and, hence, the price ratio less sensitive to changes in posterior precisions (thereby weakening the two-way interaction between price informativeness and the price ratio). Making the output exogenous ($\kappa = \infty$) leaves the impact of the bond signal largely unchanged.

Finally, Panel D demonstrates that our results remain qualitatively unchanged when investors have CRRA preferences. The case of log utility is particularly instructive as it implies a constant wealth-consumption ratio (because the income effect perfectly offsets the substitution effect). Hence, the signal error originating from aggregate consumption ($\bar{C}_1$) in the bond market signal (14) vanishes, thereby bringing the model closer to the illustrative model discussed in Section III. Quantitatively, the impact of the variations in bond supply is weaker for low levels of relative risk aversion (i.e., for the log case) as a lower risk aversion limits variation in the price ratio (similar to the case of CARA utility).

V. Extensions: Multiple Signals

To illustrate the general applicability of our key economic mechanism, we now explore two extensions of our main framework, both of which feature *multiple signals*. As in our main model, investors consume in both periods; the interest rate is determined endogenously, with investors learning from it; and output is endogenous.

V.A. Multiple Risky Assets

In our first extension, we allow for *multiple risky assets* and focus on the cross-sectional implications of learning from the interest rate. Beyond generality, this extension brings our theoretical measure of informational efficiency (so far based on a single firm) closer to the empirical measure in Section I.

The setting is identical to our main model's, except for the addition of a second stock. Specifically, this setting features two stocks, $k \in \{1, 2\}$, with prices $P^{(k)}$, and a risk-free bond with (endogenous) price $1/R_f$. All assets are in finite supplies, denoted by $\bar{X}^{\mathcal{S}_k}$ and $\bar{X}^{\mathcal{B}}$, respectively. The stocks are modeled as claims to the output of two corresponding firms, $k \in \{1, 2\}$, which employ the linear production technology (3). $F_1^{(k)}$ and $F^{(k)}$ denote output in periods 1 and 2, $z^{(k)} \sim \mathcal{N}(0, \tau_z)$ productivity shocks, and $I^{(k)}$ real investment. Investors, $i \in [0, 1]$, have CARA utility (4) and receive private signals about each firm's productivity: $S_i^{(k)} = z^{(k)} + \varepsilon_i^{(k)}$, with $\varepsilon_i^{(k)} \sim \mathcal{N}(0, \tau_\varepsilon)$. Finally, noise traders operate in all three markets, with demands $u^{\mathcal{S}_k} \sim \mathcal{N}(0, \tau_{u^{\mathcal{S}}})$ and $u^{\mathcal{B}} \sim \mathcal{N}(0, \tau_{u^{\mathcal{B}}})$. To highlight the impact of learning from the bond market, we assume that the two stocks are *independent of one another* in terms of fundamentals, private signals, and noise traders' demand.

To understand the role that learning from the interest rate plays in this economy, note that aggregating investors' budget constraints and market clearing delivers a straightforward two-stock version of the bond market-clearing condition (14). That is, in equilibrium, the aggregate demand for the three assets plus aggregate consumption must equal aggregate endowed wealth:

$$(18)\quad \left(\bar{X}^{\mathcal{S}_1} - u^{\mathcal{S}_1}\right) P^{(1)} + \left(\bar{X}^{\mathcal{S}_2} - u^{\mathcal{S}_2}\right) P^{(2)} + \left(\bar{X}^{\mathcal{B}} - u^{\mathcal{B}}\right) R_f^{-1} + \underbrace{\int_0^1 C_{i,1}\, di}_{\equiv \bar{C}_1} = \underbrace{\int_0^1 W_{i,1}\, di}_{\equiv \bar{W}_1}.$$

Condition (18) has three important implications. First, consistent with the one-stock model, the bond market provides a signal about the stock demands $u^{\mathcal{S}_k}$ (i.e., discount rate news), which allow investors to form more precise (conditional) beliefs about fundamentals. As before, the error in this signal originates from the bond demand, $u^{\mathcal{B}}$, and aggregate consumption, $\bar{C}_1$.

Second, a higher interest rate attenuates the noise originating from noise traders' demand and, thus, improves the signal's precision. As a result, stock-price informativeness is again increasing in the bond supply. Panel A of Figure 12 illustrates this property for price informativeness defined as in (15), that is, separately for each stock. Panel B shows that the increase also holds for price informativeness measured as the (square root of the) *cross-sectional* variance of the predictable component of firms' payoffs, $F^{(k)}$, given stock prices (i.e., as in our empirical measure (2)).[36] Accordingly, allocative efficiency also improves in the supply of the bond (Panel C).[37]

Third, the bond market signal induces a *negative correlation* between an investor's beliefs about the noisy demands of the two stocks, even though the demands are uncorrelated with one another. In particular, condition (18) constrains the (weighted) *sum* of the residual supplies of the two stocks, $(\bar{X}^{\mathcal{S}_k} - u^{\mathcal{S}_k})$. Hence, conditional on the bond's noisy demands and aggregate consumption, an investor who assigns a higher value to one of the stocks' noisy

[36] As expected, the cross-sectional variance and its sensitivity to the bond supply are fairly small because we only have two (independent and symmetric) stocks in the model, rather than hundreds of correlated stocks as in the empirical analysis.

[37] Note, in our framework, with no constraint on investment, firms' investment decisions are independent of one another. If aggregate capital is scarce, then the improvement in price informativeness also leads to a better allocation of capital *across firms* and, hence, to lower dispersion in the marginal products of capital.

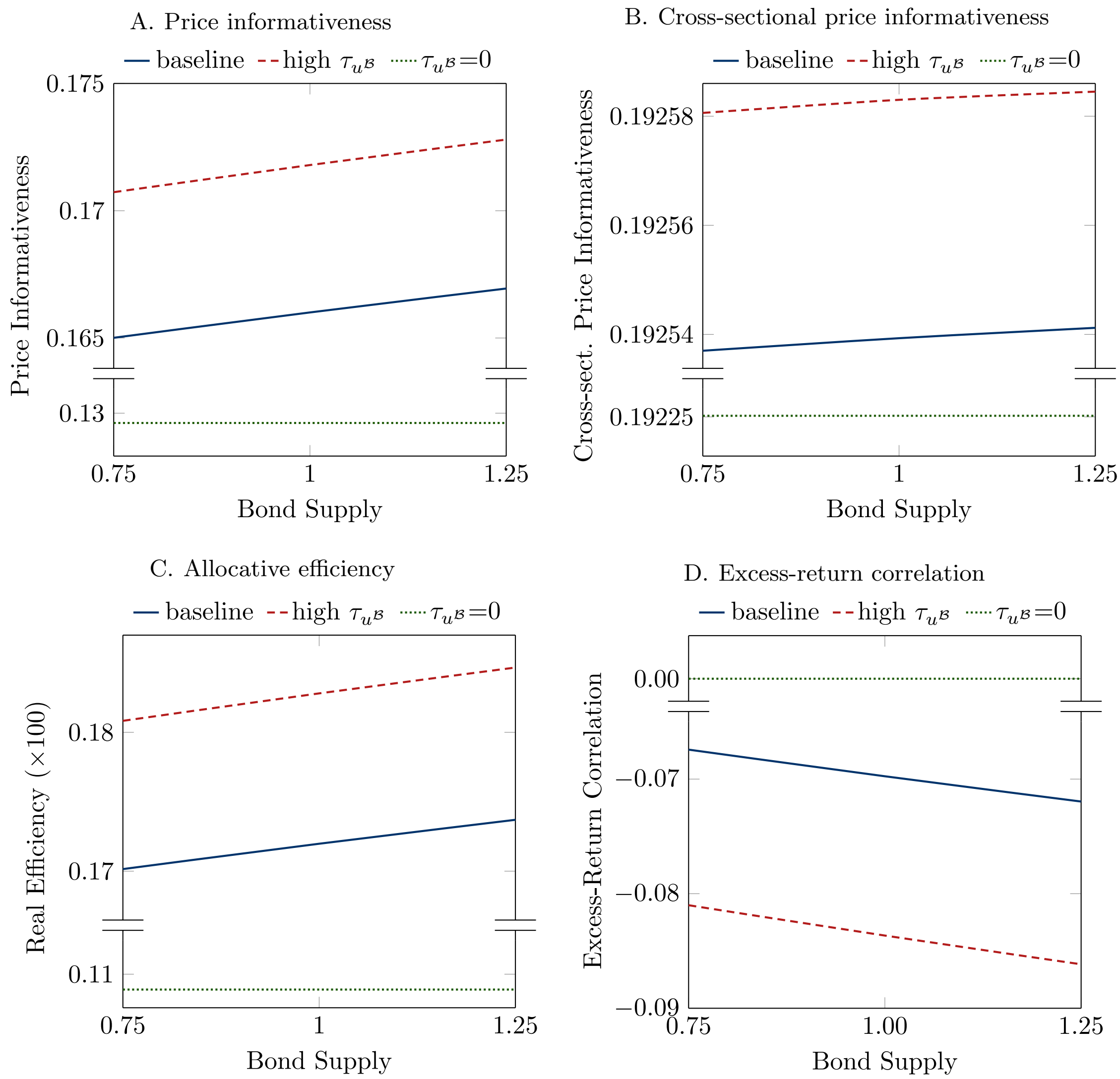

FIGURE 12. EXTENSION: TWO STOCKS

Notes: The figure plots price informativeness (Panels A and B), allocative efficiency (Panel C), and excess-return correlation (Panel D) as functions of the bond supply $\bar{X}^{\mathcal{B}}$. Panel A reports price informativeness, PI, calculated as in Equation (15), that is, separately for each stock. Panel B reports price informativeness measured as square root of the unconditional expectation of the *cross-sectional* variance of the predictable component of firms' payoffs, $F^{(k)}$, given stock prices averaged over all realizations of the state variables. Allocative efficiency is computed as in Equation (17). The excess-return correlation is calculated as the unconditional expectation of the correlation between the two stocks' returns in excess of the interest rate. The baseline parameter values are as follows: $\beta = 0.95$, $\rho = 4$, $\tau_z = 2.5^2$, $\tau_\epsilon = 0.75^2$, $\bar{X}^{\mathcal{S}} = 1$, $\tau_{u^{\mathcal{S}}} = 5^2$, $\tau_{u^{\mathcal{B}}} = 10^2$, $W_{i,1} = 3$, $K_1^{(k)} = 1/2$, $\kappa = 5$, and $\delta = 0$. High $\tau_{u^{\mathcal{B}}}$ describes an economy with a higher prior precision of the bond demand, and $\tau_{u^{\mathcal{B}}} = 0$ describes an economy in which investors do not learn from the bond-market signal.

demands rationally assigns a lower value to the demand for the other stock. This generates a negative correlation between an investor's beliefs about the payoffs of the two stocks, which, in turn, lowers the correlation of the excess returns of the two stocks. Crucially, this effect strengthens with the precision of the bond market signal. Hence, the correlation between excess returns declines (or, put differently, their dispersion rises) in the interest rate, or, equivalently, in the bond supply (Panel D).[38] Notably, in stark contrast to Admati (1985), the correlation of the excess returns of the two stocks is nonzero despite the stocks' payoffs, private signal errors, and noisy demands being independent of each other.

As expected, the higher the precision of the noise traders' bond demand, the stronger these three effects are. These mounting effects lead to improved informational and allocative efficiency (Panels A to C) and lower excess-return correlation (Panel D).

V.B. Multiple Price Signals

In our second extension, we allow for *multiple price signals*. In particular, we demonstrate that other prices, such as the good's price (i.e., the rate of inflation), also reveal discount rate news.

For that purpose, we incorporate money into our single-stock framework and, accordingly, now distinguish between nominal and real quantities. In particular, we assume that investors derive utility from the quantity of the real money balances they hold.[39] Formally, investor i maximizes $\mathcal{U}_i(C_{i,1}, C_{i,2}) - (1/\alpha)\exp\left(-\alpha(X_i^{\mathcal{M}}/P_1^G)\right)$, with parameter $\alpha > 0$, and $\mathcal{U}_i$ denoting the two-period CARA utility in (4). Here, P_t^G denotes the price of the good in period $t \in \{1, 2\}$ (with P_2^G being normalized to 1); $X_i^{\mathcal{M}}$ denotes investor i's money holdings; and $X_i^{\mathcal{M}}/P_t^G$ denotes her real money balance in period t. Investors' budget constraints, accordingly, also account for their real money holdings.[40] As with the other assets, the supply of money, $\bar{X}^{\mathcal{M}}$, is assumed to be finite. Finally, in addition to their stock and bond demands, we assume that noise traders have an uncorrelated random demand for money, $u^{\mathcal{M}} \sim \mathcal{N}(0, 1/\tau_{u^{\mathcal{M}}})$, which prevents the price system from being perfectly revealing.[41]

[38] For ease of exposition, we assume independent payoffs and demands. As a result, the conditional correlation of the two stocks' excess returns is zero absent learning from the interest rate ($\tau_{u^{\mathcal{B}}} = 0$). To accommodate a positive correlation between the two stocks (as is typically found in the data), one could simply assume positively correlated payoffs (or liquidity demands).

[39] This is a commonly used shortcut to model the usefulness of money as a medium of exchange; otherwise, money would be dominated as a store of value (to the extent that bonds strictly pay positive nominal interest). It captures the notion that, the higher the purchasing power of an investor's money holdings, the lower is the disutility cost associated with exchange, which results in higher overall utility.

[40] Equations (A21) and (A22) in Appendix B.C.2 display the exact formulation of investors' optimization problem.

[41] The equilibrium is defined as previously with the addition of the market-clearing condition for money: $\int X_i^{\mathcal{M}} di + u^{\mathcal{M}} = \bar{X}^{\mathcal{M}}$. Note that, by Walras' law, clearing in the bond, the stock, and the money markets guarantees clearing in the goods market.

To understand the role that the good's price and the interest rate play in this economy, note that investors' budget constraints, combined with market clearing, imply the following two equilibrium conditions:[42]

$$(19) \qquad \hat{s}_1 = u^{\mathcal{S}} + \frac{u^{\mathcal{B}}}{R_f\,P} + \frac{u^{\mathcal{M}}}{P_1^G\,P} - \frac{\bar{C}_1}{P} \qquad \text{and} \qquad \hat{s}_2 = u^{\mathcal{S}} + \frac{u^{\mathcal{B}}}{R_f\,P} + \frac{\rho+\alpha}{\rho}\,\frac{u^{\mathcal{M}}}{P_1^G\,P},$$

where $\hat{s}_1$ and $\hat{s}_2$ represent two distinct signals (and, as before, $\bar{C}_1 \equiv \int_0^1 C_{i,1}\,di$ denotes aggregate consumption). The first equation is the counterpart to the bond market signal (14), to which noise traders' (real) money holdings were added; it is the result of market clearing in the bond market. The second equation—absent from the model without money—can be interpreted as a money market signal, as it arises from clearing the money market.

Notably, both the bond and the money markets provide signals about the noisy stock demand, $u^{\mathcal{S}}$. Hence, not only the rate of interest but also the good's price (or, equivalently, the rate of inflation, P_2^G/P_1^G) reveal discount rate news, which, in turn, allows investors to form more precise beliefs about the fundamental (i.e., their information set has expanded to $\mathcal{F}_i = \{S_i, P, R_f, P_1^G\}$). The signal errors originate from the noisy bond demand, $u^{\mathcal{B}}$, the noisy money demand, $u^{\mathcal{M}}$, and, in the case of the bond market signal, aggregate consumption, $\bar{C}_1$.

A higher interest rate, R_f, once again attenuates the bond noise and, thus, improves both signals' precisions. The same property holds now for the price of the good, P_1^G, which attenuates the money noise. As a result, stock-price informativeness is also increasing in the money supply, $\bar{X}^{\mathcal{M}}$, as Panel A of Figure 13 illustrates. Note also that price informativeness increases in the supply of money even if its noisy demand is uninformative ($\tau_{u^{\mathcal{M}}} = 0$). This is a consequence of the increase in the real interest rate (and, hence, the price ratio $R_f P$) that is triggers as, in equilibrium, investors must be indifferent between saving through risk-free bonds and money holdings.[43] This, in turn, leads to a higher precision of the two signals. Indeed, only if the variance of the noisy demands is infinite in *both* markets ($\tau_{u^{\mathcal{M}}} = 0$ and $\tau_{u^{\mathcal{B}}} = 0$) is price informativeness independent of the money supply. As in our main framework, the higher price informativeness (thanks to a larger money supply) translates into higher allocative efficiency (Panel B).

[42]See Appendix B for the proof and the explicit expressions for $\hat{s}_1$ and $\hat{s}_2$.

[43]The positive relation between the interest rate and the period-1 good's price is easily understood when investors derive no utility from real money holdings. In that case, the first-order condition for an investor's money holdings implies that $R_f = P_1^G/P_2^G$.

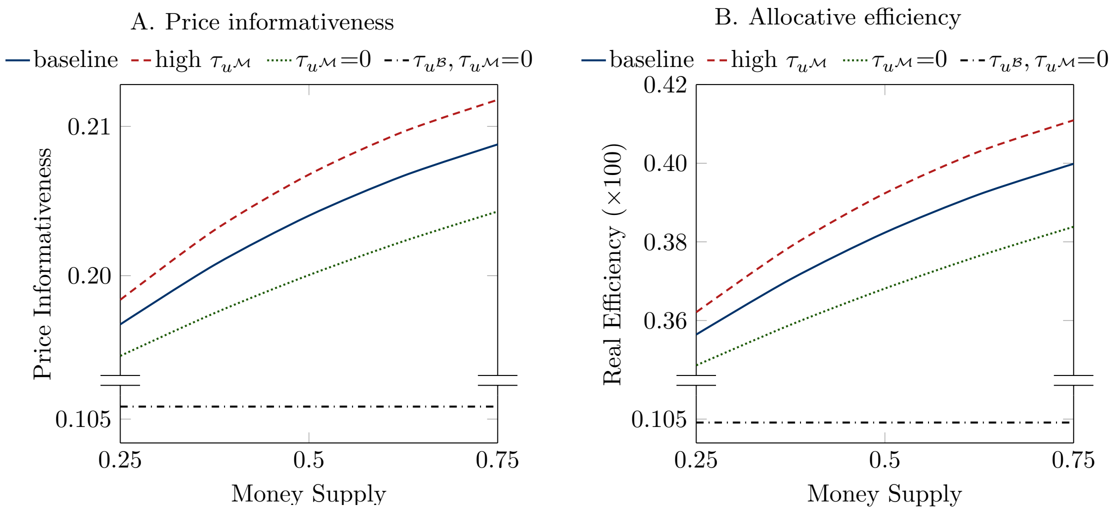

FIGURE 13. EXTENSION: MONEY

Notes: The figure plots price informativeness (Panel A) and allocative efficiency (Panel B) as functions of the money supply $\bar{X}^{\mathcal{M}}$. Price informativeness, PI, is calculated as in Equation (15) and allocative efficiency is computed as in Equation (17). The baseline parameter values are as follows: $\beta = 0.95$, $\rho = 4$, $\tau_z = 2.5^2$, $\tau_\epsilon = 0.75^2$, $\bar{X}^{\mathcal{S}} = 1$, $\tau_{u^{\mathcal{S}}} = 5^2$, $\tau_{u^{\mathcal{B}}} = 10^2$, $\tau_{u^{\mathcal{M}}} = 10^2$, $W_{i,1} = 5$, $K_1 = 1$, $\kappa = 5$, and $\delta = 0$. High $\tau_{u^{\mathcal{M}}}$ describes an economy with a higher prior precision of the money demand, $\tau_{u^{\mathcal{M}}} = 0$ describes an economy in which investors do not learn from the good's price, and $\tau_{u^{\mathcal{B}}}, \tau_{u^{\mathcal{M}}} = 0$ describes an economy in which investors do not learn from the bond-market and money-market signals.

As with bond market noise, the effect is stronger, the more precise the noisy money demand. In particular, monetary policy might be interpreted as determining not only the supply of bonds and money ($\bar{X}^{\mathcal{B}}$ and $\bar{X}^{\mathcal{M}}$) but also the precision of bonds and money ($\tau_{u^{\mathcal{B}}}$ and $\tau_{u^{\mathcal{M}}}$).[44] Indeed, to the extent that the government or central bank does not commit to (or communicate) a precise level of debt or money supplies, it adds to the noise created by liquidity traders. For instance, by disclosing a narrower range of bond or money supplies (corresponding to a more transparent policy), it enables investors to know with greater confidence what these supplies are and, thus, enhances the sensitivity of price informativeness to debt and money supplies.[45] This makes policy implementation more efficient by allowing the government to raise informativeness without actually varying supplies.

[44] Under this interpretation, monetary policy is exogenous and, accordingly, does not convey any information to investors. Our focus is on how such a policy affects the public's (as well as the government's own) ability to learn about economic fundamentals from stock prices. An interesting extension of the model could be to endogenize monetary policy, to account for its impact on stock-price informativeness.

[45] Central bank communication and monetary policy transparency comprise many aspects, and the literatures on both are extensive (for a survey, see Blinder et al. (2008)). Using the (Geraats, 2014, p. 5) classification of transparency, we consider here "policy transparency," that is, the "communication of the policy stance (including the policy decision, policy explanation and inclination with respect to future policy actions)."

VI. Conclusion

In this paper, we provide new theoretical and empirical insights into how investors use information contained in interest rates to learn about economic fundamentals and how this affects informational and allocative efficiency.

We develop a novel noisy rational expectations equilibrium model in which the interest rate is determined endogenously by supply and demand. We demonstrate that the interest rate reveals information about noise traders' stock demand (i.e., discount rate news), which, in turn, allows investors to form more precise beliefs about a stock's cashflows from observing the stock's price. The strength of this effect is positively related to the interest rate as a higher interest rate attenuates the error in the bond market signal. Consequently, both stock-price informativeness and allocative efficiency positively correlate with long-term rates or, equivalently, bond supply. The robust empirical evidence we report lends support to this prediction. This mechanism also endogenously creates countercyclicality in the price of risk as well as in the volatility and comovement of stock returns, as in the data.

More broadly, our analyses offer novel insights into the impact of fiscal and monetary policies in an environment in which information about economic fundamentals is asymmetric across investors. In particular, we show that increases in the supply of (demand for) both bonds and money improve (harm) informational and allocative efficiency. As such, our findings point towards important unintended consequences of unconventional monetary policy (QE) and "financial repression" for aggregate efficiency.[46] Our findings also highlight that more transparent policies (in the sense of more precise disclosures about or of stricter commitments to bond and money issuance) can improve the stock-market efficiency.

Finally, our findings also suggest a novel interpretation of the concomitant declines in aggregate productivity growth and real interest rates in the United States (Decker et al. 2017) and in capital allocation efficiency and real interest rates in southern Europe (Gopinath et al. 2017). Specifically, the decline in interest rates might have impaired investor learning about economic fundamentals and, hence, made the allocation of capital less efficient, thereby slowing down productivity growth.[47] We look forward to empirical work testing whether these associations are causal or mere correlations.

[46] Financial repression refers to policies that force captive domestic audiences (such as pension funds or domestic banks) to hold government debt and, hence, keep interest rates lower than would otherwise prevail (see, e.g., Reinhart, Kirkegaard and Sbrancia 2011, Chari, Dovis and Kehoe 2020).

[47] Most of this evidence is on private firms, which can be added to our model in a straightforward fashion. Assume that a private firm employs a similar production technology, with productivity that is correlated with the public firm's (e.g., through economywide shocks). The private firm's manager then learns about the public firm's productivity from the stock price and the interest rate (just as the public firm manager does), from which she learns about her own productivity. Thus, a higher interest rate facilitates learning for the private firm and increases the efficiency of the private firm's investments (as for the public firm).

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Appendix

A. Summary statistics

Variable	Obs.	Mean	Median	Std. Dev.	Min	Max	ρ_1
Price Informativeness PI_5	50	0.049	0.050	0.011	0.023	0.073	0.475
Price Informativeness PI_3	50	0.040	0.040	0.010	0.021	0.062	0.431
Debt/GDP	50	0.369	0.358	0.123	0.209	0.754	0.854
FED MBS Hold./GDP	50	0.005	0.000	0.016	0.000	0.072	0.781
FED Treasury Hold./GDP	50	0.051	0.050	0.013	0.032	0.107	0.584
S&P500 Volatility	50	0.141	0.139	0.053	0.058	0.296	0.334
Cashflow Volatility	50	0.070	0.069	0.006	0.060	0.095	0.558
Real Interest Rate	50	0.026	0.027	0.027	-0.045	0.085	0.741

TABLE A1—SUMMARY STATISTICS

Notes: The table reports summary statistics for our main variables. *Price Informativeness* PI_h refers to the coefficient, $b_{t,h}$, of the cross-sectional regression (1) multiplied by the cross-sectional standard deviation of (scaled) stock prices (for forecasting horizons $h \in \{3, 5\}$). *Debt/GDP* is the ratio of the market value of Treasury debt held by the public to U.S. GDP. *FED MBS Hold./GDP* and *FED Treasury Hold./GDP* are the ratio of the Federal Reserve banks' holdings of MBS and Treasury securities, divided by U.S. GDP. *S&P500 Volatility* and *Cashflow Volatility* are measures of volatility of, respectively, the S&P500 returns and of firms' earnings. *Real Interest Rate* is the nominal rate of long-term U.S. government bonds minus expected inflation (estimated using a random-walk model). ρ_1 denotes the first-order autocorrelation.

B. Proofs and Derivations

B.A. Proofs for Section III

With exogenous output, the fundamental F is normally distributed with mean μ_F and precision τ_F: $F \sim \mathcal{N}(\mu_F, 1/\tau_F)$. Investors receive a private signal of the form: $S_i = F + \varepsilon_i$.

Moreover, in the absence of initial consumption, the objective of each investor i is to choose her portfolio holdings in the stock, $X_i^{\mathcal{S}}$, and in the bond, $X_i^{\mathcal{B}}$, to maximize the expected utility over terminal consumption $C_{i,2}$:

$$\mathcal{U}_i(C_{i,2}) = -(1/\rho)\,\mathbb{E}\left[\exp\bigl(-\rho\, C_{i,2}\bigr) \,\middle|\, \mathcal{F}_i\right], \tag{A1}$$

subject to the budget constraints:

$$X_i^{\mathcal{S}}\, P + X_i^{\mathcal{B}}\, R_f^{-1} = W_{i,1} \quad \text{and} \quad C_{i,2} = X_i^{\mathcal{S}}\, F + X_i^{\mathcal{B}}. \tag{A2}$$

B.A.1 Main Setting: Identical Aggregate Stock Endowments across Periods

We first focus on the setting described in Section III that assumes that the aggregation of investors' stock endowments coincides with the period-1 residual stock supply (i.e., $\int X_{i,0}^{\mathcal{S}}\, di = \bar{X}^{\mathcal{S}} - u^{\mathcal{S}}$), that investors have no endowments in bonds (i.e., $X_{i,0}^{\mathcal{B}} = 0$) and that individual stock endowments reveal no information about noise traders' stock demand.

We conjecture (and later verify) that the market-clearing conditions in the bond and the stock market (6) are linear in the state variables:[48]

$$0 = b_0 + b_1\, u^{\mathcal{S}} + b_2\, u^{\mathcal{B}}, \tag{A3}$$

$$R_f\, P = a_0 + a_1\, F + a_2\, u^{\mathcal{S}}. \tag{A4}$$

Both serve as public signals for investors. Specifically, the bond-market-clearing condition (A3) provides a signal about the noisy stock demand, $u^{\mathcal{S}}$, allowing investors to form *conditional* (posterior) beliefs about $u^{\mathcal{S}}$, with posterior precision $\tau_{u^{\mathcal{S}}|R_f} \equiv \mathbb{V}\mathrm{ar}\left(u^{\mathcal{S}} \mid R_f\right)^{-1}$ and a posterior mean $\mu_{u^{\mathcal{S}}|R_f} = \mathbb{E}\left[u^{\mathcal{S}} \mid R_f\right]$:

$$\tau_{u^{\mathcal{S}}|R_f} = \tau_{u^{\mathcal{S}}} + \frac{b_1^2}{b_2^2}\, \tau_{u^{\mathcal{B}}}, \quad \text{and} \quad \mu_{u^{\mathcal{S}}|R_f} = \frac{\tau_{u^{\mathcal{B}}}}{\tau_{u^{\mathcal{S}}|R_f}}\, \frac{b_1^2}{b_2^2} \left(-\frac{b_0}{b_1}\right). \tag{A5}$$

Combining these posterior beliefs about the noisy stock demand with an investor's prior information about F, her private signal, $S_i = F + \varepsilon_i$, and the conjectured stock price signal, $R_f P$, in (A4), yields her posterior beliefs about the stock payoff, F:

$$\tau \equiv \mathbb{V}\mathrm{ar}(F \mid S_i, R_f, P) = \tau_F + \tau_\varepsilon + \frac{a_1^2}{a_2^2}\, \tau_{u^{\mathcal{S}}|R_f}; \quad \text{and} \tag{A6}$$

$$\begin{aligned} \mathbb{E}\left[F \mid S_i, R_f, P\right] &= \frac{\tau_F}{\tau}\, \mu_F + \frac{\tau_\varepsilon}{\tau}\, S_i + \frac{\tau_{u^{\mathcal{S}}|R_f}}{\tau}\, \frac{a_1^2}{a_2^2}\, \frac{R_f P - a_0 - a_2\, \mu_{u^{\mathcal{S}}|R_f}}{a_1} \\ &= \frac{1}{\tau}\left(\tau_F \mu_F - \frac{a_1}{a_2^2}\, \tau_{u^{\mathcal{S}}|R_f} \left(a_0 + a_2\, \mu_{u^{\mathcal{S}}|R_f}\right)\right) + \frac{\tau_\varepsilon}{\tau}\, S_i + \frac{\tau_{u^{\mathcal{S}}|R_f}}{\tau}\, \frac{a_1}{a_2^2}\, R_f P. \end{aligned} \tag{A7}$$

[48] Or, equivalently, if written explicitly in the form of the market-clearing conditions:

$$\frac{1}{b_2}\left(b_0 + b_1 u^{\mathcal{S}} + b_2\, \bar{X}^{\mathcal{B}}\right) + u^{\mathcal{B}} = \bar{X}^{\mathcal{B}}, \quad \text{and} \quad \frac{1}{a_2}\left(a_0 - R_f\, P + a_1\, F + a_2\, \bar{X}^{\mathcal{S}}\right) + u^{\mathcal{S}} = \bar{X}^{\mathcal{S}}.$$

Solving the period-1 budget constraint in (5) for the bond holdings, $X_i^{\mathcal{B}}$, yields

$$X_i^{\mathcal{B}} = R_f \left(W_{i,1} - X_i^{\mathcal{S}} \, P \right), \tag{A8}$$

which can be used to rewrite the period-2 budget constraint in (5) as

$$C_{i,2} = R_f \, W_{i,1} + X_i^{\mathcal{S}} \, (F - P) \, . \tag{A9}$$

Plugging the period-2 consumption (A9) into the investor's utility function and maximizing the utility with respect to $X_i^{\mathcal{S}}$ yields the traditional CARA optimal stock demand:

$$X_i^{\mathcal{S}} = \frac{\mathbb{E}[F \,|\, S_i, R_f, P] - P \, R_f}{\rho \, \mathbb{V}\text{ar}(F \,|\, S_i, R_f, P)} . \tag{A10}$$

Aggregating investors' bond demand (A8) and imposing market clearing in the bond and stock markets imply

$$\begin{aligned} \int_0^1 X_i^{\mathcal{B}} \, di + u^{\mathcal{B}} &= \int_0^1 R_f \, \left(X_{i,0}^{\mathcal{S}} \, (F_1 + P) - X_i^{\mathcal{S}} \, P \right) \, di + u^{\mathcal{B}} \\ &= R_f \, F_1 \int_0^1 X_{i,0}^{\mathcal{S}} \, di + u^{\mathcal{B}} = R_f \, F_1 \, \left(\bar{X}^{\mathcal{S}} - u^{\mathcal{S}} \right) + u^{\mathcal{B}} \triangleq \bar{X}^{\mathcal{B}}, \end{aligned}$$

where we use that, by assumption, $\int X_{i,0}^{\mathcal{S}} \, di = \bar{X}^{\mathcal{S}} - u^{\mathcal{S}}$. This verifies conjecture (A3) and (by matching coefficients) directly yields

$$b_0 = R_f \, F_1 \, \bar{X}^{\mathcal{S}} - \bar{X}^{\mathcal{B}}, \qquad b_1 = -R_f \, F_1 \qquad \text{and} \qquad b_2 = 1. \tag{A11}$$

As a result, investors' posterior mean and precision about the noisy stock demand in (A5) are given by

$$\tau_{u^{\mathcal{S}}|R_f} = \tau_{u^{\mathcal{S}}} + F_1^2 \, R_f^2 \, \tau_{u^{\mathcal{B}}}, \quad \text{and} \quad \mu_{u^{\mathcal{S}}|R_f} = \frac{\tau_{u^{\mathcal{B}}}}{\tau_{u^{\mathcal{S}}|R_f}} \, F_1 \, R_f \, \left(R_f \, F_1 \, \bar{X}^{\mathcal{S}} - \bar{X}^{\mathcal{B}} \right) . \tag{A12}$$

Plugging the investors' posterior beliefs (A6) and (A7) about payoff F (replacing b_0, b_1, and b_2 with (A11)) into the stock demand (A10), aggregating across investors, and imposing market clearing yields

$$\int_0^1 \frac{\tau}{\rho}\left\{\frac{1}{\tau}\left(\tau_F\mu_F - \frac{a_1\,\tau_{u^{\mathcal{S}}|R_f}}{a_2^2}\left(a_0 + a_2\mu_{u^{\mathcal{S}}|R_f}\right)\right) + \frac{\tau_\varepsilon}{\tau}S_i + \frac{\tau_{u^{\mathcal{S}}|R_f}}{\tau}\frac{a_1}{a_2^2}R_fP - R_fP\right\}di + u^{\mathcal{S}}$$

(A13)

$$= \frac{1}{\rho}\left(\tau_F\mu_F - \frac{a_1\tau_{u^{\mathcal{S}}|R_f}}{a_2^2}\left(a_0 + a_2\mu_{u^{\mathcal{S}}|R_f}\right)\right) + \frac{\tau_\varepsilon}{\tau}F + \frac{1}{\rho}\left(\frac{a_1\tau_{u^{\mathcal{S}}|R_f}}{a_2^2} - \tau\right)R_fP + u^{\mathcal{S}} \triangleq \bar{X}^{\mathcal{S}},$$

which verifies conjecture (A4). Finally, matching the coefficients of (A13) to the ones of the conjecture (A4) and solving the resultant equation system for a_0, a_1, and a_2, yields

$$a_0 = \frac{\tau_F}{\tau}\,\mu_F + \frac{\tau_\epsilon\,\tau_{u^{\mathcal{S}}|R_f}}{\rho\,\tau}\,\mu_{u^{\mathcal{S}}|R_f}, \tag{A14}$$

$$a_1 = \frac{\tau_\epsilon\left(\rho^2 + \tau_\epsilon\,\tau_{u^{\mathcal{S}}|R_f}\right)}{\tau\,\rho^2}, \quad \text{and} \quad a_2 = -\frac{\tau_\epsilon\left(\rho^2 + \tau_\epsilon\,\tau_{u^{\mathcal{S}}|R_f}\right)}{\tau\,\rho^2}\,\frac{\rho}{\tau_\epsilon}. \tag{A15}$$

Hence, investors' posterior precision about F in (A6) is given by

$$\tau = \tau_F + \tau_\varepsilon + \frac{\tau_\varepsilon^2}{\rho^2}\,\tau_{u^{\mathcal{S}}|R_f}. \tag{A16}$$

Theorem 1 readily follows from (a) plugging coefficients (A11) into the conjecture for the bond-market-clearing condition (A3), (b) plugging coefficients (A14) and (A15) into the conjecture for the stock-market-clearing condition (A4), (c) the optimal bond and stock demand (A8) and (A10), and (d) posterior beliefs (A12) and (A16).

Lemmas 1 and 2 immediately follow from (A12) and (A16), respectively.

B.A.2 Alternative Setting: Arbitrary (Aggregate) Endowments

We now allow for *arbitrary endowments*; in particular, assume that, in aggregate, investors are endowed with $\bar{X}_0^{\mathcal{S}} \equiv \int_0^1 X_{i,0}^{\mathcal{S}}\,di$ shares of the stock and $\bar{X}_0^{\mathcal{B}} \equiv \int_0^1 X_{i,0}^{\mathcal{B}}\,di$ units of the maturing bond.

The derivations for this case closely follow those in the preceding section. Specifically, one starts with the same conjectures (A3) and (A4) (implying that the market-clearing conditions in the bond and the stock market are linear in the state variables) and, hence, investor i's posterior beliefs are again described by (A6) and (A7).

The optimal stock demand (A10) also remains unchanged. However, investor i's demand for the bond is now given by

$$X_i^{\mathcal{B}} = X_{i,0}^{\mathcal{S}} \, (F_1 + P) + X_{i,0}^{\mathcal{B}} - X_i^{\mathcal{S}} \, P.$$

Aggregating the bond demand across investors and imposing market clearing in the bond and stock markets implies

$$\int_0^1 X_i^{\mathcal{B}} \, di + u^{\mathcal{B}} = \int_0^1 R_f \, \left(X_{i,0}^{\mathcal{S}} \, (F_1 + P) + X_{i,0}^{\mathcal{B}} - X_i^{\mathcal{S}} \, P \right) \, di + u^{\mathcal{B}}$$

$$= R_f \, (F_1 + P) \, \bar{X}_0^{\mathcal{S}} + R_f \, \bar{X}_0^{\mathcal{B}} - R_f P \, \left(\bar{X}^{\mathcal{S}} - u^{\mathcal{S}} \right) + u^{\mathcal{B}} \triangleq \bar{X}^{\mathcal{B}}. \tag{A17}$$

In particular, note that in this equation, the stock price does not cancel out, whereas it did in the setting in which we assume aggregate stock endowments are identical across periods.

This verifies the initial conjecture and (by matching coefficients) directly yields

$$b_0 = R_f \, (F_1 + P) \, \bar{X}_0^{\mathcal{S}} + R_f \, \bar{X}_0^{\mathcal{B}} - R_f \, P \, \bar{X}^{\mathcal{S}} - \bar{X}^{\mathcal{B}}, \qquad b_1 = R_f \, P \qquad \text{and} \qquad b_2 = 1.$$

Plugging these expressions into (A5) yields the following mean and precision for the noisy stock demand *conditional* on the interest rate:

$$\tau_{u^{\mathcal{S}}|R_f} = \tau_{u^{\mathcal{S}}} + (R_f \, P)^2 \, \tau_{u^{\mathcal{B}}}, \tag{A18}$$

$$\mu_{u^{\mathcal{S}}|R_f} = \frac{\tau_{u^{\mathcal{B}}}}{\tau_{u^{\mathcal{S}}|R_f}} \, P \, R_f \, \left(R_f \, (F_1 + P) \, \bar{X}_0^{\mathcal{S}} + R_f \, \bar{X}_0^{\mathcal{B}} - R_f \, P \, \bar{X}^{\mathcal{S}} - \bar{X}^{\mathcal{B}} \right). \tag{A19}$$

Notably, the conditional precision, $\tau_{u^{\mathcal{S}}|R_f}$, is increasing in the price ratio, $R_f \, P$.

Market clearing in the stock market yields the same price ratio as in (8). Note, however, that the conditional mean and precision ($\mu_{u^{\mathcal{S}}|R_f}$ and $\tau_{u^{\mathcal{S}}|R_f}$) are now given by (A18) and (A19). As a result, the right-hand side of (8) now also depends on the price ratio, $R_f P$, leading to a cubic equation in the price ratio. The cubic nature of this equation reflects the *two-way relationship* that links the price ratio and the precision of information: the higher the price ratio, the more precise is information (as (A18) shows); conversely, the more precise information, the higher is the price ratio (as Equation (8) in Theorem 1 shows).

Finally, the precision of investor i's conditional beliefs for the payoff F is given by

$$\tau \equiv \mathbb{V}\mathrm{ar}\left(F \,|\, \mathcal{F}_i\right)^{-1} = \tau_F + \tau_\varepsilon + \frac{\tau_\varepsilon^2}{\rho^2}\left(\tau_{u^{\mathcal{S}}} + \left(R_f\, P\right)^2 \tau_{u^{\mathcal{B}}}\right), \tag{A20}$$

where the last term represents the *informativeness of the stock price, which is increasing in the price ratio,* $R_f P$.

B.B. Derivations for Section IV

Solving the period-1 budget constraint in (5) for the bond holdings, $X_i^{\mathcal{B}}$, yields

$$X_i^{\mathcal{B}} = R_f\,\left(W_{i,1} - X_i^{\mathcal{S}}\, P - C_{i,1}\right).$$

Aggregating investors' bond demand and imposing market clearing in the bond and stock markets implies

$$\begin{aligned}\int_0^1 X_i^{\mathcal{B}}\, di + u^{\mathcal{B}} &= \int_0^1 R_f\,\left(W_{i,1} - X_i^{\mathcal{S}}\, P - C_{i,1}\right)\, di + u^{\mathcal{B}} \\ &= R_f\,\bar{W}_1 - R_f P\,\left(\bar{X}^{\mathcal{S}} - u^{\mathcal{S}}\right) - R_f\,\bar{C}_1 + u^{\mathcal{B}} \triangleq \bar{X}^{\mathcal{B}},\end{aligned}$$

where $\bar{W}_1 \equiv \int_0^1 W_{i,1} di$ and $\bar{C}_1 = \int_0^1 C_{i,1}\, di$ denote aggregate endowed wealth and aggregate initial consumption, respectively. This immediately yields **Equation 14**.

Taking the first-order condition of the expected firm value, $\mathbb{E}\left[v(z, I) \,|\, R_f, P\right]$, with respect to real investment, I, yields

$$-1 + \mathbb{E}\left[(1+z) - \frac{\kappa}{K_1}\, I \,\middle|\, P, R_f\right] = 0,$$

which is equivalent to **Equation 16**.

B.C. Derivations for Section V

B.C.1 Derivations for Section V.A

The objective of each investor i is to maximize expected CARA utility (4), conditional on her information set $\mathcal{F}_i = \{S_i^{(1)}, S_i^{(2)}, R_f, P^{(1)}, P^{(2)}\}$ and subject to the following budget constraints:

$$C_{i,1} + X_i^{\mathcal{S}_1}\, P^{(1)} + X_i^{\mathcal{S}_2}\, P^{(2)} + X_i^{\mathcal{B}}\, R_f^{-1} = W_{i,1}, \text{ and } C_{i,2} = X_i^{\mathcal{S}_1}\, F^{(1)} + X_i^{\mathcal{S}_2}\, F^{(2)} + X_i^{\mathcal{B}},$$

where $X_i^{\mathcal{S}_k}$ denotes the number of shares of stock k held by investor i.

Accordingly, investor i's demand for the bond can be written as

$$X_i^{\mathcal{B}} = R_f\, \left(W_{i,1} - X_i^{\mathcal{S}_1}\, P^{(1)} + X_i^{\mathcal{S}_2}\, P^{(2)} - C_{i,1}\right).$$

Aggregating this demand across investors and imposing market clearing in all three markets then yields

$$R_f\, \bar{W}_1 - R_f\, \left(\bar{X}^{\mathcal{S}_1} - u^{\mathcal{S}_1}\right)\, P^{(1)} - R_f\, \left(\bar{X}^{\mathcal{S}_2} - u^{\mathcal{S}_2}\right)\, P^{(2)} - R_f\, \bar{C}_1 + u^{\mathcal{B}} = \bar{X}^{\mathcal{B}},$$

which is equivalent to **Equation 18**.

B.C.2 Derivations for Section V.B

The objective of each investor i is to maximize expected utility

$$\text{(A21)} \qquad -\frac{1}{\rho}\, \exp\!\left(-\rho\, C_{i,1}\right) + \beta\, \mathbb{E}\left[-\frac{1}{\rho}\, \exp\!\left(-\rho\, C_{i,2}\right) \,\middle|\, \mathcal{F}_i\right] - \frac{1}{\alpha}\, \exp\left(-\alpha\, \left(X_i^{\mathcal{M}}/P_1^G\right)\right),$$

subject to the budget constraints:

$$\text{(A22)} \qquad C_{i,1} + X_i^{\mathcal{S}}\, P + X_i^{\mathcal{B}}\, R_f^{-1} + \frac{X_i^{\mathcal{M}}}{P_1^G} = W_{i,1}, \text{ and } C_{i,2} = X_i^{\mathcal{S}}\, F + X_i^{\mathcal{B}} + \frac{X_i^{\mathcal{M}}}{P_2^G}.$$

Combining the budget constraints in (A22), plugging the resultant period-2 consumption into the investor's utility function (A21), and deriving the first-order conditions with respect to period-1 consumption, $C_{i,1}$, and money holdings, $X_i^{\mathcal{M}}$, yields

$$\exp\left(-\rho C_{i,1}\right) = \beta\, R_f\, \mathbb{E}\left[\exp\left(-\rho C_{i,2}\right) \,\middle|\, \mathcal{F}_i\right], \text{ and}$$

$$\exp\left(-\alpha \frac{X_i^{\mathcal{M}}}{P_1^G}\right) = \beta\, R_f\, \mathbb{E}\left[\exp\left(-\rho C_{i,2}\right) \,\middle|\, \mathcal{F}_i\right]\, \left(1 - \frac{P_1^G}{R_f\, P_2^G}\right),$$

where the second equation can be simplified to

$$
\text{(A23)} \qquad -\alpha\,\frac{X_i^{\mathcal{M}}}{P_1^G} = -\rho\, C_{i,1} + \ln\left(1 - \frac{P_1^G}{R_f\, P_2^G}\right).
$$

Substituting out period-1 consumption from first-order condition (A23) (thanks to (A22)), aggregating across investors, and clearing the stock, bond, and goods markets yields

$$
\begin{aligned}
\text{(A24)} \qquad -\frac{\alpha}{P_1^G}\left(\bar{X}^{\mathcal{M}} - u^{\mathcal{M}}\right) &= \ln\left(1 - \frac{P_1^G}{R_f\, P_2^G}\right) \\
&\quad - \rho\left(\bar{W}_1 - \left(\bar{X}^{\mathcal{S}} - u^{\mathcal{S}}\right) P - \left(\bar{X}^{\mathcal{B}} - u^{\mathcal{B}}\right) R_f^{-1} - \left(\bar{X}^{\mathcal{M}} - u^{\mathcal{M}}\right)\left(P_1^G\right)^{-1}\right),
\end{aligned}
$$

where $\bar{W}_1 \equiv \int_0^1 W_{i,1} di$ denotes the aggregate endowed wealth.

Simplifying Equation (A24) yields

$$
\left(1 + \frac{\alpha}{\rho}\right)\frac{\bar{X}^{\mathcal{M}} - u^{\mathcal{M}}}{P_1^G} = -\frac{1}{\rho}\ln\left(1 - \frac{P_1^G}{R_f\, P_2^G}\right) + \bar{W}_1 - \left(\bar{X}^{\mathcal{S}} - u^{\mathcal{S}}\right) P - \frac{\bar{X}^{\mathcal{B}} - u^{\mathcal{B}}}{R_f},
$$

which, after defining $\hat{s}_2 \equiv \frac{\rho+\alpha}{\rho}\,\frac{\bar{X}^{\mathcal{M}}}{P_1^G\, P} + \bar{X}^{\mathcal{S}} + \frac{\bar{X}^{\mathcal{B}}}{R_f\, P} + \frac{1}{\rho}\,\frac{1}{P}\,\ln\left(1 - \frac{P_1^G}{R_f\, P_2^G}\right) - \frac{\bar{W}_1}{P}$ yields the signal $\hat{s}_2$ in (19).

In addition, aggregating investors' period-1 budget constraint in (A22) and imposing market clearing in all three markets yields

$$
\text{(A25)} \qquad \bar{C}_1 + \left(\bar{X}^{\mathcal{S}} - u^{\mathcal{S}}\right) P + \left(\bar{X}^{\mathcal{B}} - u^{\mathcal{B}}\right) R_f^{-1} + \left(\bar{X}^{\mathcal{M}} - u^{\mathcal{M}}\right)\left(P_1^G\right)^{-1} = \bar{W}_1,
$$

which, after defining $\hat{s}_1 \equiv \bar{X}^{\mathcal{S}} + \frac{\bar{X}^{\mathcal{B}}}{R_f\, P} + \frac{\bar{X}^{\mathcal{M}}}{P_1^G\, P} - \frac{1}{P}\,\bar{W}_1$, yields the signal $\hat{s}_1$ in (19).

C. Numerical Solution Approach

The main difficulty in identifying the equilibrium in the presence of intertemporal consumption choices is that the market-clearing conditions in the stock and bond markets are nonlinear functions of the state variables, with unknown functional forms. As a result, one cannot explicitly compute the investors' posterior beliefs and, hence, cannot find a closed-form solution for the equilibrium. Accordingly, the model must be solved numerically.

For that purpose, we extend the numerical solution approach presented in Breugem and Buss (2019) to allow for learning from the interest rate, two-period consumption, and endogenous output. The approach allows for arbitrary price and demand functions, that is, one does not need to parameterize (conjecture) these functions in any form. Also, it identifies the equilibrium *exactly*, up to a discretization of the state space (which can be made arbitrarily narrow). The algorithm comprises the following four key steps.

First, we discretize the state space into a grid of N_z, $N_{u^{\mathcal{S}}}$, and $N_{u^{\mathcal{B}}}$ realizations of the random variables z, $u^{\mathcal{S}}$, and $u^{\mathcal{B}}$, respectively.[49]

Second, we form, for any given grid point $\Omega = \{z_n, u^{\mathcal{S}}_m, u^{\mathcal{B}}_o\}$, $n \in \{1, \dots, N_z\}$, $m \in \{1, \dots, N_{u^{\mathcal{S}}}\}$, $o \in \{1, \dots, N_{u^{\mathcal{B}}}\}$, the system of equations that characterizes the equilibrium. The system comprises investors' first-order conditions with respect to bond and stock holdings, plus the two market-clearing conditions (6) and the optimal real investment condition (16). Specifically, to accommodate investors' dispersed signal realizations, we form N_S groups of investors ("signal realization groups") for each grid point Ω, with each group receiving a different signal S_s, $s \in \{1, \dots, N_S\}$. Thus, we arrive at an equation system with $N_S \times 2 + 3$ equations, with unknowns: $R_f(\Omega)$, $P(\Omega)$, $I(\Omega)$, and $\{X^{\mathcal{S}}_s(\Omega),\ X^{\mathcal{B}}_s(\Omega)\}$, $\forall s \in \{1, \dots, N_S\}$ (i.e., $3 + N_S \times 2$ unknowns in total).

Third, we complement the equation system with a set of equations that characterize investors' rational expectations.[50] Specifically, for each signal-realization group s and each "conjectured" level of productivity $\hat{z}_w$, $w \in \{1, \dots, N_{\hat{z}}\}$, we add equations that, under the beliefs of group s and conditional on prices, describe the aggregate demand for the two assets.[51,52] This requires solving for the optimal asset demands of all signal realization groups and all conjectured levels of productivity, conditional on prices, and aggregating the resultant demands. This adds $N_S^2 \times N_{\hat{z}} \times 2$ equations for each grid point Ω, though many of those are redundant (i.e., can be removed). Based on the conjectured aggregate demands, $\{\hat{u}^{\mathcal{S}}_w, \hat{u}^{\mathcal{B}}_w\}$, each group s can then compute her posterior probabilities (employed in

[49]We truncate the realizations of the bond demand, $u^{\mathcal{B}}$, such that $\bar{X}^{\mathcal{B}} - u^{\mathcal{B}} \geq 0$. This is needed because, under CARA preferences, the equilibrium might not exist for $\bar{X}^{\mathcal{B}} - u^{\mathcal{B}} < 0$, because of the violation of the Inada conditions.

[50]If investors' posterior probabilities were exogenous (e.g., a function of private signals or prior beliefs only), one could directly solve the equation system described in step 2. However, under rational expectations, investors' beliefs depend on the prices of the two assets. This dependence gives rise to a fixed-point problem.

[51]To distinguish between the actual values of productivity and asset demands at a given grid point, $\{z_n, u^{\mathcal{S}}_m, u^{\mathcal{B}}_o\}$, and conjectured productivity and demands, $\{\hat{z}_n, \hat{u}^{\mathcal{S}}_m, \hat{u}^{\mathcal{B}}_o\}$, we denote the latter with a hat.

[52]To allow for conjectured productivity to cover a wide range around the actual productivity z_n, we create a separate grid specific to conjectured productivity, with entries $\{\hat{z}_1, \dots, \hat{z}_{N_{\hat{z}}}\}$.

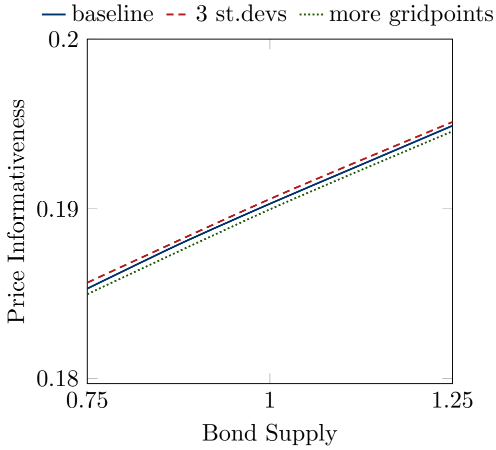

Figure A1. Grid precision

Notes: The graph plots price informativeness, calculated as in (15), as a function of the bond supply, $\bar{X}^{\mathcal{B}}$. The graph is based on the following baseline parameter values: $\beta = 0.95$, $\rho = 4$, $\tau_z = 2.5^2$, $\tau_\epsilon = 0.75^2$, $\bar{X}^{\mathcal{S}} = 1$, $\tau_{u^{\mathcal{S}}} = 5^2$, $\tau_{u^{\mathcal{B}}} = 10^2$, $W_{i,1} = 3$, $K_1 = 1$, $\kappa = 5$, and $\delta = 0$. "3 st.devs" denotes computations with a grid that spans three standard deviations for all state variables, and "more gridpoints" denotes computations with additional grid points along all dimensions.

the first-order conditions) for all conjectured levels of productivity $\{\hat{z}_w\}$, $w \in \{1, \ldots, N_{\hat{z}}\}$, using the distribution of the noise traders' bond and stock demands.[53]

Fourth, for each grid point Ω, we solve the resultant large-scale fixed-point problem using Mathematica. We thereby rely on `FindRoot`, which uses a dampened version of the Newton-Raphson method, together with finite differences to compute the Hessian.

We find that the solution of the system is very accurate for $N_z = N_{u^{\mathcal{S}}} = N_{u^{\mathcal{B}}} = 5$, $N_S = 31$, and $N_{\hat{z}} = 31$. Based on that grid, solving the system of equations for one grid point takes about 0.8 seconds on an Intel Core i7 workstation. Hence, solving it for all 729 grid points requires less than 10 minutes.[54] Further increasing the number of discretization points hardly changes the solution. Figure A1 illustrates this by plotting price informativeness as a function of the bond supply for computations with a narrower grid.

[53]Formally, the posterior probability of group s for productivity $\hat{z}_{w'}$, conditional on prices and her private signal S_s, is given by

$$\mathbb{P}\left(\hat{z}_{w'} \mid R_f, P, S_s\right) = \frac{f_{u^{\mathcal{S}}}\left(\hat{u}^{\mathcal{S}}_{w'}\right) f_{u^{\mathcal{B}}}\left(\hat{u}^{\mathcal{B}}_{w'}\right) f_z\left(\hat{z}_{w'} \mid S_s\right)}{\sum_{w=1}^{N_{\hat{z}}} f_{u^{\mathcal{S}}}\left(\hat{u}^{\mathcal{S}}_{w}\right) f_{u^{\mathcal{B}}}\left(\hat{u}^{\mathcal{B}}_{w}\right) f_z\left(\hat{z}_{w} \mid S_s\right)},$$

where f_z, $f_{u^{\mathcal{S}}}$, and $f_{u^{\mathcal{B}}}$ denote the *exact* density functions of productivity z, the noisy stock demand $u^{\mathcal{S}}$, and the noisy bond demand $u^{\mathcal{B}}$, respectively.

[54]To verify the solution approach, we (a) replicate our closed-form solution for the economy without initial consumption (see Section III), (b) replicate the Hellwig (1980) solution in an economy without learning from the interest rate and without initial consumption, and (c) confirm that the solution converges to the solution without private information as τ_ε converges to zero.